Chapter 11

1. (a) Eq. 11-6 leads to
$$\omega = \frac{d}{dt}\left(at + bt^3 - ct^4\right) = a + 3bt^2 - 4ct^3 \ .$$
   (b) And Eq. 11-8 gives
$$\alpha = \frac{d}{dt}\left(a + 3bt^2 - 4ct^3\right) = 6bt - 12ct^2 \ .$$

2. (a) The second hand of the smoothly running watch turns through 2π radians during 60 s. Thus,
$$\omega = \frac{2\pi}{60} = 0.105 \text{ rad/s} \ .$$
   (b) The minute hand of the smoothly running watch turns through 2π radians during 3600 s. Thus,
$$\omega = \frac{2\pi}{3600} = 1.75 \times 10^{-3} \text{ rad/s} \ .$$
   (c) The hour hand of the smoothly running 12-hour watch turns through 2π radians during 43200 s. Thus,
$$\omega = \frac{2\pi}{43200} = 1.45 \times 10^{-4} \text{ rad/s} \ .$$

3. (a) The time for one revolution is the circumference of the orbit divided by the speed v of the Sun: $T = 2\pi R/v$, where R is the radius of the orbit. We convert the radius:
$$R = \left(2.3 \times 10^4 \,\text{ly}\right)\left(9.46 \times 10^{12} \,\text{km/ly}\right) = 2.18 \times 10^{17} \text{ km}$$
   where the ly $\leftrightarrow$ km conversion can be found in Appendix D or figured "from basics" (knowing the speed of light). Therefore, we obtain
$$T = \frac{2\pi\left(2.18 \times 10^{17}\,\text{km}\right)}{250\,\text{km/s}} = 5.5 \times 10^{15} \text{ s} \ .$$
   (b) The number of revolutions N is the total time t divided by the time T for one revolution; that is, $N = t/T$. We convert the total time from years to seconds and obtain
$$N = \frac{\left(4.5 \times 10^9\,\text{y}\right)\left(3.16 \times 10^7\,\text{s/y}\right)}{5.5 \times 10^{15} \text{ s}} = 26 \ .$$

4. If we make the units explicit, the function is
$$\theta = (4.0\,\text{rad/s})t - \left(3.0\,\text{rad/s}^2\right)t^2 + \left(1.0\,\text{rad/s}^3\right)t^3$$
but generally we will proceed as shown in the problem – letting these units be understood. Also, in our manipulations we will generally not display the coefficients with their proper number of significant figures.

(a) Eq. 11-6 leads to

$$\omega = \frac{d}{dt}\left(4t - 3t^2 + t^3\right) = 4 - 6t + 3t^2 \ .$$

Evaluating this at $t = 2$ s yields $\omega_2 = 4.0$ rad/s.

(b) Evaluating the expression in part (a) at $t = 4$ s gives $\omega_4 = 28$ rad/s.

(c) Consequently, Eq. 11-7 gives

$$\alpha_{\text{avg}} = \frac{\omega_4 - \omega_2}{4 - 2} = 12 \text{ rad/s}^2 \ .$$

(d) And Eq. 11-8 gives

$$\alpha = \frac{d\omega}{dt} = \frac{d}{dt}\left(4 - 6t + 3t^2\right) = -6 + 6t \ .$$

Evaluating this at $t = 2$ s produces $\alpha_2 = 6.0$ rad/s^2.

(e) Evaluating the expression in part (d) at $t = 4$ s yields $\alpha_4 = 18$ rad/s^2. We note that our answer for α_{avg} does turn out to be the arithmetic average of α_2 and α_4 but point out that this will not always be the case.

5. If we make the units explicit, the function is

$$\theta = 2\,\text{rad} + \left(4\,\text{rad/s}^2\right)t^2 + \left(2\,\text{rad/s}^3\right)t^3$$

but in some places we will proceed as indicated in the problem – by letting these units be understood.

(a) We evaluate the function θ at $t = 0$ to obtain $\theta_0 = 2$ rad.

(b) The angular velocity as a function of time is given by Eq. 11-6:

$$\omega = \frac{d\theta}{dt} = \left(8\,\text{rad/s}^2\right)t + \left(6\,\text{rad/s}^3\right)t^2$$

which we evaluate at $t = 0$ to obtain $\omega_0 = 0$.

(c) For $t = 4$ s, the function found in the previous part is $\omega_4 = (8)(4) + (6)(4)^2 = 128$ rad/s. If we round this to two figures, we obtain $\omega_4 \approx 130$ rad/s.

(d) The angular acceleration as a function of time is given by Eq. 11-8:

$$\alpha = \frac{d\omega}{dt} = 8\,\text{rad/s}^2 + \left(12\,\text{rad/s}^3\right)t$$

which yields $\alpha_2 = 8 + (12)(2) = 32\,\text{rad/s}^2$ at $t = 2$ s.

(e) The angular acceleration, given by the function obtained in the previous part, depends on time; it is not constant.

6. (a) To avoid touching the spokes, the arrow must go through the wheel in not more than

$$\Delta t = \frac{1/8 \text{ rev}}{2.5\,\text{rev/s}} = 0.050 \text{ s} \ .$$

The minimum speed of the arrow is then

$$v_{\text{min}} = \frac{20\,\text{cm}}{0.050\,\text{s}} = 400\,\text{cm/s} = 4.0 \text{ m/s} \ .$$

(b) No – there is no dependence on radial position in the above computation.

7. Applying Eq. 2-15 to the vertical axis (with $+y$ downward) we obtain the free-fall time:

$$\Delta y = v_{0y}t + \frac{1}{2}gt^2 \implies t = \sqrt{\frac{2(10)}{9.8}} = 1.4 \text{ s} .$$

Thus, by Eq. 11-5, the magnitude of the average angular velocity is

$$\omega_{\text{avg}} = \frac{(2.5)(2\pi)}{1.4} = 11 \text{ rad/s} .$$

8. (a) We assume the sense of rotation is positive. Applying Eq. 11-12, we obtain

$$\omega = \omega_0 + \alpha t \implies \alpha = \frac{3000 - 1200}{12/60} = 9000 \text{ rev/min}^2 .$$

(b) And Eq. 11-15 gives

$$\theta = \frac{1}{2}(\omega_0 + \omega)\, t = \frac{1}{2}(1200 + 3000)\left(\frac{12}{60}\right)$$

which yields $\theta = 420$ rev.

9. We assume the sense of initial rotation is positive. Then, with $\omega_0 > 0$ and $\omega = 0$ (since it stops at time t), our angular acceleration is negative-valued.

(a) The angular acceleration is constant, so we can apply Eq. 11-12 ($\omega = \omega_0 + \alpha t$). To obtain the requested units, we have $t = 30/60 = 0.50\,\text{min}$. Thus,

$$\alpha = -\frac{33.33\,\text{rev/min}}{0.50\,\text{min}} = -66.7 \text{ rev/min}^2 .$$

(b) We use Eq. 11-13:

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 = (33.33)(0.50) + \frac{1}{2}(-66.7)(0.50)^2 = 8.3 \text{ rev} .$$

10. We assume the sense of initial rotation is positive. Then, with $\omega_0 = +120$ rad/s and $\omega = 0$ (since it stops at time t), our angular acceleration ("deceleration") will be negative-valued: $\alpha = -4.0$ rad/s^2.

(a) We apply Eq. 11-12 to obtain t.

$$\omega = \omega_0 + \alpha t \implies t = \frac{0 - 120}{-4.0} = 30 \text{ s} .$$

(b) And Eq. 11-15 gives

$$\theta = \frac{1}{2}(\omega_0 + \omega)\, t = \frac{1}{2}(120 + 0)\,(30)$$

which yields $\theta = 1800$ rad. Alternatively, Eq. 11-14 could be used if it is desired to only use the given information (as opposed to using the result from part (a)) in obtaining θ. If using the result of part (a) is acceptable, then any angular equation in Table 11-1 (except Eq. 11-12) can be used to find θ.

11. We apply Eq. 11-12 twice, assuming the sense of rotation is positive. We have $\omega > 0$ and $\alpha < 0$. Since the angular velocity at $t = 1$ min is $\omega_1 = (0.90)(250) = 225$ rev/min, we have

$$\omega_1 = \omega_0 + \alpha t \implies \alpha = \frac{225 - 250}{1} = -25 \text{ rev/min}^2 .$$

Next, between $t = 1$ min and $t = 2$ min we have the interval $\Delta t = 1$ min. Consequently, the angular velocity at $t = 2$ min is

$$\omega_2 = \omega_1 + \alpha \Delta t = 225 + (-25)(1) = 200 \text{ rev/min} .$$

12. We assume the sense of rotation is positive, which (since it starts from rest) means all quantities (angular displacements, accelerations, etc.) are positive-valued.

    (a) The angular acceleration satisfies Eq. 11-13:

$$25\,\text{rad} = \frac{1}{2}\alpha(5.0\,\text{s})^2 \implies \alpha = 2.0\ \text{rad/s}^2\ .$$

    (b) The average angular velocity is given by Eq. 11-5:

$$\omega_{\text{avg}} = \frac{\Delta\theta}{\Delta t} = \frac{25\,\text{rad}}{5.0\,\text{s}} = 5.0\ \text{rad/s}\ .$$

    (c) Using Eq. 11-12, the instantaneous angular velocity at $t = 5.0$ s is

$$\omega = \left(2.0\,\text{rad/s}^2\right)(5.0\,\text{s}) = 10\ \text{rad/s}\ .$$

    (d) According to Eq. 11-13, the angular displacement at $t = 10$ s is

$$\theta = \omega_0 + \frac{1}{2}\alpha t^2 = 0 + \frac{1}{2}(2.0)(10)^2 = 100\ \text{rad}\ .$$

    Thus, the displacement between $t = 5$ s and $t = 10$ s is $\Delta\theta = 100 - 25 = 75$ rad.

13. We take $t = 0$ at the start of the interval and take the sense of rotation as positive. Then at the end of the $t = 4.0\,\text{s}$ interval, the angular displacement is $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$. We solve for the angular velocity at the start of the interval:

$$\omega_0 = \frac{\theta - \frac{1}{2}\alpha t^2}{t} = \frac{120\,\text{rad} - \frac{1}{2}\left(3.0\,\text{rad/s}^2\right)(4.0\,\text{s})^2}{4.0\,\text{s}} = 24\ \text{rad/s}\ .$$

    We now use $\omega = \omega_0 + \alpha t$ (Eq. 11-12) to find the time when the wheel is at rest:

$$t = -\frac{\omega_0}{\alpha} = -\frac{24\,\text{rad/s}}{3.0\,\text{rad/s}^2} = -8.0\ \text{s}\ .$$

    That is, the wheel started from rest 8.0 s before the start of the described 4.0 s interval.

14. The wheel starts turning from rest ($\omega_0 = 0$) at $t = 0$, and accelerates uniformly at $\alpha = 2.00$ rad/s^2. Between t_1 and t_2 it turns through $\Delta\theta = 90.0$ rad, where $t_2 - t_1 = \Delta t = 3.00$ s.

    (a) We use Eq. 11-13 (with a slight change in notation) to describe the motion for $t_1 \le t \le t_2$:

$$\Delta\theta = \omega_1 \Delta t + \frac{1}{2}\alpha\,(\Delta t)^2 \implies \omega_1 = \frac{\Delta\theta}{\Delta t} - \frac{\alpha\Delta t}{2}$$

    which we plug into Eq. 11-12, set up to describe the motion during $0 \le t \le t_1$:

$$\begin{aligned} \omega_1 &= \omega_0 + \alpha t_1 \\ \frac{\Delta\theta}{\Delta t} - \frac{\alpha\Delta t}{2} &= \alpha t_1 \\ \frac{90.0}{3.00} - \frac{(2.00)(3.00)}{2} &= (2.00)t_1 \end{aligned}$$

    yielding $t_1 = 13.5$ s.

