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# Parcial 2 S.D

1)

$$I_0\ddot{\theta} + B\dot{\theta} + K\theta = \tau_a$$

$$I_0\dot{q}_2 + Bq_2 + Kq_1 = \tau_a$$

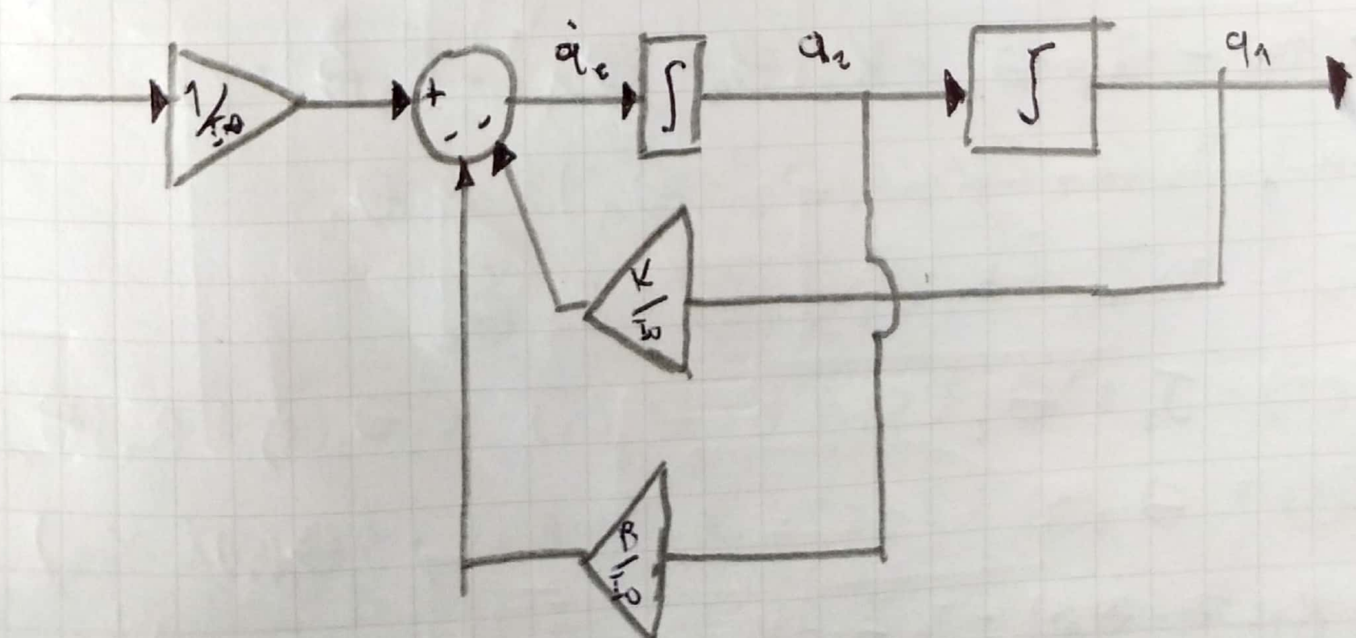
$$I_0\dot{q}_2 = \tau_a - Bq_2 - Kq_1$$

$$\begin{cases} q_1 = \theta \\ q_2 = \dot{q}_1 = \dot{\theta}_1 \\ \dot{q}_2 = \dot{q}_1 = \ddot{\theta} \end{cases}$$

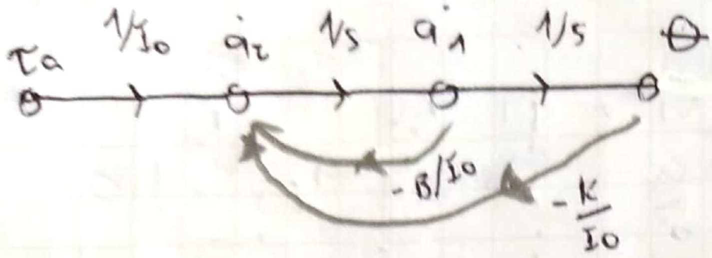
$$\begin{vmatrix} \dot{q}_1 \\ \dot{q}_2 \end{vmatrix} = \begin{vmatrix} 0 & 1 \\ -\frac{K}{I_0} & \frac{-B}{I_0} \end{vmatrix} \begin{vmatrix} q_1 \\ q_2 \end{vmatrix} + \begin{vmatrix} 0 \\ 1/I_0 \end{vmatrix} \tau_a(t)$$

