Chapter 20

1. Each atom has a mass of $m = M/N_A$, where M is the molar mass and N_A is the Avogadro constant. The molar mass of arsenic is $74.9\,\text{g/mol}$ or $74.9 \times 10^{-3}\,\text{kg/mol}$. 7.50×10^{24} arsenic atoms have a total mass of $(7.50 \times 10^{24})(74.9 \times 10^{-3}\,\text{kg/mol})/(6.02 \times 10^{23}\,\text{mol}^{-1}) = 0.933\,\text{kg}$.

2. (a) Eq. 20-3 yields $n = M_{\text{sam}}/M = 2.5/197 = 0.0127$.

   (b) The number of atoms is found from Eq. 20-2: $N = nN_A = (0.0127)(6.02 \times 10^{23}) = 7.64 \times 10^{21}$.

3. The surface area of a sphere is $4\pi R^2$, and we find the radius of Earth in Appendix C ($R_E = 6.37 \times 10^6\,\text{m} = 6.37 \times 10^8\,\text{cm}$). Therefore, the number of square "patches" (with one centimeter side length) needed to cover Earth is

$$A = 4\pi \left(6.37 \times 10^8\right)^2 = 5.1 \times 10^{18} \ .$$

The number of molecules that we want to distribute as evenly as possible among all those patches is (using Eqs. 20-2, 20-3, with $M = 18$ g/mol)

$$N = N_A \frac{M_{\text{sam}}}{M} = \left(6.02 \times 10^{23}\right) \frac{1.00\,\text{g}}{18\,\text{g/mol}} = 3.3 \times 10^{22} \ .$$

Therefore, we have $N/A = 6.56 \times 10^3$ molecules in each patch. Note: students are encouraged to figure $M = 18$ g/mol (for water) based on what they have learned in their chemistry courses, but it should be mentioned that this can also be gleaned from Table 20-1.

4. The number of molecules in $M_{\text{sam}} = 1\,\mu\text{g} = 10^{-6}\,\text{g}$ of ink is (using Eqs. 20-2, 20-3, with $M = 18$ g/mol)

$$N = N_A \frac{M_{\text{sam}}}{M} = \left(6.02 \times 10^{23}/\text{mol}\right) \left(\frac{1 \times 10^{-6}\,\text{g}}{18\,\text{g/mol}}\right) \approx 3 \times 10^{16} \ .$$

The number of creatures in our galaxy, with the assumption made in the problem, is about $N' = 5 \times 10^9 \times 10^{11} = 5 \times 10^{20}$. So the statement is wrong by a factor of about 20,000.

5. (a) We solve the ideal gas law $pV = nRT$ for n:

$$n = \frac{pV}{RT} = \frac{(100\,\text{Pa})(1.0 \times 10^{-6}\,\text{m}^3)}{(8.31\,\text{J/mol} \cdot \text{K})(220\,\text{K})} = 5.47 \times 10^{-8}\,\text{mol} \ .$$

   (b) Using Eq. 20-2, the number of molecules N is

$$N = nN_A = \left(5.47 \times 10^{-6}\,\text{mol}\right) \left(6.02 \times 10^{23}\,\text{mol}^{-1}\right) = 3.29 \times 10^{16} \text{ molecules} \ .$$

6. With $V = 1.0 \times 10^{-6}\,\text{m}^3$, $p = 1.01 \times 10^{-13}$ Pa, and $T = 293$ K, the ideal gas law gives

$$n = \frac{pV}{RT} = \frac{\left(1.01 \times 10^{-13}\right)\left(1.0 \times 10^{-6}\right)}{(8.31)(293)} = 4.1 \times 10^{-23}\,\text{mole} \ .$$

Consequently, Eq. 20-2 yields $N = nN_A = 25$ molecules. We can express this as a ratio (with V now written as $1\,\text{cm}^3$) $N/V = 25$ molecules/cm^3.

7. (a) In solving $pV = nRT$ for n, we first convert the temperature to the Kelvin scale: $T = 40.0 + 273.15 = 313.15\,\text{K}$. And we convert the volume to SI units: $1000\,\text{cm}^3 = 1000 \times 10^{-6}\,\text{m}^3$. Now, according to the ideal gas law,

$$n = \frac{pV}{RT} = \frac{(1.01 \times 10^5\,\text{Pa})(1000 \times 10^{-6}\,\text{m}^3)}{(8.31\,\text{J/mol}\cdot\text{K})(313.15\,\text{K})} = 3.88 \times 10^{-2}\,\text{mol}\ .$$

(b) The ideal gas law $pV = nRT$ leads to

$$T = \frac{pV}{nR} = \frac{(1.06 \times 10^5\,\text{Pa})(1500 \times 10^{-6}\,\text{m}^3)}{(3.88 \times 10^{-2}\,\text{mol})(8.31\,\text{J/mol}\cdot\text{K})} = 493\,\text{K}\ .$$

We note that the final temperature may be expressed in degrees Celsius as $220°\text{C}$.

8. Since (standard) air pressure is 101 kPa, then the initial (absolute) pressure of the air is $p_i = 266\,\text{kPa}$. Setting up the gas law in ratio form (where $n_i = n_f$ and thus cancels out – see Sample Problem 20-1), we have

$$\frac{p_f V_f}{p_i V_i} = \frac{T_f}{T_i} \quad\Longrightarrow\quad p_f = (266\,\text{kPa})\left(\frac{1.64 \times 10^{-2}\,\text{m}^3}{1.67 \times 10^{-2}\,\text{m}^3}\right)\left(\frac{300\,\text{K}}{273\,\text{K}}\right)$$

which yields $p_f = 287\,\text{kPa}$. Expressed as a gauge pressure, we subtract 101 kPa and obtain 186 kPa.

9. (a) With $T = 283$ K, we obtain

$$n = \frac{pV}{RT} = \frac{(100 \times 10^3\,\text{Pa})(2.50\,\text{m}^3)}{(8.31\,\frac{\text{J}}{\text{mol}\cdot\text{K}})(283\,\text{K})} = 106\,\text{mol}\ .$$

(b) We can use the answer to part (a) with the new values of pressure and temperature, and solve the ideal gas law for the new volume, or we could set up the gas law in ratio form as in Sample Problem 20-1 (where $n_i = n_f$ and thus cancels out):

$$\frac{p_f V_f}{p_i V_i} = \frac{T_f}{T_i} \quad\Longrightarrow\quad V_f = (2.50\,\text{m}^3)\left(\frac{100\,\text{kPa}}{300\,\text{kPa}}\right)\left(\frac{303\,\text{K}}{283\,\text{K}}\right)$$

which yields a final volume of $V_f = 0.892\,\text{m}^3$.

10. We write $T = 273$ K and use Eq. 20-14:

$$W = (1.00\,\text{mol})\left(8.31\,\frac{\text{J}}{\text{mol}\cdot\text{K}}\right)(273\,\text{K})\ln\left(\frac{16.8}{22.4}\right)$$

which yields $W = -653$ J. Recalling the sign conventions for work stated in Chapter 19, this means an external agent does 653 J of work *on* the ideal gas during this process.

11. Since the pressure is constant the work is given by $W = p(V_2 - V_1)$. The initial volume is $V_1 = (AT_1 - BT_1^2)/p$, where T_1 is the initial temperature. The final volume is $V_2 = (AT_2 - BT_2^2)/p$. Thus $W = A(T_2 - T_1) - B(T_2^2 - T_1^2)$.

12. The pressure p_1 due to the first gas is $p_1 = n_1 RT/V$, and the pressure p_2 due to the second gas is $p_2 = n_2 RT/V$. So the total pressure on the container wall is

$$p = p_1 + p_2 = \frac{n_1 RT}{V} + \frac{n_2 RT}{V} = (n_1 + n_2)\frac{RT}{V}\ .$$

The fraction of P due to the second gas is then

$$\frac{p_2}{p} = \frac{n_2 RT/V}{(n_1 + n_2)(RT/V)} = \frac{n_2}{n_1 + n_2} = \frac{0.5}{2 + 0.5} = \frac{1}{5}\ .$$

13. Suppose the gas expands from volume V_i to volume V_f during the isothermal portion of the process. The work it does is

$$W = \int_{V_i}^{V_f} p\, dV = nRT \int_{V_i}^{V_f} \frac{dV}{V} = nRT \ln \frac{V_f}{V_i},$$

where the ideal gas law $pV = nRT$ was used to replace p with nRT/V. Now $V_i = nRT/p_i$ and $V_f = nRT/p_f$, so $V_f/V_i = p_i/p_f$. Also replace nRT with p_iV_i to obtain

$$W = p_i V_i \ln \frac{p_i}{p_f}.$$

Since the initial gauge pressure is 1.03×10^5 Pa, $p_i = 1.03 \times 10^5$ Pa $+ 1.013 \times 10^5$ Pa $= 2.04 \times 10^5$ Pa. The final pressure is atmospheric pressure: $p_f = 1.013 \times 10^5$ Pa. Thus

$$W = (2.04 \times 10^5 \text{ Pa})(0.14\,\text{m}^3) \ln \frac{2.04 \times 10^5 \text{ Pa}}{1.013 \times 10^5 \text{ Pa}} = 2.00 \times 10^4 \text{ J}.$$

During the constant pressure portion of the process the work done by the gas is $W = p_f(V_i - V_f)$. The gas starts in a state with pressure p_f, so this is the pressure throughout this portion of the process. We also note that the volume decreases from V_f to V_i. Now $V_f = p_iV_i/p_f$, so

$$\begin{aligned} W &= p_f \left(V_i - \frac{p_i V_i}{p_f} \right) = (p_f - p_i)V_i \\ &= (1.013 \times 10^5 \text{ Pa} - 2.04 \times 10^5 \text{ Pa})(0.14\,\text{m}^3) = -1.44 \times 10^4 \text{ J}. \end{aligned}$$

The total work done by the gas over the entire process is $W = 2.00 \times 10^4$ J $- 1.44 \times 10^4$ J $= 5.6 \times 10^3$ J.

14. (a) At point a, we know enough information to compute n:

$$n = \frac{pV}{RT} = \frac{(2500\,\text{Pa})\,(1.0\,\text{m}^3)}{\left(8.31 \frac{\text{J}}{\text{mol·K}}\right)(200\,\text{K})} = 1.5 \text{ mol}.$$

(b) We can use the answer to part (a) with the new values of pressure and volume, and solve the ideal gas law for the new temperature, or we could set up the gas law as in Sample Problem 20-1 in terms of ratios (note: $n_a = n_b$ and cancels out):

$$\frac{p_b V_b}{p_a V_a} = \frac{T_b}{T_a} \implies T_b = (200\,\text{K}) \left(\frac{7.5\,\text{kPa}}{2.5\,\text{kPa}} \right) \left(\frac{3.0\,\text{m}^3}{1.0\,\text{m}^3} \right)$$

which yields an absolute temperature at b of $T_b = 1800$ K.

(c) As in the previous part, we choose to approach this using the gas law in ratio form (see Sample Problem 20-1):

$$\frac{p_c V_c}{p_a V_a} = \frac{T_c}{T_a} \implies T_c = (200\,\text{K}) \left(\frac{2.5\,\text{kPa}}{2.5\,\text{kPa}} \right) \left(\frac{3.0\,\text{m}^3}{1.0\,\text{m}^3} \right)$$

which yields an absolute temperature at c of $T_c = 600$ K.

