Chapter 31

1. The magnetic field is normal to the plane of the loop and is uniform over the loop. Thus at any instant the magnetic flux through the loop is given by $\Phi_B = AB = \pi r^2 B$, where $A = \pi r^2$ is the area of the loop. According to Faraday's law the magnitude of the emf in the loop is

$$\mathcal{E} = \frac{d\Phi_B}{dt} = \pi r^2 \frac{dB}{dt} = \pi(0.055\,\mathrm{m})^2(0.16\,\mathrm{T/s}) = 1.5 \times 10^{-3}\ \mathrm{V}\ .$$

2. The induced emf is

$$\begin{aligned}\mathcal{E} &= -\frac{d\Phi_B}{dt} = -\frac{d(BA)}{dt} = -A\frac{dB}{dt} \\ &= -A\frac{d}{dt}(\mu_0 i n) = -A\mu_0 n\frac{d}{dt}(i_0 \sin\omega t) \\ &= -A\mu_0 n i_0 \omega \cos\omega t\ .\end{aligned}$$

3. (a)

$$|\mathcal{E}| = \left|\frac{d\Phi_B}{dt}\right| = \frac{d}{dt}(6.0t^2 + 7.0t) = 12t + 7.0 = 12(2.0) + 7.0 = 31\ \mathrm{mV}\ .$$

   (b) Appealing to Lenz's law (especially Fig. 31-5(a)) we see that the current flow in the loop is clockwise. Thus, the current is from right to left through R.

4. (a) We use $\mathcal{E} = -d\Phi_B/dt = -\pi r^2 dB/dt$. For $0 < t < 2.0\,\mathrm{s}$:

$$\mathcal{E} = -\pi r^2 \frac{dB}{dt} = -\pi(0.12\,\mathrm{m})^2\left(\frac{0.5\,\mathrm{T}}{2.0\,\mathrm{s}}\right) = -1.1 \times 10^{-2}\ \mathrm{V}\ .$$

   (b) $2.0\,\mathrm{s} < t < 4.0\,\mathrm{s}$: $\mathcal{E} \propto dB/dt = 0$.

   (c) $4.0\,\mathrm{s} < t < 6.0\,\mathrm{s}$:

$$\mathcal{E} = -\pi r^2 \frac{dB}{dt} = -\pi(0.12\,\mathrm{m})^2\left(\frac{-0.5\,\mathrm{T}}{6.0\,\mathrm{s} - 4.0\,\mathrm{s}}\right) = 1.1 \times 10^{-2}\ \mathrm{V}\ .$$

5. (a) Table 27-1 gives the resistivity of copper. Thus,

$$R = \rho\frac{L}{A} = (1.68 \times 10^{-8}\,\Omega\cdot\mathrm{m})\left[\frac{\pi(0.10\,\mathrm{m})}{\pi(2.5 \times 10^{-3})^2/4}\right] = 1.1 \times 10^{-3}\,\Omega\ .$$

   (b) We use $i = |\mathcal{E}|/R = |d\Phi_B/dt|/R = (\pi r^2/R)|dB/dt|$. Thus

$$\left|\frac{dB}{dt}\right| = \frac{iR}{\pi r^2} = \frac{(10\,\mathrm{A})(1.1 \times 10^{-3}\,\Omega)}{\pi(0.05\,\mathrm{m})^2} = 1.4\ \mathrm{T/s}\ .$$

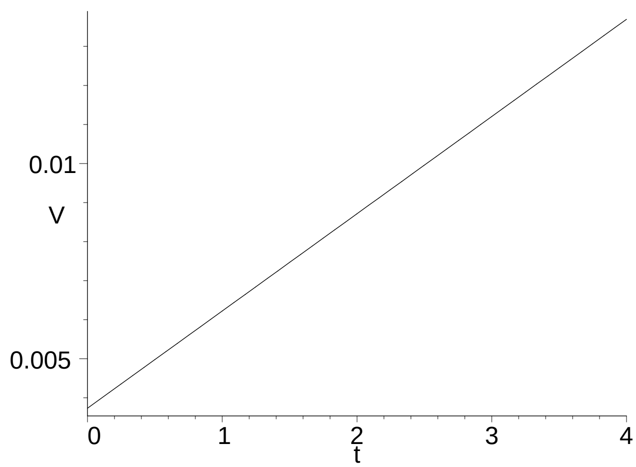
6. (a) Following Sample Problem 31-1, we have

$$\Phi_B = \mu_0 i n A \qquad \text{where} \quad A = \frac{\pi d^2}{4}$$

with $i = 3t + t^2$ (SI units and 2 significant figures understood). The magnitude of the induced emf is therefore

$$\mathcal{E} = N\frac{d\Phi_B}{dt} \quad \approx \quad 0.0012(3 + 2t)$$

where we have used the values specified in Sample Problem 31-1 for all quantities except the current. The plot is shown below.



(b) Using Ohm's law, the induced current is

$$i\Big|_{t=2.0\,\mathrm{s}} = \frac{\mathcal{E}\big|_{t=2.0\,\mathrm{s}}}{R} = \frac{0.0087\,\mathrm{V}}{0.15\,\Omega} = 0.058 \text{ A} .$$

7. The primary difference between this and the situation described in Sample Problem 31-1 is in the quantity A. The area through which there is magnetic flux is not the area of the short coil, in this case, but is the area of the solenoid (there is no field outside an ideal solenoid). Actually, because of the current (which we calculate here) in the short coil, there is a very small amount of field outside the solenoid (caused by that current) – but it may be disregarded in this calculation. The values are as indicated in Sample Problem 31-1 except that $A = \pi D^2/4$ (where $D = 0.032$ m) and $N = 120$ for the short coil. Thus, we find $\Phi_{B,i} = 3.3 \times 10^{-5}$ Wb, and the magnitude of the induced emf is 0.16 V. Ohm's law then yields $0.16\,\mathrm{V}/5.3\,\Omega = 0.030$ A.

8. Using Faraday's law, the induced emf is

$$\begin{aligned}\mathcal{E} &= -\frac{d\Phi_B}{dt} = -\frac{d(BA)}{dt} = -B\frac{dA}{dt} = -B\frac{d(\pi r^2)}{dt} = -2\pi r B\frac{dr}{dt} \\ &= -2\pi(0.12\,\mathrm{m})(0.800\,\mathrm{T})(-0.750\,\mathrm{m/s}) = 0.452 \text{ V} .\end{aligned}$$

9. (a) In the region of the smaller loop the magnetic field produced by the larger loop may be taken to be uniform and equal to its value at the center of the smaller loop, on the axis. Eq. 30-29, with $z = x$ (taken to be much greater than R), gives

$$\vec{B} = \frac{\mu_0 i R^2}{2x^3}\ \hat{\imath}$$

where the $+x$ direction is upward in Fig. 31-36. The magnetic flux through the smaller loop is, to a good approximation, the product of this field and the area (πr^2) of the smaller loop:

$$\Phi_B = \frac{\pi\mu_0 i r^2 R^2}{2x^3} \ .$$

(b) The emf is given by Faraday's law:

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\left(\frac{\pi\mu_0 i r^2 R^2}{2}\right)\frac{d}{dt}\left(\frac{1}{x^3}\right) = -\left(\frac{\pi\mu_0 i r^2 R^2}{2}\right)\left(-\frac{3}{x^4}\frac{dx}{dt}\right) = \frac{3\pi\mu_0 i r^2 R^2 v}{2x^4} \ .$$

(c) As the smaller loop moves upward, the flux through it decreases, and we have situation like that shown in Fig. 31-5(b). The induced current will be directed so as to produce a magnetic field that is upward through the smaller loop, in the same direction as the field of the larger loop. It will be counterclockwise as viewed from above, in the same direction as the current in the larger loop.

10. The flux $\Phi_B = BA\cos\theta$ does not change as the loop is rotated. Faraday's law only leads to a nonzero induced emf when the flux is changing, so the result in this instance is 0.

11. (a) Ohm's law combines with Faraday's law to give $i = -\frac{N}{R}\frac{d\Phi_B}{dt}$ where R is the resistance of the coil. In this case, $N = 1$ (it is a single loop), and we integrate to find the charge:

$$\begin{aligned}\int_0^t i\,dt &= -\frac{1}{R}\int_0^t \frac{d\Phi_B}{dt}\,dt \\ q(t) &= -\frac{1}{R}\left(\Phi_B(t) - \Phi_B(0)\right)\end{aligned}$$

which is equivalent to the expression shown in the problem statement. We have used little more than the fundamental theorem of calculus; no particular assumptions have been made about how the integrations should be performed. The result is independent of the way $\vec{B}$ has changed.

(b) If the current is identically zero for over the whole range $0 \to t$ then certainly the left-hand side of our computation, above, gives zero. But the same result can come from the current being in one direction for, say, $0 \to \frac{t}{2}$ and then in the opposite direction for $\frac{t}{2} \to t$ in such a way that $\int_0^t i\,dt = 0$. So a vanishing integral does not necessarily mean the integrand itself is identically zero.

12. (a) Eq. 30-12 gives the field at the center of the large loop with $R = 1.00$ m and current $i(t)$. This is approximately the field throughout the area ($A = 2.00 \times 10^{-4}$ m^2) enclosed by the small loop. Thus, with $B = \mu_0 i/2R$ and $i(t) = i_0 + kt$ (where $i_0 = 200$ A and $k = (-200\,\text{A} - 200\,\text{A})/1.00\,\text{s} = -400$ A/s), we find

$$\begin{aligned}B\Big|_{t=0} &= \frac{\mu_0 i_0}{2R} = \frac{(4\pi\times10^{-7}\,\text{H/m})(200\,\text{A})}{2(1.00\,\text{m})} = 1.26\times10^{-4}\,\text{T} \ , \\ B\Big|_{t=0.500\,\text{s}} &= \frac{(4\pi\times10^{-7}\,\text{H/m})[200\,\text{A} - (400\,\text{A/s})(0.500\,\text{s})]}{2(1.00\,\text{m})} = 0 \ , \\ B\big|_{t=1.00\,\text{s}} &= \frac{(4\pi\times10^{-7}\,\text{H/m})[200\,\text{A} - (400\,\text{A/s})(1.00\,\text{s})]}{2(1.00\,\text{m})} = -1.26\times10^{-4}\ \text{T} \ .\end{aligned}$$

(b) Let the area of the small loop be a. Then $\Phi_B = Ba$, and Faraday's law yields

$$\begin{aligned}\mathcal{E} &= -\frac{d\Phi_B}{dt} = -\frac{d(Ba)}{dt} = -a\frac{dB}{dt} = -a\left(\frac{\Delta B}{\Delta t}\right) \\ &= -(2.00\times10^{-4}\,\text{m}^2)\left(\frac{-1.26\times10^{-4}\,\text{T} - 1.26\times10^{-4}\,\text{T}}{1.00\,\text{s}}\right) = 5.04\times10^{-8}\ \text{V} \ .\end{aligned}$$

13. From the result of the problem 11,

$$\begin{aligned} q(t) &= \frac{1}{R}[\Phi_B(0) - \Phi_B(t)] = \frac{A}{R}[B(0) - B(t)] \\ &= \frac{1.20 \times 10^{-3}\,\text{m}^2}{13.0\,\Omega}[1.60\,\text{T} - (-1.60\,\text{T})] = 2.95 \times 10^{-2}\ \text{C} . \end{aligned}$$

14. We note that $1\,\text{gauss} = 10^{-4}$ T. Adapting the result of the problem 11,

$$\begin{aligned} q(t) &= \frac{N}{R}[BA\cos 20^\circ - (-BA\cos 20^\circ)] = \frac{2NBA\cos 20^\circ}{R} \\ &= \frac{2(1000)(0.590 \times 10^{-4}\,\text{T})\pi(0.100\,\text{m})^2(\cos 20^\circ)}{85.0\,\Omega + 140\,\Omega} = 1.55 \times 10^{-5}\,\text{C} . \end{aligned}$$

Note that the axis of the coil is at 20°, not 70°, from the magnetic field of the Earth.

15. (a) Let L be the length of a side of the square circuit. Then the magnetic flux through the circuit is $\Phi_B = L^2B/2$, and the induced emf is

$$\mathcal{E}_i = -\frac{\mathrm{d}\Phi_B}{dt} = -\frac{L^2}{2}\frac{dB}{dt} .$$

Now $B = 0.042 - 0.870t$ and $dB/dt = -0.870\,\text{T/s}$. Thus,

$$\mathcal{E}_i = \frac{(2.00\,\text{m})^2}{2}(0.870\,\text{T/s}) = 1.74\ \text{V} .$$

The magnetic field is out of the page and decreasing so the induced emf is counterclockwise around the circuit, in the same direction as the emf of the battery. The total emf is $\mathcal{E}+\mathcal{E}_i = 20.0\,\text{V}+1.74\,\text{V} = 21.7\,\text{V}$.

(b) The current is in the sense of the total emf (counterclockwise).

