Chapter 37

1. The condition for a minimum of a single-slit diffraction pattern is

$$a \sin \theta = m\lambda$$

where a is the slit width, λ is the wavelength, and m is an integer. The angle θ is measured from the forward direction, so for the situation described in the problem, it is $0.60°$ for $m = 1$. Thus

$$a = \frac{m\lambda}{\sin\theta} = \frac{633 \times 10^{-9}\,\mathrm{m}}{\sin 0.60°} = 6.04 \times 10^{-5}\ \mathrm{m}\ .$$

2. (a) $\theta = \sin^{-1}(1.50\,\mathrm{cm}/2.00\,\mathrm{m}) = 0.430°$.

   (b) For the mth diffraction minimum $a \sin\theta = m\lambda$. We solve for the slit width:

$$a = \frac{m\lambda}{\sin\theta} = \frac{2(441\,\mathrm{nm})}{\sin 0.430°} = 0.118\ \mathrm{mm}\ .$$

3. (a) The condition for a minimum in a single-slit diffraction pattern is given by $a\sin\theta = m\lambda$, where a is the slit width, λ is the wavelength, and m is an integer. For $\lambda = \lambda_a$ and $m = 1$, the angle θ is the same as for $\lambda = \lambda_b$ and $m = 2$. Thus $\lambda_a = 2\lambda_b$.

   (b) Let m_a be the integer associated with a minimum in the pattern produced by light with wavelength λ_a, and let m_b be the integer associated with a minimum in the pattern produced by light with wavelength λ_b. A minimum in one pattern coincides with a minimum in the other if they occur at the same angle. This means $m_a\lambda_a = m_b\lambda_b$. Since $\lambda_a = 2\lambda_b$, the minima coincide if $2m_a = m_b$. Consequently, every other minimum of the λ_b pattern coincides with a minimum of the λ_a pattern.

4. (a) We use Eq. 37-3 to calculate the separation between the first ($m_1 = 1$) and fifth ($m_2 = 5$) minima:

$$\Delta y = D\Delta\sin\theta = D\Delta\left(\frac{m\lambda}{a}\right) = \frac{D\lambda}{a}\Delta m = \frac{D\lambda}{a}(m_2 - m_1)\ .$$

   Solving for the slit width, we obtain

$$a = \frac{D\lambda(m_2 - m_1)}{\Delta y} = \frac{(400\,\mathrm{mm})(550 \times 10^{-6}\,\mathrm{mm})(5-1)}{0.35\,\mathrm{mm}} = 2.5\ \mathrm{mm}\ .$$

   (b) For $m = 1$,

$$\sin\theta = \frac{m\lambda}{a} = \frac{(1)(550 \times 10^{-6}\,\mathrm{mm})}{2.5\,\mathrm{mm}} = 2.2 \times 10^{-4}\ .$$

   The angle is $\theta = \sin^{-1}(2.2 \times 10^{-4}) = 2.2 \times 10^{-4}\,\mathrm{rad}$.

5. (a) A plane wave is incident on the lens so it is brought to focus in the focal plane of the lens, a distance of 70 cm from the lens.

(b) Waves leaving the lens at an angle θ to the forward direction interfere to produce an intensity minimum if $a\sin\theta = m\lambda$, where a is the slit width, λ is the wavelength, and m is an integer. The distance on the screen from the center of the pattern to the minimum is given by $y = D\tan\theta$, where D is the distance from the lens to the screen. For the conditions of this problem,

$$\sin\theta = \frac{m\lambda}{a} = \frac{(1)(590\times10^{-9}\,\text{m})}{0.40\times10^{-3}\,\text{m}} = 1.475\times10^{-3} \ .$$

This means $\theta = 1.475\times10^{-3}$ rad and $y = (70\times10^{-2}\,\text{m})\tan(1.475\times10^{-3}\,\text{rad}) = 1.03\times10^{-3}$ m.

6. Let the first minimum be a distance y from the central axis which is perpendicular to the speaker. Then $\sin\theta = y/(D^2+y^2)^{1/2} = m\lambda/a = \lambda/a$ (for $m=1$). Therefore,

$$\begin{aligned} y &= \frac{D}{\sqrt{(a/\lambda)^2-1}} = \frac{D}{\sqrt{(af/v_s)^2-1}} \\ &= \frac{100\,\text{m}}{\sqrt{[(0.300\,\text{m})(3000\,\text{Hz})/(343\,\text{m/s})]^2-1}} = 41.2 \text{ m} \ . \end{aligned}$$

7. The condition for a minimum of intensity in a single-slit diffraction pattern is $a\sin\theta = m\lambda$, where a is the slit width, λ is the wavelength, and m is an integer. To find the angular position of the first minimum to one side of the central maximum, we set $m=1$:

$$\theta_1 = \sin^{-1}\left(\frac{\lambda}{a}\right) = \sin^{-1}\left(\frac{589\times10^{-9}\,\text{m}}{1.00\times10^{-3}\,\text{m}}\right) = 5.89\times10^{-4} \text{ rad} \ .$$

If D is the distance from the slit to the screen, the distance on the screen from the center of the pattern to the minimum is

$$y_1 = D\tan\theta_1 = (3.00\,\text{m})\tan(5.89\times10^{-4}\,\text{rad}) = 1.767\times10^{-3} \text{ m} \ .$$

To find the second minimum, we set $m=2$:

$$\theta_2 = \sin^{-1}\left(\frac{2(589\times10^{-9}\,\text{m})}{1.00\times10^{-3}\,\text{m}}\right) = 1.178\times10^{-3} \text{ rad} \ .$$

The distance from the center of the pattern to this second minimum is $y_2 = D\tan\theta_2 = (3.00\,\text{m})\tan(1.178\times10^{-3}\,\text{rad}) = 3.534\times10^{-3}\,\text{m}$. The separation of the two minima is $\Delta y = y_2 - y_1 = 3.534\,\text{mm} - 1.767\,\text{mm} = 1.77\,\text{mm}$.

8. We note that nm $= 10^{-9}$ m $= 10^{-6}$ mm. From Eq. 37-4,

$$\Delta\phi = \left(\frac{2\pi}{\lambda}\right)(\Delta x\sin\theta) = \left(\frac{2\pi}{589\times10^{-6}\,\text{mm}}\right)\left(\frac{0.10\,\text{mm}}{2}\right)\sin 30^\circ = 266.7 \text{ rad} \ .$$

This is equivalent to $266.7 - 84\pi = 2.8\,\text{rad} = 160^\circ$.

9. We imagine dividing the original slit into N strips and represent the light from each strip, when it reaches the screen, by a phasor. Then, at the central maximum in the diffraction pattern, we would add the N phasors, all in the same direction and each with the same amplitude. We would find that the intensity there is proportional to N^2. If we double the slit width, we need $2N$ phasors if they are each to have the amplitude of the phasors we used for the narrow slit. The intensity at the central maximum is proportional to $(2N)^2$ and is, therefore, four times the intensity for the narrow slit. The energy reaching the screen per unit time, however, is only twice the energy reaching it per unit time when the narrow slit is in place. The energy is simply redistributed. For example, the central peak is now half as wide and the integral of the intensity over the peak is only twice the analogous integral for the narrow slit.

10. (a) $\theta = \sin^{-1}(0.011\,\text{cm}/3.5\,\text{m}) = 0.18^\circ$.

(b) We use Eq. 37-6:

$$\alpha = \left(\frac{\pi a}{\lambda}\right)\sin\theta = \frac{\pi(0.025\,\text{mm})\sin 0.18^\circ}{538\times 10^{-6}\,\text{mm}} = 0.46 \text{ rad} .$$

(c) Making sure our calculator is in radian mode, Eq. 37-5 yields

$$\frac{I(\theta)}{I_m} = \left(\frac{\sin\alpha}{\alpha}\right)^2 = 0.93 .$$

11. (a) The intensity for a single-slit diffraction pattern is given by

$$I = I_m \frac{\sin^2\alpha}{\alpha^2}$$

where $\alpha = (\pi a/\lambda)\sin\theta$, a is the slit width and λ is the wavelength. The angle θ is measured from the forward direction. We require $I = I_m/2$, so

$$\sin^2\alpha = \frac{1}{2}\alpha^2 .$$

(b) We evaluate $\sin^2\alpha$ and $\alpha^2/2$ for $\alpha = 1.39$ rad and compare the results. To be sure that 1.39 rad is closer to the correct value for α than any other value with three significant digits, we could also try 1.385 rad and 1.395 rad.

