Assignment 1 CSU33081 October 2020

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#1 E: None of the options. Should be [2 2 -6] + [1 0 2 -4] as $x^3 + 2x - 4$ doesn't have an element of degree 2.

#2 B

```
>> A=eye(3,3);
for x=1:2:3
A(1,x)=1;
end

>> disp(A)
     1     0     1
     0     1     0
     0     0     1
```

#3 C

```
>> x=[6:8;-1:1;5:7];
y=x(:,3);
size(y.')

ans =

     1     3
```

#4 E: none. $f(x) = 3 - 17x^3 \ evaluated \ at \ 2.5$ gives us -262.625.

The first derivative of $f(x)$ is $-51x^2$. The second derivative of $f(x)$ is -102x. Evaluating the first derivate $f'(2) = -204.$
$Evaluating \ the \ second \ derivative \ \ f''(2)$ = -204. Evaluating $f(2)$= -133. Hence the taylor polynomial of degree 2 about x=2 is:

$$P_2(x) = \frac{-133}{0!}(x-(2))^0 + \frac{-204}{1!}(x-(2))^1 + \frac{-204}{2!}(x-(2))^2 =$$

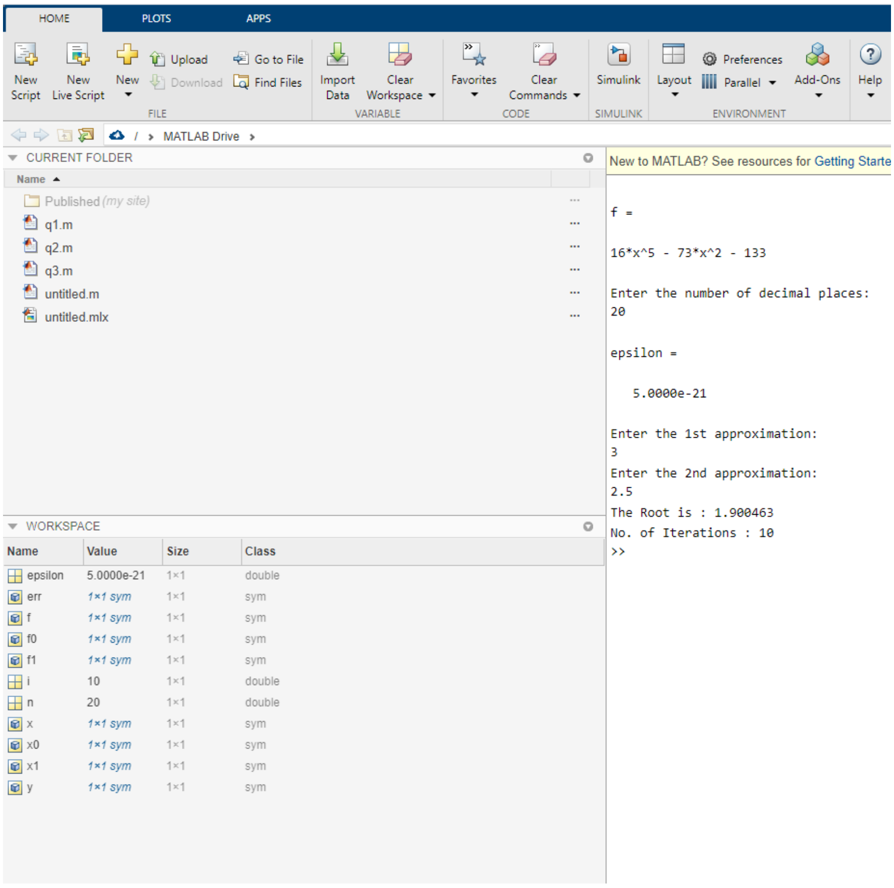
$$-133 \ - 204(x - 2) - 102(x-2)^2.$$

Truncation error = | $P_2(x) - f(x)$ | = | $P_2(2.5) - f(2.5)$ | = 2.125 = truncation error, which is not one of the options, so I answered E.

