Chapter 2

1. Assuming the horizontal velocity of the ball is constant, the horizontal displacement is

$$\Delta x = v\Delta t$$

where Δx is the horizontal distance traveled, Δt is the time, and v is the (horizontal) velocity. Converting v to meters per second, we have $160\,\text{km/h} = 44.4\,\text{m/s}$. Thus

$$\Delta t = \frac{\Delta x}{v} = \frac{18.4\,\text{m}}{44.4\,\text{m/s}} = 0.414\,\text{s}\,.$$

The velocity-unit conversion implemented above can be figured "from basics" (1000 m = 1 km, 3600 s = 1 h) or found in Appendix D.

2. Converting to SI units, we use Eq. 2-3 with d for distance.

$$\begin{aligned} s_{\text{avg}} &= \frac{d}{t} \\ (110.6\ \text{km/h})\left(\frac{1000\,\text{m/km}}{3600\ \text{s/h}}\right) &= \frac{200.0\,\text{m}}{t} \end{aligned}$$

which yields $t = 6.510$ s. We converted the speed km/h $\rightarrow$ m/s by converting each unit (km $\rightarrow$ m, h $\rightarrow$ s) individually. But we mention that the "one-step" conversion can be found in Appendix D ($1\,\text{km/h} = 0.2778\,\text{m/s}$).

3. We use Eq. 2-2 and Eq. 2-3. During a time t_c when the velocity remains a positive constant, speed is equivalent to velocity, and distance is equivalent to displacement, with $\Delta x = v\,t_c$.

   (a) During the first part of the motion, the displacement is $\Delta x_1 = 40$ km and the time interval is

   $$t_1 = \frac{(40\,\text{km})}{(30\,\text{km/h})} = 1.33\,\text{h}\ .$$

   During the second part the displacement is $\Delta x_2 = 40\,\text{km}$ and the time interval is

   $$t_2 = \frac{(40\,\text{km})}{(60\,\text{km/h})} = 0.67\,\text{h}\,.$$

   Both displacements are in the same direction, so the total displacement is $\Delta x = \Delta x_1 + \Delta x_2 = 40\,\text{km} + 40\,\text{km} = 80\,\text{km}$. The total time for the trip is $t = t_1 + t_2 = 2.00\,\text{h}$. Consequently, the average velocity is

   $$v_{\text{avg}} = \frac{(80\,\text{km})}{(2.0\,\text{h})} = 40\,\text{km/h}\ .$$

   (b) In this example, the numerical result for the average speed is the same as the average velocity $40\,\text{km/h}$.

(c) In the interest of saving space, we briefly describe the graph (with kilometers and hours understood): two contiguous line segments, the first having a slope of 30 and connecting the origin to $(t_1, x_1) = (1.33, 40)$ and the second having a slope of 60 and connecting (t_1, x_1) to $(t, x) = (2.00, 80)$. The average velocity, from the graphical point of view, is the slope of a line drawn from the origin to (t, x).

4. If the plane (with velocity v) maintains its present course, and if the terrain continues its upward slope of 4.3°, then the plane will strike the ground after traveling

$$\Delta x = \frac{h}{\tan\theta} = \frac{35\,\text{m}}{\tan 4.3^\circ} = 465.5 \text{ m} \approx 0.465 \text{ km} .$$

This corresponds to a time of flight found from Eq. 2-2 (with $v = v_{\text{avg}}$ since it is constant)

$$t = \frac{\Delta x}{v} = \frac{0.465\,\text{km}}{1300\,\text{km/h}} = 0.000358 \text{ h} \approx 1.3 \text{ s} .$$

This, then, estimates the time available to the pilot to make his correction.

5. (a) Denoting the travel time and distance from San Antonio to Houston as T and D, respectively, the average speed is

$$s_{\text{avg }1} = \frac{D}{T} = \frac{(55\,\text{km/h})\frac{T}{2} + (90\,\text{km/h})\frac{T}{2}}{T} = 72.5 \text{ km/h}$$

which should be rounded to 73 km/h.

(b) Using the fact that time = distance/speed while the speed is constant, we find

$$s_{\text{avg }2} = \frac{D}{T} = \frac{D}{\frac{D/2}{55\,\text{km/h}} + \frac{D/2}{90\,\text{km/h}}} = 68.3 \text{ km/h}$$

which should be rounded to 68 km/h.

(c) The total distance traveled ($2D$) must not be confused with the net displacement (zero). We obtain for the two-way trip

$$s_{\text{avg}} = \frac{2D}{\frac{D}{72.5\,\text{km/h}} + \frac{D}{68.3\,\text{km/h}}} = 70 \text{ km/h} .$$

(d) Since the net displacement vanishes, the average velocity for the trip in its entirety is zero.

(e) In asking for a *sketch*, the problem is allowing the student to arbitrarily set the distance D (the intent is *not* to make the student go to an Atlas to look it up); the student can just as easily arbitrarily set T instead of D, as will be clear in the following discussion. In the interest of saving space, we briefly describe the graph (with kilometers-per-hour understood for the slopes): two contiguous line segments, the first having a slope of 55 and connecting the origin to $(t_1, x_1) = (T/2, 55T/2)$ and the second having a slope of 90 and connecting (t_1, x_1) to (T, D) where $D = (55 + 90)T/2$. The average velocity, from the graphical point of view, is the slope of a line drawn from the origin to (T, D).

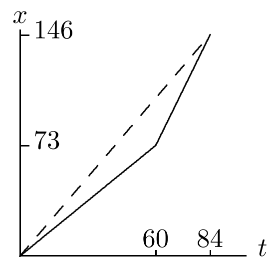
6. (a) Using the fact that time = distance/velocity while the velocity is constant, we find

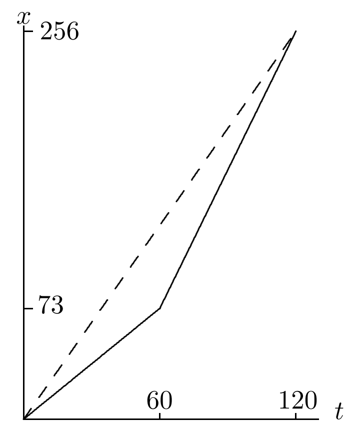
$$v_{\text{avg}} = \frac{73.2\,\text{m} + 73.2\,\text{m}}{\frac{73.2\,\text{m}}{1.22\,\text{m/s}} + \frac{73.2\,\text{m}}{3.05\,\text{m}}} = 1.74\,\text{m/s} .$$

(b) Using the fact that distance = vt while the velocity v is constant, we find

$$v_{\text{avg}} = \frac{(1.22\,\text{m/s})(60\,\text{s}) + (3.05\,\text{m/s})(60\,\text{s})}{120\,\text{s}} = 2.14\,\text{m/s} .$$

(c) The graphs are shown below (with meters and seconds understood). The first consists of two (solid) line segments, the first having a slope of 1.22 and the second having a slope of 3.05. The slope of the dashed line represents the average velocity (in both graphs). The second graph also consists of two (solid) line segments, having the same slopes as before – the main difference (compared to the first graph) being that the stage involving higher-speed motion lasts much longer.





7. Using $x = 3t - 4t^2 + t^3$ with SI units understood is efficient (and is the approach we will use), but if we wished to make the units explicit we would write $x = (3\,\text{m/s})t - (4\,\text{m/s}^2)t^2 + (1\,\text{m/s}^3)t^3$. We will quote our answers to one or two significant figures, and not try to follow the significant figure rules rigorously.

(a) Plugging in $t = 1$ s yields $x = 0$. With $t = 2$ s we get $x = -2$ m. Similarly, $t = 3$ s yields $x = 0$ and $t = 4$ s yields $x = 12$ m. For later reference, we also note that the position at $t = 0$ is $x = 0$.

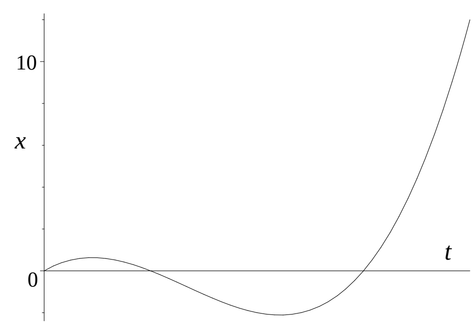
(b) The position at $t = 0$ is subtracted from the position at $t = 4$ s to find the displacement $\Delta x = 12$ m.

(c) The position at $t = 2$ s is subtracted from the position at $t = 4$ s to give the displacement $\Delta x = 14$ m. Eq. 2-2, then, leads to

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{14}{2} = 7 \text{ m/s} .$$

(d) The horizontal axis is $0 \le t \le 4$ with SI units understood.

Not shown is a straight line drawn from the point at $(t, x) = (2, -2)$ to the highest point shown (at $t = 4$ s) which would represent the answer for part (c).



8. Recognizing that the gap between the trains is closing at a constant rate of 60 km/h, the total time which elapses before they crash is $t = (60\,\text{km})/(60\,\text{km/h}) = 1.0$ h. During this time, the bird travels a distance of $x = vt = (60\,\text{km/h})(1.0\,\text{h}) = 60$ km.

9. Converting to seconds, the running times are $t_1 = 147.95$ s and $t_2 = 148.15$ s, respectively. If the runners were equally fast, then

$$s_{\text{avg }1} = s_{\text{avg }2} \implies \frac{L_1}{t_1} = \frac{L_2}{t_2} .$$

From this we obtain

$$L_2 - L_1 = \left(\frac{148.15}{147.95} - 1\right) L_1 \approx 1.35 \text{ m}$$

where we set $L_1 \approx 1000$ m in the last step. Thus, if L_1 and L_2 are no different than about 1.35 m, then runner 1 is indeed faster than runner 2. However, if L_1 is shorter than L_2 than 1.4 m then runner 2 is actually the faster.

10. The velocity (both magnitude and sign) is determined by the slope of the x versus t curve, in accordance with Eq. 2-4.

    (a) The armadillo is to the left of the coordinate origin on the axis between $t = 2.0$ s and $t = 4.0$ s.

    (b) The velocity is negative between $t = 0$ and $t = 3.0$ s.

    (c) The velocity is positive between $t = 3.0$ s and $t = 7.0$ s.

    (d) The velocity is zero at the graph minimum (at $t = 3.0$ s).

11. We use Eq. 2-4.

    (a) The velocity of the particle is

    $$v = \frac{dx}{dt} = \frac{d}{dt}\left(4 - 12t + 3t^2\right) = -12 + 6t .$$

    Thus, at $t = 1$ s, the velocity is $v = (-12 + (6)(1)) = -6$ m/s.

    (b) Since $v < 0$, it is moving in the negative x direction at $t = 1$ s.

    (c) At $t = 1$ s, the *speed* is $|v| = 6$ m/s.

    (d) For $0 < t < 2$ s, $|v|$ decreases until it vanishes. For $2 < t < 3$ s, $|v|$ increases from zero to the value it had in part (c). Then, $|v|$ is larger than that value for $t > 3$ s.

    (e) Yes, since v smoothly changes from negative values (consider the $t = 1$ result) to positive (note that as $t \to +\infty$, we have $v \to +\infty$). One can check that $v = 0$ when $t = 2$ s.

    (f) No. In fact, from $v = -12 + 6t$, we know that $v > 0$ for $t > 2$ s.

12. We use Eq. 2-2 for average velocity and Eq. 2-4 for instantaneous velocity, and work with distances in centimeters and times in seconds.

    (a) We plug into the given equation for x for $t = 2.00$ s and $t = 3.00$ s and obtain $x_2 = 21.75$ cm and $x_3 = 50.25$ cm, respectively. The average velocity during the time interval $2.00 \le t \le 3.00$ s is

    $$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{50.25\,\text{cm} - 21.75\,\text{cm}}{3.00\,\text{s} - 2.00\,\text{s}}$$

    which yields $v_{\text{avg}} = 28.5$ cm/s.

    (b) The instantaneous velocity is $v = \frac{dx}{dt} = 4.5t^2$, which yields $v = (4.5)(2.00)^2 = 18.0$ cm/s at time $t = 2.00$ s.

    (c) At $t = 3.00$ s, the instantaneous velocity is $v = (4.5)(3.00)^2 = 40.5$ cm/s.

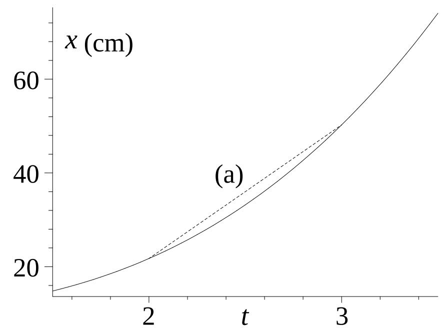
    (d) At $t = 2.50$ s, the instantaneous velocity is $v = (4.5)(2.50)^2 = 28.1$ cm/s.

    (e) Let t_m stand for the moment when the particle is midway between x_2 and x_3 (that is, when the particle is at $x_m = (x_2 + x_3)/2 = 36$ cm). Therefore,

    $$x_m = 9.75 + 1.5t_m^3 \implies t_m = 2.596$$

    in seconds. Thus, the instantaneous speed at this time is $v = 4.5(2.596)^2 = 30.3$ cm/s.

(f) The answer to part (a) is given by the slope of the straight line between $t = 2$ and $t = 3$ in this x-vs-t plot. The answers to parts (b), (c), (d) and (e) correspond to the slopes of tangent lines (not shown but easily imagined) to the curve at the appropriate points.

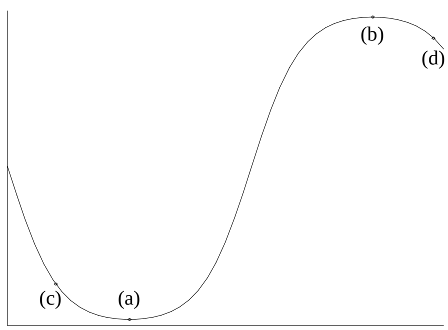


13. Since $v = \frac{dx}{dt}$ (Eq. 2-4), then $\Delta x = \int v\, dt$, which corresponds to the area under the v vs t graph. Dividing the total area A into rectangular (base×height) and triangular ($\frac{1}{2}$base×height) areas, we have

$$
\begin{aligned}
A &= A_{0<t<2} + A_{2<t<10} + A_{10<t<12} + A_{12<t<16} \\
&= \frac{1}{2}(2)(8) + (8)(8) + \left((2)(4) + \frac{1}{2}(2)(4)\right) + (4)(4)
\end{aligned}
$$

with SI units understood. In this way, we obtain $\Delta x = 100$ m.

14. From Eq. 2-4 and Eq. 2-9, we note that the sign of the velocity is the sign of the slope in an x-vs-t plot, and the sign of the acceleration corresponds to whether such a curve is concave up or concave down. In the interest of saving space, we indicate sample points for parts (a)-(d) in a single figure; this means that all points are not at $t = 1$ s (which we feel is an acceptable modification of the problem – since the datum $t = 1$ s is not used).



Any change from zero to non-zero values of $\vec{v}$ represents increasing $|\vec{v}|$ (speed). Also, $\vec{v} \parallel \vec{a}$ implies that the particle is going faster. Thus, points (a), (b) and (d) involve increasing speed.

15. We appeal to Eq. 2-4 and Eq. 2-9.

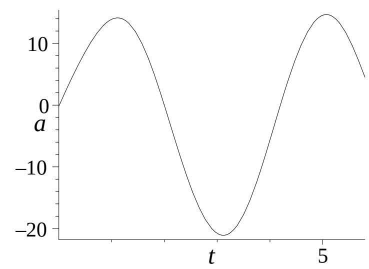
(a) This is v^2 – that is, the velocity squared.

(b) This is the acceleration a.

(c) The SI units for these quantities are $(\text{m/s})^2 = \text{m}^2/\text{s}^2$ and m/s^2, respectively.

16. Eq. 2-9 indicates that acceleration is the slope of the v-vs-t graph.

Based on this, we show here a sketch of the acceleration (in m/s^2) as a function of time. The values along the acceleration axis should not be taken too seriously.



17. We represent its initial direction of motion as the $+x$ direction, so that $v_0 = +18$ m/s and $v = -30$ m/s (when $t = 2.4$ s). Using Eq. 2-7 (or Eq. 2-11, suitably interpreted) we find

$$a_{\text{avg}} = \frac{(-30) - (+18)}{2.4} = -20 \text{ m/s}^2$$

which indicates that the average acceleration has magnitude 20 m/s^2 and is in the opposite direction to the particle's initial velocity.

18. We use Eq. 2-2 (average velocity) and Eq. 2-7 (average acceleration). Regarding our coordinate choices, the initial position of the man is taken as the origin and his direction of motion during $5\,\text{min} \le t \le 10\,\text{min}$ is taken to be the positive x direction. We also use the fact that $\Delta x = v\Delta t'$ when the velocity is constant during a time interval $\Delta t'$.