(d) The net energy added to the gas (as heat) is equal to the net work that is done as it progresses through the cycle (represented as a right triangle in the pV diagram shown in Fig. 20-19). This, in turn, is related to $\pm$ "area" inside that triangle (with area $= \frac{1}{2}$(base)(height)), where we choose the plus sign because the volume change at the largest pressure is an *increase*. Thus,

$$Q_{\text{net}} = W_{\text{net}} = \frac{1}{2} \left(2.0\,\text{m}^3 \right) \left(5.0 \times 10^3 \text{ Pa} \right) = 5000 \text{ J}.$$

15. We assume that the pressure of the air in the bubble is essentially the same as the pressure in the surrounding water. If d is the depth of the lake and ρ is the density of water, then the pressure at the bottom of the lake is $p_1 = p_0 + \rho g d$, where p_0 is atmospheric pressure. Since $p_1 V_1 = nRT_1$, the number of moles of gas in the bubble is $n = p_1 V_1 / RT_1 = (p_0 + \rho g d) V_1 / RT_1$, where V_1 is the volume of the bubble at the bottom of the lake and T_1 is the temperature there. At the surface of the lake the pressure is p_0 and the volume of the bubble is $V_2 = nRT_2/p_0$. We substitute for n to obtain

$$
\begin{aligned}
V_2 &= \frac{T_2}{T_1} \frac{p_0 + \rho g d}{p_0} V_1 \\
&= \left(\frac{293\,\text{K}}{277\,\text{K}}\right) \left(\frac{1.013 \times 10^5\,\text{Pa} + (0.998 \times 10^3\,\text{kg/m}^3)(9.8\,\text{m/s}^2)(40\,\text{m})}{1.013 \times 10^5\,\text{Pa}}\right) (20\,\text{cm}^3) \\
&= 100\,\text{cm}^3 .
\end{aligned}
$$

16. Consider the open end of the pipe. The balance of the pressures inside and outside the pipe requires that $p + \rho_w(L/2)g = p_0 + \rho_w h g$, where p_0 is the atmospheric pressure, and p is the pressure of the air inside the pipe, which satisfies $p(L/2) = p_0 L$, or $p = 2p_0$. We solve for h:

$$
h = \frac{p - p_0}{\rho_w g} + \frac{L}{2} = \frac{1.01 \times 10^5\,\text{Pa}}{(1.00 \times 10^3\,\text{kg/m}^3)\left(9.8\,\text{m/s}^2\right)} + \frac{25.0\,\text{m}}{2} = 22.8\,\text{m} .
$$

17. When the valve is closed the number of moles of the gas in container A is $n_A = p_A V_A/RT_A$ and that in container B is $n_B = 4p_B V_A/RT_B$. The total number of moles in both containers is then

$$
n = n_A + n_B = \frac{V_A}{R}\left(\frac{p_A}{T_A} + \frac{4p_B}{T_B}\right) = \text{const.}
$$

After the valve is opened the pressure in container A is $p'_A = Rn'_A T_A/V_A$ and that in container B is $p'_B = Rn'_B T_B/4V_A$. Equating p'_A and p'_B, we obtain $Rn'_A T_A/V_A = Rn'_B T_B/4V_A$, or $n'_B = (4T_A/T_B)n'_A$. Thus,

$$
n = n'_A + n'_B = n'_A\left(1 + \frac{4T_A}{T_B}\right) = n_A + n_B = \frac{V_A}{R}\left(\frac{p_A}{T_A} + \frac{4p_B}{T_B}\right) .
$$

We solve the above equation for n'_A:

$$
n'_A = \frac{V}{R} \frac{(p_A/T_A + 4p_B/T_B)}{(1 + 4T_A/T_B)} .
$$

Substituting this expression for n'_A into $p'V_A = n'_A RT_A$, we obtain the final pressure:

$$
p' = \frac{n'_A RT_A}{V_A} = \frac{p_A + 4p_B T_A/T_B}{1 + 4T_A/T_B} = 2.0 \times 10^5\,\text{Pa} .
$$

18. Appendix F gives $M = 4.00 \times 10^{-3}$ kg/mol (Table 20-1 gives this to fewer significant figures). Using Eq. 20-22, we obtain

$$
v_{\text{rms}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3\left(8.31\,\frac{\text{J}}{\text{mol·K}}\right)(1000\,\text{K})}{4.00 \times 10^{-3}\,\text{kg/mol}}} = 2.50 \times 10^3\,\text{m/s} .
$$

19. According to kinetic theory, the rms speed is

$$
v_{\text{rms}} = \sqrt{\frac{3RT}{M}}
$$

where T is the temperature and M is the molar mass. See Eq. 20-34. According to Table 20-1, the molar mass of molecular hydrogen is $2.02\,\text{g/mol} = 2.02 \times 10^{-3}\,\text{kg/mol}$, so

$$v_{\text{rms}} = \sqrt{\frac{3\,(8.31\,\text{J/mol·K})\,(2.7\,\text{K})}{2.02 \times 10^{-3}\,\text{kg/mol}}} = 180\,\text{m/s} \ .$$

20. The molar mass of argon is 39.95 g/mol. Eq. 20-22 gives

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3\left(8.31\,\frac{\text{J}}{\text{mol·K}}\right)(313\,\text{K})}{39.95 \times 10^{-3}\,\text{kg/mol}}} = 442\,\text{m/s} \ .$$

21. First we rewrite Eq. 20-22 using Eq. 20-4 and Eq. 20-7:

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3\,(kN_{\text{A}})\,T}{(m\,N_{\text{A}})}} = \sqrt{\frac{3kT}{M}} \ .$$

The mass of the electron is given in the problem, and $k = 1.38 \times 10^{-23}$ J/K is given in the textbook. With $T = 2.00 \times 10^6$ K, the above expression gives $v_{\text{rms}} = 9.53 \times 10^6$ m/s. The pressure value given in the problem is not used in the solution.

22. Table 20-1 gives $M = 28.0$ g/mol for Nitrogen. This value can be used in Eq. 20-22 with T in Kelvins to obtain the results. A variation on this approach is to set up ratios, using the fact that Table 20-1 also gives the rms speed for nitrogen gas at 300 K (the value is 517 m/s). Here we illustrate the latter approach, using v for v_{rms}:

$$\frac{v_2}{v_1} = \frac{\sqrt{\frac{3RT_2}{M}}}{\sqrt{\frac{3RT_1}{M}}} = \sqrt{\frac{T_2}{T_1}} \ .$$

   (a) With $T_2 = 20.0 + 273.15 \approx 293$ K, we obtain

   $$v_2 = (517\,\text{m/s}) \sqrt{\frac{293\,\text{K}}{300\,\text{K}}} = 511\,\text{m/s} \ .$$

   (b) In this case, we set $v_3 = \frac{1}{2}v_2$ and solve $v_3/v_2 = \sqrt{T_3/T_2}$ for T_3:

   $$T_3 = T_2 \left(\frac{v_3}{v_2}\right)^2 = (293\,\text{K})\left(\frac{1}{2}\right)^2 = 73\,\text{K}$$

   which we write as $73 - 273 = -200°\text{C}$.

   (c) Now we have $v_4 = 2v_2$ and obtain

   $$T_4 = T_2 \left(\frac{v_4}{v_2}\right)^2 = (293\,\text{K})(4) = 1.17 \times 10^3\,\text{K}$$

   which is equivalent to $899°$.

23. In the reflection process, only the normal component of the momentum changes, so for one molecule the change in momentum is $2mv\cos\theta$, where m is the mass of the molecule, v is its speed, and θ is the angle between its velocity and the normal to the wall. If N molecules collide with the wall, then the change in their total momentum is $2Nmv\cos\theta$, and if the total time taken for the collisions is Δt, then

the average rate of change of the total momentum is $2(N/\Delta t)mv\cos\theta$. This is the average force exerted by the N molecules on the wall, and the pressure is the average force per unit area:

$$
\begin{aligned}
p &= \frac{2}{A}\left(\frac{N}{\Delta t}\right)mv\cos\theta \\
&= \left(\frac{2}{2.0\times10^{-4}\,\text{m}^2}\right)(1.0\times10^{23}\,\text{s}^{-1})(3.3\times10^{-27}\,\text{kg})(1.0\times10^3\,\text{m/s})\cos55^\circ \\
&= 1.9\times10^3\,\text{Pa} .
\end{aligned}
$$

We note that the value given for the mass was converted to kg and the value given for the area was converted to m^2.

24. We can express the ideal gas law in terms of density using $n = M_{\text{sam}}/M$:

$$
pV = \frac{M_{\text{sam}}RT}{M} \quad\Longrightarrow\quad \rho = \frac{pM}{RT} .
$$

We can also use this to write the rms speed formula in terms of density:

$$
v_{\text{rms}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3(pM/\rho)}{M}} = \sqrt{\frac{3p}{\rho}} .
$$

   (a) We convert to SI units: $\rho = 1.24\times10^{-2}$ kg/m^3 and $p = 1.01\times10^3$ Pa. The rms speed is $\sqrt{3(1010)/0.0124} = 494$ m/s.

   (b) We find M from $\rho = pM/RT$ with $T = 273$ K.

$$
M = \frac{\rho RT}{p} = \frac{\left(0.0124\,\text{kg/m}^3\right)\left(8.31\,\frac{\text{J}}{\text{mol}\cdot\text{K}}\right)(273\,\text{K})}{1.01\times10^3\,\text{Pa}}
$$

   This yields $M = 0.028$ kg/mol, which converts to 28 g/mol.

25. The average translational kinetic energy is given by $K_{\text{avg}} = \frac{3}{2}kT$, where k is the Boltzmann constant $(1.38\times10^{-23}$ J/K) and T is the temperature on the Kelvin scale. Thus

$$
K_{\text{avg}} = \frac{3}{2}(1.38\times10^{-23}\,\text{J/K})(1600\,\text{K}) = 3.31\times10^{-20}\,\text{J} .
$$

26. (a) Eq. 20-24 gives
$$
K_{\text{avg}} = \frac{3}{2}\left(1.38\times10^{-23}\,\frac{\text{J}}{\text{K}}\right)(273\,\text{K}) = 5.65\times10^{-21}\,\text{J} .
$$

   (b) Similarly, for $T = 373$ K, the average translational kinetic energy is $K_{\text{avg}} = 7.72\times10^{-21}$ J.

   (c) The unit mole may be thought of as a (large) collection: 6.02×10^{23} molecules of ideal gas, in this case. Each molecule has energy specified in part (a), so the large collection has a total kinetic energy equal to

$$
K_{\text{mole}} = N_A K_{\text{avg}} = \left(6.02\times10^{23}\right)\left(5.65\times10^{-21}\,\text{J}\right) = 3.40\times10^3\,\text{J} .
$$

   (d) Similarly, the result from part (b) leads to

$$
K_{\text{mole}} = \left(6.02\times10^{23}\right)\left(7.72\times10^{-21}\,\text{J}\right) = 4.65\times10^3\,\text{J} .
$$

27. (a) We use $\epsilon = L_V/N$, where L_V is the heat of vaporization and N is the number of molecules per gram. The molar mass of atomic hydrogen is 1 g/mol and the molar mass of atomic oxygen is 16 g/mol so the molar mass of H_2O is $1+1+16 = 18$ g/mol. There are $N_A = 6.02\times10^{23}$ molecules in a mole so the number of molecules in a gram of water is $(6.02\times10^{23}\,\text{mol}^{-1})/(18\,\text{g/mol}) = 3.34\times10^{22}$ molecules/g. Thus $\epsilon = (539\,\text{cal/g})/(3.34\times10^{22}/\text{g}) = 1.61\times10^{-20}$ cal. This is $(1.61\times10^{-20}\,\text{cal})(4.186\,\text{J/cal}) = 6.76\times10^{-20}$ J.

(b) The average translational kinetic energy is

$$K_{\text{avg}} = \frac{3}{2}kT = \frac{3}{2}(1.38 \times 10^{-23}\,\text{J/K})\,[(32.0 + 273.15)\,\text{K}] = 6.32 \times 10^{-21}\,\text{J}\ .$$

The ratio ϵ/K_{avg} is $(6.76 \times 10^{-20}\,\text{J})/(6.32 \times 10^{-21}\,\text{J}) = 10.7$.

28. We express the ideal gas law in terms of density using $\rho = M_{\text{sam}}/V$ and $n = M_{\text{sam}}/M$:

$$pV = \frac{M_{\text{sam}}RT}{M} \implies p = \frac{\rho RT}{M}\ .$$

29. They are not equivalent. Avogadro's law does not tell how the pressure, volume, and temperature are related, so you cannot use it, for example, to calculate the change in volume when the pressure increases at constant temperature. The ideal gas law, however, implies Avogadro's law. It yields $N = nN_A = (pV/RT)N_A = pV/kT$, where $k = R/N_A$ was used. If the two gases have the same volume, the same pressure, and the same temperature, then pV/kT is the same for them. This implies that N is also the same.

30. We solve Eq. 20-25 for d:

$$d = \sqrt{\frac{1}{\lambda\pi\sqrt{2}\,(N/V)}} = \sqrt{\frac{1}{(0.80 \times 10^5\,\text{cm})\,\pi\sqrt{2}\,(2.7 \times 10^{19}/\text{cm}^3)}}$$

which yields $d = 3.2 \times 10^{-8}$ cm, or 0.32 nm.

31. (a) According to Eq. 20–25, the mean free path for molecules in a gas is given by

$$\lambda = \frac{1}{\sqrt{2}\pi d^2 N/V}\ ,$$

where d is the diameter of a molecule and N is the number of molecules in volume V. Substitute $d = 2.0 \times 10^{-10}$ m and $N/V = 1 \times 10^6$ molecules/m^3 to obtain

$$\lambda = \frac{1}{\sqrt{2}\pi(2.0 \times 10^{-10}\,\text{m})^2(1 \times 10^6\,\text{m}^{-3})} = 6 \times 10^{12}\,\text{m}\ .$$

(b) At this altitude most of the gas particles are in orbit around Earth and do not suffer randomizing collisions. The mean free path has little physical significance.

