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Appendix I:
Kinematics Microproject System 4

1. **Kinematics Model:** (Results shown in the report)
2. **Displacement:**

First, we decomposed the vector DE to be the following:

$$DE = DI + IE \quad (i)$$

$$DE = -0.4\ m \cdot \vec{x}_4\ ,\ IE = X \cdot \vec{x}_0$$

As I is a fixed point that is placed 0.4 meters in the $\vec{y}_0$ from point O, by using vector decomposition one can obtain that:

$$OD = OI + ID$$

$$OD = 0.6\ m \cdot \vec{y}_3,\ OI = 0.4\ m \cdot \vec{y}_0$$

$$ID = 0.6\ m \cdot \vec{y}_3 - 0.4\ m \cdot \vec{y}_0 \rightarrow DI = -0.6\ m \cdot \vec{y}_3 + 0.4\ m \cdot \vec{y}_0$$

Therefore, equation (i) becomes:

$$-0.4\ m \cdot \vec{x}_4 = -0.6\ m \cdot \vec{y}_3 + 0.4\ m \cdot \vec{y}_0 + X \cdot \vec{x}_0 \quad (i)$$

As vector $\vec{x}_4$ in terms of $\vec{x}_0$ would require an introduction of another angle other than the parameter ones given, we square equation (i) such that the result would be a magnitude that eliminates $\cdot \vec{x}_4$.

$$(-0.4 \cdot \vec{x}_4)^2 = (-0.6 \cdot \vec{y}_3 + 0.4 \cdot \vec{y}_0 + X \cdot \vec{x}_0)^2$$

$$0.16 = 0.52 - 0.48 \cdot \vec{y}_3 \cdot \vec{y}_0 - 1.2 \cdot X \cdot \vec{y}_3 \cdot \vec{x}_0 + X^2$$

$$0 = 0.36 - 0.48 \cdot cos\Psi_2 + 1.2 \cdot X \cdot sin\Psi_2 + X^2 \quad (ii)$$

As equation (ii) is a polynomial of degree 2, the quadratic equation can be used to obtain X:

$$X = \frac{-1.2 \cdot sin\Psi_2 \pm \sqrt{1.44 \cdot sin^2\Psi_2 - 4(0.36 - 0.48 \cdot cos\Psi_2)}}{2} \quad (iii)$$

3. **Input-Output angle relation**

$$AB = BC + CO + OA$$

$$-l_2 \cdot x_2 = -0.05 \cdot y_1 - 0.2 \cdot x_0 + d \cdot y_A \quad (iv)$$

To eliminate the introduction of new angles other than the parameter one, we rearranged and squared the given constraint equation. As we are squaring vectors, we had to change the basis with respect to the reference frame of Solid zero:

$$\vec{y}_1 = -sin\Psi_1 \cdot \vec{x}_0 + cos\Psi_1 \cdot \vec{y}_0$$

$$\vec{y}_A = -sin\alpha \cdot \vec{x}_3 + cos\alpha \cdot \vec{y}_3$$

$$\vec{x}_3 = cos\Psi_2 \cdot \vec{x}_0 + sin\Psi_2 \cdot y_0, \qquad y_3 = -sin\Psi_2 \cdot \vec{x}_0 + cos\Psi_2 \cdot \vec{y}_0$$

$$\Rightarrow y_A = -(sin\alpha \cdot cos\Psi_2 - cos\alpha \cdot sin\Psi_2) \cdot x_0 - (sin\alpha \cdot sin\Psi_2 + cos\alpha \cdot cos\Psi_2) \cdot y_0$$

$$= -sin(\alpha + \Psi_2) \cdot \vec{x}_0 - cos(\alpha + \Psi_2) \cdot \vec{y}_0$$

After applying the change of bases formulae, we square the result to obtain input output angle relation:

$$(-l_2 \cdot \vec{x}_2)^2 =$$

$$[-0.05(-sin\Psi_1 \cdot \vec{x}_0 + cos\Psi_1 \cdot y_0) - 0.2 \cdot \vec{x}_0 + d(-sin(\alpha + \Psi_2) \cdot \vec{x}_0 - cos(\alpha + \Psi_2) \cdot y_0)]^2 \quad (iv)$$

Where d = 0.224 meters (the distance of OA), $\boldsymbol{\alpha}$ = 0.464 rads (the angle between $\mathbf{y_3}$ and $\mathbf{y_a}$), l_2 =0.304 (the distance of AB). Giving us:

$$0.01185 - 0.0224cos(\alpha + \Psi_2 - \Psi_1) - 0.1(2sin\Psi_1 - cos\Psi_1)$$
$$+ 0.0896sin(\alpha + \Psi_2) - 0.0448cos(\alpha + \Psi_2) = 0 \quad (v)$$

To proceed we applied Euler's forward method to get an approximated version of equation (v) which we could use in equation (iii) (the displacement) as it is an inseparable function.

