Chapter 25

1. (a) An Ampere is a Coulomb per second, so

$$84\,\mathrm{A\cdot h} = \left(84\,\frac{\mathrm{C\cdot h}}{\mathrm{s}}\right)\left(3600\,\frac{\mathrm{s}}{\mathrm{h}}\right) = 3.0\times 10^5\ \mathrm{C}\ .$$

   (b) The change in potential energy is $\Delta U = q\,\Delta V = (3.0\times 10^5\,\mathrm{C})(12\,\mathrm{V}) = 3.6\times 10^6\,\mathrm{J}$.

2. The magnitude is $\Delta U = e\Delta V = 1.2\times 10^9\,\mathrm{eV} = 1.2\,\mathrm{GeV}$.

3. (a) When charge q moves through a potential difference ΔV, its potential energy changes by $\Delta U = q\,\Delta V$. In this case, $\Delta U = (30\,\mathrm{C})(1.0\times 10^9\,\mathrm{V}) = 3.0\times 10^{10}\,\mathrm{J}$.

   (b) We equate the final kinetic energy $\frac{1}{2}mv^2$ of the automobile to the energy released by the lightning, denoted by $U_{\mathrm{lightning}}$.

$$v = \sqrt{\frac{2\,U_{\mathrm{lightning}}}{m}} = \sqrt{\frac{2(3.0\times 10^{10}\,\mathrm{J})}{1000\,\mathrm{kg}}} = 7.7\times 10^3\ \mathrm{m/s}\ .$$

   (c) We equate the energy required to melt mass m of ice to the energy released by the lightning: $\Delta U = mL_F$, where L_F is the heat of fusion for ice. Thus,

$$m = \frac{\Delta U}{L_F} = \frac{3.0\times 10^{10}\,\mathrm{J}}{3.33\times 10^5\,\mathrm{J/kg}} = 9.0\times 10^4\ \mathrm{kg}\ .$$

4. (a) $V_B - V_A = \Delta U/(-e) = (3.94\times 10^{-19}\,\mathrm{J})/(-1.60\times 10^{-19}\,\mathrm{C}) = -2.46\,\mathrm{V}$.

   (b) $V_C - V_A = V_B - V_A = -2.46\,\mathrm{V}$.

   (c) $V_C - V_B = 0$ (Since C and B are on the same equipotential line).

5. The electric field produced by an infinite sheet of charge has magnitude $E = \sigma/2\varepsilon_0$, where σ is the surface charge density. The field is normal to the sheet and is uniform. Place the origin of a coordinate system at the sheet and take the x axis to be parallel to the field and positive in the direction of the field. Then the electric potential is

$$V = V_s - \int_0^x E\,dx = V_s - Ex\ ,$$

where V_s is the potential at the sheet. The equipotential surfaces are surfaces of constant x; that is, they are planes that are parallel to the plane of charge. If two surfaces are separated by Δx then their potentials differ in magnitude by $\Delta V = E\Delta x = (\sigma/2\varepsilon_0)\Delta x$. Thus,

$$\Delta x = \frac{2\varepsilon_0\,\Delta V}{\sigma} = \frac{2\left(8.85\times 10^{-12}\,\mathrm{C^2/N\cdot m^2}\right)(50\,\mathrm{V})}{0.10\times 10^{-6}\,\mathrm{C/m^2}} = 8.8\times 10^{-3}\ \mathrm{m}\ .$$

6. (a) $E = F/e = (3.9 \times 10^{-15}\,\text{N})/(1.60 \times 10^{-19}\,\text{C}) = 2.4 \times 10^4\,\text{N/C}$.

   (b) $\Delta V = E\Delta s = (2.4 \times 10^4\,\text{N/C})(0.12\,\text{m}) = 2.9 \times 10^3\,\text{V}$.

7. The potential difference between the wire and cylinder is given, not the linear charge density on the wire. We use Gauss' law to find an expression for the electric field a distance r from the center of the wire, between the wire and the cylinder, in terms of the linear charge density. Then integrate with respect to r to find an expression for the potential difference between the wire and cylinder in terms of the linear charge density. We use this result to obtain an expression for the linear charge density in terms of the potential difference and substitute the result into the equation for the electric field. This will give the electric field in terms of the potential difference and will allow you to compute numerical values for the field at the wire and at the cylinder. For the Gaussian surface use a cylinder of radius r and length ℓ, concentric with the wire and cylinder. The electric field is normal to the rounded portion of the cylinder's surface and its magnitude is uniform over that surface. This means the electric flux through the Gaussian surface is given by $2\pi r\ell E$, where E is the magnitude of the electric field. The charge enclosed by the Gaussian surface is $q = \lambda\ell$, where λ is the linear charge density on the wire. Gauss' law yields $2\pi\varepsilon_0 r\ell E = \lambda\ell$. Thus,

$$E = \frac{\lambda}{2\pi\varepsilon_0 r} .$$

Since the field is radial, the difference in the potential V_c of the cylinder and the potential V_w of the wire is

$$\Delta V = V_w - V_c = -\int_{r_c}^{r_w} E\,\mathrm{d}r = \int_{r_w}^{r_c} \frac{\lambda}{2\pi\varepsilon_0 r}\,\mathrm{d}r = \frac{\lambda}{2\pi\varepsilon_0}\ln\frac{r_c}{r_w} ,$$

where r_w is the radius of the wire and r_c is the radius of the cylinder. This means that

$$\lambda = \frac{2\pi\varepsilon_0\,\Delta V}{\ln(r_c/r_w)}$$

and

$$E = \frac{\lambda}{2\pi\varepsilon_0 r} = \frac{\Delta V}{r\ln(r_c/r_w)} .$$

(a) We substitute r_c for r to obtain the field at the surface of the wire:

$$\begin{aligned} E &= \frac{\Delta V}{r_w\ln(r_c/r_w)} = \frac{850\,\text{V}}{(0.65 \times 10^{-6}\,\text{m})\ \ln\left[(1.0 \times 10^{-2}\,\text{m})/(0.65 \times 10^{-6}\,\text{m})\right]} \\ &= 1.36 \times 10^8\ \text{V/m} . \end{aligned}$$

(b) We substitute r_c for r to find the field at the surface of the cylinder:

$$\begin{aligned} E &= \frac{\Delta V}{r_c\ln(r_c/r_w)} = \frac{850\,\text{V}}{(1.0 \times 10^{-2}\,\text{m})\ \ln\left[(1.0 \times 10^{-2}\,\text{m})/(0.65 \times 10^{-6}\,\text{m})\right]} \\ &= 8.82 \times 10^3\ \text{V/m} . \end{aligned}$$

8. (a) The potential as a function of r is

$$V(r) = V(0) - \int_0^r E(r)\,dr = 0 - \int_0^r \frac{qr}{4\pi\varepsilon_0 R^3}\,dr = -\frac{qr^2}{8\pi\varepsilon_0 R^3} .$$

   (b) $\Delta V = V(0) - V(R) = q/8\pi\varepsilon_0 R$.

   (c) Since $\Delta V = V(0) - V(R) > 0$, the potential at the center of the sphere is higher.

9. (a) We use Gauss' law to find expressions for the electric field inside and outside the spherical charge distribution. Since the field is radial the electric potential can be written as an integral of the field along a sphere radius, extended to infinity. Since different expressions for the field apply in different regions the integral must be split into two parts, one from infinity to the surface of the distribution and one from the surface to a point inside. Outside the charge distribution the magnitude of the field is $E = q/4\pi\varepsilon_0 r^2$ and the potential is $V = q/4\pi\varepsilon_0 r$, where r is the distance from the center of the distribution. This is the same as the field and potential of a point charge at the center of the spherical distribution. To find an expression for the magnitude of the field inside the charge distribution, we use a Gaussian surface in the form of a sphere with radius r, concentric with the distribution. The field is normal to the Gaussian surface and its magnitude is uniform over it, so the electric flux through the surface is $4\pi r^2 E$. The charge enclosed is qr^3/R^3. Gauss' law becomes

$$4\pi\varepsilon_0 r^2 E = \frac{qr^3}{R^3} ,$$

so

$$E = \frac{qr}{4\pi\varepsilon_0 R^3} .$$

If V_s is the potential at the surface of the distribution ($r = R$) then the potential at a point inside, a distance r from the center, is

$$V = V_s - \int_R^r E\,dr = V_s - \frac{q}{4\pi\varepsilon_0 R^3}\int_R^r r\,dr = V_s - \frac{qr^2}{8\pi\varepsilon_0 R^3} + \frac{q}{8\pi\varepsilon_0 R} .$$

The potential at the surface can be found by replacing r with R in the expression for the potential at points outside the distribution. It is $V_s = q/4\pi\varepsilon_0 R$. Thus,

$$V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{R} - \frac{r^2}{2R^3} + \frac{1}{2R}\right] = \frac{q}{8\pi\varepsilon_0 R^3}(3R^2 - r^2) .$$

(b) In problem 8 the electric potential was taken to be zero at the center of the sphere. In this problem it is zero at infinity. According to the expression derived in part (a) the potential at the center of the sphere is $V_c = 3q/8\pi\varepsilon_0 R$. Thus $V - V_c = -qr^2/8\pi\varepsilon_0 R^3$. This is the result of problem 8.

(c) The potential difference is

$$\Delta V = V_s - V_c = \frac{2q}{8\pi\varepsilon_0 R} - \frac{3q}{8\pi\varepsilon_0 R} = -\frac{q}{8\pi\varepsilon_0 R} .$$

The expression obtained in problem 8 would give this same value.

(d) Only potential differences have physical significance, not the value of the potential at any particular point. The same value can be added to the potential at every point without changing the electric field, for example. Changing the reference point from the center of the distribution to infinity changes the value of the potential at every point but it does not change any potential differences.

10. (a)

$$W = \int_i^f q_0\vec{E}\cdot d\vec{s} = \frac{q_0\sigma}{2\varepsilon_0}\int_0^z dz = \frac{q_0\sigma z}{2\varepsilon_0} .$$

(b) Since $V - V_0 = -W/q_0 = -\sigma z/2\varepsilon_0$,

$$V = V_0 - \frac{\sigma z}{2\varepsilon_0} .$$

11. (a) For $r > r_2$ the field is like that of a point charge and

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r} ,$$

where the zero of potential was taken to be at infinity.

