

# Integer arithmetic

Computer Systems  
Lecture 3

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**Based on slides by:**

Randal E. Bryant and David R. O'Hallaron

# Encoding Integers

## Unsigned

$$B2U(X) = \sum_{i=0}^{w-1} x_i \cdot 2^i$$

## Two's Complement

$$B2T(X) = -x_{w-1} \cdot 2^{w-1} + \sum_{i=0}^{w-2} x_i \cdot 2^i$$

```
short int x = 15213;  
short int y = -15213;
```

Sign  
Bit

### ■ C short 2 bytes long

	Decimal	Hex	Binary
<b>x</b>	15213	3B 6D	00111011 01101101
<b>y</b>	-15213	C4 93	11000100 10010011

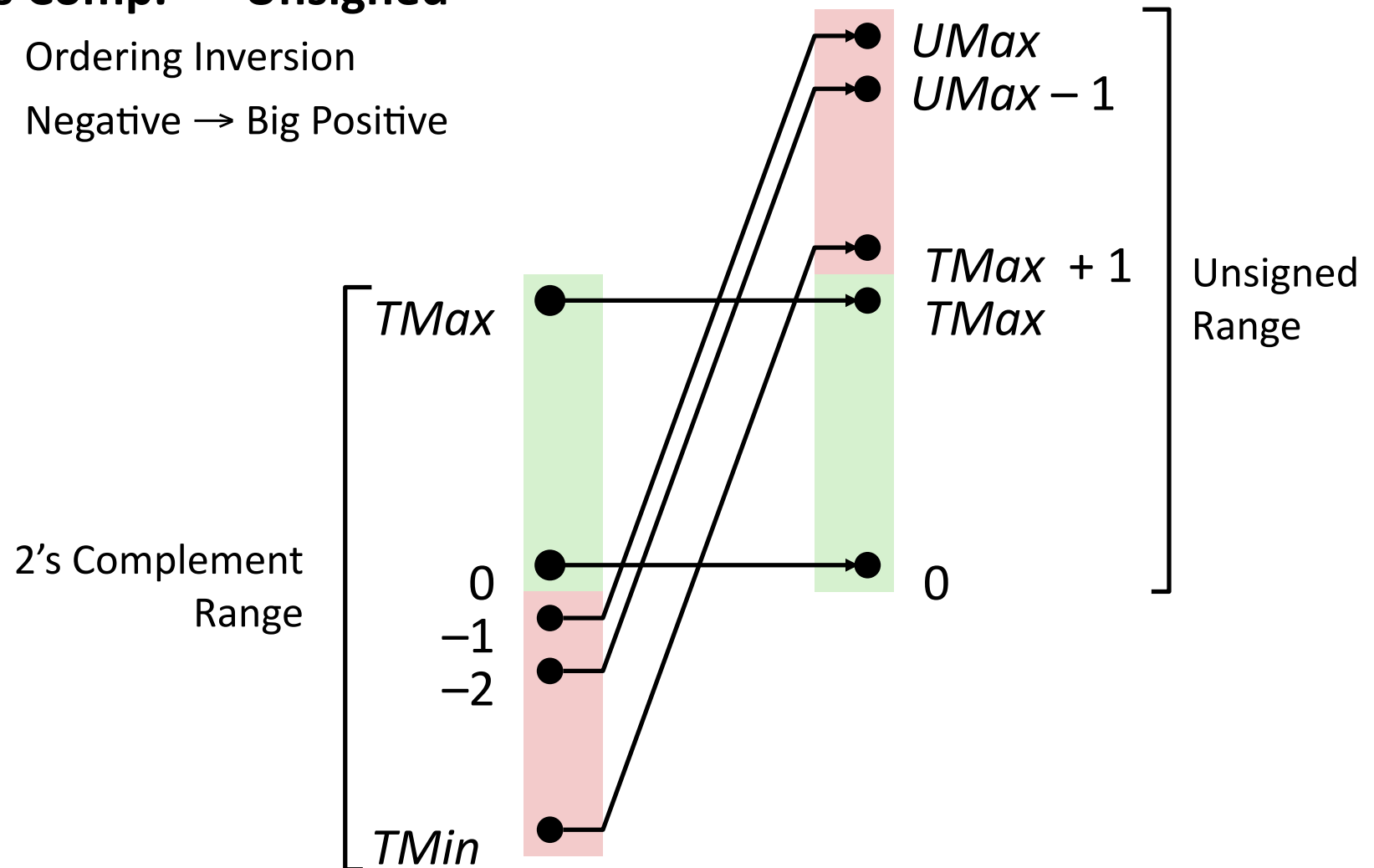
### ■ Sign Bit

- For 2's complement, most significant bit indicates sign
  - 0 for nonnegative
  - 1 for negative

