Aula Prática 6 — Pergunta 1

1. $\vdash \varphi \to (\varphi \vee \psi)$

$$\dfrac{\dfrac{\varphi^{1}}{\varphi \vee \psi}\,\vee I_d}{\varphi \to (\varphi \vee \psi)}\,\to I,1$$

2. $\vdash (\varphi \vee \varphi) \to \varphi$

$$\dfrac{\dfrac{\varphi \vee \varphi^{3} \qquad \varphi^{1} \qquad \varphi^{2}}{\varphi}\,\vee E 1,2}{(\varphi \vee \varphi) \to \varphi}\,\to I,3$$

3. $\vdash (\varphi \wedge \varphi) \to \varphi$

$$\dfrac{\dfrac{\varphi \wedge \varphi^{\,①}}{\varphi}\,\wedge E_d}{(\varphi \wedge \varphi) \to \varphi}\,\to I,1$$

4. $\vdash \varphi \to (\psi \to \varphi)$

$$\dfrac{\dfrac{\varphi^{①}}{\psi \to \varphi}\,\to I,2}{\varphi \to (\psi \to \varphi)}\,\to I,1$$

5. $\vdash ((\varphi \to \psi) \wedge (\psi \to \gamma)) \to (\varphi \to \gamma)$

$$\dfrac{\dfrac{\dfrac{\dfrac{(\varphi \to \psi) \wedge (\psi \to \gamma)^{①}}{\varphi \to \psi}\,\wedge E_d \qquad \varphi^{②}}{\psi}\,\to E \qquad \dfrac{(\varphi \to \psi) \wedge (\psi \to \gamma)^{①}}{\psi \to \gamma}\,\wedge E_e}{\gamma}\,\to I,2}{((\varphi \to \psi) \wedge (\psi \to \gamma)) \to (\varphi \to \gamma)}\,\to I,1$$

7. $\vdash (\varphi \to \psi) \to (\varphi \to (\psi \vee \gamma))$

$$\dfrac{\dfrac{\dfrac{\dfrac{(\varphi \to \psi)^{1} \quad \varphi^{2}}{\psi}\to E}{\psi \vee \gamma}\vee Id}{\varphi \to (\psi \vee \gamma)}\to I,2}{(\varphi \to \psi) \to (\varphi \to (\psi \vee \gamma))}\to I,1$$

8. $(\psi \to \gamma) \to ((\varphi \wedge \psi) \to \gamma)$

$$\dfrac{\dfrac{\dfrac{(\psi \to \gamma)^{1} \quad \dfrac{\varphi \wedge \psi^{2}}{\psi}\wedge E_e}{\gamma}\to E}{(\varphi \wedge \psi) \to \gamma}\to I,2}{(\psi \to \gamma) \to ((\varphi \wedge \psi) \to \gamma)}\to I,1$$

9. $\neg(\varphi \vee \psi) \to \neg\varphi$

$$\dfrac{\dfrac{\dfrac{\neg(\varphi \vee \psi)^{1} \quad \dfrac{\varphi^{2}}{\varphi \vee \psi}\vee Id}{\bot}\neg E}{\neg\varphi}\neg I,2}{\neg(\varphi \vee \psi) \to \neg\varphi}\to I,1$$

Ficha 6 Pergunta 2

Em todos:
Pela correção do sistema $\mathcal{N}$ temos $\vdash$

1. $\vDash \neg\varphi \to (\varphi \to \psi)$

$$\dfrac{\dfrac{\dfrac{\dfrac{\neg\varphi^{1} \quad \varphi^{2}}{\bot}\neg E}{\psi}\bot,3}{\varphi\to\psi}\to I,2}{\neg\varphi\to(\varphi\to\psi)}\to I,1$$