32. Using $v = f\lambda$ with $v = 331$ m/s (see Table 18-1) with Eq. 20-2 and Eq. 20-25 leads to

$$f = \frac{v}{\left(\frac{1}{\sqrt{2}\,\pi\,d^2(N/V)}\right)} = (331\,\text{m/s})\pi\sqrt{2}\,(3.0 \times 10^{-10}\,\text{m})^2\left(\frac{nN_A}{V}\right) = \left(8.0 \times 10^7\,\frac{\text{m}^3}{\text{s}\cdot\text{mol}}\right)\left(\frac{n}{V}\right)$$

Using the ideal gas law, we substitute $n/V = p/RT$ into the above expression and find

$$f = \left(8.0 \times 10^7\,\frac{\text{m}^3}{\text{s}\cdot\text{mol}}\right)\left(\frac{1.01 \times 10^5\,\text{Pa}}{\left(8.31\,\frac{\text{J}}{\text{mol}\cdot\text{K}}\right)(273.15\,\text{K})}\right) = 3.5 \times 10^9\,\text{Hz}\ .$$

If we instead use $v = 343$ m/s (the "default value" for speed of sound in air, used repeatedly in Ch. 18), then the answer is 3.7×10^9 Hz.

33. We substitute $d = 1.0 \times 10^{-2}$ m and $N/V = 15/(1.0 \times 10^{-3}\,\text{m}^3) = 15 \times 10^3$ beans/m^3 into Eq. 20–25

$$\lambda = \frac{1}{\sqrt{2}\pi d^2 N/V}$$

to obtain

$$\lambda = \frac{1}{\sqrt{2}\pi(1.0 \times 10^{-2}\,\text{m})^2(15 \times 10^3/\text{m}^3)} = 0.15\,\text{m}\ .$$

The conversion $1.00\,\text{L} = 1.00 \times 10^{-3}\,\text{m}^3$ is used.

34. (a) We set up a ratio using Eq. 20-25:

$$\frac{\lambda_{Ar}}{\lambda_{N_2}} = \frac{1/\left(\pi\sqrt{2}d_{Ar}^2(N/V)\right)}{1/\left(\pi\sqrt{2}d_{N_2}^2(N/V)\right)} = \left(\frac{d_{N_2}}{d_{Ar}}\right)^2 .$$

Therefore, we obtain

$$\frac{d_{Ar}}{d_{N_2}} = \sqrt{\frac{\lambda_{N_2}}{\lambda_{Ar}}} = \sqrt{\frac{27.5}{9.9}} = 1.7 .$$

(b) Using Eq. 20-2 and the ideal gas law, we substitute $N/V = N_A n/V = N_A p/RT$ into Eq. 20-25 and find

$$\lambda = \frac{RT}{\pi\sqrt{2}d^2 p N_A} .$$

Comparing (for the same species of molecule) at two different pressures and temperatures, this leads to

$$\frac{\lambda_2}{\lambda_1} = \left(\frac{T_2}{T_1}\right)\left(\frac{p_1}{p_2}\right) .$$

With $\lambda_1 = 9.9 \times 10^{-6}$ cm, $T_1 = 293$ K (the same as T_2 in this part), $p_1 = 750$ torr and $p_2 = 150$ torr, we find $\lambda_2 = 5.0 \times 10^{-5}$ cm.

(c) The ratio set up in part (b), using the same values for quantities with subscript 1, leads to $\lambda_2 = 7.9 \times 10^{-6}$ cm for $T_2 = 233$ K and $p_2 = 750$ torr.

35. (a) We use the ideal gas law $pV = nRT = NkT$, where p is the pressure, V is the volume, T is the temperature, n is the number of moles, and N is the number of molecules. The substitutions $N = nN_A$ and $k = R/N_A$ were made. Since 1 cm of mercury $= 1333$ Pa, the pressure is $p = (10^{-7})(1333) = 1.333 \times 10^{-4}$ Pa. Thus,

$$\begin{aligned} \frac{N}{V} &= \frac{p}{kT} = \frac{1.333 \times 10^{-4}\,\text{Pa}}{(1.38 \times 10^{-23}\,\text{J/K})(295\,\text{K})} \\ &= 3.27 \times 10^{16}\,\text{molecules/m}^3 = 3.27 \times 10^{10}\,\text{molecules/cm}^3 . \end{aligned}$$

(b) The molecular diameter is $d = 2.00 \times 10^{-10}$ m, so, according to Eq. 20–25, the mean free path is

$$\lambda = \frac{1}{\sqrt{2}\pi d^2 N/V} = \frac{1}{\sqrt{2}\pi(2.00 \times 10^{-10}\,\text{m})^2(3.27 \times 10^{16}\,\text{m}^{-3})} = 172\,\text{m} .$$

36. (a) The average speed is

$$v_{avg} = \frac{\sum n_i v_i}{\sum n_i} = \frac{2(1.0) + 4(2.0) + 6(3.0) + 8(4.0) + 2(5.0)}{2 + 4 + 6 + 8 + 2} = 3.2\,\text{cm/s} .$$

(b) From $v_{rms} = \sqrt{\sum n_i v_i^2 / \sum n_i}$ we get

$$v_{rms} = \sqrt{\frac{2(1.0)^2 + 4(2.0)^2 + 6(3.0)^2 + 8(4.0)^2 + 2(5.0)^2}{2 + 4 + 6 + 8 + 2}} = 3.4\,\text{cm/s} .$$

(c) There are eight particles at $v = 4.0$ cm/s, more than the number of particles at any other single speed. So 4.0 cm/s is the most probable speed.

37. (a) The average speed is

$$\overline{v} = \frac{\sum v}{N} ,$$

where the sum is over the speeds of the particles and N is the number of particles. Thus

$$\overline{v} = \frac{(2.0 + 3.0 + 4.0 + 5.0 + 6.0 + 7.0 + 8.0 + 9.0 + 10.0 + 11.0)\,\text{km/s}}{10} = 6.5\,\text{km/s} .$$

(b) The rms speed is given by

$$v_{\text{rms}} = \sqrt{\frac{\sum v^2}{N}} \ .$$

Now

$$\sum v^2 = (2.0)^2 + (3.0)^2 + (4.0)^2 + (5.0)^2 + (6.0)^2$$
$$+ (7.0)^2 + (8.0)^2 + (9.0)^2 + (10.0)^2 + (11.0)^2 = 505 \text{ km}^2/\text{s}^2$$

so

$$v_{\text{rms}} = \sqrt{\frac{505 \text{ km}^2/\text{s}^2}{10}} = 7.1 \text{ km/s} \ .$$

38. (a) The average and rms speeds are as follows:

$$v_{\text{avg}} = \frac{1}{N} \sum_{i=1}^{N} v_i = \frac{1}{10}[4(200 \text{ m/s}) + 2(500 \text{ m/s}) + 4(600 \text{ m/s})] = 420 \text{ m/s},$$

$$v_{\text{rms}} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} v_i^2} = \sqrt{\frac{1}{10}[4(200 \text{ m/s})^2 + 2(500 \text{ m/s})^2 + 4(600 \text{ m/s})^2]} = 458 \text{ m/s} \ .$$

From these results, we see that $v_{\text{rms}} > v_{\text{avg}}$.

(b) One may check the validity of the inequality $v_{\text{rms}} \geq v_{\text{avg}}$ for any speed distribution. For example, we consider a set of ten particles divided into two groups of five particles each, with the first group of particles moving at speed v_1 and the second group at v_2 where both v_1 and v_2 are positive-valued (by the definition of speed). In this case, $v_{\text{avg}} = (v_1 + v_2)/2$ and

$$v_{\text{rms}} = \sqrt{\frac{v_1^2 + v_2^2}{2}} \ .$$

To show this must be greater than (or equal to) v_{avg} we examine the difference in the squares of the quantities:

$$\begin{aligned} v_{\text{rms}}^2 - v_{\text{avg}}^2 &= \frac{v_1^2 + v_2^2}{2} - \frac{1}{4}\left(v_1^2 + v_2^2 + 2v_1v_2\right) \\ &= \frac{v_1^2 + v_2^2 - 2v_1v_2}{4} \\ &= \frac{1}{4}\left(v_1 - v_2\right)^2 \geq 0 \end{aligned}$$

which demonstrates that $v_{\text{rms}} \geq v_{\text{avg}}$ in this situation.

(c) As one can infer from our manipulation in the previous part, we will obtain $v_{\text{rms}} = v_{\text{avg}}$ if all speeds are the same (if $v_1 = v_2$ in the previous part).

39. (a) The rms speed of molecules in a gas is given by $v_{\text{rms}} = \sqrt{3RT/M}$, where T is the temperature and M is the molar mass of the gas. See Eq. 20–34. The speed required for escape from Earth's gravitational pull is $v = \sqrt{2gr_e}$, where g is the acceleration due to gravity at Earth's surface and $r_e \ (= 6.37 \times 10^6 \text{ m})$ is the radius of Earth. To derive this expression, take the zero of gravitational potential energy to be at infinity. Then, the gravitational potential energy of a particle with mass m at Earth's surface is $U = -GMm/r_e^2 = -mgr_e$, where $g = GM/r_e^2$ was used. If v is the speed of the particle, then its total energy is $E = -mgr_e + \frac{1}{2}mv^2$. If the particle is just able to travel far away, its kinetic energy must tend toward zero as its distance from Earth becomes large without bound. This means $E = 0$ and $v = \sqrt{2gr_e}$. We equate the expressions for the speeds to

obtain $\sqrt{3RT/M} = \sqrt{2gr_e}$. The solution for T is $T = 2gr_e M/3R$. The molar mass of hydrogen is 2.02×10^{-3} kg/mol, so for that gas

$$T = \frac{2(9.8\,\mathrm{m/s^2})(6.37 \times 10^6\,\mathrm{m})(2.02 \times 10^{-3}\,\mathrm{kg/mol})}{3(8.31\,\mathrm{J/mol \cdot K})} = 1.0 \times 10^4 \text{ K} .$$

(b) The molar mass of oxygen is 32.0×10^{-3} kg/mol, so for that gas

$$T = \frac{2(9.8\,\mathrm{m/s^2})(6.37 \times 10^6\,\mathrm{m})(32.0 \times 10^{-3}\,\mathrm{kg/mol})}{3(8.31\,\mathrm{J/mol \cdot K})} = 1.6 \times 10^5 \text{ K} .$$

(c) Now, $T = 2g_m r_m M/3R$, where r_m $(= 1.74 \times 10^6\,\mathrm{m})$ is the radius of the Moon and g_m $(= 0.16g)$ is the acceleration due to gravity at the Moon's surface. For hydrogen

$$T = \frac{2(0.16)(9.8\,\mathrm{m/s^2})(1.74 \times 10^6\,\mathrm{m})(2.02 \times 10^{-3}\,\mathrm{kg/mol})}{3(8.31\,\mathrm{J/mol \cdot K})} = 4.4 \times 10^2 \text{ K} .$$

For oxygen

$$T = \frac{2(0.16)(9.8\,\mathrm{m/s^2})(1.74 \times 10^6\,\mathrm{m})(32.0 \times 10^{-3}\,\mathrm{kg/mol})}{3(8.31\,\mathrm{J/mol \cdot K})} = 7.0 \times 10^3 \text{ K} .$$

(d) The temperature high in Earth's atmosphere is great enough for a significant number of hydrogen atoms in the tail of the Maxwellian distribution to escape. As a result the atmosphere is depleted of hydrogen. On the other hand, very few oxygen atoms escape.

40. We divide Eq. 20-35 by Eq. 20-22:

$$\frac{v_P}{v_{\mathrm{rms}}} = \frac{\sqrt{2RT_2/M}}{\sqrt{3RT_1/M}} = \sqrt{\frac{2T_2}{3T_1}}$$

which leads to

$$\frac{T_2}{T_1} = \frac{3}{2}\left(\frac{v_P}{v_{\mathrm{rms}}}\right)^2 = \frac{3}{2} \qquad \text{if } v_P = v_{\mathrm{rms}} .$$

41. (a) The root-mean-square speed is given by $v_{\mathrm{rms}} = \sqrt{3RT/M}$. See Eq. 20–34. The molar mass of hydrogen is 2.02×10^{-3} kg/mol, so

$$v_{\mathrm{rms}} = \sqrt{\frac{3(8.31\,\mathrm{J/mol \cdot K})(4000\,\mathrm{K})}{2.02 \times 10^{-3}\,\mathrm{kg/mol}}} = 7.0 \times 10^3\,\mathrm{m/s} .$$

(b) When the surfaces of the spheres that represent an H_2 molecule and an Ar atom are touching, the distance between their centers is the sum of their radii: $d = r_1 + r_2 = 0.5 \times 10^{-8}\,\mathrm{cm} + 1.5 \times 10^{-8}\,\mathrm{cm} = 2.0 \times 10^{-8}\,\mathrm{cm}$.

