

Chapter 22

1. Eq. 22-1 gives Coulomb's Law, $F = k \frac{|q_1||q_2|}{r^2}$, which we solve for the distance:

$$\begin{aligned} r &= \sqrt{\frac{k|q_1||q_2|}{F}} \\ &= \sqrt{\frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) (26.0 \times 10^{-6} \text{ C}) (47.0 \times 10^{-6} \text{ C})}{5.70 \text{ N}}} = 1.39 \text{ m} . \end{aligned}$$

2. The magnitude of the mutual force of attraction at $r = 0.120 \text{ m}$ is

$$F = k \frac{|q_1||q_2|}{r^2} = (8.99 \times 10^9) \frac{(3.00 \times 10^{-6}) (1.50 \times 10^{-6})}{0.120^2} = 2.81 \text{ N} .$$

3. (a) With a understood to mean the magnitude of acceleration, Newton's second and third laws lead to

$$m_2 a_2 = m_1 a_1 \implies m_2 = \frac{(6.3 \times 10^{-7} \text{ kg}) (7.0 \text{ m/s}^2)}{9.0 \text{ m/s}^2} = 4.9 \times 10^{-7} \text{ kg} .$$

- (b) The magnitude of the (only) force on particle 1 is

$$F = m_1 a_1 = k \frac{|q_1||q_2|}{r^2} = (8.99 \times 10^9) \frac{|q|^2}{0.0032^2} .$$

Inserting the values for m_1 and a_1 (see part (a)) we obtain $|q| = 7.1 \times 10^{-11} \text{ C}$.

4. The fact that the spheres are identical allows us to conclude that when two spheres are in contact, they share equal charge. Therefore, when a charged sphere (q) touches an uncharged one, they will (fairly quickly) each attain half that charge ($q/2$). We start with spheres 1 and 2 each having charge q and experiencing a mutual repulsive force $F = kq^2/r^2$. When the neutral sphere 3 touches sphere 1, sphere 1's charge decreases to $q/2$. Then sphere 3 (now carrying charge $q/2$) is brought into contact with sphere 2, a total amount of $q/2 + q$ becomes shared equally between them. Therefore, the charge of sphere 3 is $3q/4$ in the final situation. The repulsive force between spheres 1 and 2 is finally

$$F' = k \frac{(\frac{q}{2})(\frac{3q}{4})}{r^2} = \frac{3}{8} k \frac{q^2}{r^2} = \frac{3}{8} F .$$

5. We put the origin of a coordinate system at the lower left corner of the square and take $+x$ rightward and $+y$ upward. The force exerted by the charge $+q$ on the charge $+2q$ is

$$\vec{F}_1 = k \frac{q(2q)}{a^2} (-\hat{j}) .$$

The force exerted by the charge $-q$ on the $+2q$ charge is directed along the diagonal of the square and has magnitude

$$F_2 = k \frac{q(2q)}{(a\sqrt{2})^2}$$

which becomes, upon finding its components (and using the fact that $\cos 45^\circ = 1/\sqrt{2}$),

$$\vec{F}_2 = k \frac{q(2q)}{2\sqrt{2}a^2} \hat{i} + k \frac{q(2q)}{2\sqrt{2}a^2} \hat{j} .$$

Finally, the force exerted by the charge $-2q$ on $+2q$ is

$$\vec{F}_3 = k \frac{(2q)(2q)}{a^2} \hat{i} .$$

(a) Therefore, the horizontal component of the resultant force on $+2q$ is

$$\begin{aligned} F_x &= F_{1x} + F_{2x} + F_{3x} = k \frac{q^2}{a^2} \left(\frac{1}{\sqrt{2}} + 4 \right) \\ &= (8.99 \times 10^9) \frac{(1.0 \times 10^{-7})^2}{0.050^2} \left(\frac{1}{\sqrt{2}} + 4 \right) = 0.17 \text{ N} . \end{aligned}$$

(b) The vertical component of the net force is

$$F_y = F_{1y} + F_{2y} + F_{3y} = k \frac{q^2}{a^2} \left(-2 + \frac{1}{\sqrt{2}} \right) = -0.046 \text{ N} .$$

6. (a) The individual force magnitudes (acting on Q) are, by Eq. 22-1,

$$k \frac{|q_1|Q}{(-a - \frac{a}{2})^2} = k \frac{|q_2|Q}{(a - \frac{a}{2})^2}$$

which leads to $|q_1| = 9|q_2|$. Since Q is located between q_1 and q_2 , we conclude q_1 and q_2 are like-sign. Consequently, $q_1 = 9q_2$.

(b) Now we have

$$k \frac{|q_1|Q}{(-a - \frac{3a}{2})^2} = k \frac{|q_2|Q}{(a - \frac{3a}{2})^2}$$

which yields $|q_1| = 25|q_2|$. Now, Q is not located between q_1 and q_2 , one of them must push and the other must pull. Thus, they are unlike-sign, so $q_1 = -25q_2$.

7. We assume the spheres are far apart. Then the charge distribution on each of them is spherically symmetric and Coulomb's law can be used. Let q_1 and q_2 be the original charges. We choose the coordinate system so the force on q_2 is positive if it is repelled by q_1 . Then, the force on q_2 is

$$F_a = -\frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} = -k \frac{q_1 q_2}{r^2}$$

where $r = 0.500$ m. The negative sign indicates that the spheres attract each other. After the wire is connected, the spheres, being identical, acquire the same charge. Since charge is conserved, the total charge is the same as it was originally. This means the charge on each sphere is $(q_1 + q_2)/2$. The force is now one of repulsion and is given by

$$F_b = \frac{1}{4\pi\epsilon_0} \frac{\left(\frac{q_1+q_2}{2}\right)\left(\frac{q_1+q_2}{2}\right)}{r^2} = k \frac{(q_1 + q_2)^2}{4r^2} .$$

We solve the two force equations simultaneously for q_1 and q_2 . The first gives the product

$$q_1 q_2 = -\frac{r^2 F_a}{k} = -\frac{(0.500 \text{ m})^2 (0.108 \text{ N})}{8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2} = -3.00 \times 10^{-12} \text{ C}^2 ,$$

and the second gives the sum

$$q_1 + q_2 = 2r\sqrt{\frac{F_b}{k}} = 2(0.500\text{ m})\sqrt{\frac{0.0360\text{ N}}{8.99 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2}} = 2.00 \times 10^{-6}\text{ C}$$

where we have taken the positive root (which amounts to assuming $q_1 + q_2 \geq 0$). Thus, the product result provides the relation

$$q_2 = \frac{-(3.00 \times 10^{-12}\text{ C}^2)}{q_1}$$

which we substitute into the sum result, producing

$$q_1 - \frac{3.00 \times 10^{-12}\text{ C}^2}{q_1} = 2.00 \times 10^{-6}\text{ C} .$$