(b) To find the potential in the region $r_1 < r < r_2$, first use Gauss's law to find an expression for the electric field, then integrate along a radial path from r_2 to r. The Gaussian surface is a sphere of radius r, concentric with the shell. The field is radial and therefore normal to the surface. Its magnitude is uniform over the surface, so the flux through the surface is $\Phi = 4\pi r^2 E$. The volume of the shell is $(4\pi/3)(r_2^3 - r_1^3)$, so the charge density is

$$\rho = \frac{3Q}{4\pi(r_2^3 - r_1^3)} \, ,$$

and the charge enclosed by the Gaussian surface is

$$q = \left(\frac{4\pi}{3}\right)(r^3 - r_1^3)\rho = Q\left(\frac{r^3 - r_1^3}{r_2^3 - r_1^3}\right) \, .$$

Gauss' law yields

$$4\pi\varepsilon_0 r^2 E = Q\left(\frac{r^3 - r_1^3}{r_2^3 - r_1^3}\right) \implies E = \frac{Q}{4\pi\varepsilon_0}\frac{r^3 - r_1^3}{r^2(r_2^3 - r_1^3)} \, .$$

If V_s is the electric potential at the outer surface of the shell ($r = r_2$) then the potential a distance r from the center is given by

$$\begin{aligned} V &= V_s - \int_{r_2}^{r} E\,dr = V_s - \frac{Q}{4\pi\varepsilon_0}\frac{1}{r_2^3 - r_1^3}\int_{r_2}^{r}\left(r - \frac{r_1^3}{r^2}\right)dr \\ &= V_s - \frac{Q}{4\pi\varepsilon_0}\frac{1}{r_2^3 - r_1^3}\left(\frac{r^2}{2} - \frac{r_2^2}{2} + \frac{r_1^3}{r} - \frac{r_1^3}{r_2}\right) \, . \end{aligned}$$

The potential at the outer surface is found by placing $r = r_2$ in the expression found in part (a). It is $V_s = Q/4\pi\varepsilon_0 r_2$. We make this substitution and collect terms to find

$$V = \frac{Q}{4\pi\varepsilon_0}\frac{1}{r_2^3 - r_1^3}\left(\frac{3r_2^2}{2} - \frac{r^2}{2} - \frac{r_1^3}{r}\right) \, .$$

Since $\rho = 3Q/4\pi(r_2^3 - r_1^3)$ this can also be written

$$V = \frac{\rho}{3\varepsilon_0}\left(\frac{3r_2^2}{2} - \frac{r^2}{2} - \frac{r_1^3}{r}\right) \, .$$

(c) The electric field vanishes in the cavity, so the potential is everywhere the same inside and has the same value as at a point on the inside surface of the shell. We put $r = r_1$ in the result of part (b). After collecting terms the result is

$$V = \frac{Q}{4\pi\varepsilon_0}\frac{3(r_2^2 - r_1^2)}{2(r_2^3 - r_1^3)} \, ,$$

or in terms of the charge density

$$V = \frac{\rho}{2\varepsilon_0}(r_2^2 - r_1^2) \, .$$

(d) The solutions agree at $r = r_1$ and at $r = r_2$.

12. The charge is

$$q = 4\pi\varepsilon_0 RV = \frac{(10\,\mathrm{m})(-1.0\,\mathrm{V})}{8.99\times10^9\,\frac{\mathrm{N\cdot m^2}}{\mathrm{C^2}}} = -1.1\times10^{-9}\ \mathrm{C} \, .$$

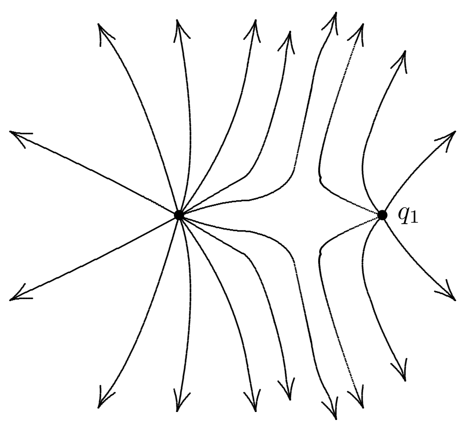
13. (a) The potential difference is

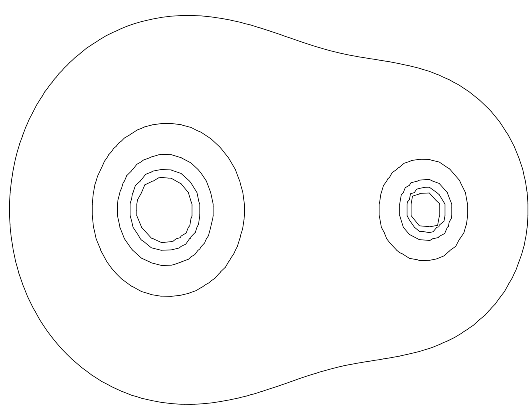
$$\begin{aligned} V_A - V_B &= \frac{q}{4\pi\varepsilon_0 r_A} - \frac{q}{4\pi\varepsilon_0 r_B} \\ &= \left(1.0 \times 10^{-6}\,\mathrm{C}\right)\left(8.99 \times 10^9\,\frac{\mathrm{N{\cdot}m^2}}{\mathrm{C^2}}\right)\left(\frac{1}{2.0\,\mathrm{m}} - \frac{1}{1.0\,\mathrm{m}}\right) = -4500\ \mathrm{V}\ . \end{aligned}$$

(b) Since $V(r)$ depends only on the magnitude of $\vec{r}$, the result is unchanged.

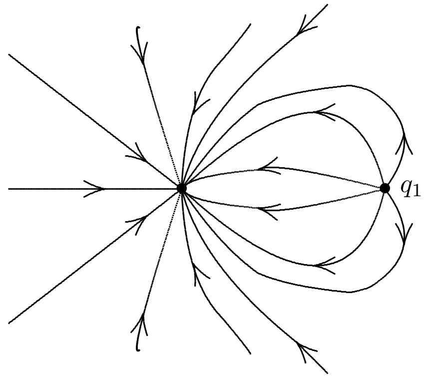
14. In the sketches shown below, the lines with the arrows are field lines and those without are the equipotentials (which become more circular the closer one gets to the individual charges) . In all pictures, q_2 is on the left and q_1 is on the right (which is reversed from the way it is shown in the textbook).

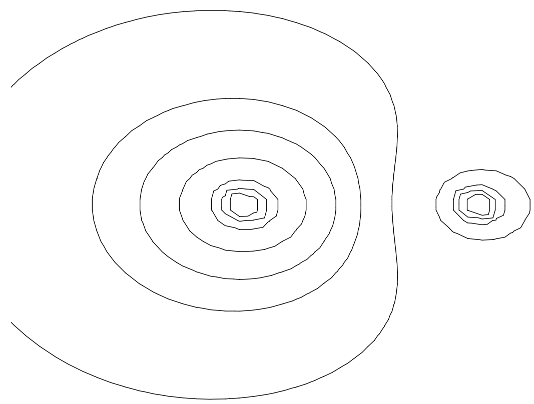
(a)





(b)





15. First, we observe that $V(x)$ cannot be equal to zero for $x > d$. In fact $V(x)$ is always negative for $x > d$. Now we consider the two remaining regions on the x axis: $x < 0$ and $0 < x < d$. For $x < 0$ the separation between q_1 and a point on the x axis whose coordinate is x is given by $d_1 = -x$; while the corresponding separation for q_2 is $d_2 = d - x$. We set

$$V(x) = k\left(\frac{q_1}{d_1} + \frac{q_2}{d_2}\right) = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{-x} + \frac{-3}{d-x}\right) = 0$$

to obtain $x = -d/2$. Similarly, for $0 < x < d$ we have $d_1 = x$ and $d_2 = d - x$. Let

$$V(x) = k\left(\frac{q_1}{d_1} + \frac{q_2}{d_2}\right) = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{x} + \frac{-3}{d-x}\right) = 0$$

and solve: $x = d/4$.

16. Since according to the problem statement there is a point in between the two charges on the x axis where the net electric field is zero, the fields at that point due to q_1 and q_2 must be directed opposite to each other. This means that q_1 and q_2 must have the same sign (i.e., either both are positive or both negative). Thus, the potentials due to either of them must be of the same sign. Therefore, the net electric potential cannot possibly be zero anywhere except at infinity.

17. (a) The electric potential V at the surface of the drop, the charge q on the drop, and the radius R of the drop are related by $V = q/4\pi\varepsilon_0 R$. Thus

$$R = \frac{q}{4\pi\varepsilon_0 V} = \frac{(8.99 \times 10^9\,\mathrm{N\cdot m^2/C^2})(30 \times 10^{-12}\,\mathrm{C})}{500\,\mathrm{V}} = 5.4 \times 10^{-4}\ \mathrm{m}\ .$$

(b) After the drops combine the total volume is twice the volume of an original drop, so the radius R' of the combined drop is given by $(R')^3 = 2R^3$ and $R' = 2^{1/3}R$. The charge is twice the charge of original drop: $q' = 2q$. Thus,

$$V' = \frac{1}{4\pi\varepsilon_0}\frac{q'}{R'} = \frac{1}{4\pi\varepsilon_0}\frac{2q}{2^{1/3}R} = 2^{2/3}V = 2^{2/3}(500\,\text{V}) \approx 790\ \text{V} .$$

18. (a) The charge on the sphere is

$$q = 4\pi\varepsilon_0 VR = \frac{(200\,\text{V})(0.15\,\text{m})}{8.99\times10^9\,\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}} = 3.3\times10^{-9}\ \text{C} .$$

(b) The (uniform) surface charge density (charge divided by the area of the sphere) is

$$\sigma = \frac{q}{4\pi R^2} = \frac{3.3\times10^{-9}\,\text{C}}{4\pi(0.15\,\text{m})^2} = 1.2\times10^{-8}\ \text{C/m}^2 .$$

19. Assume the charge on Earth is distributed with spherical symmetry. If the electric potential is zero at infinity then at the surface of Earth it is $V = q/4\pi\varepsilon_0 R$, where q is the charge on Earth and $R = 6.37\times10^6$ m is the radius of Earth. The magnitude of the electric field at the surface is $E = q/4\pi\varepsilon_0 R^2$, so $V = ER = (100\,\text{V/m})(6.37\times10^6\,\text{m}) = 6.4\times10^8\,\text{V}$.

20. The net electric potential at point P is the sum of those due to the six charges:

$$\begin{aligned} V_P &= \sum_{i=1}^{6} V_{Pi} = \sum_{i=1}^{6}\frac{q_i}{4\pi\varepsilon_0 r_i} \\ &= \frac{1}{4\pi\varepsilon_0}\left[\frac{5.0q}{\sqrt{d^2+(d/2)^2}} + \frac{-2.0q}{d/2} + \frac{-3.0q}{\sqrt{d^2+(d/2)^2}}\right. \\ &\qquad \left. + \frac{3.0q}{\sqrt{d^2+(d/2)^2}} + \frac{-2.0q}{d/2} + \frac{-5.0q}{\sqrt{d^2+(d/2)^2}}\right] \\ &= \frac{-0.94q}{4\pi\varepsilon_0 d} . \end{aligned}$$

21. A charge $-5q$ is a distance $2d$ from P, a charge $-5q$ is a distance d from P, and two charges $+5q$ are each a distance d from P, so the electric potential at P is

$$V = \frac{q}{4\pi\varepsilon_0}\left[-\frac{5}{2d} - \frac{5}{d} + \frac{5}{d} + \frac{5}{d}\right] = \frac{5q}{8\pi\varepsilon_0} .$$

The zero of the electric potential was taken to be at infinity.

