Chapter 34

1. The time for light to travel a distance d in free space is $t = d/c$, where c is the speed of light (3.00×10^8 m/s).

   (a) We take d to be 150 km $= 150 \times 10^3$ m. Then,

   $$t = \frac{d}{c} = \frac{150 \times 10^3\,\mathrm{m}}{3.00 \times 10^8\,\mathrm{m/s}} = 5.00 \times 10^{-4}\ \mathrm{s}\ .$$

   (b) At full moon, the Moon and Sun are on opposite sides of Earth, so the distance traveled by the light is $d = (1.5 \times 10^8\,\mathrm{km}) + 2(3.8 \times 10^5\,\mathrm{km}) = 1.51 \times 10^8\,\mathrm{km} = 1.51 \times 10^{11}\,\mathrm{m}$. The time taken by light to travel this distance is

   $$t = \frac{d}{c} = \frac{1.51 \times 10^{11}\,\mathrm{m}}{3.00 \times 10^8\,\mathrm{m/s}} = 500\,\mathrm{s} = 8.4\ \mathrm{min}\ .$$

   (c) We take d to be $2(1.3 \times 10^9\,\mathrm{km}) = 2.6 \times 10^{12}$ m. Then,

   $$t = \frac{d}{c} = \frac{2.6 \times 10^{12}\,\mathrm{m}}{3.00 \times 10^8\,\mathrm{m/s}} = 8.7 \times 10^3\,\mathrm{s} = 2.4\ \mathrm{h}\ .$$

   (d) We take d to be 6500 ly and the speed of light to be 1.00 ly/y. Then,

   $$t = \frac{d}{c} = \frac{6500\,\mathrm{ly}}{1.00\,\mathrm{ly/y}} = 6500\,\mathrm{y}\ .$$

   The explosion took place in the year $1054 - 6500 = -5446$ or 5446 BCE.

2. (a)

   $$f = \frac{c}{\lambda} = \frac{3.0 \times 10^8\,\mathrm{m/s}}{(1.0 \times 10^5)(6.4 \times 10^6\,\mathrm{m})} = 4.7 \times 10^{-3}\ \mathrm{Hz}\ .$$

   (b)

   $$T = \frac{1}{f} = \frac{1}{4.7 \times 10^{-3}\,\mathrm{Hz}} = 212\,\mathrm{s} = 3\,\mathrm{min}\ 32\,\mathrm{s}\ .$$

3. (a) From Fig. 34-2 we find the wavelengths in question to be about 515 nm and 610 nm.

   (b) Again from Fig. 34-2 the wavelength is about 555 nm. Therefore,

   $$f = \frac{c}{\lambda} = \frac{3.00 \times 10^8\,\mathrm{m/s}}{555\,\mathrm{nm}} = 5.41 \times 10^{14}\ \mathrm{Hz}\ ,$$

   and the period is $(5.41 \times 10^{14}\,\mathrm{Hz})^{-1} = 1.85 \times 10^{-15}$ s.

4. Since $\Delta\lambda \ll \lambda$, we find Δf is equal to

$$\left|\Delta\left(\frac{c}{\lambda}\right)\right| \approx \frac{c\Delta\lambda}{\lambda^2} = \frac{(3.0\times 10^8\,\mathrm{m/s})(0.0100\times 10^{-9}\,\mathrm{m})}{(632.8\times 10^{-9}\,\mathrm{m})^2} = 7.49\times 10^9\ \mathrm{Hz}\ .$$

5. (a) Suppose that at time t_1, the moon is starting a revolution (on the verge of going behind Jupiter, say) and that at this instant, the distance between Jupiter and Earth is ℓ_1. The time of the start of the revolution as seen on Earth is $t_1^* = t_1 + \ell_1/c$. Suppose the moon starts the next revolution at time t_2 and at that instant, the Earth-Jupiter distance is ℓ_2. The start of the revolution as seen on Earth is $t_2^* = t_2 + \ell_2/c$. Now, the actual period of the moon is given by $T = t_2 - t_1$ and the period as measured on Earth is

$$T^* = t_2^* - t_1^* = t_2 - t_1 + \frac{\ell_2}{c} - \frac{\ell_1}{c} = T + \frac{\ell_2 - \ell_1}{c}\ .$$

The period as measured on Earth is longer than the actual period. This is due to the fact that Earth moves during a revolution, and light takes a finite time to travel from Jupiter to Earth. For the situation depicted in Fig. 34-38, light emitted at the end of a revolution travels a longer distance to get to Earth than light emitted at the beginning. Suppose the position of Earth is given by the angle θ, measured from x. Let R be the radius of Earth's orbit and d be the distance from the Sun to Jupiter. The law of cosines, applied to the triangle with the Sun, Earth, and Jupiter at the vertices, yields $\ell^2 = d^2 + R^2 - 2dR\cos\theta$. This expression can be used to calculate ℓ_1 and ℓ_2. Since Earth does not move very far during one revolution of the moon, we may approximate $\ell_2 - \ell_1$ by $(d\ell/dt)T$ and T^* by $T + (d\ell/dt)(T/c)$. Now

$$\frac{d\ell}{dt} = \frac{2Rd\sin\theta}{\sqrt{d^2+R^2-2dR\cos\theta}}\,\frac{d\theta}{dt} = \frac{2vd\sin\theta}{\sqrt{d^2+R^2-2dR\cos\theta}}\ ,$$

where $v = R\,(d\theta/dt)$ is the speed of Earth in its orbit. For $\theta = 0$, $(d\ell/dt) = 0$ and $T^* = T$. Since Earth is then moving perpendicularly to the line from the Sun to Jupiter, its distance from the planet does not change appreciably during one revolution of the moon. On the other hand, when $\theta = 90°$, $d\ell/dt = vd/\sqrt{d^2+R^2}$ and

$$T^* = T\left(1 + \frac{vd}{c\sqrt{d^2+R^2}}\right)\ .$$

The Earth is now moving parallel to the line from the Sun to Jupiter, and its distance from the planet changes during a revolution of the moon.

(b) Our notation is as follows: t is the actual time for the moon to make N revolutions, and t^* is the time for N revolutions to be observed on Earth. Then,

$$t^* = t + \frac{\ell_2 - \ell_1}{c}\ ,$$

where ℓ_1 is the Earth-Jupiter distance at the beginning of the interval and ℓ_2 is the Earth-Jupiter distance at the end. Suppose Earth is at position x at the beginning of the interval, and at y at the end. Then, $\ell_1 = d - R$ and $\ell_2 = \sqrt{d^2+R^2}$. Thus,

$$t^* = t + \frac{\sqrt{d^2+R^2} - (d-R)}{c}\ .$$

A value can be found for t by measuring the observed period of revolution when Earth is at x and multiplying by N. We note that the observed period is the true period when Earth is at x. The time interval as Earth moves from x to y is t^*. The difference is

$$t^* - t = \frac{\sqrt{d^2+R^2} - (d-R)}{c}\ .$$

If the radii of the orbits of Jupiter and Earth are known, the value for $t^* - t$ can be used to compute c. Since Jupiter is much further from the Sun than Earth, $\sqrt{d^2 + R^2}$ may be approximated by d and $t^* - t$ may be approximated by R/c. In this approximation, only the radius of Earth's orbit need be known.

6. The emitted wavelength is

$$\begin{aligned}\lambda &= \frac{c}{f} = 2\pi c\sqrt{LC} \\ &= 2\pi(2.998 \times 10^8\,\text{m/s})\sqrt{(0.253 \times 10^{-6}\,\text{H})(25.0 \times 10^{-12}\,\text{F})} = 4.74\ \text{m}\ .\end{aligned}$$

7. If f is the frequency and λ is the wavelength of an electromagnetic wave, then $f\lambda = c$. The frequency is the same as the frequency of oscillation of the current in the LC circuit of the generator. That is, $f = 1/2\pi\sqrt{LC}$, where C is the capacitance and L is the inductance. Thus

$$\frac{\lambda}{2\pi\sqrt{LC}} = c\ .$$

The solution for L is

$$L = \frac{\lambda^2}{4\pi^2 C c^2} = \frac{(550 \times 10^{-9}\,\text{m})^2}{4\pi^2(17 \times 10^{-12}\,\text{F})(2.998 \times 10^8\,\text{m/s})^2} = 5.00 \times 10^{-21}\ \text{H}\ .$$

This is exceedingly small.

8. The amplitude of the magnetic field in the wave is

$$B_m = \frac{E_m}{c} = \frac{3.20 \times 10^{-4}\,\text{V/m}}{2.998 \times 10^8\,\text{m/s}} = 1.07 \times 10^{-12}\ \text{T}\ .$$

9. Since the $\vec{E}$-wave oscillates in the z direction and travels in the x direction, we have $B_x = B_z = 0$. With SI units understood, we find

$$\begin{aligned}B_y &= B_\text{m}\cos\left[\pi \times 10^{15}\left(t - \frac{x}{c}\right)\right] = \frac{2.0\cos[10^{15}\pi(t - x/c)]}{3.0 \times 10^8} \\ &= (6.7 \times 10^{-9})\cos\left[10^{15}\pi\left(t - \frac{x}{c}\right)\right]\end{aligned}$$

10. Using $\vec{S} = (1/\mu_0)\vec{E} \times \vec{B}$, we see that (on the right hand) letting the thumb be in the $\vec{E}$ direction and the index finger be in the $\vec{B}$ direction means that the middle finger (held perpendicular to the other two, making a "triad" of the thumb and two fingers) points in the direction of wave propagation (the direction of $\vec{S}$). Holding the right hand in this manner can facilitate checking the directions in the Figures. A more algebraic approach is to note that $\hat{\jmath} \times \hat{k} = \hat{\imath}$. This is especially useful for checking Figures 34-6 and 34-7.

11. If P is the power and Δt is the time interval of one pulse, then the energy in a pulse is

$$E = P\,\Delta t = (100 \times 10^{12}\,\text{W})(1.0 \times 10^{-9}\,\text{s}) = 1.0 \times 10^5\ \text{J}\ .$$

12. The intensity of the signal at Proxima Centauri is

$$I = \frac{P}{4\pi r^2} = \frac{1.0 \times 10^6\,\text{W}}{4\pi[(4.3\,\text{ly})(9.46 \times 10^{15}\,\text{m/ly})]^2} = 4.8 \times 10^{-29}\ \text{W/m}^2\ .$$

13. The region illuminated on the Moon is a circle with radius $R = r\theta/2$, where r is the Earth-Moon distance (3.82×10^8 m) and θ is the full-angle beam divergence in radians. The area A illuminated is

$$A = \pi R^2 = \frac{\pi r^2\theta^2}{4} = \frac{\pi(3.82 \times 10^8\,\mathrm{m})^2(0.880 \times 10^{-6}\,\mathrm{rad})^2}{4} = 8.88 \times 10^4\ \mathrm{m}^2\ .$$

14. The intensity is the average of the Poynting vector:

$$I = S_{\mathrm{avg}} = \frac{cB_m^2}{2\mu_0} = \frac{(3.0 \times 10^8\,\mathrm{m/s})(1.0 \times 10^{-4}\,\mathrm{T})^2}{2(1.26 \times 10^{-6}\,\mathrm{H/m})^2} = 1.2 \times 10^6\ \mathrm{W/m^2}\ .$$

15. (a) The amplitude of the magnetic field in the wave is

$$B_m = \frac{E_m}{c} = \frac{5.00\,\mathrm{V/m}}{2.998 \times 10^8\,\mathrm{m/s}} = 1.67 \times 10^{-8}\ \mathrm{T}\ .$$

(b) The intensity is the average of the Poynting vector:

$$I = S_{\mathrm{avg}} = \frac{E_m^2}{2\mu_0 c} = \frac{(5.00\,\mathrm{V/m})^2}{2(4\pi \times 10^{-7}\,\mathrm{T{\cdot}m/A})(2.998 \times 10^8\,\mathrm{m/s})} = 3.31 \times 10^{-2}\ \mathrm{W/m^2}\ .$$

