

# **Instituto Tecnológico de Costa Rica**

**Operations Research - Semester II**

## **Equipment Replacement Problem**

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# Equipment Replacement Problem

**Note:** This problem includes inflation consideration with a constant rate of 2.00% per time unit.

The equipment replacement problem is an optimization problem that involves deciding the optimal time to replace a piece of equipment (sell and re-buy) to minimize business costs. There are four main components to this problem:

- The project term (e.g. 5 years long project)
- Life span of the equipment
- Price of new equipment
- Maintenance costs (could increase as equipment ages)
- Sell prices (e.g. 4 year old equipment can be sold for \$50)

The problem involves determining the optimal number of equipment replacements and the timing of each, in order to minimize operating costs.

Given the following variables:

- $t_{max}$ : Project term
- $t_{span}$ : Lifes span of equipment
- $t$ : A given instant through the project term.
- $C_{tx}$ : The cost of buying at instant  $t$  and selling at  $x$ 
  - Cost of new equipment + maintenance costs - sell price - profits

$G(t)$  = optimal business costs from  $t$  to  $t_{max}$  (Buying new equipment at  $t$ ).

## 0.1 Trivial case

$$G(t_{max}) = 0$$

- From instant  $t_{max}$  to instant  $t_{max}$
- We don't buy any equipment

## 0.2 Optimization

If new equipment is bought at  $t$  we can sell it and buy new equipment at either  $t + 1$ ,  $t + 2$  , ..., or  $t + t_{span}$ . We should sell at the point where we get the least operation cost given the project term.

## 0.3 Bellman Equation

Knowing this, we can establish the Bellman equation:

$$G(t) = \min\{C_{tx} + G(x)\} \quad (1)$$

From  $t = t_{max}$  until  $t = 0$ .

## Equipment Life Cycle Costs

Year	Maintenance Cost	Resale Price	Profit
1	120	2000	80
2	180	1800	90
3	240	1600	75
4	300	1400	65
5	360	1200	55
6	420	1000	45
7	480	900	40
8	540	800	35
9	600	700	30
10	660	600	25

## Problem Parameters

- **Initial Equipment Cost:** 2500
- **Project Term:** 30 years
- **Equipment Lifespan:** 10 years
- **Inflation Rate:** 2.00% per time unit
- **Note:** All costs and profits will be inflated according to the time period when they occur.

## $C_{tx}$ Calculations

(with 2.00% inflation)

**Inflation Formula:** Value at time  $t$  = Base Value  $\times (1 + 0.0200)^t$

$$\text{Equipment bought at time 0: } C_{01} = 2500 + 120 \times (1 + 0.0200)^1 - 2000 \times (1 + 0.0200)^1 - 80 \times (1 + 0.0200)^1 = 500$$

$$C_{02} = 2500 + 120 \times (1 + 0.0200)^1 + 180 \times (1 + 0.0200)^2 - 1800 \times (1 + 0.0200)^2 - 80 \times (1 + 0.0200)^1 - 90 \times (1 + 0.0200)^2 = 760$$

$$C_{03} = 2500 + 120 \times (1 + 0.0200)^1 + 180 \times (1 + 0.0200)^2 + 240 \times (1 + 0.0200)^3 - 1600 \times (1 + 0.0200)^3 - 80 \times (1 + 0.0200)^1 - 90 \times (1 + 0.0200)^2 - 75 \times (1 + 0.0200)^3 = 1109$$

$$C_{04} = 2500 + 120 \times (1 + 0.0200)^1 + 180 \times (1 + 0.0200)^2 + 240 \times (1 + 0.0200)^3 + 300 \times (1 + 0.0200)^4 - 1400 \times (1 + 0.0200)^4 - 80 \times (1 + 0.0200)^1 - 90 \times (1 + 0.0200)^2 - 75 \times (1 + 0.0200)^3 - 65 \times (1 + 0.0200)^4 = 1544$$

$$C_{05} = 2500 + 120 \times (1 + 0.0200)^1 + 180 \times (1 + 0.0200)^2 + 240 \times (1 + 0.0200)^3 + 300 \times (1 + 0.0200)^4 + 360 \times (1 + 0.0200)^5 - 1200 \times (1 + 0.0200)^5 - 80 \times (1 + 0.0200)^1 - 90 \times (1 + 0.0200)^2 - 75 \times (1 + 0.0200)^3 - 65 \times (1 + 0.0200)^4 - 55 \times (1 + 0.0200)^5 = 2071$$

$$C_{06} = 2500 + 120 \times (1 + 0.0200)^1 + 180 \times (1 + 0.0200)^2 + 240 \times (1 + 0.0200)^3 + 300 \times (1 + 0.0200)^4 + 360 \times (1 + 0.0200)^5 + 420 \times (1 + 0.0200)^6 - 1000 \times (1 + 0.0200)^6 - 80 \times (1 + 0.0200)^1 - 90 \times (1 + 0.0200)^2 - 75 \times (1 + 0.0200)^3 - 65 \times (1 + 0.0200)^4 - 55 \times (1 + 0.0200)^5 - 45 \times (1 + 0.0200)^6 = 2690$$

$$C_{07} = 2500 + 120 \times (1 + 0.0200)^1 + 180 \times (1 + 0.0200)^2 + 240 \times (1 + 0.0200)^3 + 300 \times (1 + 0.0200)^4 + 360 \times (1 + 0.0200)^5 + 420 \times (1 + 0.0200)^6 + 480 \times (1 + 0.0200)^7 - 900 \times$$

$$(1 + 0.0200)^7 - 80 \times (1 + 0.0200)^1 - 90 \times (1 + 0.0200)^2 - 75 \times (1 + 0.0200)^3 - 65 \times (1 + 0.0200)^4 - 55 \times (1 + 0.0200)^5 - 45 \times (1 + 0.0200)^6 - 40 \times (1 + 0.0200)^7 = 3288$$

$$C_{08} = 2500 + 120 \times (1+0.0200)^1 + 180 \times (1+0.0200)^2 + 240 \times (1+0.0200)^3 + 300 \times (1+0.0200)^4 + 360 \times (1+0.0200)^5 + 420 \times (1+0.0200)^6 + 480 \times (1+0.0200)^7 + 540 \times (1+0.0200)^8 - 800 \times (1+0.0200)^9 - 80 \times (1+0.0200)^1 - 90 \times (1+0.0200)^2 - 75 \times (1+0.0200)^3 - 65 \times (1+0.0200)^4 - 55 \times (1+0.0200)^5 - 45 \times (1+0.0200)^6 - 40 \times (1+0.0200)^7 - 35 \times (1+0.0200)^8 = 3974$$

$$C_{09} = 2500 + 120 \times (1 + 0.0200)^1 + 180 \times (1 + 0.0200)^2 + 240 \times (1 + 0.0200)^3 + 300 \times (1 + 0.0200)^4 + 360 \times (1 + 0.0200)^5 + 420 \times (1 + 0.0200)^6 + 480 \times (1 + 0.0200)^7 + 540 \times (1 + 0.0200)^8 + 600 \times (1 + 0.0200)^9 - 700 \times (1 + 0.0200)^9 - 80 \times (1 + 0.0200)^1 - 90 \times (1 + 0.0200)^2 - 75 \times (1 + 0.0200)^3 - 65 \times (1 + 0.0200)^4 - 55 \times (1 + 0.0200)^5 - 45 \times (1 + 0.0200)^6 - 40 \times (1 + 0.0200)^7 - 35 \times (1 + 0.0200)^8 - 30 \times (1 + 0.0200)^9 = 4756$$

$$C_{010} = 2500 + 120 \times (1+0.0200)^1 + 180 \times (1+0.0200)^2 + 240 \times (1+0.0200)^3 + 300 \times (1+0.0200)^4 + 360 \times (1+0.0200)^5 + 420 \times (1+0.0200)^6 + 480 \times (1+0.0200)^7 + 540 \times (1+0.0200)^8 + 600 \times (1+0.0200)^9 + 660 \times (1+0.0200)^{10} - 600 \times (1+0.0200)^{10} - 80 \times (1+0.0200)^1 - 90 \times (1+0.0200)^2 - 75 \times (1+0.0200)^3 - 65 \times (1+0.0200)^4 - 55 \times (1+0.0200)^5 - 45 \times (1+0.0200)^6 - 40 \times (1+0.0200)^7 - 35 \times (1+0.0200)^8 - 30 \times (1+0.0200)^9 - 25 \times (1+0.0200)^{10} = 5634$$

