Design and Implementation of Small Microphone Arrays

for Acoustic and Speech Signal Processing

Jingdong Chen and Jacob Benesty

Northwestern Polytechnical University
127 Youyi West Road, Xi'an, China
jingdongchen@ieee.org

Université du Québec, INRS-EMT
Montréal, QC, Canada
benesty@emt.inrs.ca

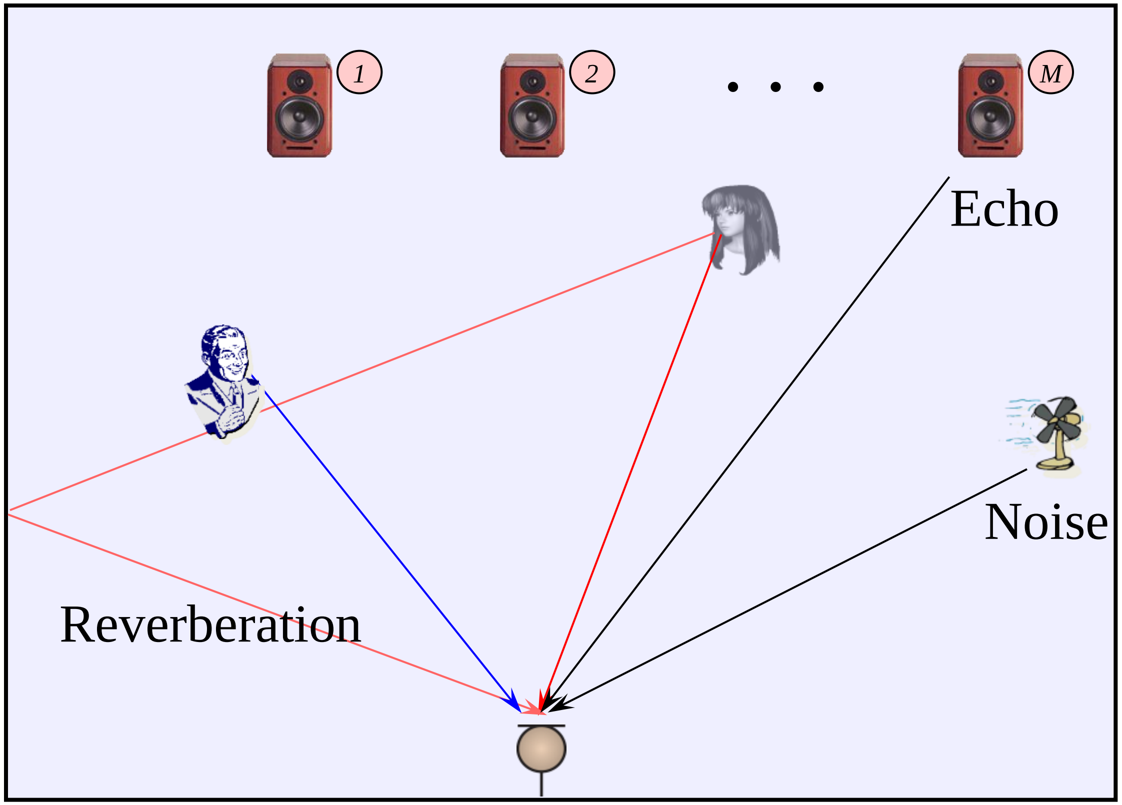
西北工業大學
NORTHWESTERN POLYTECHNICAL UNIVERSITY

Université du Québec
Institut national de la recherche scientifique

- Background

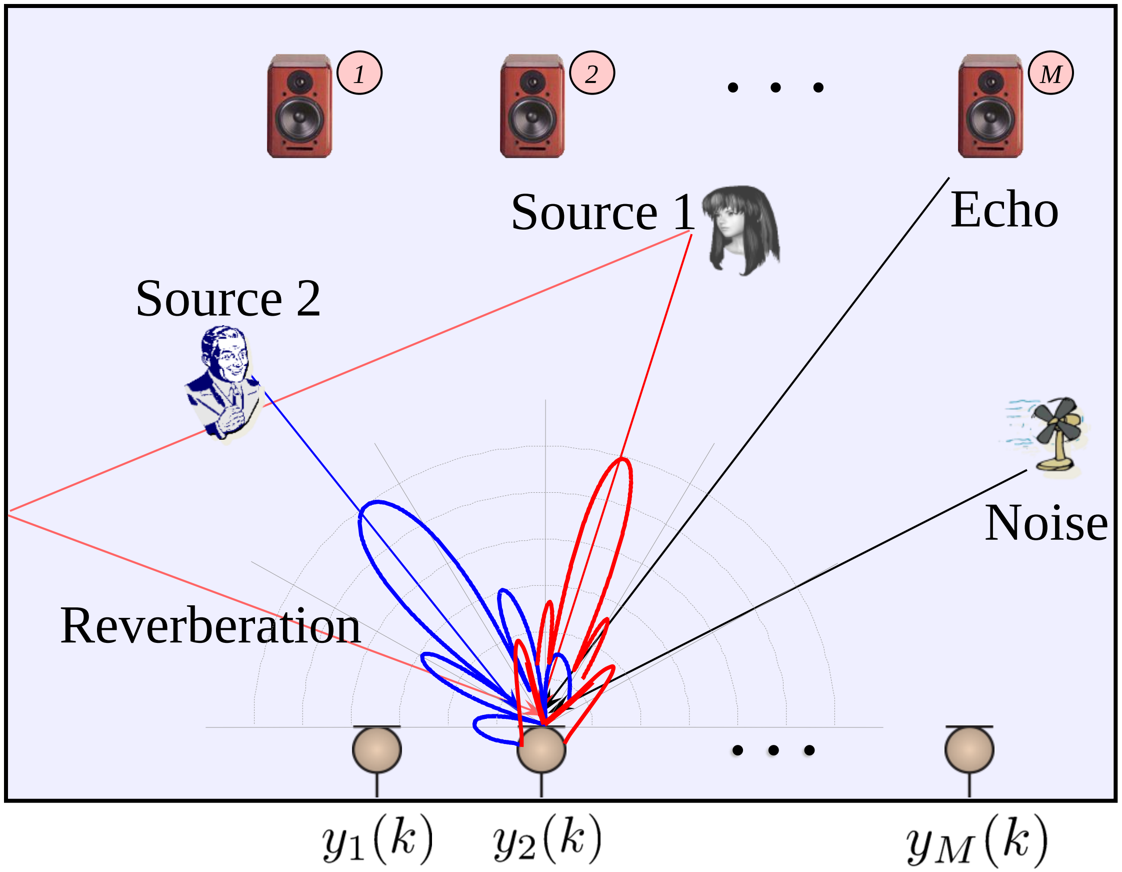
Tradition Voice Communication

- Background



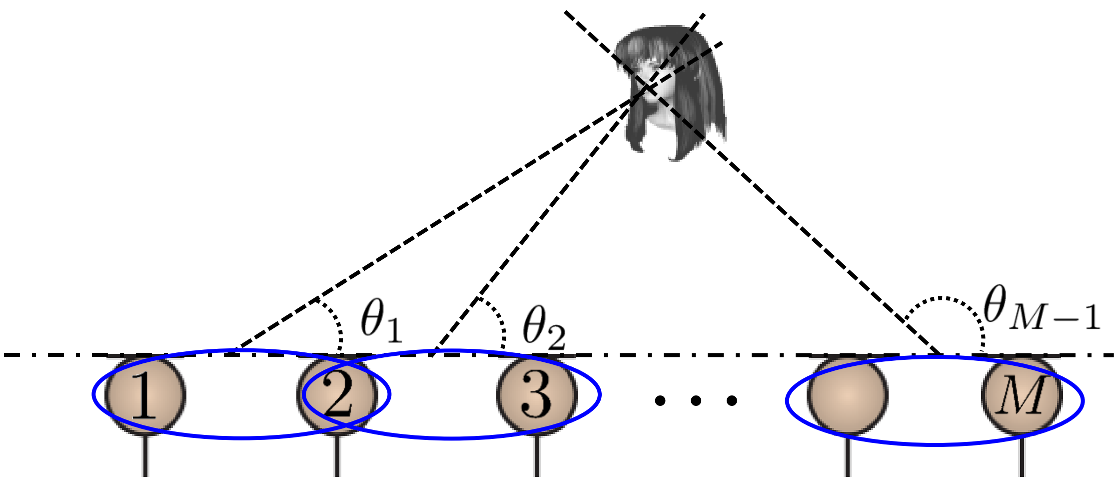
- Noise
- Reverberation
- Interference
- Echo

- Why Multiple Microphones?



- Source extraction
- Noise, interference, reverberation, and echo suppression
- Source separation

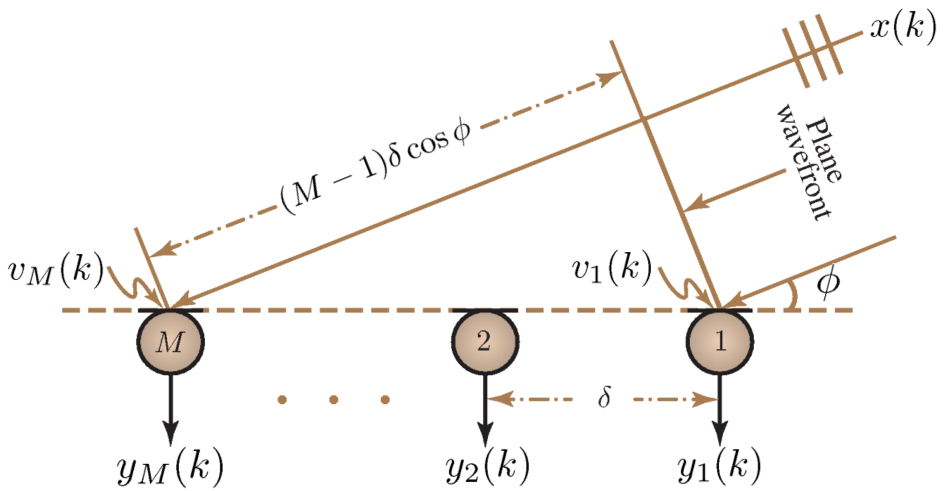
- Why Multiple Microphones?



- Source extraction
- Noise, interference, reverberation, and echo suppression
- Source separation
- DOA estimation
- Source localization

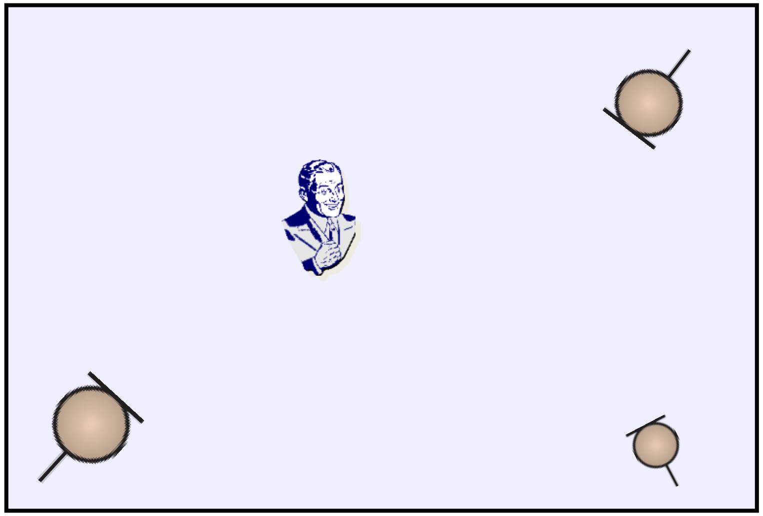
• Multiple-Microphone Systems

Organized arrays



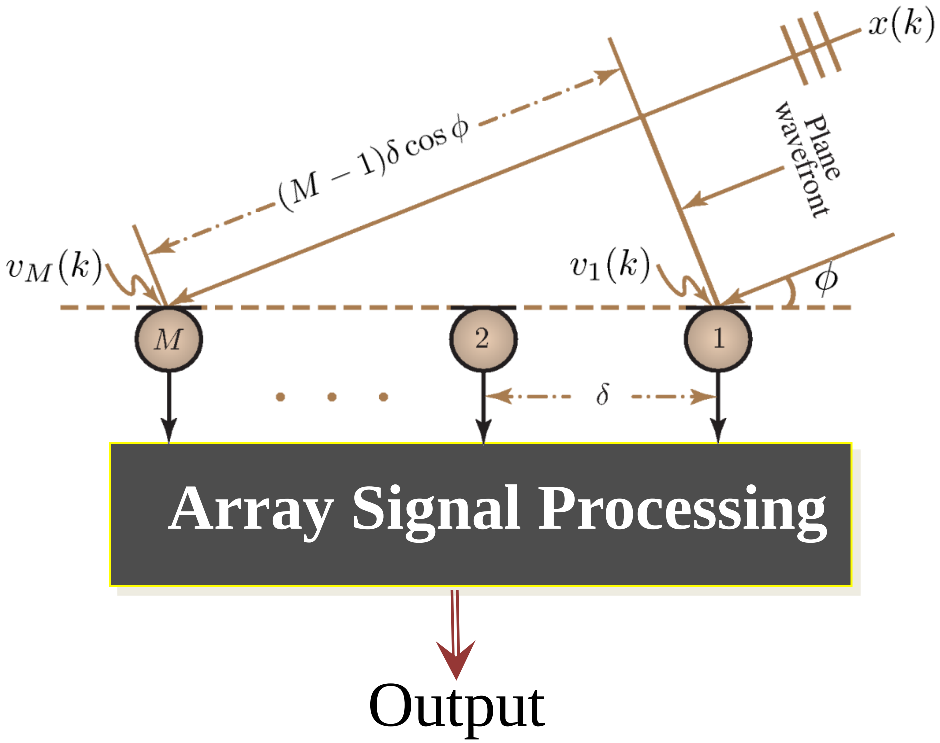
- Geometry is known
- Sensors are uniform in responses
- Sampled with the same clock

