Assignment 1 | Rvail Naveed | 17321983

Question 2.31

(a)

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Options:
    (i) 4
    (ii) 13
    (iii) 26
    (iv)  18

My answer: (ii)
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$$\begin{bmatrix} 1 & 5 & 4 \\ 2 & 3 & 6 \\ 1 & 1 & 1 \end{bmatrix} = A$$

$$\det(A) = 1\begin{vmatrix} 3 & 6 \\ 1 & 1 \end{vmatrix} - 5\begin{vmatrix} 2 & 6 \\ 1 & 1 \end{vmatrix} + \begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix}$$

$$\det(A) = (1 \times (3 \times 1) - (6 \times 1)) - (5((2 \times 1) - (6 \times 1))$$
$$+(4 \times (2 \times 1) - (3 \times 1)))$$

$$\det(A) = 13$$

(b)

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Options:
    (i) 0
    (ii) 12
    (iii) 7
    (iv) 4

My answer: (i)
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$$\begin{pmatrix}1 & 2 & 3 & 4\\ 5 & 6 & 7 & 8\\ 9 & 10 & 11 & 12\\ 13 & 14 & 15 & 16\end{pmatrix}$$

Swap matrix rows: $R_1 \leftrightarrow R_4$

$$= \begin{pmatrix}13 & 14 & 15 & 16\\ 5 & 6 & 7 & 8\\ 9 & 10 & 11 & 12\\ 1 & 2 & 3 & 4\end{pmatrix}$$

Cancel leading coefficient in row R_2 by performing $R_2 \leftarrow R_2 - \frac{5}{13} \cdot R_1$

$$= \begin{pmatrix}13 & 14 & 15 & 16\\ 0 & \frac{8}{13} & \frac{16}{13} & \frac{24}{13}\\ 9 & 10 & 11 & 12\\ 1 & 2 & 3 & 4\end{pmatrix}$$

Cancel leading coefficient in row R_3 by performing $R_3 \leftarrow R_3 - \frac{9}{13} \cdot R_1$

$$= \begin{pmatrix}13 & 14 & 15 & 16\\ 0 & \frac{8}{13} & \frac{16}{13} & \frac{24}{13}\\ 0 & \frac{4}{13} & \frac{8}{13} & \frac{12}{13}\\ 1 & 2 & 3 & 4\end{pmatrix}$$

Cancel leading coefficient in row R_4 by performing $R_4 \leftarrow R_4 - \frac{1}{13} \cdot R_1$

$$\begin{pmatrix}13 & 14 & 15 & 16\\ 0 & \frac{8}{13} & \frac{16}{13} & \frac{24}{13}\\ 0 & \frac{4}{13} & \frac{8}{13} & \frac{12}{13}\\ 0 & \frac{12}{13} & \frac{24}{13} & \frac{36}{13}\end{pmatrix} =$$

Swap matrix rows: $R_2 \leftrightarrow R_4$

$$\begin{pmatrix}13 & 14 & 15 & 16\\ 0 & \frac{12}{13} & \frac{24}{13} & \frac{36}{13}\end{pmatrix}$$

Cancel leading coefficient in row R_4 by performing $R_4 \leftarrow R_4 - \frac{2}{3} \cdot R_2$

$$= \begin{pmatrix}13 & 14 & 15 & 16\\ 0 & \frac{12}{13} & \frac{24}{13} & \frac{36}{13}\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{pmatrix}$$

The determinant of the matrix equals the diagonal product of the matrix

$\mathrm{dp}(A) = a_{11} \cdot a_{22} \cdot \ldots \cdot a_{nn} = \prod_{i=1}^{n} a_{ii}$

$= 13 \cdot \frac{12}{13} \cdot 0 \cdot 0$

Refine

$= 0$

Question 3.2

(a) Bisection Method

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Options:
    (i) 0.1241
    (ii) 0.8125
    (iii) 0.074995
    (iv) 0.003462

