

Instituto Tecnológico de Costa Rica

Operations Research - Semester II

Knapsack Problem

Members:

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Degenerate

Maximize

$$Z = 2.00x_1 + 1.00x_2$$

Subject to

$$3.00x_1 + 1.00x_2 \leq 6.00$$

$$1.00x_1 - 1.00x_2 \leq 2.00$$

$$0.00x_1 + 1.00x_2 \leq 3.00$$

Simplex Table

	Z	x_1	x_2	s_1	s_2	s_3	b
	1.00	-2.00	-1.00	0.00	0.00	0.00	0.00
	0.00	3.00	1.00	1.00	0.00	0.00	6.00
	0.00	1.00	-1.00	0.00	1.00	0.00	2.00
	0.00	0.00	1.00	0.00	0.00	1.00	3.00

Intermediate Tables

Pivoting 1

Most Negative

Column 2 (-2.00)

	Z	x_1	x_2	s_1	s_2	s_3	b
	1.00	-2.00	-1.00	0.00	0.00	0.00	0.00
	0.00	3.00	1.00	1.00	0.00	0.00	6.00
	0.00	1.00	-1.00	0.00	1.00	0.00	2.00
	0.00	0.00	1.00	0.00	0.00	1.00	3.00

Fractions

	Z	x_1	x_2	s_1	s_2	s_3	b	Frac
	1.00	-2.00	-1.00	0.00	0.00	0.00	0.00	
	0.00	3.00	1.00	1.00	0.00	0.00	6.00	2.00
	0.00	1.00	-1.00	0.00	1.00	0.00	2.00	2.00
	0.00	0.00	1.00	0.00	0.00	1.00	3.00	inf

$$6.00/3.00 = 2.00$$

$$2.00/1.00 = 2.00$$

$$3.00/0.00 = \text{inf}$$

Smallest fraction: 2.00 \rightarrow Pivot: row 3

Pivot

	Z	x_1	x_2	s_1	s_2	s_3	b
	1.00	-2.00	-1.00	0.00	0.00	0.00	0.00
	0.00	3.00	1.00	1.00	0.00	0.00	6.00
	0.00	1.00	-1.00	0.00	1.00	0.00	2.00
	0.00	0.00	1.00	0.00	0.00	1.00	3.00

Canonization

$$R_3 \leftarrow R_3/1.00$$

	Z	x_1	x_2	s_1	s_2	s_3	b
	1.00	-2.00	-1.00	0.00	0.00	0.00	0.00
	0.00	3.00	1.00	1.00	0.00	0.00	6.00
	0.00	1.00	-1.00	0.00	1.00	0.00	2.00
	0.00	0.00	1.00	0.00	0.00	1.00	3.00

$$R_1 \leftarrow R_1 + 2.00R_3$$

	Z	x_1	x_2	s_1	s_2	s_3	b
	1.00	0.00	-3.00	0.00	2.00	0.00	4.00
	0.00	3.00	1.00	1.00	0.00	0.00	6.00
	0.00	1.00	-1.00	0.00	1.00	0.00	2.00
	0.00	0.00	1.00	0.00	0.00	1.00	3.00

$$R_2 \leftarrow R_2 + -3.00R_3$$

	Z	x_1	x_2	s_1	s_2	s_3	b
	1.00	0.00	-3.00	0.00	2.00	0.00	4.00
	0.00	0.00	4.00	1.00	-3.00	0.00	0.00
	0.00	1.00	-1.00	0.00	1.00	0.00	2.00
	0.00	0.00	1.00	0.00	0.00	1.00	3.00

$$R_4 \leftarrow R_4 + -0.00R_3$$

	Z	x_1	x_2	s_1	s_2	s_3	b
	1.00	0.00	-3.00	0.00	2.00	0.00	4.00
	0.00	0.00	4.00	1.00	-3.00	0.00	0.00
	0.00	1.00	-1.00	0.00	1.00	0.00	2.00
	0.00	0.00	1.00	0.00	0.00	1.00	3.00

Pivot Result

	Z	x_1	x_2	s_1	s_2	s_3	b
	1.00	0.00	-3.00	0.00	2.00	0.00	4.00
	0.00	0.00	4.00	1.00	-3.00	0.00	0.00
	0.00	1.00	-1.00	0.00	1.00	0.00	2.00
	0.00	0.00	1.00	0.00	0.00	1.00	3.00

Degenerate Base

The variable s_1 is part of the base but has a value of 0. Therefore, this is a degenerate Basic Feasible Solution (BFS).

