

Running Time for Gap Penalties

$$M[i, j] = \text{match}(i, j) + \max \begin{cases} M[i-1, j-1] \\ X[i-1, j-1] \\ Y[i-1, j-1] \end{cases}$$

$$X[i, j] = \max \begin{cases} M[i, j-k] - \text{gap}(k) & \text{for } 1 \leq k \leq j \\ Y[i, j-k] - \text{gap}(k) & \text{for } 1 \leq k \leq j \end{cases}$$

$$Y[i, j] = \max \begin{cases} M[i-k, j] - \text{gap}(k) & \text{for } 1 \leq k \leq i \\ X[i-k, j] - \text{gap}(k) & \text{for } 1 \leq k \leq i \end{cases}$$

Final score is $\max \{M[n, m], X[n, m], Y[n, m]\}$.

How do you do the traceback?

Runtime:

- Assume $|X| = |Y| = n$ for simplicity: $3n^2$ subproblems
- $2n^2$ subproblems take $O(n)$ time to solve (because we have to try all k)

$\Rightarrow O(n^3)$ total time