

# CSC236 – Tutorial 4: Closed-Forms

## Exercise 1: Finding a Closed-Form

$$f(n) = \begin{cases} 3, & \text{if } n = 1 \\ 4f(n/2) + n, & \text{if } n \geq 2 \end{cases}$$

Assume that  $n$  is a power of 2. Carry out the five steps for repeated substitution to prove a closed-form for this recursive definition  $f$ .

## Exercise 2: Trouble with Substitution

$$f(n) = \begin{cases} 1, & \text{if } n = 1 \text{ or } n = 2 \\ f(n-1) + f(n-2), & \text{if } n \geq 3 \end{cases}$$

Write out  $k = 1, 2, 3$  for step 1 of repeated substitution. What do you notice? Why is repeated substitution going to be more difficult to use here?

## Exercise 3: Developing a Recurrence

Consider an  $m$  by  $n$  board ( $m$  rows,  $n$  columns). You begin at the upper-left square  $(m, n)$  and want to reach the lower-right square  $(1, 1)$ . On each move, you are allowed to go one square to the right or one square down. Let  $f(m, n)$  be the total number of allowed paths from  $(m, n)$  to  $(1, 1)$ . Give a recursive definition for  $f(m, n)$  and argue why it is correct.