Chapter 17

1. (a) The angular wave number is

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{1.80\,\text{m}} = 3.49\ \text{m}^{-1}\ .$$

(b) The speed of the wave is

$$v = \lambda f = \frac{\lambda\omega}{2\pi} = \frac{(1.8\,\text{m})(110\,\text{rad/s})}{2\pi} = 31.5\ \text{m/s}\ .$$

2. (a) For visible light

$$f_{\text{min}} = \frac{c}{\lambda_{\text{max}}} = \frac{3.0 \times 10^8\,\text{m/s}}{700 \times 10^{-9}\,\text{m}} = 4.3 \times 10^{14}\ \text{Hz}$$

and

$$f_{\text{max}} = \frac{c}{\lambda_{\text{min}}} = \frac{3.0 \times 10^8\,\text{m/s}}{400 \times 10^{-9}\,\text{m}} = 7.5 \times 10^{14}\ \text{Hz}\ .$$

(b) For radio waves

$$\lambda_{\text{min}} = \frac{c}{\lambda_{\text{max}}} = \frac{3.0 \times 10^8\,\text{m/s}}{300 \times 10^6\,\text{Hz}} = 1.0\,\text{m}$$

and

$$\lambda_{\text{max}} = \frac{c}{\lambda_{\text{min}}} = \frac{3.0 \times 10^8\,\text{m/s}}{1.5 \times 10^6\,\text{Hz}} = 2.0 \times 10^2\ \text{m}\ .$$

(c) For X rays

$$f_{\text{min}} = \frac{c}{\lambda_{\text{max}}} = \frac{3.0 \times 10^8\,\text{m/s}}{5.0 \times 10^{-9}\,\text{m}} = 6.0 \times 10^{16}\,\text{Hz}$$

and

$$f_{\text{max}} = \frac{c}{\lambda_{\text{min}}} = \frac{3.0 \times 10^8\,\text{m/s}}{1.0 \times 10^{-11}\,\text{m}} = 3.0 \times 10^{19}\ \text{Hz}\ .$$

3. (a) The motion from maximum displacement to zero is one-fourth of a cycle so 0.170 s is one-fourth of a period. The period is $T = 4(0.170\,\text{s}) = 0.680\,\text{s}$.

(b) The frequency is the reciprocal of the period:

$$f = \frac{1}{T} = \frac{1}{0.680\,\text{s}} = 1.47\ \text{Hz}\ .$$

(c) A sinusoidal wave travels one wavelength in one period:

$$v = \frac{\lambda}{T} = \frac{1.40\,\text{m}}{0.680\,\text{s}} = 2.06\ \text{m/s}\ .$$

4. Since the wave is traveling in the $-x$ direction, the argument of the trig function is $kx + \omega t$ instead of $kx - \omega t$ (as in Eq. 17-2).

$$\begin{aligned} y(x,t) &= y_\mathrm{m} \sin(kx + \omega t) = y_\mathrm{m} \sin\left[2\pi f\left(\frac{x}{v} + t\right)\right] \\ &= (0.010\,\mathrm{m}) \sin\left[2\pi(550\,\mathrm{Hz})\left(\frac{x}{330\,\mathrm{m/s}} + t\right)\right] \\ &= 0.010\,\mathrm{m} \sin[\pi(3.33x + 1100t)] \end{aligned}$$

where x is in meters and t is in seconds.

5. We substitute $\omega = kv$ into $y = y_m \sin(kx - \omega t)$ to obtain

$$y = y_m \sin(kx - kvt) = y_m \sin k(x - vt) \ .$$

We put $k = 2\pi/\lambda$ and $\omega = 2\pi f$ into $y = y_m \sin(kx - \omega t)$ and obtain

$$y = y_m \sin\left(\frac{2\pi x}{\lambda} - 2\pi f t\right) = y_m \sin 2\pi\left(\frac{x}{\lambda} - ft\right) \ .$$

When we substitute $k = \omega/v$ into $y = y_m \sin(kx - \omega t)$, we find

$$y = y_m \sin\left(\frac{\omega x}{v} - \omega t\right) = y_m \sin\omega\left(\frac{x}{v} - t\right) \ .$$

Finally, we substitute $k = 2\pi/\lambda$ and $\omega = 2\pi/T$ into $y = y_m \sin(kx - \omega t)$ to get

$$y = y_m \sin\left(\frac{2\pi x}{\lambda} - \frac{2\pi t}{T}\right) = y_m \sin 2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right) \ .$$

6. (a) The amplitude is $y_m = 6.0\,\mathrm{cm}$.
   (b) We find λ from $2\pi/\lambda = 0.020\pi$: $\lambda = 100\,\mathrm{cm}$.
   (c) Solving $2\pi f = \omega = 4.0\pi$, we obtain$f = 2.0\,\mathrm{Hz}$.
   (d) The wavespeed is $v = \lambda f = (100\,\mathrm{cm})(2.0\,\mathrm{Hz}) = 200\,\mathrm{cm/s}$.
   (e) The wave propagates in the negative x direction, since the argument of the trig function is $kx + \omega t$ instead of $kx - \omega t$ (as in Eq. 17-2).
   (f) The maximum transverse speed (found from the time derivative of y) is

$$u_\mathrm{max} = 2\pi f y_m = \left(4.0\pi\,\mathrm{s}^{-1}\right)(6.0\,\mathrm{cm}) = 75\ \mathrm{cm/s} \ .$$

   (g) $y(3.5\,\mathrm{cm}, 0.26\,\mathrm{s}) = (6.0\,\mathrm{cm}) \sin[0.020\pi(3.5) + 4.0\pi(0.26)] = -2.0\,\mathrm{cm}$.

7. (a) We write the expression for the displacement in the form $y(x,t) = y_m \sin(kx - \omega t)$. A negative sign is used before the ωt term in the argument of the sine function because the wave is traveling in the positive x direction. The angular wave number k is $k = 2\pi/\lambda = 2\pi/(0.10\,\mathrm{m}) = 62.8\,\mathrm{m}^{-1}$ and the angular frequency is $\omega = 2\pi f = 2\pi(400\,\mathrm{Hz}) = 2510\,\mathrm{rad/s}$. Here λ is the wavelength and f is the frequency. The amplitude is $y_m = 2.0\,\mathrm{cm}$. Thus

$$y(x,t) = (2.0\,\mathrm{cm}) \sin\left(\left(62.8\,\mathrm{m}^{-1}\right)x - \left(2510\,\mathrm{s}^{-1}\right)t\right) \ .$$

   (b) The (transverse) speed of a point on the cord is given by taking the derivative of y:

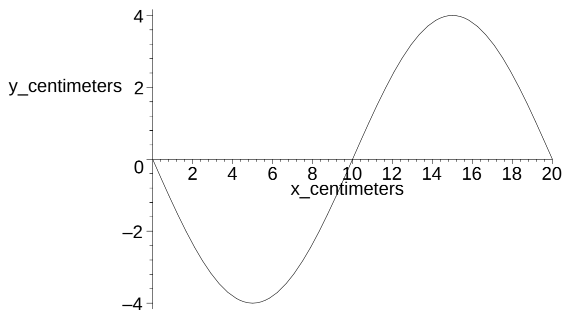
$$u(x,t) = \frac{\partial y}{\partial t} = -\omega y_m \cos(kx - \omega t)$$

   which leads to a maximum speed of $u_m = \omega y_m = (2510\,\mathrm{rad/s})(0.020\,\mathrm{m}) = 50\,\mathrm{m/s}$.

(c) The speed of the wave is

$$v = \frac{\lambda}{T} = \frac{\omega}{k} = \frac{2510\,\mathrm{rad/s}}{62.8\,\mathrm{m}^{-1}} = 40 \text{ m/s} .$$

8. (a) The figure in the book makes it clear that the period is $T = 10$ s and the amplitude is $y_m = 4.0$ cm. The phase constant ϕ is more subtly determined by that figure: what is shown is $4\sin\omega t$, yet what follows from Eq. 17-2 (without the phase constant) should be $4\sin(-\omega t)$ at $x = 0$. Thus, we need the phase constant $\phi = \pi$ since $4\sin(-\omega t + \pi) = 4\sin(\omega t)$). Therefore, we use Eq. 17-2 (modified by the inclusion of ϕ) with $k = 2\pi/\lambda = \pi/10$ (in inverse centimeters) and $\omega = 2\pi/T = \pi/5$ (in inverse seconds). In the graph below we plot the equation for $t = 0$ over the range $0 \le x \le 20$ cm, making sure our calculator is in radians mode.



(b) Since the frequency is $f = 1/T = 0.10$ s, the speed of the wave is $v = f\lambda = 2.0$ cm/s.

(c) Using the observations made in part (a), Eq. 17-2 becomes

$$y = 4.0\sin\left(\frac{\pi x}{10} - \frac{\pi t}{5} + \pi\right) = -4.0\sin\left(\frac{\pi x}{10} - \frac{\pi t}{5}\right)$$

where y and x are in centimeters and t is in seconds.

(d) Taking the derivative of y with respect to t, we find

$$u = \frac{\partial y}{\partial t} = 4.0\left(\frac{\pi}{t}\right)\cos\left(\frac{\pi x}{10} - \frac{\pi t}{5}\right)$$

which (evaluated at $(x, t) = (0, 5.0)$, making sure our calculator is in radians mode) yields $u = -2.5$ cm/s.

9. Using $v = f\lambda$, we find the length of one cycle of the wave is $\lambda = 350/500 = 0.700\,\mathrm{m} = 700$ mm. From $f = 1/T$, we find the time for one cycle of oscillation is $T = 1/500 = 2.00 \times 10^{-3}\,\mathrm{s} = 2.00$ ms.

(a) A cycle is equivalent to 2π radians, so that $\pi/3$ rad corresponds to one-sixth of a cycle. The corresponding length, therefore, is $\lambda/6 = 700/6 = 117$ mm.

(b) The interval 1.00 ms is half of T and thus corresponds to half of one cycle, or half of 2π rad. Thus, the phase difference is $(1/2)2\pi = \pi$ rad.

