Instituto Tecnológico de Costa Rica

Operations Research - Semester II

Knapsack Problem

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The Knapsack Problem

The **Knapsack Problem** is a classic optimization problem in computer science and operations research. It consists of having a set of items, each with a weight and a value. We also have a knapsack with limited capacity. The goal is to select a subset of items so that the total weight does not exceed the capacity of the knapsack, while the total value gained is maximized.

$$\max(Z) = \sum x_i v_i$$

$$\text{subject to} \quad \sum x_i c_i \leq C$$

- Z: gained value (we want to maximize it)
- C: knapsack capacity
- v_i: value of item i
- c_i: cost of item i
- x_i: amount of item i taken

0/1 Knapsack Problem

In the **0/1 Knapsack Problem**, items exist only once. We can either take it or leave it. We cannot take multiple of the same item.

Bounded Knapsack Problem

In the **Bounded Knapsack Problem**, we can take multiple copies of each item. The items are limited and we have to choose how many to take of each.

Unbounded Knapsack Problem

In the **Unbounded Knapsack Problem**, there are infinite copies of each item. We can take as much as we please of any item.

Knapsack algorithm

- We have k_i copies of item i
- We can include up to Q copies of item i

$$Q = \min(k_j, \left\lfloor \frac{i}{c_j} \right\rfloor)$$

- Now we fill up a $C \times k_i$ sized table like this:

$$\begin{aligned}\text{Table}[i][j] = \max \Big(& \text{Table}[i][j-1], \\ & 1 \cdot v_i + \text{Table}[i - 1 \cdot c_i][j-1], \\ & 2 \cdot v_i + \text{Table}[i - 2 \cdot c_i][j-1], \\ & \dots, \\ & Q \cdot v_i + \text{Table}[i - Q \cdot c_i][j-1] \Big)\end{aligned}$$

- In each cell we save the maximum possible value, and the amount of the item we should take to get it.

Problem

Maximize

$$Z = 4x_1 + 2x_2 + 3x_3 + 6x_4 + 10x_5 + 7x_6 + 4x_7 + 5x_8 + 4x_9 + 2x_10 + 3x_11$$

Subject to

$$2x_1 + x_2 + 2x_3 + 4x_4 + 8x_5 + 4x_6 + 2x_7 + 3x_8 + x_9 + x_10 + x_11 \leq 20$$
$$0 \leq x_1 \leq 5$$
$$0 \leq x_2 \leq 10$$
$$0 \leq x_3 \leq 3$$
$$0 \leq x_4 \leq 1$$
$$0 \leq x_5 \leq 1$$
$$0 \leq x_6 \leq 2$$
$$0 \leq x_7 \leq 5$$
$$0 \leq x_8 \leq 18$$
$$0 \leq x_9 \leq 2$$
$$0 \leq x_10 \leq 5$$
$$0 \leq x_11 \leq 3$$