$$\theta = [1 \quad 0] \begin{vmatrix} q_1 \\ q_2 \end{vmatrix}$$

b) Diagrama de bloques



2) Diagrama de flujo




-> F. transferencia

$$I_0 \ddot{\theta} + B\dot{\theta} + K\theta = \tau_a$$

$$I_0 s^2 \theta(s) + Bs\,\theta(s) + K\theta(s) = \tau_a(s)$$

$$\theta(s)\,(I_0 s^2 + Bs + K) = \tau_a(s)$$

$$G(s) = \frac{\theta(s)}{\tau_a(s)} = \frac{1}{I_0 s^2 + Bs + K}$$

(2) Función de transferencia entre $\theta_2$ y $\tau_a$

$$\Rightarrow \tau_a - J_2\ddot{\theta}_2 - K_2(\theta_2 - \theta_1) - B_2\dot{\theta}_2 = 0$$

$$\tau_a = J_2\ddot{\theta}_2 + K_2(\theta_2 - \theta_1) + B_2\dot{\theta}_2$$

$$\tau_a = J_2\ddot{\theta}_2 + K_2\theta_2 - K_2\theta_1 + B_2\dot{\theta}_2$$

$$\tau(s) = J_2 s^2\theta_2(s) + K_2\theta_2(s) - K_2\theta_1(s) + B_2 s\theta_2(s)$$

$$\tau(s) = \theta_2(s)\,(J_2 s^2 + K_2 + B_2 s) + \theta_1(s)(-K_2)$$

$$\Rightarrow K_2(\theta_2 - \theta_1) - J_1\ddot{\theta}_1 - K_1\theta_1 - B_1\dot{\theta}_1 = 0$$

$$K_2\theta_2 - K_2\theta_1 - J_1\ddot{\theta}_1 - K_1\theta_1 - B_1\dot{\theta}_1 = 0$$

$$K_2\Theta_2(s)-K_2\Theta_1(s)-J_1s^2\Theta_1(s)-K_1\Theta_1(s)-B_1s\Theta_1(s)=0$$

$$\Theta_1(s)(-B_1s-K_1-K_2-J_1s^2)+K_2\Theta_2(s)=0$$

$$\begin{bmatrix} -K_2 & J_2s^2+K_2+B_2s \\ -B_1s-K_1-K_2-J_1s^2 & K_2 \end{bmatrix}\begin{bmatrix}\Theta_1(s)\\ \Theta_2(s)\end{bmatrix}=\begin{bmatrix}T_s\\ 0\end{bmatrix}$$

$$\frac{\Theta_2(s)}{T(s)}=\frac{J_2s^2+K_2+B_2s}{-K_2^2(J_2s^2+K_2+B_2s)(B_1s+K_1+K_2+J_1s^2)}$$

b) en espacio de estados

$$K_2\Theta_2-K_2\Theta_1-J_1\ddot{\Theta}_1-K_1\Theta_1-B_1\dot{\Theta}_1=0$$

$\Theta_1=q_1$ ; $q_2=\dot{\Theta}_1$ ; $\dot{q}_2=\ddot{\Theta}_1$

$\Theta_2=q_3$ ; $q_4=\dot{\Theta}$ ; $\dot{q}_4=\ddot{\Theta}_2$

$$\ddot{\Theta}=\frac{K_2}{J_1}\Theta_2-\frac{(K_2+K_1)}{J_1}-\frac{K_1}{J_1}\Theta_1-\frac{B_1}{J_1}\dot{\Theta}_1$$

