Chapter 41

1. One way to think of the units of h is that, because of the equation $E = hf$ and the fact that f is in cycles/second, then the "explicit" units for h should be J·s/cycle. Then, since 2π rad/cycle is a conversion factor for cycles → radians, $\hbar = h/2\pi$ can be thought of as the Planck constant expressed in terms of radians instead of cycles. Using the precise values stated in Appendix B,

$$\begin{aligned}\hbar &= \frac{h}{2\pi} = \frac{6.62606876 \times 10^{-34}\,\text{J·s}}{2\pi} = 1.05457 \times 10^{-34}\,\text{J·s} \\ &= \frac{1.05457 \times 10^{-34}\,\text{J·s}}{1.6021765 \times 10^{-19}\,\text{J/eV}} = 6.582 \times 10^{-16}\ \text{eV·s} .\end{aligned}$$

2. For a given quantum number l there are $(2l + 1)$ different values of m_l. For each given m_l the electron can also have two different spin orientations. Thus, the total number of electron states for a given l is given by $N_l = 2(2l + 1)$.

   (a) Now $l = 3$, so $N_l = 2(2 \times 3 + 1) = 14$.

   (b) In this case, $l = 1$, which means $N_l = 2(2 \times 1 + 1) = 6$.

   (c) Here $l = 1$, so $N_l = 2(2 \times 1 + 1) = 6$.

   (d) Now $l = 0$, so $N_l = 2(2 \times 0 + 1) = 2$.

3. (a) For a given value of the principal quantum number n, the orbital quantum number ℓ ranges from 0 to $n - 1$. For $n = 3$, there are three possible values: 0, 1, and 2.

   (b) For a given value of ℓ, the magnetic quantum number m_ℓ ranges from $-\ell$ to $+\ell$. For $\ell = 1$, there are three possible values: -1, 0, and $+1$.

4. (a) We use Eq. 41-2:

$$L = \sqrt{l(l+1)}\,\hbar = \sqrt{3(3+1)}\,(1.055 \times 10^{-34}\,\text{J·s}) = 3.653 \times 10^{-34}\ \text{J·s} .$$

   (b) We use Eq. 41-7: $L_z = m_l\hbar$. For the maximum value of L_z set $m_l = l$. Thus

$$[L_z]_{\text{max}} = l\hbar = 3(1.055 \times 10^{-34}\,\text{J·s}) = 3.165 \times 10^{-34}\ \text{J·s} .$$

5. For a given quantum number n there are n possible values of l, ranging from 0 to $n - 1$. For each l the number of possible electron states is $N_l = 2(2l + 1)$ (see problem 2). Thus, the total number of possible electron states for a given n is

$$N_n = \sum_{l=0}^{n-1} N_l = 2\sum_{l=0}^{n-1}(2l+1) = 2n^2 .$$

   (a) In this case $n = 4$, which implies $N_n = 2(4^2) = 32$.

(b) Now $n = 1$, so $N_n = 2(1^2) = 2$.

(c) Here $n = 3$, and we obtain $N_n = 2(3^2) = 18$.

(d) Finally, $n = 2 \rightarrow N_n = 2(2^2) = 8$.

6. Using Table 41-1, we find for $n = 4$ and $l = 3$: $m_l = +3, +2, +1, 0, -1, -2, -3$ and $m_s = \pm\frac{1}{2}$.

7. The principal quantum number n must be greater than 3. The magnetic quantum number m_ℓ can have any of the values $-3, -2, -1, 0, +1, +2$, or $+3$. The spin quantum number can have either of the values $-\frac{1}{2}$ or $+\frac{1}{2}$.

8. Using Table 41-1, we find $l = [m_l]_{\max} = 4$ and $n = l_{\max} + 1 \geq l + 1 = 5$. And, as usual, $m_s = \pm\frac{1}{2}$.

9. The principal quantum number n must be greater than 3. The magnetic quantum number m_l can have any of the values $-3, -2, -1, 0, +1, +2$, or $+3$. The spin quantum number can have either of the values $-\frac{1}{2}$ or $+\frac{1}{2}$.

10. For a given quantum number n there are n possible values of l, ranging from 0 to $n - 1$. For each l the number of possible electron states is $N_l = 2(2l + 1)$ (see problem 2). Thus the total number of possible electron states for a given n is

$$N_n = \sum_{l=0}^{n-1} N_l = 2\sum_{l=0}^{n-1}(2l + 1) = 2n^2 \ .$$

Thus, in this problem, the total number of electron states is $N_n = 2n^2 = 2(5)^2 = 50$.

11. (a) For $\ell = 3$, the magnitude of the orbital angular momentum is $L = \sqrt{\ell(\ell + 1)}\hbar = \sqrt{3(3 + 1)}\hbar = \sqrt{12}\hbar$.

(b) The magnitude of the orbital dipole moment is $\mu_{\text{orb}} = \sqrt{\ell(\ell + 1)}\mu_B = \sqrt{12}\mu_B$.

(c) We use $L_z = m_\ell\hbar$ to calculate the z component of the orbital angular momentum, $\mu_z = -m_\ell\mu_B$ to calculate the z component of the orbital magnetic dipole moment, and $\cos\theta = m_\ell/\sqrt{\ell(\ell + 1)}$ to calculate the angle between the orbital angular momentum vector and the z axis. For $\ell = 3$, the magnetic quantum number m_ℓ can take on the values $-3, -2, -1, 0, +1, +2, +3$. Results are tabulated below.

m_ℓ	L_z	$\mu_{\text{orb},\ z}$	θ
-3	$-3\hbar$	$+3\mu_B$	150.0°
-2	$-2\hbar$	$+2\mu_B$	125°
-1	$-\hbar$	$+\mu_B$	107°
0	0	0	90.0°
1	$+\hbar$	$-\mu_B$	73.2°
2	$2\hbar$	$-2\mu_B$	54.7°
3	$3\hbar$	$-3\mu_B$	30.0°

12. (a) For $n = 3$ there are 3 possible values of l: 0, 1, and 2.

(b) We interpret this as asking for the number of distinct values for m_l (this ignores the multiplicity of any particular value). For each l there are $2l+1$ possible values of m_l. Thus the number of possible m_l's for $l=2$ is $(2l+1)=5$. Examining the $l=1$ and $l=0$ cases cannot lead to any new (distinct) values for m_l, so the answer is 5.

(c) Regardless of the values of n, l and m_l, for an electron there are always two possible values of m_s: $\pm\frac{1}{2}$.

(d) The population in the $n=3$ shell is equal to the number of electron states in the shell, or $2n^2 = 2(3^2) = 18$.

(e) Each subshell has its own value of l. Since there are three different values of l for $n=3$, there are three subshells in the $n=3$ shell.

13. Since $L^2 = L_x^2 + L_y^2 + L_z^2$, $\sqrt{L_x^2+L_y^2} = \sqrt{L^2 - L_z^2}$. Replacing L^2 with $\ell(\ell+1)\hbar^2$ and L_z with $m_\ell\hbar$, we obtain

$$\sqrt{L_x^2+L_y^2} = \hbar\sqrt{\ell(\ell+1) - m_\ell^2} \ .$$

For a given value of ℓ, the greatest that m_ℓ can be is ℓ, so the smallest that $\sqrt{L_x^2+L_y^2}$ can be is $\hbar\sqrt{\ell(\ell+1)-\ell^2} = \hbar\sqrt{\ell}$. The smallest possible magnitude of m_ℓ is zero, so the largest $\sqrt{L_x^2+L_y^2}$ can be is $\hbar\sqrt{\ell(\ell+1)}$. Thus,

$$\hbar\sqrt{\ell} \le \sqrt{L_x^2+L_y^2} \le \hbar\sqrt{\ell(\ell+1)} \ .$$

14. (a) The value of l satisfies $\sqrt{l(l+1)}\hbar \approx \sqrt{l^2}\hbar = l\hbar = L$, so $l \simeq L/\hbar \simeq 3\times10^{74}$.