10. The volume of a cylinder of height $\ell$is $V = \pi r^2\ell = \pi d^2\ell/4$. The strings are long, narrow cylinders, one of diameter d_1 and the other of diameter d_2 (and corresponding linear densities μ_1 and μ_2). The mass is the (regular) density multiplied by the volume: $m = \rho V$, so that the mass-per-unit length is

$$\mu = \frac{m}{\ell} = \frac{\rho\pi d^2\ell/4}{\ell} = \frac{\pi\rho d^2}{4}$$

and their ratio is

$$\frac{\mu_1}{\mu_2} = \frac{\pi\rho d_1^2/4}{\pi\rho d_2^2/4} = \left(\frac{d_1}{d_2}\right)^2 .$$

Therefore, the ratio of diameters is

$$\frac{d_1}{d_2} = \sqrt{\frac{\mu_1}{\mu_2}} = \sqrt{\frac{3.0}{0.29}} = 3.2 \ .$$

11. The wave speed v is given by $v = \sqrt{\tau/\mu}$, where τ is the tension in the rope and μ is the linear mass density of the rope. The linear mass density is the mass per unit length of rope: $\mu = m/L = (0.0600\,\text{kg})/(2.00\,\text{m}) = 0.0300\,\text{kg/m}$. Thus

$$v = \sqrt{\frac{500\,\text{N}}{0.0300\,\text{kg/m}}} = 129 \ \text{m/s} \ .$$

12. From $v = \sqrt{\tau/\mu}$, we have

$$\frac{v_{\text{new}}}{v_{\text{old}}} = \frac{\sqrt{\tau_{\text{new}}/\mu_{\text{new}}}}{\sqrt{\tau_{\text{old}}/\mu_{\text{old}}}} = \sqrt{2} \ .$$

13. (a) The wave speed is given by $v = \lambda/T = \omega/k$, where λ is the wavelength, T is the period, ω is the angular frequency $(2\pi/T)$, and k is the angular wave number $(2\pi/\lambda)$. The displacement has the form $y = y_m \sin(kx + \omega t)$, so $k = 2.0\,\text{m}^{-1}$ and $\omega = 30\,\text{rad/s}$. Thus $v = (30\,\text{rad/s})/(2.0\,\text{m}^{-1}) = 15\,\text{m/s}$.

    (b) Since the wave speed is given by $v = \sqrt{\tau/\mu}$, where τ is the tension in the string and μ is the linear mass density of the string, the tension is

$$\tau = \mu v^2 = \left(1.6 \times 10^{-4}\,\text{kg/m}\right)(15\,\text{m/s})^2 = 0.036 \ \text{N} \ .$$

14. (a) Comparing with Eq. 17-2, we see that $k = 20/\text{m}$ and $\omega = 600/\text{s}$. Therefore, the speed of the wave is (see Eq. 17-12) $v = \omega/k = 30$ m/s.

    (b) From Eq. 17-25, we find

$$\mu = \frac{\tau}{v^2} = \frac{15}{30^2} = 0.017\,\text{kg/m} = 17 \ \text{g/m} \ .$$

15. We write the string displacement in the form $y = y_m \sin(kx + \omega t)$. The plus sign is used since the wave is traveling in the negative x direction. The frequency is $f = 100\,\text{Hz}$, so the angular frequency is $\omega = 2\pi f = 2\pi(100\,\text{Hz}) = 628\,\text{rad/s}$. The wave speed is given by $v = \sqrt{\tau/\mu}$, where τ is the tension in the string and μ is the linear mass density of the string, so the wavelength is $\lambda = v/f = \sqrt{\tau/\mu}/f$ and the angular wave number is

$$k = \frac{2\pi}{\lambda} = 2\pi f\sqrt{\frac{\mu}{\tau}} = 2\pi(100\,\text{Hz})\sqrt{\frac{0.50\,\text{kg/m}}{10\,\text{N}}} = 141\,\text{m}^{-1} \ .$$

The amplitude is $y_m = 0.12\,\text{mm}$. Thus

$$y = (0.12\,\text{mm})\sin\left[(141\,\text{m}^{-1})x + (628\,\text{s}^{-1})t\right] \ .$$

16. Let the cross-sectional area of the wire be A and the density of steel be ρ. The tensile stress is given by τ/A where τ is the tension in the wire. Also, $\mu = \rho A$. Thus,

$$\begin{aligned} v_{\max} &= \sqrt{\frac{\tau_{\max}}{\mu}} = \sqrt{\frac{\tau_{\max}/A}{\rho}} \\ &= \sqrt{\frac{7.0 \times 10^8\,\text{N/m}^2}{7800\,\text{kg/m}^3}} = 3.0 \times 10^2 \ \text{m/s} \end{aligned}$$

which is indeed independent of the diameter of the wire.

17. (a) We take the form of the displacement to be $y(x,t) = y_m \sin(kx - \omega t)$. The speed of a point on the cord is $u(x,t) = \partial y/\partial t = -\omega y_m \cos(kx - \omega t)$ and its maximum value is $u_m = \omega y_m$. The wave speed, on the other hand, is given by $v = \lambda/T = \omega/k$. The ratio is

$$\frac{u_m}{v} = \frac{\omega y_m}{\omega/k} = k y_m = \frac{2\pi y_m}{\lambda} \ .$$

(b) The ratio of the speeds depends only on the ratio of the amplitude to the wavelength. Different waves on different cords have the same ratio of speeds if they have the same amplitude and wavelength, regardless of the wave speeds , linear densities of the cords, and the tensions in the cords.

18. (a) The general expression for $y(x,t)$ for the wave is $y(x,t) = y_m \sin(kx - \omega t)$, which, at $x = 10\,\text{cm}$, becomes $y(x = 10\,\text{cm}, t) = y_m \sin[k(10\,\text{cm} - \omega t)]$. Comparing this with the expression given, we find $\omega = 4.0\,\text{rad/s}$, or $f = \omega/2\pi = 0.64\,\text{Hz}$.

(b) Since $k(10\,\text{cm}) = 1.0$, the wave number is $k = 0.10/\text{cm}$. Consequently, the wavelength is $\lambda = 2\pi/k = 63\,\text{cm}$.

(c) Substituting the values of k and ω into the general expression for $y(x,t)$, with centimeters and seconds understood, we obtain

$$y(x,t) = 5.0 \sin(0.10x - 4.0t) \ .$$

(d) Since $v = \omega/k = \sqrt{\tau/\mu}$, the tension is

$$\tau = \frac{\omega^2 \mu}{k^2} = \frac{(4.0\,\text{g/cm})\left(4.0\,\text{s}^{-1}\right)^2}{(0.10\,\text{cm}^{-1})^2} = 6400\,\text{g}\cdot\text{cm/s}^2 = 0.064\ \text{N} \ .$$

19. (a) We read the amplitude from the graph. It is about $5.0\,\text{cm}$.

(b) We read the wavelength from the graph. The curve crosses $y = 0$ at about $x = 15\,\text{cm}$ and again with the same slope at about $x = 55\,\text{cm}$, so $\lambda = 55\,\text{cm} - 15\,\text{cm} = 40\,\text{cm} = 0.40\,\text{m}$.

(c) The wave speed is $v = \sqrt{\tau/\mu}$, where τ is the tension in the string and μ is the linear mass density of the string. Thus,

$$v = \sqrt{\frac{3.6\,\text{N}}{25 \times 10^{-3}\,\text{kg/m}}} = 12\,\text{m/s} \ .$$

(d) The frequency is $f = v/\lambda = (12\,\text{m/s})/(0.40\,\text{m}) = 30\,\text{Hz}$ and the period is $T = 1/f = 1/(30\,\text{Hz}) = 0.033\,\text{s}$.

(e) The maximum string speed is $u_m = \omega y_m = 2\pi f y_m = 2\pi(30\,\text{Hz})(5.0\,\text{cm}) = 940\,\text{cm/s} = 9.4\,\text{m/s}$.

(f) The string displacement is assumed to have the form $y(x,t) = y_m \sin(kx + \omega t + \phi)$. A plus sign appears in the argument of the trigonometric function because the wave is moving in the negative x direction. The amplitude is $y_m = 5.0 \times 10^{-2}\,\text{m}$, the angular frequency is $\omega = 2\pi f = 2\pi(30\,\text{Hz}) = 190\,\text{rad/s}$, and the angular wave number is $k = 2\pi/\lambda = 2\pi/(0.40\,\text{m}) = 16\,\text{m}^{-1}$. According to the graph, the displacement at $x = 0$ and $t = 0$ is $4.0 \times 10^{-2}\,\text{m}$. The formula for the displacement gives $y(0,0) = y_m \sin\phi$. We wish to select ϕ so that $5.0 \times 10^{-2} \sin\phi = 4.0 \times 10^{-2}$. The solution is either $0.93\,\text{rad}$ or $2.21\,\text{rad}$. In the first case the function has a positive slope at $x = 0$ and matches the graph. In the second case it has negative slope and does not match the graph. We select $\phi = 0.93\,\text{rad}$. The expression for the displacement is

$$y(x,t) = (5.0 \times 10^{-2}\,\text{m}) \sin\left[(16\,\text{m}^{-1})x + (190\,\text{s}^{-1})t + 0.93\right] \ .$$

20. (a) The tension in each string is given by $\tau = Mg/2$. Thus, the wave speed in string 1 is

$$v_1 = \sqrt{\frac{\tau}{\mu_1}} = \sqrt{\frac{Mg}{2\mu_1}} = \sqrt{\frac{(500\,\text{g})\,(9.8\,\text{m/s}^2)}{2(3.00\,\text{g/m})}} = 28.6\ \text{m/s} \ .$$

(b) And the wave speed in string 2 is

$$v_2 = \sqrt{\frac{Mg}{2\mu_2}} = \sqrt{\frac{(500\,\text{g})\,(9.8\,\text{m/s}^2)}{2(5.00\,\text{g/m})}} = 22.1 \text{ m/s} .$$

(c) Let $v_1 = \sqrt{M_1 g/(2\mu_1)} = v_2 = \sqrt{M_2 g/(2\mu_2)}$ and $M_1 + M_2 = M$. We solve for M_1 and obtain

$$M_1 = \frac{M}{1 + \mu_2/\mu_1} = \frac{500\,\text{g}}{1 + 5.00/3.00} = 187.5\,\text{g} \approx 188 \text{ g} .$$

(d) And we solve for the second mass: $M_2 = M - M_1 = 500\,\text{g} - 187.5\,\text{g} \approx 313\,\text{g}$.

21. The pulses have the same speed v. Suppose one pulse starts from the left end of the wire at time $t = 0$. Its coordinate at time t is $x_1 = vt$. The other pulse starts from the right end, at $x = L$, where L is the length of the wire, at time $t = 30\,\text{ms}$. If this time is denoted by t_0 then the coordinate of this wave at time t is $x_2 = L - v(t - t_0)$. They meet when $x_1 = x_2$, or, what is the same, when $vt = L - v(t - t_0)$. We solve for the time they meet: $t = (L + vt_0)/2v$ and the coordinate of the meeting point is $x = vt = (L + vt_0)/2$. Now, we calculate the wave speed:

$$v = \sqrt{\frac{\tau L}{m}} = \sqrt{\frac{(250\,\text{N})(10.0\,\text{m})}{0.100\,\text{kg}}} = 158\,\text{m/s} .$$

Here τ is the tension in the wire and L/m is the linear mass density of the wire. The coordinate of the meeting point is

$$x = \frac{10.0\,\text{m} + (158\,\text{m/s})(30 \times 10^{-3}\,\text{s})}{2} = 7.37\,\text{m} .$$

This is the distance from the left end of the wire. The distance from the right end is $L - x = 10\,\text{m} - 7.37\,\text{m} = 2.63\,\text{m}$.