16. (a) Since $\vec{B} = B\,\hat{\imath}$ uniformly, then only the area "projected" onto the yz plane will contribute to the flux (due to the scalar [dot] product). This "projected" area corresponds to one-fourth of a circle. Thus, the magnetic flux Φ_B through the loop is

$$\Phi_B = \int \vec{B}\cdot d\vec{A} = \frac{1}{4}\pi r^2 B .$$

Thus,

$$\begin{aligned} |\mathcal{E}| &= \left|\frac{d\Phi_B}{dt}\right| = \left|\frac{d}{dt}\left(\frac{1}{4}\pi r^2 B\right)\right| = \frac{\pi r^2}{4}\left|\frac{dB}{dt}\right| \\ &= \frac{1}{4}\pi(0.10\,\text{m})^2(3.0 \times 10^{-3}\,\text{T/s}) = 2.4 \times 10^{-5}\ \text{V} . \end{aligned}$$

(b) We have a situation analogous to that shown in Fig. 31-5(a). Thus, the current in segment bc flows from c to b (following Lenz's law).

17. (a) It should be emphasized that the result, given in terms of $\sin(2\pi ft)$, could as easily be given in terms of $\cos(2\pi ft)$ or even $\cos(2\pi ft + \phi)$ where ϕ is a phase constant as discussed in Chapter 16. The angular position θ of the rotating coil is measured from some reference line (or plane), and which line one chooses will affect whether the magnetic flux should be written as $BA\cos\theta$, $BA\sin\theta$ or $BA\cos(\theta + \phi)$. Here our choice is such that $\Phi_B = BA\cos\theta$. Since the coil is rotating steadily, θ increases linearly with time. Thus, $\theta = \omega t$ (equivalent to $\theta = 2\pi ft$) if θ is understood to be in

radians (and ω would be the angular velocity). Since the area of the rectangular coil is $A = ab$, Faraday's law leads to

$$\mathcal{E} = -N\frac{d(BA\cos\theta)}{dt} = -NBA\frac{d\cos(2\pi ft)}{dt} = NBab2\pi f\sin(2\pi ft)$$

which is the desired result, shown in the problem statement. The second way this is written ($\mathcal{E}_0\sin(2\pi ft)$) is meant to emphasize that the voltage output is sinusoidal (in its time dependence) and has an amplitude of $\mathcal{E}_0 = 2\pi fNabB$.

(b) We solve $\mathcal{E}_0 = 150\,\mathrm{V} = 2\pi fNabB$ when $f = 60.0$ rev/s and $B = 0.500$ T. The three unknowns are N, a, and b which occur in a product; thus, we obtain $Nab = 0.796\ \mathrm{m}^2$. This means, for instance, that if we wanted the coil to have a square shape and consist of 50 turns, then the side length of the square would be $a = b = 0.126$ m.

18. (a) The rotational frequency (in revolutions per second) is identical to the time-dependent voltage frequency (in cycles per second, or Hertz). This conclusion should not be considered obvious, and the calculation shown in part (b) should serve to reinforce it.

(b) First, we define angle relative to the plane of Fig. 31-41, such that the semicircular wire is in the $\theta = 0$ position and a quarter of a period (of revolution) later it will be in the $\theta = \pi/2$ position (where its midpoint will reach a distance of a above the plane of the figure). At the moment it is in the $\theta = \pi/2$ position, the area enclosed by the "circuit" will appear to us (as we look down at the figure) to that of a simple rectangle (call this area A_0 which is the area it will again appear to enclose when the wire is in the $\theta = 3\pi/2$ position). Since the area of the semicircle is $\pi a^2/2$ then the area (as it appears to us) enclosed by the circuit, as a function of our angle θ, is

$$A = A_0 + \frac{\pi a^2}{2}\cos\theta$$

where (since θ is increasing at a steady rate) the angle depends linearly on time, which we can write either as $\theta = \omega t$ or $\theta = 2\pi ft$ if we take $t = 0$ to be a moment when the arc is in the $\theta = 0$ position. Since $\vec{B}$ is uniform (in space) and constant (in time), Faraday's law leads to

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -B\frac{dA}{dt} = -B\frac{d\left(A_0 + \frac{\pi a^2}{2}\cos\theta\right)}{dt} = -B\frac{\pi a^2}{2}\,\frac{d\cos(2\pi ft)}{dt}$$

which yields $\mathcal{E} = B\pi^2a^2f\sin(2\pi ft)$. This (due to the sinusoidal dependence) reinforces the conclusion in part (a) and also (due to the factors in front of the sine) provides the voltage amplitude: $\mathcal{E}_{\max} = B\pi^2a^2f$.

19. First we write $\Phi_B = BA\cos\theta$. We note that the angular position θ of the rotating coil is measured from some reference line or plane, and we are implicitly making such a choice by writing the magnetic flux as $BA\cos\theta$ (as opposed to, say, $BA\sin\theta$). Since the coil is rotating steadily, θ increases linearly with time. Thus, $\theta = \omega t$ if θ is understood to be in radians (here, $\omega = 2\pi f$ is the angular velocity of the coil in radians per second, and $f = 1000\,\mathrm{rev/min} \approx 16.7$ rev/s is the frequency). Since the area of the rectangular coil is $A = 0.500 \times 0.300 = 0.150\ \mathrm{m}^2$, Faraday's law leads to

$$\mathcal{E} = -N\frac{d(BA\cos\theta)}{dt} = -NBA\frac{d\cos(2\pi ft)}{dt} = NBA2\pi f\sin(2\pi ft)$$

which means it has a voltage amplitude of

$$\mathcal{E}_{\max} = 2\pi fNAB = 2\pi(16.7\,\mathrm{rev/s})(100\,\mathrm{turns})(0.15\,\mathrm{m}^2)(3.5\,\mathrm{T}) = 5.50 \times 10^3\ \mathrm{V}\ .$$

20. The field (due to the current in the straight wire) is out-of-the-page in the upper half of the circle and is into the page in the lower half of the circle, producing zero net flux, at any time. There is no induced current in the circle.

21. Consider a (thin) strip of area of height dy and width $\ell = 0.020$ m. The strip is located at some $0 < y < \ell$. The element of flux through the strip is

$$d\Phi_B = B\,dA = (4t^2y)(\ell\,dy)$$

where SI units (and 2 significant figures) are understood. To find the total flux through the square loop, we integrate:

$$\Phi_B = \int d\Phi_B = \int_0^\ell \left(4t^2 y\ell\right) dy = 2t^2\ell^3 \ .$$

Thus, Faraday's law yields

$$|\mathcal{E}| = \left|\frac{d\Phi_B}{dt}\right| = 4t\ell^3 \ .$$

At $t = 2.5$ s, we find the magnitude of the induced emf is 8.0×10^{-5} V. Its "direction" (or "sense") is clockwise, by Lenz's law.

22. (a) First, we observe that a large portion of the figure contributes flux which "cancels out." The field (due to the current in the long straight wire) through the part of the rectangle above the wire is out of the page (by the right-hand rule) and below the wire it is into the page. Thus, since the height of the part above the wire is $b - a$, then a strip below the wire (where the strip borders the long wire, and extends a distance $b - a$ away from it) has exactly the equal-but-opposite flux which cancels the contribution from the part above the wire. Thus, we obtain the non-zero contributions to the flux:

$$\Phi_B = \int B\,dA = \int_{b-a}^{a} \left(\frac{\mu_0 i}{2\pi r}\right)(b\,dr) = \frac{\mu_0 i b}{2\pi} \ln\left(\frac{a}{b-a}\right) \ .$$

Faraday's law, then, (with SI units and 3 significant figures understood) leads to

$$\begin{aligned}
\mathcal{E} &= -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\left[\frac{\mu_0 i b}{2\pi}\ln\left(\frac{a}{b-a}\right)\right] \\
&= -\frac{\mu_0 b}{2\pi}\ln\left(\frac{a}{b-a}\right)\frac{di}{dt} = -\frac{\mu_0 b}{2\pi}\ln\left(\frac{a}{b-a}\right)\frac{d}{dt}\left(\frac{9}{2}t^2 - 10t\right) \\
&= \frac{-\mu_0 b(9t-10)}{2\pi}\ln\left(\frac{a}{b-a}\right) \ .
\end{aligned}$$

With $a = 0.120$ m and $b = 0.160$ m, then, at $t = 3.00$ s, the magnitude of the emf induced in the rectangular loop is

$$|\mathcal{E}| = \frac{\left(4\pi \times 10^{-7}\right)(0.16)(9(3) - 10)}{2\pi} \ln\left(\frac{0.12}{0.16 - 0.12}\right) = 5.98 \times 10^{-7} \text{ V} \ .$$

(b) We note that $\frac{di}{dt} > 0$ at $t = 3$ s. The situation is roughly analogous to that shown in Fig. 31-5(c). From Lenz's law, then, the induced emf (hence, the induced current) in the loop is counterclockwise.

23. (a) We refer to the (very large) wire length as L and seek to compute the flux per meter: Φ_B/L. Using the right-hand rule discussed in Chapter 30, we see that the net field in the region between the axes of antiparallel currents is the addition of the magnitudes of their individual fields, as given by Eq. 30-19 and Eq. 30-22. There is an evident reflection symmetry in the problem, where the plane of symmetry is midway between the two wires (at what we will call $x = \ell/2$, where $\ell = 20\,\text{mm} = 0.020\,\text{m}$); the net field at any point $0 < x < \ell/2$ is the same at its "mirror image" point $\ell - x$. The central axis of one of the wires passes through the origin, and that of the other passes through $x = \ell$. We make use of the symmetry by integrating over $0 < x < \ell/2$ and then multiplying by 2:

$$\Phi_B = 2\int_0^{\ell/2} B\,dA = 2\int_0^{d/2} B\,(L\,dx) + 2\int_{d/2}^{\ell/2} B\,(L\,dx)$$

where $d = 0.0025$ m is the diameter of each wire. We will use $R = d/2$, and r instead of x in the following steps. Thus, using the equations from Ch. 30 referred to above, we find

$$\begin{aligned}\frac{\Phi_B}{L} &= 2\int_0^R \left(\frac{\mu_0 i}{2\pi R^2} r + \frac{\mu_0 i}{2\pi(\ell - r)}\right) dr + 2\int_R^{\ell/2} \left(\frac{\mu_0 i}{2\pi r} + \frac{\mu_0 i}{2\pi(\ell - r)}\right) dr \\ &= \frac{\mu_0 i}{2\pi}\left(1 - 2\ln\left(\frac{\ell - R}{\ell}\right)\right) + \frac{\mu_0 i}{\pi}\ln\left(\frac{\ell - R}{R}\right) \\ &= 0.23\times 10^{-5}\,\mathrm{T{\cdot}m} + 1.08\times 10^{-5}\,\mathrm{T{\cdot}m}\end{aligned}$$

which yields $\Phi_B/L = 1.3\times 10^{-5}\,\mathrm{T{\cdot}m}$ or $1.3\times 10^{-5}\,\mathrm{Wb/m}$.

(b) The flux (per meter) existing within the regions of space occupied by one or the other wires was computed above to be $0.23\times 10^{-5}\,\mathrm{T{\cdot}m}$. Thus,

$$\frac{0.23\times 10^{-5}\,\mathrm{T{\cdot}m}}{1.3\times 10^{-5}\,\mathrm{T{\cdot}m}} = 0.17 = 17\% \ .$$

(c) What was described in part (a) as a symmetry plane at $x = \ell/2$ is now (in the case of parallel currents) a plane of vanishing field (the fields subtract from each other in the region between them, as the right-hand rule shows). The flux in the $0 < x < \ell/2$ region is now of opposite sign of the flux in the $\ell/2 < x < \ell$ region which causes the total flux (or, in this case, flux per meter) to be zero.

