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Computer assignment 2

How to handle the Interface?

In the first input “amplitudes_in_percent” you can add amplitudes in percent which shall be plotted. In the “amplitude_monotonic_percent” you can add a value for the amplitude of the monotonic strain loading in percent. In “load_rates” you can add different load rates with which the amplitudes in the first variable are plotted. The boolean “Triangular_Wave” can be set to true if a triangular cycle shall be used for the strain and set to false if a strain ramp shall be used for the strain. If “Plot_over_Strain” is set to true, the strain will be displayed over the strain. Otherwise, the strain will be displayed over time, which is only possible for the Chaboche-Norton model, as this is the only model with time dependence. In the “Which_Model” variable three options are available: if it is set to 1, perfect plasticity is used. For 2 the Chaboche model is used and for 3 the Chaboche-Norton Model is used.

```
amplitudes_in_percent=[0.4, 0.8]
amplitude_monotonic_percent=1.0
load_rates=[1.0, 2.0 ]
load_rate_monotonic=1.0
Triangular_Wave=true
Plot_over_Strain=true #if true-> plot over strain, else: plot over time
Which_Model=3 #1 for perfect_plasticity, 2 for chaboche, 3 for chabocheNorton
#because perfect_plasticity and chaboche are time independent,
# plotting over time is only permitted for chabocheNorton
```

Task 1

The code for a perfect plasticity material model evaluates in Figure 1.

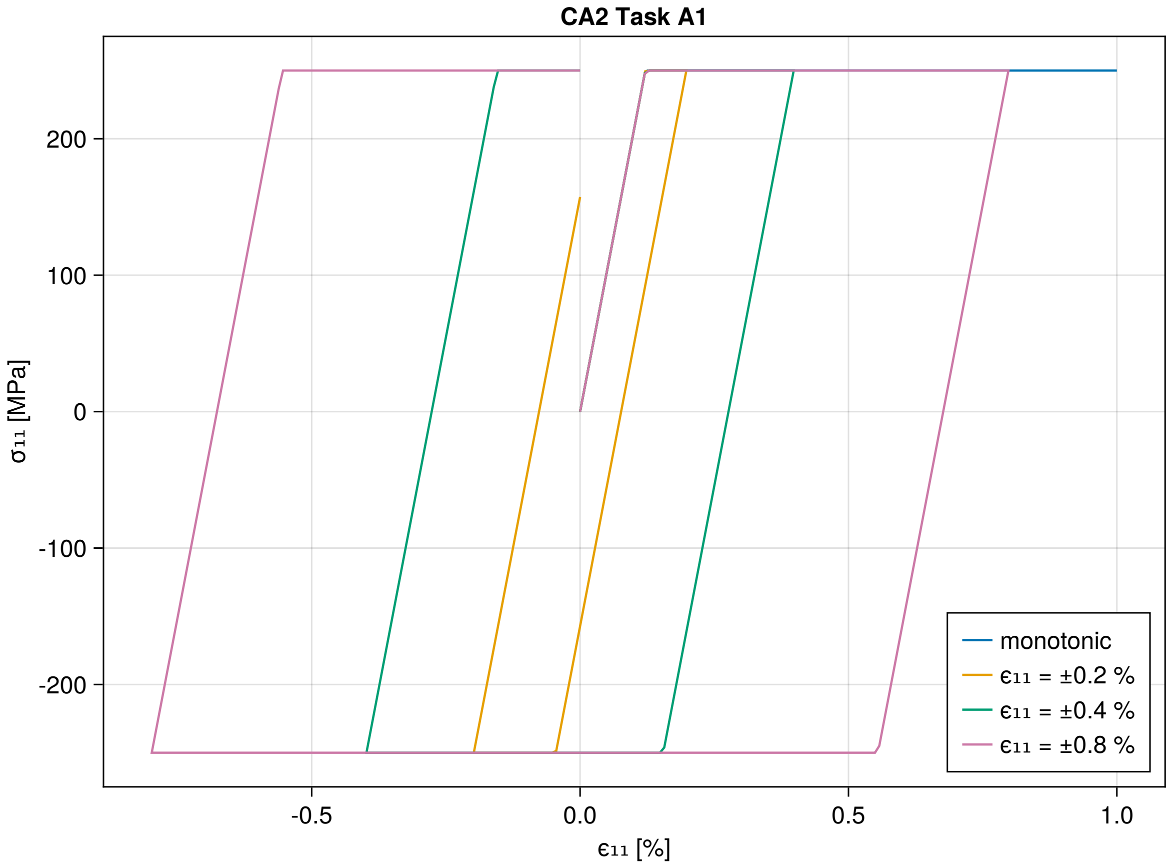


Figure 1

Task 2

To make the Chaboche model equivalent to the perfect plasticity model, one must carefully select the parameters that extend the perfect plasticity model. The Chaboche model incorporates the effects of hardening, including isotropic and kinematic hardening, which are determined by four key parameters: $\mathrm{H_{kin}}, \mathrm{H_{iso}}, \mathrm{k_\infty}, \beta_\infty$.

We can see that the hardening effect is included by the following residual function.

$$r_\beta = (\beta - \beta_{old}) - 2 * \left(\frac{\Delta\lambda}{3}\right) * H_{kin} * (v - \frac{3}{2} * \frac{\beta}{\beta_\infty})$$

$$r_\kappa = (\kappa - \kappa_{old}) - \Delta\lambda * H_{iso} * \left(1 - \frac{\kappa}{\kappa_\infty}\right)$$

By setting the parameters H_{kin}, H_{iso} to zero, we can observe that the residual function are fulfilled directly and therefore β and κ are not updated. The functions are therefore obsolete and only the other residual functions have an impact on the model.

With β and κ equal to zero the function for the yield criterion (Von Misses stress included) is equal to the perfect plasticity model.

$$r_\epsilon = (\epsilon_p - \epsilon_{p,old}) - (\Delta\lambda * v)$$

$$r_{\Phi} = f_{vM}(\sigma_{red}) - (Y_0 + \kappa)$$

This results in Figure 2.