# Conversion Visualized

## ■ 2's Comp. → Unsigned

- Ordering Inversion
- Negative → Big Positive



## Example: Decimal addition

$$\begin{array}{r} \text{,} \\ 4 \quad 6 \quad 4 \\ + 8 \quad 7 \quad 5 \\ \hline 13 \quad 3 \quad 9 \end{array}$$

## Example: Binary addition

$$\begin{array}{rcccc} & & 1 & 0 & 1 & 1 \\ + & 1 & 1 & 1 & 0 & \\ \hline & 1 & 0 & 0 & 1 & \end{array}$$

# Unsigned Addition

Operands:  $w$  bits


$u$  

$+ v$  

True Sum:  $w+1$  bits

$u + v$  

Discard Carry:  $w$  bits

$\text{UAdd}_w(u, v)$  

## ■ Standard Addition Function

- Ignores carry output

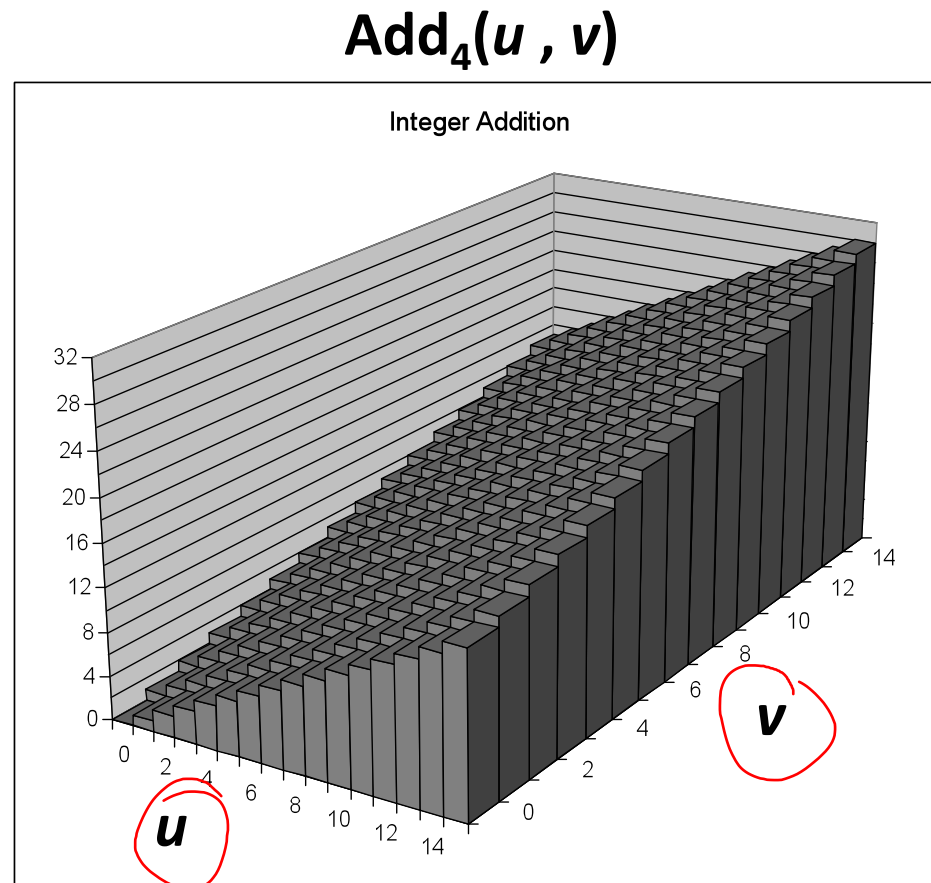
## ■ Implements Modular Arithmetic

$$s = \text{UAdd}_w(u, v) = u + v \bmod 2^w$$