(b) The number is $2l+1 \approx 2(3\times10^{74}) = 6\times10^{74}$.

(c) Since

$$\cos\theta_{\min} = \frac{m_{l\,\max}\hbar}{\sqrt{l(l+1)}\hbar} = \frac{l}{\sqrt{l(l+1)}} \approx 1 - \frac{1}{2l} = 1 - \frac{1}{2(3\times10^{74})}$$

or $\cos\theta_{\min} \simeq 1 - \theta_{\min}^2/2 \approx 1 - 10^{-74}/6$, we have $\theta_{\min} \simeq \sqrt{10^{-74}/3} = 6\times10^{-38}\,\text{rad}$. The correspondence principle requires that all the quantum effects vanish as $\hbar \to 0$. In this case $\hbar/L$ is extremely small so the quantization effects are barely existent, with $\theta_{\min} \simeq 10^{-38}\,\text{rad} \simeq 0$.

15. The magnitude of the spin angular momentum is $S = \sqrt{s(s+1)}\,\hbar = (\sqrt{3}/2)\hbar$, where $s=\frac{1}{2}$ is used. The z component is either $S_z = \hbar/2$ or $-\hbar/2$. If $S_z = +\hbar/2$, the angle θ between the spin angular momentum vector and the positive z axis is

$$\theta = \cos^{-1}\left(\frac{S_z}{S}\right) = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right) = 54.7^\circ \ .$$

If $S_z = -\hbar/2$, the angle is $\theta = 180^\circ - 54.7^\circ = 125.3^\circ$.

16. (a) From Fig. 41-10 and Eq. 41-18,

$$\Delta E = 2\mu_B B = \frac{2(9.27\times10^{-24}\,\text{J/T})(0.50\,\text{T})}{1.60\times10^{-19}\,\text{J/eV}} = 58\,\mu\text{eV} \ .$$

(b) From $\Delta E = hf$ we get

$$f = \frac{\Delta E}{h} = \frac{9.27\times10^{-24}\,\text{J}}{6.63\times10^{-34}\,\text{J}\cdot\text{s}} = 1.4\times10^{10}\,\text{Hz} = 14\ \text{GHz} \ .$$

(c) The wavelength is

$$\lambda = \frac{c}{f} = \frac{2.998\times10^{8}\,\text{m/s}}{1.4\times10^{10}\,\text{Hz}} = 2.1\ \text{cm} \ ,$$

which is in the short radio wave region.

17. The acceleration is

$$a = \frac{F}{M} = \frac{(\mu \cos\theta)\,(dB/dz)}{M}\,,$$

where M is the mass of a silver atom, μ is its magnetic dipole moment, B is the magnetic field, and θ is the angle between the dipole moment and the magnetic field. We take the moment and the field to be parallel ($\cos\theta = 1$) and use the data given in Sample Problem 41-1 to obtain

$$a = \frac{(9.27\times 10^{-24}\,\mathrm{J/T})(1.4\times 10^{3}\,\mathrm{T/m})}{1.8\times 10^{-25}\,\mathrm{kg}} = 7.21\times 10^{4}\ \mathrm{m/s^2}\ .$$

18. (a) From Eq. 41-19,

$$F = \mu_B \left|\frac{dB}{dz}\right| = (9.27\times 10^{-24}\,\mathrm{J/T})(1.6\times 10^{2}\,\mathrm{T/m}) = 1.5\times 10^{-21}\ \mathrm{N}\ .$$

(b) The vertical displacement is

$$\begin{aligned}\Delta x &= \frac{1}{2}at^2 = \frac{1}{2}\left(\frac{F}{m}\right)\left(\frac{l}{v}\right)^2 \\ &= \frac{1}{2}\left(\frac{1.5\times 10^{-21}\,\mathrm{N}}{1.67\times 10^{-27}\,\mathrm{kg}}\right)\left(\frac{0.80\,\mathrm{m}}{1.2\times 10^{5}\,\mathrm{m/s}}\right)^2 \\ &= 2.0\times 10^{-5}\ \mathrm{m}\ .\end{aligned}$$

19. The energy of a magnetic dipole in an external magnetic field $\vec{B}$ is $U = -\vec{\mu}\cdot\vec{B} = -\mu_z B$, where $\vec{\mu}$ is the magnetic dipole moment and μ_z is its component along the field. The energy required to change the moment direction from parallel to antiparallel is $\Delta E = \Delta U = 2\mu_z B$. Since the z component of the spin magnetic moment of an electron is the Bohr magneton μ_B, $\Delta E = 2\mu_B B = 2(9.274\times 10^{-24}\,\mathrm{J/T})(0.200\,\mathrm{T}) = 3.71\times 10^{-24}\,\mathrm{J}$. The photon wavelength is

$$\lambda = \frac{c}{f} = \frac{hc}{\Delta E} = \frac{(6.63\times 10^{-34}\,\mathrm{J\cdot s})(3.00\times 10^{8}\,\mathrm{m/s})}{3.71\times 10^{-24}\,\mathrm{J}} = 5.36\times 10^{-2}\ \mathrm{m}\ .$$

20. We let $\Delta E = 2\mu_B B_{\mathrm{eff}}$ (based on Fig. 41-10 and Eq. 41-18) and solve for B_{eff}:

$$B_{\mathrm{eff}} = \frac{\Delta E}{2\mu_B} = \frac{hc}{2\lambda\mu_B} = \frac{1240\,\mathrm{nm\cdot eV}}{2(21\times 10^{-7}\,\mathrm{nm})(5.788\times 10^{-5}\,\mathrm{eV/T})} = 51\ \mathrm{mT}\ .$$

21. (a) Using the result of problem 3 in Chapter 39,

$$\Delta E = hc\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right) = (1240\,\mathrm{eV\cdot nm})\left(\frac{1}{588.995\,\mathrm{nm}} - \frac{1}{589.592\,\mathrm{nm}}\right) = 2.13\ \mathrm{meV}\ .$$

(b) From $\Delta E = 2\mu_B B$ (see Fig. 41-10 and Eq. 41-18), we get

$$B = \frac{\Delta E}{2\mu_B} = \frac{2.13\times 10^{-3}\,\mathrm{eV}}{2(5.788\times 10^{-5}\,\mathrm{eV/T})} = 18\ \mathrm{T}\ .$$

22. The total magnetic field, $B = B_{\mathrm{local}} + B_{\mathrm{ext}}$, satisfies $\Delta E = hf = 2\mu B$ (see Eq. 41-22). Thus,

$$B_{\mathrm{local}} = \frac{hf}{2\mu} - B_{\mathrm{ext}} = \frac{(6.63\times 10^{-34}\mathrm{J\cdot s})(34\times 10^{6}\,\mathrm{Hz})}{2(1.41\times 10^{-26}\,\mathrm{J/T})} - 0.78\,\mathrm{T} = 19\ \mathrm{mT}\ .$$

23. Because of the Pauli principle (and the requirement that we construct a state of lowest possible total energy), two electrons fill the $n = 1, 2, 3$ levels and one electron occupies the $n = 4$ level. Thus, using Eq. 40-4,

$$\begin{aligned} E_{\text{ground}} &= 2E_1 + 2E_2 + 2E_3 + E_4 \\ &= 2\left(\frac{h^2}{8mL^2}\right)(1)^2 + 2\left(\frac{h^2}{8mL^2}\right)(2)^2 + 2\left(\frac{h^2}{8mL^2}\right)(3)^2 + \left(\frac{h^2}{8mL^2}\right)(4)^2 \\ &= (2 + 8 + 18 + 16)\left(\frac{h^2}{8mL^2}\right) = 44\left(\frac{h^2}{8mL^2}\right) . \end{aligned}$$

24. Using Eq. 40-20 (see also problem 27 in Chapter 40) we find that the lowest four levels of the rectangular corral (with this specific "aspect ratio") are non-degenerate, with energies $E_{1,1} = 1.25$, $E_{1,2} = 2.00$, $E_{1,3} = 3.25$, and $E_{2,1} = 4.25$ (all of these understood to be in "units" of $h^2/8mL^2$). Therefore, obeying the Pauli principle, we have

$$E_{\text{ground}} = 2E_{1,1} + 2E_{1,2} + 2E_{1,3} + E_{2,1} = 2(1.25) + 2(2.00) + 2(3.25) + 4.25$$

which means (putting the "unit" factor back in) that the lowest possible energy of the system is $E_{\text{ground}} = 17.25(h^2/8mL^2)$.