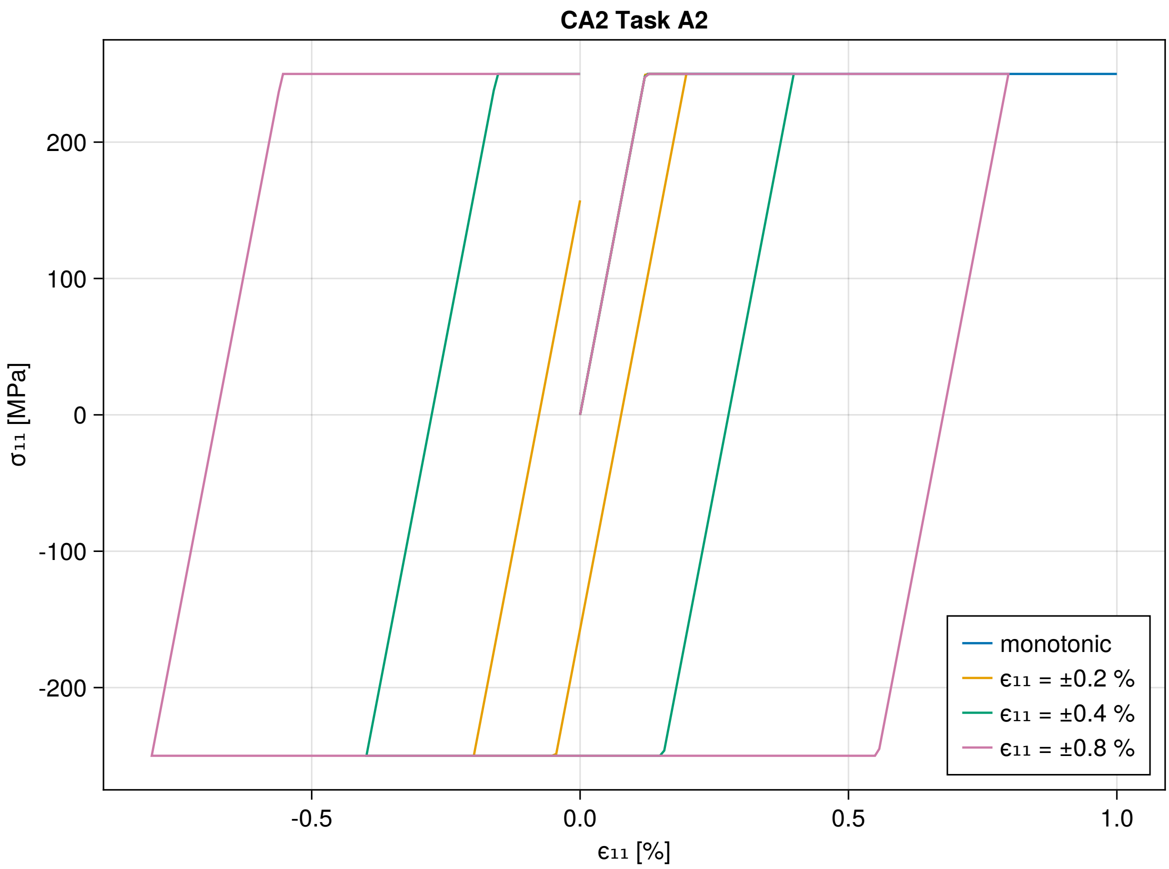


Figure 2

Task 3

When using the parameters as given in the Task the model evaluates exactly to the demo figure as seen in Figure 3.

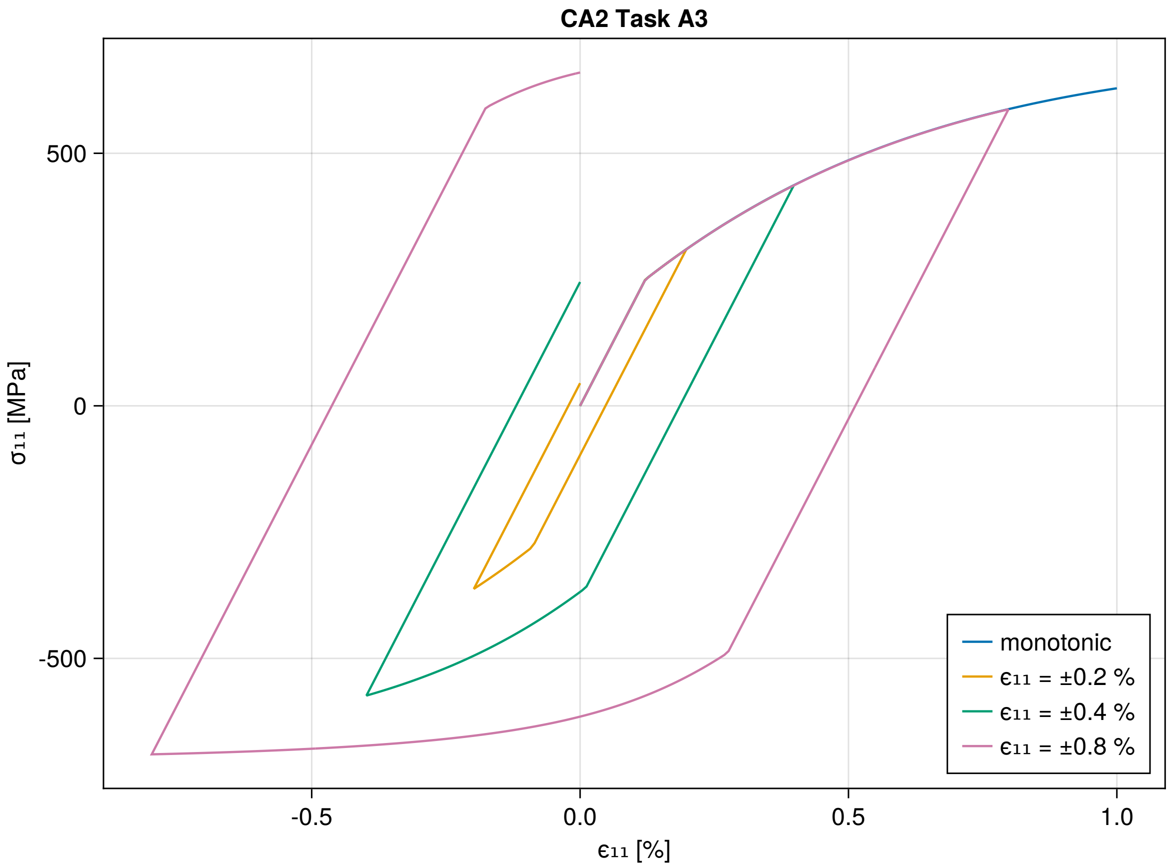


Figure 3

Now the effects of kinematic and isotropic hardening are visible since the center and the size of the yield stress have changed. The explanation follows in Task 4.

