

Pergunta 1

$$1.6 \vdash (\psi \rightarrow (\psi \rightarrow \gamma)) \rightarrow ((\psi \rightarrow \psi) \rightarrow (\psi \rightarrow \gamma))$$

$$\begin{array}{c} \begin{array}{c} \overset{1}{\psi} \quad \overset{3}{(\psi \rightarrow \psi)} \quad \rightarrow E \quad \overset{2}{(\psi \rightarrow (\psi \rightarrow \gamma))} \quad \overset{1}{\psi} \rightarrow E \\ \hline \psi \quad \psi \rightarrow \gamma \quad \rightarrow E \\ \hline \gamma \quad \rightarrow I, 1 \\ \hline \psi \rightarrow \gamma \quad \rightarrow I, 3 \\ \hline (\psi \rightarrow \psi) \rightarrow (\psi \rightarrow \gamma) \quad \rightarrow I, 2 \\ \hline (\psi \rightarrow (\psi \rightarrow \gamma)) \rightarrow ((\psi \rightarrow \psi) \rightarrow (\psi \rightarrow \gamma)) \end{array} \end{array}$$

1.10

$$\vdash (\psi \rightarrow \chi) \rightarrow ((\psi \wedge \psi) \rightarrow (\psi \wedge \chi))$$

$$\begin{array}{c} \frac{\frac{(\psi \wedge \psi)^1}{\psi} \wedge E_1 \quad \frac{\frac{(\psi \rightarrow \chi)^2 \quad \frac{(\psi \wedge \psi)^4}{\psi} \wedge E_2}{\chi} \rightarrow E}{\psi \wedge \chi} \rightarrow I, 1}{(\psi \wedge \psi) \rightarrow (\psi \wedge \chi)} \rightarrow I, 2 \\ (\psi \rightarrow \chi) \rightarrow ((\psi \wedge \psi) \rightarrow (\psi \wedge \chi)) \end{array}$$

2.5

$$\models \perp \rightarrow \varphi$$

$$\frac{\frac{\perp^*}{\varphi} \perp, 10}{\perp \rightarrow \varphi} \rightarrow I, 1$$

2.6

$$\models \top \text{ qua } \text{dizn} \models \neg \perp$$

$$\frac{\perp^*}{\neg \perp} \neg I, 1$$

2.9

$$\models \varphi \vee (\varphi \rightarrow \psi)$$

$$\frac{\frac{\frac{\frac{\frac{\frac{\perp}{\neg \varphi} \neg I, 1}{\varphi} \neg E}{\perp} \perp, 10}{\varphi \rightarrow \psi} \rightarrow I, 2}{\varphi \vee (\varphi \rightarrow \psi)} \vee I, e}{\neg(\varphi \vee (\varphi \rightarrow \psi))} \neg E}{\perp} \neg I, 2$$

$$\underline{2.10} \quad \models (\psi \rightarrow \psi) \vee (\psi \rightarrow \delta)$$

$$\frac{\frac{\psi^{\cancel{1}}}{\psi \rightarrow \psi} \rightarrow I, 10}{(\psi \rightarrow \psi) \vee (\psi \rightarrow \delta)} \vee I_d \quad \frac{\cancel{2}}{\neg((\psi \rightarrow \psi) \vee (\psi \rightarrow \delta))} \neg E$$

$$\frac{\psi^{\cancel{3}}}{\neg \psi} \neg I, 1$$

$$\frac{\perp}{\delta} \perp, 10$$

$$\frac{\delta}{\psi \rightarrow \delta} \rightarrow I, 3$$

$$\frac{(\psi \rightarrow \psi) \vee (\psi \rightarrow \delta)}{\neg((\psi \rightarrow \psi) \vee (\psi \rightarrow \delta))} \neg E \quad \frac{\cancel{2}}{\neg((\psi \rightarrow \psi) \vee (\psi \rightarrow \delta))} \neg E$$

$$\frac{\perp}{(\psi \rightarrow \psi) \vee (\psi \rightarrow \delta)} \perp, 2$$

Pergunta 2
2.8

$$\begin{array}{c}
 \frac{\psi^1 \quad \neg \psi^2}{\perp} \neg E \\
 \frac{\perp}{\psi} \perp, 10 \\
 \frac{\psi}{\psi \rightarrow \psi} \rightarrow I, 4 \\
 \frac{\psi \rightarrow \psi \quad ((\psi \rightarrow \psi) \rightarrow \psi)^3}{\psi} \rightarrow E \\
 \frac{\psi \quad \neg \psi^2}{\perp} \neg E \\
 \frac{\perp}{\psi} \perp, 2 \\
 \frac{\psi}{((\psi \rightarrow \psi) \rightarrow \psi) \rightarrow \psi} \rightarrow I, 3
 \end{array}$$