    (b) Plugging into our expression for ω_1 (in previous part) we obtain

$$\omega_1 = \frac{\Delta\theta}{\Delta t} - \frac{\alpha\Delta t}{2} = \frac{90.0}{3.00} - \frac{(2.00)(3.00)}{2} = 27.0\ \text{rad/s}\ .$$

15. The problem has (implicitly) specified the positive sense of rotation. The angular acceleration of magnitude 0.25 rad/s^2 in the negative direction is assumed to be constant over a large time interval, including negative values (for t).

    (a) We specify $\theta_{\rm max}$ with the condition $\omega = 0$ (this is when the wheel reverses from positive rotation to rotation in the negative direction). We obtain $\theta_{\rm max}$ using Eq. 11-14:

    $$\theta_{\rm max} = -\frac{\omega_{\rm o}^2}{2\alpha} = -\frac{4.7^2}{2(-0.25)} = 44 \text{ rad} .$$

    (b) We find values for t_1 when the angular displacement (relative to its orientation at $t = 0$) is $\theta_1 = 22$ rad (or 22.09 rad if we wish to keep track of accurate values in all intermediate steps and only round off on the final answers). Using Eq. 11-13 and the quadratic formula, we have

    $$\theta_1 = \omega_{\rm o} t_1 + \frac{1}{2}\alpha t_1^2 \implies t_1 = \frac{-\omega_{\rm o} \pm \sqrt{\omega_{\rm o}^2 + 2\theta_1\alpha}}{\alpha}$$

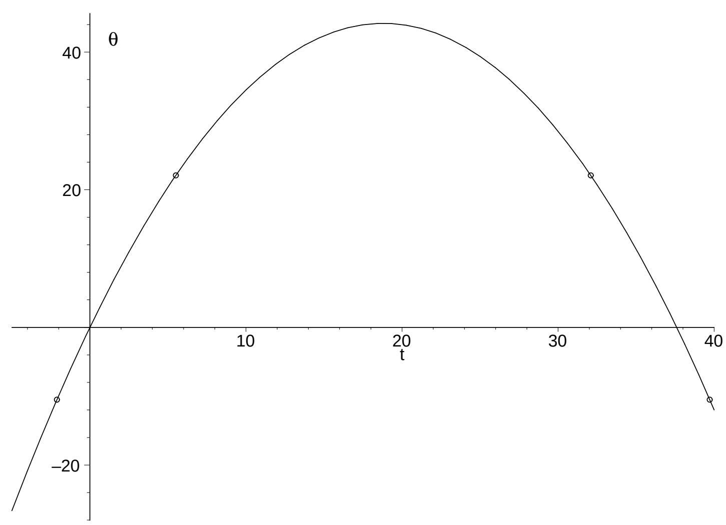
    which yields the two roots 5.5 s and 32 s.

    (c) We find values for t_2 when the angular displacement (relative to its orientation at $t = 0$) is $\theta_2 = -10.5$ rad. Using Eq. 11-13 and the quadratic formula, we have

    $$\theta_2 = \omega_{\rm o} t_2 + \frac{1}{2}\alpha t_2^2 \implies t_2 = \frac{-\omega_{\rm o} \pm \sqrt{\omega_{\rm o}^2 + 2\theta_2\alpha}}{\alpha}$$

    which yields the two roots -2.1 s and 40 s.

    (d) With radians and seconds understood, the graph of θ versus t is shown below (with the points found in the previous parts indicated as small circles).



16. The wheel starts turning from rest ($\omega_0 = 0$) at $t = 0$, and accelerates uniformly at $\alpha > 0$, which makes our choice for positive sense of rotation. At t_1 its angular velocity is $\omega_1 = +10$ rev/s, and at t_2 its angular velocity is $\omega_2 = +15$ rev/s. Between t_1 and t_2 it turns through $\Delta\theta = 60$ rev, where $t_2 - t_1 = \Delta t$.

(a) We find α using Eq. 11-14:

$$\omega_2^2 = \omega_1^2 + 2\alpha\Delta\theta \implies \alpha = \frac{15^2 - 10^2}{2(60)}$$

which yields $\alpha = 1.04$ rev/s^2 which we round off to 1.0 rev/s^2.

(b) We find Δt using Eq. 11-15:

$$\Delta\theta = \frac{1}{2}(\omega_1 + \omega_2)\,\Delta t \implies \Delta t = \frac{2(60)}{10+15} = 4.8 \text{ s} .$$

(c) We obtain t_1 using Eq. 11-12:

$$\omega_1 = \omega_0 + \alpha t_1 \implies t_1 = \frac{10}{1.04} = 9.6 \text{ s} .$$

(d) Any equation in Table 11-1 involving θ can be used to find θ_1 (the angular displacement during $0 \le t \le t_1$); we select Eq. 11-14.

$$\omega_1^2 = \omega_0^2 + 2\alpha\theta_1 \implies \theta_1 = \frac{10^2}{2(1.04)} = 48 \text{ rev} .$$

17. The wheel has angular velocity $\omega_0 = +1.5$ rad/s $= +0.239$ rev/s^2 at $t = 0$, and has constant value of angular acceleration $\alpha < 0$, which indicates our choice for positive sense of rotation. At t_1 its angular displacement (relative to its orientation at $t = 0$) is $\theta_1 = +20$ rev, and at t_2 its angular displacement is $\theta_2 = +40$ rev and its angular velocity is $\omega_2 = 0$.

(a) We obtain t_2 using Eq. 11-15:

$$\theta_2 = \frac{1}{2}(\omega_0 + \omega_2)\,t_2 \implies t_2 = \frac{2(40)}{0.239}$$

which yields $t_2 = 335$ s which we round off to $t_2 \approx 340$ s.

(b) Any equation in Table 11-1 involving α can be used to find the angular acceleration; we select Eq. 11-16.

$$\theta_2 = \omega_2 t_2 - \frac{1}{2}\alpha t_2^2 \implies \alpha = -\frac{2(40)}{335^2}$$

which yields $\alpha = -7.12 \times 10^{-4}$ rev/s^2 which we convert to $\alpha = -4.5 \times 10^{-3}$ rad/s^2.

(c) Using $\theta_1 = \omega_0 t_1 + \frac{1}{2}\alpha t_1^2$ (Eq. 11-13) and the quadratic formula, we have

$$t_1 = \frac{-\omega_0 \pm \sqrt{\omega_0^2 + 2\theta_1\alpha}}{\alpha} = \frac{-0.239 \pm \sqrt{0.239^2 + 2(20)\left(-7.12 \times 10^{-4}\right)}}{-7.12 \times 10^{-4}}$$

which yields two positive roots: 98 s and 572 s. Since the question makes sense only if $t_1 < t_2$ we conclude the correct result is $t_1 = 98$ s.

18. The wheel starts turning from rest ($\omega_0 = 0$) at $t = 0$, and accelerates uniformly at $\alpha = +4.0$ rad/s^2, which makes our choice for positive sense of rotation. At t_1 its angular displacement (relative to its orientation at $t = 0$) is θ_1, and at t_2 its angular velocity is θ_2, where $\theta_2 - \theta_1 = \Delta\theta = 80$ rad. Also, $t_2 - t_1 = \Delta t = 4.0$ s.

(a) We find the angular velocity at t_1 using Eq. 11-13 (set up to describe the interval $t_1 \le t \le t_2$).

$$\Delta\theta = \omega_1 \Delta t + \frac{1}{2}\alpha\left(\Delta t\right)^2 \implies \omega_1 = \frac{80 - \frac{1}{2}(4.0)(4.0)^2}{4.0}$$

which yields $\omega_1 = 12$ rad/s.

(b) We obtain t_1 using Eq. 11-12:

$$\omega_1 = \omega_0 + \alpha t_1 \quad \Longrightarrow \quad t_1 = \frac{12}{4.0} = 3.0 \text{ s} .$$

19. The magnitude of the acceleration is given by $a = \omega^2 r$ (Eq. 11-23) where r is the distance from the center of rotation and ω is the angular velocity. We convert the given angular velocity to rad/s:

$$\omega = \frac{(33.33\,\text{rev/min})(2\pi\,\text{rad/rev})}{60\,\text{s/min}} = 3.49 \text{ rad/s} .$$

Therefore,

$$a = \left(3.49\,\text{rad/s}^2\right)^2 (0.15\,\text{m}) = 1.8 \text{ m/s}^2 .$$

The acceleration vector is toward the center of the record.

20. (a) We obtain

$$\omega = \frac{(33.33\,\text{rev/min})(2\pi\,\text{rad/rev})}{60\,\text{s/min}} = 3.49 \text{ rad/s} .$$

(b) Using Eq. 11-18, we have

$$v = r\omega = (15)(3.49) = 52 \text{ cm/s} .$$

(c) Similarly, when $r = 7.4$ cm we find $v = r\omega = 26$ cm/s. The goal of this exercise to observe what is and is not the same at different locations on a body in rotational motion (ω is the same, v is not), as well as to emphasize the importance of radians when working with equations such as Eq. 11-18.

21. With $v = 50(1000/3600) = 13.9$ m/s, Eq. 11-18 leads to

$$\omega = \frac{v}{r} = \frac{13.9}{110} = 0.13 \text{ rad/s} .$$

22. (a) We obtain

$$\omega = \frac{(200\,\text{rev/min})(2\pi\,\text{rad/rev})}{60\,\text{s/min}} = 20.9 \text{ rad/s} .$$

(b) With $r = 1.20/2 = 0.60$ m, Eq. 11-18 leads to

$$v = r\omega = (0.60)(20.9) = 12.6 \text{ m/s} .$$

(c) With $t = 1$ min, $\omega = 1000$ rev/min and $\omega_\text{o} = 200$ rev/min, Eq. 11-12 gives

$$\alpha = \frac{\omega - \omega_\text{o}}{t} = 800 \text{ rev/min}^2 .$$

(d) With the same values used in part (c), Eq. 11-15 becomes

$$\theta = \frac{1}{2}(\omega_\text{o} + \omega)\,t = \frac{1}{2}(200 + 1000)(1) = 600 \text{ rev} .$$

23. (a) Using Eq. 11-6, the angular velocity at $t = 5.0$ s is

$$\omega = \left.\frac{d\theta}{dt}\right|_{t=5.0} = \left.\frac{d}{dt}\left(0.30t^2\right)\right|_{t=5.0} = 2(0.30)(5.0) = 3.0 \text{ rad/s} .$$

(b) Eq. 11-18 gives the linear speed at $t = 5.0$ s:

$$v = \omega r = (3.0\,\text{rad/s})(10\,\text{m}) = 30 \text{ m/s} .$$

(c) The angular acceleration is, from Eq. 11-8,

$$\alpha = \frac{d\omega}{dt} = \frac{d}{dt}(0.60t) = 0.60 \text{ rad/s}^2 .$$

Then, the tangential acceleration at $t = 5.0\,\text{s}$ is, using Eq. 11-22,

$$a_t = r\alpha = (10\,\text{m})\left(0.60\,\text{rad/s}^2\right) = 6.0 \text{ m/s}^2 .$$

(d) The radial (centripetal) acceleration is given by Eq. 11-23:

$$a_r = \omega^2 r = (3.0\,\text{rad/s})^2(10\,\text{m}) = 90 \text{ m/s}^2 .$$

24. (a) Converting from hours to seconds, we find the angular velocity (assuming it is positive) from Eq. 11-18:

$$\omega = \frac{v}{r} = \frac{\left(2.90 \times 10^4\,\text{km/h}\right)\left(\frac{1.00\,\text{h}}{3600\,\text{s}}\right)}{3.22 \times 10^3\,\text{km}} = 2.50 \times 10^{-3}\,\text{rad/s} .$$

(b) The radial (or centripetal) acceleration is computed according to Eq. 11-23:

$$a_r = \omega^2 r = \left(2.50 \times 10^{-3}\,\text{rad/s}\right)^2 \left(3.22 \times 10^6\,\text{m}\right) = 20.2 \text{ m/s}^2 .$$