Disorganized systems



- Geometry is unknown or time varying
- Sensors may be different in responses
- Clock skew

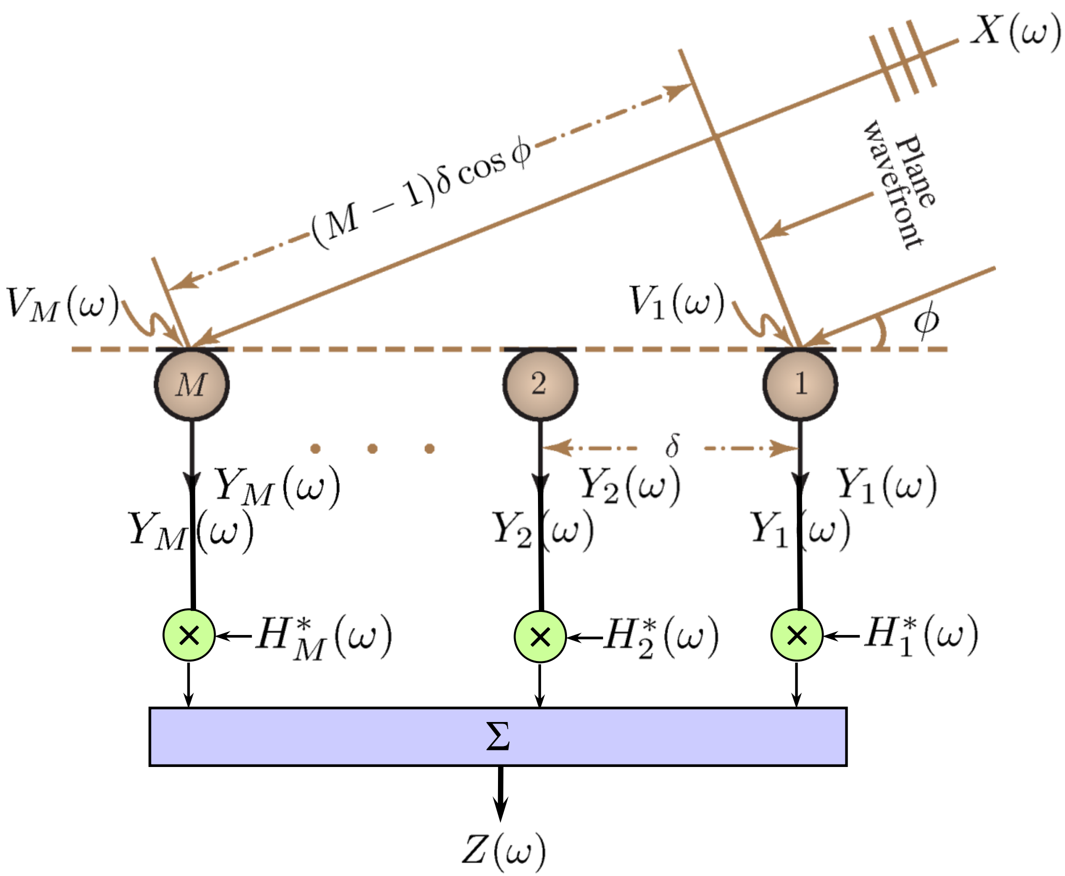
• Microphone Array and Beamforming



- Geometry
- Selection of sensors
- Calibration
- A/D

- Beamforming
- Multichannel NR
- Source Separation
- Source Localization

- Beamforming



- Beamforming

$$\boxed{Z(\omega)} = \sum_{m=1}^{M} H_m^*(\omega) Y_m(\omega) = \mathbf{h}^H(\omega)\mathbf{y}(\omega)$$

$$= \mathbf{h}^H(\omega)\mathbf{d}(\omega, \cos\phi)\boxed{X(\omega)} + \mathbf{h}^H(\omega)\mathbf{v}(\omega)$$

$$\mathbf{h}(\omega) = \begin{bmatrix} H_1(\omega) & H_2(\omega) & \cdots & H_M(\omega) \end{bmatrix}^T$$

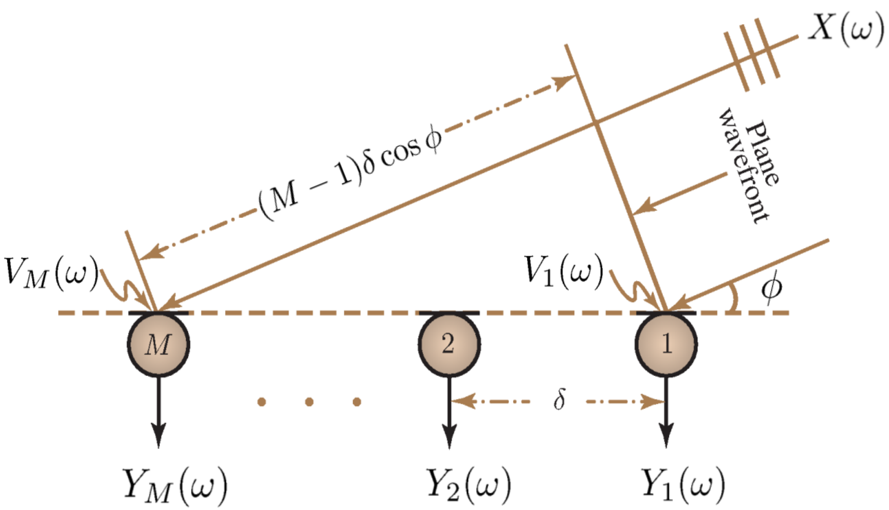
$$\mathbf{d}(\omega, \cos\theta) = \begin{bmatrix} 1 & e^{-\jmath\omega\delta\cos\phi/c} & \ldots & e^{-\jmath(M-1)\omega\delta\cos\phi/c} \end{bmatrix}^T$$

The core problem of beamforming design is to the optimal beamforming filter $\mathrm{h}(\omega)$

- Performance Criterion

Signal model: $Y_m(\omega) = e^{-\jmath(m-1)\omega\tau_0\cos\phi} X(\omega) + V_m(\omega)$

Beampattern: Describes the sensitivity of a beamformer to a plane wave impinging on the array from the direction θ)

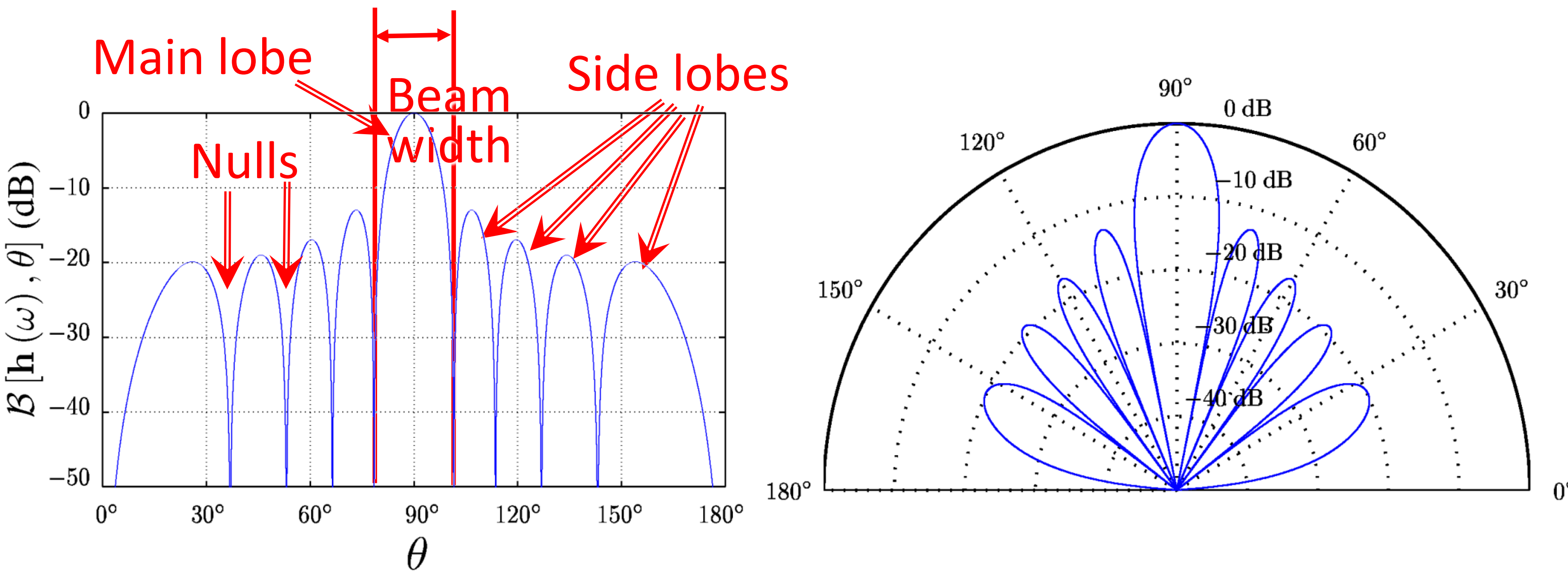


$$Z(\omega) = \mathbf{h}^H(\omega)\,\mathbf{d}(\omega, \cos\phi)\,X(\omega)$$

$$\mathcal{B}[\mathbf{h}(\omega), \theta] = \mathbf{d}^H(\omega, \cos\phi)\,\mathbf{h}(\omega) = \sum_{m=1}^{M} H_m(\omega)\, e^{\jmath(m-1)\omega\tau_0\cos\phi}$$

- Beamforming

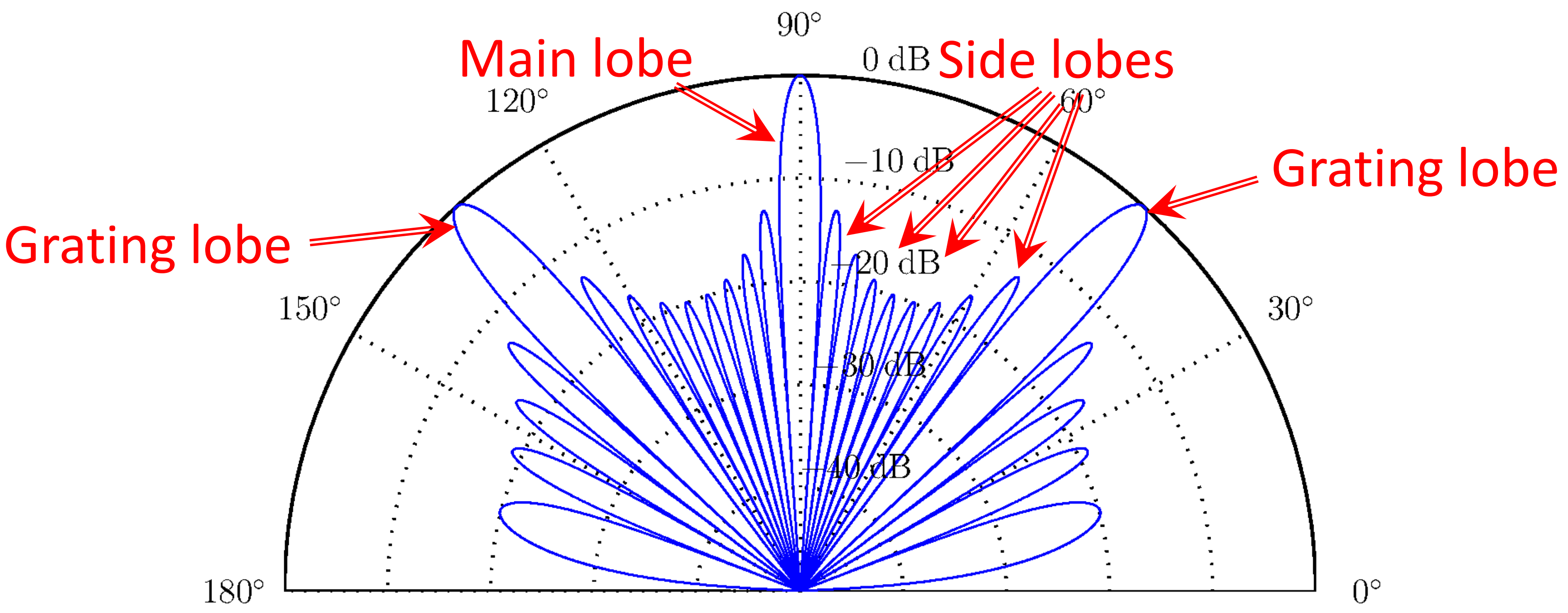
DS Beampattern: $\mathcal{B}\left[\mathbf{h}(\omega), \theta\right] = \left| \frac{\sin\left[M\omega\delta(\cos\phi - \cos\theta)/(2c)\right]}{M\sin\left[\omega\delta(\cos\phi - \cos\theta)/(2c)\right]} \right|$



