Chapter 23

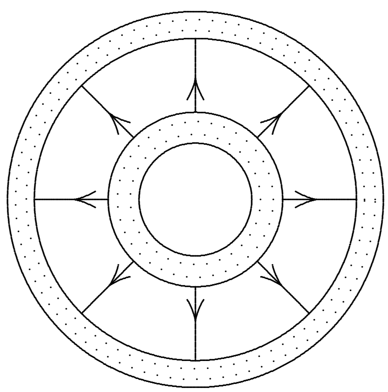
1. (a) We note that the electric field points leftward at both points. Using $\vec{F} = q_0\vec{E}$, and orienting our x axis rightward (so î points right in the figure), we find

$$\vec{F} = \left(+1.6 \times 10^{-19}\,\text{C}\right)\left(-40\,\frac{\text{N}}{\text{C}}\,\hat{\imath}\right) = -6.4 \times 10^{-18}\ \text{N}\ \ \hat{\imath}$$

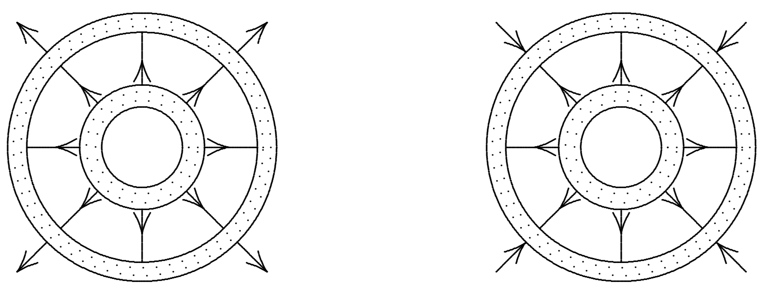
which means the magnitude of the force on the proton is 6.4×10^{-18} N and its direction ($-\hat{\imath}$) is leftward.

(b) As the discussion in §23-2 makes clear, the field strength is proportional to the "crowdedness" of the field lines. It is seen that the lines are twice as crowded at A than at B, so we conclude that $E_A = 2E_B$. Thus, $E_B = 20$ N/C.

2. We note that the symbol q_2 is used in the problem statement to mean the absolute value of the negative charge which resides on the larger shell. The following sketch is for $q_1 = q_2$.

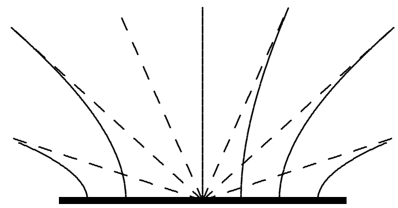


The following two sketches are for the cases $q_1 > q_2$ (left figure) and $q_1 < q_2$ (right figure).



3. The diagram below is an edge view of the disk and shows the field lines above it. Near the disk, the lines are perpendicular to the surface and since the disk is uniformly charged, the lines are uniformly distributed over the surface. Far away from the disk, the lines are like those of a single point charge (the

charge on the disk). Extended back to the disk (along the dotted lines of the diagram) they intersect at the center of the disk.



If the disk is positively charged, the lines are directed outward from the disk. If the disk is negatively charged, they are directed inward toward the disk. A similar set of lines is associated with the region below the disk.

4. We find the charge magnitude $|q|$ from $E = |q|/4\pi\varepsilon_0 r^2$:

$$q = 4\pi\varepsilon_0 E r^2 = \frac{(1.00\,\mathrm{N/C})(1.00\,\mathrm{m})^2}{8.99\times 10^9\,\frac{\mathrm{N\cdot m^2}}{\mathrm{C^2}}} = 1.11\times 10^{-10}\ \mathrm{C}\ .$$

5. Since the magnitude of the electric field produced by a point charge q is given by $E = |q|/4\pi\varepsilon_0 r^2$, where r is the distance from the charge to the point where the field has magnitude E, the magnitude of the charge is

$$|q| = 4\pi\varepsilon_0 r^2 E = \frac{(0.50\,\mathrm{m})^2(2.0\,\mathrm{N/C})}{8.99\times 10^9\,\mathrm{N\cdot m^2/C^2}} = 5.6\times 10^{-11}\ \mathrm{C}\ .$$

6. For concreteness, consider that charge 2 lies 0.15 m east of charge 1, and the point at which we are asked to evaluate their net field is $r = 0.075$ m east of charge 1 and $r = 0.075$ m west of charge 2. The values of charge are $q_1 = -q_2 = 2.0\times 10^{-7}$ C. The magnitudes and directions of the individual fields are specified:

$$\left|\vec{E}_1\right| = \frac{q_1}{4\pi\varepsilon_0 r^2} = 3.2\times 10^5\ \mathrm{N/C} \qquad \text{and} \quad \vec{E}_1 \text{ points east}$$
$$\left|\vec{E}_2\right| = \frac{|q_2|}{4\pi\varepsilon_0 r^2} = 3.2\times 10^5\ \mathrm{N/C} \qquad \text{and} \quad \vec{E}_2 \text{ points east}$$

Since they point the same direction, the magnitude of the net field is the sum of their amplitudes, $\left|\vec{E}_{\rm net}\right| = 6.4\times 10^5$ N/C, and it points east (that is, towards the negative charge).

7. Since the charge is uniformly distributed throughout a sphere, the electric field at the surface is exactly the same as it would be if the charge were all at the center. That is, the magnitude of the field is

$$E = \frac{q}{4\pi\varepsilon_0 R^2}$$

where q is the magnitude of the total charge and R is the sphere radius. The magnitude of the total charge is Ze, so

$$E = \frac{Ze}{4\pi\varepsilon_0 R^2} = \frac{(8.99\times 10^9\,\mathrm{N\cdot m^2/C^2})(94)(1.60\times 10^{-19}\,\mathrm{C})}{(6.64\times 10^{-15}\,\mathrm{m})^2} = 3.07\times 10^{21}\ \mathrm{N/C}\ .$$

The field is normal to the surface and since the charge is positive, it points outward from the surface.

8. The individual magnitudes $\left|\vec{E}_1\right|$ and $\left|\vec{E}_2\right|$ are figured from Eq. 23-3, where the absolute value signs for q are unnecessary since these charges are both positive. Whether we add the magnitudes or subtract them depends on if $\vec{E}_1$ is in the same, or opposite, direction as $\vec{E}_2$. At points to the left of q_1 (along the

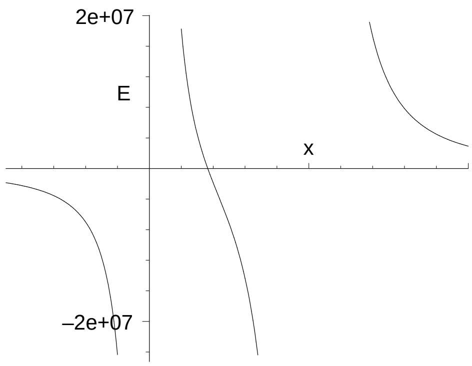
$-x$ axis) both fields point leftward, and at points right of q_2 (at $x > d$) both fields point rightward; in these regions the magnitude of the net field is the sum $\left|\vec{E}_1\right| + \left|\vec{E}_2\right|$. In the region between the charges ($0 < x < d$) $\vec{E}_1$ points rightward and $\vec{E}_2$ points leftward, so the net field in this range is $\vec{E}_{\text{net}} = \left|\vec{E}_1\right| - \left|\vec{E}_2\right|$ in the $\hat{\imath}$ direction. Summarizing, we have

$$\vec{E}_{\text{net}} = \hat{\imath}\,\frac{1}{4\pi\varepsilon_0}\begin{cases} -\frac{q_1}{x^2} - \frac{q_2}{(d+|x|)^2} & \text{for } x < 0 \\ \frac{q_1}{x^2} - \frac{q_2}{(d-x)^2} & \text{for } 0 < x < d \\ \frac{q_1}{x^2} + \frac{q_2}{(x-d)^2} & \text{for } d < x \end{cases} .$$

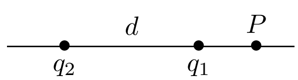
We note that these can be written as a single expression applying to all three regions:

$$\vec{E}_{\text{net}} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 x}{|x|^3} + \frac{q_2(x-d)}{|x-d|^3}\right)\hat{\imath} .$$

For $-0.09 \le x \le 0.20$ m with $d = 0.10$ m and charge values as specified in the problem, we find



9. At points between the charges, the individual electric fields are in the same direction and do not cancel. Charge q_2 has a greater magnitude than charge q_1, so a point of zero field must be closer to q_1 than to q_2. It must be to the right of q_1 on the diagram.



We put the origin at q_2 and let x be the coordinate of P, the point where the field vanishes. Then, the total electric field at P is given by

$$E = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_2}{x^2} - \frac{q_1}{(x-d)^2}\right)$$

where q_1 and q_2 are the magnitudes of the charges. If the field is to vanish,

$$\frac{q_2}{x^2} = \frac{q_1}{(x-d)^2} .$$

We take the square root of both sides to obtain $\sqrt{q_2}/x = \sqrt{q_1}/(x-d)$. The solution for x is

$$x = \left(\frac{\sqrt{q_2}}{\sqrt{q_2} - \sqrt{q_1}}\right) d$$

$$\begin{aligned}
&= \left(\frac{\sqrt{4.0q_1}}{\sqrt{4.0q_1}-\sqrt{q_1}}\right)d \\
&= \left(\frac{2.0}{2.0-1.0}\right)d = 2.0d \\
&= (2.0)(50\,\text{cm}) = 100 \text{ cm} .
\end{aligned}$$

The point is 50 cm to the right of q_1.