Multiplying by q_1 and rearranging, we obtain a quadratic equation

$$q_1^2 - (2.00 \times 10^{-6}\text{ C})q_1 - 3.00 \times 10^{-12}\text{ C}^2 = 0 .$$

The solutions are

$$q_1 = \frac{2.00 \times 10^{-6}\text{ C} \pm \sqrt{(-2.00 \times 10^{-6}\text{ C})^2 - 4(-3.00 \times 10^{-12}\text{ C}^2)}}{2} .$$

If the positive sign is used, $q_1 = 3.00 \times 10^{-6}\text{ C}$, and if the negative sign is used, $q_1 = -1.00 \times 10^{-6}\text{ C}$. Using $q_2 = (-3.00 \times 10^{-12})/q_1$ with $q_1 = 3.00 \times 10^{-6}\text{ C}$, we get $q_2 = -1.00 \times 10^{-6}\text{ C}$. If we instead work with the $q_1 = -1.00 \times 10^{-6}\text{ C}$ root, then we find $q_2 = 3.00 \times 10^{-6}\text{ C}$. Since the spheres are identical, the solutions are essentially the same: one sphere originally had charge $-1.00 \times 10^{-6}\text{ C}$ and the other had charge $+3.00 \times 10^{-6}\text{ C}$. What if we had not made the assumption, above, that $q_1 + q_2 \geq 0$? If the signs of the charges were reversed (so $q_1 + q_2 < 0$), then the forces remain the same, so a charge of $+1.00 \times 10^{-6}\text{ C}$ on one sphere and a charge of $-3.00 \times 10^{-6}\text{ C}$ on the other also satisfies the conditions of the problem.

8. With rightwards positive, the net force on q_3 is

$$k\frac{q_1q_3}{(2d)^2} + k\frac{q_2q_3}{d^2} .$$

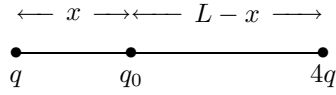
We note that each term exhibits the proper sign (positive for rightward, negative for leftward) for all possible signs of the charges. For example, the first term (the force exerted on q_3 by q_1) is negative if they are unlike charges, indicating that q_3 is being pulled toward q_1 , and it is positive if they are like charges (so q_3 would be repelled from q_1). Setting the net force equal to zero and canceling k , q_3 and d^2 leads to

$$\frac{q_1}{4} + q_2 = 0 \implies q_1 = -4q_2 .$$

9. (a) If the system of three charges is to be in equilibrium, the force on each charge must be zero. Let the third charge be q_0 . It must lie between the other two or else the forces acting on it due to the other charges would be in the same direction and q_0 could not be in equilibrium. Suppose q_0 is a distance x from q , as shown on the diagram below. The force acting on q_0 is then given by

$$F_0 = \frac{1}{4\pi\epsilon_0} \left(\frac{qq_0}{x^2} - \frac{4qq_0}{(L-x)^2} \right)$$

where the positive direction is rightward. We require $F_0 = 0$ and solve for x . Canceling common factors yields $1/x^2 = 4/(L-x)^2$ and taking the square root yields $1/x = 2/(L-x)$. The solution is $x = L/3$.



The force on q is

$$F_q = \frac{-1}{4\pi\epsilon_0} \left(\frac{qq_0}{x^2} + \frac{4q^2}{L^2} \right).$$

The signs are chosen so that a negative force value would cause q to move leftward. We require $F_q = 0$ and solve for q_0 :

$$q_0 = -\frac{4qx^2}{L^2} = -\frac{4}{9}q$$

where $x = L/3$ is used. We now examine the force on $4q$:

$$\begin{aligned} F_{4q} &= \frac{1}{4\pi\epsilon_0} \left(\frac{4q^2}{L^2} + \frac{4qq_0}{(L-x)^2} \right) = \frac{1}{4\pi\epsilon_0} \left(\frac{4q^2}{L^2} + \frac{4(-4/9)q^2}{(4/9)L^2} \right) \\ &= \frac{1}{4\pi\epsilon_0} \left(\frac{4q^2}{L^2} - \frac{4q^2}{L^2} \right) \end{aligned}$$

which we see is zero. Thus, with $q_0 = -(4/9)q$ and $x = L/3$, all three charges are in equilibrium.

- (b) If q_0 moves toward q the force of attraction exerted by q is greater in magnitude than the force of attraction exerted by $4q$. This causes q_0 to continue to move toward q and away from its initial position. The equilibrium is unstable.
10. There is no equilibrium position for q_3 *between* the two fixed charges, because it is being pulled by one and pushed by the other (since q_1 and q_2 have different signs); in this region this means the two force arrows on q_3 are in the same direction and cannot cancel. It should also be clear that off-axis (with the axis defined as that which passes through the two fixed charges) there are no equilibrium positions. On the semi-infinite region of the axis which is nearest q_2 and furthest from q_1 an equilibrium position for q_3 cannot be found because $|q_1| < |q_2|$ and the magnitude of force exerted by q_2 is everywhere (in that region) stronger than that exerted by q_1 on q_3 . Thus, we must look in the semi-infinite region of the axis which is nearest q_1 and furthest from q_2 , where the net force on q_3 has magnitude

$$\left| k \frac{|q_1 q_3|}{x^2} - k \frac{|q_2 q_3|}{(d+x)^2} \right|$$

with $d = 10$ cm and x assumed positive. We set this equal to zero, as required by the problem, and cancel k and q_3 . Thus, we obtain

$$\frac{|q_1|}{x^2} - \frac{|q_2|}{(d+x)^2} = 0 \implies \left(\frac{d+x}{x} \right)^2 = \frac{|q_2|}{|q_1|} = 3$$

which yields (after taking the square root)

$$\frac{d+x}{x} = \sqrt{3} \implies x = \frac{d}{\sqrt{3}-1} \approx 14 \text{ cm}$$

for the distance between q_3 and q_1 , so $x+d$ (the distance between q_2 and q_3) is approximately 24 cm.

