# Implementing a MIPS processor using SME

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Abstract

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## 1 Introduction

Background and terminology for:

CSP - hele konceptet med processer der kommunikerer

SME - processer, bus og clock

Machine architecture - hvertfald hvad word, bit og clock er for noget.

DIKU course

VHDL

C# - især hvorfor der er uint over det hele.

beskriv hvordan figurene fungerer, i.e. box/circle = process, black wire = data, blue wire = control, green wire = clock

Beskriv hvad MARS er, og hvordan det virker.

## 2 Getting started with SME

## 2.1 Installing required software for SME

Muligvis ryk alt det her ned i appendix, da det mest bare er fifs? monodevelop git

- 2.2 Writing first SME program
- 2.3 C# and bit hacking
- 2.4 Translating first SME program into VHDL
- 2.5 Running and verifying VHDL

## 3 Basic logic circuits

In this section, we will be looking at some basic combinatorial circuits. We use this as an entry point for hardware design, both due to the ARK course[4] at DIKU, and due to the book[3] recommending to read appendix C, which covers the basics of logic design, prior to reading chapter 4, which covers implementing a simple MIPS processor. We are only going to cover a subset of appendix C, as it should be sufficient as an introduction to connecting simpler components together into a more complex network.

We start by looking at some logic gates, which implement some basic boolean functions. Then we will combine these basic gates into more complex networks:

- A decoder, which expands an n-bit input into  $2^n$  outputs.
- A half adder, which takes two binary inputs and computes the sum and the carry of the two.
- A full adder, which does the same operation as the half adder, but with an additional third binary input.
- a *n*-bit adder, by combining a chain of a half adder followed by n-1 full adders.

For each of these combinatorial circuits, we go through the theory behind it, describe the procedure of translating the theory into an SME network and finally how to test and verify the SME networks.

#### 3.1 Basic logic gates

A logic gate is a circuit abstraction, which has inputs and outputs, and computes the logic function that corresponds to the gates name, i.e. its output values are based upon the input values. We are going to implement the following logic gates:

AND - outputs 1 iff. all of its inputs are 1, otherwise it outputs 0.

OR - outputs 1 if one or more of its inputs are 1, otherwise it outputs 0.

NOT - outputs the inverse of its input, i.e. 1 becomes 0 and 0 becomes 1.

XOR - outputs 1 iff exactly one of its inputs are 1, otherwise it outputs 0

The full truth table for all of the four logic gates with 2-bit input (except for NOT, which only uses 1 bit) can be seen in Table 1.

#### Implementation

Implementing each of these four logic gates is straightforward: There is an input bus with two 1-bit values, a process for each of the gates, and an output bus with a 1-bit value for each of the logic gates. We do not need additional input busses, as each process which uses the bus as input, receives its own copy of the input in each clock. Furthermore, we only need one output bus, as each process writes to its own bit within it. Each process takes the two bits from the input

Bit1	Bit2	AND	OR	NOT	XOR
0	0	0	0	1	0
0	1	0	1	1	1
1	0	0	1	0	1
1	1	1	1	0	0

Table 1: The truth table for the four basic logic gates. Note: NOT is only considering Bit1.

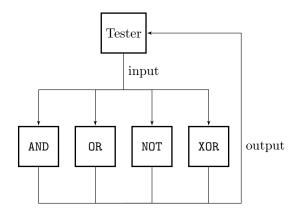


Figure 1: The structure of the test of the logic gates

bus, computes their respective logical function and sends the result out onto its bit on the output bus.

We could have made a more complex process, i.e. one that takes the two inputs, and computes each function. This would reduce the amount of bus connections, but we want to make each process as simple as possible, when constructing the processor, as this makes arguing for correctness simpler.

#### Testing

To ensure that the processes functions as expected, we construct a tester process, which sets the bits on the input bus to all of the values in the truth table, and checks whether or not each process puts the expected value from the truth table onto their bit on the output bus. We use this testing technique, as it tests the functionality of the processes, without knowing how they work internally. We could also just have printed the output values, and verified the results our selves, however automating this reduces the time needed for verification. How the processes are connected can be seen in Figure 1.

#### 3.2 Decoder

A decoder is a component, which takes an n-bit input, and produces an  $2^n$ -bit output, where the bit corresponding to the input numbers binary representation is set to 1. E.g. if the input value is the binary representation of the number 5, then the 5th output bit will be 1, and the rest will be 0. Note: the output bits

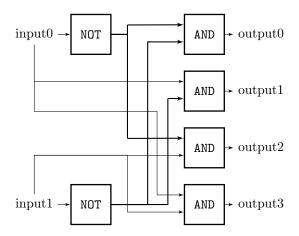


Figure 2: An 2-bit decoder made by AND and NOT gates.

are zero indexed, i.e. there is also an output bit for the binary representation of 0.

A decoder can be made from a set of NOT and AND gates[5]. We need to have n NOT gates, and  $2^n$  AND gates. For each input, we split it into two, and send the copy to a NOT gate. Then for each output, we attach an AND gate, and give it inputs corresponding to the binary representation of the number E.g. if we get the number 5, the binary representation is 101, i.e. the 5th AND gate gets input from Bit0, NOT Bit1 and Bit2. An example of a 2-bit decoder can be seen in Figure 2.

#### Implementation

To implement a 2-bit decoder, we just need to connect logic gate processes, which we already have. How to connect a 2-bit decoder can be seen in Figure 2

We could have made the decoder a single process, but we are trying to emphasize connecting multiple processes together, in order to gain functionality.

However, making a scalable decoder is not trivial, as SME requires everything to be known at compile time, and we cannot make a generic process, as these depend on the names of the different busses. To solve this problem, we can use C# templates. For each input bit, we create an input bus. Then, for each input bus, we create an NOT gate process, which takes the input bus corresponding to its index. Then, we create  $2^n$  output busses, and for each of these, we connect an AND gate with its corresponding output bus. Finally, for each of these AND gates, we connect the busses whose logical AND will produce a 1. E.g. for output0, we connect all the busses from all of the NOT gates, for output1, we connect all the NOT gates, except from the bus from the first NOT gate, as this should be the first input bus.

#### Testing

To test the 2-bit decoder, we follow the same procedure as with the logic gates. We construct a tester process, which sends input to the logic gates, and verifies

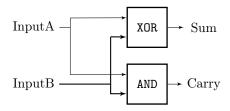


Figure 3: An half adder composed of XOR and AND gates

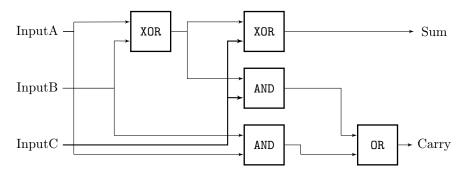


Figure 4: A full adder composed of AND, OR and XOR gates

the output from the AND gates. This is also what we need to do with the n-bit decoder. However as with the n-bit decoder, we are going to need C# templates, as the tester process also needs a variable amount of busses. Each of the two tester processes should send all possible inputs as input.

