# Chapter 35

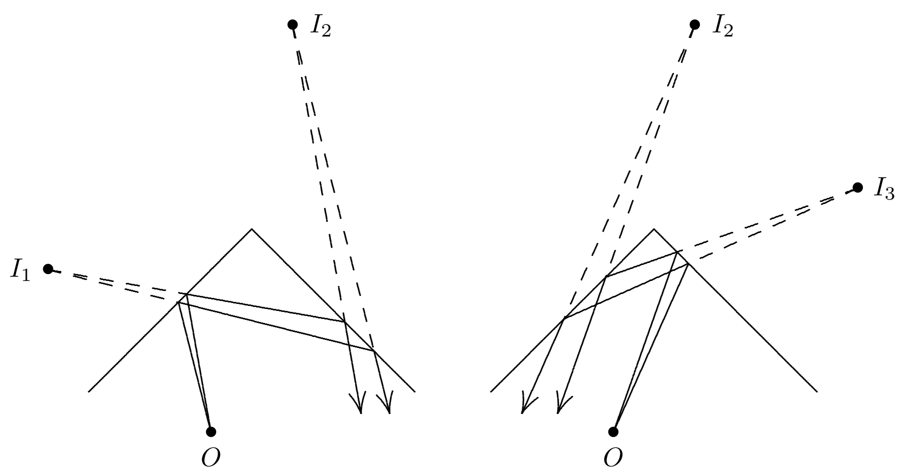
1. The image is 10 cm behind the mirror and you are 30 cm in front of the mirror. You must focus your eyes for a distance of $10\,\text{cm} + 30\,\text{cm} = 40\,\text{cm}$.

2. The bird is a distance $d_2$ in front of the mirror; the plane of its image is that same distance $d_2$ behind the mirror. The lateral distance between you and the bird is $d_3 = 5.00$ m. We denote the distance from the camera to the mirror as $d_1$, and we construct a right triangle out of $d_3$ and the distance between the camera and the image plane $(d_1 + d_2)$. Thus, the focus distance is

$$\begin{aligned} d &= \sqrt{(d_1+d_2)^2 + d_3^2} \\ &= \sqrt{(4.30\,\text{m} + 3.30\,\text{m})^2 + (5.00\,\text{m})^2} \\ &= 9.10\ \text{m}\ . \end{aligned}$$

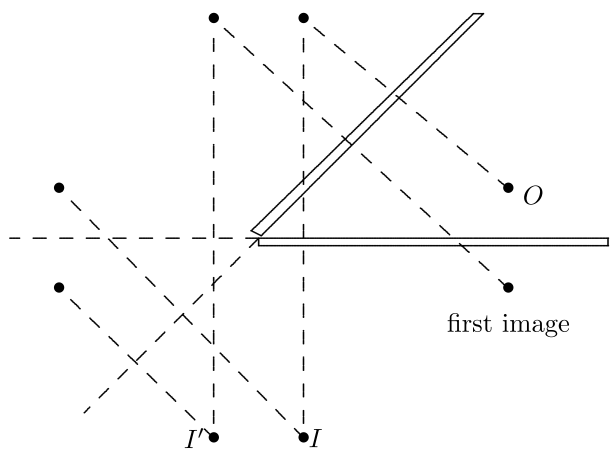
3. (a) There are three images. Two are formed by single reflections from each of the mirrors and the third is formed by successive reflections from both mirrors.

   (b) The positions of the images are shown on the two diagrams below. The diagram on the left below shows the image $I_1$, formed by reflections from the left-hand mirror. It is the same distance behind the mirror as the object $O$ is in front, and lies on the line perpendicular to the mirror and through the object. Image $I_2$ is formed by light that is reflected from both mirrors. We may consider $I_2$ to be the image of $I_1$ formed by the right-hand mirror, extended. $I_2$ is the same distance behind the line of the right-hand mirror as $I_1$ is in front and it is on the line that is perpendicular to the line of the mirror. The diagram on the right, below, shows image $I_3$, formed by reflections from the right-hand mirror. It is the same distance behind the mirror as the object is in front, and lies on the line perpendicular to the mirror and through the object. As the diagram shows, light that is first reflected from the right-hand mirror and then from the left-hand mirror forms an image at $I_2$.



4. In each case there is an object and its “first” image in the mirror closest to it (this image is the same distance behind the mirror as the object is in front of it and might be referred to as the object’s “twin”). The rest of the “figuring” consists of drawing perpendiculars from these (or imagining doing so) to the mirror-planes and constructing further images.

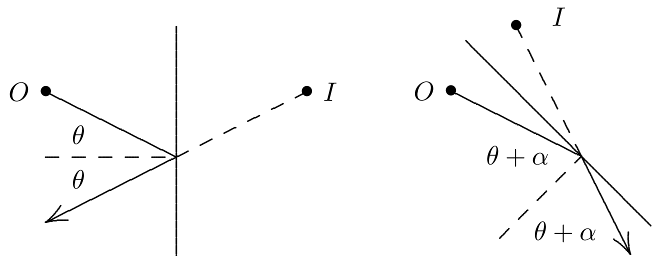
   (a) For $\theta = 45°$, we have two images in the second mirror caused by the object and its “first” image, and from these one can construct two new images $I$ and $I'$ behind the first mirror plane. Extending the second mirror plane, we can find two further images of $I$ and $I'$ which are on equal sides of the extension of the first mirror plane. This circumstance implies there are no further images, since these final images are each other’s “twins.” We show this construction in the figure below. Summarizing, we find $1 + 2 + 2 + 2 = 7$ images in this case.




   (b) For $\theta = 60°$, we have two images in the second mirror caused by the object and its “first” image, and from these one can construct two new images $I$ and $I'$ behind the first mirror plane. The images $I$ and $I'$ are each other’s “twins” in the sense that they are each other’s reflections about the extension of the second mirror plane; there are no further images. Summarizing, we find $1+2+2 = 5$ images in this case.

   (c) For $\theta = 120°$, we have two images $I_1'$ and $I_2$ behind the extension of the second mirror plane, caused by the object and its “first” image (which we refer to here as $I_1$). No further images can be constructed from $I_1'$ and $I_2$, since the method indicated above would place any further possibilities in front of the mirrors. This construction has the disadvantage of deemphasizing the actual ray-tracing, and thus any dependence on where the observer of these images is actually placing his or her eyes. It turns out in this case that the number of images that can be seen ranges from 1 to 3, depending on the locations of both the object and the observer. As an example, if the observer’s eye is collinear with $I_1$ and $I_1'$, then the observer can only see one image ($I_1$ and not the one behind it). Another observer, close to the second mirror would probably be able to see only $I_1$ and $I_2$. However, if that observer moves further back from the vertex of the two mirrors he or she should also be able to see the third image, $I_1'$, which is essentially the “twin” image formed from $I_1$ relative to the extension of the second mirror plane.

5. Consider a single ray from the source to the mirror and let $\theta$ be the angle of incidence. The angle of reflection is also $\theta$ and the reflected ray makes an angle of $2\theta$ with the incident ray. Now we rotate the mirror through the angle $\alpha$ so that the angle of incidence increases to $\theta + \alpha$. The reflected ray now makes an angle of $2(\theta + \alpha)$ with the incident ray. The reflected ray has been rotated through an angle of $2\alpha$. If the mirror is rotated so the angle of incidence is decreased by $\alpha$, then the reflected ray makes an angle of $2(\theta - \alpha)$ with the incident ray. Again it has been rotated through $2\alpha$. The diagrams below show the situation for $\alpha = 45°$. The ray from the object to the mirror is the same in both cases and the reflected rays are 90° apart.




6. When $S$ is barely able to see $B$ the light rays from $B$ must reflect to $S$ off the edge of the mirror. The angle of reflection in this case is 45°, since a line drawn from $S$ to the mirror's edge makes a 45° angle relative to the wall. By the law of reflection, we find

$$\frac{x}{d/2} = \tan 45^\circ \quad \Longrightarrow \quad x = \frac{d}{2} = \frac{3.0\,\mathrm{m}}{2} = 1.5 \text{ m} .$$

7. The intensity of light from a point source varies as the inverse of the square of the distance from the source. Before the mirror is in place, the intensity at the center of the screen is given by $I_0 = A/d^2$, where $A$ is a constant of proportionality. After the mirror is in place, the light that goes directly to the screen contributes intensity $I_0$, as before. Reflected light also reaches the screen. This light appears to come from the image of the source, a distance $d$ behind the mirror and a distance $3d$ from the screen. Its contribution to the intensity at the center of the screen is

$$I_r = \frac{A}{(3d)^2} = \frac{A}{9d^2} = \frac{I_0}{9} .$$

The total intensity at the center of the screen is

$$I = I_0 + I_r = I_0 + \frac{I_0}{9} = \frac{10}{9} I_0 .$$

The ratio of the new intensity to the original intensity is $I/I_0 = 10/9$.

