

Lógica Computacional

LEI, 2024/2025

FCT UNL

Aula Prática 3

Dedução Natural em Lógica Proposicional.

Pergunta 1. Prove as afirmações seguintes.

1. $\vdash \varphi \rightarrow (\varphi \vee \psi)$
2. $\vdash (\varphi \vee \varphi) \rightarrow \varphi$
3. $\vdash (\varphi \wedge \psi) \rightarrow \varphi$
4. $\vdash \varphi \rightarrow (\psi \rightarrow \varphi)$
5. $\vdash ((\varphi \rightarrow \psi) \wedge (\psi \rightarrow \gamma)) \rightarrow (\varphi \rightarrow \gamma)$
6. $\vdash (\varphi \rightarrow (\psi \rightarrow \gamma)) \rightarrow ((\varphi \rightarrow \psi) \rightarrow (\varphi \rightarrow \gamma))$
7. $\vdash (\varphi \rightarrow \psi) \rightarrow (\varphi \rightarrow (\psi \vee \gamma))$
8. $\vdash (\psi \rightarrow \gamma) \rightarrow ((\varphi \wedge \psi) \rightarrow \gamma)$
9. $\vdash \neg(\varphi \vee \psi) \rightarrow \neg\varphi$
10. $\vdash (\psi \rightarrow \gamma) \rightarrow ((\varphi \wedge \psi) \rightarrow (\varphi \wedge \gamma))$

Pergunta 2. Prove as afirmações seguintes usando o Sistema \mathcal{N} .

1. $\models \neg\varphi \rightarrow (\varphi \rightarrow \psi)$
2. $\models ((\varphi \rightarrow \psi) \wedge \neg\psi) \rightarrow \neg\varphi$
3. $\models \varphi \leftrightarrow (\neg\neg\varphi)$
4. $\models ((\varphi \rightarrow \psi) \leftrightarrow (\neg\psi \rightarrow \neg\varphi))$
5. $\models \perp \rightarrow \varphi$
6. $\models \top$
7. $\models \varphi \vee \neg\varphi$
8. $\models ((\varphi \rightarrow \delta) \rightarrow \varphi) \rightarrow \varphi$
9. $\models \varphi \vee (\varphi \rightarrow \psi)$
10. $\models (\varphi \rightarrow \psi) \vee (\psi \rightarrow \delta)$

Pergunta 3. Prove as afirmações seguintes usando o Sistema \mathcal{N} .

1. $\{\neg\varphi \vee \psi\} \models \varphi \rightarrow \psi$
2. $\{\varphi \rightarrow \psi\} \models \neg\varphi \vee \psi$
3. $\{\neg(\varphi \wedge \psi)\} \models (\neg\varphi \vee \neg\psi)$
4. $\{\neg\varphi \vee \neg\psi\} \models \neg(\varphi \wedge \psi)$
5. $\{\neg(\varphi \vee \psi)\} \models \neg\varphi \wedge \neg\psi$
6. $\{\neg\varphi \wedge \neg\psi\} \models \neg(\varphi \vee \psi)$
7. $\{\varphi \vee (\psi \wedge \delta)\} \models (\varphi \vee \psi) \wedge (\varphi \vee \delta)$
8. $\{(\varphi \vee \psi) \wedge (\varphi \vee \delta)\} \models \varphi \vee (\psi \wedge \delta)$
9. $\{\varphi \wedge (\psi \vee \delta)\} \models (\varphi \wedge \psi) \vee (\varphi \wedge \delta)$
10. $\{(\varphi \wedge \psi) \vee (\varphi \wedge \delta)\} \models \varphi \wedge (\psi \vee \delta)$
11. $\{\varphi \leftrightarrow \psi\} \models (\varphi \wedge \delta) \leftrightarrow (\psi \wedge \delta)$
12. $\{\neg(\neg\varphi \vee \neg\psi)\} \models \varphi \wedge \psi$