Computational Maths Assignment 3 Solutions

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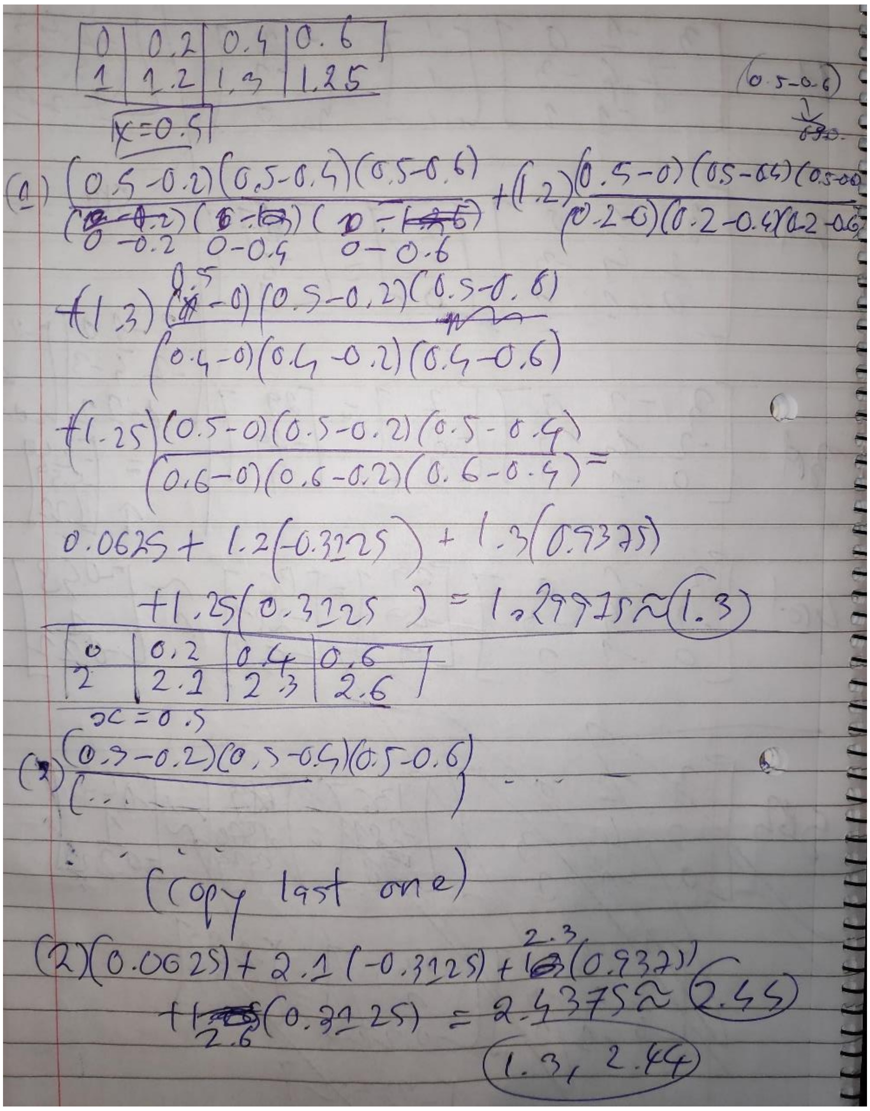
1. **B (given insert into MATLAB)**
2. **B (given insert into MATLAB)**
3. **E (given insert into MATLAB)**
4. **B**



0	0.2	0.4	0.6
1	1.2	1.3	1.25

$x = 0.5$

$$(1)\ \frac{(0.5-0.2)(0.5-0.4)(0.5-0.6)}{(0-0.2)(0-0.4)(0-0.6)} + (1.2)\frac{(0.5-0)(0.5-0.4)(0.5-0.6)}{(0.2-0)(0.2-0.4)(0.2-0.6)}$$

$$+(1.3)\frac{(0.5-0)(0.5-0.2)(0.5-0.6)}{(0.4-0)(0.4-0.2)(0.4-0.6)}$$

$$+(1.25)\frac{(0.5-0)(0.5-0.2)(0.5-0.4)}{(0.6-0)(0.6-0.2)(0.6-0.4)} =$$

$$0.0625 + 1.2(-0.3125) + 1.3(0.9375) + 1.25(0.3125) = 1.29975 \approx 1.3$$

0	0.2	0.4	0.6
2	2.1	2.3	2.6

$x = 0.5$

$$(2)\ \frac{(0.5-0.2)(0.5-0.4)(0.5-0.6)}{(\ldots)}$$

(copy last one)