25. (a) Promoting one of the electrons (described in problem 23) to a not-fully occupied higher level, we find that the configuration with the least total energy greater than that of the ground state has the $n = 1$ and 2 levels still filled, but now has only one electron in the $n = 3$ level; the remaining two electrons are in the $n = 4$ level. Thus,

$$\begin{aligned} E_{\text{first excited}} &= 2E_1 + 2E_2 + E_3 + 2E_4 \\ &= 2\left(\frac{h^2}{8mL^2}\right)(1)^2 + 2\left(\frac{h^2}{8mL^2}\right)(2)^2 + \left(\frac{h^2}{8mL^2}\right)(3)^2 + 2\left(\frac{h^2}{8mL^2}\right)(4)^2 \\ &= (2 + 8 + 9 + 32)\left(\frac{h^2}{8mL^2}\right) = 51\left(\frac{h^2}{8mL^2}\right) . \end{aligned}$$

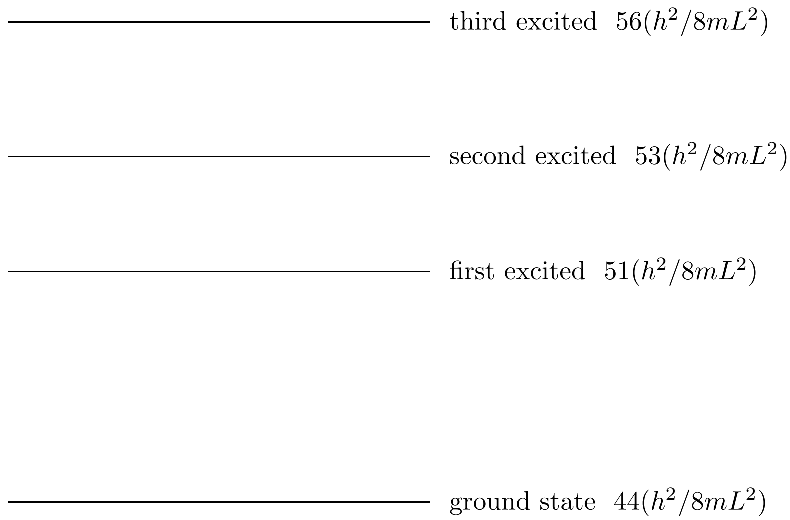
(b) Now, the configuration which provides the next higher total energy, above that found in part (a), has the bottom three levels filled (just as in the ground state configuration) and has the seventh electron occupying the $n = 5$ level:

$$\begin{aligned} E_{\text{second excited}} &= 2E_1 + 2E_2 + 2E_3 + E_5 \\ &= 2\left(\frac{h^2}{8mL^2}\right)(1)^2 + 2\left(\frac{h^2}{8mL^2}\right)(2)^2 + 2\left(\frac{h^2}{8mL^2}\right)(3)^2 + \left(\frac{h^2}{8mL^2}\right)(5)^2 \\ &= (2 + 8 + 18 + 25)\left(\frac{h^2}{8mL^2}\right) = 53\left(\frac{h^2}{8mL^2}\right) . \end{aligned}$$

(c) The third excited state has the $n = 1, 3, 4$ levels filled, and the $n = 2$ level half-filled:

$$\begin{aligned} E_{\text{third excited}} &= 2E_1 + E_2 + 2E_3 + 2E_4 \\ &= 2\left(\frac{h^2}{8mL^2}\right)(1)^2 + \left(\frac{h^2}{8mL^2}\right)(2)^2 + 2\left(\frac{h^2}{8mL^2}\right)(3)^2 + 2\left(\frac{h^2}{8mL^2}\right)(4)^2 \\ &= (2 + 4 + 18 + 32)\left(\frac{h^2}{8mL^2}\right) = 56\left(\frac{h^2}{8mL^2}\right) . \end{aligned}$$

(d) The energy states of this problem and problem 23 are suggested in the sketch below:



26. (a) Using Eq. 40-20 (see also problem 27 in Chapter 40) we find that the lowest five levels of the rectangular corral (with this specific "aspect ratio") have energies $E_{1,1} = 1.25$, $E_{1,2} = 2.00$, $E_{1,3} = 3.25$, $E_{2,1} = 4.25$, and $E_{2,2} = 5.00$ (all of these understood to be in "units" of $h^2/8mL^2$). It should be noted that the energy level we denote $E_{2,2}$ actually corresponds to two energy levels ($E_{2,2}$ and $E_{1,4}$; they are degenerate), but that will not affect our calculations in this problem. The configuration which provides the lowest system energy higher than that of the ground state has the first three levels filled, the fourth one empty, and the fifth one half-filled:

$$E_{\text{first excited}} = 2E_{1,1} + 2E_{1,2} + 2E_{1,3} + E_{2,2} = 2(1.25) + 2(2.00) + 2(3.25) + 5.00$$

which means (putting the "unit" factor back in) the energy of the first excited state is $E_{\text{first excited}} = 18.00(h^2/8mL^2)$.

(b) The configuration which provides the next higher system energy has the first two levels filled, the third one half-filled, and the fourth one filled:

$$E_{\text{second excited}} = 2E_{1,1} + 2E_{1,2} + E_{1,3} + 2E_{2,1} = 2(1.25) + 2(2.00) + 3.25 + 2(4.25)$$

which means (putting the "unit" factor back in) the energy of the second excited state is $E_{\text{second excited}} = 18.25(h^2/8mL^2)$.

(c) Now, the configuration which provides the *next* higher system energy has the first two levels filled, with the next three levels half-filled:

$$E_{\text{third excited}} = 2E_{1,1} + 2E_{1,2} + E_{1,3} + E_{2,1} + E_{2,2} = 2(1.25) + 2(2.00) + 3.25 + 4.25 + 5.00$$

which means (putting the "unit" factor back in) the energy of the third excited state is $E_{\text{third excited}} = 19.00(h^2/8mL^2)$.

(d) The energy states of this problem and problem 24 are suggested in the sketch below:

third excited $19.00(h^2/8mL^2)$

second excited $18.25(h^2/8mL^2)$

first excited $18.00(h^2/8mL^2)$

ground state $17.25(h^2/8mL^2)$

27. In terms of the quantum numbers n_x, n_y, and n_z, the single-particle energy levels are given by

$$E_{n_x,n_y,n_z} = \frac{h^2}{8mL^2}\left(n_x^2 + n_y^2 + n_z^2\right) \ .$$

The lowest single-particle level corresponds to $n_x = 1$, $n_y = 1$, and $n_z = 1$ and is $E_{1,1,1} = 3(h^2/8mL^2)$. There are two electrons with this energy, one with spin up and one with spin down. The next lowest single-particle level is three-fold degenerate in the three integer quantum numbers. The energy is $E_{1,1,2} = E_{1,2,1} = E_{2,1,1} = 6(h^2/8mL^2)$. Each of these states can be occupied by a spin up and a spin down electron, so six electrons in all can occupy the states. This completes the assignment of the eight electrons to single-particle states. The ground state energy of the system is $E_{\text{gr}} = (2)(3)(h^2/8mL^2) + (6)(6)(h^2/8mL^2) = 42(h^2/8mL^2)$.