22. (a) The wave speed is

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{k\Delta\ell}{m/(\ell + \Delta\ell)}} = \sqrt{\frac{k\Delta\ell(\ell + \Delta\ell)}{m}} .$$

(b) The time required is

$$t = \frac{2\pi(\ell + \Delta\ell)}{v} = \frac{2\pi(\ell + \Delta\ell)}{\sqrt{k\Delta\ell(\ell + \Delta\ell)/m}} = 2\pi\sqrt{\frac{m}{k}}\sqrt{1 + \frac{\ell}{\Delta\ell}} .$$

Thus if $\ell/\Delta\ell \gg 1$, then $t \propto \sqrt{\ell/\Delta\ell} \propto 1/\sqrt{\Delta\ell}$; and if $\ell/\Delta\ell \ll 1$, then $t \simeq 2\pi\sqrt{m/k}$=const.

23. (a) The wave speed at any point on the rope is given by $v = \sqrt{\tau/\mu}$, where τ is the tension at that point and μ is the linear mass density. Because the rope is hanging the tension varies from point to point. Consider a point on the rope a distance y from the bottom end. The forces acting on it are the weight of the rope below it, pulling down, and the tension, pulling up. Since the rope is in equilibrium, these forces balance. The weight of the rope below is given by $\mu g y$, so the tension is $\tau = \mu g y$. The wave speed is $v = \sqrt{\mu g y/\mu} = \sqrt{gy}$.

(b) The time $\text{d}t$ for the wave to move past a length $\text{d}y$, a distance y from the bottom end, is $\text{d}t = \text{d}y/v = \text{d}y/\sqrt{gy}$ and the total time for the wave to move the entire length of the rope is

$$t = \int_0^L \frac{\text{d}y}{\sqrt{gy}} = 2\sqrt{\frac{y}{g}}\Bigg|_0^L = 2\sqrt{\frac{L}{g}} .$$

24. Using Eq. 17-32 for the average power and Eq. 17-25 for the speed of the wave, we solve for $f = \omega/2\pi$:

$$\begin{aligned} f &= \frac{1}{2\pi y_m}\sqrt{\frac{2P_{\text{avg}}}{\mu\sqrt{\tau/\mu}}} \\ &= \frac{1}{2\pi(7.7\times 10^{-3}\,\text{m})}\sqrt{\frac{2(85\,\text{W})}{\sqrt{(36\,\text{N})(0.260\,\text{kg}/2.7\,\text{m})}}} = 198\text{ Hz} . \end{aligned}$$

25. (a) The displacement of the string is assumed to have the form $y(x,t) = y_m \sin(kx - \omega t)$. The velocity of a point on the string is $u(x,t) = \partial y/\partial t = -\omega y_m \cos(kx - \omega t)$ and its maximum value is $u_m = \omega y_m$. For this wave the frequency is $f = 120\,\text{Hz}$ and the angular frequency is $\omega = 2\pi f = 2\pi(120\,\text{Hz}) = 754\,\text{rad/s}$. Since the bar moves through a distance of 1.00 cm, the amplitude is half of that, or $y_m = 5.00\times 10^{-3}\,\text{m}$. The maximum speed is $u_m = (754\,\text{rad/s})(5.00\times 10^{-3}\,\text{m}) = 3.77\,\text{m/s}$.

(b) Consider the string at coordinate x and at time t and suppose it makes the angle θ with the x axis. The tension is along the string and makes the same angle with the x axis. Its transverse component is $\tau_{\text{trans}} = \tau\sin\theta$. Now θ is given by $\tan\theta = \partial y/\partial x = k y_m \cos(kx - \omega t)$ and its maximum value is given by $\tan\theta_m = k y_m$. We must calculate the angular wave number k. It is given by $k = \omega/v$, where v is the wave speed. The wave speed is given by $v = \sqrt{\tau/\mu}$, where τ is the tension in the rope and μ is the linear mass density of the rope. Using the data given,

$$v = \sqrt{\frac{90.0\,\text{N}}{0.120\,\text{kg/m}}} = 27.4\,\text{m/s}$$

and

$$k = \frac{754\,\text{rad/s}}{27.4\,\text{m/s}} = 27.5\,\text{m}^{-1} .$$

Thus

$$\tan\theta_m = (27.5\,\text{m}^{-1})(5.00\times 10^{-3}\,\text{m}) = 0.138$$

and $\theta = 7.83°$. The maximum value of the transverse component of the tension in the string is $\tau_{\text{trans}} = (90.0\,\text{N})\sin 7.83° = 12.3\,\text{N}$. We note that $\sin\theta$ is nearly the same as $\tan\theta$ because θ is small. We can approximate the maximum value of the transverse component of the tension by $\tau k y_m$.

(c) We consider the string at x. The transverse component of the tension pulling on it due to the string to the left is $-\tau\partial y/\partial x = -\tau k y_m \cos(kx - \omega t)$ and it reaches its maximum value when $\cos(kx - \omega t) = -1$. The wave speed is $u = \partial y/\partial t = -\omega y_m \cos(kx - \omega t)$ and it also reaches its maximum value when $\cos(kx - \omega t) = -1$. The two quantities reach their maximum values at the same value of the phase. When $\cos(kx - \omega t) = -1$ the value of $\sin(kx - \omega t)$ is zero and the displacement of the string is $y = 0$.

(d) When the string at any point moves through a small displacement Δy, the tension does work $\Delta W = \tau_{\text{trans}}\,\Delta y$. The rate at which it does work is

$$P = \frac{\Delta W}{\Delta t} = \tau_{\text{trans}}\frac{\Delta y}{\Delta t} = \tau_{\text{trans}} u .$$

P has its maximum value when the transverse component τ_{trans} of the tension and the string speed u have their maximum values. Hence the maximum power is $(12.3\,\text{N})(3.77\,\text{m/s}) = 46.4\,\text{W}$.

(e) As shown above $y = 0$ when the transverse component of the tension and the string speed have their maximum values.

(f) The power transferred is zero when the transverse component of the tension and the string speed are zero.

(g) $P = 0$ when $\cos(kx - \omega t) = 0$ and $\sin(kx - \omega t) = \pm 1$ at that time. The string displacement is $y = \pm y_m = \pm 0.50\,\text{cm}$.

26. (a) Let the phase difference be ϕ. Then from Eq. 17-39, $2y_m\cos(\phi/2) = 1.50y_m$, which gives

$$\phi = 2\cos^{-1}\left(\frac{1.50y_m}{2y_m}\right) = 82.8^\circ \ .$$

(b) Converting to radians, we have $\phi = 1.45\,\text{rad}$.

(c) In terms of wavelength (the length of each cycle, where each cycle corresponds to 2π rad), this is equivalent to $1.45\,\text{rad}/2\pi = 0.23$ wavelength.

27. The displacement of the string is given by $y = y_m \sin(kx-\omega t) + y_m \sin(kx-\omega t+\phi) = 2y_m\cos(\frac{1}{2}\phi)\sin(kx-\omega t+\frac{1}{2}\phi)$, where $\phi = \pi/2$. The amplitude is $A = 2y_m\cos(\frac{1}{2}\phi) = 2y_m\cos(\pi/4) = 1.41y_m$.

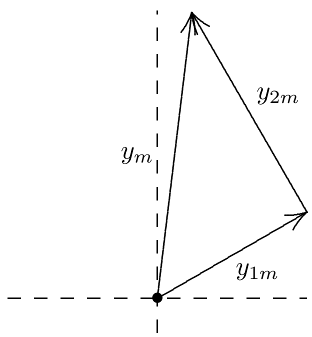
28. We compare the resultant wave given with the standard expression (Eq. 17-39) to obtain $k = 20\,\text{m}^{-1} = 2\pi/\lambda$, $2y_m\cos(\frac{1}{2}\phi) = 3.0\,\text{mm}$, and $\frac{1}{2}\phi = 0.820\,\text{rad}$.

(a) Therefore, $\lambda = 2\pi/k = 0.31\,\text{m}$.

(b) The phase difference is $\phi = 1.64\,\text{rad}$.

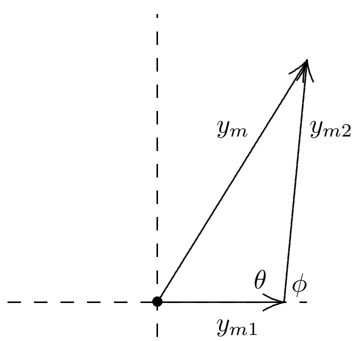
(c) And the amplitude is $y_m = 2.2\,\text{mm}$.

29. The phasor diagram is shown below: y_{1m} and y_{2m} represent the original waves and y_m represents the resultant wave. The phasors corresponding to the two constituent waves make an angle of 90° with each other, so the triangle is a right triangle. The Pythagorean theorem gives $y_m^2 = y_{1m}^2 + y_{2m}^2 = (3.0\,\text{cm})^2 + (4.0\,\text{cm})^2 = 25\,\text{cm}^2$. Thus $y_m = 5.0\,\text{cm}$.



30. The phasor diagram is shown below. We use the cosine theorem:

$$y_m^2 = y_{m1}^2 + y_{m2}^2 - 2y_{m1}y_{m2}\cos\theta = y_{m1}^2 + y_{m2}^2 + 2y_{m1}y_{m2}\cos\phi \ .$$



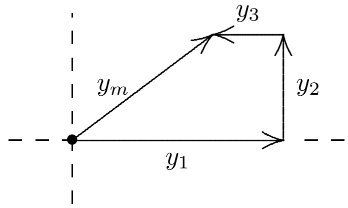
We solve for $\cos\phi$:

$$\begin{aligned}\cos\phi &= \frac{y_m^2 - y_{m1}^2 - y_{m2}^2}{2y_{m1}y_{m2}} \\ &= \frac{(9.0\,\text{mm})^2 - (5.0\,\text{mm})^2 - (7.0\,\text{mm})^2}{2(5.0\,\text{mm})(7.0\,\text{mm})} \\ &= 0.10 \ .\end{aligned}$$

The phase constant is therefore $\phi = 84°$.