(c) Assuming the angular velocity is constant, then the angular acceleration and the tangential acceleration vanish, since

$$\alpha = \frac{d\omega}{dt} = 0 \quad \text{and} \quad a_t = r\alpha = 0 .$$

25. (a) In the time light takes to go from the wheel to the mirror and back again, the wheel turns through an angle of $\theta = 2\pi/500 = 1.26 \times 10^{-2}\,\text{rad}$. That time is

$$t = \frac{2\ell}{c} = \frac{2(500\,\text{m})}{2.998 \times 10^8\,\text{m/s}} = 3.34 \times 10^{-6} \text{ s}$$

so the angular velocity of the wheel is

$$\omega = \frac{\theta}{t} = \frac{1.26 \times 10^{-2}\,\text{rad}}{3.34 \times 10^{-6}\,\text{s}} = 3.8 \times 10^3 \text{ rad/s} .$$

(b) If r is the radius of the wheel, the linear speed of a point on its rim is

$$v = \omega r = \left(3.8 \times 10^3\,\text{rad/s}\right)(0.05\,\text{m}) = 190 \text{ m/s} .$$

26. (a) The angular acceleration is

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{0 - 150\,\text{rev/min}}{(2.2\,\text{h})(60\,\text{min/1 h})} = -1.14 \text{ rev/min}^2 .$$

(b) Using Eq. 11-13 with $t = (2.2)(60) = 132$ min, the number of revolutions is

$$\begin{aligned} \theta &= \omega_0 t + \frac{1}{2}\alpha t^2 \\ &= (150\,\text{rev/min})(132\,\text{min}) + \frac{1}{2}\left(-1.14\,\text{rev/min}^2\right)(132\,\text{min})^2 \\ &= 9.9 \times 10^3 \text{ rev} . \end{aligned}$$

(c) With $r = 500$ mm, the tangential acceleration is

$$a_t = \alpha\, r = \left(-1.14\,\text{rev/min}^2\right)\left(\frac{2\pi\,\text{rad}}{1\ \text{rev}}\right)\left(\frac{1\,\text{min}}{60\,\text{s}}\right)^2 (500\,\text{mm})$$

which yields $a_t = -0.99$ mm/s^2.

(d) With $r = 0.50$ m, the radial (or centripetal) acceleration is given by Eq. 11-23:

$$a_r = \omega^2 r = \left((75\,\text{rev/min})\left(\frac{2\pi\,\text{rad/rev}}{1\,\text{min}/60\,\text{s}}\right)\right)^2 (0.50\,\text{m})$$

which yields $a_r = 31$ in SI units – and is seen to be much bigger than a_t. Consequently, the magnitude of the acceleration is

$$|\vec{a}| = \sqrt{a_r^2 + a_t^2} \approx a_r = 31\ \text{m/s}^2\ .$$

27. (a) Earth makes one rotation per day and $1\,d$ is $(24\,\text{h})(3600\,\text{s/h}) = 8.64 \times 10^4\,\text{s}$, so the angular speed of Earth is

$$\omega = \frac{2\pi\,\text{rad}}{8.64 \times 10^4\,\text{s}} = 7.27 \times 10^{-5}\ \text{rad/s}\ .$$

(b) We use $v = \omega r$, where r is the radius of its orbit. A point on Earth at a latitude of 40° moves along a circular path of radius $r = R\cos 40°$, where R is the radius of Earth (6.37×10^6 m). Therefore, its speed is

$$v = \omega\,(R\cos 40°) = \left(7.27 \times 10^{-5}\,\text{rad/s}\right)\left(6.37 \times 10^6\,\text{m}\right)\cos 40° = 355\ \text{m/s}\ .$$

(c) At the equator (and all other points on Earth) the value of ω is the same (7.27×10^{-5} rad/s).

(d) The latitude is 0° and the speed is

$$v = \omega R = \left(7.27 \times 10^{-5}\,\text{rad/s}\right)\left(6.37 \times 10^6\,\text{m}\right) = 463\ \text{m/s}\ .$$

28. (a) The tangential acceleration, using Eq. 11-22, is

$$a_t = \alpha r = \left(14.2\,\text{rad/s}^2\right)(2.83\,\text{cm}) = 40.2\ \text{cm/s}^2\ .$$

(b) In rad/s, the angular velocity is $\omega = (2760)(2\pi/60) = 289$, so

$$a_r = \omega^2 r = (289\,\text{rad/s})^2(0.0283\,\text{m}) = 2.36 \times 10^3\ \text{m/s}^2\ .$$

(c) The angular displacement is, using Eq. 11-14,

$$\theta = \frac{\omega^2}{2\alpha} = \frac{289^2}{2(14.2)} = 2.94 \times 10^3\ \text{rad}\ .$$

Then, using Eq. 11-1, the distance traveled is

$$s = r\theta = (0.0283\ \text{m})\left(2.94 \times 10^3\,\text{rad}\right) = 83.2\ \text{m}\ .$$

29. Since the belt does not slip, a point on the rim of wheel C has the same tangential acceleration as a point on the rim of wheel A. This means that $\alpha_A r_A = \alpha_C r_C$, where α_A is the angular acceleration of wheel A and α_C is the angular acceleration of wheel C. Thus,

$$\alpha_C = \left(\frac{r_A}{r_C}\right)\alpha_A = \left(\frac{10\,\text{cm}}{25\,\text{cm}}\right)(1.6\,\text{rad/s}^2) = 0.64\ \text{rad/s}^2\ .$$

Since the angular speed of wheel C is given by $\omega_C = \alpha_C t$, the time for it to reach an angular speed of $\omega = 100\,\text{rev/min} = 10.5\,\text{rad/s}$ starting from rest is

$$t = \frac{\omega_C}{\alpha_C} = \frac{10.5\,\text{rad/s}}{0.64\,\text{rad/s}^2} = 16\ \text{s}\ .$$

30. The function $\theta = \xi e^{\beta t}$ where $\xi = 0.40$ rad and $\beta = 2\,\mathrm{s}^{-1}$ is describing the angular coordinate of a line (which is marked in such a way that all points on it have the same value of angle at a given time) on the object. Taking derivatives with respect to time leads to $\frac{d\theta}{dt} = \xi\,\beta\,e^{\beta t}$ and $\frac{d^2\theta}{dt^2} = \xi\,\beta^2\,e^{\beta t}$.

   (a) Using Eq. 11-22, we have

$$a_t = \alpha r = \frac{d^2\theta}{dt^2}\,r = \;6.4\ \mathrm{cm/s^2}\ .$$

   (b) Using Eq. 11-23, we have

$$a_r = \omega^2 r = \left(\frac{d\theta}{dt}\right)^2 r = \;2.6\ \mathrm{cm/s^2}\ .$$

31. (a) A complete revolution is an angular displacement of $\Delta\theta = 2\pi$ rad, so the angular velocity in rad/s is given by $\omega = \Delta\theta/T = 2\pi/T$. The angular acceleration is given by

$$\alpha = \frac{d\omega}{dt} = -\frac{2\pi}{T^2}\frac{dT}{dt}\ .$$

   For the pulsar described in the problem, we have

$$\frac{dT}{dt} = \frac{1.26\times10^{-5}\,\mathrm{s/y}}{3.16\times10^{7}\,\mathrm{s/y}} = 4.00\times10^{-13}\ .$$

   Therefore,

$$\alpha = -\left(\frac{2\pi}{(0.033\,\mathrm{s})^2}\right)(4.00\times10^{-13}) = -2.3\times10^{-9}\,\mathrm{rad/s^2}\ .$$

   The negative sign indicates that the angular acceleration is opposite the angular velocity and the pulsar is slowing down.

   (b) We solve $\omega = \omega_0 + \alpha t$ for the time t when $\omega = 0$:

$$t = -\frac{\omega_0}{\alpha} = -\frac{2\pi}{\alpha T} = -\frac{2\pi}{(-2.3\times10^{-9}\,\mathrm{rad/s^2})(0.033\,\mathrm{s})} = 8.3\times10^{10}\,\mathrm{s}\ .$$

   This is about 2600 years.

   (c) The pulsar was born $1992 - 1054 = 938$ years ago. This is equivalent to $(938\,\mathrm{y})(3.16\times10^7\,\mathrm{s/y}) = 2.96\times10^{10}\,\mathrm{s}$. Its angular velocity at that time was

$$\omega = \omega_0 + \alpha t = \frac{2\pi}{T} + \alpha t = \frac{2\pi}{0.033\,\mathrm{s}} + (-2.3\times10^{-9}\,\mathrm{rad/s^2})(-2.96\times10^{10}\,\mathrm{s}) = 258\,\mathrm{rad/s}\ .$$

   Its period was

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{258\,\mathrm{rad/s}} = 2.4\times10^{-2}\,\mathrm{s}\ .$$

32. (a) The angular speed in rad/s is

$$\omega = \left(33\frac{1}{3}\,\mathrm{rev/min}\right)\left(\frac{2\pi\,\mathrm{rad/rev}}{60\,\mathrm{s/min}}\right) = 3.49\ \mathrm{rad/s}\ .$$

   Consequently, the radial (centripetal) acceleration is (using Eq. 11-23)

$$a = \omega^2 r = (3.49\,\mathrm{rad/s})^2\left(6.0\times10^{-2}\,\mathrm{m}\right) = 0.73\,\mathrm{m/s^2}\ .$$

   (b) Using Ch. 6 methods, we have $ma = f_s \le f_{s,\,\max} = \mu_s mg$, which is used to obtain the (minimum allowable) coefficient of friction:

$$\mu_{s,\,\min} = \frac{a}{g} = \frac{0.73}{9.8} = 0.075\ .$$

(c) The radial acceleration of the object is $a_r = \omega^2 r$, while the tangential acceleration is $a_t = \alpha r$. Thus

$$|\vec{a}| = \sqrt{a_r^2 + a_t^2} = \sqrt{(\omega^2 r)^2 + (\alpha r)^2} = r\sqrt{\omega^4 + \alpha^2} \ .$$

If the object is not to slip at any time, we require

$$f_{s,\text{max}} = \mu_s mg = ma_{\text{max}} = mr\sqrt{\omega_{\text{max}}^4 + \alpha^2} \ .$$

Thus, since $\alpha = \omega/t$ (from Eq. 11-12), we find

$$\begin{aligned}\mu_{s,\text{min}} &= \frac{r\sqrt{\omega_{\text{max}}^4 + \alpha^2}}{g} \\ &= \frac{r\sqrt{\omega_{\text{max}}^4 + (\omega_{\text{max}}/t)^2}}{g} \\ &= \frac{(0.060)\sqrt{3.49^4 + (3.49/0.25)^2}}{9.8} \\ &= 0.11 \ .\end{aligned}$$

33. The kinetic energy (in J) is given by $K = \frac{1}{2}I\omega^2$, where I is the rotational inertia (in kg·m^2) and ω is the angular velocity (in rad/s). We have

$$\omega = \frac{(602\,\text{rev/min})(2\pi\,\text{rad/rev})}{60\,\text{s/min}} = 63.0\,\text{rad/s} \ .$$

Consequently, the rotational inertia is

$$I = \frac{2K}{\omega^2} = \frac{2(24400\,\text{J})}{(63.0\,\text{rad/s})^2} = 12.3 \ \text{kg}\cdot\text{m}^2 \ .$$

34. The translational kinetic energy of the molecule is

$$K_t = \frac{1}{2}mv^2 = \frac{1}{2}\left(5.30 \times 10^{-26}\right)(500)^2 = 6.63 \times 10^{-21} \ \text{J} \ .$$

With $I = 1.94 \times 10^{-46}$ kg·m^2, we employ Eq. 11-27:

$$\begin{aligned}K_r &= \frac{2}{3}K_t \\ \frac{1}{2}I\omega^2 &= \frac{2}{3}\left(6.63 \times 10^{-21}\right)\end{aligned}$$

which leads to $\omega = 6.75 \times 10^{12}$ rad/s.