16. We use $I = E_m^2/2\mu_0 c$ to calculate E_{m}:

$$\begin{aligned} E_m &= \sqrt{2\mu_0 I c} = \sqrt{2(4\pi \times 10^{-7}\,\mathrm{T{\cdot}m/A})(1.40 \times 10^3\,\mathrm{W/m^2})(2.998 \times 10^8\,\mathrm{m/s})} \\ &= 1.03 \times 10^3\ \mathrm{V/m}\ . \end{aligned}$$

The magnetic field amplitude is therefore

$$B_m = \frac{E_m}{c} = \frac{1.03 \times 10^4\,\mathrm{V/m}}{2.998 \times 10^8\,\mathrm{m/s}} = 3.43 \times 10^{-6}\ \mathrm{T}\ .$$

17. (a) The magnetic field amplitude of the wave is

$$B_m = \frac{E_m}{c} = \frac{2.0\,\mathrm{V/m}}{2.998 \times 10^8\,\mathrm{m/s}} = 6.7 \times 10^{-9}\ \mathrm{T}\ .$$

(b) The intensity is

$$I = \frac{E_m^2}{2\mu_0 c} = \frac{(2.0\,\mathrm{V/m})^2}{2(4\pi \times 10^{-7}\,\mathrm{T{\cdot}m/A})(2.998 \times 10^8\,\mathrm{m/s})} = 5.3 \times 10^{-3}\ \mathrm{W/m^2}\ .$$

(c) The power of the source is

$$P = 4\pi r^2 I_{\mathrm{avg}} = 4\pi(10\,\mathrm{m})^2(5.3 \times 10^{-3}\,\mathrm{W/m^2}) = 6.7\ \mathrm{W}\ .$$

18. (a) The power received is

$$P_r = (1.0 \times 10^{-12}\,\mathrm{W})\frac{\pi[(1000\,\mathrm{ft})(0.3048\,\mathrm{m/ft})]^2/4}{4\pi(6.37 \times 10^6\,\mathrm{m})^2} = 1.4 \times 10^{-22}\ \mathrm{W}\ .$$

(b) The power of the source would be

$$P = 4\pi r^2 I = 4\pi[(2.2 \times 10^4\,\mathrm{ly})(9.46 \times 10^{15}\,\mathrm{m/ly})]^2\left[\frac{1.0 \times 10^{-12}\,\mathrm{W}}{4\pi(6.37 \times 10^6\,\mathrm{m})^2}\right] = 1.1 \times 10^{15}\ \mathrm{W}\ .$$

19. (a) The average rate of energy flow per unit area, or intensity, is related to the electric field amplitude E_m by $I = E_m^2/2\mu_0 c$, so

$$\begin{aligned} E_m &= \sqrt{2\mu_0 c I} = \sqrt{2(4\pi \times 10^{-7}\,\text{H/m})(2.998 \times 10^8\,\text{m/s})(10 \times 10^{-6}\,\text{W/m}^2)} \\ &= 8.7 \times 10^{-2}\ \text{V/m}\ . \end{aligned}$$

(b) The amplitude of the magnetic field is given by

$$B_m = \frac{E_m}{c} = \frac{8.7 \times 10^{-2}\,\text{V/m}}{2.998 \times 10^8\,\text{m/s}} = 2.9 \times 10^{-10}\ \text{T}\ .$$

(c) At a distance r from the transmitter, the intensity is $I = P/4\pi r^2$, where P is the power of the transmitter. Thus

$$P = 4\pi r^2 I = 4\pi(10 \times 10^3\,\text{m})^2(10 \times 10^{-6}\,\text{W/m}^2) = 1.3 \times 10^4\ \text{W}\ .$$

20. The radiation pressure is

$$p_r = \frac{I}{c} = \frac{10\,\text{W/m}^2}{2.998 \times 10^8\,\text{m/s}} = 3.3 \times 10^{-8}\ \text{Pa}\ .$$

21. The plasma completely reflects all the energy incident on it, so the radiation pressure is given by $p_r = 2I/c$, where I is the intensity. The intensity is $I = P/A$, where P is the power and A is the area intercepted by the radiation. Thus

$$p_r = \frac{2P}{Ac} = \frac{2(1.5 \times 10^9\,\text{W})}{(1.00 \times 10^{-6}\,\text{m}^2)(2.998 \times 10^8\,\text{m/s})} = 1.0 \times 10^7\,\text{Pa} = 10\ \text{MPa}\ .$$

22. (a) The radiation pressure produces a force equal to

$$\begin{aligned} F_r &= p_r(\pi R_e^2) = \left(\frac{I}{c}\right)(\pi R_e^2) \\ &= \frac{\pi(1.4 \times 10^3\,\text{W/m}^2)(6.37 \times 10^6\,\text{m})^2}{2.998 \times 10^8\,\text{m/s}} = 6.0 \times 10^8\ \text{N}\ . \end{aligned}$$

(b) The gravitational pull of the Sun on Earth is

$$\begin{aligned} F_{\text{grav}} &= \frac{GM_s M_e}{d_{es}^2} \\ &= \frac{(6.67 \times 10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2)(2.0 \times 10^{30}\,\text{kg})(5.98 \times 10^{24}\,\text{kg})}{(1.5 \times 10^{11}\,\text{m})^2} \\ &= 3.6 \times 10^{22}\ \text{N}\ , \end{aligned}$$

which is much greater than F_r.

23. Since the surface is perfectly absorbing, the radiation pressure is given by $p_r = I/c$, where I is the intensity. Since the bulb radiates uniformly in all directions, the intensity a distance r from it is given by $I = P/4\pi r^2$, where P is the power of the bulb. Thus

$$p_r = \frac{P}{4\pi r^2 c} = \frac{500\,\text{W}}{4\pi(1.5\,\text{m})^2(2.998 \times 10^8\,\text{m/s})} = 5.9 \times 10^{-8}\ \text{Pa}\ .$$

24. (a) We note that the cross section area of the beam is $\pi d^2/4$, where d is the diameter of the spot ($d = 2.00\lambda$). The beam intensity is

$$I = \frac{P}{\pi d^2/4} = \frac{5.00 \times 10^{-3}\,\text{W}}{\pi[(2.00)(633 \times 10^{-9}\,\text{m})]^2/4} = 3.97 \times 10^9 \text{ W/m}^2 .$$

(b) The radiation pressure is

$$p_r = \frac{I}{c} = \frac{3.97 \times 10^9\,\text{W/m}^2}{2.998 \times 10^8\,\text{m/s}} = 13.2 \text{ Pa} .$$

(c) In computing the corresponding force, we can use the power and intensity to eliminate the area (mentioned in part (a)). We obtain

$$F_r = \left(\frac{\pi d^2}{4}\right) p_r = \left(\frac{P}{I}\right) p_r = \frac{(5.00 \times 10^{-3}\,\text{W})(13.2\,\text{Pa})}{3.97 \times 10^9\,\text{W/m}^2} = 1.67 \times 10^{-11} \text{ N} .$$

(d) The acceleration of the sphere is

$$\begin{aligned} a &= \frac{F_r}{m} = \frac{F_r}{\rho(\pi d^3/6)} = \frac{6(1.67 \times 10^{-11}\,\text{N})}{\pi(5.00 \times 10^3\,\text{kg/m}^3)[(2.00)(633 \times 10^{-9}\,\text{m})]^3} \\ &= 3.14 \times 10^3 \text{ m/s}^2 . \end{aligned}$$

25. (a) Since $c = \lambda f$, where λ is the wavelength and f is the frequency of the wave,

$$f = \frac{c}{\lambda} = \frac{2.998 \times 10^8\,\text{m/s}}{3.0\,\text{m}} = 1.0 \times 10^8 \text{ Hz} .$$

(b) The magnetic field amplitude is

$$B_m = \frac{E_m}{c} = \frac{300\,\text{V/m}}{2.998 \times 10^8\,\text{m/s}} = 1.00 \times 10^{-6} \text{ T} .$$

$\vec{B}$ must be in the positive z direction when $\vec{E}$ is in the positive y direction in order for $\vec{E} \times \vec{B}$ to be in the positive x direction (the direction of propagation).

(c) The angular wave number is

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{3.0\,\text{m}} = 2.1\,\text{rad/m} .$$

The angular frequency is

$$\omega = 2\pi f = 2\pi(1.0 \times 10^8\,\text{Hz}) = 6.3 \times 10^8 \text{ rad/s} .$$

(d) The intensity of the wave is

$$I = \frac{E_m^2}{2\mu_0 c} = \frac{(300\,\text{V/m})^2}{2(4\pi \times 10^{-7}\,\text{H/m})(2.998 \times 10^8\,\text{m/s})} = 119 \text{ W/m}^2 .$$

(e) Since the sheet is perfectly absorbing, the rate per unit area with which momentum is delivered to it is I/c, so

$$\frac{dp}{dt} = \frac{IA}{c} = \frac{(119\,\text{W/m}^2)(2.0\,\text{m}^2)}{2.998 \times 10^8\,\text{m/s}} = 8.0 \times 10^{-7}\,\text{N} .$$

The radiation pressure is

$$p_r = \frac{dp/dt}{A} = \frac{8.0 \times 10^{-7}\,\text{N}}{2.0\,\text{m}^2} = 4.0 \times 10^{-7} \text{ Pa} .$$

26. The mass of the cylinder is $m = \rho(\pi d_1^2/4)H$, where d_1 is the diameter of the cylinder. Since it is in equilibrium

$$F_{\text{net}} = mg - F_r = \frac{\pi H d_1^2 g\rho}{4} - \left(\frac{\pi d_1^2}{4}\right)\left(\frac{2I}{c}\right) = 0 \ .$$

We solve for H:

$$\begin{aligned} H &= \frac{2I}{gc\rho} = \left(\frac{2P}{\pi d^2/4}\right)\frac{1}{gc\rho} \\ &= \frac{8(4.60\,\text{W})}{\pi(2.60\times10^{-3}\,\text{m})^2(9.8\,\text{m/s}^2)(3.0\times10^8\,\text{m/s})(1.20\times10^3\,\text{kg/m}^3)} \\ &= 4.91\times10^{-7}\ \text{m} \ . \end{aligned}$$

27. Let f be the fraction of the incident beam intensity that is reflected. The fraction absorbed is $1-f$. The reflected portion exerts a radiation pressure of

$$p_r = \frac{2fI_0}{c}$$

and the absorbed portion exerts a radiation pressure of

$$p_a = \frac{(1-f)I_0}{c} \ ,$$

where I_0 is the incident intensity. The factor 2 enters the first expression because the momentum of the reflected portion is reversed. The total radiation pressure is the sum of the two contributions:

$$p_{\text{total}} = p_r + p_a = \frac{2fI_0 + (1-f)I_0}{c} = \frac{(1+f)I_0}{c} \ .$$

To relate the intensity and energy density, we consider a tube with length ℓ and cross-sectional area A, lying with its axis along the propagation direction of an electromagnetic wave. The electromagnetic energy inside is $U = uA\ell$, where u is the energy density. All this energy passes through the end in time $t = \ell/c$, so the intensity is

$$I = \frac{U}{At} = \frac{uA\ell c}{A\ell} = uc \ .$$

Thus $u = I/c$. The intensity and energy density are positive, regardless of the propagation direction. For the partially reflected and partially absorbed wave, the intensity just outside the surface is $I = I_0 + fI_0 = (1+f)I_0$, where the first term is associated with the incident beam and the second is associated with the reflected beam. Consequently, the energy density is

$$u = \frac{I}{c} = \frac{(1+f)I_0}{c} \ ,$$

the same as radiation pressure.

