Aerial Manipulation using a Tilted-Motors Hexacopter with a Two DoF Robotic Arm

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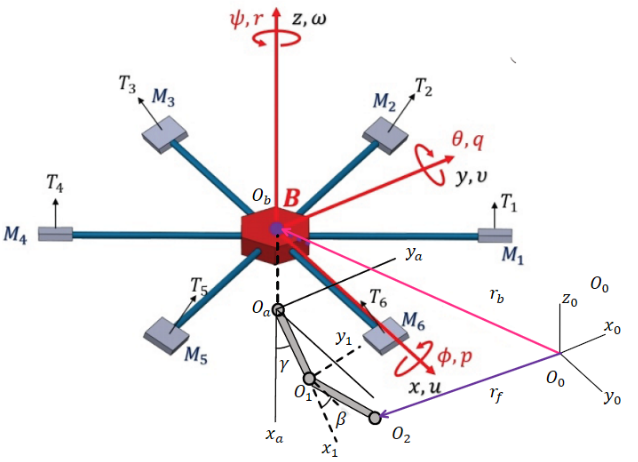

Figure 1: UAM/Arm system illustration. O_i represents the origin of a reference system with unit vectors x_i, y_i, z_i. The angle γ is taken from x_a to the first link and β from x_1 to the second link, both restricted to move in the plane (x_a, y_a) or (y_b, z_b).

I. System Modeling

In this section, modeling of the UAM presented in Fig. 1 is formulated using Euler-Lagrange method. The following assumptions are considered:

- The 2 DoF robotic arm is modeled as a double compound pendulum.
- No payload is attached to the end-effector.
- Air drag is negligible.

A. Kinematic Model

Coordinate frames of the combined system are defined in Fig. 1. The frames O_0, O_b, O_a, O_1, O_2 are namely the inertial frame, the body frame of the hexarotor, the joint between the aerial vehicle and the manipulator, the joint between both links, and the end-effector attached frame.

Let the absolute position of the UAV be denoted by $r_b = [x, y, z]^T \in \mathbb{R}^3$, while the attitude is described by the yaw-pitch-roll Euler Angles $\eta = [\phi, \theta, \psi] \in \mathbb{R}^3$. The position of the center of mass and angular velocity of link i with respect to O_0 can then be expressed by

$$r_{li} = r_b + \mathbf{R}_b r_{li}^b, \tag{1}$$
$$\omega_{li} = \omega_b + \mathbf{R}_b \omega_{li}^b, \tag{2}$$

where r_{li}^b and ω_{li}^b are the position of the center of mass and angular velocity of the link with respect to O_b, and

$$\mathbf{R}_b = \begin{bmatrix} C_\psi C_\theta & C_\psi S_\phi S_\theta - C_\phi S_\psi & S_\phi S_\psi + C_\phi C_\psi S_\theta \\ C_\theta S_\psi & C_\phi C_\psi + S_\psi S_\theta S_\phi & -C_\psi S_\phi + S_\psi S_\theta C_\phi \\ -S_\theta & C_\theta S_\phi & S_\theta C_\phi \end{bmatrix}, \tag{3}$$

is the rotation matrix denoting the orientation of O_b with respect to O_0, with $C_* = \cos(*)$, $S_* = \sin(*)$.

If the angular velocity of the hexacopter with respect to O_0 is denoted by ω_b, while ω_b^b is the angular velocity of the UAV with respect to O_b, the following relations persist

$$\omega_b = \mathbf{A}\dot{\eta}, \tag{4}$$
$$\omega_b^b = \mathbf{R}_b^T \mathbf{A}\dot{\eta} = \mathbf{W}\dot{\eta}, \tag{5}$$

where

$$\mathbf{W} = \begin{bmatrix} 1 & 0 & -S_\theta \\ 0 & C_\phi & C_\theta S_\phi \\ 0 & -S_\phi & C_\theta C_\phi \end{bmatrix}, \tag{6}$$

maps from the time derivative of the Euler Angles to the angular velocity expressed in O_b.

Consider the following expression for the linear and angular velocities of the i-th link with respect to O_b

$$\dot{r}_{li}^b = \mathbf{J}_v^{li}\dot{q}, \tag{7}$$
$$\omega_{li}^b = \mathbf{J}_\omega^{li}\dot{q}, \tag{8}$$

where $q = [\gamma, \beta]^T \in \mathbb{R}^2$ is the vector of joint variables and $\mathbf{J}_v^{li}$ and $\mathbf{J}_\omega^{li}$ are the contributions of the Jacobian columns relative to the joint velocities up to the current link i.