(c) Since $\alpha = (\pi a/\lambda)\sin\theta$,

$$\theta = \sin^{-1}\left(\frac{\alpha\lambda}{\pi a}\right) .$$

Now $\alpha/\pi = 1.39/\pi = 0.442$, so

$$\theta = \sin^{-1}\left(\frac{0.442\lambda}{a}\right) .$$

The angular separation of the two points of half intensity, one on either side of the center of the diffraction pattern, is

$$\Delta\theta = 2\theta = 2\sin^{-1}\left(\frac{0.442\lambda}{a}\right) .$$

(d) For $a/\lambda = 1.0$,

$$\Delta\theta = 2\sin^{-1}(0.442/1.0) = 0.916\,\text{rad} = 52.5^\circ ,$$

for $a/\lambda = 5.0$,

$$\Delta\theta = 2\sin^{-1}(0.442/5.0) = 0.177\,\text{rad} = 10.1^\circ ,$$

and for $a/\lambda = 10$,

$$\Delta\theta = 2\sin^{-1}(0.442/10) = 0.0884\,\text{rad} = 5.06^\circ .$$

12. Consider Huygens' explanation of diffraction phenomena. When A is in place only the Huygens' wavelets that pass through the hole get to point P. Suppose they produce a resultant electric field E_A. When B is in place, the light that was blocked by A gets to P and the light that passed through the hole in A is blocked. Suppose the electric field at P is now $\vec{E}_B$. The sum $\vec{E}_A + \vec{E}_B$ is the resultant of all waves that get to P when neither A nor B are present. Since P is in the geometric shadow, this is zero. Thus $\vec{E}_A = -\vec{E}_B$, and since the intensity is proportional to the square of the electric field, the intensity at P is the same when A is present as when B is present.

13. (a) The intensity for a single-slit diffraction pattern is given by

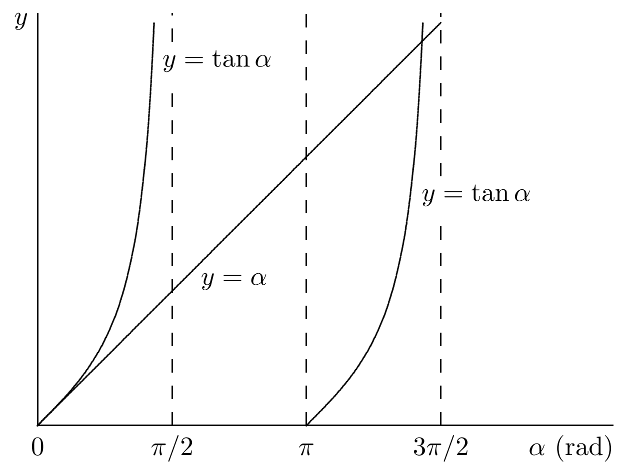
$$I = I_m \frac{\sin^2\alpha}{\alpha^2}$$

where α is described in the text (see Eq. 37-6). To locate the extrema, we set the derivative of I with respect to α equal to zero and solve for α. The derivative is

$$\frac{dI}{d\alpha} = 2I_m \frac{\sin\alpha}{\alpha^3} (\alpha\cos\alpha - \sin\alpha) \ .$$

The derivative vanishes if $\alpha \neq 0$ but $\sin\alpha = 0$. This yields $\alpha = m\pi$, where m is a nonzero integer. These are the intensity minima: $I = 0$ for $\alpha = m\pi$. The derivative also vanishes for $\alpha\cos\alpha - \sin\alpha = 0$. This condition can be written $\tan\alpha = \alpha$. These implicitly locate the maxima.

(b) The values of α that satisfy $\tan\alpha = \alpha$ can be found by trial and error on a pocket calculator or computer. Each of them is slightly less than one of the values $(m + \frac{1}{2})\pi$ rad, so we start with these values. The first few are 0, 4.4934, 7.7252, 10.9041, 14.0662, and 17.2207. They can also be found graphically. As in the diagram below, we plot $y = \tan\alpha$ and $y = \alpha$ on the same graph. The intersections of the line with the $\tan\alpha$ curves are the solutions. The first two solutions listed above are shown on the diagram.



(c) We write $\alpha = (m + \frac{1}{2})\pi$ for the maxima. For the central maximum, $\alpha = 0$ and $m = -\frac{1}{2}$. For the next, $\alpha = 4.4934$ and $m = 0.930$. For the next, $\alpha = 7.7252$ and $m = 1.959$.

14. We use Eq. 37-12 with $\theta = 2.5°/2 = 1.25°$. Thus,

$$d = \frac{1.22\lambda}{\sin\theta} = \frac{1.22(550\,\text{nm})}{\sin 1.25°} = 31\,\mu\text{m} \ .$$

15. (a) We use the Rayleigh criteria. Thus, the angular separation (in radians) of the sources must be at least $\theta_R = 1.22\lambda/d$, where λ is the wavelength and d is the diameter of the aperture. For the headlights of this problem,

$$\theta_R = \frac{1.22(550 \times 10^{-9}\,\text{m})}{5.0 \times 10^{-3}\,\text{m}} = 1.34 \times 10^{-4} \text{ rad} \ .$$

(b) If L is the distance from the headlights to the eye when the headlights are just resolvable and D is the separation of the headlights, then $D = L\theta_R$, where the small angle approximation is made. This is valid for θ_R in radians. Thus,

$$L = \frac{D}{\theta_R} = \frac{1.4\,\text{m}}{1.34 \times 10^{-4}\,\text{rad}} = 1.0 \times 10^4\,\text{m} = 10 \text{ km} \ .$$

16. (a) We use Eq. 37-14:

$$\theta_R = 1.22\,\frac{\lambda}{d} = \frac{(1.22)(540 \times 10^{-6}\,\text{mm})}{5.0\,\text{mm}} = 1.3 \times 10^{-4} \text{ rad} \ .$$

(b) The linear separation is $D = L\theta_R = (160 \times 10^3\,\text{m})(1.3 \times 10^{-4}\,\text{rad}) = 21\,\text{m}$.

17. Using the notation of Sample Problem 37-6 (which is in the textbook supplement), the minimum separation is

$$D = L\theta_R = L\left(1.22\,\frac{\lambda}{d}\right) = \left(3.82 \times 10^8\,\text{m}\right)\frac{(1.22)(550 \times 10^{-9}\,\text{m})}{5.1\,\text{m}} = 50\,\text{m}\ .$$

18. Using the notation of Sample Problem 37-6 (which is in the textbook supplement), the maximum distance is

$$L = \frac{D}{\theta_R} = \frac{D}{1.22\lambda/d} = \frac{(5.0 \times 10^{-3}\,\text{m})(4.0 \times 10^{-3}\,\text{m})}{1.22(550 \times 10^{-9}\,\text{m})} = 30\ \text{m}\ .$$

19. (a) We use the Rayleigh criteria. If L is the distance from the observer to the objects, then the smallest separation D they can have and still be resolvable is $D = L\theta_R$, where θ_R is measured in radians. The small angle approximation is made. Thus,

$$D = \frac{1.22L\lambda}{d} = \frac{1.22(8.0 \times 10^{10}\,\text{m})(550 \times 10^{-9}\,\text{m})}{5.0 \times 10^{-3}\,\text{m}} = 1.1 \times 10^7\,\text{m} = 1.1 \times 10^4\ \text{km}\ .$$

This distance is greater than the diameter of Mars; therefore, one part of the planet's surface cannot be resolved from another part.

(b) Now $d = 5.1\,\text{m}$ and

$$D = \frac{1.22(8.0 \times 10^{10}\,\text{m})(550 \times 10^{-9}\,\text{m})}{5.1\,\text{m}} = 1.1 \times 10^4\,\text{m} = 11\ \text{km}\ .$$

20. Using the notation of Sample Problem 37-6 (which is in the textbook supplement), the minimum separation is

$$D = L\theta_R = L\left(\frac{1.22\lambda}{d}\right) = \frac{(6.2 \times 10^3\,\text{m})(1.22)(1.6 \times 10^{-2}\,\text{m})}{2.3\,\text{m}} = 53\ \text{m}\ .$$

21. Eq. 37-14 gives $\theta_R = 1.22\lambda/d$, where in our case $\theta_R \approx D/L$, with $D = 60\,\mu\text{m}$ being the size of the object your eyes must resolve, and L being the maximum viewing distance in question. If $d = 3.00\,\text{mm} = 3000\,\mu\text{m}$ is the diameter of your pupil, then

$$L = \frac{Dd}{1.22\lambda} = \frac{(60\,\mu\text{m})(3000\,\mu\text{m})}{1.22(0.55\,\mu\text{m})} = 2.7 \times 10^5\ \mu\text{m} = 27\ \text{cm}\ .$$

22. Since we are considering the *diameter* of the central diffraction maximum, then we are working with *twice* the Rayleigh angle. Using notation similar to that in Sample Problem 37-6 (which is in the textbook supplement), we have $2(1.22\lambda/d) = D/L$. Therefore,

$$d = 2\frac{1.22\lambda L}{D} = 2\frac{(1.22)(500 \times 10^{-9}\,\text{m})(3.54 \times 10^5\,\text{m})}{9.1\,\text{m}} = 0.047\ \text{m}\ .$$

23. (a) The first minimum in the diffraction pattern is at an angular position θ, measured from the center of the pattern, such that $\sin\theta = 1.22\lambda/d$, where λ is the wavelength and d is the diameter of the antenna. If f is the frequency, then the wavelength is

$$\lambda = \frac{c}{f} = \frac{3.00 \times 10^8\,\text{m/s}}{220 \times 10^9\,\text{Hz}} = 1.36 \times 10^{-3}\ \text{m}\ .$$

Thus

$$\theta = \sin^{-1}\left(\frac{1.22\lambda}{d}\right) = \sin^{-1}\left(\frac{1.22(1.36 \times 10^{-3}\,\text{m})}{55.0 \times 10^{-2}\,\text{m}}\right) = 3.02 \times 10^{-3}\ \text{rad}\ .$$

The angular width of the central maximum is twice this, or $6.04 \times 10^{-3}\,\text{rad}$ $(0.346°)$.