28. We imagine the bullets (of mass m and speed v each) which will strike a surface of area A of the plane within time t to $t+\Delta t$ to be contained in a cylindrical volume at time t. Since the number of bullets contained in the cylinder is $N = n(Av\Delta t)$ and each bullet changes its momentum by $\Delta p_b = mv$, the rate of change of the total momentum for the bullets that strike the area is

$$F = \frac{\Delta P_{\text{total}}}{\Delta t} = N\frac{p_b}{\Delta t} = \frac{(Av\Delta t)nmv}{\Delta t} = Anmv^2$$

where n is the number density of the bullets (bullets per unit volume). The pressure is then

$$p_r = \frac{F}{A} = nmv^2 = 2nK \ ,$$

where $K = \frac{1}{2}mv^2$. Note that nK is the kinetic energy density. Also note that the relation between energy and momentum for a bullet is quite different from the relation between those quantities for an electromagnetic wave.

29. If the beam carries energy U away from the spaceship, then it also carries momentum $p = U/c$ away. Since the total momentum of the spaceship and light is conserved, this is the magnitude of the momentum acquired by the spaceship. If P is the power of the laser, then the energy carried away in time t is $U = Pt$. We note that there are 86400 seconds in a day. Thus, $p = Pt/c$ and, if m is mass of the spaceship, its speed is

$$v = \frac{p}{m} = \frac{Pt}{mc} = \frac{(10 \times 10^3\,\mathrm{W})(86400\,\mathrm{s})}{(1.5 \times 10^3\,\mathrm{kg})(2.998 \times 10^8\,\mathrm{m/s})} = 1.9 \times 10^{-3}\ \mathrm{m/s}\ .$$

30. We require $F_{\mathrm{grav}} = F_r$ or

$$G\frac{mM_s}{d_{es}^2} = \frac{2IA}{c}\ ,$$

and solve for the area A:

$$\begin{aligned} A &= \frac{cGmM_s}{2Id_{es}^2} = \frac{(6.67 \times 10^{-11}\,\mathrm{N\cdot m^2/kg^2})(1500\,\mathrm{kg})(1.99 \times 10^{30}\,\mathrm{kg})(2.998 \times 10^8\,\mathrm{m/s})}{2(1.40 \times 10^3\,\mathrm{W/m^2})(1.50 \times 10^{11}\,\mathrm{m})^2} \\ &= 9.5 \times 10^5\,\mathrm{m^2} = 0.95\ \mathrm{km^2}\ . \end{aligned}$$

31. (a) Let r be the radius and ρ be the density of the particle. Since its volume is $(4\pi/3)r^3$, its mass is $m = (4\pi/3)\rho r^3$. Let R be the distance from the Sun to the particle and let M be the mass of the Sun. Then, the gravitational force of attraction of the Sun on the particle has magnitude

$$F_g = \frac{GMm}{R^2} = \frac{4\pi GM\rho r^3}{3R^2}\ .$$

If P is the power output of the Sun, then at the position of the particle, the radiation intensity is $I = P/4\pi R^2$, and since the particle is perfectly absorbing, the radiation pressure on it is

$$p_r = \frac{I}{c} = \frac{P}{4\pi R^2 c}\ .$$

All of the radiation that passes through a circle of radius r and area $A = \pi r^2$, perpendicular to the direction of propagation, is absorbed by the particle, so the force of the radiation on the particle has magnitude

$$F_r = p_r A = \frac{\pi P r^2}{4\pi R^2 c} = \frac{Pr^2}{4R^2 c}\ .$$

The force is radially outward from the Sun. Notice that both the force of gravity and the force of the radiation are inversely proportional to R^2. If one of these forces is larger than the other at some distance from the Sun, then that force is larger at all distances. The two forces depend on the particle radius r differently: F_g is proportional to r^3 and F_r is proportional to r^2. We expect a small radius particle to be blown away by the radiation pressure and a large radius particle with the same density to be pulled inward toward the Sun. The critical value for the radius is the value for which the two forces are equal. Equating the expressions for F_g and F_r, we solve for r:

$$r = \frac{3P}{16\pi GM\rho c}\ .$$

(b) According to Appendix C, $M = 1.99 \times 10^{30}\,\mathrm{kg}$ and $P = 3.90 \times 10^{26}\,\mathrm{W}$. Thus,

$$\begin{aligned} r &= \frac{3(3.90 \times 10^{26}\,\mathrm{W})}{16\pi(6.67 \times 10^{-11}\,\mathrm{N \cdot m^2/kg^2})(1.99 \times 10^{30}\,\mathrm{kg})(1.0 \times 10^3\,\mathrm{kg/m^3})(3.00 \times 10^8\,\mathrm{m/s})} \\ &= 5.8 \times 10^{-7}\ \mathrm{m}\ . \end{aligned}$$

32. (a) The discussion in §17-5 regarding the argument of the sine function $(kx + \omega t)$ makes it clear that the wave is traveling in the negative y direction. Thus, $\vec{S}$ points in the $-\hat{\mathrm{j}}$ direction.

(b) Since $\vec{E} \times \vec{B} \propto \vec{S}$ and $\vec{B}$ points in the $\hat{\imath}$ direction, then we may conclude that $\vec{E}$ points in the $-\hat{k}$ direction (recall that $\hat{k} \times \hat{\imath} = \hat{j}$). Therefore, $E_x = E_y = 0$ and $E_z = -cB\sin(kx + \omega t)$.

(c) Since $E_x = E_y = 0$, the wave is polarized along the z axis.

33. (a) Since the incident light is unpolarized, half the intensity is transmitted and half is absorbed. Thus the transmitted intensity is $I = 5.0\,\mathrm{mW/m^2}$. The intensity and the electric field amplitude are related by $I = E_m^2/2\mu_0 c$, so

$$\begin{aligned} E_m &= \sqrt{2\mu_0 cI} = \sqrt{2(4\pi\times10^{-7}\,\mathrm{H/m})(3.00\times10^8\,\mathrm{m/s})(5.0\times10^{-3}\,\mathrm{W/m^2})} \\ &= 1.9\ \mathrm{V/m}\ . \end{aligned}$$

(b) The radiation pressure is $p_r = I_a/c$, where I_a is the absorbed intensity. Thus

$$p_r = \frac{5.0\times10^{-3}\,\mathrm{W/m^2}}{3.00\times10^8\,\mathrm{m/s}} = 1.7\times10^{-11}\ \mathrm{Pa}\ .$$

34. After passing through the first polarizer the initial intensity I_0 reduces by a factor of 1/2. After passing through the second one it is further reduced by a factor of $\cos^2(\pi-\theta_1-\theta_2) = \cos^2(\theta_1+\theta_2)$. Finally, after passing through the third one it is again reduced by a factor of $\cos^2(\pi-\theta_2-\theta_3) = \cos^2(\theta_2+\theta_3)$. Therefore,

$$\begin{aligned} \frac{I_f}{I_0} &= \frac{1}{2}\cos^2(\theta_1+\theta_2)\cos^2(\theta_2+\theta_3) \\ &= \frac{1}{2}\cos^2(50^\circ+50^\circ)\cos^2(50^\circ+50^\circ) = 4.5\times10^{-4}\ . \end{aligned}$$

35. Let I_0 be the intensity of the unpolarized light that is incident on the first polarizing sheet. The transmitted intensity is $I_1 = \frac{1}{2}I_0$, and the direction of polarization of the transmitted light is $\theta_1 = 40^\circ$ counterclockwise from the y axis in the diagram. The polarizing direction of the second sheet is $\theta_2 = 20^\circ$ clockwise from the y axis, so the angle between the direction of polarization that is incident on that sheet and the polarizing direction of the sheet is $40^\circ + 20^\circ = 60^\circ$. The transmitted intensity is

$$I_2 = I_1\cos^2 60^\circ = \frac{1}{2}I_0\cos^2 60^\circ\ ,$$

and the direction of polarization of the transmitted light is 20° clockwise from the y axis. The polarizing direction of the third sheet is $\theta_3 = 40^\circ$ counterclockwise from the y axis. Consequently, the angle between the direction of polarization of the light incident on that sheet and the polarizing direction of the sheet is $20^\circ + 40^\circ = 60^\circ$. The transmitted intensity is

$$I_3 = I_2\cos^2 60^\circ = \frac{1}{2}I_0\cos^4 60^\circ = 3.1\times10^{-2}\ .$$

Thus, 3.1% of the light's initial intensity is transmitted.

36. As the unpolarized beam of intensity I_0 passes the first polarizer, its intensity is reduced to $\frac{1}{2}I_0$. After passing through the second polarizer, for which the direction of polarization is at an angle θ from that of the first one, the intensity is $I = \frac{1}{2}I_0\cos^2\theta = \frac{1}{3}I_0$. Thus, $\cos^2\theta = 2/3$, which leads to $\theta = 35^\circ$.

37. The angle between the direction of polarization of the light incident on the first polarizing sheet and the polarizing direction of that sheet is $\theta_1 = 70^\circ$. If I_0 is the intensity of the incident light, then the intensity of the light transmitted through the first sheet is

$$I_1 = I_0\cos^2\theta_1 = (43\,\mathrm{W/m^2})\cos^2 70^\circ = 5.03\ \mathrm{W/m^2}\ .$$

The direction of polarization of the transmitted light makes an angle of 70° with the vertical and an angle of $\theta_2 = 20^\circ$ with the horizontal. θ_2 is the angle it makes with the polarizing direction of the second polarizing sheet. Consequently, the transmitted intensity is

$$I_2 = I_1\cos^2\theta_2 = (5.03\,\mathrm{W/m^2})\cos^2 20^\circ = 4.4\ \mathrm{W/m^2}\ .$$

38. In this case, we replace $I_0 \cos^2 70°$ by $\frac{1}{2}I_0$ as the intensity of the light after passing through the first polarizer. Therefore,

$$I_f = \frac{1}{2} I_0 \cos^2(90° - 70°) = \frac{1}{2}(43\,\mathrm{W/m^2})(\cos^2 20°) = 19\ \mathrm{W/m^2}\ .$$

39. Let I_0 be the intensity of the incident beam and f be the fraction that is polarized. Thus, the intensity of the polarized portion is fI_0. After transmission, this portion contributes $fI_0 \cos^2\theta$ to the intensity of the transmitted beam. Here θ is the angle between the direction of polarization of the radiation and the polarizing direction of the filter. The intensity of the unpolarized portion of the incident beam is $(1-f)I_0$ and after transmission, this portion contributes $(1-f)I_0/2$ to the transmitted intensity. Consequently, the transmitted intensity is

$$I = fI_0 \cos^2\theta + \frac{1}{2}(1-f)I_0\ .$$

As the filter is rotated, $\cos^2\theta$ varies from a minimum of 0 to a maximum of 1, so the transmitted intensity varies from a minimum of

$$I_{\mathrm{min}} = \frac{1}{2}(1-f)I_0$$

to a maximum of

$$I_{\mathrm{max}} = fI_0 + \frac{1}{2}(1-f)I_0 = \frac{1}{2}(1+f)I_0\ .$$

The ratio of I_{max} to I_{min} is

$$\frac{I_{\mathrm{max}}}{I_{\mathrm{min}}} = \frac{1+f}{1-f}\ .$$

Setting the ratio equal to 5.0 and solving for f, we get $f = 0.67$.

40. (a) The fraction of light which is transmitted by the glasses is

$$\frac{I_f}{I_0} = \frac{E_f^2}{E_0^2} = \frac{E_v^2}{E_v^2 + E_h^2} = \frac{E_v^2}{E_v^2 + (2.3E_v)^2} = 0.16\ .$$

(b) Since now the horizontal component of $\vec{E}$ will pass through the glasses,

$$\frac{I_f}{I_0} = \frac{E_h^2}{E_v^2 + E_h^2} = \frac{(2.3E_v)^2}{E_v^2 + (2.3E_v)^2} = 0.84\ .$$

41. (a) The rotation cannot be done with a single sheet. If a sheet is placed with its polarizing direction at an angle of 90° to the direction of polarization of the incident radiation, no radiation is transmitted. It can be done with two sheets. We place the first sheet with its polarizing direction at some angle θ, between 0 and 90°, to the direction of polarization of the incident radiation. Place the second sheet with its polarizing direction at 90° to the polarization direction of the incident radiation. The transmitted radiation is then polarized at 90° to the incident polarization direction. The intensity is $I_0 \cos^2\theta \cos^2(90° - \theta) = I_0 \cos^2\theta \sin^2\theta$, where I_0 is the incident radiation. If θ is not 0 or 90°, the transmitted intensity is not zero.