22. We use Eq. 25-20:

$$V = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^2} = \frac{\left(8.99\times10^9\,\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(1.47\times3.34\times10^{-30}\,\text{C}\cdot\text{m})}{(52.0\times10^{-9}\,\text{m})^2} = 1.63\times10^{-5}\ \text{V} .$$

23. A positive charge q is a distance $r-d$ from P, another positive charge q is a distance r from P, and a negative charge $-q$ is a distance $r+d$ from P. Sum the individual electric potentials created at P to find the total:

$$V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r-d} + \frac{1}{r} - \frac{1}{r+d}\right] .$$

We use the binomial theorem to approximate $1/(r-d)$ for r much larger than d:

$$\frac{1}{r-d} = (r-d)^{-1} \approx (r)^{-1} - (r)^{-2}(-d) = \frac{1}{r} + \frac{d}{r^2} .$$

Similarly,

$$\frac{1}{r+d} \approx \frac{1}{r} - \frac{d}{r^2} \ .$$

Only the first two terms of each expansion were retained. Thus,

$$V \approx \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r} + \frac{d}{r^2} + \frac{1}{r} - \frac{1}{r} + \frac{d}{r^2}\right] = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r} + \frac{2d}{r^2}\right] = \frac{q}{4\pi\varepsilon_0 r}\left[1 + \frac{2d}{r}\right] \ .$$

24. (a) From Eq. 25-35

$$V = 2\frac{\lambda}{4\pi\varepsilon_0}\ \ln\left[\frac{L/2 + \sqrt{(L^2/4) + d^2}}{d}\right] \ .$$

(b) The potential at P is $V = 0$ due to superposition.

25. (a) All the charge is the same distance R from C, so the electric potential at C is

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{R} - \frac{6Q}{R}\right] = -\frac{5Q}{4\pi\varepsilon_0 R} \ ,$$

where the zero was taken to be at infinity.

(b) All the charge is the same distance from P. That distance is $\sqrt{R^2+z^2}$, so the electric potential at P is

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{\sqrt{R^2+z^2}} - \frac{6Q}{\sqrt{R^2+z^2}}\right] = -\frac{5Q}{4\pi\varepsilon_0\sqrt{R^2+z^2}} \ .$$

26. The potential is

$$V_P = \frac{1}{4\pi\varepsilon_0}\int_{\text{rod}} \frac{dq}{R} = \frac{1}{4\pi\varepsilon_0 R}\int_{\text{rod}} dq = \frac{-Q}{4\pi\varepsilon_0 R} \ .$$

We note that the result is exactly what one would expect for a point-charge $-Q$ at a distance R. This "coincidence" is due, in part, to the fact that V is a scalar quantity.

27. The disk is uniformly charged. This means that when the full disk is present each quadrant contributes equally to the electric potential at P, so the potential at P due to a single quadrant is one-fourth the potential due to the entire disk. First find an expression for the potential at P due to the entire disk. We consider a ring of charge with radius r and (infinitesimal) width dr. Its area is $2\pi r\, dr$ and it contains charge $dq = 2\pi\sigma r\, dr$. All the charge in it is a distance $\sqrt{r^2+z^2}$ from P, so the potential it produces at P is

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{2\pi\sigma r\, dr}{\sqrt{r^2+z^2}} = \frac{\sigma r\, dr}{2\varepsilon_0\sqrt{r^2+z^2}} \ .$$

The total potential at P is

$$V = \frac{\sigma}{2\varepsilon_0}\int_0^R \frac{r\, dr}{\sqrt{r^2+z^2}} = \frac{\sigma}{2\varepsilon_0}\sqrt{r^2+z^2}\bigg|_0^R = \frac{\sigma}{2\varepsilon_0}\left[\sqrt{R^2+z^2} - z\right] \ .$$

The potential V_{sq} at P due to a single quadrant is

$$V_{sq} = \frac{V}{4} = \frac{\sigma}{8\varepsilon_0}\left[\sqrt{R^2+z^2} - z\right] \ .$$

28. Consider an infinitesimal segment of the rod, located between x and $x+dx$. It has length dx and contains charge $dq = \lambda\, dx$, where $\lambda = Q/L$ is the linear charge density of the rod. Its distance from P_1 is $d+x$ and the potential it creates at P_1 is

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{d+x} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dx}{d+x} \ .$$

To find the total potential at P_1, integrate over the rod:

$$V = \frac{\lambda}{4\pi\varepsilon_0}\int_0^L \frac{dx}{d+x} = \frac{\lambda}{4\pi\varepsilon_0}\ln(d+x)\Big|_0^L = \frac{Q}{4\pi\varepsilon_0 L}\ln\left(1+\frac{L}{d}\right) .$$

29. Consider an infinitesimal segment of the rod, located between x and $x+dx$. It has length dx and contains charge $dq = \lambda\,dx = cx\,dx$. Its distance from P_1 is $d+x$ and the potential it creates at P_1 is

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{d+x} = \frac{1}{4\pi\varepsilon_0}\frac{cx\,dx}{d+x} .$$

To find the total potential at P_1, integrate over the rod:

$$V = \frac{c}{4\pi\varepsilon_0}\int_0^L \frac{x\,dx}{d+x} = \frac{c}{4\pi\varepsilon_0}[x - d\,\ln(x+d)]\Big|_0^L = \frac{c}{4\pi\varepsilon_0}\left[L - d\,\ln\left(1+\frac{L}{d}\right)\right] .$$

30. The magnitude of the electric field is given by

$$|E| = \left|-\frac{\Delta V}{\Delta x}\right| = \frac{2(5.0\,\text{V})}{0.015\,\text{m}} = 6.7\times 10^2\ \text{V/m} .$$

At any point in the region between the plates, $\vec{E}$ points away from the positively charged plate, directly towards the negatively charged one.

31. We use Eq. 25-41:

$$\begin{aligned} E_x(x,y) &= -\frac{\partial V}{\partial x} = -\frac{\partial}{\partial x}\left((2.0\,\text{V/m}^2)x^2 - (3.0\,\text{V/m}^2)y^2\right) = -2(2.0\,\text{V/m}^2)x ; \\ E_y(x,y) &= -\frac{\partial V}{\partial y} = -\frac{\partial}{\partial y}\left((2.0\,\text{V/m}^2)x^2 - (3.0\,\text{V/m}^2)y^2\right) = 2(3.0\,\text{V/m}^2)y . \end{aligned}$$

We evaluate at $x = 3.0\,\text{m}$ and $y = 2.0\,\text{m}$ to obtain the magnitude of $\vec{E}$:

$$E = \sqrt{E_x^2 + E_y^2} = 17\ \text{V/m} .$$

$\vec{E}$ makes an angle θ with the positive x axis, where

$$\theta = \tan^{-1}\left(\frac{E_y}{E_x}\right) = 135^\circ .$$

32. We use Eq. 25-41. This is an ordinary derivative since the potential is a function of only one variable.

$$\begin{aligned} \vec{E} &= -\left(\frac{dV}{dx}\right)\hat{\imath} = -\frac{d}{dx}(1500x^2)\hat{\imath} = (-3000x)\hat{\imath} \\ &= (-3000\,\text{V/m}^2)(0.0130\,\text{m})\hat{\imath} = (-39\,\text{V/m})\hat{\imath} . \end{aligned}$$

33. (a) The charge on every part of the ring is the same distance from any point P on the axis. This distance is $r = \sqrt{z^2+R^2}$, where R is the radius of the ring and z is the distance from the center of the ring to P. The electric potential at P is

$$V = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{\sqrt{z^2+R^2}} = \frac{1}{4\pi\varepsilon_0}\frac{1}{\sqrt{z^2+R^2}}\int dq = \frac{1}{4\pi\varepsilon_0}\frac{q}{\sqrt{z^2+R^2}} .$$

(b) The electric field is along the axis and its component is given by

$$\begin{aligned} E &= -\frac{\partial V}{\partial z} = -\frac{q}{4\pi\varepsilon_0}\frac{\partial}{\partial z}(z^2+R^2)^{-1/2} \\ &= \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{2}\right)(z^2+R^2)^{-3/2}(2z) = \frac{q}{4\pi\varepsilon_0}\frac{z}{(z^2+R^2)^{3/2}} . \end{aligned}$$

This agrees with Eq. 23-16.

34. (a) Consider an infinitesimal segment of the rod from x to $x+dx$. Its contribution to the potential at point P_2 is

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{\lambda(x)dx}{\sqrt{x^2+y^2}} = \frac{1}{4\pi\varepsilon_0}\frac{cx}{\sqrt{x^2+y^2}}\,dx .$$

Thus,

$$V = \int_{\text{rod}} dV_P = \frac{c}{4\pi\varepsilon_0}\int_0^L \frac{x}{\sqrt{x^2+y^2}}\,dx = \frac{c}{4\pi\varepsilon_0}(\sqrt{L^2+y^2}-y) .$$

(b) The y component of the field there is

$$E_y = -\frac{\partial V_P}{\partial y} = -\frac{c}{4\pi\varepsilon_0}\frac{d}{dy}(\sqrt{L^2+y^2}-y) = \frac{c}{4\pi\varepsilon_0}\left(1-\frac{y}{\sqrt{L^2+y^2}}\right) .$$

(c) We obtained above the value of the potential at any point P strictly on the y-axis. In order to obtain $E_x(x,y)$ we need to first calculate $V(x,y)$. That is, we must find the potential for an arbitrary point located at $(x,\,y)$. Then $E_x(x,\,y)$ can be obtained from $E_x(x,\,y) = -\partial V(x,\,y)/\partial x$.

35. (a) According to the result of problem 28, the electric potential at a point with coordinate x is given by

$$V = \frac{Q}{4\pi\varepsilon_0 L}\ln\left(\frac{x-L}{x}\right) .$$

We differentiate the potential with respect to x to find the x component of the electric field:

$$\begin{aligned} E_x &= -\frac{\partial V}{\partial x} = -\frac{Q}{4\pi\varepsilon_0 L}\frac{\partial}{\partial x}\ln\left(\frac{x-L}{x}\right) = -\frac{Q}{4\pi\varepsilon_0 L}\frac{x}{x-L}\left(\frac{1}{x}-\frac{x-L}{x^2}\right) \\ &= -\frac{Q}{4\pi\varepsilon_0 x(x-L)} . \end{aligned}$$

At $x=-d$ we obtain

$$E_x = -\frac{Q}{4\pi\varepsilon_0 d(d+L)} .$$

(b) Consider two points an equal infinitesimal distance on either side of P_1, along a line that is perpendicular to the x axis. The difference in the electric potential divided by their separation gives the transverse component of the electric field. Since the two points are situated symmetrically with respect to the rod, their potentials are the same and the potential difference is zero. Thus the transverse component of the electric field is zero.

36. (a) We use Eq. 25-43 with $q_1 = q_2 = -e$ and $r = 2.00\,\text{nm}$:

$$U = k\frac{q_1q_2}{r} = k\frac{e^2}{r} = \frac{\left(8.99\times10^9\,\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(1.60\times10^{-19}\,\text{C})^2}{2.00\times10^{-9}\,\text{m}} = 1.15\times10^{-19}\ \text{J} .$$

(b) Since $U>0$ and $U\propto r^{-1}$ the potential energy U decreases as r increases.

37. We choose the zero of electric potential to be at infinity. The initial electric potential energy U_i of the system before the particles are brought together is therefore zero. After the system is set up the final potential energy is

$$\begin{aligned} U_f &= \frac{q^2}{4\pi\varepsilon_0}\left(-\frac{1}{a}-\frac{1}{a}+\frac{1}{\sqrt{2}a}-\frac{1}{a}-\frac{1}{a}+\frac{1}{\sqrt{2}a}\right) \\ &= \frac{2q^2}{4\pi\varepsilon_0 a}\left(\frac{1}{\sqrt{2}}-2\right) = -\frac{0.21q^2}{\varepsilon_0 a} \ . \end{aligned}$$

Thus the amount of work required to set up the system is given by $W = \Delta U = U_f - U_i = -0.21q^2/(\varepsilon_0 a)$.

