Instituto Tecnológico de Costa Rica

Operations Research - Semester II

Knapsack Problem

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The Knapsack Problem

The **Knapsack Problem** is a classic optimization problem in computer science and operations research. It consists of having a set of items, each with a weight and a value. We also have a knapsack with limited capacity. The goal is to select a subset of items so that the total weight does not exceed the capacity of the knapsack, while the total value gained is maximized.

$$\max(Z) = \sum x_i v_i$$

$$\text{subject to} \quad \sum x_i c_i \leq C$$

- Z: gained value (we want to maximize it)
- C: knapsack capacity
- v_i: value of item i
- c_i: cost of item i
- x_i: amount of item i taken

0/1 Knapsack Problem

In the **0/1 Knapsack Problem**, items exist only once. We can either take it or leave it. We cannot take multiple of the same item.

Bounded Knapsack Problem

In the **Bounded Knapsack Problem**, we can take multiple copies of each item. The items are limited and we have to choose how many to take of each.

Unbounded Knapsack Problem

In the **Unbounded Knapsack Problem**, there are infinite copies of each item. We can take as much as we please of any item.

Knapsack algorithm

- We have k_i copies of item i
- We can include up to Q copies of item i

$$Q = \min(k_j, \left\lfloor \frac{i}{c_j} \right\rfloor)$$

- Now we fill up a $C \times k_i$ sized table like this:

$$\begin{aligned}\text{Table}[i][j] = \max \Big(& \text{Table}[i][j-1], \\ & 1 \cdot v_i + \text{Table}[i - 1 \cdot c_i][j-1], \\ & 2 \cdot v_i + \text{Table}[i - 2 \cdot c_i][j-1], \\ & \dots, \\ & Q \cdot v_i + \text{Table}[i - Q \cdot c_i][j-1] \Big)\end{aligned}$$

- In each cell we save the maximum possible value, and the amount of the item we should take to get it.

Problem

Maximize

$$Z = 5x_1 + 8x_2 + 6x_3 + 16x_4 + 16x_5 + 15x_6 + 3x_7 + 12x_8 + 50x_9 + 27x_10$$

Subject to

$$3x_1 + 4x_2 + 4x_3 + 12x_4 + 8x_5 + 12x_6 + 2x_7 + 6x_8 + 38x_9 + 20x_10 \leq 20$$
$$0 \leq x_1$$
$$0 \leq x_2$$
$$0 \leq x_3$$
$$0 \leq x_4$$
$$0 \leq x_5$$
$$0 \leq x_6$$
$$0 \leq x_7$$
$$0 \leq x_8$$
$$0 \leq x_9$$
$$0 \leq x_10$$