(b) Consider n sheets, with the polarizing direction of the first sheet making an angle of $\theta = 90°/n$ relative to the direction of polarization of the incident radiation. The polarizing direction of each successive sheet is rotated $90°/n$ in the same sense from the polarizing direction of the previous sheet. The transmitted radiation is polarized, with its direction of polarization making an angle of 90° with the direction of polarization of the incident radiation. The intensity is $I = I_0 \cos^{2n}(90°/n)$. We want the smallest integer value of n for which this is greater than $0.60I_0$. We start with $n = 2$ and calculate $\cos^{2n}(90°/n)$. If the result is greater than 0.60, we have obtained the solution. If it is less, increase n by 1 and try again. We repeat this process, increasing n by 1 each time, until we have a value for which $\cos^{2n}(90°/n)$ is greater than 0.60. The first one will be $n = 5$.

42. The angle of incidence for the light ray on mirror B is $90° - \theta$. So the outgoing ray r' makes an angle $90° - (90° - \theta) = \theta$ with the vertical direction, and is antiparallel to the incoming one. The angle between i and r' is therefore $180°$.

43. The law of refraction states

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \ .$$

We take medium 1 to be the vacuum, with $n_1 = 1$ and $\theta_1 = 32.0°$. Medium 2 is the glass, with $\theta_2 = 21.0°$. We solve for n_2:

$$n_2 = n_1 \frac{\sin\theta_1}{\sin\theta_2} = (1.00)\left(\frac{\sin 32.0°}{\sin 21.0°}\right) = 1.48 \ .$$

44. (a) The law of refraction requires that $\sin\theta_1 / \sin\theta_2 = n_{\text{water}} = \text{const}$. We can check that this is indeed valid for any given pair of θ_1 and θ_2. For example $\sin 10°/\sin 8° = 1.3$, and $\sin 20°/\sin 15°30' = 1.3$, etc.

(b) $n_{\text{water}} = 1.3$, as shown in part (a).

45. Note that the normal to the refracting surface is vertical in the diagram. The angle of refraction is $\theta_2 = 90°$ and the angle of incidence is given by $\tan\theta_1 = w/h$, where h is the height of the tank and w is its width. Thus

$$\theta_1 = \tan^{-1}\left(\frac{w}{h}\right) = \tan^{-1}\left(\frac{1.10\,\text{m}}{0.850\,\text{m}}\right) = 52.31° \ .$$

The law of refraction yields

$$n_1 = n_2 \frac{\sin\theta_2}{\sin\theta_1} = (1.00)\left(\frac{\sin 90°}{\sin 52.31°}\right) = 1.26 \ ,$$

where the index of refraction of air was taken to be unity.

46. (a) Approximating $n = 1$ for air, we have

$$n_1 \sin\theta_1 = (1)\sin\theta_5 \implies 56.9° = \theta_5$$

and with the more accurate value for n_{air} in Table 34-1, we obtain $56.8°$.

(b) Eq. 34-44 leads to

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 = n_3 \sin\theta_3 = n_4 \sin\theta_4$$

so that

$$\theta_4 = sin^{-1}\left(\frac{n_1}{n_4}\sin\theta_1\right) = 35.3° \ .$$

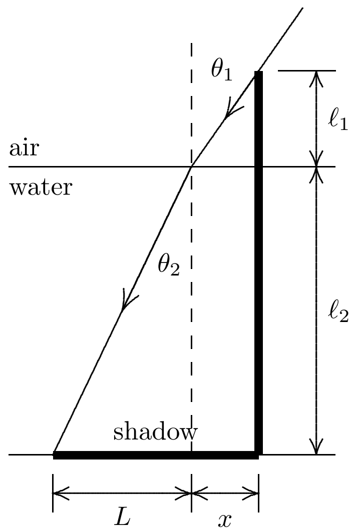
47. Consider a ray that grazes the top of the pole, as shown in the diagram below. Here $\theta_1 = 35°$, $\ell_1 = 0.50\,\text{m}$, and $\ell_2 = 1.50\,\text{m}$. The length of the shadow is $x + L$. x is given by $x = \ell_1 \tan\theta_1 = (0.50\,\text{m})\tan 35° = 0.35\,\text{m}$. According to the law of refraction, $n_2 \sin\theta_2 = n_1 \sin\theta_1$. We take $n_1 = 1$ and $n_2 = 1.33$ (from Table 34–1). Then,

$$\theta_2 = \sin^{-1}\left(\frac{\sin\theta_1}{n_2}\right) = \sin^{-1}\left(\frac{\sin 35.0°}{1.33}\right) = 25.55° \ .$$

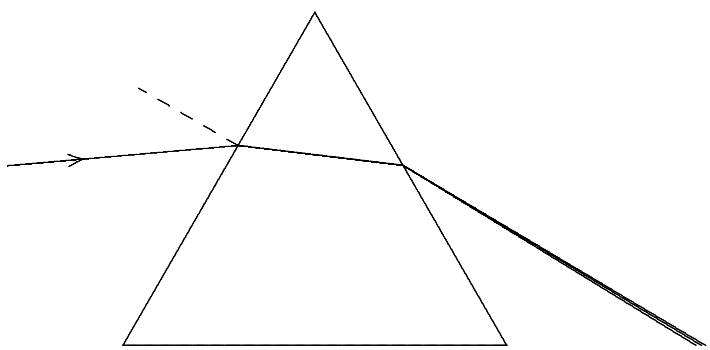
L is given by

$$L = \ell_2 \tan\theta_2 = (1.50\,\text{m})\tan 25.55° = 0.72\,\text{m} \ .$$

The length of the shadow is $0.35\,\text{m} + 0.72\,\text{m} = 1.07\,\text{m}$.



48. We use the law of refraction (assuming $n_{\rm air} = 1$) and the law of sines to determine the paths of various light rays. The index of refraction for fused quartz can be found in Fig. 34-19. We estimate $n_{\rm blue} = 1.463$, $n_{\rm y\,g} = 1.459$, and $n_{\rm red} = 1.456$. The light rays as they leave the prism (from the right side of the prism shown below) are very close together; on the scale we used below, the individual rays are difficult to resolve. Measured from the surface of the prism (at the face from which they emerge from the prism) their angles are $\theta_{\rm blue} = 28.51°, \theta_{\rm y\,g} = 28.95°$, and $\theta_{\rm red} = 29.29°$. The angle between the incident rays (on the left side of the picture) and the dashed line (the axis normal to the left face of the prism) is 35°.

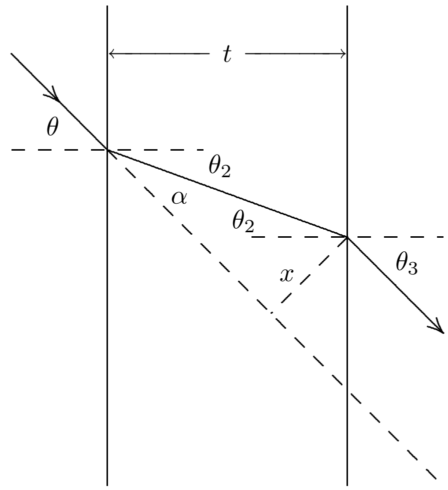


49. Let θ be the angle of incidence and θ_2 be the angle of refraction at the left face of the plate. Let n be the index of refraction of the glass. Then, the law of refraction yields $\sin\theta = n\sin\theta_2$. The angle of incidence at the right face is also θ_2. If θ_3 is the angle of emergence there, then $n\sin\theta_2 = \sin\theta_3$. Thus $\sin\theta_3 = \sin\theta$ and $\theta_3 = \theta$. The emerging ray is parallel to the incident ray. We wish to derive an expression for x in terms of θ. If D is the length of the ray in the glass, then $D\cos\theta_2 = t$ and $D = t/\cos\theta_2$. The angle α in the diagram equals $\theta - \theta_2$ and $x = D\sin\alpha = D\sin(\theta - \theta_2)$. Thus

$$x = \frac{t\sin(\theta - \theta_2)}{\cos\theta_2} \,.$$

If all the angles θ, θ_2, θ_3, and $\theta - \theta_2$ are small and measured in radians, then $\sin\theta \approx \theta$, $\sin\theta_2 \approx \theta_2$, $\sin(\theta - \theta_2) \approx \theta - \theta_2$, and $\cos\theta_2 \approx 1$. Thus $x \approx t(\theta - \theta_2)$. The law of refraction applied to the point of incidence at the left face of the plate is now $\theta \approx n\theta_2$, so $\theta_2 \approx \theta/n$ and

$$x \approx t\left(\theta - \frac{\theta}{n}\right) = \frac{(n-1)t\theta}{n} \ .$$



50. (a) An incident ray which is normal to the water surface is not refracted, so the angle at which it strikes the first mirror is $\theta_1 = 45°$. According to the law of reflection, the angle of reflection is also 45°. This means the ray is horizontal as it leaves the first mirror, and the angle of incidence at the second mirror is $\theta_2 = 45°$. Since the angle of reflection at the second mirror is also 45° the ray leaves that mirror normal again to the water surface. There is no refraction at the water surface, and the emerging ray is parallel to the incident ray.

(b) We imagine that the incident ray makes an angle θ_1 with the normal to the water surface. The angle of refraction θ_2 is found from $\sin\theta_1 = n\sin\theta_2$, where n is the index of refraction of the water. The normal to the water surface and the normal to the first mirror make an angle of 45°. If the normal to the water surface is continued downward until it meets the normal to the first mirror, the triangle formed has an interior angle of $180° - 45° = 135°$ at the vertex formed by the normal. Since the interior angles of a triangle must sum to 180°, the angle of incidence at the first mirror satisfies $\theta_3 + \theta_2 + 135° = 180°$, so $\theta_3 = 45° - \theta_2$. Using the law of reflection, the angle of reflection at the first mirror is also $45° - \theta_2$. We note that the triangle formed by the ray and the normals to the two mirrors is a right triangle. Consequently, $\theta_3 + \theta_4 + 90° = 180°$ and $\theta_4 = 90° - \theta_3 = 90° - 45° + \theta_2 = 45° + \theta_2$. The angle of reflection at the second mirror is also $45° + \theta_2$. Now, we continue the normal to the water surface downward from the exit point of the ray to the second mirror. It makes an angle of 45° with the mirror. Consider the triangle formed by the second mirror, the ray, and the normal to the water surface. The angle at the intersection of the normal and the mirror is $180° - 45° = 135°$. The angle at the intersection of the ray and the mirror is $90° - \theta_4 = 90° - (45° + \theta_2) = 45° - \theta_2$. The angle at the intersection of the ray and the water surface is θ_5. These three angles must sum to 180°, so $135° + 45° - \theta_2 + \theta_5 = 180°$. This means $\theta_5 = \theta_2$. Finally, we use the law of refraction to find θ_6:

$$\sin\theta_6 = n\sin\theta_5 \implies \sin\theta_6 = n\sin\theta_2 \ ,$$

since $\theta_5 = \theta_2$. Finally, since $\sin\theta_1 = n\sin\theta_2$, we conclude that $\sin\theta_6 = \sin\theta_1$ and $\theta_6 = \theta_1$. The exiting ray is parallel to the incident ray.