Task 4

When simulating the cyclic response for three triangular cycles for the two given strain domains you get the response depicted in Figure 4.

Notably, significant differences in stress and strain amplitudes are observed between the two triangular cycles: the 0.9% strain cycle exhibits a much larger amplitude than the 0.15% strain cycle. Moreover, you can see in both cases that the first cycle yields a smaller stress compared to the next two cycles, which both have a higher stress than the first one and are almost identical.

The primary deviation from perfect plasticity lies in the occurrence of isotropic and kinematic hardening processes during the triangular cycles. Isotropic hardening is apparent from the increase of the yield stress between the first and second/third cycles. Initially, the yield stress is set at 250 MPa (as initialized in the code), but after one cycle, the plastic regime only initiates at around 600 Mpa (as evident from the orange graph). This increase in yield stress is a result of isotropic hardening, where the yield stress changes its size.

The manifestation of kinematic hardening is observed in the shift of the center point of the yield stress. Specifically, the center point of the elastic or linear part of the graph shifts upwards in positive strain and downwards in negative strain. Additionally, the higher stress levels in the second and third cycles can be attributed to kinematic hardening. As the center of the yield stress approaches an upper limit, the maximum stress of the model is reached.

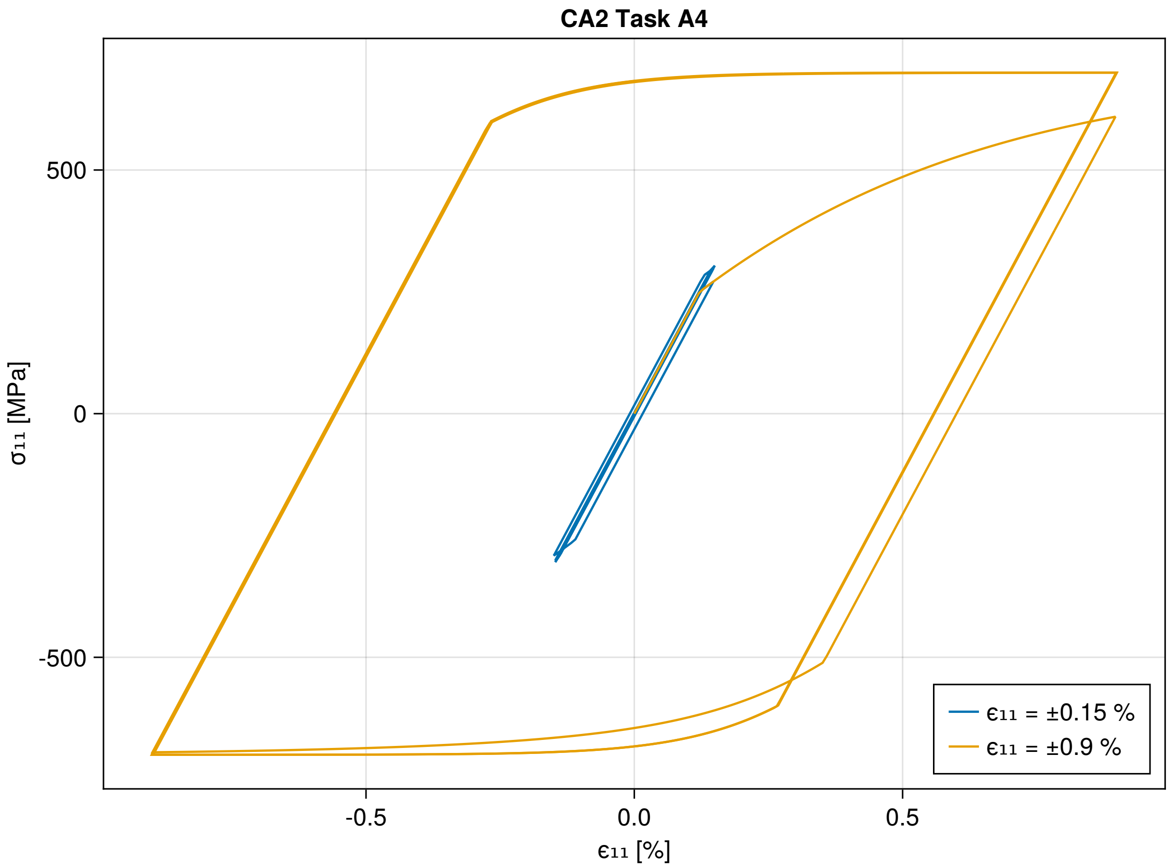


Figure 4

Task 5

The change of the plot depending on different time steps can be seen in Figure 5.

When experimenting with different time steps during the triangular cycles, several interesting observations come to light. Firstly, it is evident that the graph for 10 time steps differs significantly from the graphs with 100 or 1000 time steps, which exhibit less disparity and closely approximate the graph with theoretically infinite time steps.

The limitation of using only 10 time steps lies in the difficulty of precisely identifying when the plastic regime exactly starts. Since the first time step already falls into the plastic regime, the connection between the zero point and the first time step appears linear, leading to the wrong impression that the elastic regime extends until the first time step. This discrepancy becomes evident when comparing the graph with the other two.

Moreover, employing only 10 time steps prevents the attainment of maximum stress and strain, as the exact boundaries of the strain domain are not adequately captured within the equally spaced elements used to create the time steps.

Furthermore, the effects of kinematic and isotropic hardening are not clearly discernible due to the coarse time step discretization. These subtle hardening processes are not accurately captured with only 10 time steps, leading to potential inaccuracies and loss of valuable insights.

