Chapter 9

1. (a) We locate the coordinate origin at the center of Earth. Then the distance r_{com} of the center of mass of the Earth-Moon system is given by

$$r_{\text{com}} = \frac{m_M r_M}{m_M + m_E}$$

where m_M is the mass of the Moon, m_E is the mass of Earth, and r_M is their separation. These values are given in Appendix C. The numerical result is

$$r_{\text{com}} = \frac{(7.36 \times 10^{22}\,\text{kg})(3.82 \times 10^{8}\,\text{m})}{7.36 \times 10^{22}\,\text{kg} + 5.98 \times 10^{24}\,\text{kg}} = 4.64 \times 10^{6}\ \text{m}\ .$$

(b) The radius of Earth is $R_E = 6.37 \times 10^6\,\text{m}$, so $r_{\text{com}} = 0.73 R_E$.

2. We locate the coordinate origin at the center of the carbon atom, and we consider both atoms to be "point particles." We will use the non-SI units for mass found in Appendix F; since they will cancel they will not prevent the answer from being in SI units.

$$r_{\text{com}} = \frac{(15.9994\,\text{grams/mole})(1.131 \times 10^{-10}\,\text{m})}{12.01115\,\text{grams/mole} + 15.9994\,\text{grams/mole}} = 6.46 \times 10^{-11}\ \text{m}\ .$$

3. Our notation is as follows: $x_1 = 0$ and $y_1 = 0$ are the coordinates of the $m_1 = 3.0\,\text{kg}$ particle; $x_2 = 1.0\,\text{m}$ and $y_2 = 2.0\,\text{m}$ are the coordinates of the $m_2 = 8.0\,\text{kg}$ particle; and, $x_3 = 2.0\,\text{m}$ and $y_3 = 1.0\,\text{m}$ are the coordinates of the $m_3 = 4.0\,\text{kg}$ particle.

(a) The x coordinate of the center of mass is

$$\begin{aligned} x_{\text{com}} &= \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_2} \\ &= \frac{0 + (8.0\,\text{kg})(1.0\,\text{m}) + (4.0\,\text{kg})(2.0\,\text{m})}{3.0\,\text{kg} + 8.0\,\text{kg} + 4.0\,\text{kg}} \\ &= 1.1\ \text{m}\ . \end{aligned}$$

(b) The y coordinate of the center of mass is

$$\begin{aligned} y_{\text{com}} &= \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3} \\ &= \frac{0 + (8.0\,\text{kg})(2.0\,\text{m}) + (4.0\,\text{kg})(1.0\,\text{m})}{3.0\,\text{kg} + 8.0\,\text{kg} + 4.0\,\text{kg}} \\ &= 1.3\ \text{m}\ . \end{aligned}$$

(c) As the mass of the topmost particle is increased, the center of mass shifts toward that particle. As we approach the limit as the topmost particle is infinitely more massive than the others, the center of mass becomes infinitesimally close to the position of that particle.

4. We will refer to the arrangement as a "table." We locate the coordinate origin at the center of the tabletop and note that the center of mass of each "leg" is a distance $L/2$ below the top. With $+x$ rightward and $+y$ upward, then the center of mass of the right leg is at $(x, y) = (+L/2, -L/2)$ and the center of mass of the left leg is at $(x, y) = (-L/2, -L/2)$. Thus, the x coordinate of the (whole table) center of mass is
$$x_{\text{com}} = \frac{M\,(+L/2) + M\,(-L/2)}{M + M + 3M} = 0$$
as expected. And the y coordinate of the (whole table) center of mass is
$$y_{\text{com}} = \frac{M\,(-L/2) + M\,(-L/2)}{M + M + 3M} = -\frac{L}{5}$$
so that the whole table center of mass is a small distance $(0.2L)$ directly below the middle of the tabletop.

5. First, we imagine that the small square piece (of mass m) that was cut from the large plate is returned to it so that the large plate is again a complete $6\,\text{m} \times 6\,\text{m}$ square plate (which has its center of mass at the origin). Then we "add" a square piece of "negative mass" $(-m)$ at the appropriate location to obtain what is shown in Fig. 9-24. If the mass of the whole plate is M, then the mass of the small square piece cut from it is obtained from a simple ratio of areas:
$$m = \left(\frac{2.0\,\text{m}}{6.0\,\text{m}}\right)^2 M \implies M = 9m\ .$$
   (a) The x coordinate of the small square piece is $x = 2.0\,\text{m}$ (the middle of that square "gap" in the figure). Thus the x coordinate of the center of mass of the remaining piece is
$$x_{\text{com}} = \frac{(-m)x}{M + (-m)} = \frac{-m(2.0\,\text{m})}{9m - m} = -0.25\ \text{m}\ .$$
   (b) Since the y coordinate of the small square piece is zero, we have $y_{\text{com}} = 0$.

6. We locate the coordinate origin at the lower left corner of the iron side of the composite slab. We orient the x axis along the length of the slab (the 22.0-cm side); the y axis along the width of the slab (the 13.0-cm side); and, the z axis along the height of the slab (the 2.80-cm side). The coordinates for the opposite corner on the aluminum side are then $x = 22.0\,\text{cm}$, $y = 13.0\,\text{cm}$, and $z = 2.80\,\text{cm}$. By symmetry $y_{\text{com}} = 13.0\,\text{cm}/2 = 6.50\,\text{cm}$ and $z_{\text{com}} = 2.80\,\text{cm}/2 = 1.40\,\text{cm}$. We use Eq. 9-5 to find x_{com}:
$$\begin{aligned} x_{\text{com}} &= \frac{m_i x_{\text{com},i} + m_a x_{\text{com},a}}{m_i + m_a} = \frac{\rho_i V_i x_{\text{com},i} + \rho_a V_a x_{\text{cm},a}}{\rho_i V_i + \rho_a V_a} \\ &= \frac{(11.0\,\text{cm}/2)\left(7.85\,\text{g/cm}^3\right) + 3(11.0\,\text{cm}/2)\left(2.70\,\text{g/cm}^3\right)}{7.85\,\text{g/cm}^3 + 2.70\,\text{g/cm}^3} = 8.30\,\text{cm}\ . \end{aligned}$$
Therefore, the center of mass is at $11.0\,\text{cm} - 8.3\,\text{cm} = 2.7\,\text{cm}$ from the midpoint of the slab.

7. By symmetry the center of mass is located on the axis of symmetry of the molecule. We denote the distance between the nitrogen atom and the center of mass of NH_3 as x. Then $m_{\text{N}}x = 3m_{\text{H}}(d - x)$, where d is the distance from the nitrogen atom to the plane containing the three hydrogen atoms:
$$d = \sqrt{(10.14 \times 10^{-11}\,\text{m})^2 - (9.4 \times 10^{-11}\text{m})^2} = 3.803 \times 10^{-11}\,\text{m}\ .$$
Thus,
$$x = \frac{3m_{\text{H}}d}{m_{\text{N}} + 3m_{\text{H}}} = \frac{3(1.00797)(3.803 \times 10^{-11}\text{m})}{14.0067 + 3(1.00797)} = 6.8 \times 10^{-12}\,\text{m}$$
where Appendix F has been used to find the masses.

8. The centers of mass (with centimeters understood) for each of the five sides are as follows:

$(x_1, y_1, z_1) = (0, 20, 20)$	for the side in the yz plane
$(x_2, y_2, z_2) = (20, 0, 20)$	for the side in the xz plane
$(x_3, y_3, z_3) = (20, 20, 0)$	for the side in the xy plane
$(x_4, y_4, z_4) = (40, 20, 20)$	for the remaining side parallel to side 1
$(x_5, y_5, z_5) = (20, 40, 20)$	for the remaining side parallel to side 2

Recognizing that all sides have the same mass m, we plug these into Eq. 9-5 to obtain the results (the first two being expected based on the symmetry of the problem).

(a)

$$x_{\text{com}} = \frac{mx_1 + mx_2 + mx_3 + mx_4 + mx_5}{5m} = \frac{0 + 20 + 20 + 40 + 20}{5} = 20\,\text{cm}$$

(b)

$$y_{\text{com}} = \frac{my_1 + my_2 + my_3 + my_4 + my_5}{5m} = \frac{20 + 0 + 20 + 20 + 40}{5} = 20\,\text{cm}$$

(c)

$$z_{\text{com}} = \frac{mz_1 + mz_2 + mz_3 + mz_4 + mz_5}{5m} = \frac{20 + 20 + 0 + 20 + 20}{5} = 16\,\text{cm}$$

9. (a) Since the can is uniform, its center of mass is at its geometrical center, a distance $H/2$ above its base. The center of mass of the soda alone is at its geometrical center, a distance $x/2$ above the base of the can. When the can is full this is $H/2$. Thus the center of mass of the can and the soda it contains is a distance

$$h = \frac{M(H/2) + m(H/2)}{M + m} = \frac{H}{2}$$

above the base, on the cylinder axis.

(b) We now consider the can alone. The center of mass is $H/2$ above the base, on the cylinder axis.

(c) As x decreases the center of mass of the soda in the can at first drops, then rises to $H/2$ again.

(d) When the top surface of the soda is a distance x above the base of the can, the mass of the soda in the can is $m_p = m(x/H)$, where m is the mass when the can is full ($x = H$). The center of mass of the soda alone is a distance $x/2$ above the base of the can. Hence

$$h = \frac{M(H/2) + m_p(x/2)}{M + m_p} = \frac{M(H/2) + m(x/H)(x/2)}{M + (mx/H)} = \frac{MH^2 + mx^2}{2(MH + mx)} .$$

We find the lowest position of the center of mass of the can and soda by setting the derivative of h with respect to x equal to 0 and solving for x. The derivative is

$$\frac{dh}{dx} = \frac{2mx}{2(MH + mx)} - \frac{(MH^2 + mx^2)m}{2(MH + mx)^2} = \frac{m^2x^2 + 2MmHx - MmH^2}{2(MH + mx)^2} .$$

The solution to $m^2x^2 + 2MmHx - MmH^2 = 0$ is

$$x = \frac{MH}{m}\left(-1 + \sqrt{1 + \frac{m}{M}}\right) .$$

The positive root is used since x must be positive. Next, we substitute the expression found for x into $h = (MH^2 + mx^2)/2(MH + mx)$. After some algebraic manipulation we obtain

$$h = \frac{HM}{m}\left(\sqrt{1 + \frac{m}{M}} - 1\right) .$$

10. Since the center of mass of the two-skater system does not move, both skaters will end up at the center of mass of the system. Let the center of mass be a distance x from the 40-kg skater, then

$$(65\,\text{kg})(10\,\text{m} - x) = (40\,\text{kg})x \quad \Longrightarrow \quad x = 6.2\,\text{m}\ .$$

Thus the 40-kg skater will move by 6.2 m.

11. Let m_c be the mass of the Chrysler and v_c be its velocity. Let m_f be the mass of the Ford and v_f be its velocity. Then the velocity of the center of mass is

$$v_{\text{com}} = \frac{m_c v_c + m_f v_f}{m_c + m_f} = \frac{(2400\,\text{kg})(80\,\text{km/h}) + (1600\,\text{kg})(60\,\text{km/h})}{2400\,\text{kg} + 1600\,\text{kg}} = 72\,\text{km/h}\,.$$

We note that the two velocities are in the same direction, so the two terms in the numerator have the same sign.

12. (a) Since the center of mass of the man-balloon system does not move, the balloon will move downward with a certain speed u relative to the ground as the man climbs up the ladder. The speed of the man relative to the ground is $v_g = v - u$. Thus, the speed of the center of mass of the system is

$$v_{\text{com}} = \frac{m v_g - M u}{M + m} = \frac{m(v-u) - Mu}{M+m} = 0\ .$$

This yields $u = mv/(M+m)$.

(b) Now that there is no relative motion within the system, the speed of both the balloon and the man is equal to v_{com}, which is zero. So the balloon will again be stationary.

13. We use the constant-acceleration equations of Table 2-1 (with $+y$ downward and the origin at the release point), Eq. 9-5 for y_{com} and Eq. 9-17 for $\vec{v}_{\text{com}}$.