35. Since the rotational inertia of a cylinder is $I = \frac{1}{2}MR^2$ (Table 11-2(c)), its rotational kinetic energy is

$$K = \frac{1}{2}I\omega^2 = \frac{1}{4}MR^2\omega^2 \ .$$

For the first cylinder, we have $K = \frac{1}{4}(1.25)(0.25)^2(235)^2 = 1.1 \times 10^3$ J. For the second cylinder, we obtain $K = \frac{1}{4}(1.25)(0.75)^2(235)^2 = 9.7 \times 10^3$ J.

36. (a) Using Table 11-2(c), the rotational inertia is

$$I = \frac{1}{2}mR^2 = \frac{1}{2}(1210\,\text{kg})\left(\frac{1.21\,\text{m}}{2}\right)^2 = 221 \ \text{kg}\cdot\text{m}^2 \ .$$

(b) The rotational kinetic energy is, by Eq. 11-27,

$$\begin{aligned} K &= \frac{1}{2}I\omega^2 \\ &= \frac{1}{2}\left(2.21\times10^2\,\mathrm{kg\cdot m^2}\right)\left((1.52\,\mathrm{rev/s})(2\pi\,\mathrm{rad/rev})\right)^2 \\ &= 1.10\times10^4\ \mathrm{J}\ . \end{aligned}$$

37. The particles are treated "point-like" in the sense that Eq. 11-26 yields their rotational inertia, and the rotational inertia for the rods is figured using Table 11-2(e) and the parallel-axis theorem (Eq. 11-29).

(a) With subscript 1 standing for the rod nearest the axis and 4 for the particle farthest from it, we have

$$\begin{aligned} I &= I_1+I_2+I_3+I_4 \\ &= \left(\frac{1}{12}Md^2+M\left(\frac{1}{2}d\right)^2\right)+md^2+\left(\frac{1}{12}Md^2+M\left(\frac{3}{2}d\right)^2\right)+m(2d)^2 \\ &= \frac{8}{3}Md^2+5md^2\ . \end{aligned}$$

(b) Using Eq. 11-27, we have

$$K=\frac{1}{2}I\omega^2=\left(\frac{4}{3}Md^2+\frac{5}{2}md^2\right)\omega^2\ .$$

38. (a) The rotational inertia of the three blades (each of mass m and length L) is

$$I=3\left(\frac{1}{3}mL^2\right)=mL^2=(240\,\mathrm{kg})(5.2\,\mathrm{m})^2=6.49\times10^3\ \mathrm{kg\cdot m^2}\ .$$

(b) The rotational kinetic energy is

$$\begin{aligned} K &= \frac{1}{2}I\omega^2 \\ &= \frac{1}{2}\left(6.49\times10^3\,\mathrm{kg\cdot m^2}\right)\left((350\,\mathrm{rev/min})\left(\frac{2\pi\,\mathrm{rad/rev}}{60\,\mathrm{s/min}}\right)\right)^2 \\ &= 4.36\times10^6\ \mathrm{J}=\ 4.36\ \mathrm{MJ}\ . \end{aligned}$$

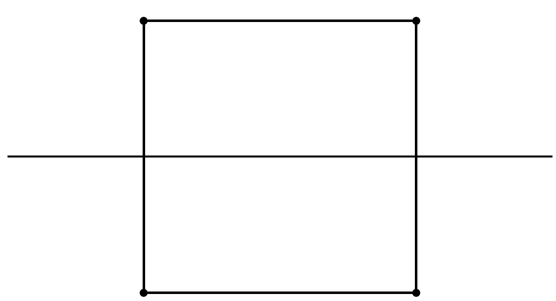
39. We use the parallel axis theorem: $I=I_{\mathrm{com}}+Mh^2$, where I_{com} is the rotational inertia about the center of mass (see Table 11-2(d)), M is the mass, and h is the distance between the center of mass and the chosen rotation axis. The center of mass is at the center of the meter stick, which implies $h=0.50\,\mathrm{m}-0.20\,\mathrm{m}=0.30\,\mathrm{m}$. We find

$$I_{\mathrm{com}}=\frac{1}{12}ML^2=\frac{1}{12}(0.56\,\mathrm{kg})(1.0\,\mathrm{m})^2=4.67\times10^{-2}\ \mathrm{kg\cdot m^2}\ .$$

Consequently, the parallel axis theorem yields

$$I=4.67\times10^{-2}\,\mathrm{kg\cdot m^2}+(0.56\,\mathrm{kg})(0.30\,\mathrm{m})^2=9.7\times10^{-2}\ \mathrm{kg\cdot m^2}\ .$$

40. (a) We show the figure with its axis of rotation (the thin horizontal line).



We note that each mass is $r = 1.0$ m from the axis. Therefore, using Eq. 11-26, we obtain

$$I = \sum m_i r_i^2 = 4(0.50 \text{ kg})(1.0 \text{ m})^2 = 2 \text{ kg}\cdot\text{m}^2 \ .$$

(b) In this case, the two masses nearest the axis are $r = 1.0$ m away from it, but the two furthest from the axis are $r = \sqrt{1.0^2 + 2.0^2}$ m from it. Here, then, Eq. 11-26 leads to

$$I = \sum m_i r_i^2 = 2(0.50 \text{ kg})(1.0 \text{ m}^2) + 2(0.50 \text{ kg})(5.0 \text{ m}^2) = 6.0 \text{ kg}\cdot\text{m}^2 \ .$$

(c) Now, two masses are on the axis (with $r = 0$) and the other two are a distance $r = \sqrt{1.0^2 + 1.0^2}$ m away. Now we obtain $I = 2.0 \text{ kg} \cdot \text{m}^2$.

41. We use the parallel-axis theorem. According to Table 11-2(i), the rotational inertia of a uniform slab about an axis through the center and perpendicular to the large faces is given by

$$I_{\text{com}} = \frac{M}{12}(a^2 + b^2) \ .$$

A parallel axis through the corner is a distance $h = \sqrt{(a/2)^2 + (b/2)^2}$ from the center. Therefore,

$$I = I_{\text{com}} + Mh^2 = \frac{M}{12}\left(a^2 + b^2\right) + \frac{M}{4}\left(a^2 + b^2\right) = \frac{M}{3}\left(a^2 + b^2\right) \ .$$

42. (a) We apply Eq. 11-26:

$$I_x = \sum_{i=1}^{4} m_i y_i^2 = 50(2.0)^2 + (25)(4.0)^2 + 25(-3.0)^2 + 30(4.0)^2 = 1.3 \times 10^3 \text{ g}\cdot\text{cm}^2 \ .$$

(b) For rotation about the y axis we obtain

$$I_y = \sum_{i=1}^{4} m_i x_i^2 = 50(2.0)^2 + (25)(0)^2 + 25(3.0)^2 + 30(2.0)^2 = 5.5 \times 10^2 \text{ g}\cdot\text{cm}^2 \ .$$

(c) And about the z axis, we find (using the fact that the distance from the z axis is $\sqrt{x^2 + y^2}$)

$$I_z = \sum_{i=1}^{4} m_i(x_i^2 + y_i^2) = I_x + I_y = 1.3 \times 10^3 + 5.5 \times 10^2 = 1.9 \times 10^2 \text{ g}\cdot\text{cm}^2 \ .$$

(d) Clearly, the answer to part (c) is $A + B$.

43. (a) According to Table 11-2, the rotational inertia formulas for the cylinder (radius R) and the hoop (radius r) are given by

$$I_C = \frac{1}{2}MR^2 \quad \text{and} \quad I_H = Mr^2 \ .$$

Since the two bodies have the same mass, then they will have the same rotational inertia if $R^2/2 = R_H^2$, or $R_H = R/\sqrt{2}$.

(b) We require the rotational inertia to be written as $I = Mk^2$, where M is the mass of the given body and k is the radius of the "equivalent hoop." It follows directly that $k = \sqrt{I/M}$.

44. (a) Using Table 11-2(c) and Eq. 11-27, the rotational kinetic energy is

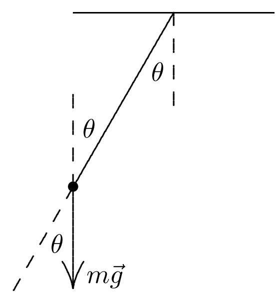
$$\begin{aligned} K &= \frac{1}{2}I\omega^2 \\ &= \frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega^2 \\ &= \frac{1}{4}(500\,\text{kg})(200\pi\,\text{rad/s})^2(1.0\,\text{m})^2 \\ &= 4.9 \times 10^7 \text{ J} \ . \end{aligned}$$

(b) We solve $P = K/t$ (where P is the average power) for the operating time t.

$$t = \frac{K}{P} = \frac{4.9 \times 10^7 \text{ J}}{8.0 \times 10^3 \text{ W}} = 6.2 \times 10^3 \text{ s}$$

which we rewrite as $t \approx 100$ min.

45. Two forces act on the ball, the force of the rod and the force of gravity. No torque about the pivot point is associated with the force of the rod since that force is along the line from the pivot point to the ball. As can be seen from the diagram, the component of the force of gravity that is perpendicular to the rod is $mg \sin\theta$. If ℓ is the length of the rod, then the torque associated with this force has magnitude $\tau = mg\ell \sin\theta = (0.75)(9.8)(1.25) \sin 30° = 4.6 \text{ N} \cdot \text{m}$. For the position shown, the torque is counterclockwise.



46. We compute the torques using $\tau = rF \sin\phi$.

$$\begin{aligned}
\tau_a &= (0.152 \text{ m})(111 \text{ N}) \sin 30° = 8.4 \text{ N}\cdot\text{m} \\
\tau_b &= (0.152 \text{ m})(111 \text{ N}) \sin 90° = 17 \text{ N}\cdot\text{m} \\
\tau_c &= (0.152 \text{ m})(111 \text{ N}) \sin 180° = 0
\end{aligned}$$

47. (a) We take a torque that tends to cause a counterclockwise rotation from rest to be positive and a torque tending to cause a clockwise rotation to be negative. Thus, a positive torque of magnitude $r_1 F_1 \sin\theta_1$ is associated with $\vec{F}_1$ and a negative torque of magnitude $r_2 F_2 \sin\theta_2$ is associated with $\vec{F}_2$. The net torque is consequently

$$\tau = r_1 F_1 \sin\theta_1 - r_2 F_2 \sin\theta_2 .$$

(b) Substituting the given values, we obtain

$$\tau = (1.30 \text{ m})(4.20 \text{ N}) \sin 75° - (2.15 \text{ m})(4.90 \text{ N}) \sin 60° = -3.85 \text{ N}\cdot\text{m} .$$

48. The net torque is

$$\begin{aligned}
\tau &= \tau_A + \tau_B + \tau_C \\
&= F_A r_A \sin\phi_A - F_B r_B \sin\phi_B + F_C r_C \sin\phi_C \\
&= (10)(8.0) \sin 135° - (16)(4.0) \sin 90° + (19)(3.0) \sin 160° \\
&= 12 \text{ N}\cdot\text{m} .
\end{aligned}$$

49. (a) We use the kinematic equation $\omega = \omega_0 + \alpha t$, where ω_0 is the initial angular velocity, ω is the final angular velocity, α is the angular acceleration, and t is the time. This gives

$$\alpha = \frac{\omega - \omega_0}{t} = \frac{6.20 \text{ rad/s}}{220 \times 10^{-3} \text{ s}} = 28.2 \text{ rad/s}^2 .$$

(b) If I is the rotational inertia of the diver, then the magnitude of the torque acting on her is

$$\tau = I\alpha = \left(12.0\,\text{kg}\cdot\text{m}^2\right)\left(28.2\,\text{rad/s}^2\right) = 3.38\times10^2\,\text{N}\cdot\text{m}\ .$$

