

Instituto Tecnológico de Costa Rica

Operations Research - Semester II

Knapsack Problem

Members:

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Maximize

$$Z = 60.00x_1 + 35.00x_2 + 20.00x_3$$

Subject to

$$\begin{aligned}8.00x_1 + 6.00x_2 + 1.00x_3 &\leq 48.00 \\4.00x_1 + 2.00x_2 + 1.50x_3 &\leq 20.00 \\2.00x_1 + 1.50x_2 + 0.50x_3 &\leq 8.00 \\0.00x_1 + 1.00x_2 + 0.00x_3 &\leq 5.00\end{aligned}$$

Simplex Table

	Z	x_1	x_2	x_3	s_1	s_2	s_3	s_4	b
	1.00	-60.00	-35.00	-20.00	0.00	0.00	0.00	0.00	0.00
	0.00	8.00	6.00	1.00	1.00	0.00	0.00	0.00	48.00
	0.00	4.00	2.00	1.50	0.00	1.00	0.00	0.00	20.00
	0.00	2.00	1.50	0.50	0.00	0.00	1.00	0.00	8.00
	0.00	0.00	1.00	0.00	0.00	0.00	0.00	1.00	5.00

Intermediate Tables

Pivoting 1

Most Negative

Column 2 (-60.00)

	Z	x_1	x_2	x_3	s_1	s_2	s_3	s_4	b
1.00	-60.00	-35.00	-20.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	8.00	6.00	1.00	1.00	0.00	0.00	0.00	0.00	48.00
0.00	4.00	2.00	1.50	0.00	1.00	0.00	0.00	0.00	20.00
0.00	2.00	1.50	0.50	0.00	0.00	1.00	0.00	0.00	8.00
0.00	0.00	1.00	0.00	0.00	0.00	0.00	1.00	0.00	5.00

Fractions

	Z	x_1	x_2	x_3	s_1	s_2	s_3	s_4	b	Frac
1.00	-60.00	-35.00	-20.00	0.00	0.00	0.00	0.00	0.00	0.00	
0.00	8.00	6.00	1.00	1.00	0.00	0.00	0.00	0.00	48.00	6.00
0.00	4.00	2.00	1.50	0.00	1.00	0.00	0.00	0.00	20.00	5.00
0.00	2.00	1.50	0.50	0.00	0.00	1.00	0.00	0.00	8.00	4.00
0.00	0.00	1.00	0.00	0.00	0.00	0.00	1.00	0.00	5.00	inf

$$48.00/8.00 = 6.00$$

$$20.00/4.00 = 5.00$$

$$8.00/2.00 = 4.00$$

$$5.00/0.00 = inf$$

Smallest fraction: 4.00 → Pivot: row 4

Pivot

	Z	x_1	x_2	x_3	s_1	s_2	s_3	s_4	b
1.00	-60.00	-35.00	-20.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	8.00	6.00	1.00	1.00	0.00	0.00	0.00	0.00	48.00
0.00	4.00	2.00	1.50	0.00	1.00	0.00	0.00	0.00	20.00
0.00	2.00	1.50	0.50	0.00	0.00	1.00	0.00	0.00	8.00
0.00	0.00	1.00	0.00	0.00	0.00	0.00	1.00	0.00	5.00

Canonization

$$R_4 \leftarrow R_4/2.00$$

Z	x_1	x_2	x_3	s_1	s_2	s_3	s_4	b
1.00	-60.00	-35.00	-20.00	0.00	0.00	0.00	0.00	0.00
0.00	8.00	6.00	1.00	1.00	0.00	0.00	0.00	48.00
0.00	4.00	2.00	1.50	0.00	1.00	0.00	0.00	20.00
0.00	1.00	0.75	0.25	0.00	0.00	0.50	0.00	4.00
0.00	0.00	1.00	0.00	0.00	0.00	0.00	1.00	5.00