In summary, increasing the number of time steps, particularly to 100 or 1000, offers more reliable results that closely align with the theoretical infinite time step scenario. It allows for better visualization of plastic regime initiation, reaching maximum stress and strain, and observing the effects of kinematic and isotropic hardening, which are critical aspects to consider for accurate simulations. The downside to an increase in the number of time steps is the increase in computational time.

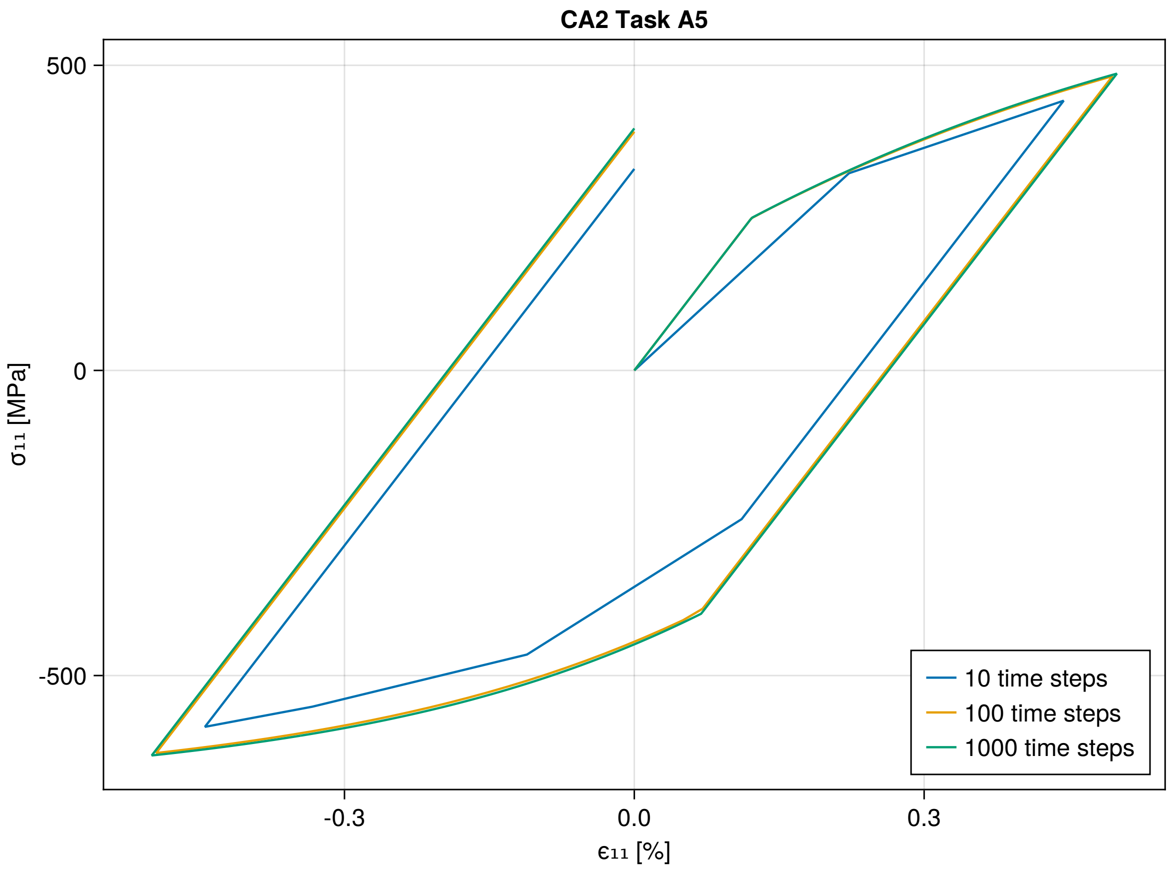


Figure 5

Task B4

The incorporation of the Norton-overstress function introduces significant changes to the model's behavior within the plastic regime, replacing the KKT-conditions. This alteration becomes evident from the formula of the Norton-overstress function, which employs Macaulay brackets around ϕ. These brackets indicate that when ϕ is below zero, representing the elastic regime, the overstress function equals zero, and no modifications are applied to the model. However, once the plastic regime is entered, the Macaulay brackets no longer yield zero, and the effects of the overstress function come into play.

The Norton-overstress function models the behavior of the plastic strain rate $\dot{\lambda}$, making the strain rate of the loading a crucial factor in evaluating the model's response. Prior to introducing the overstress function, the KKT conditions ensured that ϕ always remained less than or equal to zero, and the product of ϕ and the plastic strain rate was set to zero. With the introduction of the overstress function, the KKT conditions are discarded, and ϕ is no longer constrained to be zero. Consequently, stress-states within the plastic domain are now permissible.

Now, there is a residual function for the plastic strain rate, which we seek to determine to satisfy the nonlinear equation. The value of ϕ plays a pivotal role, as it governs the magnitude of the strain rate; larger ϕ values result in higher strain rates.

In summary, the Norton-overstress function fundamentally transforms the model's behavior in the plastic regime, enabling stress-states within that domain and introducing a residual function for the plastic strain rate. The interplay between ϕ and the plastic strain rate plays a significant role in the model's response, and the presence of the Macaulay brackets ensures that the overstress function only comes into effect when the plastic regime is encountered.

The following notes in Figure 6 represent our consideration of the paper mentioned in the task while implementing the model. First the variables from the paper were compared to the formulas derived in the lecture. Next the differences in the paper were implemented. The only difference to the lecture is that κ is added to Y_0 in the denominator.