2. $((\varphi\to\psi)\wedge\neg\psi)\to\neg\varphi$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{(\varphi\to\psi)\wedge\neg\psi^{1}}{\varphi\to\psi}\wedge E_d \quad \varphi^{2}}{\psi}\to E \quad \dfrac{(\varphi\to\psi)\wedge\neg\psi^{1}}{\neg\psi}\wedge E_e}{\bot}\neg E}{\neg\varphi}\neg I,2}{((\varphi\to\psi)\wedge\neg\psi)\to\neg\varphi}\to I,1$$

~~3~~. $\bot\to\varphi$

$$\dfrac{\dfrac{\bot^{1}}{\varphi}\bot,1}{\bot\to\varphi}\to I,1$$

3 $\vdash \varphi \leftrightarrow (\neg\neg\varphi)$

$$\dfrac{\dfrac{\dfrac{\varphi^{1} \quad \neg\varphi^{2}}{\bot}\neg E}{\neg\varphi}\neg I,2}{\varphi \to \neg\neg\varphi}\to I,1 \qquad \dfrac{\dfrac{\dfrac{\neg\neg\varphi^{3} \quad \neg\varphi^{4}}{\bot}\neg E}{\varphi}\bot,4}{\neg\neg\varphi \to \varphi}\to I,3$$

$$\dfrac{\varphi \to \neg\neg\varphi \qquad \neg\neg\varphi \to \varphi}{(\varphi \to \neg\neg\varphi) \wedge (\neg\neg\varphi \to \varphi)} \wedge I$$

4. $((\varphi \to \psi) \leftrightarrow (\neg\psi \to \neg\varphi))$

$$\dfrac{\dfrac{\dfrac{\dfrac{\varphi \to \psi^{1} \quad \varphi^{2}}{\psi}\to E \quad \neg\psi^{3}}{\bot}\neg E}{\neg\varphi}\neg I,2}{\neg\psi \to \neg\varphi}\to I,3}{(\varphi \to \psi) \to (\neg\psi \to \neg\varphi)}\to I,1$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\neg\psi \to \neg\varphi^{4} \quad \neg\psi^{5}}{\neg\varphi}\to E \quad \varphi^{6}}{\bot}\neg E}{\psi}\bot,5}{\varphi \to \psi}\to I,6}{(\neg\psi \to \neg\varphi) \to (\varphi \to \psi)}\to I,4$$

$$\dfrac{(\varphi \to \psi) \to (\neg\psi \to \neg\varphi) \qquad (\neg\psi \to \neg\varphi) \to (\varphi \to \psi)}{(\varphi \to \psi) \leftrightarrow (\neg\psi \to \neg\varphi)} \wedge I$$

6. $\vdash \top \quad \top \equiv \neg\bot \equiv \bot \to \bot$

2: $\neg\bot$

$$\dfrac{\dfrac{\bot^{1}}{\bot}\bot,2}{\bot \to \bot}\to I,1$$

7. $\varphi \vee \neg\varphi$

$$\dfrac{\dfrac{\dfrac{\neg(\varphi \vee \neg\varphi)^{1} \quad \dfrac{\varphi^{2}}{\varphi \vee \neg\varphi}\vee I,d}{\bot}\neg E}{\neg\varphi}\neg I,2}{\varphi \vee \neg\varphi}\vee I_{e} \qquad \neg(\varphi \vee \neg\varphi)^{1}}{\bot}\neg E$$

$$\dfrac{\bot}{\varphi \vee \neg\varphi}\bot,1$$

Pergunta 3

Em todas as linhas: Pela convenção do Sistema SN temos que $\vdash$

1 $\{\neg\varphi \vee \psi\} \vdash \varphi \to \psi$

$$\dfrac{\dfrac{\dfrac{\dfrac{\neg\varphi^{2} \quad \varphi^{2}}{\bot}\neg E}{\psi}\bot,8}{\varphi\to\psi}\to I,3 \qquad \dfrac{\psi^{4}}{\varphi\to\psi}\to I,9}{\neg\varphi\vee\psi^{1}}\ VE,2,4$$