(a) Here, the entire interval considered is $\Delta t = 8 - 2 = 6$ min which is equivalent to 360 s, whereas the sub-interval in which he is *moving* is only $\Delta t' = 8 - 5 = 3$ min$=$ 180 s. His position at $t = 2$ min is $x = 0$ and his position at $t = 8$ min is $x = v\Delta t' = (2.2)(180) = 396$ m. Therefore,

$$v_{\text{avg}} = \frac{396\,\text{m} - 0}{360\,\text{s}} = 1.10 \text{ m/s} .$$

(b) The man is at rest at $t = 2$ min and has velocity $v = +2.2$ m/s at $t = 8$ min. Thus, keeping the answer to 3 significant figures,

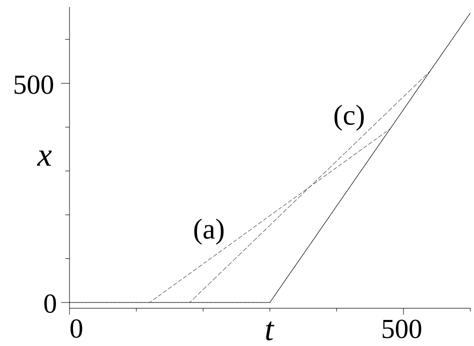
$$a_{\text{avg}} = \frac{2.2\,\text{m/s} - 0}{360\,\text{s}} = 0.00611 \text{ m/s}^2 .$$

(c) Now, the entire interval considered is $\Delta t = 9 - 3 = 6$ min (360 s again), whereas the sub-interval in which he is moving is $\Delta t' = 9 - 5 = 4$ min$=$ 240 s). His position at $t = 3$ min is $x = 0$ and his position at $t = 9$ min is $x = v\Delta t' = (2.2)(240) = 528$ m. Therefore,

$$v_{\text{avg}} = \frac{528\,\text{m} - 0}{360\,\text{s}} = 1.47 \text{ m/s} .$$

(d) The man is at rest at $t = 3$ min and has velocity $v = +2.2$ m/s at $t = 9$ min. Consequently, $a_{\text{avg}} = 2.2/360 = 0.00611$ m/s^2 just as in part (b).

(e) The horizontal line near the bottom of this x-vs-t graph represents the man standing at $x = 0$ for $0 \le t < 300$ s and the linearly rising line for $300 \le t \le 600$ s represents his constant-velocity motion. The dotted lines represent the answers to part (a) and (c) in the sense that their slopes yield those results.



The graph of v-vs-t is not shown here, but would consist of two horizontal "steps" (one at $v = 0$ for $0 \le t < 300$ s and the next at $v = 2.2$ m/s for $300 \le t \le 600$ s). The indications of the average accelerations found in parts (b) and (d) would be dotted lines connected the "steps" at the appropriate t values (the slopes of the dotted lines representing the values of a_{avg}).

19. In this solution, we make use of the notation $x(t)$ for the value of x at a particular t. Thus, $x(t) = 50t + 10t^2$ with SI units (meters and seconds) understood.

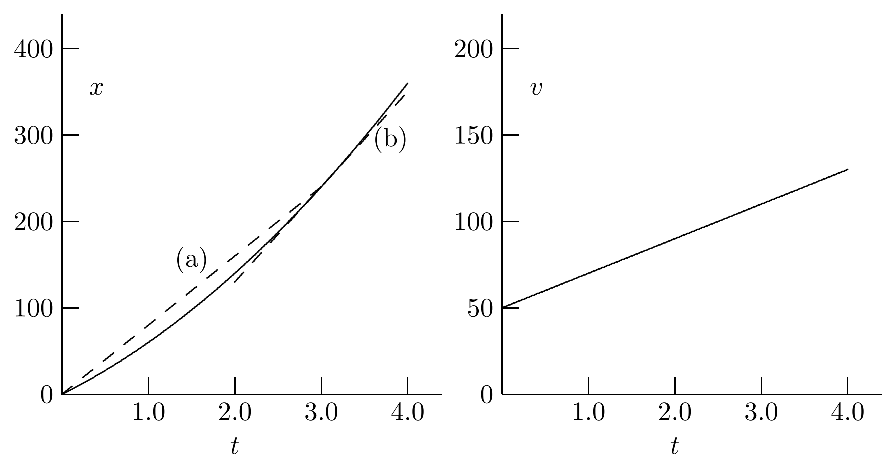
    (a) The average velocity during the first 3 s is given by

$$v_{\text{avg}} = \frac{x(3) - x(0)}{\Delta t} = \frac{(50)(3) + (10)(3)^2 - 0}{3} = 80 \text{ m/s} .$$

    (b) The instantaneous velocity at time t is given by $v = dx/dt = 50 + 20t$, in SI units. At $t = 3.0$ s, $v = 50 + (20)(3.0) = 110$ m/s.

    (c) The instantaneous acceleration at time t is given by $a = dv/dt = 20 \text{ m/s}^2$. It is constant, so the acceleration at any time is 20 m/s^2.

    (d) and (e) The graphs below show the coordinate x and velocity v as functions of time, with SI units understood. The dotted line marked (a) in the first graph runs from $t = 0$, $x = 0$ to $t = 3.0$ s, $x = 240$ m. Its slope is the average velocity during the first 3 s of motion. The dotted line marked (b) is tangent to the x curve at $t = 3.0$ s. Its slope is the instantaneous velocity at $t = 3.0$ s.



20. Using the general property $\frac{d}{dx}\exp(bx) = b\exp(bx)$, we write

$$v = \frac{dx}{dt} = \left(\frac{d\,(19t)}{dt}\right) \cdot e^{-t} + (19t) \cdot \left(\frac{d\,e^{-t}}{dt}\right) \quad .$$

If a concern develops about the appearance of an argument of the exponential $(-t)$ apparently having units, then an explicit factor of $1/T$ where $T = 1$ second can be inserted and carried through the computation (which does not change our answer). The result of this differentiation is

$$v = 16(1-t)e^{-t}$$

with t and v in SI units (s and m/s, respectively). We see that this function is zero when $t = 1$ s. Now that we know *when* it stops, we find out *where* it stops by plugging our result $t = 1$ into the given function $x = 16te^{-t}$ with x in meters. Therefore, we find $x = 5.9$ m.

21. In this solution, we make use of the notation $x(t)$ for the value of x at a particular t. The notations $v(t)$ and $a(t)$ have similar meanings.

   (a) Since the unit of ct^2 is that of length, the unit of c must be that of length/time^2, or m/s^2 in the SI system. Since bt^3 has a unit of length, b must have a unit of length/time^3, or m/s^3.

   (b) When the particle reaches its maximum (or its minimum) coordinate its velocity is zero. Since the velocity is given by $v = dx/dt = 2ct - 3bt^2$, $v = 0$ occurs for $t = 0$ and for

$$t = \frac{2c}{3b} = \frac{2(3.0\,\text{m/s}^2)}{3(2.0\,\text{m/s}^3)} = 1.0\,\text{s}\,.$$

   For $t = 0$, $x = x_0 = 0$ and for $t = 1.0\,\text{s}, x = 1.0\,\text{m} > x_0$. Since we seek the maximum, we reject the first root $(t = 0)$ and accept the second $(t = 1\,\text{s})$.

   (c) In the first 4 s the particle moves from the origin to $x = 1.0$ m, turns around, and goes back to

$$x(4\,\text{s}) = (3.0\,\text{m/s}^2)(4.0\,\text{s})^2 - (2.0\,\text{m/s}^3)(4.0\,\text{s})^3 = -80\,\text{m}\ .$$

   The total path length it travels is $1.0\,\text{m} + 1.0\,\text{m} + 80\,\text{m} = 82\,\text{m}$.

   (d) Its displacement is given by $\Delta x = x_2 - x_1$, where $x_1 = 0$ and $x_2 = -80\,\text{m}$. Thus, $\Delta x = -80\,\text{m}$.

   (e) The velocity is given by $v = 2ct - 3bt^2 = (6.0\,\text{m/s}^2)t - (6.0\,\text{m/s}^3)t^2$. Thus

$$\begin{aligned}
v(1\,\text{s}) &= (6.0\,\text{m/s}^2)(1.0\,\text{s}) - (6.0\,\text{m/s}^3)(1.0\,\text{s})^2 = 0\\
v(2\,\text{s}) &= (6.0\,\text{m/s}^2)(2.0\,\text{s}) - (6.0\,\text{m/s}^3)(2.0\,\text{s})^2 = -12\,\text{m/s}\\
v(3\,\text{s}) &= (6.0\,\text{m/s}^2)(3.0\,\text{s}) - (6.0\,\text{m/s}^3)(3.0\,\text{s})^2 = -36.0\,\text{m/s}\\
v(4\,\text{s}) &= (6.0\,\text{m/s}^2)(4.0\,\text{s}) - (6.0\,\text{m/s}^3)(4.0\,\text{s})^2 = -72\,\text{m/s}\ .
\end{aligned}$$

   (f) The acceleration is given by $a = dv/dt = 2c - 6b = 6.0\,\text{m/s}^2 - (12.0\,\text{m/s}^3)t$. Thus

$$\begin{aligned}
a(1\,\text{s}) &= 6.0\,\text{m/s}^2 - (12.0\,\text{m/s}^3)(1.0\,\text{s}) = -6.0\,\text{m/s}^2\\
a(2\,\text{s}) &= 6.0\,\text{m/s}^2 - (12.0\,\text{m/s}^3)(2.0\,\text{s}) = -18\,\text{m/s}^2\\
a(3\,\text{s}) &= 6.0\,\text{m/s}^2 - (12.0\,\text{m/s}^3)(3.0\,\text{s}) = -30\,\text{m/s}^2\\
a(4\,\text{s}) &= 6.0\,\text{m/s}^2 - (12.0\,\text{m/s}^3)(4.0\,\text{s}) = -42\,\text{m/s}^2\ .
\end{aligned}$$

22. For the automobile $\Delta v = 55 - 25 = 30\,\text{km/h}$, which we convert to SI units:

$$a = \frac{\Delta v}{\Delta t} = \frac{(30\,\text{km/h})\left(\frac{1000\,\text{m/km}}{3600\,\text{s/h}}\right)}{(0.50\,\text{min})(60\,\text{s/min})} = 0.28\ \text{m/s}^2\ .$$

   The change of velocity for the bicycle, for the same time, is identical to that of the car, so its acceleration is also 0.28 m/s^2.

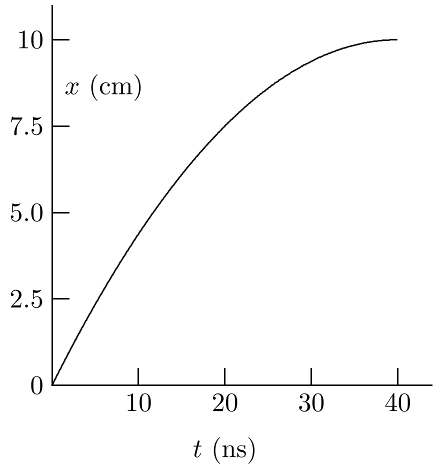
23. The constant-acceleration condition permits the use of Table 2-1.

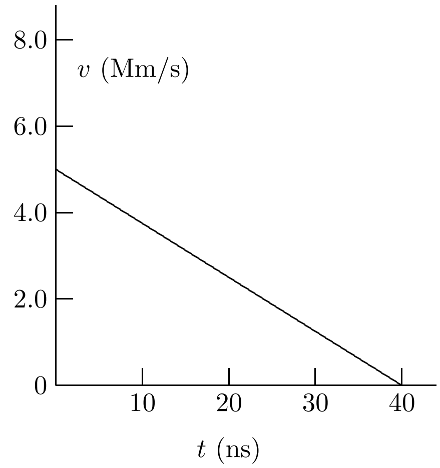
(a) Setting $v = 0$ and $x_0 = 0$ in $v^2 = v_0^2 + 2a(x - x_0)$, we find

$$x = -\frac{1}{2}\frac{v_0^2}{a} = -\frac{1}{2}\left(\frac{5.00 \times 10^6}{-1.25 \times 10^{14}}\right) = 0.100 \text{ m} .$$

Since the muon is slowing, the initial velocity and the acceleration must have opposite signs.

(b) Below are the time-plots of the position x and velocity v of the muon from the moment it enters the field to the time it stops. The computation in part (a) made no reference to t, so that other equations from Table 2-1 (such as $v = v_0 + at$ and $x = v_0 t + \frac{1}{2}at^2$) are used in making these plots.





24. The time required is found from Eq. 2-11 (or, suitably interpreted, Eq. 2-7). First, we convert the velocity change to SI units:

$$\Delta v = (100\,\text{km/h})\left(\frac{1000\,\text{m/km}}{3600\,\text{s/h}}\right) = 27.8 \text{ m/s} .$$

Thus, $\Delta t = \Delta v/a = 27.8/50 = 0.556$ s.

25. We use $v = v_0 + at$, with $t = 0$ as the instant when the velocity equals +9.6 m/s.

(a) Since we wish to calculate the velocity for a time *before* $t = 0$, we set $t = -2.5$ s. Thus, Eq. 2-11 gives

$$v = (9.6\,\text{m/s}) + \left(3.2\,\text{m/s}^2\right)(-2.5\,\text{s}) = 1.6 \text{ m/s} .$$

(b) Now, $t = +2.5$ s and we find

$$v = (9.6\,\text{m/s}) + \left(3.2\,\text{m/s}^2\right)(2.5\,\text{s}) = 18 \text{ m/s} .$$

26. The bullet starts at rest ($v_0 = 0$) and after traveling the length of the barrel ($\Delta x = 1.2$ m) emerges with the given velocity ($v = 640$ m/s), where the direction of motion is the positive direction. Turning to the constant acceleration equations in Table 2-1, we use

$$\Delta x = \frac{1}{2}(v_0 + v)\,t \quad .$$

Thus, we find $t = 0.00375$ s (about 3.8 ms).

27. The constant acceleration stated in the problem permits the use of the equations in Table 2-1.

(a) We solve $v = v_0 + at$ for the time:

$$t = \frac{v - v_0}{a} = \frac{\frac{1}{10}\left(3.0 \times 10^8\,\text{m/s}\right)}{9.8\,\text{m/s}^2} = 3.1 \times 10^6 \text{ s}$$

which is equivalent to 1.2 months.

(b) We evaluate $x = x_0 + v_0 t + \frac{1}{2}at^2$, with $x_0 = 0$. The result is

$$x = \frac{1}{2}\left(9.8\,\mathrm{m/s^2}\right)\left(3.1\times10^6\,\mathrm{s}\right)^2 = 4.7\times10^{13}\,\mathrm{m}\ .$$

28. From Table 2-1, $v^2 - v_0^2 = 2a\Delta x$ is used to solve for a. Its minimum value is

$$a_{\min} = \frac{v^2 - v_0^2}{2\Delta x_{\max}} = \frac{(360\,\mathrm{km/h})^2}{2(1.80\,\mathrm{km})} = 36000\ \mathrm{km/h^2}$$

which converts to 2.78 $\mathrm{m/s^2}$.

29. Assuming constant acceleration permits the use of the equations in Table 2-1. We solve $v^2 = v_0^2 + 2a(x - x_0)$ with $x_0 = 0$ and $x = 0.010$ m. Thus,

$$a = \frac{v^2 - v_0^2}{2x} = \frac{\left(5.7\times10^5\right)^2 - \left(1.5\times10^5\right)^2}{2(0.01)} = 1.62\times10^{15}\ \mathrm{m/s^2}\ .$$

30. The acceleration is found from Eq. 2-11 (or, suitably interpreted, Eq. 2-7).

$$a = \frac{\Delta v}{\Delta t} = \frac{(1020\,\mathrm{km/h})\left(\frac{1000\,\mathrm{m/km}}{3600\,\mathrm{s/h}}\right)}{1.4\,\mathrm{s}} = 202.4\ \mathrm{m/s^2}\ .$$

In terms of the gravitational acceleration g, this is expressed as a multiple of 9.8 $\mathrm{m/s^2}$ as follows:

$$a = \frac{202.4}{9.8}\,g = 21g\ .$$

31. We choose the positive direction to be that of the initial velocity of the car (implying that $a < 0$ since it is slowing down). We assume the acceleration is constant and use Table 2-1.

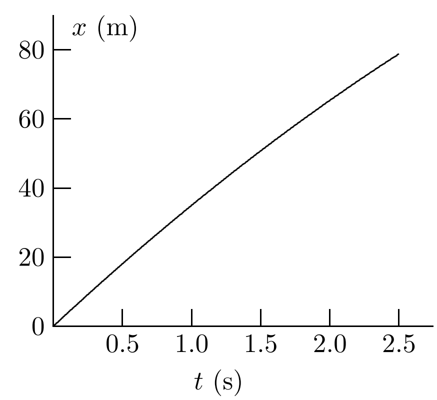
(a) Substituting $v_0 = 137\,\mathrm{km/h} = 38.1\,\mathrm{m/s}$, $v = 90\,\mathrm{km/h} = 25\,\mathrm{m/s}$, and $a = -5.2\,\mathrm{m/s^2}$ into $v = v_0 + at$, we obtain

$$t = \frac{25\,\mathrm{m/s} - 38\,\mathrm{m/s}}{-5.2\,\mathrm{m/s^2}} = 2.5\ \mathrm{s}\ .$$

(b) We take the car to be at $x = 0$ when the brakes are applied (at time $t = 0$). Thus, the coordinate of the car as a function of time is given by

$$x = (38)t + \frac{1}{2}(-5.2)t^2$$

in SI units. This function is plotted from $t = 0$ to $t = 2.5$ s on the graph to the right. We have not shown the v-vs-t graph here; it is a descending straight line from v_0 to v.