(a) The location of the first stone (of mass m_1) at $t = 300\times10^{-3}\,\text{s}$ is $y_1 = (1/2)gt^2 = (1/2)(9.8)\left(300\times10^{-3}\right)^2 = 0.44\,\text{m}$, and the location of the second stone (of mass $m_2 = 2m_1$) at $t = 300\times10^{-3}\,\text{s}$ is $y_2 = (1/2)gt^2 = (1/2)(9.8)(300\times10^{-3} - 100\times10^{-3})^2 = 0.20\,\text{m}$. Thus, the center of mass is at

$$y_{\text{com}} = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2} = \frac{m_1(0.44\,\text{m}) + 2m_1(0.20\,\text{m})}{m_1 + 2m_2} = 0.28\ \text{m}\ .$$

(b) The speed of the first stone at time t is $v_1 = gt$, while that of the second stone is $v_2 = g(t - 100\times10^{-3}\,\text{s})$. Thus, the center-of-mass speed at $t = 300\times10^{-3}\,\text{s}$ is

$$\begin{aligned} v_{\text{com}} &= \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} \\ &= \frac{m_1(9.8)\left(300\times10^{-3}\right) + 2m_1(9.8)\left(300\times10^{-3} - 100\times10^{-3}\right)}{m_1 + 2m_1} \\ &= 2.3\ \text{m/s}\ . \end{aligned}$$

14. We use the constant-acceleration equations of Table 2-1 (with the origin at the traffic light), Eq. 9-5 for x_{com} and Eq. 9-17 for $\vec{v}_{\text{com}}$. At $t = 3.0\,\text{s}$, the location of the automobile (of mass m_1) is $x_1 = \frac{1}{2}at^2 = \frac{1}{2}(4.0\,\text{m/s}^2)(3.0\,\text{s})^2 = 18\,\text{m}$, while that of the truck (of mass m_2) is $x_2 = vt = (8.0\,\text{m/s})(3.0\,\text{s}) = 24\,\text{m}$. The speed of the automobile then is $v_1 = at = \left(4.0\,\text{m/s}^2\right)(3.0\,\text{s}) = 12\,\text{m/s}$, while the speed of the truck remains $v_2 = 8.0\,\text{m/s}$.

(a) The location of their center of mass is

$$x_{\text{com}} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{(1000\,\text{kg})(18\,\text{m}) + (2000\,\text{kg})(24\,\text{m})}{1000\,\text{kg} + 2000\,\text{kg}} = 22\ \text{m}\ .$$

(b) The speed of the center of mass is

$$v_{\text{com}} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = \frac{(1000\,\text{kg})(12\,\text{m/s}) + (2000\,\text{kg})(8.0\,\text{m/s})}{1000\,\text{kg} + 2000\,\text{kg}} = 9.3\,\text{m/s}\,.$$

15. We need to find the coordinates of the point where the shell explodes and the velocity of the fragment that does not fall straight down. The coordinate origin is at the firing point, the $+x$ axis is rightward, and the $+y$ direction is upward. The y component of the velocity is given by $v = v_{0\,y} - gt$ and this is zero at time $t = v_{0\,y}/g = (v_0/g)\sin\theta_0$, where v_0 is the initial speed and θ_0 is the firing angle. The coordinates of the highest point on the trajectory are

$$x = v_{0x}t = v_0 t\cos\theta_0 = \frac{v_0^2}{g}\sin\theta_0\cos\theta_0 = \frac{(20\,\text{m/s})^2}{9.8\,\text{m/s}^2}\sin 60^\circ\cos 60^\circ = 17.7\text{ m}$$

and

$$y = v_{0\,y}\,t - \frac{1}{2}gt^2 = \frac{1}{2}\,\frac{v_0^2}{g}\sin^2\theta_0 = \frac{1}{2}\,\frac{(20\,\text{m/s})^2}{9.8\,\text{m/s}^2}\sin^2 60^\circ = 15.3\text{ m}\ .$$

Since no horizontal forces act, the horizontal component of the momentum is conserved. Since one fragment has a velocity of zero after the explosion, the momentum of the other equals the momentum of the shell before the explosion. At the highest point the velocity of the shell is $v_0\cos\theta_0$, in the positive x direction. Let M be the mass of the shell and let V_0 be the velocity of the fragment. Then $Mv_0\cos\theta_0 = MV_0/2$, since the mass of the fragment is $M/2$. This means

$$V_0 = 2v_0\cos\theta_0 = 2(20\,\text{m/s})\cos 60^\circ = 20\text{ m/s}\ .$$

This information is used in the form of initial conditions for a projectile motion problem to determine where the fragment lands. Resetting our clock, we now analyze a projectile launched horizontally at time $t = 0$ with a speed of 20 m/s from a location having coordinates $x_0 = 17.7\,\text{m}$, $y_0 = 15.3\,\text{m}$. Its y coordinate is given by $y = y_0 - \frac{1}{2}gt^2$, and when it lands this is zero. The time of landing is $t = \sqrt{2y_0/g}$ and the x coordinate of the landing point is

$$x = x_0 + V_0 t = x_0 + V_0\sqrt{\frac{2y_0}{g}} = 17.7\,\text{m} + (20\,\text{m/s})\sqrt{\frac{2(15.3\,\text{m})}{9.8\,\text{m/s}^2}} = 53\text{ m}\ .$$

16. The implication in the problem regarding $\vec{v}_0$ is that the olive and the nut start at rest. Although we could proceed by analyzing the forces on each object, we prefer to approach this using Eq. 9-14. The total force on the nut-olive system is $\vec{F}_{\mathbf{o}} + \vec{F}_{\mathbf{n}} = -\hat{\imath} + \hat{\jmath}$ with the unit newton understood. Thus, Eq. 9-14 becomes

$$-\hat{\imath} + \hat{\jmath} = M\vec{a}_{\text{com}}$$

where $M = 2.0$ kg. Thus, $\vec{a}_{\text{com}} = -\frac{1}{2}\hat{\imath} + \frac{1}{2}\hat{\jmath}$ in SI units. Each component is constant, so we apply the equations discussed in Chapters 2 and 4.

$$\Delta\vec{r}_{\text{com}} = \frac{1}{2}\vec{a}_{\text{com}}t^2 = -4.0\,\hat{\imath} + 4.0\,\hat{\jmath}$$

(in meters) when $t = 4.0$ s. It is perhaps instructive to work through this problem the *long way* (separate analysis for the olive and the nut and then application of Eq. 9-5) since it helps to point out the computational advantage of Eq. 9-14.

17. (a) We place the origin of a coordinate system at the center of the pulley, with the x axis horizontal and to the right and with the y axis downward. The center of mass is halfway between the containers, at $x = 0$ and $y = \ell$, where ℓ is the vertical distance from the pulley center to either of the containers. Since the diameter of the pulley is 50 mm, the center of mass is 25 mm from each container.

(b) Suppose 20 g is transferred from the container on the left to the container on the right. The container on the left has mass $m_1 = 480$ g and is at $x_1 = -25$ mm. The container on the right has mass $m_2 = 520$ g and is at $x_2 = +25$ mm. The x coordinate of the center of mass is then

$$x_{\text{com}} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{(480\,\text{g})(-25\,\text{mm}) + (520\,\text{g})(25\,\text{mm})}{480\,\text{g} + 520\,\text{g}} = 1.0\,\text{mm}\,.$$

The y coordinate is still ℓ. The center of mass is 26 mm from the lighter container, along the line that joins the bodies.

(c) When they are released the heavier container moves downward and the lighter container moves upward, so the center of mass, which must remain closer to the heavier container, moves downward.

(d) Because the containers are connected by the string, which runs over the pulley, their accelerations have the same magnitude but are in opposite directions. If a is the acceleration of m_2, then $-a$ is the acceleration of m_1. The acceleration of the center of mass is

$$a_{\text{com}} = \frac{m_1(-a) + m_2 a}{m_1 + m_2} = a\,\frac{m_2 - m_1}{m_1 + m_2}\ .$$

We must resort to Newton's second law to find the acceleration of each container. The force of gravity $m_1 g$, down, and the tension force of the string T, up, act on the lighter container. The second law for it is $m_1 g - T = -m_1 a$. The negative sign appears because a is the acceleration of the heavier container. The same forces act on the heavier container and for it the second law is $m_2 g - T = m_2 a$. The first equation gives $T = m_1 g + m_1 a$. This is substituted into the second equation to obtain $m_2 g - m_1 g - m_1 a = m_2 a$, so $a = (m_2 - m_1)g/(m_1 + m_2)$. Thus

$$a_{\text{com}} = \frac{g(m_2 - m_1)^2}{(m_1 + m_2)^2} = \frac{(9.8\,\text{m/s}^2)(520\,\text{g} - 480\,\text{g})^2}{(480\,\text{g} + 520\,\text{g})^2} = 1.6 \times 10^{-2}\,\text{m/s}^2\,.$$

The acceleration is downward.

18. We denote the mass of Ricardo as M_R and that of Carmelita as M_C. Let the center of mass of the two-person system (assumed to be closer to Ricardo) be a distance x from the middle of the canoe of length L and mass m. Then $M_R(L/2 - x) = mx + M_C(L/2 + x)$. Now, after they switch positions, the center of the canoe has moved a distance $2x$ from its initial position. Therefore, $x = 40\,\text{cm}/2 = 0.20\,\text{m}$, which we substitute into the above equation to solve for M_C:

$$M_C = \frac{M_R(L/2 - x) - mx}{L/2 + x} = \frac{(80)\left(\frac{3.0}{2} - 0.20\right) - (30)(0.20)}{(3.0/2) + 0.20} = 58\ \text{kg}\ .$$

19. There is no net horizontal force on the dog-boat system, so their center of mass does not move. Therefore by Eq. 9-16,

$$M\Delta x_{\text{com}} = 0 = m_b \Delta x_b + m_d \Delta x_d$$

which implies

$$|\Delta x_b| = \frac{m_d}{m_b}\,|\Delta x_d|\ .$$

Now we express the geometrical condition that *relative to the boat* the dog has moved a distance $d = 2.4$ m:

$$|\Delta x_b| + |\Delta x_d| = d$$

which accounts for the fact that the dog moves one way and the boat moves the other. We substitute for $|\Delta x_b|$ from above:

$$\frac{m_d}{m_b}\,|\Delta x_d| + |\Delta x_d| = d$$

which leads to

$$|\Delta x_d| = \frac{d}{1 + \frac{m_d}{m_b}} = \frac{2.4}{1 + \frac{4.5}{18}} = 1.92\ \text{m}\ .$$

The dog is therefore 1.9 m closer to the shore than initially (where it was 6.1 m from it). Thus, it is now 4.2 m from the shore.

20. We apply Eq. 9-22 ($p = mv$) and Eq. 7-1 ($K = \frac{1}{2}mv^2$).

(a) The speed of the VW Beetle of mass m is

$$v = \frac{p}{m} = \frac{(2650\,\text{kg})(16\,\text{km/h})}{816\,\text{kg}} = 52\,\text{km/h} \ .$$

(b) In this case, the speed of the VW Beetle must be

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(2650\,\text{kg})(16\,\text{km/h})^2/2}{816\,\text{kg}}} = 29\,\text{km/h} \ .$$

21. Using Eq. 9-22, the necessary speed v is

$$v = \frac{p}{m} = \frac{(1600\,\text{kg})(1.2\,\text{km/h})}{80\,\text{kg}} = 24\,\text{km/h} \ .$$

22. The magnitude of the ball's momentum change is

$$\Delta p = |mv_i - mv_f| = (0.70\,\text{kg})\,|5.0\,\text{m/s} - (-2.0\,\text{m/s})| = 4.9\ \text{kg}\cdot\text{m/s} \ .$$

23. (a) The change in kinetic energy is

$$\begin{aligned}\Delta K &= \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 \\ &= \frac{1}{2}(2100\,\text{kg})\left((51\,\text{km/h})^2 - (41\,\text{km/h})^2\right) \\ &= 9.66\times 10^4\,\text{kg}\cdot(\text{km/h})^2\left((10^3\,\text{m/km})(1\,\text{h}/3600\,\text{s})\right)^2 \\ &= 7.5\times 10^4\ \text{J} \ .\end{aligned}$$

(b) The magnitude of the change in velocity is

$$\begin{aligned}|\Delta\vec{v}| &= \sqrt{(-v_i)^2 + (v_f)^2} \\ &= \sqrt{(-41\,\text{km/h})^2 + (51\,\text{km/h})^2} \\ &= 65.4\ \text{km/h}\end{aligned}$$

so the magnitude of the change in momentum is

$$|\Delta\vec{p}| = m\,|\Delta\vec{v}| = (2100\,\text{kg})(65.4\,\text{km/h})\left(\frac{1000\,\text{m/km}}{3600\,\text{s/h}}\right) = 3.8\times 10^4\,\text{kg}\cdot\text{m/s} \ .$$

(c) The vector $\Delta\vec{p}$ points at an angle θ south of east, where

$$\theta = \tan^{-1}\left(\frac{v_i}{v_f}\right) = \tan^{-1}\left(\frac{41\,\text{km/h}}{51\,\text{km/h}}\right) = 39^\circ \ .$$

24. (a) Since the force of impact on the ball is in the y direction, p_x is conserved: $p_{x\,i} = mv_i \sin 30^\circ = p_{x\,f} = mv_i \sin\theta$. Thus $\theta = 30^\circ$.