$$\varphi \to \psi$$

2. $\varphi \to \psi \vdash \neg\varphi \vee \psi$

$$\dfrac{\dfrac{\dfrac{\dfrac{(\varphi\to\psi)^{1} \quad \varphi^{2}}{\psi}\to E}{\neg\varphi\vee\psi}VI,e \qquad \neg(\neg\varphi\vee\psi)^{3}}{\bot}\neg E}{\neg\varphi}\neg I,2$$

$$\dfrac{\dfrac{\neg\varphi}{\neg\varphi\vee\psi}VId \qquad \neg(\neg\varphi\vee\psi)^{3}}{\bot}\neg E$$

$$\dfrac{\bot}{\neg\varphi\vee\psi}\bot,3$$

3. $\{\neg(\varphi\wedge\psi)\} \vdash (\neg\varphi \vee \neg\psi)$

$$\dfrac{\dfrac{\dfrac{\varphi^{1} \quad \psi^{2}}{\varphi\wedge\psi}\wedge I \qquad \neg(\varphi\wedge\psi)^{3}}{\bot}\neg E}{\neg\varphi}\neg I,1 \qquad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dots}{\neg\psi}\neg I,2}{\neg\varphi\vee\neg\psi}VI,e \qquad \neg(\neg\varphi\vee\neg\psi)^{4}}{\bot}\neg}{\bot}\bot,4}{\neg\varphi\vee\neg\psi}$$

$$\dfrac{\dfrac{\neg\varphi}{\neg\varphi\vee\neg\psi}VId \qquad \neg(\neg\varphi\vee\neg\psi)^{4}}{\bot}\neg E$$

$$\dfrac{\bot}{\neg\psi}\neg I,2$$

4. $\{\neg\varphi \vee \neg\psi\} \vDash \neg(\varphi \wedge \psi)$

$$\dfrac{\neg\varphi\vee\neg\psi^{5} \qquad \dfrac{\dfrac{\neg\varphi^{1} \quad \dfrac{\varphi\wedge\psi^{3}}{\varphi}\wedge E d}{\bot}\neg E}{\neg(\varphi\wedge\psi)}\neg I,3 \qquad \dfrac{\dfrac{\neg\psi^{2} \quad \dfrac{\varphi\wedge\psi^{4}}{\psi}\wedge E e}{\bot}\neg E}{\neg(\varphi\wedge\psi)}\neg I,4}{\neg(\varphi\wedge\psi)}\vee E,1,3$$

5. $\neg(\varphi\vee\psi) \vDash \neg\varphi\wedge\neg\psi$

$$\dfrac{\dfrac{\dfrac{\neg(\varphi\vee\psi)^{1} \quad \dfrac{\varphi^{2}}{\varphi\vee\psi}\vee I d}{\bot}\neg E}{\neg\varphi}\neg I,2 \qquad \dfrac{\dfrac{\dfrac{\psi^{3}}{\varphi\vee\psi}\vee I e \quad \neg(\varphi\vee\psi)^{1}}{\bot}\neg E}{\neg\psi}\neg I,3}{\neg\varphi\wedge\neg\psi}\wedge I$$

6. $\{\neg\varphi\wedge\neg\psi\} \vDash \neg(\varphi\vee\psi)$

$$\dfrac{\varphi\vee\psi^{1} \qquad \dfrac{\dfrac{\neg\varphi\wedge\neg\psi^{2}}{\neg\varphi}\wedge E d \quad \varphi^{3}}{\bot}\neg E \qquad \dfrac{\dfrac{\neg\varphi\wedge\neg\psi^{2}}{\neg\psi}\wedge E e \quad \psi^{4}}{\bot}\neg E}{\bot}\vee E,3,4$$

$$\dfrac{\bot}{\neg(\varphi\vee\psi)}\neg I,1$$

7. $\{\varphi \vee (\psi \wedge \delta)\} \vDash (\varphi \vee \psi) \wedge (\varphi \vee \delta)$