**Equipment bought at time 1:**  $C_{12} = 2500 \times (1 + 0.0200)^1 + 120 \times (1 + 0.0200)^2 - 2000 \times (1 + 0.0200)^2 - 80 \times (1 + 0.0200)^2 = 509$

$$C_{13} = 2500 \times (1 + 0.0200)^1 + 120 \times (1 + 0.0200)^2 + 180 \times (1 + 0.0200)^3 - 1800 \times (1 + 0.0200)^3 - 80 \times (1 + 0.0200)^2 - 90 \times (1 + 0.0200)^3 = 774$$

$$C_{14} = 2500 \times (1+0.0200)^1 + 120 \times (1+0.0200)^2 + 180 \times (1+0.0200)^3 + 240 \times (1+0.0200)^4 - 1600 \times (1+0.0200)^4 - 80 \times (1+0.0200)^2 - 90 \times (1+0.0200)^3 - 75 \times (1+0.0200)^4 = 1130$$

$$C_{15} = 2500 \times (1 + 0.0200)^1 + 120 \times (1 + 0.0200)^2 + 180 \times (1 + 0.0200)^3 + 240 \times (1 + 0.0200)^4 + 300 \times (1 + 0.0200)^5 - 1400 \times (1 + 0.0200)^5 - 80 \times (1 + 0.0200)^2 - 90 \times (1 + 0.0200)^3 - 75 \times (1 + 0.0200)^4 - 65 \times (1 + 0.0200)^5 = 1575$$

$$C_{16} = 2500 \times (1 + 0.0200)^1 + 120 \times (1 + 0.0200)^2 + 180 \times (1 + 0.0200)^3 + 240 \times (1 + 0.0200)^4 + 300 \times (1 + 0.0200)^5 + 360 \times (1 + 0.0200)^6 - 1200 \times (1 + 0.0200)^6 - 80 \times (1 + 0.0200)^2 - 90 \times (1 + 0.0200)^3 - 75 \times (1 + 0.0200)^4 - 65 \times (1 + 0.0200)^5 - 55 \times (1 + 0.0200)^6 = 2112$$

$$C_{17} = 2500 \times (1 + 0.0200)^1 + 120 \times (1 + 0.0200)^2 + 180 \times (1 + 0.0200)^3 + 240 \times (1 + 0.0200)^4 + 300 \times (1 + 0.0200)^5 + 360 \times (1 + 0.0200)^6 + 420 \times (1 + 0.0200)^7 - 1000 \times (1 + 0.0200)^7 - 80 \times (1 + 0.0200)^2 - 90 \times (1 + 0.0200)^3 - 75 \times (1 + 0.0200)^4 - 65 \times (1 + 0.0200)^5 - 55 \times (1 + 0.0200)^6 - 45 \times (1 + 0.0200)^7 = 2745$$

$$C_{18} = 2500 \times (1 + 0.0200)^1 + 120 \times (1 + 0.0200)^2 + 180 \times (1 + 0.0200)^3 + 240 \times (1 + 0.0200)^4 + 300 \times (1 + 0.0200)^5 + 360 \times (1 + 0.0200)^6 + 420 \times (1 + 0.0200)^7 + 480 \times (1 + 0.0200)^8 - 900 \times (1 + 0.0200)^8 - 80 \times (1 + 0.0200)^2 - 90 \times (1 + 0.0200)^3 - 75 \times (1 + 0.0200)^4 - 65 \times (1 + 0.0200)^5 - 55 \times (1 + 0.0200)^6 - 45 \times (1 + 0.0200)^7 - 40 \times (1 + 0.0200)^8 = 3354$$

$$C_{19} = 2500 \times (1+0.0200)^1 + 120 \times (1+0.0200)^2 + 180 \times (1+0.0200)^3 + 240 \times (1+0.0200)^4 + 300 \times (1+0.0200)^5 + 360 \times (1+0.0200)^6 + 420 \times (1+0.0200)^7 + 480 \times (1+0.0200)^8 + 540 \times (1+0.0200)^9 - 800 \times (1+0.0200)^{10} - 80 \times (1+0.0200)^2 - 90 \times (1+0.0200)^3 - 75 \times (1+0.0200)^4 - 65 \times (1+0.0200)^5 - 55 \times (1+0.0200)^6 - 45 \times (1+0.0200)^7 - 40 \times (1+0.0200)^8 - 35 \times (1+0.0200)^9 = 4055$$

$$C_{110} = 2500 \times (1 + 0.0200)^1 + 120 \times (1 + 0.0200)^2 + 180 \times (1 + 0.0200)^3 + 240 \times (1 + 0.0200)^4 + 300 \times (1 + 0.0200)^5 + 360 \times (1 + 0.0200)^6 + 420 \times (1 + 0.0200)^7 + 480 \times (1 + 0.0200)^8 + 540 \times (1 + 0.0200)^9 + 600 \times (1 + 0.0200)^{10} - 700 \times (1 + 0.0200)^{10} - 80 \times (1 + 0.0200)^2 - 90 \times (1 + 0.0200)^3 - 75 \times (1 + 0.0200)^4 - 65 \times (1 + 0.0200)^5 - 55 \times (1 + 0.0200)^6 - 45 \times (1 + 0.0200)^7 - 40 \times (1 + 0.0200)^8 - 35 \times (1 + 0.0200)^9 - 30 \times (1 + 0.0200)^{10} = 4852$$

$$C_{111} = 2500 \times (1+0.0200)^1 + 120 \times (1+0.0200)^2 + 180 \times (1+0.0200)^3 + 240 \times (1+0.0200)^4 + 300 \times (1+0.0200)^5 + 360 \times (1+0.0200)^6 + 420 \times (1+0.0200)^7 + 480 \times (1+0.0200)^8 + 540 \times (1+0.0200)^9$$

$$0.0200)^9 + 600 \times (1+0.0200)^{10} + 660 \times (1+0.0200)^{11} - 600 \times (1+0.0200)^{11} - 80 \times (1+0.0200)^2 - 90 \times (1+0.0200)^3 - 75 \times (1+0.0200)^4 - 65 \times (1+0.0200)^5 - 55 \times (1+0.0200)^6 - 45 \times (1+0.0200)^7 - 40 \times (1+0.0200)^8 - 35 \times (1+0.0200)^9 - 30 \times (1+0.0200)^{10} - 25 \times (1+0.0200)^{11} = 5747$$

**Equipment bought at time 2:**  $C_{23} = 2500 \times (1+0.0200)^2 + 120 \times (1+0.0200)^3 - 2000 \times (1+0.0200)^3 - 80 \times (1+0.0200)^3 = 520$

$$C_{24} = 2500 \times (1+0.0200)^2 + 120 \times (1+0.0200)^3 + 180 \times (1+0.0200)^4 - 1800 \times (1+0.0200)^4 - 80 \times (1+0.0200)^3 - 90 \times (1+0.0200)^4 = 790$$

$$C_{25} = 2500 \times (1+0.0200)^2 + 120 \times (1+0.0200)^3 + 180 \times (1+0.0200)^4 + 240 \times (1+0.0200)^5 - 1600 \times (1+0.0200)^5 - 80 \times (1+0.0200)^3 - 90 \times (1+0.0200)^4 - 75 \times (1+0.0200)^5 = 1153$$

$$C_{26} = 2500 \times (1+0.0200)^2 + 120 \times (1+0.0200)^3 + 180 \times (1+0.0200)^4 + 240 \times (1+0.0200)^5 + 300 \times (1+0.0200)^6 - 1400 \times (1+0.0200)^6 - 80 \times (1+0.0200)^3 - 90 \times (1+0.0200)^4 - 75 \times (1+0.0200)^5 - 65 \times (1+0.0200)^6 = 1606$$

$$C_{27} = 2500 \times (1+0.0200)^2 + 120 \times (1+0.0200)^3 + 180 \times (1+0.0200)^4 + 240 \times (1+0.0200)^5 + 300 \times (1+0.0200)^6 + 360 \times (1+0.0200)^7 - 1200 \times (1+0.0200)^7 - 80 \times (1+0.0200)^3 - 90 \times (1+0.0200)^4 - 75 \times (1+0.0200)^5 - 65 \times (1+0.0200)^6 - 55 \times (1+0.0200)^7 = 2153$$

$$C_{28} = 2500 \times (1+0.0200)^2 + 120 \times (1+0.0200)^3 + 180 \times (1+0.0200)^4 + 240 \times (1+0.0200)^5 + 300 \times (1+0.0200)^6 + 360 \times (1+0.0200)^7 + 420 \times (1+0.0200)^8 - 1000 \times (1+0.0200)^8 - 80 \times (1+0.0200)^3 - 90 \times (1+0.0200)^4 - 75 \times (1+0.0200)^5 - 65 \times (1+0.0200)^6 - 55 \times (1+0.0200)^7 - 45 \times (1+0.0200)^8 = 2799$$