8. We apply the law of refraction, assuming all angles are in radians:

$$\frac{\sin\theta}{\sin\theta'} = \frac{n_w}{n_{\mathrm{air}}} ,$$

which in our case reduces to $\theta' \approx \theta/n_w$ (since both $\theta$ and $\theta'$ are small, and $n_{\mathrm{air}} \approx 1$). We refer to our figure, below. The object $O$ is a vertical distance $h_1$ above the water, and the water surface is a vertical distance $h_2$ above the mirror. We are looking for a distance $d$ (treated as a positive number) below the mirror where the image $I$ of the object is formed. In the triangle $OAB$

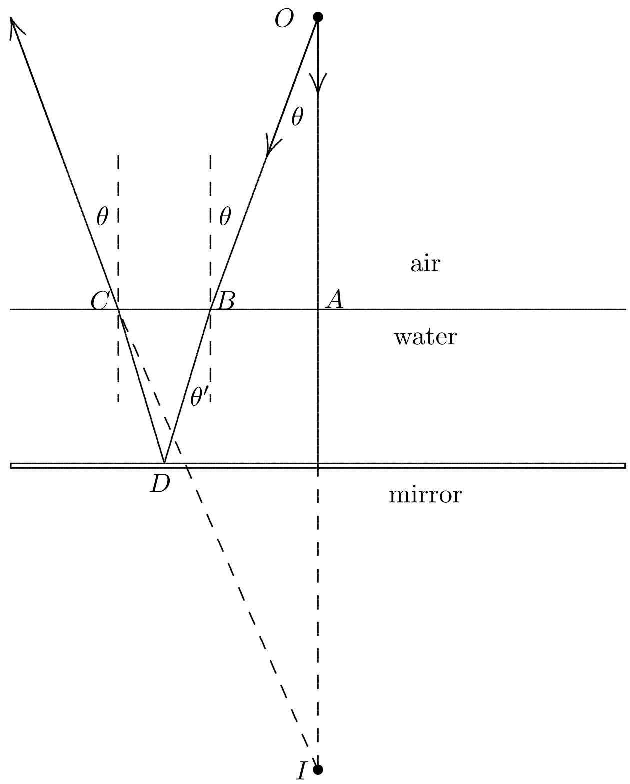
$$|AB| = h_1 \tan\theta \approx h_1\theta ,$$

and in the triangle $CBD$

$$|BC| = 2h_2 \tan\theta' \approx 2h_2\theta' \approx \frac{2h_2\theta}{n_w} .$$

Finally, in the triangle $ACI$, we have $|AI| = d + h_2$. Therefore,

$$\begin{aligned} d &= |AI| - h_2 = \frac{|AC|}{\tan\theta} - h_2 \\ &\approx \frac{|AB| + |BC|}{\theta} - h_2 \\ &= \left(\frac{h_1}{\theta} + \frac{2h_2\theta}{n_w}\right)\frac{1}{\theta} - h_2 = h_1 + \frac{2h_2}{n_w} - h_2 \\ &= 250\,\mathrm{cm} + \frac{2(200\,\mathrm{cm})}{1.33} - 200\,\mathrm{cm} = 351 \text{ cm} . \end{aligned}$$




9. We use Eqs. 35-3 and 35-4, and note that $m = -i/p$. Thus,

$$\frac{1}{p} - \frac{1}{pm} = \frac{1}{f} = \frac{2}{r} \ .$$

We solve for $p$:

$$p = \frac{r}{2}\left(1 - \frac{1}{m}\right) = \frac{35.0\,\text{cm}}{2}\left(1 - \frac{1}{2.50}\right) = 10.5 \text{ cm} \ .$$

10. (a) $f = +20\,\text{cm}$ (positive, because the mirror is concave); $r = 2f = 2(+20\,\text{cm}) = +40\,\text{cm}$; $i = (1/f - 1/p)^{-1} = (1/20\,\text{cm} - 1/10\,\text{cm})^{-1} = -20\,\text{cm}$; $m = -i/p = -(-20\,\text{cm}/10\,\text{cm}) = +2.0$. The image is virtual and upright. The ray diagram would be similar to Fig. 35-8(a) in the textbook.

(b) The fact that the magnification is 1 and the image is virtual means that the mirror is flat (plane). Flat mirrors (and flat "lenses" such as a window pane) have $f = \infty$ (or $f = -\infty$ since the sign does not matter in this extreme case), and consequently $r = \infty$ (or $r = -\infty$) by Eq. 35-3. Eq. 35-4 readily yields $i = -10\,\text{cm}$. The magnification being positive implies the image is upright; the answer is "no" (it's not inverted). The ray diagram would be similar to Fig. 35-6(a) in the textbook.

(c) Since $f > 0$, the mirror is concave. Using Eq. 35-3, we obtain $r = 2f = +40\,\text{cm}$. Eq. 35-4 readily yields $i = +60\,\text{cm}$. Substituting this (and the given object distance) into Eq. 35-6 gives $m = -2.0$. Since $i > 0$, the answer is "yes" (the image is real). Since $m < 0$, our answer is "yes" (the image is inverted). The ray diagram would be similar to Fig. 35-8(c) in the textbook.

(d) Since $m < 0$, the image is inverted. With that in mind, we examine the various possibilities in Figs. 35-6, 35-8 and 35-9, and note that an inverted image (for reflections from a single mirror) can only occur if the mirror is concave (and if $p > f$). Next, we find $i$ from Eq. 35-6 (which yields $i = 30\,\text{cm}$) and then use this value (and Eq. 35-4) to compute the focal length; we obtain $f = +20\,\text{cm}$. Then, Eq. 35-3 gives $r = +40\,\text{cm}$. As already noted, $i = +30\,\text{cm}$. Yes, the image is real (since $i > 0$). Yes, the image is inverted (as already noted). The ray diagram would be similar to Figs. 35-9(a) and (b) in the textbook.

(e) Since $r < 0$ then (by Eq. 35-3) $f < 0$, which means the mirror is convex. The focal length is $f = r/2 = -20$ cm. Eq. 35-4 leads to $p = +20$ cm, and Eq. 35-6 gives $m = +0.50$. No, the image is virtual. No, the image is upright. The ray diagram would be similar to Figs. 35-9(c) and (d) in the textbook.

(f) Since $0 < m < 1$, the image is upright but smaller than the object. With that in mind, we examine the various possibilities in Figs. 35-6, 35-8 and 35-9, and note that such an image (for reflections from a single mirror) can only occur if the mirror is convex. Thus, we must put a minus sign in front of the "20" value given for $f$. Eq. 35-3 then gives $r = -40$ cm. To solve for $i$ and $p$ we must set up Eq. 35-4 and Eq. 35-6 as a simultaneous set and solve for the two unknowns. The results are $i = -18$ cm and $p = +180$ cm. No, the image is virtual (since $i < 0$). No, the image is upright (as already noted). The ray diagram would be similar to Figs. 35-9(c) and (d) in the textbook.

(g) Knowing the mirror is convex means we must put a minus sign in front of the "40" value given for $r$. Then, Eq. 35-3 yields $f = r/2 = -20$ cm. The fact that the mirror is convex also means that we need to insert a minus sign in front of the "4.0" value given for $i$, since the image in this case must be virtual (see Figs. 35-6, 35-8 and 35-9). Eq. 35-4 leads to $p = +5.0$ cm, and Eq. 35-6 gives $m = +0.8$. No, the image is virtual. No, the image is upright. The ray diagram would be similar to Figs. 35-9(c) and (d) in the textbook.

(h) Since the image is inverted, we can scan Figs. 35-6, 35-8 and 35-9 in the textbook and find that the mirror must be concave. This also implies that we must put a minus sign in front of the "0.50" value given for $m$. To solve for $f$, we first find $i = +12$ cm from Eq. 35-6 and plug into Eq. 35-4; the result is $f = +8$ cm. Thus, $r = 2f = +16$ cm. Yes, the image is real (since $i > 0$). The ray diagram would be similar to Figs. 35-9(a) and (b) in the textbook.

11. (a) Suppose one end of the object is a distance $p$ from the mirror and the other end is a distance $p+L$. The position $i_1$ of the image of the first end is given by

$$\frac{1}{p} + \frac{1}{i_1} = \frac{1}{f}$$

where $f$ is the focal length of the mirror. Thus,

$$i_1 = \frac{fp}{p-f} .$$

The image of the other end is located at

$$i_2 = \frac{f(p+L)}{p+L-f} ,$$

so the length of the image is

$$L' = i_1 - i_2 = \frac{fp}{p-f} - \frac{f(p+L)}{p+L-f} = \frac{f^2 L}{(p-f)(p+L-f)} .$$

Since the object is short compared to $p - f$, we may neglect the $L$ in the denominator and write

$$L' = L\left(\frac{f}{p-f}\right)^2 .$$

(b) The lateral magnification is $m = -i/p$ and since $i = fp/(p-f)$, this can be written $m = -f/(p-f)$. The longitudinal magnification is

$$m' = \frac{L'}{L} = \left(\frac{f}{p-f}\right)^2 = m^2 .$$

12. (a) From Eqs. 35-3 and 35-4, we obtain $i = pf/(p - f) = pr/(2p - r)$. Differentiating both sides with respect to time and using $v_O = -dp/dt$, we find

$$v_I = \frac{di}{dt} = \frac{d}{dt}\left(\frac{pr}{2p - r}\right) = \frac{-rv_O(2p - r) + 2v_O pr}{(2p - r)^2} = \left(\frac{r}{2p - r}\right)^2 v_O \ .$$

(b) If $p = 30\,\text{cm}$, we obtain

$$v_I = \left[\frac{15\,\text{cm}}{2(30\,\text{cm}) - 15\,\text{cm}}\right]^2 (5.0\,\text{cm/s}) = 0.56 \text{ cm/s} \ .$$

(c) If $p = 8.0\,\text{cm}$, we obtain

$$v_I = \left[\frac{15\,\text{cm}}{2(8.0\,\text{cm}) - 15\,\text{cm}}\right]^2 (5.0\,\text{cm/s}) = 1.1 \times 10^3 \text{ cm/s} \ .$$

(d) If $p = 1.0\,\text{cm}$, we obtain

$$v_I = \left[\frac{15\,\text{cm}}{2(1.0\,\text{cm}) - 15\,\text{cm}}\right]^2 (5.0\,\text{cm/s}) = 6.7 \text{ cm/s} \ .$$

13. (a) We use Eq. 35-8 and note that $n_1 = n_{\text{air}} = 1.00,\ n_2 = n,\ p = \infty$, and $i = 2r$:

$$\frac{1.00}{\infty} + \frac{n}{2r} = \frac{n - 1}{r} \ .$$

We solve for the unknown index: $n = 2.00$.

