Chapter 36

1. (a) The frequency of yellow sodium light is

$$f = \frac{c}{\lambda} = \frac{2.998 \times 10^8\,\mathrm{m/s}}{589 \times 10^{-9}\,\mathrm{m}} = 5.09 \times 10^{14}\ \mathrm{Hz}\ .$$

(b) When traveling through the glass, its wavelength is

$$\lambda_n = \frac{\lambda}{n} = \frac{589\,\mathrm{nm}}{1.52} = 388\ \mathrm{nm}\ .$$

(c) The light speed when traveling through the glass is

$$v = f\lambda_n = (5.09 \times 10^{14}\,\mathrm{Hz})(388 \times 10^{-9}\,\mathrm{m}) = 1.97 \times 10^8\ \mathrm{m/s}\ .$$

2. Comparing the light speeds in sapphire and diamond, we obtain

$$\begin{aligned}\Delta v &= v_s - v_d = c\left(\frac{1}{n_s} - \frac{1}{n_d}\right) \\ &= (2.998 \times 10^8\,\mathrm{m/s})\left(\frac{1}{1.77} - \frac{1}{2.42}\right) = 4.55 \times 10^7\ \mathrm{m/s}\ .\end{aligned}$$

3. The index of refraction is found from Eq. 36-3:

$$n = \frac{c}{v} = \frac{2.998 \times 10^8\,\mathrm{m/s}}{1.92 \times 10^8\,\mathrm{m/s}} = 1.56\ .$$

4. The index of refraction of fused quartz at $\lambda = 550\,\mathrm{nm}$ is about 1.459, obtained from Fig. 34-19. Thus, from Eq. 36-3, we find

$$v = \frac{c}{n} = \frac{2.998 \times 10^8\,\mathrm{m/s}}{1.459} = 2.06 \times 10^8\ \mathrm{m/s}\ .$$

5. Applying the law of refraction, we obtain $\sin\theta/\sin 30° = v_s/v_d$. Consequently,

$$\theta = \sin^{-1}\left(\frac{v_s \sin 30°}{v_d}\right) = \sin^{-1}\left[\frac{(3.0\,\mathrm{m/s})\sin 30°}{4.0\,\mathrm{m/s}}\right] = 22°\ .$$

The angle of incidence is gradually reduced due to refraction, such as shown in the calculation above (from 30° to 22°). Eventually after many refractions, θ will be virtually zero. This is why most waves come in normal to a shore.

6. (a) The time t_2 it takes for pulse 2 to travel through the plastic is

$$t_2 = \frac{L}{c/1.55} + \frac{L}{c/1.70} + \frac{L}{c/1.60} + \frac{L}{c/1.45} = \frac{6.30L}{c}\ .$$

Similarly for pulse 1:

$$t_1 = \frac{2L}{c/1.59} + \frac{L}{c/1.65} + \frac{L}{c/1.50} = \frac{6.33L}{c} \ .$$

Thus, pulse 2 travels through the plastic in less time.

(b) The time difference (as a multiple of L/c) is

$$\Delta t = t_2 - t_1 = \frac{6.33L}{c} - \frac{6.30L}{c} = \frac{0.03L}{c} \ .$$

7. (a) We take the phases of both waves to be zero at the front surfaces of the layers. The phase of the first wave at the back surface of the glass is given by $\phi_1 = k_1L - \omega t$, where k_1 $(= 2\pi/\lambda_1)$ is the angular wave number and λ_1 is the wavelength in glass. Similarly, the phase of the second wave at the back surface of the plastic is given by $\phi_2 = k_2L - \omega t$, where k_2 $(= 2\pi/\lambda_2)$ is the angular wave number and λ_2 is the wavelength in plastic. The angular frequencies are the same since the waves have the same wavelength in air and the frequency of a wave does not change when the wave enters another medium. The phase difference is

$$\phi_1 - \phi_2 = (k_1 - k_2)L = 2\pi\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)L \ .$$

Now, $\lambda_1 = \lambda_{\text{air}}/n_1$, where λ_{air} is the wavelength in air and n_1 is the index of refraction of the glass. Similarly, $\lambda_2 = \lambda_{\text{air}}/n_2$, where n_2 is the index of refraction of the plastic. This means that the phase difference is $\phi_1 - \phi_2 = (2\pi/\lambda_{\text{air}})(n_1 - n_2)L$. The value of L that makes this 5.65 rad is

$$L = \frac{(\phi_1 - \phi_2)\lambda_{\text{air}}}{2\pi(n_1 - n_2)} = \frac{5.65(400 \times 10^{-9}\,\text{m})}{2\pi(1.60 - 1.50)} = 3.60 \times 10^{-6}\ \text{m} \ .$$

(b) 5.65 rad is less than 2π rad = 6.28 rad, the phase difference for completely constructive interference, and greater than π rad (= 3.14 rad), the phase difference for completely destructive interference. The interference is, therefore, intermediate, neither completely constructive nor completely destructive. It is, however, closer to completely constructive than to completely destructive.

8. (a) Eq. 36-11 (in absolute value) yields

$$\frac{L}{\lambda}\,|n_2 - n_1| = \frac{(8.50 \times 10^{-6}\,\text{m})}{500 \times 10^{-9}\,\text{m}}(1.60 - 1.50) = 1.70 \ .$$

(b) Similarly,

$$\frac{L}{\lambda}\,|n_2 - n_1| = \frac{(8.50 \times 10^{-6}\,\text{m})}{500 \times 10^{-9}\,\text{m}}(1.72 - 1.62) = 1.70 \ .$$

(c) In this case, we obtain

$$\frac{L}{\lambda}\,|n_2 - n_1| = \frac{(3.25 \times 10^{-6}\,\text{m})}{500 \times 10^{-9}\,\text{m}}(1.79 - 1.59) = 1.30 \ .$$

(d) Since their phase differences were identical, the brightness should be the same for (a) and (b). Now, the phase difference in (c) differs from an integer by 0.30, which is also true for (a) and (b). Thus, their effective phase differences are equal, and the brightness in case (c) should be the same as that in (a) and (b).

9. (a) We choose a horizontal x axis with its origin at the left edge of the plastic. Between $x = 0$ and $x = L_2$ the phase difference is that given by Eq. 36-11 (with L in that equation replaced with L_2). Between $x = L_2$ and $x = L_1$ the phase difference is given by an expression similar to Eq. 36-11 but with L replaced with $L_1 - L_2$ and n_2 replaced with 1 (since the top ray in Fig. 36-28 is now traveling through air, which has index of refraction approximately equal to 1). Thus, combining these phase differences and letting all lengths be in μm (so $\lambda = 0.600$), we have

$$\frac{L_2}{\lambda}\,(n_2 - n_1) + \frac{L_1 - L_2}{\lambda}\,(1 - n_1) = \frac{3.50}{0.600}\,(1.60 - 1.40) + \frac{4.00 - 3.50}{0.600}\,(1 - 1.40) = 0.833 \ .$$

(b) Since the answer in part (a) is closer to an integer than to a half-integer, then the interference is more nearly constructive than destructive.

10. (a) We wish to set Eq. 36-11 equal to $\frac{1}{2}$, since a half-wavelength phase difference is equivalent to a π radians difference. Thus,

$$L_{\text{min}} = \frac{\lambda}{2\,(n_2 - n_1)} = \frac{620\,\text{nm}}{2(1.65 - 1.45)} = 1550\,\text{nm} = 1.55\ \mu\text{m}\ .$$

(b) Since a phase difference of $\frac{3}{2}$ (wavelengths) is effectively the same as what we required in part (a), then

$$L = \frac{3\lambda}{2\,(n_2 - n_1)} = 3L_{\text{min}} = 3(1.55\mu\text{m}) = 4.65\ \mu\text{m}\ .$$

11. (a) We use Eq. 36-14 with $m = 3$:

$$\theta = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left[\frac{2(550 \times 10^{-9}\,\text{m})}{7.70 \times 10^{-6}\,\text{m}}\right] = 0.216\ \text{rad}\ .$$

(b) $\theta = (0.216)(180°/\pi) = 12.4°$.

12. Here we refer to phase difference in radians (as opposed to wavelengths or degrees). For the first dark fringe $\phi_1 = \pm\pi$, and for the second one $\phi_2 = \pm 3\pi$, etc. For the mth one $\phi_m = \pm(2m + 1)\pi$.