(b) Now $\lambda = 1.6\,\text{cm}$ and $d = 2.3\,\text{m}$, so

$$\theta = \sin^{-1}\left(\frac{1.22(1.6 \times 10^{-2}\,\text{m})}{2.3\,\text{m}}\right) = 8.5 \times 10^{-3}\ \text{rad}\ .$$

The angular width of the central maximum is $1.7 \times 10^{-2}\,\text{rad}\ (0.97°)$.

24. (a) Since $\theta = 1.22\lambda/d$, the larger the wavelength the larger the radius of the first minimum (and second maximum, etc). Therefore, the white pattern is outlined by red lights (with longer wavelength than blue lights).

(b) The diameter of a water drop is

$$d = \frac{1.22\lambda}{\theta} \approx \frac{1.22(7 \times 10^{-7}\,\text{m})}{1.5(0.50°)(\pi/180°)/2} = 1.3 \times 10^{-4}\ \text{m}\ .$$

25. (a) Using Eq. 37-14, the angular separation is

$$\theta_{\text{R}} = \frac{1.22\lambda}{d} = \frac{(1.22)(550 \times 10^{-9}\,\text{m})}{0.76\,\text{m}} = 8.8 \times 10^{-7}\ \text{rad}\ .$$

(b) Using the notation of Sample Problem 37-6 (which is in the textbook supplement), the distance between the stars is

$$D = L\theta_{\text{R}} = \frac{(10\,\text{ly})(9.46 \times 10^{12}\,\text{km/ly})(0.18)\pi}{(3600)(180)} = 8.4 \times 10^{7}\ \text{km}\ .$$

(c) The diameter of the first dark ring is

$$d = 2\theta_{\text{R}}L = \frac{2(0.18)(\pi)(14\,\text{m})}{(3600)(180)} = 2.5 \times 10^{-5}\,\text{m} = 0.025\ \text{mm}\ .$$

26. We denote the Earth-Moon separation as L. The energy of the beam of light which is projected onto the moon is concentrated in a circular spot of diameter d_1, where $d_1/L = 2\theta_{\text{R}} = 2(1.22\lambda/d_0)$, with d_0 the diameter of the mirror on Earth. The fraction of energy picked up by the reflector of diameter d_2 on the Moon is then $\eta' = (d_2/d_1)^2$. This reflected light, upon reaching the Earth, has a circular cross section of diameter d_3 satisfying $d_3/L = 2\theta_{\text{R}} = 2(1.22\lambda/d_2)$. The fraction of the reflected energy that is picked up by the telescope is then $\eta'' = (d_0/d_3)^2$. Consequently, the fraction of the original energy picked up by the detector is

$$\begin{aligned}\eta &= \eta'\eta'' = \left(\frac{d_0}{d_3}\right)^2\left(\frac{d_2}{d_1}\right)^2 = \left[\frac{d_0 d_2}{(2.44\lambda d_{em}/d_0)(2.44\lambda d_{em}/d_2)}\right]^2 = \left(\frac{d_0 d_2}{2.44\lambda d_{em}}\right)^4 \\ &= \left[\frac{(2.6\,\text{m})(0.10\,\text{m})}{2.44(0.69 \times 10^{-6}\,\text{m})(3.82 \times 10^{8}\,\text{m})}\right]^4 \approx 4 \times 10^{-13}\ .\end{aligned}$$

27. Bright interference fringes occur at angles θ given by $d\sin\theta = m\lambda$, where m is an integer. For the slits of this problem, $d = 11a/2$, so $a\sin\theta = 2m\lambda/11$ (see Sample Problem 37-4). The first minimum of the diffraction pattern occurs at the angle θ_1 given by $a\sin\theta_1 = \lambda$, and the second occurs at the angle θ_2 given by $a\sin\theta_2 = 2\lambda$, where a is the slit width. We should count the values of m for which $\theta_1 < \theta < \theta_2$, or, equivalently, the values of m for which $\sin\theta_1 < \sin\theta < \sin\theta_2$. This means $1 < (2m/11) < 2$. The values are $m = 6$, 7, 8, 9, and 10. There are five bright fringes in all.

28. In a manner similar to that discussed in Sample Problem 37-4, we find the number is $2(d/a) - 1 = 2(2a/a) - 1 = 3$.

29. (a) In a manner similar to that discussed in Sample Problem 37-4, we find the ratio should be $d/a = 4$. Our reasoning is, briefly, as follows: we let the location of the fourth bright fringe coincide with the first minimum of diffraction pattern, and then set $\sin\theta = 4\lambda/d = \lambda/a$ (so $d = 4a$).

    (b) Any bright fringe which happens to be at the same location with a diffraction minimum will vanish. Thus, if we let $\sin\theta = m_1\lambda/d = m_2\lambda/a = m_1\lambda/4a = m_2\lambda/a$, or $m_1 = 4m_2$ where $m_2 = 1, 2, 3, \cdots$. The fringes missing are the 4th, 8th, 12th, and so on. Hence, every fourth fringe is missing.

30. The angular location of the mth bright fringe is given by $d\sin\theta = m\lambda$, so the linear separation between two adjacent fringes is

$$\Delta y = \Delta(D\sin\theta) = \Delta\left(\frac{D_m\lambda}{d}\right) = \frac{D\lambda}{d}\Delta m = \frac{D\lambda}{d} .$$

31. (a) The angular positions θ of the bright interference fringes are given by $d\sin\theta = m\lambda$, where d is the slit separation, λ is the wavelength, and m is an integer. The first diffraction minimum occurs at the angle θ_1 given by $a\sin\theta_1 = \lambda$, where a is the slit width. The diffraction peak extends from $-\theta_1$ to $+\theta_1$, so we should count the number of values of m for which $-\theta_1 < \theta < +\theta_1$, or, equivalently, the number of values of m for which $-\sin\theta_1 < \sin\theta < +\sin\theta_1$. This means $-1/a < m/d < 1/a$ or $-d/a < m < +d/a$. Now $d/a = (0.150 \times 10^{-3}\,\text{m})/(30.0 \times 10^{-6}\,\text{m}) = 5.00$, so the values of m are $m = -4, -3, -2, -1, 0, +1, +2, +3$, and $+4$. There are nine fringes.

    (b) The intensity at the screen is given by

$$I = I_m\,\left(\cos^2\beta\right)\left(\frac{\sin\alpha}{\alpha}\right)^2$$

where $\alpha = (\pi a/\lambda)\sin\theta$, $\beta = (\pi d/\lambda)\sin\theta$, and I_m is the intensity at the center of the pattern. For the third bright interference fringe, $d\sin\theta = 3\lambda$, so $\beta = 3\pi$ rad and $\cos^2\beta = 1$. Similarly, $\alpha = 3\pi a/d = 3\pi/5.00 = 0.600\pi$ rad and

$$\left(\frac{\sin\alpha}{\alpha}\right)^2 = \left(\frac{\sin 0.600\pi}{0.600\pi}\right)^2 = 0.255 .$$

The intensity ratio is $I/I_m = 0.255$.

32. (a) The first minimum of the diffraction pattern is at 5.00°, so

$$a = \frac{\lambda}{\sin\theta} = \frac{0.440\,\mu\text{m}}{\sin 5.00°} = 5.05\,\mu\text{m} .$$

    (b) Since the fourth bright fringe is missing, $d = 4a = 4(5.05\,\mu\text{m}) = 20.2\,\mu\text{m}$.

    (c) For the $m = 1$ bright fringe,

$$\alpha = \frac{\pi a\sin\theta}{\lambda} = \frac{\pi(5.05\,\mu\text{m})\sin 1.25°}{0.440\,\mu\text{m}} = 0.787 \text{ rad} .$$

Consequently, the intensity of the $m = 1$ fringe is

$$I = I_m\left(\frac{\sin\alpha}{\alpha}\right)^2 = (7.0\,\text{mW/cm}^2)\left(\frac{\sin 0.787\,\text{rad}}{0.787}\right)^2 = 5.7 \text{ mW/cm}^2 ,$$

which agrees with Fig. 37-36. Similarly for $m = 2$, the intensity is $I = 2.9\,\text{mW/cm}^2$, also in agreement with Fig. 37-36.

33. (a) $d = 20.0\,\text{mm}/6000 = 0.00333\,\text{mm} = 3.33\,\mu\text{m}$.

(b) Let $d\sin\theta = m\lambda\,(m = 0, \pm 1, \pm 2, \cdots)$. We find $\theta = 0$ for $m = 0$, and

$$\theta = \sin^{-1}(\pm\lambda/d) = \sin^{-1}\left(\pm\frac{0.589\,\mu\text{m}}{3.30\,\mu\text{m}}\right) = \pm 10.2^\circ$$

for $m = \pm 1$. Similarly, we find $\pm 20.7^\circ$ for $m = \pm 2$, $\pm 32.2^\circ$ for $m = \pm 3$, $\pm 45^\circ$ for $m = \pm 4$, and $\pm 62.2^\circ$ for $m = \pm 5$. Since $|m|\lambda/d > 1$ for $|m| \geq 6$, these are all the maxima.