(b) The momentum change is

$$\begin{aligned}\Delta\vec{p} &= mv_i\cos\theta\,(-\,\hat{\jmath}\,) - mv_i\cos\theta\,(+\,\hat{\jmath}\,) \\ &= -2(0.165\,\text{kg})(2.00\,\text{m/s})(\cos 30^\circ)\,\hat{\jmath} \\ &= -0.572\,\hat{\jmath}\quad \text{kg}\cdot\text{m/s} \ .\end{aligned}$$

25. The velocity of the object is

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}\left((3500 - 160t)\,\hat{\imath} + 2700\,\hat{\jmath} + 300\,\hat{k}\right) = -160\,\hat{\imath}\ \text{m/s}\ .$$

(a) The linear momentum is

$$\vec{p} = m\vec{v} = (250)(-160\,\hat{\imath}) = -4.0 \times 10^4\,\hat{\imath}\ \ \text{kg}\cdot\text{m/s}\ .$$

(b) The object is moving west (our $-\hat{\imath}$ direction).

(c) Since the value of $\vec{p}$ does not change with time, the net force exerted on the object is zero, by Eq. 9-23.

26. We use coordinates with $+x$ horizontally toward the pitcher and $+y$ upward. Angles are measured counterclockwise from the $+x$ axis. Mass, velocity and momentum units are SI. Thus, the initial momentum can be written $\vec{p}_0 = (4.5\ \angle\ 215^\circ)$ in magnitude-angle notation.

(a) In magnitude-angle notation, the momentum change is $(6.0\ \angle -90^\circ) - (4.5\ \angle\ 215^\circ) = (5.0\ \angle -43^\circ)$ (efficiently done with a vector capable calculator in polar mode). The magnitude of the momentum change is therefore 5.0 kg·m/s.

(b) The momentum change is $(6.0\ \angle\ 0^\circ) - (4.5\ \angle\ 215^\circ) = (10\ \angle\ 15^\circ)$. Thus, the magnitude of the momentum change is 10 kg·m/s.

27. No external forces with horizontal components act on the man-stone system and the vertical forces sum to zero, so the total momentum of the system is conserved. Since the man and the stone are initially at rest, the total momentum is zero both before and after the stone is kicked. Let m_s be the mass of the stone and v_s be its velocity after it is kicked; let m_m be the mass of the man and v_m be his velocity after he kicks the stone. Then $m_s v_s + m_m v_m = 0 \ \rightarrow\ v_m = -m_s v_s/m_m$. We take the axis to be positive in the direction of motion of the stone. Then

$$v_m = -\frac{(0.068\,\text{kg})(4.0\,\text{m/s})}{91\,\text{kg}} = -3.0 \times 10^{-3}\ \text{m/s}\ .$$

The negative sign indicates that the man moves in the direction opposite to the direction of motion of the stone.

28. The fact that they are connected by a spring is not used in the solution. We use Eq. 9-17 for $\vec{v}_{\text{com}}$:

$$\begin{aligned} M\vec{v}_{\text{com}} &= m_1\vec{v}_1 + m_2\vec{v}_2 \\ 0 &= (1.0)(1.7) + (3.0)\vec{v}_2 \end{aligned}$$

which yields $|\vec{v}_2| = 0.57$ m/s. The direction of $\vec{v}_2$ is opposite that of $\vec{v}_1$ (that is, they are both headed towards the center of mass, but from opposite directions).

29. No external forces with horizontal components act on the cart-man system and the vertical forces sum to zero, so the total momentum of the system is conserved. Let m_c be the mass of the cart, v be its initial velocity, and v_c be its final velocity (after the man jumps off). Let m_m be the mass of the man. His initial velocity is the same as that of the cart and his final velocity is zero. Conservation of momentum yields $(m_m + m_c)v = m_c v_c$. Consequently, the final speed of the cart is

$$v_c = \frac{v(m_m + m_c)}{m_c} = \frac{(2.3\,\text{m/s})(75\,\text{kg} + 39\,\text{kg})}{39\,\text{kg}} = 6.7\,\text{m/s}\ .$$

The cart speeds up by $6.7 - 2.3 = 4.4\,\text{m/s}$. In order to slow himself, the man gets the cart to push backward on him by pushing forward on it, so the cart speeds up.

30. We apply Eq. 9-17, with $M = \sum m = 1.3$ kg,

$$\begin{aligned} M\vec{v}_{\rm com} &= m_A\vec{v}_A + m_B\vec{v}_B + m_C\vec{v}_C \\ (1.3)\,(-0.40\,\hat{\imath}) &= (0.50)\vec{v}_A + (0.60)\,(0.20\,\hat{\imath}) + (0.20)\,(0.30\,\hat{\imath}) \end{aligned}$$

which leads to $\vec{v}_A = -1.4\,\hat{\imath}$ in SI units (m/s).

31. Our notation is as follows: the mass of the motor is M; the mass of the module is m; the initial speed of the system is v_0; the relative speed between the motor and the module is v_r; and, the speed of the module relative to the Earth is v after the separation. Conservation of linear momentum requires $(M+m)v_0 = mv + M(v - v_r)$. Therefore,

$$v = v_0 + \frac{Mv_r}{M+m} = 4300\,{\rm km/h} + \frac{(4m)(82\,{\rm km/h})}{4m+m} = 4.4\times 10^3\,{\rm km/h}\ .$$

32. Denoting the new speed of the car as v, then the new speed of the man relative to the ground is $v - v_{\rm rel}$. Conservation of momentum requires

$$\left(\frac{W}{g} + \frac{w}{g}\right)v_0 = \left(\frac{W}{g}\right)v + \left(\frac{w}{g}\right)(v - v_{\rm rel})\ .$$

Consequently, the change of velocity is

$$\Delta\vec{v} = v - v_0 = \frac{w\,v_{\rm rel}}{W+w}\ .$$

33. We assume no external forces act on the system composed of the two parts of the last stage. Hence, the total momentum of the system is conserved. Let m_c be the mass of the rocket case and m_p be the mass of the payload. At first they are traveling together with velocity v. After the clamp is released m_c has velocity v_c and m_p has velocity v_p. Conservation of momentum yields $(m_c + m_p)v = m_cv_c + m_pv_p$.

   (a) After the clamp is released the payload, having the lesser mass, will be traveling at the greater speed. We write $v_p = v_c + v_{\rm rel}$, where $v_{\rm rel}$ is the relative velocity. When this expression is substituted into the conservation of momentum condition, the result is

$$(m_c + m_p)\,v = m_cv_c + m_pv_c + m_pv_{\rm rel}\ .$$

   Therefore,

$$\begin{aligned} v_c &= \frac{(m_c+m_p)\,v - m_pv_{\rm rel}}{m_c+m_p} \\ &= \frac{(290.0\,{\rm kg} + 150.0\,{\rm kg})(7600\,{\rm m/s}) - (150.0\,{\rm kg})(910.0\,{\rm m/s})}{290.0\,{\rm kg} + 150.0\,{\rm kg}} \\ &= 7290\ {\rm m/s}\ . \end{aligned}$$

   (b) The final speed of the payload is $v_p = v_c + v_{\rm rel} = 7290\,{\rm m/s} + 910.0\,{\rm m/s} = 8200\,{\rm m/s}$.

   (c) The total kinetic energy before the clamp is released is

$$K_i = \frac{1}{2}\,(m_c+m_p)v^2 = \frac{1}{2}(290.0\,{\rm kg} + 150.0\,{\rm kg})(7600\,{\rm m/s})^2 = 1.271\times 10^{10}\ {\rm J}\ .$$

   (d) The total kinetic energy after the clamp is released is

$$\begin{aligned} K_f &= \frac{1}{2}\,m_cv_c^2 + \frac{1}{2}\,m_pv_p^2 \\ &= \frac{1}{2}(290.0\,{\rm kg})(7290\,{\rm m/s})^2 + \frac{1}{2}(150.0\,{\rm kg})(8200\,{\rm m/s})^2 \\ &= 1.275\times 10^{10}\ {\rm J}\ . \end{aligned}$$

   The total kinetic energy increased slightly. Energy originally stored in the spring is converted to kinetic energy of the rocket parts.

34. Our $+x$ direction is east and $+y$ direction is north. The linear momenta for the two $m = 2.0\,\mathrm{kg}$ parts are then

$$\vec{p}_1 = m\vec{v}_1 = mv_1\,\hat{\jmath}$$

where $v_1 = 3.0\,\mathrm{m/s}$, and

$$\vec{p}_2 = m\vec{v}_2 = m(v_{2x}\,\hat{\imath} + v_{2y}\,\hat{\jmath}) = mv_2(\cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath})$$

where $v_2 = 5.0\,\mathrm{m/s}$ and $\theta = 30°$. The combined linear momentum of both parts is then

$$\begin{aligned}
\vec{P} &= \vec{p}_1 + \vec{p}_2 \\
&= mv_1\,\hat{\jmath} + mv_2\left(\cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath}\right) = (mv_2\cos\theta)\,\hat{\imath} + (mv_1 + mv_2\sin\theta)\,\hat{\jmath} \\
&= (2.0\,\mathrm{kg})(5.0\,\mathrm{m/s})(\cos 30°)\hat{\imath} + (2.0\,\mathrm{kg})\,(3.0\,\mathrm{m/s} + (5.0\,\mathrm{m/s})(\sin 30°))\,\hat{\jmath} \\
&= \left(8.66\,\hat{\imath} + 11\,\hat{\jmath}\right)\ \mathrm{kg{\cdot}m/s}\ .
\end{aligned}$$

From conservation of linear momentum we know that this is also the linear momentum of the whole kit before it splits. Thus the speed of the 4.0-kg kit is

$$v = \frac{P}{M} = \frac{\sqrt{P_x^2 + P_y^2}}{M} = \frac{\sqrt{(8.66\,\mathrm{kg{\cdot}m/s})^2 + (11\,\mathrm{kg{\cdot}m/s})^2}}{4.0\,\mathrm{kg}} = 3.5\ \mathrm{m/s}\ .$$

35. We establish a coordinate system with the origin at the position of initial nucleus of mass $m_{m\,i}$ (which was stationary), with the electron momentum $\vec{p}_e$ in the $-x$ direction and the neutrino momentum $\vec{p}_\nu$ in the $-y$ direction. We will use unit-vector notation, although the problem does not specifically request it.