32. From the figure, we see that $x_0 = -2.0$ m. From Table 2-1, we can apply $x - x_0 = v_0 t + \frac{1}{2}at^2$ with $t = 1.0$ s, and then again with $t = 2.0$ s. This yields two equations for the two unknowns, v_0 and a. SI units are understood.

$$\begin{aligned} 0.0 - (-2.0) &= v_0\,(1.0) + \frac{1}{2}a(1.0)^2 \\ 6.0 - (-2.0) &= v_0\,(2.0) + \frac{1}{2}a(2.0)^2\ . \end{aligned}$$

Solving these simultaneous equations yields the results $v_0 = 0.0$ and $a = 4.0\ \mathrm{m/s^2}$. The fact that the answer is positive tells us that the acceleration vector points in the $+x$ direction.

33. The problem statement (see part (a)) indicates that $a =$ constant, which allows us to use Table 2-1.

(a) We take $x_0 = 0$, and solve $x = v_0 t + \frac{1}{2}at^2$ (Eq. 2-15) for the acceleration: $a = 2(x - v_0 t)/t^2$. Substituting $x = 24.0\,\mathrm{m}$, $v_0 = 56.0\,\mathrm{km/h} = 15.55\,\mathrm{m/s}$ and $t = 2.00\,\mathrm{s}$, we find

$$a = \frac{2\,(24.0\,\mathrm{m} - (15.55\,\mathrm{m/s})(2.00\,\mathrm{s}))}{(2.00\,\mathrm{s})^2} = -3.56\,\mathrm{m/s^2}\ .$$

The negative sign indicates that the acceleration is opposite to the direction of motion of the car. The car is slowing down.

(b) We evaluate $v = v_0 + at$ as follows:

$$v = 15.55\,\mathrm{m/s} - \left(3.56\,\mathrm{m/s^2}\right)(2.00\,\mathrm{s}) = 8.43\ \mathrm{m/s}$$

which is equivalent to 30.3 km/h.

34. We take the moment of applying brakes to be $t = 0$. The deceleration is constant so that Table 2-1 can be used. Our primed variables (such as $v'_\mathrm{o} = 72$ km/h $= 20$ m/s) refer to one train (moving in the $+x$ direction and located at the origin when $t = 0$) and unprimed variables refer to the other (moving in the $-x$ direction and located at $x_0 = +950$ m when $t = 0$). We note that the acceleration vector of the unprimed train points in the *positive* direction, even though the train is slowing down; its initial velocity is $v_\mathrm{o} = -144$ km/h $= -40$ m/s. Since the primed train has the lower initial speed, it should stop sooner than the other train would (were it not for the collision). Using Eq 2-16, it should stop (meaning $v' = 0$) at

$$x' = \frac{(v')^2 - (v'_\mathrm{o})^2}{2a'} = \frac{0 - 20^2}{-2} = 200\ \mathrm{m}\ .$$

The speed of the other train, when it reaches that location, is

$$v = \sqrt{v_\mathrm{o}^2 + 2a\Delta x} = \sqrt{(-40)^2 + 2(1.0)(200 - 950)} = \sqrt{100} = 10\ \mathrm{m/s}$$

using Eq 2-16 again. Specifically, its velocity at that moment would be -10 m/s since it is still traveling in the $-x$ direction when it crashes. If the computation of v had failed (meaning that a negative number would have been inside the square root) then we would have looked at the possibility that there was no collision and examined how far apart they finally were. A concern that can be brought up is whether the primed train collides before it comes to rest; this can be studied by computing the time it stops (Eq. 2-11 yields $t = 20$ s) and seeing where the unprimed train is at that moment (Eq. 2-18 yields $x = 350$ m, still a good distance away from contact).

35. The acceleration is constant and we may use the equations in Table 2-1.

(a) Taking the first point as coordinate origin and time to be zero when the car is there, we apply Eq. 2-17 (with SI units understood):

$$x = \frac{1}{2}(v + v_0)\,t = \frac{1}{2}(15 + v_0)(6)\ .$$

With $x = 60.0$ m (which takes the direction of motion as the $+x$ direction) we solve for the initial velocity: $v_0 = 5.00$ m/s.

(b) Substituting $v = 15$ m/s, $v_0 = 5$ m/s and $t = 6$ s into $a = (v - v_0)/t$ (Eq. 2-11), we find $a = 1.67\ \mathrm{m/s^2}$.

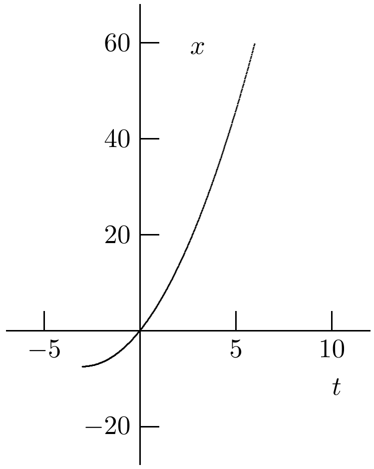
(c) Substituting $v = 0$ in $v^2 = v_0^2 + 2ax$ and solving for x, we obtain

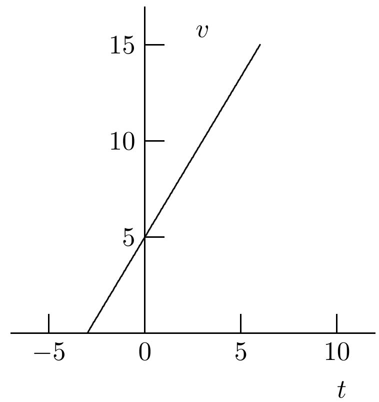
$$x = -\frac{v_0^2}{2a} = -\frac{5^2}{2(1.67)} = -7.50\ \mathrm{m}\ .$$

(d) The graphs require computing the time when $v = 0$, in which case, we use $v = v_0 + at' = 0$. Thus,

$$t' = \frac{-v_0}{a} = \frac{-5}{1.67} = -3.0 \text{ s}$$

indicates the moment the car was at rest. SI units are assumed.





36. We denote the required time as t, assuming the light turns green when the clock reads zero. By this time, the distances traveled by the two vehicles must be the same.

(a) Denoting the acceleration of the automobile as a and the (constant) speed of the truck as v then

$$\Delta x = \left(\frac{1}{2}at^2\right)_{\text{car}} = (vt)_{\text{truck}}$$

which leads to

$$t = \frac{2v}{a} = \frac{2(9.5)}{2.2} = 8.6 \text{ s} .$$

Therefore,

$$\Delta x = vt = (9.5)(8.6) = 82 \text{ m} .$$

(b) The speed of the car at that moment is

$$v_{\text{car}} = at = (2.2)(8.6) = 19 \text{ m/s} .$$

37. We denote t_r as the reaction time and t_b as the braking time. The motion during t_r is of the constant-velocity (call it v_0) type. Then the position of the car is given by

$$x = v_0 t_r + v_0 t_b + \frac{1}{2}at_b^2$$

where v_0 is the initial velocity and a is the acceleration (which we expect to be negative-valued since we are taking the velocity in the positive direction and we know the car is decelerating). *After* the brakes are applied the velocity of the car is given by $v = v_0 + at_b$. Using this equation, with $v = 0$, we eliminate t_b from the first equation and obtain

$$x = v_0 t_r - \frac{v_0^2}{a} + \frac{1}{2}\frac{v_0^2}{a} = v_0 t_r - \frac{1}{2}\frac{v_0^2}{a} .$$

We write this equation for each of the initial velocities:

$$x_1 = v_{01} t_r - \frac{1}{2}\frac{v_{01}^2}{a}$$

and

$$x_2 = v_{02}t_r - \frac{1}{2}\frac{v_{02}^2}{a} \ .$$

Solving these equations simultaneously for t_r and a we get

$$t_r = \frac{v_{02}^2 x_1 - v_{01}^2 x_2}{v_{01}v_{02}(v_{02} - v_{01})}$$

and

$$a = -\frac{1}{2}\frac{v_{02}v_{01}^2 - v_{01}v_{02}^2}{v_{02}x_1 - v_{01}x_2} \ .$$

Substituting $x_1 = 56.7\,\text{m}$, $v_{01} = 80.5\,\text{km/h} = 22.4\,\text{m/s}$, $x_2 = 24.4$ m and $v_{02} = 48.3\,\text{km/h} = 13.4$ m/s, we find

$$t_r = \frac{13.4^2(56.7) - 22.4^2(24.4)}{(22.4)(13.4)(13.4 - 22.4)} = 0.74 \text{ s}$$

and

$$a = -\frac{1}{2}\frac{(13.4)22.4^2 - (22.4)13.4^2}{(13.4)(56.7) - (22.4)(24.4)} = -6.2 \text{ m/s}^2 \ .$$

The *magnitude* of the deceleration is therefore 6.2 m/s^2. Although rounded off values are displayed in the above substitutions, what we have input into our calculators are the "exact" values (such as $v_{02} = \frac{161}{12}$ m/s).

38. In this solution we elect to wait until the last step to convert to SI units. Constant acceleration is indicated, so use of Table 2-1 is permitted. We start with Eq. 2-17 and denote the train's initial velocity as v_t and the locomotive's velocity as v_ℓ (which is also the final velocity of the train, if the rear-end collision is barely avoided). We note that the distance Δx consists of the original gap between them D as well as the forward distance traveled during this time by the locomotive $v_\ell t$. Therefore,

$$\frac{v_t + v_\ell}{2} = \frac{\Delta x}{t} = \frac{D + v_\ell t}{t} = \frac{D}{t} + v_\ell \ .$$

We now use Eq. 2-11 to eliminate time from the equation. Thus,

$$\frac{v_t + v_\ell}{2} = \frac{D}{(v_\ell - v_t)/a} + v_\ell$$

leads to

$$a = \left(\frac{v_t + v_\ell}{2} - v_\ell\right)\left(\frac{v_\ell - v_t}{D}\right) = -\frac{1}{2D}(v_\ell - v_t)^2 \ .$$

Hence,

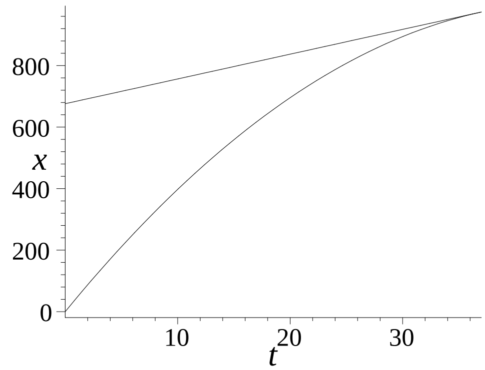
$$a = -\frac{1}{2(0.676\,\text{km})}\left(29\,\frac{\text{km}}{\text{h}} - 161\,\frac{\text{km}}{\text{h}}\right)^2 = -12888 \text{ km/h}^2$$

which we convert as follows:

$$a = \left(-12888\,\text{km/h}^2\right)\left(\frac{1000\,\text{m}}{1 \text{ km}}\right)\left(\frac{1 \text{ h}}{3600\,\text{s}}\right)^2 = -0.994 \text{ m/s}^2$$

so that its *magnitude* is 0.994 m/s^2. A graph is shown below for the case where a collision is just avoided (x along the vertical axis is in meters and t along the horizontal axis is in seconds). The top (straight) line shows the motion of the locomotive and the bottom curve shows the motion of the passenger train.

The other case (where the collision is not quite avoided) would be similar except that the slope of the bottom curve would be greater than that of the top line at the point where they meet.



39. We assume the periods of acceleration (duration t_1) and deceleration (duration t_2) are periods of constant a so that Table 2-1 can be used. Taking the direction of motion to be $+x$ then $a_1 = +1.22$ m/s^2 and $a_2 = -1.22$ m/s^2. We use SI units so the velocity at $t = t_1$ is $v = 305/60 = 5.08$ m/s.

   (a) We denote Δx as the distance moved during t_1, and use Eq. 2-16:

   $$v^2 = v_0^2 + 2a_1\Delta x \implies \Delta x = \frac{5.08^2}{2(1.22)}$$

   which yields $\Delta x = 10.59 \approx 10.6$ m.

   (b) Using Eq. 2-11, we have

   $$t_1 = \frac{v - v_0}{a_1} = \frac{5.08}{1.22} = 4.17 \text{ s} .$$

   The deceleration time t_2 turns out to be the same so that $t_1 + t_2 = 8.33$ s. The distances traveled during t_1 and t_2 are the same so that they total to $2(10.59) = 21.18$ m. This implies that for a distance of $190 - 21.18 = 168.82$ m, the elevator is traveling at constant velocity. This time of constant velocity motion is

   $$t_3 = \frac{168.82\,\text{m}}{5.08\,\text{m/s}} = 33.21 \text{ s} .$$

   Therefore, the total time is $8.33 + 33.21 \approx 41.5$ s.

40. Neglect of air resistance justifies setting $a = -g = -9.8$ m/s^2 (where *down* is our $-y$ direction) for the duration of the fall. This is constant acceleration motion, and we may use Table 2-1 (with Δy replacing Δx).

   (a) Using Eq. 2-16 and taking the negative root (since the final velocity is downward), we have

   $$v = -\sqrt{v_0^2 - 2g\Delta y} = -\sqrt{0 - 2(9.8)(-1700)} = -183$$

   in SI units. Its magnitude is therefore 183 m/s.

   (b) No, but it is hard to make a convincing case without more analysis. We estimate the mass of a raindrop to be about a gram or less, so that its mass and speed (from part (a)) would be less than that of a typical bullet, which is good news. But the fact that one is dealing with *many* raindrops leads us to suspect that this scenario poses an unhealthy situation. If we factor in air resistance, the final speed is smaller, of course, and we return to the relatively healthy situation with which we are familiar.

41. We neglect air resistance, which justifies setting $a = -g = -9.8$ m/s^2 (taking *down* as the $-y$ direction) for the duration of the fall. This is constant acceleration motion, which justifies the use of Table 2-1 (with Δy replacing Δx).

(a) Starting the clock at the moment the wrench is dropped ($v_o = 0$), then $v^2 = v_o^2 - 2g\Delta y$ leads to

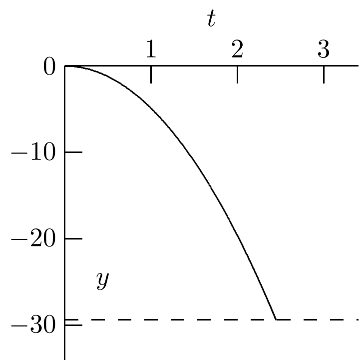
$$\Delta y = -\frac{(-24)^2}{2(9.8)} = -29.4 \text{ m}$$

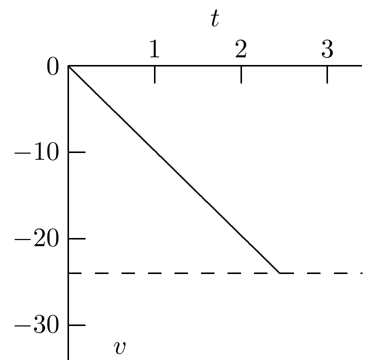
so that it fell through a height of 29.4 m.

(b) Solving $v = v_0 - gt$ for time, we find:

$$t = \frac{v_0 - v}{g} = \frac{0-(-24)}{9.8} = 2.45 \text{ s} .$$

(c) SI units are used in the graphs, and the initial position is taken as the coordinate origin. In the interest of saving space, we do not show the acceleration graph, which is a horizontal line at $-9.8\,\text{m/s}^2$.





42. We neglect air resistance, which justifies setting $a = -g = -9.8$ m/s^2 (taking *down* as the $-y$ direction) for the duration of the fall. This is constant acceleration motion, which justifies the use of Table 2-1 (with Δy replacing Δx).

(a) Noting that $\Delta y = y - y_0 = -30$ m, we apply Eq. 2-15 and the quadratic formula (Appendix E) to compute t:

$$\Delta y = v_0 t - \frac{1}{2}gt^2 \implies t = \frac{v_0 \pm \sqrt{v_0^2 - 2g\Delta y}}{g}$$

which (with $v_0 = -12$ m/s since it is downward) leads, upon choosing the positive root (so that $t > 0$), to the result:

$$t = \frac{-12 + \sqrt{(-12)^2 - 2(9.8)(-30)}}{9.8} = 1.54 \text{ s} .$$

(b) Enough information is now known that any of the equations in Table 2-1 can be used to obtain v; however, the one equation that does *not* use our result from part (a) is Eq. 2-16:

$$v = \sqrt{v_0^2 - 2g\Delta y} = 27.1 \text{ m/s}$$

where the positive root has been chosen in order to give *speed* (which is the magnitude of the velocity vector).

43. We neglect air resistance for the duration of the motion (between "launching" and "landing"), so $a = -g = -9.8$ m/s^2 (we take downward to be the $-y$ direction). We use the equations in Table 2-1 (with Δy replacing Δx) because this is $a =$ constant motion.

