Floyd's Algorithm: Shortest Path Problem

Project 1

Daniel Romero - 2023059668
Adrián Zamora - 2023083307

Escuela de Ingeniería en Computación
Instituto Tecnológico de Costa Rica
Semester II 2025

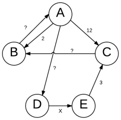
September 12, 2025

Robert W. Floyd

- American computer scientist (1936 - 2001)
- Studied physics at the University of Chicago, B.A. at age **19**
- No formal CS degree $\rightarrow$ self-taught programming and algorithms
- Worked as a math teacher, then in computing $\rightarrow$ professor at Stanford
- Published foundational papers in computational theory
- Collaborated with extbfDonald Knuth on "The Art of Computer Programming"
- Created cycle detection and **shortest paths** algorithms

Floyd Algorithm (Floyd–Warshall Algorithm)

- The Floyd Algorithm computes the **shortest paths between all pairs of nodes** in a graph with edge weights
- Works for both directed and undirected graphs
- **Time Complexity:** $O(n^3)$
  - For each new node considered as an intermediate step, an entire $n \times n$ table is updated
- **Space Complexity:** $O(n^2)$
  - All calculations are performed within the same distance table
- Based on the principle of **dynamic programming**
- It has applications in network routing and navigation systems



	A	B	C	D	E
A	0	2	10	5	7
B	3	0	13	8	10
C	7	4	0	12	14
D	12	9	5	0	X
E	10	7	3	15	0

Floyd Algorithm Overview

- There are two tables: **D** and **P**.
- **D table:** stores distances between any two nodes.
  - $D[i][i] = 0$ (distance from a node to itself).
  - If edge (i, j) exists, then $D[i][j] =$ weight of that edge, otherwise $D[i][j] = \infty$.
- **P table:** stores path reconstruction information.
  - P table is initialized with **0** on every cell (This means that there is a direct path between the two nodes)
- **Algorithm process:**
  - For each node $k = 1$ to n (considered as an intermediate node):
  - For each pair of nodes (i, j), check if going through k is shorter:

$$D(k)[i][j] = \min\{D(k-1)[i][j],\ D(k-1)[i][k] + D(k-1)[k][j]\}$$

  - Update $P[i][j]$ with the value k if there was a change in $D[i][j]$. (meaning we go through k to get from i to j)
- After all iterations:
  - D contains shortest distances.
  - P contains the information to reconstruct the shortest paths.

Graph

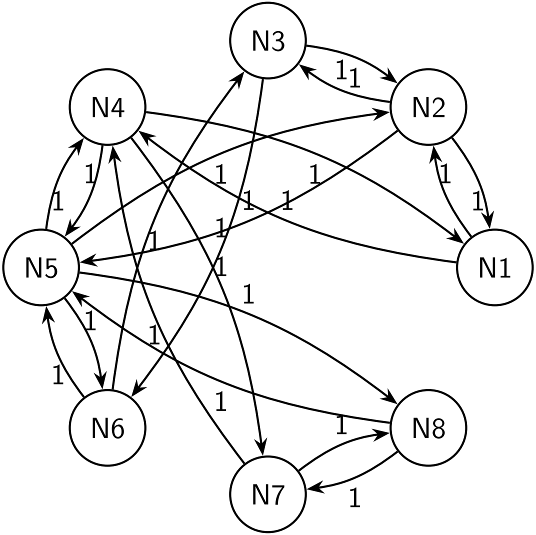


Table D(0)

	N1	N2	N3	N4	N5	N6	N7	N8
N1	0	1	∞	1	∞	∞	∞	∞
N2	1	0	1	∞	1	∞	∞	∞
N3	∞	1	0	∞	∞	1	∞	∞
N4	1	∞	∞	0	1	∞	1	∞
N5	∞	1	∞	1	0	1	∞	1
N6	∞	∞	1	∞	1	0	∞	∞
N7	∞	∞	∞	1	∞	∞	0	1
N8	∞	∞	∞	∞	1	∞	1	0

Table D(1)

