Due date: Wednesday, February 20, 2019, at 9:30 AM.

Objective: To write a python program to compute an array containing the binomial coefficients and use this array to perform computations.

Problem: First follow the example from class to compute the binomial coefficients $C(n, m) = \binom{n}{m}$ and store them in an array (in other words, a matrix). Use the binomial coefficient array to perform the following computations. To verify a formula, it is enough to print out the number on left hand side, and the number on the right hand side.

(1) For $n = 24$, verify the identity:
$$\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \cdots + \binom{n}{n} = 2^n.$$

(2) For $n = 29$, verify the identity:
$$\binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \cdots + (-1)^n \binom{n}{n} = 0.$$

(3) For $n = 24$, verify the identity:
$$\binom{n}{1} + 2\binom{n}{2} + 3\binom{n}{3} + \cdots + n\binom{n}{n} = n2^{n-1}.$$

(4) For $n = 15$, verify the identity:
$$\binom{n}{0} + 2\binom{n}{1} + 2^2\binom{n}{2} + \cdots + 2^n\binom{n}{n} = 3^n.$$

(5) For $n = 27$ and these values of m: $5, 11, 17$, verify the identity:
$$\binom{m}{m} + \binom{m+1}{m} + \binom{m+2}{m} + \cdots + \binom{n}{m} = \binom{n+1}{m+1}.$$

(6) For these values of m: $1, 2, 3, 4, 5, 10, 16, 22, 29$, verify the identity:
$$\binom{m}{0} + \binom{m-1}{1} + \binom{m-2}{2} + \binom{m-3}{3} + \cdots = F_m.$$

where F_m is the mth Fibonacci number. Notice that the summation on the left hand side is understood to stop when $m - i < i$. To compute the first N Fibonacci numbers, use one of the programs that we wrote in class.