Instituto Tecnológico de Costa Rica

Operations Research - Semester II

Knapsack Problem

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Multiple

Maximize

$$Z = 4.00x_1 + 14.00x_2$$

Subject to

$$2.00x_1 + 7.00x_2 \leq 21.00$$
$$7.00x_1 + 2.00x_2 \leq 21.00$$

Simplex Table

	Z	x_1	x_2	s_1	s_2	b
	1.00	-4.00	-14.00	0.00	0.00	0.00
	0.00	2.00	7.00	1.00	0.00	21.00
	0.00	7.00	2.00	0.00	1.00	21.00

Intermediate Tables

Pivoting 1

Most Negative

Column 3 (-14.00)

Z	x_1	x_2	s_1	s_2	b
1.00	-4.00	-14.00	0.00	0.00	0.00
0.00	2.00	7.00	1.00	0.00	21.00
0.00	7.00	2.00	0.00	1.00	21.00

Fractions

Z	x_1	x_2	s_1	s_2	b	Frac
1.00	-4.00	-14.00	0.00	0.00	0.00	
0.00	2.00	7.00	1.00	0.00	21.00	3.00
0.00	7.00	2.00	0.00	1.00	21.00	10.50

$21.00/7.00 = 3.00$
$21.00/2.00 = 10.50$

Pivot

Z	x_1	x_2	s_1	s_2	b
1.00	-4.00	-14.00	0.00	0.00	0.00
0.00	2.00	7.00	1.00	0.00	21.00
0.00	7.00	2.00	0.00	1.00	21.00

Canonization

$R_2 \leftarrow R_2/7.00$

Z	x_1	x_2	s_1	s_2	b
1.00	-4.00	-14.00	0.00	0.00	0.00
0.00	0.29	1.00	0.14	0.00	3.00
0.00	7.00	2.00	0.00	1.00	21.00

$R_1 \leftarrow R_1 + 14.00 R_2$

Z	x_1	x_2	s_1	s_2	b
1.00	0.00	0.00	2.00	0.00	42.00
0.00	0.29	1.00	0.14	0.00	3.00
0.00	7.00	2.00	0.00	1.00	21.00

$R_3 \leftarrow R_3 + -2.00R_2$

Z	x_1	x_2	s_1	s_2	b
1.00	0.00	0.00	2.00	0.00	42.00
0.00	0.29	1.00	0.14	0.00	3.00
0.00	6.43	0.00	-0.29	1.00	15.00

Pivot Result

Z	x_1	x_2	s_1	s_2	b
1.00	0.00	0.00	2.00	0.00	42.00
0.00	0.29	1.00	0.14	0.00	3.00
0.00	6.43	0.00	-0.29	1.00	15.00

Results

	Z	x_1	x_2	s_1	s_2	b
	1.00	0.00	0.00	2.00	0.00	42.00
	0.00	0.29	1.00	0.14	0.00	3.00
	0.00	6.43	0.00	-0.29	1.00	15.00

Objective Value

Z = 42.00

Variables

$x_1 = 0.00$
$x_2 = 3.00$

Slack or Surplus

$s_1 = 0.00$
$s_2 = 15.00$

Multiple Solutions

A basic variable has a 0 on its first row, allowing us to pivot again and find another optimal solution.

Pivoting 2

Column 2 (0.00)

Z	x_1	x_2	s_1	s_2	b
1.00	0.00	0.00	2.00	0.00	42.00
0.00	0.29	1.00	0.14	0.00	3.00
0.00	6.43	0.00	-0.29	1.00	15.00

Fractions

Z	x_1	x_2	s_1	s_2	b	Frac
1.00	0.00	0.00	2.00	0.00	42.00	
0.00	0.29	1.00	0.14	0.00	3.00	10.50
0.00	6.43	0.00	-0.29	1.00	15.00	2.33

$3.00/0.29 = 10.50$
$15.00/6.43 = 2.33$

Pivot

Z	x_1	x_2	s_1	s_2	b
1.00	0.00	0.00	2.00	0.00	42.00
0.00	0.29	1.00	0.14	0.00	3.00
0.00	6.43	0.00	-0.29	1.00	15.00

Canonization

$R_3 \leftarrow R_3/6.43$

Z	x_1	x_2	s_1	s_2	b
1.00	0.00	0.00	2.00	0.00	42.00
0.00	0.29	1.00	0.14	0.00	3.00
0.00	1.00	0.00	-0.04	0.16	2.33

$R_1 \leftarrow R_1 + -0.00R_3$

Z	x_1	x_2	s_1	s_2	b
1.00	0.00	0.00	2.00	0.00	42.00
0.00	0.29	1.00	0.14	0.00	3.00
0.00	1.00	0.00	-0.04	0.16	2.33

$R_2 \leftarrow R_2 + -0.29R_3$

Z	x_1	x_2	s_1	s_2	b
1.00	0.00	0.00	2.00	0.00	42.00
0.00	0.00	1.00	0.16	-0.04	2.33
0.00	1.00	0.00	-0.04	0.16	2.33

Pivot Result

Z	x_1	x_2	s_1	s_2	b
1.00	0.00	0.00	2.00	0.00	42.00
0.00	0.00	1.00	0.16	-0.04	2.33
0.00	1.00	0.00	-0.04	0.16	2.33

Results

	Z	x_1	x_2	s_1	s_2	b
	1.00	0.00	0.00	2.00	0.00	42.00
	0.00	0.00	1.00	0.16	-0.04	2.33
	0.00	1.00	0.00	-0.04	0.16	2.33

Objective Value

Z = 42.00

Variables

$x_1 = 2.33$
$x_2 = 2.33$

Slack or Surplus

$s_1 = 0.00$
$s_2 = 0.00$