	N1	N2	N3	N4	N5	N6	N7	N8
N1	0	1	∞	1	∞	∞	∞	∞
N2	1	0	1	2	1	∞	∞	∞
N3	∞	1	0	∞	∞	1	∞	∞
N4	1	2	∞	0	1	∞	1	∞
N5	∞	1	∞	1	0	1	∞	1
N6	∞	∞	1	∞	1	0	∞	∞
N7	∞	∞	∞	1	∞	∞	0	1
N8	∞	∞	∞	∞	1	∞	1	0

Table P

	N1	N2	N3	N4	N5	N6	N7	N8
N1	0	0	0	0	0	0	0	0
N2	0	0	0	1	0	0	0	0
N3	0	0	0	0	0	0	0	0
N4	0	1	0	0	0	0	0	0
N5	0	0	0	0	0	0	0	0
N6	0	0	0	0	0	0	0	0
N7	0	0	0	0	0	0	0	0
N8	0	0	0	0	0	0	0	0

Table D(2)

	N1	N2	N3	N4	N5	N6	N7	N8
N1	0	1	2	1	2	∞	∞	∞
N2	1	0	1	2	1	∞	∞	∞
N3	2	1	0	3	2	1	∞	∞
N4	1	2	3	0	1	∞	1	∞
N5	2	1	2	1	0	1	∞	1
N6	∞	∞	1	∞	1	0	∞	∞
N7	∞	∞	∞	1	∞	∞	0	1
N8	∞	∞	∞	∞	1	∞	1	0

Table P

	N1	N2	N3	N4	N5	N6	N7	N8
N1	0	0	2	0	2	0	0	0
N2	0	0	0	1	0	0	0	0
N3	2	0	0	2	2	0	0	0
N4	0	1	2	0	0	0	0	0
N5	2	0	2	0	0	0	0	0
N6	0	0	0	0	0	0	0	0
N7	0	0	0	0	0	0	0	0
N8	0	0	0	0	0	0	0	0

Table D(3)

	N1	N2	N3	N4	N5	N6	N7	N8
N1	0	1	2	1	2	3	∞	∞
N2	1	0	1	2	1	2	∞	∞
N3	2	1	0	3	2	1	∞	∞
N4	1	2	3	0	1	4	1	∞
N5	2	1	2	1	0	1	∞	1
N6	3	2	1	4	1	0	∞	∞
N7	∞	∞	∞	1	∞	∞	0	1
N8	∞	∞	∞	∞	1	∞	1	0

Table P

	N1	N2	N3	N4	N5	N6	N7	N8
N1	0	0	2	0	2	3	0	0
N2	0	0	0	1	0	3	0	0
N3	2	0	0	2	2	0	0	0
N4	0	1	2	0	0	3	0	0
N5	2	0	2	0	0	0	0	0
N6	3	3	0	3	0	0	0	0
N7	0	0	0	0	0	0	0	0
N8	0	0	0	0	0	0	0	0

Table D(4)

	N1	N2	N3	N4	N5	N6	N7	N8
N1	0	1	2	1	2	3	2	∞
N2	1	0	1	2	1	2	3	∞
N3	2	1	0	3	2	1	4	∞
N4	1	2	3	0	1	4	1	∞
N5	2	1	2	1	0	1	2	1
N6	3	2	1	4	1	0	5	∞
N7	2	3	4	1	2	5	0	1
N8	∞	∞	∞	∞	1	∞	1	0

Table P

	N1	N2	N3	N4	N5	N6	N7	N8
N1	0	0	2	0	2	3	4	0
N2	0	0	0	1	0	3	4	0
N3	2	0	0	2	2	0	4	0
N4	0	1	2	0	0	3	0	0
N5	2	0	2	0	0	0	4	0
N6	3	3	0	3	0	0	4	0
N7	4	4	4	0	4	4	0	0
N8	0	0	0	0	0	0	0	0

Table D(5)

	N1	N2	N3	N4	N5	N6	N7	N8
N1	0	1	2	1	2	3	2	3
N2	1	0	1	2	1	2	3	2
N3	2	1	0	3	2	1	4	3
N4	1	2	3	0	1	2	1	2
N5	2	1	2	1	0	1	2	1
N6	3	2	1	2	1	0	3	2
N7	2	3	4	1	2	3	0	1
N8	3	2	3	2	1	2	1	0

