Exercise Session 07

Exercise 1.
[CLRS-3 11.1-1] Suppose that a dynamic set S is represented by a direct-address table T of length m. Describe a procedure that finds the maximum element of S . What is the worst-case performance of your procedure?

To find the maximum value element in S we need to choose the largest actual key (K) from our universe of keys (U). This can also be understood as $\max(key \in K)$. As this is open addressing the maximum key must be equal to the largest element value of S

Exercise 2.
[CLRS-3 11.2-2] Demonstrate what happens when one inserts the keys $5; 28; 19; 15; 20; 33; 12; 17; 10$ into a hash table with collisions resolved by chaining. Let the table have 9 slots, and let the hash function be $h(k) = k \mod 9$.

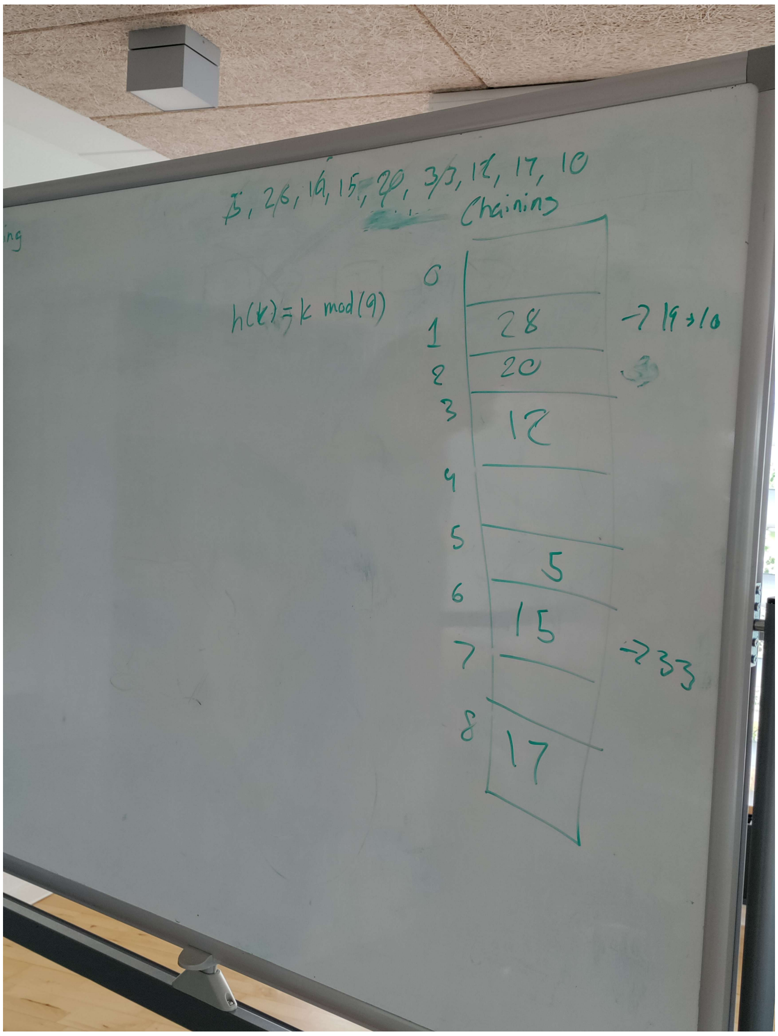

Figure 1: Caption

Exercise 3.
[CLRS-3 11.4-1] Consider inserting the keys $10; 22; 31; 4; 15; 28; 17; 88; 59$ into a hash table of length $m = 11$ using *open addressing* with the auxiliary function $h'(k) = k$. Illustrate the result of inserting these keys using linear probing, using quadratic probing with $c_1 = 1$ and $c_2 = 3$, and using double hashing with $h_1(k) = k$ and $h_2(k) = 1 + (k \mod (m - 1))$.