51. We label the light ray's point of entry A, the vertex of the prism B, and the light ray's exit point C. Also, the point in Fig. 34-49 where ψ is defined (at the point of intersection of the extrapolations of the incident and emergent rays) is denoted D. The angle indicated by ADC is the supplement of ψ, so we

denote it $\psi_s = 180° - \psi$. The angle of refraction in the glass is $\theta_2 = \frac{1}{n}\sin\theta$. The angles between the interior ray and the nearby surfaces is the complement of θ_2, so we denote it $\theta_{2c} = 90° - \theta_2$. Now, the angles in the triangle ABC must add to 180°:

$$180° = 2\theta_{2c} + \phi \implies \theta_2 = \frac{\phi}{2} .$$

Also, the angles in the triangle ADC must add to 180°:

$$180° = 2(\theta - \theta_2) + \psi_s \implies \theta = 90° + \theta_2 - \frac{1}{2}\psi_s$$

which simplifies to $\theta = \theta_2 + \frac{1}{2}\psi$. Combining this with our previous result, we find $\theta = \frac{1}{2}(\phi + \psi)$. Thus, the law of refraction yields

$$n = \frac{\sin(\theta)}{\sin(\theta_2)} = \frac{\sin\left(\frac{1}{2}(\phi + \psi)\right)}{\sin\left(\frac{1}{2}\phi\right)} .$$

52. The critical angle is

$$\theta_c = \sin^{-1}\left(\frac{1}{n}\right) = \sin^{-1}\left(\frac{1}{1.8}\right) = 34° .$$

53. Let $\theta_1 = 45°$ be the angle of incidence at the first surface and θ_2 be the angle of refraction there. Let θ_3 be the angle of incidence at the second surface. The condition for total internal reflection at the second surface is $n\sin\theta_3 \geq 1$. We want to find the smallest value of the index of refraction n for which this inequality holds. The law of refraction, applied to the first surface, yields $n\sin\theta_2 = \sin\theta_1$. Consideration of the triangle formed by the surface of the slab and the ray in the slab tells us that $\theta_3 = 90° - \theta_2$. Thus, the condition for total internal reflection becomes $1 \leq n\sin(90° - \theta_2) = n\cos\theta_2$. Squaring this equation and using $\sin^2\theta_2 + \cos^2\theta_2 = 1$, we obtain $1 \leq n^2(1 - \sin^2\theta_2)$. Substituting $\sin\theta_2 = (1/n)\sin\theta_1$ now leads to

$$1 \leq n^2\left(1 - \frac{\sin^2\theta_1}{n^2}\right) = n^2 - \sin^2\theta_1 .$$

The largest value of n for which this equation is true is the value for which $1 = n^2 - \sin^2\theta_1$. We solve for n:

$$n = \sqrt{1 + \sin^2\theta_1} = \sqrt{1 + \sin^2 45°} = 1.22 .$$

54. Reference to Fig. 34-24 may help in the visualization of why there appears to be a "circle of light" (consider revolving that picture about a vertical axis). The depth and the radius of that circle (which is from point a to point f in that figure) is related to the tangent of the angle of incidence. Thus, the diameter D of the circle in question is

$$D = 2h\tan\theta_c = 2h\tan\left[\sin^{-1}\left(\frac{1}{n_w}\right)\right] = 2(80.0\,\text{cm})\tan\left[\sin^{-1}\left(\frac{1}{1.33}\right)\right] = 182\ \text{cm} .$$

55. (a) No refraction occurs at the surface ab, so the angle of incidence at surface ac is $90° - \phi$. For total internal reflection at the second surface, $n_g\sin(90° - \phi)$ must be greater than n_a. Here n_g is the index of refraction for the glass and n_a is the index of refraction for air. Since $\sin(90° - \phi) = \cos\phi$, we want the largest value of ϕ for which $n_g\cos\phi \geq n_a$. Recall that $\cos\phi$ decreases as ϕ increases from zero. When ϕ has the largest value for which total internal reflection occurs, then $n_g\cos\phi = n_a$, or

$$\phi = \cos^{-1}\left(\frac{n_a}{n_g}\right) = \cos^{-1}\left(\frac{1}{1.52}\right) = 48.9° .$$

The index of refraction for air is taken to be unity.

(b) We now replace the air with water. If $n_w = 1.33$ is the index of refraction for water, then the largest value of ϕ for which total internal reflection occurs is

$$\phi = \cos^{-1}\left(\frac{n_w}{n_g}\right) = \cos^{-1}\left(\frac{1.33}{1.52}\right) = 29.0^\circ \ .$$

56. (a) (b) and (c) The index of refraction n for fused quartz is slightly higher on the bluish side of the visible light spectrum (with shorter wavelength). We estimate $n = 1.463$ for blue and $n = 1.456$ for red. Since $\sin\theta_c = 1/n$, the critical angle is slightly smaller for blue than it is for red: $\theta_c = 43.12^\circ$ for blue and $\theta_c = 43.38^\circ$ for red. Thus, at an angle of incidence of, say, $\theta = 43.29^\circ$, the refracted beam would be depleted of blue (and would appear to an outside observer as reddish), and the reflected beam would consequently appear to be bluish (to someone able to observe that beam, the operational details of which are not discussed here).

57. (a) The diagram below shows a cross section, through the center of the cube and parallel to a face. L is the length of a cube edge and S labels the spot. A portion of a ray from the source to a cube face is also shown. Light leaving the source at a small angle θ is refracted at the face and leaves the cube; light leaving at a sufficiently large angle is totally reflected. The light that passes through the cube face forms a circle, the radius r being associated with the critical angle for total internal reflection. If θ_c is that angle, then

$$\sin\theta_c = \frac{1}{n}$$

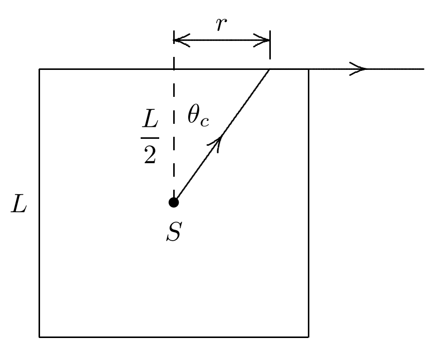
where n is the index of refraction for the glass. As the diagram shows, the radius of the circle is given by $r = (L/2)\tan\theta_c$. Now,

$$\tan\theta_c = \frac{\sin\theta_c}{\cos\theta_c} = \frac{\sin\theta_c}{\sqrt{1-\sin^2\theta_c}} = \frac{1/n}{\sqrt{1-(1/n)^2}} = \frac{1}{\sqrt{n^2-1}}$$

and the radius of the circle is

$$r = \frac{L}{2\sqrt{n^2-1}} = \frac{10\,\text{mm}}{2\sqrt{(1.5)^2-1}} = 4.47\,\text{mm} \ .$$

If an opaque circular disk with this radius is pasted at the center of each cube face, the spot will not be seen (provided internally reflected light can be ignored).



(b) There must be six opaque disks, one for each face. The total area covered by disks is $6\pi r^2$ and the total surface area of the cube is $6L^2$. The fraction of the surface area that must be covered by disks is

$$f = \frac{6\pi r^2}{6L^2} = \frac{\pi r^2}{L^2} = \frac{\pi(4.47\,\text{mm})^2}{(10\,\text{mm})^2} = 0.63 \ .$$

58. (a) We refer to the entry point for the original incident ray as point A (which we take to be on the left side of the prism, as in Fig. 34-49), the prism vertex as point B, and the point where the interior

ray strikes the right surface of the prism as point C. The angle between line AB and the interior ray is β (the complement of the angle of refraction at the first surface), and the angle between the line BC and the interior ray is α (the complement of its angle of incidence when it strikes the second surface). When the incident ray is at the minimum angle for which light is able to exit the prism, the light exits along the second face. That is, the angle of refraction at the second face is 90°, and the angle of incidence there for the interior ray is the critical angle for total internal reflection. Let θ_1 be the angle of incidence for the original incident ray and θ_2 be the angle of refraction at the first face, and let θ_3 be the angle of incidence at the second face. The law of refraction, applied to point C, yields $n\sin\theta_3 = 1$, so $\sin\theta_3 = 1/n = 1/1.60 = 0.625$ and $\theta_3 = 38.68°$. The interior angles of the triangle ABC must sum to 180°, so $\alpha + \beta = 120°$. Now, $\alpha = 90° - \theta_3 = 51.32°$, so $\beta = 120° - 51.32° = 69.68°$. Thus, $\theta_2 = 90° - \beta = 21.32°$. The law of refraction, applied to point A, yields $\sin\theta_1 = n\sin\theta_2 = 1.60\sin 21.32° = 0.5817$. Thus $\theta_1 = 35.6°$.

(b) We apply the law of refraction to point C. Since the angle of refraction there is the same as the angle of incidence at A, $n\sin\theta_3 = \sin\theta_1$. Now, $\alpha + \beta = 120°$, $\alpha = 90° - \theta_3$, and $\beta = 90° - \theta_2$, as before. This means $\theta_2 + \theta_3 = 60°$. Thus, the law of refraction leads to

$$\sin\theta_1 = n\sin(60° - \theta_2) \implies \sin\theta_1 = n\sin 60°\cos\theta_2 - n\cos 60°\sin\theta_2$$

where the trigonometric identity $\sin(A-B) = \sin A\cos B - \cos A\sin B$ is used. Next, we apply the law of refraction to point A:

$$\sin\theta_1 = n\sin\theta_2 \implies \sin\theta_2 = (1/n)\sin\theta_1$$

which yields $\cos\theta_2 = \sqrt{1-\sin^2\theta_2} = \sqrt{1-(1/n^2)\sin^2\theta_1}$. Thus,

$$\sin\theta_1 = n\sin 60°\sqrt{1-(1/n)^2\sin^2\theta_1} - \cos 60°\sin\theta_1$$

or

$$(1+\cos 60°)\sin\theta_1 = \sin 60°\sqrt{n^2 - \sin^2\theta_1}\ .$$

Squaring both sides and solving for $\sin\theta_1$, we obtain

$$\sin\theta_1 = \frac{n\sin 60°}{\sqrt{(1+\cos 60°)^2 + \sin^2 60°}} = \frac{1.60\sin 60°}{\sqrt{(1+\cos 60°)^2 + \sin^2 60°}} = 0.80$$

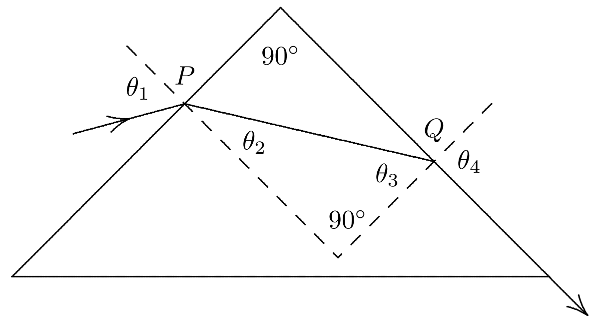
and $\theta_1 = 53.1°$.

59. (a) A ray diagram is shown below. Let θ_1 be the angle of incidence and θ_2 be the angle of refraction at the first surface. Let θ_3 be the angle of incidence at the second surface. The angle of refraction there is $\theta_4 = 90°$. The law of refraction, applied to the second surface, yields $n\sin\theta_3 = \sin\theta_4 = 1$. As shown in the diagram, the normals to the surfaces at P and Q are perpendicular to each other. The interior angles of the triangle formed by the ray and the two normals must sum to 180°, so $\theta_3 = 90° - \theta_2$ and $\sin\theta_3 = \sin(90° - \theta_2) = \cos\theta_2 = \sqrt{1-\sin^2\theta_2}$. According to the law of refraction, applied at Q, $n\sqrt{1-\sin^2\theta_2} = 1$. The law of refraction, applied to point P, yields $\sin\theta_1 = n\sin\theta_2$, so $\sin\theta_2 = (\sin\theta_1)/n$ and

$$n\sqrt{1-\frac{\sin^2\theta_1}{n^2}} = 1\ .$$

Squaring both sides and solving for n, we get

$$n = \sqrt{1+\sin^2\theta_1}\ .$$



(b) The greatest possible value of $\sin^2\theta_1$ is 1, so the greatest possible value of n is $n_{\max} = \sqrt{2} = 1.41$.

(c) For a given value of n, if the angle of incidence at the first surface is greater than θ_1, the angle of refraction there is greater than θ_2 and the angle of incidence at the second face is less than θ_3 $(= 90^\circ - \theta_2)$. That is, it is less than the critical angle for total internal reflection, so light leaves the second surface and emerges into the air.

(d) If the angle of incidence at the first surface is less than θ_1, the angle of refraction there is less than θ_2 and the angle of incidence at the second surface is greater than θ_3. This is greater than the critical angle for total internal reflection, so all the light is reflected at Q.