Euler forward method: $y_{n+1} = y_n + h \cdot f(x_n, y_n)$

To use this, we first defined the following:

- The discretisation step (h): we defined it in our Matlab script to be 0.1, this is an arbitrary amount with no real criteria used.
- The initial conditions (x_o, y_o): as the only piece of information we have on this inseparable equation is our graph from Geogebra, we made a vertical line (x=-π) and estimated graphically by plotting a point where this vertical line intersects with equation (iv). Which is point A, $x_o = -\pi, y_o = -0.23914$.
- The function $f(x_n, y_n)$: is the implicit derivative of equation (v).

$$f(\Psi_1, \Psi_2) = \frac{d\Psi_2}{d\Psi_1} = \frac{0.02cos\Psi_1 + 0.01sin\Psi_1 + 0.0224sin(\alpha+\Psi_2-\Psi_1)}{0.0224sin(\alpha+\Psi_2-\Psi_1) + 0.0896cos(\alpha+\Psi_2) + 0.0448sin(\alpha+\Psi_2)} \quad (vi)$$

4. **Velocity:**

As we already know that the velocity is the derivative with respect to time of the displacement, which means that all we had to do was derive the displacement [equation (iii)] to get the equation of velocity.

So here is the equation of velocity:

$$X = \frac{-1.2 \cdot sin\Psi_2 \pm \sqrt{1.44 \cdot sin^2\Psi_2 - 4(0.36 - 0.48 \cdot cos\Psi_2)}}{2} \quad (iii)$$

- We start by taking the constant $\frac{1}{2}$ out and we write it as 0.5
- Now we apply the difference rule where we differentiate the two parts of the equation separately
  $\frac{d}{d\psi_2}(1.2 \cdot sin(\psi_2) = 1.2 \cdot cos(\psi_2)$
  $\frac{d}{d\psi_2}(\sqrt{(1.44 \cdot sin^2(\psi_2) - 4 \cdot (0.36 - 0.48 \cdot cos(\psi_2)})$
- To differentiate this second part we apply the chain rule
  $\frac{df(u)}{dx} = \frac{df}{du} \cdot \frac{du}{dx}$
  $\frac{d}{d\psi_2}(\sqrt{(1.44 \cdot sin^2(\psi_2) - 4 \cdot (0.36 - 0.48 \cdot cos(\psi_2)}) =$
  $= \frac{1.44 \cdot sin(2 \cdot \psi_2) - 1.92 \cdot sin(\psi_2)}{2 \cdot \sqrt{1.44 \cdot sin^2(\psi_2) - 4 \cdot (0.36 - 0.48 \cdot cos(\psi_2))}}$
- And then we get the final equation (vii)

Finally, we get this equation:

$$\frac{dX}{dt} = 0.5 \times (-1.2 \cdot cos(\psi_2) + \frac{1.44 \cdot sin(2\psi_2) - 1.92 \cdot sin(\psi_2)}{2 \times \sqrt{1.44 \cdot sin^2(\psi_2) - 4 \times (0.36 - 0.48 \cdot cos(\psi_2))}}) \times \frac{d\psi_2}{dt} \quad (vii)$$

- **Angular speed ratio:**

$$0.01185 - 0.0224cos(+_2 -_1) - 0.1(2sin_1 - cos_1) + 0.0896sin(+_2) - 0.0448cos(+_2) = 0 \quad (v)$$

In order to create a better understanding of how the system works we had to also have some information on how the angular velocities are related to each other. In order to do this we derived with respect to time this initial equation of the input-output angle relationship, so we could have an equation that links the 2 angular velocities together.

- $\frac{d}{dt}(0.01185) = 0$
- $\frac{d}{dt}(-0.0224cos(\alpha + \psi_2 - \psi_1)) = [0.224sin(\alpha + \psi_2 - \psi_1)] \cdot \frac{d\psi_1}{dt}$
- $\frac{d}{dt}(-0.1(2sin\psi_2 - cos\psi_1)) = [-0.1(2cos\psi_2 + sin\psi_1)] \cdot \frac{d\psi_1}{dt}$
- $\frac{d}{dt}(0.0896sin(\alpha + \psi_2)) = [0.0896cos(\alpha + \psi_2)] \cdot \frac{d\psi_2}{dt}$
- $\frac{d}{dt}(-0.0448cos(\alpha + \psi_2)) = [0.0448sin(\alpha + \psi_2)] \cdot \frac{d\psi_2}{dt}$

Thus, we sum all of this previous derivatives and factored out their respective angular velocities:

$$\{[0.224sin(\alpha + \psi_2 - \psi_1)] + [0.1(2cos\psi_2 + sin\psi_1)]\} \cdot \frac{d\psi_1}{dt} + \{[0.0896cos(\alpha + \psi_2)] + [0.0448sin(\alpha + \psi_2)]\} \cdot \frac{d\psi_2}{dt} = 0$$

$$\therefore \frac{\frac{d\psi_2}{dt}}{\frac{d\psi_1}{dt}} = \frac{0.224sin(\alpha+\psi_2-\psi_1)+0.1(2cos\psi_2+sin\psi_1)}{0.0896cos(\alpha+\psi_2)+0.0448sin(\alpha+\psi_2)} \quad (viii)$$