(c) The argon atoms are essentially at rest so in time t the hydrogen atom collides with all the argon atoms in a cylinder of radius d and length vt, where v is its speed. That is, the number of collisions is $\pi d^2 vt N/V$, where N/V is the concentration of argon atoms. The number of collisions per unit time is

$$\frac{\pi d^2 vN}{V} = \pi(2.0 \times 10^{-10}\,\mathrm{m})^2(7.0 \times 10^3\,\mathrm{m/s})(4.0 \times 10^{25}\,\mathrm{m^{-3}}) = 3.5 \times 10^{10}\,\mathrm{collisions/s} .$$

42. We divide Eq. 20-31 by Eq. 20-22:

$$\frac{v_{\text{avg2}}}{v_{\text{rms1}}} = \frac{\sqrt{8RT/\pi M_2}}{\sqrt{3RT/M_1}} = \sqrt{\frac{8M_1}{3\pi M_2}}$$

which leads to

$$\frac{m_1}{m_2} = \frac{M_1}{M_2} = \frac{3\pi}{8}\left(\frac{v_{\text{avg2}}}{v_{\text{rms1}}}\right)^2 = \frac{3\pi}{2} \qquad \text{if } v_{\text{avg2}} = 2v_{\text{rms1}} \ .$$

43. (a) The distribution function gives the fraction of particles with speeds between v and $v + dv$, so its integral over all speeds is unity: $\int P(v)\, dv = 1$. Evaluate the integral by calculating the area under the curve in Fig. 20–22. The area of the triangular portion is half the product of the base and altitude, or $\frac{1}{2}av_0$. The area of the rectangular portion is the product of the sides, or av_0. Thus $\int P(v)\, dv = \frac{1}{2}av_0 + av_0 = \frac{3}{2}av_0$, so $\frac{3}{2}av_0 = 1$ and $a = 2/3v_0$.

(b) The number of particles with speeds between $1.5v_0$ and $2v_0$ is given by $N\int_{1.5v_0}^{2v_0} P(v)\, dv$. The integral is easy to evaluate since $P(v) = a$ throughout the range of integration. Thus the number of particles with speeds in the given range is $Na(2.0v_0 - 1.5v_0) = 0.5Nav_0 = N/3$, where $2/3v_0$ was substituted for a.

(c) The average speed is given by

$$v_{\text{avg}} = \int vP(v)\, dv \ .$$

For the triangular portion of the distribution $P(v) = av/v_0$, and the contribution of this portion is

$$\frac{a}{v_0}\int_0^{v_0} v^2\, dv = \frac{a}{3v_0}v_0^3 = \frac{av_0^2}{3} = \frac{2}{9}v_0 \ ,$$

where $2/3v_0$ was substituted for a. $P(v) = a$ in the rectangular portion, and the contribution of this portion is

$$a\int_{v_0}^{2v_0} v\, dv = \frac{a}{2}\left(4v_0^2 - v_0^2\right) = \frac{3a}{2}v_0^2 = v_0 \ .$$

Therefore,

$$v_{\text{avg}} = \frac{2}{9}v_0 + v_0 = 1.22v_0 \ .$$

(d) The mean-square speed is given by

$$v_{\text{rms}}^2 = \int v^2 P(v)\, dv \ .$$

The contribution of the triangular section is

$$\frac{a}{v_0}\int_0^{v_0} v^3\, dv = \frac{a}{4v_0}v_0^4 = \frac{1}{6}v_0^2 \ .$$

The contribution of the rectangular portion is

$$a\int_{v_0}^{2v_0} v^2\, dv = \frac{a}{3}\left(8v_0^3 - v_0^3\right) = \frac{7a}{3}v_0^3 = \frac{14}{9}v_0^2 \ .$$

Thus,

$$v_{\text{rms}} = \sqrt{\frac{1}{6}v_0^2 + \frac{14}{9}v_0^2} = 1.31v_0 \ .$$

44. The internal energy is

$$E_{\text{int}} = \frac{3}{2}nRT = \frac{3}{2}(1.0\,\text{mol})\left(8.31\,\frac{\text{J}}{\text{mol·K}}\right)(273\,\text{K}) = 3.4 \times 10^3\ \text{J} \ .$$

45. According to the first law of thermodynamics, $\Delta E_{\text{int}} = Q - W$. Since the process is isothermal $\Delta E_{\text{int}} = 0$ (the internal energy of an ideal gas depends only on the temperature) and $Q = W$. The work done by the gas as its volume expands from V_i to V_f at temperature T is

$$W = \int_{V_i}^{V_f} p\,dV = nRT \int_{V_i}^{V_f} \frac{dV}{V} = nRT\ln\frac{V_f}{V_i}$$

where the ideal gas law $pV = nRT$ was used to substitute for p. For 1 mole $Q = W = RT\ln(V_f/V_i)$.

46. (a) According to the first law of thermodynamics $Q = \Delta E_{\text{int}} + W$. When the pressure is a constant $W = p\,\Delta V$. So

$$
\begin{aligned}
\Delta E_{\text{int}} &= Q - p\Delta V \\
&= 20.9\,\text{J} - \left(1.01 \times 10^5\,\text{Pa}\right)\left(100\,\text{cm}^3 - 50\,\text{cm}^3\right)\left(\frac{1 \times 10^{-6}\,\text{m}^3}{1\,\text{cm}^3}\right) \\
&= 15.9\ \text{J} \ .
\end{aligned}
$$

(b) The molar specific heat at constant pressure is

$$
\begin{aligned}
C_p &= \frac{Q}{n\Delta T} \\
&= \frac{Q}{n\left(\frac{p\Delta V}{nR}\right)} = \frac{R}{p}\frac{Q}{\Delta V} \\
&= \frac{\left(8.31\,\frac{\text{J}}{\text{mol·K}}\right)(20.9\,\text{J})}{(1.01 \times 10^5\,\text{Pa})(50 \times 10^{-6}\,\text{m}^3)} = 34.4\ \text{J/mol·K} \ .
\end{aligned}
$$

(c) Using Eq. 20-49, $C_V = C_p - R = 26.1\,\text{J/mol·K}$.

47. When the temperature changes by ΔT the internal energy of the first gas changes by $n_1 C_1\,\Delta T$, the internal energy of the second gas changes by $n_2 C_2\,\Delta T$, and the internal energy of the third gas changes by $n_3 C_3\,\Delta T$. The change in the internal energy of the composite gas is $\Delta E_{\text{int}} = (n_1 C_1 + n_2 C_2 + n_3 C_3)\,\Delta T$. This must be $(n_1 + n_2 + n_3)C\,\Delta T$, where C is the molar specific heat of the mixture. Thus

$$C = \frac{n_1 C_1 + n_2 C_2 + n_3 C_3}{n_1 + n_2 + n_3} \ .$$

48. Two formulas (other than the first law of thermodynamics) will be of use to us. It is straightforward to show, from Eq. 20-11, that for any process that is depicted as a *straight line* on the pV diagram – the work is

$$W_{\text{straight}} = \left(\frac{p_i + p_f}{2}\right)\Delta V$$

which includes, as special cases, $W = p\Delta V$ for constant-pressure processes and $W = 0$ for constant-volume processes. Further, Eq. 20-44 with Eq. 20-51 gives

$$E_{\text{int}} = n\left(\frac{f}{2}\right)RT = \left(\frac{f}{2}\right)pV$$

where we have used the ideal gas law in the last step. We emphasize that, in order to obtain work and energy in Joules, pressure should be in Pascals (N/m^2) and volume should be in cubic meters. The degrees of freedom for a diatomic gas is $f = 5$.

(a) The internal energy change is

$$E_{\text{int }c} - E_{\text{int }a} = \frac{5}{2}(p_c V_c - p_a V_a)$$
$$= \frac{5}{2}\left((2000\,\text{Pa})\left(4.0\,\text{m}^3\right) - (5000\,\text{Pa})\left(2.0\,\text{m}^3\right)\right)$$
$$= -5000\ \text{J}\ .$$

(b) The work done during the process represented by the diagonal path is

$$W_{\text{diag}} = \left(\frac{p_a + p_c}{2}\right)(V_c - V_a) = (3500\,\text{Pa})\left(2.0\,\text{m}^3\right)$$

which yields $W_{\text{diag}} = 7000$ J. Consequently, the first law of thermodynamics gives

$$Q_{\text{diag}} = \Delta E_{\text{int}} + W_{\text{diag}} = -5000 + 7000 = 2000\ \text{J}\ .$$

(c) The fact that ΔE_{int} only depends on the initial and final states, and not on the details of the "path" between them, means we can write

$$\Delta E_{\text{int}} = E_{\text{int }c} - E_{\text{int }a} = -5000\ \text{J}$$

for the indirect path, too. In this case, the work done consists of that done during the constant pressure part (the horizontal line in the graph) plus that done during the constant volume part (the vertical line):

$$W_{\text{indirect}} = (5000\,\text{Pa})\left(2.0\,\text{m}^3\right) + 0 = 10000\ \text{J}\ .$$

Now, the first law of thermodynamics leads to

$$Q_{\text{indirect}} = \Delta E_{\text{int}} + W_{\text{indirect}} = -5000 + 10000 = 5000\ \text{J}\ .$$

49. Argon is a monatomic gas, so $f = 3$ in Eq. 20-51, which provides

$$C_V = \left(\frac{3}{2}\right)R = \left(\frac{3}{2}\right)\left(8.31\,\frac{\text{J}}{\text{mol}\cdot\text{K}}\right)\left(\frac{1\,\text{cal}}{4.186\,\text{J}}\right) = 2.98\,\frac{\text{cal}}{\text{mol}\cdot\text{C}^\circ}$$

where we have converted Joules to calories (Eq. 19-12), and taken advantage of the fact that a Celsius degree is equivalent to a unit change on the Kelvin scale. Since (for a given substance) M is effectively a conversion factor between grams and moles, we see that c_V (see units specified in the problem statement) is related to C_V by

$$C_V = c_V M \qquad \text{where}\ \ M = mN_{\text{A}}$$

where m is the mass of a single atom (see Eq. 20-4).

(a) From the above discussion, we obtain

$$m = \frac{M}{N_{\text{A}}} = \frac{C_V/c_V}{N_{\text{A}}} = \frac{2.98/0.075}{6.02 \times 10^{23}} = 6.6 \times 10^{-23}\ \text{g}\ .$$

(b) The molar mass is found to be $M = C_V/c_V = 2.98/0.075 = 39.7$ g/mol which should be rounded to 40 since the given value of c_V is specified to only two significant figures.