13. The condition for a maximum in the two-slit interference pattern is $d \sin\theta = m\lambda$, where d is the slit separation, λ is the wavelength, m is an integer, and θ is the angle made by the interfering rays with the forward direction. If θ is small, $\sin\theta$ may be approximated by θ in radians. Then, $\theta = m\lambda/d$, and the angular separation of adjacent maxima, one associated with the integer m and the other associated with the integer $m + 1$, is given by $\Delta\theta = \lambda/d$. The separation on a screen a distance D away is given by $\Delta y = D\,\Delta\theta = \lambda D/d$. Thus,

$$\Delta y = \frac{(500 \times 10^{-9}\,\text{m})(5.40\,\text{m})}{1.20 \times 10^{-3}\,\text{m}} = 2.25 \times 10^{-3}\,\text{m} = 2.25\ \text{mm}\ .$$

14. (a) For the maximum adjacent to the central one, we set $m = 1$ in Eq. 36-14 and obtain

$$\theta_1 = \sin^{-1}\left(\frac{m\lambda}{d}\right)\bigg|_{m=1} = \sin^{-1}\left[\frac{(1)(\lambda)}{100\lambda}\right] = 0.010\ \text{rad}\ .$$

(b) Since $y_1 = D\tan\theta_1$ (see Fig. 36-8(a)), we obtain $y_1 = (500\,\text{mm})\tan(0.010\,\text{rad}) = 5.0\,\text{mm}$. The separation is $\Delta y = y_1 - y_0 = y_1 - 0 = 5.0\,\text{mm}$.

15. The angular positions of the maxima of a two-slit interference pattern are given by $d\sin\theta = m\lambda$, where d is the slit separation, λ is the wavelength, and m is an integer. If θ is small, $\sin\theta$ may be approximated by θ in radians. Then, $\theta = m\lambda/d$ to good approximation. The angular separation of two adjacent maxima is $\Delta\theta = \lambda/d$. Let λ' be the wavelength for which the angular separation is 10.0% greater. Then, $1.10\lambda/d = \lambda'/d$ or $\lambda' = 1.10\lambda = 1.10(589\,\text{nm}) = 648\,\text{nm}$.

16. In Sample Problem 36-2, an experimentally useful relation is derived: $\Delta y = \lambda D/d$. Dividing both sides by D, this becomes $\Delta\theta = \lambda/d$ with θ in radians. In the steps that follow, however, we will end up with an expression where degrees may be directly used. Thus, in the present case,

$$\Delta\theta_n = \frac{\lambda_n}{d} = \frac{\lambda}{nd} = \frac{\Delta\theta}{n} = \frac{0.20°}{1.33} = 0.15°\ .$$

17. Interference maxima occur at angles θ such that $d \sin\theta = m\lambda$, where m is an integer. Since $d = 2.0\,\mathrm{m}$ and $\lambda = 0.50\,\mathrm{m}$, this means that $\sin\theta = 0.25m$. We want all values of m (positive and negative) for which $|0.25m| \le 1$. These are -4, -3, -2, -1, 0, $+1$, $+2$, $+3$, and $+4$. For each of these except -4 and $+4$, there are two different values for θ. A single value of θ ($-90°$) is associated with $m = -4$ and a single value ($+90°$) is associated with $m = +4$. There are sixteen different angles in all and, therefore, sixteen maxima.

18. Initially, source A leads source B by 90°, which is equivalent to 1/4 wavelength. However, source A also lags behind source B since r_A is longer than r_B by 100 m, which is 100 m/400 m = 1/4 wavelength. So the net phase difference between A and B at the detector is zero.

19. The maxima of a two-slit interference pattern are at angles θ given by $d \sin\theta = m\lambda$, where d is the slit separation, λ is the wavelength, and m is an integer. If θ is small, $\sin\theta$ may be replaced by θ in radians. Then, $d\theta = m\lambda$. The angular separation of two maxima associated with different wavelengths but the same value of m is $\Delta\theta = (m/d)(\lambda_2 - \lambda_1)$, and their separation on a screen a distance D away is

$$\begin{aligned}\Delta y &= D\tan\Delta\theta \approx D\,\Delta\theta = \left[\frac{mD}{d}\right](\lambda_2 - \lambda_1)\\ &= \left[\frac{3(1.0\,\mathrm{m})}{5.0\times10^{-3}\,\mathrm{m}}\right](600\times10^{-9}\,\mathrm{m} - 480\times10^{-9}\,\mathrm{m}) = 7.2\times10^{-5}\ \mathrm{m}\ .\end{aligned}$$

The small angle approximation $\tan\Delta\theta \approx \Delta\theta$ (in radians) is made.

20. Let the distance in question be x. The path difference (between rays originating from S_1 and S_2 and arriving at points on the $x > 0$ axis) is

$$\sqrt{d^2 + x^2} - x = \left(m + \frac{1}{2}\right)\lambda\ ,$$

where we are requiring destructive interference (half-integer wavelength phase differences) and $m = 0, 1, 2, \cdots$. After some algebraic steps, we solve for the distance in terms of m:

$$x = \frac{d^2}{(2m+1)\lambda} - \frac{(2m+1)\lambda}{4}\ .$$

To obtain the largest value of x, we set $m = 0$:

$$x_0 = \frac{d^2}{\lambda} - \frac{\lambda}{4} = \frac{(3.00\lambda)^2}{\lambda} - \frac{\lambda}{4} = 8.75\lambda\ .$$

21. Consider the two waves, one from each slit, that produce the seventh bright fringe in the absence of the mica. They are in phase at the slits and travel different distances to the seventh bright fringe, where they have a phase difference of $2\pi m = 14\pi$. Now a piece of mica with thickness x is placed in front of one of the slits, and an additional phase difference between the waves develops. Specifically, their phases at the slits differ by

$$\frac{2\pi x}{\lambda_m} - \frac{2\pi x}{\lambda} = \frac{2\pi x}{\lambda}(n-1)$$

where λ_m is the wavelength in the mica and n is the index of refraction of the mica. The relationship $\lambda_m = \lambda/n$ is used to substitute for λ_m. Since the waves are now in phase at the screen,

$$\frac{2\pi x}{\lambda}(n-1) = 14\pi$$

or

$$x = \frac{7\lambda}{n-1} = \frac{7(550\times10^{-9}\,\mathrm{m})}{1.58-1} = 6.64\times10^{-6}\ \mathrm{m}\ .$$

22. (a) We use $\Delta y = D\lambda/d$ (see Sample Problem 36-2). Because of the placement of the mirror in the problem $D = 2(20.0\,\text{m}) = 40.0\,\text{m}$, which we express in millimeters in the calculation below:

$$d = \frac{D\lambda}{\Delta y} = \frac{(4.00 \times 10^4\,\text{mm})(632.8 \times 10^{-6}\,\text{mm})}{100\,\text{mm}} = 0.253\ \text{mm}\ .$$

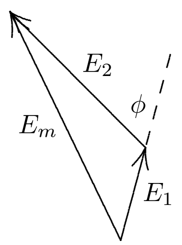
(b) In this case the interference pattern will be shifted. At the location of the original central maximum, the effective phase difference is now $\frac{1}{2}$ wavelength, so there is now a minimum instead of a maximum.

23. The phasor diagram is shown below. Here $E_1 = 1.00$, $E_2 = 2.00$, and $\phi = 60°$. The resultant amplitude E_m is given by the trigonometric law of cosines:

$$E_m^2 = E_1^2 + E_2^2 - 2E_1E_2\cos(180° - \phi)\ .$$

Thus,

$$E_m = \sqrt{(1.00)^2 + (2.00)^2 - 2(1.00)(2.00)\cos 120°} = 2.65\ .$$



24. In adding these with the phasor method (as opposed to, say, trig identities), we may set $t = 0$ (see Sample Problem 36-3) and add them as vectors:

$$\begin{aligned} y_h &= 10\cos 0° + 8.0\cos 30° = 16.9 \\ y_v &= 10\sin 0° + 8.0\sin 30° = 4.0 \end{aligned}$$

so that

$$\begin{aligned} y_R &= \sqrt{y_h^2 + y_v^2} = 17.4 \\ \beta &= \tan^{-1}\left(\frac{y_v}{y_h}\right) = 13.3°\ . \end{aligned}$$

Thus, $y = y_1 + y_2 = y_R\sin(\omega t + \beta) = 17.4\sin(\omega t + 13.3°)$.

25. In adding these with the phasor method (as opposed to, say, trig identities), we may set $t = 0$ (see Sample Problem 36-3) and add them as vectors:

$$\begin{aligned} y_h &= 10\cos 0° + 15\cos 30° + 5.0\cos(-45°) = 26.5 \\ y_v &= 10\sin 0° + 15\sin 30° + 5.0\sin(-45°) = 4.0 \end{aligned}$$

so that

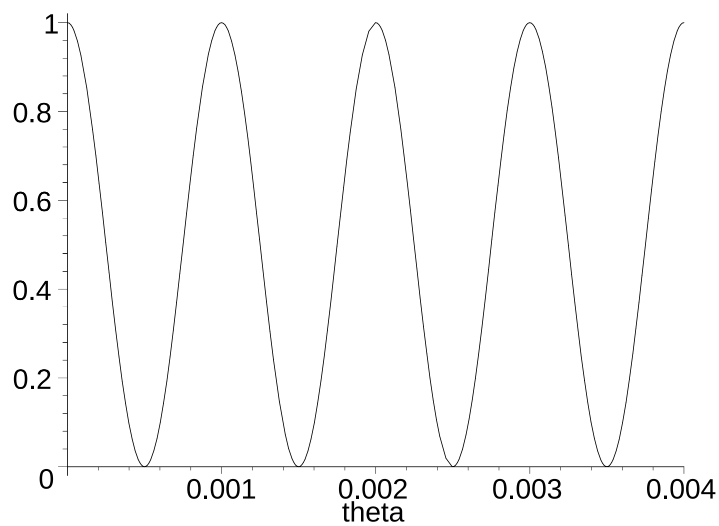
$$\begin{aligned} y_R &= \sqrt{y_h^2 + y_v^2} = 26.8 \\ \beta &= \tan^{-1}\left(\frac{y_v}{y_h}\right) = 8.5°\ . \end{aligned}$$

Thus, $y = y_1 + y_2 + y_3 = y_R\sin(\omega t + \beta) = 26.8\sin(\omega t + 8.5°)$.