(a) We find the momentum $\vec{p}_{n\,r}$ of the residual nucleus from momentum conservation.

$$\begin{aligned}
\vec{p}_{n\,i} &= \vec{p}_e + \vec{p}_\nu + \vec{p}_{n\,r} \\
0 &= -1.2\times10^{-22}\,\hat{\imath} - 6.4\times10^{-23}\,\hat{\jmath} + \vec{p}_{n\,r}
\end{aligned}$$

Thus, $\vec{p}_{n\,r} = 1.2\times10^{-22}\,\hat{\imath} + 6.4\times10^{-23}\,\hat{\jmath}$ in SI units (kg·m/s). Its magnitude is

$$|\vec{p}_{n\,r}| = \sqrt{(1.2\times10^{-22})^2 + (6.4\times10^{-23})^2} = 1.4\times10^{-22}\ \mathrm{kg{\cdot}m/s}\ .$$

(b) The angle measured from the $+x$ axis to $\vec{p}_{n\,r}$ is

$$\theta = \tan^{-1}\left(\frac{6.4\times10^{-23}}{1.2\times10^{-22}}\right) = 28°\ .$$

Therefore, the angle between $\vec{p}_e$ (which is in the $-x$ direction) and $\vec{p}_{n\,r}$ is $180° - 28° \approx 150°$.

(c) Measuring clockwise (but not using the "traditional" minus sign with that sense) we find the angle between $\vec{p}_{n\,r}$ and $\vec{p}_\nu$ (which points in the $-y$ direction) is $90° + 28° \approx 120°$.

(d) Combining the two equations $p = mv$ and $K = \frac{1}{2}mv^2$, we obtain (with $p = p_{n\,r}$ and $m = m_{n\,r}$)

$$K = \frac{p^2}{2m} = \frac{\left(1.4\times10^{-22}\right)^2}{2\,(5.8\times10^{-26})} = 1.6\times10^{-19}\ \mathrm{J}\ .$$

36. This problem involves both mechanical energy conservation

$$U_i = K_1 + K_2$$

where $U_i = 60$ J, and momentum conservation

$$0 = m_1\vec{v}_1 + m_2\vec{v}_2$$

where $m_2 = 2m_1$. From the second equation, we find $|\vec{v}_1| = 2\,|\vec{v}_2|$ which in turn implies (since $v_1 = |\vec{v}_1|$ and likewise for v_2)

$$K_1 = \frac{1}{2}m_1v_1^2 = \frac{1}{2}\left(\frac{1}{2}m_2\right)(2v_2)^2 = 2\left(\frac{1}{2}m_2v_2^2\right) = 2K_2 \ .$$

(a) We substitute $K_1 = 2K_2$ into the energy conservation relation and find

$$U_i = 2K_2 + K_2 \quad\Longrightarrow\quad K_2 = \frac{1}{3}U_i = 20 \text{ J} \ .$$

(b) And we obtain $K_1 = 2(20) = 40$ J.

37. Our notation is as follows: the mass of the original body is $M = 20.0$ kg; its initial velocity is $\vec{v}_0 = 200\hat{\imath}$ in SI units (m/s); the mass of one fragment is $m_1 = 10.0$ kg; ; its velocity is $\vec{v}_1 = 100\hat{\jmath}$ in SI units; the mass of the second fragment is $m_2 = 4.0$ kg; ; its velocity is $\vec{v}_2 = -500\hat{\imath}$ in SI units; and, the mass of the third fragment is $m_3 = 6.00$ kg.

(a) Conservation of linear momentum requires

$$M\vec{v}_0 = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3$$

which (using the above information) leads to

$$\vec{v}_3 = 1000\,\hat{\imath} - 167\,\hat{\jmath}$$

in SI units. The magnitude of $\vec{v}_3$ is $v_3 = \sqrt{1000^2 + (-167)^2} = 1.01 \times 10^3$ m/s. It points at $\tan^{-1}(-167/1000) = -9.48°$ (that is, at 9.5° measured clockwise from the $+x$ axis).

(b) We are asked to calculate ΔK or

$$\left(\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + \frac{1}{2}m_3v_3^2\right) - \frac{1}{2}Mv_0^2 = 3.23 \times 10^6 \text{ J} \ .$$

38. Our notation (and, implicitly, our choice of coordinate system) is as follows: the mass of the original body is m; its initial velocity is $\vec{v}_0 = v\,\hat{\imath}$; the mass of the less massive piece is m_1; ; its velocity is $\vec{v}_1 = 0$; and, the mass of the more massive piece is m_2. We note that the conditions $m_2 = 3m_1$ (specified in the problem) and $m_1 + m_2 = m$ generally assumed in classical physics (before Einstein) lead us to conclude

$$m_1 = \frac{1}{4}\,m \quad\text{and}\quad m_2 = \frac{3}{4}\,m \ .$$

Conservation of linear momentum requires

$$\begin{aligned} m\vec{v}_0 &= m_1\vec{v}_1 + m_2\vec{v}_2 \\ mv\,\hat{\imath} &= 0 + \frac{3}{4}\,m\,\vec{v}_2 \end{aligned}$$

which leads to

$$\vec{v}_2 = \frac{4}{3}\,v\;\hat{\imath} \ .$$

The increase in the system's kinetic energy is therefore

$$\begin{aligned} \Delta K &= \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 - \frac{1}{2}mv_0^2 \\ &= 0 + \frac{1}{2}\left(\frac{3}{4}\,m\right)\left(\frac{4}{3}\,v\right)^2 - \frac{1}{2}mv^2 \\ &= \frac{1}{6}\,m\,v^2 \ . \end{aligned}$$

39. Our notation (and, implicitly, our choice of coordinate system) is as follows: the mass of one piece is $m_1 = m$; ; its velocity is $\vec{v}_1 = -30\,\hat{\imath}$ in SI units (m/s); the mass of the second piece is $m_2 = m$; ; its velocity is $\vec{v}_2 = -30\,\hat{\jmath}$ in SI units; and, the mass of the third piece is $m_3 = 3m$. Conservation of linear momentum requires

$$\begin{aligned} m\vec{v}_0 &= m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 \\ 0 &= m\,(-30\,\hat{\imath}) + m\left(-30\,\hat{\jmath}\right) + 3m\vec{v}_3 \end{aligned}$$

which leads to

$$\vec{v}_3 = 10\,\hat{\imath} + 10\,\hat{\jmath}$$

in SI units. Its magnitude is $v_3 = 10\sqrt{2} \approx 14$ m/s and its angle is 45° counterclockwise from $+x$ (in this system where we have m_1 flying off in the $-x$ direction and m_2 flying off in the $-y$ direction).

40. One approach is to choose a *moving* coordinate system which travels the center of mass of the body, and another is to do a little extra algebra analyzing it in the original coordinate system (in which the speed of the $m = 8.0$ kg mass is $v_0 = 2$ m/s, as given). Our solution is in terms of the latter approach since we are assuming that this is the approach most students would take. Conservation of linear momentum (along the direction of motion) requires

$$\begin{aligned} mv_0 &= m_1v_1 + m_2v_2 \\ (8.0)(2.0) &= (4.0)v_1 + (4.0)v_2 \end{aligned}$$

which leads to

$$v_2 = 4 - v_1$$

in SI units (m/s). We require

$$\begin{aligned} \Delta K &= \left(\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2\right) - \frac{1}{2}mv_0^2 \\ 16 &= \left(\frac{1}{2}(4.0)v_1^2 + \frac{1}{2}(4.0)v_2^2\right) - \frac{1}{2}(8.0)(2.0)^2 \end{aligned}$$

which simplifies to

$$v_2^2 = 16 - v_1^2$$

in SI units. If we substitute for v_2 from above, we find

$$(4 - v_1)^2 = 16 - v_1^2$$

which simplifies to

$$2v_1^2 - 8v_1 = 0$$

and yields either $v_1 = 0$ or $v_1 = 4$ m/s. If $v_1 = 0$ then $v_2 = 4 - v_1 = 4$ m/s, and if $v_1 = 4$ then $v_2 = 0$. Stated more simply, one of the chunks has zero speed and the other has a velocity of 4.0 m/s (along the original direction of motion).

41. We use Eq. 9-43. Then

$$\begin{aligned} v_f &= v_i + v_{\rm rel}\ln\frac{M_i}{M_f} \\ &= 105\,\text{m/s} + (253\,\text{m/s})\ln\frac{6090\,\text{kg}}{6010\,\text{kg}} \\ &= 108\ \text{m/s}\ . \end{aligned}$$

42. (a) We use Eq. 9-42. The thrust is

$$\begin{aligned} R\,v_{\text{rel}} &= Ma \\ &= \left(4.0\times10^4\,\text{kg}\right)\left(2.0\,\text{m/s}^2\right) \\ &= 8.0\times10^4\ \text{N}\ . \end{aligned}$$

(b) Since $v_{\text{rel}} = 3000$ m/s, we see from part (a) that $R \approx 27$ kg/s.

43. (a) The thrust of the rocket is given by $T = Rv_{\text{rel}}$, where R is the rate of fuel consumption and v_{rel} is the speed of the exhaust gas relative to the rocket. For this problem $R = 480\,\text{kg/s}$ and $v_{\text{rel}} = 3.27\times10^3\,\text{m/s}$, so

$$T = (480\,\text{kg/s})(3.27\times10^3\,\text{m/s}) = 1.57\times10^6\ \text{N}\ .$$

(b) The mass of fuel ejected is given by $M_{\text{fuel}} = R\Delta t$, where Δt is the time interval of the burn. Thus, $M_{\text{fuel}} = (480\,\text{kg/s})(250\,\text{s}) = 1.20\times10^5\,\text{kg}$. The mass of the rocket after the burn is $M_f = M_i - M_{\text{fuel}} = 2.55\times10^5\,\text{kg} - 1.20\times10^5\,\text{kg} = 1.35\times10^5\,\text{kg}$.

(c) Since the initial speed is zero, the final speed is given by

$$v_f = v_{\text{rel}}\ \ln\frac{M_i}{M_f} = \left(3.27\times10^3\right)\ln\left(\frac{2.55\times10^5}{1.35\times10^5}\right) = 2.08\times10^3\ \text{m/s}\ .$$

44. We use Eq. 9-43 and simplify with $v_i = 0$, $v_f = v$, and $v_{\text{rel}} = u$.

$$v_f - v_i = v_{\text{rel}}\ \ln\frac{M_i}{M_f} \quad\Longrightarrow\quad \frac{M_i}{M_f} = e^{v/u}$$

(a) If $v = u$, we obtain $\frac{M_i}{M_f} = e^1 \approx 2.7$.

(b) If $v = 2u$, we obtain $\frac{M_i}{M_f} = e^2 \approx 7.4$.

45. We use Eq. 9-43 and simplify with $v_f - v_i = \Delta v$, and $v_{\text{rel}} = u$.

$$v_f - v_i = v_{\text{rel}}\ \ln\frac{M_i}{M_f} \quad\Longrightarrow\quad \frac{M_f}{M_i} = e^{-\Delta v/u}$$

If $\Delta v = 2.2$ m/s and $u = 1000$ m/s, we obtain $\frac{M_i - M_f}{M_i} = 1 - e^{-0.0022} \approx 0.0022$.

46. We convert mass rate to SI units: $R = 540/60 = 9.00$ kg/s. In the absence of the asked-for additional force, the car would decelerate with a magnitude given by Eq. 9-42:

$$R\,v_{\text{rel}} = M\,|a|$$

so that if $a = 0$ is desired then the additional force must have a magnitude equal to $R\,v_{\text{rel}}$ (so as to cancel that effect).

$$F = R\,v_{\text{rel}} = (9.00)(3.20) = 28.8\ \text{N}\ .$$

47. (a) We consider what must happen to the coal that lands on the faster barge during one minute ($\Delta t = 60\,\text{s}$). In that time, a total of $m = 1000\,\text{kg}$ of coal must experience a change of velocity

$$\Delta v = 20\,\text{km/h} - 10\,\text{km/h} = 10\,\text{km/h} = 2.8\ \text{m/s}$$

where rightwards is considered the positive direction. The rate of change in momentum for the coal is therefore

$$\frac{\Delta\vec{p}}{\Delta t} = \frac{m\Delta\vec{v}}{\Delta t} = \frac{(1000)(2.8)}{60} = 46\ \text{N}$$

which, by Eq. 9-23, must equal the force exerted by the (faster) barge on the coal. The processes (the shoveling, the barge motions) are constant, so there is no ambiguity in equating $\frac{\Delta p}{\Delta t}$ with $\frac{dp}{dt}$.