Viscoplastic constitutive equation:

a) choice of viscosity function

b) choice of hardening equations

a) $\dot{p} = \left\langle \frac{f}{D} \right\rangle^n = \left\langle \frac{\sigma_v}{D} \right\rangle^n \rightarrow \dot{\lambda} = \left\langle \frac{\Phi}{D} \right\rangle^n = \left\langle \frac{\Phi}{Y_0 + \mathcal{I}(\varepsilon_p)} \right\rangle^n$

n ... material parameter $n = 1$ ... „diffusional creep" of a perfect alloy

$n \gg 1$ ... low viscosity range

$3 \leq n \leq 30$ usual for engineering materials

$f = \Phi$ ... yield function

$\dot{p} = \dot{\lambda}$ ... plastic strain rate

$D = \mathbb{K} + \xi R(p)$ ... drag stress $\rightarrow D(\varepsilon_p) = Y_0 + \mathcal{I}(\varepsilon_p)$

$p = \varepsilon_p$ ... plastic strain

$\left.\begin{array}{l} \mathbb{K} \text{ ... Initial Value of Drag Stress} \\ \xi \text{ ... Weighting parameter} \end{array}\right\}$ Perzyna: $\mathbb{K} = k$, $\xi = 1$

$k = Y_0$ ... initial yield surface size

σ_v ... viscous stress

$R = \mathcal{I}$ ... yield stress increase

Figure 6

You can see the stress of the Chaboche Norton model w.r.t. the strain and time in Figure 7 and Figure 8 for a cyclic loading with different amplitudes but the same loading rates.

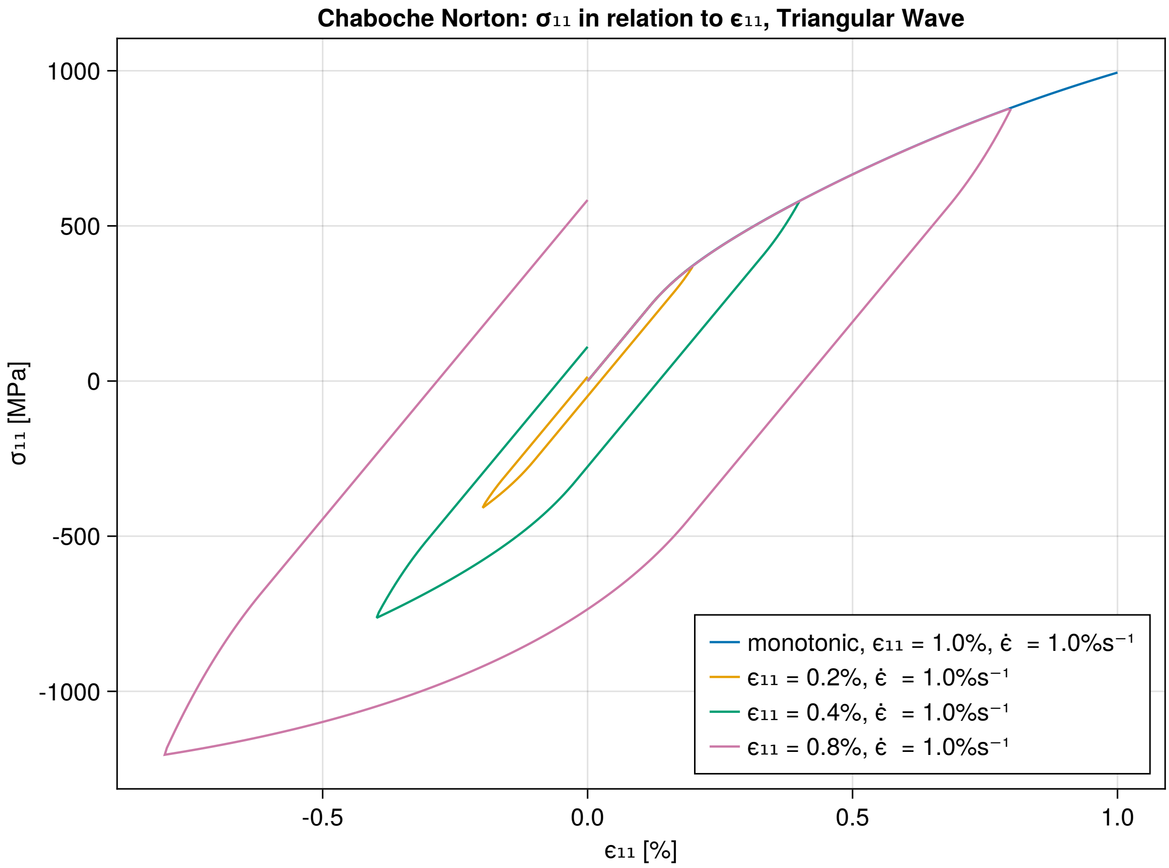

Chaboche Norton: σ₁₁ in relation to ϵ₁₁, Triangular Wave
1000
500
0
-500
-1000
σ₁₁ [MPa]
-0.5
0.0
0.5
1.0
ϵ₁₁ [%]
monotonic, ϵ₁₁ = 1.0%, ϵ̇ = 1.0%s⁻¹
ϵ₁₁ = 0.2%, ϵ̇ = 1.0%s⁻¹
ϵ₁₁ = 0.4%, ϵ̇ = 1.0%s⁻¹
ϵ₁₁ = 0.8%, ϵ̇ = 1.0%s⁻¹