34. The angular location of the mth order diffraction maximum is given by $m\lambda = d\sin\theta$. To be able to observe the fifth-order maximum, we must let $\sin\theta|_{m=5} = 5\lambda/d < 1$, or

$$\lambda < \frac{d}{5} = \frac{1.00\,\text{nm}/315}{5} = 635 \text{ nm} .$$

Therefore, all wavelengths shorter than 635 nm can be used.

35. The ruling separation is $d = 1/(400\,\text{mm}^{-1}) = 2.5 \times 10^{-3}\,\text{mm}$. Diffraction lines occur at angles θ such that $d\sin\theta = m\lambda$, where λ is the wavelength and m is an integer. Notice that for a given order, the line associated with a long wavelength is produced at a greater angle than the line associated with a shorter wavelength. We take λ to be the longest wavelength in the visible spectrum (700 nm) and find the greatest integer value of m such that θ is less than 90°. That is, find the greatest integer value of m for which $m\lambda < d$. Since $d/\lambda = (2.5 \times 10^{-6}\,\text{m})/(700 \times 10^{-9}\,\text{m}) = 3.57$, that value is $m = 3$. There are three complete orders on each side of the $m = 0$ order. The second and third orders overlap.

36. We use Eq. 37-22 for diffraction maxima: $d\sin\theta = m\lambda$. In our case, since the angle between the $m = 1$ and $m = -1$ maxima is 26°, the angle θ corresponding to $m = 1$ is $\theta = 26^\circ/2 = 13^\circ$. We solve for the grating spacing:

$$d = \frac{m\lambda}{\sin\theta} = \frac{(1)(550\,\text{nm})}{\sin 13^\circ} = 2.4\,\mu\text{m} .$$

37. (a) Maxima of a diffraction grating pattern occur at angles θ given by $d\sin\theta = m\lambda$, where d is the slit separation, λ is the wavelength, and m is an integer. The two lines are adjacent, so their order numbers differ by unity. Let m be the order number for the line with $\sin\theta = 0.2$ and $m+1$ be the order number for the line with $\sin\theta = 0.3$. Then, $0.2d = m\lambda$ and $0.3d = (m+1)\lambda$. We subtract the first equation from the second to obtain $0.1d = \lambda$, or $d = \lambda/0.1 = (600\times 10^{-9}\,\text{m})/0.1 = 6.0\times 10^{-6}\,\text{m}$.

(b) Minima of the single-slit diffraction pattern occur at angles θ given by $a\sin\theta = m\lambda$, where a is the slit width. Since the fourth-order interference maximum is missing, it must fall at one of these angles. If a is the smallest slit width for which this order is missing, the angle must be given by $a\sin\theta = \lambda$. It is also given by $d\sin\theta = 4\lambda$, so $a = d/4 = (6.0 \times 10^{-6}\,\text{m})/4 = 1.5 \times 10^{-6}\,\text{m}$.

(c) First, we set $\theta = 90^\circ$ and find the largest value of m for which $m\lambda < d\sin\theta$. This is the highest order that is diffracted toward the screen. The condition is the same as $m < d/\lambda$ and since $d/\lambda = (6.0 \times 10^{-6}\,\text{m})/(600 \times 10^{-9}\,\text{m}) = 10.0$, the highest order seen is the $m = 9$ order. The fourth and eighth orders are missing, so the observable orders are $m = 0$, 1, 2, 3, 5, 6, 7, and 9.

38. (a) For the maximum with the greatest value of $m\,(= M)$ we have $M\lambda = a\sin\theta < d$, so $M < d/\lambda = 900\,\text{nm}/600\,\text{nm} = 1.5$, or $M = 1$. Thus three maxima can be seen, with $m = 0, \pm 1$.

(b) From Eq. 37-25

$$\begin{aligned}\Delta\theta_{\text{hw}} &= \frac{\lambda}{Nd\cos\theta} = \frac{d\sin\theta}{Nd\cos\theta} = \frac{\tan\theta}{N} = \frac{1}{N}\tan\left[\sin^{-1}\left(\frac{\lambda}{d}\right)\right] \\ &= \frac{1}{1000}\tan\left[\sin^{-1}\left(\frac{600\,\text{nm}}{900\,\text{nm}}\right)\right] = 0.051^\circ .\end{aligned}$$

39. The angular positions of the first-order diffraction lines are given by $d\sin\theta = \lambda$. Let λ_1 be the shorter wavelength (430 nm) and θ be the angular position of the line associated with it. Let λ_2 be the longer wavelength (680 nm), and let $\theta+\Delta\theta$ be the angular position of the line associated with it. Here $\Delta\theta = 20°$. Then, $d\sin\theta = \lambda_1$ and $d\sin(\theta+\Delta\theta) = \lambda_2$. We write $\sin(\theta+\Delta\theta)$ as $\sin\theta\cos\Delta\theta + \cos\theta\sin\Delta\theta$, then use the equation for the first line to replace $\sin\theta$ with λ_1/d, and $\cos\theta$ with $\sqrt{1-\lambda_1^2/d^2}$. After multiplying by d, we obtain

$$\lambda_1\cos\Delta\theta + \sqrt{d^2-\lambda_1^2}\sin\Delta\theta = \lambda_2 \ .$$

Solving for d, we find

$$\begin{aligned} d &= \sqrt{\frac{(\lambda_2-\lambda_1\cos\Delta\theta)^2+(\lambda_1\sin\Delta\theta)^2}{\sin^2\Delta\theta}} \\ &= \sqrt{\frac{[(680\,\text{nm})-(430\,\text{nm})\cos 20°]^2+[(430\,\text{nm})\sin 20°]^2}{\sin^2 20°}} \\ &= 914\,\text{nm} = 9.14\times 10^{-4}\ \text{mm} \ . \end{aligned}$$

There are $1/d = 1/(9.14\times 10^{-4}\,\text{mm}) = 1090$ rulings per mm.

40. We use Eq. 37-22. For $m = \pm 1$

$$\lambda = \frac{d\sin\theta}{m} = \frac{(1.73\mu\text{m})\sin(\pm 17.6°)}{\pm 1} = 523\ \text{nm} \ ,$$

and for $m = \pm 2$

$$\lambda = \frac{(1.73\mu\text{m})\sin(\pm 37.3°)}{\pm 2} = 524\,\text{nm} \ .$$

Similarly, we may compute the values of λ corresponding to the angles for $m = \pm 3$. The average value of these λ's is 523 nm.

41. Consider two of the rays shown in Fig. 37-37, one just above the other. The extra distance traveled by the lower one may be found by drawing perpendiculars from where the top ray changes direction (point P) to the incident and diffracted paths of the lower one. Where these perpendiculars intersect the lower ray's paths are here referred to as points A and C. Where the bottom ray changes direction is point B. We note that angle $\angle APB$ is the same as ψ, and angle BPC is the same as θ (see Fig. 37-37). The difference in path lengths between the two adjacent light rays is $\Delta x = |AB| + |BC| = d\sin\psi + d\sin\theta$. The condition for bright fringes to occur is therefore

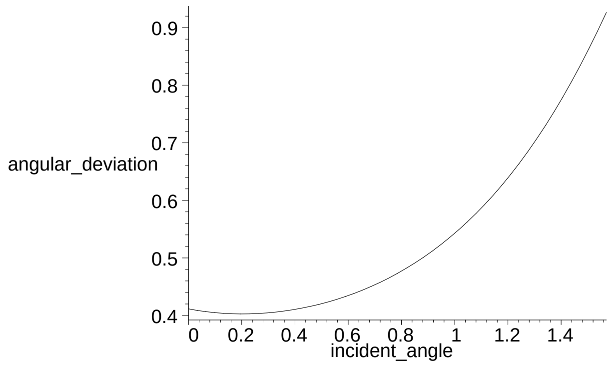
$$\Delta x = d\,(\sin\psi + \sin\theta) = m\lambda$$

where $m = 0, 1, 2, \cdots$. If we set $\psi = 0$ then this reduces to Eq. 37-22.

42. Referring to problem 41, we note that the angular deviation of a diffracted ray (the angle between the forward extrapolation of the incident ray and its diffracted ray) is $\psi + \theta$. For $m = 1$, this becomes

$$\psi + \theta = \psi + \sin^{-1}\left(\frac{\lambda}{d} - \sin\psi\right)$$

where the ratio $\lambda/d = 0.40$ using the values given in the problem statement. The graph of this is shown below (with radians used along both axes).