60. (a) We use Eq. 34-49: $\theta_{\rm B} = \tan^{-1} n_w = \tan^{-1}(1.33) = 53.1^\circ$.

(b) Yes, since n_w depends on the wavelength of the light.

61. The angle of incidence θ_B for which reflected light is fully polarized is given by Eq. 34–48 of the text. If n_1 is the index of refraction for the medium of incidence and n_2 is the index of refraction for the second medium, then $\theta_B = \tan^{-1}(n_2/n_1) = \tan^{-1}(1.53/1.33) = 63.8^\circ$.

62. From Fig. 34-19 we find $n_{\max} = 1.470$ for $\lambda = 400\,\text{nm}$ and $n_{\min} = 1.456$ for $\lambda = 700\,\text{nm}$. The corresponding Brewster's angles are $\theta_{\rm B,max} = \tan^{-1} n_{\max} = \tan^{-1}(1.470) = 55.77^\circ$ and $\theta_{\rm B,min} = \tan^{-1}(1.456) = 55.52^\circ$.

63. (a) The Sun is far enough away that we approximate its rays as "parallel" in this Figure. That is, if the sunray makes angle θ from horizontal when the bird is in one position, then it makes the same angle θ when the bird is any other position. Therefore, its shadow on the ground moves as the bird moves: at 15 m/s.

(b) If the bird is in a position, a distance $x > 0$ from the wall, such that its shadow is on the wall at a distance $0 \ge y \ge h$ from the top of the wall, then it is clear from the Figure that $tan\theta = y/x$. Thus,

$$\frac{dy}{dt} = \frac{dx}{dt}\tan\theta = (-15\,\text{m/s})\tan 30^\circ = -8.7\text{ m/s} ,$$

which means that the distance y (which was measured as a positive number downward from the top of the wall) is shrinking at the rate of 8.7 m/s.

(c) Since $\tan\theta$ grows as $0 \le \theta < 90^\circ$ increases, then a larger value of $|dy/dt|$ implies a larger value of θ. The Sun is higher in the sky when the hawk glides by.

(d) With $|dy/dt| = 45$ m/s, we find

$$v_{\rm hawk} = \left|\frac{dx}{dt}\right| = \frac{\left|\frac{dy}{dt}\right|}{\tan\theta}$$

so that we obtain $\theta = 72^\circ$ if we assume $v_{\rm hawk} = 15$ m/s.

64. (a) The 63.00 ns arrival times are consistent with the top of the tomb being 31.50 ns (pulse travel time) away from the surface. Since the pulses travel at 10.0 cm/ns in the soil, this travel time corresponds to a distance equal to $315\,\text{cm} = 3.15\,\text{m}$.

(b) We are told that the locations in Fig. 34-54 are 2.0 m apart. Return pulses are registered at stations 2 through 7, but the returns from stations 2 and 7 are not "robust." The tomb's horizontal length is therefore at least 9 m long, and very probably less than 12 m in length.

(c) As demonstrated in part (a), we divide the travel times by 2 to infer depth. Thus, at station 3: the top of the tomb is 31.50 ns (pulse travel time in soil) from the surface; the top stone slab is 1.885 ns thick (pulse travel time in stone); the interior of the tomb is 8.00 ns high (pulse travel time in air); and the bottom stone slab is 1.885 ns thick (pulse travel time in stone). Since the pulse travels at 30 cm/s in the air, the interior of the tomb under station 3 (at the west end of the tomb) is 240 cm = 2.40 m high. At the east end (under, say, station 5), the corresponding time difference is

$$\frac{74.77\,\text{ns} - 66.77\,\text{ns}}{2} = 4.00 \text{ ns}$$

which corresponds to an interior height equal to $(4.00\,\text{ns})(30\,\text{cm/s}) = 120\,\text{cm/s} = 1.20$ m.

65. Since the layers are parallel, the angle of refraction regarding the first surface is the same as the angle of incidence regarding the second surface (as is suggested by the notation in Fig. 34-55). We recall that as part of the derivation of Eq. 34-49 (Brewster's angle), the textbook shows that the refracted angle is the complement of the incident angle:

$$\theta_2 = (\theta_1)_\text{c} = 90^\circ - \theta_1 \ .$$

We apply Eq. 34-49 to both refractions, setting up a product:

$$\begin{aligned} \left(\frac{n_2}{n_1}\right)\left(\frac{n_3}{n_2}\right) &= (\tan\theta_{\text{B}\,1\to 2})\,(\tan\theta_{\text{B}\,2\to 3}) \\ \frac{n_3}{n_1} &= (\tan\theta_1)\,(\tan\theta_2) \ . \end{aligned}$$

Now, since θ_2 is the complement of θ_1 we have

$$\tan\theta_2 = \tan(\theta_1)_\text{c} = \frac{1}{\tan\theta_1} \ .$$

Therefore, the product of tangents cancel and we obtain $n_3/n_1 = 1$. Consequently, the third medium is air: $n_3 = 1.0$.

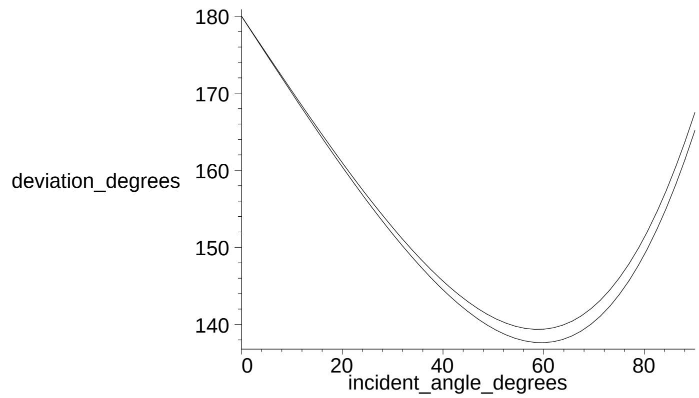
66. In air, light travels at roughly $c = 3.0 \times 10^8$ m/s. Therefore, for $t = 1.0$ ns, we have a distance of

$$d = ct = \left(3.0 \times 10^8\,\text{m/s}\right)\left(1.0 \times 10^{-9}\,\text{s}\right) = 0.30 \text{ m} \ .$$

67. (a) The first contribution to the overall deviation is at the first refraction: $\delta\theta_1 = \theta_i - \theta_r$. The next contribution to the overall deviation is the reflection. Noting that the angle between the ray right before reflection and the axis normal to the back surface of the sphere is equal to θ_r, and recalling the law of reflection, we conclude that the angle by which the ray turns (comparing the direction of propagation before and after the reflection) is $\delta\theta_2 = 180^\circ - 2\theta_r$. The final contribution is the refraction suffered by the ray upon leaving the sphere: $\delta\theta_3 = \theta_i - \theta_r$ again. Therefore,

$$\theta_\text{dev} = \delta\theta_1 + \delta\theta_2 + \delta\theta_3 = 180^\circ + 2\theta_i - 4\theta_r \ .$$

(b) We substitute $\theta_r = \sin^{-1}(\frac{1}{n}\sin\theta_i)$ into the expression derived in part (a), using the two given values for n. The higher curve is for the blue light.



(c) We can expand the graph and try to estimate the minimum, or search for it with a more sophisticated numerical procedure. We find that the θ_{dev} minimum for red light is 137.63°, and this occurs at $\theta_i = 59.52°$.

(d) For blue light, we find that the θ_{dev} minimum is 139.35°, and this occurs at $\theta_i = 59.52°$.

(e) The difference in θ_{dev} in the previous two parts is 1.72°.

68. (a) The first contribution to the overall deviation is at the first refraction: $\delta\theta_1 = \theta_i - \theta_r$. The next contribution(s) to the overall deviation is (are) the reflection(s). Noting that the angle between the ray right before reflection and the axis normal to the back surface of the sphere is equal to θ_r, and recalling the law of reflection, we conclude that the angle by which the ray turns (comparing the direction of propagation before and after [each] reflection) is $\delta\theta_r = 180° - 2\theta_r$. Thus, for k reflections, we have $\delta\theta_2 = k\theta_r$ to account for these contributions. The final contribution is the refraction suffered by the ray upon leaving the sphere: $\delta\theta_3 = \theta_i - \theta_r$ again. Therefore,

$$\theta_{\text{dev}} = \delta\theta_1 + \delta\theta_2 + \delta\theta_3 = 2\,(\theta_i - \theta_r) + k\,(180° - 2\theta_r) = k\,(180°) + 2\theta_i - 2(k+1)\theta_r \ .$$

(b) For $k = 2$ and $n = 1.331$ (given in problem 67), we search for the second-order rainbow angle numerically. We find that the θ_{dev} minimum for red light is 230.37°, and this occurs at $\theta_i = 71.90°$.

(c) Similarly, we find that the second-order θ_{dev} minimum for blue light (for which $n = 1.343$) is 233.48°, and this occurs at $\theta_i = 71.52°$.

(d) The difference in θ_{dev} in the previous two parts is 3.11°.

(e) Setting $k = 3$, we search for the third-order rainbow angle numerically. We find that the θ_{dev} minimum for red light is 317.53°, and this occurs at $\theta_i = 76.88°$.

(f) Similarly, we find that the third-order θ_{dev} minimum for blue light is 321.89°, and this occurs at $\theta_i = 76.62°$.

(g) The difference in θ_{dev} in the previous two parts is 4.37°.

69. Reference to Fig. 34-24 may help in the visualization of why there appears to be a "circle of light" (consider revolving that picture about a vertical axis). The depth and the radius of that circle (which is from point a to point f in that figure) is related to the tangent of the angle of incidence. The diameter of the circle in question is given by $d = 2h\tan\theta_c$. For water $n = 1.33$, so Eq. 34-47 gives $\sin\theta_c = 1/1.33$, or $\theta_c = 48.75°$. Thus,

$$d = 2h\tan\theta_c = 2(2.00\,\text{m})(\tan 48.75°) = 4.56 \text{ m} \ .$$

70. We apply Eq. 34-42 (twice) to obtain

$$I = I_0 \cos^2\theta_1 \cos^2\theta_2$$

where $\theta_1 = 20°$ and $\theta_2 = (20° + \theta)$. Since $I/I_0 = 0.200$, we find $\cos\theta_2 = \sqrt{0.2265}$ which leads to $\theta_2 = 62°$ and consequently to $\theta = 42°$.

71. (a) The electric field amplitude is $E_m = \sqrt{2}\,E_{\text{rms}} = 70.7$ V/m, so that the magnetic field amplitude is $B_m = 2.36 \times 10^{-7}$ T by Eq. 34-5. Since the direction of propagation, $\vec{E}$, and $\vec{B}$ are mutually perpendicular, we infer that the only non-zero component of $\vec{B}$ is B_x, and note that the direction of propagation being along the $-z$ axis means the spatial and temporal parts of the wave function argument are of like sign (see §17-5). Also, from $\lambda = 250$ nm, we find that $f = c/\lambda = 1.20 \times 10^{15}$ Hz, which leads to $\omega = 2\pi f = 7.53 \times 10^{15}$ rad/s. Also, we note that $k = 2\pi/\lambda = 2.51 \times 10^7$ m^{-1}. Thus, assuming some "initial condition" (that, say the field is zero, with its derivative positive, at $z = 0$ when $t = 0$), we have

$$B_x = 2.36 \times 10^{-7}\ \sin\left(\left(2.51 \times 10^7\right) z + \left(7.53 \times 10^{15}\right) t\right)$$

in SI units.