38. The electric potential energy is

$$\begin{aligned} U &= k\sum_{i\neq j}\frac{q_i q_j}{r_{ij}} = \frac{1}{4\pi\varepsilon_0 d}\left(q_1q_2 + q_1q_3 + q_2q_4 + q_3q_4 + \frac{q_1q_4}{\sqrt{2}} + \frac{q_2q_3}{\sqrt{2}}\right) \\ &= \frac{\left(8.99\times 10^9\ \frac{\mathrm{N\cdot m^2}}{\mathrm{C^2}}\right)}{1.3\,\mathrm{m}}\Bigg[(12)(-24) + (12)(31) + (-24)(17) + (31)(17) \\ &\qquad + \frac{(12)(17)}{\sqrt{2}} + \frac{(-24)(31)}{\sqrt{2}}\Bigg](10^{-19}\,\mathrm{C})^2 \\ &= -1.2\times 10^{-6}\ \mathrm{J} \ . \end{aligned}$$

39. (a) Let $\ell = 0.15\,\mathrm{m}$ be the length of the rectangle and $w = 0.050\,\mathrm{m}$ be its width. Charge q_1 is a distance ℓ from point A and charge q_2 is a distance w, so the electric potential at A is

$$\begin{aligned} V_A &= \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{\ell}+\frac{q_2}{w}\right] \\ &= \left(8.99\times 10^9\,\mathrm{N\cdot m^2/C^2}\right)\left[\frac{-5.0\times 10^{-6}\,\mathrm{C}}{0.15\,\mathrm{m}} + \frac{2.0\times 10^{-6}\,\mathrm{C}}{0.050\,\mathrm{m}}\right] \\ &= 6.0\times 10^4\ \mathrm{V} \ . \end{aligned}$$

(b) Charge q_1 is a distance w from point b and charge q_2 is a distance ℓ, so the electric potential at B is

$$\begin{aligned} V_B &= \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{w}+\frac{q_2}{\ell}\right] \\ &= \left(8.99\times 10^9\,\mathrm{N\cdot m^2/C^2}\right)\left[\frac{-5.0\times 10^{-6}\,\mathrm{C}}{0.050\,\mathrm{m}} + \frac{2.0\times 10^{-6}\,\mathrm{C}}{0.15\,\mathrm{m}}\right] \\ &= -7.8\times 10^5\ \mathrm{V} \ . \end{aligned}$$

(c) Since the kinetic energy is zero at the beginning and end of the trip, the work done by an external agent equals the change in the potential energy of the system. The potential energy is the product of the charge q_3 and the electric potential. If U_A is the potential energy when q_3 is at A and U_B is the potential energy when q_3 is at B, then the work done in moving the charge from B to A is $W = U_A - U_B = q_3(V_A - V_B) = (3.0\times 10^{-6}\,\mathrm{C})(6.0\times 10^4\,\mathrm{V} + 7.8\times 10^5\,\mathrm{V}) = 2.5\,\mathrm{J}$.

(d) The work done by the external agent is positive, so the energy of the three-charge system increases.

(e) and (f) The electrostatic force is conservative, so the work is the same no matter which path is used.

40. The work required is

$$W = \Delta U = \frac{1}{4\pi\varepsilon_0}\left[\frac{(4q)(5q)}{2d} + \frac{(5q)(-2q)}{d}\right] = 0 \ .$$

41. The particle with charge $-q$ has both potential and kinetic energy, and both of these change when the radius of the orbit is changed. We first find an expression for the total energy in terms of the orbit radius r. Q provides the centripetal force required for $-q$ to move in uniform circular motion. The magnitude of the force is $F = Qq/4\pi\varepsilon_0 r^2$. The acceleration of $-q$ is v^2/r, where v is its speed. Newton's second law yields

$$\frac{Qq}{4\pi\varepsilon_0 r^2} = \frac{mv^2}{r} \implies mv^2 = \frac{Qq}{4\pi\varepsilon_0 r} ,$$

and the kinetic energy is $K = \frac{1}{2}mv^2 = Qq/8\pi\varepsilon_0 r$. The potential energy is $U = -Qq/4\pi\varepsilon_0 r$, and the total energy is

$$E = K + U = \frac{Qq}{8\pi\varepsilon_0 r} - \frac{Qq}{4\pi\varepsilon_0 r} = -\frac{Qq}{8\pi\varepsilon_0 r} .$$

When the orbit radius is r_1 the energy is $E_1 = -Qq/8\pi\varepsilon_0 r_1$ and when it is r_2 the energy is $E_2 = -Qq/8\pi\varepsilon_0 r_2$. The difference $E_2 - E_1$ is the work W done by an external agent to change the radius:

$$W = E_2 - E_1 = -\frac{Qq}{8\pi\varepsilon_0}\left(\frac{1}{r_2} - \frac{1}{r_1}\right) = \frac{Qq}{8\pi\varepsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) .$$

42. (a) The potential is

$$\begin{aligned} V(r) &= \frac{1}{4\pi\varepsilon_0}\frac{e}{r} \\ &= \frac{\left(8.99 \times 10^9 \, \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(1.60 \times 10^{-19}\,\text{C})}{5.29 \times 10^{-11}\,\text{m}} = 27.2\ \text{V} . \end{aligned}$$

(b) The potential energy is $U = -eV(r) = -27.2\,\text{eV}$.

(c) Since $m_e v^2/r = -e^2/4\pi\varepsilon_0 r^2$,

$$K = \frac{1}{2}mv^2 = -\frac{1}{2}\left(\frac{e^2}{4\pi\varepsilon_0 r}\right) = -\frac{1}{2}V(r) = \frac{27.2\,\text{eV}}{2} = 13.6\ \text{eV} .$$

(d) The energy required is

$$\Delta E = 0 - [V(r) + K] = 0 - (-27.2\,\text{eV} + 13.6\,\text{eV}) = 13.6\ \text{eV} .$$

43. We use the conservation of energy principle. The initial potential energy is $U_i = q^2/4\pi\varepsilon_0 r_1$, the initial kinetic energy is $K_i = 0$, the final potential energy is $U_f = q^2/4\pi\varepsilon_0 r_2$, and the final kinetic energy is $K_f = \frac{1}{2}mv^2$, where v is the final speed of the particle. Conservation of energy yields

$$\frac{q^2}{4\pi\varepsilon_0 r_1} = \frac{q^2}{4\pi\varepsilon_0 r_2} + \frac{1}{2}mv^2 .$$

The solution for v is

$$\begin{aligned} v &= \sqrt{\frac{2q^2}{4\pi\varepsilon_0 m}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)} \\ &= \sqrt{\frac{(8.99 \times 10^9 \text{N}\cdot\text{m}^2/\text{C}^2)(2)(3.1 \times 10^{-6}\,\text{C})^2}{20 \times 10^{-6}\ \text{kg}}\left(\frac{1}{0.90 \times 10^{-3}\,\text{m}} - \frac{1}{2.5 \times 10^{-3}\,\text{m}}\right)} \\ &= 2.5 \times 10^3\ \text{m/s} . \end{aligned}$$

44. Let $r = 1.5\,\text{m}$, $x = 3.0\,\text{m}$, $q_1 = -9.0\,\text{nC}$, and $q_2 = -6.0\,\text{pC}$. The work done by an external agent is given by

$$\begin{aligned} W &= \Delta U = \frac{q_1 q_2}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{\sqrt{r^2+x^2}}\right) \\ &= (-9.0\times10^{-9}\,\text{C})(-6.0\times10^{-12}\,\text{C})\left(8.99\times10^{9}\,\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\cdot\left[\frac{1}{1.5\,\text{m}} - \frac{1}{\sqrt{(1.5\,\text{m})^2+(3.0\,\text{m})^2}}\right] \\ &= 1.8\times10^{-10}\ \text{J}\ . \end{aligned}$$

45. (a) The potential energy is

$$U = \frac{q^2}{4\pi\varepsilon_0 d} = \frac{(8.99\times10^{9}\,\text{N}\cdot\text{m}^2/\text{C}^2)(5.0\times10^{-6}\,\text{C})^2}{1.00\,\text{m}} = 0.225\ \text{J}$$

relative to the potential energy at infinite separation.

(b) Each sphere repels the other with a force that has magnitude

$$F = \frac{q^2}{4\pi\varepsilon_0 d^2} = \frac{(8.99\times10^{9}\,\text{N}\cdot\text{m}^2/\text{C}^2)(5.0\times10^{-6}\,\text{C})^2}{(1.00\,\text{m})^2} = 0.225\ \text{N}\ .$$

According to Newton's second law the acceleration of each sphere is the force divided by the mass of the sphere. Let m_A and m_B be the masses of the spheres. The acceleration of sphere A is

$$a_A = \frac{F}{m_A} = \frac{0.225\,\text{N}}{5.0\times10^{-3}\,\text{kg}} = 45.0\ \text{m/s}^2$$

and the acceleration of sphere B is

$$a_B = \frac{F}{m_B} = \frac{0.225\,\text{N}}{10\times10^{-3}\,\text{kg}} = 22.5\ \text{m/s}^2\ .$$

(c) Energy is conserved. The initial potential energy is $U = 0.225\,\text{J}$, as calculated in part (a). The initial kinetic energy is zero since the spheres start from rest. The final potential energy is zero since the spheres are then far apart. The final kinetic energy is $\frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2$, where v_A and v_B are the final velocities. Thus,

$$U = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2\ .$$

Momentum is also conserved, so

$$0 = m_A v_A + m_B v_B\ .$$

These equations may be solved simultaneously for v_A and v_B. Substituting $v_B = -(m_A/m_B)v_A$, from the momentum equation into the energy equation, and collecting terms, we obtain $U = \frac{1}{2}(m_A/m_B)(m_A+m_B)v_A^2$. Thus,

$$\begin{aligned} v_A &= \sqrt{\frac{2Um_B}{m_A(m_A+m_B)}} \\ &= \sqrt{\frac{2(0.225\,\text{J})(10\times10^{-3}\,\text{kg})}{(5.0\times10^{-3}\,\text{kg})(5.0\times10^{-3}\,\text{kg}+10\times10^{-3}\,\text{kg})}} = 7.75\ \text{m/s}\ . \end{aligned}$$

We thus obtain

$$v_B = -\frac{m_A}{m_B}v_A = -\left(\frac{5.0\times10^{-3}\,\text{kg}}{10\times10^{-3}\,\text{kg}}\right)(7.75\,\text{m/s}) = -3.87\ \text{m/s}\ .$$

46. The change in electric potential energy of the electron-shell system as the electron starts from its initial position and just reaches the shell is $\Delta U = (-e)(-V) = eV$. Thus from $\Delta U = K = \frac{1}{2}m_e v_i^2$ we find the initial electron speed to be

$$v_i = \sqrt{\frac{2\Delta U}{m_e}} = \sqrt{\frac{2eV}{m_e}} \ .$$

47. We use conservation of energy, taking the potential energy to be zero when the moving electron is far away from the fixed electrons. The final potential energy is then $U_f = 2e^2/4\pi\varepsilon_0 d$, where d is half the distance between the fixed electrons. The initial kinetic energy is $K_i = \frac{1}{2}mv^2$, where m is the mass of an electron and v is the initial speed of the moving electron. The final kinetic energy is zero. Thus $K_i = U_f$ or $\frac{1}{2}mv^2 = 2e^2/4\pi\varepsilon_0 d$. Hence

$$v = \sqrt{\frac{4e^2}{4\pi\varepsilon_0\, d\, m}} = \sqrt{\frac{(8.99\times10^9\,\mathrm{N\cdot m^2/C^2})(4)(1.60\times10^{-19}\,\mathrm{C})^2}{(0.010\,\mathrm{m})(9.11\times10^{-31}\,\mathrm{kg})}} = 3.2\times10^2\ \mathrm{m/s}\ .$$