$$C_{29} = 2500 \times (1+0.0200)^2 + 120 \times (1+0.0200)^3 + 180 \times (1+0.0200)^4 + 240 \times (1+0.0200)^5 + 300 \times (1+0.0200)^6 + 360 \times (1+0.0200)^7 + 420 \times (1+0.0200)^8 + 480 \times (1+0.0200)^9 - 900 \times (1+0.0200)^9 - 80 \times (1+0.0200)^3 - 90 \times (1+0.0200)^4 - 75 \times (1+0.0200)^5 - 65 \times (1+0.0200)^6 - 55 \times (1+0.0200)^7 - 45 \times (1+0.0200)^8 - 40 \times (1+0.0200)^9 = 3420$$

$$C_{210} = 2500 \times (1+0.0200)^2 + 120 \times (1+0.0200)^3 + 180 \times (1+0.0200)^4 + 240 \times (1+0.0200)^5 + 300 \times (1+0.0200)^6 + 360 \times (1+0.0200)^7 + 420 \times (1+0.0200)^8 + 480 \times (1+0.0200)^9 + 540 \times (1+0.0200)^{10} - 800 \times (1+0.0200)^{10} - 80 \times (1+0.0200)^3 - 90 \times (1+0.0200)^4 - 75 \times (1+0.0200)^5 - 65 \times (1+0.0200)^6 - 55 \times (1+0.0200)^7 - 45 \times (1+0.0200)^8 - 40 \times (1+0.0200)^9 - 35 \times (1+0.0200)^{10} = 4135$$

$$C_{211} = 2500 \times (1+0.0200)^2 + 120 \times (1+0.0200)^3 + 180 \times (1+0.0200)^4 + 240 \times (1+0.0200)^5 + 300 \times (1+0.0200)^6 + 360 \times (1+0.0200)^7 + 420 \times (1+0.0200)^8 + 480 \times (1+0.0200)^9 + 540 \times (1+0.0200)^{10} + 600 \times (1+0.0200)^{11} - 700 \times (1+0.0200)^{11} - 80 \times (1+0.0200)^3 - 90 \times (1+0.0200)^4 - 75 \times (1+0.0200)^5 - 65 \times (1+0.0200)^6 - 55 \times (1+0.0200)^7 - 45 \times (1+0.0200)^8 - 40 \times (1+0.0200)^9 - 35 \times (1+0.0200)^{10} - 30 \times (1+0.0200)^{11} = 4948$$

$$C_{212} = 2500 \times (1+0.0200)^2 + 120 \times (1+0.0200)^3 + 180 \times (1+0.0200)^4 + 240 \times (1+0.0200)^5 + 300 \times (1+0.0200)^6 + 360 \times (1+0.0200)^7 + 420 \times (1+0.0200)^8 + 480 \times (1+0.0200)^9 + 540 \times (1+0.0200)^{10} + 600 \times (1+0.0200)^{11} + 660 \times (1+0.0200)^{12} - 600 \times (1+0.0200)^{12} - 80 \times (1+0.0200)^3 - 90 \times (1+0.0200)^4 - 75 \times (1+0.0200)^5 - 65 \times (1+0.0200)^6 - 55 \times (1+0.0200)^7 - 45 \times (1+0.0200)^8 - 40 \times (1+0.0200)^9 - 35 \times (1+0.0200)^{10} - 30 \times (1+0.0200)^{11} - 25 \times (1+0.0200)^{12} = 5863$$

**Equipment bought at time 3:**  $C_{34} = 2500 \times (1+0.0200)^3 + 120 \times (1+0.0200)^4 - 2000 \times (1+0.0200)^4 - 80 \times (1+0.0200)^4 = 530$

$$C_{35} = 2500 \times (1+0.0200)^3 + 120 \times (1+0.0200)^4 + 180 \times (1+0.0200)^5 - 1800 \times (1+0.0200)^5 - 80 \times (1+0.0200)^4 - 90 \times (1+0.0200)^5 = 805$$

$$C_{36} = 2500 \times (1+0.0200)^3 + 120 \times (1+0.0200)^4 + 180 \times (1+0.0200)^5 + 240 \times (1+0.0200)^6 - 1600 \times (1+0.0200)^6 - 80 \times (1+0.0200)^4 - 90 \times (1+0.0200)^5 - 75 \times (1+0.0200)^6 = 1176$$

$$C_{37} = 2500 \times (1+0.0200)^3 + 120 \times (1+0.0200)^4 + 180 \times (1+0.0200)^5 + 240 \times (1+0.0200)^6 + 300 \times (1+0.0200)^7 - 1400 \times (1+0.0200)^7 - 80 \times (1+0.0200)^4 - 90 \times (1+$$





$$0.0200)^{12} + 540 \times (1 + 0.0200)^{13} + 600 \times (1 + 0.0200)^{14} + 660 \times (1 + 0.0200)^{15} - 600 \times (1 + 0.0200)^{15} - 80 \times (1 + 0.0200)^6 - 90 \times (1 + 0.0200)^7 - 75 \times (1 + 0.0200)^8 - 65 \times (1 + 0.0200)^9 - 55 \times (1 + 0.0200)^{10} - 45 \times (1 + 0.0200)^{11} - 40 \times (1 + 0.0200)^{12} - 35 \times (1 + 0.0200)^{13} - 30 \times (1 + 0.0200)^{14} - 25 \times (1 + 0.0200)^{15} = 6221$$

**Equipment bought at time 6:**  $C_{67} = 2500 \times (1 + 0.0200)^6 + 120 \times (1 + 0.0200)^7 - 2000 \times (1 + 0.0200)^7 - 80 \times (1 + 0.0200)^7 = 562$

$$C_{68} = 2500 \times (1 + 0.0200)^6 + 120 \times (1 + 0.0200)^7 + 180 \times (1 + 0.0200)^8 - 1800 \times (1 + 0.0200)^8 - 80 \times (1 + 0.0200)^7 - 90 \times (1 + 0.0200)^8 = 855$$

$$C_{69} = 2500 \times (1+0.0200)^6 + 120 \times (1+0.0200)^7 + 180 \times (1+0.0200)^8 + 240 \times (1+0.0200)^9 - 1600 \times (1+0.0200)^9 - 80 \times (1+0.0200)^7 - 90 \times (1+0.0200)^8 - 75 \times (1+0.0200)^9 = 1247$$

$$C_{610} = 2500 \times (1 + 0.0200)^6 + 120 \times (1 + 0.0200)^7 + 180 \times (1 + 0.0200)^8 + 240 \times (1 + 0.0200)^9 + 300 \times (1 + 0.0200)^{10} - 1400 \times (1 + 0.0200)^{10} - 80 \times (1 + 0.0200)^7 - 90 \times (1 + 0.0200)^8 - 75 \times (1 + 0.0200)^9 - 65 \times (1 + 0.0200)^{10} = 1738$$

$$C_{611} = 2500 \times (1+0.0200)^6 + 120 \times (1+0.0200)^7 + 180 \times (1+0.0200)^8 + 240 \times (1+0.0200)^9 + 300 \times (1+0.0200)^{10} + 360 \times (1+0.0200)^{11} - 1200 \times (1+0.0200)^{11} - 80 \times (1+0.0200)^7 - 90 \times (1+0.0200)^8 - 75 \times (1+0.0200)^9 - 65 \times (1+0.0200)^{10} - 55 \times (1+0.0200)^{11} - 2230$$

$$C_{612} = 2500 \times (1 + 0.0200)^6 + 120 \times (1 + 0.0200)^7 + 180 \times (1 + 0.0200)^8 + 240 \times (1 + 0.0200)^9 + 300 \times (1 + 0.0200)^{10} + 360 \times (1 + 0.0200)^{11} + 420 \times (1 + 0.0200)^{12} - 1000 \times (1 + 0.0200)^{12} - 80 \times (1 + 0.0200)^7 - 90 \times (1 + 0.0200)^8 - 75 \times (1 + 0.0200)^9 - 65 \times (1 + 0.0200)^{10} - 55 \times (1 + 0.0200)^{11} - 45 \times (1 + 0.0200)^{12} = 3028$$

$$C_{613} = 2500 \times (1 + 0.0200)^6 + 120 \times (1 + 0.0200)^7 + 180 \times (1 + 0.0200)^8 + 240 \times (1 + 0.0200)^9 + 300 \times (1 + 0.0200)^{10} + 360 \times (1 + 0.0200)^{11} + 420 \times (1 + 0.0200)^{12} + 480 \times (1 + 0.0200)^{13} - 900 \times (1 + 0.0200)^{13} - 80 \times (1 + 0.0200)^7 - 90 \times (1 + 0.0200)^8 - 75 \times (1 + 0.0200)^9 - 65 \times (1 + 0.0200)^{10} - 55 \times (1 + 0.0200)^{11} - 45 \times (1 + 0.0200)^{12} - 40 \times (1 + 0.0200)^{13} = 3700$$