(b) Now $i = r$ so Eq. 35-8 becomes

$$\frac{n}{r} = \frac{n - 1}{r} \ ,$$

which is not valid unless $n \to \infty$ or $r \to \infty$. It is impossible to focus at the center of the sphere.

14. We remark that the sign convention for $r$ (for these refracting surfaces) is the opposite of what was used for mirrors. This point is discussed in §35-5.

(a) We use Eq. 35-8:

$$i = n_2\left(\frac{n_2 - n_1}{r} - \frac{n_1}{p}\right)^{-1} = 1.5\left(\frac{1.5 - 1.0}{30\,\text{cm}} - \frac{1.0}{10\,\text{cm}}\right)^{-1} = -18 \text{ cm} \ .$$

The image is virtual and upright. The ray diagram would be similar to Fig. 35-10(c) in the textbook.

(b) We manipulate Eq. 35-8 to find $r$:

$$r = (n_2 - n_1)\left(\frac{n_1}{p} + \frac{n_2}{i}\right)^{-1} = (1.5 - 1.0)\left(\frac{1.0}{10} + \frac{1.5}{-13}\right)^{-1} = -32.5 \text{ cm}$$

which should be rounded to two significant figures. The image is virtual and upright. The ray diagram would be similar to Fig. 35-10(e) in the textbook, but with the object and the image placed closer to the surface.

(c) We manipulate Eq. 35-8 to find $p$:

$$p = \frac{n_1}{\frac{n_2 - n_1}{r} - \frac{n_2}{i}} = \frac{1.0}{\frac{1.5 - 1.0}{30} - \frac{1.5}{600}} = 71 \text{ cm} \ .$$

The image is real and inverted. The ray diagram would be similar to Fig. 35-10(a) in the textbook.

(d) We manipulate Eq. 35-8 to separate the indices:

$$\begin{aligned} n_2\left(\frac{1}{r}-\frac{1}{i}\right) &= \left(\frac{n_1}{p}+\frac{n_1}{r}\right) \\ n_2\left(\frac{1}{-20}-\frac{1}{-20}\right) &= \left(\frac{1.0}{20}+\frac{1.0}{-20}\right) \\ n_2(0) &= 0 \end{aligned}$$

which is identically satisfied for any choice of $n_2$. The ray diagram would be similar to Fig. 35-10(d) in the textbook, but with $C$, $O$ and $I$ together at the same point. The image is virtual and upright.

(e) We manipulate Eq. 35-8 to find $r$:

$$r = (n_2 - n_1)\left(\frac{n_1}{p}+\frac{n_2}{i}\right)^{-1} = (1.0-1.5)\left(\frac{1.5}{10}+\frac{1.0}{-6.0}\right)^{-1} = 30 \text{ cm} .$$

The image is virtual and upright. The ray diagram would be similar to Fig. 35-10(f) in the textbook, but with the object and the image located closer to the surface.

(f) We manipulate Eq. 35-8 to find $p$:

$$p = \frac{n_1}{\frac{n_2-n_1}{r}-\frac{n_2}{i}} = \frac{1.5}{\frac{1.0-1.5}{-30}-\frac{1.0}{-7.5}} = 10 \text{ cm} .$$

The image is virtual and upright. The ray diagram would be similar to Fig. 35-10(d) in the textbook.

(g) We manipulate Eq. 35-8 to find the image distance:

$$i = n_2\left(\frac{n_2-n_1}{r}-\frac{n_1}{p}\right)^{-1} = 1.0\left(\frac{1.0-1.5}{30\,\text{cm}}-\frac{1.5}{70\,\text{cm}}\right)^{-1} = -26 \text{ cm} .$$

The image is virtual and upright. The ray diagram would be similar to Fig. 35-10(f) in the textbook.

(h) We manipulate Eq. 35-8 to separate the indices:

$$\begin{aligned} n_2\left(\frac{1}{r}-\frac{1}{i}\right) &= \left(\frac{n_1}{p}+\frac{n_1}{r}\right) \\ n_2\left(\frac{1}{-30}-\frac{1}{600}\right) &= \left(\frac{1.5}{100}+\frac{1.5}{-30}\right) \\ n_2(-0.035) &= -0.035 \end{aligned}$$

which implies $n_2 = 1.0$. The ray diagram would be similar to Fig. 35-10(b) in the textbook, but with $C$, $O$ and $I$ together at the same point. The image is real and inverted.

15. The water is medium 1, so $n_1 = n_w$ which we simply write as $n$. The air is medium 2, for which $n_2 \approx 1$. We refer points where the light rays strike the water surface as $A$ (on the left side of Fig. 35-32) and $B$ (on the right side of the picture). The point midway between $A$ and $B$ (the center point in the picture) is $C$. The penny $P$ is directly below $C$, and the location of the "apparent" or Virtual penny is $V$. We note that the angle $\angle CVB$ (the same as $\angle CVA$) is equal to $\theta_2$, and the angle $\angle CPB$ (the same as $\angle CPA$) is equal to $\theta_1$. The triangles $CVB$ and $CPB$ share a common side, the horizontal distance from $C$ to $B$ (which we refer to as $x$). Therefore,

$$\tan\theta_2 = \frac{x}{d_\text{a}} \quad \text{and} \quad \tan\theta_1 = \frac{x}{d} .$$

Using the small angle approximation (so a ratio of tangents is nearly equal to a ratio of sines) and the law of refraction, we obtain

$$\frac{\tan\theta_2}{\tan\theta_1} \approx \frac{\sin\theta_2}{\sin\theta_1}$$

$$\frac{\frac{x}{d_a}}{\frac{x}{d}} \approx \frac{n_1}{n_2}$$
$$\frac{d}{d_a} \approx n$$

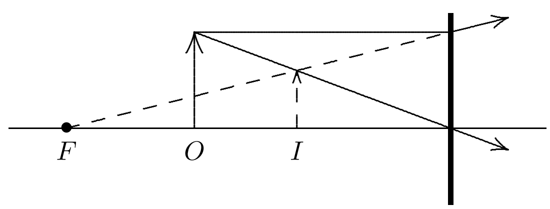
which yields the desired relation: $d_a = d/n$.

16. First, we note that – *relative to the water* – the index of refraction of the carbon tetrachloride should be thought of as $n = 1.46/1.33 = 1.1$ (this notation is chosen to be consistent with problem 15). Now, if the observer were in the water, directly above the 40 mm deep carbon tetrachloride layer, then the apparent depth of the penny as measured below the surface of the carbon tetrachloride is $d_a = 40\,\text{mm}/1.1 = 36.4\,\text{mm}$. This "apparent penny" serves as an "object" for the rays propagating upward through the 20 mm layer of water, where this "object" should be thought of as being $20\,\text{mm} + 36.4\,\text{mm} = 56.4\,\text{mm}$ from the top surface. Using the result of problem 15 again, we find the perceived location of the penny, for a person at the normal viewing position above the water, to be $56.4\,\text{mm}/1.33 = 42\,\text{mm}$ below the water surface.

17. We solve Eq. 35-9 for the image distance $i$: $i = pf/(p - f)$. The lens is diverging, so its focal length is $f = -30\,\text{cm}$. The object distance is $p = 20\,\text{cm}$. Thus,

$$i = \frac{(20\,\text{cm})(-30\,\text{cm})}{(20\,\text{cm}) - (-30\,\text{cm})} = -12\,\text{cm} .$$

The negative sign indicates that the image is virtual and is on the same side of the lens as the object. The ray diagram, drawn to scale, is shown on the right.




18. Let the diameter of the Sun be $d_s$ and that of the image be $d_i$. Then, Eq. 35-5 leads to

$$\begin{aligned} d_i &= |m| d_s = \left(\frac{i}{p}\right) d_s \approx \left(\frac{f}{p}\right) d_s \\ &= \frac{(20.0 \times 10^{-2}\,\text{m})(2)(6.96 \times 10^{8}\,\text{m})}{1.50 \times 10^{11}\,\text{m}} \\ &= 1.86 \times 10^{-3}\,\text{m} = 1.86\ \text{mm} . \end{aligned}$$

19. We use the lens maker's equation, Eq. 35–10:

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

where $f$ is the focal length, $n$ is the index of refraction, $r_1$ is the radius of curvature of the first surface encountered by the light and $r_2$ is the radius of curvature of the second surface. Since one surface has twice the radius of the other and since one surface is convex to the incoming light while the other is concave, set $r_2 = -2r_1$ to obtain

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} + \frac{1}{2r_1}\right) = \frac{3(n-1)}{2r_1} .$$

We solve for $r_1$:

$$r_1 = \frac{3(n-1)f}{2} = \frac{3(1.5-1)(60\,\text{mm})}{2} = 45\,\text{mm} .$$

The radii are 45 mm and 90 mm.