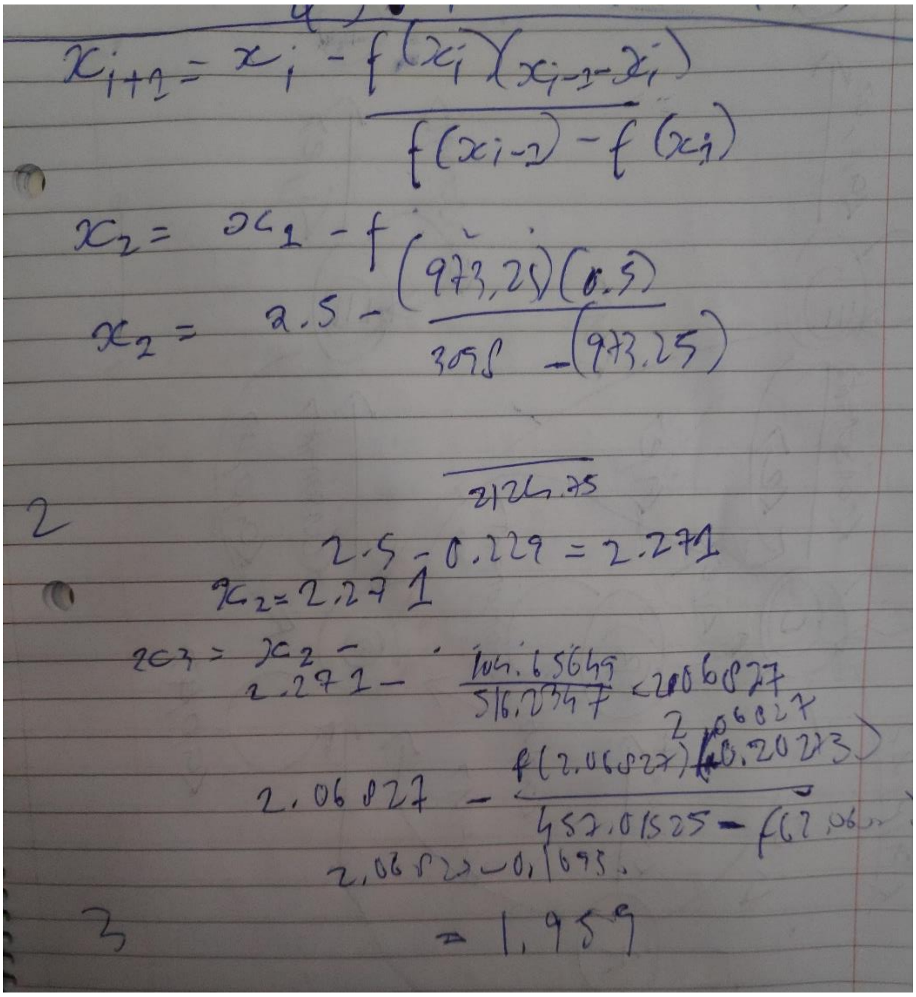
#5 C:

```
syms x;
f=16*x^5-73*x^2 -133 %The function
n=input('Enter the number of decimal places:');
epsilon = 5*10^-(n+1)
x0 = input('Enter x0:');
x1 = input('Enter x1:');
for i=1:20
 f0=vpa(subs(f,x,x0)); %Calculating the value of function at x0
    f1=vpa(subs(f,x,x1)); %Calculating the value of function at x1
y=x1-((x1-x0)/(f1-f0))*f1; %[x0,x1] is the interval of the root
err=abs(y-x1);
if err<epsilon %check the amount of error at each iteration
break
end
x0=x1;
x1=y;
end
y = y - rem(y,10^-n); %Displaying upto required decimal places
fprintf('The Root is : %f \n',y);
fprintf('No. of Iterations : %d\n',i);
```

I used a function in matlab to compute this for me as it was taking many many iterations on paper .


HOME
PLOTS
APPS
New Script
New Live Script
New
Upload
Download
Go to File
Find Files
FILE
Import Data
Clear Workspace
VARIABLE
Favorites
Clear Commands
CODE
Simulink
SIMULINK
Layout
Preferences
Parallel
Add-Ons
ENVIRONMENT
Help
MATLAB Drive
CURRENT FOLDER
Name
Published (my site)
q1.m
q2.m
q3.m
untitled.m
untitled.mlx
WORKSPACE
Name Value Size Class
epsilon 5.0000e-21 1×1 double
err 1×1 sym 1×1 sym
f 1×1 sym 1×1 sym
f0 1×1 sym 1×1 sym
f1 1×1 sym 1×1 sym
i 10 1×1 double
n 20 1×1 double
x 1×1 sym 1×1 sym
x0 1×1 sym 1×1 sym
x1 1×1 sym 1×1 sym
y 1×1 sym 1×1 sym
New to MATLAB? See resources for Getting Starte
f =
16*x^5 - 73*x^2 - 133
Enter the number of decimal places:
20
epsilon =
5.0000e-21
Enter the 1st approximation:
3
Enter the 2nd approximation:
2.5
The Root is : 1.900463
No. of Iterations : 10
>>

