Computational Maths Assignment #2

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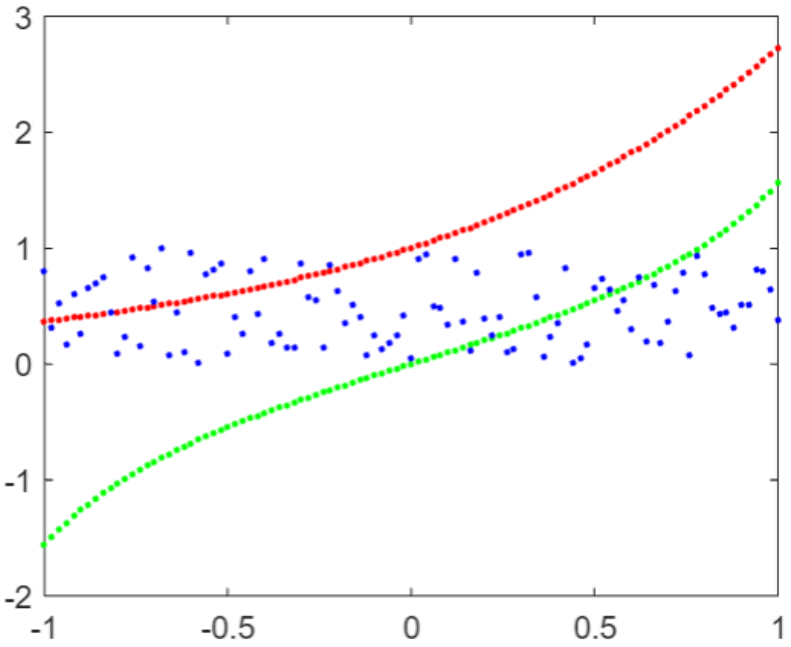
1. C

```
>> FUN = @ (x) 8 - 4.5 * (x - sin(x))
sol = fzero(FUN, 2)

FUN =

  function_handle with value:

    @(x)8-4.5*(x-sin(x))



sol =

    2.4305
```

2. A

```
>> A = ones(4,4)+2*eye(4,4);
>> A=A-2;
>> A(:,3:4)=A(:,3:4)+2;
>> disp(A)
     1    -1     1     1
    -1     1     1     1
    -1    -1     3     1
    -1    -1     1     3
```

3. A



COMMAND WINDOW

```
>> plot(x,y1,'r.',x,y2,'g.',x,y3,'b.');
```

4. C

$$A = \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}. \quad \det(A - \lambda I) = 0. \quad A - \lambda I = \begin{bmatrix} 1-\lambda & -3 & 3 \\ 3 & -5-\lambda & 3 \\ 6 & -6 & 4-\lambda \end{bmatrix}.$$

$$1-\lambda((-5-\lambda)(4-\lambda)-((3)(-6))+3((3)(4-\lambda)-(3)(6))+3((3)(-6) - (-5-\lambda)(6))$$

$$= -\lambda^3 + 12\lambda + 16 = -(\lambda + 2)(\lambda^2 - 2\lambda - 8)$$

$$= -(\lambda+2)(\lambda+2)(\lambda - 4)$$

$$\lambda_1 = -2 \quad \lambda_2 = 4$$

A where $\lambda = -2$, $A = \begin{bmatrix} 3 & -3 & 3 \\ 3 & -3 & 3 \\ 6 & -6 & 6 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$.

Reduced row echelon form $\begin{bmatrix} 1 & -1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$x_1 - x_2 + x_3 = 0$

$x_1 = x_2 - x_3$

$x_2 = x_2$

$x_3 = x_3$

V = $\begin{pmatrix} x_2 - x_3 \\ x_2 \\ x_3 \end{pmatrix}$.

Let $x_2 = 1, x_3 = 0$, $V_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$

Let $x_2 = 0, x_3 = 1$, $V_2 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$

A where $\lambda = 4$, A = $\begin{bmatrix} -3 & -3 & 3 \\ 3 & -9 & 3 \\ 6 & -6 & 0 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$.