20. (a) We use Eq. 35-10:

$$f = \left[(n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)\right]^{-1} = \left[(1.5-1)\left(\frac{1}{\infty} - \frac{1}{-20\,\text{cm}}\right)\right]^{-1} = +40 \text{ cm} .$$

(b) From Eq. 35-9,

$$i = \left(\frac{1}{f} - \frac{1}{p}\right)^{-1} = \left(\frac{1}{40\,\text{cm}} - \frac{1}{40\,\text{cm}}\right)^{-1} = \infty .$$

21. For a thin lens, $(1/p) + (1/i) = (1/f)$, where $p$ is the object distance, $i$ is the image distance, and $f$ is the focal length. We solve for $i$:

$$i = \frac{fp}{p-f} .$$

Let $p = f + x$, where $x$ is positive if the object is outside the focal point and negative if it is inside. Then,

$$i = \frac{f(f+x)}{x} .$$

Now let $i = f + x'$, where $x'$ is positive if the image is outside the focal point and negative if it is inside. Then,

$$x' = i - f = \frac{f(f+x)}{x} - f = \frac{f^2}{x}$$

and $xx' = f^2$.

22. We solve Eq. 35-9 for the image distance:

$$i = \left(\frac{1}{f} - \frac{1}{p}\right)^{-1} = \frac{fp}{p-f} .$$

The height of the image is thus

$$h_i = mh_p = \left(\frac{i}{p}\right) h_p = \frac{fh_p}{p-f} = \frac{(75\,\text{mm})(1.80\,\text{m})}{27\,\text{m} - 0.075\,\text{m}} = 5.0 \text{ mm} .$$

23. Using Eq. 35-9 and noting that $p + i = d = 44\,\text{cm}$, we obtain $p^2 - dp + df = 0$. Therefore,

$$p = \frac{1}{2}(d \pm \sqrt{d^2 - 4df}) = 22\,\text{cm} \pm \frac{1}{2}\sqrt{(44\,\text{cm})^2 - 4(44\,\text{cm})(11\,\text{cm})} = 22 \text{ cm} .$$

24. (a) Since this is a converging lens ("C") then $f > 0$, so we should put a plus sign in front of the "10" value given for the focal length. There is not enough information to determine $r_1$ and $r_2$. Eq. 35-9 gives

$$i = \frac{1}{\frac{1}{f} - \frac{1}{p}} = \frac{1}{\frac{1}{10} - \frac{1}{20}} = +20 \text{ cm} .$$

There is insufficient information for the determination of $n$. From Eq. 35-6, $m = -20/20 = -1.0$. The image is real (since $i > 0$) and inverted (since $m < 0$). The ray diagram would be similar to Fig. 35-14(a) in the textbook.

(b) Since $f > 0$, this is a converging lens ("C"). There is not enough information to determine $r_1$ and $r_2$. Eq. 35-9 gives

$$i = \frac{1}{\frac{1}{f} - \frac{1}{p}} = \frac{1}{\frac{1}{10} - \frac{1}{5}} = -10 \text{ cm} .$$

There is insufficient information for the determination of $n$. From Eq. 35-6, $m = -(-10)/5 = +2.0$. The image is virtual (since $i < 0$) and upright (since $m > 0$). The ray diagram would be similar to Fig. 35-14(b) in the textbook.

(c) We are told the magnification is positive and greater than 1. Scanning the single-lens-image figures in the textbook (Figs. 35-13, 35-14 and 35-16), we see that such a magnification (which implies an upright image larger than the object) is only possible if the lens is of the converging ("C") type (and if $p < f$). Thus, we should put a plus sign in front of the "10" value given for the focal length. Eq. 35-9 gives

$$i = \frac{1}{\frac{1}{f} - \frac{1}{p}} = \frac{1}{\frac{1}{10} - \frac{1}{5}} = -10 \text{ cm} ,$$

which implies the image is virtual. There is insufficient information for the determinations of $n$, $r_1$ and $r_2$. The ray diagram would be similar to Fig. 35-14(b) in the textbook.

(d) We are told the magnification is less than 1, and we note that $p < |f|$). Scanning Figs. 35-13, 35-14 and 35-16, we see that such a magnification (which implies an image smaller than the object) and object position (being fairly close to the lens) are simultaneously possible only if the lens is of the diverging ("D") type. Thus, we should put a minus sign in front of the "10" value given for the focal length. Eq. 35-9 gives

$$i = \frac{1}{\frac{1}{f} - \frac{1}{p}} = \frac{1}{\frac{1}{-10} - \frac{1}{5}} = -3.3 \text{ cm} ,$$

which implies the image is virtual (and upright). There is insufficient information for the determinations of $n$, $r_1$ and $r_2$. The ray diagram would be similar to Fig. 35-14(c) in the textbook.

(e) Eq. 35-10 yields $f = \frac{1}{n-1}(1/r_1 - 1/r_2)^{-1} = +30$ cm. Since $f > 0$, this must be a converging ("C") lens. From Eq. 35-9, we obtain

$$i = \frac{1}{\frac{1}{f} - \frac{1}{p}} = \frac{1}{\frac{1}{30} - \frac{1}{10}} = -15 \text{ cm} .$$

Eq. 35-6 yields $m = -(-15)/10 = +1.5$. Therefore, the image is virtual ($i < 0$) and upright ($m > 0$). The ray diagram would be similar to Fig. 35-14(b) in the textbook.

(f) Eq. 35-10 yields $f = \frac{1}{n-1}(1/r_1 - 1/r_2)^{-1} = -30$ cm. Since $f < 0$, this must be a diverging ("D") lens. From Eq. 35-9, we obtain

$$i = \frac{1}{\frac{1}{f} - \frac{1}{p}} = \frac{1}{\frac{1}{-30} - \frac{1}{10}} = -7.5 \text{ cm} .$$

Eq. 35-6 yields $m = -(-7.5)/10 = +0.75$. Therefore, the image is virtual ($i < 0$) and upright ($m > 0$). The ray diagram would be similar to Fig. 35-14(c) in the textbook.

(g) Eq. 35-10 yields $f = \frac{1}{n-1}(1/r_1 - 1/r_2)^{-1} = -120$ cm. Since $f < 0$, this must be a diverging ("D") lens. From Eq. 35-9, we obtain

$$i = \frac{1}{\frac{1}{f} - \frac{1}{p}} = \frac{1}{\frac{1}{-120} - \frac{1}{10}} = -9.2 \text{ cm} .$$

Eq. 35-6 yields $m = -(-9.2)/10 = +0.92$. Therefore, the image is virtual ($i < 0$) and upright ($m > 0$). The ray diagram would be similar to Fig. 35-14(c) in the textbook.

(h) We are told the absolute value of the magnification is 0.5 and that the image was upright. Thus, $m = +0.5$. Using Eq. 35-6 and the given value of $p$, we find $i = -5.0$ cm; it is a virtual image. Eq. 35-9 then yields the focal length: $f = -10$ cm. Therefore, the lens is of the diverging ("D") type. The ray diagram would be similar to Fig. 35-14(c) in the textbook. There is insufficient information for the determinations of $n$, $r_1$ and $r_2$.

(i) Using Eq. 35-6 (which implies the image is inverted) and the given value of $p$, we find $i = -mp = +5.0$ cm; it is a real image. Eq. 35-9 then yields the focal length: $f = +3.3$ cm. Therefore, the lens is of the converging ("C") type. The ray diagram would be similar to Fig. 35-14(a) in the textbook. There is insufficient information for the determinations of $n$, $r_1$ and $r_2$.

25. For an object in front of a thin lens, the object distance $p$ and the image distance $i$ are related by $(1/p)+(1/i) = (1/f)$, where $f$ is the focal length of the lens. For the situation described by the problem, all quantities are positive, so the distance $x$ between the object and image is $x = p + i$. We substitute $i = x - p$ into the thin lens equation and solve for $x$:

$$x = \frac{p^2}{p - f} .$$

To find the minimum value of $x$, we set $dx/dp = 0$ and solve for $p$. Since

$$\frac{dx}{dp} = \frac{p(p - 2f)}{(p - f)^2} ,$$

the result is $p = 2f$. The minimum distance is

$$x_{\min} = \frac{p^2}{p - f} = \frac{(2f)^2}{2f - f} = 4f .$$

This is a minimum, rather than a maximum, since the image distance $i$ becomes large without bound as the object approaches the focal point.

26. (a) (b) (c) and (d) Our first step is to form the image from the first lens. With $p_1 = 10$ cm and $f_1 = -15$ cm, Eq. 35-9 leads to

$$\frac{1}{p_1} + \frac{1}{i_1} = \frac{1}{f_1} \implies i_1 = -6 \text{ cm} .$$

The corresponding magnification is $m_1 = -i_1/p_1 = 0.6$. This image serves the role of "object" for the second lens, with $p_2 = 12 + 6 = 18$ cm, and $f_2 = 12$ cm. Now, Eq. 35-9 leads to

$$\frac{1}{p_2} + \frac{1}{i_2} = \frac{1}{f_2} \implies i_2 = 36 \text{ cm}$$

with a corresponding magnification of $m_2 = -i_2/p_2 = -2$, resulting in a net magnification of $m = m_1 m_2 = -1.2$. The fact that $m$ is positive means that the orientation of the final image is inverted with respect to the (original) object. The height of the final image is (in absolute value) $(1.2)(1.0\,\text{cm}) = 1.2$ cm. The fact that $i_2$ is positive means that the final image is real.