Linear probing:

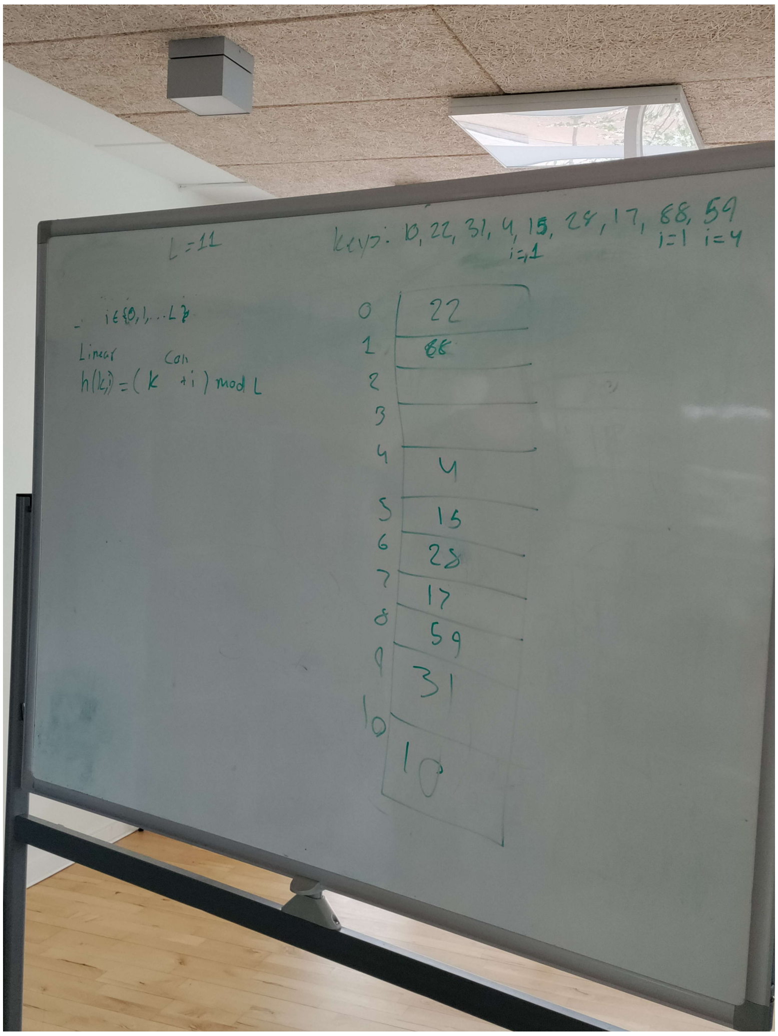

Figure 2: Caption

Quadratic probing:

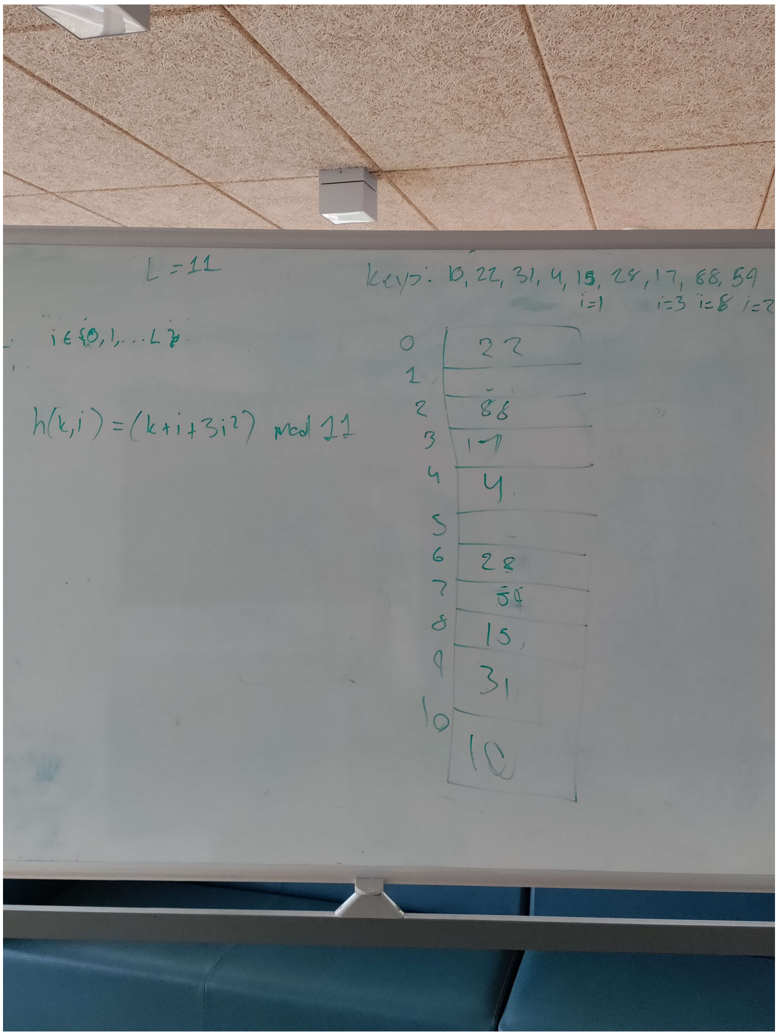

Figure 3: Caption

Double hashing:

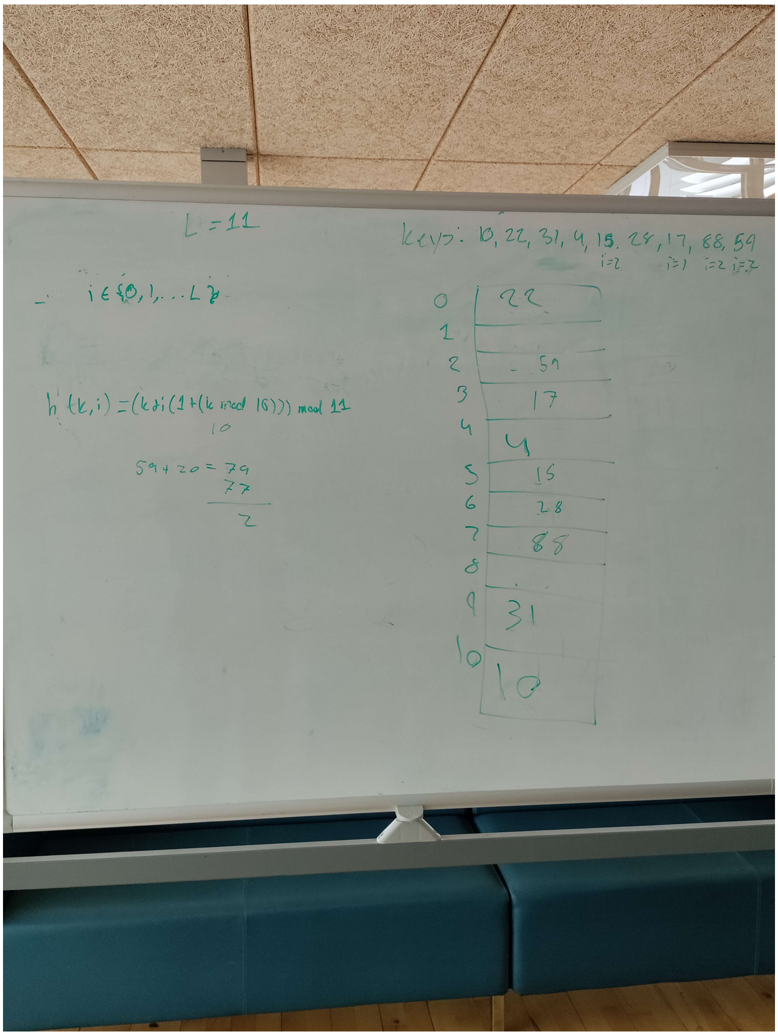

Figure 4: Caption

Exercise 4.

Consider an open-address hash table with uniform hashing. Give upper bounds on the expected number of probes in an unsuccessful search and on the expected number of probes in a successful search when the load factor is 3/4 and when it is 7/8.

Upper bound for 3/4 load factor: $\frac{1}{1-\alpha} = \frac{1}{1-\frac{3}{4}} = \frac{1}{\frac{1}{4}} = 4$

Upper bound for 7/8 load factor: $\frac{1}{1-\alpha} = \frac{1}{1-\frac{7}{8}} = \frac{1}{\frac{1}{8}} = 8$

Expected number of probes in successful search:

$\frac{3}{4} = \frac{1}{\frac{3}{4}\ln\frac{1}{1-\frac{3}{4}}} = \frac{4}{3}\ln 4 \approx 1.85$

$\frac{7}{8} = \frac{1}{\frac{7}{8}\ln\frac{1}{1-\frac{7}{8}}} = \frac{8}{7}\ln 8 \approx 2.38$

★ **Exercise 5.**

[CLRS-3 11.2-5] Suppose that we are storing a set of n keys into a hash table of size m. Show that if the keys are drawn from a universe U with $|U| > nm$, then U has a subset of size n consisting of keys that all hash to the same slot, so that the worst-case searching time for hashing with chaining is $\Theta(n)$.

Hint: The Dirichlet's box principle —a.k.a. pigeon hole principle— states that for $n, m \in \mathbb{N}$, if $nm+1$ objects are distributed among m sets, then at least one of the sets will contain at least $n+1$ objects.