$$\dfrac{\dfrac{\dfrac{\varphi^{2}}{\varphi \vee \psi}\vee I,d \quad \dfrac{\varphi^{2}}{\varphi \vee \delta}\vee Id}{(\varphi \vee \psi) \wedge (\varphi \vee \delta)}\wedge I \qquad \dfrac{\dfrac{\dfrac{\psi \wedge \delta^{3}}{\psi}\wedge Ed}{\varphi \vee \psi} \qquad \dfrac{\dfrac{\psi \wedge \delta^{3}}{\delta}\wedge Ee}{\varphi \vee \delta}\vee Ie}{(\varphi \vee \psi) \wedge (\varphi \vee \delta)}\wedge I}{(\varphi \vee \psi) \wedge (\varphi \vee \delta)}\vee E,2,3$$

$\varphi \vee (\psi \wedge \delta)^{1}$

8. $\{(\varphi \vee \psi) \wedge (\varphi \vee \delta)\} \vDash \varphi \vee (\psi \wedge \delta)$

$$\dfrac{\dfrac{\dfrac{(\varphi \vee \psi) \wedge (\varphi \vee \delta)^{1}}{\varphi \vee \psi}\wedge Ed \quad \dfrac{\varphi^{2}}{\varphi \vee (\psi \wedge \delta)}\vee I,2}{\varphi \vee (\psi \wedge \delta)}\vee E \qquad \dfrac{\dfrac{(\varphi \vee \psi) \wedge (\varphi \vee \delta)^{1}}{\varphi \vee \delta} \qquad \dfrac{\varphi^{3} \quad \dfrac{\varphi^{4} \quad \delta^{5}}{\psi \wedge \delta}\wedge I}{\varphi \vee (\psi \wedge \delta)}\vee Ie}{\varphi \vee (\psi \wedge \delta)}\vee E\ 3,5}{\varphi \vee (\psi \wedge \delta)}\vee E\ 1,4$$

9. $\{\varphi \wedge (\psi \vee \delta)\} \vDash (\varphi \wedge \psi) \vee (\varphi \wedge \delta)$

$$\dfrac{\dfrac{\dfrac{\varphi \wedge (\psi \vee \delta)^{1}}{\psi \vee \delta}\wedge Ee \quad \dfrac{\dfrac{\dfrac{\varphi \wedge (\psi \vee \delta)^{1}}{\varphi}\wedge Ed \quad \varphi^{2}}{\varphi \wedge \psi}\wedge I}{(\varphi \wedge \psi) \vee (\varphi \wedge \delta)}\vee Id}{(\varphi \wedge \psi) \vee (\varphi \wedge \delta)}\vee E\ 2,3 \qquad \dfrac{\dfrac{\varphi \wedge (\psi \vee \delta)^{1}}{\varphi}\wedge Ed \quad \delta^{3}}{\varphi \wedge \delta}\wedge I}{(\varphi \wedge \psi) \vee (\varphi \wedge \delta)}\vee Ie$$

10. $\{(\varphi \wedge \psi) \vee (\varphi \wedge \delta)\} \vDash \varphi \wedge (\psi \vee \delta)$

$$\dfrac{\dfrac{\dfrac{(\varphi \wedge \psi) \vee (\varphi \wedge \delta)^{1}}{\varphi \wedge (\psi \vee \delta)}\vee E\ 2,3 \qquad \dfrac{\dfrac{\varphi \wedge \psi^{2}}{\varphi}\wedge Ed \quad \dfrac{\dfrac{\varphi \wedge \psi^{2}}{\psi}\wedge Ee}{\psi \vee \delta}\vee Id}{\varphi \wedge (\psi \vee \delta)}\wedge I}{\varphi \wedge (\psi \vee \delta)}\vee E\ 2,3 \qquad \dfrac{\dfrac{\varphi \wedge \delta^{3}}{\varphi}\wedge Ed \quad \dfrac{\dfrac{\varphi \wedge \delta^{3}}{\delta}\wedge Ee}{\psi \vee \delta}\vee Ie}{\varphi \wedge (\psi \vee \delta)}\wedge I}{\varphi \wedge (\psi \vee \delta)}\vee E\ 2,3$$