11. (a) The magnitudes of the gravitational and electrical forces must be the same:

$$\frac{1}{4\pi\epsilon_0} \frac{q^2}{r^2} = G \frac{mM}{r^2}$$

where q is the charge on either body, r is the center-to-center separation of Earth and Moon, G is the universal gravitational constant, M is the mass of Earth, and m is the mass of the Moon. We solve for q :

$$q = \sqrt{4\pi\epsilon_0 GmM}.$$

According to Appendix C of the text, $M = 5.98 \times 10^{24} \text{ kg}$, and $m = 7.36 \times 10^{22} \text{ kg}$, so (using $4\pi\epsilon_0 = 1/k$) the charge is

$$q = \sqrt{\frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(7.36 \times 10^{22} \text{ kg})(5.98 \times 10^{24} \text{ kg})}{8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2}} = 5.7 \times 10^{13} \text{ C} .$$

We note that the distance r cancels because both the electric and gravitational forces are proportional to $1/r^2$.

- (b) The charge on a hydrogen ion is $e = 1.60 \times 10^{-19} \text{ C}$, so there must be

$$\frac{q}{e} = \frac{5.7 \times 10^{13} \text{ C}}{1.6 \times 10^{-19} \text{ C}} = 3.6 \times 10^{32} \text{ ions} .$$

Each ion has a mass of $1.67 \times 10^{-27} \text{ kg}$, so the total mass needed is

$$(3.6 \times 10^{32})(1.67 \times 10^{-27} \text{ kg}) = 6.0 \times 10^5 \text{ kg} .$$

12. (a) The distance between q_1 and q_2 is

$$r_{12} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-0.020 - 0.035)^2 + (0.015 - 0.005)^2} = 0.0559 \text{ m} .$$

The magnitude of the force exerted by q_1 on q_2 is

$$F_{21} = k \frac{|q_1 q_2|}{r_{12}^2} = \frac{(8.99 \times 10^9) (3.0 \times 10^{-6}) (4.0 \times 10^{-6})}{0.0559^2} = 34.5 \text{ N} .$$

The vector \vec{F}_{21} is directed towards q_1 and makes an angle θ with the $+x$ axis, where

$$\theta = \tan^{-1} \left(\frac{y_2 - y_1}{x_2 - x_1} \right) = \tan^{-1} \left(\frac{1.5 - 0.5}{-2.0 - 3.5} \right) = -10.3^\circ .$$

- (b) Let the third charge be located at (x_3, y_3) , a distance r from q_2 . We note that q_1 , q_2 and q_3 must be colinear; otherwise, an equilibrium position for any one of them would be impossible to find. Furthermore, we cannot place q_3 on the same side of q_2 where we also find q_1 , since in that region both forces (exerted on q_2 by q_3 and q_1) would be in the same direction (since q_2 is attracted to both of them). Thus, in terms of the angle found in part (a), we have $x_3 = x_2 - r \cos \theta$ and $y_3 = y_2 - r \sin \theta$ (which means $y_3 > y_2$ since θ is negative). The magnitude of force exerted on q_2 by q_3 is $F_{23} = k|q_2 q_3|/r^2$, which must equal that of the force exerted on it by q_1 (found in part (a)). Therefore,

$$k \frac{|q_2 q_3|}{r^2} = k \frac{|q_1 q_2|}{r_{12}^2} \implies r = r_{12} \sqrt{\frac{q_3}{q_1}} = 0.0645 \text{ cm} .$$

Consequently, $x_3 = x_2 - r \cos \theta = -2.0 \text{ cm} - (6.45 \text{ cm}) \cos(-10.3^\circ) = -8.4 \text{ cm}$ and $y_3 = y_2 - r \sin \theta = 1.5 \text{ cm} - (6.45 \text{ cm}) \sin(-10.3^\circ) = 2.7 \text{ cm}$.

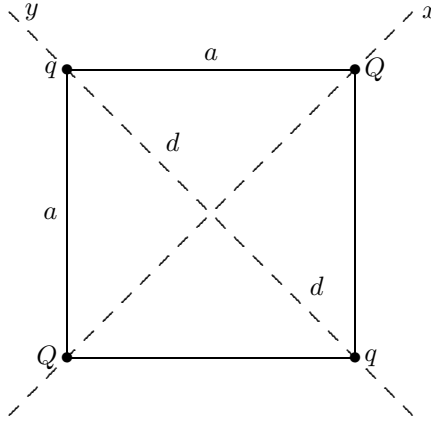
13. The magnitude of the force of either of the charges on the other is given by

$$F = \frac{1}{4\pi\epsilon_0} \frac{q(Q - q)}{r^2}$$

where r is the distance between the charges. We want the value of q that maximizes the function $f(q) = q(Q - q)$. Setting the derivative df/dq equal to zero leads to $Q - 2q = 0$, or $q = Q/2$.

14. (a) We choose the coordinate axes as shown in the diagram below. For ease of presentation (of the computations below) we assume $Q > 0$ and $q < 0$ (although the final result does not depend on this particular choice). The repulsive force between the diagonally opposite Q 's is along our (tilted)

x axis. The attractive force between each pair of Q and q is along the sides (of length a). In our drawing, the distance between the center to the corner is d , where $d = a/\sqrt{2}$, and the diagonal itself is therefore of length $2d = a\sqrt{2}$.



Since the angle between each attractive force and the x axis is 45° (note: $\cos 45^\circ = 1/\sqrt{2}$), then the net force on Q is

$$\begin{aligned} F_x &= \frac{1}{4\pi\epsilon_0} \left(\frac{(Q)(Q)}{(2d)^2} - 2 \frac{(|q|)(Q)}{a^2} \cos 45^\circ \right) \\ &= \frac{1}{4\pi\epsilon_0} \left(\frac{Q^2}{2a^2} - 2 \frac{|q| \cdot Q}{a^2} \frac{1}{\sqrt{2}} \right) \end{aligned}$$

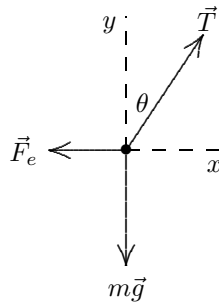
which (upon requiring $F_x = 0$) leads to $|q| = Q/2\sqrt{2}$ or $q = -\frac{Q}{2\sqrt{2}}$.

(b) The net force on q , examined along the y axis is

$$\begin{aligned} F_y &= \frac{1}{4\pi\epsilon_0} \left(\frac{q^2}{(2d)^2} - 2 \frac{(|q|)(Q)}{a^2} \sin 45^\circ \right) \\ &= \frac{1}{4\pi\epsilon_0} \left(\frac{q^2}{2a^2} - 2 \frac{|q| \cdot Q}{a^2} \frac{1}{\sqrt{2}} \right) \end{aligned}$$

which (if we demand $F_y = 0$) leads to $q = -2Q\sqrt{2}$ which is inconsistent with the result of part (a). Thus, we are unable to construct an equilibrium configuration with this geometry, where the only forces acting are given by Eq. 22-1.