Pivoting 2

Most Negative

Column 3 (-3.00)

	Z	x_1	x_2	s_1	s_2	s_3	b
	1.00	0.00	-3.00	0.00	2.00	0.00	4.00
	0.00	0.00	4.00	1.00	-3.00	0.00	0.00
	0.00	1.00	-1.00	0.00	1.00	0.00	2.00
	0.00	0.00	1.00	0.00	0.00	1.00	3.00

Fractions

	Z	x_1	x_2	s_1	s_2	s_3	b	Frac
	1.00	0.00	-3.00	0.00	2.00	0.00	4.00	
	0.00	0.00	4.00	1.00	-3.00	0.00	0.00	0.00
	0.00	1.00	-1.00	0.00	1.00	0.00	2.00	-2.00
	0.00	0.00	1.00	0.00	0.00	1.00	3.00	3.00

$$0.00/4.00 = 0.00$$

$$2.00/-1.00 = -2.00$$

$$3.00/1.00 = 3.00$$

Smallest fraction: 0.00 \rightarrow Pivot: row 2

Pivot

	Z	x_1	x_2	s_1	s_2	s_3	b
	1.00	0.00	-3.00	0.00	2.00	0.00	4.00
	0.00	0.00	4.00	1.00	-3.00	0.00	0.00
	0.00	1.00	-1.00	0.00	1.00	0.00	2.00
	0.00	0.00	1.00	0.00	0.00	1.00	3.00

Canonization

$$R_2 \leftarrow R_2/4.00$$

	Z	x_1	x_2	s_1	s_2	s_3	b
	1.00	0.00	-3.00	0.00	2.00	0.00	4.00
	0.00	0.00	1.00	0.25	-0.75	0.00	0.00
	0.00	1.00	-1.00	0.00	1.00	0.00	2.00
	0.00	0.00	1.00	0.00	0.00	1.00	3.00

$$R_1 \leftarrow R_1 + 3.00R_2$$

	Z	x_1	x_2	s_1	s_2	s_3	b
	1.00	0.00	0.00	0.75	-0.25	0.00	4.00
	0.00	0.00	1.00	0.25	-0.75	0.00	0.00
	0.00	1.00	-1.00	0.00	1.00	0.00	2.00
	0.00	0.00	1.00	0.00	0.00	1.00	3.00

$$R_3 \leftarrow R_3 + 1.00R_2$$

	Z	x_1	x_2	s_1	s_2	s_3	b
	1.00	0.00	0.00	0.75	-0.25	0.00	4.00
	0.00	0.00	1.00	0.25	-0.75	0.00	0.00
	0.00	1.00	0.00	0.25	0.25	0.00	2.00
	0.00	0.00	1.00	0.00	0.00	1.00	3.00

$$R_4 \leftarrow R_4 + -1.00R_2$$

	Z	x_1	x_2	s_1	s_2	s_3	b
	1.00	0.00	0.00	0.75	-0.25	0.00	4.00
	0.00	0.00	1.00	0.25	-0.75	0.00	0.00
	0.00	1.00	0.00	0.25	0.25	0.00	2.00
	0.00	0.00	0.00	-0.25	0.75	1.00	3.00

Pivot Result

	Z	x_1	x_2	s_1	s_2	s_3	b
	1.00	0.00	0.00	0.75	-0.25	0.00	4.00
	0.00	0.00	1.00	0.25	-0.75	0.00	0.00
	0.00	1.00	0.00	0.25	0.25	0.00	2.00
	0.00	0.00	0.00	-0.25	0.75	1.00	3.00

Degenerate Base

The variable x_2 is part of the base but has a value of 0. Therefore, this is a degenerate Basic Feasible Solution (BFS).