26. Fig. 36-9 in the textbook is plotted versus the phase difference (in radians), whereas this problem requests that we plot the intensity versus the physical angle θ (defined in Fig. 36-8). The values given in the problem imply $d\lambda = 1000$. Combining this with Eq. 36-22 and Eq. 36-21, we solve for the (normalized) intensity:

$$\frac{I}{4I_0} = \cos^2(1000\pi \sin\theta) \ .$$

This is plotted over $0 \le \theta \le 0.0040$ rad:



27. (a) To get to the detector, the wave from S_1 travels a distance x and the wave from S_2 travels a distance $\sqrt{d^2 + x^2}$. The phase difference (in terms of wavelengths) between the two waves is

$$\sqrt{d^2 + x^2} - x = m\lambda \qquad m = 0, 1, 2, \ldots$$

where we are requiring constructive interference. The solution is

$$x = \frac{d^2 - m^2\lambda^2}{2m\lambda} \ .$$

The largest value of m that produces a positive value for x is $m = 3$. This corresponds to the maximum that is nearest S_1, at

$$x = \frac{(4.00\,\text{m})^2 - 9(1.00\,\text{m})^2}{(2)(3)(1.00\,\text{m})} = 1.17 \text{ m} \ .$$

For the next maximum, $m = 2$ and $x = 3.00\,\text{m}$. For the third maximum, $m = 1$ and $x = 7.50\,\text{m}$.

(b) Minima in intensity occur where the phase difference is π rad; the intensity at a minimum, however, is not zero because the amplitudes of the waves are different. Although the amplitudes are the same at the sources, the waves travel different distances to get to the points of minimum intensity and each amplitude decreases in inverse proportion to the distance traveled.

28. Setting $I = 2I_0$ in Eq. 36-21 and solving for the smallest (in absolute value) two roots for $\phi/2$, we find

$$\phi = 2\cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = \pm\frac{\pi}{2} \text{ rad} \ .$$

Now, for small θ in radians, Eq. 36-22 becomes $\phi = 2\pi d\theta/\lambda$. This leads to two corresponding angle values:

$$\theta = \pm\frac{\lambda}{4d} \ .$$

The difference between these two values is $\Delta\theta = \frac{\lambda}{4d} - \left(-\frac{\lambda}{4d}\right) = \frac{\lambda}{2d}$.

29. We take the electric field of one wave, at the screen, to be

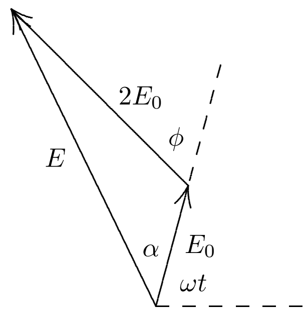
$$E_1 = E_0 \sin(\omega t)$$

and the electric field of the other to be

$$E_2 = 2E_0 \sin(\omega t + \phi) \ ,$$

where the phase difference is given by

$$\phi = \left(\frac{2\pi d}{\lambda}\right) \sin\theta \ .$$



Here d is the center-to-center slit separation and λ is the wavelength. The resultant wave can be written $E = E_1 + E_2 = E\sin(\omega t + \alpha)$, where α is a phase constant. The phasor diagram is shown above. The resultant amplitude E is given by the trigonometric law of cosines:

$$E^2 = E_0^2 + (2E_0)^2 - 4E_0^2\cos(180^\circ - \phi) = E_0^2(5 + 4\cos\phi) \ .$$

The intensity is given by $I = I_0(5 + 4\cos\phi)$, where I_0 is the intensity that would be produced by the first wave if the second were not present. Since $\cos\phi = 2\cos^2(\phi/2) - 1$, this may also be written $I = I_0\left[1 + 8\cos^2(\phi/2)\right]$.

30. The fact that wave W_2 reflects two additional times has no substantive effect on the calculations, since two reflections amount to a $2(\lambda/2) = \lambda$ phase difference, which is effectively not a phase difference at all. The substantive difference between W_2 and W_1 is the extra distance $2L$ traveled by W_2.

(a) For wave W_2 to be a half-wavelength "behind" wave W_1, we require $2L = \lambda/2$, or $L = \lambda/4 = 155$ nm using the wavelength value given in the problem.

(b) Destructive interference will again appear if W_2 is $\frac{3}{2}\lambda$ "behind" the other wave. In this case, $2L' = 3\lambda/2$, and the difference is

$$L' - L = \frac{3\lambda}{4} - \frac{\lambda}{4} = \frac{\lambda}{2} = 310 \text{ nm} \ .$$

31. The wave reflected from the front surface suffers a phase change of π rad since it is incident in air on a medium of higher index of refraction. The phase of the wave reflected from the back surface does not change on reflection since the medium beyond the soap film is air and has a lower index of refraction than the film. If L is the thickness of the film, this wave travels a distance $2L$ farther than the wave reflected from the front surface. The phase difference of the two waves is $2L(2\pi/\lambda_f) - \pi$, where λ_f is the wavelength in the film. If λ is the wavelength in vacuum and n is the index of refraction of the soap film, then $\lambda_f = \lambda/n$ and the phase difference is

$$2nL\left(\frac{2\pi}{\lambda}\right) - \pi = 2(1.33)(1.21\times10^{-6}\,\text{m})\left(\frac{2\pi}{585\times10^{-9}\,\text{m}}\right) - \pi = 10\pi \text{ rad} \ .$$

Since the phase difference is an even multiple of π, the interference is completely constructive.

32. In contrast to the initial conditions of problem 30, we now consider waves W_2 and W_1 with an initial effective phase difference (in wavelengths) equal to $\frac{1}{2}$, and seek positions of the sliver which cause the wave to constructively interfere (which corresponds to an integer-valued phase difference in wavelengths). Thus, the extra distance $2L$ traveled by W_2 must amount to $\frac{1}{2}\lambda$, $\frac{3}{2}\lambda$, and so on. We may write this requirement succinctly as

$$L = \frac{2m+1}{4}\,\lambda \qquad \text{where} \;\; m = 0, 1, 2, \ldots \;\; .$$

33. For constructive interference, we use Eq. 36-34: $2n_2L = (m + 1/2)\lambda$. For the two smallest values of L, let $m = 0$ and 1:

$$\begin{aligned} L_0 &= \frac{\lambda/2}{2n_2} = \frac{624\,\text{nm}}{4(1.33)} = 117\,\text{nm} = 0.117\,\mu\text{m} \\ L_1 &= \frac{(1+1/2)\lambda}{2n_2} = \frac{3\lambda}{2n_2} = 3L_0 = 3(0.1173\mu\text{m}) = 0.352\,\mu\text{m}\ . \end{aligned}$$

34. We use the formula obtained in Sample Problem 36-5:

$$L_{\min} = \frac{\lambda}{4n_2} = \frac{\lambda}{4(1.25)} = 0.200\lambda\ .$$

35. Light reflected from the front surface of the coating suffers a phase change of π rad while light reflected from the back surface does not change phase. If L is the thickness of the coating, light reflected from the back surface travels a distance $2L$ farther than light reflected from the front surface. The difference in phase of the two waves is $2L(2\pi/\lambda_c) - \pi$, where λ_c is the wavelength in the coating. If λ is the wavelength in vacuum, then $\lambda_c = \lambda/n$, where n is the index of refraction of the coating. Thus, the phase difference is $2nL(2\pi/\lambda) - \pi$. For fully constructive interference, this should be a multiple of 2π. We solve

$$2nL\left(\frac{2\pi}{\lambda}\right) - \pi = 2m\pi$$

for L. Here m is an integer. The solution is

$$L = \frac{(2m+1)\lambda}{4n}\ .$$

To find the smallest coating thickness, we take $m = 0$. Then,

$$L = \frac{\lambda}{4n} = \frac{560 \times 10^{-9}\,\text{m}}{4(2.00)} = 7.00 \times 10^{-8}\ \text{m}\ .$$

36. Let the thicknesses (which appear in Fig. 36-31 as different heights h) of the structure be $h = kL$, where k is a pure number. In section (b), for example, $k = 2$. Using Eq. 36-34, the condition for constructive interference becomes

$$2h = 2(kL) = \frac{(m+1/2)\lambda}{n_2} \qquad \text{where} \;\; m = 0, 1, 2, \ldots$$

which leads to

$$k = \frac{(m+1/2)\lambda}{2n_2L} = \frac{(m+1/2)(600\,\text{nm})}{2(1.50)(4.00 \times 10^3\,\text{nm})} = \frac{2m+1}{40}\ ,$$

or $40k - 1 = 2m$. This means that $40k - 1$ would have to be an even integer. One can check that none of the given values of k (1, 2, $\frac{1}{2}$, 3, $\frac{1}{10}$) will satisfy this condition. Therefore, none of the sections provides the right thickness for constructive interference.