Beampattern of a DS beamformer with a ten-sensor array when $\phi = 90°$, $\delta = 8$ cm, and $f = 2$ kHz

- Beamforming

DS Beampattern: $$\mathcal{B}\left[\mathbf{h}\left(\omega\right),\theta\right] = \left|\frac{\sin\left[M\omega d(\cos\phi - \cos\theta)/(2c)\right]}{M\sin\left[\omega\delta(\cos\phi - \cos\theta)/(2c)\right]}\right|$$



Beampattern of a DS beamformer with a ten-sensor array when $\phi = 90°$, $\delta = 24$ cm, and $f = 2$ kHz

- Performance Measures (Cont'd)
  - **SNR Gain**

Signal model: $Y_m(\omega) = e^{-\jmath(m-1)\omega\tau_0\cos\phi} X(\omega) + V_m(\omega)$

Input SNR: $\mathrm{iSNR}(\omega) = \dfrac{\phi_X(\omega)}{\phi_{V_1}(\omega)}$

BF output: $Z(\omega) = \mathbf{h}^H(\omega)\,\mathbf{y}(\omega) = \mathbf{h}^H(\omega)\,\mathbf{x}(\omega) + \mathbf{h}^H(\omega)\,\mathbf{v}(\omega)$

Output

$$\boldsymbol{\Phi}_\mathbf{v}(\omega) = E\left[\mathbf{v}(\omega)\,\mathbf{v}^H(\omega)\right]$$

$$\boldsymbol{\Gamma}_\mathbf{v}(\omega) = \frac{\boldsymbol{\Phi}_\mathbf{v}(\omega)}{\phi_{V_1}(\omega)}$$

$$\omega) \quad \frac{\left|\mathbf{h}^H(\omega)\,\mathbf{d}(\omega, \cos 0^\circ)\right|^2}{\mathbf{h}^H(\omega)\,\boldsymbol{\Phi}_\mathbf{v}(\omega)\,\mathbf{h}(\omega)}$$

$$\frac{\left|\mathbf{h}^H(\omega)\,\mathbf{d}(\omega, \cos 0^\circ)\right|^2}{\mathbf{h}^H(\omega)\,\boldsymbol{\Gamma}_\mathbf{v}(\omega)\,\mathbf{h}(\omega)}$$

- Performance Measures (Cont'd)

  - **SNR Gain**

SNR Gain: $$\mathcal{G}\left[\mathbf{h}\left(\omega\right)\right] = \frac{\text{oSNR}\left[\mathbf{h}\left(\omega\right)\right]}{\text{iSNR}\left(\omega\right)} = \frac{\left|\mathbf{h}^{H}\left(\omega\right)\mathbf{d}\left(\omega, \cos 0^{\circ}\right)\right|^{2}}{\mathbf{h}^{H}\left(\omega\right)\boldsymbol{\Gamma}_{\mathbf{v}}\left(\omega\right)\mathbf{h}\left(\omega\right)}$$

SNR gain depends on the noise pseudo-coherence matrix

  - **White Noise Gain (SNR gain in white noise)**

White Noise: $$\boldsymbol{\Gamma}_{\mathbf{v}}\left(\omega\right) = \mathbf{I}_{M}$$

WNG: $$\mathcal{G}_{\text{wn}}\left[\mathbf{h}\left(\omega\right)\right] = \frac{\left|\mathbf{h}^{H}\left(\omega\right)\mathbf{d}\left(\omega, \cos 0^{\circ}\right)\right|^{2}}{\mathbf{h}^{H}\left(\omega\right)\mathbf{h}\left(\omega\right)}$$

- Performance Measures (Cont'd)
  - **SNR Gain**

SNR Gain: $$\mathcal{G}\left[\mathbf{h}(\omega)\right] = \frac{\mathrm{oSNR}\left[\mathbf{h}(\omega)\right]}{\mathrm{iSNR}(\omega)} = \frac{\left|\mathbf{h}^H(\omega)\,\mathbf{d}(\omega, \cos 0^\circ)\right|^2}{\mathbf{h}^H(\omega)\,\mathbf{\Gamma}_{\mathbf{v}}(\omega)\,\mathbf{h}(\omega)}$$

  - **Directivity Index (SNR gain in diffuse noise)**

Diffuse Noise: $$\left[\mathbf{\Gamma}_{\mathbf{v}}(\omega)\right]_{ij} = \frac{\sin\left[\omega(j-i)\tau_0\right]}{\omega(j-i)\tau_0} = \mathrm{sinc}\left[\omega(j-i)\tau_0\right]$$

Directivity Index: $$\mathcal{G}\left[\mathbf{h}(\omega)\right] \longrightarrow \mathcal{G}_{\mathrm{dn}}\left[\mathbf{h}(\omega)\right]$$

$$\mathcal{D}\left[\mathbf{h}(\omega)\right] = 10\log_{10}\mathcal{G}_{\mathrm{dn}}\left[\mathbf{h}(\omega)\right]$$

- Performance Measures (Cont'd)

  - **SNR Gain**

SNR Gain: $$\mathcal{G}\left[\mathbf{h}(\omega)\right] = \frac{\mathrm{oSNR}\left[\mathbf{h}(\omega)\right]}{\mathrm{iSNR}(\omega)} = \frac{\left|\mathbf{h}^H(\omega)\,\mathbf{d}(\omega, \cos 0^\circ)\right|^2}{\mathbf{h}^H(\omega)\,\mathbf{\Gamma}_\mathbf{v}(\omega)\,\mathbf{h}(\omega)}$$

  - **SNR gain in point-source noise**

Point Noise: $$\mathbf{\Gamma}_\mathbf{v}(\omega) = \mathbf{d}(\omega, \cos\theta_\mathrm{n})\,\mathbf{d}^H(\omega, \cos\theta_\mathrm{n})$$

SNR Gain: $$\mathcal{G}\left[\mathbf{h}(\omega)\right] \longrightarrow \mathcal{G}_\mathrm{pn}\left[\mathbf{h}(\omega), \theta_\mathrm{n}\right]$$

$$\mathcal{G}_\mathrm{pn}\left[\mathbf{h}(\omega), \theta_\mathrm{n}\right] = \frac{1}{\left|\mathcal{B}\left(\mathbf{h}(\omega), \theta_\mathrm{n}\right)\right|^2}$$

- Beamforming

Microphone Array Beamforming

Noise Field?