24. (a) We assume the flux is entirely due to the field generated by the long straight wire (which is given by Eq. 30-19). We integrate according to Eq. 31-3, not worrying about the possibility of an overall minus sign since we are asked to find the absolute value of the flux.

$$|\Phi_B| = \int_{r-b/2}^{r+b/2} \left(\frac{\mu_0 i}{2\pi r}\right)(a\,dr) = \frac{\mu_0 i a}{2\pi}\ln\left(\frac{r + \frac{b}{2}}{r - \frac{b}{2}}\right) \ .$$

(b) Implementing Faraday's law involves taking a derivative of the flux in part (a), and recognizing that $\frac{dr}{dt} = v$. The magnitude of the induced emf divided by the loop resistance then gives the induced current:

$$i_{\text{loop}} = \left|\frac{\mathcal{E}}{R}\right| = -\frac{\mu_0 i a}{2\pi R}\left|\frac{d}{dt}\ln\left(\frac{r + \frac{b}{2}}{r - \frac{b}{2}}\right)\right| = \frac{\mu_0 i a b v}{2\pi R\,(r^2 - (b/2)^2)} \ .$$

25. Thermal energy is generated at the rate $P = \mathcal{E}^2/R$ (see Eq. 27-23). Using Eq. 27-16, the resistance is given by $R = \rho L/A$, where the resistivity is $1.69\times 10^{-8}\,\Omega{\cdot}\mathrm{m}$ (by Table 27-1) and $A = \pi d^2/4$ is the cross-sectional area of the wire ($d = 0.00100$ m is the wire thickness). The area *enclosed* by the loop is

$$A_{\text{loop}} = \pi r_{\text{loop}}^2 = \pi\left(\frac{L}{2\pi}\right)^2$$

since the length of the wire ($L = 0.500$ m) is the circumference of the loop. This enclosed area is used in Faraday's law (where we ignore minus signs in the interest of finding the magnitudes of the quantities):

$$\mathcal{E} = \frac{d\Phi_B}{dt} = A_{\text{loop}}\frac{dB}{dt} = \frac{L^2}{4\pi}\frac{dB}{dt}$$

where the rate of change of the field is $dB/dt = 0.0100$ T/s. Consequently, we obtain

$$P = \frac{\left(\frac{L^2}{4\pi}\frac{dB}{dt}\right)^2}{4\rho L/\pi d^2} = \frac{d^2 L^3}{64\pi\rho}\left(\frac{dB}{dt}\right)^2 = 3.68\times 10^{-6}\ \mathrm{W} \ .$$

26. Noting that $|\Delta B| = B$, we find the thermal energy is

$$P_{\text{thermal}}\Delta t = \frac{\mathcal{E}^2\Delta t}{R} = \frac{1}{R}\left(-\frac{d\Phi_B}{dt}\right)^2 \Delta t = \frac{1}{R}\left(-A\frac{\Delta B}{\Delta t}\right)^2 \Delta t = \frac{A^2B^2}{R\Delta t} .$$

27. (a) Eq. 31-10 leads to

$$\mathcal{E} = BLv = (0.350\,\mathrm{T})(0.250\,\mathrm{m})(0.550\,\mathrm{m/s}) = 0.0481\ \mathrm{V} .$$

(b) By Ohm's law, the induced current is $i = 0.0481\,\mathrm{V}/18.0\,\Omega = 0.00267\,\mathrm{A}$. By Lenz's law, the current is clockwise in Fig. 31-46.

(c) Eq. 27-22 leads to $P = i^2R = 0.000129$ W.

28. Noting that $F_{\text{net}} = BiL - mg = 0$, we solve for the current:

$$i = \frac{mg}{BL} = \frac{|\mathcal{E}|}{R} = \frac{1}{R}\left|\frac{d\Phi_B}{dt}\right| = \frac{B}{R}\left|\frac{dA}{dt}\right| = \frac{Bv_tL}{R} ,$$

which yields $v_t = mgR/B^2L^2$.

29. (a) By Lenz's law, the induced emf is clockwise. In the rod itself, we would say the emf is directed up the page. Eq. 31-10 leads to

$$\mathcal{E} = BLv = (1.2\,\mathrm{T})(0.10\,\mathrm{m})(5.0\,\mathrm{m/s}) = 0.60\ \mathrm{V} .$$

(b) By Ohm's law, the (clockwise) induced current is $i = 0.60\,\mathrm{V}/0.40\,\Omega = 1.5\,\mathrm{A}$.

(c) Eq. 27-22 leads to $P = i^2R = 0.90$ W.

(d) From Eq. 29-2, we find that the force on the rod associated with the uniform magnetic field is directed rightward and has magnitude

$$F = iLB = (1.5\,\mathrm{A})(0.10\,\mathrm{m})(1.2\,\mathrm{T}) = 0.18\ \mathrm{N} .$$

To keep the rod moving at constant velocity, therefore, a leftward force (due to some external agent) having that same magnitude must be continuously supplied to the rod.

(e) Using Eq. 7-48, we find the power associated with the force being exerted by the external agent: $P = Fv = (0.18\,\mathrm{N})(5.0\,\mathrm{m/s}) = 0.90$ W, which is the same as our result from part (c).

30. (a) The "height" of the triangular area enclosed by the rails and bar is the same as the distance traveled in time v: $d = vt$, where $v = 5.20\,\mathrm{m/s}$. We also note that the "base" of that triangle (the distance between the intersection points of the bar with the rails) is $2d$. Thus, the area of the triangle is

$$A = \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}(2vt)(vt) = v^2t^2 .$$

Since the field is a uniform $B = 0.350\,\mathrm{T}$, then the magnitude of the flux (in SI units) is $\Phi_B = BA = (0.350)(5.20)^2t^2 = 9.46t^2$. At $t = 3.00$ s, we obtain $\Phi_B = 85.2\,\mathrm{Wb}$.

(b) The magnitude of the emf is the (absolute value of) Faraday's law:

$$\mathcal{E} = \frac{d\Phi_B}{dt} = 9.46\,\frac{dt^2}{dt} = 18.9t$$

in SI units. At $t = 3.00$ s, this yields $\mathcal{E} = 56.8$ V.

(c) Our calculation in part (b) shows that $n = 1$.

31. (a) Letting x be the distance from the right end of the rails to the rod, we find an expression for the magnetic flux through the area enclosed by the rod and rails. By Eq. 30-19, the field is $B = \mu_0 i/2\pi r$, where r is the distance from the long straight wire. We consider an infinitesimal horizontal strip of length x and width dr, parallel to the wire and a distance r from it; it has area $A = x\,dr$ and the flux $d\Phi_B = (\mu_0 ix/2\pi r)dr$. By Eq. 31-3, the total flux through the area enclosed by the rod and rails is

$$\Phi_B = \frac{\mu_0 ix}{2\pi}\int_a^{a+L}\frac{dr}{r} = \frac{\mu_0 ix}{2\pi}\ln\left(\frac{a+L}{a}\right) .$$

According to Faraday's law the emf induced in the loop is

$$\begin{aligned}\mathcal{E} &= \frac{d\Phi_B}{dt} = \frac{\mu_0 i}{2\pi}\frac{dx}{dt}\ln\left(\frac{a+L}{a}\right) = \frac{\mu_0 iv}{2\pi}\ln\left(\frac{a+L}{a}\right)\\ &= \frac{(4\pi\times 10^{-7}\,\mathrm{T\cdot m/A})(100\,\mathrm{A})(5.00\,\mathrm{m/s})}{2\pi}\ln\left(\frac{1.00\,\mathrm{cm}+10.0\,\mathrm{cm}}{1.00\,\mathrm{cm}}\right)\\ &= 2.40\times 10^{-4}\ \mathrm{V} .\end{aligned}$$

(b) By Ohm's law, the induced current is $i_\ell = \mathcal{E}/R = (2.40\times 10^{-4}\,\mathrm{V})/(0.400\,\Omega) = 6.00\times 10^{-4}\,\mathrm{A}$. Since the flux is increasing the magnetic field produced by the induced current must be into the page in the region enclosed by the rod and rails. This means the current is clockwise.

(c) Thermal energy is being generated at the rate $P = i_\ell^2 R = (6.00\times 10^{-4}\mathrm{A})^2(0.400\,\Omega) = 1.44\times 10^{-7}\mathrm{W}$.

(d) Since the rod moves with constant velocity, the net force on it is zero. The force of the external agent must have the same magnitude as the magnetic force and must be in the opposite direction. The magnitude of the magnetic force on an infinitesimal segment of the rod, with length dr at a distance r from the long straight wire, is $dF_B = i_\ell B\,dr = (\mu_0 i_\ell i/2\pi r)\,dr$. We integrate to find the magnitude of the total magnetic force on the rod:

$$\begin{aligned}F_B &= \frac{\mu_0 i_\ell i}{2\pi}\int_a^{a+L}\frac{dr}{r} = \frac{\mu_0 i_\ell i}{2\pi}\ln\left(\frac{a+L}{a}\right)\\ &= \frac{(4\pi\times 10^{-7}\,\mathrm{T\cdot m/A})(6.00\times 10^{-4}\,\mathrm{A})(100\,\mathrm{A})}{2\pi}\ln\left(\frac{1.00\,\mathrm{cm}+10.0\,\mathrm{cm}}{1.00\,\mathrm{cm}}\right)\\ &= 2.87\times 10^{-8}\ \mathrm{N} .\end{aligned}$$

Since the field is out of the page and the current in the rod is upward in the diagram, the force associated with the magnetic field is toward the right. The external agent must therefore apply a force of $2.87\times 10^{-8}\,\mathrm{N}$, to the left.

(e) By Eq. 7-48, the external agent does work at the rate $P = Fv = (2.87\times 10^{-8}\,\mathrm{N})(5.00\,\mathrm{m/s}) = 1.44\times 10^{-7}\,\mathrm{W}$. This is the same as the rate at which thermal energy is generated in the rod. All the energy supplied by the agent is converted to thermal energy.

32.

$$\begin{aligned}\oint_1 \vec{E}\cdot d\vec{s} &= -\frac{d\vec{\Phi}_{B1}}{dt} = \frac{d}{dt}(B_1A_1) = A_1\frac{dB_1}{dt} = \pi r_1^2\frac{dB_1}{dt}\\ &= \pi(0.200\,\mathrm{m})^2(-8.50\times 10^{-3}\,\mathrm{T/s}) = -1.07\times 10^{-3}\ \mathrm{V}\end{aligned}$$

$$\begin{aligned}\oint_2 \vec{E}\cdot d\vec{s} &= -\frac{d\vec{\Phi}_{B2}}{dt} = \pi r_2^2\frac{dB_2}{dt}\\ &= \pi(0.300\,\mathrm{m})^2(-8.50\times 10^{-3}\,\mathrm{T/s}) = -2.40\times 10^{-3}\ \mathrm{V}\end{aligned}$$

$$\oint_3 \vec{E}\cdot d\vec{s} = \oint_1 \vec{E}\cdot d\vec{s} - \oint_2 \vec{E}\cdot d\vec{s} = -1.07\times 10^{-3}\,\mathrm{V} - (-2.4\times 10^{-3}\,\mathrm{V}) = 1.33\times 10^{-3}\ \mathrm{V}$$

33. (a) The point at which we are evaluating the field is inside the solenoid, so Eq. 31-27 applies. The magnitude of the induced electric field is

$$E = \frac{1}{2}\frac{dB}{dt}r = \frac{1}{2}(6.5 \times 10^{-3}\,\mathrm{T/s})(0.0220\,\mathrm{m}) = 7.15 \times 10^{-5}\ \mathrm{V/m}\ .$$

(b) Now point at which we are evaluating the field is outside the solenoid and Eq. 31-29 applies. The magnitude of the induced field is

$$E = \frac{1}{2}\frac{dB}{dt}\frac{R^2}{r} = \frac{1}{2}(6.5 \times 10^{-3}\,\mathrm{T/s})\frac{(0.0600\,\mathrm{m})^2}{(0.0820\,\mathrm{m})} = 1.43 \times 10^{-4}\ \mathrm{V/m}\ .$$

34. The magnetic field B can be expressed as

$$B(t) = B_0 + B_1 \sin(\omega t + \phi_0)\ ,$$

where $B_0 = (30.0\,\mathrm{T} + 29.6\,\mathrm{T})/2 = 29.8\,\mathrm{T}$ and $B_1 = (30.0\,\mathrm{T} - 29.6\,\mathrm{T})/2 = 0.200\,\mathrm{T}$. Then from Eq. 31-27

$$E = \frac{1}{2}\left(\frac{dB}{dt}\right)r = \frac{r}{2}\frac{d}{dt}[B_0 + B_1\sin(\omega t + \phi_0)] = \frac{1}{2}B_1\omega r\cos(\omega t + \phi_0)\ .$$

We note that $\omega = 2\pi f$ and that the factor in front of the cosine is the maximum value of the field. Consequently,

$$E_{\max} = \frac{1}{2}B_1(2\pi f)r = \frac{1}{2}(0.200\,\mathrm{T})(2\pi)(15\,\mathrm{Hz})(1.6 \times 10^{-2}\,\mathrm{m}) = 0.15\ \mathrm{V/m}\ .$$

35. We use Faraday's law in the form $\oint \vec{E}\cdot d\vec{s} = -(d\Phi_B/dt)$, integrating along the dotted path shown in the Figure. At all points on the upper and lower sides the electric field is either perpendicular to the side or else it vanishes. We assume it vanishes at all points on the right side (outside the capacitor). On the left side it is parallel to the side and has constant magnitude. Thus, direct integration yields $\oint \vec{E}\cdot d\vec{s} = EL$, where L is the length of the left side of the rectangle. The magnetic field is zero and remains zero, so $d\Phi_B/dt = 0$. Faraday's law leads to a contradiction: $EL = 0$, but neither E nor L is zero. Therefore, there must be an electric field along the right side of the rectangle.