31. (a) The phasor diagram is shown to the right: y_1, y_2, and y_3 represent the original waves and y_m represents the resultant wave. The horizontal component of the resultant is $y_{mh} = y_1 - y_3 = y_1 - y_1/3 = 2y_1/3$. The vertical component is $y_{mv} = y_2 = y_1/2$. The amplitude of the resultant is

$$y_m = \sqrt{y_{mh}^2 + y_{mv}^2} = \sqrt{\left(\frac{2y_1}{3}\right)^2 + \left(\frac{y_1}{2}\right)^2} = \frac{5}{6}y_1 = 0.83y_1 \ .$$



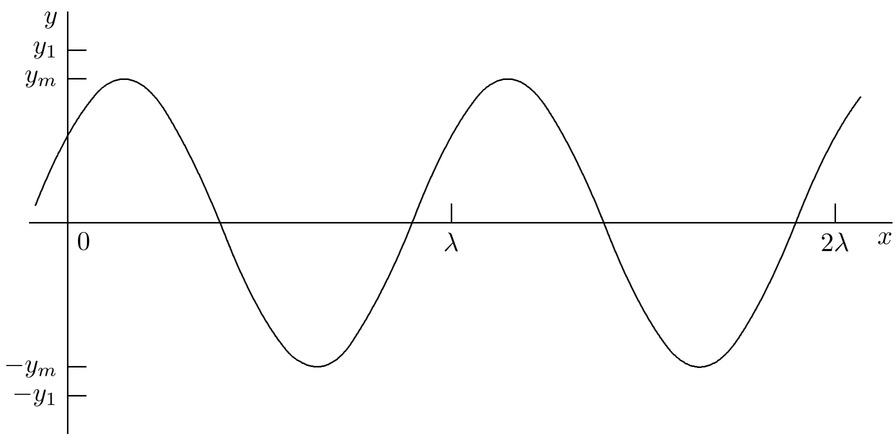
(b) The phase constant for the resultant is

$$\phi = \tan^{-1}\frac{y_{mv}}{y_{mh}} = \tan^{-1}\left(\frac{y_1/2}{2y_1/3}\right) = \tan^{-1}\frac{3}{4} = 0.644\,\text{rad} = 37° \ .$$

(c) The resultant wave is

$$y = \frac{5}{6}y_1 \sin(kx - \omega t + 0.644\,\text{rad}) \ .$$

The graph below shows the wave at time $t = 0$. As time goes on it moves to the right with speed $v = \omega/k$.



32. Use Eq. 17-53 (for the resonant frequencies) and Eq. 17-25 ($v = \sqrt{\tau/\mu}$) to find f_n:

$$f_n = \frac{nv}{2L} = \frac{n}{2L}\sqrt{\frac{\tau}{\mu}}$$

which gives $f_3 = (3/2L)\sqrt{\tau_i/\mu}$.

(a) When $\tau_f = 4\tau_i$, we get the new frequency

$$f_3' = \frac{3}{2L}\sqrt{\frac{\tau_f}{\mu}} = 2f_3 \ .$$

(b) And we get the new wavelength

$$\lambda_3' = \frac{v'}{f_3'} = \frac{2L}{3} = \lambda_3 \ .$$

33. (a) Eq. 17-25 gives the speed of the wave:

$$v = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{150\,\mathrm{N}}{7.2 \times 10^{-3}\,\mathrm{kg/m}}} = 1.4 \times 10^2 \ \mathrm{m/s} \ .$$

(b) From the Figure, we find the wavelength of the standing wave to be $\lambda = (2/3)(90\,\mathrm{cm}) = 60\,\mathrm{cm}$.

(c) The frequency is

$$f = \frac{v}{\lambda} = \frac{1.4 \times 10^2\,\mathrm{m/s}}{0.60\,\mathrm{m}} = 2.4 \times 10^2 \ \mathrm{Hz} \ .$$

34. The string is flat each time the particles passes through its equilibrium position. A particle may travel up to its positive amplitude point and back to equilibrium during this time. This describes *half* of one complete cycle, so we conclude $T = 2(0.50\,\mathrm{s}) = 1.0\,\mathrm{s}$. Thus, $f = 1/T = 1.0$ Hz, and the wavelength is

$$\lambda = \frac{v}{f} = \frac{10\,\mathrm{cm/s}}{1.0\,\mathrm{Hz}} = 10 \ \mathrm{cm} \ .$$

35. (a) The wave speed is given by $v = \sqrt{\tau/\mu}$, where τ is the tension in the string and μ is the linear mass density of the string. Since the mass density is the mass per unit length, $\mu = M/L$, where M is the mass of the string and L is its length. Thus

$$v = \sqrt{\frac{\tau L}{M}} = \sqrt{\frac{(96.0\,\mathrm{N})(8.40\,\mathrm{m})}{0.120\,\mathrm{kg}}} = 82.0 \ \mathrm{m/s} \ .$$

(b) The longest possible wavelength λ for a standing wave is related to the length of the string by $L = \lambda/2$, so $\lambda = 2L = 2(8.40\,\mathrm{m}) = 16.8\,\mathrm{m}$.

(c) The frequency is $f = v/\lambda = (82.0\,\mathrm{m/s})/(16.8\,\mathrm{m}) = 4.88\,\mathrm{Hz}$.

36. (a) The wave speed is given by

$$v = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{7.00\,\mathrm{N}}{2.00 \times 10^{-3}\,\mathrm{kg/1.25m}}} = 66.1 \ \mathrm{m/s} \ .$$

(b) The wavelength of the wave with the lowest resonant frequency f_1 is $\lambda_1 = 2L$, where $L = 125\,\mathrm{cm}$. Thus,

$$f_1 = \frac{v}{\lambda_1} = \frac{66.1\,\mathrm{m/s}}{2(1.25\,\mathrm{m})} = 26.4 \ \mathrm{Hz} \ .$$

37. Possible wavelengths are given by $\lambda = 2L/n$, where L is the length of the wire and n is an integer. The corresponding frequencies are given by $f = v/\lambda = nv/2L$, where v is the wave speed. The wave speed is given by $v = \sqrt{\tau/\mu} = \sqrt{\tau L/M}$, where τ is the tension in the wire, μ is the linear mass density of the wire, and M is the mass of the wire. $\mu = M/L$ was used to obtain the last form. Thus

$$f = \frac{n}{2L}\sqrt{\frac{\tau L}{M}} = \frac{n}{2}\sqrt{\frac{\tau}{LM}} = \frac{n}{2}\sqrt{\frac{250\,\mathrm{N}}{(10.0\,\mathrm{m})(0.100\,\mathrm{kg})}} = n(7.91\,\mathrm{Hz}) \ .$$

For $n = 1$, $f = 7.91\,\mathrm{Hz}$; for $n = 2$, $f = 15.8\,\mathrm{Hz}$; and for $n = 3$, $f = 23.7\,\mathrm{Hz}$.

38. The n^{th} resonant frequency of string A is

$$f_{n,A} = \frac{v_A}{2l_A} n = \frac{n}{2L}\sqrt{\frac{\tau}{\mu}} \, ,$$

while for string B it is

$$f_{n,B} = \frac{v_B}{2l_B} n = \frac{n}{8L}\sqrt{\frac{\tau}{\mu}} = \frac{1}{4} f_{n,A} \, .$$

Thus, we see $f_{1,A} = f_{4,B}$ and $f_{2,A} = f_{8,B}$.

39. (a) The resonant wavelengths are given by $\lambda = 2L/n$, where L is the length of the string and n is an integer, and the resonant frequencies are given by $f = v/\lambda = nv/2L$, where v is the wave speed. Suppose the lower frequency is associated with the integer n. Then, since there are no resonant frequencies between, the higher frequency is associated with $n+1$. That is, $f_1 = nv/2L$ is the lower frequency and $f_2 = (n+1)v/2L$ is the higher. The ratio of the frequencies is

$$\frac{f_2}{f_1} = \frac{n+1}{n} \, .$$

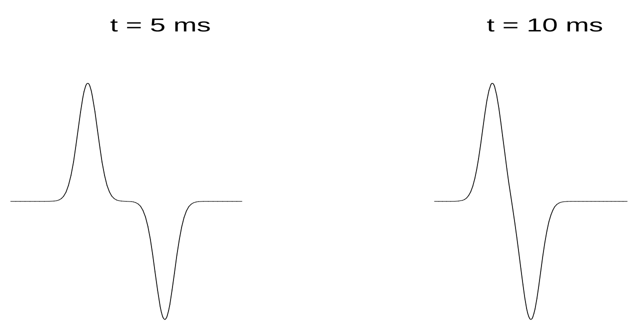
The solution for n is

$$n = \frac{f_1}{f_2 - f_1} = \frac{315\,\text{Hz}}{420\,\text{Hz} - 315\,\text{Hz}} = 3 \, .$$

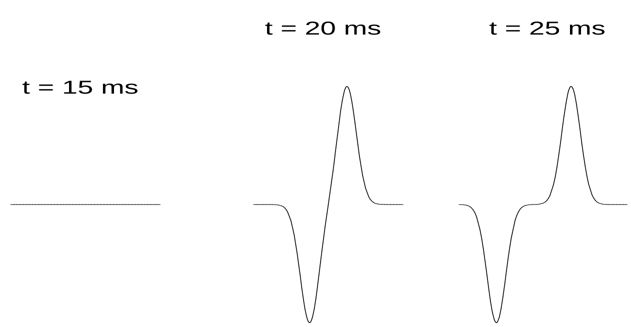
The lowest possible resonant frequency is $f = v/2L = f_1/n = (315\,\text{Hz})/3 = 105\,\text{Hz}$.

(b) The longest possible wavelength is $\lambda = 2L$. If f is the lowest possible frequency then $v = \lambda f = 2Lf = 2(0.75\,\text{m})(105\,\text{Hz}) = 158\,\text{m/s}$.

40. (a) We note that each pulse travels 1 cm during each $\Delta t = 5$ ms interval. Thus, in these first two pictures, their peaks are closer to each other by 2 cm, successively.



And the next pictures show the (momentary) complete cancellation of the visible pattern at $t = 15$ ms, and the pulses moving away from each other after that.



(b) The particles of the string are moving rapidly as they pass (transversely) through their equilibrium positions; the energy at $t = 15$ ms is purely kinetic.

41. (a) The amplitude of each of the traveling waves is half the maximum displacement of the string when the standing wave is present, or 0.25 cm.

(b) Each traveling wave has an angular frequency of $\omega = 40\pi$ rad/s and an angular wave number of $k = \pi/3\,\text{cm}^{-1}$. The wave speed is $v = \omega/k = (40\pi\,\text{rad/s})/(\pi/3\,\text{cm}^{-1}) = 120\,\text{cm/s}$.

(c) The distance between nodes is half a wavelength: $d = \lambda/2 = \pi/k = \pi/(\pi/3\,\text{cm}^{-1}) = 3.0\,\text{cm}$. Here $2\pi/k$ was substituted for λ.

(d) The string speed is given by $u(x,t) = \partial y/\partial t = -\omega y_m \sin(kx)\sin(\omega t)$. For the given coordinate and time,

$$u = -(40\pi\,\text{rad/s})(0.50\,\text{cm})\sin\left[\left(\frac{\pi}{3}\,\text{cm}^{-1}\right)(1.5\,\text{cm})\right]\sin\left[\left(40\pi\,\text{s}^{-1}\right)\left(\frac{9}{8}\,\text{s}\right)\right] = 0\ .$$

42. Repeating the steps of Eq. 17-34 $\longrightarrow$ Eq. 17-40, but applying

$$\cos\alpha + \cos\beta = 2\,\cos\left(\frac{\alpha+\beta}{2}\right)\,\cos\left(\frac{\alpha-\beta}{2}\right)$$

(see Appendix E) instead of Eq. 17-37, we obtain

$$y' = [0.10\cos\pi x]\cos 4\pi t$$

with SI units understood.