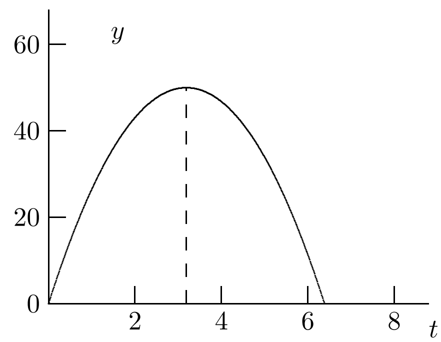
(a) At the highest point the velocity of the ball vanishes. Taking $y_0 = 0$, we set $v = 0$ in $v^2 = v_0^2 - 2gy$, and solve for the initial velocity: $v_0 = \sqrt{2gy}$. Since $y = 50$ m we find $v_0 = 31$ m/s.

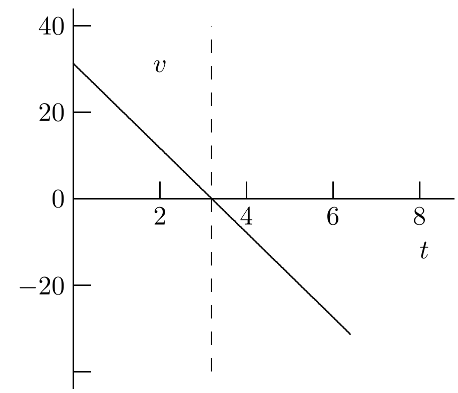
(b) It will be in the air from the time it leaves the ground until the time it returns to the ground ($y = 0$). Applying Eq. 2-15 to the entire motion (the rise and the fall, of total time $t > 0$) we have

$$y = v_0 t - \frac{1}{2} g t^2 \implies t = \frac{2v_0}{g}$$

which (using our result from part (a)) produces $t = 6.4$ s. It is possible to obtain this without using part (a)'s result; one can find the time just for the rise (from ground to highest point) from Eq. 2-16 and then double it.

(c) SI units are understood in the x and v graphs shown. In the interest of saving space, we do not show the graph of a, which is a horizontal line at $-9.8\,\text{m/s}^2$.





44. There is no air resistance, which makes it quite accurate to set $a = -g = -9.8$ m/s^2 (where downward is the $-y$ direction) for the duration of the fall. We are allowed to use Table 2-1 (with Δy replacing Δx) because this is constant acceleration motion; in fact, when the acceleration changes (during the process of catching the ball) we will again assume constant acceleration conditions; in this case, we have $a_2 = +25g = 245$ m/s^2.

(a) The time of fall is given by Eq. 2-15 with $v_0 = 0$ and $y = 0$. Thus,

$$t = \sqrt{\frac{2y_0}{g}} = \sqrt{\frac{2(145)}{9.8}} = 5.44 \text{ s} .$$

(b) The final velocity for its free-fall (which becomes the initial velocity during the catching process) is found from Eq. 2-16 (other equations can be used but they would use the result from part (a)).

$$v = -\sqrt{v_0^2 - 2g\,(y - y_0)} = -\sqrt{2gy_0} = -53.3 \text{ m/s}$$

where the negative root is chosen since this is a downward velocity.

(c) For the catching process, the answer to part (b) plays the role of an *initial* velocity ($v_0 = -53.3$ m/s) and the final velocity must become zero. Using Eq. 2-16, we find

$$\Delta y_2 = \frac{v^2 - v_0^2}{2a_2} = \frac{-(-53.3)^2}{2(245)} = -5.80 \text{ m}$$

where the negative value of Δy_2 signifies that the distance traveled while arresting its motion is downward.

45. Taking the $+y$ direction *downward* and $y_0 = 0$, we have $y = v_0 t + \frac{1}{2} g t^2$ which (with $v_0 = 0$) yields $t = \sqrt{2y/g}$.

(a) For this part of the motion, $y = 50$ m so that

$$t = \sqrt{\frac{2(50)}{9.8}} = 3.2 \text{ s} .$$

(b) For this next part of the motion, we note that the total displacement is $y = 100$ m. Therefore, the total time is

$$t = \sqrt{\frac{2(100)}{9.8}} = 4.5 \text{ s} .$$

The difference between this and the answer to part (a) is the time required to fall through that second 50 m distance: $4.5 - 3.2 = 1.3$ s.

46. We neglect air resistance, which justifies setting $a = -g = -9.8$ m/s^2 (taking down as the $-y$ direction) for the duration of the motion. We are allowed to use Table 2-1 (with Δy replacing Δx) because this is constant acceleration motion. The ground level is taken to correspond to $y = 0$.

(a) With $y_0 = h$ and v_0 replaced with $-v_0$, Eq. 2-16 leads to

$$v = \sqrt{(-v_0)^2 - 2g\,(y - y_0)} = \sqrt{v_0^2 + 2gh} .$$

The positive root is taken because the problem asks for the speed (the *magnitude* of the velocity).

(b) We use the quadratic formula to solve Eq. 2-15 for t, with v_0 replaced with $-v_0$,

$$\Delta y = -v_0 t - \frac{1}{2} g t^2 \quad \Longrightarrow \quad t = \frac{-v_0 + \sqrt{(-v_0)^2 - 2g\Delta y}}{g}$$

where the positive root is chosen to yield $t > 0$. With $y = 0$ and $y_0 = h$, this becomes

$$t = \frac{\sqrt{v_0^2 + 2gh} - v_0}{g} .$$

(c) If it were thrown upward with that speed from height h then (in the absence of air friction) it would return to height h with that same downward speed and would therefore yield the same final speed (before hitting the ground) as in part (a). An important perspective related to this is treated later in the book (in the context of energy conservation) .

(d) Having to travel up before it starts its descent certainly requires more time than in part (b). The calculation is quite similar, however, except for now having $+v_0$ in the equation where we had put in $-v_0$ in part (b). The details follow:

$$\Delta y = v_0 t - \frac{1}{2} g t^2 \quad \Longrightarrow \quad t = \frac{v_0 + \sqrt{v_0^2 - 2g\Delta y}}{g}$$

with the positive root again chosen to yield $t > 0$. With $y = 0$ and $y_0 = h$, we obtain

$$t = \frac{\sqrt{v_0^2 + 2gh} + v_0}{g} .$$

47. We neglect air resistance, which justifies setting $a = -g = -9.8$ m/s^2 (taking *down* as the $-y$ direction) for the duration of the motion. We are allowed to use Table 2-1 (with Δy replacing Δx) because this is constant acceleration motion. The ground level is taken to correspond to the origin of the y axis.

(a) Using $y = v_0 t - \frac{1}{2} g t^2$, with $y = 0.544$ m and $t = 0.200$ s, we find

$$v_0 = \frac{y + \frac{1}{2} g t^2}{t} = \frac{0.544 + \frac{1}{2}(9.8)(0.200)^2}{0.200} = 3.70 \text{ m/s} .$$

(b) The velocity at $y = 0.544$ m is

$$v = v_0 - gt = 3.70 - (9.8)(0.200) = 1.74 \text{ m/s} .$$

(c) Using $v^2 = v_0^2 - 2gy$ (with different values for y and v than before), we solve for the value of y corresponding to maximum height (where $v = 0$).

$$y = \frac{v_0^2}{2g} = \frac{3.7^2}{2(9.8)} = 0.698 \text{ m} .$$

Thus, the armadillo goes $0.698 - 0.544 = 0.154$ m higher.

48. We neglect air resistance, which justifies setting $a = -g = -9.8$ m/s^2 (taking *down* as the $-y$ direction) for the duration of the motion. We are allowed to use Table 2-1 (with Δy replacing Δx) because this is constant acceleration motion. The ground level is taken to correspond to the origin of the y axis. The total time of fall can be computed from Eq. 2-15 (using the quadratic formula).

$$\Delta y = v_0 t - \frac{1}{2}gt^2 \implies t = \frac{v_0 + \sqrt{v_0^2 - 2g\Delta y}}{g}$$

with the positive root chosen. With $y = 0$, $v_0 = 0$ and $y_0 = h = 60$ m, we obtain

$$t = \frac{\sqrt{2gh}}{g} = \sqrt{\frac{2h}{g}} = 3.5 \text{ s} .$$

Thus, "1.2 s earlier" means we are examining where the rock is at $t = 2.3$ s:

$$y - h = v_0(2.3) - \frac{1}{2}g(2.3)^2 \implies y = 34 \text{ m}$$

where we again use the fact that $h = 60$ m and $v_0 = 0$.

49. The speed of the boat is constant, given by $v_b = d/t$. Here, d is the distance of the boat from the bridge when the key is dropped (12 m) and t is the time the key takes in falling. To calculate t, we put the origin of the coordinate system at the point where the key is dropped and take the y axis to be positive in the *downward* direction. Taking the time to be zero at the instant the key is dropped, we compute the time t when $y = 45$ m. Since the initial velocity of the key is zero, the coordinate of the key is given by $y = \frac{1}{2}gt^2$. Thus

$$t = \sqrt{\frac{2y}{g}} = \sqrt{\frac{2(45\,\text{m})}{9.8\,\text{m/s}^2}} = 3.03 \text{ s} .$$

Therefore, the speed of the boat is

$$v_b = \frac{12\,\text{m}}{3.03\,\text{s}} = 4.0 \text{ m/s} .$$

50. With $+y$ upward, we have $y_0 = 36.6$ m and $y = 12.2$ m. Therefore, using Eq. 2-18 (the last equation in Table 2-1), we find

$$y - y_0 = vt + \frac{1}{2}gt^2 \implies v = -22 \text{ m/s}$$

at $t = 2.00$ s. The term *speed* refers to the magnitude of the velocity vector, so the answer is $|v| = 22.0$ m/s.

51. We first find the velocity of the ball just before it hits the ground. During contact with the ground its average acceleration is given by

$$a_{\text{avg}} = \frac{\Delta v}{\Delta t}$$

where Δv is the change in its velocity during contact with the ground and $\Delta t = 20.0 \times 10^{-3}$ s is the duration of contact. Now, to find the velocity just *before* contact, we put the origin at the point where the ball is dropped (and take $+y$ upward) and take $t = 0$ to be when it is dropped. The ball strikes the ground at $y = -15.0$ m. Its velocity there is found from Eq. 2-16: $v^2 = -2gy$. Therefore,

$$v = -\sqrt{-2gy} = -\sqrt{-2(9.8)(-15.0)} = -17.1 \text{ m/s}$$

where the negative sign is chosen since the ball is traveling downward at the moment of contact. Consequently, the average acceleration during contact with the ground is

$$a_{\text{avg}} = \frac{0 - (-17.1)}{20.0 \times 10^{-3}} = 857 \text{ m/s}^2 .$$

The fact that the result is positive indicates that this acceleration vector points upward. In a later chapter, this will be directly related to the magnitude and direction of the force exerted by the ground on the ball during the collision.

52. The y axis is arranged so that ground level is $y = 0$ and $+y$ is upward.

(a) At the point where its fuel gets exhausted, the rocket has reached a height of

$$y' = \frac{1}{2}at^2 = \frac{(4.00)(6.00)^2}{2} = 72.0 \text{ m} .$$

From Eq. 2-11, the speed of the rocket (which had started at rest) at this instant is

$$v' = at = (4.00)(6.00) = 24.0 \text{ m/s} .$$

The additional height Δy_1 the rocket can attain (beyond y') is given by Eq. 2-16 with vanishing final speed: $0 = v'^{\,2} - 2g\Delta y_1$. This gives

$$\Delta y_1 = \frac{v'^{\,2}}{2g} = \frac{(24.0)^2}{2(9.8)} = 29.4 \text{ m} .$$

Recalling our value for y', the total height the rocket attains is seen to be $72.0 + 29.4 = 101$ m.

(b) The time of free-fall flight (from y' until it returns to $y = 0$) after the fuel gets exhausted is found from Eq. 2-15:

$$-y' = v't - \frac{1}{2}gt^2 \implies -72.0 = (24.0)t - \frac{9.80}{2}t^2 .$$

Solving for t (using the quadratic formula) we obtain $t = 7.00$ s. Recalling the upward acceleration time used in part (a), we see the total time of flight is $7.00 + 6.00 = 13.0$ s.

53. The average acceleration during contact with the floor is given by $a_{\text{avg}} = (v_2 - v_1)/\Delta t$, where v_1 is its velocity just before striking the floor, v_2 is its velocity just as it leaves the floor, and Δt is the duration of contact with the floor (12×10^{-3} s). Taking the y axis to be positively upward and placing the origin at the point where the ball is dropped, we first find the velocity just before striking the floor, using $v_1^2 = v_0^2 - 2gy$. With $v_0 = 0$ and $y = -4.00$ m, the result is

$$v_1 = -\sqrt{-2gy} = -\sqrt{-2(9.8)(-4.00)} = -8.85 \text{ m/s}$$

where the negative root is chosen because the ball is traveling downward. To find the velocity just after hitting the floor (as it ascends without air friction to a height of 2.00 m), we use $v^2 = v_2^2 - 2g(y - y_0)$ with $v = 0$, $y = -2.00$ m (it ends up two meters *below* its initial drop height), and $y_0 = -4.00$ m. Therefore,

$$v_2 = \sqrt{2g(y - y_0)} = \sqrt{2(9.8)(-2.00 + 4.00)} = 6.26 \text{ m/s} .$$

Consequently, the average acceleration is

$$a_{\text{avg}} = \frac{v_2 - v_1}{\Delta t} = \frac{6.26 + 8.85}{12.0 \times 10^{-3}} = 1.26 \times 10^3 \text{ m/s}^2 \ .$$

The positive nature of the result indicates that the acceleration vector points upward. In a later chapter, this will be directly related to the magnitude and direction of the force exerted by the ground on the ball during the collision.

54. The height reached by the player is $y = 0.76$ m (where we have taken the origin of the y axis at the floor and $+y$ to be upward).

(a) The initial velocity v_0 of the player is

$$v_0 = \sqrt{2gy} = \sqrt{2(9.8)(0.76)} = 3.86 \text{ m/s} \ .$$

This is a consequence of Eq. 2-16 where velocity v vanishes. As the player reaches $y_1 = 0.76 - 0.15 = 0.61$ m, his speed v_1 satisfies $v_0^2 - v_1^2 = 2gy_1$, which yields

$$v_1 = \sqrt{v_0^2 - 2gy_1} = \sqrt{(3.86)^2 - 2(9.80)(0.61)} = 1.71 \text{ m/s} \ .$$

The time t_1 that the player spends *ascending* in the top $\Delta y_1 = 0.15$ m of the jump can now be found from Eq. 2-17:

$$\Delta y_1 = \frac{1}{2}\left(v_1 + v\right) t_1 \quad \Longrightarrow \quad t_1 = \frac{2(0.15)}{1.71 + 0} = 0.175 \text{ s}$$

which means that the total time spend in that top 15 cm (both ascending and descending) is $2(0.17) = 0.35$ s $= 350$ ms.

(b) The time t_2 when the player reaches a height of 0.15 m is found from Eq. 2-15:

$$0.15 = v_0 t_2 - \frac{1}{2} g t_2^2 = (3.86) t_2 - \frac{9.8}{2} t_2^2 \ ,$$

which yields (using the quadratic formula, taking the smaller of the two positive roots) $t_2 = 0.041$ s $= 41$ ms, which implies that the total time spend in that bottom 15 cm (both ascending and descending) is $2(41) = 82$ ms.

55. We neglect air resistance, which justifies setting $a = -g = -9.8 \text{ m/s}^2$ (taking *down* as the $-y$ direction) for the duration of the motion. We are allowed to use Table 2-1 (with Δy replacing Δx) because this is constant acceleration motion. The ground level is taken to correspond to the origin of the y axis. The time drop 1 leaves the nozzle is taken as $t = 0$ and its time of landing on the floor t_1 can be computed from Eq. 2-15, with $v_0 = 0$ and $y_1 = -2.00$ m.

$$y_1 = -\frac{1}{2} g t_1^2 \quad \Longrightarrow \quad t_1 = \sqrt{\frac{-2y}{g}} = \sqrt{\frac{-2(-2.00)}{9.8}} = 0.639 \text{ s} \ .$$

At that moment,the fourth drop begins to fall, and from the regularity of the dripping we conclude that drop 2 leaves the nozzle at $t = 0.639/3 = 0.213$ s and drop 3 leaves the nozzle at $t = 2(0.213) = 0.426$ s. Therefore, the time in free fall (up to the moment drop 1 lands) for drop 2 is $t_2 = t_1 - 0.213 = 0.426$ s and the time in free fall (up to the moment drop 1 lands) for drop 3 is $t_3 = t_1 - 0.426 = 0.213$ s. Their positions at that moment are

$$\begin{aligned} y_2 &= -\frac{1}{2} g t_2^2 = -\frac{1}{2}(9.8)(0.426)^2 = -0.889 \text{ m} \\ y_3 &= -\frac{1}{2} g t_3^2 = -\frac{1}{2}(9.8)(0.213)^2 = -0.222 \text{ m} \ , \end{aligned}$$

respectively. Thus, drop 2 is 89 cm below the nozzle and drop 3 is 22 cm below the nozzle when drop 1 strikes the floor.

56. The graph shows $y = 25$ m to be the highest point (where the speed momentarily vanishes). The neglect of "air friction" (or whatever passes for that on the distant planet) is certainly reasonable due to the symmetry of the graph.