50. Referring to Table 20-3, Eq. 20-45 and Eq. 20-46, we have

$$\Delta E_{\text{int}} = nC_V\Delta T = \frac{5}{2}nR\Delta T$$
$$\text{and} \qquad Q = nC_p\Delta T = \frac{7}{2}nR\Delta T\ .$$

Dividing the equations, we obtain

$$\frac{\Delta E_{int}}{Q} = \frac{5}{7} \ .$$

Thus, the given value $Q = 70$ J leads to

$$\Delta E_{int} = 50 \text{ J} \ .$$

51. The fact that they rotate but do not oscillate means that the value of f given in Table 20-3 is relevant. Thus, Eq. 20-46 leads to

$$Q = nC_p\Delta T = n\left(\frac{7}{2}R\right)(T_f - T_i) = nRT_i\left(\frac{7}{2}\right)\left(\frac{T_f}{T_i} - 1\right)$$

where $T_i = 273$ K and $n = 1$ mol. The ratio of absolute temperatures is found from the gas law in ratio form (see Sample Problem 20-1). With $p_f = p_i$ we have

$$\frac{T_f}{T_i} = \frac{V_f}{V_i} = 2 \ .$$

Therefore, the energy added as heat is

$$Q = (1 \text{ mol})\left(8.31\,\frac{\text{J}}{\text{mol} \cdot \text{K}}\right)(273 \text{ K})\left(\frac{7}{2}\right)(2 - 1) \approx 8 \times 10^3 \text{ J} \ .$$

52. (a) Using $M = 32.0$ g/mol from Table 20-1 and Eq. 20-3, we obtain

$$n = \frac{M_{sam}}{M} = \frac{12.0\,\text{g}}{32.0\,\text{g/mol}} = 0.375 \text{ mol} \ .$$

(b) This is a constant pressure process with a diatomic gas, so we use Eq. 20-46 and Table 20-3. We note that a change of Kelvin temperature is numerically the same as a change of Celsius degrees.

$$\begin{aligned} Q &= nC_p\Delta T = n\left(\frac{7}{2}R\right)\Delta T \\ &= (0.375 \text{ mol})\left(\frac{7}{2}\right)\left(8.31\,\frac{\text{J}}{\text{mol} \cdot \text{K}}\right)(100 \text{ K}) \\ &= 1.09 \times 10^3 \text{ J} \ . \end{aligned}$$

(c) We could compute a value of ΔE_{int} from Eq. 20-45 and divide by the result from part (b), or perform this manipulation algebraically to show the generality of this answer (that is, many factors will be seen to cancel). We illustrate the latter approach:

$$\frac{\Delta E_{int}}{Q} = \frac{n\left(\frac{5}{2}R\right)\Delta T}{n\left(\frac{7}{2}R\right)\Delta T} = \frac{5}{7} \approx 0.714 \ .$$

53. (a) Since the process is at constant pressure energy transferred as heat to the gas is given by $Q = nC_p\,\Delta T$, where n is the number of moles in the gas, C_p is the molar specific heat at constant pressure, and ΔT is the increase in temperature. For a diatomic ideal gas $C_p = \frac{7}{2}R$. Thus

$$Q = \frac{7}{2}nR\,\Delta T = \frac{7}{2}(4.00 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(60.0 \text{ K}) = 6.98 \times 10^3 \text{ J} \ .$$

(b) The change in the internal energy is given by $\Delta E_{int} = nC_V\,\Delta T$, where C_V is the specific heat at constant volume. For a diatomic ideal gas $C_V = \frac{5}{2}R$, so

$$\Delta E_{int} = \frac{5}{2}nR\,\Delta T = \frac{5}{2}(4.00 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(60.0 \text{ K}) = 4.99 \times 10^3 \text{ J} \ .$$

(c) According to the first law of thermodynamics, $\Delta E_{\text{int}} = Q - W$, so

$$W = Q - \Delta E_{\text{int}} = 6.98 \times 10^3 \text{ J} - 4.99 \times 10^3 \text{ J} = 1.99 \times 10^3 \text{ J} .$$

(d) The change in the total translational kinetic energy is

$$\Delta K = \frac{3}{2} n R \, \Delta T = \frac{3}{2}(4.00 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(60.0 \text{ K}) = 2.99 \times 10^3 \text{ J} .$$

54. (a) We use Eq. 20-54 with $V_f/V_i = \frac{1}{2}$ for the gas (assumed to obey the ideal gas law).

$$p_i V_i^\gamma = p_f V_f^\gamma \quad \Longrightarrow \quad \frac{p_f}{p_i} = \left(\frac{V_i}{V_f}\right)^\gamma = 2^{1.3}$$

which yields $p_f = (2.46)(1.0 \text{ atm}) = 2.5$ atm. Similarly, Eq. 20-56 leads to

$$T_f = T_i \left(\frac{V_i}{V_f}\right)^{\gamma-1} = (273 \text{ K})(1.23) = 336 \text{ K} .$$

(b) We use the gas law in ratio form (see Sample Problem 20-1) and note that when $p_1 = p_2$ then the ratio of volumes is equal to the ratio of (absolute) temperatures. Consequently, with the subscript 1 referring to the situation (of small volume, high pressure, and high temperature) the system is in at the end of part (a), we obtain

$$\frac{V_2}{V_1} = \frac{T_2}{T_1} = \frac{273 \text{ K}}{336 \text{ K}} = 0.81 .$$

The volume V_1 is half the original volume of one liter, so

$$V_2 = 0.81(0.50 \text{ L}) = 0.41 \text{ L} .$$

55. (a) Let p_i, V_i, and T_i represent the pressure, volume, and temperature of the initial state of the gas. Let p_f, V_f, and T_f represent the pressure, volume, and temperature of the final state. Since the process is adiabatic $p_i V_i^\gamma = p_f V_f^\gamma$, so

$$p_f = \left(\frac{V_i}{V_f}\right)^\gamma p_i = \left(\frac{4.3 \text{ L}}{0.76 \text{ L}}\right)^{1.4} (1.2 \text{ atm}) = 13.6 \text{ atm} .$$

We note that since V_i and V_f have the same units, their units cancel and p_f has the same units as p_i.

(b) The gas obeys the ideal gas law $pV = nRT$, so $p_i V_i/p_f V_f = T_i/T_f$ and

$$T_f = \frac{p_f V_f}{p_i V_i} T_i = \left[\frac{(13.6 \text{ atm})(0.76 \text{ L})}{(1.2 \text{ atm})(4.3 \text{ L})}\right] (310 \text{ K}) = 620 \text{ K} .$$

56. The fact that they rotate but do not oscillate means that the value of f given in Table 20-3 is relevant. In §20-11, it is noted that $\gamma = C_p/C_V$ so that we find $\gamma = 7/5$ in this case. In the state described in the problem, the volume is

$$V = \frac{nRT}{p} = \frac{(2.0 \text{ mol}) \left(8.31 \frac{\text{J}}{\text{mol} \cdot \text{K}}\right)(300 \text{ K})}{1.01 \times 10^5 \text{ N/m}^2}$$

which yields $V = 0.049 \text{ m}^3$. Consequently,

$$pV^\gamma = \left(1.01 \times 10^5 \text{ N/m}^2\right)\left(0.049 \text{ m}^3\right)^{1.4} = 1.5 \times 10^3 \text{ N} \cdot \text{m}^{2.2} .$$

57. We use the first law of thermodynamics: $\Delta E_{\text{int}} = Q - W$. The change in internal energy is $\Delta E_{\text{int}} = nC_V(T_2 - T_1)$, where C_V is the molar heat capacity for a constant volume process. Since the process is adiabatic $Q = 0$. Thus, $W = -\Delta E_{\text{int}} = nC_V(T_1 - T_2)$.

58. (a) Differentiating Eq. 20-53, we obtain

$$\frac{dp}{dV} = (\text{constant})\frac{-\gamma}{V^{\gamma+1}} \implies B = --V\frac{dp}{dV} = (\text{constant})\frac{\gamma}{V^\gamma}$$

which produces the desired result upon using Eq. 20-53 again ($p = (\text{constant})/V^\gamma$).

(b) Due to the fact that $v = \sqrt{B/\rho}$ (from Chapter 18) and $p = nRT/V = (M_{\text{sam}}/M)RT/V$ (from this chapter) with $\rho = M_{\text{sam}}/V$ (the definition of density), the speed of sound in an ideal gas becomes

$$v = \sqrt{\frac{\gamma p}{\rho}} = \sqrt{\frac{\gamma (M_{\text{sam}}/M)RT/V}{M_{\text{sam}}/V}} = \sqrt{\frac{\gamma RT}{M}} .$$

59. With $p = 1.01 \times 10^5$ Pa and $\rho = 1.29$ kg/m^3, we use the result of part (b) of the previous problem to obtain

$$\gamma = \frac{\rho v^2}{p} = \frac{(1.29\,\text{kg/m}^3)\,(331\,\text{m/s})^2}{1.01 \times 10^5\,\text{Pa}} = 1.40 .$$

60. (a) In the free expansion from state 0 to state 1 we have $Q = W = 0$, so $\Delta E_{\text{int}} = 0$, which means that the temperature of the ideal gas has to remain unchanged. Thus the final pressure is

$$p_1 = \frac{p_0 V_0}{V_1} = \frac{p_0 V_0}{3V_0} = \frac{1}{3}p_0 .$$

(b) For the adiabatic process from state 1 to 2 we have $p_1 V_1^\gamma = p_2 V_2^\gamma$, i.e.,

$$\frac{1}{3}p_0(3V_0)^\gamma = (3.00)^{\frac{1}{3}}p_0 V_0^\gamma$$

which gives $\gamma = 4/3$. The gas is therefore polyatomic.

(c) From $T = pV/nR$ we get

$$\frac{\bar{K}_2}{\bar{K}_1} = \frac{T_2}{T_1} = \frac{p_2}{p_1} = (3.00)^{\frac{1}{3}} = 1.44 .$$

61. In the following $C_V = \frac{3}{2}R$ is the molar specific heat at constant volume, $C_p = \frac{5}{2}R$ is the molar specific heat at constant pressure, ΔT is the temperature change, and n is the number of moles.

(a) The process $1 \to 2$ takes place at constant volume. The heat added is

$$\begin{aligned} Q &= nC_V \Delta T = \frac{3}{2}nR\Delta T \\ &= \frac{3}{2}(1.00\,\text{mol})(8.31\,\text{J/mol·K})(600\,\text{K} - 300\,\text{K}) = 3.74 \times 10^3\,\text{J} . \end{aligned}$$

Since the process takes place at constant volume the work W done by the gas is zero, and the first law of thermodynamics tells us that the change in the internal energy is

$$\Delta E_{\text{int}} = Q = 3.74 \times 10^3\,\text{J} .$$

The process $2 \to 3$ is adiabatic. The heat added is zero. The change in the internal energy is

$$\begin{aligned} \Delta E_{\text{int}} &= nC_V \Delta T = \frac{3}{2}nR\Delta T \\ &= \frac{3}{2}(1.00\,\text{mol})(8.31\,\text{J/mol} \cdot \text{K})(455\,\text{K} - 600\,\text{K}) = -1.81 \times 10^3\,\text{J} . \end{aligned}$$

According to the first law of thermodynamics the work done by the gas is

$$W = Q - \Delta E_{\text{int}} = +1.81 \times 10^3 \text{ J} \ .$$

The process $3 \to 1$ takes place at constant pressure. The heat added is

$$
\begin{aligned}
Q &= nC_p \, \Delta T = \frac{5}{2} nR \, \Delta T \\
&= \frac{5}{2}(1.00 \, \text{mol})(8.31 \, \text{J/mol} \cdot \text{K})(300 \, \text{K} - 455 \, \text{K}) = -3.22 \times 10^3 \text{ J} \ .
\end{aligned}
$$

The change in the internal energy is

$$
\begin{aligned}
\Delta E_{\text{int}} &= nC_V \, \Delta T = \frac{3}{2} nR \, \Delta T \\
&= \frac{3}{2}(1.00 \, \text{mol})(8.31 \, \text{J/mol} \cdot \text{K})(300 \, \text{K} - 455 \, \text{K}) = -1.93 \times 10^3 \text{ J} \ .
\end{aligned}
$$

According to the first law of thermodynamics the work done by the gas is

$$W = Q - \Delta E_{\text{int}} = -3.22 \times 10^3 \text{ J} + 1.93 \times 10^3 \text{ J} = -1.29 \times 10^3 \text{ J} \ .$$

For the entire process the heat added is

$$Q = 3.74 \times 10^3 \text{ J} + 0 - 3.22 \times 10^3 \text{ J} = 520 \text{ J} \ ,$$

the change in the internal energy is

$$\Delta E_{\text{int}} = 3.74 \times 10^3 \text{ J} - 1.81 \times 10^3 \text{ J} - 1.93 \times 10^3 \text{ J} = 0 \ ,$$

and the work done by the gas is

$$W = 0 + 1.81 \times 10^3 \text{ J} - 1.29 \times 10^3 \text{ J} = 520 \text{ J} \ .$$

(b) We first find the initial volume. Use the ideal gas law $p_1 V_1 = nRT_1$ to obtain

$$V_1 = \frac{nRT_1}{p_1} = \frac{(1.00 \, \text{mol})(8.31 \, \text{J/mol} \cdot \text{K})(300 \, \text{K})}{(1.013 \times 10^5 \, \text{Pa})} = 2.46 \times 10^{-2} \, \text{m}^3 \ .$$

Since $1 \to 2$ is a constant volume process $V_2 = V_1 = 2.46 \times 10^{-2} \, \text{m}^3$. The pressure for state 2 is

$$p_2 = \frac{nRT_2}{V_2} = \frac{(1.00 \, \text{mol})(8.31 \, \text{J/mol} \cdot \text{K})(600 \, \text{K})}{2.46 \times 10^{-2} \, \text{m}^3} = 2.02 \times 10^5 \, \text{Pa} \ .$$

This is equivalent to 1.99 atm. Since $3 \to 1$ is a constant pressure process, the pressure for state 3 is the same as the pressure for state 1: $p_3 = p_1 = 1.013 \times 10^5 \, \text{Pa}$ (1.00 atm). The volume for state 3 is

$$V_3 = \frac{nRT_3}{p_3} = \frac{(1.00 \, \text{mol})(8.31 \, \text{J/mol} \cdot \text{K})(455 \, \text{K})}{1.013 \times 10^5 \, \text{Pa}} = 3.73 \times 10^{-2} \, \text{m}^3 \ .$$

62. We note that $\Delta K = n \left(\frac{3}{2} R \right) \Delta T$ according to the discussion in §20-5 and §20-9. Also, $\Delta E_{\text{int}} = nC_V \Delta T$ can be used for each of these processes (since we are told this is an ideal gas). Finally, we note that Eq. 20-49 leads to $C_p = C_V + R \approx 8.0 \, \text{cal/mol} \cdot \text{K}$ after we convert Joules to calories in the ideal gas constant value (Eq. 20-6): $R \approx 2.0 \, \text{cal/mol} \cdot \text{K}$. The first law of thermodynamics $Q = \Delta E_{\text{int}} + W$ applies to each process.