15. (a) A force diagram for one of the balls is shown below. The force of gravity $m\vec{g}$ acts downward, the electrical force \vec{F}_e of the other ball acts to the left, and the tension in the thread acts along the thread, at the angle θ to the vertical. The ball is in equilibrium, so its acceleration is zero. The y component of Newton's second law yields $T \cos \theta - mg = 0$ and the x component yields $T \sin \theta - F_e = 0$. We solve the first equation for T and obtain $T = mg/\cos \theta$. We substitute the result into the second to obtain $mg \tan \theta - F_e = 0$.



Examination of the geometry of Figure 22-19 leads to

$$\tan \theta = \frac{x/2}{\sqrt{L^2 - (x/2)^2}} .$$

If L is much larger than x (which is the case if θ is very small), we may neglect $x/2$ in the denominator and write $\tan \theta \approx x/2L$. This is equivalent to approximating $\tan \theta$ by $\sin \theta$. The magnitude of the electrical force of one ball on the other is

$$F_e = \frac{q^2}{4\pi\epsilon_0 x^2}$$

by Eq. 22-4. When these two expressions are used in the equation $mg \tan \theta = F_e$, we obtain

$$\frac{mgx}{2L} \approx \frac{1}{4\pi\epsilon_0} \frac{q^2}{x^2} \implies x \approx \left(\frac{q^2 L}{2\pi\epsilon_0 mg} \right)^{1/3} .$$

(b) We solve $x^3 = 2kq^2 L/mg$ for the charge (using Eq. 22-5):

$$q = \sqrt[3]{\frac{mgx^3}{2kL}} = \sqrt[3]{\frac{(0.010 \text{ kg})(9.8 \text{ m/s}^2)(0.050 \text{ m})^3}{2(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(1.20 \text{ m})}} = \pm 2.4 \times 10^{-8} \text{ C} .$$

16. If one of them is discharged, there would no electrostatic repulsion between the two balls and they would both come to the position $\theta = 0$, making contact with each other. A redistribution of the remaining charge would then occur, with each of the balls getting $q/2$. Then they would again be separated due to electrostatic repulsion, which results in the new equilibrium separation

$$x' = \left[\frac{(q/2)^2 L}{2\pi\epsilon_0 mg} \right]^{1/3} = \left(\frac{1}{4} \right)^{1/3} x = \left(\frac{1}{4} \right)^{1/3} (5.0 \text{ cm}) = 3.1 \text{ cm} .$$

17. (a) Since the rod is in equilibrium, the net force acting on it is zero, and the net torque about any point is also zero. We write an expression for the net torque about the bearing, equate it to zero, and solve for x . The charge Q on the left exerts an upward force of magnitude $(1/4\pi\epsilon_0)(qQ/h^2)$, at a distance $L/2$ from the bearing. We take the torque to be negative. The attached weight exerts a downward force of magnitude W , at a distance $x - L/2$ from the bearing. This torque is also negative. The charge Q on the right exerts an upward force of magnitude $(1/4\pi\epsilon_0)(2qQ/h^2)$, at a distance $L/2$ from the bearing. This torque is positive. The equation for rotational equilibrium is

$$\frac{-1}{4\pi\epsilon_0} \frac{qQ}{h^2} \frac{L}{2} - W \left(x - \frac{L}{2} \right) + \frac{1}{4\pi\epsilon_0} \frac{2qQ}{h^2} \frac{L}{2} = 0 .$$

The solution for x is

$$x = \frac{L}{2} \left(1 + \frac{1}{4\pi\epsilon_0} \frac{qQ}{h^2 W} \right) .$$

- (b) If N is the magnitude of the upward force exerted by the bearing, then Newton's second law (with zero acceleration) gives

$$W - \frac{1}{4\pi\epsilon_0} \frac{qQ}{h^2} - \frac{1}{4\pi\epsilon_0} \frac{2qQ}{h^2} - N = 0 .$$

We solve for h so that $N = 0$. The result is

$$h = \sqrt{\frac{1}{4\pi\epsilon_0} \frac{3qQ}{W}} .$$

18. The magnitude of the force is

$$F = k \frac{e^2}{r^2} = \left(8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \right) \frac{(1.60 \times 10^{-19} \text{ C})^2}{(2.82 \times 10^{-10} \text{ m})^2} = 2.89 \times 10^{-9} \text{ N} .$$

19. The mass of an electron is $m = 9.11 \times 10^{-31} \text{ kg}$, so the number of electrons in a collection with total mass $M = 75.0 \text{ kg}$ is

$$N = \frac{M}{m} = \frac{75.0 \text{ kg}}{9.11 \times 10^{-31} \text{ kg}} = 8.23 \times 10^{31} \text{ electrons} .$$

The total charge of the collection is

$$q = -Ne = -(8.23 \times 10^{31})(1.60 \times 10^{-19} \text{ C}) = -1.32 \times 10^{13} \text{ C} .$$

20. There are two protons (each with charge $q = +e$) in each molecule, so

$$Q = N_A q = (6.02 \times 10^{23})(2)(1.60 \times 10^{-19} \text{ C}) = 1.9 \times 10^5 \text{ C} = 0.19 \text{ MC} .$$

21. (a) The magnitude of the force between the (positive) ions is given by

$$F = \frac{(q)(q)}{4\pi\epsilon_0 r^2} = k \frac{q^2}{r^2}$$

where q is the charge on either of them and r is the distance between them. We solve for the charge:

$$q = r \sqrt{\frac{F}{k}} = (5.0 \times 10^{-10} \text{ m}) \sqrt{\frac{3.7 \times 10^{-9} \text{ N}}{8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2}} = 3.2 \times 10^{-19} \text{ C} .$$

- (b) Let N be the number of electrons missing from each ion. Then, $Ne = q$, or

$$N = \frac{q}{e} = \frac{3.2 \times 10^{-19} \text{ C}}{1.6 \times 10^{-19} \text{ C}} = 2 .$$

22. (a) Eq. 22-1 gives

$$F = \frac{\left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2 \right) (1.00 \times 10^{-16} \text{ C})^2}{(1.00 \times 10^{-2} \text{ m})^2} = 8.99 \times 10^{-19} \text{ N} .$$

- (b) If n is the number of excess electrons (of charge $-e$ each) on each drop then

$$n = -\frac{q}{e} = -\frac{-1.00 \times 10^{-16} \text{ C}}{1.60 \times 10^{-19} \text{ C}} = 625 .$$

23. Eq. 22-11 (in absolute value) gives

$$n = \frac{|q|}{e} = \frac{1.0 \times 10^{-7} \text{ C}}{1.6 \times 10^{-19} \text{ C}} = 6.3 \times 10^{11} .$$

24. With $F = m_e g$, Eq. 22-1 leads to

$$r^2 = \frac{ke^2}{m_e g} = \frac{\left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2 \right) (1.60 \times 10^{-19} \text{ C})^2}{(9.11 \times 10^{-31} \text{ kg}) (9.8 \text{ m/s}^2)}$$

which leads to $r = 5.1 \text{ m}$. The second electron should be below the first one, so that the repulsive force (acting on the first) is in the direction opposite to the pull of Earth's gravity.