(b) The exposed area of the triangular chip is $A = \sqrt{3}\,\ell^2/8$, where $\ell = 2.00 \times 10^{-6}$ m. The intensity of the wave is

$$I = \frac{1}{c\mu_0}\,E_{\text{rms}}^2 = 6.64 \text{ W/m}^2 .$$

Thus, Eq. 34-33 leads to

$$F = \frac{2IA}{c} = 3.83 \times 10^{-20} \text{ N} .$$

72. We follow Sample Problem 34-2 in computing the sunlight intensity at the sail's location.

$$I = \frac{P_S}{4\pi r^2} = \frac{3.9 \times 10^{26} \text{ W}}{4\pi \left(3.0 \times 10^{11} \text{ m}\right)^2} = 345 \text{ W/m}^2$$

With $A = (2.0 \text{ m})^2$, we use Eq. 34-33 to obtain the radiation force:

$$F = \frac{2IA}{c} = 9.2 \times 10^{-6} \text{ N} .$$

73. (a) Eq. 34-5 gives $E = cB$, which relates the field values at any instant – and so relates rms values to rms values, and amplitude values to amplitude values, as the case may be. Thus, $E_{\text{rms}} = cB_{\text{rms}} = 16.8$ V/m. Multiplying by $\sqrt{2}$ yields the electric field amplitude $E_m = 23.7$ V/m.

(b) We use Eq. 34-26:

$$I = \frac{1}{\mu_0\, c}\,E_{\text{rms}}^2 = 0.748 \text{ W/m}^2 .$$

74. Consider two wavelengths, λ_1 and λ_2, whose corresponding frequencies are f_1 and f_2. Then $\lambda_1 = C/f_1$ and $\lambda_2 = C/f_2$. If $\lambda_1/\lambda_2 = 10$, then

$$\frac{\lambda_1}{\lambda_2} = \frac{C/f_1}{C/f_2} = \frac{f_2}{f_1} = 10 .$$

The spaces are the same on both scales.

75. We take the derivative with respect to x of both sides of Eq. 34-11:

$$\frac{\partial}{\partial x}\left(\frac{\partial E}{\partial x}\right) = \frac{\partial^2 E}{\partial x^2} = \frac{\partial}{\partial x}\left(-\frac{\partial B}{\partial t}\right) = -\frac{\partial^2 B}{\partial x \partial t} .$$

Now we differentiate both sides of Eq. 34-18 with respect to t:

$$\frac{\partial}{\partial t}\left(-\frac{\partial B}{\partial x}\right) = -\frac{\partial^2 B}{\partial x \partial t} = \frac{\partial}{\partial t}\left(\varepsilon_0\mu_0\frac{\partial E}{\partial t}\right) = \varepsilon_0\mu_0\frac{\partial^2 E}{\partial t^2} .$$

Substituting $\partial^2 E/\partial x^2 = -\partial^2 B/\partial x \partial t$ from the first equation above into the second one, we get

$$\varepsilon_0 \mu_0 \frac{\partial^2 E}{\partial t^2} = \frac{\partial^2 E}{\partial x^2} ,$$

or

$$\frac{\partial^2 E}{\partial t^2} = \frac{1}{\varepsilon_0 \mu_0} \frac{\partial^2 E}{\partial x^2} = c^2 \frac{\partial^2 E}{\partial x^2} .$$

Similarly, we differentiate both sides of Eq. 34-11 with respect to t

$$\frac{\partial^2 E}{\partial x \partial t} = -\frac{\partial^2 B}{\partial t^2} ,$$

and differentiate both sides of Eq. 34-18 with respect to x

$$-\frac{\partial^2 B}{\partial x^2} = \varepsilon_0 \mu_0 \frac{\partial^2 E}{\partial x \partial t} .$$

Combining these two equations, we get

$$\frac{\partial^2 B}{\partial t^2} = \frac{1}{\varepsilon_0 \mu_0} \frac{\partial^2 B}{\partial x^2} = c^2 \frac{\partial^2 B}{\partial x^2} .$$

76. The energy density of an electromagnetic wave is given by $u = u_E + u_B$. From the discussion in §34-4, $u_E = u_B = \frac{1}{2}\varepsilon_0 E^2$, so $u = 2u_E = \varepsilon_0 E^2$. Upon averaging over time this becomes

$$u_{\text{avg}} = \varepsilon_0 \overline{E^2} = \varepsilon_0 E_{\text{rms}}^2 .$$

Combining this equation with Eq. 34-26 in the textbook, we obtain

$$I = \frac{1}{c\mu_0} E_{\text{rms}}^2 = \frac{1}{c\mu_0} \frac{u_{\text{avg}}}{\varepsilon_0} = \frac{c^2 u_{\text{avg}}}{c} = c u_{\text{avg}}$$

where $c^2 = 1/\varepsilon_0 \mu_0$ is used.

77. (a) Assuming complete absorption, the radiation pressure is

$$p_r = \frac{I}{c} = \frac{1.4 \times 10^3 \, \text{W/m}^2}{3.0 \times 10^8 \, \text{m/s}} = 4.7 \times 10^{-6} \, \text{N/m}^2 .$$

(b) We compare values by setting up a ratio:

$$\frac{p_r}{p_0} = \frac{4.7 \times 10^{-6} \, \text{N/m}^2}{1.0 \times 10^5 \, \text{N/m}^2} = 4.7 \times 10^{-11} .$$

78. (a) Suppose there are a total of N transparent layers ($N = 5$ in our case). We label these layers from left to right with indices 1, 2, ..., N. Let the index of refraction of the air be n_0. We denote the initial angle of incidence of the light ray upon the air-layer boundary as θ_i and the angle of the emerging light ray as θ_f. We note that, since all the boundaries are parallel to each other, the angle of incidence θ_j at the boundary between the j-th and the $(j+1)$-th layers is the same as the angle between the transmitted light ray and the normal in the j-th layer. Thus, for the first boundary (the one between the air and the first layer)

$$\frac{n_1}{n_0} = \frac{\sin\theta_i}{\sin\theta_1} ,$$

for the second boundary

$$\frac{n_2}{n_1} = \frac{\sin\theta_1}{\sin\theta_2} \ ,$$

and so on. Finally, for the last boundary

$$\frac{n_0}{n_N} = \frac{\sin\theta_N}{\sin\theta_f} \ .$$

Multiplying these equations, we obtain

$$\left(\frac{n_1}{n_0}\right)\left(\frac{n_2}{n_1}\right)\left(\frac{n_3}{n_2}\right)\cdots\left(\frac{n_0}{n_N}\right) = \left(\frac{\sin\theta_i}{\sin\theta_1}\right)\left(\frac{\sin\theta_1}{\sin\theta_2}\right)\left(\frac{\sin\theta_2}{\sin\theta_3}\right)\cdots\left(\frac{\sin\theta_N}{\sin\theta_f}\right) \ .$$

We see that the L.H.S. of the equation above can be reduced to n_0/n_0 while the R.H.S. is equal to $\sin\theta_i/\sin\theta_f$. Equating these two expressions, we find

$$\sin\theta_f = \left(\frac{n_0}{n_0}\right)\sin\theta_i = \sin\theta_i \ ,$$

which gives $\theta_i = \theta_f$. So for the two light rays in the problem statement, the angle of the emerging light rays are both the same as their respective incident angles. Thus, $\theta_f = 0$ for ray a and $\theta_f = 20°$ for ray b.

(b) In this case, all we need to do is to change the value of n_0 from 1.0 (for air) to 1.5 (for glass). This does not change the result above. Note that the result of this problem is fairly general. It is independent of the number of layers and the thickness and index of refraction of each layer.

79. We use the result of the problem 51 to solve for ψ. Note that $\phi = 60.0°$ in our case. Thus, from

$$n = \frac{\sin\frac{1}{2}(\psi+\phi)}{\sin\frac{1}{2}\phi} \ ,$$

we get

$$\sin\frac{1}{2}(\psi+\phi) = n\sin\frac{1}{2}\phi = (1.31)\sin\left(\frac{60.0°}{2}\right) = 0.655 \ ,$$

which gives $\frac{1}{2}(\psi+\phi) = \sin^{-1}(0.655) = 40.9°$. Thus, $\psi = 2(40.9°) - \phi = 2(40.9°) - 60.0° = 21.8°$.

80. (a) The light that passes through the surface of the lake is within a cone of apex angle $2\theta_c$ making a "circle of light" there; reference to Fig. 34-24 may help in visualizing this (consider revolving that picture about a vertical axis). Since the source is point-like, its energy spreads out with perfect spherical symmetry, until reaching the surface and other boundaries of the lake. The problem asks us to assume there are no partial reflections at the surface, only the total reflections outside the "circle of light." Thus, of the full sphere of light (of area $A_s = 4\pi R^2$) emitted by the source, only a fraction of it – coinciding with the cone of apex angle $2\theta_c$ – enters the air above. If we label the area of that portion of the sphere which reaches the air above as A, then the fraction of the total energy emitted that passes through the surface is

$$frac = \frac{A}{4\pi R^2} \qquad \text{where} \qquad R = \frac{h}{\cos\theta_c}$$

is the distance from the point-source to the edge of the "circle of light." Now, the area A of the spherical cap of height H bounded by that circle is

$$A = 2\pi RH = 2\pi R(R-h)$$

may be looked up in a number of references, or can be derived from $A = 2\pi R^2\int_0^{\theta_c}\sin\theta\,d\theta$. Consequently,

$$frac = \frac{2\pi R(R-h)}{4\pi R^2} = \frac{1}{2}\left(1-\frac{h}{R}\right) = \frac{1}{2}\left(1-\cos\theta_c\right) \ .$$

The critical angle is given by $\sin\theta_c = 1/n$, which implies $\cos\theta_c = \sqrt{1-\sin^2\theta_c} = \sqrt{1-1/n^2}$. When this expression is substituted into our result above, we obtain

$$frac = \frac{1}{2}\left(1-\sqrt{1-\frac{1}{n^2}}\right) .$$

(b) For $n = 1.33$,

$$frac = \frac{1}{2}\left(1-\sqrt{1-\frac{1}{(1.33)^2}}\right) = 0.170 .$$

81. We apply Eq. 34-40 (once) and Eq. 34-42 (twice) to obtain

$$I = \frac{1}{2} I_0 \cos^2\theta_1 \cos^2\theta_2 .$$

With $\theta_1 = 60° - 20° = 40°$ and $\theta_2 = 40° + 30° = 70°$, this yields $I/I_0 = 0.034$.

82. (a) From $kc = \omega$ where $k = 1.00 \times 10^6\ \mathrm{m}^{-1}$, we obtain $\omega = 3.00 \times 10^{14}$ rad/s. The magnetic field amplitude is, from Eq. 34-5, $B = (5.00\ \mathrm{V/m})/c = 1.67 \times 10^{-8}$ T. From the fact that $-\hat{\mathrm{k}}$ (the direction of propagation), $\vec{E} = E_y\hat{\mathrm{j}}$, and $\vec{B}$ are mutually perpendicular, we conclude that the only non-zero component of $\vec{B}$ is B_x, so that we have (in SI units)

$$B_x = 1.67 \times 10^{-8}\ \sin\left(\left(1.00 \times 10^6\right) z + \left(3.00 \times 10^{14}\right) t\right) .$$

(b) The wavelength is $\lambda = 2\pi/k = 6.28 \times 10^{-6}$ m.

(c) The period is $T = 2\pi/\omega = 2.09 \times 10^{-14}$ s.

(d) The intensity is

$$I = \frac{1}{c\mu_0}\left(\frac{5.00\ \mathrm{V/m}}{\sqrt{2}}\right)^2 = 0.0332\ \mathrm{W/m^2} .$$

(e) As noted in part (a), the only nonzero component of $\vec{B}$ is B_x. The magnetic field oscillates along the x axis.

(f) The wavelength found in part (b) places this in the infrared portion of the spectrum.

83. We write $m = \rho\mathcal{V}$ where $\mathcal{V} = 4\pi R^3/3$ is the volume. Plugging this into $F = ma$ and then into Eq. 34-32 (with $A = \pi R^2$, assuming the light is in the form of plane waves), we find

$$\rho\,\frac{4\pi R^3}{3}\,a = \frac{I\pi R^2}{c} .$$

This simplifies to

$$a = \frac{3I}{4\rho c R}$$

which yields $a = 1.5 \times 10^{-9}\ \mathrm{m/s^2}$.