28. We use the results of problem 28 in Chapter 40. The Pauli principle requires that no more than two electrons be in the lowest energy level (at $E_{1,1,1} = 3(h^2/8mL^2)$), but – due to their degeneracies – as many as six electrons can be in the next three levels ($E' = E_{1,1,2} = E_{1,2,1} = E_{2,1,1} = 6(h^2/8mL^2)$, $E'' = E_{1,2,2} = E_{2,2,1} = E_{2,1,2} = 9(h^2/8mL^2)$, and $E''' = E_{1,1,3} = E_{1,3,1} = E_{3,1,1} = 11(h^2/8mL^2)$). Using Eq. 40-21, the level above those can only hold two electrons: $E_{2,2,2} = (2^2 + 2^2 + 2^2)(h^2/8mL^2) = 12(h^2/8mL^2)$. And the next higher level can hold as much as twelve electrons (see part (e) of problem 28 in Chapter 40) and has energy $E'''' = 14(h^2/8mL^2)$.

(a) The configuration which provides the lowest system energy higher than that of the ground state has the first level filled, the second one with one vacancy, and the third one with one occupant:

$$E_{\text{first excited}} = 2E_{1,1,1} + 5E' + E'' = 2(3) + 5(6) + 9$$

which means (putting the "unit" factor back in) the energy of the first excited state is $E_{\text{first excited}} = 45(h^2/8mL^2)$.

(b) The configuration which provides the next higher system energy has the first level filled, the second one with one vacancy, the third one empty, and the fourth one with one occupant:

$$E_{\text{second excited}} = 2E_{1,1,1} + 5E' + E'' = 2(3) + 5(6) + 11$$

which means (putting the "unit" factor back in) the energy of the second excited state is $E_{\text{second excited}} = 47(h^2/8mL^2)$.

(c) Now, there are a couple of configurations which provides the *next* higher system energy. One has the first level filled, the second one with one vacancy, the third and fourth ones empty, and the fifth one with one occupant:

$$E_{\text{third excited}} = 2E_{1,1,1} + 5E' + E''' = 2(3) + 5(6) + 12$$

which means (putting the "unit" factor back in) the energy of the third excited state is $E_{\text{third excited}} = 48(h^2/8mL^2)$. The other configuration with this same total energy has the first level filled, the second one with two vacancies, and the third one with one occupant.

(d) The energy states of this problem and problem 27 are suggested in the sketch below:

third excited $48(h^2/8mL^2)$

second excited $47(h^2/8mL^2)$

first excited $45(h^2/8mL^2)$

ground state $42(h^2/8mL^2)$

29. For a given shell with quantum number n the total number of available electron states is $2n^2$. Thus, for the first four shells ($n = 1$ through 4) the number of available states are 2, 8, 18, and 32 (see Appendix G). Since $2 + 8 + 18 + 32 = 60 < 63$, according to the "logical" sequence the first four shells would be completely filled in an europium atom, leaving $63 - 60 = 3$ electrons to partially occupy the $n = 5$ shell. Two of these three electrons would fill up the $5s$ subshell, leaving only one remaining electron in the only partially filled subshell (the $5p$ subshell). In chemical reactions this electron would have the tendency to be transferred to another element, leaving the remaining 62 electrons in chemically stable, completely filled subshells. This situation is very similar to the case of sodium, which also has only one electron in a partially filled shell (the $3s$ shell).

30. The first three shells ($n = 1$ through 3), which can accommodate a total of $2 + 8 + 18 = 28$ electrons, are completely filled. For selenium ($Z = 34$) there are still $34 - 28 = 6$ electrons left. Two of them go to the $4s$ subshell, leaving the remaining four in the highest occupied subshell, the $4p$ subshell. Similarly, for bromine ($Z = 35$) the highest occupied subshell is also the $4p$ subshell, which contains five electrons; and for krypton ($Z = 36$) the highest occupied subshell is also the $4p$ subshell, which now accommodates six electrons.

31. Without the spin degree of freedom the number of available electron states for each shell would be reduced by half. So the values of Z for the noble gas elements would become half of what they are now: $Z = 1, 5, 9, 18, 27$, and 43. Of this set of numbers, the only one which coincides with one of the familiar noble gas atomic numbers ($Z = 2, 10, 18, 36, 54$, and 86) is 18. Thus, argon would be the only one that would remain "noble."

32. When a helium atom is in its ground state, both of its electrons are in the $1s$ state. Thus, for each of the electrons, $n = 1$, $l = 0$, and $m_l = 0$. One of the electrons is spin up ($m_s = +\frac{1}{2}$), while the other is spin down ($m_s = -\frac{1}{2}$).

33. (a) All states with principal quantum number $n = 1$ are filled. The next lowest states have $n = 2$. The orbital quantum number can have the values $\ell = 0$ or 1 and of these, the $\ell = 0$ states have the lowest energy. The magnetic quantum number must be $m_\ell = 0$ since this is the only possibility if $\ell = 0$. The spin quantum number can have either of the values $m_s = -\frac{1}{2}$ or $+\frac{1}{2}$. Since there is no external magnetic field, the energies of these two states are the same. Therefore, in the ground state, the quantum numbers of the third electron are either $n = 2$, $\ell = 0$, $m_\ell = 0$, $m_s = -\frac{1}{2}$ or $n = 2$, $\ell = 0$, $m_\ell = 0$, $m_s = +\frac{1}{2}$.

(b) The next lowest state in energy is an $n = 2$, $\ell = 1$ state. All $n = 3$ states are higher in energy. The magnetic quantum number can be $m_\ell = -1$, 0, or $+1$; the spin quantum number can be $m_s = -\frac{1}{2}$ or $+\frac{1}{2}$. If both external and internal magnetic fields can be neglected, all these states have the same energy.

34. (a) The number of different m_l's is $2l+1=3$, and the number of different m_s's is 2. Thus, the number of combinations is $N=(3\times 2)^2/2=18$.

(b) There are six states disallowed by the exclusion principle, in which both electrons share the quantum numbers

$$(n,l,m_l,m_s)=\left(2,1,1,\frac{1}{2}\right),\left(2,1,1,-\frac{1}{2}\right),\left(2,1,0,\frac{1}{2}\right),\left(2,1,0,-\frac{1}{2}\right),\left(2,1,-1,\frac{1}{2}\right),\left(2,1,-1,-\frac{1}{2}\right).$$

35. For a given value of the principal quantum number n, there are n possible values of the orbital quantum number ℓ, ranging from 0 to $n-1$. For any value of ℓ, there are $2\ell+1$ possible values of the magnetic quantum number m_ℓ, ranging from $-\ell$ to $+\ell$. Finally, for each set of values of ℓ and m_ℓ, there are two states, one corresponding to the spin quantum number $m_s=-\frac{1}{2}$ and the other corresponding to $m_s=+\frac{1}{2}$. Hence, the total number of states with principal quantum number n is

$$N=2\sum_0^{n-1}(2\ell+1)\ .$$

Now

$$\sum_0^{n-1}2\ell=2\sum_0^{n-1}\ell=2\frac{n}{2}(n-1)=n(n-1)\,,$$

since there are n terms in the sum and the average term is $(n-1)/2$. Furthermore,

$$\sum_0^{n-1}1=n\ .$$

Thus $N=2\left[n(n-1)+n\right]=2n^2$.

36. The kinetic energy gained by the electron is eV, where V is the accelerating potential difference. A photon with the minimum wavelength (which, because of $E=hc/\lambda$, corresponds to maximum photon energy) is produced when all of the electron's kinetic energy goes to a single photon in an event of the kind depicted in Fig. 41-15. Thus, using the result of problem 3 in Chapter 39,

$$eV=\frac{hc}{\lambda_{\rm min}}=\frac{1240\,{\rm eV\cdot nm}}{0.10\,{\rm nm}}=1.24\times 10^4\ {\rm eV}\ .$$

Therefore, the accelerating potential difference is $V=1.24\times 10^4\,{\rm V}=12.4\,{\rm kV}$.