(a) For non-negative x, the smallest value to produce $\cos\pi x = 0$ is $x = 1/2$, so the answer is $x = 0.50$ m.

(b) Taking the derivative,

$$u' = \frac{dy'}{dt} = [0.10\cos\pi x]\,(-4\pi\sin 4\pi t)$$

We observe that the last factor is zero when $t = 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \ldots$ which leads to the answers $t = 0$, $t = 0.25$ s, and $t = 0.50$ s.

43. (a) Since the standing wave has three loops, the string is three half-wavelengths long: $L = 3\lambda/2$, or $\lambda = 2L/3$. If v is the wave speed, then the frequency is

$$f = \frac{v}{\lambda} = \frac{3v}{2L} = \frac{3(100\,\text{m/s})}{2(3.0\,\text{m})} = 50\ \text{Hz}\ .$$

(b) The waves have the same amplitude, the same angular frequency, and the same angular wave number, but they travel in opposite directions. We take them to be $y_1 = y_m\sin(kx - \omega t)$ and $y_2 = y_m\sin(kx + \omega t)$. The amplitude y_m is half the maximum displacement of the standing wave, or 5.0×10^{-3} m. The angular frequency is the same as that of the standing wave, or $\omega = 2\pi f = 2\pi(50\,\text{Hz}) = 314\,\text{rad/s}$. The angular wave number is $k = 2\pi/\lambda = 2\pi/(2.0\,\text{m}) = 3.14\,\text{m}^{-1}$. Thus,

$$y_1 = (5.0\times10^{-3}\,\text{m})\sin\left[\left(3.14\,\text{m}^{-1}\right)x - \left(314\,\text{s}^{-1}\right)t\right]$$

and

$$y_2 = \left(5.0\times10^{-3}\,\text{m}\right)\sin\left[\left(3.14\,\text{m}^{-1}\right)x + \left(314\,\text{s}^{-1}\right)t\right]\ .$$

44. To oscillate in four loops means $n = 4$ in Eq. 17-52 (treating both ends of the string as effectively "fixed'). Thus, $\lambda = 2(0.90\,\text{m})/4 = 0.45\,\text{m}$. Therefore, the speed of the wave is $v = f\lambda = 27$ m/s. The mass-per-unit-length is $\mu = m/L = (0.044\,\text{kg})/(0.90\,\text{m}) = 0.049$ kg/m. Thus, using Eq. 17-25, we obtain the tension: $\tau = v^2\mu = (27)^2(0.049) = 36$ N.

45. (a) Since the string has four loops its length must be two wavelengths. That is, $\lambda = L/2$, where λ is the wavelength and L is the length of the string. The wavelength is related to the frequency f and wave speed v by $\lambda = v/f$, so $L/2 = v/f$ and $L = 2v/f = 2(400\,\mathrm{m/s})/(600\,\mathrm{Hz}) = 1.3\,\mathrm{m}$.

(b) We write the expression for the string displacement in the form $y = y_m \sin(kx)\cos(\omega t)$, where y_m is the maximum displacement, k is the angular wave number, and ω is the angular frequency. The angular wave number is $k = 2\pi/\lambda = 2\pi f/v = 2\pi(600\,\mathrm{Hz})/(400\,\mathrm{m/s}) = 9.4\,\mathrm{m}^{-1}$ and the angular frequency is $\omega = 2\pi f = 2\pi(600\,\mathrm{Hz}) = 3800\,\mathrm{rad/s}$. y_m is 2.0 mm. The displacement is given by

$$y(x,t) = (2.0\,\mathrm{mm})\sin[(9.4\,\mathrm{m}^{-1})x]\cos\left[(3800\,\mathrm{s}^{-1})t\right] \ .$$

46. Since the rope is fixed at both ends, then the phrase "second-harmonic standing wave pattern" describes the oscillation shown in Figure 17-21(b), where

$$\lambda = L \qquad \text{and} \qquad f = \frac{v}{L}$$

(see Eq. 17-52 and Eq. 17-53).

(a) Comparing the given function with Eq. 17-47, we obtain $k = \pi/2$ and $\omega = 12\pi$ (SI units understood). Since $k = 2\pi/\lambda$ then

$$\frac{2\pi}{\lambda} = \frac{\pi}{2} \implies \lambda = 4 \text{ m} \implies L = 4 \text{ m} \ .$$

(b) Since $\omega = 2\pi f$ then

$$2\pi f = 12\pi \implies f = 6 \text{ Hz} \implies v = f\lambda = 24 \text{ m/s} \ .$$

(c) Using Eq. 17-25, we have

$$\begin{aligned} v &= \sqrt{\frac{\tau}{\mu}} \\ 24 &= \sqrt{\frac{200}{m/L}} \end{aligned}$$

with leads to $m = 1.4$ kg.

(d) Now, "third-harmonic ... pattern" draws our attention to Figure 17-22(c), where

$$f = \frac{3v}{2L} = \frac{3(24)}{2(4)} = 9 \text{ Hz}$$

so that $T = 1/f = 0.11$ s.

47. (a) The angular frequency is $\omega = 8.0\pi/2 = 4.0\pi\,\mathrm{rad/s}$, so the frequency is $f = \omega/2\pi = (4.0\pi\,\mathrm{rad/s})/2\pi = 2.0\,\mathrm{Hz}$.

(b) The angular wave number is $k = 2.0\pi/2 = 1.0\pi\,\mathrm{m}^{-1}$, so the wavelength is $\lambda = 2\pi/k = 2\pi/(1.0\pi\,\mathrm{m}^{-1}) = 2.0\,\mathrm{m}$.

(c) The wave speed is

$$v = \lambda f = (2.0\,\mathrm{m})(2.0\,\mathrm{Hz}) = 4.0\,\mathrm{m/s} \ .$$

(d) We need to add two cosine functions. First convert them to sine functions using $\cos\alpha = \sin(\alpha+\pi/2)$, then apply Eq. 42. The steps are as follows:

$$\begin{aligned} \cos\alpha + \cos\beta &= \sin\left(\alpha + \frac{\pi}{2}\right) + \sin\left(\beta + \frac{\pi}{2}\right) = 2\sin\left(\frac{\alpha+\beta+\pi}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right) \\ &= 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right) \end{aligned}$$

Letting $\alpha = kx$ and $\beta = \omega t$, we find

$$y_m \cos(kx + \omega t) + y_m \cos(kx - \omega t) = 2y_m \cos(kx)\cos(\omega t) \ .$$

Nodes occur where $\cos(kx) = 0$ or $kx = n\pi + \pi/2$, where n is an integer (including zero). Since $k = 1.0\pi\,\mathrm{m}^{-1}$, this means $x = (n + \frac{1}{2})(1.0\,\mathrm{m})$. Nodes occur at $x = 0.50\,\mathrm{m}$, $1.5\,\mathrm{m}$, $2.5\,\mathrm{m}$, etc.

(e) The displacement is a maximum where $\cos(kx) = \pm 1$. This means $kx = n\pi$, where n is an integer. Thus, $x = n(1.0\,\mathrm{m})$. Maxima occur at $x = 0$, $1.0\,\mathrm{m}$, $2.0\,\mathrm{m}$, $3.0\,\mathrm{m}$, etc.

48. (a) The nodes are located from vanishing of the spatial factor $\sin 5\pi x = 0$ for which the solutions are

$$5\pi x = 0, \pi, 2\pi, 3\pi, \ldots \quad \Longrightarrow \quad x = 0, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \ldots$$

so that the values of x lying in the allowed range are $x = 0$, $x = 0.20$ m, and $x = 0.40$ m.

(b) Every point (except at a node) is in simple harmonic motion of frequency $f = \omega/2\pi = 40\pi/2\pi =$ 20 Hz. Therefore, the period of oscillation is $T = 1/f = 0.050$ s.

(c) Comparing the given function with Eq. 17-45 through Eq. 17-47, we obtain

$$y_1 = 0.020\sin(5\pi x - 40\pi t) \quad \text{and} \quad y_2 = 0.020\sin(5\pi x + 40\pi t)$$

for the two traveling waves. Thus, we infer from these that the speed is $v = \omega/k = 40\pi/5\pi =$ 8.0 m/s.

(d) And we see the amplitude is $y_m = 0.020$ m.

(e) The derivative of the given function with respect to time is

$$u = \frac{\partial y}{\partial t} = -\,(0.040)\,(40\pi)\sin(5\pi x)\sin(40\pi t)$$

which vanishes (for all x) at times such $\sin(40\pi t) = 0$. Thus,

$$40\pi t = 0, \pi, 2\pi, 3\pi, \ldots \quad \Longrightarrow \quad t = 0, \frac{1}{40}, \frac{2}{40}, \frac{3}{40}, \ldots$$

so that the values of t lying in the allowed range are $t = 0$, $t = 0.025$ s, and $t = 0.050$ s.

49. We consider an infinitesimal segment of a string oscillating in a standing wave pattern. Its length is $\mathrm{d}x$ and its mass is $\mathrm{d}m = \mu\,\mathrm{d}x$, where μ is its linear mass density. If it is moving with speed u its kinetic energy is $\mathrm{d}K = \frac{1}{2}u^2\,\mathrm{d}m = \frac{1}{2}\mu u^2\,\mathrm{d}x$. If the segment is located at x its displacement at time t is $y = 2y_m \sin(kx)\cos(\omega t)$ and its velocity is $u = \partial y/\partial t = -2\omega y_m \sin(kx)\sin(\omega t)$, so its kinetic energy is

$$\mathrm{d}K = \left(\frac{1}{2}\right)\left(4\mu\omega^2 y_m^2\right)\sin^2(kx)\sin^2(\omega t) = 2\mu\omega^2 y_m^2 \sin^2(kx)\sin^2(\omega t) \ .$$

Here y_m is the amplitude of each of the traveling waves that combine to form the standing wave. The infinitesimal segment has maximum kinetic energy when $\sin^2(\omega t) = 1$ and the maximum kinetic energy is given by the differential amount

$$dK_m = 2\mu\omega^2 y_m^2 \sin^2(kx) \ .$$

Note that every portion of the string has its maximum kinetic energy at the same time although the values of these maxima are different for different parts of the string. If the string is oscillating with n loops, the length of string in any one loop is L/n and the kinetic energy of the loop is given by the integral

$$K_m = 2\mu\omega^2 y_m^2 \int_0^{L/n} \sin^2(kx)\,dx \ .$$

We use the trigonometric identity $\sin^2(kx) = \frac{1}{2}[1 + 2\cos(2kx)]$ to obtain

$$K_m = \mu\omega^2 y_m^2 \int_0^{L/n} [1 + 2\cos(2kx)]\,\mathrm{d}x = \mu\omega^2 y_m^2 \left[\frac{L}{n} + \frac{1}{k}\sin\frac{2kL}{n}\right] .$$

For a standing wave of n loops the wavelength is $\lambda = 2L/n$ and the angular wave number is $k = 2\pi/\lambda = n\pi/L$, so $2kL/n = 2\pi$ and $\sin(2kL/n) = 0$, no matter what the value of n. Thus,

$$K_m = \frac{\mu\omega^2 y_m^2 L}{n} .$$

To obtain the expression given in the problem statement, we first make the substitutions $\omega = 2\pi f$ and $L/n = \lambda/2$, where f is the frequency and λ is the wavelength. This produces $K_m = 2\pi^2\mu y_m^2 f^2\lambda$. We now substitute the wave speed v for $f\lambda$ and obtain $K_m = 2\pi^2\mu y_m^2 fv$.