Reduced row echelon form $\begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & -\frac{1}{2} \\ 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$$\begin{cases} x_1 - \frac{x_3}{2} = 0 \\ x_2 - \frac{x_3}{2} = 0 \end{cases}$$

$x_1 = \frac{x_3}{2}$

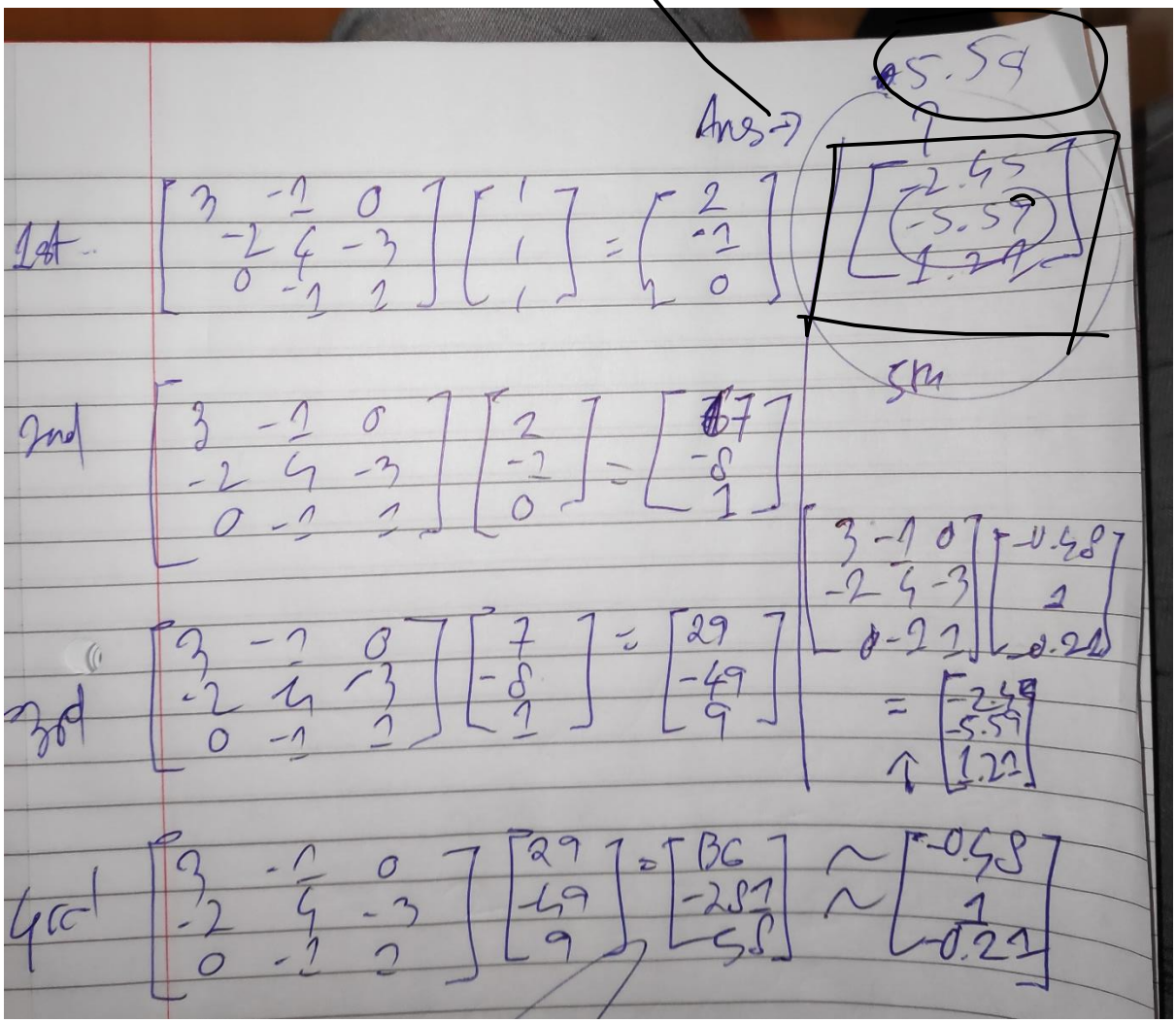
$x_2 = \frac{x_3}{2}$

$x_3 = x_3$

V = $\begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ 1 \end{pmatrix}$

none of them

5. E



6. B

x	y	$x - \bar{x}$	$(x - \bar{x})^2$	$y - \bar{y}$	$(x - \bar{x})(y - \overline{y})$
30	70	-28.33	802.59	-85	2408.05
40	90	-18.33	335.99	-75	1374.75
40	100	-18.33	335.99	-55	1008.15
50	120	-8.33	69.39	-35	291.55
50	130	-8.33	69.39	-25	208.25
50	150	-8.33	69.39	-5	41.65
60	160	1.67	2.79	5	8.35
70	190	11.67	136.19	35	408.45
70	200	11.67	136.19	45	525.15
80	200	21.67	469.59	45	975.15
80	220	21.67	469.59	65	1408.55
80	230	21.67	469.59	75	1625.25
$\sum x = 700$	$\sum y = 1860$		$\sum(x - \bar{x})^2$ = 3366.68		$\sum(x - \bar{x})(y - \overline{y})$ = 10284.3

$\bar{x}$ *is the mean of the* x *values.* $\bar{y}$ *is the mean of the* y *values.*

$\bar{x} = \sum x$ / 12 = 58.33. $\bar{y} = \sum y$ / 12 = 155

y = mx + c

m = $\sum(x - \bar{x})(y - \overline{y})$ / $(x - \bar{x})^2$ = 3.05

c = 155-58.33(3.05) = 22.9

y = 3.05x - 22.9 so roughly y = 3x-20

(i) 35kg -> 85m