27. Without the diverging lens (lens 2), the real image formed by the converging lens (lens 1) is located at a distance

$$i_1 = \left(\frac{1}{f_1} - \frac{1}{p_1}\right)^{-1} = \left(\frac{1}{20\,\text{cm}} - \frac{1}{40\,\text{cm}}\right)^{-1} = 40 \text{ cm}$$

to the right of lens 1. This image now serves as an object for lens 2, with $p_2 = -(40\,\text{cm} - 10\,\text{cm}) = -30\,\text{cm}$. So

$$i_2 = \left(\frac{1}{f_2} - \frac{1}{p_2}\right)^{-1} = \left(\frac{1}{-15\,\text{cm}} - \frac{1}{-30\,\text{cm}}\right)^{-1} = -30 \text{ cm} .$$

Thus, the image formed by lens 2 is located 30 cm to the left of lens 2. It is virtual (since $i_2 < 0$). The magnification is $m = (-i_1/p_1) \times (-i_2/p_2) = +1$, so the image has the same size and orientation as the object.

28. (a) For the image formed by the first lens

$$i_1 = \left(\frac{1}{f_1} - \frac{1}{p_1}\right)^{-1} = \left(\frac{1}{10\,\text{cm}} - \frac{1}{20\,\text{cm}}\right)^{-1} = 20 \text{ cm} .$$

For the subsequent image formed by the second lens $p_2 = 30\,\text{cm} - 20\,\text{cm} = 10\,\text{cm}$, so

$$i_2 = \left(\frac{1}{f_2} - \frac{1}{p_2}\right)^{-1} = \left(\frac{1}{12.5\,\text{cm}} - \frac{1}{10\,\text{cm}}\right)^{-1} = -50 \text{ cm} .$$

Thus, the final image is 50 cm to the left of the second lens, which means that it coincides with the object. The magnification is

$$m = \left(\frac{i_1}{p_1}\right)\left(\frac{i_2}{p_2}\right) = \left(\frac{20\,\text{cm}}{20\,\text{cm}}\right)\left(\frac{-50\,\text{cm}}{10\,\text{cm}}\right) = -5.0 ,$$

which means that the final image is five times larger than the original object.

(b) The ray diagram would be very similar to Fig. 35-17 in the textbook, except that the final image would be directly underneath the original object.

(c) and (d) It is virtual and inverted.

29. We place an object far away from the composite lens and find the image distance $i$. Since the image is at a focal point, $i = f$, where $f$ equals the effective focal length of the composite. The final image is produced by two lenses, with the image of the first lens being the object for the second. For the first lens, $(1/p_1) + (1/i_1) = (1/f_1)$, where $f_1$ is the focal length of this lens and $i_1$ is the image distance for the image it forms. Since $p_1 = \infty$, $i_1 = f_1$. The thin lens equation, applied to the second lens, is $(1/p_2) + (1/i_2) = (1/f_2)$, where $p_2$ is the object distance, $i_2$ is the image distance, and $f_2$ is the focal length. If the thicknesses of the lenses can be ignored, the object distance for the second lens is $p_2 = -i_1$. The negative sign must be used since the image formed by the first lens is beyond the second lens if $i_1$ is positive. This means the object for the second lens is virtual and the object distance is negative. If $i_1$ is negative, the image formed by the first lens is in front of the second lens and $p_2$ is positive. In the thin lens equation, we replace $p_2$ with $-f_1$ and $i_2$ with $f$ to obtain

$$-\frac{1}{f_1} + \frac{1}{f} = \frac{1}{f_2}$$

or

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{f_1 + f_2}{f_1 f_2} .$$

Thus,

$$f = \frac{f_1 f_2}{f_1 + f_2} .$$

30. (a) A convex (converging) lens, since a real image is formed.

(b) Since $i = d - p$ and $i/p = 1/2$,

$$p = \frac{2d}{3} = \frac{2(40.0\,\text{cm})}{3} = 26.7 \text{ cm} .$$

(c) The focal length is

$$f = \left(\frac{1}{i} + \frac{1}{p}\right)^{-1} = \left(\frac{1}{d/3} + \frac{1}{2d/3}\right)^{-1} = \frac{2d}{9} = \frac{2(40.0\,\text{cm})}{9} = 8.89\,\text{cm} .$$

31. (a) If the object distance is $x$, then the image distance is $D - x$ and the thin lens equation becomes

$$\frac{1}{x} + \frac{1}{D-x} = \frac{1}{f} .$$

We multiply each term in the equation by $fx(D - x)$ and obtain $x^2 - Dx + Df = 0$. Solving for $x$, we find that the two object distances for which images are formed on the screen are

$$x_1 = \frac{D - \sqrt{D(D-4f)}}{2} \quad \text{and} \quad x_2 = \frac{D + \sqrt{D(D-4f)}}{2} .$$

The distance between the two object positions is

$$d = x_2 - x_1 = \sqrt{D(D-4f)} .$$

(b) The ratio of the image sizes is the same as the ratio of the lateral magnifications. If the object is at $p = x_1$, the magnitude of the lateral magnification is

$$|m_1| = \frac{i_1}{p_1} = \frac{D - x_1}{x_1} .$$

Now $x_1 = \frac{1}{2}(D - d)$, where $d = \sqrt{D(D-f)}$, so

$$|m_1| = \frac{D - (D-d)/2}{(D-d)/2} = \frac{D+d}{D-d} .$$

Similarly, when the object is at $x_2$, the magnitude of the lateral magnification is

$$|m_2| = \frac{I_2}{p_2} = \frac{D - x_2}{x_2} = \frac{D - (D+d)/2}{(D+d)/2} = \frac{D-d}{D+d} .$$

The ratio of the magnifications is

$$\frac{m_2}{m_1} = \frac{(D-d)/(D+d)}{(D+d)/(D-d)} = \left(\frac{D-d}{D+d}\right)^2 .$$

32. The minimum diameter of the eyepiece is given by

$$d_{\text{ey}} = \frac{d_{\text{ob}}}{m_\theta} = \frac{75\,\text{mm}}{36} = 2.1 \text{ mm} .$$

33. (a) If $L$ is the distance between the lenses, then according to Fig. 35-17, the tube length is $s = L - f_{\text{ob}} - f_{\text{ey}} = 25.0\,\text{cm} - 4.00\,\text{cm} - 8.00\,\text{cm} = 13.0\,\text{cm}$.

(b) We solve $(1/p) + (1/i) = (1/f_{\text{ob}})$ for $p$. The image distance is $i = f_{\text{ob}} + s = 4.00\,\text{cm} + 13.0\,\text{cm} = 17.0\,\text{cm}$, so

$$p = \frac{i f_{\text{ob}}}{i - f_{\text{ob}}} = \frac{(17.0\,\text{cm})(4.00\,\text{cm})}{17.0\,\text{cm} - 4.00\,\text{cm}} = 5.23 \text{ cm} .$$

(c) The magnification of the objective is

$$m = -\frac{i}{p} = -\frac{17.0\,\text{cm}}{5.23\,\text{cm}} = -3.25 .$$

(d) The angular magnification of the eyepiece is

$$m_\theta = \frac{25\,\text{cm}}{f_{\text{ey}}} = \frac{25\,\text{cm}}{8.00\,\text{cm}} = 3.13 .$$

(e) The overall magnification of the microscope is

$$M = mm_\theta = (-3.25)(3.13) = -10.2 \ .$$

34. (a) Without the magnifier, $\theta = h/P_n$ (see Fig. 35-16). With the magnifier, letting $p = P_n$ and $i = -|i| = -P_n$, we obtain

$$\frac{1}{p} = \frac{1}{f} - \frac{1}{i} = \frac{1}{f} + \frac{1}{|i|} = \frac{1}{f} + \frac{1}{P_n} \ .$$

Consequently,

$$m_\theta = \frac{\theta'}{\theta} = \frac{h/p}{h/P_n} = \frac{1/f + 1/P_n}{1/P_n} = 1 + \frac{P_n}{f} = 1 + \frac{25\,\text{cm}}{f} \ .$$

(b) Now $i = -|i| \to -\infty$, so $1/p + 1/i = 1/p = 1/f$ and

$$m_\theta = \frac{\theta'}{\theta} = \frac{h/p}{h/P_n} = \frac{1/f}{1/P_n} = \frac{P_n}{f} = \frac{25\,\text{cm}}{f} \ .$$

(c) For $f = 10\,\text{cm}$,

$$m_\theta = 1 + \frac{25\,\text{cm}}{10\,\text{cm}} = 3.5 \text{ (case (a))} \quad \text{and} \quad \frac{25\,\text{cm}}{10\,\text{cm}} = 2.5 \text{ (case (b))} \ .$$

35. (a) When the eye is relaxed, its lens focuses far-away objects on the retina, a distance $i$ behind the lens. We set $p = \infty$ in the thin lens equation to obtain $1/i = 1/f$, where $f$ is the focal length of the relaxed effective lens. Thus, $i = f = 2.50\,\text{cm}$. When the eye focuses on closer objects, the image distance $i$ remains the same but the object distance and focal length change. If $p$ is the new object distance and $f'$ is the new focal length, then

$$\frac{1}{p} + \frac{1}{i} = \frac{1}{f'} \ .$$

We substitute $i = f$ and solve for $f'$:

$$f' = \frac{pf}{f+p} = \frac{(40.0\,\text{cm})(2.50\,\text{cm})}{40.0\,\text{cm} + 2.50\,\text{cm}} = 2.35 \text{ cm} \ .$$

(b) Consider the lensmaker's equation

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

where $r_1$ and $r_2$ are the radii of curvature of the two surfaces of the lens and $n$ is the index of refraction of the lens material. For the lens pictured in Fig. 35-34, $r_1$ and $r_2$ have about the same magnitude, $r_1$ is positive, and $r_2$ is negative. Since the focal length decreases, the combination $(1/r_1) - (1/r_2)$ must increase. This can be accomplished by decreasing the magnitudes of both radii.