#### 3.3 Adder

An adder is an component, which adds two binary numbers together. As with the decoder, an adder can be constructed by a combination of AND, OR and XOR gates[5]. An n-bit adder is a chain of two major components: an half adder and a full adder.

The half adder is the initial component in the chain. It takes two binary inputs, and outputs the sum and the carry of the addition (See Figure 3). The rest of the *n*-bit adder consists of a chain of full adders, that take three inputs, A, B, and the carry from the previous link in the chain, and outputs the sum and the carry of the addition (See Figure 4). The combination of the components can be seen in Figure 5.

#### Implementation

The half adder and the full adder is made like the decoder, in which we have the basic logic gate processes, and connect them as specified in Figure 3 and Figure 4.

To make the n-bit adder, we have to use C# templates like we did when constructing the n-bit decoder. We program it so that each of the input bits has its own bus, each of the output sums has its own bus, and each of the carrys have their own bus. Then we start by making a half adder, which has the inputs: input A bit 0 and input B bit 0. The initial half adder outputs its sum on sum

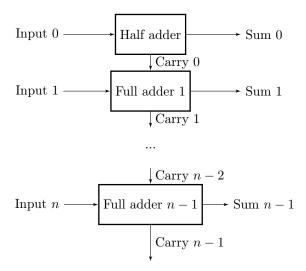


Figure 5: An n-bit adder composed of a half adder, and n-1 full adders. Note: Input A and B are both inside the inputs for simplicity.

0, and outputs the carry on carry 0. Then we construct n-1 full adders, where full adder i (1-indexed) has the inputs: input A bit i, input B bit i and carry i-1. Each full adder has output i and carry i as output.

#### Testing

To test the adder, we construct a tester process as before. We start by testing the two subcomponents, the half and the full adder, by giving each every possible input, and verifying the output. For the n-bit adder, we make a function that takes an integer as input, and sends it along the corresponding input wires. We also make a similar function that takes the values from the output wires and packs it into an integer. This makes the test more human readable, and reduces the chance of failure, when hand encoding/decoding binary numbers. There are two tests: the simplest addition: 2+2, adding two numbers together that produce an overflow and finally a series of adding random numbers together.

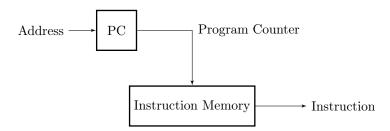


Figure 6: The Instruction Memory

### 4 Core components

In this section, we will be looking at the major components of the MIPS processor. We look at the components before wiring them together into a processor, as it keeps each step more isolated and simple.

For each component, we will go through the theory of the component, then we will look at translating theory into an SME implementation, and finally the verification of the implementations. When we are verifying the implementation, we should construct a tester process, just as with the basic logic circuits. The order in which the components are mentioned is derived from chapter 4.3 in the book[3], and should be the order of implementation.

By the end of this section, we should have all of the required resources for building our single cycle MIPS processor using SME.

#### 4.1 Instruction Memory

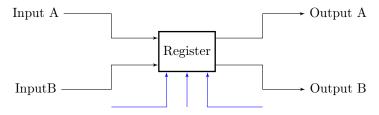
The first component that we need is a memory unit to hold the instructions of a program, and to supply instructions at a given address. We will also need a register holding the current address in the program called the Program Counter (PC). The memory unit has one input: the address from the PC, and one output: the read instruction. The PC has one input: the input address. The Instruction Memory, PC and their connections can be seen in Figure 6.

#### Implementation & Testing

Both the PC and the Instruction Memory should be SME processes. We could have combined the two components into a single SME process, but we are trying to keep each process as simple as possible. The input bus for the PC, the Program Counter bus and the instruction bus, should all contain an uint value.

The PC should have a single uint value holding the current address, and on each tick, it should output its stored value, and store the new input value. It should also be clocked, as this is the starting point of every instruction.

The Instruction Memory should have an byte array, as this will make indexing into the array simpler. Usually, the Instruction Memory and the Memory Unit share the same address space. However, for simplicity, we give each component its own memory. On each tick, the Instruction Memory should take the input address, read the byte at the given index and the following 3 bytes, and finally pack all 4 bytes together into an uint, and place it on the instruction bus.



Write register Write data Write enabled

Figure 7: The register file

We could have had the memory as an uint array, but then we would have to divide the address by 4, since the addresses in the MIPS processor are byte addressed. We could modify the processor to deal with addresses in an indexing manner, however this would make programs harder to translate, as the compiled programs can have absolute addresses.

Testing the Instruction Memory is very trivial. The program should be hardcoded into the byte array in the memory. The tester process should then send values to the PC, and verify that the output on the instruction bus, matches the value at the given address.

#### 4.2 Register file

The MIPS processor has 32 general-purpose 32-bit registers, which is stored in a structure called the Register File. Each of the registers can be read from or written to, except for register 0, which is immutable and always 0. It is the first step in a memory hierarchy, and is thus the fastest memory available. The registers are divided into groups based on their usage, but this does not matter from a hardware perspective.

The Register File has 5 inputs: Read Address A, Read Address B, Write Enabled (RegWrite), Write Address and Write Data. The Register File has two outputs: Output A and Output B. There are two stages of the Register File: reading and writing. In the reading stage, it takes the value in the register at the address of Read Address A, and outputs it on Output A, and vice versa for Read Address B and Output B. In the writing stage, if the Write Enabled flag is set, it takes the value from the Write Data bus, and stores it in the register with the address given in the Write Address bus.

We need to be careful of the order in which we read and write from the Register File. We need to make sure that when an instruction reads from the Register File, it always gets the latest data, i.e. if an instruction reads from the same register as a previous instruction writes to, it should get the newly written value. This is easy to fix in the single cycle processor, as we just need to write before reading. We cannot do it in the reverse order, as the Register File might then output old values. The Register File and its inputs and outputs can be seen in Figure 7.

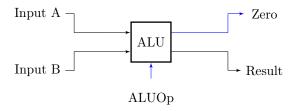


Figure 8: The ALU

#### Implementation & Testing

The Register File should be an SME process. The collection of registers should be an uint array of length 32. The address busses should all contain an byte value, as the number of addressable registers never exceed  $2^8 = 256$ . The output busses and the Write Data bus should all contain an uint value. Finally, the RegWrite bus should contain a bool.