(ii) 85kg->235m
(iii) 100kg->280m

7. D

x	y	$x - \bar{x}$	$(x - \bar{x})^2$	$y - \bar{y}$	$(x - \bar{x})(y - \overline{y})$
0.32	1.15	-0.64	0.4096	0.0875	-0.056
0.64	1.10	-0.32	0.1024	0.0375	-0.012
1.28	1.05	0.32	0.1024	-0.0125	-0.004
1.60	0.95	0.64	0.4096	-0.1125	-0.072
$\sum x = 3.84$	$\sum y = 4.25$		$\sum(x - \bar{x})^2$ = 1.024		$\sum(x - \bar{x})(y - \overline{y})$ = -0.144

$\bar{x}$ is the mean of the x values. $\bar{y}$ is the mean of the y values.

$\bar{x} = \sum x$ / 4 =0.96. $\bar{y} = \sum y$ / 12 = 1.0625

using laws of logarithms

$\ln y = -k_2 h + \ln k_1$

Which is in the form y = mx + c

We are given k_2 to be 0.1315

Solve for c (k_1): 1.0625-0.96(-0.1315) = 1.19

$k_1 = 1.19$

Atmospheric pressure @ sea level = $1.19\ kg/m^3$ and atmospheric pressure at top of the atmosphere is given to be 1.19/1000 = 0.00119 kg/m^3

0.00119 / 1.19 = $e^{-0.1315h}$

ln(0.00119 / 1.19) = -0.1315h

h = 52.53m

8. C

Time (s)	0	15	18	22	24
Velocity (ms^{-1})	22	24	37	25	123

Choose the data points closest to the point we want to estimate

So we choose (15,24), (18,37) and (22,25) as these timestamps are closest to 16s.

v(t) = $a_0 + a_1 t + a_2 t^2$

we get three equations

1. $24 = a_0 + a_1(15) + a_2(15)^2$
2. $37 = a_0 + a_1(18) + a_2(18)^2$
3. $25 = a_0 + a_1(22) + a_2(22)^2$