$$(2)(0.0625) + 2.1(-0.3125) + 2.3(0.9375) + 2.6(0.3125) = 2.4375 \approx 2.45$$

$(1.3, 2.44)$

5. **C**

Time (s)	10	15	20	22
Velocity (ms^{-1})	22	36	57	10

Choose the data points closest to the point we want to estimate

So we choose (15,36), (20,57) and (22,10) as these timestamps are closest to 17s.

v(t) = $a_0 + a_1 t \; + \; a_2 t^2$

we get three equations

1. 36 = $a_0 + a_1(15) + \; a_2(15)^2$
2. 57 = $a_0 + a_1(20) \; + \; a_2(20)^2$
3. 10 = $a_0 + a_1(22) \; + \; a_2(22)^2$

$$\begin{pmatrix} 1 & 15 & 225 \\ 1 & 20 & 400 \\ 1 & 22 & 484 \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} 36 \\ 57 \\ 10 \end{pmatrix}$$

$Solving\ gives\ us\ a_0 \;=\; -\frac{8499}{7},\ a_1 \;=\; \frac{1427}{10},\ a_2 \;=\; -\frac{277}{70}$

$\mathrm{v(t)} \;=\; -\frac{8499}{7} + \frac{1427}{10}\mathrm{t} \; - \frac{277}{70}\mathrm{t}^2$

Differentiation of v(t) with respect to t will give us acceleration

$v'(t) \;=\; \frac{1427}{10} \; - \frac{277}{35}\mathrm{t}$

Subbing in 17 gives us 8.157142

6. **D**

   Newtons

	x	y	b_0	b_1	b_2
x_0	-1	9		$\frac{5-9}{1+1} = -2$	$\frac{7+2}{2+1} = 3$
x_1	1	5		$\frac{12-5}{2-1} = 7$	
x_2	2	12			

$$f(x) = b_0 + b_1(x - x_0) + b_2(x - x_0)(x - x_1)$$
$$= 9 + (-2)(x+1) + 3(x+1)(x-1)$$
$$= 9 - 2x - 2 + 3x^2 - 3$$
$$\boxed{= 3x^2 - 2x + 4}$$

Legrange

6 Legrange

(i)

-1	1	2
9	5	12

$$(9)\frac{(x-1)(x-2)}{(-1-1)(-1-2)}+(5)\frac{(x+1)(x-2)}{(1+1)(1-2)}$$

$$+(12)\frac{(x+1)(x-1)}{(2+1)(2-1)}$$

$$=9\cdot\frac{x^2-3x+2}{-2\cdot-3}+5\cdot\frac{x^2-x-2}{2\cdot-1}$$

$$+12\cdot\frac{x^2-1}{3\cdot 1}$$

$$=\frac{9x^2-27x+18}{6}+\frac{5x^2-5x-10}{-2}+\frac{12x^2-12}{3}$$

$$=\frac{9x^2-27x+18-15x^2+15x+30+24x^2-24}{6}$$

$$=\frac{18x^2-12x+24}{6}$$

$$\boxed{=3x^2-2x+4}$$

7. D

$$\left.\frac{df}{dx}\right|_{x=x_0}=\frac{f(x_{i+1})-f(x_{i-1})}{x_{i+1}-x_{i-1}}$$

$$=\frac{f(x_1)-f(x_{-1})}{(x_0+h_2)-(x_0-h_1)}$$

$$=\frac{f(x_1)-f(x_{-1})}{h_2+h_1}\qquad(h_2=h_1=h)$$

$$=\frac{f(x_1)-f(x_{-1})}{2h}$$

8. E

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$$f_i(x) = \frac{a_i}{6h_i}(x_{i+1}-x)^3 + \frac{a_{i+1}}{6h_i}(x-x_i)^3$$
$$\left(\frac{y_i}{h_i} - \frac{a_i h_i}{6}\right)(x_{i+1}-x) + \left(\frac{y_{i+1}}{h_i} - \frac{a_{i+1}h_i}{6}\right)(x-x_i)$$

a_1 & $a_4 = 0$ for cubic spline.

