Lógica Computacional

LEI, 2023/2024
FCT UNL

Aula Prática 8

Dedução Natural em Lógica de Primeira Ordem

Pergunta 1. Prove as seguintes afirmações. Nota: φ e ψ são fórmulas.

1. $\{\forall_x P(x) \vee \forall_x Q(x)\} \vdash \forall_x (P(x) \vee Q(x))$
2. $\{\forall_x (P(x) \wedge Q(x))\} \vdash \forall_x P(x) \wedge \forall_x Q(x)$
3. $\vdash (\forall_x P(x) \wedge \forall_x Q(x)) \to \forall_x (P(x) \wedge Q(x))$
4. $\{\exists_x (P(x) \wedge Q(x))\} \vdash \exists_x P(x) \wedge \exists_x Q(x)$
5. $\vdash (\exists_x P(x) \vee \exists_x Q(x)) \to \exists_x (P(x) \vee Q(x))$
6. $\{\exists_x (P(x) \vee Q(x))\} \vdash \exists_x P(x) \vee \exists_x Q(x)$
7. $\{\forall_x (P(x) \to Q(x))\} \vdash \forall_x P(x) \to \forall_x Q(x)$
8. $\{\exists_y \forall_x \varphi\} \vdash \forall_x \exists_y \varphi$
9. $\vdash \exists_x \neg P(x) \to \neg\forall_x P(x)$
10. $\vdash \neg\forall_x P(x) \to \exists_x \neg P(x)$
11. $\vdash \forall_x \neg P(x) \to \neg\exists_x P(x)$
12. $\vdash \neg\exists_x P(x) \to \forall_x \neg P(x)$
13. $\vdash \exists_x \varphi \to \neg\forall_x \neg\varphi$
14. $\{\neg\forall_x \neg\varphi\} \vdash \exists_x \varphi$
15. $\vdash \forall_x \varphi \to \neg\exists_x \neg\varphi$
16. $\{\neg\exists_x \neg\varphi\} \vdash \forall_x \varphi$
17. $\vdash (\forall_x \varphi \wedge \psi) \leftrightarrow \forall_x (\varphi \wedge \psi)$, se $x \notin \mathsf{VL}(\psi)$
18. $\vdash (\forall_x \varphi \vee \psi) \leftrightarrow \forall_x (\varphi \vee \psi)$, se $x \notin \mathsf{VL}(\psi)$
19. $\vdash (\exists_x \varphi \wedge \psi) \leftrightarrow \exists_x (\varphi \wedge \psi)$, se $x \notin \mathsf{VL}(\psi)$
20. $\vdash (\exists_x \varphi \vee \psi) \leftrightarrow \exists_x (\varphi \vee \psi)$, se $x \notin \mathsf{VL}(\psi)$
21. $\vdash \forall_x (\psi \to \varphi) \leftrightarrow (\psi \to \forall_x \varphi)$, se $x \notin \mathsf{VL}(\psi)$
22. $\vdash \exists_x (\psi \to \varphi) \leftrightarrow (\psi \to \exists_x \varphi)$, se $x \notin \mathsf{VL}(\psi)$
23. $\vdash \forall_x (\varphi \to \psi) \leftrightarrow (\exists_x \varphi \to \psi)$, se $x \notin \mathsf{VL}(\psi)$
24. $\vdash \exists_x (\varphi \to \psi) \leftrightarrow (\forall_x \varphi \to \psi)$, se $x \notin \mathsf{VL}(\psi)$

Pergunta 2. Prove as seguintes afirmações.

1. $\{\exists_x(T(x) \wedge S(x)),\, \forall_x(S(x) \rightarrow L(x,b))\} \vdash \exists_x\exists_y L(x,y)$
2. $\{\forall_y(C(y) \vee D(y)),\, \forall_x(C(x) \rightarrow L(x)),\, \exists_x \neg L(x)\} \vdash \exists_x D(x)$
3. $\{\forall_x(C(x) \rightarrow S(x)),\, \forall_x(\neg A(x,b) \rightarrow \neg S(x))\} \vdash \forall_x((C(x) \vee S(x)) \rightarrow A(x,b))$
4. $\{L(a,b),\, \forall_x(\exists_y(L(y,x) \vee L(x,y)) \rightarrow L(x,x))\} \vdash \exists_x L(x,a)$
5. $\{\forall_x\forall_y(L(x,y) \rightarrow L(y,x)),\, \exists_x\forall_y L(x,y)\} \vdash \forall_x\exists_y L(x,y)$
6. $\{\forall_x(S(x) \rightarrow C(x)),\, \exists_x \neg C(x) \rightarrow \exists_x S(x)\} \vdash \exists_x C(x)$
7. $\{\neg\exists_x(T(x) \wedge S(x)),\, \forall_y(S(y) \vee M(y))\} \vdash \forall_x(T(x) \rightarrow M(x))$
8. $\{\forall_x(D(x) \rightarrow S(x,a)),\, S(a,c),\, \forall_x\forall_y\forall_z((S(x,y) \wedge S(y,z)) \rightarrow S(x,z))\} \vdash \forall_x(D(x) \rightarrow S(x,c))$