Table P

	N1	N2	N3	N4	N5	N6	N7	N8
N1	0	0	2	0	2	3	4	5
N2	0	0	0	1	0	3	4	5
N3	2	0	0	2	2	0	4	5
N4	0	1	2	0	0	5	0	5
N5	2	0	2	0	0	0	4	0
N6	3	3	0	5	0	0	5	5
N7	4	4	4	0	4	5	0	0
N8	5	5	5	5	0	5	0	0

Table D(6)

	N1	N2	N3	N4	N5	N6	N7	N8
N1	0	1	2	1	2	3	2	3
N2	1	0	1	2	1	2	3	2
N3	2	1	0	3	2	1	4	3
N4	1	2	3	0	1	2	1	2
N5	2	1	2	1	0	1	2	1
N6	3	2	1	2	1	0	3	2
N7	2	3	4	1	2	3	0	1
N8	3	2	3	2	1	2	1	0

Table P

	N1	N2	N3	N4	N5	N6	N7	N8
N1	0	0	2	0	2	3	4	5
N2	0	0	0	1	0	3	4	5
N3	2	0	0	2	2	0	4	5
N4	0	1	2	0	0	5	0	5
N5	2	0	2	0	0	0	4	0
N6	3	3	0	5	0	0	5	5
N7	4	4	4	0	4	5	0	0
N8	5	5	5	5	0	5	0	0

Table D(7)

	N1	N2	N3	N4	N5	N6	N7	N8
N1	0	1	2	1	2	3	2	3
N2	1	0	1	2	1	2	3	2
N3	2	1	0	3	2	1	4	3
N4	1	2	3	0	1	2	1	2
N5	2	1	2	1	0	1	2	1
N6	3	2	1	2	1	0	3	2
N7	2	3	4	1	2	3	0	1
N8	3	2	3	2	1	2	1	0

Table P

	N1	N2	N3	N4	N5	N6	N7	N8
N1	0	0	2	0	2	3	4	5
N2	0	0	0	1	0	3	4	5
N3	2	0	0	2	2	0	4	5
N4	0	1	2	0	0	5	0	5
N5	2	0	2	0	0	0	4	0
N6	3	3	0	5	0	0	5	5
N7	4	4	4	0	4	5	0	0
N8	5	5	5	5	0	5	0	0

Table D(8)

	N1	N2	N3	N4	N5	N6	N7	N8
N1	0	1	2	1	2	3	2	3
N2	1	0	1	2	1	2	3	2
N3	2	1	0	3	2	1	4	3
N4	1	2	3	0	1	2	1	2
N5	2	1	2	1	0	1	2	1
N6	3	2	1	2	1	0	3	2
N7	2	3	4	1	2	3	0	1
N8	3	2	3	2	1	2	1	0

Table P

	N1	N2	N3	N4	N5	N6	N7	N8
N1	0	0	2	0	2	3	4	5
N2	0	0	0	1	0	3	4	5
N3	2	0	0	2	2	0	4	5
N4	0	1	2	0	0	5	0	5
N5	2	0	2	0	0	0	4	0
N6	3	3	0	5	0	0	5	5
N7	4	4	4	0	4	5	0	0
N8	5	5	5	5	0	5	0	0

Shortest Paths from N1

- to N2 (1): N1 $\rightarrow$ N2
- to N3 (2): N1 $\rightarrow$ N2 $\rightarrow$ N3
- to N4 (1): N1 $\rightarrow$ N4
- to N5 (2): N1 $\rightarrow$ N2 $\rightarrow$ N5
- to N6 (3): N1 $\rightarrow$ N3 $\rightarrow$ N6
- to N7 (2): N1 $\rightarrow$ N4 $\rightarrow$ N7
- to N8 (3): N1 $\rightarrow$ N5 $\rightarrow$ N8

