

Instituto Tecnológico de Costa Rica

Operations Research - Semester II

Knapsack Problem

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Unbounded Problem

Maximize

$$Z = 36.00x_1 + 30.00x_2 - 3.00x_3 - 4.00x_4$$

Subject to

$$\begin{aligned} 1.00x_1 + 1.00x_2 - 1.00x_3 + 0.00x_4 &\leq 5.00 \\ 6.00x_1 + 5.00x_2 + 0.00x_3 - 1.00x_4 &\leq 10.00 \end{aligned}$$

Simplex Table

	Z	x ₁	x ₂	x ₃	x ₄	s ₁	s ₂	b
	1.00	-36.00	-30.00	3.00	4.00	0.00	0.00	0.00
	0.00	1.00	1.00	-1.00	0.00	1.00	0.00	5.00
	0.00	6.00	5.00	0.00	-1.00	0.00	1.00	10.00

Intermediate Tables

Pivoting 1

Most Negative

Column 2 (-36.00)

	Z	x1	x2	x3	x4	s ₁	s ₂	b
	1.00	-36.00	-30.00	3.00	4.00	0.00	0.00	0.00
	0.00	1.00	1.00	-1.00	0.00	1.00	0.00	5.00
	0.00	6.00	5.00	0.00	-1.00	0.00	1.00	10.00

Fractions

	Z	x1	x2	x3	x4	s ₁	s ₂	b	Frac
	1.00	-36.00	-30.00	3.00	4.00	0.00	0.00	0.00	
	0.00	1.00	1.00	-1.00	0.00	1.00	0.00	5.00	5.00
	0.00	6.00	5.00	0.00	-1.00	0.00	1.00	10.00	1.67

$$5.00/1.00 = 5.00$$

$$10.00/6.00 = 1.67$$

Smallest fraction: 1.67 \rightarrow Pivot: row 3

Pivot

	Z	x1	x2	x3	x4	s ₁	s ₂	b
	1.00	-36.00	-30.00	3.00	4.00	0.00	0.00	0.00
	0.00	1.00	1.00	-1.00	0.00	1.00	0.00	5.00
	0.00	6.00	5.00	0.00	-1.00	0.00	1.00	10.00

Canonization

$$R_3 \leftarrow R_3/6.00$$

	Z	x1	x2	x3	x4	s ₁	s ₂	b
	1.00	-36.00	-30.00	3.00	4.00	0.00	0.00	0.00
	0.00	1.00	1.00	-1.00	0.00	1.00	0.00	5.00
	0.00	1.00	0.83	0.00	-0.17	0.00	0.17	1.67

$$R_1 \leftarrow R_1 + 36.00R_3$$

	Z	x1	x2	x3	x4	s ₁	s ₂	b
	1.00	0.00	0.00	3.00	-2.00	0.00	6.00	60.00
	0.00	1.00	1.00	-1.00	0.00	1.00	0.00	5.00
	0.00	1.00	0.83	0.00	-0.17	0.00	0.17	1.67

$$R_2 \leftarrow R_2 + -1.00R_3$$

	Z	x1	x2	x3	x4	s ₁	s ₂	b
	1.00	0.00	0.00	3.00	-2.00	0.00	6.00	60.00
	0.00	0.00	0.17	-1.00	0.17	1.00	-0.17	3.33
	0.00	1.00	0.83	0.00	-0.17	0.00	0.17	1.67

Pivot Result

	Z	x1	x2	x3	x4	s ₁	s ₂	b
	1.00	0.00	0.00	3.00	-2.00	0.00	6.00	60.00
	0.00	0.00	0.17	-1.00	0.17	1.00	-0.17	3.33
	0.00	1.00	0.83	0.00	-0.17	0.00	0.17	1.67

Pivoting 2

Most Negative

Column 5 (-2.00)

	Z	x1	x2	x3	x4	s ₁	s ₂	b
	1.00	0.00	0.00	3.00	-2.00	0.00	6.00	60.00
	0.00	0.00	0.17	-1.00	0.17	1.00	-0.17	3.33
	0.00	1.00	0.83	0.00	-0.17	0.00	0.17	1.67

Fractions

	Z	x1	x2	x3	x4	s ₁	s ₂	b	Frac
	1.00	0.00	0.00	3.00	-2.00	0.00	6.00	60.00	
	0.00	0.00	0.17	-1.00	0.17	1.00	-0.17	3.33	20.00
	0.00	1.00	0.83	0.00	-0.17	0.00	0.17	1.67	-10.00

$$3.33/0.17 = 20.00$$

$$1.67/-0.17 = -10.00$$

Smallest fraction: 20.00 \rightarrow Pivot: row 2

Pivot

	Z	x1	x2	x3	x4	s ₁	s ₂	b
	1.00	0.00	0.00	3.00	-2.00	0.00	6.00	60.00
	0.00	0.00	0.17	-1.00	0.17	1.00	-0.17	3.33
	0.00	1.00	0.83	0.00	-0.17	0.00	0.17	1.67

Canonization

$$R_2 \leftarrow R_2/0.17$$

	Z	x1	x2	x3	x4	s ₁	s ₂	b
	1.00	0.00	0.00	3.00	-2.00	0.00	6.00	60.00
	0.00	0.00	1.00	-6.00	1.00	6.00	-1.00	20.00
	0.00	1.00	0.83	0.00	-0.17	0.00	0.17	1.67

$$R_1 \leftarrow R_1 + 2.00R_2$$

	Z	x1	x2	x3	x4	s ₁	s ₂	b
	1.00	0.00	2.00	-9.00	0.00	12.00	4.00	100.00
	0.00	0.00	1.00	-6.00	1.00	6.00	-1.00	20.00
	0.00	1.00	0.83	0.00	-0.17	0.00	0.17	1.67

$$R_3 \leftarrow R_3 + 0.17R_2$$

	Z	x1	x2	x3	x4	s ₁	s ₂	b
	1.00	0.00	2.00	-9.00	0.00	12.00	4.00	100.00
	0.00	0.00	1.00	-6.00	1.00	6.00	-1.00	20.00
	0.00	1.00	1.00	-1.00	0.00	1.00	0.00	5.00

Pivot Result

	Z	x1	x2	x3	x4	s ₁	s ₂	b
	1.00	0.00	2.00	-9.00	0.00	12.00	4.00	100.00
	0.00	0.00	1.00	-6.00	1.00	6.00	-1.00	20.00
	0.00	1.00	1.00	-1.00	0.00	1.00	0.00	5.00

Pivoting 3

Most Negative

Column 4 (-9.00)

	Z	x1	x2	x3	x4	s ₁	s ₂	b
	1.00	0.00	2.00	-9.00	0.00	12.00	4.00	100.00
	0.00	0.00	1.00	-6.00	1.00	6.00	-1.00	20.00
	0.00	1.00	1.00	-1.00	0.00	1.00	0.00	5.00

Fractions

	Z	x1	x2	x3	x4	s ₁	s ₂	b	Frac
	1.00	0.00	2.00	-9.00	0.00	12.00	4.00	100.00	
	0.00	0.00	1.00	-6.00	1.00	6.00	-1.00	20.00	-3.33
	0.00	1.00	1.00	-1.00	0.00	1.00	0.00	5.00	-5.00

$$20.00 / -6.00 = -3.33$$

$$5.00 / -1.00 = -5.00$$

Unbounded Problem!

This problem is unbounded. The variable x3 can be infinitely decreased, making Z have an infinitely negative value. This can be solved by adding restrictions that cap the value of x3. Please re-model it and try again!