Stationary, Known before the design

Fixed Beamformers

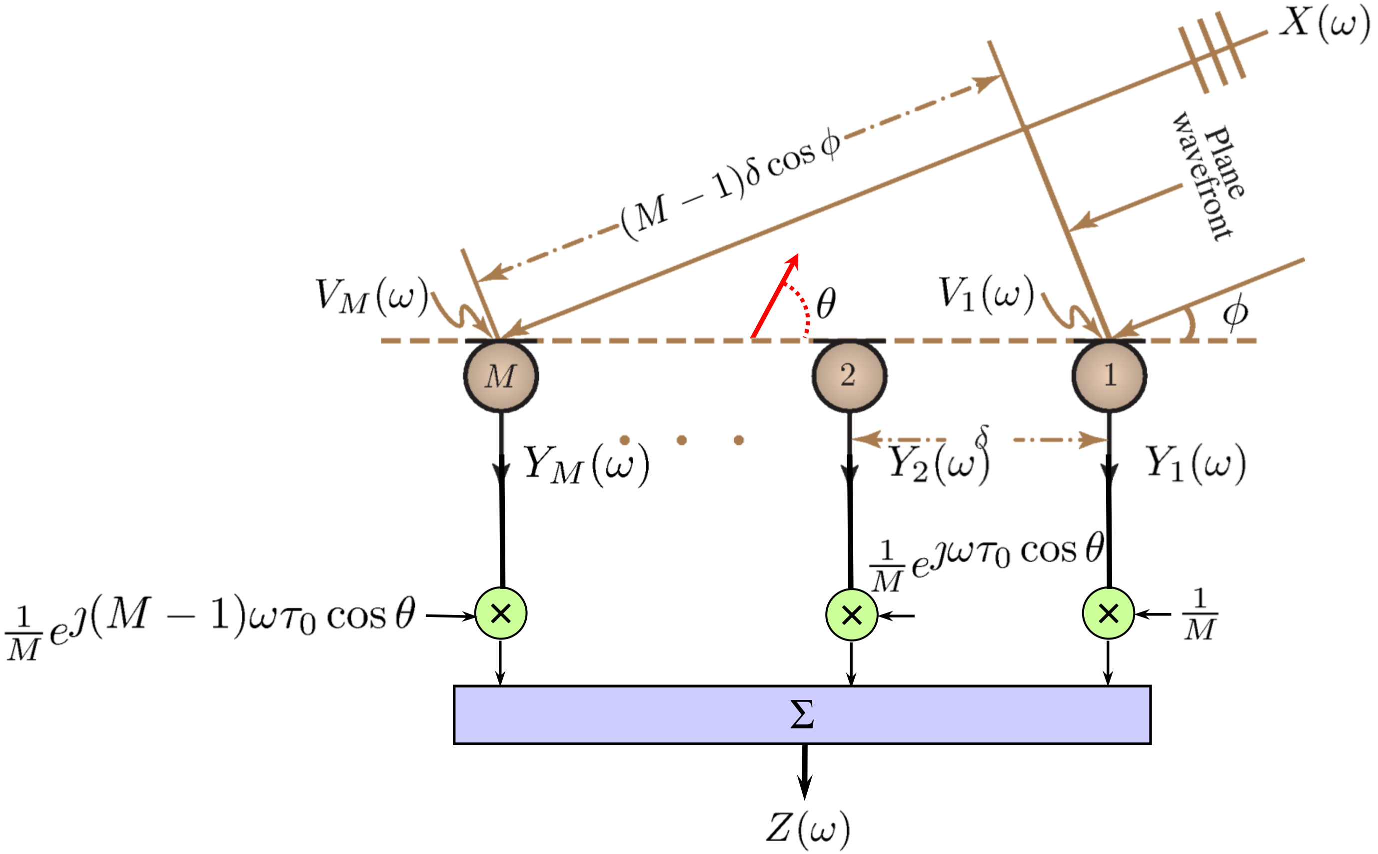
Isotropic noise
generally assumed

Delay-and-Sum

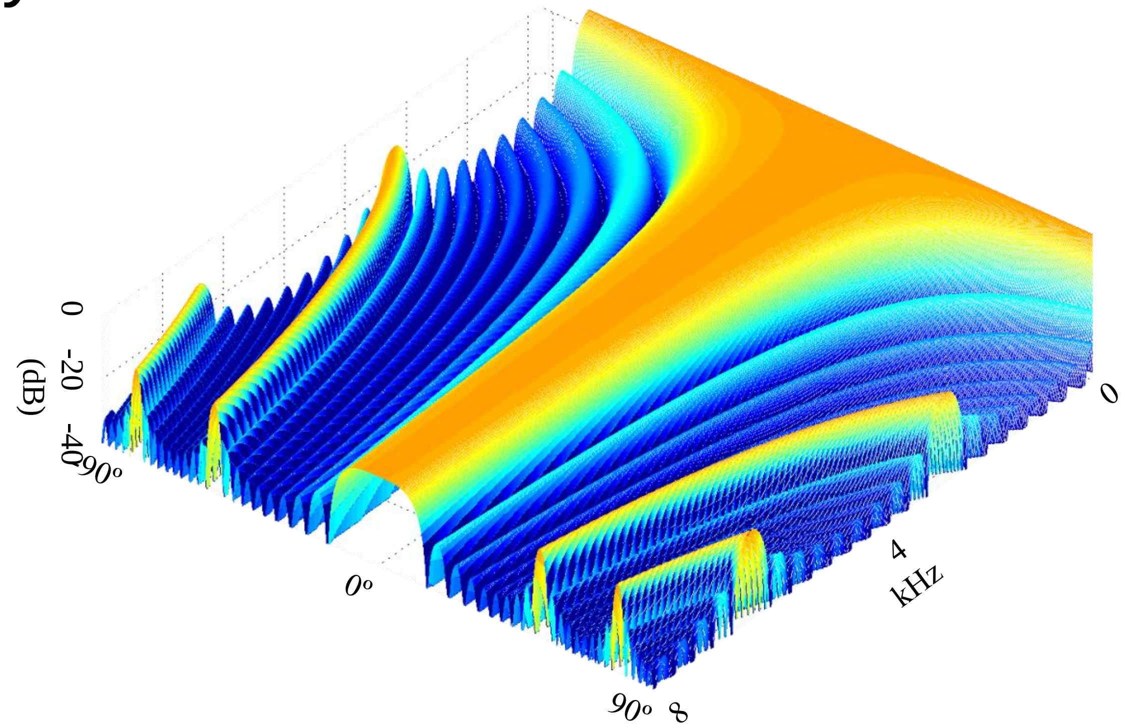
Delay-and-Sum

- Simple
- Non-uniform directional responses over a wide spectrum of frequencies

- Delay and Sum



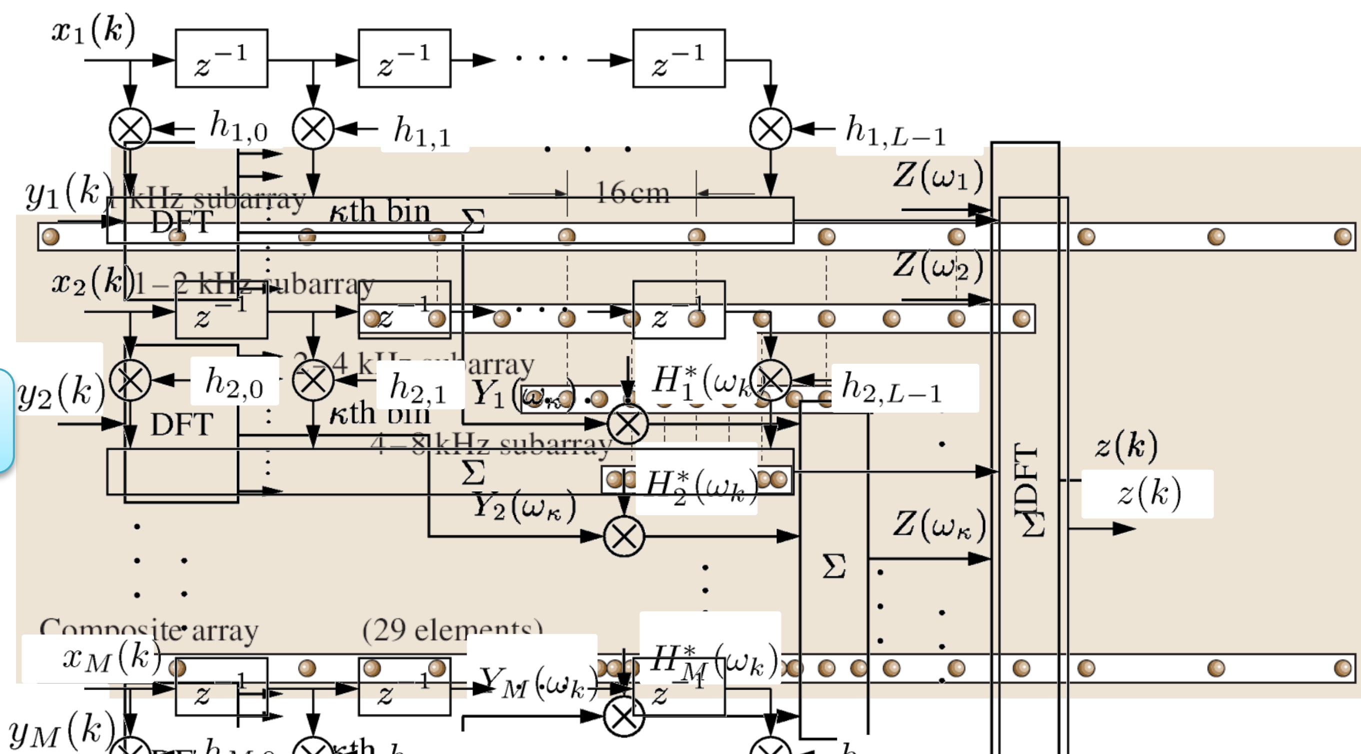
- Delay and Sum



– Nonuniform beam width (spectral tilt, lowpass filtering the desired speech signal)
– Not very effective in reducing the reverberation effect.

- Constant Beamwidth Beamforming

Filter-and-Sum:



G. W. Elko and J. Meyer, "Microphone Arrays," in *Springer Handbook on Speech Processing and Speech Communication*, J. Benesty, M. M. Sondhi, and Y. Huang, Eds., Berlin: Springer-Verlag, 2007.

- Beamforming

Microphone Array Beamformers

Noise Field?

Stationary, Known before the design

Fixed Beamformers

Isotropic noise generally assumed

Delay-and-Sum

Filter-and-Sum

Delay-and-Sum

- Simple
- Non-uniform directional responses over a wide spectrum of frequencies

Filter-and-Sum

- Uniform directional responses over a wide spectrum of frequencies: good for wideband signals, like speech

- Beamforming

Microphone Array Beamformers

Noise Field?

Stationary, Known before the design → **Fixed Beamformers**

Time Varying, Unknown → **Adaptive Beamformers**

Isotropic noise generally assumed

Fixed Beamformers → **Delay-and-Sum**, **Filter-and-Sum**

Adaptive Beamformers — Reverberation? — Not Concerned → **MVDR (Capon)**; Significant → **LCMV (Frost)/GSC**

Super Gain

(assuming noise Correlation matrix is known)

Delay-and-Sum

- Simple
- Non-uniform directional responses over a wide spectrum of frequencies

Filter-and-Su

- Complicate
- Uniform di responses c spectrum o frequencies wideband signals, like speech

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- Reverberation causes the signal cancellation problem.
- Time-domain or frequency-domain

LCMV (Frost)/GSC

- The impulse responses (IRs) from the source to the microphones have to be known or estimated.
- Errors in the IRs lead to the signal cancellation problem.

- Performance consistency over frequencies
- Performance consistency in different environments
- Working with other Processors
- Working with modern devices

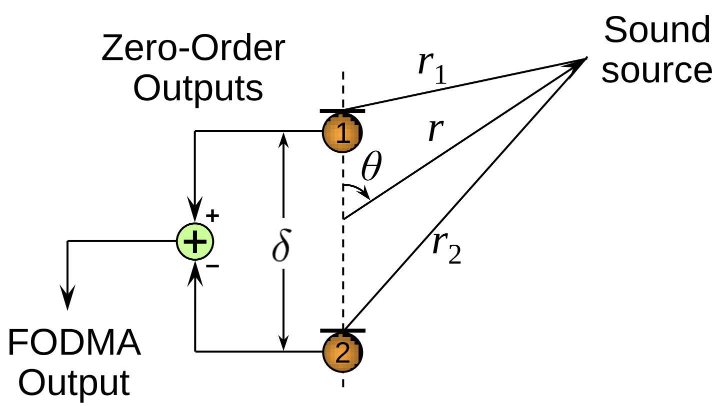
- Small Microphone Arrays

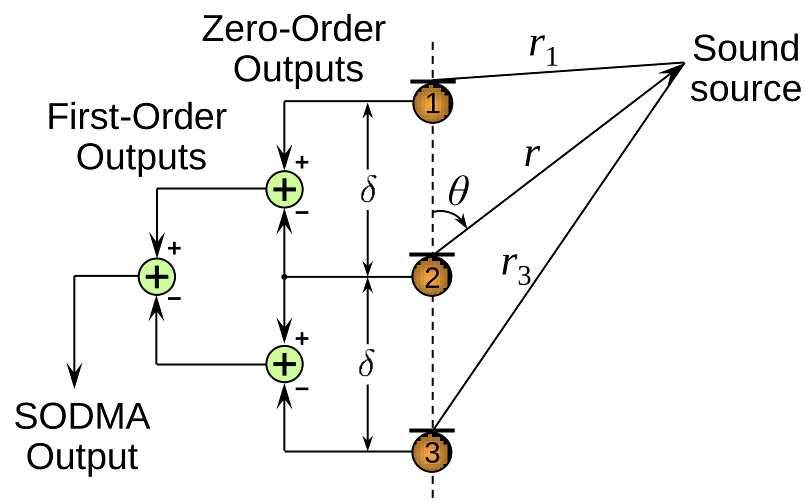
Differential Microphone Arrays

- Additive Array vs Differential Array
  - Both sensors and the array are responsive to the pressure field
  - Size is large (spacing from a couple of centimeters to a few decimeters)
  - Optimal gain is on the broadside
  - Signal extraction is achieved by steering the main lobe to the signal direction

- Additive Array vs Differential Array
  - Sensors measure the pressure field; while the array is responsive to the spatial derivatives of the acoustic pressure field
  - Size is small: the sensor spacing, δ, is much smaller than the acoustic wavelength, so that the true acoustic pressure differentials can be approximated by finite differences of the microphones' outputs.
  - Optimal gain is on the endfire direction

• Traditional Design of DMA



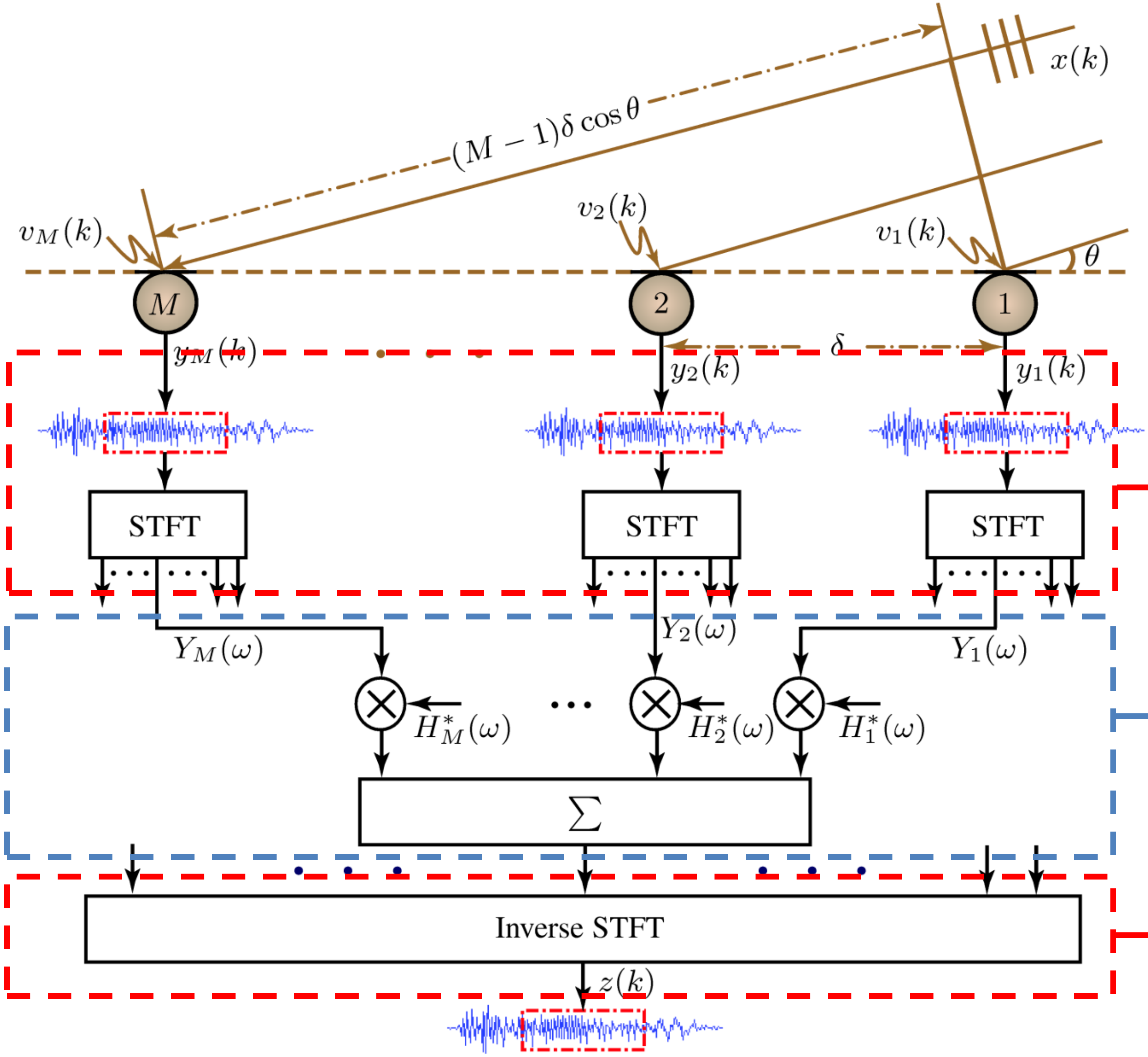


1st-Order DMA

2nd-Order DMA

- An Nth order DMA is formed by subtractively combining the outputs of two DMAs of order N-1
- Not flexible to design different beampatterns
- Not flexible in dealing with white noise amplification