Taking the time derivative of 1 and substituting 7 yields

$$\dot{r}_{li} = \dot{r}_b - S(\mathbf{R}_b r_{li}^b)\omega_b + \mathbf{R}_b \mathbf{J}_v^{li}\dot{q}, \tag{9}$$

where $S(v)$ is the skew-symmetric matrix related to the vector v. By substituting 8 in 2 a similar expression for the angular velocity can be obtained

$$\omega_{li} = \omega_b + \mathbf{R}_b \mathbf{J}_\omega^{li}\dot{q}. \tag{10}$$

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The full kinematic model can then be rewritten in matrix form

$$\dot{r}_b = [\mathbf{I}_{3\times3}, \mathbf{0}_{3\times3}, \mathbf{0}_{3\times2}]\,\dot{\xi} = \mathbf{D}_r^b\dot{\xi} \tag{11}$$

$$\omega_b = [\mathbf{I}_{3\times3}, \mathbf{A}, \mathbf{0}_{3\times2}]\,\dot{\xi} = \mathbf{D}_\omega^b\dot{\xi} \tag{12}$$

$$\dot{r}_{li} = \left[\mathbf{I}_{3\times3}, -S(\mathbf{R}_b r_{li}^b \mathbf{A}), \mathbf{R}_b\mathbf{J}_v^{li}\right]\dot{\xi} = \mathbf{D}_r^{li}\dot{\xi} \tag{13}$$

$$\omega_{li} = \left[\mathbf{0}_{3\times3}, \mathbf{A}, \mathbf{R}_b\mathbf{J}_\omega^{li}\right]\dot{\xi} = \mathbf{D}_\omega^{li}\dot{\xi} \tag{14}$$

where

$$\xi = [x, y, z, \phi, \theta, \psi, \gamma, \beta]^T \in \mathbb{R}^8, \tag{15}$$

is the vector of generalized coordinates, $\mathbf{I}_{3\times3}$ is the (3×3) identity matrix, and $\mathbf{0}_{n\times m}$ is the $(n\times m)$ zero matrix.

Furthermore, the manipulator system is showed in 2. Hence, the rotation and translation operators up to the body frame are defined as follows

$$\mathbf{R}_2^1 = \mathbf{Z}(\beta), \quad (16) \qquad d_2^1 = l_2\begin{bmatrix} C_\beta \\ S_\beta \\ 0 \end{bmatrix}, \quad (17)$$

$$\mathbf{R}_1^a = \mathbf{Z}(\gamma), \quad (18) \qquad d_1^a = l_1\begin{bmatrix} C_\gamma \\ S_\gamma \\ 0 \end{bmatrix}, \quad (19)$$

$$\mathbf{R}_a^b = \mathbf{Y}\left(\frac{\pi}{2}\right), \quad (20) \qquad d_a^b = a\begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}, \quad (21)$$

where $\mathbf{R}_j^i$ and d_j^i represent the rotation and translation from frame O_j to O_i, respectively, $\mathbf{Y}(*)$ and $\mathbf{Z}(*)$ are the rotation matrix about the y and z axis, respectively, a is the distance from O_b to O_a, and l_i the total length of i-th link. Note that $d_b = r_b = [x, y, z]^T$ is the position of the hexacopter with respect to the inertial frame.

The double compound pendulum system is a really well studied problem, the position of the center of mass of each link with respect to O_a can derived by

$$r_{l1}^a = \left(c_1\begin{bmatrix} 0 \\ C_\gamma \\ S_\gamma \end{bmatrix}\right), \tag{22}$$

$$r_{l2}^a = \mathbf{R}_1^a\left(c_2\begin{bmatrix} 0 \\ C_\beta \\ S_\beta \end{bmatrix}\right) + d_1^a, \tag{23}$$

where c_i is the distance from the beginning of link i to its center of mass. Then, adding the rotation about the y axis $\mathbf{R}_a^b$ and translation d_a^b yields

$$\begin{aligned} r_{l1}^b &= \mathbf{R}_a^b r_{l1}^a + d_a^b \\ &= \begin{bmatrix} 0 \\ c_1 S_\gamma \\ -a - c_1 C_\gamma \end{bmatrix}, \end{aligned} \tag{24}$$

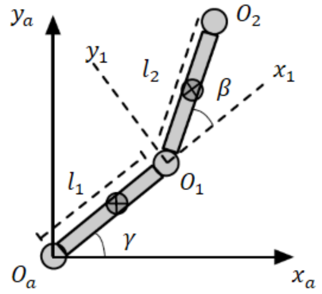

Figure 2: Double compound pendulum system.

$$\begin{aligned} r_{l2}^b &= \mathbf{R}_a^b r_{l2}^a + d_a^b \\ &= \begin{bmatrix} 0 \\ l_1 S_\gamma + c_2 S_{\gamma\beta} \\ -a - l_1 C_\gamma - c_2 C_{\gamma\beta} \end{bmatrix}, \end{aligned} \tag{25}$$

where $C_{\gamma\beta} = \cos(\gamma + \beta)$ and $S_{\gamma\beta} = \sin(\gamma + \beta)$.