$$R_1 \leftarrow R_1 + 60.00R_4$$

Z	x_1	x_2	x_3	s_1	s_2	s_3	s_4	b
1.00	0.00	10.00	-5.00	0.00	0.00	30.00	0.00	240.00
0.00	8.00	6.00	1.00	1.00	0.00	0.00	0.00	48.00
0.00	4.00	2.00	1.50	0.00	1.00	0.00	0.00	20.00
0.00	1.00	0.75	0.25	0.00	0.00	0.50	0.00	4.00
0.00	0.00	1.00	0.00	0.00	0.00	0.00	1.00	5.00

$$R_2 \leftarrow R_2 + -8.00R_4$$

Z	x_1	x_2	x_3	s_1	s_2	s_3	s_4	b
1.00	0.00	10.00	-5.00	0.00	0.00	30.00	0.00	240.00
0.00	0.00	0.00	-1.00	1.00	0.00	-4.00	0.00	16.00
0.00	4.00	2.00	1.50	0.00	1.00	0.00	0.00	20.00
0.00	1.00	0.75	0.25	0.00	0.00	0.50	0.00	4.00
0.00	0.00	1.00	0.00	0.00	0.00	0.00	1.00	5.00

$$R_3 \leftarrow R_3 + -4.00R_4$$

Z	x_1	x_2	x_3	s_1	s_2	s_3	s_4	b
1.00	0.00	10.00	-5.00	0.00	0.00	30.00	0.00	240.00
0.00	0.00	0.00	-1.00	1.00	0.00	-4.00	0.00	16.00
0.00	0.00	-1.00	0.50	0.00	1.00	-2.00	0.00	4.00
0.00	1.00	0.75	0.25	0.00	0.00	0.50	0.00	4.00
0.00	0.00	1.00	0.00	0.00	0.00	0.00	1.00	5.00

$$R_5 \leftarrow R_5 + -0.00R_4$$

Z	x_1	x_2	x_3	s_1	s_2	s_3	s_4	b
1.00	0.00	10.00	-5.00	0.00	0.00	30.00	0.00	240.00
0.00	0.00	0.00	-1.00	1.00	0.00	-4.00	0.00	16.00
0.00	0.00	-1.00	0.50	0.00	1.00	-2.00	0.00	4.00
0.00	1.00	0.75	0.25	0.00	0.00	0.50	0.00	4.00
0.00	0.00	1.00	0.00	0.00	0.00	0.00	1.00	5.00

Pivot Result

Z	x_1	x_2	x_3	s_1	s_2	s_3	s_4	b
1.00	0.00	10.00	-5.00	0.00	0.00	30.00	0.00	240.00
0.00	0.00	0.00	-1.00	1.00	0.00	-4.00	0.00	16.00
0.00	0.00	-1.00	0.50	0.00	1.00	-2.00	0.00	4.00
0.00	1.00	0.75	0.25	0.00	0.00	0.50	0.00	4.00
0.00	0.00	1.00	0.00	0.00	0.00	0.00	1.00	5.00

Pivoting 2

Most Negative

Column 4 (-5.00)

Z	x_1	x_2	x_3	s_1	s_2	s_3	s_4	b
1.00	0.00	10.00	-5.00	0.00	0.00	30.00	0.00	240.00
0.00	0.00	0.00	-1.00	1.00	0.00	-4.00	0.00	16.00
0.00	0.00	-1.00	0.50	0.00	1.00	-2.00	0.00	4.00
0.00	1.00	0.75	0.25	0.00	0.00	0.50	0.00	4.00
0.00	0.00	1.00	0.00	0.00	0.00	0.00	1.00	5.00

Fractions

Z	x_1	x_2	x_3	s_1	s_2	s_3	s_4	b	Frac
1.00	0.00	10.00	-5.00	0.00	0.00	30.00	0.00	240.00	
0.00	0.00	0.00	-1.00	1.00	0.00	-4.00	0.00	16.00	-16.00
0.00	0.00	-1.00	0.50	0.00	1.00	-2.00	0.00	4.00	8.00
0.00	1.00	0.75	0.25	0.00	0.00	0.50	0.00	4.00	16.00
0.00	0.00	1.00	0.00	0.00	0.00	0.00	1.00	5.00	inf