48. The initial speed v_i of the electron satisfies $K_i = \frac{1}{2}m_e v_i^2 = e\Delta V$, which gives

$$v_i = \sqrt{\frac{2e\Delta V}{m_e}} = \sqrt{\frac{2(1.60\times10^{-19}\,\mathrm{J})(625\,\mathrm{V})}{9.11\times10^{-31}\,\mathrm{kg}}} = 1.48\times10^7\ \mathrm{m/s}\ .$$

49. Let the distance in question be r. The initial kinetic energy of the electron is $K_i = \frac{1}{2}m_e v_i^2$, where $v_i = 3.2\times10^5\,\mathrm{m/s}$. As the speed doubles, K becomes $4K_i$. Thus

$$\Delta U = \frac{-e^2}{4\pi\varepsilon_0 r} = -\Delta K = -(4K_i - K_i) = -3K_i = -\frac{3}{2}m_e v_i^2\ ,$$

or

$$\begin{aligned} r &= \frac{2e^2}{3(4\pi\varepsilon_0)m_e v_i^2} = \frac{2(1.6\times10^{-19}\,\mathrm{C})^2\left(8.99\times10^9\,\frac{\mathrm{N\cdot m^2}}{\mathrm{C^2}}\right)}{3(9.11\times10^{-19}\,\mathrm{kg})(3.2\times10^5\,\mathrm{m/s})^2} \\ &= 1.6\times10^{-9}\ \mathrm{m}\ . \end{aligned}$$

50. Since the electric potential throughout the entire conductor is a constant, the electric potential at its center is also $+400\,\mathrm{V}$.

51. If the electric potential is zero at infinity, then the potential at the surface of the sphere is given by $V = q/4\pi\varepsilon_0 r$, where q is the charge on the sphere and r is its radius. Thus

$$q = 4\pi\varepsilon_0 rV = \frac{(0.15\,\mathrm{m})(1500\,\mathrm{V})}{8.99\times10^9\,\mathrm{N\cdot m^2/C^2}} = 2.5\times10^{-8}\ \mathrm{C}\ .$$

52. (a) Since the two conductors are connected V_1 and V_2 must be the same.

(b) Let $V_1 = q_1/4\pi\varepsilon_0 R_1 = V_2 = q_2/4\pi\varepsilon_0 R_2$ and note that $q_1 + q_2 = q$ and $R_2 = 2R_1$. We solve for q_1 and q_2: $q_1 = q/3$, $q_2 = 2q/3$.

(c) The ratio of surface charge densities is

$$\frac{\sigma_1}{\sigma_2} = \frac{q_1/4\pi R_1^2}{q_2/4\pi R_2^2} = \left(\frac{q_1}{q_2}\right)\left(\frac{R_2}{R_1}\right)^2 = 2\ .$$

53. (a) The electric potential is the sum of the contributions of the individual spheres. Let q_1 be the charge on one, q_2 be the charge on the other, and d be their separation. The point halfway between them is the same distance $d/2$ ($= 1.0\,\mathrm{m}$) from the center of each sphere, so the potential at the halfway point is

$$V = \frac{q_1 + q_2}{4\pi\varepsilon_0 d/2} = \frac{(8.99\times10^9\,\mathrm{N\cdot m^2/C^2})(1.0\times10^{-8}\,\mathrm{C} - 3.0\times10^{-8}\,\mathrm{C})}{1.0\,\mathrm{m}} = -1.80\times10^2\ \mathrm{V}\ .$$

(b) The distance from the center of one sphere to the surface of the other is $d - R$, where R is the radius of either sphere. The potential of either one of the spheres is due to the charge on that sphere and the charge on the other sphere. The potential at the surface of sphere 1 is

$$\begin{aligned} V_1 &= \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{R} + \frac{q_2}{d-R}\right] \\ &= (8.99 \times 10^9\,\mathrm{N \cdot m^2/C^2})\left[\frac{1.0 \times 10^{-8}\,\mathrm{C}}{0.030\,\mathrm{m}} - \frac{3.0 \times 10^{-8}\,\mathrm{C}}{2.0\,\mathrm{m} - 0.030\,\mathrm{m}}\right] \\ &= 2.9 \times 10^3\ \mathrm{V}\ . \end{aligned}$$

The potential at the surface of sphere 2 is

$$\begin{aligned} V_2 &= \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{d-R} + \frac{q_2}{R}\right] \\ &= (8.99 \times 10^9\,\mathrm{N \cdot m^2/C^2})\left[\frac{1.0 \times 10^{-8}\,\mathrm{C}}{2.0\,\mathrm{m} - 0.030\,\mathrm{m}} - \frac{3.0 \times 10^{-8}\,\mathrm{C}}{0.030\,\mathrm{m}}\right] \\ &= -8.9 \times 10^3\ \mathrm{V}\ . \end{aligned}$$

54. (a) The magnitude of the electric field is

$$E = \frac{\sigma}{\varepsilon_0} = \frac{q}{4\pi\varepsilon_0 R^2} = \frac{(3.0 \times 10^{-8}\,\mathrm{C})\left(8.99 \times 10^9\,\frac{\mathrm{N \cdot m^2}}{\mathrm{C^2}}\right)}{(0.15\,\mathrm{m})^2} = 1.2 \times 10^4\,\mathrm{N/C}\ .$$

(b) $V = RE = (0.15\,\mathrm{m})(1.2 \times 10^4\,\mathrm{N/C}) = 1.8 \times 10^3\,\mathrm{V}$.

(c) Let the distance be x. Then

$$\Delta V = V(x) - V = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{R+x} - \frac{1}{R}\right) = -500\,\mathrm{V}\ ,$$

which gives

$$x = \frac{R\Delta V}{-V - \Delta V} = \frac{(0.15\,\mathrm{m})(-500\,\mathrm{V})}{-1800\,\mathrm{V} + 500\,\mathrm{V}} = 5.8 \times 10^{-2}\ \mathrm{m}\ .$$

55. (a) The potential would be

$$\begin{aligned} V_e &= \frac{Q_e}{4\pi\varepsilon_0 R_e} = \frac{4\pi R_e^2\sigma_e}{4\pi\varepsilon_0 R_e} = 4\pi R_e\sigma_e k \\ &= 4\pi(6.37 \times 10^6\,\mathrm{m})(1.0\,\mathrm{electron/\,m^2})(-1.6 \times 10^{-19}\,\mathrm{C/\,electron})\left(8.99 \times 10^9\,\frac{\mathrm{N \cdot m^2}}{\mathrm{C^2}}\right) \\ &= -0.12\ \mathrm{V}\ . \end{aligned}$$

(b) The electric field is

$$E = \frac{\sigma_e}{\varepsilon_0} = \frac{V_e}{R_e} = -\frac{0.12\,\mathrm{V}}{6.37 \times 10^6\,\mathrm{m}} = -1.8 \times 10^{-8}\ \mathrm{N/C}\ ,$$

where the minus sign indicates that $\vec{E}$ is radially inward.

56. Since the charge distribution is spherically symmetric we may write

$$E(r) = \frac{1}{4\pi\varepsilon_0}\frac{q_{\mathrm{encl}}}{r}\ ,$$

where q_{encl} is the charge enclosed in a sphere of radius r centered at the origin. Also, Eq. 25-18 is implemented in the form: $V(r) - V(r') = \int_r^{r'} E(r)\,dr$. The results are as follows: For $r > R_2 > R_1$

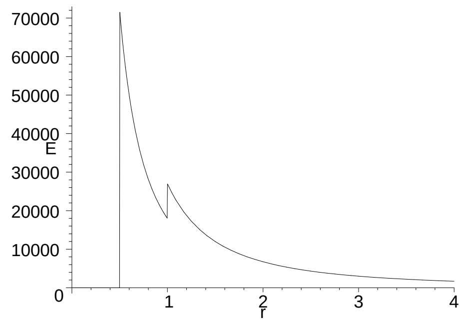
$$V(r) = \frac{q_1 + q_2}{4\pi\varepsilon_0 r} \quad \text{and} \quad E(r) = \frac{q_1 + q_2}{4\pi\varepsilon_0 r^2} \ .$$

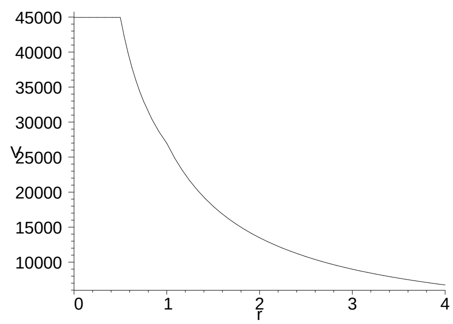
For $R_2 > r > R_1$

$$V(r) = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r} + \frac{q_2}{R_2}\right) \quad \text{and} \quad E(r) = \frac{q_1}{4\pi\varepsilon_0 r^2} \ .$$

Finally, for $R_2 > R_1 > r$

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{R_1} + \frac{q_2}{R_2}\right) \quad \text{and} \quad E = 0 \ .$$





57. (a) We use Eq. 25-18 to find the potential:

$$\begin{aligned} V_{\text{wall}} - V &= -\int_r^R E\,dr \\ 0 - V &= -\int_r^R \left(\frac{\rho r}{2\varepsilon_0}\right) \\ -V &= -\frac{\rho}{4\varepsilon_0}\left(R^2 - r^2\right) \ . \end{aligned}$$

Consequently, $V = \frac{\rho}{4\varepsilon_0}\left(R^2 - r^2\right)$.

(b) The value at $r = 0$ is

$$V_{\text{center}} = \frac{-1.1 \times 10^{-3}\,\text{C/m}^3}{4\,(8.85 \times 10^{-12}\,\text{C/V}{\cdot}\text{m})}\left((0.05\,\text{m})^2 - 0\right) = -7.8 \times 10^4 \text{ V} \ .$$

58. We treat the system as a superposition of a disk of surface charge density σ and radius R and a smaller, oppositely charged, disk of surface charge density $-\sigma$ and radius r. For each of these, Eq 25-37 applies (for $z > 0$)

$$V = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{z^2 + R^2} - z\right) + \frac{-\sigma}{2\varepsilon_0}\left(\sqrt{z^2 + r^2} - z\right) \ .$$

This expression does vanish as $r \to \infty$, as the problem requires. Substituting $r = R/5$ and $z = 2R$ and simplifying, we obtain

$$V = \frac{\sigma R}{\varepsilon_0}\left(\frac{5\sqrt{5} - \sqrt{101}}{10}\right) \approx \frac{\sigma R}{\varepsilon_0}(0.113) \ .$$

59. We use $q = 1.37 \times 10^5$ C from Sample Problem 22-7 and $k = 1/4\pi\varepsilon_0$ to find the potential:

$$V = \frac{q}{4\pi\varepsilon_0 R_e} = \frac{(1.37 \times 10^5\,\text{C})\left(8.99 \times 10^9\,\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)}{6.37 \times 10^6\,\text{m}} = 1.93 \times 10^8 \text{ V} \ .$$

60. (a) The potential on the surface is

$$V = \frac{q}{4\pi\varepsilon_0 R} = \frac{\left(4.0 \times 10^{-6}\,\text{C}\right)\left(8.99 \times 10^9\,\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)}{0.10\,\text{m}} = 3.6 \times 10^5\ \text{V}\ .$$

(b) The field just outside the sphere would be

$$E = \frac{q}{4\pi\varepsilon_0 R^2} = \frac{V}{R} = \frac{3.6 \times 10^5\,\text{V}}{0.10\,\text{m}} = 3.6 \times 10^6\ \text{V/m}\ ,$$

which would have exceeded 3.0 MV/m. So this situation cannot occur.