25. The unit Ampere is discussed in §22-4. The proton flux is given as 1500 protons per square meter per second, where each proton provides a charge of $q = +e$. The current through the spherical area $4\pi R^2 = 4\pi(6.37 \times 10^6 \text{ m})^2 = 5.1 \times 10^{14} \text{ m}^2$ would be

$$i = (5.1 \times 10^{14} \text{ m}^2) \left(1500 \frac{\text{protons}}{\text{s} \cdot \text{m}^2} \right) (1.6 \times 10^{-19} \text{ C/proton}) = 0.122 \text{ A} .$$

26. The volume of 250 cm^3 corresponds to a mass of 250 g since the density of water is 1.0 g/cm^3 . This mass corresponds to $250/18 = 14$ moles since the molar mass of water is 18. There are ten protons (each with charge $q = +e$) in each molecule of H_2O , so

$$Q = 14N_A q = 14(6.02 \times 10^{23})(10)(1.60 \times 10^{-19} \text{ C}) = 1.3 \times 10^7 \text{ C} = 13 \text{ MC} .$$

27. (a) Every cesium ion at a corner of the cube exerts a force of the same magnitude on the chlorine ion at the cube center. Each force is a force of attraction and is directed toward the cesium ion that exerts it, along the body diagonal of the cube. We can pair every cesium ion with another, diametrically positioned at the opposite corner of the cube. Since the two ions in such a pair exert forces that have the same magnitude but are oppositely directed, the two forces sum to zero and, since every cesium ion can be paired in this way, the total force on the chlorine ion is zero.
- (b) Rather than remove a cesium ion, we superpose charge $-e$ at the position of one cesium ion. This neutralizes the ion, and as far as the electrical force on the chlorine ion is concerned, it is equivalent to removing the ion. The forces of the eight cesium ions at the cube corners sum to zero, so the only force on the chlorine ion is the force of the added charge.

The length of a body diagonal of a cube is $\sqrt{3}a$, where a is the length of a cube edge. Thus, the distance from the center of the cube to a corner is $d = (\sqrt{3}/2)a$. The force has magnitude

$$F = k \frac{e^2}{d^2} = \frac{ke^2}{(3/4)a^2} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(1.60 \times 10^{-19} \text{ C})^2}{(3/4)(0.40 \times 10^{-9} \text{ m})^2} = 1.9 \times 10^{-9} \text{ N} .$$

Since both the added charge and the chlorine ion are negative, the force is one of repulsion. The chlorine ion is pushed away from the site of the missing cesium ion.

28. If the relative difference between the proton and electron charges (in absolute value) were

$$\frac{q_p - |q_e|}{e} = 0.0000010$$

then the actual difference would be

$$q_p - |q_e| = 1.6 \times 10^{-25} \text{ C} .$$

Amplified by a factor of $29 \times 3 \times 10^{22}$ as indicated in the problem, this amounts to a deviation from perfect neutrality of

$$\Delta q = (29 \times 3 \times 10^{22}) (1.6 \times 10^{-25} \text{ C}) = 0.14 \text{ C}$$

in a copper penny. Two such pennies, at $r = 1.0 \text{ m}$, would therefore experience a very large force. Eq. 22-1 gives

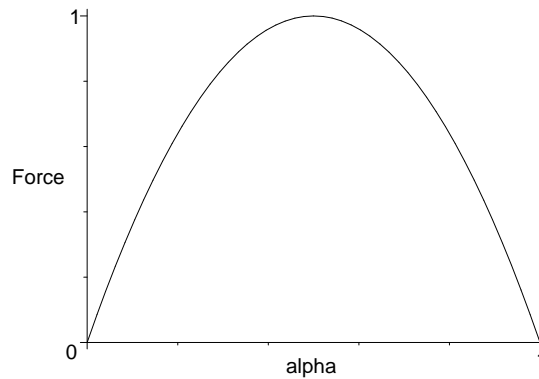
$$F = k \frac{(\Delta q)^2}{r^2} = 1.7 \times 10^8 \text{ N} .$$

29. None of the reactions given include a beta decay, so the number of protons, the number of neutrons, and the number of electrons are each conserved. Atomic numbers (numbers of protons and numbers of electrons) and molar masses (combined numbers of protons and neutrons) can be found in Appendix F of the text.

- (a) ${}^1\text{H}$ has 1 proton, 1 electron, and 0 neutrons and ${}^9\text{Be}$ has 4 protons, 4 electrons, and $9 - 4 = 5$ neutrons, so X has $1 + 4 = 5$ protons, $1 + 4 = 5$ electrons, and $0 + 5 - 1 = 4$ neutrons. One of the neutrons is freed in the reaction. X must be boron with a molar mass of $5 + 4 = 9$ g/mol: ${}^9\text{B}$.
- (b) ${}^{12}\text{C}$ has 6 protons, 6 electrons, and $12 - 6 = 6$ neutrons and ${}^1\text{H}$ has 1 proton, 1 electron, and 0 neutrons, so X has $6 + 1 = 7$ protons, $6 + 1 = 7$ electrons, and $6 + 0 = 6$ neutrons. It must be nitrogen with a molar mass of $7 + 6 = 13$ g/mol: ${}^{13}\text{N}$.
- (c) ${}^{15}\text{N}$ has 7 protons, 7 electrons, and $15 - 7 = 8$ neutrons; ${}^1\text{H}$ has 1 proton, 1 electron, and 0 neutrons; and ${}^4\text{He}$ has 2 protons, 2 electrons, and $4 - 2 = 2$ neutrons; so X has $7 + 1 - 2 = 6$ protons, 6 electrons, and $8 + 0 - 2 = 6$ neutrons. It must be carbon with a molar mass of $6 + 6 = 12$: ${}^{12}\text{C}$.
30. (a) The two charges are $q = \alpha Q$ (where α is a pure number presumably less than 1 and greater than zero) and $Q - q = (1 - \alpha)Q$. Thus, Eq. 22-4 gives

$$F = \frac{1}{4\pi\epsilon_0} \frac{(\alpha Q)((1 - \alpha)Q)}{d^2} = \frac{Q^2\alpha(1 - \alpha)}{4\pi\epsilon_0 d^2}.$$

- (b) The graph below, of F versus α , has been scaled so that the maximum is 1. In actuality, the maximum value of the force is $F_{\max} = Q^2/16\pi\epsilon_0 d^2$.