# Visualizing (Mathematical) Integer Addition

## ■ Integer Addition

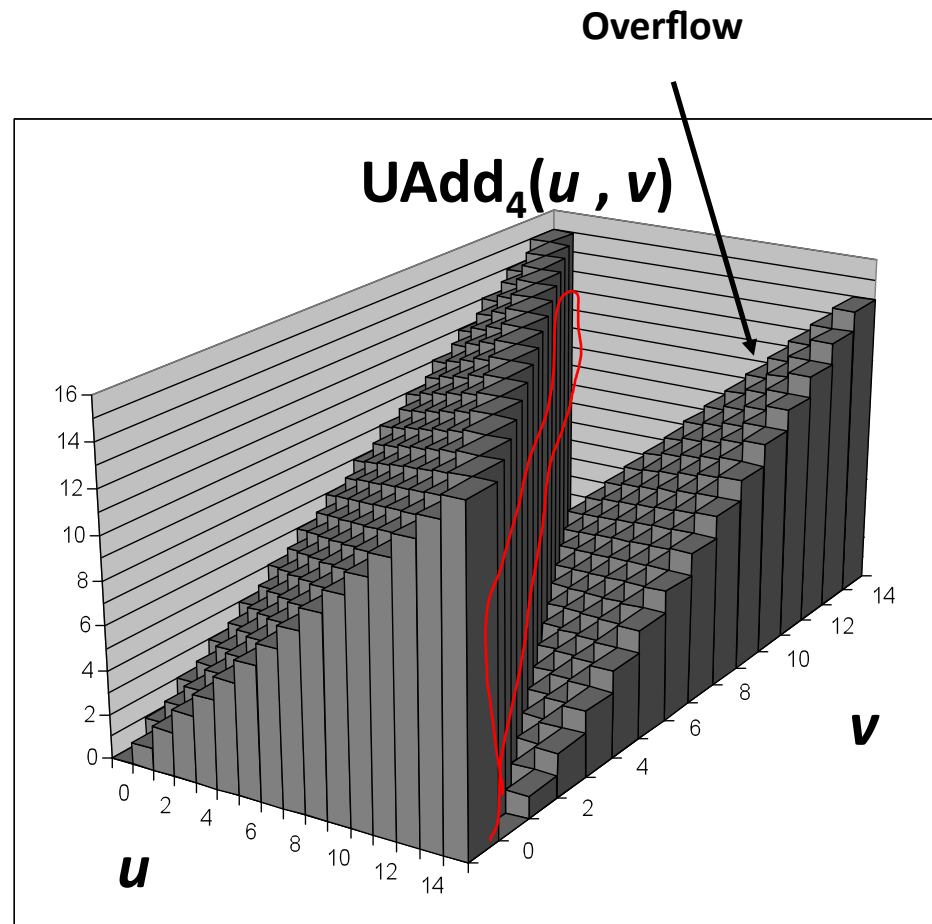
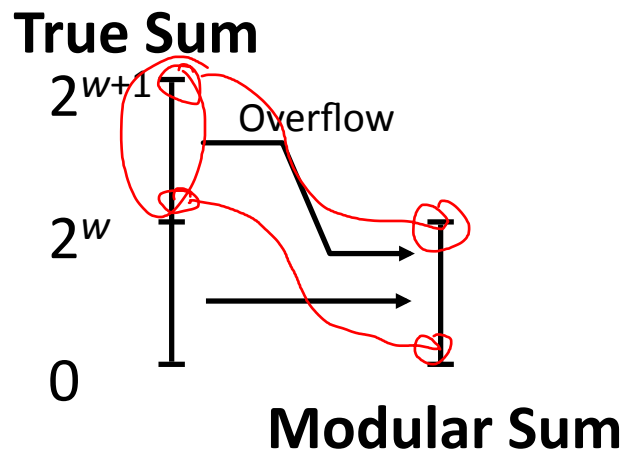
- 4-bit integers  $u, v$
- Compute true sum  $\text{Add}_4(u, v)$
- Values increase linearly with  $u$  and  $v$
- Forms planar surface



# Visualizing Unsigned Addition

## ■ Wraps Around

- If true sum  $\geq 2^w$
- At most once





# Two's Complement Addition

Operands:  $w$  bits



True Sum:  $w+1$  bits



Discard Carry:  $w$  bits



## ■ TAdd and UAdd have Identical Bit-Level Behavior

- Signed vs. unsigned addition in C:

```
int s, t, u, v;
```

```
s = (int) ((unsigned) u + (unsigned) v);
```