10. The individual magnitudes $\left|\vec{E}_1\right|$ and $\left|\vec{E}_2\right|$ are figured from Eq. 23-3, where the absolute value signs for q_2 are unnecessary since this charge is positive. Whether we add the magnitudes or subtract them depends on if $\vec{E}_1$ is in the same, or opposite, direction as $\vec{E}_2$. At points left of q_1 (on the $-x$ axis) the fields point in opposite directions, but there is no possibility of cancellation (zero net field) since $\left|\vec{E}_1\right|$ is everywhere bigger than $\left|\vec{E}_2\right|$ in this region. In the region between the charges ($0 < x < d$) both fields point leftward and there is no possibility of cancellation. At points to the right of q_2 (where $x > d$), $\vec{E}_1$ points leftward and $\vec{E}_2$ points rightward so the net field in this range is

$$\vec{E}_{\text{net}} = \left|\vec{E}_2\right| - \left|\vec{E}_1\right| \quad \text{in the } \hat{\imath} \text{ direction.}$$

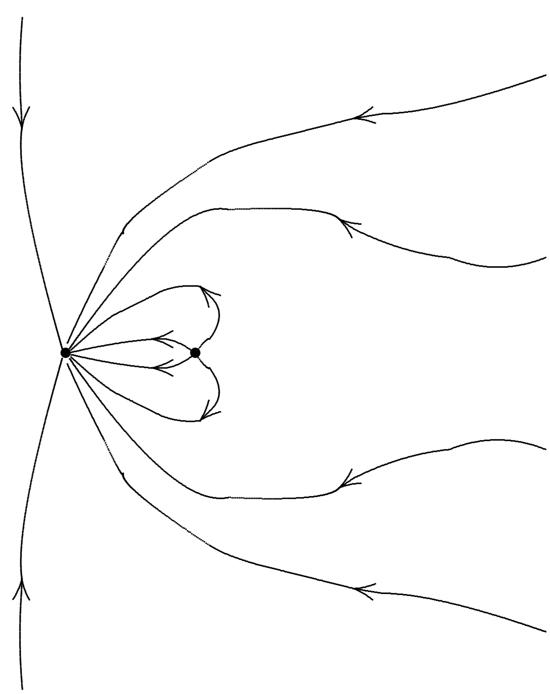
Although $|q_1| > q_2$ there is the possibility of $\vec{E}_{\text{net}} = 0$ since these points are closer to q_2 than to q_1. Thus, we look for the zero net field point in the $x > d$ region:

$$\begin{aligned}
\left|\vec{E}_1\right| &= \left|\vec{E}_2\right| \\
\frac{1}{4\pi\varepsilon_0}\frac{|q_1|}{x^2} &= \frac{1}{4\pi\varepsilon_0}\frac{q_2}{(x-d)^2}
\end{aligned}$$

which leads to

$$\frac{x-d}{x} = \sqrt{\frac{q_2}{|q_1|}} = \sqrt{\frac{2}{5}} \ .$$

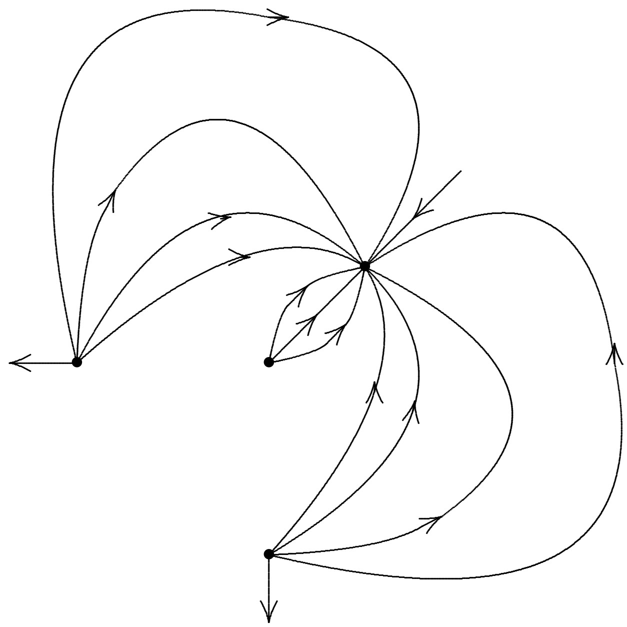
Thus, we obtain $x = \frac{d}{1-\sqrt{2/5}} \approx 2.7d$. A sketch of the field lines is shown below.



11. We place the origin of our coordinate system at point P and orient our y axis in the direction of the $q_4 = -12q$ charge (passing through the $q_3 = +3q$ charge). The x axis is perpendicular to the y axis, and thus passes through the identical $q_1 = q_2 = +5q$ charges. The individual magnitudes $\left|\vec{E}_1\right|$, $\left|\vec{E}_2\right|$, $\left|\vec{E}_3\right|$, and $\left|\vec{E}_4\right|$ are figured from Eq. 23-3, where the absolute value signs for q_1, q_2, and q_3 are unnecessary since those charges are positive (assuming $q > 0$). We note that the contribution from q_1 cancels that of q_2 (that is, $\left|\vec{E}_1\right| = \left|\vec{E}_2\right|$), and the net field (if there is any) should be along the y axis, with magnitude equal to

$$\vec{E}_{\text{net}} = \frac{1}{4\pi\varepsilon_0}\left(\frac{|q_4|}{(2d)^2} - \frac{q_3}{d^2}\right)\hat{\jmath} \quad = \quad \frac{1}{4\pi\varepsilon_0}\left(\frac{12q}{4\,d^2} - \frac{3q}{d^2}\right)\hat{\jmath}$$

which is seen to be zero. A rough sketch of the field lines is shown below:



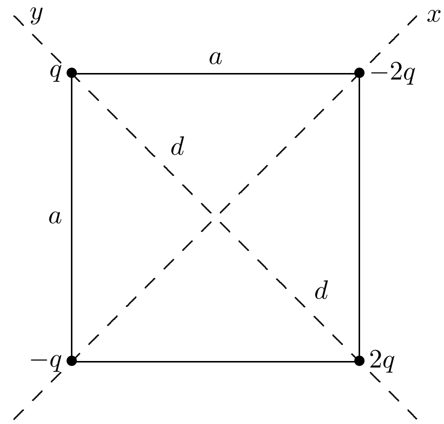
12. By symmetry we see the contributions from the $+q$ charges cancel each other, and we simply use Eq. 23-3 to compute magnitude of the field due to the $+2q$ charge (this field points at 45°, which is clear from the figure in the textbook).

$$\left|\vec{E}_{\text{net}}\right| = \frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2}$$

where $r = a/\sqrt{2}$. Thus, we obtain $\left|\vec{E}_{\text{net}}\right| = q/\pi\varepsilon_0 a^2$.

13. We choose the coordinate axes as shown on the diagram below. At the center of the square, the electric fields produced by the charges at the lower left and upper right corners are both along the x axis and each points away from the center and toward the charge that produces it. Since each charge is a distance $d = \sqrt{2}a/2 = a/\sqrt{2}$ away from the center, the net field due to these two charges is

$$\begin{aligned} E_x &= \frac{1}{4\pi\varepsilon_0}\left(\frac{2q}{a^2/2} - \frac{q}{a^2/2}\right) = \frac{1}{4\pi\varepsilon_0}\frac{q}{a^2/2} \\ &= \frac{\left(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.0\times10^{-8}\,\text{C}\right)}{(0.050\,\text{m})^2/2} = 7.19\times10^4\ \text{N/C}\ . \end{aligned}$$



At the center of the square, the field produced by the charges at the upper left and lower right corners are both along the y axis and each points away from the charge that produces it. The net field produced at the center by these charges is

$$E_y = \frac{1}{4\pi\varepsilon_0}\left[\frac{2q}{a^2/2} - \frac{q}{a^2/2}\right] = \frac{1}{4\pi\varepsilon_0}\,\frac{q}{a^2/2} = 7.19 \times 10^4 \text{ N/C} .$$

The magnitude of the field is

$$E = \sqrt{E_x^2 + E_y^2} = \sqrt{2(7.19 \times 10^4\,\text{N/C})^2} = 1.02 \times 10^5\,\text{N/C}$$

and the angle it makes with the x axis is

$$\theta = \tan^{-1}\frac{E_y}{E_x} = \tan^{-1}(1) = 45^\circ .$$

It is upward in the diagram, from the center of the square toward the center of the upper side.

14. Since both charges are positive (and aligned along the z axis) we have

$$\left|\vec{E}_{\text{net}}\right| = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{(z-d/2)^2} + \frac{q}{(z+d/2)^2}\right] .$$

For $z \gg d$ we have $(z \pm d/2)^{-2} \approx z^{-2}$, so

$$\left|\vec{E}_{\text{net}}\right| \approx \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{z^2} + \frac{q}{z^2}\right) = \frac{2q}{4\pi\varepsilon_0 z^2} .$$

15. The magnitude of the dipole moment is given by $p = qd$, where q is the positive charge in the dipole and d is the separation of the charges. For the dipole described in the problem, $p = (1.60 \times 10^{-19}\,\text{C})(4.30 \times 10^{-9}\,\text{m}) = 6.88 \times 10^{-28}\,\text{C}\cdot\text{m}$. The dipole moment is a vector that points from the negative toward the positive charge.

