Algorithm computing the Eilenberg–Moore spectral sequence

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Abstract

In this paper we describe the steps of an algorithm for computing the Eilenberg–Moore spectral sequence of a fibration $F \hookrightarrow E \to B$. It is a supplementary note for our work [1].

Algorithm 1: Eilenberg–Moore spectral sequence

Input: A fibration $F \hookrightarrow E \to B$, with B a 1-reduced simplicial set and equivalences $C_*(B) \Leftarrow DB_* \Rightarrow HB_*$ and $C_*(E) \Leftarrow DE_* \Rightarrow HE_*$, where HB_* and HE_* are effective chain complexes.

Output: All the components of the associated Eilenberg–Moore spectral sequence, that is, the groups $E^r_{p,q}$ and the differential maps $d^r_{p,q}$ for all $p, q \in \mathbb{Z}$ and $r \geq 1$, and the convergence level for each degree.

4 Construct a reduction $\mathrm{Cobar}^{C_*(B)}(C_*(E), \mathbb{Z}) \Rightarrow \mathrm{Cobar}^{C_*(B)}(C_*(F) \otimes_t C_*(B), \mathbb{Z})$ by means of the twisted Eilenberg–Zilber reduction.

5 Construct the effective homology of $\mathrm{Cobar}^{C_*(B)}(C_*(F) \otimes_t C_*(B), \mathbb{Z})$ by using the effective homologies of B and E, as a particular application of the computation of the effective homology of a bicomplex. The right chain complex in the equivalence, $\widetilde{\mathrm{Cobar}}^{HB_*}(HE_*, \mathbb{Z})$, is a chain complex of finite type.

6 Define a canonical filtration on the chain complex $\widetilde{\mathrm{Cobar}}^{HB_*}(HE_*, \mathbb{Z})$ so that the associated spectral sequence is isomorphic to the Eilenberg–Moore spectral sequence defined by the bicomplex $\mathrm{Cobar}^{C_*(B)}(C_*(E), \mathbb{Z})$.

7 Compute the groups and the differential maps of the spectral sequence associated to the chain complex $\mathrm{Cobar}^{C_*(B)}(C_*(E), \mathbb{Z})$ by means of diagonalization algorithms on matrices.

References

[1] A. Romero, J. Rubio, F. Sergeraert and M. Szymik. *A new Kenzo module for computing the Eilenberg–Moore spectral sequence*. To appear in ISSAC 2020 Software presentation session.