The angular velocity of each link with respect to the body frame can be expressed as

$$\omega_{l1}^b = \mathbf{R}_a^b\begin{bmatrix} 0 \\ 0 \\ \dot{\gamma} \end{bmatrix} = \begin{bmatrix} \dot{\gamma} \\ 0 \\ 0 \end{bmatrix}, \tag{26}$$

$$\omega_{l2}^b = \mathbf{R}_a^b\begin{bmatrix} 0 \\ 0 \\ \dot{\gamma} + \dot{\beta} \end{bmatrix} = \begin{bmatrix} \dot{\gamma} + \dot{\beta} \\ 0 \\ 0 \end{bmatrix}. \tag{27}$$

By equating the time derivative of 24 and 25 with 7, the Jacobian contributions can be defined

$$\mathbf{J}_v^{l1} = \begin{bmatrix} 0 & 0 \\ c_1 C_\gamma & 0 \\ c_1 S_\gamma & 0 \end{bmatrix}, \tag{28}$$

$$\mathbf{J}_v^{l2} = \begin{bmatrix} 0 & 0 \\ l_1 C_\gamma + c_2 C_{\gamma\beta} & c_2 C_{\gamma\beta} \\ l_1 S_\gamma + c_2 S_{\gamma\beta} & c_2 S_{\gamma\beta} \end{bmatrix}. \tag{29}$$

Also 26 and 26 with 8

$$\mathbf{J}_\omega^{l1} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad (30) \qquad \mathbf{J}_\omega^{l2} = \begin{bmatrix} 1 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}. \quad (31)$$

B. Dynamic Model

The system is described by the general Matrix Form of Euler-Lagrange Equations

$$\mathbf{D}(\xi)\ddot{\xi} + \mathbf{C}(\xi, \dot{\xi})\dot{\xi} + \mathbf{G}(\xi) = \mathbf{u}, \tag{32}$$

For the Euler-Lagrange method, it is required to establish the kinetic and potential energies of the system, thus obtaining the Lagrangian defined by

$$\mathcal{L} = \mathcal{K} - \mathcal{U} \tag{33}$$

where $\mathcal{K}$ is the total kinetic energy and $\mathcal{U}$ is the total potential energy. Let the contribution of the vehicle to the kinetic energy be denoted by $\mathcal{K}_H$ and the contribution from each link $\mathcal{K}_{li}$, such that

$$\mathcal{K} = \mathcal{K}_H + \sum_{i=1}^{2} \mathcal{K}_{li}. \tag{34}$$

The contribution of the UAV is given by

$$\mathcal{K}_H = \frac{1}{2} m_b \dot{r}_b^T \dot{r}_b + \frac{1}{2} \omega_b^T \mathbf{R}_b \mathbf{H}_b \mathbf{R}_b^T \omega_b, \tag{35}$$

in which m_b is the mass of the vehicle and $\mathbf{H}_b$ is the inertia tensor expressed in the body frame, where, by symmetry, $\mathbf{H}_b$ is diagonal and constant.

The contribution of each link is given by

$$\mathcal{K}_{li} = \frac{1}{2} m_{li} \dot{r}_{li}^T \dot{r}_{li} + \frac{1}{2} \omega_{li}^T \mathbf{R}_b \mathbf{R}_{li}^b \mathbf{H}_{li} (\mathbf{R}_{li}^b)^T \mathbf{R}_b^T \omega_{li}, \tag{36}$$

where $\mathbf{H}_{li}$ is the constant inertia tensor of the link i expressed in a frame O_{li} attached to its center of mass and with the axes being coincident with the inertia central axes, while $\mathbf{R}_{li}^b$ is the rotation matrix that transforms from O_{li} to O_b. From Fig. 2, the frames that are parallel to O_{l1} and O_{l2} are O_1 and O_2, respectively. Thus, the rotation matrices can be defined as

$$\mathbf{R}_{l1}^b = \mathbf{R}_a^b \mathbf{R}_1^a, \tag{37}$$ $$\mathbf{R}_{l2}^b = \mathbf{R}_a^b \mathbf{R}_1^a \mathbf{R}_2^1. \tag{38}$$

