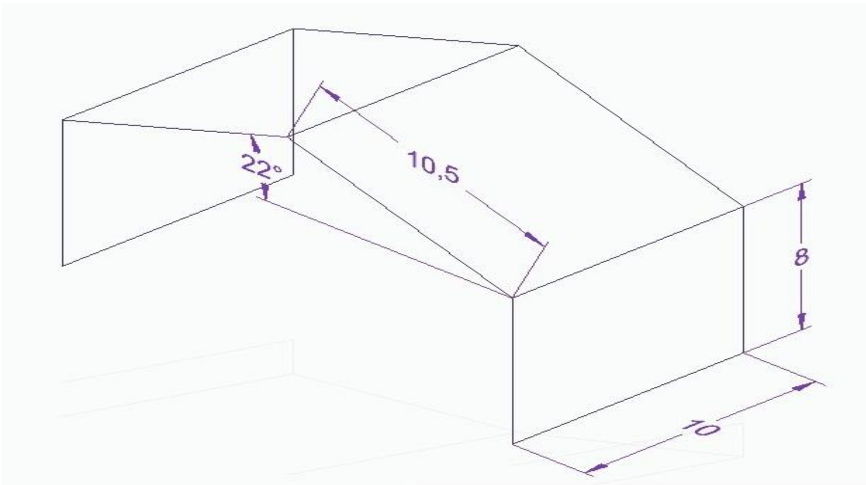
STRENGTH OF MATERIALS - Preliminary design of a shed

1. Specifications:



The structure of the frame of a shed (schematized above) must be designed such that it can support the following loading as well as other conditions:

- The weight of the roof and possible accumulation of snow, where the snow has a density of 500 $\frac{kg}{m^3}$ and with a maximum possible thickness of 15 cm . As there is a spacing of 10 meters between the frames, the load per frame results in:
$$q = 9.81 \times 500 \times 0.15 \times 10 = 10{,}300\frac{N}{m}.$$
- The weight of the central beam and anything hanged on it: $F = 5{,}000\ N$
- The weight of the frame if it has any explicit effect on the design.

Material characteristics (Steel):

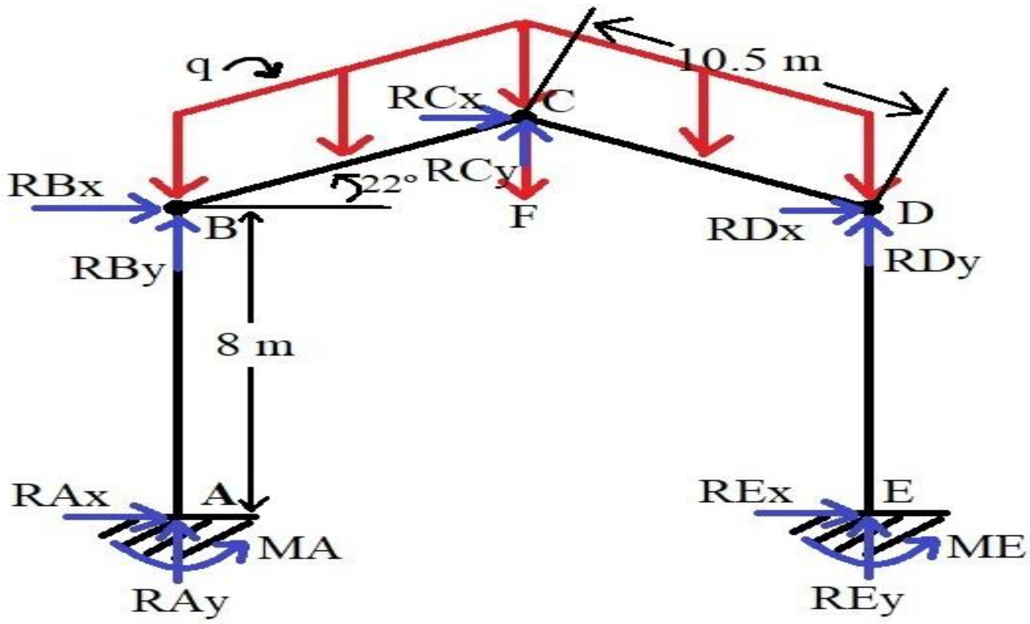
- Young's modulus: $E = 210\ GPa$
- Poisson's ratio: $\upsilon = 0.3$
- Density: $\rho = 7850\frac{kg}{m^3}$
- Allowable stress: $\sigma_y = 250MPa$