16. From the figure below it is clear that the net electric field at point P points in the $-\hat{\jmath}$ direction. Its magnitude is

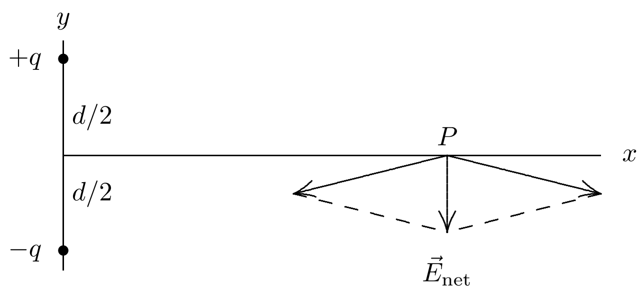
$$\begin{aligned}\left|\vec{E}_{\text{net}}\right| &= 2E_1\sin\theta = 2\left[k\frac{q}{(d/2)^2+r^2}\right]\frac{d/2}{\sqrt{(d/2)^2+r^2}} \\ &= k\frac{qd}{[(d/2)^2+r^2]^{3/2}}\end{aligned}$$

where we use k for $1/4\pi\varepsilon_0$ for brevity. For $r \gg d$, we write $[(d/2)^2 + r^2]^{3/2} \approx r^3$ so the expression above reduces to

$$\left|\vec{E}_{\text{net}}\right| \approx k\frac{qd}{r^3} \ .$$

Since $\vec{p} = (qd)\hat{\text{j}}$,

$$\vec{E}_{\text{net}} \approx -k\frac{\vec{p}}{r^3} \ .$$



17. Think of the quadrupole as composed of two dipoles, each with dipole moment of magnitude $p = qd$. The moments point in opposite directions and produce fields in opposite directions at points on the quadrupole axis. Consider the point P on the axis, a distance z to the right of the quadrupole center and take a rightward pointing field to be positive. Then, the field produced by the right dipole of the pair is $qd/2\pi\varepsilon_0(z - d/2)^3$ and the field produced by the left dipole is $-qd/2\pi\varepsilon_0(z + d/2)^3$. Use the binomial expansions $(z - d/2)^{-3} \approx z^{-3} - 3z^{-4}(-d/2)$ and $(z + d/2)^{-3} \approx z^{-3} - 3z^{-4}(d/2)$ to obtain

$$E = \frac{qd}{2\pi\varepsilon_0}\left[\frac{1}{z^3} + \frac{3d}{2z^4} - \frac{1}{z^3} + \frac{3d}{2z^4}\right] = \frac{6qd^2}{4\pi\varepsilon_0 z^4} \ .$$

Let $Q = 2qd^2$. Then,

$$E = \frac{3Q}{4\pi\varepsilon_0 z^4} \ .$$

18. We use Eq. 23-3, assuming both charges are positive.

$$\begin{aligned} E_{\text{left ring}} &= E_{\text{right ring}} \qquad \text{evaluated at } P \\ \frac{q_1 R}{4\pi\varepsilon_0\left(R^2 + R^2\right)^{3/2}} &= \frac{q_2(2R)}{4\pi\varepsilon_0\left((2R)^2 + R^2\right)^{3/2}} \end{aligned}$$

Simplifying, we obtain

$$\frac{q_1}{q_2} = 2\left(\frac{2}{5}\right)^{3/2} \approx 0.51 \ .$$

19. The electric field at a point on the axis of a uniformly charged ring, a distance z from the ring center, is given by

$$E = \frac{qz}{4\pi\varepsilon_0(z^2 + R^2)^{3/2}}$$

where q is the charge on the ring and R is the radius of the ring (see Eq. 23–16). For q positive, the field points upward at points above the ring and downward at points below the ring. We take the positive direction to be upward. Then, the force acting on an electron on the axis is

$$F = -\frac{eqz}{4\pi\varepsilon_0(z^2 + R^2)^{3/2}} \ .$$

For small amplitude oscillations $z \ll R$ and z can be neglected in the denominator. Thus,

$$F = -\frac{eqz}{4\pi\varepsilon_0 R^3} \ .$$

The force is a restoring force: it pulls the electron toward the equilibrium point $z = 0$. Furthermore, the magnitude of the force is proportional to z, just as if the electron were attached to a spring with spring constant $k = eq/4\pi\varepsilon_0 R^3$. The electron moves in simple harmonic motion with an angular frequency given by

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{eq}{4\pi\varepsilon_0 m R^3}}$$

where m is the mass of the electron.

20. From symmetry, we see that the net field at P is twice the field caused by the upper semicircular charge $+q = \lambda \cdot \pi R$ (and that it points downward). Adapting the steps leading to Eq. 23-21, we find

$$\vec{E}_{\text{net}} = 2\left(-\hat{\jmath}\right)\frac{\lambda}{4\pi\varepsilon_0 R}\Big[\sin\theta\Big]_{-90^\circ}^{90^\circ} = -\frac{q}{\varepsilon_0 \pi^2 R^2}\,\hat{\jmath}\ .$$

21. Studying Sample Problem 23-3, we see that the field evaluated at the center of curvature due to a charged distribution on a circular arc is given by

$$\vec{E} = \frac{\lambda}{4\pi\varepsilon_0 r}\Big[\sin\theta\Big]_{-\theta/2}^{\theta/2} \qquad \text{along the symmetry axis}$$

where $\lambda = q/r\theta$ with θ in radians. In this problem, each charged quarter-circle produces a field of magnitude

$$\left|\vec{E}\right| = \frac{|q|}{r\pi/2}\frac{1}{4\pi\varepsilon_0 r}\Big[\sin\theta\Big]_{-\pi/4}^{\pi/4} = \frac{|q|}{\varepsilon_0 \pi^2 r^2\sqrt{2}}\ .$$

That produced by the positive quarter-circle points at $-45°$, and that of the negative quarter-circle points at $+45°$. By symmetry, we conclude that their net field is horizontal (and rightward in the textbook figure) with magnitude

$$E_x = 2\left(\frac{|q|}{\varepsilon_0\pi^2 r^2\sqrt{2}}\right)\cos 45° = \frac{|q|}{\varepsilon_0\pi^2 r^2}\ .$$

22. We find the maximum by differentiating Eq. 23-16 and setting the result equal to zero.

$$\frac{d}{dz}\left(\frac{qz}{4\pi\varepsilon_0\left(z^2+R^2\right)^{3/2}}\right) = \frac{q}{4\pi\varepsilon_0}\frac{R^2-2z^2}{\left(z^2+R^2\right)^{5/2}} = 0$$

which leads to $z = R/\sqrt{2}$.

23. (a) The linear charge density is the charge per unit length of rod. Since the charge is uniformly distributed on the rod, $\lambda = -q/L$.

(b) We position the x axis along the rod with the origin at the left end of the rod, as shown in the diagram. Let dx be an infinitesimal length of rod at x. The charge in this segment is $dq = \lambda\, dx$. The charge dq may be considered to be a point charge. The electric field it produces at point P has only an x component and this component is given by

$$dE_x = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dx}{(L+a-x)^2}\ .$$

The total electric field produced at P by the whole rod is the integral

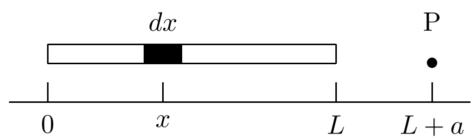
$$E_x = \frac{\lambda}{4\pi\varepsilon_0}\int_0^L \frac{dx}{(L+a-x)^2}$$

$$
\begin{aligned}
&= \frac{\lambda}{4\pi\varepsilon_0} \frac{1}{L+a-x}\Bigg|_0^L \\
&= \frac{\lambda}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{L+a}\right) \\
&= \frac{\lambda}{4\pi\varepsilon_0} \frac{L}{a(L+a)} \; .
\end{aligned}
$$

When $-q/L$ is substituted for λ the result is

$$E_x = -\frac{1}{4\pi\varepsilon_0} \frac{q}{a(L+a)} \; .$$

The negative sign indicates that the field is toward the rod.



(c) If a is much larger than L, the quantity $L+a$ in the denominator can be approximated by a and the expression for the electric field becomes

$$E_x = -\frac{q}{4\pi\varepsilon_0 a^2} \; .$$

This is the expression for the electric field of a point charge at the origin.

24. We assume $q > 0$. Using the notation $\lambda = q/L$ we note that the (infinitesimal) charge on an element dx of the rod contains charge $dq = \lambda\, dx$. By symmetry, we conclude that all horizontal field components (due to the dq's) cancel and we need only "sum" (integrate) the vertical components. Symmetry also allows us to integrate these contributions over only half the rod ($0 \le x \le L/2$) and then simply double the result. In that regard we note that $\sin\theta = y/r$ where $r = \sqrt{x^2+y^2}$. Using Eq. 23-3 (with the 2 and $\sin\theta$ factors just discussed) we obtain

$$
\begin{aligned}
\left|\vec{E}\right| &= 2\int_0^{L/2} \left(\frac{dq}{4\pi\varepsilon_0 r^2}\right)\sin\theta \\
&= \frac{2}{4\pi\varepsilon_0}\int_0^{L/2}\left(\frac{\lambda dx}{x^2+y^2}\right)\left(\frac{y}{\sqrt{x^2+y^2}}\right) \\
&= \frac{\lambda\, y}{2\pi\varepsilon_0}\int_0^{L/2}\frac{dx}{(x^2+y^2)^{3/2}} \\
&= \frac{(q/L)y}{2\pi\varepsilon_0}\left[\frac{x}{y^2\sqrt{x^2+y^2}}\right]_0^{L/2} \\
&= \frac{q}{2\pi\varepsilon_0 L y}\frac{L/2}{\sqrt{(L/2)^2+y^2}} \\
&= \frac{q}{2\pi\varepsilon_0\, y}\frac{1}{\sqrt{L^2+4y^2}}
\end{aligned}
$$

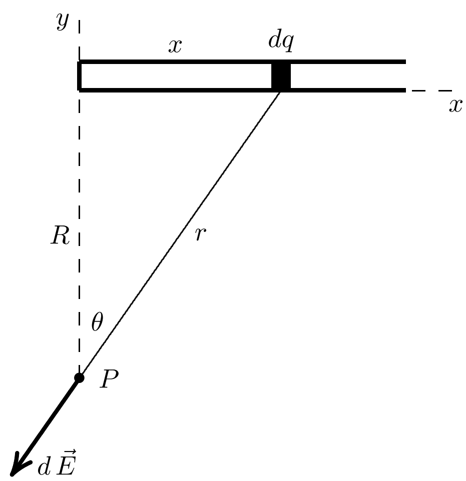
where the integral may be evaluated by elementary means or looked up in Appendix E (item #19 in the list of integrals).

25. Consider an infinitesimal section of the rod of length dx, a distance x from the left end, as shown in the diagram below. It contains charge $dq = \lambda\, dx$ and is a distance r from P. The magnitude of the field it produces at P is given by

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dx}{r^2} .$$

The x component is $\quad dE_x = -\dfrac{1}{4\pi\varepsilon_0}\dfrac{\lambda\, dx}{r^2}\sin\theta$

and the y component is $\quad dE_y = -\dfrac{1}{4\pi\varepsilon_0}\dfrac{\lambda\, dx}{r^2}\cos\theta$.