- New Method of DMA Design



The new paradigm is for processing nonstationary broadband signals like speech

Analysis (STFT)

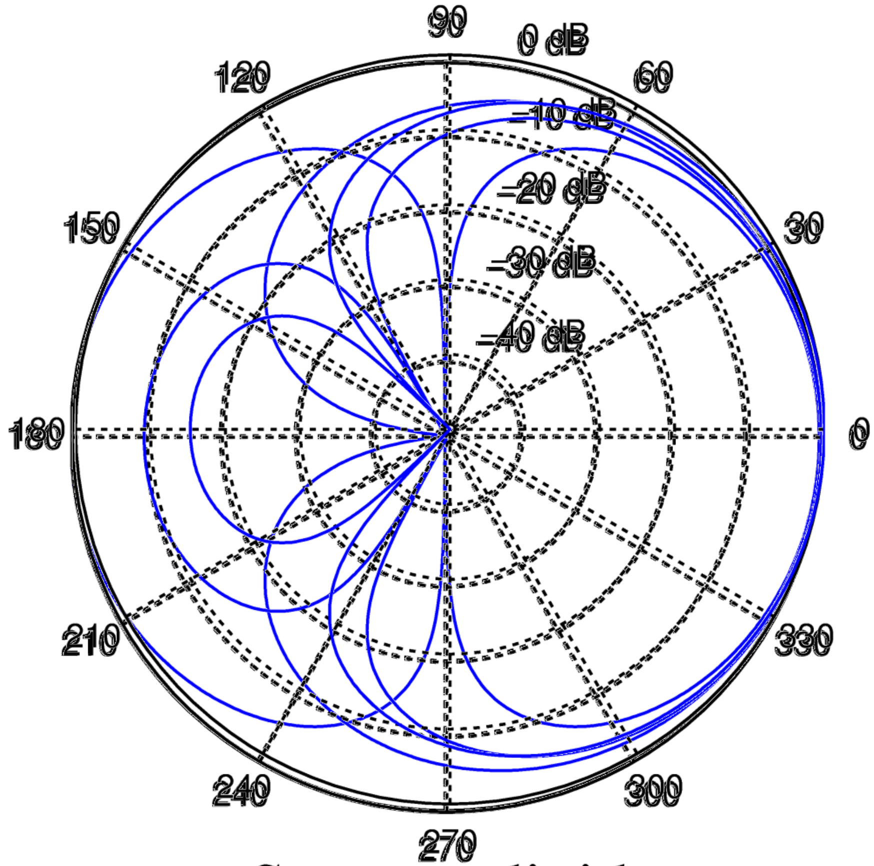
DMA Processing

Synthesis (ISTFT)

- What is a DMA
  - DMA Beampatterns: $\mathcal{B}_N(\theta) = \sum_{n=0}^{N} a_{N,n} \cos^n(\theta)$

- What is a DMA
  - 1st-order DMA: $\mathcal{B}_1(\theta) = (1 - a_{1,1}) + a_{1,1}\cos\theta$

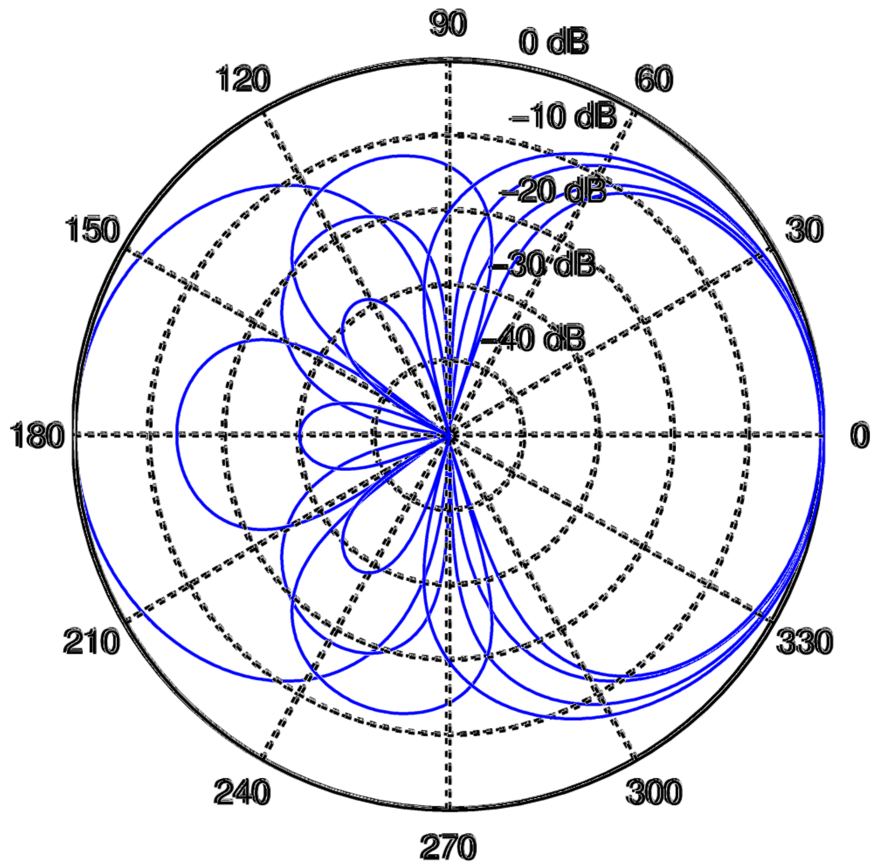
$$a_{1,1} = 2 - \sqrt{2}$$



Supercardioid

- What is a DMA
  - 2nd-order DMA: $\mathcal{B}_2(\theta) = (1 - a_{2,1} - a_{2,2}) + a_{2,1}\cos\theta + a_{2,2}\cos^2\theta$

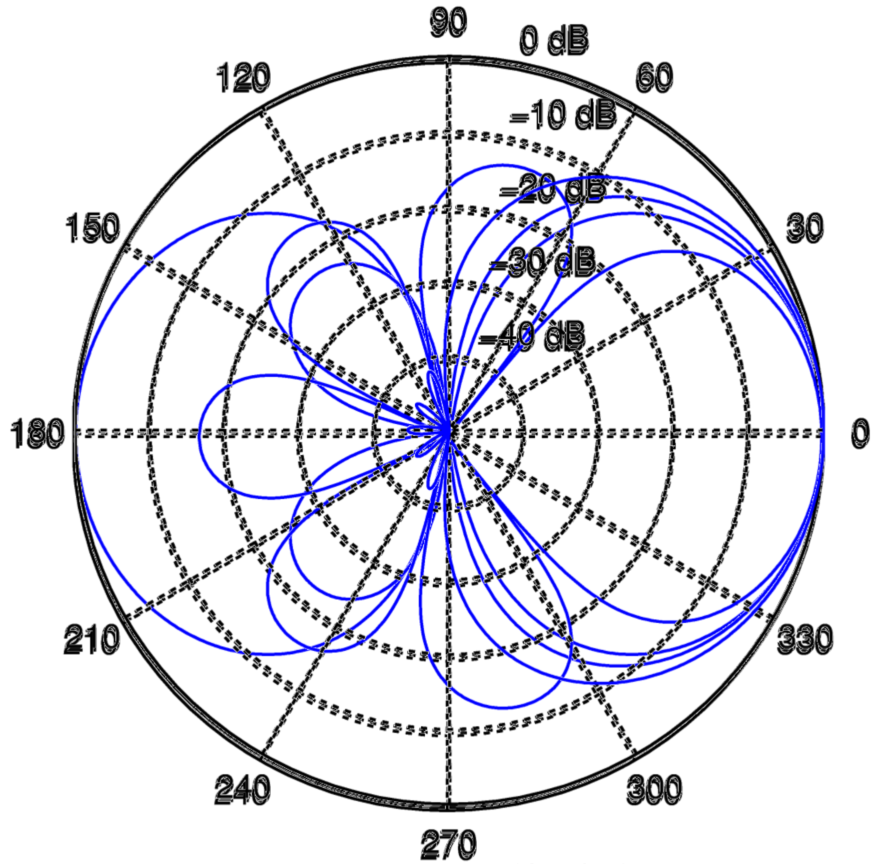
$$a_{2,1} \approx 0.484$$
$$a_{2,2} \approx 0.413$$



Supercardioid

- What is a DMA
  - 3rd-order DMA: $\mathcal{B}_3(\theta) = (1 - a_{3,1} - a_{3,2} - a_{3,3}) + a_{3,1}\cos\theta + a_{3,2}\cos^2\theta + a_{3,3}\cos^3\theta$

$$a_{3,1} = 0.217$$
$$a_{3,2} = 0.475$$
$$a_{3,3} = 0.286$$



Supercardioid

- DMA Beamforming
  - Ideal DMA Beampatterns:

$$\mathcal{B}_N(\theta) = \sum_{n=0}^{N} a_{N,n} \cos^n(\theta)$$

  - Beampattern with $\mathbf{h}(\omega)$

$$\mathcal{B}[\mathbf{h}(\omega), \theta] = \mathbf{d}^H(\omega, \cos\theta)\,\mathbf{h}(\omega)$$

Given *M*, *N*, and the array geometry, finding coefficients in $\mathbf{h}(\omega)$ so that

$$\mathcal{B}[\mathbf{h}(\omega), \theta] \rightarrow \mathcal{B}(\theta)$$

- DMA Beamforming for a 2^{nd}-Order Cardioid

Beampattern: $\mathcal{B}\left[\mathbf{h}\left(\omega\right), \theta\right] = \mathbf{d}^H\left(\omega, \cos\theta\right)\mathbf{h}\left(\omega\right)$

$$\mathbf{h}\left(\omega\right) = \begin{bmatrix} H_1\left(\omega\right) & H_2\left(\omega\right) & H_3\left(\omega\right) \end{bmatrix}^T$$

$$\mathbf{d}\left(\omega, \cos\theta\right) = \begin{bmatrix} 1 & e^{-\jmath\omega\delta\cos\theta/c} & e^{-\jmath 2\omega\delta\cos\theta/c} \end{bmatrix}^T$$

- DMA Beamforming for a 2nd-Order Cardioid

$$\mathbf{d}^H\left(\omega, \cos 0^\circ\right) \mathbf{h}\left(\omega\right) = 1$$

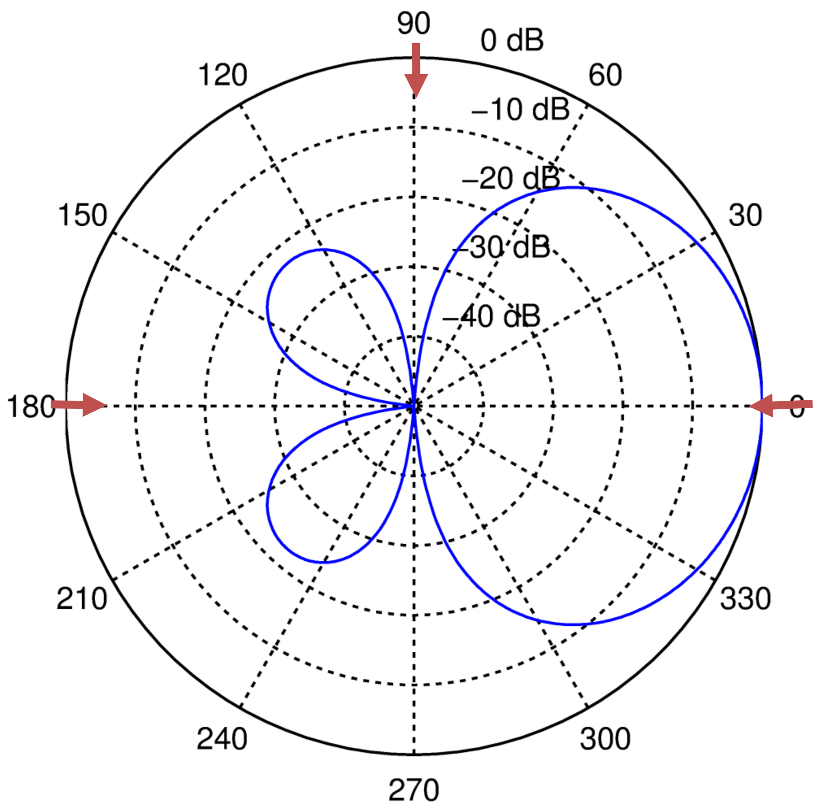
$$\mathbf{d}^H\left(\omega, \cos 90^\circ\right) \mathbf{h}\left(\omega\right) = 0$$

$$\mathbf{d}^H\left(\omega, \cos 180^\circ\right) \mathbf{h}\left(\omega\right) = 0$$

$$\boxed{\mathbf{D}\left(\omega, \boldsymbol{\alpha}\right) \mathbf{h}\left(\omega\right) = \boldsymbol{\beta}}$$

$$\mathbf{D}\left(\omega, \boldsymbol{\alpha}\right) = \begin{bmatrix} \mathbf{d}^H\left(\omega, \cos 0^\circ\right) \\ \mathbf{d}^H\left(\omega, \cos 90^\circ\right) \\ \mathbf{d}^H\left(\omega, \cos 180^\circ\right) \end{bmatrix}$$

$$\boldsymbol{\alpha} = \begin{bmatrix} \cos 0^\circ \\ \cos 90^\circ \\ \cos 180^\circ \end{bmatrix} \qquad \boldsymbol{\beta} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$



2nd-order Cardioid

- DMA Beamforming for a 2nd-Order Cardioid
  - Is the design beampattern the same as the ideal beampattern?
  - Is the method generalizable?