The function after x_3 was 1.959, and was converging on the answer I got using my matlab program.



$$x_{i+1} = x_i - \frac{f(x_i)(x_{i-1} - x_i)}{f(x_{i-1}) - f(x_i)}$$

$$x_2 = x_1 - f$$

$$x_2 = 2.5 - \frac{(973.25)(0.5)}{3098 - (973.25)}$$

$$2124.75$$

2

$$2.5 - 0.229 = 2.271$$

$$x_2 = 2.271$$

$$x_3 = x_2 -$$

$$2.271 - \frac{104.5649}{516.2747} < 2.06827$$

$$2.06827$$

$$2.06827 - \frac{f(2.06827)(0.20273)}{452.01525 - f(2.06\ldots)}$$

$$2.06827 - 0.1093$$

$$= 1.959$$

3

#6 A:

$f(x) = x^6 - x - 1.$ *The derivative* $f'(x) = 6x^5 - 1.$ $x_0 = 1.5.$

1st iteration:

f(x$_0$*)=f*(1.5)=1.56-1.5-1=8.890625.

f'(x$_0$*)=f'*(1.5)=6·1.55-1=44.5625.

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$ = 1.300491.

2^{nd} iteration:

f(x_1)=*f*(1.300491)=1.3004916-1.300491-1=2.537264

f′(x_1)=*f*′(1.300491)=6·1.3004915-1=21.319672

$x_2 \;=\; x_1 \;-\; \frac{f(x_1)}{f\prime(x_1)}$ = 1.18148

3^{rd} iteration:

f(x_2)=*f*(1.18148)=1.181486-1.18148-1=0.538459

f′(x_2)=*f*′(1.18148)=6·1.181485-1=12.812869

$x_3 \;=\; x_2 \;-\; \frac{f(x_2)}{f\prime(x_2)}$ = 1.139456.

This is the solution A.

#7 B

Find the Jacobian matrix of the system of equations.

$$x^2 + xy = 10$$

$$y + 3xy^2 = 57$$

$$J_{1,1} \;=\; \frac{\partial f1}{\partial x} \;=\; 2x + y.$$

$$J_{1,2} \;=\; \frac{\partial f1}{\partial y} \;=\; x.$$

$$J_{2,1} \;=\; \frac{\partial f2}{\partial x} \;=\; 3y^2.$$

$$J_{2,2} \;=\; \frac{\partial f2}{\partial y} \;=\; 1 \;+\; 6xy.$$

J = $\begin{bmatrix} 2x + y & x \\ 3y^2 & 1 \;+\; 6xy \end{bmatrix}$

The formula for the next iteration is as follows

$$\begin{bmatrix} x_{n+1} \\ y_{n+1} \end{bmatrix} = \begin{bmatrix} x_n \\ y_n \end{bmatrix} \;-\; J^{-1}\; x\; f\left(\begin{bmatrix} x_n \\ y_n \end{bmatrix}\right).$$

So for the first iteration n = 0, x = 1.5, y = 3.5

$\begin{bmatrix}1.5\\3.5\end{bmatrix} - \begin{bmatrix}6.5 & 1.5\\36.75 & 32.5\end{bmatrix} x\ f\left(\begin{bmatrix}1.5\\3.5\end{bmatrix}\right).$

= $\begin{bmatrix}1.5\\3.5\end{bmatrix} - \begin{bmatrix}6.5 & 1.5\\36.75 & 32.5\end{bmatrix} x\ f\left(\begin{bmatrix}-2.5\\1.625\end{bmatrix}\right)$

= $\begin{bmatrix}2.036\\2.8439\end{bmatrix}$

2nd iteration, n = 1 x = 2.036, y = 2.8439

$\begin{bmatrix}2.036\\2.8439\end{bmatrix} - \begin{bmatrix}6.9159 & 2.036\\24.2629 & 35.7413\end{bmatrix} x\ f\left(\begin{bmatrix}2.036\\2.8439\end{bmatrix}\right).$