   (a) To find the acceleration due to gravity g_p on that planet, we use Eq. 2-15 (with $+y$ up)

$$y - y_0 = vt + \frac{1}{2}g_p t^2 \quad \Longrightarrow \quad 25 - 0 = (0)(2.5) + \frac{1}{2}g_p(2.5)^2$$

   so that $g_p = 8.0$ m/s^2.

   (b) That same (max) point on the graph can be used to find the initial velocity.

$$y - y_0 = \frac{1}{2}(v_0 + v)\,t \quad \Longrightarrow \quad 25 - 0 = \frac{1}{2}(v_0 + 0)\,(2.5)$$

   Therefore, $v_0 = 20$ m/s.

57. Taking $+y$ to be upward and placing the origin at the point from which the objects are dropped, then the location of diamond 1 is given by $y_1 = -\frac{1}{2}gt^2$ and the location of diamond 2 is given by $y_2 = -\frac{1}{2}g(t-1)^2$. We are starting the clock when the first object is dropped. We want the time for which $y_2 - y_1 = 10$ m. Therefore,

$$-\frac{1}{2}g(t-1)^2 + \frac{1}{2}gt^2 = 10 \quad \Longrightarrow \quad t = (10/g) + 0.5 = 1.5 \text{ s} .$$

58. We neglect air resistance, which justifies setting $a = -g = -9.8$ m/s^2 (taking *down* as the $-y$ direction) for the duration of the motion. We are allowed to use Table 2-1 (with Δy replacing Δx) because this is constant acceleration motion. When something is thrown straight up and is caught at the level it was thrown from (with a trajectory similar to that shown in Fig. 2-25), the time of flight t is half of its time of ascent t_a, which is given by Eq. 2-18 with $\Delta y = H$ and $v = 0$ (indicating the maximum point).

$$H = vt_a + \frac{1}{2}gt_a^2 \quad \Longrightarrow \quad t_a = \sqrt{\frac{2H}{g}}$$

   Writing these in terms of the total time in the air $t = 2t_a$ we have

$$H = \frac{1}{8}gt^2 \quad \Longrightarrow \quad t = 2\sqrt{\frac{2H}{g}} \; .$$

   We consider two throws, one to height H_1 for total time t_1 and another to height H_2 for total time t_2, and we set up a ratio:

$$\frac{H_2}{H_1} = \frac{\frac{1}{8}gt_2^2}{\frac{1}{8}gt_1^2} = \left(\frac{t_2}{t_1}\right)^2$$

   from which we conclude that if $t_2 = 2t_1$ (as is required by the problem) then $H_2 = 2^2 H_1 = 4H_1$.

59. We neglect air resistance, which justifies setting $a = -g = -9.8$ m/s^2 (taking *down* as the $-y$ direction) for the duration of the motion. We are allowed to use Table 2-1 (with Δy replacing Δx) because this is constant acceleration motion. We placing the coordinate origin on the ground. We note that the initial velocity of the package is the same as the velocity of the balloon, $v_0 = +12$ m/s and that its initial coordinate is $y_0 = +80$ m.

   (a) We solve $y = y_0 + v_0 t - \frac{1}{2}gt^2$ for time, with $y = 0$, using the quadratic formula (choosing the positive root to yield a positive value for t).

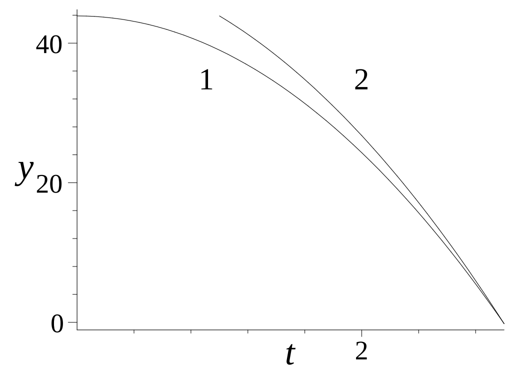
$$t = \frac{v_0 + \sqrt{v_0^2 + 2gy_0}}{g} = \frac{12 + \sqrt{12^2 + 2(9.8)(80)}}{9.8} = 5.4 \text{ s}$$

(b) If we wish to avoid using the result from part (a), we could use Eq. 2-16, but if that is not a concern, then a variety of formulas from Table 2-1 can be used. For instance, Eq. 2-11 leads to $v = v_0 - gt = 12 - (9.8)(5.4) = -41$ m/s. Its final *speed* is 41 m/s.

60. We neglect air resistance, which justifies setting $a = -g = -9.8$ m/s^2 (taking *down* as the $-y$ direction) for the duration of the motion. We are allowed to use Eq. 2-15 (with Δy replacing Δx) because this is constant acceleration motion. We use primed variables (except t) with the first stone, which has zero initial velocity, and unprimed variables with the second stone (with initial downward velocity $-v_0$, so that v_0 is being used for the initial *speed*). SI units are used throughout.

$$\begin{aligned}\Delta y' &= 0(t) - \frac{1}{2}gt^2 \\ \Delta y &= (-v_0)(t-1) - \frac{1}{2}g(t-1)^2\end{aligned}$$

Since the problem indicates $\Delta y' = \Delta y = -43.9$ m, we solve the first equation for t (finding $t = 2.99$ s) and use this result to solve the second equation for the initial speed of the second stone:



$$-43.9 = (-v_0)(1.99) - \frac{1}{2}(9.8)(1.99)^2$$

which leads to $v_0 = 12.3$ m/s.

61. We neglect air resistance, which justifies setting $a = -g = -9.8$ m/s^2 (taking *down* as the $-y$ direction) for the duration of the motion of the shot ball. We are allowed to use Table 2-1 (with Δy replacing Δx) because the ball has constant acceleration motion. We use primed variables (except t) with the constant-velocity elevator (so $v' = 20$ m/s), and unprimed variables with the ball (with initial velocity $v_0 = v' + 10 = 30$ m/s, relative to the ground). SI units are used throughout.

(a) Taking the time to be zero at the instant the ball is shot, we compute its maximum height y (relative to the ground) with $v^2 = v_0^2 - 2g(y - y_o)$, where the highest point is characterized by $v = 0$. Thus,

$$y = y_o + \frac{v_0^2}{2g} = 76 \text{ m}$$

where $y_o = y'_o + 2 = 30$ m (where $y'_o = 28$ m is given in the problem) and $v_0 = 30$ m/s relative to the ground as noted above.

(b) There are a variety of approaches to this question. One is to continue working in the frame of reference adopted in part (a) (which treats the ground as motionless and "fixes" the coordinate origin to it); in this case, one describes the elevator motion with $y' = y'_o + v't$ and the ball motion with Eq. 2-15, and solves them for the case where they reach the same point at the same time. Another is to work in the frame of reference of the elevator (the boy in the elevator might be oblivious to the fact the elevator is moving since it isn't accelerating), which is what we show here in detail:

$$\Delta y_e = v_{0e}t - \frac{1}{2}gt^2 \implies t = \frac{v_{0e} + \sqrt{v_{0e}^2 - 2g\Delta y_e}}{g}$$

where $v_{0e} = 20$ m/s is the initial velocity of the ball relative to the elevator and $\Delta y_e = -2.0$ m is the ball's displacement relative to the floor of the elevator. The positive root is chosen to yield a positive value for t ; the result is $t = 4.2$ s.

62. We neglect air resistance, which justifies setting $a = -g = -9.8$ m/s^2 (taking *down* as the $-y$ direction) for the duration of the stone's motion. We are allowed to use Table 2-1 (with Δx replaced by y) because the ball has constant acceleration motion (and we choose $y_o = 0$).

    (a) We apply Eq. 2-16 to both measurements, with SI units understood.

$$\begin{array}{lll} v_B^2 = v_0^2 - 2gy_B & \Longrightarrow & \left(\frac{1}{2}v\right)^2 + 2g(y_A + 3) = v_0^2 \\ v_A^2 = v_0^2 - 2gy_A & \Longrightarrow & v^2 + 2gy_A = v_0^2 \end{array}$$

    We equate the two expressions that each equal v_0^2 and obtain

$$\frac{1}{4}v^2 + 2gy_A + 2g(3) = v^2 + 2gy_A \quad \Longrightarrow \quad 2g(3) = \frac{3}{4}v^2$$

    which yields $v = \sqrt{2g(4)} = 8.85$ m/s.

    (b) An object moving upward at A with speed $v = 8.85$ m/s will reach a maximum height $y - y_A = v^2/2g = 4.00$ m above point A (this is again a consequence of Eq. 2-16, now with the "final" velocity set to zero to indicate the highest point). Thus, the top of its motion is 1.00 m above point B.

63. The object, once it is dropped ($v_0 = 0$) is in free-fall ($a = -g = -9.8$ m/s^2 if we take *down* as the $-y$ direction), and we use Eq. 2-15 repeatedly.

    (a) The (positive) distance D from the lower dot to the mark corresponding to a certain reaction time t is given by $\Delta y = -D = -\frac{1}{2}gt^2$, or $D = gt^2/2$. Thus for $t_1 = 50.0\,\text{ms}$

$$D_1 = \frac{(9.8\,\text{m/s}^2)(50.0 \times 10^{-3}\,\text{s})^2}{2} = 0.0123\,\text{m} = 1.23\,\text{cm} \ .$$

    (b) For $t_2 = 100\,\text{ms}$

$$D_2 = \frac{(9.8\,\text{m/s}^2)(100 \times 10^{-3}\,\text{s})^2}{2} = 0.049\,\text{m} = 4D_1 \ ;$$

    for $t_3 = 150\,\text{ms}$

$$D_3 = \frac{(9.8\,\text{m/s}^2)(150 \times 10^{-3}\,\text{s})^2}{2} = 0.11\,\text{m} = 9D_1 \ ;$$

    for $t_4 = 200\,\text{ms}$

$$D_4 = \frac{(9.8\,\text{m/s}^2)(200 \times 10^{-3}\,\text{s})^2}{2} = 0.196\,\text{m} = 16D_1 \ ;$$

    and for $t_4 = 250\,\text{ms}$

$$D_5 = \frac{(9.8\,\text{m/s}^2)(250 \times 10^{-3}\,\text{s})^2}{2} = 0.306\,\text{m} = 25D_1 \ .$$

64. During free fall, we ignore the air resistance and set $a = -g = -9.8$ m/s^2 where we are choosing *down* to be the $-y$ direction. The initial velocity is zero so that Eq. 2-15 becomes $\Delta y = -\frac{1}{2}gt^2$ where Δy represents the *negative* of the distance d she has fallen. Thus, we can write the equation as $d = \frac{1}{2}gt^2$ for simplicity.

(a) The time t_1 during which the parachutist is in free fall is (using Eq. 2-15) given by

$$d_1 = 50\,\text{m} = \frac{1}{2}gt_1^2 = \frac{1}{2}\left(9.80\,\text{m/s}^2\right)t_1^2$$

which yields $t_1 = 3.2\,\text{s}$. The *speed* of the parachutist just before he opens the parachute is given by the positive root $v_1^2 = 2gd_1$, or

$$v_1 = \sqrt{2gh_1} = \sqrt{(2)(9.80\,\text{m/s}^2)(50\,\text{m})} = 31\,\text{m/s}\,.$$

If the final speed is v_2, then the time interval t_2 between the opening of the parachute and the arrival of the parachutist at the ground level is

$$t_2 = \frac{v_1 - v_2}{a} = \frac{31\,\text{m/s} - 3.0\,\text{m/s}}{2\,\text{m/s}^2} = 14\,\text{s}\ .$$

This is a result of Eq. 2-11 where *speeds* are used instead of the (negative-valued) velocities (so that final-velocity minus initial-velocity turns out to equal initial-speed minus final-speed); we also note that the acceleration vector for this part of the motion is positive since it points upward (opposite to the direction of motion – which makes it a deceleration). The total time of flight is therefore $t_1 + t_2 = 17\,\text{s}$.

(b) The distance through which the parachutist falls after the parachute is opened is given by

$$d = \frac{v_1^2 - v_2^2}{2a} = \frac{(31\,\text{m/s})^2 - (3.0\,\text{m/s})^2}{(2)(2.0\,\text{m/s}^2)} \approx 240\,\text{m}\,.$$

In the computation, we have used Eq. 2-16 with both sides multiplied by -1 (which changes the negative-valued Δy into the positive d on the left-hand side, and switches the order of v_1 and v_2 on the right-hand side). Thus the fall begins at a height of $h = 50 + d \approx 290\,\text{m}$.

65. The time t the pot spends passing in front of the window of length $L = 2.0$ m is 0.25 s each way. We use v for its velocity as it passes the top of the window (going up). Then, with $a = -g = -9.8\ \text{m/s}^2$ (taking *down* to be the $-y$ direction), Eq. 2-18 yields

$$L = vt - \frac{1}{2}gt^2 \implies v = \frac{L}{t} - \frac{1}{2}gt\ .$$

The distance H the pot goes above the top of the window is therefore (using Eq. 2-16 with the *final velocity* being zero to indicate the highest point)

$$H = \frac{v^2}{2g} = \frac{(L/t - gt/2)^2}{2g} = \frac{(2.00/0.25 - (9.80)(0.25)/2)^2}{(2)\,(9.80)} = 2.34\,\text{m}\ .$$

66. The time being considered is 6 years and roughly 235 days, which is approximately $\Delta t = 2.1 \times 10^7$ s. Using Eq. 2-3, we find the average speed to be

$$\frac{30600 \times 10^3\,\text{m}}{2.1 \times 10^7\,\text{s}} = 0.15\ \text{m/s}\ .$$

67. We assume constant velocity motion and use Eq. 2-2 (with $v_{\text{avg}} = v > 0$). Therefore,

$$\Delta x = v\Delta t = \left(303\,\frac{\text{km}}{\text{h}}\left(\frac{1000\,\text{m/km}}{3600\,\text{s/h}}\right)\right)\left(100 \times 10^{-3}\,\text{s}\right) = 8.4\,\text{m}\ .$$

68. For each rate, we use distance $d = vt$ and convert to SI using 0.0254 cm = 1 inch (from which we derive the factors appearing in the computations below).

(a) The total distance d comes from summing

$$\begin{aligned}
d_1 &= \left(120\,\frac{\text{steps}}{\text{min}}\right)\left(\frac{0.762\,\text{m/step}}{60\,\text{s/min}}\right)(5\,\text{s}) = 7.62\,\text{m} \\
d_2 &= \left(120\,\frac{\text{steps}}{\text{min}}\right)\left(\frac{0.381\,\text{m/step}}{60\,\text{s/min}}\right)(5\,\text{s}) = 3.81\,\text{m} \\
d_3 &= \left(180\,\frac{\text{steps}}{\text{min}}\right)\left(\frac{0.914\,\text{m/step}}{60\,\text{s/min}}\right)(5\,\text{s}) = 13.72\,\text{m} \\
d_4 &= \left(180\,\frac{\text{steps}}{\text{min}}\right)\left(\frac{0.457\,\text{m/step}}{60\,\text{s/min}}\right)(5\,\text{s}) = 6.86\,\text{m}
\end{aligned}$$

so that $d = d_1 + d_2 + d_3 + d_4 = 32$ m.

(b) Average velocity is computed using Eq. 2-2: $v_{\text{avg}} = 32/20 = 1.6$ m/s, where we have used the fact that the total time is 20 s.

(c) The total time t comes from summing

$$\begin{aligned}
t_1 &= \frac{8\,\text{m}}{\left(120\,\frac{\text{steps}}{\text{min}}\right)\left(\frac{0.762\,\text{m/step}}{60\,\text{s/min}}\right)} = 5.25\,\text{s} \\
t_2 &= \frac{8\,\text{m}}{\left(120\,\frac{\text{steps}}{\text{min}}\right)\left(\frac{0.381\,\text{m/step}}{60\,\text{s/min}}\right)} = 10.5\,\text{s} \\
t_3 &= \frac{8\,\text{m}}{\left(180\,\frac{\text{steps}}{\text{min}}\right)\left(\frac{0.914\,\text{m/step}}{60\,\text{s/min}}\right)} = 2.92\,\text{s} \\
t_4 &= \frac{8\,\text{m}}{\left(180\,\frac{\text{steps}}{\text{min}}\right)\left(\frac{0.457\,\text{m/step}}{60\,\text{s/min}}\right)} = 5.83\,\text{s}
\end{aligned}$$

so that $t = t_1 + t_2 + t_3 + t_4 = 24.5$ s.

(d) Average velocity is computed using Eq. 2-2: $v_{\text{avg}} = 32/24.5 = 1.3$ m/s, where we have used the fact that the total distance is $4(8) = 32$ m.

69. The statement that the stoneflies have "constant speed along a straight path" means we are dealing with constant velocity motion (Eq. 2-2 with v_{avg} replaced with v_{s} or v_{ns}, as the case may be).