- DMA Beamforming with Distinct Nulls

  – *N*+1 Constraints:

$$\alpha_{N,n} = \cos\theta_{N,n}$$

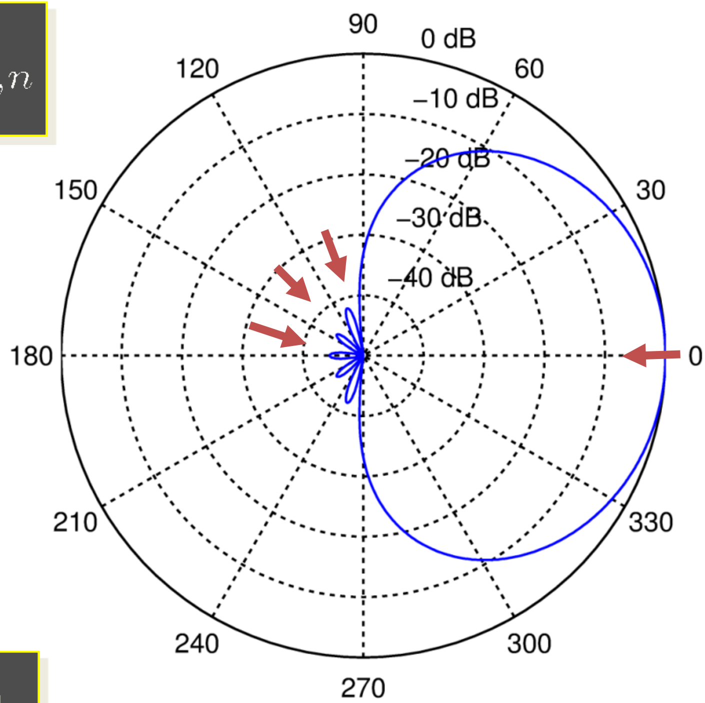
$$\boldsymbol{\alpha} = [1 \quad \alpha_{N,1} \quad \cdots \quad \alpha_{N,N}]^T$$

$$\boldsymbol{\beta} = [1 \quad 0 \quad \cdots \quad 0]^T$$

  – Linear system with *N*+1 equations

$$\mathbf{D}(\omega, \boldsymbol{\alpha})\,\mathbf{h}(\omega) = \boldsymbol{\beta}$$

$$\mathbf{D}(\omega, \boldsymbol{\alpha}) = \begin{bmatrix} \mathbf{d}^H(\omega, 1) \\ \mathbf{d}^H(\omega, \alpha_{N,1}) \\ \vdots \\ \mathbf{d}^H(\omega, \alpha_{N,N}) \end{bmatrix}$$



- DMA Beamforming with Distinct Nulls: Examples

  – DMA filter: $\mathbf{h}(\omega) = \mathbf{D}^{-1}(\omega, \boldsymbol{\alpha})\boldsymbol{\beta}$

  – 1st-order DMA filter

$$\begin{bmatrix} 1 & e^{\jmath\omega\tau_0} \\ 1 & e^{\jmath\omega\tau_0\alpha_{1,1}} \end{bmatrix} \mathbf{h}(\omega) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

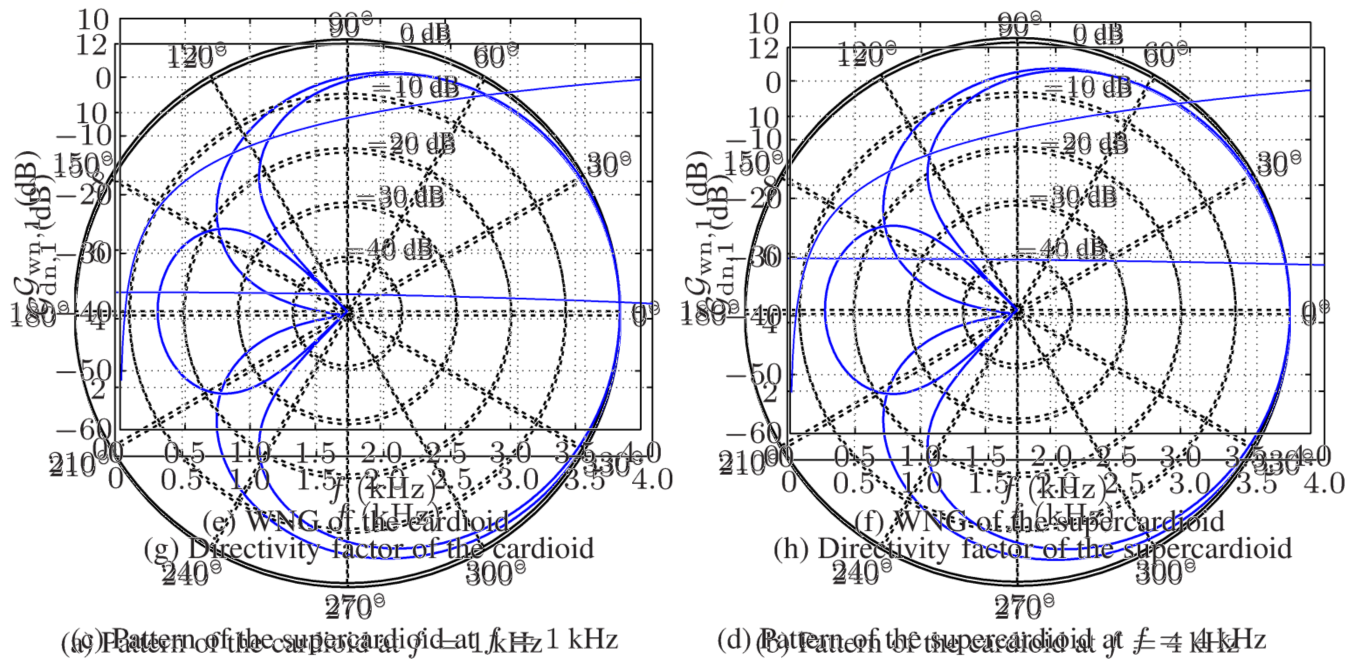
$$\mathbf{h}(\omega) = \frac{1}{1 - e^{\jmath\omega\tau_0(1-\alpha_{1,1})}} \begin{bmatrix} 1 \\ -e^{-\jmath\omega\tau_0\alpha_{1,1}} \end{bmatrix}$$

$$\tau_0 = \delta/c$$

- 1st-order DMA filter:

$$\mathbf{h}(\omega) = \frac{1}{1 - e^{\jmath\omega\tau_0(1-\alpha_{1,1})}} \begin{bmatrix} 1 \\ -e^{-\jmath\omega\tau_0\alpha_{1,1}} \end{bmatrix}$$

WNG



—2nd-order DMA filter

$$\begin{bmatrix} 1 & e^{\jmath\omega\tau_0} & e^{\jmath 2\omega\tau_0} \\ 1 & e^{\jmath\omega\tau_0\alpha_{2,1}} & e^{\jmath 2\omega\tau_0\alpha_{2,1}} \\ 1 & e^{\jmath\omega\tau_0\alpha_{2,2}} & e^{\jmath 2\omega\tau_0\alpha_{2,2}} \end{bmatrix} \mathbf{h}(\omega) = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\mathbf{h}(\omega) = \frac{1}{\left[1 - e^{\jmath\omega\tau_0(1-\alpha_{2,1})}\right]\left[1 - e^{\jmath\omega\tau_0(1-\alpha_{2,2})}\right]} \begin{bmatrix} 1 \\ -e^{-\jmath\omega\tau_0\alpha_{2,1}} - e^{-\jmath\omega\tau_0\alpha_{2,2}} \\ e^{-\jmath\omega\tau_0(\alpha_{2,1}+\alpha_{2,2})} \end{bmatrix}$$

Cardioid: $\alpha_{2,1} = 0$, $\alpha_{2,2} = -1$ (i.e., $\theta_{2,1} = 90^\circ$, $\theta_{2,2} = 180^\circ$)

Hypercardioid: $\alpha_{2,1} = 0.31$, $\alpha_{2,2} = -0.81$ (i.e., $\theta_{2,1} \approx 72^\circ$, $\theta_{2,2} \approx 144^\circ$)

Supercardioid: $\alpha_{2,1} = -0.28$, $\alpha_{2,2} = -0.89$ (i.e., $\theta_{2,1} \approx 106^\circ$, $\theta_{2,2} \approx 153^\circ$)

Quadrupole: $\alpha_{2,1} = \sqrt{2}/2$, $\alpha_{2,2} = -\sqrt{2}/2$ (i.e., $\theta_{2,1} = 45^\circ$, $\theta_{2,2} = 135^\circ$)

—3rd-order DMA filter

$$\begin{bmatrix} \mathbf{d}^H(\omega, 1) \\ \mathbf{d}^H(\omega, \alpha_{3,1}) \\ \mathbf{d}^H(\omega, \alpha_{3,2}) \\ \mathbf{d}^H(\omega, \alpha_{3,3}) \end{bmatrix} \mathbf{h}(\omega) = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\mathbf{h}(\omega) = \frac{1}{\left[1 - e^{\jmath\omega\tau_0(1-\alpha_{3,1})}\right]\left[1 - e^{\jmath\omega\tau_0(1-\alpha_{3,2})}\right]\left[1 - e^{\jmath\omega\tau_0(1-\alpha_{3,3})}\right]} \begin{bmatrix} 1 \\ \gamma_1 \\ \gamma_2 \\ \gamma_3 \end{bmatrix}$$

$$\gamma_1 = -e^{-\jmath\omega\tau_0\alpha_{3,1}} - e^{-\jmath\omega\tau_0\alpha_{3,2}} - e^{-\jmath\omega\tau_0\alpha_{3,3}}$$

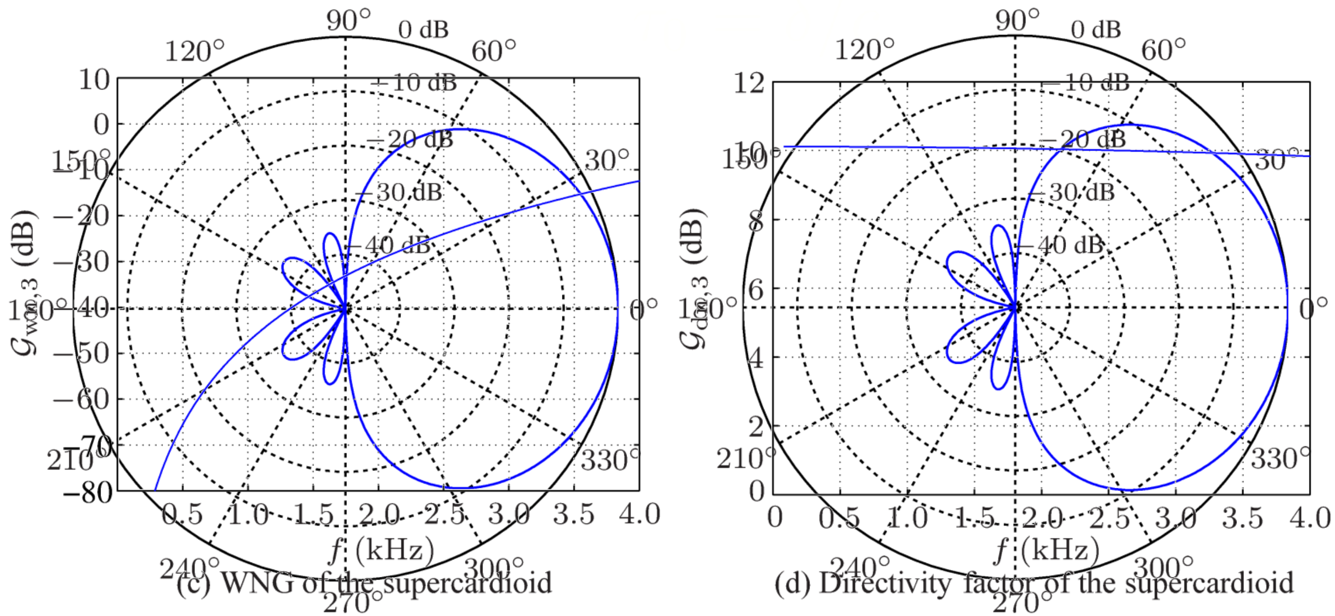
$$\gamma_2 = e^{-\jmath\omega\tau_0(\alpha_{3,1}+\alpha_{3,2})} + e^{-\jmath\omega\tau_0(\alpha_{3,2}+\alpha_{3,3})} + e^{-\jmath\omega\tau_0(\alpha_{3,1}+\alpha_{3,3})}$$