On each clock tick, the register file should check if the RegWrite flag is set, in which case it should take the value from the Write Data bus, and store in the register at the address from the Write Address bus. Then it should take the value in the register at the address from the Read Address A bus, and output onto the Output A bus, and analougously for the Read Address B bus and the Output B bus.

It should be noted that if the write address is 0, then the write should be ignored, as register 0 is immutable.

Testing the register file is very trivial. We start by sending some values on the write data bus, along with some addresses and the RegWrite flag set.

Then we just try to send some addresses on the Read address A and Read address B buses, and verify that the register file outputs the values stored at these addresses. It is also important to verify the behaviour of register zero.

#### 4.3 ALU

The ALU (Arithmetic Logic Unit) is the part of the processor, which makes the actual computation. It takes three inputs: InputA, InputB and an ALU Opcode indicating which computation to perform. It has two outputs: The result of the computation, and a zero flag indicating whether or not the result of the computation was 0. The overview of the component and its inputs and outputs can be seen in Figure 8.

The ALU starts by looking at the value in the ALU Opcode, as this determines which operation to perform. Then it reads the values from Input A and Input B, and performs the operation specified by the ALU Opcode. Finally, it outputs the result, and a flag indicating whether the result was 0.

#### Implementation & Testing

To implement the ALU, we start by making an enum, so that the code becomes more human readable. Each entry in the enum corresponds to a computation that the ALU should perform. We could have followed the ALU Opcode given in

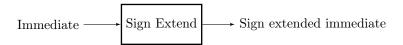


Figure 9: The Sign Extend

the book[3], however it states that its encoding of the ALU Opcode is generated using a CAD tool, and as such we will generate it ourselves too.

We start by implementing the same instructions as the book[3] proposes in chapter 4.1: add, sub, and, or, slt, sw, lw and beq. To perform these instructions, the ALU should be able to perform addition, subtraction, logical AND, logical OR and the comparison less than.

The two input busses and the Result bus should all contain an uint value. The ALU Opcode bus should contain a byte value. Finally, the Zero bus should contain a bool value.

Constructing the ALU process in SME is straightforward, it reads the ALU Opcode from the ALUOp bus, and then it performs a switch on the ALU Opcode. For the instructions that it accepts, it takes the input from the two input busses, do the computation, and output the result on the Result bus. SME will handle the the actual computation, and as such we do not need to do anything but do the right computation in C#. Note: it is important to cast the uint input to int, if the operation is signed.

Finally, the ALU should set the flag on the Zero bus, depending on whether or not the result of the computation was 0. How the ALU opcode is encoded is described in Section 4.8.

There are no special cases to consider when testing the ALU, we should just test whether or not it can compute the right result, based on the inputs.

#### 4.4 Sign Extend

The Sign Extend is used for extracting the 16-bit values from the instruction, called the Immediate. It has one input: the lower 16 bits from the instruction, and one output: the 32-bit sign extended value. It takes its input, which is 16-bit, and converts it into a 32-bit value, extending the sign if present. The Sign Extend can be seen in Figure 9

#### Implementation & Testing

To implement the Sign Extend, we construct an SME process. The input bus should contain a short value, and the output bus should contain an uint value. On each clock tick, the SME proces takes the 16-bit immediate, and outputs it on its 32-bit output bus. We could manually do the sign extending, but it is simple to let C# handle the sign extension for us.

Testing the Sign Extend is straightforward: send values on the input bus, and verify that the same number is on the output bus.

#### 4.5 Memory unit

The Memory Unit is the main memory of the processor. It is the second step of the memory hierarchy, and is thus slower than the Register File, but can contain

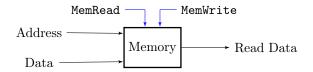


Figure 10: The Memory Unit

a lot more data. The processor can either read or write to the Memory Unit. The addresses for the memory are byte addresses and word aligned, i.e. in the 32-bit processor, the word size is 32 bit, and therefore the address should be dividable by 4.

The Memory Unit has four inputs: Address, Data, MemRead and MemWrite. It has a single output: Read Data. In one clock cycle, the Memory Unit either reads or writes. The Memory Unit can be seen in Figure 10.

#### Implementation & Testing

To implement the Memory Unit, we construct an SME process. The Address, Data and Output busses should all contain an uint value. The two control busses MemRead and MemWrite should both contain a bool value.

As with the Instruction Memory, we are going to need a chunk of memory. Like the Instruction Memory, the memory chunk should be a byte array, and should read and write in the same manner, i.e. either packing or unpacking 4 bytes, from and to memory.

On each clock tick, the process should check if the MemRead flag is set, in which case it should read the value on the Address bus, and output the value stored in memory at the read address.

If that was not the case, it should check if the MemWrite flag is set, in which case it should read the value at the Address bus and the Data bus, store the read data in memory at the read address.

To test the Memory Unit, we should just try to store some values, and check if we can fetch the same values again.

#### 4.6 Splitter

The book[3] takes the instruction read from the Instruction Memory, and splits it out to the different components. One way of handling this could be to send the instruction bus to all the components that needs something from the instruction. However, then the resulting VHDL might send all 32 bits to multiple components, when 5 bits might have been sufficient. As such, we construct our own component for splitting up the instruction.

The splitter is a very simple component: It takes the instruction, which has been fetched from memory, and divides it into chunks for the different parts of the decoding. The instruction is partitioned as followed (the bits are inclusive):

- Opcode bits 26-31
- Read Address A bits 21-25

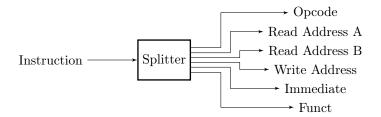


Figure 11: The Splitter

- Read Address B bits 16-20
- Write Address bits 11-15
- Immediate bits 0-15
- Funct bits 0-5

The Splitter can be seen in Figure 11

#### Implementation & Testing

The implementation is straightforward: We construct an SME process, which takes the instruction coming from the Instruction Memory, and extract the bits at the indices, by using C# bit hacking. The Input bus should contain an uint value. The Immediate bus should contain an short value. The rest of the busses should contain an byte value. Finally, the process should output the extracted values on the respective output busses.

Testing the Splitter requires no special procedures.

#### 4.7 Jump Unit

The book[3] describes how to implement beq, by the use of multiple components. However, we should combine these components into a single unit, the Jump Unit, to simplify the processor.

The Jump Unit is the one controlling which instruction to load next. It takes four inputs: sign extend, Zero, beq and the PC. It produces one output: the new PC.