36. Refer to Fig. 35-17. For the intermediate image $p = 10\,\text{mm}$ and $i = (f_\text{ob}+s+f_\text{ey}) - f_\text{ey} = 300\,\text{m} - 50\,\text{mm} = 250\,\text{mm}$, so

$$\frac{1}{f_\text{ob}} = \frac{1}{i} + \frac{1}{p} = \frac{1}{250\,\text{mm}} + \frac{1}{10\,\text{mm}} \implies f_\text{ob} = 9.62 \text{ mm} \ ,$$

and $s = (f_\text{ob} + s + f_\text{ey}) - f_\text{ob} - f_\text{ey} = 300\,\text{mm} - 9.62\,\text{mm} - 50\,\text{mm} = 240\,\text{mm}$. Then from Eq. 35-14,

$$M = -\frac{s}{f_\text{ob}}\frac{25\,\text{cm}}{f_\text{ey}} = -\left(\frac{240\,\text{mm}}{9.62\,\text{mm}}\right)\left(\frac{150\,\text{mm}}{50\,\text{mm}}\right) = -125 \ .$$

37. (a) Now, the lens-film distance is

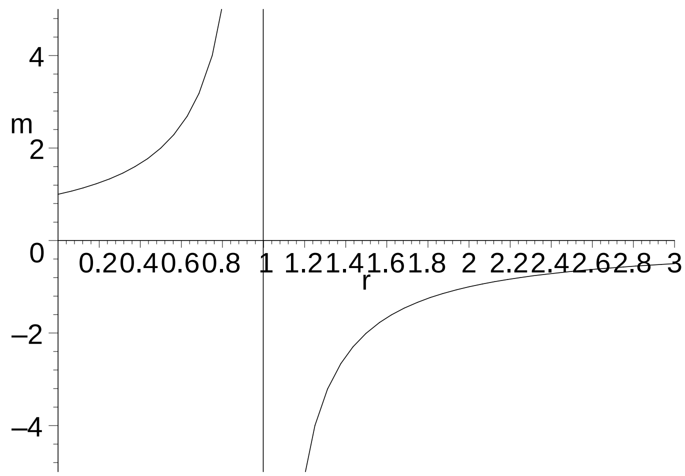
$$i = \left(\frac{1}{f} - \frac{1}{p}\right)^{-1} = \left(\frac{1}{5.0\,\text{cm}} - \frac{1}{100\,\text{cm}}\right)^{-1} = 5.3 \text{ cm} .$$

(b) The change in the lens-film distance is $5.3\,\text{cm} - 5.0\,\text{cm} = 0.30\,\text{cm}$.

38. We combine Eq. 35-4 and Eq. 35-6 and arrive at

$$m = -\frac{pf/(p-f)}{p} = \frac{1}{1-r} \qquad \text{where} \quad r = \frac{p}{f}$$

We emphasize that this $r$ (for ratio) is not the radius of curvature. The magnification as a function of $r$ is graphed below:




39. (a) The discussion in the textbook of the refracting telescope (a subsection of §35-7) applies to the Newtonian arrangement if we replace the objective lens of Fig. 35-18 with an objective mirror (with the light incident on it from the right). This might suggest that the incident light would be blocked by the person's head in Fig. 35-18, which is why Newton added the mirror $M'$ in his design (to move the head and eyepiece out of the way of the incoming light). The beauty of the idea of characterizing both lenses and mirrors by focal lengths is that it is easy, in a case like this, to simply carry over the results of the objective-lens telescope to the objective-mirror telescope, so long as we replace a positive $f$ device with another positive $f$ device. Thus, the converging lens serving as the objective of Fig. 35-18 must be replaced (as Newton has done in Fig. 35-44) with a concave mirror. With this change of language, the discussion in the textbook leading up to Eq. 35-15 applies equally as well to the Newtonian telescope: $m_\theta = -f_{\text{ob}}/f_{\text{ey}}$.

(b) A meter stick (held perpendicular to the line of sight) at a distance of 2000 m subtends an angle of

$$\theta_{\text{stick}} \approx \frac{1\,\text{m}}{2000\,\text{m}} = 0.0005 \text{ rad} .$$

Multiplying this by the mirror focal length gives $(16.8\,\text{m})(0.0005) = 8.4$ mm for the size of the image.

(c) With $r = 10$ m, Eq. 35-3 gives $f_{\text{ob}} = 5$ m. Plugging this into (the absolute value of) Eq. 35-15 leads to $f_{\text{ey}} = 5/200 = 2.5$ cm.

40. (a) The "object" for the mirror which results in that box-image is equally in front of the mirror (4 cm). This object is actually the first image formed by the system (produced by the first transmission through the lens); in those terms, it corresponds to $i_1 = 10 - 4 = 6$ cm. Thus, with $f_1 = 2$ cm, Eq. 35-9 leads to

$$\frac{1}{p_1} + \frac{1}{i_1} = \frac{1}{f_1} \implies p_1 = 3.00 \text{ cm} .$$

(b) The previously mentioned box-image (4 cm behind the mirror) serves as an "object" (at $p_3 = 14$ cm) for the return trip of light through the lens ($f_3 = f_1 = 2$ cm). This time, Eq. 35-9 leads to

$$\frac{1}{p_3} + \frac{1}{i_3} = \frac{1}{f_3} \implies i_3 = 2.33 \text{ cm} .$$

41. (a) In this case $m > +1$ and we know we are dealing with a converging lens (producing a virtual image), so that our result for focal length should be positive. Since $|p + i| = 20$ cm and $i = -2p$, we find $p = 20$ cm and $i = -40$ cm. Substituting these into Eq. 35-9,

$$\frac{1}{p} + \frac{1}{i} = \frac{1}{f}$$

leads to $f = +40$ cm, which is positive as we expected.

(b) In this case $0 < m < 1$ and we know we are dealing with a diverging lens (producing a virtual image), so that our result for focal length should be negative. Since $|p + i| = 20$ cm and $i = -p/2$, we find $p = 40$ cm and $i = -20$ cm. Substituting these into Eq. 35-9 leads to $f = -40$ cm, which is negative as we expected.

42. (a) The first image is figured using Eq. 35-8, with $n_1 = 1$ (using the rounded-off value for air) and $n_2 = 8/5$.

$$\frac{1}{p} + \frac{8}{5i} = \frac{1.6 - 1}{r}$$

For a "flat lens" $r = \infty$, so we obtain $i = -8p/5 = -64/5$ (with the unit cm understood) for that object at $p = 10$ cm. Relative to the second surface, this image is at a distance of $3 + 64/5 = 79/5$. This serves as an object in order to find the final image, using Eq. 35-8 again (and $r = \infty$) but with $n_1 = 8/5$ and $n_2 = 4/3$.

$$\frac{8}{5p'} + \frac{4}{3i'} = 0$$

which produces (for $p' = 79/5$) $i' = -5p/6 = -79/6 \approx -13.2$. This means the observer appears $13.2 + 6.8 = 20$ cm from the fish.

(b) It is straightforward to "reverse" the above reasoning, the result being that the final fish-image is 7.0 cm to the right of the air-wall interface, and thus 15 cm from the observer.

43. (a) (b) and (c) Since $m = +0.250$, we have $i = -0.25p$ which indicates that the image is virtual (as well as being diminished in size). We conclude from this that the mirror is convex and that $f < 0$; in fact, $f = -2.00$ cm. Substituting $i = -p/4$ into Eq. 35-4 produces

$$\frac{1}{p} - \frac{4}{p} = -\frac{3}{p} = \frac{1}{f}$$

Therefore, we find $p = 6.00$ cm and $i = -1.50$ cm.

44. (a) A parallel ray of light focuses at the focal point behind the lens. In the case of farsightedness we need to bring the focal point closer. That is, we need to reduce the focal length. From problem 29, we know that we need to use a converging lens of certain focal length $f_1 > 0$ which, when combined with the eye of focal length $f_2$, gives $f = f_1 f_2/(f_1 + f_2) < f_2$. Similarly, we see that in the case of nearsightedness we need to do a similar computation but with a diverging ($f_1 < 0$) lens.

(b) In this case, the unaided eyes are able to accommodate rays of light coming from distant (and medium-range) sources, but not from close ones. The person (not wearing glasses) is able to see far (not near), so the person is farsighted.

(c) The bifocal glasses can provide suitable corrections for different types of visual defects that prove a hindrance in different situations, such as reading (difficult for the farsighted individual) and viewing a distant object (difficult for a nearsighted individual).

45. (a) We use Eq. 35-10, with the conventions for signs discussed in §35-5 and §35-6.

(b) For the bi-convex (or double convex) case, we have

$$f = \left[(n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)\right]^{-1} = \left[(1.5-1)\left(\frac{1}{40\,\text{cm}} - \frac{1}{-40\,\text{cm}}\right)\right]^{-1} = 40 \text{ cm} .$$

Since $f > 0$ the lens forms a real image of the Sun.