- Constant volume process with $\Delta T = 50$ K and $n = 3.0$ mol.
  $\Delta K = (3.0) \left(\frac{3}{2}(2.0) \right) (50) = 450$ cal
  $\Delta E_{\text{int}} = (3.0)(6.00)(50) = 900$ cal
  $W = 0$ for constant volume processes since the application of force (from the pressure) is not associated with an displacements (see §7-2 and §7-3).
  The first law gives $Q = 900 + 0 = 900$ cal.

- Constant pressure process with $\Delta T = 50$ K and $n = 3.0$ mol.
  $\Delta K = (3.0)\left(\frac{3}{2}(2.0)\right)(50) = 450$ cal.
  $\Delta E_{int} = (3.0)(6.00)(50) = 900$ cal
  $W = p\Delta V$ for constant pressure processes, so (using the ideal gas law) $W = nR\Delta T = (3.0)(2.0)(50) = 300$ cal.
  The first law gives $Q = 900 + 300 = 1200$ cal.

- Adiabatic process with $\Delta T = 50$ K and $n = 3.0$ mol.
  $\Delta K = (3.0)\left(\frac{3}{2}(2.0)\right)(50) = 450$ cal.
  $\Delta E_{int} = (3.0)(6.00)(50) = 900$ cal
  $Q = 0$ by definition of "adiabatic."
  The first law leads to $W = Q - E_{int} = 0 - 900 = -900$ cal.

63. (a) We use $p_i V_i^\gamma = p_f V_f^\gamma$ to compute γ:

$$\gamma = \frac{\log(p_i/p_f)}{\log(V_f/V_i)} = \frac{\log(1.0\,\text{atm}/1.0 \times 10^5\,\text{atm})}{\log(1.0 \times 10^3\,\text{L}/1.0 \times 10^6\,\text{L})} = \frac{5}{3} \ .$$

Therefore the gas is monatomic.

(b) Using the gas law in ratio form (see Sample Problem 20-1), the final temperature is

$$T_f = T_i \frac{p_f V_f}{p_i V_i} = (273\,\text{K})\frac{(1.0 \times 10^5\,\text{atm})(1.0 \times 10^3\,\text{L})}{(1.0\,\text{atm})(1.0 \times 10^6\,\text{L})} = 2.7 \times 10^4\,\text{K} \ .$$

(c) The number of moles of gas present is

$$n = \frac{p_i V_i}{RT_i} = \frac{(1.01 \times 10^5\,\text{Pa})\left(1.0 \times 10^3\,\text{cm}^3\right)}{\left(8.31\,\frac{\text{J}}{\text{mol K}}\right)(273\,\text{K})} = 4.5 \times 10^4\,\text{mol} \ .$$

(d) The total translational energy per mole before the compression is

$$K_i = \frac{3}{2}RT_i = \frac{3}{2}\left(8.31\,\frac{\text{J}}{\text{mol} \cdot \text{K}}\right)(273\,\text{K}) = 3.4 \times 10^3\,\text{J} \ .$$

After the compression,

$$K_f = \frac{3}{2}RT_f = \frac{3}{2}\left(8.31\,\frac{\text{J}}{\text{mol} \cdot \text{K}}\right)\left(2.7 \times 10^4\,\text{K}\right) = 3.4 \times 10^5\,\text{J} \ .$$

(e) Since $v_{\text{rms}}^2 \propto T$, we have

$$\frac{v_{\text{rms},i}^2}{v_{\text{rms},f}^2} = \frac{T_i}{T_f} = \frac{273\,\text{K}}{2.7 \times 10^4\,\text{K}} = 0.01 \ .$$

64. (a) For the isothermal process the final temperature of the gas is $T_f = T_i = 300$ K. The final pressure is

$$p_f = \frac{p_i V_i}{V_f} = \frac{(32\,\text{atm})(1.0\,\text{L})}{4.0\,\text{L}} = 8.0\,\text{atm} \ ,$$

and the work done is

$$\begin{aligned}
W &= nRT_i \ln\left(\frac{V_f}{V_i}\right) = p_i V_i \ln\left(\frac{V_f}{V_i}\right) \\
&= (32\,\text{atm})(1.01 \times 10^5\,\text{Pa/atm})(1.0 \times 10^{-3}\,\text{m}^3)\ln\left(\frac{4.0\,\text{L}}{1.0\,\text{L}}\right) \\
&= 4.4 \times 10^3\,\text{J} \ .
\end{aligned}$$

(b) For the adiabatic process $p_i V_i^\gamma = p_f V_f^\gamma$. Thus,

$$
\begin{aligned}
p_f &= p_f \left(\frac{V_i}{V_f}\right)^\gamma = (32\,\text{atm}) \left(\frac{1.0\,\text{L}}{4.0\,\text{L}}\right)^{5/3} = 3.2\,\text{atm} , \\
T_f &= \frac{p_f V_f T_i}{p_i V_i} = \frac{(3.2\,\text{atm})(4.0\,\text{L})(300\,\text{K})}{(32\,\text{atm})(1.0\,\text{L})} = 120\,\text{K} , \quad \text{and} \\
W &= Q - \Delta E_{\text{int}} = -\Delta E_{\text{int}} = -\frac{3}{2} n R \Delta T = -\frac{3}{2}(p_f V_f - p_i V_i) \\
&= -\frac{3}{2}[(3.2\,\text{atm})(4.0\,\text{L}) - (32\,\text{atm})(1.0\,\text{L})](1.01 \times 10^5\,\text{Pa/atm})(10^{-3}\,\text{m}^3/\text{L}) \\
&= 2.9 \times 10^3 \text{ J} .
\end{aligned}
$$

(c) Now, $\gamma = 1.4$ so

$$
\begin{aligned}
p_f &= p_i \left(\frac{V_i}{V_f}\right)^\gamma = (32\,\text{atm}) \left(\frac{1.0\,\text{L}}{4.0\,\text{L}}\right)^{1.4} = 4.6\text{ atm} , \\
T_f &= \frac{p_f V_f T_i}{p_i V_i} = \frac{(4.6\,\text{atm})(4.0\,\text{L})(300\,\text{K})}{(32\,\text{atm})(1.0\,\text{L})} = 170\text{ K} , \quad \text{and} \\
W &= Q - \Delta E_{\text{int}} = -\frac{5}{2} n R \Delta T = -\frac{5}{2}(p_f V_f - p_i V_i) \\
&= -\frac{5}{2}[(4.6\,\text{atm})(4.0\,\text{L}) - (32\,\text{atm})(1.0\,\text{L})](1.01 \times 10^5\,\text{Pa/atm})(10^{-3}\,\text{m}^3/\text{L}) \\
&= 3.4 \times 10^3 \text{ J} .
\end{aligned}
$$

65. We label the various states of the ideal gas as follows: it starts expanding adiabatically from state 1 until it reaches state 2, with $V_2 = 4\,\text{m}^3$; then continues onto state 3 isothermally, with $V_3 = 10\,\text{m}^3$; and eventually getting compressed adiabatically to reach state 4, the final state. For the adiabatic process $1 \to 2$ $p_1 V_1^\gamma = p_2 V_2^\gamma$, for the isothermal process $2 \to 3$ $p_2 V_2 = p_3 V_3$, and finally for the adiabatic process $3 \to 4$ $p_3 V_3^\gamma = p_4 V_4^\gamma$. These equations yield

$$
p_4 = p_3 \left(\frac{V_3}{V_4}\right)^\gamma = p_2 \left(\frac{V_2}{V_3}\right)\left(\frac{V_3}{V_4}\right)^\gamma = p_1 \left(\frac{V_1}{V_2}\right)^\gamma \left(\frac{V_2}{V_3}\right)\left(\frac{V_3}{V_4}\right)^\gamma .
$$

We substitute this expression for p_4 into the equation $p_1 V_1 = p_4 V_4$ (since $T_1 = T_4$) to obtain $V_1 V_3 = V_2 V_4$. Solving for V_4 we obtain

$$
V_4 = \frac{V_1 V_3}{V_2} = \frac{(2\,\text{m}^3)(10\,\text{m}^3)}{4\,\text{m}^3} = 5 \text{ m}^3 .
$$

66. We use the result of exercise 58 to set up the ratio

$$
\frac{v_1}{v_2} = \frac{\sqrt{\gamma_1 RT/M_1}}{\sqrt{\gamma_2 RT/M_2}} = \sqrt{\frac{M_2}{M_1}} \quad \text{if } \gamma_1 = \gamma_2 .
$$

That final condition (equality of the γ's) is reasonable if we are comparing diatomic gas to diatomic gas, or monatomic gas to monatomic gas. That is, all diatomic gases have $\gamma = 1.4$ (or very nearly so), and all monatomic gases have $\gamma \approx 1.7$.

67. (a) We use the result of exercise 58 to express γ in terms of the speed of sound $v = f\lambda$.

$$
\gamma = \frac{Mv^2}{RT} = \frac{M\lambda^2 f^2}{RT} .
$$

The distance between nodes is half of a wavelength $\lambda = 2 \times 0.0677\,\text{m}$, and the molar mass in SI units is $M = 0.127$ kg/mol. Consequently,

$$
\gamma = \frac{(0.127)(2 \times 0.0677)^2(1400)^2}{(8.31)(400)} = 1.37 .
$$

(b) Using Table 20-3, we find $\gamma = 5/3 \approx 1.7$ for monatomic gases, $\gamma = 7/5 = 1.4$ for diatomic gases, and $\gamma = 4/3 \approx 1.3$ for polyatomic gases. Our result in part (a) suggests that iodine is a diatomic gas.

68. We assume this to be an ideal gas, so that $C_p = C_V + R = 6.0R$. Therefore, $\gamma = C_p/C_V = 1.2$, and the result of exercise 58 divided by Eq. 20-2 becomes

$$\frac{v_s}{v_{\text{rms}}} = \frac{\sqrt{\gamma RT/M}}{\sqrt{3RT/M}} = \sqrt{\frac{\gamma}{3}} = \sqrt{0.40} = 0.63 .$$

69. The initial data concerning the balloon is indicated by the subscript 1. As in Sample Problem 1, we use the gas law in ratio form:

$$\frac{p_1 V_1}{p_2 V_2} = \frac{T_1}{T_2} \implies V_2 = (2.2\,\text{m}^3)\left(\frac{760\,\text{torr}}{380\,\text{torr}}\right)\left(\frac{225\,\text{K}}{293\,\text{K}}\right) = 3.4\,\text{m}^3 .$$

70. (a) We use $pV = nRT$. The volume of the tank is

$$V = \frac{nRT}{p} = \frac{\left(\frac{300\,\text{g}}{17\,\text{g/mol}}\right)(8.31\,\frac{\text{J}}{\text{mol·K}})(350\,\text{K})}{1.35 \times 10^6\,\text{Pa}}$$
$$= 3.8 \times 10^{-2}\,\text{m}^3 = 38\,\text{L} .$$

(b) The number of moles of the remaining gas is

$$n' = \frac{p'V}{RT'} = \frac{(8.7 \times 10^5\,\text{Pa})(3.8 \times 10^{-2}\,\text{m}^3)}{(8.31\,\frac{\text{J}}{\text{mol·K}})(293\,\text{K})} = 13.5\,\text{mol} .$$

The mass of the gas that leaked out is then $\Delta m = 300\,\text{g} - (13.5\,\text{mol})(17\,\text{g/mol}) = 71\,\text{g}$.