43. The derivation is similar to that used to obtain Eq. 37-24. At the first minimum beyond the mth principal maximum, two waves from adjacent slits have a phase difference of $\Delta\phi = 2\pi m + (2\pi/N)$, where N is the number of slits. This implies a difference in path length of $\Delta L = (\Delta\phi/2\pi)\lambda = m\lambda + (\lambda/N)$. If θ_m is the angular position of the mth maximum, then the difference in path length is also given by $\Delta L = d\sin(\theta_m + \Delta\theta)$. Thus $d\sin(\theta_m + \Delta\theta) = m\lambda + (\lambda/N)$. We use the trigonometric identity $\sin(\theta_m + \Delta\theta) = \sin\theta_m \cos\Delta\theta + \cos\theta_m \sin\Delta\theta$. Since $\Delta\theta$ is small, we may approximate $\sin\Delta\theta$ by $\Delta\theta$ in radians and $\cos\Delta\theta$ by unity. Thus $d\sin\theta_m + d\,\Delta\theta\cos\theta_m = m\lambda + (\lambda/N)$. We use the condition $d\sin\theta_m = m\lambda$ to obtain $d\,\Delta\theta\cos\theta_m = \lambda/N$ and

$$\Delta\theta = \frac{\lambda}{Nd\cos\theta_m} .$$

44. At the point on the screen where we find the inner edge of the hole, we have $\tan\theta = 5.0\,\text{cm}/30\,\text{cm}$, which gives $\theta = 9.46°$. We note that d for the grating is equal to $1.0\,\text{mm}/350 = 1.0\times10^6\,\text{nm}/350$. From $m\lambda = d\sin\theta$, we find

$$m = \frac{d\sin\theta}{\lambda} = \frac{\left(\frac{1.0\times10^6\,\text{nm}}{350}\right)(0.1644)}{\lambda} = \frac{470\,\text{nm}}{\lambda} .$$

Since for white light $\lambda > 400\,\text{nm}$, the only integer m allowed here is $m = 1$. Thus, at one edge of the hole, $\lambda = 470\,\text{nm}$. However, at the other edge, we have $\tan\theta' = 6.0\,\text{cm}/30\,\text{cm}$, which gives $\theta' = 11.31°$. This leads to

$$\lambda' = d\sin\theta' = \left(\frac{1.0\times10^6\,\text{nm}}{350}\right)\sin 11.31° = 560\text{ nm} .$$

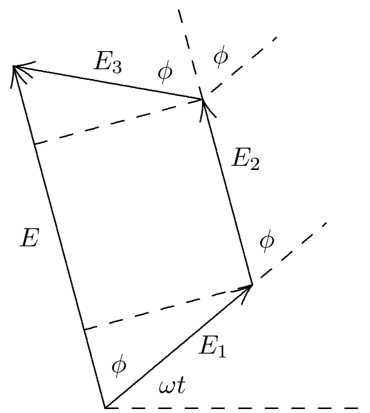
Consequently, the range of wavelength is from 470 to 560 nm.

45. Since the slit width is much less than the wavelength of the light, the central peak of the single-slit diffraction pattern is spread across the screen and the diffraction envelope can be ignored. Consider three waves, one from each slit. Since the slits are evenly spaced, the phase difference for waves from the first and second slits is the same as the phase difference for waves from the second and third slits. The electric fields of the waves at the screen can be written $E_1 = E_0\sin(\omega t)$, $E_2 = E_0\sin(\omega t + \phi)$, and $E_3 = E_0\sin(\omega t + 2\phi)$, where $\phi = (2\pi d/\lambda)\sin\theta$. Here d is the separation of adjacent slits and λ is the wavelength. The phasor diagram is shown below. It yields

$$E = E_0\cos\phi + E_0 + E_0\cos\phi = E_0(1 + 2\cos\phi)$$

for the amplitude of the resultant wave. Since the intensity of a wave is proportional to the square of the electric field, we may write $I = AE_0^2(1+2\cos\phi)^2$, where A is a constant of proportionality. If I_m is the intensity at the center of the pattern, for which $\phi = 0$, then $I_m = 9AE_0^2$. We take A to be $I_m/9E_0^2$ and obtain

$$I = \frac{I_m}{9}\,(1+2\cos\phi)^2 = \frac{I_m}{9}\left(1 + 4\cos\phi + 4\cos^2\phi\right) .$$



46. Letting $R = \lambda/\Delta\lambda = Nm$, we solve for N:

$$N = \frac{\lambda}{m\Delta\lambda} = \frac{(589.6\,\text{nm} + 589.0\,\text{nm})/2}{2(589.6\,\text{nm} - 589.0\,\text{nm})} = 491 \ .$$

47. If a grating just resolves two wavelengths whose average is λ_{avg} and whose separation is $\Delta\lambda$, then its resolving power is defined by $R = \lambda_{\text{avg}}/\Delta\lambda$. The text shows this is Nm, where N is the number of rulings in the grating and m is the order of the lines. Thus $\lambda_{\text{avg}}/\Delta\lambda = Nm$ and

$$N = \frac{\lambda_{\text{avg}}}{m\,\Delta\lambda} = \frac{656.3\,\text{nm}}{(1)(0.18\,\text{nm})} = 3650\,\text{rulings} \ .$$

48. (a) We find $\Delta\lambda$ from $R = \lambda/\Delta\lambda = Nm$:

$$\Delta\lambda = \frac{\lambda}{Nm} = \frac{500\,\text{nm}}{(600/\text{mm})(5.0\,\text{mm})(3)} = 0.056\,\text{nm} = 56 \ \text{pm} \ .$$

(b) Since $\sin\theta = m_{\text{max}}\lambda/d < 1$,

$$m_{\text{max}} < \frac{d}{\lambda} = \frac{1}{(600/\text{mm})(500 \times 10^{-6}\,\text{mm})} = 3.3 \ .$$

Therefore, $m_{\text{max}} = 3$. No higher orders of maxima can be seen.

49. The dispersion of a grating is given by $D = d\theta/d\lambda$, where θ is the angular position of a line associated with wavelength λ. The angular position and wavelength are related by $\mathbf{d}\sin\theta = m\lambda$, where $\mathbf{d}$ is the slit separation (which we made boldfaced in order not to confuse it with the d used in the derivative, below) and m is an integer. We differentiate this expression with respect to θ to obtain

$$\frac{d\theta}{d\lambda}\,\mathbf{d}\,\cos\theta = m \ ,$$

or

$$D = \frac{d\theta}{d\lambda} = \frac{m}{\mathbf{d}\cos\theta} \ .$$

Now $m = (\mathbf{d}/\lambda)\sin\theta$, so

$$D = \frac{\mathbf{d}\sin\theta}{\mathbf{d}\lambda\cos\theta} = \frac{\tan\theta}{\lambda} \ .$$

50. (a) From $d\sin\theta = m\lambda$ we find

$$d = \frac{m\lambda_{\text{avg}}}{\sin\theta} = \frac{3(589.3\,\text{nm})}{\sin 10^\circ} = 1.0 \times 10^4\,\text{nm} = 10\,\mu\text{m} \ .$$

(b) The total width of the ruling is

$$L = Nd = \left(\frac{R}{m}\right) d = \frac{\lambda_{\rm avg} d}{m\Delta\lambda} = \frac{(589.3\,{\rm nm})(10\,\mu{\rm m})}{3(589.59\,{\rm nm} - 589.00\,{\rm nm})} = 3.3\times 10^3\mu{\rm m} = 3.3\ {\rm mm}\ .$$

51. (a) Since the resolving power of a grating is given by $R = \lambda/\Delta\lambda$ and by Nm, the range of wavelengths that can just be resolved in order m is $\Delta\lambda = \lambda/Nm$. Here N is the number of rulings in the grating and λ is the average wavelength. The frequency f is related to the wavelength by $f\lambda = c$, where c is the speed of light. This means $f\,\Delta\lambda + \lambda\,\Delta f = 0$, so

$$\Delta\lambda = -\frac{\lambda}{f}\,\Delta f = -\frac{\lambda^2}{c}\,\Delta f$$

where $f = c/\lambda$ is used. The negative sign means that an increase in frequency corresponds to a decrease in wavelength. We may interpret Δf as the range of frequencies that can be resolved and take it to be positive. Then,

$$\frac{\lambda^2}{c}\,\Delta f = \frac{\lambda}{Nm}$$

and

$$\Delta f = \frac{c}{Nm\lambda}\ .$$

(b) The difference in travel time for waves traveling along the two extreme rays is $\Delta t = \Delta L/c$, where ΔL is the difference in path length. The waves originate at slits that are separated by $(N-1)d$, where d is the slit separation and N is the number of slits, so the path difference is $\Delta L = (N-1)d\sin\theta$ and the time difference is

$$\Delta t = \frac{(N-1)d\sin\theta}{c}\ .$$

If N is large, this may be approximated by $\Delta t = (Nd/c)\sin\theta$. The lens does not affect the travel time.

(c) Substituting the expressions we derived for Δt and Δf, we obtain

$$\Delta f\,\Delta t = \left(\frac{c}{Nm\lambda}\right)\left(\frac{Nd\sin\theta}{c}\right) = \frac{d\sin\theta}{m\lambda} = 1\ .$$

The condition $d\sin\theta = m\lambda$ for a diffraction line is used to obtain the last result.

52. (a) From the expression for the half-width $\Delta\theta_{\rm hw}$ (given by Eq. 37-25) and that for the resolving power R (given by Eq. 37-29), we find the product of $\Delta\theta_{\rm hw}$ and R to be

$$\Delta\theta_{\rm hw} R = \left(\frac{\lambda}{Nd\cos\theta}\right) Nm = \frac{m\lambda}{d\cos\theta} = \frac{d\sin\theta}{d\cos\theta} = \tan\theta\,,$$

where we used $m\lambda = d\sin\theta$ (see Eq. 37-22).