(c) For the planar convex lens, we find

$$f = \left[(1.5-1)\left(\frac{1}{\infty} - \frac{1}{-40\,\text{cm}}\right)\right]^{-1} = 80 \text{ cm} ,$$

and the image formed is real (since $f > 0$).

(d) Now

$$f = \left[(1.5-1)\left(\frac{1}{40\,\text{cm}} - \frac{1}{60\,\text{cm}}\right)\right]^{-1} = 240 \text{ cm} ,$$

and the image formed is real (since $f > 0$).

(e) For the bi-concave lens, the focal length is

$$f = \left[(1.5-1)\left(\frac{1}{-40\,\text{cm}} - \frac{1}{40\,\text{cm}}\right)\right]^{-1} = -40 \text{ cm} ,$$

and the image formed is virtual (since $f < 0$).

(f) In this case,

$$f = \left[(1.5-1)\left(\frac{1}{\infty} - \frac{1}{40\,\text{cm}}\right)\right]^{-1} = -80 \text{ cm} ,$$

and the image formed is virtual (since $f < 0$).

(g) Now

$$f = \left[(1.5-1)\left(\frac{1}{60\,\text{cm}} - \frac{1}{40\,\text{cm}}\right)\right]^{-1} = -240 \text{ cm} ,$$

and the image formed is virtual (since $f < 0$).

46. Of course, the shortest possible path between $A$ and $B$ is the straight line path which does not go to the mirror at all. In this problem, we are concerned with only those paths which do strike the mirror. The problem statement suggests that we turn our attention to the mirror-image point of $A$ (call it $A'$) and requests that we construct a proof without calculus. We can see that the length of any line segment $AP$ drawn from $A$ to the mirror (at point $P$ on the mirror surface) is the same as the length of its "mirror segment" $A'P$ drawn from $A'$ to that point $P$. Thus, the total length of the light path from $A$ to $P$ to $B$ is the same as the total length of segments drawn from $A'$ to $P$ to $B$. Now, we dismissed (in the first sentence of this solution) the possibility of a straight line path directly from $A$ to $B$ because it does not strike the mirror. However, we *can* construct a straight line path from $A'$ to $B$ which does intersect the mirror surface! Any other pair of segments ($A'P$ and $PB$) would give greater total length than the straight path (with $A'P$ and $PB$ collinear), so if the straight path $A'B$ obeys the law of reflection,

then we have our proof. Now, since $A'P$ is the mirror-twin of $AP$, then they both approach the mirror surface with the same angle $\alpha$ (one from the front side and the other from the back side). And since $A'P$ is collinear with $PB$, then $PB$ also makes the same angle $\alpha$ with respect to the mirror surface (by vertex angles). If $AP$ and $PB$ are each $\alpha$ degrees away from the front of the mirror, then they are each $\theta$ degrees (where $\theta$ is the complement of $\alpha$) measured from the normal axis. Thus, the law of reflection is consistent with the concept of the shortest light path.

47. (a) (b) and (c) Our first step is to form the image from the first lens. With $p_1 = 4$ cm and $f_1 = -4$ cm, Eq. 35-9 leads to

$$\frac{1}{p_1} + \frac{1}{i_1} = \frac{1}{f_1} \implies i_1 = -2 \text{ cm} .$$

The corresponding magnification is $m_1 = -i_1/p_1 = 1/2$. This image serves the role of "object" for the second lens, with $p_2 = 10 + 2 = 12$ cm, and $f_2 = -4$ cm. Now, Eq. 35-9 leads to

$$\frac{1}{p_2} + \frac{1}{i_2} = \frac{1}{f_2} \implies i_2 = -3.00 \text{ cm}$$

with a corresponding magnification of $m_2 = -i_2/p_2 = 1/4$, resulting in a net magnification of $m = m_1m_2 = 1/8$. The fact that $m$ is positive means that the orientation of the final image is the same as the (original) object. The fact that $i_2$ is negative means that the final image is virtual.

48. (a) (b) (c) and (d) Our first step is to form the image from the first lens. With $p_1 = 3$ cm and $f_1 = +4$ cm, Eq. 35-9 leads to

$$\frac{1}{p_1} + \frac{1}{i_1} = \frac{1}{f_1} \implies i_1 = -12 \text{ cm} .$$

The corresponding magnification is $m_1 = -i_1/p_1 = 4$. This image serves the role of "object" for the second lens, with $p_2 = 8 + 12 = 20$ cm, and $f_2 = -4$ cm. Now, Eq. 35-9 leads to

$$\frac{1}{p_2} + \frac{1}{i_2} = \frac{1}{f_2} \implies i_2 = -3.33 \text{ cm}$$

with a corresponding magnification of $m_2 = -i_2/p_2 = 1/6$, resulting in a net magnification of $m = m_1m_2 = 2/3$. The fact that $m$ is positive means that the orientation of the final image is the same as the (original) object. The fact that $i_2$ is negative means that the final image is virtual (and therefore to the left of the second lens).

49. Since $0 < m < 1$, we conclude the lens is of the diverging type (so $f = -40$ cm). Thus, substituting $i = -3p/10$ into Eq. 35-9 produces

$$\frac{1}{p} - \frac{10}{3p} = -\frac{7}{3p} = \frac{1}{f} .$$

Therefore, we find $p = 93.3$ cm and $i = -28.0$ cm.

50. (a) We use Eq. 35-8 (and Fig. 35-10(b) is useful), with $n_1 = 1$ (using the rounded-off value for air) and $n_2 = 1.5$ .

$$\frac{1}{p} + \frac{1.5}{i} = \frac{1.5 - 1}{r}$$

Using the sign convention for $r$ stated in the paragraph following Eq. 35-8 (so that $r = +6.0$ cm), we obtain $i = -90$ cm for objects at $p = 10$ cm. Thus, the object and image are 80 cm apart.

(b) The image distance $i$ is negative with increasing magnitude as $p$ increases from very small values to some value $p_0$ at which point $i \to -\infty$. Since $1/(-\infty) = 0$, the above equation yields

$$\frac{1}{p_0} = \frac{1.5 - 1}{r} \implies p_0 = 2r.$$

Thus, the range for producing virtual images is $0 < p \leq 12$ cm.

51. (a) Since $m = +0.200$, we have $i = -0.2p$ which indicates that the image is virtual (as well as being diminished in size). We conclude from this that the mirror is convex (and that $f = -40.0$ cm).

(b) Substituting $i = -p/5$ into Eq. 35-4 produces

$$\frac{1}{p} - \frac{5}{p} = -\frac{4}{p} = \frac{1}{f} \ .$$

Therefore, we find $p = 160$ cm.

52. (a) First, the lens forms a real image of the object located at a distance

$$i_1 = \left(\frac{1}{f_1} - \frac{1}{p_1}\right)^{-1} = \left(\frac{1}{f_1} - \frac{1}{2f_1}\right)^{-1} = 2f_1$$

to the right of the lens, or at $p_2 = 2(f_1 + f_2) - 2f_1 = 2f_2$ in front of the mirror. The subsequent image formed by the mirror is located at a distance

$$i_2 = \left(\frac{1}{f_2} - \frac{1}{p_2}\right)^{-1} = \left(\frac{1}{f_2} - \frac{1}{2f_2}\right)^{-1} = 2f_2$$

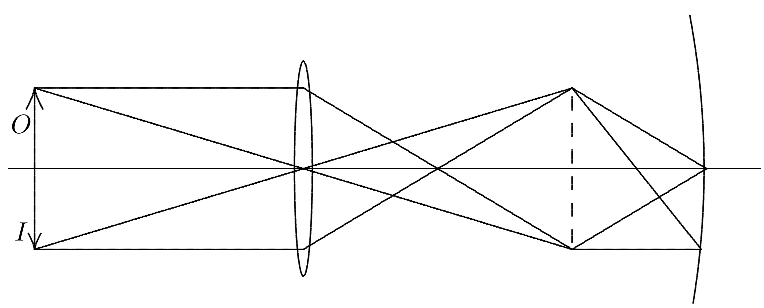
to the left of the mirror, or at $p_1' = 2(f_1 + f_2) - 2f_2 = 2f_1$ to the right of the lens. The final image formed by the lens is that at a distance $i_1'$ to the left of the lens, where

$$i_1' = \left(\frac{1}{f_1} - \frac{1}{p_1'}\right)^{-1} = \left(\frac{1}{f_1} - \frac{1}{2f_1}\right)^{-1} = 2f_1 \ .$$

This turns out to be the same as the location of the original object. The final image is real and inverted. The lateral magnification is

$$m = \left(-\frac{i_1}{p_1}\right)\left(-\frac{i_2}{p_2}\right)\left(-\frac{i_1'}{p_1'}\right) = \left(-\frac{2f_1}{2f_1}\right)\left(-\frac{2f_2}{2f_2}\right)\left(-\frac{2f_1}{2f_1}\right) = -1.0 \ .$$

(b) The ray diagram is shown below. We set the ratio $f_2/f_1 = 1/2$ for the purposes of this sketch. The intermediate images are not shown explicitly, but they are both located on the plane indicated by the dashed line.




53. From Eq. 35-10, if

$$f \propto \left(\frac{1}{r_1} - \frac{1}{r_2}\right)^{-1} = \frac{r_1 r_2}{r_2 - r_1}$$

is positive (that is, if $r_2 > r_1$), then the lens is converging. Otherwise it is diverging.

(a) Converging, since $r_2 \to \infty$ and $r_1$ is finite (so $r_2 > r_1$).

(b) Diverging, since $r_1 \to \infty$ and $r_2$ is finite (so $r_2 < r_1$).