(b) The problem states that the frictional forces acting on the barges does not depend on mass, so the loss of mass from the slower barge does not affect its motion (so no extra force is required as a result of the shoveling).

48. (a) The thrust is Rv_{rel} where $v_{\text{rel}} = 1200$ m/s. For this to equal the weight Mg where $M = 6100$ kg, we must have $R = (6100)(9.8)/1200 \approx 50$ kg/s.

(b) Using Eq. 9-42 with the additional effect due to gravity, we have

$$Rv_{\text{rel}} - Mg = Ma$$

so that requiring $a = 21$ m/s^2 leads to $R = (6100)(9.8 + 21)/1200 = 1.6 \times 10^2$ kg/s.

49. (a) We assume his mass is between $m_1 = 50$ kg and $m_2 = 70$ kg (corresponding to a weight between 110 lb and 154 lb). His increase in gravitational potential energy is therefore in the range

$$\begin{aligned} m_1 gh \le \quad & \Delta U \quad \le m_2 gh \\ 2 \times 10^5 \le \quad & \Delta U \quad \le 3 \times 10^5 \end{aligned}$$

in SI units (J), where $h = 443$ m.

(b) The problem only asks for the amount of internal energy which converts into gravitational potential energy, so this result is the same as in part (a). But if we were to consider his *total* internal energy "output" (much of which converts to heat) we can expect that external climb is quite different from taking the stairs.

50. (a) The (internal) energy the climber must convert to gravitational potential energy is

$$\Delta U = mgh = (90)(9.8)(8850) = 7.8 \times 10^6 \text{ J} .$$

(b) The number of candy bars this corresponds to is

$$N = \frac{7.8 \times 10^6 \text{ J}}{1.25 \times 10^6 \text{ J/bar}} \approx 6 \text{ bars} .$$

51. (a) The acceleration of the sprinter is (using Eq. 2-15)

$$a = \frac{2\Delta x}{t^2} = \frac{(2)(7.0\,\text{m})}{(1.6\,\text{s})^2} = 5.47 \text{ m/s}^2 .$$

Consequently, the speed at $t = 1.6$ s is

$$v = at = \left(5.47\,\text{m/s}^2\right)(1.6\,\text{s}) = 8.8 \text{ m/s} .$$

Alternatively, Eq. 2-17 could be used.

(b) The kinetic energy of the sprinter (of weight w and mass $m = w/g$) is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}\left(\frac{w}{g}\right)v^2 = \frac{(670)(8.8)^2}{2(9.8)} = 2.6 \times 10^3 \text{ J} .$$

(c) The average power is

$$P_{\text{avg}} = \frac{\Delta K}{\Delta t} = \frac{2.6 \times 10^3\,\text{J}}{1.6\,\text{s}} = 1.6 \times 10^3 \text{ W} .$$

52. We use $P = Fv$ (Eq. 7-48) to compute the force:

$$F = \frac{P}{v} = \frac{92 \times 10^6\,\text{W}}{(32.5\,\text{knot})\left(1.852\,\frac{\text{km/h}}{\text{knot}}\right)\left(\frac{1000\,\text{m/km}}{3600\,\text{s/h}}\right)} = 5.5 \times 10^6\,\text{N} .$$

53. To swim at constant velocity the swimmer must push back against the water with a force of 110 N. Relative to him the water is going at 0.22 m/s toward his rear, in the same direction as his force. Using Eq. 7-48, his power output is obtained:

$$P = \vec{F} \cdot \vec{v} = Fv = (110\,\text{N})(0.22\,\text{m/s}) = 24\ \text{W}\ .$$

54. The initial kinetic energy of the automobile of mass m moving at speed v_i is $K_i = \frac{1}{2}mv_i^2$, where $m = 16400/9.8 = 1673$ kg. Using Eq. 8-29 and Eq. 8-31, this relates to the effect of friction force f in stopping the auto over a distance d by

$$K_i = fd$$

where the road is assumed level (so $\Delta U = 0$). Thus,

$$d = \frac{K_i}{f} = \frac{mv_i^2}{2f} = \frac{(1673\,\text{kg})\left((113\,\text{km/h})\left(\frac{1000\,\text{m/km}}{3600\,\text{s/h}}\right)\right)^2}{2(8230\,\text{N})} = 100\ \text{m}\ .$$

55. (a) By combining Newton's second law $F - mg = ma$ (where F is the force exerted up on her by the floor) and Eq. 2-16 $v^2 = 2ad_1$ (where $d_1 = 0.90 - 0.40 = 0.50$ m is the distance her center of mass moves while her feet are on the floor) it is straightforward to derive the equation

$$K_{\text{launch}} = (F - mg)d_1$$

where $K_{\text{launch}} = \frac{1}{2}mv^2$ is her kinetic energy as her feet leave the floor. We mention this method of deriving that equation (which also follows from the work-kinetic energy theorem Eq. 7-10, or – suitably interpreted – from energy conservation as expressed by Eq. 8-31) since the energy approaches might seem paradoxical (one might sink into the quagmire of questions such as "how can the floor possibly provide energy to the person?"); the Newton's law approach leads to no such quandaries. Next, her feet leave the floor and this kinetic energy is converted to gravitational potential energy. Then mechanical energy conservation leads straightforwardly to

$$K_{\text{launch}} = mgd_2$$

where $d_2 = 1.20 - 0.90 = 0.30$ m is the distance her center of mass rises from the time her feet leave the floor to the time she reaches the top of her leap. Now we combine these two equations and solve $(F - mg)d_1 = mgd_2$ for the force:

$$F = \frac{mg(d_1 + d_2)}{d_1} = \frac{(55\,\text{kg})(9.8\,\text{m/s}^2)(0.50\,\text{m} + 0.30\,\text{m})}{0.50\,\text{m}} = 860\ \text{N}\ .$$

(b) She has her maximum speed at the time her feet leave the floor (this is her "launch" speed). Consequently, the equation derived above becomes

$$\frac{1}{2}mv^2 = (F - mg)d_1$$

from which we obtain

$$\begin{aligned} v &= \sqrt{\frac{2(F - mg)d_1}{m}} \\ &= \sqrt{\frac{2\left(860\,\text{N} - (55\,\text{kg})\left(9.8\,\text{m/s}^2\right)\right)(0.50\,\text{m})}{55\,\text{kg}}} \\ &= 2.4\ \text{m/s}\ . \end{aligned}$$

56. (a) The kinetic energy K of the automobile of mass m at $t = 30\,\mathrm{s}$ is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(1500\,\mathrm{kg})\left((72\,\mathrm{km/h})\left(\frac{1000\,\mathrm{m/km}}{3600\,\mathrm{s/h}}\right)\right)^2 = 3.0\times 10^5\ \mathrm{J}\ .$$

(b) The average power required is

$$P_{\mathrm{avg}} = \frac{\Delta K}{\Delta t} = \frac{3.0\times 10^5\,\mathrm{J}}{30\,\mathrm{s}} = 1.0\times 10^4\ \mathrm{W}\ .$$

(c) We use Eq. 7-48 ($P = Fv$) for the instantaneous power delivered at t. Since the acceleration a is constant, the power is $P = Fv = mav = ma(at) = ma^2t$, using Eq. 2-11. By contrast, from part (b), the average power is $P_{\mathrm{avg}} = \frac{mv^2}{2t}$ which becomes $\frac{1}{2}ma^2t$ when $v = at$ is again utilized. Thus, the instantaneous power at the end of the interval is twice the average power during it:

$$P = 2P_{\mathrm{avg}} = (2)\left(1.0\times 10^4\,\mathrm{W}\right) = 2.0\times 10^4\ \mathrm{W}\ .$$

57. (a) With $P = 1.5\,\mathrm{MW} = 1.5\times 10^6\,\mathrm{W}$ (assumed constant) and $t = 6.0\,\mathrm{min} = 360\,\mathrm{s}$, the work-kinetic energy theorem (along with Eq. 7-48) becomes

$$W = Pt = \Delta K = \frac{1}{2}m\left(v_f^2 - v_i^2\right)\ .$$

The mass of the locomotive is then

$$m = \frac{2Pt}{v_f^2 - v_i^2} = \frac{(2)(1.5\times 10^6\,\mathrm{W})(360\,\mathrm{s})}{(25\,\mathrm{m/s})^2 - (10\,\mathrm{m/s})^2} = 2.1\times 10^6\,\mathrm{kg}\,.$$

(b) With t arbitrary, we use $Pt = \frac{1}{2}m\left(v^2 - v_i^2\right)$ to solve for the speed $v = v(t)$ as a function of time and obtain

$$v(t) = \sqrt{v_i^2 + \frac{2Pt}{m}} = \sqrt{(10)^2 + \frac{(2)(1.5\times 10^6)t}{2.1\times 10^6}} = \sqrt{100 + 1.5t}$$

in SI units (v in m/s and t in s).

(c) Using Eq. 7-48, the force $F(t)$ as a function of time is

$$F(t) = \frac{P}{v(t)} = \frac{1.5\times 10^6}{\sqrt{100 + 1.5t}}$$

in SI units (F in N and t in s).

(d) The distance d the train moved is given by

$$d = \int_0^t v(t')\,dt' = \int_0^{360}\left(100 + \frac{3}{2}t\right)^{\frac{1}{2}}\,dt = \frac{4}{9}\left(100 + \frac{3}{2}t\right)^{\frac{3}{2}}\Bigg|_0^{360}$$

which yields 6.7×10^3 m.

58. We work this in SI units and convert to horsepower in the last step. Thus,

$$v = (80\,\mathrm{km/h})\left(\frac{1000\,\mathrm{m/km}}{3600\,\mathrm{s/h}}\right) = 22.2\ \mathrm{m/s}\ .$$

The force F_P needed to propel the car (of weight w and mass $m = w/g$) is found from Newton's second law:

$$F_{\mathrm{net}} = F_P - F = ma = \frac{wa}{g}$$

where $F = 300 + 1.8v^2$ in SI units. Therefore, the power required is

$$\begin{aligned} P &= \vec{F}_P \cdot \vec{v} \\ &= \left(F + \frac{wa}{g}\right) v \\ &= \left(300 + 1.8(22.2)^2 + \frac{(12000)(0.92)}{9.8}\right)(22.2) \\ &= 5.14 \times 10^4 \text{ W} \\ &= \left(5.14 \times 10^4 \text{ W}\right)\left(\frac{1 \text{ hp}}{746 \text{ W}}\right) = 69 \text{ hp} . \end{aligned}$$

59. The third-to-last statement in the problem about the peeling-off rate of the top layer and the thickening rate of the bottom layer is best interpreted, we feel, in the rest frame of the layer. Thus, imagining that we are in a reference frame moving up at v_t, then it is clear from the uniform nature of the described peeling-off of the top and thickening of the bottom that *in this moving reference frame* the center of mass of the layer must move downward with a speed $2v_f$ (if the rates were denoted R and were different then this would be $R_{\text{bottom}} + R_{\text{top}}$). Returning to the original reference frame, where we see the trapped bubbles rising at v_t, we find (with $+y$ upward) the center of mass velocity is

$$v_{\text{com}} = v_t - 2v_f = -1.5 \text{ cm/s} .$$

60. (a) Since the initial momentum is zero, then the final momenta must add (in the vector sense) to 0. Therefore, with SI units understood, we have

$$\begin{aligned} \vec{p}_3 &= -\vec{p}_1 - \vec{p}_2 \\ &= -m_1\vec{v}_1 - m_2\vec{v}_2 \\ &= -\left(16.7 \times 10^{-27}\right)\left(6.00 \times 10^6\,\hat{\imath}\right) - \left(8.35 \times 10^{-27}\right)\left(-8.00 \times 10^6\,\hat{\jmath}\right) \\ &= -1.00 \times 10^{-19}\,\hat{\imath} + 0.67 \times 10^{-19}\,\hat{\jmath} \ \text{kg}\cdot\text{m/s} . \end{aligned}$$

(b) Dividing by $m_3 = 11.7 \times 10^{-27}$ kg and using Pythagorean's theorem we find the speed of the third particle to be $v_3 = 1.03 \times 10^7$ m/s. The total amount of kinetic energy is

$$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + \frac{1}{2}m_3v_3^2 = 1.19 \times 10^{-12} \text{ J} .$$

61. By conservation of momentum, the final speed v of the sled satisfies

$$(2900 \text{ kg})(250 \text{ m/s}) = (2900 \text{ kg} + 920 \text{ kg})v$$

which gives $v = 190$ m/s.