Specifications:

- Rigid enough, where the max displacement anywhere is lower than the length of the current beam divided by 200: $\delta_{MAX} < \frac{L}{200}$
- Resistant to the loading with a factor of safety of 1.5:
$$\sigma_{ult.} = \frac{\sigma_y}{F.o.S} = \frac{250 \times 10^6}{1.5} = 166.667\ MPa$$
- Free of buckling: The Euler critical load is lower than force to which the beams are subjected to.
- Made with standard elements (circular, square, or rectangular cross sections, solid or hollow and I, U or other kinds of profiles).

2. Calculations of forces:

To begin the design, firstly one must understand the effect the external forces have on the structure. In other words, calculations have to be made to be able to determine the information needed to make an adequate beam design.



(Free Body Diagram of entire shed)

Beams from point A to B and from point E to D will now be called as the **vertical beams**, while the beams from point B to C and from point D to C will be called the **upper beams**.

The structure's link to the ground are clamped and all other links are considered as pin connections. No matter what kind of links are made (as long as the ones to ground are clamped), this structure would result in a hyper static scenario where there are too many unknowns and not enough equations. Therefore to proceed, it is easier to solve with pin connections as the initial conditions given by the pins allows for an easier method of calculating the different properties caused by the loading (such as its bending moment).

To simplify the calculation process of the reaction forces on the y-axis, only the ground forces and the loads will be considered. Additionally since the structure is symmetrical: $R_{Ay} = R_{Ey}$

$$\sum F(y){:}\ 0 = R_{Ay} - 2 \cdot q \cdot L \cdot cos\theta - F + R_{Ay} \rightarrow R_{Ey} = \frac{2 \cdot q \cdot L \cdot cos\theta + F}{2}$$

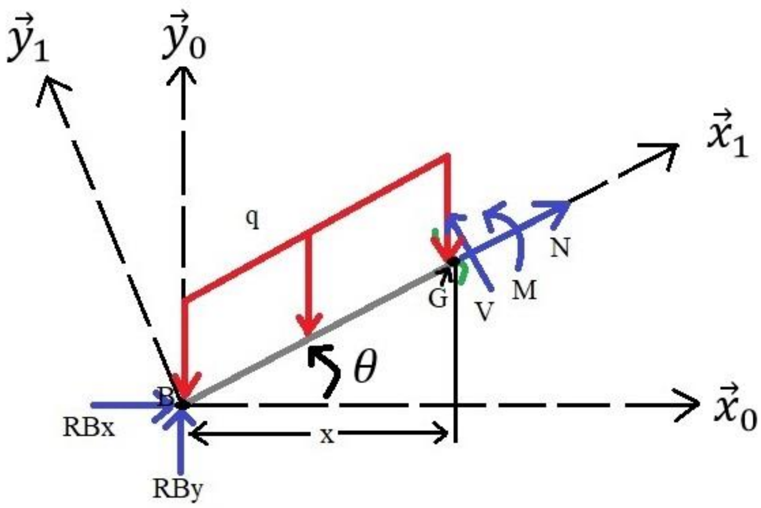
$$R_{Ay} = \frac{2 \cdot 10300 \cdot 10.5 \cdot cos22 + 500}{2} = 102774.934\ N \cdot \vec{y}_0$$

As there are only two forces acting on the x, one would firstly determine that they are 0. However, they are not as they are what counteracts both the normal and shearing force produced by the given load on the upper beams. Thus: $R_{Ax} = -R_{Ex}$

The reaction force on the y-axis of pin B is the same as the one on the ground. Thus:

$$R_{Ay} = R_{By} \qquad R_{Ey} = R_{Dy}$$

To obtain R_{Bx}, the following calculations were done from the FBD of the vertical beam from point B to G. Where G is at an x distance (a variable), the "cut".