36. (a) We interpret the question as asking for N multiplied by the flux through one turn:

$$\Phi_{\text{turns}} = N\Phi_B = NBA = NB(\pi r^2) = (30.0)(2.60 \times 10^{-3}\,\mathrm{T})(\pi)(0.100\,\mathrm{m})^2 = 2.45 \times 10^{-3}\ \mathrm{Wb}\ .$$

(b) Eq. 31-35 leads to

$$L = \frac{N\Phi_B}{i} = \frac{2.45 \times 10^{-3}\,\mathrm{Wb}}{3.80\,\mathrm{A}} = 6.45 \times 10^{-4}\ \mathrm{H}\ .$$

37. Since $N\Phi_B = Li$, we obtain

$$\Phi_B = \frac{Li}{N} = \frac{(8.0 \times 10^{-3}\,\mathrm{H})(5.0 \times 10^{-3}\,\mathrm{A})}{400} = 1.0 \times 10^{-7}\ \mathrm{Wb}\ .$$

38. (a) We imagine dividing the one-turn solenoid into N small circular loops placed along the width W of the copper strip. Each loop carries a current $\Delta i = i/N$. Then the magnetic field inside the solenoid is $B = \mu_0 n\Delta i = \mu_0(N/W)(i/N) = \mu_0 i/W$.

(b) Eq. 31-35 leads to

$$L = \frac{\Phi_B}{i} = \frac{\pi R^2 B}{i} = \frac{\pi R^2(\mu_0 i/W)}{i} = \frac{\pi\mu_0 R^2}{W}\ .$$

39. We refer to the (very large) wire length as ℓ and seek to compute the flux per meter: Φ_B/ℓ. Using the right-hand rule discussed in Chapter 30, we see that the net field in the region between the axes of antiparallel currents is the addition of the magnitudes of their individual fields, as given by Eq. 30-19 and Eq. 30-22. There is an evident reflection symmetry in the problem, where the plane of symmetry is midway between the two wires (at $x = d/2$); the net field at any point $0 < x < d/2$ is the same at its "mirror image" point $d - x$. The central axis of one of the wires passes through the origin, and that of the other passes through $x = d$. We make use of the symmetry by integrating over $0 < x < d/2$ and then multiplying by 2:

$$\Phi_B = 2\int_0^{d/2} B\,dA = 2\int_0^a B\,(\ell\,dx) + 2\int_a^{d/2} B\,(\ell\,dx)$$

where $d = 0.0025$ m is diameter of each wire. We will r instead of x in the following steps. Thus, using the equations from Ch. 30 referred to above, we find

$$\begin{aligned}\frac{\Phi_B}{\ell} &= 2\int_0^a \left(\frac{\mu_0 i}{2\pi a^2}r + \frac{\mu_0 i}{2\pi(d-r)}\right)dr + 2\int_a^{d/2}\left(\frac{\mu_0 i}{2\pi r} + \frac{\mu_0 i}{2\pi(d-r)}\right)dr \\ &= \frac{\mu_0 i}{2\pi}\left(1 - 2\ln\left(\frac{d-a}{d}\right)\right) + \frac{\mu_0 i}{\pi}\ln\left(\frac{d-a}{a}\right)\end{aligned}$$

where the first term is the flux within the wires and will be neglected (as the problem suggests). Thus, the flux is approximately $\Phi_B \approx \mu_0 i\ell/\pi \ln((d-a)/a)$. Now, we use Eq. 31-35 (with $N = 1$) to obtain the inductance:

$$L = \frac{\Phi_B}{i} = \frac{\mu_0\ell}{\pi}\ln\left(\frac{d-a}{a}\right) .$$

40. (a) Speaking anthropomorphically, the coil wants to fight the changes – so if it wants to push current rightward (when the current is already going rightward) then i must be in the process of decreasing.

(b) From Eq. 31-37 (in absolute value) we get

$$L = \left|\frac{\mathcal{E}}{di/dt}\right| = \frac{17\,\text{V}}{2.5\,\text{kA/s}} = 6.8\times10^{-4}\ \text{H} .$$

41. Since $\mathcal{E} = -L(di/dt)$, we may obtain the desired induced emf by setting

$$\frac{di}{dt} = -\frac{\mathcal{E}}{L} = -\frac{60\,\text{V}}{12\,\text{H}} = -5.0\ \text{A/s} .$$

We might, for example, uniformly reduce the current from 2.0 A to zero in 40 ms.

42. During periods of time when the current is varying linearly with time, Eq. 31-37 (in absolute values) becomes $|\mathcal{E}| = L\left|\frac{\Delta i}{\Delta t}\right|$. For simplicity, we omit the absolute value signs in the following.

(a) For $0 < t < 2$ ms

$$\mathcal{E} = L\frac{\Delta i}{\Delta t} = \frac{(4.6\,\text{H})(7.0\,\text{A} - 0)}{2.0\times10^{-3}\,\text{s}} = 1.6\times10^4\ \text{V} .$$

(b) For $2\,\text{ms} < t < 5\,\text{ms}$

$$\mathcal{E} = L\frac{\Delta i}{\Delta t} = \frac{(4.6\,\text{H})(5.0\,\text{A} - 7.0\,\text{A})}{(5.0-2.0)10^{-3}\,\text{s}} = 3.1\times10^3\ \text{V} .$$

(c) For $5\,\text{ms} < t < 6\,\text{ms}$

$$\mathcal{E} = L\frac{\Delta i}{\Delta t} = \frac{(4.6\,\text{H})(0 - 5.0\,\text{A})}{(6.0-5.0)10^{-3}\,\text{s}} = 2.3\times10^4\ \text{V} .$$

43. (a) Voltage is proportional to inductance (by Eq. 31-37) just as, for resistors, it is proportional to resistance. Since the (independent) voltages for series elements add ($V_1 + V_2$), then inductances in series must *add* just as was the case for resistances.

(b) To ensure the independence of the voltage values, it is important that the inductors not be too close together (the related topic of mutual inductance is treated in §31-12). The requirement is that magnetic field lines from one inductor should not have significant significant presence in any other.

(c) Just as with resistors, $L_{\rm eq} = \sum_{n=1}^{N} L_n$.

44. (a) Voltage is proportional to inductance (by Eq. 31-37) just as, for resistors, it is proportional to resistance. Now, the (independent) voltages for parallel elements are equal ($V_1 = V_2$), and the currents (which are generally functions of time) add ($i_1(t) + i_2(t) = i(t)$). This leads to the Eq. 28-21 for resistors. We note that this condition on the currents implies

$$\frac{di_1(t)}{dt} + \frac{di_2(t)}{dt} = \frac{di(t)}{dt} \ .$$

Thus, although the inductance equation Eq. 31-37 involves the rate of change of current, as opposed to current itself, the conditions that led to the parallel resistor formula also applies to inductors. Therefore,

$$\frac{1}{L_{\rm eq}} = \frac{1}{L_1} + \frac{1}{L_2} \ .$$

(b) To ensure the independence of the voltage values, it is important that the inductors not be too close together (the related topic of mutual inductance is treated in §31-12). The requirement is that the field of one inductor not have significant influence (or "coupling") in the next.

(c) Just as with resistors, $\frac{1}{L_{\rm eq}} = \sum_{n=1}^{N} \frac{1}{L_n}$.

45. Starting with zero current at $t = 0$ (the moment the switch is closed) the current in the circuit increases according to

$$i = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau_L}\right) \ ,$$

where $\tau_L = L/R$ is the inductive time constant and $\mathcal{E}$ is the battery emf. To calculate the time at which $i = 0.9990\mathcal{E}/R$, we solve for t:

$$0.9990\frac{\mathcal{E}}{R} = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau_L}\right) \quad\Longrightarrow\quad \ln(0.0010) = -(t/\tau) \quad\Longrightarrow\quad t = 6.91\tau_L \ .$$

46. The steady state value of the current is also its maximum value, $\mathcal{E}/R$, which we denote as i_m. We are told that $i = i_m/3$ at $t_0 = 5.00\,{\rm s}$. Eq. 31-43 becomes $i = i_m(1 - e^{-t_0/\tau_L})$, which leads to

$$\tau_L = -\frac{t_0}{\ln(1 - i/i_m)} = -\frac{5.00\,{\rm s}}{\ln(1 - 1/3)} = 12.3 \text{ s} \ .$$

47. The current in the circuit is given by $i = i_0 e^{-t/\tau_L}$, where i_0 is the current at time $t = 0$ and τ_L is the inductive time constant (L/R) . We solve for τ_L. Dividing by i_0 and taking the natural logarithm of both sides, we obtain

$$\ln\left(\frac{i}{i_0}\right) = -\frac{t}{\tau_L} \ .$$

This yields

$$\tau_L = -\frac{t}{\ln(i/i_0)} = -\frac{1.0\,{\rm s}}{\ln((10 \times 10^{-3}\,{\rm A})/(1.0\,{\rm A}))} = 0.217 \text{ s} \ .$$

Therefore, $R = L/\tau_L = 10\,{\rm H}/0.217\,{\rm s} = 46\,\Omega$.

48. (a) Immediately after the switch is closed $\mathcal{E} - \mathcal{E}_L = iR$. But $i = 0$ at this instant, so $\mathcal{E}_L = \mathcal{E}$.

(b) $\mathcal{E}_L(t) = \mathcal{E}e^{-t/\tau_L} = \mathcal{E}e^{-2.0\tau_L/\tau_L} = \mathcal{E}e^{-2.0} = 0.135\mathcal{E}$.

(c) From $\mathcal{E}_L(t) = \mathcal{E}e^{-t/\tau_L}$ we obtain

$$\frac{t}{\tau_L} = \ln\left(\frac{\mathcal{E}}{\mathcal{E}_L}\right) = \ln 2 \implies t = \tau_L \ln 2 = 0.693\tau_L \ .$$

49. (a) If the battery is switched into the circuit at $t = 0$, then the current at a later time t is given by

$$i = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau_L}\right) \ ,$$

where $\tau_L = L/R$. Our goal is to find the time at which $i = 0.800\mathcal{E}/R$. This means

$$0.800 = 1 - e^{-t/\tau_L} \implies e^{-t/\tau_L} = 0.200 \ .$$

Taking the natural logarithm of both sides, we obtain $-(t/\tau_L) = \ln(0.200) = -1.609$. Thus

$$t = 1.609\tau_L = \frac{1.609L}{R} = \frac{1.609(6.30 \times 10^{-6}\,\mathrm{H})}{1.20 \times 10^3\,\Omega} = 8.45 \times 10^{-9} \text{ s} \ .$$

(b) At $t = 1.0\tau_L$ the current in the circuit is

$$i = \frac{\mathcal{E}}{R}\left(1 - e^{-1.0}\right) = \left(\frac{14.0\,\mathrm{V}}{1.20 \times 10^3\,\Omega}\right)\left(1 - e^{-1.0}\right) = 7.37 \times 10^{-3} \text{ A} \ .$$

50. Applying the loop theorem

$$\mathcal{E} - L\left(\frac{di}{dt}\right) = iR \ ,$$

we solve for the (time-dependent) emf, with SI units understood:

$$\begin{aligned} \mathcal{E} &= L\frac{di}{dt} + iR = L\frac{d}{dt}(3.0 + 5.0t) + (3.0 + 5.0t)R \\ &= (6.0)(5.0) + (3.0 + 5.0t)(4.0) \\ &= (42 + 20t) \end{aligned}$$

in volts if t is in seconds.

51. Taking the time derivative of both sides of Eq. 31-43, we obtain

$$\begin{aligned} \frac{di}{dt} &= \frac{d}{dt}\left[\frac{\mathcal{E}}{R}\left(1 - e^{-Rt/\tau_L}\right)\right] = \frac{\mathcal{E}}{L}e^{-RT/L} \\ &= \left(\frac{45.0\,\mathrm{V}}{50.0 \times 10^{-3}\,\mathrm{H}}\right)e^{-(180\,\Omega)(1.20\times10^{-3}\,\mathrm{s})/50.0\times10^{-3}\,\mathrm{H}} = 12.0 \text{ A/s} \ . \end{aligned}$$

52. (a) Our notation is as follows: h is the height of the toroid, a its inner radius, and b its outer radius. Since it has a square cross section, $h = b - a = 0.12\,\mathrm{m} - 0.10\,\mathrm{m} = 0.02\,\mathrm{m}$.. We derive the flux using Eq. 30-26 and the self-inductance using Eq. 31-35:

$$\Phi_B = \int_a^b B\,dA = \int_a^b \left(\frac{\mu_0 Ni}{2\pi r}\right) h\,dr = \frac{\mu_0 Nih}{2\pi} \ln\left(\frac{b}{a}\right)$$

and $L = N\Phi_B/i = (\mu_0 N^2 h/2\pi)\ln(b/a)$. We note that the formulas for Φ_B and L can also be found in the Supplement for the chapter, in Sample Problem 31-11. Now, since the inner circumference of the toroid is $l = 2\pi a = 2\pi(10\,\text{cm}) \approx 62.8\,\text{cm}$, the number of turns of the toroid is roughly $N \approx 62.8\,\text{cm}/1.0\,\text{mm} = 628$. Thus

$$\begin{aligned} L &= \frac{\mu_0 N^2 h}{2\pi} \ln\left(\frac{b}{a}\right) \\ &\approx \frac{\left(4\pi \times 10^{-7}\,\text{H/m}\right)(628)^2(0.02\,\text{m})}{2\pi} \ln\left(\frac{12}{10}\right) \\ &= 2.9 \times 10^{-4}\ \text{H}\ . \end{aligned}$$