For the simple single cycle MIPS processor, it should only have support for normal program traversal (i.e. execute the instructions in order) and the beq instruction. We will be adding support for more branch and jump instructions later.

For the simple traversal, the Jump Unit takes the previous PC, and increments it by 4. For the beq instruction, it takes the value from the Sign Extend, shifts it left by 2, and add that value to the incremented PC. Finally, it chooses between the incremented PC and the added sign extend, based on the beq and Zero flags. The Jump Unit and its internals can be seen in Figure 12.

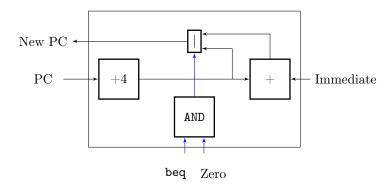


Figure 12: The Jump Unit

#### Implementation & Testing

We have the all of the logic described in the background. To implement the Jump Unit, we implement all of the logic components as SME processes. We are going to need 4 components: an +4 incrementer, an adder, an AND gate and a multiplexor.

The Incrementer takes the PC bus as input, and produces a single output on the inced bus. The two busses should both contain an uint value. On each clock tick, it should take the value from the PC bus, add 4 to it and put it on the inc bus.

The Adder takes the inc bus and the immediate bus from the Sign Extend as input. It has a single output bus: the added bus. All of the busses should contain an **uint** value. On each clock tick, the Adder should add its two inputs together, and put the result on the add bus.

The AND gate is like in the logic gates example, but with the beq and Zero as inputs, and its output should be on a new bus: anded. All of its busses should contain a bool value.

Finally, we have the multiplexor, which takes the inced, added and anded busses as input, and produces a single output: the new PC. The new PC bus should contain an uint value. On each clock tick, if the flag from the anded bus is set to 1, then it should put the value from the added bus onto the new PC bus. Otherwise it should output the value from the inced bus.

We could have made all the logic in C#, however this makes it easier to extend, as each subcomponent is very simple.

We test the Jump Unit by initially verifying that the PC increments as it should, if the two control signals are 0. Then it should be tested, if the correct branching addresses are constructed.

#### 4.8 ALU control

The ALU control is used for generating the ALU Operation control code, which the ALU uses for selecting which operation to perform. It takes two inputs: the ALUOp code from the control unit, and the funct code from the instruction. It produces a single output: The ALU Opcode. The ALU Control can be seen in Figure 13.

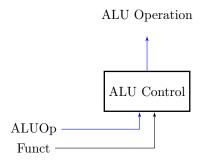


Figure 13: The ALU Control

If the ALUOp indicates that the instruction is an R format instruction, it uses the funct code for selecting the operation. Otherwise it bases its output on the ALUOp code. We will return to this component, when we need to extend the instruction set.

#### Implementation & Testing

The book[3] describes the logic to implement the ALU Control. However, like the ALU, it has been generated, and is difficult to extend. As such, we construct our own ALU Control, and let the VHDL generator handle the logic generation.

To make the source code more human readable, we should have two additional enums: one for the ALUOp code and one for the funct. Then we construct an SME process, which has the connections as specified in the background section. All of its busses should contain byte values.

On each clock tick, the ALU Control checks if the ALUOp code indicate R format instruction or not. If the instruction is an R format, the process should switch on the funct code. If not, it should switch on the ALUOp code. In all cases in both switchs, the ALU Control should output the ALU Operation corresponding to the computation that the instruction expects.

To test the ALU Control, the tester process should try all possible combinations, as there should not be too many combinations.

#### 4.9 Control Unit

The control unit is part of the decoding step. It takes the opcode of the instruction, and based on the opcode, it sets control flags used throughout the processor. All of the control flags mentioned throughout the other components are set by the Control Unit. It sets the following control flags:

**RegDst** Controls which part of the instruction that indicates which B register to read from.

Branch Controls whether or not the instruction is a branch instruction.

**MemRead** Controls whether or not there should be read from memory.

**MemtoReg** Controls whether or not the value from memory should be stored in the register file.

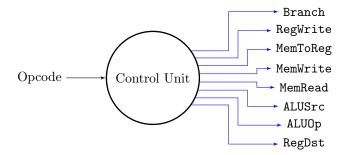


Figure 14: The Control Unit

**ALUOp** Opcode indicating which operation should be performed in the ALU. It is send to the ALU Control for further processing.

MemWrite Controls whether or not data should be written to memory.

**ALUSrc** Controls whether the B input for the ALU should be the value read from the register file, or if it should be the value extracted from the instruction.

RegWrite Controls whether or not data should be written to the register file.

Each of the control flags goes to their respective part of the processor. We will return to this component, when we have to extend it to handle more instructions. The control unit and its connections can be seen in Figure 14.

#### Implementation & Testing

As with the ALU Control, the book[3] describes the logic needed to implement this unit. However, again it is not trivial to extend later on, so we construct our own Control Unit, and let the VHDL generator handle the logic generation.

We construct an SME process, which has all of the busses described in the background section. The input opcode bus and the ALUOp code bus should both contain an byte value. The rest of the busses should all contain an bool value

As with the ALU Control, we should have an enum on the opcode, to make it more human readable. We do not need one for the ALUOp, as the ALU Control already described it.

On each clock tick, the Control Unit reads the opcode from the input bus. Then it should switch on the read opcode, and set all of the flags accordingly. How the flags should be set, depends on the instruction, and should be straightforward, but time demanding.

As with the ALU Control, testing the Control Unit should be done by trying every opcode available.

#### 4.10 Write back

The final stage of the processor is the Write Back. Here, the values are sent to the Register File for storing.

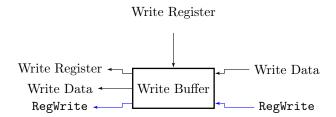


Figure 15: The Write Buffer

#### Implementation & Testing

Usually in the single cycle MIPS processor, there is nothing special in the Write Back stage. However, in SME we are not allowed to have unclocked cycles, and there is a cycle from the Register File, through the ALU and the Memory Unit, and back to the Register File.

To solve this, we introduce a Write Buffer. The write buffer should be clocked, and takes the Write Data, Write Register and WriteEnabled as input, and produces the same output. On each clock tick, it should output its stored values, and store its input values.

We could also have made one of the previous components clocked, however we do not want to this, as this specific problem will be solved when we pipeline the processor in section 6, and then the Write Buffer can be removed again.

The Write Buffer and its connections can be seen in Figure 15. Testing the Write Buffer is very trivial: verify that it outputs its input from the previous clock tick.