50. The rotational inertia is found from Eq. 11-37.

$$I = \frac{\tau}{\alpha} = \frac{32.0}{25.0} = 1.28\ \text{kg}{\cdot}\text{m}^2$$

51. (a) We use $\tau = I\alpha$, where τ is the net torque acting on the shell, I is the rotational inertia of the shell, and α is its angular acceleration. Therefore,

$$I = \frac{\tau}{\alpha} = \frac{960\,\text{N}\cdot\text{m}}{6.20\,\text{rad/s}^2} = 155\,\text{kg}\cdot\text{m}^2\ .$$

(b) The rotational inertia of the shell is given by $I = (2/3)MR^2$ (see Table 11-2 of the text). This implies

$$M = \frac{3I}{2R^2} = \frac{3(155\,\text{kg}\cdot\text{m}^2)}{2(1.90\,\text{m})^2} = 64.4\,\text{kg}\ .$$

52. According to the sign conventions used in the book, the magnitude of the net torque exerted on the cylinder of mass m and radius R_2 is

$$\begin{aligned}\tau_{\text{net}} &= F_1R_2 - F_2R_2 - F_3R_1\\ &= (6.0\,\text{N})(0.12\,\text{m}) - (4.0\,\text{N})(0.12\,\text{m}) - (2.0\,\text{N})(0.05\,\text{m})\\ &= 71\ \text{N}{\cdot}\text{m}\ .\end{aligned}$$

The resulting angular acceleration of the cylinder (with $I = \frac{1}{2}MR^2$ according to Table 11-2(c)) is

$$\begin{aligned}\alpha &= \frac{\tau_{\text{net}}}{I}\\ &= \frac{71\ \text{N}{\cdot}\text{m}}{\frac{1}{2}(2.0\,\text{kg})(0.12\,\text{m})^2}\\ &= 9.7\ \text{rad/s}^2\end{aligned}$$

and is counterclockwise (which is the positive sense of rotation).

53. We use $\tau = Fr = I\alpha$, where α satisfies $\theta = \frac{1}{2}\alpha t^2$ (Eq. 11-13). Here $\theta = 90° = \frac{\pi}{2}$ rad and $t = 30\,\text{s}$. The force needed is consequently

$$F = \frac{I\alpha}{r} = \frac{I\left(2\theta/t^2\right)}{r} = \frac{(8.7\times10^4)\left(2(\pi/2)/30^2\right)}{2.4} = 1.3\times10^2\,\text{N}\ .$$

54. With rightward positive for the block and clockwise negative for the wheel (as is conventional), then we note that the tangential acceleration of the wheel is of opposite sign from the block's acceleration (which we simply denote as a); that is, $a_t = -a$. Applying Newton's second law to the block leads to

$$P - T = ma \qquad\qquad \text{where } m = 2.0\ \text{kg}\ .$$

Applying Newton's second law (for rotation) to the wheel leads to

$$-TR = I\alpha \qquad\qquad \text{where } I = 0.050\ \text{kg}\cdot\text{m}^2\ .$$

Noting that $R\alpha = a_t = -a$, we multiply this equation by R and obtain

$$-TR^2 = -Ia \implies T = a\,\frac{I}{R^2}\ .$$

Adding this to the above equation (for the block) leads to

$$P = \left(m + \frac{I}{R^2}\right) a \ .$$

Thus, $a = 0.92 \text{ m/s}^2$ and therefore $\alpha = -4.6 \text{ rad/s}^2$, where the negative sign should not be mistaken for a deceleration (it simply indicates the clockwise sense to the motion).

55. (a) We use constant acceleration kinematics. If down is taken to be positive and a is the acceleration of the heavier block, then its coordinate is given by $y = \frac{1}{2}at^2$, so

$$a = \frac{2y}{t^2} = \frac{2(0.750\,\text{m})}{(5.00\,\text{s})^2} = 6.00 \times 10^{-2} \text{ m/s}^2 \ .$$

The lighter block has an acceleration of $6.00 \times 10^{-2}\,\text{m/s}^2$ upward.

(b) Newton's second law for the heavier block is $m_h g - T_h = m_h a$, where m_h is its mass and T_h is the tension force on the block. Thus,

$$T_h = m_h(g - a) = (0.500\,\text{kg})\left(9.8\,\text{m/s}^2 - 6.00 \times 10^{-2}\,\text{m/s}^2\right) = 4.87 \text{ N} \ .$$

(c) Newton's second law for the lighter block is $m_l g - T_l = -m_l a$, where T_l is the tension force on the block. Thus,

$$T_l = m_l(g + a) = (0.460\,\text{kg})\left(9.8\,\text{m/s}^2 + 6.00 \times 10^{-2}\,\text{m/s}^2\right) = 4.54\,\text{N} \ .$$

(d) Since the cord does not slip on the pulley, the tangential acceleration of a point on the rim of the pulley must be the same as the acceleration of the blocks, so

$$\alpha = \frac{a}{R} = \frac{6.00 \times 10^{-2}\,\text{m/s}^2}{5.00 \times 10^{-2}\,\text{m}} = 1.20 \text{ rad/s}^2 \ .$$

(e) The net torque acting on the pulley is $\tau = (T_h - T_l)R$. Equating this to $I\alpha$ we solve for the rotational inertia:

$$\begin{aligned} I &= \frac{(T_h - T_l)R}{\alpha} \\ &= \frac{(4.87\,\text{N} - 4.54\,\text{N})(5.00 \times 10^{-2}\,\text{m})}{1.20\,\text{rad/s}^2} \\ &= 1.38 \times 10^{-2} \text{ kg}\cdot\text{m}^2 \ . \end{aligned}$$

56. Since the force acts tangentially at $r = 0.10$ m, the angular acceleration (presumed positive) is

$$\alpha = \frac{\tau}{I} = \frac{Fr}{I} = \frac{\left(0.5t + 0.3t^2\right)(0.10)}{1.0 \times 10^{-3}} = 50t + 30t^2$$

in SI units (rad/s^2).

(a) At $t = 3$ s, the above expression becomes $\alpha = 420 \text{ rad/s}^2$.

(b) We integrate the above expression, noting that $\omega_o = 0$, to obtain the angular speed at $t = 3$ s:

$$\omega = \int_0^3 \alpha\, dt = \left(25t^2 + 10t^3\right)\Big|_0^3 = 5.0 \times 10^2\,\text{rad/s} \ .$$

57. With counterclockwise positive, the angular acceleration α for both masses satisfies $\tau = mgL_1 - mgL_2 = I\alpha = (mL_1^2 + mL_2^2)\alpha$, by combining Eq. 11-37 with Eq. 11-32 and Eq. 11-26. Therefore, using SI units,

$$\alpha = \frac{g\,(L_1 - L_2)}{L_1^2 + L_2^2} = \frac{(9.8)(0.20 - 0.80)}{0.80^2 + 0.20^2} = -8.65\ \text{rad/s}^2$$

where the negative sign indicates the system starts turning in the clockwise sense. The magnitude of the acceleration vector involves no radial component (yet) since it is evaluated at $t = 0$ when the instantaneous velocity is zero. Thus, for the two masses, we apply Eq. 11-22 and obtain the respective answers for parts (a) and (b):

$$\begin{aligned} |\vec{a}_1| = |\alpha|L_1 &= \left(8.65\,\text{rad/s}^2\right)(0.80\,\text{m}) = 6.9\ \text{m/s}^2 \\ |\vec{a}_2| = |\alpha|L_2 & \\ &= \left(8.65\,\text{rad/s}^2\right)(0.20\,\text{m}) \\ &= 1.7\ \text{m/s}^2 . \end{aligned}$$

58. (a) The speed of v of the mass m after it has descended $d = 50\,\text{cm}$ is given by $v^2 = 2ad$ (Eq. 2-16) where a is calculated as in Sample Problem 11-7 except that here we choose $+y$ downward (so $a > 0$). Thus, using $g = 980\ \text{cm/s}^2$, we have

$$v = \sqrt{2ad} = \sqrt{\frac{2(2mg)d}{M + 2m}} = \sqrt{\frac{4(50)(980)(50)}{400 + 2(50)}} = 1.4 \times 10^2\,\text{cm/s} .$$

(b) The answer is still $1.4 \times 10^2\,\text{cm/s} = 1.4\,\text{m/s}$, since it is independent of R.

59. With $\omega = (1800)(2\pi/60) = 188.5$ rad/s, we apply Eq. 11-47:

$$P = \tau\omega \implies \tau = \frac{74600\,\text{W}}{188.5\,\text{rad/s}}$$

which yields $\tau = 396$ N·m.

60. The initial angular speed is $\omega = (280)(2\pi/60) = 29.3$ rad/s. We use Eq. 11-44 for the work and Eq. 7-42 for the average power.

(a) Since the rotational inertia is (Table 11-2(a)) $I = (32)(1.2)^2 = 46.1\ \text{kg·m}^2$, the work done is

$$W = \Delta K = 0 - \frac{1}{2}I\omega^2 = -\frac{1}{2}(46.1)(29.3)^2$$

which yields $|W| = 19.8 \times 10^3$ J.

(b) The average power (in absolute value) is therefore

$$|P| = \frac{|W|}{\Delta t} = \frac{19.8 \times 10^3}{15} = 1.32 \times 10^3\,\text{W} .$$

61. (a) We apply Eq. 11-27:

$$K = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{3}mL^2\right)\omega^2 = \frac{1}{6}mL^2\omega^2 .$$

(b) Simple conservation of mechanical energy leads to $K = mgh$. Consequently, the center of mass rises by

$$h = \frac{K}{mg} = \frac{mL^2\omega^2}{6mg} = \frac{L^2\omega^2}{6g} .$$

62. (a) The angular speed ω associated with Earth's spin is $\omega = 2\pi/T$, where $T = 86400\,\text{s}$ (one day). Thus

$$\omega = \frac{2\pi}{86400\,\text{s}} = 7.27 \times 10^{-5}\,\text{rad/s}$$

and the angular acceleration α required to accelerate the Earth from rest to ω in one day is $\alpha = \omega/T$. The torque needed is then

$$\tau = I\alpha = \frac{I\omega}{T} = \frac{\left(9.71 \times 10^{27}\right)\left(7.27 \times 10^{-5}\right)}{86400} = 8.17 \times 10^{28}\ \text{N}\cdot\text{m}$$

where we used

$$I = \frac{2}{5}MR^2 = \frac{2}{5}\left(5.98 \times 10^{24}\right)\left(6.37 \times 10^{6}\right)^2$$

for Earth's rotational inertia.

(b) Using the values from part (a), the kinetic energy of the Earth associated with its rotation about its own axis is $K = \frac{1}{2}I\omega^2 = 2.57 \times 10^{29}\,\text{J}$. This is how much energy would need to be supplied to bring it (starting from rest) to the current angular speed.