(b) Noting that the perimeter of a square is four times its sides, the total length ℓ of the wire is $\ell = (628)4(2.0\,\text{cm}) = 50\,\text{m}$, the resistance of the wire is $R = (50\,\text{m})(0.02\,\Omega/\text{m}) = 1.0\,\Omega$. Thus

$$\tau_L = \frac{L}{R} = \frac{2.9 \times 10^{-4}\,\text{H}}{1.0\,\Omega} = 2.9 \times 10^{-4}\ \text{s}\ .$$

53. (a) The inductor prevents a fast build-up of the current through it, so immediately after the switch is closed, the current in the inductor is zero. It follows that

$$i_1 = i_2 = \frac{\mathcal{E}}{R_1 + R_2} = \frac{100\,\text{V}}{10.0\,\Omega + 20.0\,\Omega} = 3.33\ \text{A}\ .$$

(b) After a suitably long time, the current reaches steady state. Then, the emf across the inductor is zero, and we may imagine it replaced by a wire. The current in R_3 is $i_1 - i_2$. Kirchhoff's loop rule gives

$$\mathcal{E} - i_1 R_1 - i_2 R_2 = 0 \quad \text{and} \quad \mathcal{E} - i_1 R_1 - (i_1 - i_2) R_3 = 0\ .$$

We solve these simultaneously for i_1 and i_2. The results are

$$\begin{aligned} i_1 &= \frac{\mathcal{E}\,(R_2 + R_3)}{R_1 R_2 + R_1 R_3 + R_2 R_3} \\ &= \frac{(100\,\text{V})(20.0\,\Omega + 30.0\,\Omega)}{(10.0\,\Omega)(20.0\,\Omega) + (10.0\,\Omega)(30.0\,\Omega) + (20.0\,\Omega)(30.0\,\Omega)} \\ &= 4.55\ \text{A}\ , \end{aligned}$$

and

$$\begin{aligned} i_2 &= \frac{\mathcal{E} R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3} \\ &= \frac{(100\,\text{V})(30.0\,\Omega)}{(10.0\,\Omega)(20.0\,\Omega) + (10.0\,\Omega)(30.0\,\Omega) + (20.0\,\Omega)(30.0\,\Omega)} \\ &= 2.73\ \text{A}\ . \end{aligned}$$

(c) The left-hand branch is now broken. We take the current (immediately) as zero in that branch when the switch is opened (that is, $i_1 = 0$). The current in R_3 changes less rapidly because there is an inductor in its branch. In fact, immediately after the switch is opened it has the same value that it had before the switch was opened. That value is $4.55\,\text{A} - 2.73\,\text{A} = 1.82\,\text{A}$. The current in R_2 is the same as that in R_3 (1.82 A).

(d) There are no longer any sources of emf in the circuit, so all currents eventually drop to zero.

54. (a) When switch S is just closed (case I), $V_1 = \mathcal{E}$ and $i_1 = \mathcal{E}/R_1 = 10\,\text{V}/5.0\,\Omega = 2.0\,\text{A}$. After a long time (case II) we still have $V_1 = \mathcal{E}$, so $i_1 = 2.0\,\text{A}$.

(b) Case I: since now $\mathcal{E}_L = \mathcal{E}$, $i_2 = 0$; case II: since now $\mathcal{E}_L = 0$, $i_2 = \mathcal{E}/R_2 = 10\,\text{V}/10\,\Omega = 1.0\,\text{A}$.

(c) Case I: $i = i_1 + i_2 = 2.0\,\text{A} + 0 = 2.0\,\text{A}$; case II: $i = i_1 + i_2 = 2.0\,\text{A} + 1.0\,\text{A} = 3.0\,\text{A}$.

(d) Case I: since $\mathcal{E}_L = \mathcal{E}$, $V_2 = \mathcal{E} - \mathcal{E}_L = 0$; case II: since $\mathcal{E}_L = 0$, $V_2 = \mathcal{E} - \mathcal{E}_L = \mathcal{E} = 10\,\mathrm{V}$.

(e) Case I: $\mathcal{E}_L = \mathcal{E} = 10\,\mathrm{V}$; case II: $\mathcal{E}_L = 0$.

(f) Case I: $di_2/dt = \mathcal{E}_L/L = \mathcal{E}/L = 10\,\mathrm{V}/5.0\,\mathrm{H} = 2.0\,\mathrm{A/s}$; case II: $di_2/dt = \mathcal{E}_L/L = 0$.

55. (a) We assume i is from left to right through the closed switch. We let i_1 be the current in the resistor and take it to be downward. Let i_2 be the current in the inductor, also assumed downward. The junction rule gives $i = i_1 + i_2$ and the loop rule gives $i_1 R - L(di_2/dt) = 0$. According to the junction rule, $(di_1/dt) = -(di_2/dt)$. We substitute into the loop equation to obtain

$$L\frac{di_1}{dt} + i_1 R = 0 \; .$$

This equation is similar to Eq. 31-48, and its solution is the function given as Eq. 31-49:

$$i_1 = i_0 e^{-Rt/L} \; ,$$

where i_0 is the current through the resistor at $t = 0$, just after the switch is closed. Now just after the switch is closed, the inductor prevents the rapid build-up of current in its branch, so at that moment $i_2 = 0$ and $i_1 = i$. Thus $i_0 = i$, so

$$i_1 = ie^{-Rt/L} \qquad \text{and} \qquad i_2 = i - i_1 = i\left(1 - e^{-Rt/L}\right) \; .$$

(b) When $i_2 = i_1$,

$$e^{-Rt/L} = 1 - e^{-Rt/L} \implies e^{-Rt/L} = \frac{1}{2} \; .$$

Taking the natural logarithm of both sides (and using $\ln(1/2) = -\ln 2$) we obtain

$$\left(\frac{Rt}{L}\right) = \ln 2 \implies t = \frac{L}{R}\ln 2 \; .$$

56. Let $U_B(t) = \frac{1}{2}Li^2(t)$. We require the energy at time t to be half of its final value: $U(t) = \frac{1}{2}U_B(t \to \infty) = \frac{1}{4}Li_f^2$. This gives $i(t) = i_f/\sqrt{2}$. But $i(t) = i_f(1 - e^{-t/\tau_L})$, so

$$1 - e^{-t/\tau_L} = \frac{1}{\sqrt{2}} \implies t = -\tau_L \ln\left(1 - \frac{1}{\sqrt{2}}\right) = 1.23\tau_L \; .$$

57. From Eq. 31-51 and Eq. 31-43, the rate at which the energy is being stored in the inductor is

$$\begin{aligned}
\frac{dU_B}{dt} &= \frac{d\left(\frac{1}{2}Li^2\right)}{dt} = L\,i\,\frac{di}{dt} \\
&= L\left(\frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau_L}\right)\right)\left(\frac{\mathcal{E}}{R}\frac{1}{\tau_L}\,e^{-t/\tau_L}\right) \\
&= \frac{\mathcal{E}^2}{R}\left(1 - e^{-t/\tau_L}\right)e^{-t/\tau_L}
\end{aligned}$$

where $\tau_L = L/R$ has been used. From Eq. 27-22 and Eq. 31-43, the rate at which the resistor is generating thermal energy is

$$P_{\text{thermal}} = i^2 R = \frac{\mathcal{E}^2}{R^2}\left(1 - e^{-t/\tau_L}\right)^2 R = \frac{\mathcal{E}^2}{R}\left(1 - e^{-t/\tau_L}\right)^2 \; .$$

We equate this to dU_B/dt, and solve for the time:

$$\frac{\mathcal{E}^2}{R}\left(1 - e^{-t/\tau_L}\right)^2 = \frac{\mathcal{E}^2}{R}\left(1 - e^{-t/\tau_L}\right)e^{-t/\tau_L} \implies t = \tau_L \ln 2 = (37.0\,\mathrm{ms})\ln 2 = 25.6\ \mathrm{ms} \; .$$

58. (a) From Eq. 31-51 and Eq. 31-43, the rate at which the energy is being stored in the inductor is

$$\begin{aligned}\frac{dU_B}{dt} &= \frac{d\left(\frac{1}{2}Li^2\right)}{dt} = L\,i\,\frac{di}{dt}\\ &= L\left(\frac{\mathcal{E}}{R}\left(1-e^{-t/\tau_L}\right)\right)\left(\frac{\mathcal{E}}{R}\frac{1}{\tau_L}\,e^{-t/\tau_L}\right)\\ &= \frac{\mathcal{E}^2}{R}\left(1-e^{-t/\tau_L}\right)e^{-t/\tau_L}\ .\end{aligned}$$

Now, $\tau_L = L/R = 2.0\,\mathrm{H}/10\,\Omega = 0.20\,\mathrm{s}$ and $\mathcal{E} = 100\,\mathrm{V}$, so the above expression yields $dU_B/dt = 2.4\times10^2$ W when $t = 0.10$ s.

(b) From Eq. 27-22 and Eq. 31-43, the rate at which the resistor is generating thermal energy is

$$P_{\text{thermal}} = i^2R = \frac{\mathcal{E}^2}{R^2}\left(1-e^{-t/\tau_L}\right)^2 R = \frac{\mathcal{E}^2}{R}\left(1-e^{-t/\tau_L}\right)^2\ .$$

At $t = 0.10$ s, this yields $P_{\text{thermal}} = 1.5\times10^2$ W.

(c) By energy conservation, the rate of energy being supplied to the circuit by the battery is

$$P_{\text{battery}} = P_{\text{thermal}} + \frac{dU_B}{dt} = 3.9\times10^2\ \mathrm{W}\ .$$

We note that this could result could alternatively have been found from Eq. 28-14 (with Eq. 31-43).

59. (a) If the battery is applied at time $t = 0$ the current is given by

$$i = \frac{\mathcal{E}}{R}\left(1-e^{-t/\tau_L}\right)\ ,$$

where $\mathcal{E}$ is the emf of the battery, R is the resistance, and τ_L is the inductive time constant (L/R). This leads to

$$e^{-t/\tau_L} = 1-\frac{iR}{\mathcal{E}} \implies -\frac{t}{\tau_L} = \ln\left(1-\frac{iR}{\mathcal{E}}\right)\ .$$

Since

$$\ln\left(1-\frac{iR}{\mathcal{E}}\right) = \ln\left[1-\frac{(2.00\times10^{-3}\,\mathrm{A})(10.0\times10^3\,\Omega)}{50.0\,\mathrm{V}}\right] = -0.5108\ ,$$

the inductive time constant is $\tau_L = t/0.5108 = (5.00\times10^{-3}\,\mathrm{s})/0.5108 = 9.79\times10^{-3}\,\mathrm{s}$ and the inductance is

$$L = \tau_L R = (9.79\times10^{-3}\,\mathrm{s})(10.0\times10^3\,\Omega) = 97.9\ \mathrm{H}\ .$$

(b) The energy stored in the coil is

$$U_B = \frac{1}{2}Li^2 = \frac{1}{2}(97.9\,\mathrm{H})(2.00\times10^{-3}\,\mathrm{A})^2 = 1.96\times10^{-4}\ \mathrm{J}\ .$$

60. (a) The energy delivered by the battery is the integral of Eq. 28-14 (where we use Eq. 31-43 for the current):

$$\begin{aligned}\int_0^t P_{\text{battery}}\,dt &= \int_0^t \frac{\mathcal{E}^2}{R}\left(1-e^{-Rt/L}\right)dt = \frac{\mathcal{E}^2}{R}\left[t+\frac{L}{R}\left(e^{-Rt/L}-1\right)\right]\\ &= \frac{(10.0\,\mathrm{V})^2}{6.70\,\Omega}\left[2.00\,\mathrm{s}+\frac{(5.50\,\mathrm{H})\left(e^{-(6.70\,\Omega)(2.00\,\mathrm{s})/5.50\,\mathrm{H}}-1\right)}{6.70\,\Omega}\right]\\ &= 18.7\ \mathrm{J}\ .\end{aligned}$$

(b) The energy stored in the magnetic field is given by Eq. 31-51:

$$\begin{aligned} U_B &= \frac{1}{2}Li^2(t) = \frac{1}{2}L\left(\frac{\mathcal{E}}{R}\right)^2 (1-e^{-Rt/L})^2 \\ &= \frac{1}{2}(5.50\,\mathrm{H})\left(\frac{10.0\,\mathrm{V}}{6.70\,\Omega}\right)^2 \left[1-e^{-(6.70\,\Omega)(2.00\,\mathrm{s})/5.50\,\mathrm{H}}\right]^2 \\ &= 5.10\ \mathrm{J}\ . \end{aligned}$$

(c) The difference of the previous two results gives the amount "lost" in the resistor: $18.7\,\mathrm{J} - 5.10\,\mathrm{J} = 13.6\,\mathrm{J}$.