Figure 7

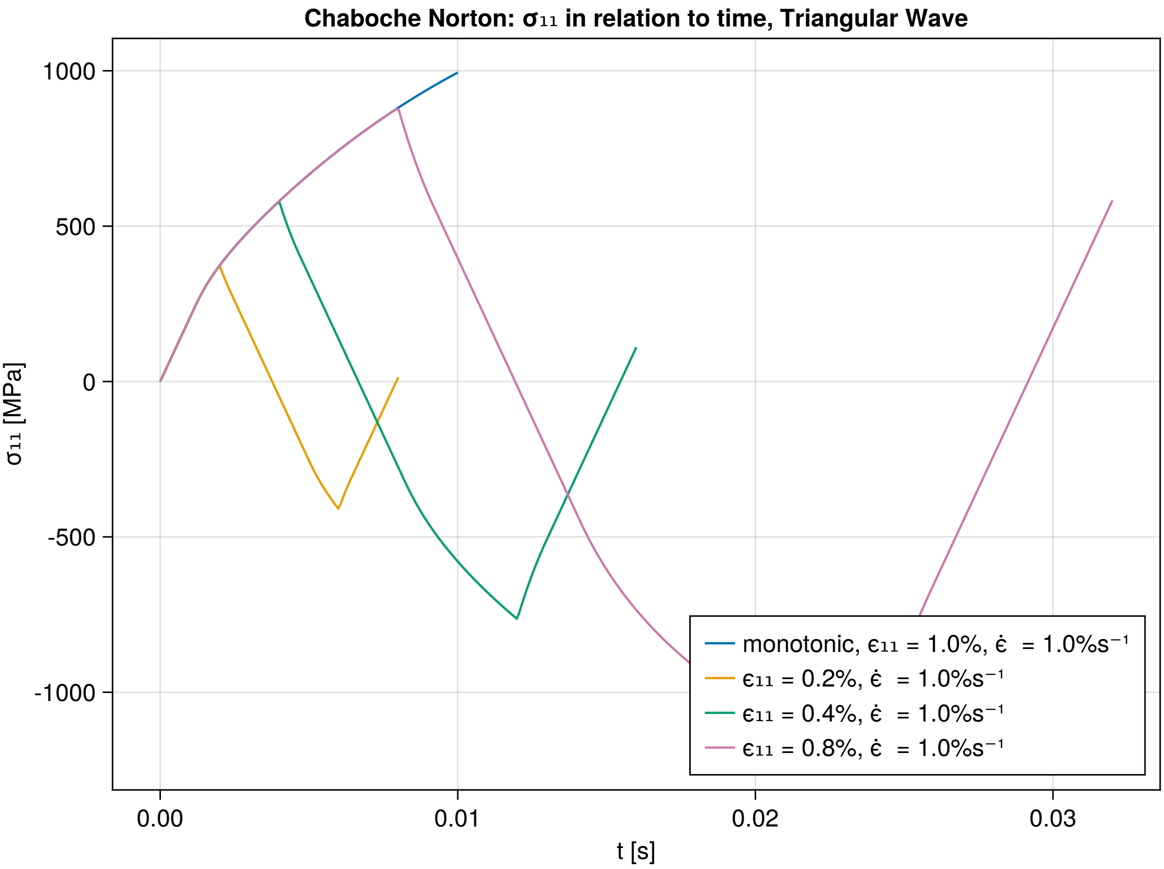
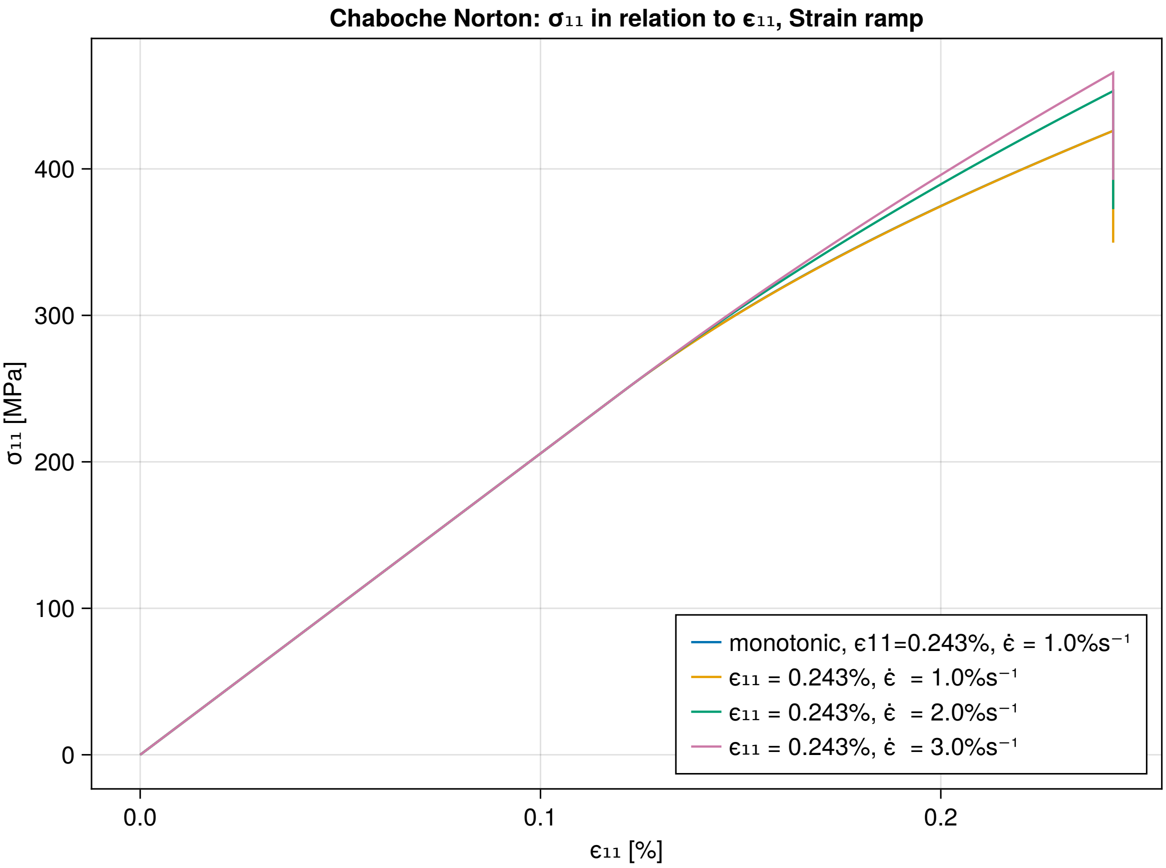


Figure 8

If we apply a strain ramp, i.e., a strain ramp up to twice the yield stress and then a constant strain at that value, we can observe the effect of relaxation. Relaxation happens, because some part of the elastic strain is transformed into plastic strain and therefore the stress is decreased, because the stress is a function of the difference between the actual strain and the plastic strain. If the first one is constant and the second one rises, the stress is decreased. You can see the stress of the Chaboche Norton model w.r.t. the strain and time in Figure 9 and Figure 10 for a constant loading followed by a constant strain.