37. We use $eV=hc/\lambda_{\rm min}$ (see Eq. 41-23 and Eq. 39-4):

$$h=\frac{eV\lambda_{\rm min}}{c}=\frac{(1.60\times 10^{-19}\,{\rm C})(40.0\times 10^3\,{\rm eV})(31.1\times 10^{-12}\,{\rm m})}{2.998\times 10^8\,{\rm m/s}}=6.63\times 10^{-34}\ {\rm J\cdot s}\ .$$

38. Letting $eV=hc/\lambda_{\rm min}$ (see Eq. 41-23 and Eq. 39-4), we get

$$\lambda_{\rm min}=\frac{hc}{eV}=\frac{1240\,{\rm nm\cdot eV}}{eV}=\frac{1240\,{\rm pm\cdot keV}}{eV}=\frac{1240\,{\rm pm}}{V}$$

where V is measured in kV.

39. The initial kinetic energy of the electron is 50.0 keV. After the first collision, the kinetic energy is 25 keV; after the second, it is 12.5 keV; and after the third, it is zero. The energy of the photon produced in the first collision is 50.0 keV − 25.0 keV = 25.0 keV. The wavelength associated with this photon is

$$\lambda=\frac{1240\,{\rm eV\cdot nm}}{25.0\times 10^3\,{\rm eV}}=4.96\times 10^{-2}\,{\rm nm}=49.6\,{\rm pm}$$

where the result of Exercise 3 of Chapter 39 is used. The energies of the photons produced in the second and third collisions are each 12.5 keV and their wavelengths are

$$\lambda = \frac{1240\,\text{eV}\cdot\text{nm}}{12.5 \times 10^3\,\text{eV}} = 9.92 \times 10^{-2}\,\text{nm} = 99.2\,\text{pm} .$$

40. (a) and (b) Let the wavelength of the two photons be λ_1 and $\lambda_2 = \lambda_1 + \Delta\lambda$. Then,

$$eV = \frac{hc}{\lambda_1} + \frac{hc}{\lambda_1 + \Delta\lambda} ,$$

or

$$\lambda_1 = \frac{-(\Delta\lambda/\lambda_0 - 2) \pm \sqrt{(\Delta\lambda/\lambda_0)^2 + 4}}{2/\Delta\lambda} .$$

Here, $\Delta\lambda = 130\,\text{pm}$ and $\lambda_0 = hc/eV = 1240\,\text{keV}\cdot\text{pm}/20\,\text{keV} = 62\,\text{pm}$. The result of problem 3 in Chapter 39 is adapted to these units ($hc = 1240\,\text{eV}\cdot\text{nm} = 1240\,\text{keV}\cdot\text{pm}$). We choose the plus sign in the expression for λ_1 (since $\lambda_1 > 0$) and obtain

$$\lambda_1 = \frac{-(130\,\text{pm}/62\,\text{pm} - 2) + \sqrt{(130\,\text{pm}/62\,\text{pm})^2 + 4}}{2/62\,\text{pm}} = 87\ \text{pm} ,$$

and

$$\lambda_2 = \lambda_1 + \Delta\lambda = 87\,\text{pm} + 130\,\text{pm} = 2.2 \times 10^2\ \text{pm} .$$

The energy of the electron after its first deceleration is

$$K = K_i - \frac{hc}{\lambda_1} = 20\,\text{keV} - \frac{1240\,\text{keV}\cdot\text{pm}}{87\,\text{pm}} = 5.7\ \text{keV}.$$

The energies of the two photons are

$$E_1 = \frac{hc}{\lambda_1} = \frac{1240\,\text{keV}\cdot\text{pm}}{87\,\text{pm}} = 14\ \text{keV}$$

and

$$E_2 = \frac{hc}{\lambda_2} = \frac{1240\,\text{keV}\cdot\text{pm}}{130\,\text{pm}} = 5.7\ \text{keV} .$$

41. Suppose an electron with total energy E and momentum $\mathbf{p}$ spontaneously changes into a photon. If energy is conserved, the energy of the photon is E and its momentum has magnitude E/c. Now the energy and momentum of the electron are related by $E^2 = (pc)^2 + (mc^2)^2$, so $pc = \sqrt{E^2 - (mc^2)^2}$. Since the electron has non-zero mass, E/c and p cannot have the same value. Hence, momentum cannot be conserved. A third particle must participate in the interaction, primarily to conserve momentum. It does, however, carry off some energy.

42. (a) We use $eV = hc/\lambda_{\text{min}}$ (see Eq. 41-23 and Eq. 39-4). The result of problem 3 in Chapter 39 is adapted to these units ($hc = 1240\,\text{eV}\cdot\text{nm} = 1240\,\text{keV}\cdot\text{pm}$).

$$\lambda_{\text{min}} = \frac{hc}{eV} = \frac{1240\,\text{keV}\cdot\text{pm}}{50.0\,\text{keV}} = 24.8\ \text{pm} .$$

(b) and (c) The values of λ for the K_α and K_β lines do not depend on the external potential and are therefore unchanged.

43. (a) The cut-off wavelength λ_{min} is characteristic of the incident electrons, not of the target material. This wavelength is the wavelength of a photon with energy equal to the kinetic energy of an incident electron. According to the result of Exercise 3 of Chapter 39,

$$\lambda_{\text{min}} = \frac{1240\,\text{eV}\cdot\text{nm}}{35 \times 10^3\,\text{eV}} = 3.54 \times 10^{-2}\,\text{nm} = 35.4\ \text{pm} .$$

(b) A K_α photon results when an electron in a target atom jumps from the L-shell to the K-shell. The energy of this photon is $25.51\,\text{keV} - 3.56\,\text{keV} = 21.95\,\text{keV}$ and its wavelength is $\lambda_{\text{K}\alpha} = (1240\,\text{eV}\cdot\text{nm})/(21.95 \times 10^3\,\text{eV}) = 5.65 \times 10^{-2}\,\text{nm} = 56.5\,\text{pm}$.

(c) A K_β photon results when an electron in a target atom jumps from the M-shell to the K-shell. The energy of this photon is $25.51\,\text{keV} - 0.53\,\text{keV} = 24.98\,\text{keV}$ and its wavelength is $\lambda_{\text{K}\beta} = (1240\,\text{eV}\cdot\text{nm})/(24.98 \times 10^3\,\text{eV}) = 4.96 \times 10^{-2}\,\text{nm} = 49.6\,\text{pm}$.

44. The result of problem 3 in Chapter 39 is adapted to these units ($hc = 1240\,\text{eV}\cdot\text{nm} = 1240\,\text{keV}\cdot\text{pm}$). For the K_α line from iron

$$\Delta E = \frac{hc}{\lambda} = \frac{1240\,\text{keV}\cdot\text{pm}}{193\,\text{pm}} = 6.4\text{ keV} .$$

We remark that for the hydrogen atom the corresponding energy difference is

$$\Delta E_{12} = -(13.6\,\text{eV})\left(\frac{1}{2^2} - \frac{1}{1^1}\right) = 10\text{ eV}.$$

That this difference is much greater in iron is due to the fact that its atomic nucleus contains 26 protons, exerting a much greater force on the K- and L-shell electrons than that provided by the single proton in hydrogen.

45. Since the frequency of an x-ray emission is proportional to $(Z-1)^2$, where Z is the atomic number of the target atom, the ratio of the wavelength λ_{Nb} for the K_α line of niobium to the wavelength λ_{Ga} for the K_α line of gallium is given by $\lambda_{\text{Nb}}/\lambda_{\text{Ga}} = (Z_{\text{Ga}}-1)^2/(Z_{\text{Nb}}-1)^2$, where Z_{Nb} is the atomic number of niobium (41) and Z_{Ga} is the atomic number of gallium (31). Thus $\lambda_{\text{Nb}}/\lambda_{\text{Ga}} = (30)^2/(40)^2 = 9/16$.