- (c) It is clear that $\alpha = \frac{1}{2}$ gives the maximum value of F .
- (d) Seeking the half-height points on the graph is difficult without grid lines or some of the special tracing features found in a variety of modern calculators. It is not difficult to algebraically solve for the half-height points (this involves the use of the quadratic formula). The results are

$$\begin{aligned}\alpha_1 &= \frac{1}{2} \left(1 - \frac{1}{\sqrt{2}} \right) \approx 0.15 \quad \text{and} \\ \alpha_2 &= \frac{1}{2} \left(1 + \frac{1}{\sqrt{2}} \right) \approx 0.85.\end{aligned}$$

31. (a) Eq. 22-11 (in absolute value) gives

$$n = \frac{|q|}{e} = \frac{2.00 \times 10^{-6} \text{ C}}{1.60 \times 10^{-19} \text{ C}} = 1.25 \times 10^{13} \text{ electrons}.$$

- (b) Since you have the excess electrons (and electrons are lighter and more mobile than protons) then the electrons “leap” from you to the faucet instead of protons moving from the faucet to you (in the process of neutralizing your body).
- (c) Unlike charges attract, and the faucet (which is grounded and is able to gain or lose any number of electrons due to its contact with Earth’s large reservoir of mobile charges) becomes positively charged, especially in the region closest to your (negatively charged) hand, just before the spark.

- (d) The cat is positively charged (before the spark), and by the reasoning given in part (b) the flow of charge (electrons) is from the faucet to the cat.
- (e) If we think of the nose as a conducting sphere, then the side of the sphere closest to the fur is of one sign (of charge) and the side furthest from the fur is of the opposite sign (which, additionally, is oppositely charged from your bare hand which had stroked the cat's fur). The charges in your hand and those of the furthest side of the "sphere" therefore attract each other, and when close enough, manage to neutralize (due to the "jump" made by the electrons) in a painful spark.

32. (a) Using Coulomb's law, we obtain

$$F = \frac{q_1 q_2}{4\pi\epsilon_0 r^2} = \frac{kq^2}{r^2} = \frac{\left(8.99 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right) (1.00 \text{ C})^2}{(1.00 \text{ m})^2} = 8.99 \times 10^9 \text{ N} .$$

- (b) If $r = 1000 \text{ m}$, then

$$F = \frac{q_1 q_2}{4\pi\epsilon_0 r^2} = \frac{kq^2}{r^2} = \frac{\left(8.99 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right) (1.00 \text{ C})^2}{(1.00 \times 10^3 \text{ m})^2} = 8.99 \times 10^3 \text{ N} .$$

33. The unit Ampere is discussed in §22-4. Using i for current, the charge transferred is

$$q = it = (2.5 \times 10^4 \text{ A}) (20 \times 10^{-6} \text{ s}) = 0.50 \text{ C} .$$

34. Let the two charges be q_1 and q_2 . Then $q_1 + q_2 = Q = 5.0 \times 10^{-5} \text{ C}$. We use Eq. 22-1:

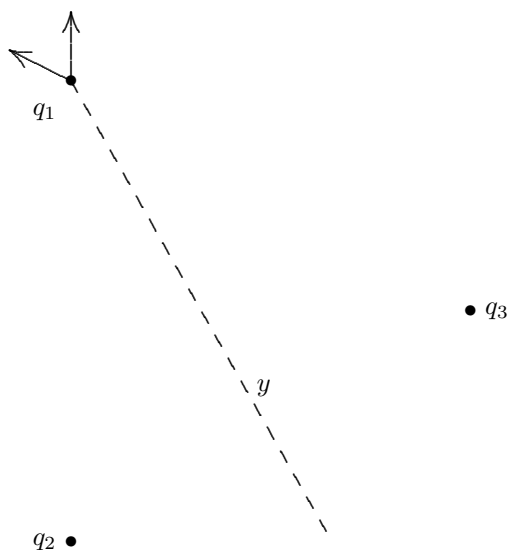
$$1.0 \text{ N} = \frac{\left(8.99 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right) q_1 q_2}{(2.0 \text{ m})^2} .$$

We substitute $q_2 = Q - q_1$ and solve for q_1 using the quadratic formula. The two roots obtained are the values of q_1 and q_2 , since it does not matter which is which. We get $1.2 \times 10^{-5} \text{ C}$ and $3.8 \times 10^{-5} \text{ C}$.

35. (a) Eq. 22-1 gives

$$F_{12} = k \frac{q_1 q_2}{d^2} = \left(8.99 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right) \frac{(20.0 \times 10^{-6} \text{ C})^2}{(1.50 \text{ m})^2} = 1.60 \text{ N} .$$

- (b) A force diagram is shown as well as our choice of y axis (the dashed line).



The y axis is meant to bisect the line between q_2 and q_3 in order to make use of the symmetry in the problem (equilateral triangle of side length d , equal-magnitude charges $q_1 = q_2 = q_3 = q$). We see that the resultant force is along this symmetry axis, and we obtain

$$|F_y| = 2 \left(k \frac{q^2}{d^2} \right) \cos 30^\circ = 2.77 \text{ N} .$$

36. (a) Since $q_A = -2Q$ and $q_C = +8Q$, Eq. 22-4 leads to

$$|\vec{F}_{AC}| = \frac{|(-2Q)(+8Q)|}{4\pi\epsilon_0 d^2} = \frac{4Q^2}{\pi\epsilon_0 d^2} .$$

- (b) After making contact with each other, both A and B have a charge of

$$\left(\frac{-2Q + (-4Q)}{2} \right) = -3Q .$$

When B is grounded its charge is zero. After making contact with C , which has a charge of $+8Q$, B acquires a charge of $[0 + (-8Q)]/2 = -4Q$, which charge C has as well. Finally, we have $Q_A = -3Q$ and $Q_B = Q_C = -4Q$. Therefore,

$$|\vec{F}_{AC}| = \frac{|(-3Q)(-4Q)|}{4\pi\epsilon_0 d^2} = \frac{3Q^2}{\pi\epsilon_0 d^2} .$$

- (c) We also obtain

$$|\vec{F}_{BC}| = \frac{|(-4Q)(-4Q)|}{4\pi\epsilon_0 d^2} = \frac{4Q^2}{\pi\epsilon_0 d^2} .$$