37. For complete destructive interference, we want the waves reflected from the front and back of the coating to differ in phase by an odd multiple of π rad. Each wave is incident on a medium of higher index of refraction from a medium of lower index, so both suffer phase changes of π rad on reflection. If L is the thickness of the coating, the wave reflected from the back surface travels a distance $2L$ farther than the wave reflected from the front. The phase difference is $2L(2\pi/\lambda_c)$, where λ_c is the wavelength in the coating. If n is the index of refraction of the coating, $\lambda_c = \lambda/n$, where λ is the wavelength in vacuum, and the phase difference is $2nL(2\pi/\lambda)$. We solve

$$2nL\left(\frac{2\pi}{\lambda}\right) = (2m+1)\pi$$

for L. Here m is an integer. The result is

$$L = \frac{(2m+1)\lambda}{4n} .$$

To find the least thickness for which destructive interference occurs, we take $m = 0$. Then,

$$L = \frac{\lambda}{4n} = \frac{600 \times 10^{-9}\,\text{m}}{4(1.25)} = 1.2 \times 10^{-7}\ \text{m} .$$

38. Eqs. 36-14 and 36-16 treat the interference of reflections, and here we are concerned with interference of the transmitted light. Maxima in the reflections should, reasonably enough, correspond to minima in the transmissions, and vice versa. So we might expect to apply those equations to this case if we switch the designations "maxima" and "minima," *if* we are careful with the phase shifts that occur at the points of reflection (which depend on the relative values of n). Now, if the expression $2L = m\lambda/n_2$ is to give the condition for constructive interference for the transmitted light, then the situation should be similar to that which led in the textbook to Eqs. 36-14 and 36-16; namely, the thin film should be surrounded by two higher-index or two lower-index media. Such is the case for Fig. 36-32(a) and Fig. 36-32(c), but not for the others.

39. The situation is analogous to that treated in Sample Problem 36-5, in the sense that the incident light is in a low index medium, the thin film has somewhat higher $n = n_2$, and the last layer has the highest refractive index. To see very little or no reflection, according to the Sample Problem, the condition

$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2} \qquad \text{where} \quad m = 0, 1, 2, \ldots$$

must hold. The value of L which corresponds to no reflection corresponds, reasonably enough, to the value which gives maximum transmission of light (into the highest index medium – which in this problem is the water).

(a) If $2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2}$ (Eq. 36-34) gives zero reflection in this type of system, then we might reasonably expect that its counterpart, Eq. 36-35, gives maximum reflection here. A more careful analysis such as that given in §36-7 bears this out. We disregard the $m = 0$ value (corresponding to $L = 0$) since there is *some* oil on the water. Thus, for $m = 1, 2, \ldots$ maximum reflection occurs for wavelengths

$$\lambda = \frac{2n_2L}{m} = \frac{2(1.20)(460\,\text{nm})}{m} = 1104\,\text{nm}\,,\ 552\,\text{nm}\,,\ 368\,\text{nm}\ \ldots$$

We note that only the 552 nm wavelength falls within the visible light range.

(b) As remarked above, maximum transmission into the water occurs for wavelengths given by

$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2} \quad\Longrightarrow\quad \lambda = \frac{4n_2L}{2m+1}$$

which yields $\lambda = 2208\,\text{nm}\,,\ 736\,\text{nm}\,,\ 442\,\text{nm}\ \ldots$ for the different values of m. We note that only the 442 nm wavelength (blue) is in the visible range, though we might expect some red contribution since the 736 nm is very close to the visible range.

40. The situation is analogous to that treated in Sample Problem 36-5, in the sense that the incident light is in a low index medium, the thin film of oil has somewhat higher $n = n_2$, and the last layer (the glass plate) has the highest refractive index. To see very little or no reflection, according to the Sample Problem, the condition

$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2} \qquad \text{where} \quad m = 0, 1, 2, \ldots$$

must hold. With $\lambda = 500$ nm and $n_2 = 1.30$, the possible answers for L are

$$L = 96\,\text{nm}\,,\ 288\,\text{nm}\,,\ 481\,\text{nm}\,,\ 673\,\text{nm}\,,\ 865\,\text{nm}\,,\ \ldots$$

And, with $\lambda = 700$ nm and the same value of n_2, the possible answers for L are

$$L = 135\,\text{nm}\,,\ 404\,\text{nm}\,,\ 673\,\text{nm}\,,\ 942\,\text{nm}\,,\ \ldots$$

The lowest number these lists have in common is $L = 673$ nm.

41. Light reflected from the upper oil surface (in contact with air) changes phase by π rad. Light reflected from the lower surface (in contact with glass) changes phase by π rad if the index of refraction of the oil is less than that of the glass and does not change phase if the index of refraction of the oil is greater than that of the glass.

- First, suppose the index of refraction of the oil is greater than the index of refraction of the glass. The condition for fully destructive interference is $2n_od = m\lambda$, where d is the thickness of the oil film, n_o is the index of refraction of the oil, λ is the wavelength in vacuum, and m is an integer. For the shorter wavelength, $2n_od = m_1\lambda_1$ and for the longer, $2n_od = m_2\lambda_2$. Since λ_1 is less than λ_2, m_1 is greater than m_2, and since fully destructive interference does not occur for any wavelengths between, $m_1 = m_2 + 1$. Solving $(m_2 + 1)\lambda_1 = m_2\lambda_2$ for m_2, we obtain

$$m_2 = \frac{\lambda_1}{\lambda_2 - \lambda_1} = \frac{500\,\text{nm}}{700\,\text{nm} - 500\,\text{nm}} = 2.50\ .$$

Since m_2 must be an integer, the oil cannot have an index of refraction that is greater than that of the glass.

- Now suppose the index of refraction of the oil is less than that of the glass. The condition for fully destructive interference is then $2n_od = (2m+1)\lambda$. For the shorter wavelength, $2m_od = (2m_1+1)\lambda_1$, and for the longer, $2n_od = (2m_2+1)\lambda_2$. Again, $m_1 = m_2 + 1$, so $(2m_2+3)\lambda_1 = (2m_2+1)\lambda_2$. This means the value of m_2 is

$$m_2 = \frac{3\lambda_1 - \lambda_2}{2(\lambda_2 - \lambda_1)} = \frac{3(500\,\text{nm}) - 700\,\text{nm}}{2(700\,\text{nm} - 500\,\text{nm})} = 2.00\ .$$

This is an integer. Thus, the index of refraction of the oil is less than that of the glass.

42. We solve Eq. 36-34 with $n_2 = 1.33$ and $\lambda = 600$ nm for $m = 1, 2, 3, \ldots$:

$$L = 113\,\text{nm}\,,\ 338\,\text{nm}\,,\ 564\,\text{nm}\,,\ 789\,\text{nm}\,,\ \ldots$$

And, we similarly solve Eq. 36-35 with the same n_2 and $\lambda = 450$ nm:

$$L = 0\,,\ 169\,\text{nm}\,,\ 338\,\text{nm}\,,\ 508\,\text{nm}\,,\ 677\,\text{nm}\,,\ \ldots$$

The lowest number these lists have in common is $L = 338$ nm.

43. Consider the interference of waves reflected from the top and bottom surfaces of the air film. The wave reflected from the upper surface does not change phase on reflection but the wave reflected from the bottom surface changes phase by π rad. At a place where the thickness of the air film is L, the condition

for fully constructive interference is $2L = (m + \frac{1}{2})\lambda$, where λ ($= 683\,\text{nm}$) is the wavelength and m is an integer. This is satisfied for $m = 140$:

$$L = \frac{(m + \frac{1}{2})\lambda}{2} = \frac{(140.5)(683 \times 10^{-9}\,\text{m})}{2} = 4.80 \times 10^{-5}\,\text{m} = 0.048\ \text{mm}\ .$$

At the thin end of the air film, there is a bright fringe. It is associated with $m = 0$. There are, therefore, 140 bright fringes in all.

44. (a) At the left end, the plates touch, so $L = 0$ there, which is clearly consistent with Eq. 36-35 (the destructive interference or "dark fringe" equation) for $m = 0$.

    (b) Eq. 36-35 shows a simple proportionality between L and λ. So as we slowly increase L (from zero – its value in part (a)), the smallest nonzero value of L for which the equation (which specifies destructive interference) is satisfied occurs for the lowest possible value of λ. Wavelengths for blue light are the shortest of the visible portion of the spectrum.