Data Table

	DASD	Basdg	Tsdsd	Hasd	Kaygsd	Iahsbd	ajaYYs
0	$0/x_0 = 0$	$0/x_1 = 0$	$0/x_2 = 0$	$0/x_3 = 0$	$0/x_4 = 0$	$0/x_5 = 0$	$0/x_6 = 0$
1	$0/x_0 = 0$	$0/x_1 = 0$	$0/x_2 = 0$	$0/x_3 = 0$	$0/x_4 = 0$	$0/x_5 = 0$	$0/x_6 = 0$
2	$0/x_0 = 0$	$0/x_1 = 0$	$0/x_2 = 0$	$0/x_3 = 0$	$0/x_4 = 0$	$0/x_5 = 0$	$3/x_6 = 1$
3	$5/x_0 = 1$	$5/x_1 = 0$	$5/x_2 = 0$	$5/x_3 = 0$	$5/x_4 = 0$	$5/x_5 = 0$	$5/x_6 = 0$
4	$5/x_0 = 1$	$8/x_1 = 1$	$8/x_2 = 0$	$8/x_3 = 0$	$8/x_4 = 0$	$8/x_5 = 0$	$8/x_6 = 0$
5	$5/x_0 = 1$	$8/x_1 = 1$	$8/x_2 = 0$	$8/x_3 = 0$	$8/x_4 = 0$	$8/x_5 = 0$	$8/x_6 = 1 - 0$
6	$10/x_0 = 2$	$10/x_1 = 0$	$10/x_2 = 0$	$10/x_3 = 0$	$10/x_4 = 0$	$10/x_5 = 0$	$11/x_6 = 1$
7	$10/x_0 = 2$	$13/x_1 = 1$	$13/x_2 = 0$	$13/x_3 = 0$	$13/x_4 = 0$	$13/x_5 = 0$	$13/x_6 = 0$
8	$10/x_0 = 2$	$16/x_1 = 2$	$16/x_2 = 0$	$16/x_3 = 0$	$16/x_4 = 1 - 0$	$16/x_5 = 0$	$16/x_6 = 0$
9	$15/x_0 = 3$	$16/x_1 = 2$	$16/x_2 = 0$	$16/x_3 = 0$	$16/x_4 = 1 - 0$	$16/x_5 = 0$	$16/x_6 = 1 - 0$
10	$15/x_0 = 3$	$18/x_1 = 1$	$18/x_2 = 0$	$18/x_3 = 0$	$18/x_4 = 0$	$18/x_5 = 0$	$19/x_6 = 1$
11	$15/x_0 = 3$	$21/x_1 = 2$	$21/x_2 = 0$	$21/x_3 = 0$	$21/x_4 = 1 - 0$	$21/x_5 = 0$	$21/x_6 = 0$
12	$20/x_0 = 4$	$24/x_1 = 3$	$24/x_2 = 0$	$24/x_3 = 0$	$24/x_4 = 1 - 0$	$24/x_5 = 0$	$24/x_6 = 0$
13	$20/x_0 = 4$	$24/x_1 = 3$	$24/x_2 = 0$	$24/x_3 = 0$	$24/x_4 = 1 - 0$	$24/x_5 = 0$	$24/x_6 = 1 - 0$
14	$20/x_0 = 4$	$26/x_1 = 2$	$26/x_2 = 0$	$26/x_3 = 0$	$26/x_4 = 1 - 0$	$26/x_5 = 0$	$27/x_6 = 1$
15	$25/x_0 = 5$	$29/x_1 = 3$	$29/x_2 = 0$	$29/x_3 = 0$	$29/x_4 = 1 - 0$	$29/x_5 = 0$	$29/x_6 = 0$
16	$25/x_0 = 5$	$32/x_1 = 4$	$32/x_2 = 0$	$32/x_3 = 0$	$32/x_4 = 2 - 0$	$32/x_5 = 0$	$32/x_6 = 0$
17	$25/x_0 = 5$	$32/x_1 = 4$	$32/x_2 = 0$	$32/x_3 = 0$	$32/x_4 = 2 - 0$	$32/x_5 = 0$	$32/x_6 = 1 - 0$
18	$30/x_0 = 6$	$34/x_1 = 3$	$34/x_2 = 0$	$34/x_3 = 0$	$34/x_4 = 1 - 0$	$34/x_5 = 0$	$35/x_6 = 1$
19	$30/x_0 = 6$	$37/x_1 = 4$	$37/x_2 = 0$	$37/x_3 = 0$	$37/x_4 = 2 - 0$	$37/x_5 = 0$	$37/x_6 = 0$
20	$30/x_0 = 6$	$40/x_1 = 5$	$40/x_2 = 0$	$40/x_3 = 0$	$40/x_4 = 2 - 0$	$40/x_5 = 0$	$40/x_6 = 0$

Optimal Solution

Z = 40
$x_1 0 = 0$
$x_9 = 0$
$x_8 = 0$
$x_7 = 0$
$x_6 = 0$
$x_5 = 0$
$x_4 = 0$
$x_3 = 0$
$x_2 = 5$
$x_1 = 0$

Optimal Solution

Z = 40
$x_1 0 = 0$
$x_9 = 0$
$x_8 = 0$
$x_7 = 0$
$x_6 = 0$
$x_5 = 2$
$x_4 = 0$
$x_3 = 0$
$x_2 = 1$
$x_1 = 0$

Optimal Solution

Z = 40
$x_1 0 = 0$
$x_9 = 0$
$x_8 = 2$
$x_7 = 0$
$x_6 = 0$
$x_5 = 0$
$x_4 = 0$
$x_3 = 0$
$x_2 = 2$
$x_1 = 0$

Optimal Solution

Z = 40
$x_1 0 = 0$
$x_9 = 0$
$x_8 = 2$
$x_7 = 0$
$x_6 = 0$
$x_5 = 1$
$x_4 = 0$
$x_3 = 0$
$x_2 = 0$
$x_1 = 0$