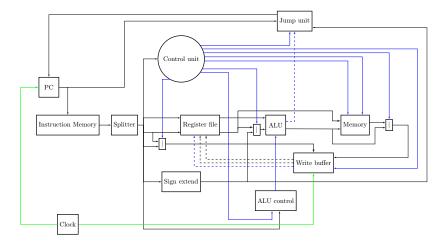


Figure 16: Simple single cycle MIPS processor. The units with | indicate multiplexors. The Clock is not an SME process, but has been added to emphasize which processes that are clocked.

## 5 Single cycle MIPS processor

In this section, we will be combining the core components into a single cycle MIPS processor, i.e. a processor where exactly one instruction is executed per clock cycle. When it is in place, we will be writing the first program, and compiling it into the processor, and running it.

Following the single cycle MIPS processor, we will be extending the processor so that it can handle more instructions. Along each added instruction, we will be extending our first program, in order to verify that the added instruction works.

Finally, we will be writing two larger programs, and look into compiling them into a series of hex values, that we can copy straight into the Instruction Memory.

#### 5.1 Wiring up the processor

With all the components in place, wiring up the single cycle MIPS processor is straightforward. We just need to declare the busses with the corresponding names, and then SME handles the wiring process. Note that as previously mentioned, the Write Buffer and the PC register should be clocked processes. The wiring of the single cycle MIPS processor can be seen in Figure 16.

#### 5.2 Writing the first program

As mentioned before, the first single cycle MIPS processor should be able to handle add, sub, and, or, slt, sw, lw and beq. As such, the first program should consist of these. The program and the different parts of each of the instructions can be seen in Table 2.

Inserting this program into the Instruction Memory is straightforward: just create an byte array, which is initialized to the hex values from the instruction

Address	Instruction	opcode	rs	rt	$\mathrm{rd/imm}$	$\operatorname{shmt}$	funct	hex
0x00	add \$3 \$1 \$2	0x00	0x01	0x02	0x03	0x00	0x20	0x00221820
0x04	sub \$4 \$3 \$2	0x00	0x03	0x02	0x04	0x00	0x22	0x00622022
80x0	and \$5 \$3 \$1	0x00	0x03	0x03	0x05	0x00	0x24	0x00612824
0x0C	and \$6 \$3 \$1	0x00	0x03	0x03	0x06	00x0	0x25	0x00613025
0x10	slt \$7 \$6 \$5	0x00	0x06	0x05	0x07	00x0	0x2A	0x00C5382A
0x14	sw \$6 0x0(\$0)	0x2B	0x00	0x06	0x0000	-	-	0xAC060000
0x18	lw \$8 0x0(\$0)	0x23	0x00	0x07	0x0000	-	-	0x8C070000
0x1C	beq \$5 \$4 0x4	0x04	0x05	0x04	0x0004	-	-	0x10A40004
0x20	add \$9 \$8 \$6	0x00	80x0	0x06	0x09	00x0	0x20	0x01064820
0x24	add \$10 \$8 \$6	0x00	80x0	0x06	OxOA	0x00	0x20	0x01065020

Table 2: The first MIPS program. Note that the instruction at 0x20 should be skipped due to the beq at 0x1C.

Address	0	1	2	3	4	5	6	7	8	9	10
Value	0	5	2	7	5	5	7	1	7	0	14

Table 3: The register file after the first program has finished.

column in Table 2. Since we do not have any way of feeding values into the Register File at this moment, we are going to hardcode registers 1 and 2 with the values 5 and 2 respectively. When the program has finished, the contents of the register file should be as in Table 3.

#### 5.3 Extending the accepted instruction set

#### Additional simple R format instructions

The simplest instructions to add, are the remaining simple R format instructions. These are addu, subu, xor, nor and sltu. The only modifications are to extend the ALU Operation enum, and then add the remaining cases in the ALU Control and the ALU. As described in section 4.3, it is important to cast the input uint to int before making the computation, if the operation is signed.

To test each of these instructions, we can just append them to our initial program.

#### Immediate instructions

The next instruction we want to add is the ori instruction, as this would allow us to feed values into the processor without hardcoding it. While we are doing this, we should also add addi, addiu, slti, sltiu, andi and xori, as these requires the same amount of work.

For the logical immediates, it is important, that the Sign Extend does not sign extend. As such, we add another signal to the Control Unit: LogicalImmediate, which goes to the Sign Extend. The Sign Extend should then, in the case the LogicalImmediate flag is 1, cast its input as unsigned, such that any potential sign bits are not extended. Then we just need to extend

the opcode and ALUOp enum, and add the remaining cases in the Control Unit and the ALU Control.

Once the processor have been extended, we can prepend instructions at the beginning of the program. Furthermore, we can remove our previous hardcoding of initial values in the Register File.

#### Jump instruction

We are going to introduce another format: the J format. This format is used for executing the j instruction. The first step is to extend the Splitter, as it should now send the lower 26 bits to the Jump Unit. Then the Control Unit should be extended, both in the opcode enum, and it should have another control signal output: jump, which should be connected to the Jump Unit.

Finally, the Jump Unit should be extended. It should take the 26 bits from the Splitter, and left shift it by 2, such that it becomes a 28 bit number. Then, it should take the 4 most significant bits from the PC+4, and prepend them to the extended number. Finally, we are going to add a multiplexor, which takes this newly computed address, the previous output address, and the jump control signal. If the control signal is 0, then the original output should be output, otherwise the newly computed jump address.

Then the program can be extended with a j instruction, in the same manner as the beq instruction was tested before, i.e. trying to jump over some instructions.

#### Branching instructions

Then we are going to add the remaining branching instructions bne, blez and bgtz instructions.

To implement the bne instruction, we are going to add another signal from the Control Unit to the Jump Unit. Then we should split the Zero signal into two, where one of the signals goes to a newly added NOT gate. Then, we should add a multiplexor, which has the bne signal from the Control Unit as the control signal, and the Zero and the NOT Zero as inputs. If the bne control signal is 0, then the original Zero should be put on the output, otherwise the NOT Zero. The output from the multiplexor should go into the AND gate, where the Zero signal originally went.

TODO blez TODO bgtz

#### Remaining jump instructions

Then we are going to add the remaining jump instructions: jr, jal and jalr, as these are useful when writing the larger programs later.

We start with jr. The instruction is in R format, so we do not know it is jr, until it has reached the ALU Control. As such, we are going to need a control signal from the ALU Control to the Jump Unit. We are also going to forward Output A from the Register File to the Jump Unit, as this is the address that the processor should jump to in the jr instruction. The Jump Unit should not compute the new address in the same manner as with the j instruction. This is due to the registers being a full 32 bit, and thus can contain the whole address

space. The Jump Unit should also have a multiplexor, controlling whether the jump address should be the immediate value, or if it should be the value from the instruction.