45. Assume the wedge-shaped film is in air, so the wave reflected from one surface undergoes a phase change of π rad while the wave reflected from the other surface does not. At a place where the film thickness is L, the condition for fully constructive interference is $2nL = (m + \frac{1}{2})\lambda$, where n is the index of refraction of the film, λ is the wavelength in vacuum, and m is an integer. The ends of the film are bright. Suppose the end where the film is narrow has thickness L_1 and the bright fringe there corresponds to $m = m_1$. Suppose the end where the film is thick has thickness L_2 and the bright fringe there corresponds to $m = m_2$. Since there are ten bright fringes, $m_2 = m_1 + 9$. Subtract $2nL_1 = (m_1 + \frac{1}{2})\lambda$ from $2nL_2 = (m_1 + 9 + \frac{1}{2})\lambda$ to obtain $2n\,\Delta L = 9\lambda$, where $\Delta L = L_2 - L_1$ is the change in the film thickness over its length. Thus,

$$\Delta L = \frac{9\lambda}{2n} = \frac{9(630 \times 10^{-9}\,\text{m})}{2(1.50)} = 1.89 \times 10^{-6}\ \text{m}\ .$$

46. The situation is analogous to that treated in Sample Problem 36-5, in the sense that the incident light is in a low index medium, the thin film of acetone has somewhat higher $n = n_2$, and the last layer (the glass plate) has the highest refractive index. To see very little or no reflection, according to the Sample Problem, the condition

$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2} \qquad \text{where} \quad m = 0, 1, 2, \ldots$$

must hold. This is the same as Eq. 36-34 which was developed for the opposite situation (constructive interference) regarding a thin film surrounded on both sides by air (a very different context than the one in this problem). By analogy, we expect Eq. 36-35 to apply in this problem to reflection *maxima*. A more careful analysis such as that given in §36-7 bears this out. Thus, using Eq. 36-35 with $n_2 = 1.25$ and $\lambda = 700$ nm yields

$$L = 0\,,\ 280\,\text{nm}\,,\ 560\,\text{nm}\,,\ 840\,\text{nm}\,,\ 1120\,\text{nm}\,,\ \ \ldots$$

for the first several m values. And the equation shown above (equivalent to Eq. 36-34) gives, with $\lambda = 600$ nm,

$$L = 120\,\text{nm}\,,\ 360\,\text{nm}\,,\ 600\,\text{nm}\,,\ 840\,\text{nm}\,,\ 1080\,\text{nm}\,,\ \ \ldots$$

for the first several m values. The lowest number these lists have in common is $L = 840$ nm.

47. We use Eq. 36-34:

$$\begin{aligned} L_{16} &= \left(16 + \frac{1}{2}\right)\frac{\lambda}{2n_2} \\ L_6 &= \left(6 + \frac{1}{2}\right)\frac{\lambda}{2n_2} \end{aligned}$$

The difference between these, using the fact that $n_2 = n_{\text{air}} = 1.0$, is

$$L_{16} - L_6 = (10)\frac{480\,\text{nm}}{2(1.0)} = 2400 \text{ nm} .$$

48. We apply Eq. 36-25 to both scenarios: $m = 4001$ and $n_2 = n_{\text{air}}$, and $m = 4000$ and $n_2 = n_{\text{vacuum}} = 1.00000$:

$$2L = (4001)\frac{\lambda}{n_{\text{air}}} \qquad \text{and} \qquad 2L = (4000)\frac{\lambda}{1.00000} .$$

Since the $2L$ factor is the same in both cases, we set the right hand sides of these expressions equal to each other and cancel the wavelength. Finally, we obtain

$$n_{\text{air}} = (1.00000)\frac{4001}{4000} = 1.00025 .$$

We remark that this same result can be obtained starting with Eq. 36-41 (which is developed in the textbook for a somewhat different situation) and using Eq. 36-40 to eliminate the $2L/\lambda$ term.

49. Consider the interference pattern formed by waves reflected from the upper and lower surfaces of the air wedge. The wave reflected from the lower surface undergoes a π rad phase change while the wave reflected from the upper surface does not. At a place where the thickness of the wedge is d, the condition for a maximum in intensity is $2d = (m + \frac{1}{2})\lambda$, where λ is the wavelength in air and m is an integer. Thus, $d = (2m+1)\lambda/4$. As the geometry of Fig. 36-34 shows, $d = R - \sqrt{R^2 - r^2}$, where R is the radius of curvature of the lens and r is the radius of a Newton's ring. Thus, $(2m+1)\lambda/4 = R - \sqrt{R^2 - r^2}$. First, we rearrange the terms so the equation becomes

$$\sqrt{R^2 - r^2} = R - \frac{(2m+1)\lambda}{4} .$$

Next, we square both sides, rearrange to solve for r^2, then take the square root. We get

$$r = \sqrt{\frac{(2m+1)R\lambda}{2} - \frac{(2m+1)^2\lambda^2}{16}} .$$

If R is much larger than a wavelength, the first term dominates the second and

$$r = \sqrt{\frac{(2m+1)R\lambda}{2}} .$$

50. (a) We find m from the last formula obtained in problem 49:

$$m = \frac{r^2}{R\lambda} - \frac{1}{2} = \frac{(10 \times 10^{-3}\,\text{m})^2}{(5.0\,\text{m})(589 \times 10^{-9}\,\text{m})} - \frac{1}{2}$$

which (rounding down) yields $m = 33$. Since the first bright fringe corresponds to $m = 0$, $m = 33$ corresponds to the thirty-fourth bright fringe.

(b) We now replace λ by $\lambda_n = \lambda/n_w$. Thus,

$$m_n = \frac{r^2}{R\lambda_n} - \frac{1}{2} = \frac{n_w r^2}{R\lambda} - \frac{1}{2} = \frac{(1.33)(10 \times 10^{-3}\,\text{m})^2}{(5.0\,\text{m})(589 \times 10^{-9}\,\text{m})} - \frac{1}{2} = 45 .$$

This corresponds to the forty-sixth bright fringe (see remark at the end of our solution in part (a)).

51. We solve for m using the formula $r = \sqrt{(2m+1)R\lambda/2}$ obtained in problem 49 and find $m = r^2/R\lambda - 1/2$. Now, when m is changed to $m + 20$, r becomes r', so $m + 20 = r'^2/R\lambda - 1/2$. Taking the difference between the two equations above, we eliminate m and find

$$R = \frac{r'^2 - r^2}{20\lambda} = \frac{(0.368\,\text{cm})^2 - (0.162\,\text{cm})^2}{20(546 \times 10^{-7}\,\text{cm})} = 100 \text{ cm} .$$

52. (a) The binomial theorem (Appendix E) allows us to write

$$\sqrt{k(1+x)} = \sqrt{k}\left(1 + \frac{x}{2} + \frac{x^2}{8} + \frac{3x^3}{48} + \cdots\right) \approx \sqrt{k} + \frac{x}{2}\sqrt{k}$$

for $x \ll 1$. Thus, the end result from the solution of problem 49 yields

$$r_m = \sqrt{R\lambda m\left(1 + \frac{1}{2m}\right)} \approx \sqrt{R\lambda m} + \frac{1}{4m}\sqrt{R\lambda m}$$

and

$$r_{m+1} = \sqrt{R\lambda m\left(1 + \frac{3}{2m}\right)} \approx \sqrt{R\lambda m} + \frac{3}{4m}\sqrt{R\lambda m}$$

for very large values of m. Subtracting these, we obtain

$$\Delta r = \frac{3}{4m}\sqrt{R\lambda m} - \frac{1}{4m}\sqrt{R\lambda m} = \frac{1}{2}\sqrt{\frac{R\lambda}{m}} \ .$$

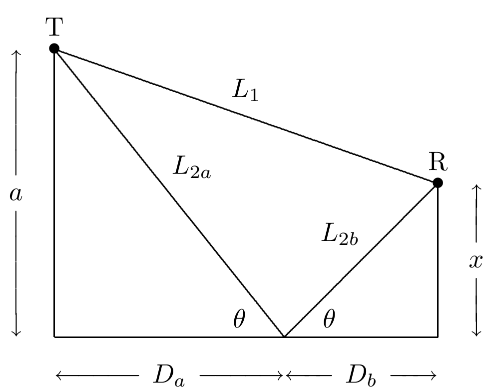
(b) We take the differential of the area: $dA = d(\pi r^2) = 2\pi r\, dr$, and replace dr with Δr in anticipation of using the result from part (a). Thus, the area between adjacent rings for large values of m is

$$2\pi r_m(\Delta r) \approx 2\pi\left(\sqrt{R\lambda m} + \frac{1}{4m}\sqrt{R\lambda m}\right)\left(\frac{1}{2}\sqrt{\frac{R\lambda}{m}}\right) \approx 2\pi\left(\sqrt{R\lambda m}\right)\left(\frac{1}{2}\sqrt{\frac{R\lambda}{m}}\right)$$

which simplifies to the desired result ($\pi\lambda R$).