Pivoting 3

Most Negative

Column 5 (-0.25)

	Z	x_1	x_2	s_1	s_2	s_3	b
	1.00	0.00	0.00	0.75	-0.25	0.00	4.00
	0.00	0.00	1.00	0.25	-0.75	0.00	0.00
	0.00	1.00	0.00	0.25	0.25	0.00	2.00
	0.00	0.00	0.00	-0.25	0.75	1.00	3.00

Fractions

	Z	x_1	x_2	s_1	s_2	s_3	b	Frac
	1.00	0.00	0.00	0.75	-0.25	0.00	4.00	
	0.00	0.00	1.00	0.25	-0.75	0.00	0.00	-0.00
	0.00	1.00	0.00	0.25	0.25	0.00	2.00	8.00
	0.00	0.00	0.00	-0.25	0.75	1.00	3.00	4.00

$$0.00 / -0.75 = -0.00$$

$$2.00 / 0.25 = 8.00$$

$$3.00 / 0.75 = 4.00$$

Smallest fraction: 4.00 \rightarrow Pivot: row 4

Pivot

	Z	x_1	x_2	s_1	s_2	s_3	b
	1.00	0.00	0.00	0.75	-0.25	0.00	4.00
	0.00	0.00	1.00	0.25	-0.75	0.00	0.00
	0.00	1.00	0.00	0.25	0.25	0.00	2.00
	0.00	0.00	0.00	-0.25	0.75	1.00	3.00

Canonization

$$R_4 \leftarrow R_4 / 0.75$$

	Z	x_1	x_2	s_1	s_2	s_3	b
	1.00	0.00	0.00	0.75	-0.25	0.00	4.00
	0.00	0.00	1.00	0.25	-0.75	0.00	0.00
	0.00	1.00	0.00	0.25	0.25	0.00	2.00
	0.00	0.00	0.00	-0.33	1.00	1.33	4.00

$$R_1 \leftarrow R_1 + 0.25R_4$$

	Z	x_1	x_2	s_1	s_2	s_3	b
	1.00	0.00	0.00	0.67	0.00	0.33	5.00
	0.00	0.00	1.00	0.25	-0.75	0.00	0.00
	0.00	1.00	0.00	0.25	0.25	0.00	2.00
	0.00	0.00	0.00	-0.33	1.00	1.33	4.00

$$R_2 \leftarrow R_2 + 0.75R_4$$

	Z	x_1	x_2	s_1	s_2	s_3	b
	1.00	0.00	0.00	0.67	0.00	0.33	5.00
	0.00	0.00	1.00	0.00	0.00	1.00	3.00
	0.00	1.00	0.00	0.25	0.25	0.00	2.00
	0.00	0.00	0.00	-0.33	1.00	1.33	4.00

$$R_3 \leftarrow R_3 + -0.25R_4$$

	Z	x_1	x_2	s_1	s_2	s_3	b
	1.00	0.00	0.00	0.67	0.00	0.33	5.00
	0.00	0.00	1.00	0.00	0.00	1.00	3.00
	0.00	1.00	0.00	0.33	0.00	-0.33	1.00
	0.00	0.00	0.00	-0.33	1.00	1.33	4.00

Pivot Result

	Z	x_1	x_2	s_1	s_2	s_3	b
	1.00	0.00	0.00	0.67	0.00	0.33	5.00
	0.00	0.00	1.00	0.00	0.00	1.00	3.00
	0.00	1.00	0.00	0.33	0.00	-0.33	1.00
	0.00	0.00	0.00	-0.33	1.00	1.33	4.00

Results

	Z	x_1	x_2	s_1	s_2	s_3	b
	1.00	0.00	0.00	0.67	0.00	0.33	5.00
	0.00	0.00	1.00	0.00	0.00	1.00	3.00
	0.00	1.00	0.00	0.33	0.00	-0.33	1.00
	0.00	0.00	0.00	-0.33	1.00	1.33	4.00

Objective Value

$$Z = 5.00$$

Variables

$$x_1 = 1.00$$

$$x_2 = 3.00$$

Slack or Surplus

$$s_1 = 0.00$$

$$s_2 = 4.00$$

$$s_3 = 0.00$$