$x_2 - x_1 = h_1 = 1$
$x_3 - x_2 = h_2 = 1$
$x_4 - x_3 = h_3 = 2$

$$a_1 + 2(1+h_2)(a_2) + h_2 a_3 = 6\left(\frac{y_3-y_2}{h_2} - \frac{y_2-y_1}{h_1}\right)$$

$4a_2 + a_3 \approx 1.74$

$$h_2 a_2 + 2(h_2+h_3)(a_3) + h_3 a_4 = 6\left(\frac{y_4-y_3}{h_3} - \frac{y_3-y_2}{h_2}\right)$$

$a_2 + 6a_3 = -1.83$

$$\begin{vmatrix} 4 & 1 \\ 1 & 6 \end{vmatrix} \begin{vmatrix} a_2 \\ a_3 \end{vmatrix} = \begin{vmatrix} 1.74 \\ -1.86 \end{vmatrix}$$

$a_2 \approx 0.5348$
$a_3 \approx -0.3777$

$f_3(x), \ldots$

$$f_3(x) = -0.0332583(5-x)^3 + 0.468833(5-x) + 0.455(x-3)$$

E

9. C

$i = 0$

$x_{i+1} = x_i + h \qquad x_{i-1} = x_i - h$

$$f(x_1) = f(x_0) + f'(x_0)h + \frac{f''(x_0)}{2!}h^2 + \frac{f'''(x_0)}{3!}h^3 + \frac{f^{iv}(\xi_1)}{4!}h^4$$

$$f(x_{-1}) = f(x_0) - f'(x_0)h + \frac{f''(x_0)}{2!}h^2 - \frac{f'''(x_0)}{3!}h^3 + \frac{f^{iv}(\xi_2)}{4!}h^4$$

$\xi_1 \in \,]x_0, x_0+h[\qquad \xi_2 \in \,]x_0, x_0-h[$

$$f(x_1) + f(x_{-1}) = 2f(x_0) + 2\frac{f''(x_0)}{2!}h^2 + \frac{f^{(iv)}(\xi_1)}{4!}h^4 + \frac{f^{(iv)}(\xi_2)}{4!}h^4$$

$f''(x_0)$

$$= 2f(x_0) + f''(x_0)h^2 + \frac{f^{(iv)}(\xi_1+\xi_2)h^4}{24}$$

$$= f(x_0+h) + f(x_0-h)$$

$\cancel{2f(x_0\, f''(x_0) + h^3}$

$$f''(x_0)h^2 = f(x_0+h) + f(x_0-h) - 2f(x_0) - \frac{f^{(iv)}(\xi_1+\xi_2)h^4}{24}$$

$$f''(x_0)h^2 = f(x_0-h) - 2f(x_0) + f(x_0+h) - (h^4)$$

$$f''(x_0) = \frac{f(x_0-h) - 2f(x_0) + f(x_0+h)}{h^2} - \frac{f^{(iv)}(\xi_1+\xi_2)}{24}$$

$\xi_1 + \xi_2 = 2x_0$

$$-\frac{f^{(iv)}(2fx_0)(h^2)}{24}$$

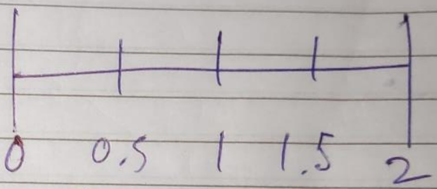
$$\boxed{-\frac{f^{iv}(x_0)}{12}}$$

extract last part of RH equation

10. B

Q10

$\int_0^2 \cosh(x)\,dx$



$\Delta x = \frac{2-0}{4} = 0.5$

$\cancel{S_4 \approx \frac{0.5}{3}(}$

$$S_4 \quad \frac{0.5}{3}\big(\cosh(0)+4\cosh(0.5)+2\cosh(1)$$
$$+4\cosh(1.5)+\cosh(2)\big)$$

$$S_4 = \frac{1}{6}\big(1+4(1.12763)+2(1.54308)$$
$$+4(2.35241)+3.762197\big)$$

$$S_4 = 3.63$$