We use θ as the variable of integration and substitute $r = R/\cos\theta$, $x = R\tan\theta$ and $dx = (R/\cos^2\theta)\, d\theta$. The limits of integration are 0 and $\pi/2$ rad. Thus,

$$E_x = -\frac{\lambda}{4\pi\varepsilon_0 R}\int_0^{\pi/2}\sin\theta\, d\theta = \frac{\lambda}{4\pi\varepsilon_0 R}\cos\theta\Big|_0^{\pi/2} = -\frac{\lambda}{4\pi\varepsilon_0 R}$$

and

$$E_y = -\frac{\lambda}{4\pi\varepsilon_0 R}\int_0^{\pi/2}\cos\theta\, d\theta = -\frac{\lambda}{4\pi\varepsilon_0 R}\sin\theta\Big|_0^{\pi/2} = -\frac{\lambda}{4\pi\varepsilon_0 R} .$$

We notice that $E_x = E_y$ no matter what the value of R. Thus, $\vec{E}$ makes an angle of 45° with the rod for all values of R.

26. From Eq. 23-26

$$\begin{aligned} E &= \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right) \\ &= \frac{5.3\times10^{-6}\,\mathrm{C/m^2}}{2\left(8.85\times10^{-12}\,\frac{\mathrm{C^2}}{\mathrm{N\cdot m^2}}\right)}\left[1 - \frac{12\,\mathrm{cm}}{\sqrt{(12\,\mathrm{cm})^2+(2.5\,\mathrm{cm})^2}}\right] = 6.3\times10^3\ \mathrm{N/C} . \end{aligned}$$

27. At a point on the axis of a uniformly charged disk a distance z above the center of the disk, the magnitude of the electric field is

$$E = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{z}{\sqrt{z^2+R^2}}\right]$$

where R is the radius of the disk and σ is the surface charge density on the disk. See Eq. 23-26. The magnitude of the field at the center of the disk ($z = 0$) is $E_c = \sigma/2\varepsilon_0$. We want to solve for the value of

z such that $E/E_c = 1/2$. This means

$$1 - \frac{z}{\sqrt{z^2 + R^2}} = \frac{1}{2} \implies \frac{z}{\sqrt{z^2 + R^2}} = \frac{1}{2} .$$

Squaring both sides, then multiplying them by z^2+R^2, we obtain $z^2 = (z^2/4)+(R^2/4)$. Thus, $z^2 = R^2/3$ and $z = R/\sqrt{3}$.

28. Eq. 23-28 gives

$$\vec{E} = \frac{\vec{F}}{q} = \frac{m\vec{a}}{(-e)} = -\left(\frac{m}{e}\right)\vec{a}$$

using Newton's second law. Therefore, with *east* being the î direction,

$$\vec{E} = -\left(\frac{9.11 \times 10^{-31}\,\text{kg}}{1.60 \times 10^{-19}\,\text{C}}\right)\left(1.80 \times 10^{9}\,\text{m/s}^2\,\hat{\imath}\right) = -0.0102\ \text{N/C}\ \hat{\imath}$$

which means the field has a magnitude of 0.0102 N/C and is directed westward.

29. The magnitude of the force acting on the electron is $F = eE$, where E is the magnitude of the electric field at its location. The acceleration of the electron is given by Newton's second law:

$$a = \frac{F}{m} = \frac{eE}{m} = \frac{(1.60 \times 10^{-19}\,\text{C})(2.00 \times 10^{4}\,\text{N/C})}{9.11 \times 10^{-31}\,\text{kg}} = 3.51 \times 10^{15}\ \text{m/s}^2 .$$

30. Vertical equilibrium of forces leads to the equality

$$q\left|\vec{E}\right| = mg \implies \left|\vec{E}\right| = \frac{mg}{2e} .$$

Using the mass given in the problem, we obtain $\left|\vec{E}\right| = 2.03 \times 10^{-7}$ N/C. Since the force of gravity is downward, then $q\vec{E}$ must point upward. Since $q > 0$ in this situation, this implies $\vec{E}$ must itself point upward.

31. We combine Eq. 23-9 and Eq. 23-28 (in absolute values).

$$F = |q|E = |q|\left(\frac{p}{2\pi\varepsilon_0 z^3}\right) = \frac{2ke\,p}{z^3}$$

where we use Eq. 22-5 in the last step. Thus, we obtain

$$F = \frac{2\left(8.99 \times 10^{9}\,\text{N·m}^2/\text{C}^2\right)\left(1.60 \times 10^{-19}\,\text{C}\right)\left(3.6 \times 10^{-29}\,\text{C·m}\right)}{\left(25 \times 10^{-9}\,\text{m}\right)^3}$$

which yields a force of magnitude 6.6×10^{-15} N. If the dipole is oriented such that $\vec{p}$ is in the $+z$ direction, then $\vec{F}$ points in the $-z$ direction.

32. (a) $F_e = Ee = (3.0 \times 10^{6}\,\text{N/C})(1.6 \times 10^{-19}\,\text{C}) = 4.8 \times 10^{-13}\,\text{N}$.

    (b) $F_i = Eq_{\text{ion}} = Ee = 4.8 \times 10^{-13}\,\text{N}$.

33. (a) The magnitude of the force on the particle is given by $F = qE$, where q is the magnitude of the charge carried by the particle and E is the magnitude of the electric field at the location of the particle. Thus,

$$E = \frac{F}{q} = \frac{3.0 \times 10^{-6}\,\text{N}}{2.0 \times 10^{-9}\,\text{C}} = 1.5 \times 10^{3}\ \text{N/C} .$$

The force points downward and the charge is negative, so the field points upward.

(b) The magnitude of the electrostatic force on a proton is

$$F_e = eE = (1.60 \times 10^{-19}\,\text{C})(1.5 \times 10^3\,\text{N/C}) = 2.4 \times 10^{-16}\ \text{N}\ .$$

A proton is positively charged, so the force is in the same direction as the field, upward.

(c) The magnitude of the gravitational force on the proton is

$$F_g = mg = (1.67 \times 10^{-27}\,\text{kg})(9.8\,\text{m/s}^2) = 1.64 \times 10^{-26}\,\text{N}\ .$$

The force is downward.

(d) The ratio of the forces is

$$\frac{F_e}{F_g} = \frac{2.4 \times 10^{-16}\,\text{N}}{1.64 \times 10^{-26}\,\text{N}} = 1.5 \times 10^{10}\ .$$

34. (a) Since $\vec{E}$ points down and we need an upward electric force (to cancel the downward pull of gravity), then we require the charge of the sphere to be negative. The magnitude of the charge is found by working with the absolute value of Eq. 23-28:

$$|q| = \frac{F}{E} = \frac{mg}{E} = \frac{4.4\,\text{N}}{150\,\text{N/C}} = 0.029\ \text{C}\ .$$

(b) The feasibility of this experiment may be studied by using Eq. 23-3 (using k for $1/4\pi\varepsilon_0$).

$$E = k\frac{|q|}{r^2} \qquad\qquad \text{where} \quad \rho_{\text{sulfur}}\left(\frac{4}{3}\pi r^3\right) = m_{\text{sphere}}$$

Since the mass of the sphere is $4.4/9.8 \approx 0.45$ kg and the density of sulfur is about 2.1×10^3 kg/m^3 (see Appendix F), then we obtain

$$r = \left(\frac{3m_{\text{sphere}}}{4\pi\rho_{\text{sulfur}}}\right)^{1/3} = 0.037\,\text{m} \quad\Longrightarrow\quad E = k\frac{|q|}{r^2} \approx 2 \times 10^{11}\ \text{N/C}$$

which is much too large a field to maintain in air (see problem #32).

35. (a) The magnitude of the force acting on the proton is $F = eE$, where E is the magnitude of the electric field. According to Newton's second law, the acceleration of the proton is $a = F/m = eE/m$, where m is the mass of the proton. Thus,

$$a = \frac{(1.60 \times 10^{-19}\,\text{C})(2.00 \times 10^4\,\text{N/C})}{1.67 \times 10^{-27}\,\text{kg}} = 1.92 \times 10^{12}\ \text{m/s}^2\ .$$

(b) We assume the proton starts from rest and use the kinematic equation $v^2 = v_0^2 + 2ax$ (or else $x = \frac{1}{2}at^2$ and $v = at$) to show that

$$v = \sqrt{2ax} = \sqrt{2(1.92 \times 10^{12}\,\text{m/s}^2)(0.0100\,\text{m})} = 1.96 \times 10^5\ \text{m/s}\ .$$

36. (a) The initial direction of motion is taken to be the $+x$ direction (this is also the direction of $\vec{E}$). We use $v_f^2 - v_i^2 = 2a\Delta x$ with $v_f = 0$ and $\vec{a} = \vec{F}/m = -e\vec{E}/m_e$ to solve for distance Δx:

$$\Delta x = \frac{-v_i^2}{2a} = \frac{-m_e v_i^2}{-2eE} = \frac{-(9.11 \times 10^{-31}\,\text{kg})(5.00 \times 10^6\,\text{m/s})^2}{-2(1.60 \times 10^{-19}\,\text{C})(1.00 \times 10^3\,\text{N/C})} = 7.12 \times 10^{-2}\ \text{m}\ .$$

(b) Eq. 2-17 leads to

$$t = \frac{\Delta x}{v_{\text{avg}}} = \frac{2\Delta x}{v_i} = \frac{2(7.12 \times 10^{-2}\,\text{m})}{5.00 \times 10^6\,\text{m/s}} = 2.85 \times 10^{-8}\ \text{s}\ .$$

(c) Using $\Delta v^2 = 2a\Delta x$ with the new value of Δx, we find

$$\begin{aligned}\frac{\Delta K}{K_i} &= \frac{\Delta(\frac{1}{2}m_e v^2)}{\frac{1}{2}m_e v_i^2} = \frac{\Delta v^2}{v_i^2} = \frac{2a\Delta x}{v_i^2} = \frac{-2eE\Delta x}{m_e v_i^2} \\ &= \frac{-2(1.60\times10^{-19}\,\mathrm{C})(1.00\times10^{3}\,\mathrm{N/C})(8.00\times10^{-3}\,\mathrm{m})}{(9.11\times10^{-31}\,\mathrm{kg})(5.00\times10^{6}\,\mathrm{m/s})^2} = -11.2\%\ .\end{aligned}$$

37. When the drop is in equilibrium, the force of gravity is balanced by the force of the electric field: $mg = qE$, where m is the mass of the drop, q is the charge on the drop, and E is the magnitude of the electric field. The mass of the drop is given by $m = (4\pi/3)r^3\rho$, where r is its radius and ρ is its mass density. Thus,

$$\begin{aligned}q &= \frac{mg}{E} = \frac{4\pi r^3 \rho g}{3E} \\ &= \frac{4\pi(1.64\times10^{-6}\,\mathrm{m})^3(851\,\mathrm{kg/m^3})(9.8\,\mathrm{m/s^2})}{3(1.92\times10^{5}\,\mathrm{N/C})} = 8.0\times10^{-19}\ \mathrm{C}\end{aligned}$$

and $q/e = (8.0\times10^{-19}\,\mathrm{C})/(1.60\times10^{-19}\,\mathrm{C}) = 5$.

