Win probabilities with dynamic programming

CMSC423

Name(s):
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Serena Williams Win Probability

Could Serena Williams be a *more* dominant player if she played best-out-of-five-sets instead of best-out-of-three?

Five-thirty-eight thinks so

How can we use dynamic programming to address this question?

Let's use this notation:

- p: probability that Serena wins a set over another player (say, Olympic gold medalist Monica Puig)
- $q = (1 - p)$: probability that the other player, Monica Puig, wins a set

Suppose Serena and Monica play a game until one player wins n sets.

Denote $P(i, j)$ as the probability that Serena wins the game when:
- Serena needs to win i more sets, and
- Monica needs to win j more sets

Using this notation, $P(n, n)$ is the probability Serena wins the game. For example, the probability Serena wins a best-out-of-five game is $P(3, 3)$, and a best-of-three game is $P(2, 2)$.

Your goal is to write a dynamic programming algorithm that computes this probability.

Step 1) Write a recursive solution to compute the probability:

a) What are base cases $P(0, j)\,\forall j = 0 \ldots n$ and $P(i, 0)\,\forall i = 0 \ldots n$.

b) Write an expression for $P(i, j)$ in terms of p, q, $P(i-1, j)$ and $P(i, j-1)$. Why don't we use $P(i-1, j-1)$ in this expression?

Step 2) Write pseudo code for a dynamic programming algorithm that computes $P(n, n)$.