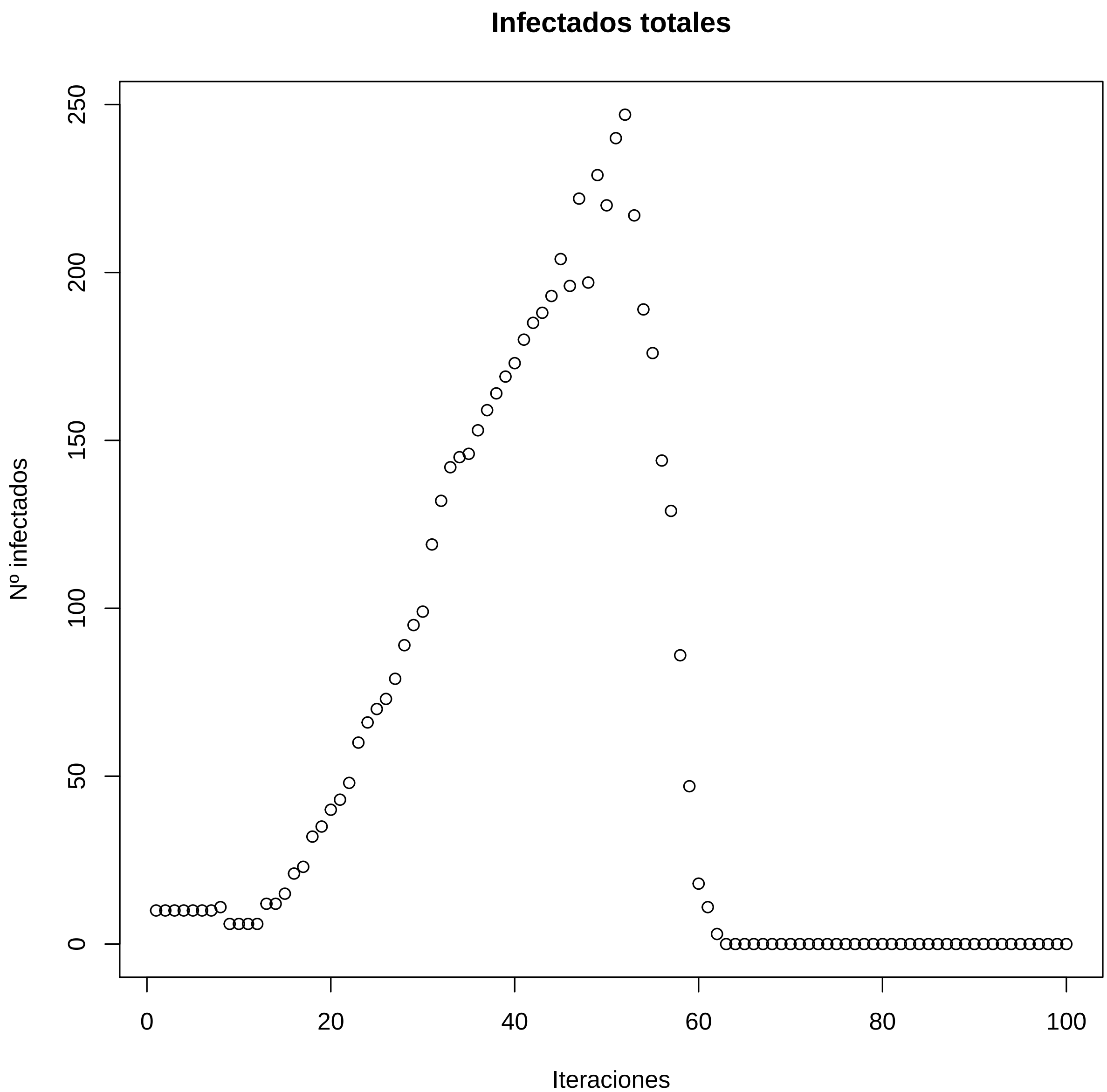
Handout 4

1) Generate an undirected random graph using the model of "preferential attachment" (sample_pa( )) of 1000 nodes. With $\beta = 0.1$, $\gamma = 0.1$, generate a SIR pandemic (iterative method). The initial infected nodes should be the 10 highest using the eigen_centrality( ). Compare the results to when the initial nodes are at random. Reduce or increase β and compare.

```r
library(igraph)
set.seed(2021)
G = sample_pa(1000, directed=F)
A = as_adjacency_matrix(G)
beta=0.1
gamma = 0.1

positions = order(eigen_centrality(G)$vector, decreasing=TRUE)[1:10]
x0 = rep(0,1000)
x0[positions] = 1

rand_positions = sample(V(G),10)
x0_ = rep(0,1000)
x0_[rand_positions] = 1

I = rep(1, 1000)
newx <- function (x,beta,gamma) {
  r = gamma*x
  s = I - x - r
  return ( x + beta * s* (A %*% x) - matrix(gamma*x) )
}
n=1000
nIter=100
RX=matrix(0,nrow=n,ncol=nIter)
RX[,1]=x0
for (i in 2:(nIter) ){
  RX[,i] = as.vector(newx(RX[,i-1],beta,gamma))
}

RX_=matrix(0,nrow=n,ncol=nIter)
RX_[,1]=x0_
for (i in 2:(nIter) ){
  RX_[,i] = as.vector(newx(RX_[,i-1],beta,gamma))
}

#Matriz en la cual si tienes más de 0.5 -> 1 en caso contrario 0
chart <- matrix(0,nrow=n,ncol=nIter)
for (i in 1:n){
  for(j in 1:nIter){
    chart[i,j] <- ifelse(RX_[i,j]>0.5,1,0)
  }
```

```
}
#Transformamos en dataframe
chart <- as.data.frame(chart)
#Sumamos los casos infectados por iteración
infect <-colSums(chart, na.rm = TRUE)
plot(infect, main= "Infectados totales", xlab = "Iteraciones", ylab = "Nº infectados")
```



2) Consider the random graph generated in the previous exercice.

a) Plot its degrees distribution in linear and in log-log scale. Which is more helpful to understand this distribution?

b) Does the degree distribution follows a Power Law? And if we consider only the nodes with degree above 5? (or 10? or 100?)

c) Find the best line that approximates the degree distribution after degree 10 (or 5?) using linear regression (lm()) on the log-log plane. Don't worry, it is almost all done in the following code. Explain in detail each line of the following code:

```
D=degree_distribution(GRAPH)
xx=which(D>0)[-(1:10)]  # remove the first 10 prob values
lyy=log(D[xx])
lxx=log(xx)
LMI=lm(lyy~lxx)$coefficients  # line coefficients
plot(D,pch=20,cex=0.7,xlab="Degree",ylab="Frequencies",main="degrees",log="xy")
points(exp(lxx),exp(LMI[1]+LMI[2]*lxx),col="red",type="l",lwd=2)
```

d) What is the exponent of the Power Law for the degree probabilities?

3) Use the routine sample_pa( ) to generate a rich-get-richer (preferential attachment) graph with similar degree distribution of the *directed* facebook graph of the file **facebook_sample_anon.txt**. Use the code similar to: sample_pa(n.GRAPH, out.seq=degree(GRAPH,mode="out")) Plot the degree distribution of the generated graph (log-log). What is the exponent of the power law of the generated graph for the in-degrees?