If you simulate the response to a triangular strain cycle using the Chaboche Norton model, you can see the effect of the strain rate clearly. If the strain rate is high, the maximum stress is increased and the stress grows faster in the plastic regime. In the elastic regime no changes to the Chaboche Model are visible as expected from the theory. The constant t^* is a parameter which influences the relaxation time. The parameter n models the viscosity of the material. For $n = 1$ a diffusional creep of a perfect alloy is modeled and for large n the model acts in the low viscosity range. For engineering materials, we use a n between three and thirty. You can see the stress of the Chaboche Norton model w.r.t. the strain and time in Figure 11 and Figure 12 for a cyclic loading with different amplitudes and different loading rates.



Chaboche Norton: σ₁₁ in relation to ϵ₁₁, Strain ramp
σ₁₁ [MPa]
400
300
200
100
0
0.0
0.1
0.2
ϵ₁₁ [%]
monotonic, ϵ11=0.243%, ϵ̇ = 1.0%s⁻¹
ϵ₁₁ = 0.243%, ϵ̇ = 1.0%s⁻¹
ϵ₁₁ = 0.243%, ϵ̇ = 2.0%s⁻¹
ϵ₁₁ = 0.243%, ϵ̇ = 3.0%s⁻¹

Figure 9

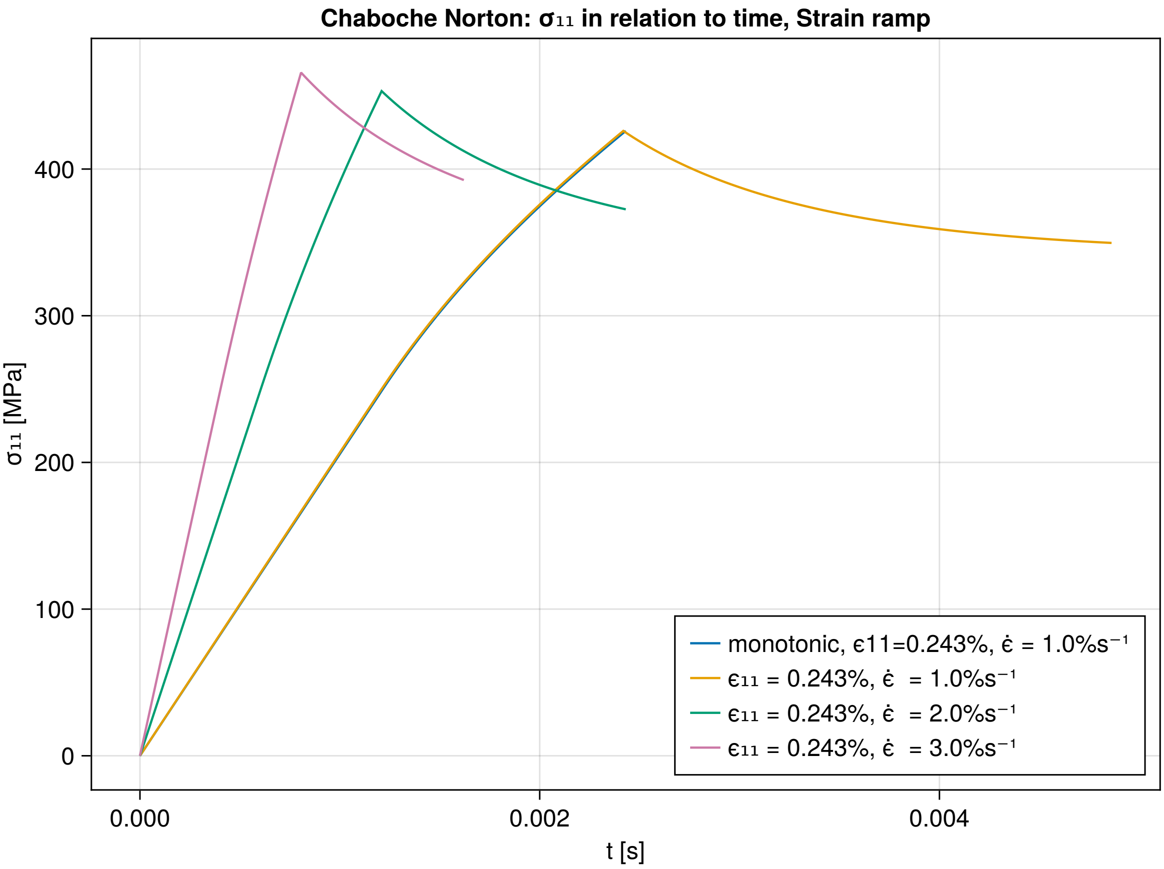
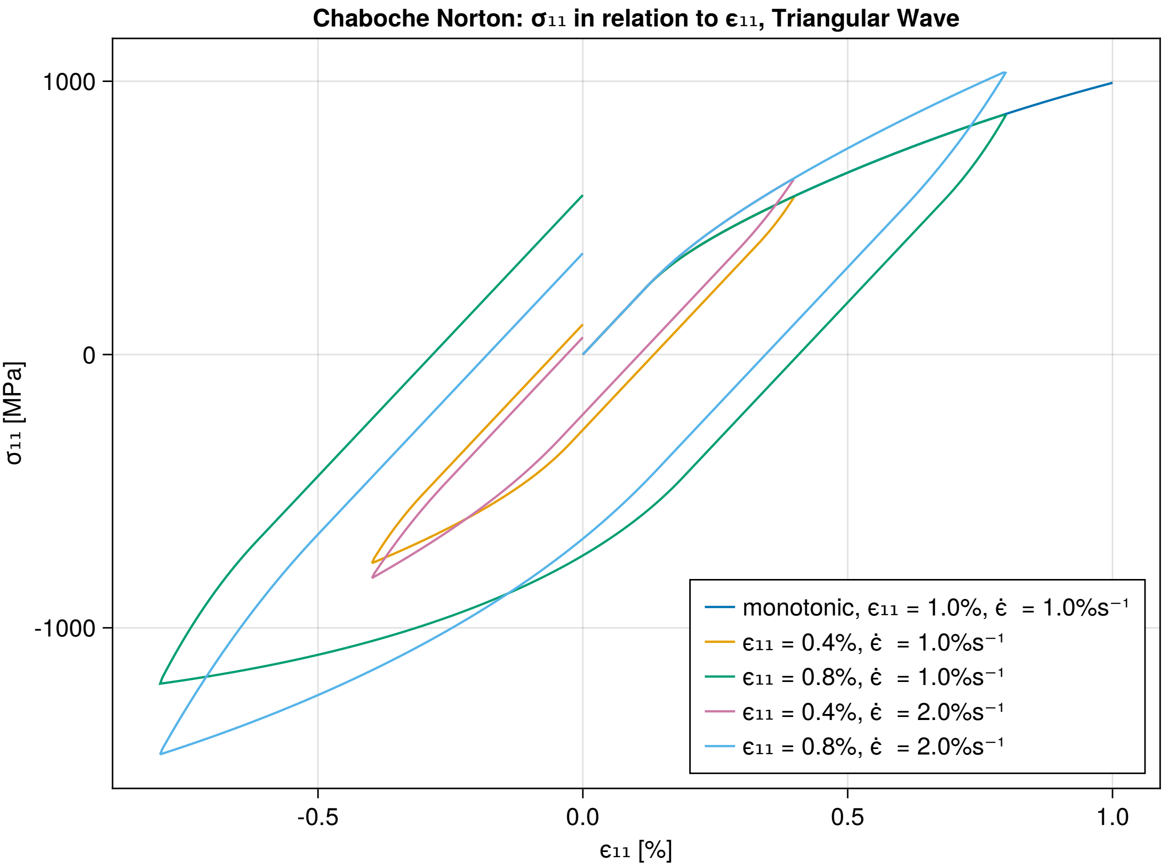

