

Exercise Session 08

Exercise 1.

(CLRS 12.3-1) Implement a recursive variant of the TREE-INSERT procedure.

TREE-INSERT(c, x)

```
1  if x.leftchild = null && x.key  $\geq$  c.key
2      Insert(c, x.leftchild)
3  if x.rigthchild = null && x.key  $\leq$  c.key
4      Insert(c, c.rigthchild)
5  if x.key  $\geq$  c.key
6      Tree-Insert(c, x.leftchild)
7  if x.key  $\leq$  c.key
8      Tree-Insert(c, x.rigthchild)
```

Exercise 2.

(CLRS 12.3-3) We can sort a sequence of n numbers by iteratively inserting each number in a binary search tree and then performing an inorder tree walk. Write the pseudocode of this algorithm. What are the worst-case and best-case running times for this sorting algorithm?

T is an empty tree

CREATEANDSORT-TREE(A, T)

```
1  T.root = A[1]
2  for i = 2 to A.length
3      Tree-Insert(A[i], T.root)
4  Inorder-Tree-Walk(T.root)
```

$$\begin{aligned}T(n) &= c_1 + (n - 1) \cdot (c_2 + c_3) + c_4 \\T(c) &= 1 + n - 1 + (n - 1) \cdot (n \cdot \log(n)) + n \\T(c) &= n + n^2 \log(n) + n = n^2 \cdot \log(n)\end{aligned}$$

Exercise 3.

Consider the binary search tree T depicted in Figure 2. Delete the node with $key = 10$ from T by applying the procedure TREE-DELETE(T, z) as described in CLRS.

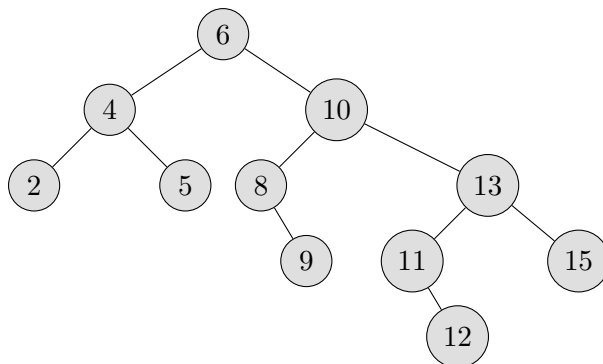


Figure 1: Binary Tree

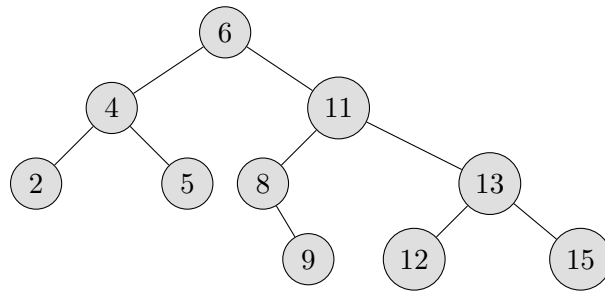


Figure 2: Binary Tree after deletion of 10

Exercise 4.

Show the red-black trees that result after successively inserting the keys 41; 38; 31; 12; 19; 8 into an initially empty red-black tree.

Exercise 5.

Consider the red-black tree T depicted in Figure 3. Insert first a node with $key = 15$ in T , then delete the node with $key = 8$. Show all the intermediate transformations of the red-black tree with particular emphasis on the rotations.

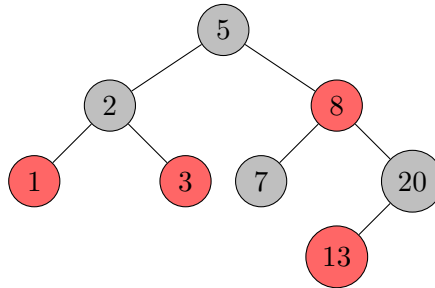


Figure 3: RB-Tree (NIL leaf nodes are omitted from the drawing)