

# **Instituto Tecnológico de Costa Rica**

**Operations Research - Semester II**

## **Equipment Replacement Problem**

Members:

Adrián Zamora Chavarría  
Daniel Romero Murillo

Date: September 24, 2025

# Equipment Replacement Problem

The equipment replacement problem is an optimization problem that involves deciding the optimal time to replace a piece of equipment (sell and re-buy) to minimize business costs. There are four main components to this problem:

- The project term (e.g. 5 years long project)
- Life span of the equipment
- Price of new equipment
- Maintenance costs (could increase as equipment ages)
- Sell prices (e.g. 4 year old equipment can be sold for \$50)

The problem involves determining the optimal number of equipment replacements and the timing of each, in order to minimize operating costs.

Given the following variables:

- $t_{max}$ : Project term
- $t_{span}$ : Lifes span of equipment
- $t$ : A given instant through the project term.
- $C_{tx}$ : The cost of buying at instant  $t$  and selling at  $x$ 
  - Cost of new equipment + maintenance costs - sell price - profits

$G(t)$  = optimal business costs from  $t$  to  $t_{max}$  (Buying new equipment at  $t$ ).

## 0.1 Trivial case

$$G(t_{max}) = 0$$

- From instant  $t_{max}$  to instant  $t_{max}$
- We don't buy any equipment

## 0.2 Optimization

If new equipment is bought at  $t$  we can sell it and buy new equipment at either  $t + 1$ ,  $t + 2$ , ..., or  $t + t_{span}$ . We should sell at the point where we get the least operation cost given the project term.

## 0.3 Bellman Equation

Knowing this, we can establish the Bellman equation:

$$G(t) = \min\{C_{tx} + G(x)\} \quad (1)$$

From  $t = t_{max}$  until  $t = 0$ .

## Equipment Life Cycle Costs

Year	Maintenance Cost	Resale Price
1	1	2
2	2	3
3	4	5

## $C_{tx}$ Calculations

$$\mathbf{1 \text{ year(s): } C_{t,t+1} = 500 + 1 - 2 = 499}$$

$$\mathbf{2 \text{ year(s): } C_{t,t+2} = 500 + 1 + 2 - 3 = 500}$$

$$\mathbf{3 \text{ year(s): } C_{t,t+3} = 500 + 1 + 2 + 4 - 5 = 502}$$

## $C_{tx}$ Table

t/x	1	2	3	4	5
0	499	500	502	—	—
1	—	499	500	502	—
2	—	—	499	500	502
3	—	—	—	499	500
4	—	—	—	—	499

## Step by Step Calculations

$$G(5) = 0 \quad (\text{Base case})$$

$$G(4) = \min\{C_{45} + G(5)\} = (500 + 1 - 2) + 0 = 499 = 499$$

**Optimal choice:** sell at time 5

$$G(3) = \min\{C_{34} + G(4), C_{35} + G(5)\} = (500 + 1 - 2) + 499 = 998, (500 + 1 + 2 - 3) + 0 = 500 = 500$$

**Optimal choice:** sell at time 5

$$G(2) = \min\{C_{23} + G(3), C_{24} + G(4), C_{25} + G(5)\} = (500 + 1 - 2) + 500 = 999, (500 + 1 + 2 - 3) + 499 = 999, (500 + 1 + 2 + 4 - 5) + 0 = 502 = 502$$

**Optimal choice:** sell at time 5

$$G(1) = \min\{C_{12} + G(2), C_{13} + G(3), C_{14} + G(4)\} = (500 + 1 - 2) + 502 = 1001, (500 + 1 + 2 - 3) + 500 = 1000, (500 + 1 + 2 + 4 - 5) + 499 = 1001 = 1000$$

**Optimal choice:** sell at time 3

$$G(0) = \min\{C_{01} + G(1), C_{02} + G(2), C_{03} + G(3)\} = (500 + 1 - 2) + 1000 = 1499, (500 + 1 + 2 - 3) + 502 = 1002, (500 + 1 + 2 + 4 - 5) + 500 = 1002 = 1002$$

**Multiple optimal choices:** sell at time 2, sell at time 3 (tie)

## Result Table (Analysis table)

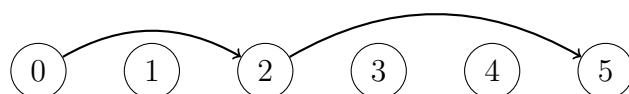
t	G(t)	Next Replacement
0	1002	2, 3
1	1000	3
2	502	5
3	500	5
4	499	5
5	0	-

## Optimal Solution

**Minimum Total Cost:** 1002

**All Optimal Replacement Plans:**

- **Plan 1:** Buy at 0, sell at 2 → Buy at 2, sell at 5



- **Plan 2:** Buy at 0, sell at 3 → Buy at 3, sell at 5