62. We denote the mass of the car as M and that of the sumo wrestler as m. Let the initial velocity of the sumo wrestler be $v_0 > 0$ and the final velocity of the car be v. We apply the momentum conservation law.

(a) From $mv_0 = (M + m)v$ we get

$$v = \frac{mv_0}{M + m} = \frac{(242 \text{ kg})(5.3 \text{ m/s})}{2140 \text{ kg} + 242 \text{ kg}} = 0.54 \text{ m/s} .$$

(b) Since $v_{\text{rel}} = v_0$, we have

$$mv_0 = Mv + m\left(v + v_{\text{rel}}\right) = mv_0 + (M + m)v$$

and obtain $v = 0$ for the final speed of the flatcar.

(c) Now $mv_0 = Mv + m(v - v_{\rm rel})$, which leads to

$$v = \frac{m(v_0 + v_{\rm rel})}{m+M} = \frac{(242\,{\rm kg})(5.3\,{\rm m/s} + 5.3\,{\rm m/s})}{242\,{\rm kg} + 2140\,{\rm kg}} = 1.1 \text{ m/s} .$$

63. (a) We use coordinates with $+x$ eastward and $+y$ northward, and employ magnitude-angle notation which is well suited for computations with vector-capable calculators. Positive angles are measured counterclockwise from the $+x$ axis (negative angles are clockwise). Length is in meters and time is in seconds. The mass of each piece is designated m. Thus, the conservation of momentum becomes

$$\begin{aligned} \vec{p}_0 &= \vec{p}_1 + \vec{p}_2 + \vec{p}_3 \\ \vec{p}_0 &= m(7.0 \angle 90^\circ) + m(4.0 \angle 210^\circ) + m(4.0 \angle -30^\circ) \\ \vec{p}_0 &= m(3.0 \angle 90^\circ) \end{aligned}$$

which implies that the velocity of the package had magnitude $|\vec{p}|/(3m) = 1.0$ m/s and was directed north.

(b) The center of mass proceeds at $1.0\,$m/s unaffected by the explosion. Its displacement during the 3.0 s interval is $(1.0\,{\rm m/s})(3.0\,{\rm s}) = 3.0$ m. The displacement is directed north, in accordance with its velocity.

64. The width ℓ of the pyramid measured at variable height z is seen to decrease from L at the base (where $z = 0$) to zero at the top (where $z = H$). This is a linear decrease, so we must have

$$\ell = L\left(1 - \frac{z}{H}\right) .$$

If we imagine the pyramid layered into a large number N of horizontal (square) slabs (each of thickness Δz) then the volume of each slab is $V' = \ell^2 \Delta z$ and the mass of each slab is $m' = \rho V' = \rho \ell^2 \Delta z$. If we make the continuum approximation ($N \to \infty$ while $\Delta z \to dz$) and substitute from above for ℓ, the mass element becomes

$$dm = \rho L^2 \left(1 - \frac{z}{H}\right)^2 dz .$$

We note, for later use, that the total mass M is given by $\rho L^2 H/3$ using the volume relation mentioned in the problem, but this can also be derived by integrating the above expression for dm.

(a) Using Eq. 9-9 we find

$$z_{\rm com} = \frac{1}{M}\int z\, dm = \frac{3}{\rho L^2 H}\int_0^H z\rho L^2 \left(1 - \frac{z}{H}\right)^2 dz$$

where ρ and L^2 are constants (and, in fact, cancel) so we obtain

$$z_{\rm com} = \frac{3}{H}\int_0^H \left(z - \frac{2z^2}{H} + \frac{z^3}{H}\right) dz = \frac{H}{4} = 36.8 \text{ m} .$$

(b) Although we could do the integral $\int dU = \int gz\, dm$ to find the work done against gravity, it is easier to use the conclusion drawn in the book that this should be equivalent to lifting a point mass M to height $z_{\rm com}$.

$$W = \Delta U = Mgz_{\rm com} = \left(\frac{\rho L^2 H}{3}\right) g\,\frac{H}{4} = 1.7 \times 10^{12} \text{ J} .$$

65. Although it is expected that the boat will have a slight downward recoil (of brief duration) from the upward component of the father's leap, the problem's intent is to concentrate only on the horizontal components, since – if the effects of friction are small – the boat can continue moving horizontally for a significant time. Mass, velocity and momentum units are SI. We use coordinates with $+x$ eastward and

$+y$ northward. Angles are positive if measured counterclockwise from the $+x$ axis. Using magnitude-angle notation, momentum conservation is expressed as

$$\begin{aligned} \vec{p}_0 &= \vec{p}_\mathrm{c} + \vec{p}_\mathrm{f} + \vec{p}_\mathrm{b} \\ (0 \angle 0°) &= (80 \angle 0°) + (90 \angle -90°) + \vec{p}_\mathrm{b} \end{aligned}$$

where it must be stressed that the relevant component of the father's momentum is $\vec{p}_\mathrm{f} = (75)(1.5)\cos 37°$ *south* (represented as $(90 \angle -90°)$ in the expression above). Thus, we obtain $\vec{p}_\mathrm{b} = (120 \angle 132°)$, which implies that the boat's (horizontal) velocity is $|\vec{p}|/m = 120/100 = 1.2$ m/s at an angle of 132° counterclockwise from east; this can also be expressed as 48° north of west.

66. (a) Ignoring air friction amounts to assuming that the ball has the same speed v when it returns to its original height.

$$K_i = K_f = \frac{1}{2}mv^2 = \frac{1}{2}(0.050\,\mathrm{kg})(16\,\mathrm{m/s})^2 = 6.4\ \mathrm{J}\ .$$

(b) The momentum at the moment it is thrown (taking $+y$ upward) is

$$|\vec{p}_i| = |\vec{p}_f| = mv = (0.050\,\mathrm{kg})(16\,\mathrm{m/s}) = 0.80\ \mathrm{kg\cdot m/s}\ .$$

The vector $\vec{p}_i$ is $\theta = 30°$ above the horizontal, while $\vec{p}_f$ is 30° below the horizontal (since the vertical component is now downward). We note for later reference that the magnitude of the change in momentum is

$$|\Delta\vec{p}| = |\vec{p}_f - \vec{p}_i| = 2mv\sin\theta = 0.80\ \mathrm{kg\cdot m/s}$$

and $\Delta\vec{p}$ points vertically downward.

(c) The time of flight for the ball is $t = 2v_i \sin\theta/g$, thus

$$mgt = mg\left(\frac{2v\sin\theta}{g}\right) = 2mv\sin\theta = 2p_i\sin\theta = 0.80\ \mathrm{kg\cdot m/s}$$

which (recalling our result in part (b)) illustrates the relation $|\Delta p| = Ft$ where $F = mg$.

67. Choosing downward as the $+y$ direction and placing the coordinate origin at the top of the building, we apply the equations from Table 2-1 to this two-block system:

$$\begin{aligned} y_1 &= \frac{1}{2}gt^2 && \text{for} \quad 0 \le t \le 5 \\ y_2 &= \frac{1}{2}g(t-1)^2 && \text{for} \quad 1 \le t \le 6 \\ v_1 &= gt && \text{for} \quad 0 \le t \le 5 \\ v_2 &= g(t-1) && \text{for} \quad 1 \le t \le 6 \end{aligned}$$

with SI units understood.

(a) With $m_1 = 2.00$ kg and $m_2 = 3.00$ kg, Eq. 9-5 provides

$$y_\mathrm{com} = \frac{m_1y_1 + m_2y_2}{m_1 + m_2} = \frac{1}{2}gt^2 - \frac{3}{5}gt + \frac{3}{10}g$$

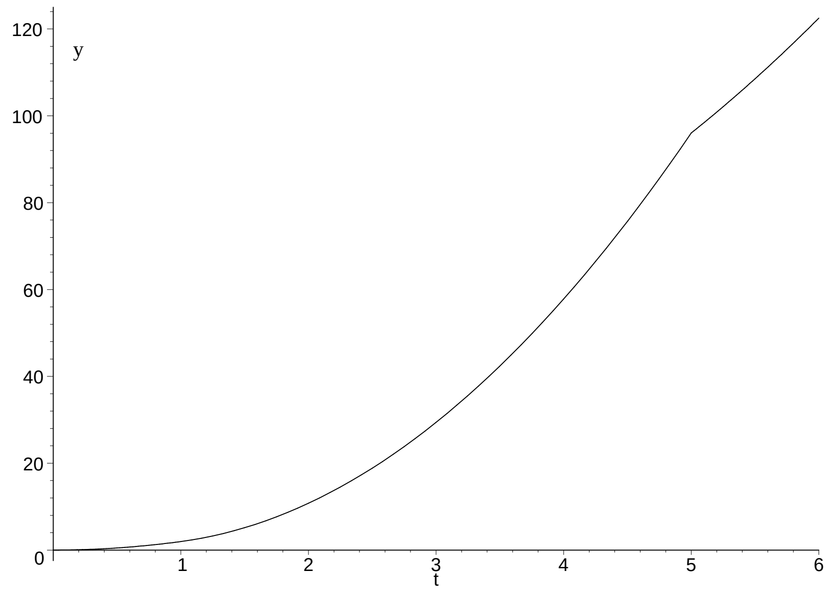
while they are both in free fall ($1 \le t \le 5$). But during the interval when m_2 is "waiting" at the top of the building, we have

$$y_\mathrm{com} = \frac{m_1y_1 + m_2(0)}{m_1 + m_2} = \frac{1}{5}gt^2 \quad \text{for } 0 \le t \le 1$$

and during the interval where m_1 is sitting on the ground (at $y = \frac{1}{2}(9.8)(5)^2$) we have

$$y_{\text{com}} = \frac{m_1\left(\frac{25g}{2}\right) + m_2 y_2}{m_1 + m_2} = \frac{3}{10}gt^2 - \frac{3}{5}gt + \frac{53}{10}g$$

for $5 \le t \le 6$. This behavior is plotted below, with y_{com} in meters and t in seconds.



(b) We turn now to Eq. 9-17 which gives

$$v_{\text{com}} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = gt - \frac{3}{5}g$$

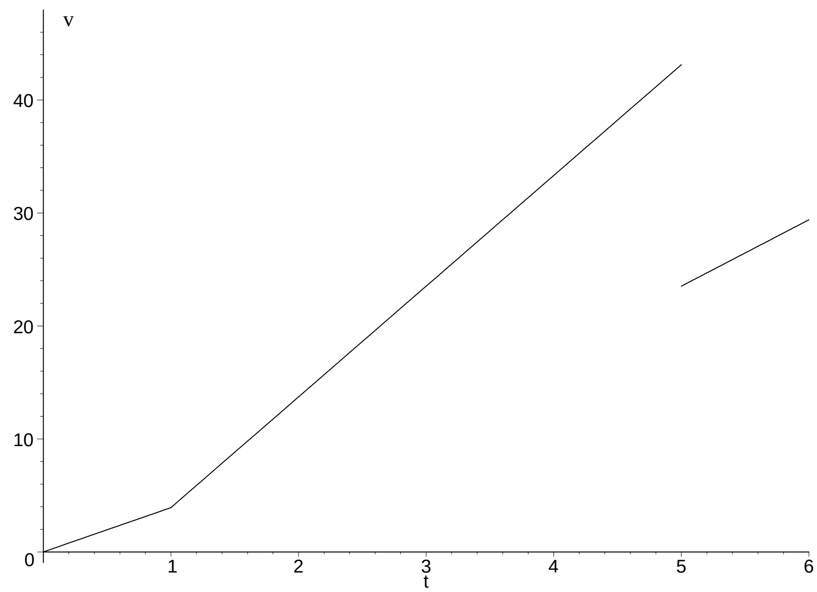
while they are both in free fall ($1 \le t \le 5$). We note that we could have easily gotten this by taking the derivative of the corresponding y_{com} expression in part (a). During the interval when m_2 sits on the top of the building, we have

$$v_{\text{com}} = \frac{m_1 v_1 + m_2(0)}{m_1 + m_2} = \frac{2}{5}gt \quad \text{for } 0 \le t \le 1$$

and during the interval where m_1 sits on the ground we have

$$v_{\text{com}} = \frac{m_1(0) + m_2 v_2}{m_1 + m_2} = \frac{3}{5}gt - \frac{3}{5}g$$

for $5 \le t \le 6$. This behavior is plotted below, with v_{com} in m/s and t in s. The sudden drop at $t = 5$ s is understandable, since m_1 stops, but it should be noted that we are ignoring the dynamics of how the ground decelerates that block – the effects of which might be to (slightly) smooth out that transition.