53. The wave that goes directly to the receiver travels a distance L_1 and the reflected wave travels a distance L_2. Since the index of refraction of water is greater than that of air this last wave suffers a phase change on reflection of half a wavelength. To obtain constructive interference at the receiver, the difference $L_2 - L_1$ must be an odd multiple of a half wavelength. Consider the diagram below. The right triangle on the left, formed by the vertical line from the water to the transmitter T, the ray incident on the water, and the water line, gives $D_a = a/\tan\theta$. The right triangle on the right, formed by the vertical line from the water to the receiver R, the reflected ray, and the water line leads to $D_b = x/\tan\theta$. Since $D_a + D_b = D$,

$$\tan\theta = \frac{a+x}{D} \ .$$



We use the identity $\sin^2\theta = \tan^2\theta/(1+\tan^2\theta)$ to show that $\sin\theta = (a+x)/\sqrt{D^2+(a+x)^2}$. This means

$$L_{2a} = \frac{a}{\sin\theta} = \frac{a\sqrt{D^2+(a+x)^2}}{a+x}$$

and

$$L_{2b} = \frac{x}{\sin\theta} = \frac{x\sqrt{D^2 + (a+x)^2}}{a+x} \; .$$

Therefore,

$$L_2 = L_{2a} + L_{2b} = \frac{(a+x)\sqrt{D^2 + (a+x)^2}}{a+x} = \sqrt{D^2 + (a+x)^2} \; .$$

Using the binomial theorem, with D^2 large and $a^2 + x^2$ small, we approximate this expression: $L_2 \approx D + (a+x)^2/2D$. The distance traveled by the direct wave is $L_1 = \sqrt{D^2 + (a-x)^2}$. Using the binomial theorem, we approximate this expression: $L_1 \approx D + (a-x)^2/2D$. Thus,

$$L_2 - L_1 \approx D + \frac{a^2 + 2ax + x^2}{2D} - D - \frac{a^2 - 2ax + x^2}{2D} = \frac{2ax}{D} \; .$$

Setting this equal to $(m + \frac{1}{2})\lambda$, where m is zero or a positive integer, we find $x = (m + \frac{1}{2})(D/2a)\lambda$.

54. According to Eq. 36-41, the number of fringes shifted (ΔN) due to the insertion of the film of thickness L is $\Delta N = (2L/\lambda)(n-1)$. Therefore,

$$L = \frac{\lambda \Delta N}{2(n-1)} = \frac{(589\,\text{nm})(7.0)}{2(1.40-1)} = 5.2\,\mu\text{m} \; .$$

55. A shift of one fringe corresponds to a change in the optical path length of one wavelength. When the mirror moves a distance d the path length changes by $2d$ since the light traverses the mirror arm twice. Let N be the number of fringes shifted. Then, $2d = N\lambda$ and

$$\lambda = \frac{2d}{N} = \frac{2(0.233 \times 10^{-3}\,\text{m})}{792} = 5.88 \times 10^{-7}\,\text{m} = 588 \text{ nm} \; .$$

56. We denote the two wavelengths as λ and λ', respectively. We apply Eq. 36-40 to both wavelengths and take the difference:

$$N' - N = \frac{2L}{\lambda'} - \frac{2L}{\lambda} = 2L\left(\frac{1}{\lambda'} - \frac{1}{\lambda}\right) \; .$$

We now require $N' - N = 1$ and solve for L:

$$\begin{aligned} L &= \frac{1}{2}\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right)^{-1} \\ &= \frac{1}{2}\left(\frac{1}{589.10\,\text{nm}} - \frac{1}{589.59\,\text{nm}}\right)^{-1} \\ &= 3.54 \times 10^5\,\text{nm} = 354\,\mu\text{m} \; . \end{aligned}$$

57. Let ϕ_1 be the phase difference of the waves in the two arms when the tube has air in it, and let ϕ_2 be the phase difference when the tube is evacuated. These are different because the wavelength in air is different from the wavelength in vacuum. If λ is the wavelength in vacuum, then the wavelength in air is λ/n, where n is the index of refraction of air. This means

$$\phi_1 - \phi_2 = 2L\left[\frac{2\pi n}{\lambda} - \frac{2\pi}{\lambda}\right] = \frac{4\pi(n-1)L}{\lambda}$$

where L is the length of the tube. The factor 2 arises because the light traverses the tube twice, once on the way to a mirror and once after reflection from the mirror. Each shift by one fringe corresponds to a change in phase of 2π rad, so if the interference pattern shifts by N fringes as the tube is evacuated,

$$\frac{4\pi(n-1)L}{\lambda} = 2N\pi$$

and

$$n = 1 + \frac{N\lambda}{2L} = 1 + \frac{60(500 \times 10^{-9}\,\text{m})}{2(5.0 \times 10^{-2}\,\text{m})} = 1.00030 \; .$$

58. Let the position of the mirror measured from the point at which $d_1 = d_2$ be x. We assume the beam-splitting mechanism is such that the two waves interfere constructively for $x = 0$ (with some beam-splitters, this would not be the case). We can adapt Eq. 36-22 to this situation by incorporating a factor of 2 (since the interferometer utilizes directly reflected light in contrast to the double-slit experiment) and eliminating the $\sin\theta$ factor. Thus, the phase difference between the two light paths is $\Delta\phi = 2(2\pi x/\lambda) = 4\pi x/\lambda$. Then from Eq. 36-21 (writing $4I_0$ as I_m) we find

$$I = I_m \cos^2\left(\frac{\Delta\phi}{2}\right) = I_m \cos^2\left(\frac{2\pi x}{\lambda}\right) .$$

59. (a) To get to the detector, the wave from S_1 travels a distance x and the wave from S_2 travels a distance $\sqrt{d^2 + x^2}$. The phase difference (in terms of wavelengths) between the two waves is

$$\sqrt{d^2 + x^2} - x = m\lambda \qquad m = 0, 1, 2, \ldots$$

where we are requiring constructive interference. The solution is

$$x = \frac{d^2 - m^2\lambda^2}{2m\lambda} .$$

We see that setting $m = 0$ in this expression produces $x = \infty$; hence, the phase difference between the waves when P is very far away is 0.

(b) The result of part (a) implies that the waves constructively interfere at P.

(c) As is particularly evident from our results in part (d), the phase difference increases as x decreases.

(d) We can use our formula from part (a) for the 0.5λ, 1.50λ, etc differences by allowing m in our formula to take on half-integer values. The half- integer values, though, correspond to destructive interference. Using the values $\lambda = 0.500\,\mu$m and $d = 2.00\,\mu$m, we find $x = 7.88\,\mu$m for $m = \frac{1}{2}$, $x = 3.75\,\mu$m for $m = 1$, $x = 2.29\,\mu$m for $m = \frac{3}{2}$, $x = 1.50\,\mu$m for $m = 2$, and $x = 0.975\,\mu$m for $m = \frac{5}{2}$.

60. (a) In a reference frame fixed on Earth, the ether travels leftward with speed v. Thus, the speed of the light beam in this reference frame is $c - v$ as the beam travels rightward from M to M_1, and $c + v$ as it travels back from M_1 to M. The total time for the round trip is therefore given by

$$t_1 = \frac{d_1}{c - v} + \frac{d_1}{c + v} = \frac{2cd_1}{c^2 - v^2} .$$

(b) In a reference frame fixed on the ether, the mirrors travel rightward with speed v, while the speed of the light beam remains c. Thus, in this reference frame, the total distance the beam has to travel is given by

$$d_2{}' = 2\sqrt{d_2^2 + \left[v\left(\frac{t_2}{2}\right)\right]^2}$$

[see Fig. 36-37(h)-(j)]. Thus,

$$t_2 = \frac{d_2{}'}{c} = \frac{2}{c}\sqrt{d_2^2 + \left[v\left(\frac{t_2}{2}\right)\right]^2} ,$$

which we solve for t_2:

$$t_2 = \frac{2d_2}{\sqrt{c^2 - v^2}} .$$

(c) We use the binomial expansion (Appendix E)

$$(1+x)^n = 1 + nx + \cdots \approx 1 + nx \qquad (|x| \ll 1) \; .$$

In our case let $x = v/c \ll 1$, then

$$L_1 = \frac{2c^2 d_1}{c^2 - v^2} = 2d_1 \left[1 - \left(\frac{v}{c}\right)^2\right]^{-1} \approx 2d_1 \left[1 + \left(\frac{v}{c}\right)^2\right] ,$$

and

$$L_2 = \frac{2cd_2}{\sqrt{c^2 - v^2}} = 2d_2 \left[1 - \left(\frac{v}{c}\right)^2\right]^{-1/2} \approx 2d_2 \left[1 + \frac{1}{2}\left(\frac{v}{c}\right)^2\right] .$$

Thus, if $d_1 = d_2 = d$ then

$$\Delta L = L_1 - L_2 \approx 2d \left[1 + \left(\frac{v}{c}\right)^2\right] - 2d \left[1 + \frac{1}{2}\left(\frac{v}{c}\right)^2\right] = \frac{dv^2}{c^2} .$$

(d) In terms of the wavelength, the phase difference is given by

$$\frac{\Delta L}{\lambda} = \frac{dv^2}{\lambda c^2} .$$

(e) We now must reverse the indices 1 and 2, so the new phase difference is

$$\frac{-\Delta L}{\lambda} = -\frac{dv^2}{\lambda c^2} .$$

The shift in phase difference between these two cases is

$$\text{shift} = \left(\frac{\Delta L}{\lambda}\right) - \left(-\frac{\Delta L}{\lambda}\right) = \frac{2dv^2}{\lambda c^2} .$$

(f) Assume that v is about the same as the orbital speed of the Earth, so that $v \approx 29.8\,\text{km/s}$ (see Appendix C). Thus,

$$\text{shift} = \frac{2dv^2}{\lambda c^2} = \frac{2(10\,\text{m})(29.8 \times 10^3\,\text{m/s})^2}{(500 \times 10^{-9}\,\text{m})(2.998 \times 10^8\,\text{m/s})^2} = 0.40 .$$

61. (a) Every time one more destructive (constructive) fringe appears the increase in thickness of the air gap is $\lambda/2$. Now that there are 6 more destructive fringes in addition to the one at point A, the thickness at B is $t_B = 6(\lambda/2) = 3(600\,\text{nm}) = 1.80\,\mu\text{m}$.