$$16.00 / -1.00 = -16.00$$

$$4.00 / 0.50 = 8.00$$

$$4.00 / 0.25 = 16.00$$

$$5.00 / 0.00 = \text{inf}$$

Smallest fraction: 8.00 → Pivot: row 3

Pivot

Z	x_1	x_2	x_3	s_1	s_2	s_3	s_4	b
1.00	0.00	10.00	-5.00	0.00	0.00	30.00	0.00	240.00
0.00	0.00	0.00	-1.00	1.00	0.00	-4.00	0.00	16.00
0.00	0.00	-1.00	0.50	0.00	1.00	-2.00	0.00	4.00
0.00	1.00	0.75	0.25	0.00	0.00	0.50	0.00	4.00
0.00	0.00	1.00	0.00	0.00	0.00	0.00	1.00	5.00

Canonization

$$R_3 \leftarrow R_3 / 0.50$$

Z	x_1	x_2	x_3	s_1	s_2	s_3	s_4	b
1.00	0.00	10.00	-5.00	0.00	0.00	30.00	0.00	240.00
0.00	0.00	0.00	-1.00	1.00	0.00	-4.00	0.00	16.00
0.00	0.00	-2.00	1.00	0.00	2.00	-4.00	0.00	8.00
0.00	1.00	0.75	0.25	0.00	0.00	0.50	0.00	4.00
0.00	0.00	1.00	0.00	0.00	0.00	0.00	1.00	5.00

$$R_1 \leftarrow R_1 + 5.00R_3$$

	Z	x_1	x_2	x_3	s_1	s_2	s_3	s_4	b
	1.00	0.00	0.00	0.00	0.00	10.00	10.00	0.00	280.00
	0.00	0.00	0.00	-1.00	1.00	0.00	-4.00	0.00	16.00
	0.00	0.00	-2.00	1.00	0.00	2.00	-4.00	0.00	8.00
	0.00	1.00	0.75	0.25	0.00	0.00	0.50	0.00	4.00
	0.00	0.00	1.00	0.00	0.00	0.00	0.00	1.00	5.00

$$R_2 \leftarrow R_2 + 1.00R_3$$

	Z	x_1	x_2	x_3	s_1	s_2	s_3	s_4	b
	1.00	0.00	0.00	0.00	0.00	10.00	10.00	0.00	280.00
	0.00	0.00	-2.00	0.00	1.00	2.00	-8.00	0.00	24.00
	0.00	0.00	-2.00	1.00	0.00	2.00	-4.00	0.00	8.00
	0.00	1.00	0.75	0.25	0.00	0.00	0.50	0.00	4.00
	0.00	0.00	1.00	0.00	0.00	0.00	0.00	1.00	5.00

$$R_4 \leftarrow R_4 + -0.25R_3$$

	Z	x_1	x_2	x_3	s_1	s_2	s_3	s_4	b
	1.00	0.00	0.00	0.00	0.00	10.00	10.00	0.00	280.00
	0.00	0.00	-2.00	0.00	1.00	2.00	-8.00	0.00	24.00
	0.00	0.00	-2.00	1.00	0.00	2.00	-4.00	0.00	8.00
	0.00	1.00	1.25	0.00	0.00	-0.50	1.50	0.00	2.00
	0.00	0.00	1.00	0.00	0.00	0.00	0.00	1.00	5.00

$$R_5 \leftarrow R_5 + -0.00R_3$$

	Z	x_1	x_2	x_3	s_1	s_2	s_3	s_4	b
	1.00	0.00	0.00	0.00	0.00	10.00	10.00	0.00	280.00
	0.00	0.00	-2.00	0.00	1.00	2.00	-8.00	0.00	24.00
	0.00	0.00	-2.00	1.00	0.00	2.00	-4.00	0.00	8.00
	0.00	1.00	1.25	0.00	0.00	-0.50	1.50	0.00	2.00
	0.00	0.00	1.00	0.00	0.00	0.00	0.00	1.00	5.00