For the jal instruction, we are going to need an extra unit following the ALU. In the case of a jal instruction, we should store the PC+4 address in register 31 (which is called the \$ra register). There should be an additional control signal from the Control Unit: the jal signal. The new JAL Unit should take three inputs: the ALU Result, the PC+4 and the jal control signal. It should produce two outputs: the Write Address for the Write Buffer, and the value to store. If the jal signal is 1, the JAL Unit should output the PC+4 on the value bus, and 31 on the address bus. Otherwise it should output the regular ALU Result, and the regular Write Address.

TODO jalr

#### Shift instructions

Then we are going to add the shift instructions: sll, slr, sra, sllv, srlv and srav, as shifting is often useful.

The shifting itself is performed in the ALU, and modifying the ALU to handle these is straightforward. The problem is that in an R format instruction, which the shift operations are, the shift amount (shamt) is stored in its own field within the instruction. As such, the splitter should extract these 5 bits, and send them to a new multiplexor, which also takes Output A from the Register File. As with the jr instruction, we do not know it is a shifting instruction until it reaches the ALU Control. So to control the new multiplexor, we need a Control signal from the ALU Control, indicating whether the multiplexor should output either the shamt or Output A from the Register File.

#### Multiplication and Division instructions

Then we are going to add the multiplication and division instructions: mult, multu, div and divu. All of these instructions put their result in two special registers: HI and LO. As such, to get the results from them, we are also going to need the mfhi, mthi, mflo and mtlo instructions.

We start by adding the special registers. Since they are performed in the ALU, we might as well put them there. Then, when we are doing the computation, we should just put the values there, and since the instructions do not write to registers, touch memory or change the PC register, it does not matter what is put on the ALU Result or Zero busses.

The instructions handling the moving to and from the HI and LO registers are fairly simple to implement: just either input or output the corresponding register, to or from the ALU.

The layout for the fully extended single cycle MIPS processor can be seen in Figure 17

#### 5.4 Larger MIPS programs

To test the full single cycle MIPS processor, we are going to implement two programs in MIPS assembly: Quicksort and Towers Of Hanoi. We choose these

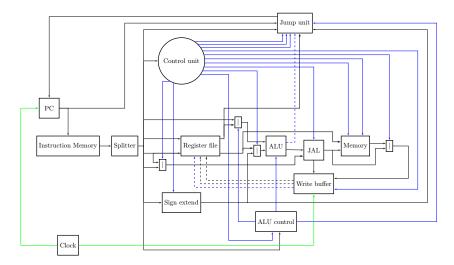


Figure 17: Full single cycle MIPS processor. The units with | indicate multiplexors. Black wires indicate data wires. Blue wires indicate control wires. Green wires indicate clock

two examples, as both are simple to implement, and they do not require anything special from the environment.

We are not going to give the implementation in MIPS assembly, as this would not be readable. Instead, we are going to construct some low level C code, which should be easy translatable into MIPS assembly, and easilier verifiable on a real machine. Once we have made the assembly, we can easily dump it into MARS, which can then produce a ascii hex dump, which we can then paste into our instruction memory.

#### Quicksort

Quicksort is a sorting algorithm, for doing comparison based sorting. The algorithm is heavily described in the algorithm book[6], and as such we wont go into details about the algorithm. We could have chosen one of the other sorting algorithms from the book, but quicksort should not be too hard to implement.

There are three parts of the Quicksort program: Loading data into memory, the partition function and the quicksort function. We are going to use the partition function from the algorithm book, and the quicksort function, also from the algorithm book, as both are simple to implement.

We could have implemented the Hoare partitioning algorithm. However, it is not as simple, and performance is not the target of the program, but rather correctness of the processor, i.e. that our processor implementation produce the same result as MARS.

Loading data into memory We start by loading data into memory, so that our quicksort program has some data to sort. Which numbers we are going to use is not important, rather the amount of numbers, as the quicksort algorithm uses it as argument. We construct a function called load, which inserts 6 numbers into the given memory address.

```
void load(int *a) {
    *(a) = 5;
    *(a+1) = 8;
    *(a+2) = 2;
    *(a+3) = 9;
    *(a+4) = 1;
    *(a+5) = 3;
}
```

The partitioning function Then we implement the partition function as described in the algorithm book[6]. Note that statements in the book have been expanded to more closely resemble assembly.

```
int partition(int *a, int p, int r) {
1
2
        int x, i, j, tmp1, tmp2, *addr1, *addr2;
3
        addr1 = a + r;
4
        x = *(addr1);
5
        i = p - 1;
6
        for (j = p; j < r; j++) {
7
            addr1 = a + j;
            if (*(addr1) <= x) {
8
9
                 i++;
10
                 addr1 = a + i;
                 addr2 = a + j;
11
12
                 tmp1 = *(addr1);
13
                 tmp2 = *(addr2);
                 *(addr1) = tmp2;
14
15
                 *(addr2) = tmp1;
16
            }
        }
17
18
        addr1 = a + i + 1;
19
        addr2 = a + r;
20
        tmp1 = *(addr1);
21
        tmp2 = *(addr2);
22
        *(addr1) = tmp2;
        *(addr2) = tmp1;
23
        return i + 1;
24
25
```

The quicksort function Finally, we implement the quicksort function as described in the algorithms book.

```
void quicksort(int *a, int p, int r) {
    if (p < r) {
        int q = partition(a, p, r);
        quicksort(a, p, q-1);
        quicksort(a, q+1, r);
    }
}</pre>
```

**Initial call to the algorithm** To emphasize the order of calling, and the arguments, we also construct a main function.

```
1  int main() {
2    int arr[6], i;
3    load(arr);
4    quicksort(arr, 0, 5);
5  }
```

Do note that the arguments to both the quicksort and partition functions, are inclusive. To verify the correctness, we should look at the memory, where the data now should be in sorted order.

#### Towers Of Hanoi

Towers Of Hanoi is a puzzle, where one has to move a tower of discs, from one peg, to another, with one additional auxiliary peg, by only moving one disc at the time.

By searching, we find a pseudo approach to the problem, by using recursion[7]. We are going to represent the three pegs as an array, which is three times the size of the tower. As such, each peg is just one third of the array. Furthermore, as with quicksort, we start by loading data into memory, have a tower function, which performs the move, and finally correct way of calling the functions.