(b) We must now replace λ by $\lambda' = \lambda/n_w$. Since t_B is unchanged $t_B = N(\lambda'/2) = N(\lambda/2n_w)$, or

$$N = \frac{2t_B n_w}{\lambda} = \frac{2(3\lambda)n_w}{\lambda} = 6n_w = 6(1.33) = 8 .$$

62. We adapt Eq. 36-21 to the non-reflective coating on a glass lens: $I = I_{\max}\cos^2(\phi/2)$, where $\phi = (2\pi/\lambda)(2n_2L) + \pi$. At $\lambda = 450\,\text{nm}$

$$\begin{aligned} \frac{I}{I_{\max}} &= \cos^2\left(\frac{\phi}{2}\right) = \cos^2\left(\frac{2\pi n_2 L}{\lambda} + \frac{\pi}{2}\right) \\ &= \cos^2\left[\frac{2\pi(1.38)(99.6\,\text{nm})}{450\,\text{nm}} + \frac{\pi}{2}\right] = 0.883 , \end{aligned}$$

and at $\lambda = 650\,\text{nm}$

$$\frac{I}{I_{\max}} = \cos^2\left[\frac{2\pi(1.38)(99.6\,\text{nm})}{650\,\text{nm}} + \frac{\pi}{2}\right] = 0.942 .$$

63. For the fifth maximum $y_5 = D\sin\theta_5 = D(5\lambda/d)$, and for the seventh minimum $y_7' = D\sin\theta_7' = D[(6 + 1/2)\lambda/d]$. Thus,

$$\begin{aligned}\Delta y &= y_7' - y_5 = D\left[\frac{(6+1/2)\lambda}{d}\right] - D\left(\frac{5\lambda}{d}\right) = \frac{3\lambda D}{2d} \\ &= \frac{3(546\times10^{-9}\,\text{m})(20\times10^{-2}\,\text{m})}{2(0.10\times10^{-3}\,\text{m})} \\ &= 1.6\times10^{-3}\,\text{m} = 1.6\ \text{mm}\ .\end{aligned}$$

64. Let the $m = 10$ bright fringe on the screen be a distance y from the central maximum. Then from Fig. 36-8(a)

$$r_1 - r_2 = \sqrt{(y+d/2)^2 + D^2} - \sqrt{(y-d/2)^2 + D^2} = 10\lambda\ ,$$

from which we may solve for y. To the order of $(d/D)^2$ we find

$$y = y_0 + \frac{y(y^2 + d^2/4)}{2D^2}\ ,$$

where $y_0 = 10D\lambda/d$. Thus, we find the percent error as follows:

$$\frac{y_0(y_0^2 + d^2/4)}{2y_0D^2} = \frac{1}{2}\left(\frac{10\lambda}{D}\right)^2 + \frac{1}{8}\left(\frac{d}{D}\right)^2 = \frac{1}{2}\left(\frac{5.89\,\mu\text{m}}{2000\,\mu\text{m}}\right)^2 + \frac{1}{8}\left(\frac{2.0\,\text{mm}}{40\,\text{mm}}\right)^2$$

which yields 0.03%.

65. $v_{\min} = c/n = (2.998\times10^8\,\text{m/s})/1.54 = 1.95\times10^8\,\text{m/s}$.

66. With phasor techniques, this amounts to a vector addition problem $\vec{R} = \vec{A}+\vec{B}+\vec{C}$ where (in magnitude-angle notation) $\vec{A} = (10\ \angle\ 0°)$, $\vec{B} = (5\ \angle\ 45°)$, and $\vec{C} = (5\ \angle -45°)$, where the magnitudes are understood to be in μV/m. We obtain the resultant (especially efficient on a vector capable calculator in polar mode):

$$\vec{R} = (10\ \angle\ 0°) + (5\ \angle\ 45°) + (5\ \angle -45°) = (17.1\ \angle\ 0°)$$

which leads to

$$E_R = (17.1\ \mu\text{V/m})\sin(\omega t)$$

where $\omega = 2.0\times10^{14}\,\text{rad/s}$.

67. (a) and (b) Dividing Eq. 36-12 by the wavelength, we obtain

$$N = \frac{\Delta L}{\lambda} = \frac{d}{\lambda}\sin\theta = 39.6$$

wavelengths. This is close to a half-integer value (destructive interference), so that the correct response is "intermediate illumination but closer to darkness."

68. To explore one quadrant of the circle, we look for angles where Eq. 36-14 is satisfied.

$$\theta = \sin^{-1}\frac{m\lambda}{d} \qquad \text{for } m = 0, 1, 2...$$

where $m\lambda/d$ cannot exceed unity. For $m = 1...7$ we have solutions that are "mirrored" in every other quadrant; so there are $4\times7 = 28$ of these. The solutions at $m = 0$ and $m = 8$ are "special" in that they have twins (at 180° and 270°, respectively) and their multiplicity is 2, not 4. Thus, we have $28+2(2) = 32$ points of maxima.

69. In this case the path traveled by ray no. 2 is longer than that of ray no. 1 by $2L/\cos\theta_r$, instead of $2L$. Here $\sin\theta_i/\sin\theta_r = n_2$, or $\theta_r = \sin^{-1}(\sin\theta_i/n_2)$. So if we replace $2L$ by $2L/\cos\theta_r$ in Eqs. 36-34 and 36-35, we obtain

$$\frac{2n_2L}{\cos\theta_r} = \left(m + \frac{1}{2}\right)\lambda \qquad m = 0,\ 1,\ 2, \cdots$$

for the maxima, and

$$\frac{2n_2L}{cos\theta_r} = m\lambda \qquad m = 0,\ 1,\ 2, \cdots$$

for the minima.

70. (a) and (b) Straightforward application of Eq. 36-3 and $v = \Delta x/\Delta t$ yields the result: pistol 1 with a time equal to 42.03×10^{-12} s; pistol 2 with a time equal to 42.3×10^{-12} s; pistol 3 with a time equal to 43.2×10^{-12} s; and, pistol 4 with a time equal to 41.96×10^{-12} s. We see that the blast from pistol 1 arrives first.

71. We use Eq. 36-34 for constructive interference: $2n_2L = (m + 1/2)\lambda$, or

$$\lambda = \frac{2n_2L}{m+1/2} = \frac{2(1.50)(410\,\text{nm})}{m+1/2} = \frac{1230\,\text{nm}}{m+1/2},$$

where $m = 0, 1, 2, \cdots$. The only value of m which, when substituted into the equation above, would yield a wavelength which falls within the visible light range is $m = 1$. Therefore,

$$\lambda = \frac{1230\,\text{nm}}{1+1/2} = 492 \text{ nm} .$$

72. For the first maximum $m = 0$ and for the tenth one $m = 9$. The separation is $\Delta y = (D\lambda/d)\Delta m = 9D\lambda/d$. We solve for the wavelength:

$$\lambda = \frac{d\Delta y}{9D} = \frac{(0.15\times10^{-3}\,\text{m})(18\times10^{-3}\,\text{m})}{9(50\times10^{-2}\,\text{m})} = 6.0\times10^{-7}\,\text{m} = 600 \text{ nm} .$$

73. In the case of a distant screen the angle θ is close to zero so $\sin\theta \approx \theta$. Thus from Eq. 36-14,

$$\Delta\theta \approx \Delta\sin\theta = \Delta\left(\frac{m\lambda}{d}\right) = \frac{\lambda}{d}\Delta m = \frac{\lambda}{d} ,$$

or $d \approx \lambda/\Delta\theta = 589\times10^{-9}\,\text{m}/0.018\,\text{rad} = 3.3\times10^{-5}\,\text{m} = 33\,\mu\text{m}$.