(c) Converging, since $r_2 > r_1$.

(d) Diverging, since $r_2 < r_1$.

54. We refer to Fig. 35-2 in the textbook. Consider the two light rays, $r$ and $r'$, which are closest to and on either side of the normal ray (the ray that reverses when it reflects). Each of these rays has an angle of incidence equal to $\theta$ when they reach the mirror. Consider that these two rays reach the top and bottom edges of the pupil after they have reflected. If ray $r$ strikes the mirror at point $A$ and ray $r'$ strikes the mirror at $B$, the distance between $A$ and $B$ (call it $x$) is

$$x = 2d_o \tan\theta$$

where $d_o$ is the distance from the mirror to the object. We can construct a right triangle starting with the image point of the object (a distance $d_o$ behind the mirror; see $I$ in Fig. 35-2). One side of the triangle follows the extended normal axis (which would reach from $I$ to the middle of the pupil), and the hypotenuse is along the extension of ray $r$ (after reflection). The distance from the pupil to $I$ is $d_{\text{ey}} + d_o$, and the small angle in this triangle is again $\theta$. Thus,

$$\tan\theta = \frac{R}{d_{\text{ey}} + d_o}$$

where $R$ is the pupil radius (2.5 mm). Combining these relations, we find

$$x = 2d_o \frac{R}{d_{\text{ey}} + d_o} = 2(100\,\text{mm}) \frac{2.5\,\text{mm}}{300\,\text{mm} + 100\,\text{mm}}$$

which yields $x = 1.67$ mm. Now, $x$ serves as the diameter of a circular area $A$ on the mirror, in which all rays that reflect will reach the eye. Therefore,

$$A = \frac{1}{4}\pi x^2 = \frac{\pi}{4}(1.67\,\text{mm})^2 = 2.2\ \text{mm}^2\ .$$

55. The sphere (of radius 0.35 m) is a convex mirror with focal length $f = -0.175$ m. We adopt the approximation that the rays are close enough to the central axis for Eq. 35-4 to be applicable. We also take the "1.0 m in front of ... [the] sphere" to mean $p = 1.0$ m (measured from the front surface as opposed to being measured from the center-point of the sphere).

(a) The equation $1/p + 1/i = 1/f$ yields $i = -0.15$ m, which means the image is 15 cm from the front surface, appearing to be *inside* the sphere.

(b) and (c) The lateral magnification is $m = -i/p$ which yields $m = 0.15$. Therefore, the image distance is $(0.15)(2.0\,\text{m}) = 0.30$ m; that this is a positive value implies the image is erect (upright).

56. (a) The mirror has focal length $f = 12$ cm. With $m = +3$, we have $i = -3p$. We substitute this into Eq. 35-4:

$$\begin{aligned} \frac{1}{p} + \frac{1}{i} &= \frac{1}{f} \\ \frac{1}{p} + \frac{1}{-3p} &= \frac{1}{12} \\ \frac{2}{3p} &= \frac{1}{12} \end{aligned}$$

with the unit cm understood. Consequently, we find $p = 2(12)/3 = 8.0$ cm.

(b) With $m = -3$, we have $i = +3p$, which we substitute into Eq. 35-4:

$$\begin{aligned} \frac{1}{p} + \frac{1}{i} &= \frac{1}{f} \\ \frac{1}{p} + \frac{1}{3p} &= \frac{1}{12} \\ \frac{4}{3p} &= \frac{1}{12} \end{aligned}$$

with the unit cm understood. Consequently, we find $p = 4(12)/3 = 16$ cm.

(c) With $m = -1/3$, we have $i = p/3$. Thus, Eq. 35-4 leads to

$$\begin{aligned} \frac{1}{p} + \frac{1}{i} &= \frac{1}{f} \\ \frac{1}{p} + \frac{3}{p} &= \frac{1}{12} \\ \frac{4}{p} &= \frac{1}{12} \end{aligned}$$

with the unit cm understood. Consequently, we find $p = 4(12) = 48$ cm.

57. Since $m = -2$ and $p = 4$ cm, then $i = 8$ cm (and is real). Eq. 35-9 is

$$\frac{1}{p} + \frac{1}{i} = \frac{1}{f}$$

and leads to $f = 2.67$ cm (which is positive, as it must be for a converging lens).

58. We use Eq. 35-8 (and Fig. 35-10(d) is useful), with $n_1 = 1.6$ and $n_2 = 1$ (using the rounded-off value for air).

$$\frac{1.6}{p} + \frac{1}{i} = \frac{1 - 1.6}{r}$$

Using the sign convention for $r$ stated in the paragraph following Eq. 35-8 (so that $r = -5.0$ cm), we obtain $i = -2.4$ cm for objects at $p = 3.0$ cm. Returning to Fig. 35-52 (and noting the location of the observer), we conclude that the tabletop seems 7.4 cm away.

59. The fact that it is inverted implies $m < 0$. Therefore, with $m = -1/2$, we have $i = p/2$, which we substitute into Eq. 35-4:

$$\begin{aligned} \frac{1}{p} + \frac{1}{i} &= \frac{1}{f} \\ \frac{1}{p} + \frac{2}{p} &= \frac{1}{f} \\ \frac{3}{30.0} &= \frac{1}{f} \end{aligned}$$

with the unit cm understood. Consequently, we find $f = 30/3 = 10.0$ cm. The fact that $f > 0$ implies the mirror is concave.

60. (a) Suppose that the lens is placed to the left of the mirror. The image formed by the converging lens is located at a distance

$$i = \left(\frac{1}{f} - \frac{1}{p}\right)^{-1} = \left(\frac{1}{0.50\,\mathrm{m}} - \frac{1}{1.0\,\mathrm{m}}\right)^{-1} = 1.0 \text{ m}$$

to the right of the lens, or $2.0\,\mathrm{m} - 1.0\,\mathrm{m} = 1.0\,\mathrm{m}$ in front of the mirror. The image formed by the mirror for this real image is then at $1.0\,\mathrm{m}$ to the right of the the mirror, or $2.0\,\mathrm{m} + 1.0\,\mathrm{m} = 3.0\,\mathrm{m}$ to the right of the lens. This image then results in another image formed by the lens, located at a distance

$$i' = \left(\frac{1}{f} - \frac{1}{p'}\right)^{-1} = \left(\frac{1}{0.50\,\mathrm{m}} - \frac{1}{3.0\,\mathrm{m}}\right)^{-1} = 6.0 \text{ m}$$

to the left of the lens (that is, $2.6\,\mathrm{cm}$ from the mirror).

(b) The final image is real since $i' > 0$.

(c) It also has the same orientation as the object, as one can verify by drawing a ray diagram or finding the product of the magnifications (see the next part, which shows $m > 0$).

(d) The lateral magnification is

$$m = \left(-\frac{i}{p}\right)\left(-\frac{i'}{p'}\right) = \left(-\frac{1.0\,\mathrm{m}}{1.0\,\mathrm{m}}\right)\left(-\frac{0.60\,\mathrm{m}}{3.0\,\mathrm{m}}\right) = +0.20 \ .$$

61. (a) Parallel rays are bent by positive-$f$ lenses to their focal points $F_1$, and rays that come from the focal point positions $F_2$ in front of positive-$f$ lenses are made to emerge parallel. The key, then, to this type of beam expander is to have the rear focal point $F_1$ of the first lens coincide with the front focal point $F_2$ of the second lens. Since the triangles that meet at the coincident focal point are similar (they share the same angle; they are vertex angles), then $W_2/f_2 = W_1/f_1$ follows immediately.

(b) The previous argument can be adapted to the first lens in the expanding pair being of the diverging type, by ensuring that the front focal point of the first lens coincides with the front focal point of the second lens. The distance between the lenses in this case is $f_2 - |f_1|$ (where we assume $f_2 > |f_1|$), which we can write as $f_2 + f_1$ just as in part (a).

62. The area is proportional to $W^2$, so the result of problem 61 plus the definition of intensity (power $P$ divided by area) leads to

$$\frac{I_2}{I_1} = \frac{P/W_2^2}{P/W_1^2} = \frac{W_1^2}{W_2^2} = \frac{f_1^2}{f_2^2} \ .$$

63. (a) Virtual, since the image is formed by plane mirrors.

(b) Same. One can easily verify this by locating, for example, the images of two points, one at the head of the penguin and the other at its feet.

(c) Same, since the image formed by any plane mirror retains the original shape and size of an object.

(d) The image of the penguin formed by the top mirror is located a distance $D$ above the top mirror, or $L + D$ above the bottom one. Therefore, the final image of the penguin, formed by the bottom mirror, is a distance $L + D$ from the bottom mirror.

64. In the closest mirror, the "first" image $I_1$ is 10 cm behind the mirror and therefore 20 cm from the object $O$. There are images from both $O$ and $I_1$ in the more distant mirror: an image $I_2$ which is 30 cm behind that mirror (since $O$ is 30 cm in front of it), and an image $I_3$ which is 50 cm behind the mirror (since $I_1$ is 50 cm in front of it). We note that $I_2$ is 60 cm from $O$, and $I_3$ is 80 cm from $O$. Returning to the closer mirror, we find images of $I_2$ and $I_3$, as follows: an image $I_4$ which is 70 cm behind the mirror (since $I_2$ is 70 cm in front of it) and an image $I_5$ which is 90 cm behind the mirror (since $I_3$ is 90 cm in front of it). The distances (measured from $O$) for $I_4$ and $I_5$ are 80 cm and 100 cm, respectively.