61. Suppose that the switch had been in position a for a long time so that the current had reached the steady-state value i_0. The energy stored in the inductor is $U_B = \frac{1}{2}Li_0^2$. Now, the switch is thrown to position b at time $t = 0$. Thereafter the current is given by

$$i = i_0 e^{-t/\tau_L}\ ,$$

where τ_L is the inductive time constant, given by $\tau_L = L/R$. The rate at which thermal energy is generated in the resistor is given by

$$P = i^2R = i_0^2 R e^{-2t/\tau_L}\ .$$

Over a long time period the energy dissipated is

$$\int_0^\infty P\,dt = i_0^2 R \int_0^\infty e^{-2t/\tau_L}\,dt = -\frac{1}{2} i_0^2 R\tau_L e^{-2t/\tau_L}\Big|_0^\infty = \frac{1}{2}i_0^2 R\tau_L\ .$$

Upon substitution of $\tau_L = L/R$ this becomes $\frac{1}{2}Li_0^2$, the same as the total energy originally stored in the inductor.

62. The magnetic energy stored in the toroid is given by $U_B = \frac{1}{2}Li^2$, where L is its inductance and i is the current. By Eq. 31-56, the energy is also given by $U_B = u_B\mathcal{V}$, where u_B is the average energy density and $\mathcal{V}$ is the volume. Thus

$$i = \sqrt{\frac{2u_B\mathcal{V}}{L}} = \sqrt{\frac{2\left(70.0\,\mathrm{J/m^3}\right)(0.0200\,\mathrm{m^3})}{90.0\times10^{-3}\,\mathrm{H}}} = 5.58\ \mathrm{A}\ .$$

63. (a) At any point the magnetic energy density is given by $u_B = B^2/2\mu_0$, where B is the magnitude of the magnetic field at that point. Inside a solenoid $B = \mu_0 ni$, where n, for the solenoid of this problem, is $(950\,\text{turns})/(0.850\,\mathrm{m}) = 1.118\times10^3\,\mathrm{m^{-1}}$. The magnetic energy density is

$$u_B = \frac{1}{2}\mu_0 n^2 i^2 = \frac{1}{2}\left(4\pi\times10^{-7}\,\mathrm{T{\cdot}m/A}\right)\left(1.118\times10^3\,\mathrm{m^{-1}}\right)^2(6.60\,\mathrm{A})^2 = 34.2\ \mathrm{J/m^3}\ .$$

(b) Since the magnetic field is uniform inside an ideal solenoid, the total energy stored in the field is $U_B = u_B\mathcal{V}$, where $\mathcal{V}$ is the volume of the solenoid. $\mathcal{V}$ is calculated as the product of the cross-sectional area and the length. Thus

$$U_B = (34.2\,\mathrm{J/m^3})(17.0\times10^{-4}\,\mathrm{m^2})(0.850\,\mathrm{m}) = 4.94\times10^{-2}\ \mathrm{J}\ .$$

64. We use $1\,\mathrm{ly} = 9.46\times10^{15}\,\mathrm{m}$, and use the symbol $\mathcal{V}$ for volume.

$$U_B = \mathcal{V}u_B = \frac{\mathcal{V}B^2}{2\mu_0} = \frac{(9.46\times10^{15}\,\mathrm{m})^3(1\times10^{-10}\,\mathrm{T})^2}{2\,(4\pi\times10^{-7}\,\mathrm{H/m})} = 3\times10^{36}\ \mathrm{J}\ .$$

65. We set $u_E = \frac{1}{2}\varepsilon_0 E^2 = u_B = \frac{1}{2}B^2/\mu_0$ and solve for the magnitude of the electric field:

$$E = \frac{B}{\sqrt{\varepsilon_0\mu_0}} = \frac{0.50\,\mathrm{T}}{\sqrt{(8.85\times10^{-12}\,\mathrm{F/m})\,(4\pi\times10^{-7}\,\mathrm{H/m})}} = 1.5\times10^{8}\ \mathrm{V/m}\ .$$

66. (a) The magnitude of the magnetic field at the center of the loop, using Eq. 30-11, is

$$B = \frac{\mu_0 i}{2R} = \frac{\left(4\pi\times10^{-7}\,\mathrm{H/m}\right)(100\,\mathrm{A})}{2(50\times10^{-3}\,\mathrm{m})} = 1.3\times10^{-3}\ \mathrm{T}\ .$$

(b) The energy per unit volume in the immediate vicinity of the center of the loop is

$$u_B = \frac{B^2}{2\mu_0} = \frac{(1.3\times10^{-3}\,\mathrm{T})^2}{2\,(4\pi\times10^{-7}\,\mathrm{H/m})} = 0.63\ \mathrm{J/m^3}\ .$$

67. (a) The energy per unit volume associated with the magnetic field is

$$u_B = \frac{B^2}{2\mu_0} = \frac{1}{2\mu_0}\left(\frac{\mu_0 i}{2R}\right)^2 = \frac{\mu_0 i^2}{8R^2} = \frac{\left(4\pi\times10^{-7}\,\mathrm{H/m}\right)(10\,\mathrm{A})^2}{8(2.5\times10^{-3}\,\mathrm{m}/2)^2} = 1.0\ \mathrm{J/m^3}\ .$$

(b) The electric energy density is

$$\begin{aligned} u_E &= \frac{1}{2}\varepsilon_0 E^2 = \frac{\epsilon_0}{2}(\rho J)^2 = \frac{\varepsilon_0}{2}\left(\frac{iR}{\ell}\right)^2 \\ &= \frac{1}{2}(8.85\times10^{-12}\,\mathrm{F/m})\left[(10\,\mathrm{A})(3.3\,\Omega/10^3\,\mathrm{m})\right]^2 \\ &= 4.8\times10^{-15}\ \mathrm{J/m^3}\ . \end{aligned}$$

Here we used $J = i/A$ and $R = \rho\ell/A$ to obtain $\rho J = iR/\ell$.

68. (a) The flux in coil 1 is

$$\frac{L_1 i_1}{N_1} = \frac{(25\,\mathrm{mH})(6.0\,\mathrm{mA})}{100} = 1.5\ \mu\mathrm{Wb}\ ,$$

and the magnitude of the self-induced emf is

$$L_1\frac{di_1}{dt} = (25\,\mathrm{mH})(4.0\,\mathrm{A/s}) = 100\ \mathrm{mV}\ .$$

(b) In coil 2, we find

$$\Phi_{21} = \frac{Mi_1}{N_2} = \frac{(3.0\,\mathrm{mH})(6.0\,\mathrm{mA})}{200} = 90\ \mathrm{nWb}\ ,$$

$$\mathcal{E}_{21} = M\frac{di_1}{dt} = (3.0\,\mathrm{mH})(4.0\,\mathrm{A/s}) = 12\ \mathrm{mV}\ .$$

69. (a) Eq. 31-67 yields

$$M = \frac{\mathcal{E}_1}{|di_2/dt|} = \frac{25.0\,\mathrm{mV}}{15.0\,\mathrm{A/s}} = 1.67\ \mathrm{mH}\ .$$

(b) Eq. 31-62 leads to

$$N_2\Phi_{21} = Mi_1 = (1.67\,\mathrm{mH})(3.60\,\mathrm{A}) = 6.00\ \mathrm{mWb}\ .$$

70. We use $\mathcal{E}_2 = -M\,di_1/dt \approx M|\Delta i/\Delta t|$ to find M:

$$M = \left|\frac{\mathcal{E}}{\Delta i_1/\Delta t}\right| = \frac{30\times10^3\,\mathrm{V}}{6.0\,\mathrm{A}/(2.5\times10^{-3}\,\mathrm{s})} = 13\ \mathrm{H}\ .$$

71. (a) We assume the current is changing at (nonzero) rate di/dt and calculate the total emf across both coils. First consider the coil 1. The magnetic field due to the current in that coil points to the right. The magnetic field due to the current in coil 2 also points to the right. When the current increases, both fields increase and both changes in flux contribute emf's in the same direction. Thus, the induced emf's are

$$\mathcal{E}_1 = -\left(L_1 + M\right)\frac{di}{dt} \quad \text{and} \quad \mathcal{E}_2 = -\left(L_2 + M\right)\frac{di}{dt} \ .$$

Therefore, the total emf across both coils is

$$\mathcal{E} = \mathcal{E}_1 + \mathcal{E}_2 = -\left(L_1 + L_2 + 2M\right)\frac{di}{dt}$$

which is exactly the emf that would be produced if the coils were replaced by a single coil with inductance $L_{\text{eq}} = L_1 + L_2 + 2M$.

(b) We imagine reversing the leads of coil 2 so the current enters at the back of coil rather than the front (as pictured in the diagram). Then the field produced by coil 2 at the site of coil 1 is opposite to the field produced by coil 1 itself. The fluxes have opposite signs. An increasing current in coil 1 tends to increase the flux in that coil, but an increasing current in coil 2 tends to decrease it. The emf across coil 1 is

$$\mathcal{E}_1 = -\left(L_1 - M\right)\frac{di}{dt} \ .$$

Similarly, the emf across coil 2 is

$$\mathcal{E}_2 = -\left(L_2 - M\right)\frac{di}{dt} \ .$$

The total emf across both coils is

$$\mathcal{E} = -\left(L_1 + L_2 - 2M\right)\frac{di}{dt} \ .$$

This the same as the emf that would be produced by a single coil with inductance $L_{\text{eq}} = L_1 + L_2 - 2M$.

72. The coil-solenoid mutual inductance is

$$M = M_{cs} = \frac{N\Phi_{cs}}{i_s} = \frac{N(\mu_0 i_s n \pi R^2)}{i_s} = \mu_0 \pi R^2 n N \ .$$

As long as the magnetic field of the solenoid is entirely contained within the cross-section of the coil we have $\Phi_{sc} = B_s A_s = B_s \pi R^2$, regardless of the shape, size, or possible lack of close-packing of the coil.

73. Letting the current in solenoid 1 be i, we calculate the flux linkage in solenoid 2. The mutual inductance, then, is this flux linkage divided by i. The magnetic field inside solenoid 1 is parallel to the axis and has uniform magnitude $B = \mu_0 i n_1$, where n_1 is the number of turns per unit length of the solenoid. The cross-sectional area of the solenoid is πR_1^2. Since $\vec{B}$ is normal to the cross section, the flux here is

$$\Phi = AB = \pi R_1^2 \mu_0 n_1 i \ .$$

Since the magnetic field is zero outside the solenoid, this is also the flux through a cross section of solenoid 2. The number of turns in a length ℓ of solenoid 2 is $N_2 = n_2 \ell$, and the flux linkage is

$$N_2 \Phi = n_2 \ell \pi R_1^2 \mu_0 n_1 i \ .$$

The mutual inductance is

$$M = \frac{N_2 \Phi}{i} = \pi R_1^2 \ell \mu_0 n_1 n_2 \ .$$

M does not depend on R_2 because there is no magnetic field in the region between the solenoids. Changing R_2 does not change the flux through solenoid 2, but changing R_1 does.

74. We use the expression for the flux Φ_B over the toroid cross-section derived in our solution to problem 52 obtain the coil-toroid mutual inductance:

$$M_{ct} = \frac{N_c \Phi_{ct}}{i_t} = \frac{N_c}{i_t} \frac{\mu_0 i_t N_t h}{2\pi} \ln\left(\frac{b}{a}\right) = \frac{\mu_0 N_1 N_2 h}{2\pi} \ln\left(\frac{b}{a}\right)$$

where $N_t = N_1$ and $N_c = N_2$. We note that the formula for Φ_B can also be found in the Supplement for the chapter, in Sample Problem 31-11.

75. (a) The flux over the loop cross section due to the current i in the wire is given by

$$\Phi = \int_a^{a+b} B_{\text{wire}} l \, dr = \int_a^{a+b} \frac{\mu_0 i l}{2\pi r} \, dr = \frac{\mu_0 i l}{2\pi} \ln\left(1 + \frac{b}{a}\right) .$$

Thus,

$$M = \frac{N\Phi}{i} = \frac{N\mu_0 l}{2\pi} \ln\left(1 + \frac{b}{a}\right) .$$

(b) From the formula for M obtained above

$$M = \frac{(100)\left(4\pi \times 10^{-7}\,\text{H/m}\right)(0.30\,\text{m})}{2\pi} \ln\left(1 + \frac{8.0}{1.0}\right) = 1.3 \times 10^{-5}\ \text{H} .$$

76. For $t < 0$, no current goes through L_2, so $i_2 = 0$ and $i_1 = \mathcal{E}/R$. As the switch is opened there will be a very brief sparking across the gap. i_1 drops while i_2 increases, both very quickly. The loop rule can be written as

$$\mathcal{E} - i_1 R - L_1 \frac{di_1}{dt} - i_2 R - L_2 \frac{di_2}{dt} = 0 ,$$

where the initial value of i_1 at $t = 0$ is given by $\mathcal{E}/R$ and that of i_2 at $t = 0$ is 0. We consider the situation shortly after $t = 0$. Since the sparking is very brief, we can reasonably assume that both i_1 and i_2 get equalized quickly, before they can change appreciably from their respective initial values. Here, the loop rule requires that $L_1(di_1/dt)$, which is large and negative, must roughly cancel $L_2(di_2/dt)$, which is large and positive:

$$L_1 \frac{di_1}{dt} \approx -L_2 \frac{di_2}{dt} .$$

Let the common value reached by i_1 and i_2 be i, then

$$\frac{di_1}{dt} \approx \frac{\Delta i_1}{\Delta t} = \frac{i - \mathcal{E}/R}{\Delta t}$$

and

$$\frac{di_2}{dt} \approx \frac{\Delta i_2}{\Delta t} = \frac{i - 0}{\Delta t} .$$

The equations above yield

$$L_1 \left(i - \frac{\mathcal{E}}{R}\right) = -L_2(i - 0) \quad \Longrightarrow \quad i = \frac{\mathcal{E} L_1}{L_2 R_1 + L_1 R_2} = \frac{L_1}{L_1 + L_2} \frac{\mathcal{E}}{R} .$$

77. (a) $i_0 = \mathcal{E}/R = 100\,\text{V}/10\,\Omega = 10\,\text{A}$.