**Loading data into memory** We need to initialize the memory. We are going to fill the first third of the array, i.e. the first peg, with descending numbers. Each number indicate which disc it is. The rest of the pegs with 0, indicating no disc.

```
void init(int num, int *from, int *to, int *aux) {
   int i;
   for (i = 0; i < num; i++) {
       *(from+i) = num - i;
       *(to+i) = 0;
       *(aux+i) = 0;
}</pre>
```

The tower function This is the tower function, which moves the discs from peg to peg. We use the same approach as the pseudo code[7]. As with quicksort, the statements have been expanded to more closely resemble assembly. We make the three arguments as stack pointers to each of the pegs. To preserve each of the pointers, we have pointers to each of them, i.e. pointer pointer.

```
void tower(int num, int **from, int **to, int **aux) {
1
2
       int *t, *f;
3
        if (num == 0) {
            t = *to;
4
5
            f = *from;
6
            f--;
            *t = *f;
7
            t++;
9
            *f = 0;
```

```
*to = t;
10
11
            *from = f;
12
        } else {
13
            tower(num-1, from, aux, to);
14
            t = *to;
15
            f = *from;
            f--;
16
            *t = *f;
17
18
            t++;
            *f = 0;
19
            *to = t;
20
21
            *from = f;
22
            tower(num-1, aux, to, from);
23
        }
^{24}
```

Calling the algorithm We construct a main function, to emphasize how to make the arguments.

```
int main() {
    int num = 5;
    int *arr = int[num*3];
    init(num, arr, arr+num, arr+(2*num));
    int *f=num, *t=num, *a=2*num;
    tower(num-1, &f, &a, &t);
}
```

When the program has finished, the last third of the array should contain the numbers in descending order.

## 6 Pipelining

In this section, we will be looking at pipelining our single cycle MIPS processor, and handle the problems, which are introduced by pipelining.

We start by going through the background and motivation for pipelining, and then proceed on how to extend our single cycle MIPS processor to have pipes.

As mentioned, pipelining introduces new problems to handle in the processor, and we will solve it by adding two new components: the Forwarding Unit, which forwards results from previous instructions to later instructions, and the Hazard Detection Unit, which controls when to stall the pipeline.

Throughout each step, we will also be writing programs, in order to verify that our processor behaves as specified.

#### 6.1 Introducing the pipes

We have the single cycle MIPS processor, which accepts the core integer instruction set. However, it is not very efficient, as the clock rate of the processor is determined by the longest possible path in the processor, so in order to increase the clock rate, we must decrease the longest path in the processor. We solve this by introducing pipes.

Pipes are registers in the processor, where we temporarely store all the values computed so far. It takes all of its inputs, and holds them until the next clock tick, where it will forward the values it is holding. This ensures that the data does not have to travel as far, until it have reached a safe state.

In order to know where to put the pipes, we divide the processor into stages. We will use the same stages, as proposed in the book[3], i.e. divide the processor into 5 stages: Instruction Fetch (IF), Instruction Decode (ID), Execution (EX), Memory (MEM) and Write Back (WB). A simplified overview of the processor and its stages can be seen in Figure 18. In between each stage, we are going to insert a pipe, i.e. we are going to insert 4 pipes. The simplified processor with pipes can be seen in figure 19.

Introducing pipes also introduces additional problems, which we will discuss further in sections 6.2 and 6.3

#### Implementation

There are two ways to implement pipes in SME: by using clocked busses, or by using clocked processes. If the bus only traverses 2 stages, then we can use clocked busses, as the semantics of a clocked bus is exactly the same as a pipe. However, if the bus traverses more than 2 stages, we are going to need additional processes, as the clocked bus only stores its value for one clock. Furthermore, if the only change to the bus is the given ClockedBus attribute, then determining whether or not a bus is part of a pipe can be problematic. As such, we should have the pipe and its busses in its own class, as it then receives its own prefix, and the code then becomes more readable.

Introducing the pipes in this manner is fairly straightforward, as we just add 4 classes, each with the same set of busses as the 'previous' stage outputs to the 'next' stage, and each with a register process, which forwards all the values from the 'previous' stages busses, onto its own busses with the same name.

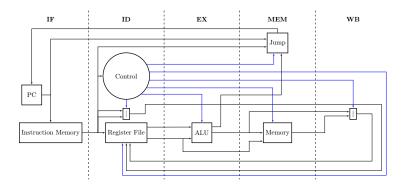


Figure 18: The simplified single cycle processor, and its stages. The stage names are highlighted in **bold**. The stages are divided by dashed lines.

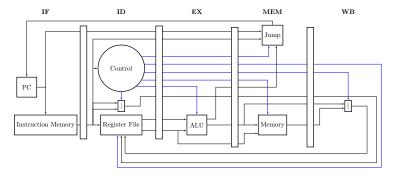


Figure 19: The simplified pipelined processor. The long bars in between each state is the pipes.

Let us take a subset of the IF/ID pipe, as an example. We assume that each state is in its own classes, i.e. IF and ID, and that the IF stage has the Instruction bus, which the ID stage reads from. Then, by adding a subclass to the IF stage, the name of the bus that the ID class calls is converted from IF.Instruction to IF.Pipe.Instruction, emphasizing that the bus is now piped.

This process can be repeated for all of the required pipes. By following the division of the components, as proposed in the book [3], there is only one really tricky part of pipelining: the Jump Unit. We dont know whether we should jump until the MEM stage, as the computations are made in the EX stage, and the conditions are computed in the MEM stage. However, the Program Counter has to increment, regardless of whether or not we should jump. As such, we should divide the Jump Unit into its subcomponents, and place some of the logic in the IF, some in the EX stage and finally some of it in the MEM stage.

For the IF stage, the incrementer and a multiplexor should be placed inside the stage. The multiplexor should take an address computed from the MEM stage, and the incremented Program Counter, and based on whether or not the instruction that has reached the MEM stage was a branch or jump instruction, it should choose the computed address.

The core computation of the jump and branch instructions should be placed in the EX stage. As such, the EX stage should compute both the branch address and the jump address.

Finally, the MEM stage should hold the decision logic. First, it should determine if the instruction was an branch instruction, and whether or not the condition has been satisfied, in which case, the address forwarded to the IF stage should be the branch address. If this was not the case, the computed jump address should be the one forwarded.

Finally, in the single cycle MIPS processor, we introduced the Write Buffer, in order to remove the cycle from and to the Register File. However, by introducing pipes, we have introduced a new buffer, and thus the Write Buffer can be removed.

#### Testing

To test the processor, we can use any of the programs, that we have previously written. However, since we have pipelined the processor, we need to insert bubbles, in order for data to be available for each instruction. A bubble is a No Operation (nop) instruction, which performs no operation, and does not modify neither the Register File nor the Memory.