(a) We set up the ratio and simplify (using d for the common distance).

$$\frac{v_{\text{s}}}{v_{\text{ns}}} = \frac{d/t_{\text{s}}}{d/t_{\text{ns}}} = \frac{t_{\text{ns}}}{t_{\text{s}}} = \frac{25.0}{7.1} = 3.52$$

(b) We examine Δt and simplify until we are left with an expression having numbers and no variables other than v_{s}. Distances are understood to be in meters.

$$\begin{aligned}
t_{\text{ns}} - t_{\text{s}} &= \frac{2}{v_{\text{ns}}} - \frac{2}{v_{\text{s}}} \\
&= \frac{2}{(v_{\text{s}}/3.52)} - \frac{2}{v_{\text{s}}} \\
&= \frac{2}{v_{\text{s}}}(3.52 - 1) \\
&\approx \frac{5}{v_{\text{s}}}
\end{aligned}$$

70. We orient $+x$ along the direction of motion (so a will be negative-valued, since it is a deceleration), and we use Eq. 2-7 with $a_{\text{avg}} = -3400g = -3400(9.8) = -3.33 \times 10^4$ m/s^2 and $v = 0$ (since the recorder finally comes to a stop).

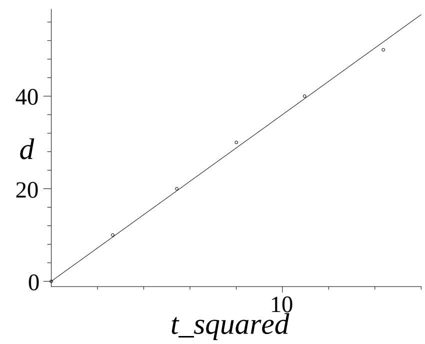
$$a_{\text{avg}} = \frac{v - v_0}{\Delta t} \implies v_0 = \left(3.33 \times 10^4\,\text{m/s}^2\right)\left(6.5 \times 10^{-3}\,\text{s}\right)$$

which leads to $v_0 = 217$ m/s.

71. (a) It is the intent of this problem to treat the $v_0 = 0$ condition rigidly. In other words, we are not fitting the distance to just any second-degree polynomial in t; rather, we requiring $d = At^2$ (which meets the condition that d and its derivative is zero when $t = 0$). If we perform a leastsquares fit with this expression, we obtain $A = 3.587$ (SI units understood). We return to this discussion in part (c). Our expectation based on Eq. 2-15, assuming no error in starting the clock at the moment the acceleration begins, is $d = \frac{1}{2}at^2$ (since he started at the coordinate origin, the location of which presumably is something we can be fairly certain about).

(b) The graph (d on the vertical axis, SI units understood) is shown.

The horizontal axis is t^2 (as indicated by the problem statement) so that we have a straight line instead of a parabola.



(c) Comparing our two expressions for d, we see the parameter A in our fit should correspond to $\frac{1}{2}a$, so $a = 2(3.587) \approx 7.2\,\text{m/s}^2$. Now, other approaches might be considered (trying to fit the data with $d = Ct^2 + B$ for instance, which leads to $a = 2C = 7.0\,\text{m/s}^2$ and $B \neq 0$), and it might be useful to have the class discuss the assumptions made in each approach.

72. (a) We estimate $x \approx 2$ m at $t = 0.5$ s, and $x \approx 12$ m at $t = 4.5$ s. Hence, using the definition of average velocity Eq. 2-2, we find

$$v_{\text{avg}} = \frac{12 - 2}{4.5 - 0.5} = 2.5 \text{ m/s} .$$

(b) In the region $4.0 \le t \le 5.0$, the graph depicts a straight line, so its slope represents the instantaneous velocity for any point in that interval. Its slope is the average velocity between $t = 4.0$ s and $t = 5.0$ s:

$$v_{\text{avg}} = \frac{16.0 - 8.0}{5.0 - 4.0} = 8.0 \text{ m/s} .$$

Thus, the instantaneous velocity at $t = 4.5$ s is 8.0 m/s. (Note: similar reasoning leads to a value needed in the next part: the slope of the $0 \le t \le 1$ region indicates that the instantaneous velocity at $t = 0.5$ s is 4.0 m/s.)

(c) The average acceleration is defined by Eq. 2-7:

$$a_{\text{avg}} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{8.0 - 4.0}{4.5 - 0.5} = 1.0 \text{ m/s}^2 .$$

(d) The instantaneous acceleration is the instantaneous rate-of-change of the velocity, and the constant x vs. t slope in the interval $4.0 \le t \le 5.0$ indicates that the velocity is constant during that interval. Therefore, $a = 0$ at $t = 4.5$ s.

73. We use the functional notation $x(t)$, $v(t)$ and $a(t)$ and find the latter two quantities by differentiating:

$$v(t) = \frac{dx(t)}{t} = 6.0t^2 \quad \text{and} \quad a(t) = \frac{dv(t)}{dt} = 12t$$

with SI units understood. This expressions are used in the parts that follow.

(a) Using the definition of average velocity, Eq. 2-2, we find

$$v_{\text{avg}} = \frac{x(2) - x(1)}{2.0 - 1.0} = \frac{2(2)^3 - 2(1)^3}{1.0} = 14 \text{ m/s} .$$

(b) The average acceleration is defined by Eq. 2-7:

$$a_{\text{avg}} = \frac{v(2) - v(1)}{2.0 - 1.0} = \frac{6(2)^2 - 6(1)^2}{1.0} = 18 \text{ m/s}^2 .$$

(c) The value of $v(t)$ when $t = 1.0$ s is $v(1) = 6(1)^2 = 6.0$ m/s.

(d) The value of $a(t)$ when $t = 1.0$ s is $a(1) = 12(1) = 12$ m/s^2.

(e) The value of $v(t)$ when $t = 2.0$ s is $v(2) = 6(2)^2 = 24$ m/s.

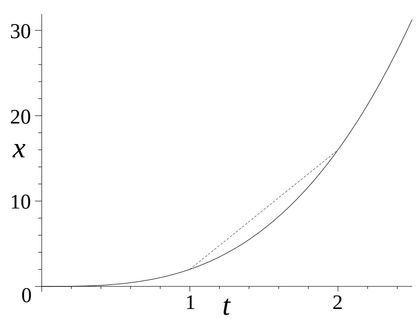
(f) The value of $a(t)$ when $t = 2.0$ s is $a(2) = 12(2) = 24$ m/s^2.

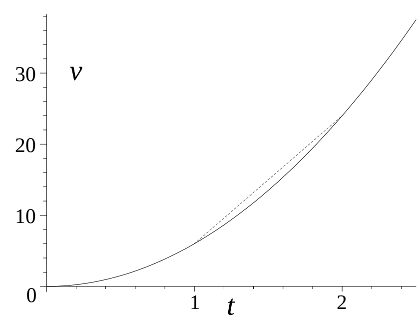
(g) We don't expect average values of a quantity, say, heights of trees, to equal any particular height for a specific tree, but we are sometimes surprised at the different kinds of averaging that can be performed. Now, the acceleration is a linear function (of time) so its average as defined by Eq. 2-7 is, not surprisingly, equal to the arithmetic average of its $a(1)$ and $a(2)$ values. The velocity is not a linear function so the result of part (a) is not equal to the arithmetic average of parts (c) and (e) (although it is fairly close). This reminds us that the calculus-based definition of the average a function (equivalent to Eq. 2-2 for v_{avg}) is not the same as the simple idea of an arithmetic average of two numbers; in other words,

$$\frac{1}{t' - t} \int_t^{t'} f(\tau)\, d\tau \neq \frac{f(t') + f(t)}{2}$$

except in very special cases (like with linear functions).

(h) The graphs are shown below, $x(t)$ on the left and $v(t)$ on the right. SI units are understood. We do not show the tangent lines (representing instantaneous slope values) at $t = 1$ and $t = 2$, but we do show line segments representing the average quantities computed in parts (a) and (b).





74. We choose *down* as the $+y$ direction and set the coordinate origin at the point where it was dropped (which is when we start the clock). We denote the 1.00 s duration mentioned in the problem as $t - t'$ where t is the value of time when it lands and t' is one second prior to that. The corresponding distance is $y - y' = 0.50h$, where y denotes the location of the ground. In these terms, y is the same as h, so we have $h - y' = 0.50h$ or $0.50h = y'$.

(a) We find t' and t from Eq. 2-15 (with $v_0 = 0$):

$$\begin{aligned} y' &= \frac{1}{2}gt'^2 \implies t' = \sqrt{\frac{2y'}{g}} \\ y &= \frac{1}{2}gt^2 \implies t = \sqrt{\frac{2y}{g}} \,. \end{aligned}$$

Plugging in $y = h$ and $y' = 0.50h$, and dividing these two equations, we obtain

$$\frac{t'}{t} = \sqrt{\frac{2(0.50h)/g}{2h/g}} = \sqrt{0.50} \,.$$

Letting $t' = t - 1.00$ (SI units understood) and cross-multiplying, we find

$$t - 1.00 = t\sqrt{0.50} \implies t = \frac{1.00}{1 - \sqrt{0.50}}$$

which yields $t = 3.41$ s.

(b) Plugging this result into $y = \frac{1}{2}gt^2$ we find $h = 57$ m.

(c) In our approach, we did not use the quadratic formula, but we did "choose a root" when we assumed (in the last calculation in part (a)) that $\sqrt{0.50} = +2.236$ instead of -2.236. If we had instead let $\sqrt{0.50} = -2.236$ then our answer for t would have been roughly 0.6 s which would imply that $t' = t - 1$ would equal a negative number (indicating a time *before* it was dropped) which certainly does not fit with the physical situation described in the problem.

75. (a) Let the height of the diving board be h. We choose *down* as the $+y$ direction and set the coordinate origin at the point where it was dropped (which is when we start the clock). Thus, $y = h$ designates the location where the ball strikes the water. Let the depth of the lake be D, and the total time for the ball to descend be T. The speed of the ball as it reaches the surface of the lake is then $v = \sqrt{2gh}$ (from Eq. 2-16), and the time for the ball to fall from the board to the lake surface is $t_1 = \sqrt{2h/g}$ (from Eq. 2-15). Now, the time it spends descending in the lake (at constant velocity v) is

$$t_2 = \frac{D}{v} = \frac{D}{\sqrt{2gh}} \,.$$

Thus, $T = t_1 + t_2 = \sqrt{\frac{2h}{g}} + \frac{D}{\sqrt{2gh}}$, which gives

$$D = T\sqrt{2gh} - 2h = (4.80)\sqrt{(2)(9.80)(5.20)} - (2)(5.20) = 38.1 \text{ m} \,.$$

(b) Using Eq. 2-2, the average velocity is

$$v_{\text{avg}} = \frac{D + h}{T} = \frac{38.1 + 5.20}{4.80} = 9.02 \text{ m/s}$$

where (recalling our coordinate choices) the positive sign means that the ball is going downward (if, however, upwards had been chosen as the positive direction, then this answer would turn out negative-valued).

(c) We find v_0 from $\Delta y = v_0 t + \frac{1}{2}gt^2$ with $t = T$ and $\Delta y = h + D$. Thus,

$$v_0 = \frac{h + D}{T} - \frac{gT}{2} = \frac{5.20 + 38.1}{4.80} - \frac{(9.8)(4.80)}{2} = -14.5 \text{ m/s}$$

where (recalling our coordinate choices) the negative sign means that the ball is being thrown upward.

76. The time Δt is $2(60) + 41 = 161$ min and the displacement $\Delta x = 370$ cm. Thus, Eq. 2-2 gives

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{370}{161} = 2.3 \text{ cm/min} .$$

77. We use the functional notation $x(t)$, $v(t)$ and $a(t)$ and find the latter two quantities by differentiating:

$$v(t) = \frac{dx(t)}{t} = -15t^2 + 20 \quad \text{and} \quad a(t) = \frac{dv(t)}{dt} = -30t$$

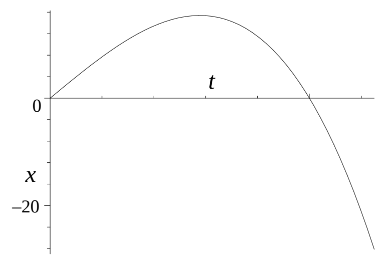
with SI units understood. This expressions are used in the parts that follow.

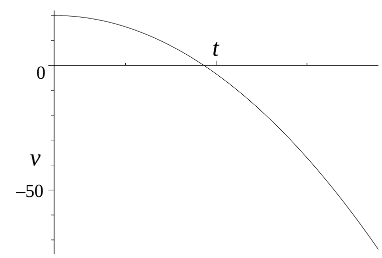
(a) From $0 = -15t^2 + 20$, we see that the only positive value of t for which the particle is (momentarily) stopped is $t = \sqrt{20/15} = 1.2$ s.

(b) From $0 = -30t$, we find $a(0) = 0$ (that is, it vanishes at $t = 0$).

(c) It is clear that $a(t) = -30t$ is negative for $t > 0$ and positive for $t < 0$.

(d) We show the two of the graphs below (the third graph, $a(t)$, which is a straight line through the origin with slope $= -30$ is omitted in the interest of saving space). SI units are understood.





78. (a) It follows from Eq. 2-8 that $v - v_0 = \int a\, dt$, which has the geometric interpretation of being the area under the graph. Thus, with $v_0 = 2.0$ m/s and that area amounting to 3.0 m/s (adding that of a triangle to that of a square, over the interval $0 \le t \le 2$ s), we find $v = 2.0 + 3.0 = 5.0$ m/s (which we will denote as v_2 in the next part). The information given that $x_0 = 4.0$ m is not used in this solution.

(b) During $2 < t \le 4$ s, the graph of a is a straight line with slope 1.0 m/s^3. Extrapolating, we see that the intercept of this line with the a axis is zero. Thus, with SI units understood,

$$v = v_2 + \int_{2.0}^{t} a\; d\tau = 5.0 + \int_{2.0}^{t} (1.0)\tau\, d\tau = 5.0 + \frac{(1.0)t^2 - (1.0)(2.0)^2}{2}$$

which yields $v = 3.0 + 0.50t^2$ in m/s.

79. We assume the train accelerates from rest ($v_0 = 0$ and $x_0 = 0$) at $a_1 = +1.34$ m/s^2 until the midway point and then decelerates at $a_2 = -1.34$ m/s^2 until it comes to a stop ($v_2 = 0$) at the next station. The velocity at the midpoint is v_1 which occurs at $x_1 = 806/2 = 403$ m.

(a) Eq. 2-16 leads to

$$v_1^2 = v_0^2 + 2a_1x_1 \implies v_1 = \sqrt{2(1.34)(403)}$$

which yields $v_1 = 32.9$ m/s.

(b) The time t_1 for the accelerating stage is (using Eq. 2-15)

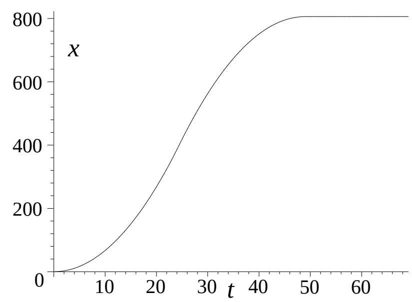
$$x_1 = v_0t_1 + \frac{1}{2}a_1t_1^2 \implies t_1 = \sqrt{\frac{2(403)}{1.34}}$$

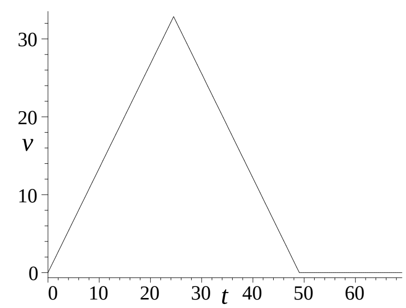
which yields $t_1 = 24.53$ s. Since the time interval for the decelerating stage turns out to be the same, we double this result and obtain $t = 49.1$ s for the travel time between stations.

(c) With a "dead time" of 20 s, we have $T = t + 20 = 69.1$ s for the total time between start-ups. Thus, Eq. 2-2 gives

$$v_{\text{avg}} = \frac{806\,\text{m}}{69.1\,\text{s}} = 11.7 \text{ m/s} .$$

(d) We show the two of the graphs below. The third graph, $a(t)$, is not shown to save space; it consists of three horizontal "steps" – one at 1.34 during $0 < t < 24.53$ and the next at -1.34 during $24.53 < t < 49.1$ and the last at zero during the "dead time" $49.1 < t < 69.1$). SI units are understood.





80. Average speed, as opposed to average velocity, relates to the total distance, as opposed to the net displacement. The distance D up the hill is, of course, the same as the distance down the hill, and since the speed is constant (during each stage of the motion) we have speed$= D/t$. Thus, the average speed is

$$\frac{D_{\text{up}} + D_{\text{down}}}{t_{\text{up}} + t_{\text{down}}} = \frac{2D}{\frac{D}{v_{\text{up}}} + \frac{D}{v_{\text{down}}}}$$

which, after canceling D and plugging in $v_{\text{up}} = 40$ km/h and $v_{\text{down}} = 60$ km/h, yields 48 km/h for the average speed.

81. During T_r the velocity v_0 is constant (in the direction we choose as $+x$) and obeys $v_0 = D_r/T_r$ where we note that in SI units the velocity is $v_0 = 200(1000/3600) = 55.6$ m/s. During T_b the acceleration is opposite to the direction of v_0 (hence, for us, $a < 0$) until the car is stopped ($v = 0$).