(b) $U_B = \frac{1}{2} L i_0^2 = \frac{1}{2}(2.0\,\text{H})(10\,\text{A})^2 = 100\,\text{J}$.

78. We write $i = i_0 e^{-t/\tau_L}$ and note that $i = 10\%\, i_0$. We solve for t:

$$t = \tau_L \ln\left(\frac{i_0}{i}\right) = \frac{L}{R} \ln\left(\frac{i_0}{i}\right) = \frac{2.00\,\text{H}}{3.00\,\Omega} \ln\left(\frac{i_0}{0.100\, i_0}\right) = 1.54\ \text{s} .$$

79. (a) The energy density at any point is given by $u_B = B^2/2\mu_0$, where B is the magnitude of the magnetic field. The magnitude of the field inside a toroid, a distance r from the center, is given by Eq. 30-26: $B = \mu_0 iN/2\pi r$, where N is the number of turns and i is the current. Thus

$$u_B = \frac{1}{2\mu_0}\left(\frac{\mu_0 iN}{2\pi r}\right)^2 = \frac{\mu_0 i^2 N^2}{8\pi^2 r^2} \ .$$

(b) We evaluate the integral $U_B = \int u_B \, d\mathcal{V}$ over the volume of the toroid. A circular strip with radius r, height h, and thickness dr has volume $d\mathcal{V} = 2\pi r h \, dr$, so

$$U_B = \frac{\mu_0 i^2 N^2}{8\pi^2} 2\pi h \int_a^b \frac{dr}{r} = \frac{\mu_0 i^2 N^2 h}{4\pi} \ln\left(\frac{b}{a}\right) \ .$$

Substituting in the given values, we find

$$\begin{aligned} U_B &= \frac{(4\pi \times 10^{-7}\mathrm{T{\cdot}m/A})(0.500\mathrm{A})^2(1250)^2(13 \times 10^{-3}\,\mathrm{m})}{4\pi} \ln\left(\frac{95\,\mathrm{mm}}{52\,\mathrm{mm}}\right) \\ &= 3.06 \times 10^{-4}\ \mathrm{J} \ . \end{aligned}$$

(c) The inductance is given in Sample Problem 31-11:

$$L = \frac{\mu_0 N^2 h}{2\pi} \ln\left(\frac{b}{a}\right)$$

so the energy is given by

$$U_B = \frac{1}{2} L i^2 = \frac{\mu_0 N^2 i^2 h}{4\pi} \ln\left(\frac{b}{a}\right) \ .$$

This the exactly the same as the expression found in part (b) and yields the same numerical result.

80. If the solenoid is long and thin, then when it is bent into a toroid $(b-a)/a$ is much less than 1. Therefore,

$$L_{\text{toroid}} = \frac{\mu_0 N^2 h}{2\pi} \ln\left(\frac{b}{a}\right) = \frac{\mu_0 N^2 h}{2\pi} \ln\left(1 + \frac{b-a}{b}\right) \approx \frac{\mu_0 N^2 h(b-a)}{2\pi b} \ .$$

Since $A = h(b-a)$ is the cross-sectional area and $l = 2\pi b$ is the length of the toroid, we may rewrite this expression for the toroid self-inductance as

$$\frac{L_{\text{toroid}}}{l} \approx \frac{\mu_0 N^2 A}{l^2} = \mu_0 n^2 A \ ,$$

which indeed reduces to that of a long solenoid. Note that the approximation $\ln(1+x) \approx x$ is used for very small $|x|$.

81. Using Eq. 31-43

$$i = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau_L}\right)$$

where $\tau_L = 2.0$ ns, we find

$$t = \tau_L \ln\left(\frac{1}{1 - \frac{iR}{\mathcal{E}}}\right) \approx 1.0\ \mathrm{ns} \ .$$

82. We note that $n = 100\,\mathrm{turns/cm} = 10000\,\mathrm{turns/m}$. The induced emf is

$$\begin{aligned} \mathcal{E} &= -\frac{d\Phi_B}{dt} = -\frac{d(BA)}{dt} = -A\frac{d}{dt}(\mu_0 n i) = -\mu_0 n \pi r^2 \frac{di}{dt} \\ &= -(4\pi \times 10^{-7}\,\mathrm{T{\cdot}m/A})(10000\,\mathrm{turn/m})(\pi)(25 \times 10^{-3}\,\mathrm{m})^2 \left(\frac{0.50\,\mathrm{A} - 1.0\,\mathrm{A}}{10 \times 10^{-3}\,\mathrm{s}}\right) \\ &= 1.2 \times 10^{-3}\ \mathrm{V} \ . \end{aligned}$$

Note that since $\vec{B}$ only appears inside the solenoid, the area A is be the cross-sectional area of the solenoid, not the (larger) loop.

83. With $\tau_L = L/R = 0.0010$ s, we find the current at $t = 0.0020$ s from Eq. 31-43:

$$i = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau_L}\right) = 0.86 \text{ A} .$$

Consequently, the energy stored, from Eq. 31-51, is

$$U_B = \frac{1}{2}Li^2 = 3.7 \times 10^{-3} \text{ J} .$$

84. (a) The magnitude of the average induced emf is

$$\mathcal{E}_{\text{avg}} = \left|\frac{-d\Phi_B}{dt}\right| = \left|\frac{\Delta\Phi_B}{\Delta t}\right| = \frac{BA_i}{t} = \frac{(2.0\,\text{T})(0.20\,\text{m})^2}{0.20\,\text{s}} = -0.40 \text{ V} .$$

(b) The average induced current is

$$i_{\text{avg}} = \frac{\mathcal{E}_{\text{avg}}}{R} = \frac{0.40\,\text{V}}{20 \times 10^{-3}\,\Omega} = 20 \text{ A} .$$

85. (a) As the switch closes at $t = 0$, the current being zero in the inductor serves as an initial condition for the building-up of current in the circuit. Thus, at $t = 0$ any current through the battery is also that through the 20 Ω and 10 Ω resistors. Hence,

$$i = \frac{\mathcal{E}}{30\ \Omega} = 0.40 \text{ A}$$

which results in a voltage drop across the 10 Ω resistor equal to (0.40)(10) = 4.0 V. The inductor must have this same voltage across it $|\mathcal{E}_L|$, and we use (the absolute value of) Eq. 31-37:

$$\frac{di}{dt} = \frac{|\mathcal{E}_L|}{L} = \frac{4.0}{0.010} = 400 \text{ A/s} .$$

(b) Applying the loop rule to the outer loop, we have

$$\mathcal{E} - (0.50 \text{ A})(20\ \Omega) - |\mathcal{E}_L| = 0 \quad .$$

Therefore, $|\mathcal{E}_L| = 2.0$ V, and Eq. 31-37 leads to

$$\frac{di}{dt} = \frac{|\mathcal{E}_L|}{L} = \frac{2.0}{0.010} = 200 \text{ A/s} .$$

(c) As $t \to \infty$, the inductor has $\mathcal{E}_L = 0$ (since the current is no longer changing). Thus, the loop rule (for the outer loop) leads to

$$\mathcal{E} - i\,(20\ \Omega) - |\mathcal{E}_L| = 0 \quad\Longrightarrow\quad i = 0.60 \text{ A} .$$

86. (a) $L = \Phi/i = 26 \times 10^{-3}\,\text{Wb}/5.5\,\text{A} = 4.7 \times 10^{-3}\,\text{H}$.

(b) We use Eq. 31-43 to solve for t:

$$\begin{aligned} t &= -\tau_L \ln\left(1 - \frac{iR}{\mathcal{E}}\right) = -\frac{L}{R}\ln\left(1 - \frac{iR}{\mathcal{E}}\right) \\ &= -\frac{4.7 \times 10^{-3}\,\text{H}}{0.75\,\Omega}\ln\left[1 - \frac{(2.5\,\text{A})(0.75\,\Omega)}{6.0\,\text{V}}\right] = 2.4 \times 10^{-3} \text{ s} . \end{aligned}$$

87. (a) We use $U_B = \frac{1}{2}Li^2$ to solve for the self-inductance:

$$L = \frac{2U_B}{i^2} = \frac{2(25.0 \times 10^{-3}\,\text{J})}{(60.0 \times 10^{-3}\,\text{A})^2} = 13.9 \text{ H} .$$

(b) Since $U_B \propto i^2$, for U_B to increase by a factor of 4, i must increase by a factor of 2. Therefore, i should be increased to $2(60.0\,\text{mA}) = 120\,\text{mA}$.

88. (a) The self-inductance per meter is

$$\frac{L}{\ell} = \mu_0 n^2 A = \left(4\pi \times 10^{-7}\,\text{H/m}\right)(100\,\text{turns/cm})^2(\pi)(1.6\,\text{cm})^2 = 0.10\ \text{H/m}\ .$$

(b) The induced emf per meter is

$$\frac{\mathcal{E}}{\ell} = \frac{L}{\ell}\frac{di}{dt} = (0.10\,\text{H/m})(13\,\text{A/s}) = 1.3\ \text{V/m}\ .$$

89. (a) The energy needed is

$$U_E = u_E V = \frac{1}{2}\epsilon_0 E^2 V = \frac{1}{2}(8.85 \times 10^{-12}\,\text{F/m})(100\,\text{kV/m})^2(10\,\text{cm})^3 = 4.4 \times 10^{-5}\ \text{J}\ .$$

(b) The energy needed is

$$U_B = u_B V = \frac{1}{2\mu_0}B^2 V = \frac{(1.0\,\text{T})^2}{2\left(4\pi \times 10^{-7}\,\text{H/m}\right)}(10\,\text{cm})^3 = 4.0 \times 10^2\ \text{J}\ .$$

(c) Obviously, since $U_B > U_E$ greater amounts of energy can be stored in the magnetic field.

90. The induced electric field E as a function of r is given by $E(r) = (r/2)(dB/dt)$. So

$$\begin{aligned} a_c &= a_a = \frac{eE}{m} = \frac{er}{2m}\left(\frac{dB}{dt}\right) \\ &= \frac{(1.60 \times 10^{-19}\,\text{C})(5.0 \times 10^{-2}\,\text{m})(10 \times 10^{-3}\,\text{T/s})}{2(9.11 \times 10^{-27}\,\text{kg})} = 4.4 \times 10^7\ \text{m/s}^2\ . \end{aligned}$$

With regard to the directions, $\vec{a}_a$ points to the right and $\vec{a}_c$ points to the left. At point b we have $a_b \propto r_b = 0$.

91. Using Eq. 31-43, we find

$$i = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau_L}\right) \implies \tau_L = \frac{t}{\ln\left(\frac{1}{1-\frac{iR}{\mathcal{E}}}\right)} = 22.4\ \text{s}\ .$$

Thus, from Eq. 31-44 (the definition of the time constant), we obtain $L = (22.4\,\text{s})(2.0\,\Omega) = 45$ H.