$$\gamma_3 = -e^{-\jmath\omega\tau_0(\alpha_{3,1}+\alpha_{3,2}+\alpha_{3,3})}$$

- DMA Beamforming with Distinct Nulls: Examples

–3rd-order DMA filter

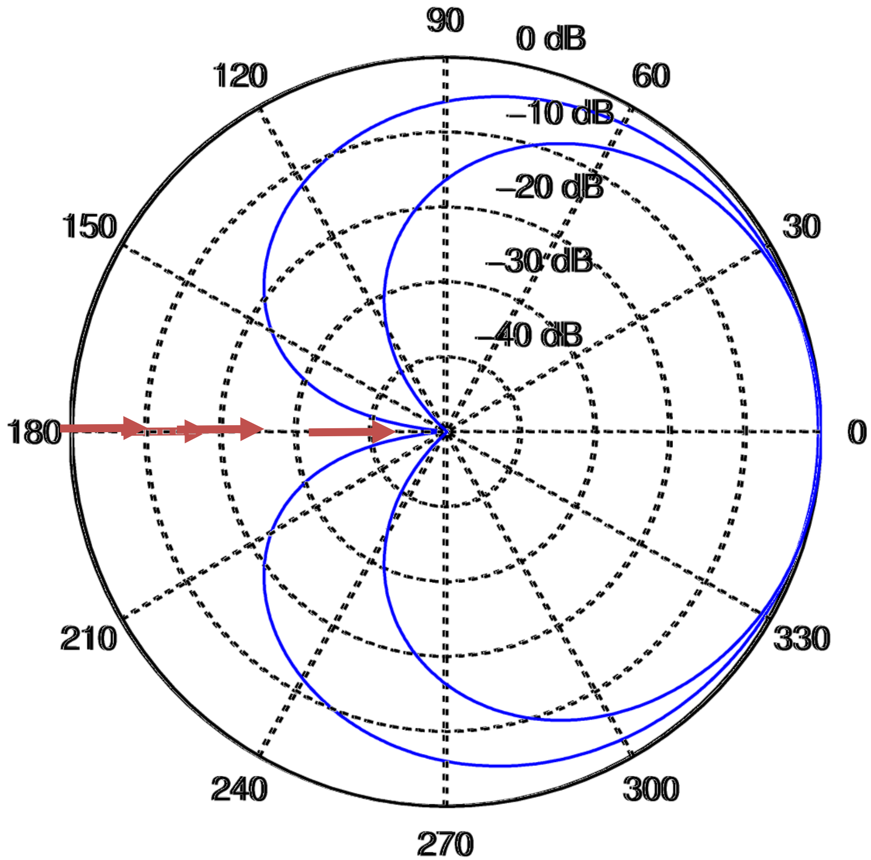
$$\alpha_{3,1} = 0$$
$$\alpha_{3,2} = -1/2$$
$$\alpha_{3,3} = -1$$
$$\delta = 1\ \mathrm{cm}$$



(a) Pattern of the supercardioid at $f = 1$ kHz

(b) Pattern of the supercardioid at $f = 4$ kHz

- DMA Beamforming with Nulls of Multiplicity More Than One



3rd-order Cardioid

1st-order Cardioid

- DMA Beamforming with Nulls of Multiplicity More Than One

An Nth-order DMA pattern that has N nulls but one of multiplicity P ($1 \leq P \leq N$) at $\alpha_{N,n} = \cos\theta_{N,n}$

$$\mathcal{B}_N(\theta) = \mathcal{B}_{N-P}(\theta) \times (\cos\theta - \alpha_{N,n})^P$$

$$\left.\frac{\partial^p \mathcal{B}_N(\theta)}{\partial \alpha^p}\right|_{\alpha=\alpha_{N,n}} = 0, \; p = 1, 2, \ldots, P-1$$

- DMA Beamforming with Nulls of Multiplicity More Than One

— Beampattern with $\mathbf{h}(\omega)$

$$\frac{\partial^p \mathcal{B}\left[\mathbf{h}(\omega), \theta\right]}{\partial \alpha^p} = \frac{\partial^p \left[\mathbf{d}^H(\omega, \alpha)\, \mathbf{h}(\omega)\right]}{\partial \alpha^p}$$

$$= (\jmath \omega \tau_0)^p \left[\boldsymbol{\Sigma}^p \mathbf{d}(\omega, \alpha)\right]^H \mathbf{h}(\omega)$$

$$\boldsymbol{\Sigma} = \mathrm{diag}\left(0,\ 1,\ \cdots,\ M-1\right)$$

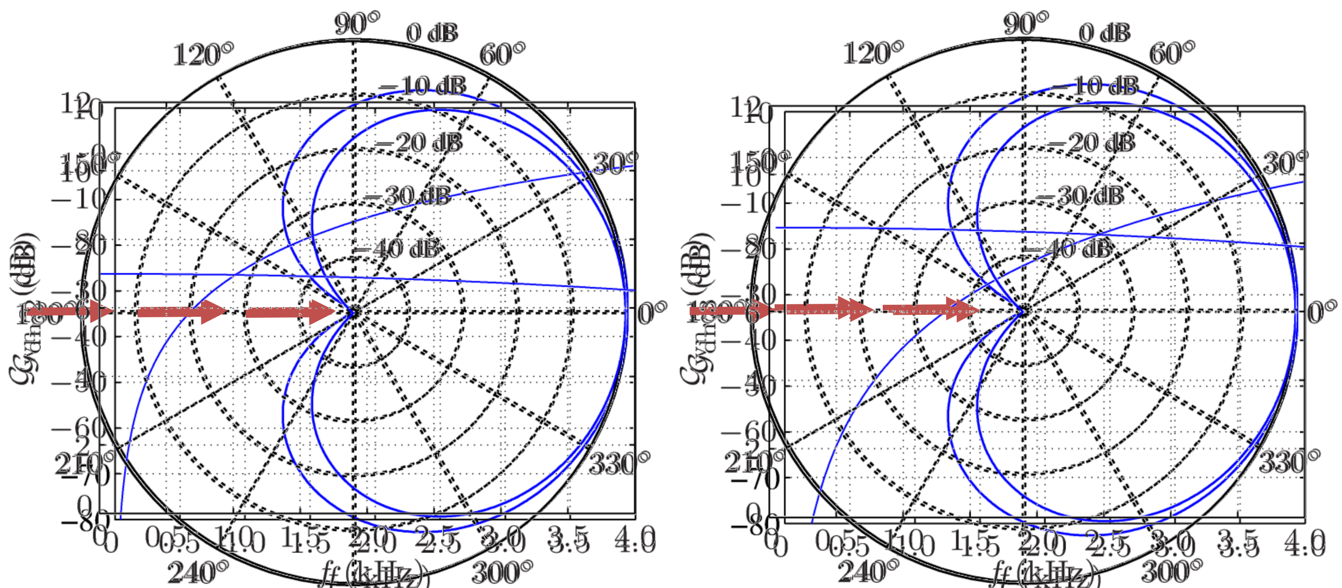
- DMA Beamforming with Nulls of Multiplicity More Than One

$$\mathbf{D}'(\omega, \boldsymbol{\alpha})\,\mathbf{h}(\omega) = \boldsymbol{\beta}$$

$$\mathbf{D}'(\omega, \boldsymbol{\alpha}) = \begin{bmatrix} \mathbf{d}^H(\omega, 1) \\ \mathbf{d}^H(\omega, \alpha_{N,1}) \\ \vdots \\ \mathbf{d}^H(\omega, \alpha_{N,n}) \\ \mathbf{d}^H(\omega, \alpha_{N,n})\,\boldsymbol{\Sigma} \\ \vdots \\ \mathbf{d}^H(\omega, \alpha_{N,n})\,\boldsymbol{\Sigma}^{P-1} \\ \mathbf{d}^H(\omega, \alpha_{N,n+P}) \\ \vdots \\ \mathbf{d}^H(\omega, \alpha_{N,N}) \end{bmatrix} \qquad \boldsymbol{\beta} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

- DMA Beamforming with Nulls of Multiplicity More Than One (Examples)

Directivity Factor



(a) Pattern of the third-order cardioid at $f = 1$ kHz

(b) Pattern of the third-order cardioid at $f = 4$ kHz

- DMA Beamforming with Ideal Pattern Information
  - N+1 Constraints:

$$\alpha_{N,n} = \cos\theta_{N,n}$$

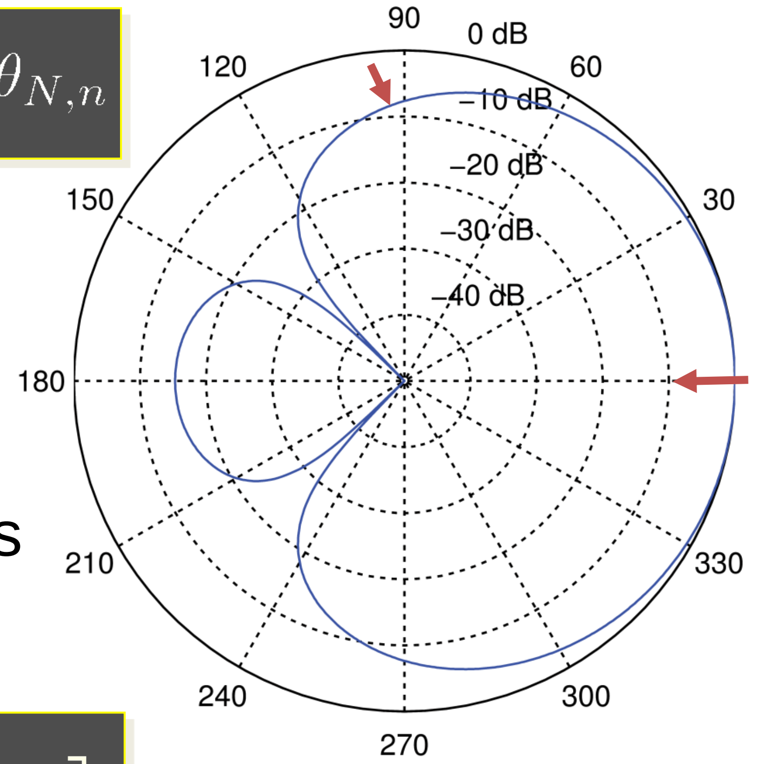
$$\boldsymbol{\alpha} = \begin{bmatrix} 1 & \alpha_{N,1} & \cdots & \alpha_{N,N} \end{bmatrix}^T$$

$$\boldsymbol{\beta} = \begin{bmatrix} 1 & \beta_{N,1} & \cdots & \beta_{N,N} \end{bmatrix}^T$$

  - Linear system with N+1 equations

$$\mathbf{D}(\omega, \boldsymbol{\alpha})\,\mathbf{h}(\omega) = \boldsymbol{\beta}$$

$$\mathbf{D}(\omega, \boldsymbol{\alpha}) = \begin{bmatrix} \mathbf{d}^H(\omega, 1) \\ \mathbf{d}^H(\omega, \alpha_{N,1}) \\ \vdots \\ \mathbf{d}^H(\omega, \alpha_{N,N}) \end{bmatrix}$$



- DMA Beamforming with Ideal Pattern Information
  - Linear system with *N*+1 equations

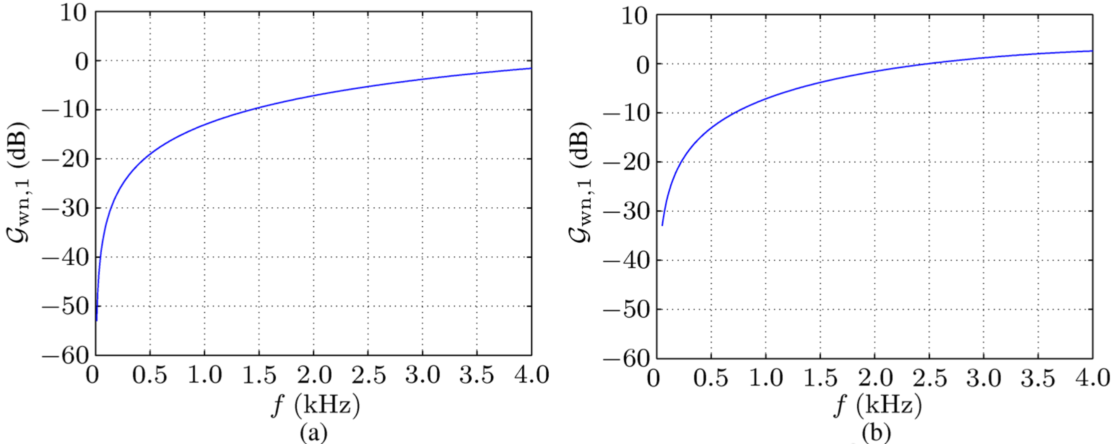
$$\mathbf{D}(\omega, \boldsymbol{\alpha})\,\mathbf{h}(\omega) = \boldsymbol{\beta}$$