61. If the electric potential is zero at infinity then at the surface of a uniformly charged sphere it is $V = q/4\pi\varepsilon_0 R$, where q is the charge on the sphere and R is the sphere radius. Thus $q = 4\pi\varepsilon_0 RV$ and the number of electrons is

$$N = \frac{|q|}{e} = \frac{4\pi\varepsilon_0 R|V|}{e} = \frac{(1.0 \times 10^{-6}\,\text{m})(400\text{V})}{(8.99 \times 10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(1.60 \times 10^{-19}\,\text{C})} = 2.8 \times 10^5\ .$$

62. This can be approached more than one way, but the simplest is to observe that the net potential (using Eq. 25-27) due to the $+2q$ and $-2q$ charges is zero at both the initial and final positions of the movable charge ($+5q$). This implies that no work is necessary to effect its change of position, which, in turn, implies there is no resulting change in potential energy of the configuration. Hence, the ratio is unity.

63. We imagine moving all the charges on the surface of the sphere to the center of the the sphere. Using Gauss' law, we see that this would not change the electric field *outside* the sphere. The magnitude of the electric field E of the uniformly charged sphere as a function of r, the distance from the center of the sphere, is thus given by $E(r) = q/(4\pi\varepsilon_0 r^2)$ for $r > R$. Here R is the radius of the sphere. Thus, the potential V at the surface of the sphere (where $r = R$) is given by

$$\begin{aligned} V(R) &= V\Big|_{r=\infty} + \int_R^\infty E(r)\,dr = \int_\infty^R \frac{q}{4\pi\varepsilon_0 r^2}\,dr = \frac{q}{4\pi\varepsilon_0 R} \\ &= \frac{\left(8.99 \times 10^9\,\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(1.50 \times 10^8\,\text{C}\right)}{0.160\,\text{m}} = 8.43 \times 10^2\ \text{V}\ . \end{aligned}$$

64. We use $E_x = -dV/dx$, where dV/dx is the local slope of the V vs. x curve depicted in Fig. 25-54. The results are: $E_x(ab) = -6.0\,\text{V/m}$, $E_x(bc) = 0$, $E_x(cd) = E_x(de) = 3.0\,\text{V/m}$, $E_x(ef) = 15\,\text{V/m}$, $E_x(fg) = 0$, $E_x(gh) = -3.0\,\text{V/m}$. Since these values are constant during their respective time-intervals, their graph consists of several disconnected line-segments (horizontal) and is not shown here in the interest of saving space.

65. On the dipole axis $\theta = 0$ or π, so $|\cos\theta| = 1$. Therefore, magnitude of the electric field is

$$|E(r)| = \left|-\frac{\partial V}{\partial r}\right| = \frac{p}{4\pi\varepsilon_0}\left|\frac{d}{dr}\left(\frac{1}{r^2}\right)\right| = \frac{p}{2\pi\varepsilon_0 r^3}\ .$$

66. (a) We denote the surface charge density of the disk as σ_1 for $0 < r < R/2$, and as σ_2 for $R/2 < r < R$. Thus the total charge on the disk is given by

$$\begin{aligned} q &= \int_{\text{disk}} dq = \int_0^{R/2} 2\pi\sigma_1 r\,dr + \int_{R/2}^R 2\pi\sigma_2 r\,dr = \frac{\pi}{4}R^2(\sigma_1 + 3\sigma_2) \\ &= \frac{\pi}{4}(2.20 \times 10^{-2}\,\text{m})^2[1.50 \times 10^{-6}\,\text{C/m}^2 + 3(8.00 \times 10^{-7}\,\text{C/m}^2)] \\ &= 1.48 \times 10^{-9}\,\text{C}\ . \end{aligned}$$

(b) We use Eq. 25-36:

$$\begin{aligned} V(z) &= \int_{\text{disk}} dV = k\left[\int_0^{R/2} \frac{\sigma_1(2\pi R')dR'}{\sqrt{z^2+R'^2}} + \int_{R/2}^{R} \frac{\sigma_2(2\pi R')dR'}{\sqrt{z^2+R'^2}}\right] \\ &= \frac{\sigma_1}{2\varepsilon_0}\left(\sqrt{z^2+\frac{R^2}{4}}-z\right) + \frac{\sigma_2}{2\varepsilon_0}\left(\sqrt{z^2+R^2}-\sqrt{z^2+\frac{R^2}{4}}\right) . \end{aligned}$$

Substituting the numerical values of σ_1, σ_2, R and z, we obtain $V(z) = 7.95 \times 10^2$ V.

67. From the previous chapter, we know that the radial field due to an infinite line-source is

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

which integrates, using Eq. 25-18, to obtain

$$V_i = V_f + \frac{\lambda}{2\pi\varepsilon_0}\int_{r_i}^{r_f} \frac{dr}{r} = V_f + \frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{r_f}{r_i}\right) .$$

The subscripts i and f are somewhat arbitrary designations, and we let $V_i = V$ be the potential of some point P at a distance $r_i = r$ from the wire and $V_f = V_\text{o}$ be the potential along some reference axis (which will be the z axis described in this problem) at a distance $r_f = a$ from the wire. In the "end-view" presented below, the wires and the z axis appear as points as they intersect the xy plane. The potential due to the wire on the left (intersecting the plane at $x = -a$) is

$$V_{\text{negative wire}} = V_\text{o} + \frac{(-\lambda)}{2\pi\varepsilon_0}\ln\left(\frac{a}{\sqrt{(x+a)^2+y^2}}\right) ,$$

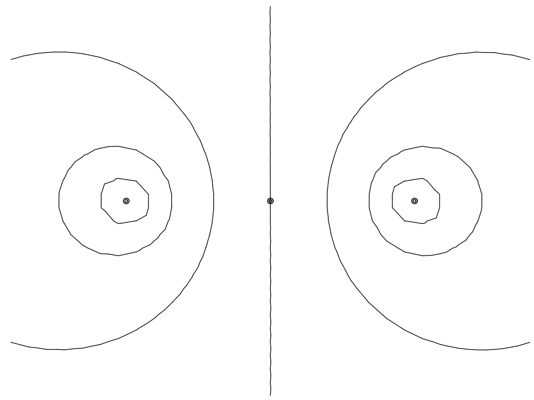
and the potential due to the wire on the right (intersecting the plane at $x = +a$) is

$$V_{\text{positive wire}} = V_\text{o} + \frac{(+\lambda)}{2\pi\varepsilon_0}\ln\left(\frac{a}{\sqrt{(x-a)^2+y^2}}\right) .$$

Since potential is a scalar quantity, the net potential at point P is the addition of $V_{-\lambda}$ and $V_{+\lambda}$ which simplifies to

$$V_{\text{net}} = 2V_\text{o} + \frac{\lambda}{2\pi\varepsilon_0}\left(\ln\left(\frac{a}{\sqrt{(x-a)^2+y^2}}\right) - \ln\left(\frac{a}{\sqrt{(x+a)^2+y^2}}\right)\right) = \frac{\lambda}{4\pi\varepsilon_0}\ln\left(\frac{(x+a)^2+y^2}{(x-a)^2+y^2}\right)$$

where we have set the potential along the z axis equal to zero ($V_\text{o} = 0$) in the last step (which we are free to do). This is the expression used to obtain the equipotentials shown below. The center dot in the figure is the intersection of the z axis with the xy plane, and the dots on either side are the intersections of the wires with the plane.



68. The potential difference is $\Delta V = E\Delta s = (1.92 \times 10^5\,\text{N/C})(0.0150\,\text{m}) = 2.90 \times 10^3\,\text{V}$.

69. Since the charge distribution on the arc is equidistant from the point where V is evaluated, its contribution is identical to that of a point charge at that distance. We assume $V \to 0$ as $r \to \infty$ and apply Eq. 25-27:

$$V = \frac{1}{4\pi\varepsilon_0}\frac{+Q}{R} + \frac{1}{4\pi\varepsilon_0}\frac{+4Q}{2R} + \frac{1}{4\pi\varepsilon_0}\frac{-2Q}{R}$$

which simplifies to $Q/4\pi\varepsilon_0 R$.

70. From the previous chapter, we know that the radial field due to an infinite line-source is

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

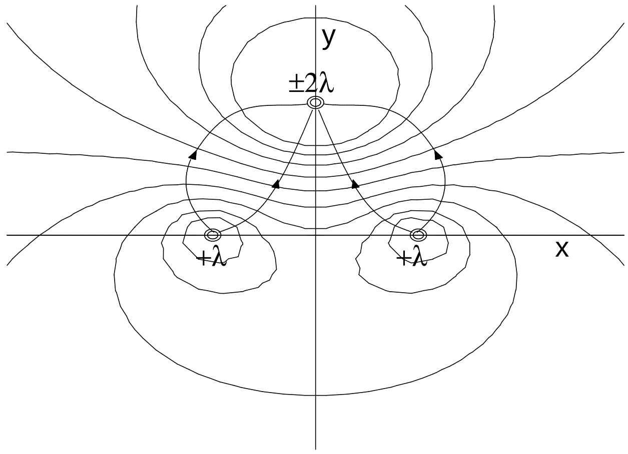
which integrates, using Eq. 25-18, to obtain

$$V_i = V_f + \frac{\lambda}{2\pi\varepsilon_0}\int_{r_i}^{r_f}\frac{dr}{r} = V_f + \frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{r_f}{r_i}\right) .$$

The subscripts i and f are somewhat arbitrary designations, and we let $V_i = V$ be the potential of some point P at a distance $r_i = r$ from the wire and $V_f = V_\text{o}$ be the potential along some reference axis (which intersects the plane of our figure, shown below, at the xy coordinate origin, placed midway between the bottom two line charges – that is, the midpoint of the bottom side of the equilateral triangle) at a distance $r_f = a$ from each of the bottom wires (and a distance $a\sqrt{3}$ from the topmost wire). Thus, each side of the triangle is of length $2a$. Skipping some steps, we arrive at an expression for the net potential created by the three wires (where we have set $V_\text{o} = 0$):

$$V_\text{net} = \frac{\lambda}{4\pi\varepsilon_0}\ln\left(\frac{(x^2 + (y - a\sqrt{3})^2)^2}{((x+a)^2 + y^2)((x-a)^2 + y^2)}\right)$$

which forms the basis of our contour plot shown below. On the same plot we have shown four electric field lines, which have been sketched (as opposed to rigorously calculated) and are not meant to be as accurate as the equipotentials. The $\pm 2\lambda$ by the top wire in our figure should be -2λ (the $\pm$ typo is an artifact of our plotting routine).