We are going to implement a simple program, which is easy to verify: a small loop, which computes n fibonacci numbers, and places them in memory. As with the simple cycle, we are going to give pseudo low level C code:

```
void init(int *arr) {
    *(arr) = 1;
    *(arr+1) = 1;

void loop(int *arr, int n) {
    int i, tmp1, tmp2, tmp3;
    for (i = 0; i < n; i++) {
</pre>
```

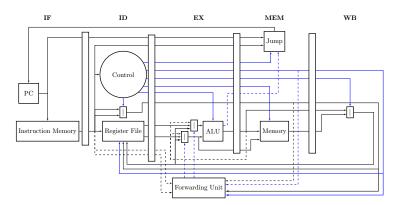


Figure 20: The simplified pipelined processor with forwarding.

Note that for verification in C, the size of the array should be n+2, due to initialization. Furthermore, when we port it to MIPS assembly, after each instruction, we should insert 4 nop's, to ensure the data is ready for the next instruction. When the program has run, the n+2 fibonacci numbers should be in memory, at the given address.

#### 6.2 Forwarding

By introducing pipes, we also introduced data hazards and control hazards. We will go through control hazards in section 6.3. Data hazards are when one instruction writes to a register, that a following instruction reads from. This is not a problem in the single cycle processor, as all data have been written to registers in the same clock cycle. This is not the case in the pipelined processor, e.g. the data might reside in the MEM stage, when it is needed in the EX stage.

We can eliminate some of the data hazards by implementing an additional unit: the Forwarding Unit. The rest of the data hazards will be handled by hazard detection in section 6.3. The Forwarding Unit looks at the register addresses used by the instruction in the EX stage, and checks if it corresponds with the register write address of either the instruction in the MEM stage, or the instruction in the WB stage. If they correspond, it forwards the value from either the MEM or the WB stage to the EX stage. The overview of the processor with the forwarding unit can be seen in Figure 20.

We could also have gone with the bubble approach, i.e. insert bubbles every time there is a data hazard. However, this would not be optimal, as there would be several clock cycles, where the processor is idle.

#### **Implementation**

Implementing the Forwarding Unit should be done with an SME process. As it interferes with the EX stage, it should be put in the EX stage. The unit should be controlling two multiplexors. The first multiplexor should control whether Output A should come from the ID pipe, the write data from the MEM stage or the write data from the WB stage. The other multiplexor is analogous, but with Output B. The two multiplexors should be controlled by the Forwarding Unit, i.e. it should generate the control signals for the multiplexors based on the two register read addresses of the current instruction in the EX stage, the register write address and write enabled signal both from the MEM and the WB stages.

#### Testing

Testing is straightforward: we can just remove all of the nop's from the fibonacci program, except for those which come after either a load, a branch or a jump, as these hazards are not handled yet.

#### 6.3 Hazard Detection

As mentioned in section 6.2, we cannot avoid all data hazards with forwarding. This is due to forwarding only forwarding to the EX stage. However, if the instruction prior to the instruction in EX is a load, then the data wont be available, until the load instruction has reached the WB stage. To handle this, the processor needs to detect the hazard and insert a bubble, as this will delay the instruction, so that when it has reached the EX stage, the load instruction will be in the WB stage. When inserting a bubble in the middle of the pipeline, we have to stall some of the pipes and registers. Stalling is the action of outputting what is stored in the register, but not updating it, i.e. outputting the same data in the next clock cycle.

We also need to be able to handle an additional type of hazard: control hazards. Control hazards are when either jump or branch instructions are executed. The problem is that the branch or jump is not performed until the MEM stage, which means that the pipeline following the jump or branch instruction may have been filled up by instructions, which should not be executed. To solve this, we should detect the hazard, and in such case flush the pipeline. Flushing the pipeline, is the action of resetting the registers in the pipes, so that they output a nop instruction. The overview of the processor with the Hazard Detection Unit can be seen in Figure 21.

#### Implementation

The hazard detection should be implemented in its own SME process: the Hazard Detection Unit.

To solve the data hazards, it needs to read the memread control flag from the EX stage, the register write address from the EX stage and the two source register addresses from the ID stage. If the memread flag has been set, and either of the source registers match the destination register, then the ID/EX pipe should be flushed, the IF/ID pipe should be stalled, and the PC register should be stalled.

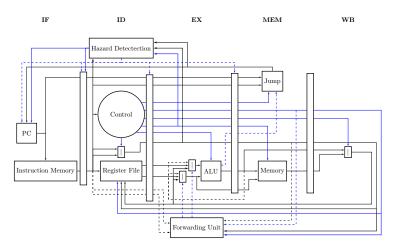


Figure 21: The simplified pipelined processor with forwarding and hazard detection.

To solve the control hazards, it should read the PCSrc flag from the MEM stage (the one used by the PC register, to multiplex between the incremented address, and the jump/branch address), and if it is set, then the IF/ID, ID/EX and EX/MEM pipes should be flushed. Note: the PC register should read normally in the case of a flush, even though it might have to stall, as the stall is detected in the to be flushed part of the pipeline.

#### Testing

Testing the new hazard detection is straightforward, as the processor should now be able to handle all of the previous programs, albeit in a larger amount of clock ticks, compared to the single cycle processor.

Note: if these programs are tested against the single cycle processor, the performance will be worse, when simulating. This is due to more processes and busses, which gives the simulation more work. There should not be any performance hit, when the processor is run on hardware.

#### 6.4 Branch prediction?

PERFORMANCE HIT! nævn at det sker i simulering TODO få noget ala linux til at køre? scratchpad memory som agere cache? flere kerner? scalar processor?

## 7 Synthesizing SME

In this section, I will be describing the steps required to implement an SME network onto an actual FPGA. I start with the initial logic gates design, as it is very simple to verify, and because it does not have any requirements regarding clocking.

Then I will be implementing the single cycle MIPS processor, and finally show how to communicate with the hardware implemented on the FPGA. This section will be very hardware specific, as I have not had time to verify or develop an approach, which will run on additional hardware.

#### 7.1 General workflow

TODO figur! SME > ghdl > vivado func > vivado post-impl > bitstream > interface func > export > SDK.

In order to implement hardware onto an FPGA, one must describe the hardware using a hardware description language such as VHDL or Verilog. As mentioned before, SME can be transpiled into VHDL, and as such provides a high level approach for hardware design. Therefore, the first thing to do is to write an SME network, verify that it runs as expected, and then checking that the transpiler does not fail to transpile.

Once the VHDL has been generated, the VHDL can be verified by using ghdl. ghdl is an commandline simulator for VHDL. SME provides a testbench and a Makefile for running the testbench using ghdl. As such, to verify the generated VHDL, one only needs to run make inside the output vhdl/ folder.

Processor is clocked at 5mhz!

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