

Instituto Tecnológico de Costa Rica

Operations Research - Semester II

Equipment Replacement Problem

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Equipment Replacement Problem

The equipment replacement problem is an optimization problem that involves deciding the optimal time to replace a piece of equipment (sell and re-buy) to minimize business costs. There are four main components to this problem:

- The project term (e.g. 5 years long project)
- Life span of the equipment
- Price of new equipment
- Maintenance costs (could increase as equipment ages)
- Sell prices (e.g. 4 year old equipment can be sold for \$50)

The problem involves determining the optimal number of equipment replacements and the timing of each, in order to minimize operating costs.

Given the following variables:

- t_{max} : Project term
- t_{span} : Lifespan of equipment
- t : A given instant through the project term.
- C_{tx} : The cost of buying at instant t and selling at x
 - Cost of new equipment + maintenance costs - sell price - profits

$G(t)$ = optimal business costs from t to t_{max} (Buying new equipment at t).

0.1 Trivial case

$$G(t_{max}) = 0$$

- From instant t_{max} to instant t_{max}
- We don't buy any equipment

0.2 Optimization

If new equipment is bought at t we can sell it and buy new equipment at either $t + 1$, $t + 2$, ..., or $t + t_{span}$. We should sell at the point where we get the least operation cost given the project term.

0.3 Bellman Equation

Knowing this, we can establish the Bellman equation:

$$G(t) = \min\{C_{tx} + G(x)\} \tag{1}$$

From $t = t_{max}$ until $t = 0$.

Equipment Life Cycle Costs

Year	Maintenance Cost	Resale Price
1	1	2
2	2	3
3	4	5

C_{tx} Calculations

1 year(s): $C_{t,t+1} = 500 + 1 - 2 = 499$

2 year(s): $C_{t,t+2} = 500 + 1 + 2 - 3 = 500$

3 year(s): $C_{t,t+3} = 500 + 1 + 2 + 4 - 5 = 502$

C_{tx} Table

t/x	1	2	3	4	5
0	499	500	502	—	—
1	—	499	500	502	—
2	—	—	499	500	502
3	—	—	—	499	500
4	—	—	—	—	499

Step by Step Calculations

$G(5) = 0$ (Base case)

$$G(4) = \min\{C_{45} + G(5) = (500 + 1 - 2) + 0 = 499\} = 499$$

Optimal choice: sell at time 5

$$G(3) = \min\{C_{34} + G(4) = (500 + 1 - 2) + 499 = 998, C_{35} + G(5) = (500 + 1 + 2 - 3) + 0 = 500\} = 500$$

Optimal choice: sell at time 5

$$G(2) = \min\{C_{23} + G(3) = (500 + 1 - 2) + 500 = 999, C_{24} + G(4) = (500 + 1 + 2 - 3) + 499 = 999, C_{25} + G(5) = (500 + 1 + 2 + 4 - 5) + 0 = 502\} = 502$$

Optimal choice: sell at time 5

$$G(1) = \min\{C_{12} + G(2) = (500 + 1 - 2) + 502 = 1001, C_{13} + G(3) = (500 + 1 + 2 - 3) + 500 = 1000, C_{14} + G(4) = (500 + 1 + 2 + 4 - 5) + 499 = 1001\} = 1000$$

Optimal choice: sell at time 3

$$G(0) = \min\{C_{01} + G(1) = (500 + 1 - 2) + 1000 = 1499, C_{02} + G(2) = (500 + 1 + 2 - 3) + 502 = 1002, C_{03} + G(3) = (500 + 1 + 2 + 4 - 5) + 500 = 1002\} = 1002$$

Multiple optimal choices: sell at time 2, sell at time 3 (tie)

Result Table (Analysis table)

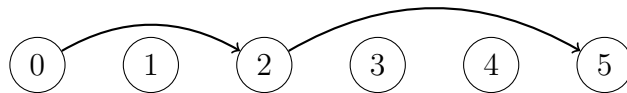
t	G(t)	Next Replacement
0	1002	2, 3
1	1000	3
2	502	5
3	500	5
4	499	5
5	0	—

Optimal Solution

Minimum Total Cost: 1002

All Optimal Replacement Plans:

- **Plan 1:** Buy at 0, sell at 2 → Buy at 2, sell at 5



- **Plan 2:** Buy at 0, sell at 3 → Buy at 3, sell at 5