$$C_{614} = 2500 \times (1 + 0.0200)^6 + 120 \times (1 + 0.0200)^7 + 180 \times (1 + 0.0200)^8 + 240 \times (1 + 0.0200)^9 + 300 \times (1 + 0.0200)^{10} + 360 \times (1 + 0.0200)^{11} + 420 \times (1 + 0.0200)^{12} + 480 \times (1 + 0.0200)^{13} + 540 \times (1 + 0.0200)^{14} - 800 \times (1 + 0.0200)^{14} - 80 \times (1 + 0.0200)^7 - 90 \times (1 + 0.0200)^8 - 75 \times (1 + 0.0200)^9 - 65 \times (1 + 0.0200)^{10} - 55 \times (1 + 0.0200)^{11} - 45 \times (1 + 0.0200)^{12} - 40 \times (1 + 0.0200)^{13} - 35 \times (1 + 0.0200)^{14} = 4474$$

$$C_{615} = 2500 \times (1 + 0.0200)^6 + 120 \times (1 + 0.0200)^7 + 180 \times (1 + 0.0200)^8 + 240 \times (1 + 0.0200)^9 + 300 \times (1 + 0.0200)^{10} + 360 \times (1 + 0.0200)^{11} + 420 \times (1 + 0.0200)^{12} + 480 \times (1 + 0.0200)^{13} + 540 \times (1 + 0.0200)^{14} + 600 \times (1 + 0.0200)^{15} - 700 \times (1 + 0.0200)^{15} - 80 \times (1 + 0.0200)^7 - 90 \times (1 + 0.0200)^8 - 75 \times (1 + 0.0200)^9 - 65 \times (1 + 0.0200)^{10} - 55 \times (1 + 0.0200)^{11} - 45 \times (1 + 0.0200)^{12} - 40 \times (1 + 0.0200)^{13} - 35 \times (1 + 0.0200)^{14} - 30 \times (1 + 0.0200)^{15} = 5353$$

$$C_{616} = 2500 \times (1 + 0.0200)^6 + 120 \times (1 + 0.0200)^7 + 180 \times (1 + 0.0200)^8 + 240 \times (1 + 0.0200)^9 + 300 \times (1 + 0.0200)^{10} + 360 \times (1 + 0.0200)^{11} + 420 \times (1 + 0.0200)^{12} + 480 \times (1 + 0.0200)^{13} + 540 \times (1 + 0.0200)^{14} + 600 \times (1 + 0.0200)^{15} + 660 \times (1 + 0.0200)^{16} - 600 \times (1 + 0.0200)^{16} - 80 \times (1 + 0.0200)^7 - 90 \times (1 + 0.0200)^8 - 75 \times (1 + 0.0200)^9 - 65 \times (1 + 0.0200)^{10} - 55 \times (1 + 0.0200)^{11} - 45 \times (1 + 0.0200)^{12} - 40 \times (1 + 0.0200)^{13} - 35 \times (1 + 0.0200)^{14} - 30 \times (1 + 0.0200)^{15} - 25 \times (1 + 0.0200)^{16} = 6343$$

**Equipment bought at time 7:**  $C_{78} = 2500 \times (1 + 0.0200)^7 + 120 \times (1 + 0.0200)^8 - 2000 \times (1 + 0.0200)^8 - 80 \times (1 + 0.0200)^8 = 573$

$$C_{79} = 2500 \times (1 + 0.0200)^7 + 120 \times (1 + 0.0200)^8 + 180 \times (1 + 0.0200)^9 - 1800 \times (1 + 0.0200)^9 - 80 \times (1 + 0.0200)^8 - 90 \times (1 + 0.0200)^9 = 872$$

$$C_{710} = 2500 \times (1 + 0.0200)^7 + 120 \times (1 + 0.0200)^8 + 180 \times (1 + 0.0200)^9 + 240 \times (1 + 0.0200)^{10} - 1600 \times (1 + 0.0200)^{10} - 80 \times (1 + 0.0200)^8 - 90 \times (1 + 0.0200)^9 - 75 \times (1 + 0.0200)^{10} = 1273$$

$$C_{711} \equiv 2500 \times (1 + 0.0200)^7 + 120 \times (1 + 0.0200)^8 + 180 \times (1 + 0.0200)^9 + 240 \times (1 + 0.0200)^{10}$$

























$$0.0200)^{29} - 900 \times (1+0.0200)^{29} - 80 \times (1+0.0200)^{23} - 90 \times (1+0.0200)^{24} - 75 \times (1+0.0200)^{25} - 65 \times (1+0.0200)^{26} - 55 \times (1+0.0200)^{27} - 45 \times (1+0.0200)^{28} - 40 \times (1+0.0200)^{29} = 5085$$

$$C_{2230} = 2500 \times (1+0.0200)^{22} + 120 \times (1+0.0200)^{23} + 180 \times (1+0.0200)^{24} + 240 \times (1+0.0200)^{25} + 300 \times (1+0.0200)^{26} + 360 \times (1+0.0200)^{27} + 420 \times (1+0.0200)^{28} + 480 \times (1+0.0200)^{29} + 540 \times (1+0.0200)^{30} - 800 \times (1+0.0200)^{30} - 80 \times (1+0.0200)^{23} - 90 \times (1+0.0200)^{24} - 75 \times (1+0.0200)^{25} - 65 \times (1+0.0200)^{26} - 55 \times (1+0.0200)^{27} - 45 \times (1+0.0200)^{28} - 40 \times (1+0.0200)^{29} - 35 \times (1+0.0200)^{30} = 6148$$

**Equipment bought at time 23:**  $C_{2324} = 2500 \times (1+0.0200)^{23} + 120 \times (1+0.0200)^{24} - 2000 \times (1+0.0200)^{24} - 80 \times (1+0.0200)^{24} = 789$

$$C_{2325} = 2500 \times (1+0.0200)^{23} + 120 \times (1+0.0200)^{24} + 180 \times (1+0.0200)^{25} - 1800 \times (1+0.0200)^{25} - 80 \times (1+0.0200)^{24} - 90 \times (1+0.0200)^{25} = 1199$$

$$C_{2326} = 2500 \times (1+0.0200)^{23} + 120 \times (1+0.0200)^{24} + 180 \times (1+0.0200)^{25} + 240 \times (1+0.0200)^{26} - 1600 \times (1+0.0200)^{26} - 80 \times (1+0.0200)^{24} - 90 \times (1+0.0200)^{25} - 75 \times (1+0.0200)^{26} = 1750$$

$$C_{2327} = 2500 \times (1+0.0200)^{23} + 120 \times (1+0.0200)^{24} + 180 \times (1+0.0200)^{25} + 240 \times (1+0.0200)^{26} + 300 \times (1+0.0200)^{27} - 1400 \times (1+0.0200)^{27} - 80 \times (1+0.0200)^{24} - 90 \times (1+0.0200)^{25} - 75 \times (1+0.0200)^{26} - 65 \times (1+0.0200)^{27} = 2439$$

$$C_{2328} = 2500 \times (1+0.0200)^{23} + 120 \times (1+0.0200)^{24} + 180 \times (1+0.0200)^{25} + 240 \times (1+0.0200)^{26} + 300 \times (1+0.0200)^{27} + 360 \times (1+0.0200)^{28} - 1200 \times (1+0.0200)^{28} - 80 \times (1+0.0200)^{24} - 90 \times (1+0.0200)^{25} - 75 \times (1+0.0200)^{26} - 65 \times (1+0.0200)^{27} - 55 \times (1+0.0200)^{28} - 45 \times (1+0.0200)^{29} = 3269$$

$$C_{2329} = 2500 \times (1+0.0200)^{23} + 120 \times (1+0.0200)^{24} + 180 \times (1+0.0200)^{25} + 240 \times (1+0.0200)^{26} + 300 \times (1+0.0200)^{27} + 360 \times (1+0.0200)^{28} + 420 \times (1+0.0200)^{29} - 1000 \times (1+0.0200)^{29} - 80 \times (1+0.0200)^{24} - 90 \times (1+0.0200)^{25} - 75 \times (1+0.0200)^{26} - 65 \times (1+0.0200)^{27} - 55 \times (1+0.0200)^{28} - 45 \times (1+0.0200)^{29} = 4248$$

$$C_{2330} = 2500 \times (1+0.0200)^{23} + 120 \times (1+0.0200)^{24} + 180 \times (1+0.0200)^{25} + 240 \times (1+0.0200)^{26} + 300 \times (1+0.0200)^{27} + 360 \times (1+0.0200)^{28} + 420 \times (1+0.0200)^{29} + 480 \times (1+0.0200)^{30} - 900 \times (1+0.0200)^{30} - 80 \times (1+0.0200)^{24} - 90 \times (1+0.0200)^{25} - 75 \times (1+0.0200)^{26} - 65 \times (1+0.0200)^{27} - 55 \times (1+0.0200)^{28} - 45 \times (1+0.0200)^{29} - 40 \times (1+0.0200)^{30} = 5189$$