38. Our approach (based on Eq. 23-29) consists of several steps. The first is to find an *approximate* value of e by taking differences between all the given data. The smallest difference is between the fifth and sixth values: $18.08\times10^{-19}\,\mathrm{C} - 16.48\times10^{-19}\,\mathrm{C} = 1.60\times10^{-19}\,\mathrm{C}$ which we denote e_{approx}. The goal at this point is to assign integers n using this approximate value of e:

$$\begin{array}{lcl}
\text{datum 1} & \dfrac{6.563\times10^{-19}\,\mathrm{C}}{e_{\mathrm{approx}}} = 4.10 & \Longrightarrow\quad n_1 = 4 \\
\text{datum 2} & \dfrac{8.204\times10^{-19}\,\mathrm{C}}{e_{\mathrm{approx}}} = 5.13 & \Longrightarrow\quad n_2 = 5 \\
\text{datum 3} & \dfrac{11.50\times10^{-19}\,\mathrm{C}}{e_{\mathrm{approx}}} = 7.19 & \Longrightarrow\quad n_3 = 7 \\
\text{datum 4} & \dfrac{13.13\times10^{-19}\,\mathrm{C}}{e_{\mathrm{approx}}} = 8.21 & \Longrightarrow\quad n_4 = 8 \\
\text{datum 5} & \dfrac{16.48\times10^{-19}\,\mathrm{C}}{e_{\mathrm{approx}}} = 10.30 & \Longrightarrow\quad n_5 = 10 \\
\text{datum 6} & \dfrac{18.08\times10^{-19}\,\mathrm{C}}{e_{\mathrm{approx}}} = 11.30 & \Longrightarrow\quad n_6 = 11 \\
\text{datum 7} & \dfrac{19.71\times10^{-19}\,\mathrm{C}}{e_{\mathrm{approx}}} = 12.32 & \Longrightarrow\quad n_7 = 12 \\
\text{datum 8} & \dfrac{22.89\times10^{-19}\,\mathrm{C}}{e_{\mathrm{approx}}} = 14.31 & \Longrightarrow\quad n_8 = 14 \\
\text{datum 9} & \dfrac{26.13\times10^{-19}\,\mathrm{C}}{e_{\mathrm{approx}}} = 16.33 & \Longrightarrow\quad n_9 = 16
\end{array}$$

Next, we construct a new data set $(e_1, e_2, e_3 \ldots)$ by dividing the given data by the respective exact integers n_i (for $i = 1, 2, 3 \ldots$):

$$(e_1, e_2, e_3 \ldots) = \left(\frac{6.563\times10^{-19}\,\mathrm{C}}{n_1}, \frac{8.204\times10^{-19}\,\mathrm{C}}{n_2}, \frac{11.50\times10^{-19}\,\mathrm{C}}{n_3} \ldots\right)$$

which gives (carrying a few more figures than are significant)

$$\left(1.64075\times10^{-19}\,\mathrm{C},\ 1.6408\times10^{-19}\,\mathrm{C},\ 1.64286\times10^{-19}\,\mathrm{C} \ldots\right)$$

as the new data set (our experimental values for e). We compute the average and standard deviation of this set, obtaining

$$e_{\text{exptal}} = e_{\text{avg}} \pm \Delta e = (1.641 \pm 0.004) \times 10^{-19} \text{ C}$$

which does not agree (to within one standard deviation) with the modern accepted value for e. The lower bound on this spread is $e_{\text{avg}} - \Delta e = 1.637 \times 10^{-19}$ C which is still about 2% too high.

39. (a) We use $\Delta x = v_{\text{avg}} t = vt/2$:

$$v = \frac{2\Delta x}{t} = \frac{2(2.0 \times 10^{-2}\,\text{m})}{1.5 \times 10^{-8}\,\text{s}} = 2.7 \times 10^{6} \text{ m/s} .$$

(b) We use $\Delta x = \frac{1}{2}at^2$ and $E = F/e = ma/e$:

$$E = \frac{ma}{e} = \frac{2\Delta x m}{et^2} = \frac{2(2.0 \times 10^{-2}\,\text{m})(9.11 \times 10^{-31}\,\text{kg})}{(1.60 \times 10^{-19}\,\text{C})(1.5 \times 10^{-8}\,\text{s})^2} = 1.0 \times 10^{3} \text{ N/C} .$$

40. We assume there are no forces or force-components along the x direction. We combine Eq. 23-28 with Newton's second law, then use Eq. 4-21 to determine time t followed by Eq. 4-23 to determine the final velocity (with $-g$ replaced by the a_y of this problem); for these purposes, the velocity components *given* in the problem statement are re-labeled as v_{0x} and v_{0y} respectively.

(a) We have $\vec{a} = \frac{q\vec{E}}{m} = -\left(\frac{e}{m}\right)\vec{E}$ which leads to

$$\vec{a} = -\left(\frac{1.60 \times 10^{-19}\,\text{C}}{9.11 \times 10^{-31}\,\text{kg}}\right)\left(120\ \frac{\text{N}}{\text{C}}\right)\hat{\jmath} = -2.1 \times 10^{13}\,\text{m/s}^2\ \hat{\jmath} .$$

(b) Since $v_x = v_{0x}$ in this problem (that is, $a_x = 0$), we obtain

$$\begin{aligned} t &= \frac{\Delta x}{v_{0x}} = \frac{0.020\,\text{m}}{1.5 \times 10^{5}\,\text{m/s}} = 1.3 \times 10^{-7} \text{ s} \\ v_y &= v_{0y} + a_y t = 3.0 \times 10^{3}\,\text{m/s} + \left(-2.1 \times 10^{13}\,\text{m/s}^2\right)\left(1.3 \times 10^{-7}\,\text{s}\right) \end{aligned}$$

which leads to $v_y = -2.8 \times 10^{6}$ m/s. Therefore, in unit vector notation (with SI units understood) the final velocity is

$$\vec{v} = 1.5 \times 10^{5}\,\hat{\imath} - 2.8 \times 10^{6}\,\hat{\jmath} .$$

41. We take the positive direction to be to the right in the figure. The acceleration of the proton is $a_p = eE/m_p$ and the acceleration of the electron is $a_e = -eE/m_e$, where E is the magnitude of the electric field, m_p is the mass of the proton, and m_e is the mass of the electron. We take the origin to be at the initial position of the proton. Then, the coordinate of the proton at time t is $x = \frac{1}{2}a_p t^2$ and the coordinate of the electron is $x = L + \frac{1}{2}a_e t^2$. They pass each other when their coordinates are the same, or $\frac{1}{2}a_p t^2 = L + \frac{1}{2}a_e t^2$. This means $t^2 = 2L/(a_p - a_e)$ and

$$\begin{aligned} x &= \frac{a_p}{a_p - a_e}L = \frac{eE/m_p}{(eE/m_p) + (eE/m_e)}L = \frac{m_e}{m_e + m_p}L \\ &= \frac{9.11 \times 10^{-31}\,\text{kg}}{9.11 \times 10^{-31}\,\text{kg} + 1.67 \times 10^{-27}\,\text{kg}}(0.050\,\text{m}) \\ &= 2.7 \times 10^{-5} \text{ m} . \end{aligned}$$

42. (a) Using Eq. 23-28, we find

$$\begin{aligned} \vec{F} &= (8.00 \times 10^{-5}\,\text{C})(3.00 \times 10^{3}\,\text{N/C})\hat{\imath} + (8.00 \times 10^{-5}\,\text{C})(-600\,\text{N/C})\hat{\jmath} \\ &= (0.240\,\text{N})\hat{\imath} - (0.0480\,\text{N})\hat{\jmath} . \end{aligned}$$

Therefore, the force has magnitude equal to

$$F = \sqrt{(0.240\,\mathrm{N})^2 + (0.0480\,\mathrm{N})^2} = 0.245 \text{ N} ,$$

and makes an angle θ (which, if negative, means clockwise) measured from the $+x$ axis, where

$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{-0.0480\,\mathrm{N}}{0.240\,\mathrm{N}}\right) = -11.3^\circ .$$

(b) With $m = 0.0100$ kg, the coordinates $(x,\, y)$ at $t = 3.00\,$s are found by combining Newton's second law with the kinematics equations of Chapters 2-4:

$$\begin{aligned} x &= \frac{1}{2}a_x t^2 = \frac{F_x t^2}{2m} = \frac{(0.240)(3.00)^2}{2(0.0100)} = 108 \text{ m} , \\ y &= \frac{1}{2}a_y t^2 = \frac{F_y t^2}{2m} = \frac{(-0.0480)(3.00)^2}{2(0.0100)} = -21.6 \text{ m} . \end{aligned}$$

43. (a) The electric field is upward in the diagram and the charge is negative, so the force of the field on it is downward. The magnitude of the acceleration is $a = eE/m$, where E is the magnitude of the field and m is the mass of the electron. Its numerical value is

$$a = \frac{(1.60 \times 10^{-19}\,\mathrm{C})(2.00 \times 10^3\,\mathrm{N/C})}{9.11 \times 10^{-31}\,\mathrm{kg}} = 3.51 \times 10^{14} \text{ m/s}^2 .$$

We put the origin of a coordinate system at the initial position of the electron. We take the x axis to be horizontal and positive to the right; take the y axis to be vertical and positive toward the top of the page. The kinematic equations are

$$x = v_0 t \cos\theta , \quad y = v_0 t \sin\theta - \frac{1}{2}at^2 , \quad \text{and} \;\; v_y = v_0 \sin\theta - at .$$

First, we find the greatest y coordinate attained by the electron. If it is less than d, the electron does not hit the upper plate. If it is greater than d, it will hit the upper plate if the corresponding x coordinate is less than L. The greatest y coordinate occurs when $v_y = 0$. This means $v_0 \sin\theta - at = 0$ or $t = (v_0/a)\sin\theta$ and

$$\begin{aligned} y_{\max} &= \frac{v_0^2 \sin^2\theta}{a} - \frac{1}{2}a\frac{v_0^2\sin^2\theta}{a^2} = \frac{1}{2}\frac{v_0^2\sin^2\theta}{a} \\ &= \frac{(6.00 \times 10^6\,\mathrm{m/s})^2 \sin^2 45^\circ}{2(3.51 \times 10^{14}\,\mathrm{m/s}^2)} = 2.56 \times 10^{-2}\,\mathrm{m} . \end{aligned}$$

Since this is greater than $d = 2.00\,$cm, the electron might hit the upper plate.