(a) Using Eq. 2-16 (with $\Delta x_b = 170$ m) we find

$$v^2 = v_0^2 + 2a\Delta x_b \implies a = -\frac{v_0^2}{2\Delta x_b}$$

which yields $|a| = 9.08$ m/s^2.

(b) We express this as a multiple of g by setting up a ratio:

$$a = \left(\frac{9.08}{9.8}\right) 9 = 0.926g .$$

(c) We use Eq. 2-17 to obtain the braking time:

$$\Delta x_b = \frac{1}{2}(v_0 + v)\,T_b \implies T_b = \frac{2(170)}{55.6} = 6.12 \text{ s} .$$

(d) We express our result for T_b as a multiple of the reaction time T_r by setting up a ratio:

$$T_b = \left(\frac{6.12}{400 \times 10^{-3}}\right) T_r = 15.3T_r \quad .$$

(e) We are only asked what the *increase* in distance D is, due to $\Delta T_r = 0.100$ s, so we simply have

$$\Delta D = v_0 \Delta T_r = (55.6)(0.100) = 5.56 \text{ m} .$$

82. We take $+x$ in the direction of motion. We use subscripts 1 and 2 for the data. Thus, $v_1 = +30$ m/s, $v_2 = +50$ m/s and $x_2 - x_1 = +160$ m.

(a) Using these subscripts, Eq. 2-16 leads to

$$a = \frac{v_2^2 - v_1^2}{2\,(x_2 - x_1)} = \frac{50^2 - 30^2}{2(160)} = 5.0 \text{ m/s}^2 .$$

(b) We find the time interval corresponding to the displacement $x_2 - x_1$ using Eq. 2-17:

$$t_2 - t_1 = \frac{2\,(x_2 - x_1)}{v_1 + v_2} = \frac{2(160)}{30 + 50} = 4.0 \text{ s} .$$

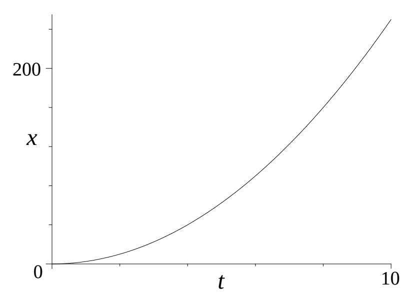
(c) Since the train is at rest ($v_0 = 0$) when the clock starts, we find the value of t_1 from Eq. 2-11:

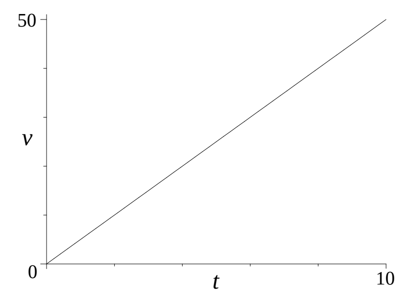
$$v_1 = v_0 + at_1 \implies t_1 = \frac{30}{5.0} = 6.0 \text{ s} .$$

(d) The coordinate origin is taken to be the location at which the train was initially at rest (so $x_0 = 0$). Thus, we are asked to find the value of x_1. Although any of several equations could be used, we choose Eq. 2-17:

$$x_1 = \frac{1}{2}\,(v_0 + v_1)\,t_1 = \frac{1}{2}(30)(6.0) = 90 \text{ m} .$$

(e) The graphs are shown below, with SI units assumed.





83. Direction of $+x$ is implicit in the problem statement. The initial position (when the clock starts) is $x_0 = 0$ (where $v_0 = 0$), the end of the speeding-up motion occurs at $x_1 = 1100/2 = 550$ m, and the subway comes to a halt ($v_2 = 0$) at $x_2 = 1100$ m.

(a) Using Eq. 2-15, the subway reaches x_1 at

$$t_1 = \sqrt{\frac{2x_1}{a_1}} = \sqrt{\frac{2(550)}{1.2}} = 30.3 \text{ s} .$$

The time interval $t_2 - t_1$ turns out to be the same value (most easily seen using Eq. 2-18 so the total time is $t_2 = 2(30.3) = 60.6$ s.

(b) Its maximum speed occurs at t_1 and equals

$$v_1 = v_0 + a_1 t_1 = 36.3 \text{ m/s} .$$

(c) The graphs are not shown here, in the interest of saving space. They are very similar to those shown in the solution for problem 79, above.

84. We note that the running time for Bill Rodgers is $\Delta t_R = 2(3600) + 10(60) = 7800$ s. We also note that the magnitude of the average velocity (Eq. 2-2) and Eq. 2-3 (for average speed) agree in this exercise (which is not usually the case).

   (a) Denoting the Lewis' average velocity as v_L (similarly for Rodgers), we find

$$v_L = \frac{100\,\text{m}}{10\,\text{s}} = 10 \text{ m/s} \quad v_R = \frac{42000\,\text{m}}{7800\,\text{s}} = 5.4 \text{ m/s} .$$

   (b) If Lewis continued at this rate, he would covered $D = 42000$ m in

$$\Delta t_L = \frac{D}{v_L} = \frac{42000}{10} = 4200 \text{ s}$$

   which is equivalent to 1 h and 10 min.

85. We choose *down* as the $+y$ direction and use the equations of Table 2-1 (replacing x with y) with $a = +g$, $v_0 = 0$ and $y_0 = 0$. We use subscript 2 for the elevator reaching the ground and 1 for the halfway point.

   (a) Eq. 2-16, $v_2^2 = v_0^2 + 2a(y_2 - y_0)$, leads to

$$v_2 = \sqrt{2gy_2} = \sqrt{2(9.8)(120)} = 48.5 \text{ m/s} .$$

   (b) The time at which it strikes the ground is (using Eq. 2-15)

$$t_2 = \sqrt{\frac{2y_2}{g}} = \sqrt{\frac{2(120)}{9.8}} = 4.95 \text{ s} .$$

   (c) Now Eq. 2-16, in the form $v_1^2 = v_0^2 + 2a(y_1 - y_0)$, leads to

$$v_1 = \sqrt{2gy_1} = \sqrt{2(9.8)(60)} = 34.2 \text{ m/s} .$$

   (d) The time at which it reaches the halfway point is (using Eq. 2-15)

$$t_1 = \sqrt{\frac{2y_1}{g}} = \sqrt{\frac{2(60)}{9.8}} = 3.50 \text{ s} .$$

86. To find the "launch" velocity of the rock, we apply Eq. 2-11 to the maximum height (where the speed is momentarily zero)

$$v = v_0 - gt \implies 0 = v_0 - (9.8)(2.5)$$

   so that $v_0 = 24.5$ m/s (with $+y$ up). Now we use Eq. 2-15 to find the height of the tower (taking $y_0 = 0$ at the ground level)

$$y - y_0 = v_0 t + \frac{1}{2}at^2 \implies y - 0 = (24.5)(1.5) - \frac{1}{2}(9.8)(1.5)^2 \quad .$$

   Thus, we obtain $y = 26$ m.

87. We take the direction of motion as $+x$, so $a = -5.18$ m/s^2, and we use SI units, so $v_0 = 55(1000/3600) = 15.28$ m/s.

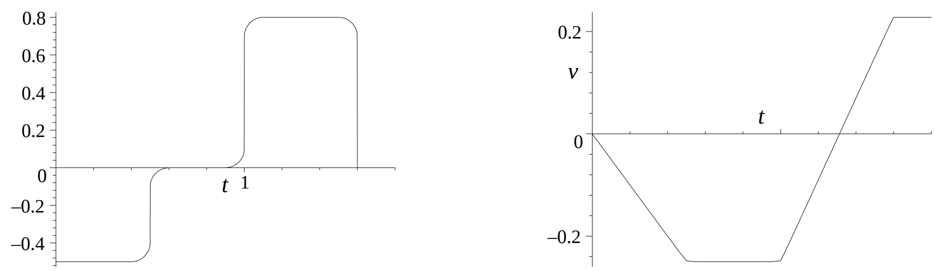
   (a) The velocity is constant during the reaction time T, so the distance traveled during it is $d_r = v_0 T - (15.28)(0.75) = 11.46$ m. We use Eq. 2-16 (with $v = 0$) to find the distance d_b traveled during braking:

$$v^2 = v_0^2 + 2ad_b \implies d_b = -\frac{15.28^2}{2(-5.18)}$$

   which yields $d_b = 22.53$ m. Thus, the total distance is $d_r + d_b = 34.0$ m, which means that the driver *is* able to stop in time. And if the driver were to continue at v_0, the car would enter the intersection in $t = (40\,\text{m})/(15.28\,\text{m/s}) = 2.6$ s which is (barely) enough time to enter the intersection before the light turns, which many people would consider an acceptable situation.

(b) In this case, the total distance to stop (found in part (a) to be 34 m) is greater than the distance to the intersection, so the driver cannot stop without the front end of the car being a couple of meters into the intersection. And the time to reach it at constant speed is $32/15.28 = 2.1$ s, which is too long (the light turns in 1.8 s). The driver is caught between a rock and a hard place.

88. We assume $v_0 = 0$ and integrate the acceleration to find the velocity. In the graphs below (the first is the acceleration, like Fig. 2-35 but with some numbers we adopted, and the second is the velocity) we modeled the curve in the textbook with straight lines and circular arcs for the rounded corners, and literally integrated it. The intent of the textbook was not, however, to go through such an involved procedure, and one should be able to obtain a close approximation to the shape of the velocity graph below (the one on the right) just by applying the idea that constant nonzero acceleration means a linearly changing velocity.



89. We take the direction of motion as $+x$, take $x_0 = 0$ and use SI units, so $v = 1600(1000/3600) = 444$ m/s.

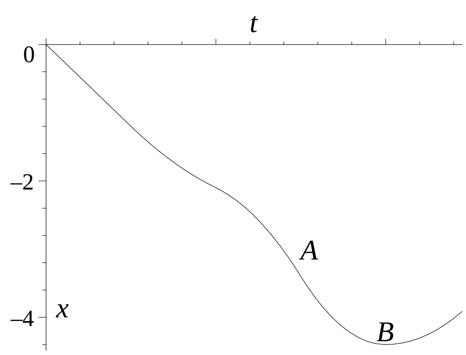
(a) Eq. 2-11 gives $444 = a(1.8)$ or $a = 247$ m/s^2. We express this as a multiple of g by setting up a ratio:

$$a = \left(\frac{247}{9.8}\right) g = 25g\ .$$

(b) Eq. 2-17 readily yields

$$x = \frac{1}{2}\left(v_0 + v\right) t = \frac{1}{2}(444)(1.8) = 400 \text{ m}\ .$$

90. The graph is shown below. We assumed each interval described in the problem was one time unit long. A marks where the curve is steepest and B is where it is least steep (where it, in fact, has zero slope).



91. We use the functional notation $x(t)$, $v(t)$ and $a(t)$ in this solution, where the latter two quantities are obtained by differentiation:

$$v(t) = \frac{dx(t)}{dt} = -12t \quad \text{and} \quad a(t) = \frac{dv(t)}{dt} = -12$$

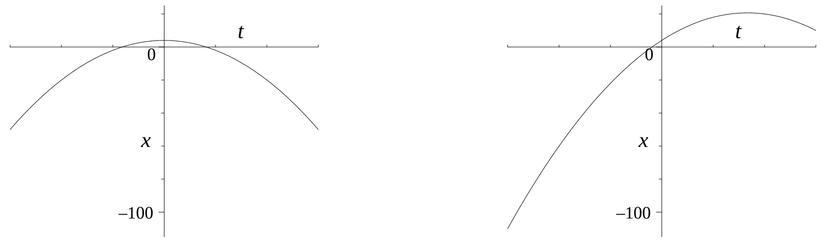
with SI units understood.

(a) From $v(t) = 0$ we find it is (momentarily) at rest at $t = 0$.

(b) We obtain $x(0) = 4.0$ m

(c) Requiring $x(t) = 0$ in the expression $x(t) = 4.0 - 6.0t^2$ leads to $t = \pm 0.82$ s for the times when the particle can be found passing through the origin.

(d) We show both the asked-for graph (on the left) as well as the "shifted" graph which is relevant to part (e). In both cases, the time axis is given by $-3 \le t \le 3$ (SI units understood).



(e) We arrived at the graph on the right (shown above) by adding $20t$ to the $x(t)$ expression.

(f) Examining where the slopes of the graphs become zero, it is clear that the shift causes the $v = 0$ point to correspond to a larger value of x (the top of the second curve shown in part (d) is higher than that of the first).

92. (a) The slope of the graph (at a point) represents the velocity there, and the up-or-down concavity of the curve there indicates the $\pm$ sign of the acceleration. Thus, during AB we have $v > 0$ and $a = 0$ (since it is a straight line). During BC, we still have $v > 0$ but there is some curvature and a downward concavity is indicated (so $a < 0$). The segment CD is horizontal, implying the particle remains at the same position for some time; thus, $v = a = 0$ during CD. Clearly, the slope is negative during DE (so $v < 0$) but whether or not the graph is curved is less clear; we believe it is, with an upward concavity ($a > 0$).

(b) The key word is "obviously." Since it seems plausible to us that the curved portions can be "fit" with parabolic arcs (indications of constant acceleration by Eq. 2-15), then our answer is "no."

(c) Neither signs of slopes nor the sign of the concavity depends on a global shift in one axis or another (or, for that matter, on rescalings of the axes themselves) so the answer again is "no."

93. (a) The slope of the graph (at a point) represents the velocity there, and the up-or-down concavity of the curve there indicates the $\pm$ sign of the acceleration. Thus, during AB we have positive slope ($v > 0$) and $a < 0$ (since it is concave downward). The segment BC is horizontal, implying the particle remains at the same position for some time; thus, $v = a = 0$ during BC. During CD we have $v > 0$ and $a > 0$ (since it is concave upward). Clearly, the slope is positive during DE (so $v > 0$) but whether or not the graph is curved is less clear; we believe it is not, so $a = 0$.

(b) The key word is "obviously." Since it seems plausible to us that the curved portions can be "fit" with parabolic arcs (indications of constant acceleration by Eq. 2-15), then our answer is "no."

(c) Neither signs of slopes nor the sign of the concavity depends on a global shift in one axis or another (or, for that matter, on rescalings of the axes themselves) so the answer again is "no."

94. This problem consists of two parts: part 1 with constant acceleration (so that the equations in Table 2-1 apply), $v_0 = 0$, $v = 11.0$ m/s, $x = 12.0$ m, and $x_0 = 0$ (adopting the starting line as the coordinate origin); and, part 2 with constant velocity (so that $x - x_0 = vt$ applies) with $v = 11.0$ m/s, $x_0 = 12.0$, and $x = 100.0$ m.

(a) We obtain the time for part 1 from Eq. 2-17

$$x - x_0 = \frac{1}{2}(v_0 + v)\,t_1 \implies 12.0 - 0 = \frac{1}{2}(0 + 11.0)t_1$$

so that $t_1 = 2.2$ s, and we find the time for part 2 simply from $88.0 = (11.0)t_2 \rightarrow t_2 = 8.0$ s. Therefore, the total time is $t_1 + t_2 = 10.2$ s.

(b) Here, the total time is required to be 10.0 s, and we are to locate the point x_p where the runner switches from accelerating to proceeding at constant speed. The equations for parts 1 and 2, used above, therefore become

$$\begin{aligned} x_p - 0 &= \frac{1}{2}(0 + 11.0)t_1 \\ 100.0 - x_p &= (11.0)(10.0 - t_1) \end{aligned}$$

where in the latter equation, we use the fact that $t_2 = 10.0 - t_1$. Solving the equations for the two unknowns, we find that $t_1 = 1.8$ s and $x_p = 10.0$ m.

95. We take $+x$ in the direction of motion, so $v_0 = +24.6$ m/s and $a = -4.92$ m/s^2. We also take $x_0 = 0$.

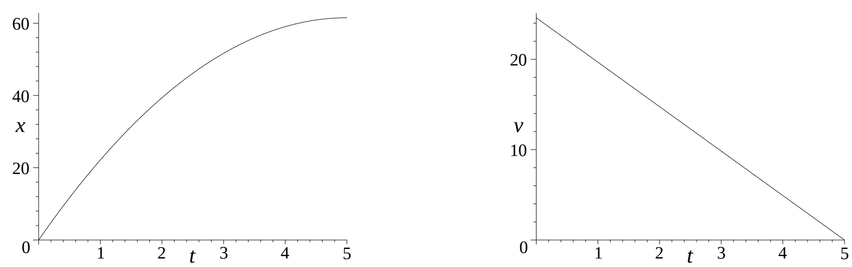
(a) The time to come to a halt is found using Eq. 2-11:

$$0 = v_0 + at \implies t = -\frac{24.6}{-4.92} = 5.00 \text{ s} .$$

(b) Although several of the equations in Table 2-1 will yield the result, we choose Eq. 2-16 (since it does not depend on our answer to part (a)).

$$0 = v_0^2 + 2ax \implies x = -\frac{24.6^2}{2(-4.92)} = 61.5 \text{ m} .$$

(c) Using these results, we plot $v_0 t + \frac{1}{2}at^2$ (the x graph, shown below, on the left) and $v_0 + at$ (the v graph, below right) over $0 \le t \le 5$ s, with SI units understood.