68. The velocity of the first particle (of mass $m_1 = 3.0$ kg) is $\vec{v}_1 = -6.0\hat{\jmath}$ m/s while that of the second one (of mass $m_2 = 4.0$ kg) is $\vec{v}_2 = 7.0\,\hat{\imath}$ m/s. The center-of-mass velocity is then

$$\vec{v}_{\text{com}} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2} = \frac{(3.0)(-6.0\,\hat{\jmath}\,) + (4.0)(7.0\,\hat{\imath}\,)}{3.0 + 4.0} = -2.6\,\hat{\imath} + 4.0\,\hat{\jmath}$$

in SI units. The corresponding speed is

$$v_{\text{com}} = \sqrt{v_x^2 + v_y^2} = \sqrt{(-2.6)^2 + (4.0)^2} = 4.8 \text{ m/s} \ .$$

69. We use Eq. 9-17, or – equivalently – we differentiate Eq. 9-5.

$$\begin{aligned} v_{\text{com}\,x} &= \frac{1}{M}\left((1500\,\text{kg})(0 \text{ m/s}) + (4000\,\text{kg})v_{\text{truck}}\right) \\ v_{\text{com}\,y} &= \frac{1}{M}\left((1500\,\text{kg})v_{\text{car}} + (4000\,\text{kg})(0\,\text{m/s})\right) \end{aligned}$$

where $M = 5500$ kg. From $v_{\text{com}\,x} = (11)\cos 55° = 6.3$ m/s and $v_{\text{com}\,y} = (11)\sin 55° = 9.0$ m/s, we get the following results for v_{truck} and v_{car} from the above formulas.

(a) $v_{\text{car}} = 33$ m/s.

(b) $v_{\text{truck}} = 8.7$ m/s.

70. (a) We use Eq. 7-48:

$$P = Fv \quad \Longrightarrow \quad F = \frac{16.0\,\text{kW}}{15.0\,\text{m/s}} = 1.07 \text{ kN} \ .$$

(b) We add to our previous result the downhill pull of gravity $mg\sin\theta$ where $\theta = \tan^{-1}(8/100)$.

$$F' = 1.07 \times 10^3 + (1710)(9.8)\sin 4.57° = 2.40 \times 10^3$$

in SI units (N). Therefore,

$$P' = F'v = (2.40\,\text{kN}))(15.0\,\text{m/s}) = 36 \text{ kW} \ .$$

(c) For the engine to be off but the (downhill) velocity to remain constant, the downhill component of gravity must equal the magnitude of the retarding forces:

$$mg\sin\theta = F\ .$$

Using F from part (a), we find $\theta = 3.65°$ which corresponds to $\tan\theta = 0.0638 \approx 6.4\%$.

71. Using Eq. 2-15 for both object j (the jelly jar) and object p (the peanut butter), with $y = 0$ designating the base of the building in both cases, we have

$$\begin{aligned} y_j &= 40t - \frac{1}{2}gt^2 \\ y_p - 50 &= 0 - \frac{1}{2}gt^2 \end{aligned}$$

with SI units understood. Thus, using Eq. 9-5, the center of mass of this system is at

$$y_{\text{com}} = \frac{1}{3.0\text{ kg}}\left((1.0\text{ kg})y_j + (2.0\text{ kg})y_p\right) = \frac{100}{3} + \frac{40}{3}t - \frac{1}{2}gt^2\ .$$

(a) With $t = 3.0$ s, the above equation gives $y_{\text{com}} = 29$ m.

(b) We maximize y_{com} by working through the condition

$$\frac{dy_{\text{com}}}{dt} = 0 = \frac{40}{3} - gt\ .$$

Thus, we find $t = 1.4$ s, which produces $y_{\text{com}} = 42$ m as its highest value.

72. (a) We denote the mass of the car (and cannon) as M (excluding that of the cannonballs) and the mass of all the cannonballs as m. For concreteness, we assume that before firing all the cannonballs are at the front (left side of Fig. 9-52) of the car, which we choose to be the origin of the x axis; we choose $+x$ rightward. The coordinate of the center of mass of the car-cannonball system is

$$x_{\text{com}} = \frac{(0)m + \left(\frac{L}{2}\right)M}{M+m} = \frac{LM}{2(M+m)}\ .$$

After the firing, we assume all the cannonballs are at the other end of the car; the train will have moved (in the negative x direction) by a distance d, at which time

$$x_{\text{com}} = \frac{\left(\frac{L}{2} - d\right)M + (L-d)m}{M+m}\ .$$

Equating the two expressions, we obtain $d = \frac{mL}{M+m} < L$. If $m \gg M$, the distance d can be very close to (but can never exceed) L. Thus $d_{\text{max}} = L$.

(b) After each impact, there is no relative motion in the system; thus, the final speed of the car is equal to that of the center of mass of the system, which is zero.

73. Let the velocity of the shell (of mass m_s) relative to the ground be $\vec{v}_s$, the recoiling velocity of the cannon (of mass m_c) be $\vec{v}_c$ (pointed in our $-x$ direction), and the velocity of the shell relative to the muzzle be $\vec{v}_s'$, where $\vec{v}_s\,' + \vec{v}_c = \vec{v}_s$. In component form, this becomes

$$\begin{aligned} v_s'\cos 39.0° - v_c &= v_{sx} \\ v_s'\sin 39.0° &= v_{sy} \end{aligned}$$

where $v_c = |\vec{v}_c|$. Conservation of linear momentum in the horizontal direction provides us with the additional relation $m_s v_{sx} = m_c v_c$. We solve these equations for the components of $\vec{v}_s$:

$$\begin{aligned} v_{sx} &= \frac{m_c v_s'\cos 39.0°}{m_s + m_c} = \frac{(1400\,\text{kg})(556\,\text{m/s})\cos 39.0°}{1400\,\text{kg} + 70.0\,\text{kg}} = 412\,\text{m/s} \\ v_{sy} &= v_s'\sin 39.0° = (556\,\text{m/s})(\sin 39.0°) = 350\,\text{m/s}\ . \end{aligned}$$

(a) The speed of the shell relative to the Earth is then

$$v_s = \sqrt{v_{sx}^2 + v_{sy}^2} = \sqrt{412^2 + 350^2} = 540 \text{ m/s} .$$

(b) The angle (relative to a stationary observer) at which the shell is fired is given by

$$\theta = \tan^{-1}\left(\frac{v_{sy}}{v_{sx}}\right) = \tan^{-1}\left(\frac{350}{412}\right) = 40.4^\circ .$$

74. The value 0.368 comes from rounding off e^{-1}. We will use e^{-1} in our solution. The speed of the rocket v as a function of the instantaneous rocket mass M' is given by $v = v_{\text{rel}} \ln(M/M')$ (Eq. 9-43 with $v_i = 0$). Thus, when $M' = e^{-1} M$, the speed of the fuel as measured by observers in the initial reference frame (defined when the rocket was at rest with $M' = M$) is

$$v_{\text{fuel}} = v - v_{\text{rel}} = v_{\text{rel}}\left(\ln\frac{M}{M'} - 1\right) = v(\ln\left(\frac{1}{e^{-1}}\right) - 1) = 0 .$$

75. We use momentum conservation choosing $+x$ forward and recognizing that the initial momentum is zero. We analyze this from the point of view of an observer at rest on the ice.

(a) If $v_{1\text{ and }2}$ is the speed of the stones, then the speeds are related by $v_{1\text{ and }2} + v_{\text{boat}} = v_{\text{rel}}$. Thus, with $m_1 = 2m_2$ and $M = 12m_2$, we obtain

$$\begin{aligned} 0 &= (m_1 + m_2)(-v_{1\text{ and }2}) + Mv_{\text{boat}} \\ &= (2m_2 + m_2)(-v_{\text{rel}} + v_{\text{boat}}) + 12m_2v_{\text{boat}} \\ &= -3m_2v_{\text{rel}} + 15m_2v_{\text{boat}} \end{aligned}$$

which yields $v_{\text{boat}} = \frac{1}{5}v_{\text{rel}} = 0.2000v_{\text{rel}}$.

(b) Using $v_1 + v'_{\text{boat}} = v_{\text{rel}}$, we find – as a result of the first throw – the boat's speed:

$$\begin{aligned} 0 &= m_1(-v_1) + (M + m_2)v'_{\text{boat}} \\ &= 2m_2(-v_{\text{rel}} + v'_{\text{boat}}) + (12m_2 + m_2)v'_{\text{boat}} \\ &= -2m_2v_{\text{rel}} + 15m_2v'_{\text{boat}} \end{aligned}$$

which yields $v'_{\text{boat}} = \frac{2}{15}v_{\text{rel}} \approx 0.133v_{\text{rel}}$. Then, using $v_2 + v_{\text{boat}} = v_{\text{rel}}$, we consider the second throw:

$$\begin{aligned} (M + m_2)v'_{\text{boat}} &= m_2(-v_2) + Mv_{\text{boat}} \\ (12m_2 + m_2)\left(\frac{2}{15}v_{\text{rel}}\right) &= m_2(-v_{\text{rel}} + v_{\text{boat}}) + 12m_2v_{\text{boat}} \\ \frac{26}{15}m_2v_{\text{rel}} &= -m_2v_{\text{rel}} + 13m_2v_{\text{boat}} \end{aligned}$$

which yields $v_{\text{boat}} = \frac{41}{195}v_{\text{rel}} \approx 0.2103v_{\text{rel}}$.

(c) Finally, using $v_2 + v'_{\text{boat}} = v_{\text{rel}}$, we find – as a result of the first throw – the boat's speed:

$$\begin{aligned} 0 &= m_2(-v_2) + (M + m_1)v'_{\text{boat}} \\ &= m_2(-v_{\text{rel}} + v'_{\text{boat}}) + (12m_2 + 2m_2)v'_{\text{boat}} \\ &= -m_2v_{\text{rel}} + 15m_2v'_{\text{boat}} \end{aligned}$$

which yields $v'_{\text{boat}} = \frac{1}{15}v_{\text{rel}} \approx 0.0673v_{\text{rel}}$. Then, using $v_1 + v_{\text{boat}} = v_{\text{rel}}$, we consider the second throw:

$$\begin{aligned} (M + m_1)v'_{\text{boat}} &= m_1(-v_1) + Mv_{\text{boat}} \\ (12m_2 + 2m_2)\left(\frac{1}{15}v_{\text{rel}}\right) &= 2m_2(-v_{\text{rel}} + v_{\text{boat}}) + 12m_2v_{\text{boat}} \\ \frac{14}{15}m_2v_{\text{rel}} &= -2m_2v_{\text{rel}} + 14m_2v_{\text{boat}} \end{aligned}$$

which yields $v_{\text{boat}} = \frac{22}{105}\, v_{\text{rel}} \approx 0.2095 v_{\text{rel}}$.

76. (a) It is clear from the problem that $\vec{v}_{\text{air,plane}} = -180\,\hat{\imath}$ m/s where $+\hat{\imath}$ is the plane's direction of motion (relative to the ground).