  - Fundamental constraints

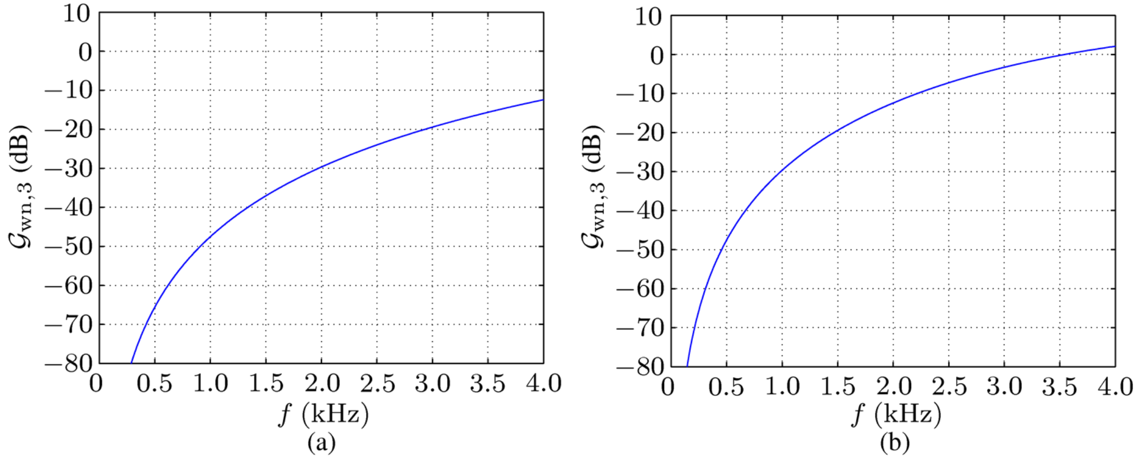
The N coefficients $\alpha_{N,n}$ should be chosen in such a way that the $\mathbf{D}(\omega, \boldsymbol{\alpha})$ matrix is well conditioned so that its inverse can be computed without any numerical problem.

The N pairs of coefficients $(\alpha_{N,n}, \beta_{N,n})$ should take values from a desired "ideal" DMA beampattern. In general, They should correspond to the nulls of the desired "ideal" DMA beampattern; but they can take other values as well. For example, if the "ideal" beampatten has multiple nulls at the same angle, we should choose a different angle. Then $\beta_{N,n}$ will no longer be 0.

- Problem of White Noise Amplification



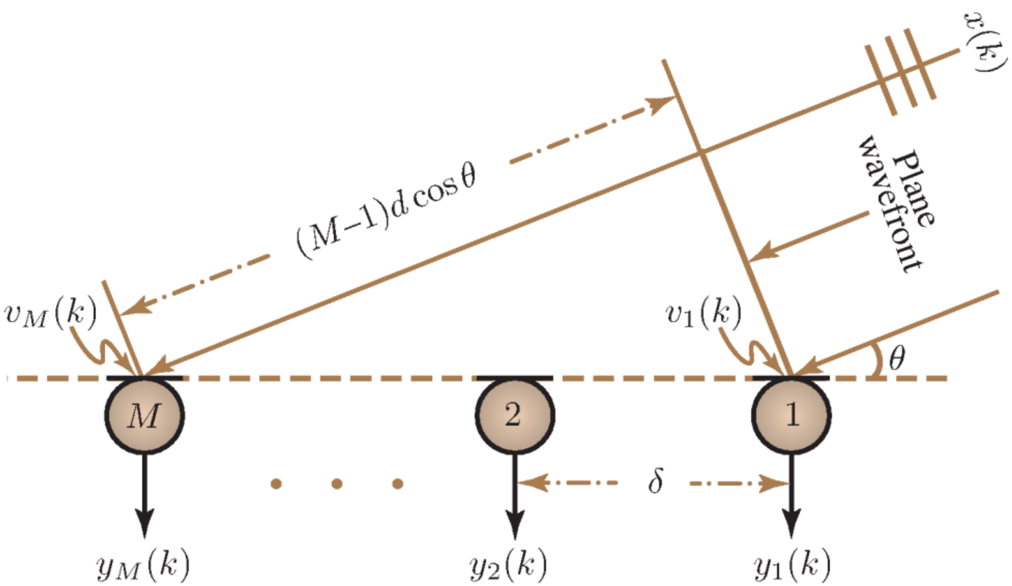
WNG of a 1st-order supercardioid at: (a) $\delta = 1$ cm and (b) $\delta = 2$ cm

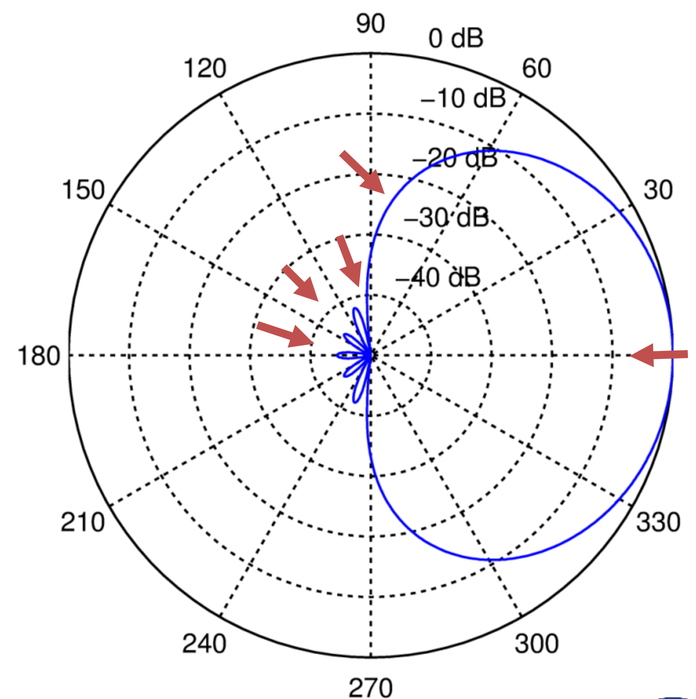


WNG of a 3rd-order supercardioid at: (a) $\delta = 1$ cm and (b) $\delta = 2$ cm

- WNG Improvement

Use more than N+1 microphones to design a DMA with order of less than N (i.e., M > N+1).





- WNG Improvement
  - Linear system with M ($\geq N+1$) equations

$$\mathbf{D}(\omega, \boldsymbol{\alpha})\,\mathbf{h}(\omega) = \boldsymbol{\beta}$$

$$\mathbf{D}(\omega, \boldsymbol{\alpha}) = \begin{bmatrix} \mathbf{d}^H(\omega, 1) \\ \mathbf{d}^H(\omega, \alpha_{N,1}) \\ \vdots \\ \mathbf{d}^H(\omega, \alpha_{N,N}) \end{bmatrix}_{(N+1)\times M}$$

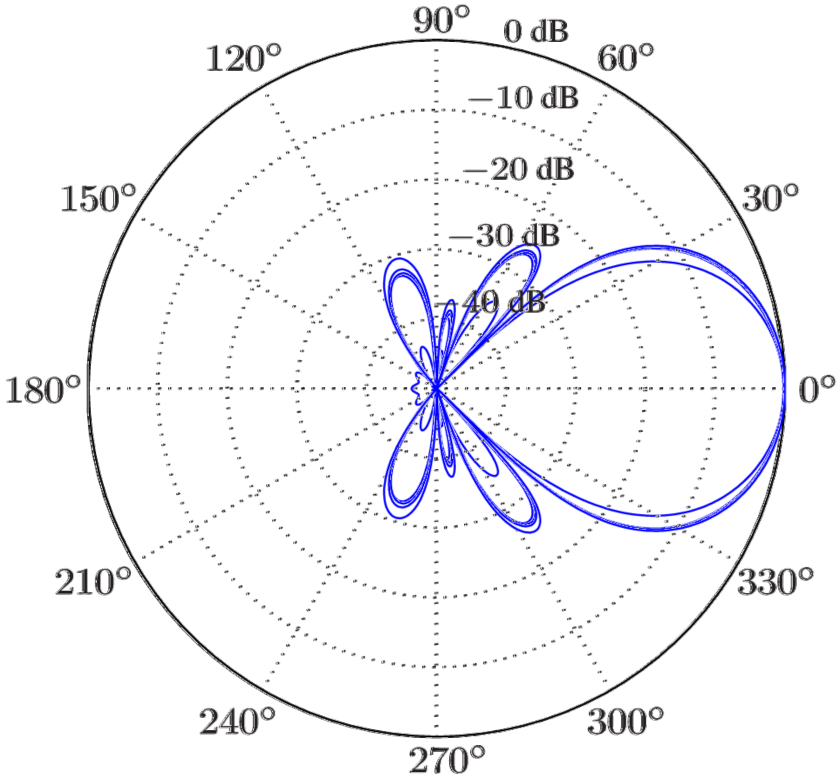
$$\mathbf{d}(\omega, \alpha_{N,n}) = \begin{bmatrix} 1 & e^{-\jmath\omega\tau_0\alpha_{N,n}} & \cdots & e^{-\jmath(M-1)\omega\tau_0\alpha_{N,n}} \end{bmatrix}^T$$

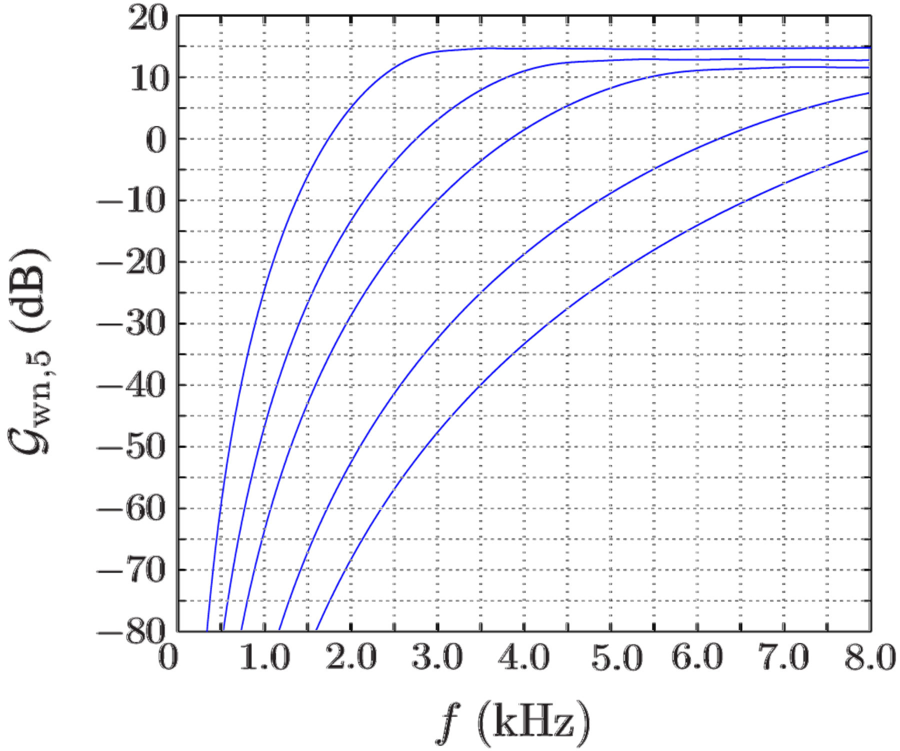
- WNG Improvement
  - Maximization of the WNG

$$\max_{\mathbf{h}(\omega)} \frac{1}{\mathbf{h}^H(\omega)\,\mathbf{h}(\omega)} \quad \text{subject to} \quad \mathbf{D}(\omega, \boldsymbol{\alpha})\,\mathbf{h}(\omega) = \boldsymbol{\beta}$$

$$\mathbf{h}_{\mathrm{MaxWNG}}(\omega) = \mathbf{D}^H(\omega, \boldsymbol{\alpha}) \left[\mathbf{D}(\omega, \boldsymbol{\alpha})\,\mathbf{D}^H(\omega, \boldsymbol{\alpha})\right]^{-1} \boldsymbol{\beta}$$

- WNG Improvement (Examples: a 5th-order DMA, $\delta = 1$ cm)





30 microphones

- References

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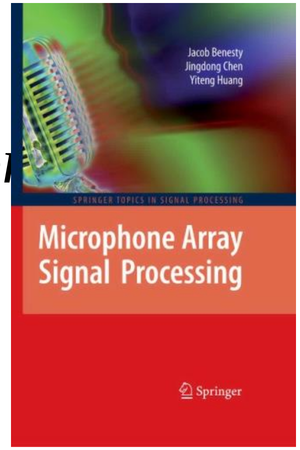

- Microphone arrays, particularly the ones with small aperture are more and more popularly used.
- Consistency in performance and flexibility in working with other processors are very important.
- A general approach to DMA Implementation is discussed, which converts the DMA design into a linear system solving problem. This method is very flexible and can design any desired beampattern.
- A robust method is discussed, that can use more microphones to design a given order DMA with less white noise amplification.