**Equipment bought at time 24:**  $C_{2425} = 2500 \times (1+0.0200)^{24} + 120 \times (1+0.0200)^{25} - 2000 \times (1+0.0200)^{25} - 80 \times (1+0.0200)^{25} = 803$

$$C_{2426} = 2500 \times (1+0.0200)^{24} + 120 \times (1+0.0200)^{25} + 180 \times (1+0.0200)^{26} - 1800 \times (1+0.0200)^{26} - 80 \times (1+0.0200)^{25} - 90 \times (1+0.0200)^{26} = 1222$$

$$C_{2427} = 2500 \times (1+0.0200)^{24} + 120 \times (1+0.0200)^{25} + 180 \times (1+0.0200)^{26} + 240 \times (1+0.0200)^{27} - 1600 \times (1+0.0200)^{27} - 80 \times (1+0.0200)^{25} - 90 \times (1+0.0200)^{26} - 75 \times (1+0.0200)^{27} = 1783$$

$$C_{2428} = 2500 \times (1+0.0200)^{24} + 120 \times (1+0.0200)^{25} + 180 \times (1+0.0200)^{26} + 240 \times (1+0.0200)^{27} + 300 \times (1+0.0200)^{28} - 1400 \times (1+0.0200)^{28} - 80 \times (1+0.0200)^{25} - 90 \times (1+0.0200)^{26} - 75 \times (1+0.0200)^{27} - 65 \times (1+0.0200)^{28} = 2485$$

$$C_{2429} = 2500 \times (1+0.0200)^{24} + 120 \times (1+0.0200)^{25} + 180 \times (1+0.0200)^{26} + 240 \times (1+0.0200)^{27} + 300 \times (1+0.0200)^{28} + 360 \times (1+0.0200)^{29} - 1200 \times (1+0.0200)^{29} - 80 \times (1+0.0200)^{25} - 90 \times (1+0.0200)^{26} - 75 \times (1+0.0200)^{27} - 65 \times (1+0.0200)^{28} - 55 \times (1+0.0200)^{29} = 3332$$

$$C_{2430} = 2500 \times (1+0.0200)^{24} + 120 \times (1+0.0200)^{25} + 180 \times (1+0.0200)^{26} + 240 \times (1+0.0200)^{27} + 300 \times (1+0.0200)^{28} + 360 \times (1+0.0200)^{29} + 420 \times (1+0.0200)^{30} - 1000 \times (1+0.0200)^{30} - 80 \times (1+0.0200)^{25} - 90 \times (1+0.0200)^{26} - 75 \times (1+0.0200)^{27} - 65 \times (1+0.0200)^{28} - 55 \times (1+0.0200)^{29} = 4330$$

**Equipment bought at time 25:**  $C_{2526} = 2500 \times (1+0.0200)^{25} + 120 \times (1+0.0200)^{26} -$

$$2000 \times (1 + 0.0200)^{26} - 80 \times (1 + 0.0200)^{26} = 820$$

$$C_{2527} = 2500 \times (1 + 0.0200)^{25} + 120 \times (1 + 0.0200)^{26} + 180 \times (1 + 0.0200)^{27} - 1800 \times (1 + 0.0200)^{27} - 80 \times (1 + 0.0200)^{26} - 90 \times (1 + 0.0200)^{27} = 1247$$

$$C_{2528} = 2500 \times (1 + 0.0200)^{25} + 120 \times (1 + 0.0200)^{26} + 180 \times (1 + 0.0200)^{27} + 240 \times (1 + 0.0200)^{28} - 1600 \times (1 + 0.0200)^{28} - 80 \times (1 + 0.0200)^{26} - 90 \times (1 + 0.0200)^{27} - 75 \times (1 + 0.0200)^{28} = 1820$$

$$C_{2529} = 2500 \times (1 + 0.0200)^{25} + 120 \times (1 + 0.0200)^{26} + 180 \times (1 + 0.0200)^{27} + 240 \times (1 + 0.0200)^{28} + 300 \times (1 + 0.0200)^{29} - 1400 \times (1 + 0.0200)^{29} - 80 \times (1 + 0.0200)^{26} - 90 \times (1 + 0.0200)^{27} - 75 \times (1 + 0.0200)^{28} - 65 \times (1 + 0.0200)^{29} = 2535$$

$$C_{2530} = 2500 \times (1 + 0.0200)^{25} + 120 \times (1 + 0.0200)^{26} + 180 \times (1 + 0.0200)^{27} + 240 \times (1 + 0.0200)^{28} + 300 \times (1 + 0.0200)^{29} + 360 \times (1 + 0.0200)^{30} - 1200 \times (1 + 0.0200)^{30} - 80 \times (1 + 0.0200)^{26} - 90 \times (1 + 0.0200)^{27} - 75 \times (1 + 0.0200)^{28} - 65 \times (1 + 0.0200)^{29} - 55 \times (1 + 0.0200)^{30} = 3400$$

**Equipment bought at time 26:**  $C_{2627} = 2500 \times (1 + 0.0200)^{26} + 120 \times (1 + 0.0200)^{27} - 2000 \times (1 + 0.0200)^{27} - 80 \times (1 + 0.0200)^{27} = 836$

$$C_{2628} = 2500 \times (1 + 0.0200)^{26} + 120 \times (1 + 0.0200)^{27} + 180 \times (1 + 0.0200)^{28} - 1800 \times (1 + 0.0200)^{28} - 80 \times (1 + 0.0200)^{27} - 90 \times (1 + 0.0200)^{28} = 1272$$

$$C_{2629} = 2500 \times (1 + 0.0200)^{26} + 120 \times (1 + 0.0200)^{27} + 180 \times (1 + 0.0200)^{28} + 240 \times (1 + 0.0200)^{29} - 1600 \times (1 + 0.0200)^{29} - 80 \times (1 + 0.0200)^{27} - 90 \times (1 + 0.0200)^{28} - 75 \times (1 + 0.0200)^{29} = 1856$$

$$C_{2630} = 2500 \times (1 + 0.0200)^{26} + 120 \times (1 + 0.0200)^{27} + 180 \times (1 + 0.0200)^{28} + 240 \times (1 + 0.0200)^{29} + 300 \times (1 + 0.0200)^{30} - 1400 \times (1 + 0.0200)^{30} - 80 \times (1 + 0.0200)^{27} - 90 \times (1 + 0.0200)^{28} - 75 \times (1 + 0.0200)^{29} - 65 \times (1 + 0.0200)^{30} = 2587$$

**Equipment bought at time 27:**  $C_{2728} = 2500 \times (1 + 0.0200)^{27} + 120 \times (1 + 0.0200)^{28} - 2000 \times (1 + 0.0200)^{28} - 80 \times (1 + 0.0200)^{28} = 852$

$$C_{2729} = 2500 \times (1 + 0.0200)^{27} + 120 \times (1 + 0.0200)^{28} + 180 \times (1 + 0.0200)^{29} - 1800 \times (1 + 0.0200)^{29} - 80 \times (1 + 0.0200)^{28} - 90 \times (1 + 0.0200)^{29} = 1297$$

$$C_{2730} = 2500 \times (1 + 0.0200)^{27} + 120 \times (1 + 0.0200)^{28} + 180 \times (1 + 0.0200)^{29} + 240 \times (1 + 0.0200)^{30} - 1600 \times (1 + 0.0200)^{30} - 80 \times (1 + 0.0200)^{28} - 90 \times (1 + 0.0200)^{29} - 75 \times (1 + 0.0200)^{30} = 1893$$

**Equipment bought at time 28:**  $C_{2829} = 2500 \times (1 + 0.0200)^{28} + 120 \times (1 + 0.0200)^{29} - 2000 \times (1 + 0.0200)^{29} - 80 \times (1 + 0.0200)^{29} = 870$

$$C_{2830} = 2500 \times (1 + 0.0200)^{28} + 120 \times (1 + 0.0200)^{29} + 180 \times (1 + 0.0200)^{30} - 1800 \times (1 + 0.0200)^{30} - 80 \times (1 + 0.0200)^{29} - 90 \times (1 + 0.0200)^{30} = 1323$$