(c) The associated power is

$$P = \frac{K}{T} = \frac{2.57 \times 10^{29}\,\text{J}}{86400\,\text{s}} = 2.97 \times 10^{24}\ \text{W}\ .$$

63. We use ℓ to denote the length of the stick. Since its center of mass is $\ell/2$ from either end, its initial potential energy is $\frac{1}{2}mg\ell$, where m is its mass. Its initial kinetic energy is zero. Its final potential energy is zero, and its final kinetic energy is $\frac{1}{2}I\omega^2$, where I is its rotational inertia about an axis passing through one end of the stick and ω is the angular velocity just before it hits the floor. Conservation of energy yields

$$\frac{1}{2}mg\ell = \frac{1}{2}I\omega^2 \quad\Longrightarrow\quad \omega = \sqrt{\frac{mg\ell}{I}}\ .$$

The free end of the stick is a distance ℓ from the rotation axis, so its speed as it hits the floor is (from Eq. 11-18)

$$v = \omega\ell = \sqrt{\frac{mg\ell^3}{I}}\ .$$

Using Table 11-2 and the parallel-axis theorem, the rotational inertial is $I = \frac{1}{3}m\ell^2$, so

$$v = \sqrt{3g\ell} = \sqrt{3\left(9.8\,\text{m/s}^2\right)(1.00\,\text{m})} = 5.42\ \text{m/s}\ .$$

64. (a) We use the parallel-axis theorem to find the rotational inertia:

$$\begin{aligned} I &= I_{\text{com}} + Mh^2 = \frac{1}{2}MR^2 + Mh^2 \\ &= \frac{1}{2}(20\,\text{kg})(0.10\,\text{m})^2 + (20\,\text{kg})(0.50\,\text{m})^2 \\ &= 0.15\ \text{kg}\cdot\text{m}^2\ . \end{aligned}$$

(b) Conservation of energy requires that $Mgh = \frac{1}{2}I\omega^2$, where ω is the angular speed of the cylinder as it passes through the lowest position. Therefore,

$$\omega = \sqrt{\frac{2Mgh}{I}} = \sqrt{\frac{2(20)(9.8)(0.050)}{0.15}} = 11\ \text{rad/s}\ .$$

65. We use conservation of mechanical energy. The center of mass is at the midpoint of the cross bar of the **H** and it drops by $L/2$, where L is the length of any one of the rods. The gravitational potential energy decreases by $MgL/2$, where M is the mass of the body. The initial kinetic energy is zero and the final kinetic energy may be written $\frac{1}{2}I\omega^2$, where I is the rotational inertia of the body and ω is its angular velocity when it is vertical. Thus

$$0 = -MgL/2 + \frac{1}{2}I\omega^2 \implies \omega = \sqrt{MgL/I} .$$

Since the rods are thin the one along the axis of rotation does not contribute to the rotational inertia. All points on the other leg are the same distance from the axis of rotation, so that leg contributes $(M/3)L^2$, where $M/3$ is its mass. The cross bar is a rod that rotates around one end, so its contribution is $(M/3)L^2/3 = ML^2/9$. The total rotational inertia is $I = (ML^2/3) + (ML^2/9) = 4ML^2/9$. Consequently, the angular velocity is

$$\omega = \sqrt{\frac{MgL}{I}} = \sqrt{\frac{MgL}{4ML^2/9}} = \sqrt{\frac{9g}{4L}} .$$

66. From Table 11-2, the rotational inertia of the spherical shell is $2MR^2/3$, so the kinetic energy (after the object has descended distance h) is

$$K = \frac{1}{2}\left(\frac{2}{3}MR^2\right)\omega_{\text{sphere}}^2 + \frac{1}{2}I\omega_{\text{pulley}}^2 + \frac{1}{2}mv^2 .$$

Since it started from rest, then this energy must be equal (in the absence of friction) to the potential energy mgh with which the system started. We substitute v/r for the pulley's angular speed and v/R for that of the sphere and solve for v.

$$v = \sqrt{\frac{mgh}{\frac{1}{2}m + \frac{1}{2}\frac{I}{r^2} + \frac{M}{3}}} = \sqrt{\frac{2gh}{1 + (I/mr^2) + (2M/3m)}}$$

67. (a) We use conservation of mechanical energy to find an expression for ω^2 as a function of the angle θ that the chimney makes with the vertical. The potential energy of the chimney is given by $U = Mgh$, where M is its mass and h is the altitude of its center of mass above the ground. When the chimney makes the angle θ with the vertical, $h = (H/2)\cos\theta$. Initially the potential energy is $U_i = Mg(H/2)$ and the kinetic energy is zero. The kinetic energy is $\frac{1}{2}I\omega^2$ when the chimney makes the angle θ with the vertical, where I is its rotational inertia about its bottom edge. Conservation of energy then leads to

$$MgH/2 = Mg(H/2)\cos\theta + \frac{1}{2}I\omega^2 \implies \omega^2 = (MgH/I)(1 - \cos\theta) .$$

The rotational inertia of the chimney about its base is $I = MH^2/3$ (found using Table 11-2(e) with the parallel axis theorem). Thus

$$\omega = \sqrt{\frac{3g}{H}(1 - \cos\theta)} .$$

(b) The radial component of the acceleration of the chimney top is given by $a_r = H\omega^2$, so $a_r = 3g(1 - \cos\theta)$.

(c) The tangential component of the acceleration of the chimney top is given by $a_t = H\alpha$, where α is the angular acceleration. We are unable to use Table 11-1 since the acceleration is not uniform. Hence, we differentiate $\omega^2 = (3g/H)(1 - \cos\theta)$ with respect to time, replacing $d\omega/dt$ with α, and $d\theta/dt$ with ω, and obtain

$$\frac{d\omega^2}{dt} = 2\omega\alpha = (3g/H)\omega\sin\theta \implies \alpha = (3g/2H)\sin\theta .$$

Consequently, $a_t = H\alpha = \frac{3g}{2}\sin\theta$.

(d) The angle θ at which $a_t = g$ is the solution to $\frac{3g}{2}\sin\theta = g$. Thus, $\sin\theta = 2/3$ and we obtain $\theta = 41.8°$.

68. (a) The longitudinal separation between Helsinki and the explosion site is $\Delta\theta = 102° - 25° = 77°$. The spin of the earth is constant at

$$\omega = \frac{1\,\text{rev}}{1\,\text{day}} = \frac{360°}{24\,\text{h}}$$

so that an angular displacement of $\Delta\theta$ corresponds to a time interval of

$$\Delta t = (77°)\left(\frac{24\,\text{h}}{360°}\right) = 5.1 \text{ h} .$$

(b) Now $\Delta\theta = 102° - (-20°) = 122°$ so the required time shift would be

$$\Delta t = (122°)\left(\frac{24\,\text{h}}{360°}\right) = 8.1 \text{ h} .$$

69. Analyzing the forces tending to drag the $M = 5124$ kg stone down the oak beam, we find

$$F = Mg\,(\sin\theta + \mu_s\cos\theta)$$

where $\mu_s = 0.22$ (static friction is assumed to be at its maximum value) and the incline angle θ for the oak beam is $\sin^{-1}(3.9/10) = 23°$ (but the incline angle for the spruce log is the complement of that). We note that the component of the weight of the workers (N of them) which is perpendicular to the spruce log is $Nmg\cos(90° - \theta) = Nmg\sin\theta$, where $m = 85$ kg. The corresponding torque is therefore $Nmg\ell\sin\theta$ where $\ell = 4.5 - 0.7 = 3.8$ m (see figure). This must (at least) equal the magnitude of torque due to F, so with $r = 0.7$ m, we have

$$Mgr\,(\sin\theta + \mu_s\cos\theta) = Ngm\ell\sin\theta .$$

This expression yields $N \approx 17$ for the number of workers.

70. (a) We apply Eq. 11-18, using the subscript J for the Jeep.

$$\omega = \frac{v_J}{r_J} = \frac{114\,\text{km/h}}{0.100\,\text{km}}$$

which yields 1140 rad/h or (dividing by 3600) 0.32 rad/s for the value of the angular speed ω.

(b) Since the cheetah has the same angular speed, we again apply Eq. 11-18, using the subscript c for the cheetah.

$$v_c = r_c\omega = (92\,\text{m})(1140\,\text{rad/h})$$

which yields 1.05×10^5 m/h or 105 km/h for the cheetah's speed.

71. The *Hint* given in the problem would make the computation in part (a) very straightforward (without doing the integration as we show here), but we present this further level of detail in case that hint is not obvious or – simply – in case one wishes to see how the calculus supports our intuition.

(a) The (centripetal) force exerted on an infinitesimal portion of the blade with mass dm located a distance r from the rotational axis is (Newton's second law) $dF = (dm)\omega^2 r$, where dm can be written as $(M/L)dr$ and the angular speed is $\omega = (320)(2\pi/60) = 33.5$ rad/s. Thus for the entire blade of mass M and length L the total force is given by

$$\begin{aligned} F &= \int dF = \int \omega^2\, r\, dm \\ &= \frac{M}{L}\int_0^L \omega^2\, r\, dr \end{aligned}$$

$$\begin{aligned}
&= \left.\frac{M\omega^2 r^2}{2L}\right|_0^L = \frac{M\omega^2 L}{2} \\
&= \frac{(110\,\text{kg})(33.5\,\text{rad/s})^2(7.80\,\text{m})}{2} \\
&= 4.8 \times 10^5 \text{ N} .
\end{aligned}$$

(b) About its center of mass, the blade has $I = ML^2/12$ according to Table 11-2(e), and using the parallel-axis theorem to "move" the axis of rotation to its end-point, we find the rotational inertia becomes $I = ML^2/3$. Using Eq. 11-37, the torque (assumed constant) is

$$\begin{aligned}
\tau &= I\alpha \\
&= \left(\frac{1}{3}ML^2\right)\left(\frac{\Delta\omega}{\Delta t}\right) \\
&= \frac{1}{3}(110\,\text{kg})(7.8\,\text{m})^2\left(\frac{33.5\,\text{rad/s}}{6.7\,\text{s}}\right) \\
&= 1.1 \times 10^4 \text{ N}\cdot\text{m} .
\end{aligned}$$

(c) Using Eq. 11-44, the work done is

$$\begin{aligned}
W &= \Delta K = \frac{1}{2}I\omega^2 - 0 \\
&= \frac{1}{2}\left(\frac{1}{3}ML^2\right)\omega^2 \\
&= \frac{1}{6}(110\,\text{kg})(7.80\,\text{m})^2(33.5\,\text{rad/s})^2 \\
&= 1.3 \times 10^6 \text{ J} .
\end{aligned}$$

72. (a) Constant angular acceleration kinematics can be used to compute the angular acceleration α. If ω_0 is the initial angular velocity and t is the time to come to rest, then

$$0 = \omega_0 + \alpha t \quad \Longrightarrow \quad \alpha = -\frac{\omega_0}{t}$$

which yields $-39/32 = -1.2$ rev/s or (multiplying by 2π) $-7.66\,\text{rad/s}^2$ for the value of α.

(b) We use $\tau = I\alpha$, where τ is the torque and I is the rotational inertia. The contribution of the rod to I is $M\ell^2/12$ (Table 11-2(e)), where M is its mass and ℓ is its length. The contribution of each ball is $m(\ell/2)^2$, where m is the mass of a ball. The total rotational inertia is

$$I = \frac{M\ell^2}{12} + 2\frac{m\ell^2}{4} = \frac{(6.40\,\text{kg})(1.20\,\text{m})^2}{12} + \frac{(1.06\,\text{kg})(1.20\,\text{m})^2}{2}$$

which yields $I = 1.53$ kg$\cdot$m^2. The torque, therefore, is

$$\tau = \left(1.53\,\text{kg}\cdot\text{m}^2\right)\left(-7.66\,\text{rad/s}^2\right) = -11.7 \text{ N}\cdot\text{m} .$$

(c) Since the system comes to rest the mechanical energy that is converted to thermal energy is simply the initial kinetic energy

$$K_i = \frac{1}{2}I\omega_0^2 = \frac{1}{2}\left(1.53\,\text{kg}\cdot\text{m}^2\right)((2\pi)(39)\,\text{rad/s})^2 = 4.59 \times 10^4 \text{ J} .$$

(d) We apply Eq. 11-13:

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 = ((2\pi)(39)\,\text{rad/s})\,(32.0\,\text{s}) + \frac{1}{2}\left(-7.66\,\text{rad/s}^2\right)(32.0\,\text{s})^2$$

which yields 3920 rad or (dividing by 2π) 624 rev for the value of angular displacement θ.

(e) Only the mechanical energy that is converted to thermal energy can still be computed without additional information. It is 4.59×10^4 J no matter how τ varies with time, as long as the system comes to rest.