(Free body diagram of "cut")

(i): $\sum$M(G): $0 = -\left(-R_{Bx} \cdot sin\theta + R_{By} \cdot cos\theta\right) \cdot L + q \cdot cos\theta \cdot L \cdot cos\theta \cdot \frac{x}{2} + M$

The sum of the moments around point C will now be calculated to get R_{Bx}:

(i) →$\sum$M(C): $0 = -\left(-R_{Bx} \cdot sin\theta + R_{By} \cdot cos\theta\right) \cdot L + q \cdot cos\theta \cdot L \cdot cos\theta \cdot \frac{x}{2} + M$

$$M = \left(-R_{Bx} \cdot sin\theta + R_{By} \cdot cos\theta\right) \cdot L - q \cdot cos\theta \cdot L \cdot cos\theta \cdot \frac{x}{2}$$

By using the following equation (i) becomes: $L = \frac{x}{cos\theta} \rightarrow x = L \cdot cos\theta$

$$(i) \rightarrow M = (-R_{Bx} \cdot tan\theta + R_{By}) \cdot x - q \cdot \frac{x^2}{2}$$

As point C is a pin connection, its internal moment is 0, thus M=0 and thus equation (i):

$$R_{Bx} = \frac{2 \cdot R_{By} - q \cdot x^2}{2 \cdot tan\theta} = \frac{2 \cdot 102775 - 10300 \cdot (10.5 \cdot cos22)^2}{2 \cdot tan22}$$
$$= +130282.691\ N \cdot \vec{x}_0$$

The following will be calculated to simplify equation (i):

$$-R_{Bx} \cdot tan\theta + R_{By} = -\left(\frac{2 \cdot R_{By} - q \cdot x^2}{2 \cdot tan\theta}\right) \cdot tan\theta + R_{By} = \frac{q}{2} \cdot L \cdot cos\theta$$

$$\therefore (i) \rightarrow M = \frac{q}{2} \cdot (L \cdot cos\theta \cdot x - x^2)$$

The shearing forces as well as the normal force in terms of a varying x can now be obtained:

$$(ii) \rightarrow \sum \mathrm{F}(\vec{x}_1): 0 = (R_{Bx} \cdot cos\theta + R_{By} \cdot sin\theta) - q \cdot sin\theta \cdot x + N$$

$$N = -(R_{Bx} \cdot cos\theta + R_{By} \cdot sin\theta) + q \cdot sin\theta \cdot x$$

Normal Force (N):

$$(ii) \rightarrow N = -159296.175 + 3858.448 \cdot x$$

$$(iii) \rightarrow \sum \mathrm{F}(y_1): 0 = (-R_{Bx} \cdot sin\theta + R_{By} \cdot cos\theta) - q \cdot cos\theta \cdot x + V$$

$$V = (R_{Bx} \cdot sin\theta - R_{By} \cdot cos\theta) + q \cdot cos\theta \cdot x$$

Shearing force (V):

$$(iii) \rightarrow V = -46486.504 + 9545.994 \cdot x$$

3. Design:

The overall objective of these calculations is to obtain the second moment of inertia, from which by using a data sheet of Universal Beams from British Steel one can obtain the correct beam design needed to withstand the given specifications.

First the characteristics of the upper beams will be defined and later that of the vertical beams which support the upper ones.

Upper Beams:

To obtain the second moment of inertia the following was used:

$E \cdot I \cdot \frac{dy^2}{d^2x} = M = \frac{q}{2} \cdot (L \cdot cos\theta \cdot x - x^2) \rightarrow$**Bending Moment**

$E \cdot I \cdot \frac{dy}{dx} = \int M \cdot dx = \frac{q}{2} \cdot \left(\frac{L \cdot cos\theta \cdot x^2}{2} - \frac{x^3}{3}\right) + C_1 \rightarrow$**Angle of Deflection**

$E \cdot I \cdot y = \int M \cdot dx \cdot dx = \frac{q}{2} \cdot \left(\frac{L \cdot cos\theta \cdot x^3}{6} - \frac{x^4}{12}\right) + C_1 \cdot x + C_2 \rightarrow$**Displacement**

As there is no none information on the angle of deflection, the following properties were used on the **Displacement** formula:

$At\ x = 0\ and\ y = 0 \rightarrow \therefore C_2 = 0$

The following assumption was made: $x = L \cdot cos\theta \rightarrow y = 0$

$$0 = \frac{q}{2} \cdot \left(\frac{L \cdot cos\theta \cdot x^3}{6} - \frac{x^4}{12}\right) + C_1 \cdot x \rightarrow C_1 = \frac{q}{2} \cdot \left(\frac{L \cdot cos\theta \cdot x^2}{6} - \frac{x^3}{12}\right)$$

$$C_1 = \frac{q}{2} \cdot \left(\frac{(L \cdot cos\theta)^3}{6} - \frac{(L \cdot cos\theta)^3}{12}\right) = -\frac{q \cdot (L \cdot cos\theta)^3}{24}$$

Now that the constants are known for the equations, by using the known specification that the maximum displacement must be a one over two hundred of the length of the current beam:

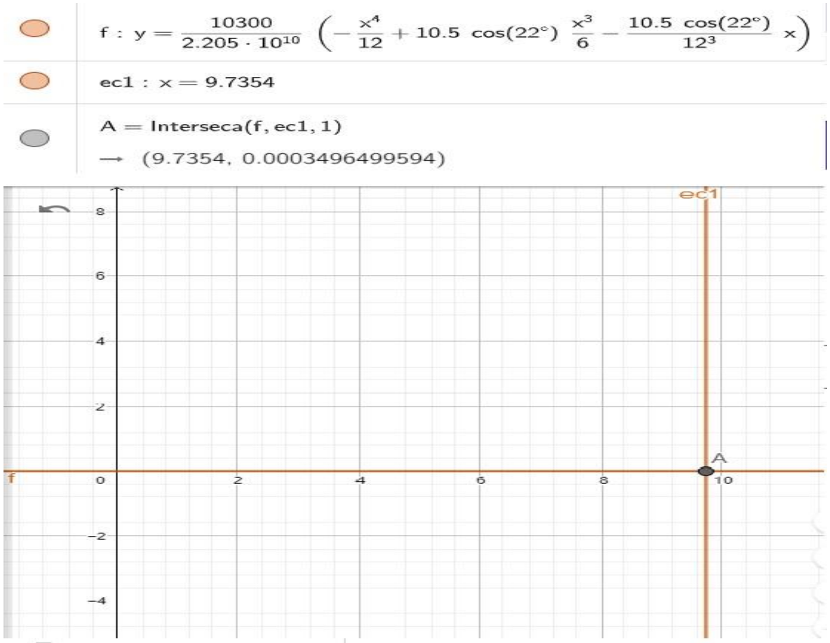
$$\delta_{MAX} = \frac{10.5}{200} = 0.0525 = y$$

$$E \cdot I \cdot y = \frac{q}{2} \cdot \left(\frac{L \cdot cos\theta \cdot x^3}{6} - \frac{x^4}{12} - \frac{(L \cdot cos\theta)^3 \cdot x}{12} \right)$$

$$\therefore I = \frac{q}{2 \cdot E \cdot y} \cdot \left(\frac{L \cdot cos\theta \cdot x^3}{6} - \frac{x^4}{12} - \frac{(L \cdot cos\theta)^3 \cdot x}{12} \right)$$

$$I = \frac{10300}{2 \cdot (210 \times 10^9) \cdot 0.0525} \cdot \left(\frac{10.5 \cdot cos22 \cdot x^3}{6} - \frac{x^4}{12} - \frac{(10.5 \cdot cos22)^3 \cdot x}{12} \right)$$

This equation represents the values for which I exists at a certain x distance with the given load and beam length. To obtain an adequate I beam value, this equation was graphed using the graphing calculator website Geogebra. The value for this equation at $x = 10.5 \cdot cos22$ will give the result which was searched for, as this is the maximum length of this beam.

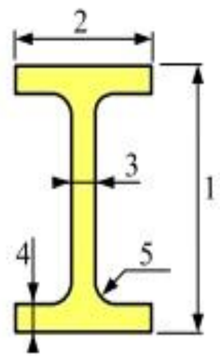


This ultimately gives:

$$I = 3.4965 \times 10^{-4}\ m^4 = 34965\ cm^4$$

The standard element for which to use for the design was considered to that which companies make, thus the design options were taken from British Steel and their universal beams.