50. From the $x = 0$ plot (and the requirement of an antinode at $x = 0$), we infer a standing wave function of the form

$$y = -(0.04)\cos(kx)\sin(\omega t) \quad \text{where} \quad \omega = \frac{2\pi}{T} = \pi \text{ rad/s}$$

with length in meters and time in seconds. The parameter k is determined by the existence of the node at $x = 0.10$ (presumably the *first* node that one encounters as one moves from the origin in the positive x direction). This implies $k(0.10) = \pi/2$ so that $k = 5\pi$ rad/m.

(a) With the parameters determined as discussed above and $t = 0.50$ s, we find

$$y = -0.04\,\cos(kx)\sin(\omega t) = 0.04 \text{ m} \quad \text{at } x = 0.20 \text{ m} .$$

(b) The above equation yields zero at $x = 0.30$ m.

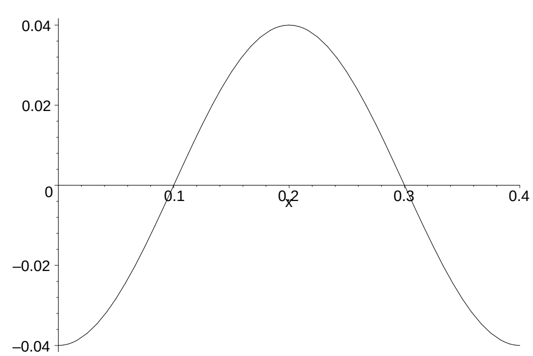
(c) We take the derivative with respect to time and obtain

$$u = \frac{dy}{dt} = -0.04\,\omega\,\cos(kx)\cos(\omega t) = 0 \quad \text{at } t = 0.50 \text{ s}$$

where $x = 0.20$ m.

(d) The above equation yields $u = -0.126$ m/s at $t = 1.0$ s.

(e) The sketch of this function at $t = 0.50$ s for $0 \le x \le 0.40$ m is shown.



51. (a) The frequency of the wave is the same for both sections of the wire. The wave speed and wavelength, however, are both different in different sections. Suppose there are n_1 loops in the aluminum section of the wire. Then, $L_1 = n_1\lambda_1/2 = n_1 v_1/2f$, where λ_1 is the wavelength and v_1 is the wave speed in that section. In this consideration, we have substituted $\lambda_1 = v_1/f$, where f is the frequency. Thus $f = n_1 v_1/2L_1$. A similar expression holds for the steel section: $f = n_2 v_2/2L_2$. Since the frequency is the same for the two sections, $n_1 v_1/L_1 = n_2 v_2/L_2$. Now the wave speed in the aluminum section is given by $v_1 = \sqrt{\tau/\mu_1}$, where μ_1 is the linear mass density of the aluminum wire. The mass of aluminum in the wire is given by $m_1 = \rho_1 A L_1$, where ρ_1 is the mass density (mass per unit volume) for aluminum and A is the cross-sectional area of the wire. Thus $\mu_1 = \rho_1 A L_1/L_1 = \rho_1 A$ and $v_1 = \sqrt{\tau/\rho_1 A}$. A similar expression holds for the wave speed in the steel section: $v_2 = \sqrt{\tau/\rho_2 A}$. We note that the cross-sectional area and the tension are the same for the two sections. The equality of the frequencies for the two sections now leads to $n_1/L_1\sqrt{\rho_1} = n_2/L_2\sqrt{\rho_2}$, where A has been canceled from both sides. The ratio of the integers is

$$\frac{n_2}{n_1} = \frac{L_2\sqrt{\rho_2}}{L_1\sqrt{\rho_1}} = \frac{(0.866\,\mathrm{m})\sqrt{7.80\times10^3\,\mathrm{kg/m^3}}}{(0.600\,\mathrm{m})\sqrt{2.60\times10^3\,\mathrm{kg/m^3}}} = 2.5 \ .$$

The smallest integers that have this ratio are $n_1 = 2$ and $n_2 = 5$. The frequency is $f = n_1 v_1/2L_1 = (n_1/2L_1)\sqrt{\tau/\rho_1 A}$. The tension is provided by the hanging block and is $\tau = mg$, where m is the mass of the block. Thus

$$f = \frac{n_1}{2L_1}\sqrt{\frac{mg}{\rho_1 A}} = \frac{2}{2(0.600\,\mathrm{m})}\sqrt{\frac{(10.0\,\mathrm{kg})(9.8\,\mathrm{m/s^2})}{(2.60\times10^3\,\mathrm{kg/m^3})(1.00\times10^{-6}\,\mathrm{m^2})}} = 324\,\mathrm{Hz} \ .$$

(b) The standing wave pattern has two loops in the aluminum section and five loops in the steel section, or seven loops in all. There are eight nodes, counting the end points.

52. (a) This distance is determined by the longitudinal speed:

$$d_\ell = v_\ell t = (2000\,\mathrm{m/s})\left(40\times10^{-6}\,\mathrm{s}\right) = 8.0\times10^{-2}\ \mathrm{m} \ .$$

(b) Assuming the acceleration is constant (justified by the near-straightness of the curve $a = 300/40 \times 10^{-6}$) we find the stopping distance d:

$$v^2 = v_o^2 + 2ad \implies d = \frac{(300)^2\left(40\times10^{-6}\right)}{2(300)}$$

which gives $d = 6.0\times10^{-3}\,\mathrm{m}$. This and the radius r form the legs of a right triangle (where r is opposite from $\theta = 60°$). Therefore,

$$\tan 60° = \frac{r}{d} \implies r = d\tan 60° = 1.0\times10^{-2}\ \mathrm{m} \ .$$

53. We refer to the points where the rope is attached as A and B, respectively. When A and B are not displaced horizontal, the rope is in its initial state (neither stretched (under tension) nor slack). If they are displaced away from each other, the rope is clearly stretched. When A and B are displaced in the same direction, by amounts (in absolute value) $|\xi_A|$ and $|\xi_B|$, then if $|\xi_A| < |\xi_B|$ then the rope is stretched, and if $|\xi_A| > |\xi_B|$ the rope is slack. We must be careful about the case where one is displaced but the other is not, as will be seen below.

(a) The standing wave solution for the shorter cable, appropriate for the initial condition $\xi = 0$ at $t = 0$, and the boundary conditions $\xi = 0$ at $x = 0$ and $x = L$ (the x axis runs vertically here), is $\xi_A = \xi_m \sin(k_A x)\sin(\omega_A t)$. The angular frequency is $\omega_A = 2\pi/T_A$, and the wave number is $k_A = 2\pi/\lambda_A$ where $\lambda_A = 2L$ (it begins oscillating in its fundamental mode) where the point of attachment is $x = L/2$. The displacement of what we are calling point A at time $t = \eta T_A$ (where η is a pure number) is

$$\xi_A = \xi_m \sin\left(\frac{2\pi}{2L}\frac{L}{2}\right)\sin\left(\frac{2\pi}{T_A}\eta T_A\right) = \xi_m \sin(2\pi\eta) \ .$$

The fundamental mode for the longer cable has wavelength $\lambda_B = 2\lambda_A = 2(2L) = 4L$, which implies (by $v = f\lambda$ and the fact that both cables support the same wave speed v) that $f_B = \frac{1}{2}f_A$ or $\omega_B = \frac{1}{2}\omega_A$. Thus, the displacement for point B is

$$\xi_B = \xi_m \sin\left(\frac{2\pi}{4L}\frac{L}{2}\right)\sin\left(\frac{1}{2}\left(\frac{2\pi}{T_A}\right)\eta T_A\right) = \frac{\xi_m}{\sqrt{2}}\sin(\pi\eta) \ .$$

Running through the possibilities ($\eta = \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4}, \frac{3}{2}, \frac{7}{4}$, and 2) we find the rope is under tension in the following cases. The first case is one we must be very careful in our reasoning, since A is not displaced but B is displaced in the positive direction; we interpret that as the direction *away from* A (rightwards in the figure) – thus making the rope stretch.

$$\begin{array}{lll}
\eta = \frac{1}{2} & \xi_A = 0 & \xi_B = \frac{\xi_m}{\sqrt{2}} > 0 \\
\eta = \frac{3}{4} & \xi_A = -\xi_m < 0 & \xi_B = \frac{\xi_m}{2} > 0 \\
\eta = \frac{7}{4} & \xi_A = -\xi_m < 0 & \xi_B = -\frac{\xi_m}{2} < 0
\end{array}$$

where in the last case they are both displaced leftward but A more so than B so that the rope is indeed stretched.

(b) The values of η (where we have defined $\eta = t/T_A$) which reproduce the initial state are

$$\begin{array}{llll}
\eta = 1 & \xi_A = 0 & \xi_B = 0 & \text{and} \\
\eta = 2 & \xi_A = 0 & \xi_B = 0 & .
\end{array}$$

(c) The values of η for which the rope is slack are given below. In the first case, both displacements are to the right, but point A is farther to the right than B. In the second case, they are displaced towards each other.

$$\begin{array}{lll}
\eta = \frac{1}{4} & \xi_A = x_m > 0 & \xi_B = \frac{\xi_m}{\sqrt{2}} > 0 \\
\eta = \frac{5}{4} & \xi_A = \xi_m > 0 & \xi_B = -\frac{\xi_m}{2} < 0 \\
\eta = \frac{3}{2} & \xi_A = 0 & \xi_B = -\frac{\xi_m}{\sqrt{2}} < 0
\end{array}$$

where in the third case B is displaced leftward toward the undisplaced point A.

(d) The first design works effectively to damp fundamental modes of vibration in the two cables (especially in the shorter one which would have an antinode at that point), whereas the second one only damps the fundamental mode in the longer cable.