Shortest Paths from N2

- to N1 (1): N2 $\rightarrow$ N1
- to N3 (1): N2 $\rightarrow$ N3
- to N4 (2): N2 $\rightarrow$ N1 $\rightarrow$ N4
- to N5 (1): N2 $\rightarrow$ N5
- to N6 (2): N2 $\rightarrow$ N3 $\rightarrow$ N6
- to N7 (3): N2 $\rightarrow$ N4 $\rightarrow$ N7
- to N8 (2): N2 $\rightarrow$ N5 $\rightarrow$ N8

Shortest Paths from N3

- to N1 (2): N3 $\rightarrow$ N2 $\rightarrow$ N1
- to N2 (1): N3 $\rightarrow$ N2
- to N4 (3): N3 $\rightarrow$ N2 $\rightarrow$ N1 $\rightarrow$ N4
- to N5 (2): N3 $\rightarrow$ N2 $\rightarrow$ N5
- to N6 (1): N3 $\rightarrow$ N6
- to N7 (4): N3 $\rightarrow$ N4 $\rightarrow$ N7
- to N8 (3): N3 $\rightarrow$ N5 $\rightarrow$ N8

Shortest Paths from N4

- to N1 (1): N4 $\rightarrow$ N1
- to N2 (2): N4 $\rightarrow$ N1 $\rightarrow$ N2
- to N3 (3): N4 $\rightarrow$ N2 $\rightarrow$ N3
- to N5 (1): N4 $\rightarrow$ N5
- to N6 (2): N4 $\rightarrow$ N5 $\rightarrow$ N6
- to N7 (1): N4 $\rightarrow$ N7
- to N8 (2): N4 $\rightarrow$ N5 $\rightarrow$ N8

Shortest Paths from N5

- to N1 (2): N5 $\rightarrow$ N2 $\rightarrow$ N1
- to N2 (1): N5 $\rightarrow$ N2
- to N3 (2): N5 $\rightarrow$ N2 $\rightarrow$ N3
- to N4 (1): N5 $\rightarrow$ N4
- to N6 (1): N5 $\rightarrow$ N6
- to N7 (2): N5 $\rightarrow$ N4 $\rightarrow$ N7
- to N8 (1): N5 $\rightarrow$ N8

Shortest Paths from N6

- to N1 (3): N6 $\rightarrow$ N3 $\rightarrow$ N2 $\rightarrow$ N1
- to N2 (2): N6 $\rightarrow$ N3 $\rightarrow$ N2
- to N3 (1): N6 $\rightarrow$ N3
- to N4 (2): N6 $\rightarrow$ N5 $\rightarrow$ N4
- to N5 (1): N6 $\rightarrow$ N5
- to N7 (3): N6 $\rightarrow$ N5 $\rightarrow$ N4 $\rightarrow$ N7
- to N8 (2): N6 $\rightarrow$ N5 $\rightarrow$ N8

Shortest Paths from N7

- to N1 (2): N7 $\rightarrow$ N4 $\rightarrow$ N1
- to N2 (3): N7 $\rightarrow$ N4 $\rightarrow$ N1 $\rightarrow$ N2
- to N3 (4): N7 $\rightarrow$ N4 $\rightarrow$ N2 $\rightarrow$ N3
- to N4 (1): N7 $\rightarrow$ N4
- to N5 (2): N7 $\rightarrow$ N4 $\rightarrow$ N5
- to N6 (3): N7 $\rightarrow$ N5 $\rightarrow$ N6
- to N8 (1): N7 $\rightarrow$ N8

Shortest Paths from N8

- to N1 (3): N8 $\rightarrow$ N5 $\rightarrow$ N2 $\rightarrow$ N1
- to N2 (2): N8 $\rightarrow$ N5 $\rightarrow$ N2
- to N3 (3): N8 $\rightarrow$ N5 $\rightarrow$ N2 $\rightarrow$ N3
- to N4 (2): N8 $\rightarrow$ N5 $\rightarrow$ N4
- to N5 (1): N8 $\rightarrow$ N5
- to N6 (2): N8 $\rightarrow$ N5 $\rightarrow$ N6
- to N7 (1): N8 $\rightarrow$ N7