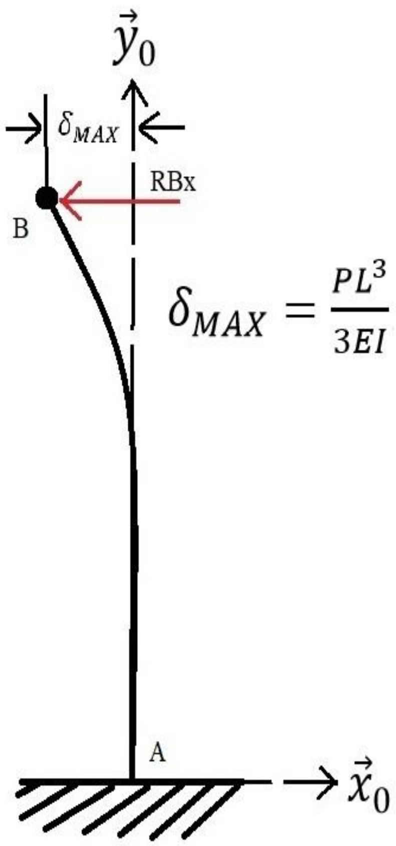
This company provides to the public a description of their designation and characteristics, more importantly what the geometrical features of these universal beams have with their respective second moment of inertia thus the adequate design can be chosen. This will be done by choosing the two closest values for Ix-x in the data sheet, which led to the choice of the following two designations:



Designation	457 × 152 × 74	457 × 152 × 82
Second Moment of Inertia (Ix-x) (in cm^4)	32891	36806
Depth of Section (1) (in mm)	462	465.8
Width of Section (2) (in mm)	154.4	155.3
Thickness of Web (3) (in mm)	9.6	10.5
Thickness of Flange (4) (in mm)	17	18.9
Root Radius (5) (in mm)	12.7	12.7

Vertical Beams:

These beams have a much simpler process to determine their second moment of inertia as they are not subjected to the same forces as the previous beams, the Free Body Diagram can be simplified to the following:



Where the only force taken into consideration is the reaction force produce by the given forces on the upper beams on the pin connection at point B. This beam can be considered as a cantilever beam, and the following equation can be used:

$$\delta_{MAX} = \frac{PL^3}{3EI}$$

Where $P = R_{Bx} = +130282\ N \cdot \vec{x}_0$

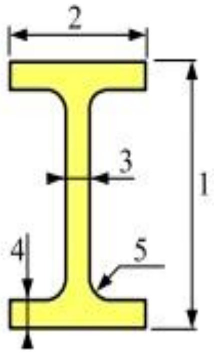
and $\delta_{MAX} = \frac{8}{200} = 0.04\ m$

$$\therefore I = \frac{R_{Bx} \cdot L^3}{3 \cdot E \cdot \delta_{MAX}} = \frac{130282.691 \cdot 8}{3 \cdot (210 \times 10^9) \cdot 0.04}$$

$$I = 2.64701 \times 10^{-3}\ m^4$$

$$I = 264701\ cm^4$$

The same thought process was followed as in the upper beams and as such values close to what was calculated were chosen. This led to the following two choices:



Designation	$838 \times 292 \times 176$	$838 \times 292 \times 194$
Second Moment of Inertia (Ix-x) (in cm^4)	247120	280274
Depth of Section (1) (in mm)	834.9	840.7
Width of Section (2) (in mm)	291.7	292.4
Thickness of Web (3) (in mm)	14	14.7
Thickness of Flange (4) (in mm)	18.8	21.7
Root Radius (5) (in mm)	20	20

4. RDM7:

To verify that all the previous calculations will indeed provide a shed that can withstand the specifications explained as well as determine which one of the choices made to use for the final design, the free-use software RDM7 was used. More specifically the "Ossatures" (Frames) simulator.

After verifying with the program, all the equation for the normal, shearing and bending moment were all correct as well as the reaction forces of the ground when pin connections are considered on all other links except the ones for the ground. These will be attached as an appendix in a .txt format.

Disclaimer*: The values for the second moment of inertia of the .txt of the pin connection setup were done with a random configuration and should be disregarded.