= $\begin{bmatrix}2.036\\2.8439\end{bmatrix} - \begin{bmatrix}6.9159 & 2.036\\24.2629 & 35.7413\end{bmatrix} x\ f\left(\begin{bmatrix}-2.5\\1.625\end{bmatrix}\right)$

= $\begin{bmatrix}1.9987\\3.0023\end{bmatrix}$

3rd iteration, n = 2, x = 1.9987, y = 3.0023

$\begin{bmatrix}1.9987\\3.0023\end{bmatrix} - \begin{bmatrix}6.9997 & 1.9987\\27.0412 & 37.0041\end{bmatrix} x\ f\left(\begin{bmatrix}1.9987\\3.0023\end{bmatrix}\right).$

= $\begin{bmatrix}1.9987\\3.0023\end{bmatrix} - \begin{bmatrix}6.9997 & 1.9987\\27.0412 & 37.0041\end{bmatrix} x\ f\left(\begin{bmatrix}-0.0045\\0.0496\end{bmatrix}\right)$

= $\begin{bmatrix}2\\3\end{bmatrix}$.

Answer is $\begin{bmatrix}2\\3\end{bmatrix}$.

#8 D

1. $\begin{pmatrix}0 & -3 & -2 & | & 1 & 0 & 0\\1 & -4 & -2 & | & 0 & 1 & 0\\-3 & -4 & 1 & | & 0 & 0 & 1\end{pmatrix}$ 2. $\begin{pmatrix}1 & -4 & -2 & | & 0 & 1 & 0\\0 & -3 & -2 & | & 1 & 0 & 0\\-3 & 4 & 1 & | & 0 & 0 & 1\end{pmatrix}$

3 $\begin{pmatrix}-3 & 12 & 6 & | & 0 & -3 & 0\\0 & -3 & -2 & | & 1 & 0 & 0\\-3 & 4 & 1 & | & 0 & 0 & 1\end{pmatrix}$ 4. $\begin{pmatrix}1 & -4 & -2 & | & 0 & 1 & 0\\0 & -3 & -2 & | & 1 & 0 & 0\\0 & -8 & -5 & | & 0 & 3 & 1\end{pmatrix}$

5 $\begin{pmatrix}1 & -4 & -2 & | & 0 & 1 & 0\\0 & 1 & \frac{2}{3} & | & -\frac{1}{3} & 0 & 0\\0 & -8 & -5 & | & 0 & 3 & 1\end{pmatrix}$ 6. $\begin{pmatrix}1 & -4 & -2 & | & 0 & 1 & 0\\0 & -4 & -\frac{8}{3} & | & \frac{4}{3} & 0 & 0\\0 & -8 & -5 & | & 0 & 3 & 1\end{pmatrix}$

7. $\begin{pmatrix} 1 & 0 & \frac{2}{3} & | & -\frac{4}{3} & 1 & 0 \\ 0 & 1 & \frac{2}{3} & | & -\frac{1}{3} & 0 & 0 \\ 0 & -8 & -5 & | & 0 & 3 & 1 \end{pmatrix}$ 8. $\begin{pmatrix} 1 & 0 & \frac{2}{3} & | & -\frac{4}{3} & 1 & 0 \\ 0 & 1 & \frac{2}{3} & | & -\frac{1}{3} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & | & -\frac{8}{3} & 3 & 1 \end{pmatrix}$

9. $\begin{pmatrix} 1 & 0 & \frac{2}{3} & | & -\frac{4}{3} & 1 & 0 \\ 0 & 1 & \frac{2}{3} & | & -\frac{1}{3} & 0 & 0 \\ 0 & 0 & 1 & | & -8 & 9 & 3 \end{pmatrix}$ 10. $\begin{pmatrix} 1 & 0 & \frac{2}{3} & | & -\frac{4}{3} & 1 & 0 \\ 0 & 1 & \frac{2}{3} & | & -\frac{1}{3} & 0 & 0 \\ 0 & -8 & \frac{2}{3} & | & -\frac{16}{3} & 6 & 2 \end{pmatrix}$

11. $\begin{pmatrix} 1 & 0 & 0 & | & 4 & -5 & -2 \\ 0 & 1 & \frac{2}{3} & | & -\frac{1}{3} & 0 & 0 \\ 0 & 0 & \frac{2}{3} & | & -\frac{16}{3} & 6 & 2 \end{pmatrix}$

12. $\begin{pmatrix} 1 & 0 & 0 & | & 4 & -5 & -2 \\ 0 & 1 & 0 & | & 5 & -6 & -2 \\ 0 & 0 & 1 & | & -8 & 9 & 3 \end{pmatrix}$

We have reduced the left had side to the identity matrix of a 3x3 square matrix *I*, and hence the right hand side is the inverse of the original matrix.

$$\text{Answer:} \begin{pmatrix} 4 & -5 & -2 \\ 5 & -6 & -2 \\ -8 & 9 & 3 \end{pmatrix}.$$

#9 C

(x, y, z) = (1, 3, 5).

