Algorithm computing a spectral system on the Cobar complex

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Abstract

In this paper we describe the steps of an algorithm for computing a spectral system on the Cobar complex of a fibration $F \hookrightarrow E \to B$.

Algorithm 1: Spectral system on the cobar complex

Input: A fibration $F \hookrightarrow E \to B$, with B a 1-reduced simplicial set and equivalences $C_*(B) \Leftarrow DB_* \Rightarrow HB_*$ and $C_*(E) \Leftarrow DE_* \Rightarrow HE_*$, where HB_* and HE_* are effective chain complexes.

Output: All the components of a spectral system over $\mathbb{Z}^2$ defined on $\mathrm{Cobar}^{C_*(B)}(\mathbb{Z}, C_*(E))$, that is, groups $S[z,s,p,b]_n$ for $z,s,p,b \in \mathbb{Z}^2$ and $n \in \mathbb{N}$ and differential maps defined on them.

4 Define constructively the isomorphism $\mathrm{Cobar}^{C_*(B)}(C_*(E), \mathbb{Z}) \cong \mathrm{Cobar}^{C_*(B)}(\mathbb{Z}, \mathbb{Z}) \otimes_t C_*(E)$.

5 Construct the effective homology of $\mathrm{Cobar}^{C_*(B)}(\mathbb{Z}, \mathbb{Z}) \otimes_t C_*(E)$ by using the effective homologies of B and E, as a particular application of the computation of the effective homology of a bicomplex. The right chain complex in the equivalence, $\widetilde{\mathrm{Cobar}}^{HB_*}(HE_*, \mathbb{Z})$, is a chain complex of finite type.

6 Define a canonical filtration on the chain complex $\widetilde{\mathrm{Cobar}}^{HB_*}(HE_*, \mathbb{Z})$, denoted F^{EM}, so that the associated spectral sequence is isomorphic to the Eilenberg–Moore spectral sequence defined by the bicomplex $\mathrm{Cobar}^{C_*(B)}(C_*(E), \mathbb{Z})$.

7 Define a second filtration on the chain complex $\widetilde{\mathrm{Cobar}}^{HB_*}(HE_*, \mathbb{Z})$, denoted F^S and given by the degree over the complex $C_*(E)$ in the tensor product.

8 Define a filtration over $\mathbb{Z}^2$ as $F_{(k_1,k_2)} := F^S_{k_1} \cap F^{EM}_{k_2}$.

9 Compute the groups and the differential maps of the spectral system associated to the chain complex $\widetilde{\mathrm{Cobar}}^{HB_*}(HE_*, \mathbb{Z})$, which are isomorphic to those of $\mathrm{Cobar}^{C_*(B)}(C_*(E), \mathbb{Z})$.