73. We assume the given rate of 1.2×10^{-3} m/y is the linear speed of the top; it is also possible to interpret it as just the horizontal component of the linear speed but the difference between these interpretations is arguably negligible. Thus, Eq. 11-18 leads to

$$\omega = \frac{1.2 \times 10^{-3}\,\text{m/y}}{55\,\text{m}} = 2.18 \times 10^{-5} \text{ rad/y}$$

which we convert (since there are about 3.16×10^7 s in a year) to $\omega = 6.9 \times 10^{-13}$ rad/s.

74. The rotational inertia of the passengers is (to a good approximation) given by Eq. 11-26: $I = \sum mR^2 = NmR^2$ where N is the number of people and m is the (estimated) mass per person. We apply Eq. 11-44:

$$W = \frac{1}{2} I\omega^2 = \frac{1}{2} NmR^2\omega^2 \ .$$

where $R = 38$ m and $N = 36 \times 60 = 2160$ persons. The rotation rate is constant so that $\omega = \theta/t$ which leads to $\omega = 2\pi/120 = 0.052$ rad/s. The mass (in kg) of the average person is probably in the range $50 \le m \le 100$, so the work should be in the range

$$\begin{aligned} \frac{1}{2}(2160)(50)(38)^2(0.052)^2 \ \le \ & W \ \le \ \frac{1}{2}(2160)(100)(38)^2(0.052)^2 \\ 2 \times 10^5 \text{ J} \ \le \ & W \ \le \ 4 \times 10^5 \text{ J} \ . \end{aligned}$$

75. (a) The axis of rotation is at the bottom right edge of the rod along the ground, a horizontal distance of $d_3 + d_2 + d_1/2$ from the middle of the table assembly (mass $m = 90$ kg). The linebacker's center of mass at that critical moment was a horizontal distance of $d_4 + d_5$ from the axis of rotation. For the clockwise torque caused by the linebacker (mass M) to overcome the counterclockwise torque of the table assembly, we require (using Eq. 11-33)

$$Mg\,(d_4 + d_5) > mg\left(d_3 + d_2 + \frac{d_1}{2}\right) \ .$$

With the values given in the problem, we do indeed find the inequality is satisfied.

(b) Replacing our inequality with an equality and solving for M, we obtain

$$M = m\,\frac{d_3 + d_2 + \frac{1}{2}d_1}{d_4 + d_5} = 114 \text{ kg} \ .$$

76. We choose positive coordinate directions (different choices for each item) so that each is accelerating positively, which will allow us to set $a_1 = a_2 = R\alpha$ (for simplicity, we denote this as a). Thus, we choose upward positive for m_1, downward positive for m_2 and (somewhat unconventionally) clockwise for positive sense of disk rotation. Applying Newton's second law to m_1, m_2 and (in the form of Eq. 11-37) to M, respectively, we arrive at the following three equations.

$$\begin{aligned} T_1 - m_1 g &= m_1 a_1 \\ m_2 g - T_2 &= m_2 a_2 \\ T_2 R - T_1 R &= I\alpha \end{aligned}$$

(a) The rotational inertia of the disk is $I = \frac{1}{2}MR^2$ (Table 11-2(c)), so we divide the third equation (above) by R, add them all, and use the earlier equality among accelerations – to obtain:

$$m_2 g - m_1 g = \left(m_1 + m_2 + \frac{1}{2}M\right) a$$

which yields $a = \frac{4}{25}\,g = 1.6$ m/s^2.

(b) Plugging back in to the first equation, we find $T_1 = \frac{29}{24}m_1 g = 4.6$ N (where it is important in this step to have the mass in SI units: $m_1 = 0.40$ kg).

(c) Similarly, with $m_2 = 0.60$ kg, we find $T_2 = \frac{5}{6}m_2 g = 4.9$ N.

77. We employ energy methods in this solution; thus, considerations of positive versus negative sense (regarding the rotation of the wheel) are not relevant.

(a) The speed of the box is related to the angular speed of the wheel by $v = R\omega$, so that

$$K_{\text{box}} = \frac{1}{2}m_{\text{box}}v^2 \quad \Longrightarrow \quad v = \sqrt{\frac{2K_{\text{box}}}{m_{\text{box}}}} = 1.41 \text{ m/s}$$

implies that the angular speed is $\omega = 1.41/0.20 = 0.71$ rad/s. Thus, the kinetic energy of rotation is $\frac{1}{2}I\omega^2 = 10.0$ J.

(b) Since it was released from rest at what we will consider to be the reference position for gravitational potential, then (with SI units understood) energy conservation requires

$$\begin{aligned} K_0 + U_0 &= K + U \\ 0 + 0 &= (6.0 + 10.0) + m_{\text{box}}g(-h) \ . \end{aligned}$$

Therefore, $h = 16.0/58.8 = 0.27$ m.

78. The distances from P to the particles are as follows:

$$\begin{aligned} r_1 &= a \quad \text{for} \quad m_1 = 2M \quad \text{(lower left)} \\ r_2 &= \sqrt{b^2 - a^2} \quad \text{for} \quad m_2 = M \quad \text{(top)} \\ r_3 &= a \quad \text{for} \quad m_1 = 2M \quad \text{(lower right)} \end{aligned}$$

The rotational inertia of the system about P is

$$I = \sum_{i=1}^{3} m_i r_i^2 = \left(3a^2 + b^2\right) M$$

which yields $I = 0.208$ kg·m^2 for $M = 0.40$ kg, $a = 0.30$ m and $b = 0.50$ m. Applying Eq. 11-44, we find

$$W = \frac{1}{2}I\omega^2 = \frac{1}{2}(0.208)(5.0)^2 = 2.6 \text{ J} \ .$$

79. We choose positive coordinate directions (different choices for each item) so that each is accelerating positively, which will allow us to set $a_2 = a_1 = R\alpha$ (for simplicity, we denote this as a). Thus, we choose rightward positive for $m_2 = M$ (the block on the table), downward positive for $m_1 = M$ (the block at the end of the string) and (somewhat unconventionally) clockwise for positive sense of disk rotation. This means that we interpret θ given in the problem as a positive-valued quantity. Applying Newton's second law to m_1, m_2 and (in the form of Eq. 11-37) to M, respectively, we arrive at the following three equations (where we allow for the possibility of friction f_2 acting on m_2).

$$\begin{aligned} m_1 g - T_1 &= m_1 a_1 \\ T_2 - f_2 &= m_2 a_2 \\ T_1 R - T_2 R &= I\alpha \end{aligned}$$

(a) From Eq. 11-13 (with $\omega_0 = 0$) we find

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 \quad \Longrightarrow \quad \alpha = \frac{2\theta}{t^2} \ .$$

(b) From the fact that $a = R\alpha$ (noted above), we obtain $a = 2R\theta/t^2$.

(c) From the first of the above equations, we find

$$T_1 = m_1 (g - a_1) = M\left(g - \frac{2R\theta}{t^2}\right) .$$

(d) From the last of the above equations, we obtain the second tension:

$$T_2 = T_1 - \frac{I\alpha}{R} = M\left(g - \frac{2R\theta}{t^2}\right) - \frac{2I\theta}{Rt^2}$$

80. (a) With $r = 0.780$ m, the rotational inertia is

$$I = Mr^2 = (1.30\,\text{kg})(0.780\,\text{m})^2 = 0.791 \text{ kg}\cdot\text{m}^2 .$$

(b) The torque that must be applied to counteract the effect of the drag is

$$\tau = rf = (0.780\,\text{m})(2.30 \times 10^{-2}\,\text{N}) = 1.79 \times 10^{-2} \text{ N}\cdot\text{m} .$$

81. (a) The rotational inertia relative to the specified axis is

$$I = \sum m_i r_i^2 = (2M)L^2 + (2M)L^2 + M(2L)^2$$

which is found to be $I = 4.6$ kg·m^2. Then, with $\omega = 1.2$ rad/s, we obtain the kinetic energy from Eq. 11-27:

$$K = \frac{1}{2}I\omega^2 = 3.3 \text{ J} .$$

(b) In this case the axis of rotation would appear as a standard y axis with origin at P. Each of the $2M$ balls are a distance of $r = L\cos 30^\circ$ from that axis. Thus, the rotational inertia in this case is

$$I = \sum m_i r_i^2 = (2M)r^2 + (2M)r^2 + M(2L)^2$$

which is found to be $I = 4.0$ kg·m^2. Again, from Eq. 11-27 we obtain the kinetic energy

$$K = \frac{1}{2}I\omega^2 = 2.9 \text{ J} .$$

82. We make use of Table 11-2(e) as well as the parallel-axis theorem, Eq. 11-27, where needed. We use ℓ (as a subscript) to refer to the long rod and s to refer to the short rod.

(a) The rotational inertia is

$$I = I_s + I_\ell = \frac{1}{12}m_s L_s^2 + \frac{1}{3}m_\ell L_\ell^2 = 0.019 \text{ kg}\cdot\text{m}^2 .$$

(b) We note that the center of the short rod is a distance of $h = 0.25$ m from the axis. The rotational inertia is

$$I = I_s + I_\ell = \frac{1}{12}m_s L_s^2 + m_s h^2 + \frac{1}{12}m_\ell L_\ell^2$$

which again yields $I = 0.019$ kg·m^2.

83. This may be derived from Eq. 11-28 or (suitably interpreted) from Eq. 11-26. Since every element of the hoop has the same radius $r = R$, the integration (or summation, if preferred) is trivial: $I = \int r^2\,dm = R^2 \int dm = MR^2$.

84. (a) Using Eq. 11-15 with $\omega = 0$, we have

$$\theta = \frac{\omega_0 + \omega}{2}\, t = 2.8 \text{ rad} .$$

(b) One ingredient in this calculation is $\alpha = (0 - 3.5\,\text{rad/s})/(1.6\,\text{s}) = -2.2\,\text{rad/s}^2$, so that the tangential acceleration is $r\alpha = 0.33$ m/s^2. Another ingredient is $\omega = \omega_0 + \alpha t = 1.3$ rad/s for $t = 1.0$ s, so that the radial (centripetal) acceleration is $\omega^2 r = 0.26\,\text{m/s}^2$. Thus, the magnitude of the acceleration is

$$|\vec{a}| = \sqrt{0.33^2 + 0.26^2} = 0.42 \text{ m/s}^2 .$$

85. (a) Using $T = 1$ yr $= 3.16 \times 10^7$ s for the time to make one full revolution (or 2π rad), we obtain

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{3.16 \times 10^7} = 2.0 \times 10^{-7}\,\text{rad/s} .$$

(b) The radius r of Earth's orbit can be found in Appendix C or the inside front cover. Eq. 11-18 gives

$$v = \alpha r = (2.0 \times 10^{-7}\,\text{rad/s})(1.49 \times 10^{11}\,\text{m}) = 3.0 \times 10^4\,\text{m/s} .$$

(c) The (radial, or centripetal) acceleration is

$$a = \omega^2 r = \left(2.0 \times 10^{-7}\,\text{rad/s}\right)^2 (1.49 \times 10^{11}\,\text{m}) = 5.9 \times 10^{-3}\,\text{m/s}^2 .$$

The direction of $\vec{a}$ is toward the sun.

86. Using Eq. 11-12, we have

$$\omega = \omega_0 + \alpha t \implies \alpha = \frac{2.6 - 8.0}{3.0}$$

which yields $\alpha = -1.8$ rad/s^2. Using this value in Eq. 11-14 leads to

$$\omega^2 = \omega_0^2 + 2\alpha\theta \implies \theta = \frac{0^2 - 8.0^2}{2\,(-1.8)} = 18 \text{ rad} .$$

87. The motion consists of two stages. The first, the interval $0 \le t \le 20$ s, consists of constant angular acceleration given by

$$\alpha = \frac{5.0\,\text{rad/s}}{2.0\,\text{s}} = 2.5 \text{ rad/s}^2 .$$

The second stage, $20 < t \le 40$ s, consists of constant angular velocity $\omega = \Delta\theta/\Delta t$. Analyzing the first stage, we find

$$\begin{aligned} \theta_1 &= \left.\frac{1}{2}\alpha t^2\right|_{t=20} = 500 \text{ rad} \\ \omega &= \left.\alpha t\right|_{t=20} = 50 \text{ rad/s} . \end{aligned}$$

Analyzing the second stage, we obtain

$$\theta_2 = \theta_1 + \omega\Delta t = 500 + (50)(20) = 1500 \text{ rad} .$$

88. (a) Eq. 11-12 leads to $\alpha = -\omega_o/t = -25.0/20.0 = -1.25\,\text{rad/s}^2$.