37. The net charge carried by John whose mass is m is roughly

$$\begin{aligned} q &= (0.0001) \frac{m N_A Z e}{M} \\ &= (0.0001) \frac{(90 \text{ kg})(6.02 \times 10^{23} \text{ molecules/mol})(18 \text{ electron proton pairs/molecule})(1.6 \times 10^{-19} \text{ C})}{0.018 \text{ kg/mol}} \\ &= 8.7 \times 10^5 \text{ C} , \end{aligned}$$

and the net charge carried by Mary is half of that. So the electrostatic force between them is estimated to be

$$F \approx k \frac{q(q/2)}{d^2} = \left(8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \right) \frac{(8.7 \times 10^5 \text{ C})^2}{2(30 \text{ m})^2} \approx 4 \times 10^{18} \text{ N} .$$

38. Letting $kq^2/r^2 = mg$, we get

$$r = q \sqrt{\frac{k}{mg}} = (1.60 \times 10^{-19} \text{ C}) \sqrt{\frac{8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}}{(1.67 \times 10^{-27} \text{ kg})(9.8 \text{ m/s}^2)}} = 0.119 \text{ m} .$$

39. Coulomb's law gives

$$F = \frac{|q| \cdot |q|}{4\pi\epsilon_0 r^2} = \frac{k(e/3)^2}{r^2} = \frac{\left(8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \right) (1.60 \times 10^{-19} \text{ C})^2}{9(2.6 \times 10^{-15} \text{ m})^2} = 3.8 \text{ N} .$$

40. We are concerned with the charges in the nucleus (not the “orbiting” electrons, if there are any). The nucleus of Helium has 2 protons and that of Thorium has 90.

(a) Eq. 22-1 gives

$$F = k \frac{q^2}{r^2} = \frac{(8.99 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}) (2(1.60 \times 10^{-19} \text{ C})) (90(1.60 \times 10^{-19} \text{ C}))}{(9.0 \times 10^{-15} \text{ m})^2} = 5.1 \times 10^2 \text{ N} .$$

(b) Estimating the helium nucleus mass as that of 4 protons (actually, that of 2 protons and 2 neutrons, but the neutrons have approximately the same mass), Newton's second law leads to

$$a = \frac{F}{m} = \frac{5.1 \times 10^2 \text{ N}}{4(1.67 \times 10^{-27} \text{ kg})} = 7.7 \times 10^{28} \text{ m/s}^2 .$$

41. Charge $q_1 = -80 \times 10^{-6} \text{ C}$ is at the origin, and charge $q_2 = +40 \times 10^{-6} \text{ C}$ is at $x = 0.20 \text{ m}$. The force on $q_3 = +20 \times 10^{-6} \text{ C}$ is due to the attractive and repulsive forces from q_1 and q_2 , respectively. In symbols, $\vec{F}_{3 \text{ net}} = \vec{F}_{31} + \vec{F}_{32}$, where

$$|\vec{F}_{31}| = k \frac{q_3 |q_1|}{r_{31}^2} \quad \text{and} \quad |\vec{F}_{32}| = k \frac{q_3 q_2}{r_{32}^2} .$$

- (a) In this case $r_{31} = 0.40 \text{ m}$ and $r_{32} = 0.20 \text{ m}$, with \vec{F}_{31} directed towards $-x$ and \vec{F}_{32} directed in the $+x$ direction. Using the value of k in Eq. 22-5, we obtain $\vec{F}_{3 \text{ net}} = 89.9 \approx 90 \text{ N}$ in the $+x$ direction.
- (b) In this case $r_{31} = 0.80 \text{ m}$ and $r_{32} = 0.60 \text{ m}$, with \vec{F}_{31} directed towards $-x$ and \vec{F}_{32} towards $+x$. Now we obtain $\vec{F}_{3 \text{ net}} = 2.5 \text{ N}$ in the $-x$ direction.
- (c) Between the locations treated in parts (a) and (b), there must be one where $\vec{F}_{3 \text{ net}} = 0$. Writing $r_{31} = x$ and $r_{32} = x - 0.20 \text{ m}$, we equate $|\vec{F}_{31}|$ and $|\vec{F}_{32}|$, and after canceling common factors, arrive at

$$\frac{|q_1|}{x^2} = \frac{q_2}{(x - 0.2)^2} .$$

This can be further simplified to

$$\frac{(x - 0.2)^2}{x^2} = \frac{q_2}{|q_1|} = \frac{1}{2} .$$

Taking the (positive) square root and solving, we obtain $x = 0.68 \text{ m}$. If one takes the negative root and 'solves', one finds the location where the net force *would* be zero *if* q_1 and q_2 were of like sign (which is not the case here).

42. (a) Charge $Q_1 = +80 \times 10^{-9} \text{ C}$ is on the y axis at $y = 0.003 \text{ m}$, and charge $Q_2 = +80 \times 10^{-9} \text{ C}$ is on the y axis at $y = -0.003 \text{ m}$. The force on particle 3 (which has a charge of $q = +18 \times 10^{-9} \text{ C}$) is due to the vector sum of the repulsive forces from Q_1 and Q_2 . In symbols, $\vec{F}_{31} + \vec{F}_{32} = \vec{F}_{3 \text{ net}}$, where

$$|\vec{F}_{31}| = k \frac{q_3 |q_1|}{r_{31}^2} \quad \text{and} \quad |\vec{F}_{32}| = k \frac{q_3 q_2}{r_{32}^2} .$$

Using the Pythagorean theorem, we have $r_{31} = r_{32} = 0.005 \text{ m}$. In magnitude-angle notation (particularly convenient if one uses a vector capable calculator in polar mode), the indicated vector addition becomes

$$(0.518 \angle -37^\circ) + (0.518 \angle 37^\circ) = (0.829 \angle 0^\circ) .$$

Therefore, the net force is 0.829 N in the $+x$ direction.

- (b) Switching the sign of Q_2 amounts to reversing the direction of its force on q . Consequently, we have

$$(0.518 \angle -37^\circ) + (0.518 \angle -143^\circ) = (0.621 \angle -90^\circ) .$$

Therefore, the net force is 0.621 N in the $-y$ direction.

43. (a) For the net force to be in the $+x$ direction, the y components of the individual forces must cancel. The angle of the force exerted by the $q_1 = 40 \mu\text{C}$ charge on $q = 20 \mu\text{C}$ is 45° , and the angle of force exerted on q by Q is at $-\theta$ where

$$\theta = \tan^{-1} \left(\frac{2.0}{3.0} \right) = 33.7^\circ .$$

Therefore, cancellation of y components requires

$$k \frac{q_1 q}{(0.02\sqrt{2})^2} \sin 45^\circ = k \frac{|Q| q}{(\sqrt{0.03^2 + 0.02^2})^2} \sin \theta$$

from which we obtain $|Q| = 82.9 \mu\text{C}$. Charge Q is “pulling” on q , so (since $q > 0$) we conclude $Q = -82.9 \mu\text{C}$.