46. The result of problem 3 in Chapter 39 is adapted to these units ($hc = 1240\,\text{eV}\cdot\text{nm} = 1240\,\text{keV}\cdot\text{pm}$). The energy difference $E_L - E_M$ for the x-ray atomic energy levels of molybdenum is

$$\Delta E = E_L - E_M = \frac{hc}{\lambda_L} - \frac{hc}{\lambda_M} = \frac{1240\,\text{keV}\cdot\text{pm}}{63.0\,\text{pm}} - \frac{1240\,\text{keV}\cdot\text{pm}}{71.0\,\text{pm}} = 2.2\text{ keV} .$$

47. From the data given in the problem, we calculate frequencies (using Eq. 39-1), take their square roots, look up the atomic numbers (see Appendix F), and do a least-squares fit to find the slope: the result is 5.02×10^7 with the odd-sounding unit of a square root of a Hertz. We remark that the least squares procedure also returns a value for the y-intercept of this statistically determined "best-fit" line; that result is negative and would appear on a graph like Fig. 41-17 to be at about -0.06 on the vertical axis. Also, we can estimate the slope of the Moseley line shown in Fig. 41-17:

$$\frac{(1.95 - 0.50)10^9\,\text{Hz}^{1/2}}{40 - 11} \approx 5.0 \times 10^7\,\text{Hz}^{1/2} .$$

These are in agreement with the discussion in §41-10.

48. (a) From Fig. 41-14 we estimate the wavelengths corresponding to the K_α and K_β lines to be $\lambda_\alpha = 70.0\,\text{pm}$ and $\lambda_\beta = 63.0\,\text{pm}$, respectively. Using the result of problem 3 in Chapter 39, adapted to these units ($hc = 1240\,\text{eV}\cdot\text{nm} = 1240\,\text{keV}\cdot\text{pm}$),

$$E_\alpha = \frac{hc}{\lambda_\alpha} = \frac{1240\,\text{keV}\cdot\text{pm}}{70.0\,\text{pm}} = 17.7\text{ keV} ,$$

and $E_\beta = (1240\,\text{keV}\cdot\text{nm})/(63.0\,\text{pm}) = 19.7\,\text{keV}$.

(b) Both Zr and Nb can be used, since $E_\alpha < 18.00\,\text{eV} < E_\beta$ and $E_\alpha < 18.99\,\text{eV} < E_\beta$. According to the hint given in the problem statement, Zr is the better choice.

49. (a) An electron must be removed from the K-shell, so that an electron from a higher energy shell can drop. This requires an energy of $69.5\,\text{keV}$. The accelerating potential must be at least $69.5\,\text{kV}$.

(b) After it is accelerated, the kinetic energy of the bombarding electron is 69.5 keV. The energy of a photon associated with the minimum wavelength is 69.5 keV, so its wavelength is

$$\lambda_{\min} = \frac{1240\,\text{eV}\cdot\text{nm}}{69.5\times 10^3\,\text{eV}} = 1.78\times 10^{-2}\,\text{nm} = 17.8\,\text{pm}\ .$$

(c) The energy of a photon associated with the K_α line is $69.5\,\text{keV} - 11.3\,\text{keV} = 58.2\,\text{keV}$ and its wavelength is $\lambda_{K\alpha} = (1240\,\text{eV}\cdot\text{nm})/(58.2\times 10^3\,\text{eV}) = 2.13\times 10^{-2}\,\text{nm} = 21.3\,\text{pm}$. The energy of a photon associated with the K_β line is $69.5\,\text{keV} - 2.30\,\text{keV} = 67.2\,\text{keV}$ and its wavelength is $\lambda_{K\beta} = (1240\,\text{eV}\cdot\text{nm})/(67.2\times 10^3\,\text{eV}) = 1.85\times 10^{-2}\,\text{nm} = 18.5\,\text{pm}$. The result of Exercise 3 of Chapter 39 is used.

50. We use Eq. 37-31, Eq. 40-6, and the result of problem 3 in Chapter 39, adapted to these units ($hc = 1240\,\text{eV}\cdot\text{nm} = 1240\,\text{keV}\cdot\text{pm}$). Letting $2d\sin\theta = m\lambda = mhc/\Delta E$, where $\theta = 74.1°$, we solve for d:

$$d = \frac{mhc}{2\Delta E\sin\theta} = \frac{(1)(1240\,\text{keV}\cdot\text{nm})}{2(8.979\,\text{keV} - 0.951\,\text{keV})(\sin 74.1°)} = 80.3\ \text{pm}\ .$$

51. (a) According to Eq. 41-26, $f \propto (Z-1)^2$, so the ratio of energies is (using Eq. 39-2) $f/f' = [(Z-1)/(Z'-1)]^2$.

(b) We refer to Appendix F. Applying the formula from part (a) to $Z = 92$ and $Z' = 13$, we obtain

$$\frac{E}{E'} = \frac{f}{f'} = \left(\frac{Z-1}{Z'-1}\right)^2 = \left(\frac{92-1}{13-1}\right)^2 = 57.5\ .$$

(c) Applying this to $Z = 92$ and $Z' = 3$, we obtain

$$\frac{E}{E'} = \left(\frac{92-1}{3-1}\right)^2 = 2070\ .$$

52. (a) The transition is from $n = 2$ to $n = 1$, so Eq. 41-26 combined with Eq. 41-24 yields

$$f = \left(\frac{m_e e^4}{8\varepsilon_0^2 h^3}\right)\left(\frac{1}{1^2} - \frac{1}{2^2}\right)(Z-1)^2$$

so that the constant in Eq. 41-27 is

$$C = \sqrt{\frac{3m_e e^4}{32\varepsilon_0^2 h^3}} = 4.9673\times 10^7\ \text{Hz}^{1/2}$$

using the values in the next-to-last column in the Table in Appendix B (but note that the power of ten is given in the middle column).

(b) We are asked to compare the results of Eq. 41-27 (squared, then multiplied by the accurate values of h/e found in Appendix B to convert to x ray energies) with those in the table of K_α energies (in eV) given at the end of the problem. We look up the corresponding atomic numbers in Appendix F. An example is shown below (for Nitrogen):

$$E_{\text{theory}} = \frac{h}{e}C^2(Z-1)^2 = \frac{6.6260688\times 10^{-34}\,\text{J}\cdot\text{s}}{1.6021765\times 10^{-19}\,\text{J/eV}}\left(4.9673\times 10^7\ \text{Hz}^{1/2}\right)^2(7-1)^2 = 367.35\ \text{eV}$$

which is 6.4% lower than the experimental value of 392.4 eV. Progressing through the list, from Lithium to Magnesium , we find all the theoretical values are lower than the experimental ones by these percentages: 24.8%, 15.4%, 10.9%, 7.9%, 6.4%, 4.7%, 3.5%, 2.6%, 2.0%, and 1.5%.

(c) The trend is clear from the list given above: the agreement between theory and experiment becomes better as Z increases. One might argue that the most questionable step in §41-10 is the replacement $e^4 \to (Z-1)^2e^4$ and ask why this could not equally well be $e^4 \to (Z-.9)^2e^4$ or $e^4 \to (Z-.8)^2e^4$? For large Z, these subtleties would not matter so much as they do for small Z, since $Z - \xi \approx Z$ for $Z \gg \xi$.

