Instituto Tecnológico de Costa Rica

Operations Research - Semester II

Equipment Replacement Problem

Members:

Adrián Zamora Chavarría
Daniel Romero Murillo

Date: September 21, 2025

Equipment Replacement Problem

Aqui va toda la yappeada del que es el replacement problem

C_{tx} is defined as the cost of buying equipment at time t and selling it at time x:

$$C_{tx} = \text{Initial Cost} + \sum_{i=1}^{x-t} \text{Maintenance}_i - \text{Resale}_{x-t}$$

The Bellman equation is:

$$G(t) = \min\{C_{tx} + G(x)\}$$

where $G(t)$ is the minimum cost from time t to the end of the project.

Problem Formulation

Given:

- Initial equipment cost: 500
- Project term: 5 years
- Equipment useful life: 3 years
- Maintenance costs for each year of equipment life
- Resale prices for each year of equipment life

Equipment Life Cycle Costs

Year	Maintenance Cost	Resale Price
1	30	400
2	40	300
3	60	250

C_{tx} Calculations

1 year(s): $C_{t,t+1} = 500 + 30 - 400 - 50 = 80$

2 year(s): $C_{t,t+2} = 500 + 30 + 40 - 300 - 50 - 50 = 170$

3 year(s): $C_{t,t+3} = 500 + 30 + 40 + 60 - 250 - 50 - 50 - 50 = 230$

C_{tx} Table

t/x	1	2	3	4	5
0	80	170	230	–	–
1	–	80	170	230	–
2	–	–	80	170	230
3	–	–	–	80	170
4	–	–	–	–	80

Step by Step Calculations

$G(5) = 0$ (Base case)

$G(4) = \min\{C_{45} + G(5) = (500 + 30 - 400 - 50) + 0 = 80\} = 80$

Optimal choice: sell at time 5

$G(3) = \min\{C_{34} + G(4) = (500 + 30 - 400 - 50) + 80 = 160, C_{35} + G(5) = (500 + 30 + 40 - 300 - 50 - 50) + 0 = 170\} = 160$

Optimal choice: sell at time 4

$G(2) = \min\{C_{23} + G(3) = (500 + 30 - 400 - 50) + 160 = 240, C_{24} + G(4) = (500 + 30 + 40 - 300 - 50 - 50) + 80 = 250, C_{25} + G(5) = (500 + 30 + 40 + 60 - 250 - 50 - 50 - 50) + 0 = 230\} = 230$

Optimal choice: sell at time 5

$G(1) = \min\{C_{12} + G(2) = (500 + 30 - 400 - 50) + 230 = 310, C_{13} + G(3) = (500 + 30 + 40 - 300 - 50 - 50) + 160 = 330, C_{14} + G(4) = (500 + 30 + 40 + 60 - 250 - 50 - 50 - 50) + 80 = 310\} = 310$

Multiple optimal choices: sell at time 2, sell at time 4 (tie)

$G(0) = \min\{C_{01} + G(1) = (500 + 30 - 400 - 50) + 310 = 390, C_{02} + G(2) = (500 + 30 + 40 - 300 - 50 - 50) + 230 = 400, C_{03} + G(3) = (500 + 30 + 40 + 60 - 250 - 50 - 50 - 50) + 160 = 390\} = 390$

Multiple optimal choices: sell at time 1, sell at time 3 (tie)

Result Table (Analysis table)

t	G(t)	Next Replacement
0	390	1, 3
1	310	2, 4
2	230	5
3	160	4
4	80	5
5	0	–

Optimal Solution

Minimum Total Cost: 390

All Optimal Replacement Plans:

- **Plan 1:** Buy at 0, sell at 1 → Buy at 1, sell at 2 → Buy at 2, sell at 5
- **Plan 2:** Buy at 0, sell at 1 → Buy at 1, sell at 4 → Buy at 4, sell at 5
- **Plan 3:** Buy at 0, sell at 3 → Buy at 3, sell at 4 → Buy at 4, sell at 5