$$\dot{q}_2=\frac{K_1}{J_1}q_3-\frac{(K_2+K_1)}{J_1}q_1-\frac{B_1}{J_1}q_2$$

$$\tau_a=J_2\ddot{\Theta}_2+K_2(\Theta_2-\Theta_1)B_2\dot{\Theta}_2$$

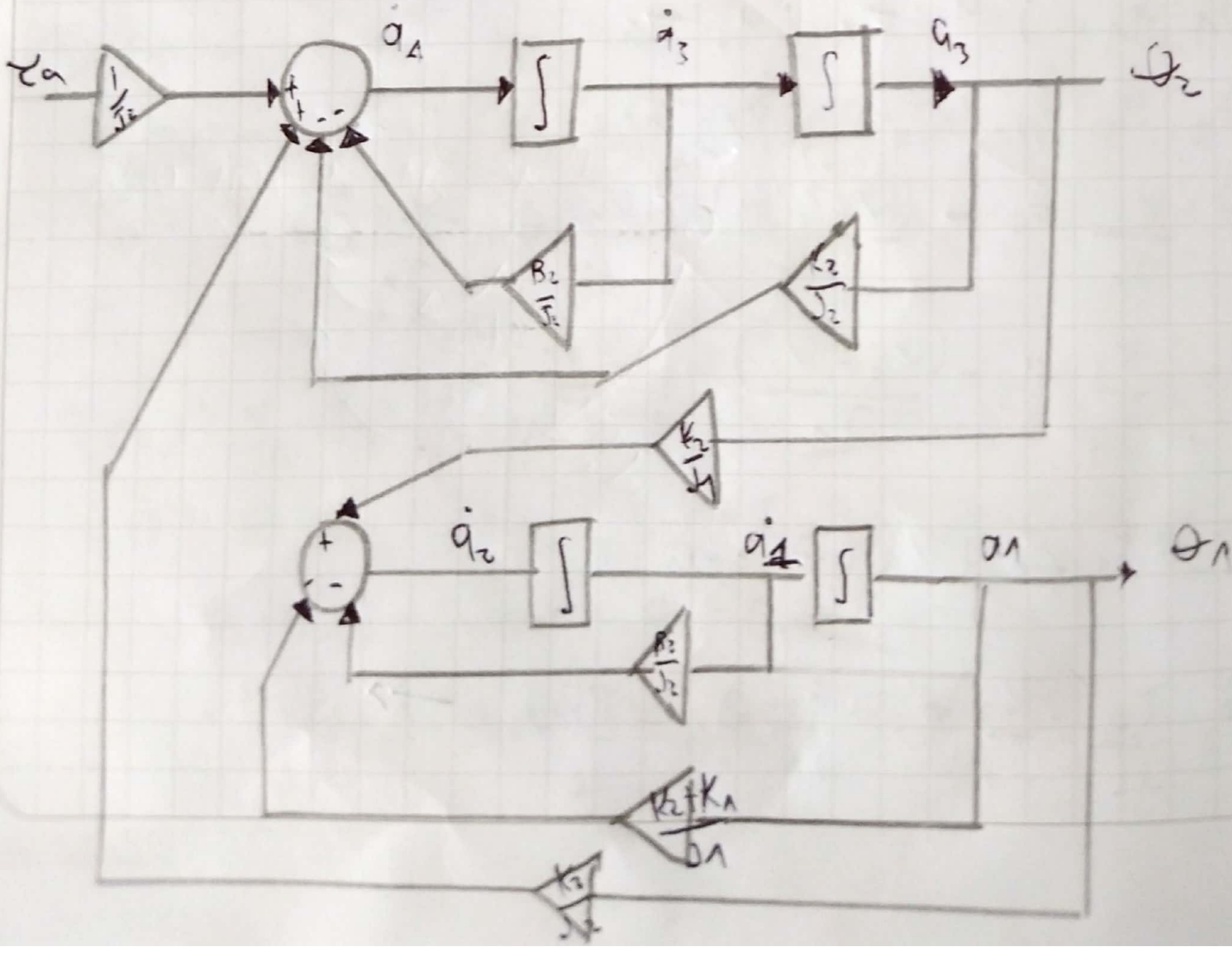
$$\ddot{\Theta}_2=\frac{\tau_a}{J_2}-\frac{K_2}{J_2}\Theta_2+\frac{K_2}{J_2}\Theta_1-\frac{B_2}{J_2}\dot{\Theta}_2 \longrightarrow$$