(b) Now, we find the x coordinate of the position of the electron when $y = d$. Since

$$v_0 \sin\theta = (6.00 \times 10^6\,\mathrm{m/s})\sin 45^\circ = 4.24 \times 10^6\,\mathrm{m/s}$$

and

$$2ad = 2(3.51 \times 10^{14}\,\mathrm{m/s}^2)(0.0200\,\mathrm{m}) = 1.40 \times 10^{13}\,\mathrm{m}^2/\mathrm{s}^2$$

the solution to $d = v_0 t \sin\theta - \frac{1}{2}at^2$ is

$$\begin{aligned} t &= \frac{v_0 \sin\theta - \sqrt{v_0^2 \sin^2\theta - 2ad}}{a} \\ &= \frac{4.24 \times 10^6\,\mathrm{m/s} - \sqrt{(4.24 \times 10^6\,\mathrm{m/s})^2 - 1.40 \times 10^{13}\,\mathrm{m}^2/\mathrm{s}^2}}{3.51 \times 10^{14}\,\mathrm{m/s}^2} \\ &= 6.43 \times 10^{-9} \text{ s} . \end{aligned}$$

The negative root was used because we want the *earliest* time for which $y = d$. The x coordinate is

$$\begin{aligned} x &= v_0 t \cos\theta \\ &= (6.00 \times 10^6\,\mathrm{m/s})(6.43 \times 10^{-9}\,\mathrm{s}) \cos 45^\circ = 2.72 \times 10^{-2}\ \mathrm{m}\ . \end{aligned}$$

This is less than L so the electron hits the upper plate at $x = 2.72\,\mathrm{cm}$.

44. (a) The magnitude of the dipole moment is

$$p = qd = \left(1.50 \times 10^{-9}\,\mathrm{C}\right)\left(6.20 \times 10^{-6}\,\mathrm{m}\right) = 9.30 \times 10^{-15}\ \mathrm{C{\cdot}m}\ .$$

(b) Following the solution to part (c) of Sample Problem 23-5, we find

$$U(180^\circ) - U(0) = 2pE = 2\left(9.30 \times 10^{-15}\right)(1100) = 2.05 \times 10^{-11}\ \mathrm{J}\ .$$

45. (a) Eq. 23-33 leads to $\tau = pE \sin 0^\circ = 0$.

(b) With $\theta = 90^\circ$, the equation gives

$$\tau = pE = \left(2(1.6 \times 10^{-19}\,\mathrm{C})(0.78 \times 10^{-9}\,\mathrm{m})\right)\left(3.4 \times 10^6\,\mathrm{N/C}\right) = 8.5 \times 10^{-22}\ \mathrm{N{\cdot}m}\ .$$

(c) Now the equation gives $\tau = pE \sin 180^\circ = 0$.

46. Following the solution to part (c) of Sample Problem 23-5, we find

$$W = U(\theta_0 + \pi) - U(\theta_0) = -pE\left(\cos(\theta_0 + \pi) - \cos(\theta_0)\right) = 2pE \cos\theta_0\ .$$

47. Eq. 23-35 ($\tau = -pE\sin\theta$) captures the sense as well as the magnitude of the effect. That is, this is a restoring torque, trying to bring the tilted dipole back to its aligned equilibrium position. If the amplitude of the motion is small, we may replace $\sin\theta$ with θ in radians. Thus, $\tau \approx -pE\theta$. Since this exhibits a simple negative proportionality to the angle of rotation, the dipole oscillates in simple harmonic motion, like a torsional pendulum with torsion constant $\kappa = pE$. The angular frequency ω is given by

$$\omega^2 = \frac{\kappa}{I} = \frac{pE}{I}$$

where I is the rotational inertia of the dipole. The frequency of oscillation is

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{pE}{I}}\ .$$

48. (a) Using $k = 1/4\pi\varepsilon_0$, we estimate the field at $r = 0.02$ m using Eq. 23-3:

$$E = k\frac{q}{r^2} = \left(8.99 \times 10^9\,\frac{\mathrm{N\cdot m^2}}{\mathrm{C^2}}\right)\frac{45 \times 10^{-12}\,\mathrm{C}}{(0.02\,\mathrm{m})^2} \approx 1 \times 10^3\ \mathrm{N/C}\ .$$

(b) The field described by Eq. 23-3 is nonuniform.

(c) As the positively charged bee approaches the grain, a concentration of negative charge is induced on the closest side of the grain, leading to a force of attraction which makes the grain jump to the bee. Although in physical contact, it is not in electrical contact with the bee, or else it would acquire a net positive charge causing it to be repelled from the bee. As the bee (with grain) approaches the stigma, a concentration of negative charge is induced on the closest side of the stigma which is presumably highly nonuniform. In some configurations, the field from the stigma (acting on the positive side of the grain) will overcome the field from the bee acting on the negative side, and the grain will jump to the stigma.

49. We consider pairs of diametrically opposed charges. The net field due to just the charges in the one o'clock ($-q$) and seven o'clock ($-7q$) positions is clearly equivalent to that of a single $-6q$ charge sitting at the seven o'clock position. Similarly, the net field due to just the charges in the six o'clock ($-6q$) and twelve o'clock ($-12q$) positions is the same as that due to a single $-6q$ charge sitting at the twelve o'clock position. Continuing with this line of reasoning, we see that there are six equal-magnitude electric field vectors pointing at the seven o'clock, eight o'clock ... twelve o'clock positions. Thus, the resultant field of all of these points, by symmetry, is directed toward the position midway between seven and twelve o'clock. Therefore, $\vec{E}_{\text{resultant}}$ points towards the nine-thirty position.

50. (a) From Eq. 23-38 (and the facts that $\hat{\imath}\cdot\hat{\imath} = 1$ and $\hat{\jmath}\cdot\hat{\imath} = 0$), the potential energy is

$$\begin{aligned} U &= -\vec{p}\cdot\vec{E} = -[(3.00\hat{\imath} + 4.00\hat{\jmath})(1.24\times 10^{-30}\,\text{C}\cdot\text{m})]\cdot[(4000\,\text{N/C})\hat{\imath}] \\ &= -1.49\times 10^{-26}\ \text{J}\ . \end{aligned}$$

(b) From Eq. 23-34 (and the facts that $\hat{\imath}\times\hat{\imath} = 0$ and $\hat{\jmath}\times\hat{\imath} = -\hat{k}$), the torque is

$$\begin{aligned} \vec{\tau} &= \vec{p}\times\vec{E} = [(3.00\hat{\imath} + 4.00\hat{\jmath})(1.24\times 10^{-30}\,\text{C}\cdot\text{m})]\times[(4000\,\text{N/C})\hat{\imath}] \\ &= (-1.98\times 10^{-26}\,\text{N}\cdot\text{m})\hat{k}\ . \end{aligned}$$

(c) The work done is

$$\begin{aligned} W &= \Delta U = \Delta(-\vec{p}\cdot\vec{E}) = (\vec{p}_i - \vec{p}_f)\cdot\vec{E} \\ &= [(3.00\hat{\imath} + 4.00\hat{\jmath}) - (-4.00\hat{\imath} + 3.00\hat{\jmath})](1.24\times 10^{-30}\,\text{C}\cdot\text{m})]\cdot[(4000\,\text{N/C})\hat{\imath}] \\ &= 3.47\times 10^{-26}\ \text{J}\ . \end{aligned}$$

51. The point at which we are evaluating the net field is denoted by P. The contributions to the net field caused by the two electrons nearest P (the two electrons on the side of the triangle shared by P) are seen to cancel, so that we only need to compute the field (using Eq. 23-3) caused by the electron at the far corner, at a distance $r = 0.17$ m from P. Using $1/4\pi\varepsilon_0 = k$, we obtain

$$\left|\vec{E}_{\text{net}}\right| = k\frac{e}{r^2} = 4.8\times 10^{-8}\ \text{N/C}\ .$$

52. Let q_1 denote the charge at $y = d$ and q_2 denote the charge at $y = -d$. The individual magnitudes $\left|\vec{E}_1\right|$ and $\left|\vec{E}_2\right|$ are figured from Eq. 23-3, where the absolute value signs for q are unnecessary since these charges are both positive. The distance from q_1 to a point on the x axis is the same as the distance from q_2 to a point on the x axis: $r = \sqrt{x^2 + d^2}$. By symmetry, the y component of the net field along the x axis is zero. The x component of the net field, evaluated at points on the positive x axis, is

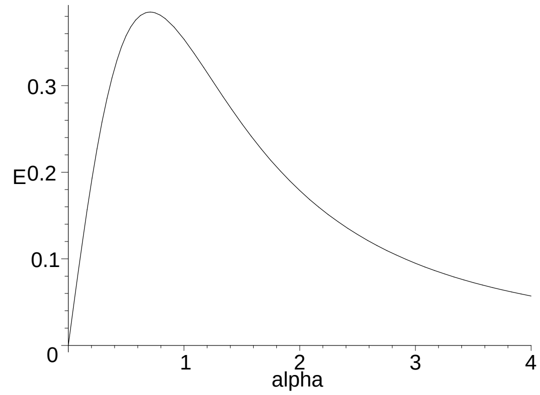
$$E_x = 2\left(\frac{1}{4\pi\varepsilon_0}\right)\left(\frac{q}{x^2 + d^2}\right)\left(\frac{x}{\sqrt{x^2 + d^2}}\right)$$

where the last factor is $\cos\theta = x/r$ with θ being the angle for each individual field as measured from the x axis.