54. (a) The frequency is $f = 1/T = 1/4$ Hz, so $v = f\lambda = 5.0$ cm/s.

(b) We refer to the graph to see that the maximum transverse speed (which we will refer to as u_m) is 5.0 cm/s. Recalling from Ch. 12 the simple harmonic motion relation $u_m = y_m\omega = y_m 2\pi f$, we have

$$5.0 = y_m\left(2\pi\frac{1}{4}\right) \implies y_m = 3.2 \text{ cm} \ .$$

(c) As already noted, $f = 0.25$ Hz.

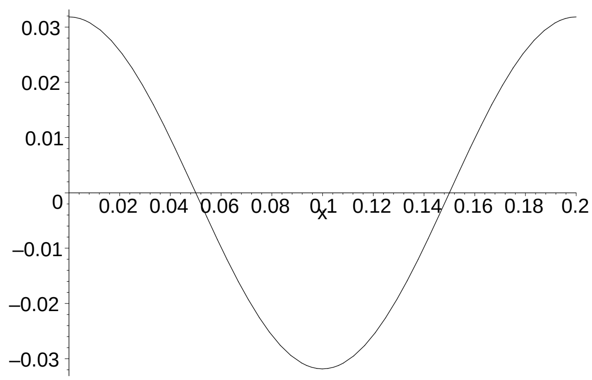
(d) Since $k = 2\pi/\lambda$, we have $k = 10\pi$ rad/m. There must be a sign difference between the t and x terms in the argument in order for the wave to travel to the right. The figure shows that at $x = 0$, the transverse velocity function is $0.050 \sin \frac{\pi}{2} t$. Therefore, the function $u(x,t)$ is

$$u = 0.050 \sin\left(\frac{\pi}{2} t - 10\pi x\right)$$

with lengths in meters and time in seconds. Integrating this with respect to time yields

$$y = -\frac{2(0.050)}{\pi} \cos\left(\frac{\pi}{2} t - 10\pi x\right) + C$$

where C is an integration constant (which we will assume to be zero). The sketch of this function at $t = 2.0$ s for $0 \le x \le 0.20$ m is shown.



55. Using Eq. 17-37, we have

$$y' = \left[0.60 \cos\frac{\pi}{6}\right] \sin\left(5\pi x - 200\pi t + \frac{\pi}{6}\right)$$

with length in meters and time in seconds (see Eq. 17-42 for comparison).

(a) The amplitude is seen to be

$$0.60 \cos\frac{\pi}{6} = 0.3\sqrt{3} = 0.52 \text{ m} \quad .$$

(b) Since $k = 5\pi$ and $\omega = 200\pi$, then (using Eq. 17-11)

$$v = \frac{\omega}{k} = 40 \text{ m/s} \quad .$$

(c) $k = 2\pi/\lambda$ leads to $\lambda = 0.40$ m.

56. We orient one phasor along the x axis with length 4.0 mm and angle 0 and the other at 0.8π rad $= 144°$ (in the second quadrant) with length 7.0 mm. Adding the components, we obtain

$$\begin{aligned} 4.0 + 7.0\cos(144°) &= -1.66 \text{ mm} \qquad &\text{along } x \text{ axis} \\ 7.0\sin(144°) &= 4.11 \text{ mm} \qquad &\text{along } y \text{ axis} \; . \end{aligned}$$

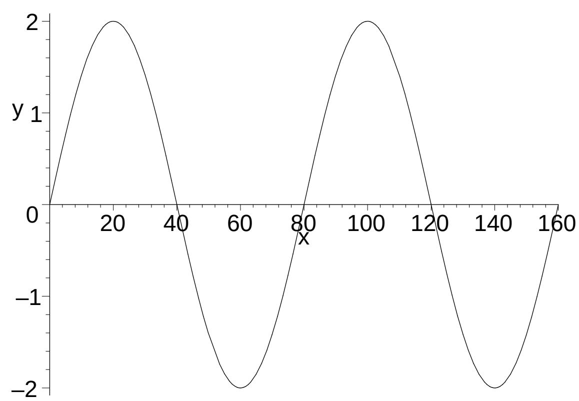
(a) The amplitude of the resultant wave is consequently

$$\sqrt{(-1.66)^2 + 4.11^2} = 4.4 \text{ mm} .$$

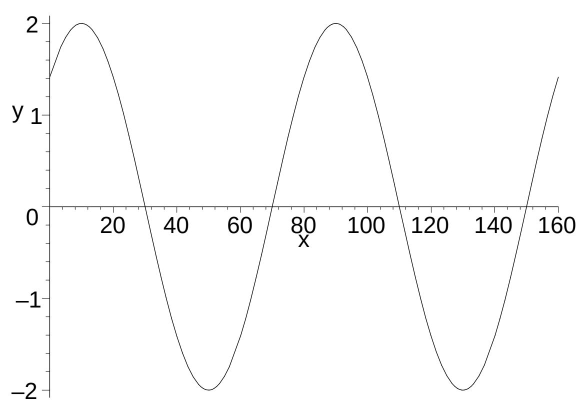
(b) And the phase constant (an angle, measured counterclockwise from the $+x$ axis) is

$$180^\circ + \tan^{-1}\left(\frac{4.11}{-1.66}\right) = 112^\circ .$$

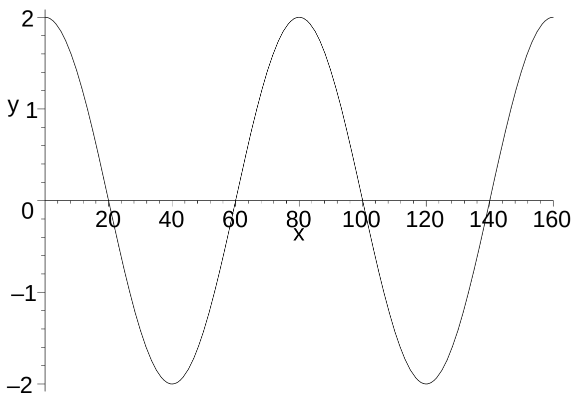
57. (a) Centimeters are to be understood as the length unit and seconds as the time unit. Making sure our (graphing) calculator is in radians mode, we find



(b) The previous graph is at $t = 0$, and this next one is at $t = 0.050$ s.



And the final one, shown below, is at $t = 0.010$ s.



(c) These graphs (as well as the discussion in the textbook) make it clear that the wave is traveling in the $-x$ direction.

58. We use Eq. 17-2, Eq. 17-5, Eq. 17-9, Eq. 17-12, and take the derivative to obtain the transverse speed u.

(a) The amplitude is $y_m = 2.0$ mm.

(b) Since $\omega = 600$ rad/s, the frequency is found to be $f = 600/2\pi \approx 95$ Hz.

(c) Since $k = 20$ rad/m, the velocity of the wave is $v = \omega/k = 600/20 = 30$ m/s in the $+x$ direction.

(d) The wavelength is $\lambda = 2\pi/k \approx 0.31$ m, or 31 cm.

(e) We obtain

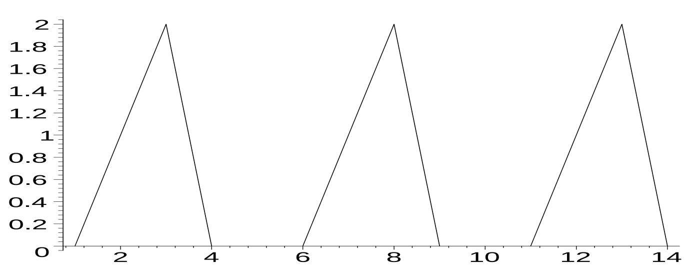
$$u = \frac{dy}{dt} = -\omega y_m \cos(kx - \omega t) \implies u_m = \omega y_m$$

so that the maximum transverse speed is $u_m = (600)(2.0) = 1200$ mm/s, or 1.2 m/s.

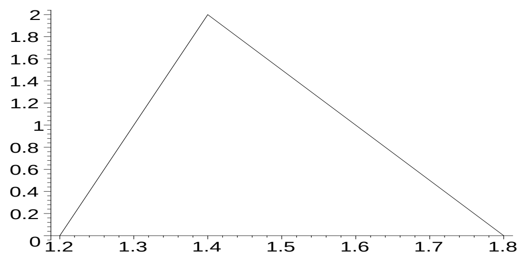
59. (a) Recalling the discussion in §17-5, we see that the speed of the wave given by a function with argument $x - 5t$ (where x is in centimeters and t is in seconds) must be 5 cm/s.

(b) In part (c), we show several "snapshots" of the wave: the one on the left is as shown in Figure 17-47 (at $t = 0$), the middle one is at $t = 1.0$ s, and the rightmost one is at $t = 2.0$ s. It is clear that the wave is traveling to the right (the $+x$ direction).

(c) The third picture in the sequence below shows the pulse at 2 s. The horizontal scale (and, presumably, the vertical one also) is in centimeters.



(d) The leading edge of the pulse reaches $x = 10$ cm at $t = (10 - 4)/5 = 1.2$ s. The particle (say, of the string that carries the pulse) at that location reaches a maximum displacement $h = 2$ cm at $t = (10 - 3)/5 = 1.4$ s. Finally, the the trailing edge of the pulse departs from $x = 10$ cm at $t = (10 - 1)/5 = 1.8$ s. Thus, we find for $h(t)$ at $x = 10$ cm (with the horizontal axis, t, in seconds):



60. We use $P = \frac{1}{2}\mu v \omega^2 y_m^2 \propto v f^2 \propto \sqrt{\tau} f^2$.

   (a) If the tension is quadrupled, then

$$P_2 = P_1\sqrt{\frac{\tau_2}{\tau_1}} = P_1\sqrt{\frac{4\tau_1}{\tau_1}} = 2P_1 \ .$$

   (b) If the frequency is halved, then

$$P_2 = P_1\left(\frac{f_2}{f_1}\right)^2 = P_1\left(\frac{f_1/2}{f_1}\right)^2 = \frac{1}{4}P_1 \ .$$

61. We use $v = \sqrt{\tau/\mu} \propto \sqrt{\tau}$ to obtain

$$\tau_2 = \tau_1\left(\frac{v_2}{v_1}\right)^2 = (120\,\mathrm{N})\left(\frac{180\,\mathrm{m/s}}{170\,\mathrm{m/s}}\right)^2 = 135\ \mathrm{N} \ .$$

62. (a) The wave speed is

$$v = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{120\,\mathrm{N}}{8.70\times10^{-3}\,\mathrm{kg}/1.50\,\mathrm{m}}} = 144\ \mathrm{m/s} \ .$$

   (b) For the one-loop standing wave we have $\lambda_1 = 2L = 2(1.50\,\mathrm{m}) = 3.00\,\mathrm{m}$. For the two-loop standing wave $\lambda_2 = L = 1.50\,\mathrm{m}$.