71. The charges are equidistant from the point where we are evaluating the potential – which is computed using Eq. 25-27 (or its integral equivalent). Eq. 25-27 implicitly assumes $V \to 0$ as $r \to \infty$. Thus, we have

$$V = \frac{1}{4\pi\varepsilon_0}\frac{+Q}{R} + \frac{1}{4\pi\varepsilon_0}\frac{-2Q}{R} + \frac{1}{4\pi\varepsilon_0}\frac{+3Q}{R}$$

which simplifies to $Q/2\pi\varepsilon_0 R$.

72. The radius of the cylinder (0.020 m, the same as r_B) is denoted R, and the field magnitude there (160 N/C) is denoted E_B. The electric field beyond the surface of the sphere follows Eq. 24-12, which expresses inverse proportionality with r:

$$\frac{|\vec{E}|}{E_B} = \frac{R}{r} \qquad \text{for } r \geq R\ .$$

(a) Thus, if $r = r_C = 0.050$ m, we obtain $|\vec{E}| = (160)(0.020)/(0.050) = 64$ N/C.

(b) Integrating the above expression (where the variable to be integrated, r, is now denoted ϱ) gives the potential difference between V_B and V_C.

$$V_B - V_C = \int_R^r \frac{E_B\, R}{\varrho}\, d\varrho = E_B\, R\, \ln\left(\frac{r}{R}\right) = 2.9 \text{ V}\ .$$

(c) The electric field throughout the conducting volume is zero, which implies that the potential there is constant and equal to the value it has on the surface of the charged cylinder: $V_A - V_B = 0$.

73. The net potential (at point A or B) is computed using Eq. 25-27. Thus, using k for $1/4\pi\varepsilon_0$, the difference is

$$\begin{aligned} V_B - V_A &= \left(\frac{kq}{2d} + \frac{k(-5q)}{2d}\right) - \left(\frac{kq}{d} + \frac{k(-5q)}{5d}\right) \\ &= -\frac{4kq}{2d} \end{aligned}$$

which simplifies to $-q/2\pi\varepsilon_0$ in SI units (with $d = 1$ m).

74. Eq. 25-32 applies with $dq = \lambda\, dx = bx\, dx$ (along $0 \leq x \leq 0.20$ m).

(a) Here $r = x > 0$, so that

$$V = \frac{1}{4\pi\varepsilon_0}\int_0^{0.20} \frac{bx\,dx}{x} = \frac{b(0.20)}{4\pi\varepsilon_0}$$

which yields $V = 36$ V.

(b) Now $r = \sqrt{x^2 + d^2}$ where $d = 0.15$ m, so that

$$V = \frac{1}{4\pi\varepsilon_0}\int_0^{0.20} \frac{bx\,dx}{\sqrt{x^2+d^2}} = \frac{b}{4\pi\varepsilon_0}\left(\sqrt{x^2+d^2}\right)\Big|_0^{0.20}$$

which yields $V = 18$ V.

75. (a) Using Eq. 25-26, we calculate the radius r of the sphere representing the 30 V equipotential surface:

$$r = \frac{q}{4\pi\varepsilon_0 V} = 4.5 \text{ m} .$$

(b) If the potential were a linear function of r then it would have equally spaced equipotentials, but since $V \propto 1/r$ they are spaced more and more widely apart as r increases.

76. We denote $q = 25 \times 10^{-9}$ C, $y = 0.6$ m, $x = 0.8$ m, with $V =$ the net potential (assuming $V \to 0$ as $r \to \infty$). Then,

$$\begin{aligned} V_A &= \frac{1}{4\pi\varepsilon_0}\frac{q}{y} + \frac{1}{4\pi\varepsilon_0}\frac{(-q)}{x} \\ V_B &= \frac{1}{4\pi\varepsilon_0}\frac{q}{x} + \frac{1}{4\pi\varepsilon_0}\frac{(-q)}{y} \end{aligned}$$

leads to

$$V_B - V_A = \frac{2}{4\pi\varepsilon_0}\frac{q}{x} - \frac{2}{4\pi\varepsilon_0}\frac{q}{y} = \frac{q}{2\pi\varepsilon_0}\left(\frac{1}{x} - \frac{1}{y}\right)$$

which yields $\Delta V = -187 \approx -190$ V.

77. (a) By Eq. 25-18, the change in potential is the negative of the "area" under the curve. Thus, using the area-of-a-triangle formula, we have

$$V - 10 = -\int_0^{x=2} \vec{E}\cdot d\vec{s} = \frac{1}{2}(2)(20)$$

which yields $V = 30$ V.

(b) For any region within $0 < x < 3$ m, $-\int \vec{E}\cdot d\vec{s}$ is positive, but for any region for which $x > 3$ m it is negative. Therefore, $V = V_{\max}$ occurs at $x = 3$ m.

$$V - 10 = -\int_0^{x=3} \vec{E}\cdot d\vec{s} = \frac{1}{2}(3)(20)$$

which yields $V_{\max} = 40$ V.

(c) In view of our result in part (b), we see that now (to find $V = 0$) we are looking for some $X > 3$ m such that the "area" from $x = 3$ m to $x = X$ is 40 V. Using the formula for a triangle ($3 < x < 4$) and a rectangle ($4 < x < X$), we require

$$\frac{1}{2}(1)(20) + (X-4)(20) = 40 \quad .$$

Therefore, $X = 5.5$ m.

78. In the "inside" region between the plates, the individual fields (given by Eq. 24.13) are in the same direction $(-\hat{\imath})$:

$$\vec{E}_{\rm in} = -\left(\frac{50\times 10^{-9}}{2\varepsilon_0} + \frac{25\times 10^{-9}}{2\varepsilon_0}\right)\hat{\imath} = -4.2\times 10^3\,\hat{\imath}$$

in SI units (N/C or V/m). And in the "outside" region where $x > 0.5$ m, the individual fields point in opposite directions:

$$\vec{E}_{\rm out} = -\frac{50\times 10^{-9}}{2\varepsilon_0}\hat{\imath} + \frac{25\times 10^{-9}}{2\varepsilon_0}\hat{\imath} = -1.4\times 10^3\,\hat{\imath}\ .$$

Therefore, by Eq. 25-18, we have

$$\begin{aligned}\Delta V = -\int_0^{0.8}\vec{E}\cdot d\vec{s} &= -\int_0^{0.5}|\vec{E}|_{\rm in}\,dx - \int_{0.5}^{0.8}|\vec{E}|_{\rm out}\,dx \\ &= -\left(4.2\times 10^3\right)(0.5) - \left(1.4\times 10^3\right)(0.3) \\ &= 2.5\times 10^3\ \text{V}\ .\end{aligned}$$

79. We connect A to the origin with a line along the y axis, along which there is no change of potential (Eq. 25-18: $\int \vec{E}\cdot d\vec{s} = 0$). Then, we connect the origin to B with a line along the x axis, along which the change in potential is

$$\Delta V = -\int_0^{x=4}\vec{E}\cdot d\vec{s} = -4.00\int_0^4 x\,dx = -4.00\left(\frac{4^2}{2}\right)$$

which yields $V_B - V_A = -32$ V.

80. (a) The charges are equal and are the same distance from C. We use the Pythagorean theorem to find the distance $r = \sqrt{(d/2)^2 + (d/2)^2} = d/\sqrt{2}$. The electric potential at C is the sum of the potential due to the individual charges but since they produce the same potential, it is twice that of either one:

$$\begin{aligned}V &= \frac{2q}{4\pi\varepsilon_0}\frac{\sqrt{2}}{d} = \frac{2\sqrt{2}q}{4\pi\varepsilon_0 d} \\ &= \frac{(8.99\times 10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(2)\sqrt{2}(2.0\times 10^{-6}\,\text{C})}{0.020\,\text{m}} = 2.54\times 10^6\ \text{V}\ .\end{aligned}$$

(b) As you move the charge into position from far away the potential energy changes from zero to qV, where V is the electric potential at the final location of the charge. The change in the potential energy equals the work you must do to bring the charge in:

$$W = qV = \left(2.0\times 10^{-6}\,\text{C}\right)\left(2.54\times 10^6\,\text{V}\right) = 5.1\ \text{J}\ .$$

(c) The work calculated in part (b) represents the potential energy of the interactions between the charge brought in from infinity and the other two charges. To find the total potential energy of the three-charge system you must add the potential energy of the interaction between the fixed charges. Their separation is d so this potential energy is $q^2/4\pi\varepsilon_0 d$. The total potential energy is

$$\begin{aligned}U &= W + \frac{q^2}{4\pi\varepsilon_0 d} \\ &= 5.1\,\text{J} + \frac{(8.99\times 10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(2.0\times 10^{-6}\,\text{C})^2}{0.020\,\text{m}} = 6.9\ \text{J}\ .\end{aligned}$$

81. (a) Let the quark-quark separation be r. To "naturally" obtain the eV unit, we only plug in for one of the e values involved in the computation:

$$\begin{aligned} U_{\text{up-up}} &= \frac{1}{4\pi\varepsilon_0}\frac{\left(\frac{2e}{3}\right)\left(\frac{2e}{3}\right)}{r} = \frac{4ke}{9r}e \\ &= \frac{4\left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(1.60\times10^{-19}\ \text{C}\right)}{9\left(1.32\times10^{-15}\ \text{m}\right)}e \\ &= 4.84\times10^5\ \text{eV} = 0.484\ \text{MeV} . \end{aligned}$$

(b) The total consists of all pair-wise terms:

$$U = \frac{1}{4\pi\varepsilon_0}\left[\frac{\left(\frac{2e}{3}\right)\left(\frac{2e}{3}\right)}{r} + \frac{\left(\frac{-e}{3}\right)\left(\frac{2e}{3}\right)}{r} + \frac{\left(\frac{-e}{3}\right)\left(\frac{2e}{3}\right)}{r}\right] = 0 .$$

82. (a) At the smallest center-to-center separation r_{min} the initial kinetic energy K_i of the proton is entirely converted to the electric potential energy between the proton and the nucleus. Thus,

$$K_i = \frac{1}{4\pi\varepsilon_0}\frac{eq_{\text{lead}}}{r_{\text{min}}} = \frac{82e^2}{4\pi\varepsilon_0 r_{\text{min}}} .$$

In solving for r_{min} using the eV unit, we note that a factor of e cancels in the middle line:

$$\begin{aligned} r_{\text{min}} &= \frac{82e^2}{4\pi\varepsilon_0 K_i} = k\frac{82e^2}{4.80\times10^6\ e\text{V}} \\ &= \left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{82(1.6\times10^{-19}\ \text{C})}{4.80\times10^6\ \text{V}} \\ &= 2.5\times10^{-14}\ \text{m} = 25\ \text{fm} . \end{aligned}$$

It is worth recalling that a volt is a Newton·meter/Coulomb, in making sense of the above manipulations.

(b) An alpha particle has 2 protons (as well as 2 neutrons). Therefore, using r'_{min} for the new separation, we find

$$K_i = \frac{1}{4\pi\varepsilon_0}\frac{q_\alpha q_{\text{lead}}}{r'_{\text{min}}} = 2\left(\frac{82e^2}{4\pi\varepsilon_0 r'_{\text{min}}}\right) = \frac{82e^2}{4\pi\varepsilon_0 r_{\text{min}}}$$

which leads to $r'_{\text{min}} = 2r_{\text{min}} = 50\,\text{fm}$.