Table

	Agua	Medias	Gallet	Repele	Bear
0	$0/x_0 = 0$	$0/x_1 = 0$	$0/x_2 = 0$	$0/x_3 = 0$	$0/x_4 = 0$
1	$0/x_0 = 0$	$2/x_1 = 1$	$2/x_2 = 0$	$2/x_3 = 0$	$2/x_4 = 0$
2	$4/x_0 = 1$	$4/x_1 = 2 - 0$	$4/x_2 = 0$	$4/x_3 = 0$	$4/x_4 = 0$
3	$4/x_0 = 1$	$6/x_1 = 3$	$6/x_2 = 0$	$6/x_3 = 0$	$6/x_4 = 0$
4	$8/x_0 = 2$	$8/x_1 = 4 - 0$	$8/x_2 = 0$	$8/x_3 = 0$	$8/x_4 = 0$
5	$8/x_0 = 2$	$10/x_1 = 5$	$10/x_2 = 0$	$10/x_3 = 0$	$10/x_4 = 0$
6	$12/x_0 = 3$	$12/x_1 = 6 - 0$	$12/x_2 = 0$	$12/x_3 = 0$	$12/x_4 = 0$
7	$12/x_0 = 3$	$14/x_1 = 7$	$14/x_2 = 0$	$14/x_3 = 0$	$14/x_4 = 0$
8	$16/x_0 = 4$	$16/x_1 = 8 - 0$	$16/x_2 = 0$	$16/x_3 = 0$	$16/x_4 = 0$
9	$16/x_0 = 4$	$18/x_1 = 9$	$18/x_2 = 0$	$18/x_3 = 0$	$18/x_4 = 0$
10	$20/x_0 = 5$	$20/x_1 = 10 - 0$	$20/x_2 = 0$	$20/x_3 = 0$	$20/x_4 = 0$
11	$20/x_0 = 5$	$22/x_1 = 9$	$22/x_2 = 0$	$22/x_3 = 0$	$22/x_4 = 0$
12	$20/x_0 = 5$	$24/x_1 = 10$	$24/x_2 = 0$	$24/x_3 = 0$	$24/x_4 = 0$
13	$20/x_0 = 5$	$26/x_1 = 9$	$26/x_2 = 0$	$26/x_3 = 0$	$26/x_4 = 0$
14	$20/x_0 = 5$	$28/x_1 = 10$	$28/x_2 = 0$	$28/x_3 = 0$	$28/x_4 = 0$
15	$20/x_0 = 5$	$30/x_1 = 9$	$30/x_2 = 0$	$30/x_3 = 0$	$30/x_4 = 0$
16	$20/x_0 = 5$	$32/x_1 = 10$	$32/x_2 = 0$	$32/x_3 = 0$	$32/x_4 = 0$
17	$20/x_0 = 5$	$34/x_1 = 9$	$34/x_2 = 0$	$34/x_3 = 0$	$34/x_4 = 0$
18	$20/x_0 = 5$	$36/x_1 = 10$	$36/x_2 = 0$	$36/x_3 = 0$	$36/x_4 = 0$
19	$20/x_0 = 5$	$38/x_1 = 9$	$38/x_2 = 0$	$38/x_3 = 0$	$38/x_4 = 0$
20	$20/x_0 = 5$	$40/x_1 = 10$	$40/x_2 = 0$	$40/x_3 = 0$	$40/x_4 = 0$

Table

	Sogas	Tronad	Birra	Mota	Bolsas
0	$0/x_5 = 0$	$0/x_6 = 0$	$0/x_7 = 0$	$0/x_8 = 0$	$0/x_9 = 0$
1	$2/x_5 = 0$	$2/x_6 = 0$	$2/x_7 = 0$	$4/x_8 = 1$	$4/x_9 = 0$
2	$4/x_5 = 0$	$4/x_6 = 1 - 0$	$4/x_7 = 0$	$8/x_8 = 2$	$8/x_9 = 0$
3	$6/x_5 = 0$	$6/x_6 = 1 - 0$	$6/x_7 = 0$	$10/x_8 = 2$	$10/x_9 = 1 - 0$
4	$8/x_5 = 0$	$8/x_6 = 2 - 0$	$8/x_7 = 0$	$12/x_8 = 2$	$12/x_9 = 2 - 0$
5	$10/x_5 = 0$	$10/x_6 = 2 - 0$	$10/x_7 = 0$	$14/x_8 = 2$	$14/x_9 = 3 - 0$
6	$12/x_5 = 0$	$12/x_6 = 3 - 0$	$12/x_7 = 0$	$16/x_8 = 2$	$16/x_9 = 4 - 0$
7	$14/x_5 = 0$	$14/x_6 = 3 - 0$	$14/x_7 = 0$	$18/x_8 = 2$	$18/x_9 = 5 - 0$
8	$16/x_5 = 0$	$16/x_6 = 4 - 0$	$16/x_7 = 0$	$20/x_8 = 2$	$20/x_9 = 5 - 0$
9	$18/x_5 = 0$	$18/x_6 = 4 - 0$	$18/x_7 = 0$	$22/x_8 = 2$	$22/x_9 = 5 - 0$
10	$20/x_5 = 0$	$20/x_6 = 5 - 0$	$20/x_7 = 0$	$24/x_8 = 2$	$24/x_9 = 5 - 0$
11	$22/x_5 = 0$	$22/x_6 = 5 - 0$	$22/x_7 = 0$	$26/x_8 = 2$	$26/x_9 = 5 - 0$
12	$24/x_5 = 0$	$24/x_6 = 5 - 0$	$24/x_7 = 0$	$28/x_8 = 2$	$28/x_9 = 5 - 0$
13	$26/x_5 = 0$	$26/x_6 = 5 - 0$	$26/x_7 = 0$	$30/x_8 = 2$	$30/x_9 = 5 - 0$
14	$28/x_5 = 0$	$28/x_6 = 5 - 0$	$28/x_7 = 0$	$32/x_8 = 2$	$32/x_9 = 5 - 0$
15	$30/x_5 = 0$	$30/x_6 = 5 - 0$	$30/x_7 = 0$	$34/x_8 = 2$	$34/x_9 = 5 - 0$
16	$32/x_5 = 0$	$32/x_6 = 5 - 0$	$32/x_7 = 0$	$36/x_8 = 2$	$36/x_9 = 5 - 0$
17	$34/x_5 = 0$	$34/x_6 = 5 - 0$	$34/x_7 = 0$	$38/x_8 = 2$	$38/x_9 = 5 - 0$
18	$36/x_5 = 0$	$36/x_6 = 5 - 0$	$36/x_7 = 0$	$40/x_8 = 2$	$40/x_9 = 5 - 0$
19	$38/x_5 = 0$	$38/x_6 = 5 - 0$	$38/x_7 = 0$	$42/x_8 = 2$	$42/x_9 = 5 - 0$
20	$40/x_5 = 0$	$40/x_6 = 5 - 0$	$40/x_7 = 0$	$44/x_8 = 2$	$44/x_9 = 5 - 0$