Even though the calculations were done with pin connections in mind, after trial and error, a rigid connection between all the beams seems to produce a structure which not only has a lower displacement but also an overall lower stress produced by both the Normal forces and the Bending Moments. Thus, the entire structure is now considered rigid in RDM7. To verify that the specifications are met, the results from RDM7 will be compared with the specifications and the values which are closest to them will indicate the adequate design. Additionally, another load has been added to create a stricter criteria for selection. This being that gravity will now be taken into consideration, thus the weight of the beams taken into account in the program simulation.

Combination of designations	$838 \times 292 \times 176 + 457 \times 152 \times 74$	$838 \times 292 \times 194 + 457 \times 152 \times 82$
Rigidity	Maximal displacement = 2.531E-002 m [Node C]	Maximal displacement = 2.435E-002 m [Node C]
Loading	Max stress (N+Mf)= 169.33 MPa	Max stress (N+Mf)= 153.50 Mpa

For the buckling criteria, each indidivual beams critical load was calculated using the following:

$$F_{CR} = \frac{\pi^2 \cdot E \cdot I}{(K \cdot L)^2} \rightarrow \sigma_{CR} = \frac{F_{CR}}{A}$$

This formula will be used on all the beams to determine the maximum stress that can occur before it buckles. Also, the column effective length factor (K) will be considered as 0.5 as both ends of all beams are rigid thus fixed.

	Upper beams L=10.5 m		**Vertical Beams** L=8 m	
Designation	$457 \times 152 \times 74$	$457 \times 152 \times 74$	$838 \times 292 \times 176$	$838 \times 292 \times 194$
F_{CR} **in MN**	24.773	27.677	320.116	363.063
A in m^2	0.0094969	0.0105028	0.0224735	0.024738
σ_{CR} **in GPa**	2.604	2.635	13.244	14.667

No value for the critical stress exceeds the calculated maximum stress, thus any of the structures are free from buckling. These values were verified with the RDM7 buckling simulation option.

Ultimately, the choice for the best beam design was the combination of beam designations: $838 \times 292 \times 194$ + $457 \times 152 \times 82$ (the "bigger" option). This was due to the maximum value for the stress obtained with it was 153.9 MPa which is lower than the ultimate stress allowed given by the specifications of 166.667 MPa, while the other combination exceeds by a small amount. Caution was the determining factor.

5. Weight:

The purpose of this project was to create a structure that would not only withstand the given forces but do so at the least amount of weight needed. Therefore with the ultimately decided beam combination, the weight ends up being the density multiplied by the volume which consists of the area of each beam times its respective length:

$$m = \rho \cdot V = \rho \cdot (2 \cdot A_1 \cdot L_1 + 2 \cdot A_2 \cdot L_2)$$

$$m = 7850 \cdot (2 \cdot 0.0105028 \cdot 10.5 + 2 \cdot 0.024738 \cdot 8) = 4838.479\ kg$$

Even though the weight of the beam configuration which exceeds the maximum stress by a small amount (being 4388.235 kg) is much smaller than that of the ultimately chosen design, the other version (the "bigger" option) will be the final one chosen. What this means is that a combination of those two options for the upper and vertical beams will provide that perfect configuration for this scenario. However they will be omitted as they are simple trial and error.

The final designs' geometrical details will be included as a .txt from RDM7.

6. Conclusion:

In conclusion, the concepts taught in the Strength of Materials class were applied in a very specific scenario which omits multiple real life factors for the design of a shed. This being, for example, the wind, the possibility of seismic activity, material selection, etc. Therefore this was a simplified process of designing a shed meant to simulate what it would be like in real life.

I believe that RDM7 was a powerful tool that allowed me to verify whether or not my calculations were on the right track and I believed that in this kind of classes they should be used more often as it would help students verify their results in a more assured manner.

7. Appendices:

- **RDM7 Loading Values for Structure with pin connection links. (.txt)**
- **RDM7 Geometrical Values for Final Design (.txt)**
- **RDM7 Loading Values for Fully Rigid Structure. (.txt)**

8. References:

- British Steel Universal Beam Data Sheet, retrieved from **https://britishsteel.co.uk/media/40515/british-steel-universal-beams-ub-datasheet.pdf** the 17th of January of 2020.
- Yves Debard (2006), RDM – Ossatures Manuel d'utilisation, retrieved from **http://iut.univ-lemans.fr/ydlogi/doc/rdmoss.pdf** the 16th of January of 2020.