84. Since intensity is power divided by area (and the area is spherical in the isotropic case), then the intensity at a distance of $r = 20$ m from the source is

$$I = \frac{P}{4\pi r^2} = 0.040\ \mathrm{W/m^2} .$$

as illustrated in Sample Problem 34-2. Now, in Eq. 34-32 for a totally absorbing area A, we note that the exposed area of the small sphere is that on a flat circle $A = \pi(0.020\,\mathrm{m})^2 = 0.0013\ \mathrm{m}^2$. Therefore,

$$F = \frac{IA}{c} = \frac{(0.040)(0.0013)}{3 \times 10^8} = 1.7 \times 10^{-13}\ \mathrm{N} .$$

85. Eq. 34-5 gives $B = E/c$, which relates the field values at any instant – and so relates rms values to rms values, and amplitude values to amplitude values, as the case may be. Thus, the rms value of the magnetic field is $0.2/3 \times 10^8 = 6.7 \times 10^{-10}$ T, which (upon multiplication by $\sqrt{2}$) yields an amplitude value of magnetic field equal to 9.4×10^{-10} T.

86. (a) From Eq. 34-1,

$$\frac{\partial^2 E}{\partial t^2} = \frac{\partial^2}{\partial t^2}[E_m \sin(kx - \omega t)] = -\omega^2 E_m \sin(kx - \omega t)\,,$$

and

$$c^2 \frac{\partial^2 E}{\partial x^2} = c^2 \frac{\partial^2}{\partial x^2}[E_{\rm m} \sin(kx - \omega t)] = -k^2 c^2 \sin(kx - \omega t) = -\omega^2 E_m \sin(kx - \omega t)\ .$$

Consequently,

$$\frac{\partial^2 E}{\partial t^2} = c^2 \frac{\partial^2 E}{\partial x^2}$$

is satisfied. Analogously, one can show that Eq. 34-2 satisfies

$$\frac{\partial^2 B}{\partial t^2} = c^2 \frac{\partial^2 B}{\partial x^2}\ .$$

(b) From $E = E_m f(kx \pm \omega t)$,

$$\frac{\partial^2 E}{\partial t^2} = E_m \frac{\partial^2 f(kx \pm \omega t)}{\partial t^2} = \omega^2 E_m \left.\frac{d^2 f}{du^2}\right|_{u=kx\pm\omega t}$$

and

$$c^2 \frac{\partial^2 E}{\partial x^2} = c^2 E_m \frac{\partial^2 f(kx \pm \omega t)}{\partial t^2} = c^2 E_m k^2 \left.\frac{d^2 f}{du^2}\right|_{u=kx\pm\omega t}\ .$$

Since $\omega = ck$ the right-hand sides of these two equations are equal. Therefore,

$$\frac{\partial^2 E}{\partial t^2} = c^2 \frac{\partial^2 E}{\partial x^2}\ .$$

Changing E to B and repeating the derivation above shows that $B = B_m f(kx \pm \omega t)$ satisfies

$$\frac{\partial^2 B}{\partial t^2} = c^2 \frac{\partial^2 B}{\partial x^2}\ .$$

87. $\vec{E} \times \vec{B} = \mu_0 \vec{S}$, where $\vec{E} = E\,\hat{\rm k}$ and $\vec{S} = S(-\hat{\rm j})$. One can verify easily that since $\hat{\rm k} \times (-\hat{\rm i}) = -\hat{\rm j}$, $\vec{B}$ has to be in the negative x direction. Also,

$$B = \frac{E}{c} = \frac{100\,{\rm V/m}}{3.0 \times 10^8\,{\rm m/s}} = 3.3 \times 10^{-7}\ {\rm T}\ .$$

88. (a) At $r = 40$ m, the intensity is

$$\begin{aligned} I &= \frac{P}{\pi d^2/4} = \frac{P}{\pi(\theta r)^2/4} \\ &= \frac{4(3.0 \times 10^{-3}\,{\rm W})}{\pi[(0.17 \times 10^{-3}\,{\rm rad})(40\,{\rm m})]^2} \\ &= 83\ {\rm W/m^2}\ . \end{aligned}$$

(b) $P' = 4\pi r^2 I = 4\pi (40\,{\rm m})^2 (83\,{\rm W/m^2}) = 1.7 \times 10^6\,{\rm W}$.

89. Using Eqs. 34-40 and 34-42, we obtain

$$\frac{I_{\text{final}}}{I_0} = \frac{\left(\frac{1}{2}I_0\right)\left(\cos^2 45^\circ\right)\left(\cos^2 45^\circ\right)}{I_0} = \frac{1}{8} \ .$$

90. We use the result obtained in problem 51:

$$n = \frac{\sin\frac{1}{2}(\phi+\psi)}{\sin\frac{1}{2}\phi} = \frac{\sin\left[\frac{1}{2}(60.0^\circ+30.0^\circ)\right]}{\sin\left[\frac{1}{2}(60.0^\circ)\right]} = 1.41 \ .$$

91. (a) and (b) At the Brewster angle, $\theta_{\text{incident}} + \theta_{\text{refracted}} = \theta_{\text{B}} + 32.0^\circ = 90.0^\circ$, so $\theta_{\text{B}} = 58.0^\circ$ and $n_{\text{glass}} = \tan\theta_{\text{B}} = \tan 58.0^\circ = 1.60$.

92. (a) In the notation of this problem, Eq. 34-47 becomes

$$\theta_c = \sin^{-1}\frac{n_3}{n_2}$$

which yields $n_3 = 1.39$ for $\theta_c = \phi = 60^\circ$.

(b) Applying Eq. 34-44 law to the interface between material 1 and material 2, we have

$$n_2 \sin 30^\circ = n_1 \sin\theta$$

which yields $\theta = 28.1^\circ$.

(c) Decreasing θ will increase ϕ and thus cause the ray to strike the interface (between materials 2 and 3) at an angle larger than θ_c. Therefore, no transmission of light into material 3 can occur.

93. We apply Eq. 34-40 (once) and Eq. 34-42 (twice) to obtain

$$I = \frac{1}{2}\,I_0\,\cos^2\theta_1\,\cos^2\theta_2 \ .$$

With $\theta_1 = 110^\circ$ and $\theta_2 = 50^\circ$, this yields $I/I_0 = 0.024$.

94. (a) The wave is traveling in the $-y$ direction (see §17-5 for the significance of the relative sign between the spatial and temporal arguments of the wave function).

(b) Figure 34-5 may help in visualizing this. The direction of propagation (along the y axis) is perpendicular to $\vec{B}$ (presumably along the x axis, since the problem gives B_x and no other component) and both are perpendicular to $\vec{E}$ (which determines the axis of polarization). Thus, the wave is z-polarized.

(c) Since the magnetic field amplitude is $B_m = 4.00\ \mu$T, then (by Eq. 34-5) $E_m = 1199$ V/m. Dividing by $\sqrt{2}$ yields $E_{\text{rms}} = 848$ V/m. Then, Eq. 34-26 gives

$$I = \frac{1}{c\mu_0}\,E_{\text{rms}}^2 = 1.91\times10^3\ \text{W/m}^2 \ .$$

(d) Since $kc = \omega$ (equivalent to $c = f\lambda$), we have

$$k = \frac{2.00\times10^{15}}{c} = 6.67\times10^6\ \text{m}^{-1} \ .$$

Summarizing the information gathered so far, we have (with SI units understood)

$$E_z = 1199\sin\left(\left(6.67\times10^6\right)y + \left(2.00\times10^{15}\right)t\right) \ .$$

(e) and (f) Since $\lambda = 2\pi/k = 942$ nm, we see that this is infrared light.

95. From Eq. 34-26, we have $E_{\text{rms}} = \sqrt{\mu_0 \, c \, I} = 1941$ V/m, which implies (using Eq. 34-5) that $B_{\text{rms}} = 1941/c = 6.47 \times 10^{-6}$ T. Multiplying by $\sqrt{2}$ yields the magnetic field amplitude $B_m = 9.16 \times 10^{-6}$ T.

96. (a) The frequency is

$$f = \frac{c}{\lambda} = \frac{3.0 \times 10^8 \, \text{m/s}}{0.067 \times 10^{-15} \, \text{m}} = 4.5 \times 10^{24} \text{ Hz} .$$

(b) In this case, the (very long) wavelength is

$$\lambda = \frac{c}{f} = \frac{3.0 \times 10^8 \, \text{m/s}}{30 \, \text{Hz}} = 1.0 \times 10^7 \, \text{m}$$

which is about 1.6 Earth radii.

97. The fraction is

$$\frac{\pi R_e^2}{4\pi d_{es}^2} = \frac{1}{4} \left(\frac{6.37 \times 10^6 \, \text{m}}{1.50 \times 10^{11} \, \text{m}} \right)^2 = 4.51 \times 10^{-10} .$$

98. (a) When examining Fig. 34-73, it is important to note that the angle (measured from the central axis) for the light ray in air, θ, is not the angle for the ray in the glass core, which we denote θ'. The law of refraction leads to

$$\sin\theta' = \frac{1}{n_1} \sin\theta \qquad \text{assuming} \quad n_{\text{air}} = 1 .$$

The angle of incidence for the light ray striking the coating is the complement of θ', which we denote as θ'_{comp} and recall that

$$\sin\theta'_{\text{comp}} = \cos\theta' = \sqrt{1 - \sin^2\theta'} .$$

In the critical case, θ'_{comp} must equal θ_c specified by Eq. 34-47. Therefore,

$$\frac{n_2}{n_1} = \sin\theta'_{\text{comp}} = \sqrt{1 - \sin^2\theta'} = \sqrt{1 - \left(\frac{1}{n_1} \sin\theta\right)^2}$$

which leads to the result: $\sin\theta = \sqrt{n_1^2 - n_2^2}$.

(b) With $n_1 = 1.58$ and $n_2 = 1.53$, we obtain

$$\theta = \sin^{-1}\left(1.58^2 - 1.53^2\right) = 23.2^\circ .$$

99. (a) In our solution here, we assume the reader has looked at our solution for problem 98. A light ray traveling directly along the central axis reaches the end in time

$$t_{\text{direct}} = \frac{L}{v_1} = \frac{n_1 L}{c} .$$

For the ray taking the critical zig-zag path, only its velocity component along the core axis direction contributes to reaching the other end of the fiber. That component is $v_1 \cos\theta'$, so the time of travel for this ray is

$$t_{\text{zig zag}} = \frac{L}{v_1 \cos\theta'} = \frac{n_1 L}{c\sqrt{1 - \left(\frac{1}{n_1} \sin\theta\right)^2}}$$

using results from the previous solution. Plugging in $\sin\theta = \sqrt{n_1^2 - n_2^2}$ and simplifying, we obtain

$$t_{\text{zig zag}} = \frac{n_1 L}{c(n_2/n_1)} = \frac{n_1^2 L}{n_2 c} .$$

The difference $t_{\text{zig zag}} - t_{\text{direct}}$ readily yields the result shown in the problem statement.

(b) With $n_1 = 1.58$, $n_2 = 1.53$ and $L = 300$ m, we obtain $\Delta t = 52$ ns.

100. (a) The condition (in Eq. 34-44) required in the critical angle calculation is $\theta_3 = 90°$. Thus (with $\theta_2 = \theta_c$, which we don't compute here),

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 = n_3 \sin\theta_3$$

leads to $\theta_1 = \theta = \sin^{-1} n_3/n_1 = 54.3°$.

(b) Reducing θ leads to a reduction of θ_2 so that it becomes less than the critical angle; therefore, there will be some transmission of light into material 3.

101. (a) We note that the complement of the angle of refraction (in material 2) is the critical angle. Thus,

$$n_1 \sin\theta = n_2 \cos\theta_c = n_2\sqrt{1 - \left(\frac{n_3}{n_2}\right)^2} = \sqrt{n_2^2 - n_3^2}$$

leads to $\theta = 51.1°$.

(b) Reducing θ leads to an increase of the angle with which the light strikes the interface between materials 2 and 3, so it becomes greater than the critical angle. Therefore, there will be no transmission of light into material 3.