(b) Let ΔM_a be the mass of air taken in and ejected and let ΔM_f be the mass of fuel ejected in time Δt. From the viewpoint of ground-based observers, the initial velocity of the air is zero and its final velocity is $v - u$, where u is the exhaust speed (labeled v_{rel} in the textbook) and v is the velocity of the plane. The initial velocity of the fuel is v and its final velocity is $v - u$. The velocity of the plane changes from v to $v + \Delta v$ over this time interval. The change in the total momentum of the plane-fuel-air system is $\Delta P = M_p \Delta v + \Delta M_f(u) + \Delta M_a(u - v)$ so the net external force is

$$\frac{\Delta P}{\Delta t} = M_p \frac{\Delta v}{\Delta t} - u\,\frac{\Delta M_f}{\Delta t} + (v - u)\,\frac{\Delta M_a}{\Delta t} \ .$$

We examine some of these terms individually. The $v\,\Delta M_a/\Delta t$ term gives the magnitude of the force on the plane due to air intake (most easily seen from the point of view of observers on the plane) and is equal to $(180)(70) = 1.3 \times 10^4$ N.

(c) We interpret the question as asking for the force due to ejection of both the air and the combustion products due to consuming the fuel. This corresponds then to the $u\,\Delta M_a/\Delta t$ and $u\,\Delta M_f/\Delta t$ terms above, and is equal to $(490)(70 + 2.9) = 3.6 \times 10^4$ N.

(d) We require $\Delta P/\Delta t = 0$ since this (the air, plane and fuel) forms an isolated system (Eq. 9-29). Therefore, our equation above leads to

$$M_p \frac{\Delta v}{\Delta t} = u\,\frac{\Delta M_f}{\Delta t} + (u - v)\,\frac{\Delta M_a}{\Delta t}$$

with all the terms on the right hand side constituting the net thrust (compare Eq. 9-42). These are the values (with appropriate signs) found in parts (b) and (c), so we obtain $3.6 \times 10^4 - 1.3 \times 10^4 = 2.3 \times 10^4$ N.

(e) Using Eq. 7-48, we multiply the net thrust by the plane speed and obtain $\left(2.3 \times 10^4\right)(180) = 4.2 \times 10^6$ W.

77. Using Eq. 9-5, we have

$$\begin{aligned} x_{\text{com}} &= 0 = \frac{1}{M}\left((4.0\text{ kg})(0\text{ m}) + (3.0\text{ kg})(3.0\text{ m}) + (2.0\text{ kg})x\right) \\ y_{\text{com}} &= 0 = \frac{1}{M}\left((4.0\text{ kg})(2.0\text{ m}) + (3.0\text{ kg})(1.0\text{ m}) + (2.0\text{ kg})y\right) \end{aligned}$$

where $M = 9.0$ kg.

(a) Evaluating the above, we find $x = -4.5$ m.

(b) And we find $y = -5.5$ m.

78. (a) We use Eq. 9-5 to compute the center of mass coordinates.

$$\begin{aligned} x_{\text{com}} &= \frac{(4\,\text{kg})(0) + (3\,\text{kg})(7\,\text{m}) + (5\,\text{kg})(3\,\text{m})}{4\,\text{kg} + 3\,\text{kg} + 5\,\text{kg}} = 3.00\text{ m} \\ y_{\text{com}} &= \frac{(4\,\text{kg})(0) + (3\,\text{kg})(3\,\text{m}) + (5\,\text{kg})(2\,\text{m})}{4\,\text{kg} + 3\,\text{kg} + 5\,\text{kg}} = 1.58\text{ m} \end{aligned}$$

(b) Using Eq. 9-17 and SI units, we obtain

$$\begin{aligned} \vec{v} &= \frac{(4\,\text{kg})\left(1.5\,\hat{\imath} - 2.5\,\hat{\jmath}\right) + (3\,\text{kg})(0) + (5\,\text{kg})\left(2.0\,\hat{\imath} - 1.0\,\hat{\jmath}\right)}{4\,\text{kg} + 3\,\text{kg} + 5\,\text{kg}} \\ &= 1.33\,\hat{\imath} - 1.25\,\hat{\jmath}\ \text{ m/s} \ . \end{aligned}$$

(c) Multiplying the previous result by the total mass yields $\vec{P} = 16.0\,\hat{\imath} - 15.0\,\hat{\jmath}$ in SI units (kg·m/s). This can also be gotten by adding up the individual momenta.

79. Although we do not show graphs here, we do jot down down an idea for each part.

(a) Find the center of mass of a rod in which the density is not uniform. If the rod extends along the x axis from the origin to $x = 5$ m, then with mass-per-unit-length (as a function of x) equal to e^{-x} in SI units, use Eq. 9-9 to find x_{com}. *A sketch of the solution is*

$$x_{\text{com}} = \frac{1}{M}\int_0^5 xe^{-x}dx \approx 37 \text{ m}$$

where M was figured from $\int_0^5 e^{-x}dx \approx 1$ kg.

(b) A firecracker is dropped from a height of 20 m. Halfway down it explodes into two identical pieces. As a result of the explosion, one of the pieces is (momentarily) at rest. What is the speed of the other piece immediately after the explosion? *A sketch of the solution is*

$$v_{\text{firecrack}} = \sqrt{2g(10\,\text{m})} = 14 \text{ m/s}$$

and we use momentum conservation:

$$mv_{\text{firecrack}} = \frac{m}{2}v_{\text{piece}} \quad\Longrightarrow\quad v_{\text{piece}} = 28 \text{ m/s} .$$

(c) An 80 kg person is climbing a ladder at a steady rate of 25 cm/s. If we assume his total power output P is three times his rate of gaining gravitational potential energy, then compute P. *The solution is*

$$P = 3\frac{\Delta U}{\Delta t} = 3\frac{mg\Delta y}{\Delta t} \approx 590 \text{ J}$$

where $\Delta t = 1$ s and $\Delta y = 0.25$ m.

(d) Unlike the ideal physics of point particles moving through a vacuum, a runner cannot continue at a constant velocity effortlessly. If a runner's total power output is 650 W while running at 5.8 m/s, then what is the force retarding him (which includes several friction-related effects)? *The solution is*

$$P = Fv = \implies F = \frac{P}{v} \approx 110 \text{ N} .$$

80. (First problem in **Cluster**)

(a) The length of each of the tall sides is $\ell = \sqrt{H^2 + (B/2)^2}$, so that the total length of the wire is $L = 2\sqrt{H^2 + (B/2)^2} + B$. If A is the cross-section area and ρ is the density, then the total mass of the wire is $M = \rho AL$ and the mass of each of the tall sides is

$$m_\ell = \rho A\sqrt{H^2 + (B/2)^2} = M\frac{\sqrt{H^2 + (B/2)^2}}{2\sqrt{H^2 + (B/2)^2} + B} .$$

It is clear by symmetry that $x_{\text{com}} = B/2$ for the system, but the value of y_{com} is not obvious. Note that the base does not contribute to this computation:

$$y_{\text{com}} = \frac{1}{M}\left(m_\ell\frac{H}{2} + m_\ell\frac{H}{2}\right)$$

which can be 'simplified' to the following form.

$$y_{\text{com}} = \frac{H}{2 + \frac{B}{\sqrt{H^2+(B/2)^2}}}$$

(b) The element of mass on the left-hand tall side is related to $d\ell = \sqrt{dx^2 + dy^2}$ and ultimately to the individual coordinate elements (since $dy = (2H/B)dx$):

$$dm_\ell = \rho A d\ell = \begin{cases} \rho A\sqrt{1 + (2H/B)^2}\, dx \\ \rho A\sqrt{(B/2H)^2 + 1}\, dy \end{cases}$$

where $\rho A = m_\ell/\sqrt{H^2 + (B/2)^2}$ (see part (a)). Therefore, using Eq. 9-9, we have

$$\begin{aligned} x_{\text{com}} &= \frac{1}{m_\ell}\int_0^{B/2} x\,\frac{m_\ell}{\sqrt{H^2+(B/2)^2}}\sqrt{1+(2H/B)^2}\,dx \\ &= \frac{\sqrt{1+(2H/B)^2}}{\sqrt{H^2+(B/2)^2}}\int_0^{B/2} x\,dx \\ &= \frac{2}{B}\frac{\sqrt{(B/2)^2+H^2}}{\sqrt{H^2+(B/2)^2}}\left(\frac{B^2}{8}-0\right) \\ &= \frac{B}{4} \quad \text{and} \\ y_{\text{com}} &= \frac{1}{m_\ell}\int_0^{H} y\,\frac{m_\ell}{\sqrt{H^2+(B/2)^2}}\sqrt{(B/2H)^2+1}\,dy \\ &= \frac{\sqrt{(B/2H)^2+1}}{\sqrt{H^2+(B/2)^2}}\int_0^{H} y\,dy \\ &= \frac{1}{H}\frac{\sqrt{(B/2)^2+H^2}}{\sqrt{H^2+(B/2)^2}}\left(\frac{H^2}{2}-0\right) \\ &= \frac{H}{2}\ . \end{aligned}$$

81. (Second problem in **Cluster**)
It is clear by symmetry that $x_{\text{com}} = B/2$ for the system, but the value of y_{com} is not obvious. If the thickness is Δz and the density is ρ, then the relation between the mass element dm and a height element dy is

$$dm = \rho\Delta z \ell_y\, dy = \frac{M}{A_\triangle}\ell_y\, dy$$

where the area of the triangle is $A_\triangle = \frac{1}{2}BH$ and the length of each horizontal "strip" at height y is $\ell_y = B(1 - y/H)$. Therefore, using Eq. 9-9, we have

$$\begin{aligned} y_{\text{com}} &= \frac{1}{M}\int_0^H y\,\frac{M}{A_\triangle}B\left(1-\frac{y}{H}\right)dy \\ &= \frac{B}{\frac{1}{2}BH}\int_0^H y\left(1-\frac{y}{H}\right)dy \\ &= \frac{2}{H}\left(\frac{H^2}{2}-\frac{H^3}{3H}\right) \\ &= \frac{H}{3}\ . \end{aligned}$$

82. (Third problem in **Cluster**)
It is clear by symmetry that $x_{\text{com}} = B/2$ for the system, but the value of y_{com} is not obvious. If the cross-section area of the wire is A and the density is ρ, then in one quadrant the relation between the mass element dm and height element dy is

$$dm = \rho A\frac{R}{\sqrt{R^2-y^2}}\,dy = \frac{M}{\ell_\cap}\frac{R}{\sqrt{R^2-y^2}}\,dy$$

where the length of the semicircle is $\ell_\cap = \pi R$. To include the contributions from both quadrants shown, we multiply by 2, and Eq. 9-9 becomes

$$\begin{aligned} y_{\rm com} &= \frac{2}{M}\int_0^R y\,\frac{M}{\ell_\cap}\,\frac{R}{\sqrt{R^2-y^2}}\,dy \\ &= \frac{2}{\pi}\int_0^R \frac{y}{\sqrt{R^2-y^2}}\,dy \\ &= \frac{2}{\pi}\left[-\sqrt{R^2-y^2}\right]_0^R \\ &= \frac{2R}{\pi}\,. \end{aligned}$$

83. (Fourth problem in **Cluster**)
It is clear by symmetry that $x_{\rm com} = B/2$ for the system. The value of $y_{\rm com}$ is found as follows. If the thickness is Δz and the density is ρ, then the relation between the mass element dm and a height element dy is

$$dm = \rho\Delta z \ell_y\,dy = \frac{M}{A}\ell_y\,dy$$

where the area of the semicircle is $A = \frac{1}{2}\pi R^2$ and the length of each horizontal "strip" at height y is $\ell_y = 2\sqrt{R^2-y^2}$. Therefore, using Eq. 9-9, we find

$$\begin{aligned} y_{\rm com} &= \frac{1}{M}\int_0^R y\,\frac{M}{A}\,2\sqrt{R^2-y^2}\,dy \\ &= \frac{2}{\frac{1}{2}\pi R^2}\int_0^R y\,\sqrt{R^2-y^2}\,dy \\ &= \frac{4}{\pi R^2}\left[-\frac{1}{3}\left(R^2-y^2\right)^{3/2}\right]_0^R \\ &= \frac{4R}{3\pi}\,. \end{aligned}$$