CSC236 – Tutorial 3: Runtime Analysis

Exercise 1: Theta Proofs

A. Prove that $\sqrt{7x^2 + 4x} = \Theta(x)$.

B. Prove that $(\sqrt{2})^{\log n} = \Theta(\sqrt{n})$.

Exercise 2: Big O Proof

Evaluate this proof attempt that $\frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right) = O(5^n)$.

$$
\begin{aligned}
\frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right) &\leq \frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^n\right) && \text{((a))} \\
&\leq \frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{9}}{2}\right)^n\right) && \text{((b))} \\
&\leq \frac{1}{\sqrt{5}}\left((5)^n\right) && \text{((c))} \\
&\leq \left((5)^n\right) && \text{((d))} \\
&= O(5^n) && \text{((e))}
\end{aligned}
$$

Exercise 3: Iterative Runtime

Give a sum for the number of steps taken by the following algorithm. Then, bound the sum from above and below to prove a $\Theta(n^3)$ bound.

```
def f(n):
  total = 0
  for i in range(1, n+1):
    for j in range(i, n+1):
      for k in range(i, j+1):
        total = total + 1
  return total
```

Exercise 4: Recurrences

Solve the recurrence

$$F(n) = \begin{cases} 2, & \text{if } n = 0 \\ F(n-1) + 3n^2 - 5n^3, & \text{if } n \geq 1 \end{cases}$$