71. (a) Since an ideal gas is involved, then $\Delta E_{\text{int}} = 0$ implies $T_1 = T_0$ (see Eq. 20-62). Consequently, the ideal gas law leads to

$$p_1 = p_0\left(\frac{V_0}{V_1}\right) = \frac{p_0}{5}$$

for the pressure at the end of the sudden expansion. Now, the (slower) adiabatic process is described by Eq. 20-54:

$$p_2 = p_1\left(\frac{V_1}{V_2}\right)^{\gamma} = p_1\,(5^{\gamma})$$

as a result of the fact that $V_2 = V_0$. Therefore,

$$p_2 = \left(\frac{p_0}{5}\right)(5^{\gamma}) = \left(5^{\gamma-1}\right)p_0$$

which is compared with the problem requirement that $p_2 = 5^{0.4}\,p_0$. Thus, we find that $\gamma = 1.4 = \frac{7}{5}$. Since $\gamma = C_p/C_V$, we see from Table 20-3 that this is a diatomic gas with rotation of the molecules.

(b) The direct connection between E_{int} and K_{avg} is explained at the beginning of §20-8. Since $\Delta E_{\text{int}} = 0$ in the free expansion, then $K_1 = K_0$.

(c) In the (slower) adiabatic process, Eq. 20-56 indicates

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = 5^{0.4}\,T_0 \implies \frac{(E_{\text{int}})_2}{(E_{\text{int}})_0} = \frac{T_2}{T_0} = 5^{0.4} \approx 1.9 .$$

Therefore, $K_2 = 1.9 K_0$.

72. A molecule with speed v_{avg} will (typically) suffer a collision after a time $t = \lambda/v_{\text{avg}}$ by definition of the mean free path λ. Thus, we think of $1/t$ as the collision frequency f and use Eq. 20-25.

$$f = \frac{v_{\text{avg}}}{\lambda} = \frac{v_{\text{avg}}}{\frac{1}{\pi\sqrt{2}d^2 N/V}} = \pi\sqrt{2}d^2 v_{\text{avg}} \left(\frac{N}{V}\right) \ .$$

73. From Table 20-3, $C_V = \frac{3}{2}R = 12.5 \frac{\text{J}}{\text{mol·K}}$ for a monatomic gas such as helium. To obtain the desired result c_V we need to effectively "convert" mol $\rightarrow$ kg, which can be done using the molar mass M expressed in kilograms per mole. Although we could look up M for helium in Table 20-1 or Appendix F, the problem gives us m so that we can use Eq. 20-4 to find M. That is,

$$M = mN_A = \left(6.66 \times 10^{-27}\,\text{kg}\right)\left(6.02 \times 10^{23}/\text{mol}\right) = 4.01 \times 10^{-3}\,\frac{\text{kg}}{\text{mol}} \ .$$

Therefore, $c_V = C_V/M = 3.11 \times 10^3\,\text{J/kg·K}$.

74. (a) When $n = 1$, $V = V_m = RT/p$, where V_m is the molar volume of the gas. So

$$V_m = \frac{RT}{p} = \frac{(8.31\,\text{J/mol·K})(273.15\,\text{K})}{1.01 \times 10^5\,\text{Pa}} = 22.5\,\text{L} \ .$$

(b) We use $v_{\text{rms}} = \sqrt{3RT/M}$. The ratio is given by

$$\frac{v_{\text{rms,He}}}{v_{\text{rms,Ne}}} = \sqrt{\frac{M_{\text{Ne}}}{M_{\text{He}}}} = \sqrt{\frac{20\,\text{g}}{4.0\,\text{g}}} = 2.25 \ .$$

(c) We use $\lambda_{\text{He}} = (\sqrt{2}\pi d^2 N/V)^{-1}$, where the number of particles per unit volume is given by $N/V = N_A n/V = N_A p/RT = p/kT$. So

$$
\begin{aligned}
\lambda_{\text{He}} &= \frac{1}{\sqrt{2}\pi d^2 (p/kT)} = \frac{kT}{\sqrt{2}\pi d^2 p} \\
&= \frac{(1.38 \times 10^{-23}\,\text{J/K})(273.15\,\text{K})}{1.414\pi(1 \times 10^{-10}\,\text{m})^2(1.01 \times 10^5\,\text{Pa})} = 0.84\,\mu\text{m} \ .
\end{aligned}
$$

(d) $\lambda_{\text{Ne}} = \lambda_{\text{He}} = 0.84\,\mu\text{m}$.

75. Since ΔE_{int} does not depend on the type of process,

$$(\Delta E_{\text{int}})_{\text{path 2}} = (\Delta E_{\text{int}})_{\text{path 1}} \ .$$

Also, since (for an ideal gas) it only depends on the temperature variable (so $\Delta E_{\text{int}} = 0$ for isotherms), then

$$(\Delta E_{\text{int}})_{\text{path 1}} = \sum (\Delta E_{\text{int}})_{\text{adiabat}} \ .$$

Finally, since $Q = 0$ for adiabatic processes, then (for path 1)

$$
\begin{aligned}
(\Delta E_{\text{int}})_{\text{adiabatic expansion}} &= -W = -40\,\text{J} \qquad \text{and} \\
(\Delta E_{\text{int}})_{\text{adiabatic compression}} &= -W = -(-25) = 25\,\text{J} \ .
\end{aligned}
$$

Therefore,

$$(\Delta E_{\text{int}})_{\text{path 2}} = -40\,\text{J} + 25\,\text{J} = -15\,\text{J} \ .$$

76. For convenience, the "int" subscript for the internal energy will be omitted in this solution. Recalling Eq. 19-28, we note that

$$\sum_{\text{cycle}} E \;=\; 0$$

$$\Delta E_{A\rightarrow B} + \Delta E_{B\rightarrow C} + \Delta E_{C\rightarrow D} + \Delta E_{D\rightarrow E} + \Delta E_{E\rightarrow A} \;=\; 0 \;.$$

Since a gas is involved (assumed to be ideal), then the internal energy does not change when the temperature does not change, so

$$\Delta E_{A\rightarrow B} = \Delta E_{D\rightarrow E} = 0 \;.$$

Now, with $\Delta E_{E\rightarrow A} = 8.0$ J given in the problem statement, we have

$$\Delta E_{B\rightarrow C} + \Delta E_{C\rightarrow D} + 8.0 = 0 \;.$$

In an adiabatic process, $\Delta E = -W$, which leads to

$$-5.0 + \Delta E_{C\rightarrow D} + 8.0 = 0 \;,$$

and we obtain $\Delta E_{C\rightarrow D} = -3.0$ J.

77. We solve

$$\sqrt{\frac{3RT}{M_{\text{helium}}}} = \sqrt{\frac{3R(293\,\text{K})}{M_{\text{hydrogen}}}}$$

for T. With the molar masses found in Table 20-1, we obtain

$$T = (293\,\text{K}) \left(\frac{4.0}{2.02} \right) = 580 \text{ K}$$

which is equivalent to 307°C.

78. It is straightforward to show, from Eq. 20-11, that for any process that is depicted as a straight line on the pV diagram, the work is

$$W_{\text{straight}} = \left(\frac{p_i + p_f}{2} \right) \Delta V$$

which includes, as special cases, $W = p\Delta V$ for constant-pressure processes and $W = 0$ for constant-volume processes. Also, from the ideal gas law in ratio form (see Sample Problem 1), we find the final temperature:

$$T_2 = T_1 \left(\frac{p_2}{p_1} \right) \left(\frac{V_2}{V_1} \right) = 4T_1 \;.$$

(a) With $\Delta V = V_2 - V_1 = 2V_1 - V_1 = V_1$ and $p_1 + p_2 = p_1 + 2p_1 = 3p_1$, we obtain

$$W_{\text{straight}} = \frac{3}{2} \left(p_1 V_1 \right) = \frac{3}{2} nRT_1$$

where the ideal gas law is used in that final step.

(b) With $\Delta T = T_2 - T_1 = 4T_1 - T_1 = 3T_1$ and $C_V = \frac{3}{2}R$, we find

$$\Delta E_{\text{int}} = n \left(\frac{3}{2}R \right) (3T_1) = \frac{9}{2} nRT_1 \;.$$

(c) The energy added as heat is $Q = \Delta E_{\text{int}} + W_{\text{straight}} = 6nRT_1$.

(d) The molar specific heat for this process may be defined by

$$C_{\text{straight}} = \frac{Q}{n\Delta T} = \frac{6nRT_1}{n\,(3T_1)} = 2R \;.$$

79. (a) The ideal gas law leads to

$$V = \frac{nRT}{p} = \frac{(1.00\,\text{mol})\left(8.31\,\frac{\text{J}}{\text{mol·K}}\right)(273\,\text{K})}{1.01 \times 10^5\,\text{Pa}}$$

which yields $V = 0.0225\,\text{m}^3 = 22.5$ L. If we use the standard pressure value given in Appendix D, $1\,\text{atm} = 1.013 \times 10^5\,\text{Pa}$, then our answer rounds more properly to 22.4 L.

(b) From Eq. 20-2, we have $N = 6.02 \times 10^{23}$ molecules in the volume found in part (a) (which may be expressed as $V = 2.24 \times 10^4\,\text{cm}^3$), so that

$$\frac{N}{V} = \frac{6.02 \times 10^{23}}{2.24 \times 10^4\,\text{cm}^3} = 2.69 \times 10^{19}\ \text{molecules/cm}^3 \ .$$

80. The gas law in ratio form (see Sample Problem 20-1) leads to

$$p_2 = p_1\left(\frac{V_1}{V_2}\right)\left(\frac{T_2}{T_1}\right) = (5.67\,\text{Pa})\left(\frac{4.00\,\text{m}^3}{7.00\,\text{m}^3}\right)\left(\frac{313\,\text{K}}{217\,\text{K}}\right) = 4.67\,\text{Pa} \ .$$

81. It is recommended to look over §20-7 before doing this problem.

(a) We normalize the distribution function as follows:

$$\int_0^{v_\text{o}} P(v)\,dv = 1 \implies C = \frac{3}{v_\text{o}^3} \ .$$

(b) The average speed is

$$\int_0^{v_\text{o}} v\,P(v)\,dv = \int_0^{v_\text{o}} v\left(\frac{3v^2}{v_\text{o}^3}\right)dv = \frac{3}{4}\,v_\text{o} \ .$$

(c) The rms speed is the square root of

$$\int_0^{v_\text{o}} v^2\,P(v)\,dv = \int_0^{v_\text{o}} v^2\left(\frac{3v^2}{v_\text{o}^3}\right)dv = \frac{3}{5}\,v_\text{o}^2 \ .$$

Therefore, $v_\text{rms} = \sqrt{3/5}\,v_\text{o} \approx 0.775v_\text{o}$.

82. (a) From Table 20-3, $C_V = \frac{5}{2}R$ and $C_p = \frac{7}{2}R$. Thus, Eq. 20-46 yields

$$Q = nC_p\Delta T = (3.00)\left(\frac{7}{2}(8.31)\right)(40.0) = 3490\,\text{J} \ .$$

(b) Eq. 20-45 leads to

$$\Delta E_\text{int} = nC_V\Delta T = (3.00)\left(\frac{5}{2}(8.31)\right)(40.0) = 2493 \approx 2490\,\text{J} \ .$$

(c) From either $W = Q - \Delta E_\text{int}$ or $W = p\Delta T = nR\Delta T$, we find $W = 997$ J.

(d) Eq. 20-24 is written in more convenient form (for this problem) in Eq. 20-38. Thus, we obtain

$$\Delta K_\text{trans} = \Delta\left(NK_\text{avg}\right) = n\left(\frac{3}{2}R\right)\Delta T \approx 1500\,\text{J} \ .$$

83. The average kinetic energy is related to the absolute temperature by

$$K_\text{avg} = \frac{3}{2}kT$$

$$4.0 \times 10^{-19}\,\text{J} = \frac{3}{2}\left(1.38 \times 10^{-23}\,\text{J/K}\right)T$$

which yields $T = 19.3 \times 10^3$ K.