74. Using the relations of §36-7, we find that the (vertical) change between the center of one dark band and the next is

$$\Delta y = \lambda 2 = 2.5\times10^{-4} \text{ mm} .$$

Thus, with the (horizontal) separation of dark bands given by $\Delta x = 1.2$ mm, we have

$$\theta \approx \tan\theta = \frac{\Delta y}{\Delta x} = 2.08\times10^{-4} \text{ rad} .$$

Converting this angle into degrees, we arrive at $\theta = 0.012°$.

75. (a) A path length difference of $\lambda/2$ produces the first dark band, of $3\lambda/2$ produces the second dark band, and so on. Therefore, the fourth dark band corresponds to a path length difference of $7\lambda/2 = 1750$ nm.

(b) In the small angle approximation (which we assume holds here), the fringes are equally spaced, so that if Δy denotes the distance from one maximum to the next, then the distance from the middle of the pattern to the fourth dark band must be $16.8\,\text{mm} = 3.5\Delta y$. Therefore, we obtain $\Delta y = 16.8/3.5 = 4.8$ mm.

76. (a) With $\lambda = 0.5\,\mu$m, Eq. 36-14 leads to

$$\theta = \sin^{-1}\frac{(3)(0.5\,\mu\text{m})}{2.00\,\mu\text{m}} = 48.6^\circ \ .$$

(b) Decreasing the frequency means increasing the wavelength – which implies y increases. Qualitatively, this is easily seen with Eq. 36-17. One should exercise caution in appealing to Eq. 36-17 here, due to the fact the small angle approximation is not justified in this problem. The new wavelength is $0.5/0.9 = 0.556\,\mu$m, which produces a new angle of

$$\theta = \sin^{-1}\frac{(3)(0.556\,\mu\text{m})}{2.00\,\mu\text{m}} = 56.4^\circ \ .$$

Using $y = D\tan\theta$ for the old and new angles, and subtracting, we find

$$\Delta y = D\,(\tan 56.4^\circ - \tan 48.6^\circ) = 1.49 \text{ m} \ .$$

77. (a) Following Sample Problem 36-1, we have

$$N_2 - N_1 = \frac{L}{\lambda}\,(n_2 - n_1) = 1.87$$

which represents a meaningful difference of 0.87 wavelength.

(b) The result in part (a) is closer to 1 wavelength (constructive interference) than it is to $\frac{1}{2}$ wavelength (destructive interference) so the latter choice applies.

(c) This would insert a $\pm\frac{1}{2}$ wavelength into the previous result – resulting in a meaningful difference (between the two rays) equal to $0.87 - 0.50 = 0.37$ wavelength, which is closer to the destructive interference condition. Thus, there is intermediate illumination but closer to darkness.

78. (a) Straightforward application of Eq. 36-3 and $v = \Delta x/\Delta t$ yields the result: film 1 with a traversal time equal to 4.0×10^{-15} s.

(b) Use of Eq. 36-9 leads to the number of wavelengths:

$$N = \frac{L_1 n_1 + L_2 n_2 + L_3 n_3}{\lambda} = 7.5 \ .$$

79. (a) In this case, we are dealing with the situation that leads in the textbook to Eq. 36-35 for minima in reflected light from a thin film. The smallest non-zero answer, then, is for $m = 1$: $L = \lambda/2n_2$.

(b) Now, we are dealing with a situation exactly like that treated in Sample Problem 36-5, where the relation $L = \lambda/4n_2$ is derived.

(c) The indices bear the same relation here as in part (b), but we are looking now for the "opposite" result (maximum reflection instead of maximum transmission). We adapt the treatment in Sample Problem 36-5 by requiring $2L = m\lambda/n_2$ instead of $(m + \frac{1}{2})\lambda/2$. The smallest nonzero result in this case is for $m = 1$: $L = \lambda/2n_2$.

80. (a) Since $n_2 > n_3$, this case has no π-phase shift, and the condition for constructive interference is $m\lambda = 2Ln_2$. We solve for L:

$$L = \frac{m\lambda}{2n_2} = \frac{m(525\,\text{nm})}{2(1.55)} = (169\,\text{nm})m \ .$$

For the minimum value of L, let $m = 1$ to obtain $L_{\min} = 169\,$nm.

(b) The light of wavelength λ (other than 525 nm) that would also be preferentially transmitted satisfies $m'\lambda = 2n_2L$, or

$$\lambda = \frac{2n_2L}{m'} = \frac{2(1.55)(169\,\text{nm})}{m'} = \frac{525\,\text{nm}}{m'} \ .$$

Here $m' = 2, 3, 4, \ldots$ (note that $m' = 1$ corresponds to the $\lambda = 525$ nm light, so it should not be included here). Since the minimum value of m' is 2, one can easily verify that no m' will give a value of λ which falls into the visible light range. So no other parts of the visible spectrum will be preferentially transmitted. They are, in fact, reflected.

(c) For a sharp reduction of transmission let

$$\lambda = \frac{2n_2L}{m'+1/2} = \frac{525\,\text{nm}}{m'+1/2} \ ,$$

where $m' = 0, 1, 2, 3, \cdots$. In the visible light range $m' = 1$ and $\lambda = 350$ nm. This corresponds to the blue-violet light.

81. We adapt the result of problem 21. Now, the phase difference in radians is

$$\frac{2\pi t}{\lambda}(n_2 - n_1) = 2m\pi \ .$$

The problem implies $m = 5$, so the thickness is

$$t = \frac{m\lambda}{n_2 - n_1} = \frac{5(480\,\text{nm})}{1.7 - 1.4} = 8.0 \times 10^3\,\text{nm} = 8.0\,\mu\text{m} \ .$$

82. In Sample Problem 36-2, the relation $\Delta y = \lambda D/d$ is derived. Thus, to prevent Δy from changing, then (since $\Delta y \propto D/d$) we need to double D if d is doubled.

83. (a) In this case, the film has a smaller index material on one side (air) and a larger index material on the other (glass), and we are dealing (in part (a)) with strongly transmitted light, so the condition is given by Eq. 36-35 (which would give dark *reflection* in this scenario)

$$L = \frac{\lambda}{2n_2}\left(m + \frac{1}{2}\right) = 110 \text{ nm}$$

for $n_2 = 1.25$ and $m = 0$.

(b) Now, we are dealing with strongly reflected light, so the condition is given by Eq. 36-34 (which would give no *transmission* in this scenario)

$$L = \frac{m\lambda}{2n_2} = 220 \text{ nm}$$

for $n_2 = 1.25$ and $m = 1$ (the $m = 0$ option is excluded in the problem statement).

84. We infer from Sample Problem 36-2, that (with angle in radians)

$$\Delta\theta = \frac{\lambda}{d}$$

for adjacent fringes. With the wavelength change ($\lambda' = \lambda/n$ by Eq. 36-8), this equation becomes

$$\Delta\theta' = \frac{\lambda'}{d} \ .$$

Dividing one equation by the other, the requirement of *radians* can now be relaxed and we obtain

$$\frac{\Delta\theta'}{\Delta\theta} = \frac{\lambda'}{\lambda} = \frac{1}{n} \ .$$

Therefore, with $n = 1.33$ and $\Delta\theta = 0.30°$, we find $\Delta\theta' = 0.23°$.

85. Using Eq. 36-16 with the small-angle approximation (illustrated in Sample Problem 36-2), we arrive at

$$y = \frac{\left(m + \frac{1}{2}\right) \lambda D}{d}$$

for the position of the $(m+1)^{\text{th}}$ dark band (a simple way to get this is by averaging the expressions in Eq. 36-17 and Eq. 36-18). Thus, with $m = 1$, $y = 0.012$ m and $d = 800\lambda$, we find $D = 6.4$ m.

86. (a) The path length difference between Rays 1 and 2 is $7d - 2d = 5d$. For this to correspond to a half-wavelength requires $5d = \lambda/2$, so that $d = 50.0$ nm.

    (b) The above requirement becomes $5d = \lambda/2n$ in the presence of the solution, with $n = 1.38$. Therefore, $d = 36.2$ nm.

87. (a) The path length difference is $0.5\,\mu$m $= 500$ nm, which is represents $500/400 = 1.25$ wavelengths – that is, a meaningful difference of 0.25 wavelengths. In angular measure, this corresponds to a phase difference of $(0.25)2\pi = \pi/2$ radians.

    (b) When a difference of index of refraction is involved, the approach used in Eq. 36-9 is quite useful. In this approach, we count the wavelengths between S_1 and the origin

$$N_1 = \frac{Ln}{\lambda} + \frac{L'n'}{\lambda}$$

where $n = 1$ (rounding off the index of air), $L = 5.0\,\mu$m, $n' = 1.5$ and $L' = 1.5\,\mu$m. This yields $N_1 = 18.125$ wavelengths. The number of wavelengths between S_2 and the origin is (with $L_2 = 6.0\,\mu$m) given by

$$N_2 = \frac{L_2 n}{\lambda} = 15.000 \; .$$

Thus, $N_1 - N_2 = 3.125$ wavelengths, which gives us a meaningful difference of 0.125 wavelength and which "converts" to a phase of $\pi/4$ radian.