Pivot Result

	Z	x_1	x_2	x_3	s_1	s_2	s_3	s_4	b
	1.00	0.00	0.00	0.00	0.00	10.00	10.00	0.00	280.00
	0.00	0.00	-2.00	0.00	1.00	2.00	-8.00	0.00	24.00
	0.00	0.00	-2.00	1.00	0.00	2.00	-4.00	0.00	8.00
	0.00	1.00	1.25	0.00	0.00	-0.50	1.50	0.00	2.00
	0.00	0.00	1.00	0.00	0.00	0.00	0.00	1.00	5.00

Results

	Z	x_1	x_2	x_3	s_1	s_2	s_3	s_4	b
	1.00	0.00	0.00	0.00	0.00	10.00	10.00	0.00	280.00
	0.00	0.00	-2.00	0.00	1.00	2.00	-8.00	0.00	24.00
	0.00	0.00	-2.00	1.00	0.00	2.00	-4.00	0.00	8.00
	0.00	1.00	1.25	0.00	0.00	-0.50	1.50	0.00	2.00
	0.00	0.00	1.00	0.00	0.00	0.00	0.00	1.00	5.00

Objective Value

$$Z = 280.00$$

Variables

$$x_1 = 2.00$$

$$x_2 = 0.00$$

$$x_3 = 8.00$$

Slack or Surplus

$$s_1 = 24.00$$

$$s_2 = 0.00$$

$$s_3 = 0.00$$

$$s_4 = 5.00$$

Multiple Solutions

A non-basic variable has a 0 on its first row, allowing us to pivot again and find another optimal solution.

Pivoting 3

Column 3 (0.00)

	Z	x_1	x_2	x_3	s_1	s_2	s_3	s_4	b
	1.00	0.00	0.00	0.00	0.00	10.00	10.00	0.00	280.00
	0.00	0.00	-2.00	0.00	1.00	2.00	-8.00	0.00	24.00
	0.00	0.00	-2.00	1.00	0.00	2.00	-4.00	0.00	8.00
	0.00	1.00	1.25	0.00	0.00	-0.50	1.50	0.00	2.00
	0.00	0.00	1.00	0.00	0.00	0.00	0.00	1.00	5.00

Fractions

	Z	x_1	x_2	x_3	s_1	s_2	s_3	s_4	b	Frac
	1.00	0.00	0.00	0.00	0.00	10.00	10.00	0.00	280.00	
	0.00	0.00	-2.00	0.00	1.00	2.00	-8.00	0.00	24.00	-12.00
	0.00	0.00	-2.00	1.00	0.00	2.00	-4.00	0.00	8.00	-4.00
	0.00	1.00	1.25	0.00	0.00	-0.50	1.50	0.00	2.00	1.60
	0.00	0.00	1.00	0.00	0.00	0.00	0.00	1.00	5.00	5.00

$$24.00 / -2.00 = -12.00$$

$$8.00 / -2.00 = -4.00$$

$$2.00 / 1.25 = 1.60$$

$$5.00 / 1.00 = 5.00$$

Smallest fraction: 1.60 → Pivot: row 4

Pivot

	Z	x_1	x_2	x_3	s_1	s_2	s_3	s_4	b
	1.00	0.00	0.00	0.00	0.00	10.00	10.00	0.00	280.00
	0.00	0.00	-2.00	0.00	1.00	2.00	-8.00	0.00	24.00
	0.00	0.00	-2.00	1.00	0.00	2.00	-4.00	0.00	8.00
	0.00	1.00	1.25	0.00	0.00	-0.50	1.50	0.00	2.00
	0.00	0.00	1.00	0.00	0.00	0.00	0.00	1.00	5.00