**Equipment bought at time 29:**  $C_{2930} = 2500 \times (1 + 0.0200)^{29} + 120 \times (1 + 0.0200)^{30} - 2000 \times (1 + 0.0200)^{30} - 80 \times (1 + 0.0200)^{30} = 888$

## $C_{tx}$ Table

Table 1/3 - Columns 1 to 10:

t/x	1	2	3	4	5	6	7	8	9	10
0	500	760	1109	1544	2071	2690	3288	3974	4756	5634
1	—	509	774	1130	1575	2112	2745	3354	4055	4852
2	—	—	520	790	1153	1606	2153	2799	3420	4135
3	—	—	—	530	805	1176	1638	2197	2854	3488
4	—	—	—	—	540	821	1199	1670	2240	2912
5	—	—	—	—	—	551	838	1224	1705	2286
6	—	—	—	—	—	—	562	855	1247	1738
7	—	—	—	—	—	—	—	573	872	1273
8	—	—	—	—	—	—	—	—	585	890
9	—	—	—	—	—	—	—	—	—	597
10	—	—	—	—	—	—	—	—	—	—
11	—	—	—	—	—	—	—	—	—	—
12	—	—	—	—	—	—	—	—	—	—
13	—	—	—	—	—	—	—	—	—	—
14	—	—	—	—	—	—	—	—	—	—
15	—	—	—	—	—	—	—	—	—	—
16	—	—	—	—	—	—	—	—	—	—
17	—	—	—	—	—	—	—	—	—	—
18	—	—	—	—	—	—	—	—	—	—
19	—	—	—	—	—	—	—	—	—	—
20	—	—	—	—	—	—	—	—	—	—
21	—	—	—	—	—	—	—	—	—	—
22	—	—	—	—	—	—	—	—	—	—
23	—	—	—	—	—	—	—	—	—	—
24	—	—	—	—	—	—	—	—	—	—
25	—	—	—	—	—	—	—	—	—	—
26	—	—	—	—	—	—	—	—	—	—
27	—	—	—	—	—	—	—	—	—	—
28	—	—	—	—	—	—	—	—	—	—
29	—	—	—	—	—	—	—	—	—	—

**Table 2/3 - Columns 11 to 20:**

t/x	11	12	13	14	15	16	17	18	19	20
0	—	—	—	—	—	—	—	—	—	—
1	5747	—	—	—	—	—	—	—	—	—
2	4948	5863	—	—	—	—	—	—	—	—
3	4218	5046	5977	—	—	—	—	—	—	—
4	3557	4301	5147	6098	—	—	—	—	—	—
5	2971	3630	4389	5251	6221	—	—	—	—	—
6	2330	3028	3700	4474	5353	6343	—	—	—	—
7	1775	2380	3092	3778	4567	5464	6472	—	—	—
8	1299	1810	2426	3153	3853	4658	5573	6603	—	—
9	907	1324	1845	2475	3217	3930	4751	5685	6734	—
10	609	926	1351	1882	2523	3280	4007	4846	5799	6869
11	—	621	944	1378	1920	2575	3346	4088	4943	5914
12	—	—	633	963	1407	1960	2627	3413	4170	5042
13	—	—	—	647	983	1435	1999	2681	3483	4255
14	—	—	—	—	659	1002	1463	2039	2733	3553
15	—	—	—	—	—	672	1022	1491	2079	2787
16	—	—	—	—	—	—	685	1043	1522	2120
17	—	—	—	—	—	—	—	699	1063	1552
18	—	—	—	—	—	—	—	—	713	1085
19	—	—	—	—	—	—	—	—	—	729
20	—	—	—	—	—	—	—	—	—	—
21	—	—	—	—	—	—	—	—	—	—
22	—	—	—	—	—	—	—	—	—	—
23	—	—	—	—	—	—	—	—	—	—
24	—	—	—	—	—	—	—	—	—	—
25	—	—	—	—	—	—	—	—	—	—
26	—	—	—	—	—	—	—	—	—	—
27	—	—	—	—	—	—	—	—	—	—
28	—	—	—	—	—	—	—	—	—	—
29	—	—	—	—	—	—	—	—	—	—

**Table 3/3 - Columns 21 to 30:**

t/x	21	22	23	24	25	26	27	28	29	30
0	—	—	—	—	—	—	—	—	—	—
1	—	—	—	—	—	—	—	—	—	—
2	—	—	—	—	—	—	—	—	—	—
3	—	—	—	—	—	—	—	—	—	—
4	—	—	—	—	—	—	—	—	—	—
5	—	—	—	—	—	—	—	—	—	—
6	—	—	—	—	—	—	—	—	—	—
7	—	—	—	—	—	—	—	—	—	—
8	—	—	—	—	—	—	—	—	—	—
9	—	—	—	—	—	—	—	—	—	—
10	—	—	—	—	—	—	—	—	—	—
11	7007	—	—	—	—	—	—	—	—	—
12	6033	7147	—	—	—	—	—	—	—	—
13	5144	6154	7290	—	—	—	—	—	—	—
14	4340	5247	6278	7436	—	—	—	—	—	—
15	3622	4426	5351	6403	7584	—	—	—	—	—
16	2843	3695	4513	5457	6529	7735	—	—	—	—
17	2163	2899	3769	4605	5567	6661	7891	—	—	—
18	1583	2206	2958	3844	4697	5679	6795	8050	—	—
19	1107	1617	2253	3020	3925	4795	5797	6935	8215	—
20	741	1128	1646	2295	3077	3998	4885	5908	7068	8374
21	—	758	1152	1682	2344	3141	4082	4987	6028	7212
22	—	—	772	1174	1714	2389	3203	4162	5085	6148
23	—	—	—	789	1199	1750	2439	3269	4248	5189
24	—	—	—	—	803	1222	1783	2485	3332	4330
25	—	—	—	—	—	820	1247	1820	2535	3400
26	—	—	—	—	—	—	836	1272	1856	2587
27	—	—	—	—	—	—	—	852	1297	1893
28	—	—	—	—	—	—	—	—	870	1323
29	—	—	—	—	—	—	—	—	—	888

## Step by Step Calculations

$$G(30) = 0 \quad (\text{Base case})$$

$$G(29) = \min\{C_{2930} + G(30) = 888 + 0 = 888\} = 888$$

**Optimal choice:** sell at time 30

$$G(28) = \min\{C_{2829} + G(29) = 870 + 888 = 1758, C_{2830} + G(30) = 1323 + 0 = 1323\} = 1323$$

**Optimal choice:** sell at time 30

$$G(27) = \min\{C_{2728} + G(28) = 852 + 1323 = 2175, C_{2729} + G(29) = 1297 + 888 = 2185, C_{2730} + G(30) = 1893 + 0 = 1893\} = 1893$$

**Optimal choice:** sell at time 30

$$G(26) = \min\{C_{2627} + G(27) = 836 + 1893 = 2729, C_{2628} + G(28) = 1272 + 1323 = 2595, C_{2629} + G(29) = 1856 + 888 = 2744, C_{2630} + G(30) = 2587 + 0 = 2587\} = 2587$$

**Optimal choice:** sell at time 30

$$G(25) = \min\{C_{2526} + G(26) = 820 + 2587 = 3407, C_{2527} + G(27) = 1247 + 1893 = 3140, C_{2528} + G(28) = 1820 + 1323 = 3143, C_{2529} + G(29) = 2535 + 888 = 3423, C_{2530} + G(30) = 3400 + 0 = 3400\} = 3140$$

**Optimal choice:** sell at time 27

$$G(24) = \min\{C_{2425} + G(25) = 803 + 3140 = 3943, C_{2426} + G(26) = 1222 + 2587 = 3809, C_{2427} + G(27) = 1783 + 1893 = 3676, C_{2428} + G(28) = 2485 + 1323 = 3808, C_{2429} + G(29) = 3332 + 888 = 4220, C_{2430} + G(30) = 4330 + 0 = 4330\} = 3676$$

**Optimal choice:** sell at time 27

$$G(23) = \min\{C_{2324} + G(24) = 789 + 3676 = 4465, C_{2325} + G(25) = 1199 + 3140 = 4339, C_{2326} + G(26) = 1750 + 2587 = 4337, C_{2327} + G(27) = 2439 + 1893 = 4332, C_{2328} + G(28) = 3269 + 1323 = 4592, C_{2329} + G(29) = 4248 + 888 = 5136, C_{2330} + G(30) = 5189 + 0 = 5189\} = 4332$$

