Instituto Tecnológico de Costa Rica

Operations Research - Semester II

Equipment Replacement Problem

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Equipment Replacement Problem

The equipment replacement problem is an optimization problem that involves deciding the optimal time to replace a piece of equipment (sell and re-buy) to minimize business costs. There are four main components to this problem:

- The project term (e.g. 5 years long project)
- Life span of the equipment
- Price of new equipment
- Maintenance costs (could increase as equipment ages)
- Sell prices (e.g. 4 year old equipment can be sold for $50)

The problem involves determining the optimal number of equipment replacements and the timing of each, in order to minimize operating costs.

Given the following variables:

- t_{max}: Project term
- t_{span}: Lifes span of equipment
- t: A given instant through the project term.
- C_{tx}: The cost of buying at instant t and selling at x
  - Cost of new equipment + maintenance costs - sell price - profits

G(t) = optimal business costs from t to t_{max} (Buying new equipment at t).

0.1 Trivial case

$G(t_{max}) = 0$

- From instant t_{max} to instant t_{max}
- We don't buy any equipment

0.2 Optimization

If new equipment is bought at t we can sell it and buy new equipment at either $t + 1$, $t + 2$, ..., or $t + t_{span}$. We should sell at the point where we get the least operation cost given the project term.

0.3 Bellman Equation

Knowing this, we can establish the Bellman equation:

$$G(t) = \min\{C_{tx} + G(x)\} \tag{1}$$

From $t = t_{max}$ until $t = 0$.

Equipment Life Cycle Costs

Year	Maintenance Cost	Resale Price
1	30	400
2	40	300
3	60	250

C_{tx} Calculations

1 year(s): $C_{t,t+1} = 500 + 30 - 400 = 130$

2 year(s): $C_{t,t+2} = 500 + 30 + 40 - 300 = 270$

3 year(s): $C_{t,t+3} = 500 + 30 + 40 + 60 - 250 = 380$

C_{tx} Table

t/x	1	2	3	4	5
0	130	270	380	–	–
1	–	130	270	380	–
2	–	–	130	270	380
3	–	–	–	130	270
4	–	–	–	–	130

Step by Step Calculations

$G(5) = 0$ (Base case)

$G(4) = \min\{C_{45} + G(5) = (500 + 30 - 400) + 0 = 130\} = 130$

Optimal choice: sell at time 5

$G(3) = \min\{C_{34} + G(4) = (500 + 30 - 400) + 130 = 260, C_{35} + G(5) = (500 + 30 + 40 - 300) + 0 = 270\} = 260$

Optimal choice: sell at time 4

$G(2) = \min\{C_{23} + G(3) = (500 + 30 - 400) + 260 = 390, C_{24} + G(4) = (500 + 30 + 40 - 300) + 130 = 400, C_{25} + G(5) = (500 + 30 + 40 + 60 - 250) + 0 = 380\} = 380$

Optimal choice: sell at time 5

$G(1) = \min\{C_{12} + G(2) = (500 + 30 - 400) + 380 = 510, C_{13} + G(3) = (500 + 30 + 40 - 300) + 260 = 530, C_{14} + G(4) = (500 + 30 + 40 + 60 - 250) + 130 = 510\} = 510$

Multiple optimal choices: sell at time 2, sell at time 4 (tie)

$G(0) = \min\{C_{01} + G(1) = (500 + 30 - 400) + 510 = 640, C_{02} + G(2) = (500 + 30 + 40 - 300) + 380 = 650, C_{03} + G(3) = (500 + 30 + 40 + 60 - 250) + 260 = 640\} = 640$

Multiple optimal choices: sell at time 1, sell at time 3 (tie)

Result Table (Analysis table)

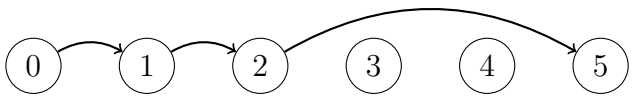
t	G(t)	Next Replacement
0	640	1, 3
1	510	2, 4
2	380	5
3	260	4
4	130	5
5	0	–

Optimal Solution

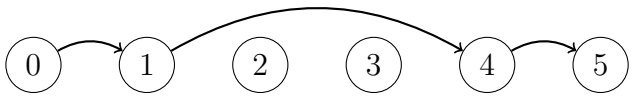
Minimum Total Cost: 640

All Optimal Replacement Plans:

- **Plan 1:** Buy at 0, sell at 1 → Buy at 1, sell at 2 → Buy at 2, sell at 5



- **Plan 2:** Buy at 0, sell at 1 → Buy at 1, sell at 4 → Buy at 4, sell at 5



- **Plan 3:** Buy at 0, sell at 3 → Buy at 3, sell at 4 → Buy at 4, sell at 5