(b) For first order $m = 1$, so the corresponding angle θ_1 satisfies $d\sin\theta_1 = m\lambda = \lambda$. Thus the product in question is given by

$$\begin{aligned}
\tan\theta_1 &= \frac{\sin\theta_1}{\cos\theta_1} = \frac{\sin\theta_1}{\sqrt{1-\sin^2\theta_1}} \\
&= \frac{1}{\sqrt{(1/\sin\theta_1)^2 - 1}} = \frac{1}{\sqrt{(d/\lambda)^2 - 1}} \\
&= \frac{1}{\sqrt{(900\,{\rm nm}/600\,{\rm nm})^2 - 1}} = 0.89\ .
\end{aligned}$$

53. Bragg's law gives the condition for a diffraction maximum:

$$2d \sin \theta = m\lambda$$

where d is the spacing of the crystal planes and λ is the wavelength. The angle θ is measured from the surfaces of the planes. For a second-order reflection $m = 2$, so

$$d = \frac{m\lambda}{2 \sin \theta} = \frac{2(0.12 \times 10^{-9}\,\text{m})}{2 \sin 28°} = 2.56 \times 10^{-10}\,\text{m} = 256\ \text{pm}\ .$$

54. We use Eq. 37-31. From the peak on the left at angle 0.75° (estimated from Fig. 37-38), we have

$$\lambda_1 = 2d \sin \theta_1 = 2(0.94\,\text{nm}) \sin(0.75°) = 0.025\,\text{nm} = 25\ \text{pm}\ .$$

This estimation should be viewed as reliable to within $\pm 2\,\text{pm}$. We now consider the next peak:

$$\lambda_2 = 2d \sin \theta_2 = 2(0.94\,\text{nm}) \sin 1.15° = 0.038\,\text{nm} = 38\ \text{pm}\ .$$

One can check that the third peak from the left is the second-order one for λ_1.

55. The x ray wavelength is $\lambda = 2d \sin \theta = 2(39.8\,\text{pm}) \sin 30.0° = 39.8\,\text{pm}$.

56. (a) For the first beam $2d \sin \theta_1 = \lambda_A$ and for the second one $2d \sin \theta_2 = 3\lambda_B$. The values of d and λ_A can then be determined:

$$d = \frac{3\lambda_B}{2 \sin \theta_2} = \frac{3(97\,\text{pm})}{2 \sin 60°} = 1.7 \times 10^2\ \text{pm}\ .$$

(b)

$$\lambda_A = 2d \sin \theta_1 = 2(1.7 \times 10^2\,\text{pm})(\sin 23°) = 1.3 \times 10^2\ \text{pm}\ .$$

57. There are two unknowns, the x-ray wavelength λ and the plane separation d, so data for scattering at two angles from the same planes should suffice. The observations obey Bragg's law, so

$$2d \sin \theta_1 = m_1 \lambda$$

and

$$2d \sin \theta_2 = m_2 \lambda\ .$$

However, these cannot be solved for the unknowns. For example, we can use the first equation to eliminate λ from the second. We obtain

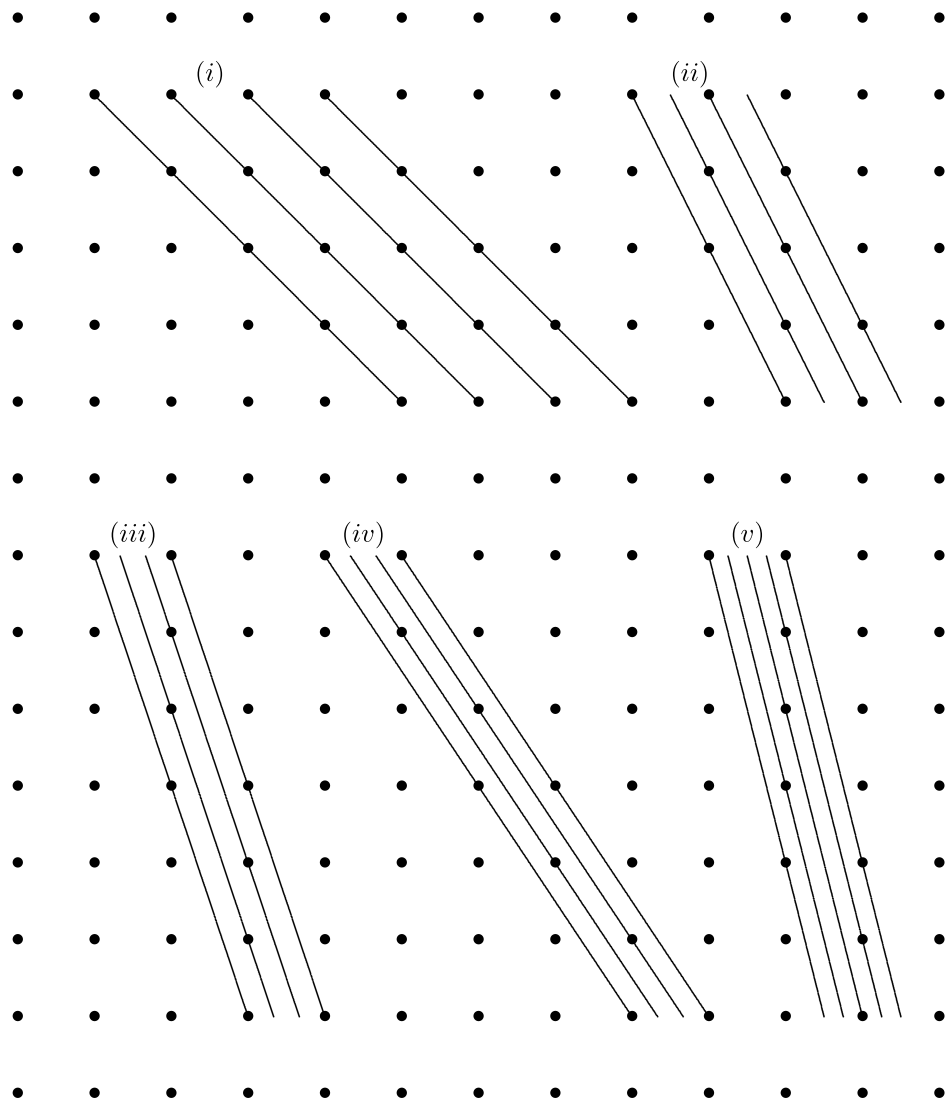
$$m_2 \sin \theta_1 = m_1 \sin \theta_2\ ,$$

an equation that does not contain either of the unknowns.

58. The angle of incidence on the reflection planes is $\theta = 63.8° - 45.0° = 18.8°$, and the plane-plane separation is $d = a_0/\sqrt{2}$. Thus, using $2d \sin \theta = \lambda$, we get

$$a_0 = \sqrt{2}d = \frac{\sqrt{2}\lambda}{2 \sin \theta} = \frac{0.260\,\text{nm}}{\sqrt{2} \sin 18.8°} = 0.570\ \text{nm}\ .$$

59. (a) The sets of planes with the next five smaller interplanar spacings (after a_0) are shown in the diagram below.



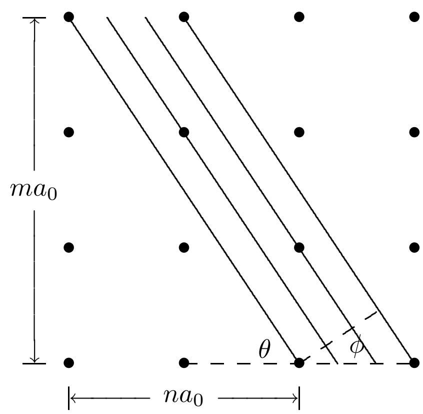
In terms of a_0, the spacings are:

$$\begin{aligned}
(i):&\quad a_0/\sqrt{2} = 0.7071a_0\\
(ii):&\quad a_0/\sqrt{5} = 0.4472a_0\\
(iii):&\quad a_0/\sqrt{10} = 0.3162a_0\\
(iv):&\quad a_0/\sqrt{13} = 0.2774a_0\\
(v):&\quad a_0/\sqrt{17} = 0.2425a_0
\end{aligned}$$

(b) Since a crystal plane passes through lattice points, its slope can be written as the ratio of two integers. Consider a set of planes with slope m/n, as shown in the diagram below. The first and last planes shown pass through adjacent lattice points along a horizontal line and there are $m-1$ planes between. If h is the separation of the first and last planes, then the interplanar spacing is $d = h/m$. If the planes make the angle θ with the horizontal, then the normal to the planes (shown dotted) makes the angle $\phi = 90° - \theta$. The distance h is given by $h = a_0 \cos\phi$ and the interplanar spacing is $d = h/m = (a_0/m)\cos\phi$. Since $\tan\theta = m/n$, $\tan\phi = n/m$ and

$\cos\phi = 1/\sqrt{1+\tan^2\phi} = m/\sqrt{n^2+m^2}$. Thus,

$$d = \frac{h}{m} = \frac{a_0\cos\phi}{m} = \frac{a_0}{\sqrt{n^2+m^2}} \ .$$



60. The wavelengths satisfy $m\lambda = 2d\sin\theta = 2(275\,\text{pm})(\sin 45°) = 389\,\text{pm}$. In the range of wavelengths given, the allowed values of m are $m = 3$, 4, with the corresponding wavelengths being $389\,\text{pm}/3 = 130\,\text{pm}$ and $389\,\text{pm}/4 = 97.2\,\text{pm}$, respectively.