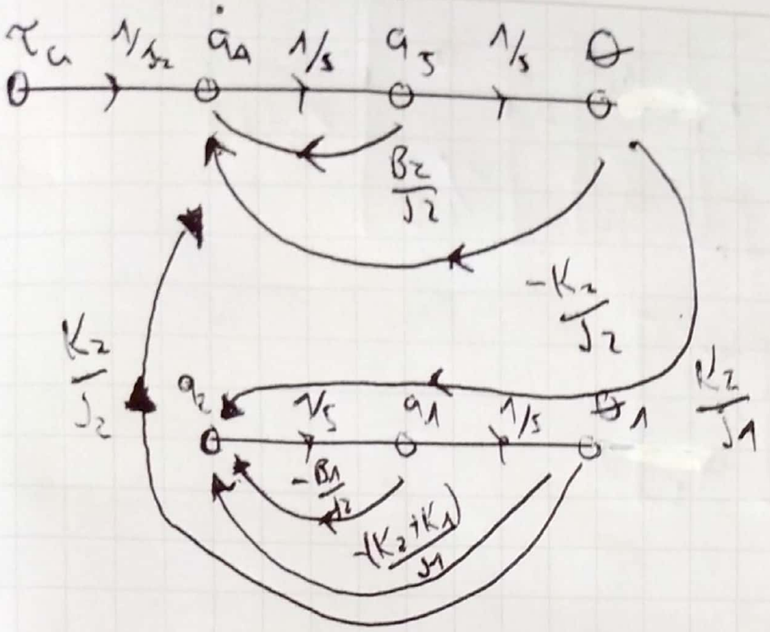
$$\rightarrow \dot{q}_4 = \frac{\tau_a}{J_2} - \frac{K_2}{J_2} q_3 + \frac{K_2}{J_2} q_1 - \frac{B_2}{J_2} q_4$$

$$\begin{vmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{vmatrix} = \begin{vmatrix} 0 & 1 & 0 & 0 \\ \frac{-(K_2+K_1)}{J_1} & \frac{-B_1}{J_1} & \frac{K_2}{J_1} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{K_2}{J_2} & 0 & \frac{-K_2}{J_2} & \frac{-B_2}{J_2} \end{vmatrix} \begin{vmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{vmatrix} + \begin{vmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{J_2} \end{vmatrix} \tau_a$$

$\theta_2 = q_3$

$$y = [0 \quad 0 \quad 1 \quad 0] \begin{vmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{vmatrix}$$






4) $I = ml^2$ $\quad \tau_a(t) - mgl\,sen\,\theta$

$ML^2\ddot{\theta} + B\dot{\theta} + MgL\,Sen\,\theta$

$\dot{\theta} = \omega \qquad \ddot{\theta} = \dot{\omega}$

$\dot{\omega} = \frac{1}{ML^2}\left[-MgL\,sen\,\theta - B\omega + \tau a(t)\right] \quad \theta = 0.5\ rad$

$ML^2\ddot{\theta} + B\dot{\theta} + MgL\theta = \tau a(t)$

$\dot{\theta} = \omega \qquad \theta = q_1 \qquad q_2 = \dot{q}_1 = \dot{\theta}\omega$

$\dot{q}_2 = \ddot{\theta} = \dot{\omega}$

$\dot{\omega} = \frac{1}{ML_1}\left[-MgL\theta - B\omega + \tau a(t)\right]$

$\rightarrow \dot{q}_2 = \frac{1}{ML^2}\left[-MgLq_1 - Bq_2 + Fa(t)\right]$

$\dot{q}_2 = -\frac{g}{L}q_1 - \frac{B}{ML^2}q_2 + \frac{\tau a(t)}{ML^2}$

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{g}{L} & -\frac{B}{ML^2} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/ML^2 \end{bmatrix} T_a(t)$$

$$\theta = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$$