Table

	Banano
0	$0/x_10 = 0$
1	$4/x_10 = 0$
2	$8/x_10 = 0$
3	$11/x_10 = 1$
4	$14/x_10 = 2$
5	$17/x_10 = 3$
6	$19/x_10 = 3$
7	$21/x_10 = 3$
8	$23/x_10 = 3$
9	$25/x_10 = 3$
10	$27/x_10 = 3$
11	$29/x_10 = 3$
12	$31/x_10 = 3$
13	$33/x_10 = 3$
14	$35/x_10 = 3$
15	$37/x_10 = 3$
16	$39/x_10 = 3$
17	$41/x_10 = 3$
18	$43/x_10 = 3$
19	$45/x_10 = 3$
20	$47/x_10 = 3$

Optimal Solution

Z = 47
$x_1 1 = 3$
$x_1 0 = 0$
$x_9 = 2$
$x_8 = 0$
$x_7 = 0$
$x_6 = 0$
$x_5 = 0$
$x_4 = 0$
$x_3 = 0$
$x_2 = 9$
$x_1 = 3$

Optimal Solution

Z = 47
$x_1 1 = 3$
$x_1 0 = 0$
$x_9 = 2$
$x_8 = 0$
$x_7 = 5$
$x_6 = 0$
$x_5 = 0$
$x_4 = 0$
$x_3 = 0$
$x_2 = 5$
$x_1 = 0$

Optimal Solution

Z = 47
$x_1 1 = 3$
$x_1 0 = 5$
$x_9 = 2$
$x_8 = 0$
$x_7 = 0$
$x_6 = 0$
$x_5 = 0$
$x_4 = 0$
$x_3 = 0$
$x_2 = 0$
$x_1 = 5$

Optimal Solution

$Z = 47$
$x_1 1 = 3$
$x_1 0 = 5$
$x_9 = 2$
$x_8 = 0$
$x_7 = 0$
$x_6 = 0$
$x_5 = 0$
$x_4 = 0$
$x_3 = 0$
$x_2 = 10$
$x_1 = 0$

Optimal Solution

Z = 47
$x_1 1 = 3$
$x_1 0 = 5$
$x_9 = 2$
$x_8 = 0$
$x_7 = 5$
$x_6 = 0$
$x_5 = 0$
$x_4 = 0$
$x_3 = 0$
$x_2 = 0$
$x_1 = 0$