53. (a) The length of the pulse's wave train is given by $L = c\Delta t = (2.998 \times 10^8\,\text{m/s})(10 \times 10^{-15}\,\text{s}) = 3.0 \times 10^{-6}\,\text{m}$. Thus, the number of wavelengths contained in the pulse is

$$N = \frac{L}{\lambda} = \frac{3.0 \times 10^{-6}\,\text{m}}{500 \times 10^{-9}\,\text{m}} = 6.0 \ .$$

(b) We solve for X from $10\,\text{fm}/1\,\text{m} = 1\,\text{s}/X$:

$$X = \frac{(1\,\text{s})(1\,\text{m})}{10 \times 10^{-15}\,\text{m}} = \frac{1\,\text{s}}{(10 \times 10^{-15})(3.15 \times 10^7\,\text{s/y})} = 3.2 \times 10^6\text{y} \ .$$

54. According to Sample Problem 41-6, $N_x/N_0 = 1.3 \times 10^{-38}$. Let the number of moles of the lasing material needed be n; then $N_0 = nN_A$, where N_A is the Avogadro constant. Also $N_x = 10$. We solve for n:

$$n = \frac{N_x}{(1.3 \times 10^{-38})\,N_A} = \frac{10}{(1.3 \times 10^{-38})(6.02 \times 10^{23})} = 1.3 \times 10^{15}\ \text{mol} \ .$$

55. The number of atoms in a state with energy E is proportional to $e^{-E/kT}$, where T is the temperature on the Kelvin scale and k is the Boltzmann constant. Thus the ratio of the number of atoms in the thirteenth excited state to the number in the eleventh excited state is

$$\frac{n_{13}}{n_{11}} = e^{-\Delta E/kT} \ ,$$

where ΔE is the difference in the energies: $\Delta E = E_{13} - E_{11} = 2(1.2\,\text{eV}) = 2.4\,\text{eV}$. For the given temperature, $kT = (8.62 \times 10^{-2}\,\text{eV/K})(2000\,\text{K}) = 0.1724\,\text{eV}$. Hence,

$$\frac{n_{13}}{n_{11}} = e^{-2.4/0.1724} = 9.0 \times 10^{-7} \ .$$

56. (a) The distance from the Earth to the Moon is $d_{em} = 3.82 \times 10^8\,\text{m}$ (see Appendix C). Thus, the time required is given by

$$t = \frac{2d_{em}}{c} = \frac{2(3.82 \times 10^8\,\text{m})}{2.998 \times 10^8\,\text{m/s}} = 2.55\ \text{s} \ .$$

(b) We denote the uncertainty in time measurement as δt and let $2\delta d_{es} = 15\,\text{cm}$. Then, since $d_{em} \propto t$, $\delta t/t = \delta d_{em}/d_{em}$. We solve for δt:

$$\delta t = \frac{t\delta d_{em}}{d_{em}} = \frac{(2.55\,\text{s})(0.15\,\text{m})}{2(3.82 \times 10^8\,\text{m})} = 5.0 \times 10^{-10}\ \text{s} \ .$$

57. From Eq. 41-29, $N_2/N_1 = e^{-(E_2-E_1)/kT}$. We solve for T:

$$T = \frac{E_2 - E_1}{k\ln(N_1/N_2)} = \frac{3.2\,\text{eV}}{(1.38 \times 10^{-23}\,\text{J/K})\ln(2.5 \times 10^{15}/6.1 \times 10^{13})} = 10000\ \text{K} \ .$$

58. Consider two levels, labeled 1 and 2, with $E_2 > E_1$. Since $T = -|T| < 0$,

$$\frac{N_2}{N_1} = e^{-(E_2-E_1)/kT} = e^{-|E_2-E_1|/(-k|T|)} = e^{|E_2-E_1|/k|T|} > 1 \ .$$

Thus, $N_2 > N_1$; this is population inversion. We solve for T:

$$T = -|T| = -\frac{E_2 - E_1}{k\ln(N_2/N_1)} = -\frac{2.26\,\text{eV}}{(8.62 \times 10^{-5}\,\text{eV/K})\ln(1 + 0.100)} = -2.75 \times 10^5\ \text{K} \ .$$

59. (a) If t is the time interval over which the pulse is emitted, the length of the pulse is $L = ct = (3.00 \times 10^8\,\mathrm{m/s})(1.20 \times 10^{-11}\,\mathrm{s}) = 3.60 \times 10^{-3}\,\mathrm{m}$.

(b) If E_p is the energy of the pulse, E is the energy of a single photon in the pulse, and N is the number of photons in the pulse, then $E_p = NE$. The energy of the pulse is $E_p = (0.150\,\mathrm{J})/(1.602 \times 10^{-19}\,\mathrm{J/eV}) = 9.36 \times 10^{17}\,\mathrm{eV}$ and the energy of a single photon is $E = (1240\,\mathrm{eV \cdot nm})/(694.4\,\mathrm{nm}) = 1.786\,\mathrm{eV}$. Hence,

$$N = \frac{E_p}{E} = \frac{9.36 \times 10^{17}\,\mathrm{eV}}{1.786\,\mathrm{eV}} = 5.24 \times 10^{17}\,\text{photons} .$$

60. Let the power of the laser beam be P and the energy of each photon emitted be E. Then, the rate of photon emission is

$$\begin{aligned} R &= \frac{P}{E} = \frac{P}{hc/\lambda} = \frac{P\lambda}{hc} \\ &= \frac{(2.3 \times 10^{-3}\,\mathrm{W})(632.8 \times 10^{-9}\,\mathrm{m})}{(6.63 \times 10^{-34}\,\mathrm{J \cdot s})(2.998 \times 10^8\,\mathrm{m/s})} \\ &= 7.3 \times 10^{15}\ \mathrm{s}^{-1} . \end{aligned}$$

61. The Moon is a distance $R = 3.82 \times 10^8\,\mathrm{m}$ from Earth (see Appendix C). We note that the "cone" of light has apex angle equal to 2θ. If we make the small angle approximation (equivalent to using Eq. 37-14), then the diameter D of the spot on the Moon is

$$\begin{aligned} D &= 2R\theta = 2R\left(\frac{1.22\lambda}{d}\right) \\ &= \frac{2(3.82 \times 10^8\,\mathrm{m})(1.22)(600 \times 10^{-9}\,\mathrm{m})}{0.12\,\mathrm{m}} \\ &= 4.7 \times 10^3\,\mathrm{m} = 4.7\ \mathrm{km} . \end{aligned}$$

62. Let the range of frequency of the microwave be Δf. Then the number of channels that could be accommodated is

$$N = \frac{\Delta f}{10\,\mathrm{MHz}} = \frac{(2.998 \times 10^8\,\mathrm{m/s})[(450\,\mathrm{nm})^{-1} - (650\,\mathrm{nm})^{-1}]}{10\,\mathrm{MHz}} = 2.1 \times 10^7 .$$

The higher frequencies of visible light would allow many more channels to be carried compared with using the microwave.