(a) If we simplify the above expression, and plug in $x = \alpha d$, we obtain

$$E_x = \frac{q}{2\pi\varepsilon_0\, d^2}\left(\frac{\alpha}{(\alpha^2 + 1)^{3/2}}\right)\ .$$

(b) The graph of $E = E_x$ versus α is shown below. For the purposes of graphing, we set $d = 1$ m and $q = 5.56\times 10^{-11}$ C.



(c) From the graph, we estimate $E_{\max}$ occurs at about $\alpha = 0.7$. More accurate computation shows that the maximum occurs at $\alpha = 1/\sqrt{2}$.

(d) The graph suggests that "half-height" points occur at $\alpha \approx 0.2$ and $\alpha \approx 1.9$. Further numerical exploration leads to the values: $\alpha = 0.2047$ and $\alpha = 1.9864$.

53. (a) We combine Eq. 23-28 (in absolute value) with Newton's second law:

$$a = \frac{|q|E}{m} = \left(\frac{1.60 \times 10^{-19}\,\mathrm{C}}{9.11 \times 10^{-31}\,\mathrm{kg}}\right)\left(1.40 \times 10^{6}\,\frac{\mathrm{N}}{\mathrm{C}}\right) = 2.46 \times 10^{17}\ \mathrm{m/s^2}\ .$$

(b) With $v = \frac{c}{10} = 3.00 \times 10^7$ m/s, we use Eq. 2-11 to find

$$t = \frac{v - v_o}{a} = \frac{3.00 \times 10^7}{2.46 \times 10^{17}} = 1.22 \times 10^{-10}\ \mathrm{s}\ .$$

(c) Eq. 2-16 gives

$$\Delta x = \frac{v^2 - v_o^2}{2a} = \frac{\left(3.00 \times 10^7\right)^2}{2\left(2.46 \times 10^{17}\right)} = 1.83 \times 10^{-3}\ \mathrm{m}\ .$$

54. Studying Sample Problem 23-3, we see that the field evaluated at the center of curvature due to a charged distribution on a circular arc is given by

$$\vec{E} = \frac{\lambda}{4\pi\varepsilon_0 r}\Big[\sin\theta\Big]_{-\theta/2}^{\theta/2} \qquad \text{along the symmetry axis}$$

where $\lambda = q/\ell = q/r\theta$ with θ in radians. Here ℓ is the length of the arc, given as $\ell = 4.0$ m. Therefore, $\theta = \ell/r = 4.0/2.0 = 2.0$ rad. Thus, with $q = 20 \times 10^{-9}$ C, we obtain

$$\left|\vec{E}\right| = \frac{q}{\ell}\,\frac{1}{4\pi\varepsilon_0 r}\Big[\sin\theta\Big]_{-1.0\,\mathrm{rad}}^{1.0\,\mathrm{rad}} = 38\ \mathrm{N/C}\ .$$

55. A small section of the distribution has charge dq is $\lambda\,dx$, where $\lambda = 9.0 \times 10^{-9}$ C/m. Its contribution to the field at $x_P = 4.0$ m is

$$d\vec{E} = \frac{dq}{4\pi\,\varepsilon_0\,(x - x_P)^2}$$

pointing in the $+x$ direction. Thus, we have

$$\vec{E} = \int_0^{3.0\,\mathrm{m}} \frac{\lambda\,dx}{4\pi\,\varepsilon_0\,(x - x_P)^2}\ \hat{\imath}$$

which becomes, using the substitution $u = x - x_P$,

$$\vec{E} = \frac{\lambda}{4\pi\,\varepsilon_0}\int_{-4.0\,\mathrm{m}}^{-1.0\,\mathrm{m}} \frac{du}{u^2}\;\hat{\imath} \;\; = \frac{\lambda}{4\pi\,\varepsilon_0}\left(\frac{-1}{-1.0\,\mathrm{m}} - \frac{-1}{-4.0\,\mathrm{m}}\right)\hat{\imath}$$

which yields 61 N/C in the $+x$ direction.

56. Let $q_1 = -4Q < 0$ and $q_2 = +2Q > 0$ (where we make the assumption that $Q > 0$). Also, let $d = 2.00$ m, the distance that separates the charges. The individual magnitudes $\left|\vec{E}_1\right|$ and $\left|\vec{E}_2\right|$ are figured from Eq. 23-3, where the absolute value signs for q_2 are unnecessary since this charge is positive. Whether we add the magnitudes or subtract them depends on if $\vec{E}_1$ is in the same, or opposite, direction as $\vec{E}_2$. At points left of q_1 (on the $-x$ axis) the fields point in opposite directions, but there is no possibility of cancellation (zero net field) since $\left|\vec{E}_1\right|$ is everywhere bigger than $\left|\vec{E}_2\right|$ in this region. In the region between the charges ($0 < x < d$) both fields point leftward and there is no possibility of cancellation. At points to the right of q_2 (where $x > d$), $\vec{E}_1$ points leftward and $\vec{E}_2$ points rightward so the net field in this range is

$$\vec{E}_{\mathrm{net}} = \left|\vec{E}_2\right| - \left|\vec{E}_1\right| \quad \text{in the } \hat{\imath} \text{ direction.}$$

Although $|q_1| > q_2$ there is the possibility of $\vec{E}_{\mathrm{net}} = 0$ since these points are closer to q_2 than to q_1. Thus, we look for the zero net field point in the $x > d$ region:

$$\begin{aligned}\left|\vec{E}_1\right| &= \left|\vec{E}_2\right| \\ \frac{1}{4\pi\varepsilon_0}\frac{|q_1|}{x^2} &= \frac{1}{4\pi\varepsilon_0}\frac{q_2}{(x-d)^2}\end{aligned}$$

which leads to

$$\frac{x-d}{x} = \sqrt{\frac{q_2}{|q_1|}} = \sqrt{\frac{1}{2}}\ .$$

Therefore, $x = \frac{d\sqrt{2}}{\sqrt{2}-1} = 6.8$ m specifies the position where $\vec{E}_{\mathrm{net}} = 0$.

57. We note that the contributions to the field from the pair of $-2q$ charges exactly cancel, and we are left with the (opposing) contributions from the $4q$ (at $r = 2d$) and $-q$ (at $r = d$) charges. Therefore, using $k = 1/4\pi\varepsilon_0$

$$|\vec{E}_{\mathrm{net}}| = k\,\frac{4q}{(2d)^2} - k\,\frac{q}{d^2} = 0\ .$$

The net field at P vanishes completely.

58. The field of each charge has magnitude

$$E = k\,\frac{e}{(0.020\,\mathrm{m})^2} = 3.6\times10^{-6}\ \mathrm{N/C}\ .$$

The directions are indicated in standard format below. We use the magnitude-angle notation (convenient if one is using a vector capable calculator in polar mode) and write (starting with the proton on the left and moving around clockwise) the contributions to $\vec{E}_{\mathrm{net}}$ as follows:

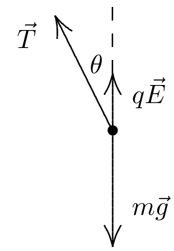
$$(E\,\angle-20^\circ) + (E\,\angle\,130^\circ) + (E\,\angle-100^\circ) + (E\,\angle-150^\circ) + (E\,\angle\,0^\circ)\ .$$

This yields $(3.93\times10^{-6}\,\angle-76.4^\circ)$, with the N/C unit understood.

59. Eq. 23-38 gives $U = -\vec{p}\cdot\vec{E} = -pE\cos\theta$. We note that $\theta_i = 110^\circ$ and $\theta_f = 70^\circ$. Therefore,

$$\Delta U = -pE\,(\cos 70^\circ - \cos 110^\circ) = -3.3\times10^{-21}\ \mathrm{J}\ .$$

60. (a) Suppose the pendulum is at the angle θ with the vertical. The force diagram is shown to the right. $\vec{T}$ is the tension in the thread, mg is the magnitude of the force of gravity, and qE is the magnitude of the electric force. The field points upward and the charge is positive, so the force is upward. Taking the angle shown to be positive, then the torque on the sphere about the point where the thread is attached to the upper plate is $\tau = -(mg - qE)\ell \sin\theta$. If $mg > qE$ then the torque is a restoring torque; it tends to pull the pendulum back to its equilibrium position.



If the amplitude of the oscillation is small, $\sin\theta$ can be replaced by θ in radians and the torque is $\tau = -(mg - qE)\ell\theta$. The torque is proportional to the angular displacement and the pendulum moves in simple harmonic motion. Its angular frequency is $\omega = \sqrt{(mg - qE)\ell/I}$, where I is the rotational inertia of the pendulum. Since $I = m\ell^2$ for a simple pendulum,

$$\omega = \sqrt{\frac{(mg - qE)\ell}{m\ell^2}} = \sqrt{\frac{g - qE/m}{\ell}}$$

and the period is

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{\ell}{g - qE/m}} \ .$$

If $qE > mg$ the torque is not a restoring torque and the pendulum does not oscillate.