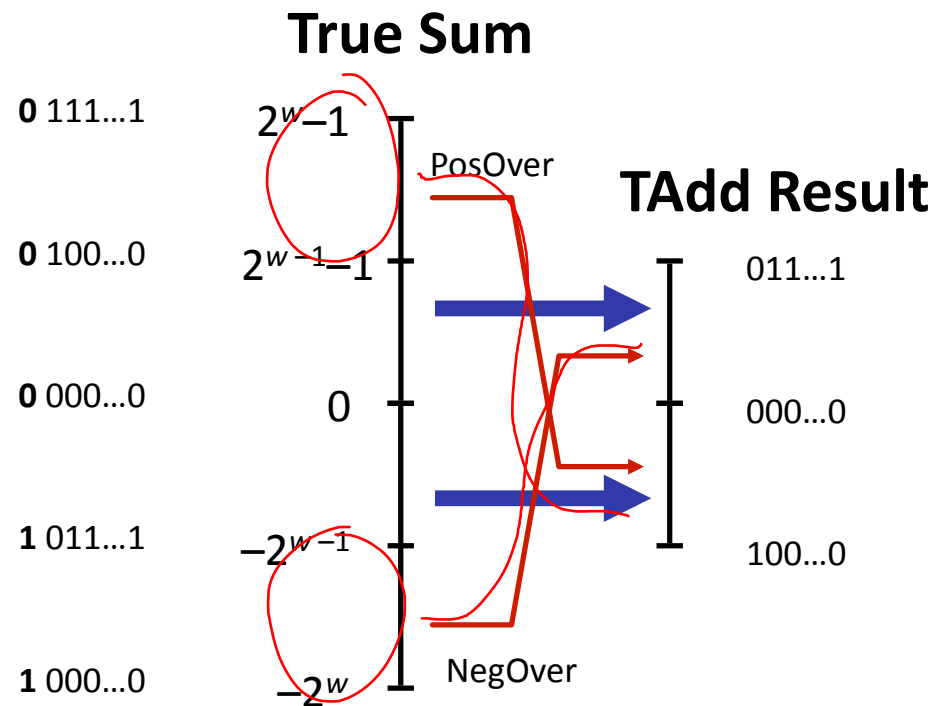
```
t = u + v
```

- Will give `s == t`

# TAdd Overflow

## ■ Functionality

- True sum requires  $w+1$  bits
- Drop off MSB
- Treat remaining bits as 2's comp. integer



# Visualizing 2's Complement Addition

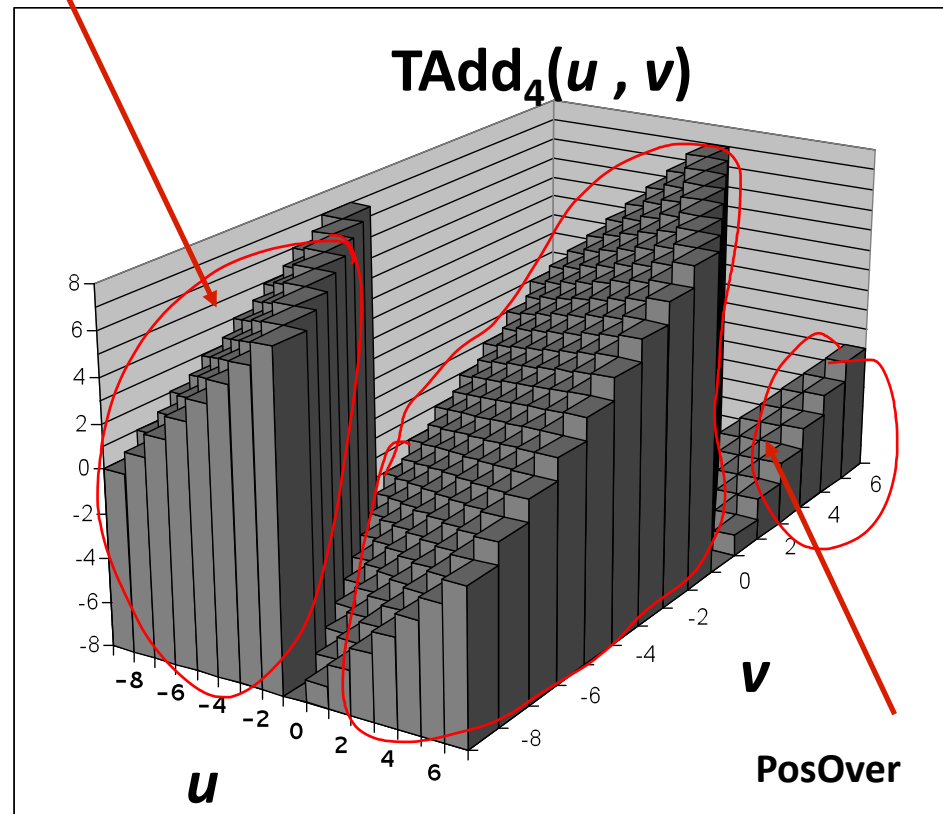
## ■ Values

- 4-bit two's comp.
- Range from -8 to +7

## ■ Wraps Around

- If  $\text{sum} \geq 2^{w-1}$ 
  - Becomes negative
  - At most once
- If  $\text{sum} < -2^{w-1}$ 
  - Becomes positive
  - At most once

NegOver



# Play the game

- <http://bit.ly/integer-arithmetic>