61. We want the reflections to obey the Bragg condition $2d\sin\theta = m\lambda$, where θ is the angle between the incoming rays and the reflecting planes, λ is the wavelength, and m is an integer. We solve for θ:

$$\theta = \sin^{-1}\left(\frac{m\lambda}{2d}\right) = \sin^{-1}\left(\frac{(0.125\times10^{-9}\,\text{m})m}{2(0.252\times10^{-9}\,\text{m})}\right) = 0.2480m \ .$$

For $m = 1$ this gives $\theta = 14.4°$. The crystal should be turned $45° - 14.4° = 30.6°$ clockwise. For $m = 2$ it gives $\theta = 29.7°$. The crystal should be turned $45° - 29.7° = 15.3°$ clockwise. For $m = 3$ it gives $\theta = 48.1°$. The crystal should be turned $48.1° - 45° = 3.1°$ counterclockwise. For $m = 4$ it gives $\theta = 82.8°$. The crystal should be turned $82.8° - 45° = 37.8°$ counterclockwise. There are no intensity maxima for $m > 4$ as one can verify by noting that $m\lambda/2d$ is greater than 1 for m greater than 4.

62. (a) Eq. 37-3 and Eq. 37-12 imply smaller angles for diffraction for smaller wavelengths. This suggests that diffraction effects in general would decrease.

(b) Using Eq. 37-3 with $m = 1$ and solving for 2θ (the angular width of the central diffraction maximum), we find

$$2\theta = 2\sin^{-1}\left(\frac{\lambda}{a}\right) = 2\sin^{-1}\left(\frac{0.50\,\text{m}}{5.0\,\text{m}}\right) = 11° \ .$$

(c) A similar calculation yields 0.23° for $\lambda = 0.010$ m.

63. (a) Using the notation of Sample Problem 37-6 (which is in the textbook supplement), the minimum separation is

$$D = L\theta_R = L\left(\frac{1.22\lambda}{d}\right) = \frac{(400\times10^3\,\text{m})(1.22)(550\times10^{-9}\,\text{m})}{(0.005\,\text{m})} \approx 50 \text{ m} \ .$$

(b) The Rayleigh criterion suggests that the astronaut will not be able to discern the Great Wall (see the result of part (a)).

(c) The signs of intelligent life would probably be, at most, ambiguous on the sunlit half of the planet. However, while passing over the half of the planet on the opposite side from the Sun, the astronaut would be able to notice the effects of artificial lighting.

64. Consider two light rays crossing each other at the middle of the lens (see Fig. 37-42(c)). The rays come from opposite sides of the circular dot of diameter D, a distance L from the eyes, so we are using the same notation found in Sample Problem 37-6 (which is in the textbook supplement). Those two rays reach the retina a distance L' behind the lens, striking two points there which are a distance D' apart. Therefore,

$$\frac{D}{L} = \frac{D'}{L'}$$

where $D = 2\,\text{mm}$ and $L' = 20\,\text{mm}$. If we estimate $L \approx 450\,\text{mm}$, we find $D' \approx 0.09\,\text{mm}$. Turning our attention to Fig. 37-42(d), we see

$$\theta = \tan^{-1}\left(\frac{\frac{1}{2}D'}{x}\right)$$

which we wish to set equal to the angle in Eq. 37-12. We could use the small angle approximation $\sin\theta \approx \tan\theta$ to relate these directly, or we could be "exact" – as we show below:

$$\text{If} \quad \tan\phi = \frac{b}{a} \;, \quad \text{then} \quad \sin\phi = \frac{b}{\sqrt{a^2 + b^2}} \quad .$$

Therefore, this "exact" use of Eq. 37-12 leads to

$$1.22\frac{\lambda}{d} = \sin\theta = \frac{\frac{1}{2}D'}{\sqrt{x^2 + (D'/2)^2}}$$

where $\lambda = 550 \times 10^{-6}\,\text{mm}$ and $1\,\text{mm} \le x \le 15\,\text{mm}$. Using the value of D' found above, this leads to a range of d values: $0.015\,\text{mm} \le d \le 0.23\,\text{mm}$.

65. Using the same notation found in Sample Problem 37-6,

$$\frac{D}{L} = \theta_{\text{R}} = 1.22\frac{\lambda}{d}$$

where we will assume a "typical" wavelength for visible light: $\lambda \approx 550 \times 10^{-9}$ m.

(a) With $L = 400 \times 10^3$ m and $D = 0.85$ m, the above relation leads to $d = 0.32$ m.

(b) Now with $D = 0.10$ m, the above relation leads to $d = 2.7$ m.

(c) The military satellites do not use Hubble Telescope-sized apertures. A great deal of very sophisticated optical filtering and digital signal processing techniques go into the final product, for which there is not space for us to describe here.

66. Assuming all $N = 2000$ lines are uniformly illuminated, we have

$$\frac{\lambda_{\text{av}}}{\Delta\lambda} = Nm$$

from Eq. 37-28 and Eq. 37-29. With $\lambda_{\text{av}} = 600$ nm and $m = 2$, we find $\Delta\lambda = 0.15$ nm.

67. The central diffraction envelope spans the range $-\theta_1 < \theta < +\theta_1$ where

$$\theta_1 = \sin^{-1}\frac{\lambda}{a} \; .$$

The maxima in the double-slit pattern are located at

$$\theta_m = \sin^{-1}\frac{m\lambda}{d} \; ,$$

so that our range specification becomes

$$-\sin^{-1}\frac{\lambda}{a} \;<\; \sin^{-1}\frac{m\lambda}{d} < \; +\sin^{-1}\frac{\lambda}{a} \; ,$$

which we change (since sine is a monotonically increasing function in the fourth and first quadrants, where all these angles lie) to

$$-\frac{\lambda}{a} < \frac{m\lambda}{d} < +\frac{\lambda}{a} .$$

Rewriting this as $-d/a < m < +d/a$, we find $-6 < m < +6$, or, since m is an integer, $-5 \le m \le +5$. Thus, we find eleven values of m that satisfy this requirement.

68. Employing Eq. 37-3, we find (with $m = 3$ and all lengths in μm)

$$\theta = \sin^{-1}\frac{m\lambda}{a} = \sin^{-1}\frac{(3)(0.5)}{2}$$

which yields $\theta = 48.6°$. Now, we use the experimental geometry ($\tan\theta = y/D$ where y locates the minimum relative to the middle of the pattern) to find

$$y = D\tan\theta = 2.27 \text{ m} .$$

69. (a) From $R = \lambda/\Delta\lambda = Nm$ we find

$$N = \frac{\lambda}{m\Delta\lambda} = \frac{(415.496\,\text{nm} + 415.487\,\text{nm})/2}{2(415.96\,\text{nm} - 415.487\,\text{nm})} = 23100 .$$

(b) We note that $d = (4.0 \times 10^7\,\text{nm})/23100 = 1732\,\text{nm}$. The maxima are found at

$$\theta = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left[\frac{(2)(415.5\,\text{nm})}{1732\,\text{nm}}\right] = 28.7° .$$

70. We use Eq. 37-31. For smallest value of θ, we let $m = 1$. Thus,

$$\theta_{\text{min}} = \sin^{-1}\left(\frac{m\lambda}{2d}\right) = \sin^{-1}\left[\frac{(1)(30\,\text{pm})}{2(0.30 \times 10^3\,\text{pm})}\right] = 2.9° .$$

71. (a) We use Eq. 37-12:

$$\begin{aligned}\theta &= \sin^{-1}\left(\frac{1.22\lambda}{d}\right) = \sin^{-1}\left[\frac{1.22(v_s/f)}{d}\right] \\ &= \sin^{-1}\left[\frac{(1.22)(1450\,\text{m/s})}{(25 \times 10^3\,\text{Hz})(0.60\,\text{m})}\right] = 6.8° .\end{aligned}$$

(b) Now $f = 1.0 \times 10^3\,\text{Hz}$ so

$$\frac{1.22\lambda}{d} = \frac{(1.22)(1450\,\text{m/s})}{(1.0 \times 10^3\,\text{Hz})(0.60\,\text{m})} = 2.9 > 1 .$$

Since $\sin\theta$ cannot exceed 1 there is no minimum.

72. From Eq. 37-3,

$$\frac{a}{\lambda} = \frac{m}{\sin\theta} = \frac{1}{\sin 45.0°} = 1.41 .$$

73. (a) Use of Eq. 37-22 for the limit-wavelengths ($\lambda_1 = 700$ nm and $\lambda_2 = 550$ nm) leads to the condition

$$m_1\lambda_1 \ge m_2\lambda_2$$

for $m_1 + 1 = m_2$ (the low end of a high-order spectrum is what is overlapping with the high end of the next-lower-order spectrum). Assuming equality in the above equation, we can solve for "m_1" (realizing it might not be an integer) and obtain $m_1 \approx 4$ where we have rounded *up*. It is the fourth order spectrum that is the lowest-order spectrum to overlap with the next higher spectrum.

(b) The problem specifies $d = 1/200$ using the mm unit, and we note there are no refraction angles greater than 90°. We concentrate on the largest wavelength $\lambda = 700$ nm $= 7 \times 10^{-4}$ mm and solve Eq. 37-22 for "m_{max}" (realizing it might not be an integer):

$$m_{\text{max}} = \frac{d \sin 90°}{\lambda} = \frac{1}{(200)\,(7 \times 10^{-4})} \approx 7$$

where we have rounded down. There are no values of m (for the appearance of the full spectrum) greater than $m = 7$.