84. Using the ideal gas law, one mole occupies a volume equal to

$$V = \frac{nRT}{p} = \frac{(1)(8.31)(50)}{1 \times 10^{-8}} = 4.2 \times 10^{10} \text{ m}^3 \ .$$

Therefore, the number of molecules per unit volume is

$$\frac{N}{V} = \frac{nN_A}{V} = \frac{(1)\left(6.02 \times 10^{23}\right)}{4.2 \times 10^{10}} = 1.4 \times 10^{13} \ \frac{\text{molecules}}{\text{m}^3} \ .$$

Using $d = 20.0 \times 10^{-9}$ m, Eq. 20-25 yields

$$\lambda = \frac{1}{\sqrt{2}\,\pi d^2 \left(\frac{N}{V}\right)} = 39 \text{ m} \ .$$

85. The mass of hot air is $M_{\text{hot}} = nM$ by Eq. 20-3, where the number of moles contained within the envelope is

$$n = \frac{pV}{RT} = \frac{\left(1.01 \times 10^5\right)\left(2.18 \times 10^3\right)}{(8.31)T} = \frac{2.65 \times 10^7}{T}$$

with SI units understood. The magnitude of the gravitational force acting on the balloon is

$$F_g = (M_{\text{envelope}} + M_{\text{basket}} + M_{\text{hot}})\,g = \left(249 + M\frac{2.65 \times 10^7}{T}\right)(9.8)$$

with SI units, again, understood (which implies $M = 0.028$). The problem requires that the buoyant force (equal to the weight of the displaced air of density $\rho = 1.21\,\text{kg/m}^3$) is equal to 2700 N plus the magnitude of the gravitational force. Therefore,

$$\rho V g = 2700 + \left(249 + (0.028)\frac{2.65 \times 10^7}{T}\right)(9.8) \qquad \text{where } V = 2.18 \times 10^3 \ .$$

Solving this for the temperature, we obtain

$$T = \frac{(0.028)\left(2.65 \times 10^7\right)}{\frac{(1.21)(2.18 \times 10^3)(9.8) - 2700}{9.8} - 249} = 351 \text{ K}$$

which is equivalent to 78°.

86. (a) The temperature is $10°\text{C} \rightarrow T = 283$ K. Then, with $n = 3.5$ mol and $V_f/V_0 = 3/4$, we use Eq. 20-14:

$$W = nRT \ln\left(\frac{V_f}{V_0}\right) = -2369 \text{ J} \approx -2.4 \text{ kJ} \ .$$

(b) The internal energy change ΔE_{int} vanishes (for an ideal gas) when $\Delta T = 0$ so that the First Law of Thermodynamics leads to $Q = W = -2.4$ kJ. The negative value implies that the heat transfer is from the sample to its environment.

87. (a) Since $n/V = p/RT$, the number of molecules per unit volume is

$$\frac{N}{V} = \frac{nN_A}{V} = N_A\left(\frac{p}{RT}\right)\left(6.02 \times 10^{23}\right)\frac{1.01 \times 10^5\,\text{Pa}}{\left(8.31\,\frac{\text{J}}{\text{mol K}}\right)(293\,\text{K})} = 2.5 \times 10^{25} \ \frac{\text{molecules}}{\text{m}^3} \ .$$

(b) Three-fourths of the 2.5×10^{25} value found in part (a) are nitrogen molecules with $M = 28.0$ g/mol (using Table 20-1), and one-fourth of that value are oxygen molecules with $M = 32.0$ g/mol. Consequently, we generalize the $M_{\text{sam}} = NM/N_A$ expression for these two species of molecules and write

$$\frac{3}{4}\left(2.5 \times 10^{25}\right)\frac{28.0}{6.02 \times 10^{23}} + \frac{1}{4}\left(2.5 \times 10^{25}\right)\frac{32.0}{6.02 \times 10^{23}} = 1.2 \times 10^3 \text{ g} \ .$$

88.   • Using a ruler, we find the diameter of the period D to be roughly $0.5\,\mathrm{mm}$. Therefore, its area is $A = \pi D^2/4 \approx 2 \times 10^{-7}\,\mathrm{m}^2$. Meanwhile, we estimate the diameter d of an air molecule to be roughly $2 \times 10^{-10}\,\mathrm{m}$ (this is "in the ballpark" of the value used in Sample Problem 20-4). So the area an air molecule covers is $a = \pi d^2/4 \approx 3 \times 10^{-20}\,\mathrm{m}^2$. Thus

$$\frac{A}{a} \approx \frac{2 \times 10^{-7}}{3 \times 10^{-20}} \approx 10^{13} \ .$$

This tells us that rouhgly 10^{13} air molecules are needed to cover the period.

•   Assume that every second there are N air molecules which collide with the period. If each one of them bounces back elastically after the collision then the change in linear momentum per molecule per collision is $2mv_x$, where m is the molecular mass and v_x is the component of the molecular velocity in the direction perpendicular to the surface of the paper containing the period. We take v_x to mean the *average* velocity x-component. Thus, the pressure exerted by the air molecules on the period is

$$p = \frac{2mNv_x}{A\Delta t} \qquad \text{where} \ \ \Delta t = 1\,\mathrm{s}$$

and $v_x \approx v_{\mathrm{rms}}/\sqrt{3}$ (see the discussion immediately preceding Eq. 20-20). Also we have $m = M/N_A$, where M is the average molar mass of the air molecules. We solve for N:

$$\begin{aligned} N &= \frac{\sqrt{3}pAN_A\Delta t}{2Mv_{\mathrm{rms}}} = \frac{pAN_A\Delta t}{2\sqrt{MRT}} \\ &= \frac{(1.01 \times 10^5\,\mathrm{Pa})(2 \times 10^{-7}\,\mathrm{m}^2)(6.02 \times 10^{23}/\mathrm{mol})(1\,\mathrm{s})}{2\sqrt{(0.028\,\mathrm{kg/mol})\left(8.31\,\frac{\mathrm{J}}{\mathrm{mol \cdot K}}\right)(300\,\mathrm{K})}} \approx 7 \times 10^{20} \ . \end{aligned}$$

89.   (a) The work done in a constant-pressure process is $W = p\Delta V$. Therefore,

$$W = \left(25\,\mathrm{N/m}^2\right)\left(1.8\,\mathrm{m}^3 - 3.0\,\mathrm{m}^3\right) = -30 \ \mathrm{J} \ .$$

The sign conventions discussed in the textbook for Q indicate that we should write -75 J for the energy which leaves the system in the form of heat. Therefore, the first law of thermodynamics leads to

$$\Delta E_{\mathrm{int}} = Q - W = (-75\,\mathrm{J}) - (-30\,\mathrm{J}) = -45 \ \mathrm{J} \ .$$

(b) Since the pressure is constant (and the number of moles is presumed constant), the ideal gas law in ratio form (see Sample Problem 20-1) leads to

$$T_2 = T_1 \left(\frac{V_2}{V_1}\right) = (300\,\mathrm{K})\left(\frac{1.8\,\mathrm{m}^3}{3.0\,\mathrm{m}^3}\right) = 180 \ \mathrm{K} \ .$$

It should be noted that this is consistent with the gas being monatomic (that is, if one assumes $C_V = \frac{3}{2}R$ and uses Eq. 20-45, one arrives at this same value for the final temperature).

90. In a constant-pressure process, the work done is $W = p\Delta V$. Using the ideal gas law (assuming the number of moles is constant) this becomes $W = nR\Delta T$. Therefore,

$$W = (3.00 \ \mathrm{mol})\left(8.31 \ \frac{\mathrm{J}}{\mathrm{mol \cdot K}}\right)(-75K) \approx -1870 \ \mathrm{J} \ .$$

Now, the First Law of Thermodynamics (Eq.19-24) yields

$$\Delta E_{\mathrm{int}} = Q - W = (-4670) - (-1870) = -2800 \ \mathrm{J} \ .$$

91. Since no heat is transferred in an adiabatic process, then

$$Q_{\text{total}} = Q_{\text{isotherm}} = W_{\text{isotherm}} = nRT \ln\left(\frac{3}{12}\right)$$

where the First Law of Thermodynamics (with $\Delta E_{\text{int}} = 0$ during the isothermal process) and Eq. 20-14 have been used. With $n = 2.0$ mol and $T = 300$ K, we obtain $Q = -6912$ J ≈ -6.9 kJ.

92. (a) We recall the sign convention for heat exchange developed in Chapter 19: the value of Q is positive when the system absorbs heat and negative when it releases heat. Thus, in part (a) we have $Q = -300$ kJ, which is used in $Q = c_{\text{liquid}}\, m\, \Delta T$ to produce $\Delta T = -18.75$ C° so that the resulting temperature is $T_{\text{a}} = T_0 + \Delta T = 61.25°$C ("block P").

   (b) With $Q = -400$ kJ in $Q = c_{\text{liquid}}\, m\, \Delta T$, we find $\Delta T = -25$ C°, yielding $T_{\text{b}} = T_{\text{a}} + \Delta T = 36.25°$C ("block S").

   (c) With $Q = -820$ kJ in $Q = c_{\text{liquid}}\, m\, \Delta T$, we find $\Delta T = -51.25$ C°, yielding $T_{\text{c}} = T_{\text{b}} + \Delta T = -15.00°$C ("block X").

   (d) We adapt the change-of-state equation to the sign convention adopted for Q (so that the equation is generally $Q = \pm L\, m$). With $Q = -820$ kJ in $Q = -L_F\, m'$, we find $m' = 1.67$ kg, so that not quite half the material has solidified (still in "block X" at $-15.00°$C).

   (e) As a result of part (d), there is $m'' = m - m' = 4.00 - 1.67 = 2.33$ kg of liquid material which remains to solidify before the system may continue lowering temperature (as a solid). With $Q = -670.0$ kJ in $Q = -L_F\, m'' + c_{\text{solid}}\, m\, \Delta T$, we find $\Delta T = -40$ C°, yielding $T_{\text{e}} = T_{\text{d}} + \Delta T = -55.00°$C ("block BB").

   (f) Now the system is absorbing heat: with $Q = 1240.0$ kJ and $\Delta T' = 40$ C° in $Q = c_{\text{solid}}\, m\, \Delta T' + L_F\, m + c_{\text{liquid}}\, m\, \Delta T$, we find $\Delta T = 20$ C°, yielding $T_{\text{f}} = T_{\text{e}} + \Delta T' + \Delta T = 5.00°$C ("block V").

   (g) With $Q = 1280$ kJ in $Q = c_{\text{liquid}}\, m\, \Delta T$, we find $\Delta T = 80$ C°, yielding $T_{\text{g}} = T_{\text{f}} + \Delta T = 85.00°$C ("block N").

   (h) With $Q = 820.0$ kJ and $\Delta T = 20$ C° in $Q = c_{\text{liquid}}\, m\, \Delta T + L_V\, m'$, we find $m' = 1.00$ kg, so one-fourth of the material has vaporized at $T_{\text{h}} = T_{\text{g}} + \Delta T = 105.0°$C ("block L").

   (i) With $Q = 1000$ kJ in $Q = L_V\, m''$, we find $m'' = 2.00$ kg, so three-fourths of the material has now vaporized at $T_{\text{i}} = T_{\text{h}} = 105.0°$C ("block L").

   (j) We are careful to interpret the given "Molar mass = 3.000" as that of the element (the single atoms), so that for a diatomic gaseous configuration we must use 6.000 g/mol when converting between mass m and moles n. Note that the temperature has not reached the point where rotational modes are excited, so $C_V = (3/2)R$. In the equation that follows, m''' is the mass (at the end of the preceding step) remaining to vaporize (1.000 kg) and $n = m/(6.000) = 666.7$ mol. With $Q = 583.1$ kJ in $Q = L_V\, m''' + nC_V\Delta T$, we find $\Delta T = 10$ C°, yielding $T_{\text{j}} = T_{\text{i}} + \Delta T = 115.0°$C ("block K").

   (k) With $Q = 166.2$ kJ in $Q = nC_V\Delta T$, we find $\Delta T = 20$ C°, yielding $T_{\text{k}} = T_{\text{j}} + \Delta T = 135.0°$C ("block I").

   (l) Note that the temperature is now in the range where rotational modes are excited, so $C_V = (5/2)R$. With $Q = 277.0$ kJ in $Q = nC_V\Delta T$, we find $\Delta T = 20$ C°, yielding $T_{\text{l}} = T_{\text{k}} + \Delta T = 155.0°$C ("block G").

   (m) With the temperature in the range where rotational modes are excited *and* expanding at constant pressure, we have $C_p = (7/2)R$. With $Q = 581.7$ kJ in $Q = nC_p\Delta T$, we find $\Delta T = 30$ C°, yielding $T_{\text{m}} = T_{\text{l}} + \Delta T = 185.0°$ C ("block D").

   (n) Finally, we are in the temperature range where vibrational modes are excited (and expanding at constant pressure), so that we have $C_p = (9/2)R$. With $Q = 249.3$ kJ in $Q = nC_p\Delta T$, we find $\Delta T = 10$ C°, yielding $T_{\text{n}} = T_{\text{m}} + \Delta T = 195.0°$C ("block C").