   (c) The frequency for the one-loop wave is $f_1 = v/\lambda_1 = (144\,\mathrm{m/s})/(3.00\,\mathrm{m}) = 48.0\,\mathrm{Hz}$ and that for the two-loop wave is $f_2 = v/\lambda_2 = (144\,\mathrm{m/s})/(1.50\,\mathrm{m}) = 96.0\,\mathrm{Hz}$.

63. (a) At $x = 2.3\,\mathrm{m}$ and $t = 0.16\,\mathrm{s}$ the displacement is

$$y(x,t) = 0.15\sin[(0.79)(2.3) - 13(0.16)]\,\mathrm{m} = -0.039\,\mathrm{m} \,.$$

   (b) The wave we are looking for must be traveling in $-x$ direction with the same speed and frequency. Thus, its general form is $y'(x,t) = y_m\sin(0.79x + 13t + \phi)$, where y_m is its amplitude and ϕ is its initial phase. In particular, if $y_m = 0.15\,\mathrm{m}$, then there would be nodes (where the wave amplitude is zero) in the string as a result.

   (c) In the special case when $y_m = 0.15\,\mathrm{m}$ and $\phi = 0$, the displacement of the standing wave at $x = 2.3\,\mathrm{m}$ and $t = 0.16\,\mathrm{s}$ is

$$y(x,t) = -0.039\,\mathrm{m} + (0.15\,\mathrm{m})\sin[(0.79)(2.3) + 13(0.16)] = -0.14\ \mathrm{m} \ .$$

64. (a) Let the displacements of the wave at (y,t) be $z(y,t)$. Then $z(y,t) = z_m\sin(ky - \omega t)$, where $z_m = 3.0\,\mathrm{mm}$, $k = 60\,\mathrm{cm}^{-1}$, and $\omega = 2\pi/T = 2\pi/0.20\,\mathrm{s} = 10\pi\,\mathrm{s}^{-1}$. Thus

$$z(y,t) = (3.0\,\mathrm{mm})\sin\left[\left(60\,\mathrm{cm}^{-1}\right)y - \left(10\pi\,\mathrm{s}^{-1}\right)t\right] \ .$$

(b) The maximum transverse speed is

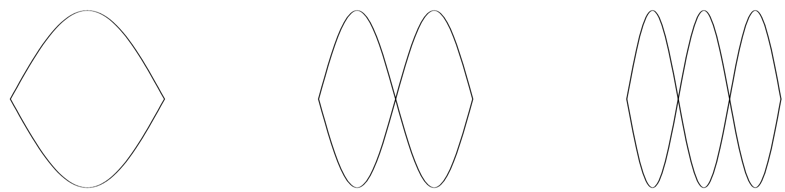
$$u_m = \omega z_m = (2\pi/0.20\,\mathrm{s})(3.0\,\mathrm{mm}) = 94\ \mathrm{mm/s}\ .$$

65. (a) Using Eq. 17-52 with $L = 120$ cm, we find

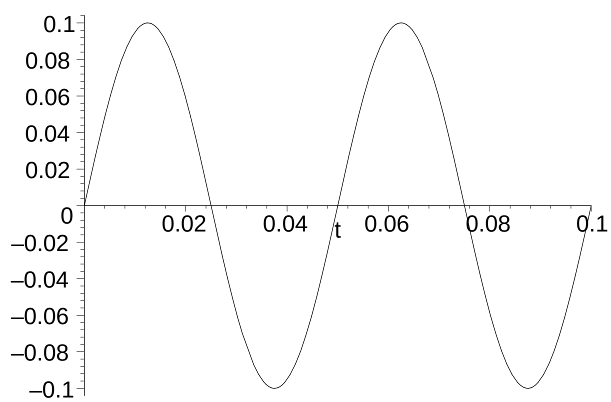
$$\lambda_1 = \frac{2L}{1} = 240 \qquad \lambda_2 = \frac{2L}{2} = 120 \qquad \lambda_3 = \frac{2L}{3} = 80$$

with all values understood to be in centimeters.

(b) The three standing waves are shown below.



66. It is certainly possible to simplify (in the trigonometric sense) the expressions at $x = 3$ m (since $k = 1/2$ in inverse-meters), but there is no particular need to do so, if the goal is to plot the time-dependence of the wave superposition at this value of x. Still, it is worth mentioning the end result of such simplification if it provides some insight into the nature of the graph (shown below): $y_1 + y_2 = (0.10\,\mathrm{m})\sin(40\pi t)$ with t in seconds.



67. By Eq. 17-53, the higher frequencies are integer multiples of the lowest (the fundamental). Therefore, $f_2 = 2(440) = 880$ Hz and $f_3 = 3(440) = 1320$ Hz are the second and third harmonics, respectively.

68. (a) Using $v = f\lambda$, we obtain

$$f = \frac{240\,\mathrm{m/s}}{3.2\,\mathrm{m}} = 75\ \mathrm{Hz}\ .$$

(b) Since frequency is the reciprocal of the period, we find

$$T = \frac{1}{f} = \frac{1}{75\,\mathrm{Hz}} = 0.0133\,\mathrm{s} \approx 13\ \mathrm{ms}\ .$$

69. (a) With length in centimeters and time in seconds, we have

$$u = \frac{dy}{dt} = -60\pi\cos\left(\frac{\pi x}{8} - 4\pi t\right)\ .$$

Thus, when $x = 6$ and $t = \frac{1}{4}$, we obtain

$$u = -60\pi \cos\frac{-\pi}{4} = \frac{-60\pi}{\sqrt{2}} = -133$$

so that the *speed* there is 1.33 m/s.

(b) The numerical coefficient of the cosine in the expression for u is -60π. Thus, the maximum *speed* is 1.88 m/s.

(c) Taking another derivative,

$$a = \frac{du}{dt} = -240\pi^2 \sin\left(\frac{\pi x}{8} - 4\pi t\right)$$

so that when $x = 6$ and $t = \frac{1}{4}$ we obtain $a = -240\pi^2 \sin\frac{-\pi}{4}$ which yields $a = 16.7\ \text{m/s}^2$.

(d) The numerical coefficient of the sine in the expression for a is $-240\pi^2$. Thus, the maximum acceleration is $23.7\ \text{m/s}^2$.

70. (a) Recalling from Ch. 12 the simple harmonic motion relation $u_m = y_m\omega$, we have

$$\omega = \frac{16}{0.04} = 400\ \text{rad/s}\ .$$

Since $\omega = 2\pi f$, we obtain $f = 64$ Hz.

(b) Using $v = f\lambda$, we find $\lambda = 80/64 = 1.26$ m.

(c) Now, $k = 2\pi/\lambda = 5$ rad/m, so the function describing the wave becomes

$$y = 0.04 \sin(5x - 400t + \phi)$$

where distances are in meters and time is in seconds. We adjust the phase constant ϕ to satisfy the condition $y = 0.04$ at $x = t = 0$. Therefore, $\sin\phi = 1$, for which the "simplest" root is $\phi = \pi/2$. Consequently, the answer is

$$y = 0.04 \sin\left(5x - 400t + \frac{\pi}{2}\right)\ .$$

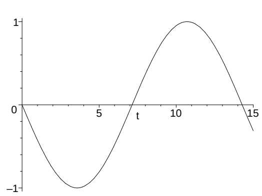
71. We orient one phasor along the x axis with length 3.0 mm and angle 0 and the other at 70° (in the first quadrant) with length 5.0 mm. Adding the components, we obtain

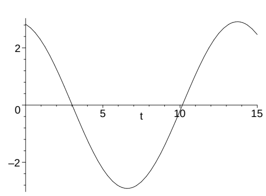
$$\begin{aligned} 3.0 + 5.0\cos(70^\circ) &= 4.71\ \text{mm} \qquad && \text{along } x \text{ axis} \\ 5.0\sin(70^\circ) &= 4.70\ \text{mm} \qquad && \text{along } y \text{ axis}\ . \end{aligned}$$

(a) Thus, amplitude of the resultant wave is $\sqrt{4.71^2 + 4.70^2} = 6.7$ mm.

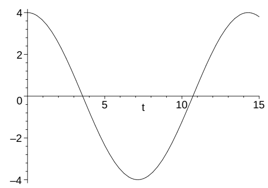
(b) And the angle (phase constant) is $\tan^{-1}(4.70/4.71) = 45^\circ$.

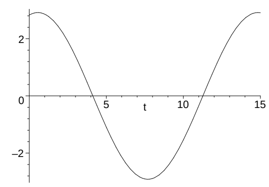
72. (a) The wave number for each wave is $k = 25.1$/m, which means $\lambda = 2\pi/k = 250$ mm. The angular frequency is $\omega = 440$/s; therefore, the period is $T = 2\pi/\omega = 14.3$ ms. We plot the superposition of the two waves $y = y_1 + y_2$ over the time interval $0 \le t \le 15$ ms. The first two graphs below show the oscillatory behavior at $x = 0$ (the graph on the left) and at $x = \lambda/8 \approx 31$ mm. The time unit is understood to be the millisecond and vertical axis (y) is in millimeters.

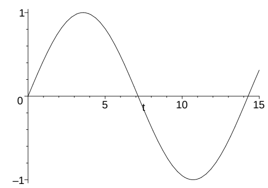




The following three graphs show the oscillation at $x = \lambda/4 \approx 63$ mm (graph on the left), at $x = 3\lambda/8 \approx 94$ mm (middle graph), and at $x = \lambda/2 \approx 125$ mm.







(b) If we think of wave y_1 as being made of two smaller waves going in the same direction, a wave y_{1a} of amplitude 1.50 mm (the same as y_2) and a wave y_{1b} of amplitude 1.00 mm. It is made clear in §17-11 that two equal-magnitude oppositely-moving waves form a standing wave pattern. Thus, waves y_{1a} and y_2 form a standing wave, which leaves y_{1b} as the remaining traveling wave. Since the argument of y_{1b} involves the subtraction $kx - \omega t$, then y_{1b} travels in the $+x$ direction.

(c) If y_2 (which travels in the $-x$ direction, which for simplicity will be called "leftward") had the larger amplitude, then the system would consist of a standing wave plus a leftward moving wave. A simple way to obtain such a situation would be to interchange the amplitudes of the given waves.

(d) Examining carefully the vertical axes, the graphs above certainly suggest that the largest amplitude of oscillation is $y_{\max} = 4.0$ mm and occurs at $x = \lambda/4$, and the smallest amplitude of oscillation is $y_{\min} = 1.0$ mm and occurs at $x = 0$ (and at $x = \lambda/2$).

(e) The largest and smallest amplitudes can be related to the amplitudes of y_1 and y_2 in a simple way: $y_{\max} = y_{1m} + y_{2m}$ and $y_{\min} = y_{1m} - y_{2m}$, where $y_{1m} = 2.5$ mm and $y_{2m} = 1.5$ mm are the amplitudes of the original traveling waves.