My answer: (ii)
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Root of $f(x) = x - 2e^{-x}$
Interval $= [a, b] = [0, 1]$
first estimate of x_{NS_1} :
$x_{NS_1} = \frac{(a+1)}{2} = \frac{1}{2}$
$f(0) = 0 - 2e^{-0} = -2$
$f(0.5) = 0.5 - 2e^{-0.5} = -0.71$
$f(a) * f(x, Ns) = -2 * -0 \cdot 71 = 1.42 > 0$

Interval $= [0.5, 1]$
$x_2 = \frac{0.5+1}{2} = 0.75$
$f(0.5) = 0.5 - 2e^{-0.5} = -0.71$
$f(0.75) = 0.75 - 2e^{-0.75} = -0.19$
$f(a) * f(x, 2) = -0.71 * 0.19 = 0.1349 > 0$

Interval $= [0.75, 1]$
$x_3 = 0.875$
$f(0.75) = -0.19$
$f(0.875) = 0.04$

$$f(a) * f(x_3) = -0.1947 * 0.04127 = -0.0080 < 0$$
$$\text{Interval } = [0.75, 0.875]$$
$$x_4 = 0.8125$$

(b) Secant Method

```
Options:
    (i) 0.72481
    (ii) 0.85261
    (iii) 0.62849
    (iv) 0.17238

My answer: (ii)
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$$x_{i+1} = x_i - \frac{f(x_i)(x_i - 1 - x_i)}{f(x_i - 1) - f(x_i)}$$

$$x_1 = 0, x_2 = 1$$
$$f(0) = -2, f(1) = 0.26$$
$$x_3 = 1 - \frac{0.26(0-1)}{-2-0.26} = 0.88$$
$$f(0.88) = 0.05$$

$$f(0.88) = 0.05$$
$$x_4 = 0.88 - \frac{0.05(1-0.88)}{0.26-0.05} = 0.85$$
$$f(0.85) = -0.005$$
$$\times 5 = 0.85 - \frac{-0.005(0.88 - 0.85)}{0.05 + 0.005} = 0.85$$

(c) Newton Method

```
Options:
    (i) 0.65782
    (ii) 0.59371
    (iii) 0.45802
    (iv) 0.85261

My answer: (iv)
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$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

$$f'(x) = -2\left(\frac{d}{dx}e^{-x}\right) + \frac{d}{dx}x$$

$$f'(x) = -2(e^{-x})\left(\frac{d}{dx} - x\right) + 1$$

$$f'(x) = 2e^{-x} + 1$$

$$x_1 = 1$$

$$x_2 = 1 - \frac{0.26}{1.74} = 0.85$$

$$x_3 = 0.85 - \frac{-0.005}{1.85} = 0.85$$

$$x_4 = 0.85$$

Question 4.24

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My answer: (i)
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Inverse(a)

$$= \begin{bmatrix} -1 & 2 & 1 & | & 1 & 0 & 0 \\ 2 & 2 & -4 & | & 0 & 1 & 0 \\ 0.2 & 1 & 0.5 & | & 0 & 0 & 1 \end{bmatrix}$$

- Augment with identity matrix
- Convert matrix on left to identity matrix on right by reducing to reduced row echelon form

$$= \begin{bmatrix} 1 & 0 & 0 & -0.71428\ldots & 0 & 1.42857\ldots \\ 0 & 1 & 0 & 0.25714\ldots & 0.1 & 0.28571\ldots \\ 0 & 0 & 1 & -0.22857\ldots & -0.2 & 0.85714\ldots \end{bmatrix}$$

$$= \begin{pmatrix} -0.71428\ldots & 0 & 1.42857\ldots \\ 0.25714\ldots & 0.1 & 0.28571\ldots \\ -0.22857\ldots & -0.2 & 0.85714\ldots \end{pmatrix}$$

- Inverse now lies on the right side

Inverse(b)

$$\begin{bmatrix} 1.6667 & 2.8889 & -2.2222 & 1.0000 \\ 0.0 & 0.3333 & -0.3333 & 0.0 \\ -0.3333 & -0.4444 & 0.1111 & 0.0 \\ 1.5000 & 2.0000 & -1.5000 & 0.5000 \end{bmatrix}$$

- Same method as above