**Optimal choice:** sell at time 27

$$G(22) = \min\{C_{2223} + G(23) = 772 + 4332 = 5104, C_{2224} + G(24) = 1174 + 3676 = 4850, C_{2225} + G(25) = 1714 + 3140 = 4854, C_{2226} + G(26) = 2389 + 2587 = 4976, C_{2227} + G(27) = 3203 + 1893 = 5096, C_{2228} + G(28) = 4162 + 1323 = 5485, C_{2229} + G(29) = 5085 + 888 = 5973, C_{2230} + G(30) = 6148 + 0 = 6148\} = 4850$$

**Optimal choice:** sell at time 24

$$G(21) = \min\{C_{2122} + G(22) = 758 + 4850 = 5608, C_{2123} + G(23) = 1152 + 4332 = 5484, C_{2124} + G(24) = 1682 + 3676 = 5358, C_{2125} + G(25) = 2344 + 3140 = 5484, C_{2126} + G(26) = 3141 + 2587 = 5728, C_{2127} + G(27) = 4082 + 1893 = 5975, C_{2128} + G(28) = 4987 + 1323 = 6310, C_{2129} + G(29) = 6028 + 888 = 6916, C_{2130} + G(30) = 7212 + 0 = 7212\} = 5358$$

**Optimal choice:** sell at time 24

$$G(20) = \min\{C_{2021} + G(21) = 741 + 5358 = 6099, C_{2022} + G(22) = 1128 + 4850 = 5978, C_{2023} + G(23) = 1646 + 4332 = 5978, C_{2024} + G(24) = 2295 + 3676 = 5971, C_{2025} + G(25) = 3077 + 3140 = 6217, C_{2026} + G(26) = 3998 + 2587 = 6585, C_{2027} + G(27) = 4885 + 1893 = 6778, C_{2028} + G(28) = 5908 + 1323 = 7231, C_{2029} + G(29) = 7068 + 888 = 7956, C_{2030} + G(30) = 8374 + 0 = 8374\} = 5971$$

**Optimal choice:** sell at time 24

$$G(19) = \min\{C_{1920} + G(20) = 729 + 5971 = 6700, C_{1921} + G(21) = 1107 + 5358 = 6465, C_{1922} + G(22) = 1617 + 4850 = 6467, C_{1923} + G(23) = 2253 + 4332 = 6585, C_{1924} + G(24) = 3020 + 3676 = 6696, C_{1925} + G(25) = 3925 + 3140 = 7065, C_{1926} + G(26) = 4795 + 2587 = 7382, C_{1927} + G(27) = 5797 + 1893 = 7690, C_{1928} + G(28) = 6935 + 1323 = 8258, C_{1929} + G(29) = 8215 + 888 = 9103\} = 6465$$

**Optimal choice:** sell at time 21

$$G(18) = \min\{C_{1819} + G(19) = 713 + 6465 = 7178, C_{1820} + G(20) = 1085 + 5971 = 7056, C_{1821} + G(21) = 1583 + 5358 = 6941, C_{1822} + G(22) = 2206 + 4850 = 7056, C_{1823} + G(23) = 2958 + 4332 = 7290, C_{1824} + G(24) = 3844 + 3676 = 7520, C_{1825} + G(25) = 4697 + 3140 = 7837, C_{1826} + G(26) = 5679 + 2587 = 8266, C_{1827} + G(27) = 6795 + 1893 = 8688, C_{1828} + G(28) = 8050 + 1323 = 9373\} = 6941$$

**Optimal choice:** sell at time 21

$$G(17) = \min\{C_{1718} + G(18) = 699 + 6941 = 7640, C_{1719} + G(19) = 1063 + 6465 = 7528, C_{1720} + G(20) = 1552 + 5971 = 7523, C_{1721} + G(21) = 2163 + 5358 = 7521, C_{1722} + G(22) = 2899 + 4850 = 7749, C_{1723} + G(23) = 3769 + 4332 = 8101, C_{1724} + G(24) = 4605 + 3676 = 8281, C_{1725} + G(25) = 5567 + 3140 = 8707, C_{1726} + G(26) = 6661 + 2587 = 9248, C_{1727} + G(27) = 7891 + 1893 = 9784\} = 7521$$

**Optimal choice:** sell at time 21

$$G(16) = \min\{C_{1617} + G(17) = 685 + 7521 = 8206, C_{1618} + G(18) = 1043 + 6941 = 7984, C_{1619} + G(19) = 1522 + 6465 = 7987, C_{1620} + G(20) = 2120 + 5971 = 8091, C_{1621} + G(21) = 2843 + 5358 = 8201, C_{1622} + G(22) = 3695 + 4850 = 8545, C_{1623} + G(23) = 4513 + 4332 = 8845, C_{1624} + G(24) = 5457 + 3676 = 9133, C_{1625} + G(25) = 6529 + 3140 = 9669, C_{1626} + G(26) = 7735 + 2587 = 10322\} = 7984$$

**Optimal choice:** sell at time 18

$$G(15) = \min\{C_{1516} + G(16) = 672 + 7984 = 8656, C_{1517} + G(17) = 1022 + 7521 = 8543, C_{1518} + G(18) = 1491 + 6941 = 8432, C_{1519} + G(19) = 2079 + 6465 = 8544, C_{1520} + G(20) = 2787 + 5971 = 8758, C_{1521} + G(21) = 3622 + 5358 = 8980, C_{1522} + G(22) = 4426 + 4850 = 9276, C_{1523} + G(23) = 5351 + 4332 = 9683, C_{1524} + G(24) = 6403 + 3676 = 10079, C_{1525} + G(25) = 7584 + 3140 = 10724\} = 8432$$

**Optimal choice:** sell at time 18

$$G(14) = \min\{C_{1415} + G(15) = 659 + 8432 = 9091, C_{1416} + G(16) = 1002 + 7984 = 8986, C_{1417} + G(17) = 1463 + 7521 = 8984, C_{1418} + G(18) = 2039 + 6941 = 8980, C_{1419} + G(19) = 2733 + 6465 = 9198, C_{1420} + G(20) = 3553 + 5971 = 9524, C_{1421} + G(21) = 4340 + 5358 = 9698, C_{1422} + G(22) = 5247 + 4850 = 10097, C_{1423} + G(23) = 6278 + 4332 = 10610, C_{1424} + G(24) = 7436 + 3676 = 11112\} = 8980$$

**Optimal choice:** sell at time 18

$$G(13) = \min\{C_{1314} + G(14) = 647 + 8980 = 9627, C_{1315} + G(15) = 983 + 8432 = 9415, C_{1316} + G(16) = 1435 + 7984 = 9419, C_{1317} + G(17) = 1999 + 7521 = 9520, C_{1318} + G(18) = 2681 + 6941 = 9622, C_{1319} + G(19) = 3483 + 6465 = 9948, C_{1320} + G(20) = 4255 + 5971 = 10226, C_{1321} + G(21) = 5144 + 5358 = 10502, C_{1322} + G(22) = 6154 + 4850 = 11004, C_{1323} + G(23) = 7290 + 4332 = 11622\} = 9415$$

**Optimal choice:** sell at time 15

$$G(12) = \min\{C_{1213} + G(13) = 633 + 9415 = 10048, C_{1214} + G(14) = 963 + 8980 = 9943, C_{1215} + G(15) = 1407 + 8432 = 9839, C_{1216} + G(16) = 1960 + 7984 = 9944, C_{1217} + G(17) = 2627 + 7521 = 10148, C_{1218} + G(18) = 3413 + 6941 = 10354, C_{1219} + G(19) = 4170 + 6465 = 10635, C_{1220} + G(20) = 5042 + 5971 = 11013, C_{1221} + G(21) = 6033 + 5358 = 11391, C_{1222} + G(22) = 7147 + 4850 = 11997\} = 9839$$

**Optimal choice:** sell at time 15

$$G(11) = \min\{C_{1112} + G(12) = 621 + 9839 = 10460, C_{1113} + G(13) = 944 + 9415 = 10359, C_{1114} + G(14) = 1378 + 8980 = 10358, C_{1115} + G(15) = 1920 + 8432 = 10352, C_{1116} + G(16) = 2575 + 7984 = 10559, C_{1117} + G(17) = 3346 + 7521 = 10867, C_{1118} + G(18) = 4088 + 6941 = 11029, C_{1119} + G(19) = 4943 + 6465 = 11408, C_{1120} + G(20) = 5914 + 5971 = 11885, C_{1121} + G(21) = 7007 + 5358 = 12365\} = 10352$$