- (b) Now, we require that the x components cancel, and we note that in this case, the angle of force on q exerted by Q is $+\theta$ (it is repulsive, and Q is positive-valued). Therefore,

$$k \frac{q_1 q}{(0.02\sqrt{2})^2} \cos 45^\circ = k \frac{Q q}{(\sqrt{0.03^2 + 0.02^2})^2} \cos \theta$$

from which we obtain $Q = 55.2 \mu\text{C}$.

44. We are looking for a charge q which, when placed at the origin, experiences $\vec{F}_{\text{net}} = 0$, where

$$\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 .$$

The magnitude of these individual forces are given by Coulomb’s law, Eq. 22-1, and without loss of generality we assume $q > 0$. The charges q_1 ($+6 \mu\text{C}$), q_2 ($-4 \mu\text{C}$), and q_3 (unknown), are located on the $+x$ axis, so that we know \vec{F}_1 points towards $-x$, \vec{F}_2 points towards $+x$, and \vec{F}_3 points towards $-x$ if $q_3 > 0$ and points towards $+x$ if $q_3 < 0$. Therefore, with $r_1 = 8 \text{ m}$, $r_2 = 16 \text{ m}$ and $r_3 = 24 \text{ m}$, we have

$$0 = -k \frac{q_1 q}{r_1^2} + k \frac{|q_2| q}{r_2^2} - k \frac{q_3 q}{r_3^2} .$$

Simplifying, this becomes

$$0 = -\frac{6}{8^2} + \frac{4}{16^2} - \frac{q_3}{24^2}$$

where q_3 is now understood to be in μC . Thus, we obtain $q_3 = -45 \mu\text{C}$.

45. The magnitude of the net force on the $q = 42 \times 10^{-6} \text{ C}$ charge is

$$k \frac{q_1 q}{0.28^2} + k \frac{|q_2| q}{0.44^2}$$

where $q_1 = 30 \times 10^{-9} \text{ C}$ and $|q_2| = 40 \times 10^{-9} \text{ C}$. This yields 0.22 N . Using Newton’s second law, we obtain

$$m = \frac{F}{a} = \frac{0.22 \text{ N}}{100 \times 10^3 \text{ m/s}^2} = 2.2 \times 10^{-6} \text{ kg} .$$

46. The charge dq within a thin shell of thickness dr is $\rho A dr$ where $A = 4\pi r^2$. Thus, with $\rho = b/r$, we have

$$q = \int dq = 4\pi b \int_{r_1}^{r_2} r dr = 2\pi b (r_2^2 - r_1^2) .$$

With $b = 3.0 \mu\text{C/m}^2$, $r_2 = 0.06 \text{ m}$ and $r_1 = 0.04 \text{ m}$, we obtain $q = 0.038 \mu\text{C}$.

47. The charge dq within a thin section of the rod (of thickness dx) is $\rho A dx$ where $A = 4.00 \times 10^{-4} \text{ m}^2$ and ρ is the charge per unit volume. The number of (excess) electrons in the rod (of length $L = 2.00 \text{ m}$) is $N = q/(-e)$ where e is given in Eq. 22-14.

(a) In the case where $\rho = -4.00 \times 10^{-6} \text{ C/m}^3$, we have

$$N = \frac{q}{-e} = \frac{\rho A}{-e} \int_0^L dx = \frac{|\rho| A L}{e}$$

which yields $N = 2.00 \times 10^{10}$.

(b) With $\rho = bx^2$ ($b = -2.00 \times 10^{-6} \text{ C/m}^5$) we obtain

$$N = \frac{b A}{-e} \int_0^L x^2 dx = \frac{|b| A L^3}{3 e} = 1.33 \times 10^{10} .$$

48. When sphere C touches sphere A , they divide up their total charge ($Q/2$ plus Q) equally between them. Thus, sphere A now has charge $3Q/4$, and the magnitude of the force of attraction between A and B becomes

$$F = k \frac{\left(\frac{3Q}{4}\right) \left(\frac{Q}{4}\right)}{d^2} = 4.68 \times 10^{-19} \text{ N} .$$

49. In experiment 1, sphere C first touches sphere A , and they divided up their total charge ($Q/2$ plus Q) equally between them. Thus, sphere A and sphere C each acquired charge $3Q/4$. Then, sphere C touches B and those spheres split up their total charge ($3Q/4$ plus $-Q/4$) so that B ends up with charge equal to $Q/4$. The force of repulsion between A and B is therefore

$$F_1 = k \frac{\left(\frac{3Q}{4}\right) \left(\frac{Q}{4}\right)}{d^2}$$

at the end of experiment 1. Now, in experiment 2, sphere C first touches B which leaves each of them with charge $Q/8$. When C next touches A , sphere A is left with charge $9Q/16$. Consequently, the force of repulsion between A and B is

$$F_2 = k \frac{\left(\frac{9Q}{16}\right) \left(\frac{Q}{8}\right)}{d^2}$$

at the end of experiment 2. The ratio is

$$\frac{F_2}{F_1} = \frac{\left(\frac{9}{16}\right) \left(\frac{1}{8}\right)}{\left(\frac{3}{4}\right) \left(\frac{1}{4}\right)} = 0.375 .$$

50. Regarding the forces on q_3 exerted by q_1 and q_2 , one must “push” and the other must “pull” in order that the net force is zero; hence, q_1 and q_2 have opposite signs. For individual forces to cancel, their magnitudes must be equal:

$$k \frac{|q_1| |q_3|}{(3d)^2} = k \frac{|q_2| |q_3|}{(2d)^2}$$

which simplifies to

$$\frac{|q_1|}{9} = \frac{|q_2|}{4} .$$

Therefore, $q_1 = -\frac{9}{4}q_2$.

51. The individual force magnitudes are found using Eq. 22-1, with SI units (so $a = 0.02 \text{ m}$) and k as in Eq. 22-5. We use magnitude-angle notation (convenient if ones uses a vector capable calculator in polar mode), listing the forces due to $+4.00q$, $+2.00q$, and $-2.00q$ charges:

$$(4.60 \times 10^{-24} \angle 180^\circ) + (2.30 \times 10^{-24} \angle -90^\circ) + (1.02 \times 10^{-24} \angle -145^\circ) = (6.16 \times 10^{-24} \angle -152^\circ)$$

Therefore, the net force has magnitude $6.16 \times 10^{-24} \text{ N}$ and is at an angle of -152° (or 208° measured counterclockwise from the $+x$ axis).