Canonization

$$R_4 \leftarrow R_4 / 1.25$$

Z	x_1	x_2	x_3	s_1	s_2	s_3	s_4	b
1.00	0.00	0.00	0.00	0.00	10.00	10.00	0.00	280.00
0.00	0.00	-2.00	0.00	1.00	2.00	-8.00	0.00	24.00
0.00	0.00	-2.00	1.00	0.00	2.00	-4.00	0.00	8.00
0.00	0.80	1.00	0.00	0.00	-0.40	1.20	0.00	1.60
0.00	0.00	1.00	0.00	0.00	0.00	0.00	1.00	5.00

$$R_1 \leftarrow R_1 + -0.00R_4$$

Z	x_1	x_2	x_3	s_1	s_2	s_3	s_4	b
1.00	0.00	0.00	0.00	0.00	10.00	10.00	0.00	280.00
0.00	0.00	-2.00	0.00	1.00	2.00	-8.00	0.00	24.00
0.00	0.00	-2.00	1.00	0.00	2.00	-4.00	0.00	8.00
0.00	0.80	1.00	0.00	0.00	-0.40	1.20	0.00	1.60
0.00	0.00	1.00	0.00	0.00	0.00	0.00	1.00	5.00

$$R_2 \leftarrow R_2 + 2.00R_4$$

Z	x_1	x_2	x_3	s_1	s_2	s_3	s_4	b
1.00	0.00	0.00	0.00	0.00	10.00	10.00	0.00	280.00
0.00	1.60	0.00	0.00	1.00	1.20	-5.60	0.00	27.20
0.00	0.00	-2.00	1.00	0.00	2.00	-4.00	0.00	8.00
0.00	0.80	1.00	0.00	0.00	-0.40	1.20	0.00	1.60
0.00	0.00	1.00	0.00	0.00	0.00	0.00	1.00	5.00

$$R_3 \leftarrow R_3 + 2.00R_4$$

Z	x_1	x_2	x_3	s_1	s_2	s_3	s_4	b
1.00	0.00	0.00	0.00	0.00	10.00	10.00	0.00	280.00
0.00	1.60	0.00	0.00	1.00	1.20	-5.60	0.00	27.20
0.00	1.60	0.00	1.00	0.00	1.20	-1.60	0.00	11.20
0.00	0.80	1.00	0.00	0.00	-0.40	1.20	0.00	1.60
0.00	0.00	1.00	0.00	0.00	0.00	0.00	1.00	5.00

$$R_5 \leftarrow R_5 + -1.00R_4$$

Z	x_1	x_2	x_3	s_1	s_2	s_3	s_4	b
1.00	0.00	0.00	0.00	0.00	10.00	10.00	0.00	280.00
0.00	1.60	0.00	0.00	1.00	1.20	-5.60	0.00	27.20
0.00	1.60	0.00	1.00	0.00	1.20	-1.60	0.00	11.20
0.00	0.80	1.00	0.00	0.00	-0.40	1.20	0.00	1.60
0.00	-0.80	0.00	0.00	0.00	0.40	-1.20	1.00	3.40

Pivot Result

	Z	x_1	x_2	x_3	s_1	s_2	s_3	s_4	b
	1.00	0.00	0.00	0.00	0.00	10.00	10.00	0.00	280.00
	0.00	1.60	0.00	0.00	1.00	1.20	-5.60	0.00	27.20
	0.00	1.60	0.00	1.00	0.00	1.20	-1.60	0.00	11.20
	0.00	0.80	1.00	0.00	0.00	-0.40	1.20	0.00	1.60
	0.00	-0.80	0.00	0.00	0.00	0.40	-1.20	1.00	3.40

Results

Z	x_1	x_2	x_3	s_1	s_2	s_3	s_4	b
1.00	0.00	0.00	0.00	0.00	10.00	10.00	0.00	280.00
0.00	1.60	0.00	0.00	1.00	1.20	-5.60	0.00	27.20
0.00	1.60	0.00	1.00	0.00	1.20	-1.60	0.00	11.20
0.00	0.80	1.00	0.00	0.00	-0.40	1.20	0.00	1.60
0.00	-0.80	0.00	0.00	0.00	0.40	-1.20	1.00	3.40

Objective Value

$$Z = 280.00$$

Variables

$$x_1 = 0.00$$

$$x_2 = 1.60$$

$$x_3 = 11.20$$

Slack or Surplus

$$s_1 = 27.20$$

$$s_2 = 0.00$$

$$s_3 = 0.00$$

$$s_4 = 3.40$$