63. Let the power of the laser beam be P and the energy of each photon emitted be E. Then, the rate of photon emission is

$$\begin{aligned} R &= \frac{P}{E} = \frac{P}{hc/\lambda} = \frac{P\lambda}{hc} \\ &= \frac{(5.0 \times 10^{-3}\,\mathrm{W})(0.80 \times 10^{-6}\,\mathrm{m})}{(6.63 \times 10^{-34}\,\mathrm{J \cdot s})(2.998 \times 10^8\,\mathrm{m/s})} \\ &= 2.0 \times 10^{16}\ \mathrm{s}^{-1} . \end{aligned}$$

64. For the nth harmonic of the standing wave of wavelength λ in the cavity of width L we have $n\lambda = 2L$, so $n\Delta\lambda + \lambda\Delta n = 0$. Let $\Delta n = \pm 1$ and use $\lambda = 2L/n$ to obtain

$$|\Delta\lambda| = \frac{\lambda|\Delta n|}{n} = \frac{\lambda}{n} = \lambda\left(\frac{\lambda}{2L}\right) = \frac{(533\,\mathrm{nm})^2}{2(8.0 \times 10^7\,\mathrm{nm})} = 1.8 \times 10^{-12}\,\mathrm{m} = 1.8\ \mathrm{pm} .$$

65. (a) If both mirrors are perfectly reflecting, there is a node at each end of the crystal. With one end partially silvered, there is a node very close to that end. We assume nodes at both ends, so there are an integer number of half-wavelengths in the length of the crystal. The wavelength in the crystal is $\lambda_c = \lambda/n$, where λ is the wavelength in a vacuum and n is the index of refraction of ruby. Thus $N(\lambda/2n) = L$, where N is the number of standing wave nodes, so

$$N = \frac{2nL}{\lambda} = \frac{2(1.75)(0.0600\,\mathrm{m})}{694 \times 10^{-9}\,\mathrm{m}} = 3.03 \times 10^5 \ .$$

(b) Since $\lambda = c/f$, where f is the frequency, $N = 2nLf/c$ and $\Delta N = (2nL/c)\,\Delta f$. Hence,

$$\Delta f = \frac{c\,\Delta N}{2nL} = \frac{(2.998 \times 10^8\,\mathrm{m/s})(1)}{2(1.75)(0.0600\,\mathrm{m})} = 1.43 \times 10^9\,\mathrm{Hz} \ .$$

(c) The speed of light in the crystal is c/n and the round-trip distance is $2L$, so the round-trip travel time is $2nL/c$. This is the same as the reciprocal of the change in frequency.

(d) The frequency is $f = c/\lambda = (2.998 \times 10^8\,\mathrm{m/s})/(694 \times 10^{-9}\,\mathrm{m}) = 4.32 \times 10^{14}\,\mathrm{Hz}$ and the fractional change in the frequency is $\Delta f/f = (1.43 \times 10^9\,\mathrm{Hz})/(4.32 \times 10^{14}\,\mathrm{Hz}) = 3.31 \times 10^{-6}$.

66. (a) We denote the upper level as level 1 and the lower one as level 2. From $N_1/N_2 = e^{-(E_1-E_2)/kT}$ we get (using the result of problem 3 in Chapter 39)

$$\begin{aligned} N_1 &= N_2 e^{-(E_1-E_2)/kT} = N_2 e^{-hc/\lambda kT} \\ &= (4.0 \times 10^{20}) e^{-(1240\,\mathrm{eV\cdot nm})/[(580\,\mathrm{nm})(8.62\times10^{-5}\,\mathrm{eV/K})(300\,\mathrm{K})]} \\ &= 5.0 \times 10^{-16} \ll 1 \ , \end{aligned}$$

so practically no electron occupies the upper level.

(b) With $N_1 = 3.0 \times 10^{20}$ atoms emitting photons and $N_2 = 1.0 \times 10^{20}$ atoms absorbing photons, then the net energy output is

$$\begin{aligned} E &= (N_1 - N_2)\,E_{\mathrm{photon}} = (N_1 - N_2)\,\frac{hc}{\lambda} \\ &= \left(2.0 \times 10^{20}\right) \frac{(6.63 \times 10^{-34}\,\mathrm{J\cdot s})(2.998 \times 10^8\,\mathrm{m/s})}{580 \times 10^{-9}\,\mathrm{m}} \\ &= 68\ \mathrm{J} \ . \end{aligned}$$

67. (a) The intensity at the target is given by $I = P/A$, where P is the power output of the source and A is the area of the beam at the target. We want to compute I and compare the result with $10^8\,\mathrm{W/m^2}$. The beam spreads because diffraction occurs at the aperture of the laser. Consider the part of the beam that is within the central diffraction maximum. The angular position of the edge is given by $\sin\theta = 1.22\lambda/d$, where λ is the wavelength and d is the diameter of the aperture (see Exercise 61). At the target, a distance D away, the radius of the beam is $r = D\tan\theta$. Since θ is small, we may approximate both $\sin\theta$ and $\tan\theta$ by θ, in radians. Then, $r = D\theta = 1.22D\lambda/d$ and

$$\begin{aligned} I &= \frac{P}{\pi r^2} = \frac{Pd^2}{\pi(1.22D\lambda)^2} \\ &= \frac{(5.0 \times 10^6\,\mathrm{W})(4.0\,\mathrm{m})^2}{\pi\left[1.22(3000 \times 10^3\,\mathrm{m})(3.0 \times 10^{-6}\,\mathrm{m})\right]^2} \\ &= 2.1 \times 10^5\ \mathrm{W/m^2} \ , \end{aligned}$$

not great enough to destroy the missile.

(b) We solve for the wavelength in terms of the intensity and substitute $I = 1.0 \times 10^8\,\mathrm{W/m^2}$:

$$\begin{aligned}\lambda &= \frac{d}{1.22D}\sqrt{\frac{P}{\pi I}} = \frac{4.0\,\mathrm{m}}{1.22(3000 \times 10^3\,\mathrm{m})}\sqrt{\frac{5.0 \times 10^6\,\mathrm{W}}{\pi(1.0 \times 10^8\,\mathrm{W/m^2})}} \\ &= 1.4 \times 10^{-7}\,\mathrm{m} = 140\ \mathrm{nm}\ .\end{aligned}$$

68. (a) The radius of the central disk is

$$R = \frac{1.22 f\lambda}{d} = \frac{(1.22)(3.50\,\mathrm{cm})(515\,\mathrm{nm})}{3.00\,\mathrm{mm}} = 7.33\,\mu\mathrm{m}\ .$$

(b) The average power flux density in the incident beam is

$$\frac{P}{\pi d^2/4} = \frac{4(5.00\,\mathrm{W})}{\pi(3.00\,\mathrm{mm})^2} = 707\ \mathrm{kW/m^2}\ .$$

(c) The average power flux density in the central disk is

$$\frac{(0.84)P}{\pi R^2} = \frac{(0.84)(5.00\,\mathrm{W})}{\pi(7.33\,\mu\mathrm{m})^2} = 24.9\ \mathrm{GW/m^2}\ .$$

69. (a) In the lasing action the molecules are excited from energy level E_0 to energy level E_2. Thus the wavelength λ of the sunlight that causes this excitation satisfies

$$\Delta E = E_2 - E_0 = \frac{hc}{\lambda}\ ,$$

which gives (using the result of problem 3 in Chapter 39)

$$\lambda = \frac{hc}{E_2 - E_0} = \frac{1240\,\mathrm{eV\cdot nm}}{0.289\,\mathrm{eV} - 0} = 4.29 \times 10^3\,\mathrm{nm} = 4.29\,\mu\mathrm{m}\ .$$

(b) Lasing occurs as electrons jump down from the higher energy level E_2 to the lower level E_1. Thus the lasing wavelength λ' satisfies

$$\Delta E' = E_2 - E_1 = \frac{hc}{\lambda'}\ ,$$

which gives

$$\lambda' = \frac{hc}{E_2 - E_1} = \frac{1240\,\mathrm{eV\cdot nm}}{0.289\,\mathrm{eV} - 0.165\,\mathrm{eV}} = 1.00 \times 10^4\,\mathrm{nm} = 10.0\,\mu\mathrm{m}\ .$$

(c) Both λ and λ' belong to the infrared region of the electromagnetic spectrum.

70. (a) The energy difference between the two states 1 and 2 was equal to the energy of the photon emitted. Since the photon frequency was $f = 1666\,\mathrm{MHz}$, its energy was given by $hf = (4.14 \times 10^{-15}\,\mathrm{eV\cdot s})(1666\,\mathrm{MHz}) = 6.90 \times 10^{-6}\,\mathrm{eV}$. Thus,

$$E_2 - E_1 = hf = 6.9 \times 10^{-6}\,\mathrm{eV} = 6.9\,\mu\mathrm{eV}\ .$$

(b) The emission was in the *radio* region of the electromagnetic spectrum.