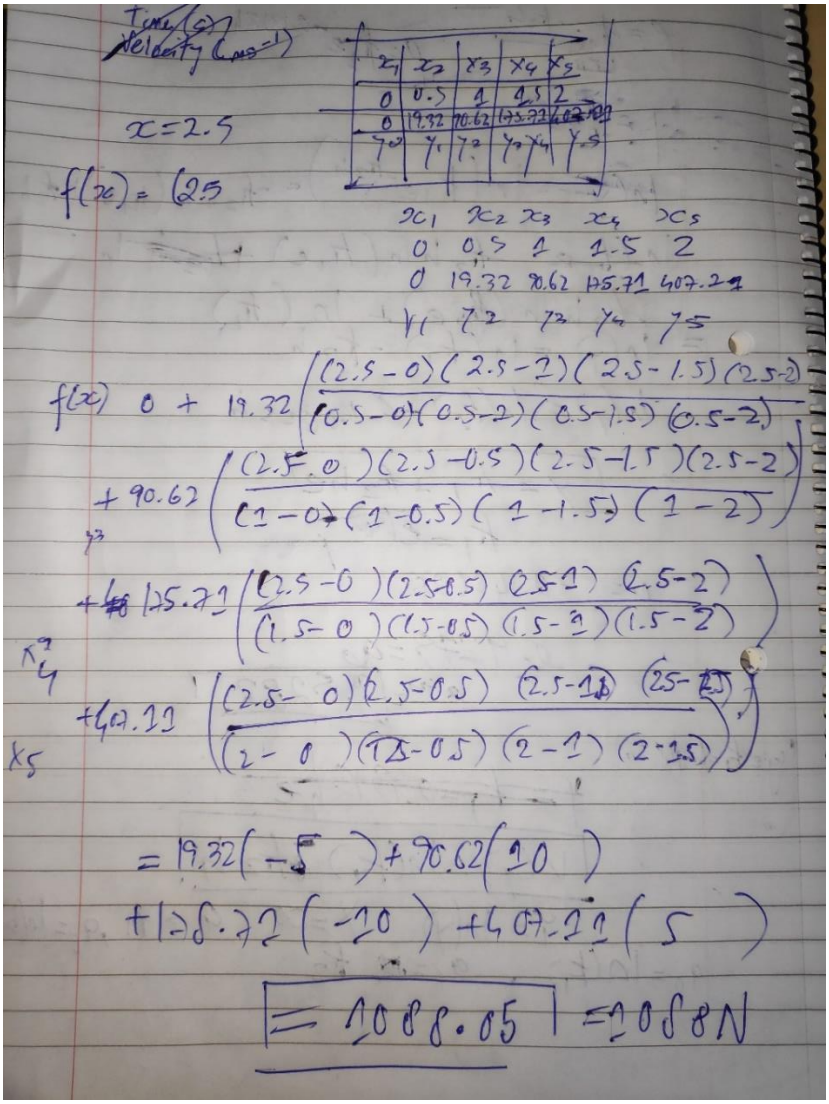
$$\begin{pmatrix} 1 & 15 & 225 \\ 1 & 18 & 324 \\ 1 & 22 & 484 \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} 24 \\ 37 \\ 25 \end{pmatrix}$$

$Solving\ gives\ us\ a_0 = -\frac{2267}{7}, a_1 = \frac{817}{21}, a_2 = -\frac{22}{21}$

$$v(t) = -\frac{2267}{7} + \frac{817}{21}t - \frac{22}{21}t^2$$

Substituting in t = 16s in the above equation we get v to be 30.43ms^{-1}

9. C



10. D

I got $b_2 = -0.017882$, *and once a created the polynomial using it,*

I estimated $\sqrt{2.5} = 1.577604$.

$$\frac{\sqrt{3}-\sqrt{2}}{3-2} = 0.31784. \frac{\sqrt{7}-\sqrt{3}}{7-3} = 0.22843. \quad \frac{0.22843-0.31784}{7-2} = -0.017882$$