83. The potential energy of the two-charge system is

$$\begin{aligned} U &= \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1q_2}{\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}}\right] \\ &= \frac{\left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(3.0\times10^{-6}\ \text{C})(-4.0\times10^{-6}\ \text{C})}{\sqrt{(3.5+2.0)^2+(0.50-1.5)^2}\ \text{cm}} \end{aligned}$$

$$= -1.9\ \text{J} .$$

Thus, $-1.9\,\text{J}$ of work is needed.

84. For a point on the axis of the ring the potential (assuming $V\to0$ as $r\to\infty$) is

$$V = \frac{q}{4\pi\varepsilon_0\sqrt{z^2+R^2}}$$

where $q = 16\times10^{-6}$ C and $R = 0.030$ m. Therefore,

$$V_B - V_A = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{\sqrt{z_B^2+R^2}} - \frac{1}{R}\right)$$

where $z_B = 0.040$ m. The result is -1.92×10^6 V.

85. We apply Eq. 25-41:

$$\begin{aligned} E_x &= -\frac{\partial V}{\partial x} = -2yz^2 \\ E_y &= -\frac{\partial V}{\partial y} = -2xz^2 \\ E_z &= -\frac{\partial V}{\partial z} = -4xyz \end{aligned}$$

which, at $(x, y, z) = (3, -2, 4)$, gives $(E_x, E_y, E_z) = (64, -96, 96)$ in SI units. The magnitude of the field is therefore

$$\left|\vec{E}\right| = \sqrt{E_x^2 + E_y^2 + E_z^2} = 150 \text{ V/m} = 150 \text{ N/C} .$$

86. We note that for two points on a circle, separated by angle θ (in radians), the direct-line distance between them is $r = 2R\sin(\theta/2)$. Using this fact, distinguishing between the cases where $N =$ odd and $N =$ even, and counting the pair-wise interactions very carefully, we arrive at the following results for the total potential energies. We use $k = 1/4\pi\varepsilon_0$. For configuration 1 (where all N electrons are on the circle), we have

$$\begin{aligned} U_{1,N=\text{even}} &= \frac{Nke^2}{2R}\left(\sum_{j=1}^{\frac{N}{2}-1}\frac{1}{\sin(j\theta/2)} + \frac{1}{2}\right) \\ U_{1,N=\text{odd}} &= \frac{Nke^2}{2R}\left(\sum_{j=1}^{\frac{N-1}{2}}\frac{1}{\sin(j\theta/2)}\right) \end{aligned}$$

where $\theta = \frac{2\pi}{N}$. For configuration 2, we find

$$\begin{aligned} U_{2,N=\text{even}} &= \frac{(N-1)ke^2}{2R}\left(\sum_{j=1}^{\frac{N}{2}-1}\frac{1}{\sin(j\theta'/2)} + 2\right) \\ U_{2,N=\text{odd}} &= \frac{(N-1)ke^2}{2R}\left(\sum_{j=1}^{\frac{N-3}{2}}\frac{1}{\sin(j\theta'/2)} + \frac{5}{2}\right) \end{aligned}$$

where $\theta' = \frac{2\pi}{N-1}$. The results are all of the form

$$U_{1 \text{ or } 2} = \frac{ke^2}{2R} \times \text{ a pure number} .$$

In our table, below, we have the results for those "pure numbers" as they depend on N and on which configuration we are considering. The values listed in the U rows are the potential energies divided by $ke^2/2R$.

N	4	5	6	7	8	9	10	11	12	13	14	15
U_1	3.83	6.88	10.96	16.13	22.44	29.92	38.62	48.58	59.81	72.35	86.22	101.5
U_2	4.73	7.83	11.88	16.96	23.13	30.44	39.92	48.62	59.58	71.81	85.35	100.2

We see that the potential energy for configuration 2 is greater than that for configuration 1 for $N < 12$, but for $N \geq 12$ it is configuration 1 that has the greatest potential energy.

(a) Configuration 1 has the smallest U for $2 \leq N \leq 11$, and configuration 2 has the smallest U for $12 \leq N \leq 15$.

(b) $N = 12$ is the smallest value such that $U_2 < U_1$.

(c) For $N = 12$, configuration 2 consists of 11 electrons distributed at equal distances around the circle, and one electron at the center. A specific electron e_0 on the circle is R distance from the one in the center, and is

$$r = 2R\sin\left(\frac{\pi}{11}\right) \quad \approx \quad 0.56R$$

distance away from its nearest neighbors on the circle (of which there are two – one on each side). Beyond the nearest neighbors, the next nearest electron on the circle is

$$r = 2R\sin\left(\frac{2\pi}{11}\right) \quad \approx \quad 1.1R$$

distance away from e_0. Thus, we see that there are only two electrons closer to e_0 than the one in the center.

87. (First problem of **Cluster**)

(a) The field between the plates is uniform; we apply Eq. 25-42 to find the magnitude of the (horizontal) field: $|\vec{E}| = \Delta V/D$ (assuming $\Delta V > 0$). This produces a horizontal acceleration from Eq. 23-1 and Newton's second law (applied along the x axis):

$$a_x = \frac{|\vec{F}_x|}{m} = \frac{q|\vec{E}|}{m} = \frac{q\Delta V}{m\,D}$$

where $q > 0$ has been assumed; the problem indicates that the acceleration is rightward, which constitutes our choice for the $+x$ direction. If we choose upward as the $+y$ direction then $a_y = -g$, and we apply the free-fall equations of Chapter 2 to the y motion while applying the constant (a_x) acceleration equations of Table 2-1 to the x motion. The displacement is defined by $\Delta x = +D/2$ and $\Delta y = -d$, and the initial velocity is zero. Simultaneous solution of

$$\begin{aligned} \Delta x &= v_{0\mathrm{x}}\,t + \frac{1}{2}\,a_x\,t^2 \qquad \text{and} \\ \Delta y &= v_{0\mathrm{y}}\,t + \frac{1}{2}\,a_y\,t^2 \ , \end{aligned}$$

leads to

$$d = \frac{gD}{2a_x} = \frac{gmD^2}{2q\Delta V} \ .$$

(b) We can continue along the same lines as in part (a) (using Table 2-1) to find v, or we can use energy conservation – which we feel is more instructive. The gain in kinetic energy derives from two potential energy changes: from gravity comes mgd and from electric potential energy comes $q|\vec{E}|\Delta x = q\Delta V/2$. Consequently,

$$\frac{1}{2}mv^2 = mgd + \frac{1}{2}q\Delta V$$

which (upon using the expression for d above) yields

$$v = \sqrt{\frac{mg^2D^2}{q\Delta V} + \frac{q\Delta V}{m}} \ .$$

(c) and (d) Using SI units (so $q = 1.0 \times 10^{-10}$ C, $m = 1.0 \times 10^{-9}$ kg) we plug into our results to obtain $d = 0.049$ m and $v = 1.4$ m/s.

88. (Second problem of **Cluster**)

(a) We argue by symmetry that of the total potential energy in the initial configuration, a third converts into the kinetic energy of each of the particles. And, because the total potential energy consists of three equal contributions

$$U = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{d}$$

then any of the particle's final kinetic energy is equal to this U. Therefore, using k for$1/4\pi\varepsilon_0$, we obtain

$$v = \sqrt{\frac{2U}{m}} = |q|\sqrt{\frac{2k}{m\,d}} \quad .$$

(b) In this case, two of the U contributions to the total potential energy are converted into a single kinetic term:

$$v = \sqrt{\frac{2(2U)}{m}} = 2|q|\sqrt{\frac{k}{m\,d}} \quad .$$

(c) Now it is clear that the one remaining U contribution is converted into a particle's kinetic energy:

$$v = \sqrt{\frac{2U}{m}} = |q|\sqrt{\frac{2k}{m\,d}} \quad .$$

(d) This leaves no potential energy to convert into kinetic for the last particle that is released. It maintains zero speed.

89. (Third problem of **Cluster**)

(a) By momentum conservation we see that their final speeds are the same. We use energy conservation (where the "final" subscript refers to when they are infinitely far away from each other):

$$\begin{aligned} U_i &= K_f \\ \frac{1}{4\pi\varepsilon_0}\frac{2Q^2}{D} &= 2\left(\frac{1}{2}mv^2\right) \end{aligned}$$

which (using $k = 1/4\pi\varepsilon_0$) yields

$$v = |Q|\sqrt{\frac{2k}{m\,D}} \quad .$$

(b) As noted above, this result is the same as that of part (a).

(c) We use energy conservation (where the "final" subscript refers to when their surfaces have made contact):

$$\begin{aligned} U_i &= K_f + U_f \\ \frac{1}{4\pi\varepsilon_0}\frac{-2Q^2}{D} &= 2\left(\frac{1}{2}mv^2\right) + \frac{1}{4\pi\varepsilon_0}\frac{-2Q^2}{2r} \end{aligned}$$

which (using $k = 1/4\pi\varepsilon_0$) yields

$$v = |Q|\sqrt{\frac{k}{m\,r} - \frac{2k}{m\,D}} \approx |Q|\sqrt{\frac{k}{m\,r}}$$

since $r \ll D$.

(d) As before, the speeds of the particles are equal (by momentum conservation).

(e) and (f) The collision being elastic means no kinetic energy is lost (or gained), so they are able to return to their original positions (climbing back up that potential "hill") whereupon their potential energy is again U_i and their kinetic energies (hence, speeds) are zero.

90. (Fourth problem of **Cluster**)

(a) At its displaced position, its potential energy (using $k = 1/4\pi\varepsilon_0$) is

$$U_i = k\frac{qQ}{d - x_0} + k\frac{qQ}{d + x_0} \quad = \quad \frac{2kqQd}{d^2 - x_0^2} \ .$$

And at A, the potential energy is

$$U_A = 2\left(k\frac{qQ}{d}\right) \ .$$

Setting this difference equal to the kinetic energy of the particle ($\frac{1}{2}mv^2$) and solving for the speed yields

$$v = \sqrt{\frac{2\,(U_i - U_A)}{m}} \quad = \quad 2\,x_0\sqrt{\frac{k\,q\,Q}{m\,d\,(d^2 - x_0^2)}} \ .$$

(b) It is straightforward to consider small x_0 (more precisely, $x_0/d \ll 1$) in the above expression (so that $d^2 - x_0^2 \approx d^2$). The result is

$$v \approx 2\,\frac{x_0}{d}\sqrt{\frac{k\,q\,Q}{m\,d}} \ .$$

(c) Plugging in the given values (converted to SI units) yields $v \approx 19$ m/s.

(d) Using the Pythagorean theorem, we now have

$$U_i = 2k\frac{-qQ}{\sqrt{d^2 + x_0^2}} \ .$$

Therefore, (with U_A in this part equal to the negative of U_A in the previous part)

$$v = \sqrt{\frac{2\,(U_i - U_A)}{m}} \quad = \quad 2\sqrt{\frac{k\,q\,Q}{m}\left(\frac{1}{d} - \frac{1}{\sqrt{d^2 + x_0^2}}\right)} \ .$$

To simplify, the binomial theorem (Appendix E) is employed:

$$\frac{1}{\sqrt{d^2 + x_0^2}} \quad \approx \quad \frac{1}{d}\left(1 - \frac{1}{2}\frac{x_0^2}{d^2}\right)$$

which leads to

$$v \quad \approx \quad \frac{x_0}{d}\sqrt{\frac{2k\,q\,Q}{m\,d}} \ .$$