Adding each kinetic energy contribution, the total kinetic energy 34 can be expressed as

$$\mathcal{K} = \frac{1}{2} \dot{\xi}^T \mathbf{D}(\xi) \dot{\xi}, \tag{39}$$

where $\mathbf{D}(\xi) \in \mathbb{R}^{8\times 8}$ is the symmetric and positive definite inertia matrix from 32, which can be written using 11-14

$$\mathbf{D}(\xi) = m_b (\mathbf{D}_r^b)^T \mathbf{D}_r^b + (\mathbf{D}_\omega^b)^T \mathbf{R}_b \mathbf{H}_b \mathbf{R}_b \mathbf{D}_\omega^b + \tag{40}$$

$$\sum_{i=1}^{2} (m_{li} (\mathbf{D}_r^{li})^T \mathbf{D}_r^{li} + (\mathbf{D}_\omega^{li})^T \mathbf{R}_b \mathbf{H}_{li}^b (\mathbf{R}_b)^T \mathbf{D}_\omega^{li}) \tag{41}$$

in which $\mathbf{H}_{li}^b = \mathbf{R}_{li}^b \mathbf{H}_{li} (\mathbf{R}_{li}^b)^T$.

The Coriolis Matrix $\mathbf{C}(\xi, \dot{\xi})$ from 32 can the be obtained from the Christoffel Symbols of the First Kind using the inertia matrix, hence, the k, j-th element of $\mathbf{C}(\xi, \dot{\xi}) \in \mathbb{R}^{8\times 8}$ is defined by

$$C_{kj} = \sum_{i=1}^{8} \frac{1}{2} \left[\frac{\partial d_{kj}}{\partial \xi_i} + \frac{\partial d_{ki}}{\partial \xi_j} - \frac{\partial d_{ij}}{\partial \xi_k} \right] \dot{\xi}_i, \tag{42}$$

where d_{kj} is the k-j-th element of $\mathbf{D}(\xi)$ and ξ_i is the i-th element of ξ.

On the other hand, the potential energy of the system can be defined as

$$\mathcal{U} = m_b g \left(\hat{z}^T r_b \right) + \sum_{i=1}^{2} [m_{li} g (\hat{z}^T r_{li})], \tag{43}$$

where $\hat{z}$ is the unit vector for the z axis of O_0. Then the gravity vector $\mathbf{G}(\xi)$ k-th element is

$$G_k = \frac{\partial \mathcal{U}}{\partial \xi_k}. \tag{44}$$

According to [1, 2] the generalized torque related to the joint variable q_i can be actuated directly through joint actuators. However, the generalized torques in x, y, z, ϕ, θ and ψ need to be transformed. Hence, the input vector $\mathbf{u}$ is

$$\mathbf{u} = \mathbf{B}(\xi) \begin{bmatrix} F_M \\ \tau_q \end{bmatrix}, \tag{45}$$

where $\tau_q = [\tau_\gamma, \tau_\beta]^T$ is the torque acting on each joint variable,

$$\mathbf{B}(\xi) = \begin{bmatrix} \mathbf{R}_b & 0 & 0 \\ 0 & \mathbf{W}^{-1} & 0 \\ 0 & 0 & \mathbf{I}_{2\times 2} \end{bmatrix} \in \mathbb{R}^{8\times 8}, \tag{46}$$

is the matrix presented in [3] but extended to implement the torques related to q, and F_M is the vector of distributed forces produced by the motors

$$F_M = \begin{bmatrix} F_x & F_y & F_z & \tau_\phi & \tau_\theta & \tau_\psi \end{bmatrix}^T. \tag{47}$$

Each motor provides a thrust T_i and torque Q_i related to its angular rate, thus, a simplified motor model is considered

$$T_i \approx k_T \Omega_i^2, \tag{48}$$

$$Q_i \approx k_Q \Omega_i^2, \tag{49}$$

where k_T and k_Q are the thrust and torque factor respectively, while Ω_i denotes the angular rate of the i-th motor.

In the hexacopter with tilted-rotors, the forces and torques provided by each motor have components over all of the axis. The total mapping of the forces for the six motors is arranged in the matrix

$$\mathbf{M}_\varphi = \begin{bmatrix} M_{\varphi 11} & \dots & M_{\varphi 16} \\ \vdots & \ddots & \vdots \\ M_{\varphi 61} & \dots & M_{\varphi 66} \end{bmatrix} \in \mathbb{R}^{6\times 6}, \tag{50}$$

which depends on the inclination φ and the angle α_i that each motor has with respect to the y_b axis. The derivation and whole components are shown in [4]. Finally, the relation between the angular rates and the distributed forces is given by

$$F_M = \mathbf{M}_\varphi \mathbf{\Omega}^2, \tag{51}$$

where $\mathbf{\Omega}^2$ is the vector containing the square of the angular rates.

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[3] J. M. Arizaga, H. Castañeda, and P. Castillo, "Payload swing attenuation of a fully-actuated hexacopter via extended high gain observer based adaptive sliding control," in *2021 International Conference on Unmanned Aircraft Systems (ICUAS)*, 2021, pp. 901–908.

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