92. (a) As the switch closes at $t = 0$, the current being zero in the inductors serves as an initial condition for the building-up of current in the circuit. Thus, the current through any element of this circuit is also zero at that instant. Consequently, the loop rule requires the emf ($\mathcal{E}_{L1}$) of the $L_1 = 0.30$ H inductor to cancel that of the battery. We now apply (the absolute value of) Eq. 31-37

$$\frac{di}{dt} = \frac{|\mathcal{E}_{L1}|}{L_1} = \frac{6.0}{0.30} = 20\ \text{A/s}\ .$$

(b) What is being asked for is essentially the current in the battery when the emf's of the inductors vanish (as $t \to \infty$). Applying the loop rule to the outer loop, with $R_1 = 8.0\ \Omega$, we have

$$\mathcal{E} - i\,R_1 - |\mathcal{E}_{L1}| - |\mathcal{E}_{L2}| = 0 \implies i = \frac{6.0\,\text{V}}{R_1} = 0.75\ \text{A}\ .$$

93. The magnetic flux is

$$\Phi_B = \vec{B}\cdot\vec{A} = BA\cos 57^\circ = \left(4.2\times 10^{-6}\,\mathrm{T}\right)\left(2.5\,\mathrm{m}^2\right)\cos 57^\circ = 5.7\times 10^{-5}\ \mathrm{Wb}\ .$$

94. From the given information, we find

$$\frac{dB}{dt} = \frac{0.030\ \mathrm{T}}{0.015\ \mathrm{s}} = 2.0\ \mathrm{T/s}\ \ .$$

Thus, with $N = 1$ and $\cos 30^\circ = \sqrt{3}/2$, and using Faraday's law with Ohm's law, we have

$$i = \frac{|\mathcal{E}|}{R} = \frac{N\pi r^2}{R}\,\frac{\sqrt{3}}{2}\,\frac{dB}{dt} = 0.021\ \mathrm{A}\ .$$

95. Before the fuse blows, the current through the resistor remains zero. We apply the loop theorem to the battery-fuse-inductor loop: $\mathcal{E} - L\,di/dt = 0$. So $i = \mathcal{E}t/L$. As the fuse blows at $t = t_0$, $i = i_0 = 3.0\,\mathrm{A}$. Thus,

$$t_0 = \frac{i_0 L}{\mathcal{E}} = \frac{(3.0\,\mathrm{A})(5.0\,\mathrm{H})}{10\,\mathrm{V}} = 1.5\ \mathrm{s}\ .$$

We do not show the graph here; qualitatively, it would be similar to Fig. 31-14.

96. We write (as functions of time) $V_L(t) = \mathcal{E}e^{-t/\tau_l}$. Considering the first two data points, (V_{L1}, t_1) and (V_{L2}, t_2), satisfying $V_{Li} = \mathcal{E}e^{-t_i/\tau_L}$ $(i = 1, 2)$, we have $V_{L1}/V_{L2} = \mathcal{E}e^{-(t_1-t_2)/\tau_L}$, which gives

$$\tau_L = \frac{t_1 - t_2}{\ln(V_2/V_1)} = \frac{1.0\,\mathrm{ms} - 2.0\,\mathrm{ms}}{\ln(13.8/18.2)} = 3.6\,\mathrm{ms}\ .$$

Therefore, $\mathcal{E} = V_{L1}e^{t_1/\tau_L} = (18.2\,\mathrm{V})e^{1.0\,\mathrm{ms}/3.6\,\mathrm{ms}} = 24\,\mathrm{V}$. One can easily check that the values of τ_L and $\mathcal{E}$ are consistent with the rest of the data points.

97. (a) The energy density is

$$u_B = \frac{B_e^2}{2\mu_0} = \frac{(50\times 10^{-6}\,\mathrm{T})^2}{2\,(4\pi\times 10^{-7}\,\mathrm{H/m})} = 1.0\times 10^{-3}\ \mathrm{J/m^3}\ .$$

(b) The volume of the shell of thickness h is $\mathcal{V} \approx 4\pi R_e^2 h$, where R_e is the radius of the Earth. So

$$U_B \approx \mathcal{V}u_B \approx 4\pi(6.4\times 10^6\,\mathrm{m})^2(16\times 10^3\,\mathrm{m})(1.0\times 10^{-3}\,\mathrm{J/m^3}) = 8.4\times 10^{15}\ \mathrm{J}\ .$$

98. (a) $N = 2.0\,\mathrm{m}/2.5\,\mathrm{mm} = 800$.

(b) $L/l = \mu_0 n^2 A = \left(4\pi\times 10^{-7}\,\mathrm{H/m}\right)(800/2.0\,\mathrm{m})^2(\pi)(0.040\,\mathrm{m})^2/4 = 2.5\times 10^{-4}\,\mathrm{H}$.

99. The self-inductance and resistance of the coil may be treated as a "pure" inductor in series with a "pure" resistor, in which case the situation described in the problem is equivalent to that analyzed in §31-9 with solution Eq. 31-43. The derivative of that solution is

$$\frac{di}{dt} = \frac{\mathcal{E}}{R\,\tau_L}\,e^{-t/\tau_L} = \frac{\mathcal{E}}{L}\,e^{-t/\tau_L}\quad .$$

With $\tau_L = 0.28$ ms (by Eq. 31-44), $L = 0.050$ H and $\mathcal{E} = 45$ V, we obtain $di/dt = 12$ A/s when $t = 1.2$ ms.

100. (a) We apply Newton's second law to the rod

$$m\frac{dv}{dt} = iBL\ ,$$

and integrate to obtain

$$v = \frac{iBLt}{m}\ .$$

The velocity $\vec{v}$ points away from the generator G.

(b) When the current i in the rod becomes zero, the rod will no longer be accelerated by a force $F = iBL$ and will therefore reach a constant terminal velocity. This occurs when $|\mathcal{E}_{\text{induced}}| = \mathcal{E}$. Specifically,

$$|\mathcal{E}_{\text{induced}}| = \left|\frac{d\Phi_B}{dt}\right| = \left|\frac{d(BA)}{dt}\right| = B\left|\frac{dA}{dt}\right| = BvL = \mathcal{E} \ .$$

Thus, $\vec{v} = \mathcal{E}/BL$, leftward.

(c) In case (a) electric energy is supplied by the generator and is transferred into the kinetic energy of the rod. In the case considered now the battery initially supplies electric energy to the rod, causing its kinetic energy to increase to a maximum value of $\frac{1}{2}mv^2 = \frac{1}{2}(\mathcal{E}/BL)^2$. Afterwards, there is no further energy transfer from the battery to the rod, and the kinetic energy of the rod remains constant.

101. (a) At $t = 0.50$ s and $t = 1.5$ s, the magnetic field is decreasing at a rate of 3/2 mT/s, leading to

$$i = \frac{|\mathcal{E}|}{R} = \frac{A\,|dB/dt|}{R} = \frac{(3.0)(3/2)}{9.0} = 0.50 \text{ mA}$$

with a counterclockwise sense (by Lenz's law).

(b) See the results of part (a).

(c) and (d) For $t > 2.0$ s, there is no change in flux and therefore no induced current.

102. The magnetic flux is

$$\begin{aligned}\Phi_B &= BA = \left(\frac{\mu_0 i_0 N}{2\pi r}\right) A \\ &= \frac{\left(4\pi \times 10^{-7}\,\text{H/m}\right)(0.800\,\text{A})(500)(5.00 \times 10^{-2}\,\text{m})^2}{2\pi(0.150\,\text{m} + 0.0500\,\text{m}/2)} \\ &= 1.15 \times 10^{-6} \text{ Wb} \ .\end{aligned}$$

103. (a) As the switch closes at $t = 0$, the current being zero in the inductor serves as an initial condition for the building-up of current in the circuit. Thus, at $t = 0$ the current through the battery is also zero.

(b) With no current anywhere in the circuit at $t = 0$, the loop rule requires the emf of the inductor $\mathcal{E}_L$ to cancel that of the battery ($\mathcal{E} = 40$ V). Thus, the absolute value of Eq. 31-37 yields

$$\frac{di}{dt} = \frac{|\mathcal{E}_L|}{L} = \frac{40}{0.050} = 800 \text{ A/s} \ .$$

(c) This circuit becomes equivalent to that analyzed in §31-9 when we replace the parallel set of 20000 Ω resistors with $R = 10000\ \Omega$. Now, with $\tau_L = L/R = 5 \times 10^{-6}$ s, we have $t/\tau_L = 3/5$, and we apply Eq. 31-43:

$$i = \frac{\mathcal{E}}{R}\left(1 - e^{-3/5}\right) \approx 1.8 \times 10^{-3} \text{ A} \ .$$

(d) The rate of change of the current is figured from the loop rule (and Eq. 31-37):

$$\mathcal{E} - i\,R - |\mathcal{E}_L| = 0 \ .$$

Using the values from part (c), we obtain $|\mathcal{E}_L| \approx 22$ V. Then,

$$\frac{di}{dt} = \frac{|\mathcal{E}_L|}{L} = \frac{22}{0.050} \approx 440 \text{ A/s} \ .$$

(e) and (f) As $t \to \infty$, the circuit reaches a steady state condition, so that $di/dt = 0$ and $\mathcal{E}_L = 0$. The loop rule then leads to

$$\mathcal{E} - i\,R - |\mathcal{E}_L| = 0 \implies i = \frac{40}{10000} = 4.0 \times 10^{-3}\ \text{A} .$$

104. The magnetic flux Φ_B through the loop is given by $\Phi_B = 2B(\pi r^2/2)(\cos 45°) = \pi r^2 B/\sqrt{2}$. Thus

$$\begin{aligned} \mathcal{E} &= -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\left(\frac{\pi r^2 B}{\sqrt{2}}\right) = -\frac{\pi r^2}{\sqrt{2}}\left(\frac{\Delta B}{\Delta t}\right) \\ &= -\frac{\pi(3.7 \times 10^{-2}\,\text{m})^2}{\sqrt{2}}\left(\frac{0 - 76 \times 10^{-3}\,\text{T}}{4.5 \times 10^{-3}\,\text{s}}\right) \\ &= 5.1 \times 10^{-2}\ \text{V} . \end{aligned}$$

The direction of the induced current is clockwise when viewed along the direction of $\vec{B}$.

105. The area enclosed by any turn of the coil is πr^2 where $r = 0.15$ m, and the coil has $N = 50$ turns. Thus, the magnitude of the induced emf, using Eq. 31-7, is

$$|\mathcal{E}| = N\pi r^2 \left|\frac{dB}{dt}\right| = \left(3.53\ \text{m}^2\right)\left|\frac{dB}{dt}\right|$$

where $\left|\frac{dB}{dt}\right| = (0.0126\ \text{T/s})|\cos\omega t|$. Thus, using Ohm's law, we have

$$i = \frac{|\mathcal{E}|}{R} = \frac{(3.53)(0.0126)}{4.0}\,|\cos\omega t| \quad .$$

When $t = 0.020$ s, this yields $i = 0.011$ A.

106. (First problem of **Cluster**)
Combining Ohm's and Faraday's laws, the current magnitude is

$$i = \frac{|\mathcal{E}|}{R} = \frac{BLv}{R}$$

for this "one-loop" circuit, where the version of Faraday's law expressed in Eq. 31-10 (often called "motional emf") has been used. Here, $B = |\vec{B}| = 0.200$ T, $L = 0.300$ m and $v = 12.0$ m/s. Reasoning with Lenz's law, the sense of the induced current is *counterclockwise* (to produce field in its interior out of the page, "fighting" the increasing inward pointed flux due to the applied field).

(a) With $R = 5.00\ \Omega$, this yields $i = 0.144$ A.

(b) With $R = 7.00\ \Omega$, we obtain $i = 0.103$ A.

107. (Second problem of **Cluster**)

(a) With $L = 0.50$ m and $R = 5.00\ \Omega$, we combine Ohm's and Faraday's laws, so that the current magnitude is

$$i = \frac{|\mathcal{E}|}{R} = \frac{BLv}{R} = 0.240\ \text{A} \quad .$$

The direction is counterclockwise, as explained in the solution to the previous problem.

(b) The area in the loop is $A = \frac{1}{2}(L_0 + L)x$ where $x = vt$ and $L_0 = 0.300$ m. But the value of L depends on the distance from the resistor x:

$$\begin{aligned} L &= 30\ \text{cm} + \left(\frac{20\ \text{cm}}{1\ \text{m}}\right)x \\ &= L_0 + 0.200(vt) \end{aligned}$$

where $x = vt$ has been used. Therefore, the area becomes

$$A = L_0\, vt + 0.100\, v^2 t^2 \quad .$$

The induced emf is, from Faraday's law,

$$\mathcal{E} = \frac{d\Phi}{dt} = B\,\frac{dA}{dt} = B\left(L_0\, v + 2(0.100)v^2 t\right)$$

and the induced current is

$$i = \frac{\mathcal{E}}{R} = 0.144 + 1.152t$$

in SI units and is counterclockwise (for reasons given in previous solution).

108. (Third problem of **Cluster**)

(a) , (b) and (c) The area enclosed by the loop is that of a rectangle with one side (x) expanding. With $B_0 = 0.200$ T and $\xi = 0.050$ T/s (the rate of field increase), we have

$$\begin{aligned} \Phi &= BA = (B_0 + \xi t)\,(Lx) \\ &= B_0 Lvt + \xi Lvt^2 \end{aligned}$$

where $x = vt$ has been used. Thus, from Faraday's and Ohm's laws, the induced current is

$$i = \frac{\mathcal{E}}{R} = \frac{B_0 Lv}{R} + 2\frac{\xi Lv}{R}\,t$$

and is counterclockwise (to produce field in the loop's interior pointing out of the page, "fighting" the increasing inward pointed flux due to the applied field). Therefore, the current at $t = 0$ is $B_0 Lv/R = 0.144$ A. And its value at $t = 1.00$ s is $(B_0 + 2\xi)Lv/R = 0.216$ A.