74. The central diffraction envelope spans the range $-\theta_1 < \theta < +\theta_1$ where

$$\theta_1 = \sin^{-1} \frac{\lambda}{a} \; .$$

The maxima in the double-slit pattern are at

$$\theta_m = \sin^{-1} \frac{m\lambda}{d} \; ,$$

so that our range specification becomes

$$-\sin^{-1} \frac{\lambda}{a} \; < \; \sin^{-1} \frac{m\lambda}{d} < \; +\sin^{-1} \frac{\lambda}{a} \; ,$$

which we change (since sine is a monotonically increasing function in the fourth and first quadrants, where all these angles lie) to

$$-\frac{\lambda}{a} \; < \; \frac{m\lambda}{d} \; < \; +\frac{\lambda}{a} \; .$$

Rewriting this as $-d/a < m < +d/a$ we arrive at the result $m_{\text{max}} < d/a \le m_{\text{max}} + 1$. Due to the symmetry of the pattern, the multiplicity of the m values is $2m_{\text{max}} + 1 = 17$ so that $m_{\text{max}} = 8$, and the result becomes

$$8 < \frac{d}{a} \le 9$$

where these numbers are as accurate as the experiment allows (that is, "9" means "9.000" if our measurements are that good).

75. As a slit is narrowed, the pattern spreads outward, so the question about "minimum width" suggests that we are looking at the lowest possible values of m (the label for the minimum produced by light $\lambda = 600$ nm) and m' (the label for the minimum produced by light $\lambda' = 500$ nm). Since the angles are the same, then Eq. 37-3 leads to

$$m\lambda = m'\lambda'$$

which leads to the choices $m = 5$ and $m' = 6$. We find the slit width from Eq. 37-3:

$$a = \frac{m\lambda}{\sin\theta} \approx \frac{m\lambda}{\theta}$$

which yields $a = 3.0$ mm.

76. (a) We note that $d = (76 \times 10^6\,\text{nm})/40000 = 1900\,\text{nm}$. For the first order maxima $\lambda = d \sin\theta$, which leads to

$$\theta = \sin^{-1}\left(\frac{\lambda}{d}\right) = \sin^{-1}\left(\frac{589\,\text{nm}}{1900\,\text{nm}}\right) = 18° \; .$$

Now, substituting $m = d\sin\theta/\lambda$ into Eq. 37-27 leads to $D = \tan\theta/\lambda = \tan 18°/589\,\text{nm} = 5.5 \times 10^{-4}\,\text{rad/nm} = 0.032°/\text{nm}$. Similarly for $m = 2$ and $m = 3$, we have $\theta = 38°$ and $68°$, and the corresponding values of dispersion are $0.076°/\text{nm}$ and $0.24°/\text{nm}$, respectively.

(b) $R = Nm = 40000\,m = 40000$ (for $m = 1$); 80000 (for $m = 2$); and, $120,000$ (for $m = 3$).

77. Letting $d\sin\theta = (L/N)\sin\theta = m\lambda$, we get

$$\lambda = \frac{(L/N)\sin\theta}{m} = \frac{(1.0\times 10^7\,\text{nm})(\sin 30^\circ)}{(1)(10000)} = 500\ \text{nm}\ .$$

78. (a) Using the notation of Sample Problem 37-6,

$$L = \frac{D}{1.22\lambda/d} = \frac{2(50\times 10^{-6}\,\text{m})(1.5\times 10^{-3}\,\text{m})}{1.22(650\times 10^{-9}\,\text{m})} = 0.19\ \text{m}\ .$$

(b) The wavelength of the blue light is shorter so $L_{\text{max}} \propto \lambda^{-1}$ will be larger.

79. From $y = m\lambda D/a$ we get

$$\Delta y = \Delta\left(\frac{m\lambda D}{a}\right) = \frac{\lambda D}{a}\Delta m = \frac{(632.8\,\text{nm})(2.60)}{1.37\,\text{mm}}[10-(-10)] = 24.0\ \text{mm}\ .$$

80. For $\lambda = 0.10$ nm, we have scattering for order m, and for $\lambda' = 0.075$ nm, we have scattering for order m'. From Eq. 37-31, we see that we must require

$$m\lambda = m'\lambda'$$

which suggests (looking for the smallest integer solutions) that $m = 3$ and $m' = 4$. Returning with this result and with $d = 0.25$ nm to Eq. 37-31, we obtain

$$\theta = \sin^{-1}\frac{m\lambda}{2d} = 37^\circ\ .$$

Studying Figure 37-26, we conclude that the angle between incident and scattered beams is $180^\circ - 2\theta = 106^\circ$.

81. (a) We express all lengths in mm, and since $1/d = 180$, we write Eq. 37-22 as

$$\theta = \sin^{-1}\left(\frac{1}{d}m\lambda\right) = \sin^{-1}(180)(2)\lambda$$

where $\lambda_1 = 4\times 10^{-4}$ and $\lambda_2 = 5\times 10^{-4}$ (in mm). Thus, $\Delta\theta = \theta_2 - \theta_1 = 2.1^\circ$.

(b) Use of Eq. 37-22 for each wavelength leads to the condition

$$m_1\lambda_1 = m_2\lambda_2$$

for which the smallest possible choices are $m_1 = 5$ and $m_2 = 4$. Returning to Eq. 37-22, then, we find

$$\theta = \sin^{-1}\left(\frac{1}{d}m_1\lambda_1\right) = 21^\circ\ .$$

(c) There are no refraction angles greater than 90°, so we can solve for "m_{max}" (realizing it might not be an integer):

$$m_{\text{max}} = \frac{d\sin 90^\circ}{\lambda_2} = 11$$

where we have rounded down. There are no values of m (for light of wavelength λ_2) greater than $m = 11$.

82. Following Sample Problem 37-6, we use Eq. 37-35:

$$L = \frac{Dd}{1.22\lambda} = 164\ \text{m}\ .$$

83. (a) Employing Eq. 37-3 with the small angle approximation ($\sin\theta \approx \tan\theta = y/D$ where y locates the minimum relative to the middle of the pattern), we find (with $m = 1$ and all lengths in mm)

$$D = \frac{ya}{m\lambda} = \frac{(0.9)(0.4)}{4.5 \times 10^{-4}} = 800$$

which places the screen 80 cm away from the slit.

(b) The above equation gives for the value of y (for $m = 3$)

$$y = \frac{(3)\lambda D}{a} = 2.7 \text{ mm} .$$

Subtracting this from the first minimum position $y = 0.9$ mm, we find the result $\Delta y = 1.8$ mm.

84. (a) We require that $\sin\theta = m\lambda_{1,2}/d \leq \sin 30°$, where $m = 1, 2$ and $\lambda_1 = 500\,\text{nm}$. This gives

$$d \geq \frac{2\lambda_s}{\sin 30°} = \frac{2(600\,\text{nm})}{\sin 30°} = 2400\,\text{nm} .$$

For a grating of given total width L we have $N = L/d \propto d^{-1}$, so we need to minimize d to maximize $R = mN \propto d^{-1}$. Thus we choose $d = 2400\,\text{nm}$.

(b) Let the third-order maximum for $\lambda_2 = 600\,\text{nm}$ be the first minimum for the single-slit diffraction profile. This requires that $d\sin\theta = 3\lambda_2 = a\sin\theta$, or $a = d/3 = 2400\,\text{nm}/3 = 800\,\text{nm}$.

(c) Letting $\sin\theta = m_{\text{max}}\lambda_2/d \leq 1$, we obtain

$$m_{\text{max}} \leq \frac{d}{\lambda_2} = \frac{2400\,\text{nm}}{800\,\text{nm}} = 3 .$$

Since the third order is missing the only maxima present are the ones with $m = 0$, 1 and 2.

85. (a) Letting $d\sin\theta = m\lambda$, we solve for λ:

$$\lambda = \frac{d\sin\theta}{m} = \frac{(1.0\,\text{mm}/200)(\sin 30°)}{m} = \frac{2500\,\text{nm}}{m}$$

where $m = 1, 2, 3\cdots$. In the visible light range m can assume the following values: $m_1 = 4$, $m_2 = 5$ and $m_3 = 6$. The corresponding wavelengths are $\lambda_1 = 2500\,\text{nm}/4 = 625\,\text{nm}$, $\lambda_2 = 2500\,\text{nm}/5 = 500\,\text{nm}$, and $\lambda_3 = 2500\,\text{nm}/6 = 416\,\text{nm}$.

(b) The colors are orange (for $\lambda_1 = 625\,\text{nm}$), blue-green (for $\lambda_2 = 500\,\text{nm}$), and violet (for $\lambda_3 = 416\,\text{nm}$).

86. Using the notation of Sample Problem 37-6,

$$L = \frac{D}{\theta_{\text{R}}} = \frac{D}{1.22\lambda/d} = \frac{(5.0 \times 10^{-2}\,\text{m})(4.0 \times 10^{-3}\,\text{m})}{1.22(0.10 \times 10^{-9}\,\text{m})} = 1.6 \times 10^{6}\,\text{m} = 1600 \text{ km} .$$

87. The condition for a minimum in a single-slit diffraction pattern is given by Eq. 37-3, which we solve for the wavelength:

$$\lambda = \frac{a\sin\theta}{m} = \frac{(0.022\,\text{mm})\sin 1.8°}{1} = 6.9 \times 10^{-4}\,\text{mm} = 690 \text{ nm} .$$