(b) Eq. 11-15 leads to $\theta = \frac{1}{2}\omega_o t = \frac{1}{2}(25.0)(20.0) = 250\,\text{rad}$.

(c) Dividing the previous result by 2π we obtain $\theta = 39.8$ rev.

89. (a) We integrate the angular acceleration (as a function of τ) with respect to τ to find the angular velocity as a function of $t > 0$.

$$\omega = \omega_0 + \int_0^t \left(4a\tau^3 - 3b\tau^2\right) d\tau = \omega_0 + at^4 - bt^3 .$$

(b) We integrate the angular velocity (as a function of τ) with respect to τ to find the angular position as a function of $t > 0$.

$$\theta = \theta_0 + \int_0^t \left(4a\tau^3 - 3b\tau^2\right) d\tau = \theta_0 + \omega_0 t + \frac{a}{5}t^5 - \frac{b}{4}t^4 .$$

90. (a) The particle at A has $r = 0$ with respect to the axis of rotation. The particle at B is $r = L = 0.50$ m from the axis; similarly for the particle directly above A in the figure. The particle diagonally opposite A is a distance $r = \sqrt{2}\,L = 0.71$ m from the axis. Therefore,

$$I = \sum m_i r_i^2 = 2mL^2 + m\left(\sqrt{2}\,L\right)^2 = 0.20\ \text{kg}\cdot\text{m}^2 .$$

(b) One imagines rotating the figure (about point A) clockwise by $90°$ and noting that the center of mass has fallen a distance equal to L as a result. If we let our reference position for gravitational potential be the height of the center of mass at the instant AB swings through vertical orientation, then

$$\begin{aligned} K_0 + U_0 &= K + U \\ 0 + (4m)gh_0 &= K + 0 . \end{aligned}$$

Since $h_0 = L = 0.50$ m, we find $K = 3.9$ J. Then, using Eq. 11-27, we obtain

$$K = \frac{1}{2} I_A \omega^2 \implies \omega = 6.3\ \frac{\text{rad}}{\text{s}} .$$

91. The center of mass is initially at height $h = \frac{L}{2}\sin 40°$ when the system is released (where $L = 2.0$ m). The corresponding potential energy Mgh (where $M = 1.5$ kg) becomes rotational kinetic energy $\frac{1}{2}I\omega^2$ as it passes the horizontal position (where I is the rotational inertia about the pin). Using Table 11-2(e) and the parallel axis theorem, we find $I = \frac{1}{12}ML^2 + M(L/2)^2 = \frac{1}{3}ML^2$. Therefore,

$$Mg\frac{L}{2}\sin 40° = \frac{1}{2}\left(\frac{1}{3}ML^2\right)\omega^2 \implies \omega = \sqrt{\frac{3g\sin 40°}{L}}$$

which yields $\omega = 3.1$ rad/s.

92. We choose $\pm$ directions such that the initial angular velocity is $\omega_0 = -317$ rad/s and the values for α, τ and F are positive.

(a) Combining Eq. 11-12 with Eq. 11-37 and Table 11-2(f) (and using the fact that $\omega = 0$) we arrive at the expression

$$\tau = \left(\frac{2}{5}MR^2\right)\left(-\frac{\omega_0}{t}\right) = -\frac{2}{5}\frac{MR^2\omega_0}{t} .$$

With $t = 15.5$ s, $R = 0.226$ m and $M = 1.65$ kg, we obtain $\tau = 0.689$ N·m.

(b) From Eq. 11-32, we find $F = \tau/R = 3.05$ N.

(c) Using again the expression found in part (a), but this time with $R = 0.854$ m, we get $\tau = 9.84$ N·m.

(d) Now, $F = \tau/R = 11.5$ N.

93. We choose positive coordinate directions so that each is accelerating positively, which will allow us to set $a_{\text{box}} = R\alpha$ (for simplicity, we denote this as a). Thus, we choose downhill positive for the $m = 2.0$ kg box and (as is conventional) counterclockwise for positive sense of wheel rotation. Applying Newton's second law to the box and (in the form of Eq. 11-37) to the wheel, respectively, we arrive at the following two equations (using θ as the incline angle 20°, not as the angular displacement of the wheel).

$$\begin{aligned} mg\sin\theta - T &= ma \\ TR &= I\alpha \end{aligned}$$

Since the problem gives $a = 2.0$ m/s^2, the first equation gives the tension $T = m(g\sin\theta - a) = 2.7$ N. Plugging this and $R = 0.20$ m into the second equation (along with the fact that $\alpha = a/R$) we find the rotational inertia $I = TR^2/a = 0.054\,\text{kg}\cdot\text{m}^2$.

94. Eq. 11-32 leads to $\tau = mgr = (70)(9.8)(0.20)$ in SI units, which yields $\tau = 1.4 \times 10^2$ N.

95. The disk centered on A has $I = \frac{1}{2}MR^2$ (Table 11-2(c)) about that point, but the rotational inertia of the other disk is found using the parallel-axis theorem $I = \frac{1}{2}MR^2 + M(2R)^2 = \frac{9}{2}MR^2$ about point A. Adding these two results, we obtain

$$\frac{1}{2}MR^2 + \frac{9}{2}MR^2 = 5MR^2 = 5(4.0)(0.40)^2$$

which yields $3.2\,\text{kg}\cdot\text{m}^2$.

96. (a) One particle is on the axis, so $r = 0$ for it. For each of the others, the distance from the axis is $r = (0.60\text{ m})\sin 60° = 0.52$ m. Therefore, the rotational inertia is $I = \sum m_i r_i^2 = 0.27\text{ kg}\cdot\text{m}^2$.

(b) The two particles that are nearest the axis are each a distance of $r = 0.30$ m from it. The particle "opposite" from that side is a distance $r = (0.60\text{ m})\sin 60° = 0.52$ m from the axis. Thus, the rotational inertia is $I = \sum m_i r_i^2 = 0.22\text{ kg}\cdot\text{m}^2$.

(c) The distance from the axis for each of the particles is $r = \frac{1}{2}(0.60\,\text{m})\sin 60°$. Now, $I = 3(0.50\,\text{kg})(0.26\,\text{m})^2 = 0.10\,\text{kg}\cdot\text{m}^2$.

97. The parallel axis theorem gives $I = I_{\text{com}} + Mh^2$ for the rotational inertia about any axis (parallel to the axis used to compute I_{com}). Let us assume that an axis has already been chosen through the center of mass of the body such that I_{com} is as small as it possibly can be. Since $Mh^2 >$ for all nonzero values of h, then $I > I_{\text{com}}$ from the parallel axis theorem as long as $h \neq 0$. Thus, with $h = 0$ we get $I = I_{\text{com}}$ and therefore the smallest possible value of rotational inertia.

98. (a) The linear speed at $t = 15.0\,\text{s}$ is

$$v = a_t t = \left(0.500\,\text{m/s}^2\right)(15.0\,\text{s}) = 7.50\text{ m/s}\ .$$

The radial (centripetal) acceleration at that moment is

$$a_r = \frac{v^2}{r} = \frac{(7.50\,\text{m/s})^2}{30.0\,\text{m}} = 1.875\text{ m/s}^2\ .$$

Thus, the net acceleration has magnitude:

$$a = \sqrt{a_t^2 + a_r^2} = \sqrt{\left(0.500\,\text{m/s}^2\right)^2 + \left(1.875\,\text{m/s}^2\right)^2} \;=\; 1.94\text{ m/s}^2\ .$$

(b) We note that $\vec{a}_t \parallel \vec{v}$. Therefore, the angle between $\vec{v}$ and $\vec{a}$ is

$$\tan^{-1}\left(\frac{a_r}{a_t}\right) = \tan^{-1}\left(\frac{1.875}{0.5}\right) = 75.1°$$

so that the vector is pointing more toward the center of the track than in the direction of motion.

99. First, we convert the angular velocity: $\omega = (2000)(2\pi/60) = 209$ rad/s. Also, we convert the plane's speed to SI units: $(480)(1000/3600) = 133$ m/s. We use Eq. 11-18 in part (a) and (implicitly) Eq. 4-39 in part (b).

    (a) The speed of the tip as seen by the pilot is

    $$v_t = \omega r = (209\,\mathrm{rad/s})(1.5\,\mathrm{m}) = 314 \text{ m/s}$$

    which (since the radius is given to only two significant figures) we write as $v = 3.1 \times 10^2$ m/s.

    (b) The plane's velocity $\vec{v}_p$ and the velocity of the tip $\vec{v}_t$ (found in the plane's frame of reference), in any of the tip's positions, must be perpendicular to each other. Thus, the speed as seen by an observer on the ground is

    $$v = \sqrt{v_p^2 + v_t^2} = \sqrt{(133\,\mathrm{m/s})^2 + (314\,\mathrm{m/s})^2} = 3.4 \times 10^2 \text{ m/s} .$$

100. Using Eq. 11-7 and Eq. 11-18, the average angular acceleration is

$$\alpha_{\text{avg}} = \frac{\Delta\omega}{\Delta t} = \frac{\Delta v}{r\Delta t} = \frac{25 - 12}{(0.75/2)(6.2)} = 5.6 \text{ rad/s}^2 .$$

101. (a) Eq. 11-15 gives

    $$90\,\mathrm{rev} = \frac{1}{2}\left(\omega_0 + 10\,\mathrm{rev/s}\right)(15\,\mathrm{s})$$

    which leads to $\omega_0 = 2.0\,\mathrm{rev/s}$.

    (b) From Eq. 11-12, the angular acceleration is

    $$\alpha = \frac{10\,\mathrm{rev/s} - 2.0\,\mathrm{rev/s}}{15\,\mathrm{s}} = 0.53 \text{ rev/s}^2 .$$

    Using the equation again (with the same value for α) we seek a *negative* value of t (meaning an earlier time than that when $\omega_0 = 2.0$ rev/s) such that $\omega = 0$. Thus,

    $$t = -\frac{\omega_0}{\alpha} = -\frac{2.0\,\mathrm{rev/s}}{0.53\,\mathrm{rev/s}^2} = -3.8 \text{ s}$$

    which means that the wheel was at rest 3.8 s before the 15 s interval began.

102. (a) Using Eq. 11-1, the angular displacement is

    $$\theta = \frac{5.6\,\mathrm{m}}{8.0 \times 10^{-2}\,\mathrm{m}} = 1.4 \times 10^2 \text{ rad} .$$

    (b) We use $\theta = \frac{1}{2}\alpha t^2$ (Eq. 11-13) to obtain t:

    $$t = \sqrt{\frac{2\theta}{\alpha}} = \sqrt{\frac{2(1.4 \times 10^2\,\mathrm{rad})}{1.5\,\mathrm{rad/s}^2}} = 14 \text{ s} .$$

103. The problem asks us to assume v_{com} and ω are constant. For consistency of units, we write

$$v_{\text{com}} = (85\,\mathrm{mi/h})\left(\frac{5280\,\mathrm{ft/mi}}{60\,\mathrm{min/h}}\right) = 7480 \text{ ft/min} .$$

Thus, with $\Delta x = 60$ ft, the time of flight is $t = \Delta x / v_{\text{com}} = 60/7480 = 0.00802$ min. During that time, the angular displacement of a point on the ball's surface is

$$\theta = \omega t = (1800\,\mathrm{rev/min})(0.00802\,\mathrm{min}) \approx 14 \text{ rev} .$$