(b) The force of the electric field is now downward and the torque on the pendulum is $\tau = -(mg+qE)\ell\theta$ if the angular displacement is small. The period of oscillation is

$$T = 2\pi\sqrt{\frac{\ell}{g + qE/m}} \ .$$

61. (a) Using the density of water ($\rho = 1000\,\text{kg/m}^3$), the weight mg of the spherical drop (of radius $r = 6.0 \times 10^{-7}$ m) is

$$W = \rho V g = \left(1000\,\text{kg/m}^3\right)\left(\frac{4\pi}{3}(6.0 \times 10^{-7}\,\text{m})^3\right)\left(9.8\,\text{m/s}^2\right) = 8.87 \times 10^{-15}\ \text{N} \ .$$

(b) Vertical equilibrium of forces leads to $mg = qE = neE$, which we solve for n, the number of excess electrons:

$$n = \frac{mg}{eE} = \frac{8.87 \times 10^{-15}\,\text{N}}{(1.60 \times 10^{-19}\,\text{C})(462\,\text{N/C})} = 120 \ .$$

62. (a) Let $E = \sigma/2\varepsilon_0 = 3 \times 10^6\,\text{N/C}$. With $\sigma = |q|/A$, this leads to

$$|q| = \pi R^2 \sigma = 2\pi\varepsilon_0 R^2 E = \frac{R^2 E}{2k} = \frac{\left(2.5 \times 10^{-2}\,\text{m}\right)^2 \left(3.0 \times 10^6\,\text{N/C}\right)}{2\left(8.99 \times 10^9\,\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)} = 1.0 \times 10^{-7}\ \text{C} \ .$$

(b) Setting up a simple proportionality (with the areas), the number of atoms is estimated to be

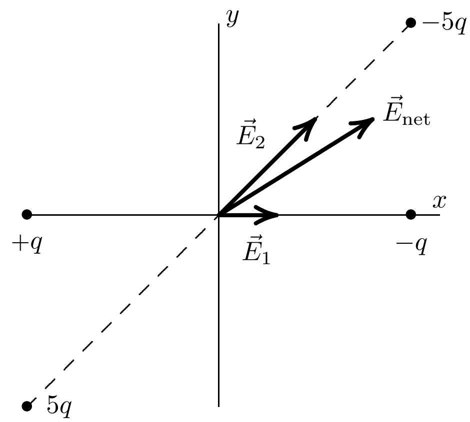
$$N = \frac{\pi(2.5 \times 10^{-2}\,\text{m})^2}{0.015 \times 10^{-18}\,\text{m}^2} = 1.3 \times 10^{17} \ .$$

(c) Therefore, the fraction is

$$\frac{q}{Ne} = \frac{1.0 \times 10^{-7}\,\text{C}}{(1.3 \times 10^{17})(1.6 \times 10^{-19}\,\text{C})} \approx 5 \times 10^{-6} \ .$$

63. On the one hand, the conclusion (that $Q = +1.0\,\mu$C) is clear from symmetry. If a more in-depth justification is desired, one should use Eq. 23-3 for the electric field magnitudes of the three charges (each at the same distance $r = a/\sqrt{3}$ from C) and then find field components along suitably chosen axes, requiring each component-sum to be zero. If the y axis is vertical, then (assuming $Q > 0$) the component-sum along that axis leads to $2kq\sin 30°/r^2 = kQ/r^2$ where q refers to either of the charges at the bottom corners. This yields $Q = 2q\sin 30° = q$ and thus to the conclusion mentioned above.

64. From symmetry, the only two pairs of charges which produce a non-vanishing field $\vec{E}_{\rm net}$ are: pair 1, which is in the middle of the two vertical sides of the square (the $+q$, $-2q$ pair); and pair 2, the $+5q$, $-5q$ pair. We denote the electric fields produced by each pair as $\vec{E}_1$ and $\vec{E}_2$, respectively. We set up a coordinate system as shown to the right, with the origin at the center of the square. Now,



$$E_1 = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{d^2} + \frac{2q}{d^2}\right) = \frac{3q}{4\pi\varepsilon_0 d^2} \qquad \text{and} \qquad E_2 = k\left[\frac{5q}{(\sqrt{2}d)^2} + \frac{5q}{(\sqrt{2}d)^2}\right] = \frac{5q}{4\pi\varepsilon_0 d^2}\ .$$

Therefore, the components of $\vec{E}_{\rm net}$ are given by

$$\begin{aligned} E_x &= E_{1x} + E_{2x} = E_1 + E_2\cos 45° \\ &= \frac{3q}{4\pi\varepsilon_0 d^2} + \left(\frac{5q}{4\pi\varepsilon_0 d^2}\right)\cos 45° = 6.536\left(\frac{q}{4\pi\varepsilon_0 d^2}\right)\ , \end{aligned}$$

and

$$E_y = E_{1y} + E_{2y} = E_2\sin 45° = \left(\frac{5q}{4\pi\varepsilon_0 d^2}\right)\sin 45° = 3.536\left(\frac{q}{4\pi\varepsilon_0 d^2}\right)\ .$$

Thus, the magnitude of $\vec{E}_{\rm net}$ is

$$E = \sqrt{E_x^2 + E_y^2} = \sqrt{(6.536)^2 + (3.536)^2}\left(\frac{q}{4\pi\varepsilon_0 d^2}\right) = \frac{7.43q}{4\pi\varepsilon_0 d^2}\ ,$$

and $\vec{E}_{\rm net}$ makes an angle θ with the positive x axis, where

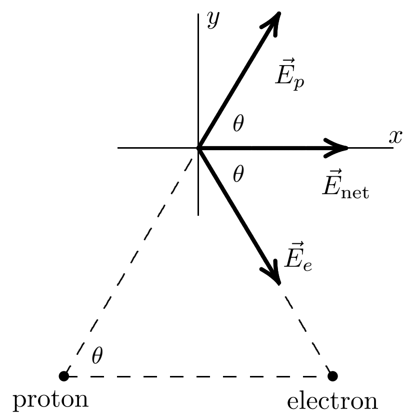
$$\theta = \tan^{-1}\left(\frac{E_y}{E_x}\right) = \tan^{-1}\left(\frac{3.536}{6.536}\right) = 28.4°\ .$$

65. We denote the electron with subscript e

and the proton with p. From the figure to the right we see that

$$\left|\vec{E}_e\right| = \left|\vec{E}_p\right| = \frac{e}{4\pi\varepsilon_0 d^2}$$

where $d = 2.0 \times 10^{-6}$ m. We note that the components along the y axis cancel during the vector summation. With $k = 1/4\pi\varepsilon_0$ and $\theta = 60°$, the magnitude of the net electric field is obtained as follows:



$$\begin{aligned}\left|\vec{E}_{\rm net}\right| &= E_x = 2E_e\cos\theta \\ &= 2\left(\frac{e}{4\pi\varepsilon_0 d^2}\right)\cos\theta = 2k\left[\frac{e}{d^2}\right]\cos\theta \\ &= 2\left(8.99\times10^9\,\frac{\rm N\cdot m^2}{\rm C^2}\right)\left[\frac{(1.6\times10^{-19}\,{\rm C})}{(2.0\times10^{-6}\,{\rm m})^2}\right]\cos 60° \\ &= 3.6\times10^2\ {\rm N/C}\ .\end{aligned}$$

66. (a) Since the two charges in question are of the same sign, the point $x = 2.0$ mm should be located in between them (so that the field vectors point in the opposite direction). Let the coordinate of the second particle be x' ($x' > 0$). Then, the magnitude of the field due to the charge $-q_1$ evaluated at x is given by $E = q_1/4\pi\varepsilon_0 x^2$, while that due to the second charge $-4q_1$ is $E' = 4q_1/4\pi\varepsilon_0(x'-x)^2$. We set the net field equal to zero:

$$\vec{E}_{\rm net} = 0 \quad\Longrightarrow\quad E = E'$$

so that

$$\frac{q_1}{4\pi\varepsilon_0 x^2} = \frac{4q_1}{4\pi\varepsilon_0(x'-x)^2}\ .$$

Thus, we obtain $x' = 3x = 3(2.0\,{\rm mm}) = 6.0\,{\rm mm}$.

(b) In this case, with the second charge now positive, the electric field vectors produced by both charges are in the negative x direction, when evaluated at $x = 2.0$ mm. Therefore, the net field points in the negative x direction.

67. The distance from Q to P is $5a$, and the distance from q to P is $3a$. Therefore, the magnitudes of the individual electric fields are, using Eq. 23-3 (writing $1/4\pi\varepsilon_0 = k$),

$$|\vec{E}_Q| = \frac{k\,|Q|}{25\,a^2}\ , \qquad |\vec{E}_q| = \frac{k\,|q|}{9\,a^2}\ .$$

We note that $\vec{E}_q$ is along the y axis (directed towards $\pm y$ in accordance with the sign of q), and $\vec{E}_Q$ has x and y components, with $\vec{E}_{Q\,x} = \pm\frac{4}{5}|\vec{E}_Q|$ and $\vec{E}_{Q\,y} = \pm\frac{3}{5}|\vec{E}_Q|$ (signs corresponding to the sign of Q). Consequently, we can write the addition of components in a simple way (basically, by dropping the absolute values):

$$\begin{aligned}\vec{E}_{{\rm net}\,x} &= \frac{4\,k\,Q}{125\,a^2} \\ \vec{E}_{{\rm net}\,y} &= \frac{3\,k\,Q}{125\,a^2} + \frac{k\,q}{9\,a^2}\end{aligned}$$

(a) Equating $\vec{E}_{\text{net}\,x}$ and $\vec{E}_{\text{net}\,y}$, it is straightforward to solve for the relation between Q and q. We obtain $Q = \frac{125}{9} q \approx 14q$.

(b) We set $\vec{E}_{\text{net}\,y} = 0$ and find the necessary relation between Q and q. We obtain $Q = -\frac{125}{27} q \approx -4.6q$.

68. (a) From the second measurement (at $(2.0,\ 0)$) we see that the charge must be somewhere on the x axis. A line passing through $(3.0,\ 3.0)$ with slope $\tan^{-1} 3/4$ will intersect the x axis at $x = -1.0$. Thus, the location of the particle is specified by the coordinates (in cm): $(-1.0,\ 0)$.

(b) Using $k = 1/4\pi\varepsilon_0$, the field magnitude measured at $(2.0,\ 0)$ (which is $r = 0.030$ m from the charge) is

$$|\vec{E}| = k\,\frac{q}{r^2} = 100 \text{ N/C} \ .$$

Therefore, $q = 1.0 \times 10^{-11}$ C.