$x_{k+1} \;=\; \frac{1}{12}(2 - 7y_k - 3_{z_k}).$

$y_{k+1} \;=\; \frac{1}{5}(-5 - x_{k+1} - z_k).$

$z_{k+1} \;=\; \frac{1}{-11}(6 - 2x_{k+1} - 3z_k).$

1st iteration:

$x_1 = \frac{1}{12}$ *[2-7(3)-3(5)]=* $\frac{1}{12}$ *[-34]=-2.8333*

$y_1 = \frac{1}{5}$ *[-5-(-2.8333)-(5)]=* $\frac{1}{5}$ *[-7.1667]=-1.4333*

z_1= $\frac{1}{-11}$ [6-2(-2.8333)-7(-1.4333)]= $\frac{1}{-11}$ [21.7]=-1.9727

2nd iteration:

x_2= $\frac{1}{12}$ [2-7(-1.4333)-3(-1.9727)]= $\frac{1}{12}$[17.9515]=1.496

y_2= $\frac{1}{5}$[-5-(1.496)-(-1.9727)]= $\frac{1}{5}$[-4.5232]=-0.9046

z_2= $\frac{1}{-11}$ [6−2(1.496)−7(−0.9046)]= $\frac{1}{-11}$ [9.3406]=-0.8491

3rd iteration:

x_3= $\frac{1}{12}$ [2-7(-0.9046)-3(-0.8491)] = $\frac{1}{12}$[10.88]=0.9067

y_3= $\frac{1}{5}$[−5−(0.9067)−(−0.8491)] = $\frac{1}{5}$[-5.0575]=-1.0115

z_3= $\frac{1}{-11}$ [6-2(0.9067)-7(-1.0115)] = $\frac{1}{-11}$ [11.2672]=-1.0243

(x_3, y_3, z_3) = (0.9067, -1.0115, -1.0243).

#10 B

First we have to decompose $\begin{pmatrix} 1 & 2 & 4 \\ 3 & 8 & 14 \\ 2 & 6 & 13 \end{pmatrix}$ into the L and U parts of the the LU decomposition. I will do so with Gauss Jordan elimination.

1. $initial\ matrix \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 4 \\ 3 & 8 & 14 \\ 2 & 6 & 13 \end{pmatrix}$

2. $R2 = R2 - 3R1 \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 4 \\ 0 & 2 & 2 \\ 2 & 6 & 13 \end{pmatrix}$

3. $R3 = R3 - 2R1 \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 4 \\ 0 & 2 & 2 \\ 0 & 2 & 5 \end{pmatrix}$

4. $R3 = R3 - R2 \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 4 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{pmatrix}$

L = $\begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & 1 & 1 \end{pmatrix}$ and U = $\begin{pmatrix} 1 & 2 & 4 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{pmatrix}$. $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 3 \\ 13 \\ 4 \end{pmatrix}$

$[L][y] = b$

$[U][x] = y$

1. $y_1 = 3$
2. $3y_1 + y_2 = 13$
3. $2y_1 + y_2 + y_3 = 4$

Substitute the value of y_1 into equation 2 and we get $3(3) + y_2 = 13$.

$y_2 = 4$

Substitute y_1 and y_2 into equation 3. $2(3) + 4 + y_3 = 4$

$y_3 = -6$

$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ -6 \end{pmatrix}$

Use these values with x for the true solution

1. $x_1 + 2x_2 + 4x_3 = 3$
2. $2x_2 + 2x_3 = 4$
3. $3x_3 = -6$

From equation 3, $x_3 = -2$

From equation 2 substituting in x_3 gives us $2x_2 + 2(-2) = 4$

$x_2 = 4$

From equation 1 substituting in x_2 *and* x_3 gives us $x_1 + 2(4) + 4(-2) = 3$

$x_1 = 3$.

$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ -2 \end{pmatrix}$. Hence the answer is B.