**Optimal choice:** sell at time 15

$$G(10) = \min\{C_{1011} + G(11) = 609 + 10352 = 10961, C_{1012} + G(12) = 926 + 9839 = 10765, C_{1013} + G(13) = 1351 + 9415 = 10766, C_{1014} + G(14) = 1882 + 8980 = 10862, C_{1015} + G(15) = 2523 + 8432 = 10955, C_{1016} + G(16) = 3280 + 7984 = 11264, C_{1017} + G(17) = 4007 + 7521 = 11528, C_{1018} + G(18) = 4846 + 6941 = 11787, C_{1019} + G(19) = 5799 + 6465 = 12264, C_{1020} + G(20) = 6869 + 5971 = 12840\} = 10765$$

**Optimal choice:** sell at time 12

$$G(9) = \min\{C_{910} + G(10) = 597 + 10765 = 11362, C_{911} + G(11) = 907 + 10352 = 11259, C_{912} + G(12) = 1324 + 9839 = 11163, C_{913} + G(13) = 1845 + 9415 = 11260, C_{914} + G(14) = 2475 + 8980 = 11455, C_{915} + G(15) = 3217 + 8432 = 11649, C_{916} + G(16) = 3930 + 7984 = 11914, C_{917} + G(17) = 4751 + 7521 = 12272, C_{918} + G(18) = 5685 + 6941 = 12626, C_{919} + G(19) = 6734 + 6465 = 13199\} = 11163$$

**Optimal choice:** sell at time 12

$$G(8) = \min\{C_{89} + G(9) = 585 + 11163 = 11748, C_{810} + G(10) = 890 + 10765 = 11655, C_{811} + G(11) = 1299 + 10352 = 11651, C_{812} + G(12) = 1810 + 9839 = 11649, C_{813} + G(13) = 2426 + 9415 = 11841, C_{814} + G(14) = 3153 + 8980 = 12133, C_{815} + G(15) = 3853 + 8432 = 12285, C_{816} + G(16) = 4658 + 7984 = 12642, C_{817} + G(17) = 5573 + 7521 = 13094, C_{818} + G(18) = 6603 + 6941 = 13544\} = 11649$$

**Optimal choice:** sell at time 12

$$G(7) = \min\{C_{78} + G(8) = 573 + 11649 = 12222, C_{79} + G(9) = 872 + 11163 = 12035, C_{710} + G(10) = 1273 + 10765 = 12038, C_{711} + G(11) = 1775 + 10352 = 12127, C_{712} + G(12) = 2380 + 9839 = 12219, C_{713} + G(13) = 3092 + 9415 = 12507, C_{714} + G(14) = 3778 + 8980 = 12758, C_{715} + G(15) = 4567 + 8432 = 12999, C_{716} + G(16) = 5464 + 7984 = 13448, C_{717} + G(17) = 6472 + 7521 = 13993\} = 12035$$

**Optimal choice:** sell at time 9

$$G(6) = \min\{C_{67} + G(7) = 562 + 12035 = 12597, C_{68} + G(8) = 855 + 11649 = 12504, C_{69} + G(9) = 1247 + 11163 = 12410, C_{610} + G(10) = 1738 + 10765 = 12503, C_{611} + G(11) = 2330 + 10352 = 12682, C_{612} + G(12) = 3028 + 9839 = 12867, C_{613} + G(13) = 3700 + 9415 = 13115, C_{614} + G(14) = 4474 + 8980 = 13454, C_{615} + G(15) = 5353 + 8432 = 13785, C_{616} + G(16) = 6343 + 7984 = 14327\} = 12410$$

**Optimal choice:** sell at time 9

$$G(5) = \min\{C_{56} + G(6) = 551 + 12410 = 12961, C_{57} + G(7) = 838 + 12035 = 12873, C_{58} + G(8) = 1224 + 11649 = 12873, C_{59} + G(9) = 1705 + 11163 = 12868, C_{510} + G(10) = 2286 + 10765 = 13051, C_{511} + G(11) = 2971 + 10352 = 13323, C_{512} + G(12) = 3630 + 9839 = 13469, C_{513} + G(13) = 4389 + 9415 = 13804, C_{514} + G(14) = 5251 + 8980 = 14231, C_{515} + G(15) = 6221 + 8432 = 14653\} = 12868$$

**Optimal choice:** sell at time 9

$$G(4) = \min\{C_{45} + G(5) = 540 + 12868 = 13408, C_{46} + G(6) = 821 + 12410 = 13231, C_{47} + G(7) = 1199 + 12035 = 13234, C_{48} + G(8) = 1670 + 11649 = 13319, C_{49} + G(9) = 2240 + 11163 = 13403, C_{410} + G(10) = 2912 + 10765 = 13677, C_{411} + G(11) = 3557 + 10352 = 13909, C_{412} + G(12) = 4301 + 9839 = 14140, C_{413} + G(13) = 5147 + 9415 = 14562, C_{414} + G(14) = 6098 + 8980 = 15078\} = 13231$$

**Optimal choice:** sell at time 6

$$G(3) = \min\{C_{34} + G(4) = 530 + 13231 = 13761, C_{35} + G(5) = 805 + 12868 = 13673, C_{36} + G(6) = 1176 + 12410 = 13586, C_{37} + G(7) = 1638 + 12035 = 13673, C_{38} + G(8) = 2197 + 11649 = 13846, C_{39} + G(9) = 2854 + 11163 = 14017, C_{310} + G(10) = 3488 + 10765 = 14253, C_{311} + G(11) = 4218 + 10352 = 14570, C_{312} + G(12) = 5046 + 9839 = 14885, C_{313} + G(13) = 5977 + 9415 = 15392\} = 13586$$

**Optimal choice:** sell at time 6

$$G(2) = \min\{C_{23} + G(3) = 520 + 13586 = 14106, C_{24} + G(4) = 790 + 13231 = 14021, C_{25} + G(5) = 1153 + 12868 = 14021, C_{26} + G(6) = 1606 + 12410 = 14016, C_{27} + G(7) = 2153 + 12035 = 14188, C_{28} + G(8) = 2799 + 11649 = 14448, C_{29} + G(9) = 3420 + 11163 = 14583, C_{210} + G(10) = 4135 + 10765 = 14900, C_{211} + G(11) = 4948 + 10352 = 15300, C_{212} + G(12) = 5863 + 9839 = 15702\} = 14016$$

**Optimal choice:** sell at time 6

$$G(1) = \min\{C_{12} + G(2) = 509 + 14016 = 14525, C_{13} + G(3) = 774 + 13586 = 14360, C_{14} + G(4) = 1130 + 13231 = 14361, C_{15} + G(5) = 1575 + 12868 = 14443, C_{16} + G(6) = 2112 + 12410 = 14522, C_{17} + G(7) = 2745 + 12035 = 14780, C_{18} + G(8) = 3354 + 11649 = 15003, C_{19} + G(9) = 4055 + 11163 = 15218, C_{110} + G(10) = 4852 + 10765 = 15617, C_{111} + G(11) = 5747 + 10352 = 16099\} = 14360$$

**Optimal choice:** sell at time 3

$$G(0) = \min\{C_{01} + G(1) = 500 + 14360 = 14860, C_{02} + G(2) = 760 + 14016 = 14776, C_{03} + G(3) = 1109 + 13586 = 14695, C_{04} + G(4) = 1544 + 13231 = 14775, C_{05} + G(5) = 2071 + 12868 = 14939, C_{06} + G(6) = 2690 + 12410 = 15100, C_{07} + G(7) = 3288 + 12035 = 15323, C_{08} + G(8) = 3974 + 11649 = 15623, C_{09} + G(9) = 4756 + 11163 = 15919, C_{010} + G(10) = 5634 + 10765 = 16399\} = 14695$$

**Optimal choice:** sell at time 3

## Result Table (Analysis table)

t	G(t)	Next Replacement
0	14695	3
1	14360	3
2	14016	6
3	13586	6
4	13231	6
5	12868	9
6	12410	9
7	12035	9
8	11649	12
9	11163	12
10	10765	12
11	10352	15
12	9839	15
13	9415	15
14	8980	18
15	8432	18
16	7984	18
17	7521	21
18	6941	21
19	6465	21
20	5971	24
21	5358	24
22	4850	24
23	4332	27
24	3676	27
25	3140	27
26	2587	30
27	1893	30
28	1323	30
29	888	30
30	0	—

## Optimal Solution

Minimum Total Cost: 14695

All Optimal Replacement Plans:

- **Plan 1:** Buy at 0, sell at 3 → Buy at 3, sell at 6 → Buy at 6, sell at 9 → Buy at 9, sell at 12 → Buy at 12, sell at 15 → Buy at 15, sell at 18 → Buy at 18, sell at 21 → Buy at 21, sell at 24 → Buy at 24, sell at 27 → Buy at 27, sell at 30