Chaboche Norton: σ11 in relation to time, Strain ramp
σ11 [MPa]
400
300
200
100
0
0.000
0.002
0.004
t [s]
monotonic, ϵ11=0.243%, ϵ̇ = 1.0%s⁻¹
ϵ11 = 0.243%, ϵ̇ = 1.0%s⁻¹
ϵ11 = 0.243%, ϵ̇ = 2.0%s⁻¹
ϵ11 = 0.243%, ϵ̇ = 3.0%s⁻¹

Figure 10



Chaboche Norton: σ₁₁ in relation to ϵ₁₁, Triangular Wave
1000
0
-1000
σ₁₁ [MPa]
-0.5
0.0
0.5
1.0
ϵ₁₁ [%]
monotonic, ϵ₁₁ = 1.0%, ϵ̇ = 1.0%s⁻¹
ϵ₁₁ = 0.4%, ϵ̇ = 1.0%s⁻¹
ϵ₁₁ = 0.8%, ϵ̇ = 1.0%s⁻¹
ϵ₁₁ = 0.4%, ϵ̇ = 2.0%s⁻¹
ϵ₁₁ = 0.8%, ϵ̇ = 2.0%s⁻¹

Figure 11

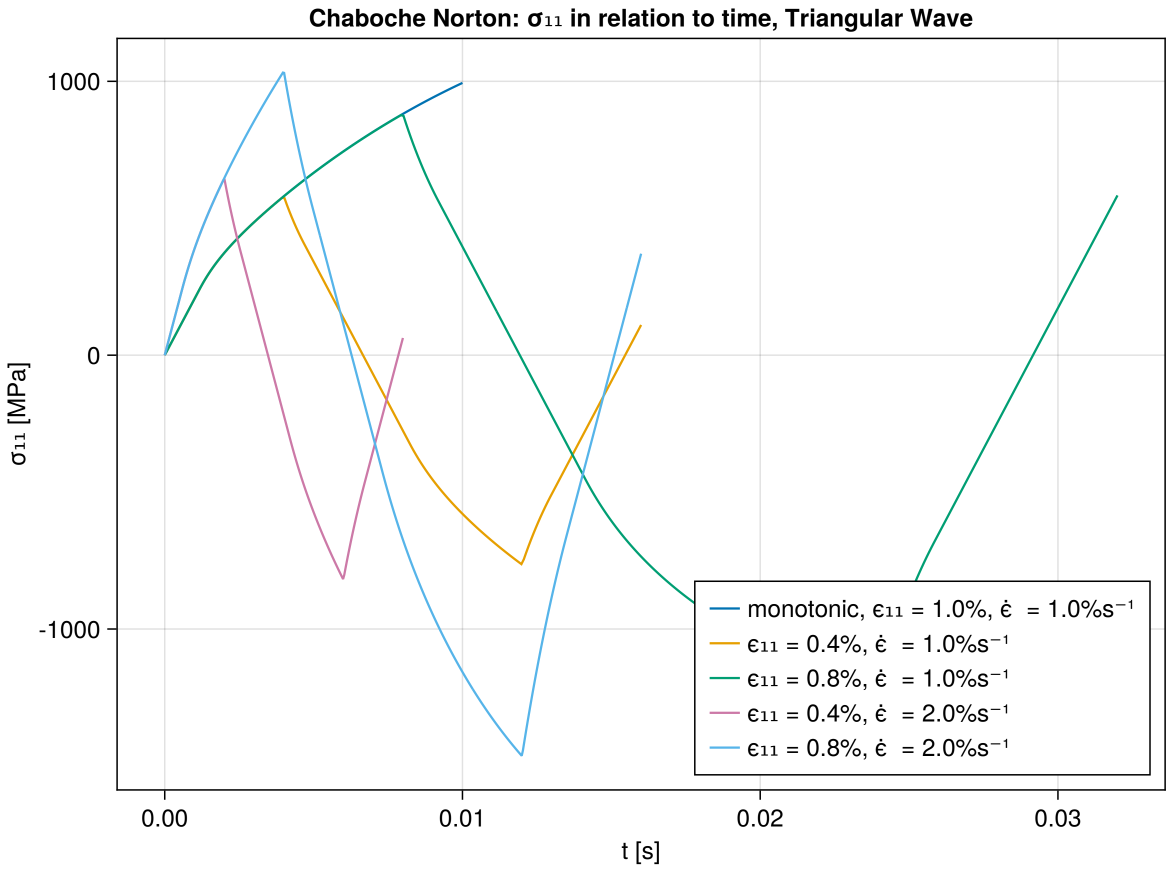


Figure 12

Individual contribution

The whole project was done as a team of two people (Daniel Nickel and Said Harb) and all tasks as well as the development of the code were done together.