96. We take $+x$ in the direction of motion, so

$$v = (60\,\text{km/h})\left(\frac{1000\,\text{m/km}}{3600\,\text{s/h}}\right) = +16.7 \text{ m/s}$$

and $a > 0$. The location where it starts from rest ($v_0 = 0$) is taken to be $x_0 = 0$.

(a) Eq. 2-7 gives $a_{\text{avg}} = (v - v_0)/t$ where $t = 5.4$ s and the velocities are given above. Thus, $a_{\text{avg}} = 3.1$ m/s^2.

(b) The assumption that $a =$constant permits the use of Table 2-1. From that list, we choose Eq. 2-17:

$$x = \frac{1}{2}(v_0 + v)\,t = \frac{1}{2}(16.7)(5.4) = 45 \text{ m} .$$

(c) We use Eq. 2-15, now with $x = 250$ m:

$$x = \frac{1}{2}at^2 \implies t = \sqrt{\frac{2x}{a}} = \sqrt{\frac{2(250)}{3.1}}$$

which yields $t = 13$ s.

97. Converting to SI units, we have $v = 3400(1000/3600) = 944$ m/s (presumed constant) and $\Delta t = 0.10$ s. Thus, $\Delta x = v\Delta t = 94$ m.

98. The (ideal) driving time before the change was $t = \Delta x/v$, and after the change it is $t' = \Delta x/v'$. The time saved by the change is therefore

$$t - t' = \Delta x\left(\frac{1}{v} - \frac{1}{v'}\right) = \Delta x\left(\frac{1}{55} - \frac{1}{65}\right) = \Delta x(0.0028\,\mathrm{h/mi})$$

which becomes, converting $\Delta x = 700/1.61 = 435$ mi (using a conversion found on the inside front cover of the textbook), $t - t' = (435)(0.0028) = 1.2$ h. This is equivalent to 1 h and 13 min.

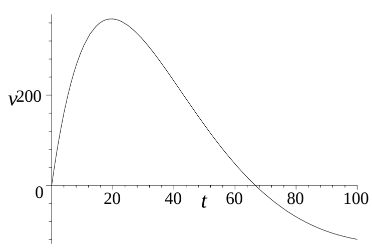
99. (a) With the understanding that these are good to three significant figures, we write the function (in SI units) as

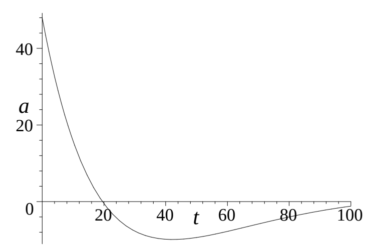
$$x(t) = -32 + 24t^2e^{-0.03t}$$

and find the velocity and acceleration functions by differentiating (calculus is reviewed Appendix E). We find

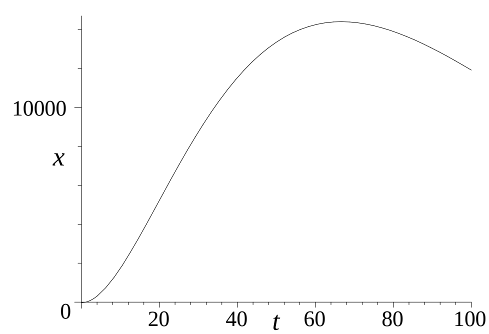
$$v(t) = 24t(2 - 0.03t)e^{-0.03t} \quad \text{and} \quad a(t) = 24\left(2 - 0.12t + 0.0009t^2\right)e^{-0.03t}\ .$$

(b) The $v(t)$ and $a(t)$ graphs are shown below (SI units understood). The time axis in both cases runs from $t = 0$ to $t = 100$ s. We include the $x(t)$ graph in the next part, accompanying our discussion of its root (which is, as suggested by the graph, a small positive value of t).





(c) We seek to find a positive value of t for which $24t^2e^{-0.03t} = 32$. We turn to the calculator or to a computer for its (numerical) solution. In this case, we ignore the roots outside the $0 \leq t \leq 100$ range (such as $t = -1.14$ s and $t = 387.77$ s) and choose $t = 1.175$ s as our answer. All of these are rounded-off values. We find $v = 53.5$ m/s and $a = 43.1$ m/s^2 at this time.



(d) It is much easier to find when $24t(2 - 0.03t)e^{-0.03t} = 0$ since the roots are clearly $t_1 = 0$ and $t_2 = 2/0.03 = 66.7$ s. We find $x(t_1) = -32.0$ m and $a(t_1) = 48.0$ m/s^2 at the first root, and we find $x(t_2) = 1.44 \times 10^4$ m and $a(t_2) = -6.50$ m/s^2 at the second root.

100. We take $+x$ in the direction of motion, so $v_0 = +30$ m/s, $v_1 = +15$ m/s and $a < 0$. The acceleration is found from Eq. 2-11: $a = (v_1 - v_0)/t_1$ where $t_1 = 3.0$ s. This gives $a = -5.0$ m/s^2. The displacement (which in this situation is the same as the distance traveled) to the point it stops ($v_2 = 0$) is, using Eq. 2-16,

$$v_2^2 = v_0^2 + 2a\Delta x \implies \Delta x = -\frac{30^2}{2(-5)} = 90 \text{ m} .$$

101. We choose the direction of motion as the positive direction. We work with the kilometer and hour units, so we write $\Delta x = 0.088$ km.

(a) Eq. 2-16 leads to

$$a = \frac{v^2 - v_0^2}{2\Delta x} = \frac{65^2 - 85^2}{2(0.088)}$$

which yields $a = -1.7 \times 10^4$ km/h^2.

(b) In this case, we obtain

$$a = \frac{60^2 - 80^2}{2(0.088)} = -1.6 \times 10^4 \text{ km/h}^2 .$$

(c) In this final situation, we find

$$a = \frac{40^2 - 50^2}{2(0.088)} = -5.1 \times 10^3 \text{ km/h}^2 .$$

102. Let the vertical distances between Jim's and Clara's feet and the jump-off level be H_J and H_C, respectively. At the instant this photo was taken, Clara has fallen for a time T_C, while Jim has fallen for T_J. Thus (using Eq. 2-15 with $v_0 = 0$) we have

$$H_J = \frac{1}{2}gT_J^2 \quad \text{and} \quad H_C = \frac{1}{2}gT_C^2 .$$

Measuring directly from the photo, we get $H_J \approx 3.6$ m and $H_C \approx 6.3$ m, which yields $T_J \approx 0.86$ s and $T_C \approx 1.13$ s. Jim's waiting time is therefore $T_C - T_J \approx 0.3$ s.

103. We choose *down* as the $+y$ direction and place the coordinate origin at the top of the building (which has height H). During its fall, the ball passes (with velocity v_1) the top of the window (which is at y_1) at time t_1, and passes the bottom (which is at y_2) at time t_2. We are told $y_2 - y_1 = 1.20$ m and $t_2 - t_1 = 0.125$ s. Using Eq. 2-15 we have

$$y_2 - y_1 = v_1 (t_2 - t_1) + \frac{1}{2}g (t_2 - t_1)^2$$

which immediately yields

$$v_1 = \frac{1.20 - \frac{1}{2}(9.8)(0.125)^2}{0.125} = 8.99 \text{ m/s} .$$

From this, Eq. 2-16 (with $v_0 = 0$) reveals the value of y_1:

$$v_1^2 = 2gy_1 \implies y_1 = \frac{8.99^2}{2(9.8)} = 4.12 \text{ m} .$$

It reaches the ground ($y_3 = H$) at t_3. Because of the symmetry expressed in the problem ("upward flight is a reverse of the fall") we know that $t_3 - t_2 = 2.00/2 = 1.00$ s. And this means $t_3 - t_1 = 1.00 + 0.125 = 1.125$ s. Now Eq. 2-15 produces

$$\begin{aligned} y_3 - y_1 &= v_1 (t_3 - t_1) + \frac{1}{2}g (t_3 - t_1)^2 \\ y_3 - 4.12 &= (8.99)(1.125) + \frac{1}{2}(9.8)(1.125)^2 \end{aligned}$$

which yields $y_3 = H = 20.4$ m.

104. (a) Using the fact that the area of a triangle is $\frac{1}{2}$(base)(height) (and the fact that the integral corresponds to area under the curve) we find, from $t = 0$ through $t = 5$ s, the integral of v with respect to t is 15 m. Since we are told that $x_0 = 0$ then we conclude that $x = 15$ m when $t = 5.0$ s.

(b) We see directly from the graph that $v = 2.0$ m/s when $t = 5.0$ s.

(c) Since $a = \frac{dv}{dt}$ = slope of the graph, we find that the acceleration during the interval $4 < t < 6$ is uniformly equal to $-2.0\,\text{m/s}^2$.

(d) Thinking of $x(t)$ in terms of accumulated area (on the graph), we note that $x(1) = 1$ m; using this and the value found in part (a), Eq. 2-2 produces

$$v_{\text{avg}} = \frac{x(5) - x(1)}{5 - 1} = \frac{15 - 1}{4} = 3.5 \text{ m/s} .$$

(e) From Eq. 2-7 and the values $v(t)$ we read directly from the graph, we find

$$a_{\text{avg}} = \frac{v(5) - v(1)}{5 - 1} = \frac{2 - 2}{4} = 0.$$

105. (First problem of **Cluster 1**)
The two parts of this problem are as follows. Part 1 (motion from A to B) consists of constant acceleration (so Table 2-1 applies) and involves the data $v_0 = 0$, $v = 10.0$ m/s, $x_0 = 0$ and $x = 40.0$ m (taking point A as the coordinate origin and orienting the positive x axis towards B and C). Part 2 (from B to C) consists of constant velocity motion (so the simple equation $\frac{\Delta x}{\Delta t} = v$ applies) with $v = 10.0$ m/s and $\Delta t = 10.0$ s.

(a) Eq. 2-16 is an efficient way of finding the part 1 acceleration:

$$v^2 = v_0^2 + 2a\,(x - x_0) \quad \Longrightarrow \quad (10.0)^2 = 0 + 2a(40.0)$$

from which we obtain $a = 1.25$ m/s^2.

(b) Using Eq. 2-17 avoids using the result from part (a) and finds the time readily.

$$x - x_0 = \frac{1}{2}\,(v_0 + v)\,t \quad \Longrightarrow \quad 40.0 - 0 = \frac{1}{2}(0 + 10.0)t$$

This leads to $t = 8.00$ s, for part 1.

(c) We find the distance traveled in part 2 with $\Delta x = v\Delta t = (10.0)(10.0) = 100$ m.

(d) The average velocity is defined by Eq. 2-2

$$v_{\text{avg}} = \frac{x_C - x_A}{t_C - t_A} = \frac{140 - 0}{18 - 0} = 7.78 \text{ m/s} .$$

106. (Second problem of **Cluster 1**)
The two parts of this problem are as follows. Part 1 (motion from A to B) consists of constant acceleration (so Table 2-1 applies) and involves the data $v_0 = 20.0$ m/s, $v = 30.0$ m/s, $x_0 = 0$ and $t_1 = 10.0$ s (taking point A as the coordinate origin, orienting the positive x axis towards B and C, and starting the clock when it passes point A). Part 2 (from B to C) also involves uniformly accelerated motion but now with the data $v_0 = 30.0$ m/s, $v = 15.0$ m/s, and $\Delta x = x - x_0 = 150$ m.

(a) The distance for part 1 is given by

$$x - x_0 = \frac{1}{2}\,(v_0 + v)\,t_1 = \frac{1}{2}(20.0 + 30.0)(10.0)$$

which yields $x = 250$ m.

(b) The time t_2 for part 2 is found from the same formula as in part (a).

$$x - x_0 = \frac{1}{2}(v_0 + v)\, t_2 \quad \Longrightarrow \quad 150 = \frac{1}{2}(30.0 + 15.0)t_2 \ .$$

This results in $t_2 = 6.67$ s.

(c) The definition of average velocity is given by Eq. 2-2:

$$v_{\text{avg}} = \frac{x_C - x_A}{t_C - t_A} = \frac{400 - 0}{16.7} = 24.0 \text{ m/s} \ .$$

(d) The definition of average acceleration is given by Eq. 2-7:

$$a_{\text{avg}} = \frac{v_C - v_A}{t_C - t_A} = \frac{15.0 - 20.0}{16.7} = -0.30 \text{ m/s}^2 \ .$$

107. (Third problem of **Cluster 1**)
The problem consists of two parts (A to B at constant velocity, then B to C with constant acceleration). The constant velocity in part 1 is 20 m/s (taking the positive direction in the direction of motion) and $t_1 = 5.0$ s. In part 2, we have $v_0 = 20$ m/s, $v = 0$, and $t_2 = 10$ s.

(a) We find the distance in part 1 from $x - x_0 = vt_1$, so we obtain $x = 100$ m (taking A to be at the origin). And the position at the end of part 2 is then found using Eq. 2-17.

$$x = x_0 + \frac{1}{2}(v_0 + v)\, t_2 = 100 + \frac{1}{2}(20 + 0)(10) = 200 \text{ m} \ .$$

(b) The acceleration in part (a) can be found using Eq. 2-11.

$$v = v_0 + at_2 \quad \Longrightarrow \quad 0 = 20 + a(10) \ .$$

Thus, we find $a = -2.0$ m/s^2. The negative sign indicates that the acceleration vector points opposite to the chosen positive direction (the direction of motion), which is what we expect of a deceleration.

108. (Fourth problem of **Cluster 1**)
The part 1 motion in this problem is simply that of constant velocity, so $x_B - x_0 = v_1 t_1$ applies with $t_1 = 5.00$ s and $x_0 = x_A = 0$ if we choose point A as the coordinate origin. Next, the part 2 motion consists of constant acceleration (so the equations of Table 2-1, such as Eq. 2-17, apply) with $x_0 = x_B$ (an unknown), $v_0 = v_B$ (also unknown, but equal to the v_1 above), $x_C = 300$ m, $v_C = 10.0$ m/s, and $t_2 = 20.0$ s. The equations describing parts 1 and 2, respectively, are therefore

$$\begin{aligned} x_B - x_A = v_1 t_1 \quad &\Longrightarrow \quad x_B = v_1(5.00) \\ x_C - x_B = \frac{1}{2}(v_B + v_C)\, t_2 \quad &\Longrightarrow \quad 300 - x_B = \frac{1}{2}(v_B + 10.0)\,(20.0) \end{aligned}$$

(a) We use the fact that $v_A = v_1 = v_B$ in solving this set of simultaneous equations. Adding equations, we obtain the result $v_1 = 13.3$ m/s.

(b) In order to find the acceleration, we use our result from part (a) as the initial velocity in Eq. 2-14 (applied to the part 2 motion):

$$v = v_0 + at_2 \quad \Longrightarrow \quad 10.0 = 13.3 + a(20.0)$$

Thus, $a = -0.167$ m/s^2.

109. (Fifth problem of **Cluster 1**)

The problem consists of two parts, where part 1 (A to B) involves constant velocity motion for $t_1 = 5.00$ s and part 2 (B to C) involves uniformly accelerated motion. Assuming the coordinate origin is at point A and the positive axis is directed towards B and C, then we have $x_C = 250$ m, $a_2 = -0.500$ m/s^2, and $v_C = 0$.

(a) We set up the uniform velocity equation for part 1 ($\Delta x = vt$) and Eq. 2-16 for part 2 ($v^2 = v_0^2 + 2a\Delta x$) as a simultaneous set of equations to be solved:

$$\begin{aligned} x_B - 0 &= v_1(5.00) \\ 0^2 &= v_B^2 + 2(-0.500)\,(250 - x_B)\ . \end{aligned}$$

Bearing in mind that $v_A = v_1 = v_B$, we can solve the equations by, for instance, substituting the first into the second – eliminating x_B and leading to a quadratic equation for v_1:

$$v_1^2 + 5v_1 - 150 = 0\ .$$

The positive root gives us $v_1 = 13.5$ m/s.

(b) We obtain the duration t_2 of part 2 from Eq. 2-11:

$$v = v_0 + at_2 \quad\Longrightarrow\quad 0 = 13.5 + (-0.500)t_2$$

which yields the value $t_2 = 27.0$ s. Therefore, the total time is $t_1 + t_2 = 32.0$ s.

110. (Sixth problem of **Cluster 1**)

Both part 1 and part 2 of this problem involve uniformly accelerated motion, but at different rates a_1 and a_2. We take the coordinate origin at point A and direct the positive axis towards B and C. In these terms, we are given $x_A = 0$, $x_C = 1300$ m, $v_A = 0$, and $v_C = 50$ m/s. Further, the time-duration for each part is given: $t_1 = 20$ s and $t_2 = 40$ s.

(a) We have enough information to apply Eq. 2-17 ($\Delta x = \frac{1}{2}(v_0 + v)t$) to parts 1 and 2 and solve the simultaneous set:

$$\begin{aligned} x_B - x_A = \frac{1}{2}\,(v_A + v_B)\,t_1 \quad &\Longrightarrow \quad x_B = \frac{1}{2}\,v_B\,(20) \\ x_C - x_B = \frac{1}{2}\,(v_B + v_C)\,t_2 \quad &\Longrightarrow \quad 1300 - x_B = \frac{1}{2}\,(v_B + 50)\,(40) \end{aligned}$$

Adding equations, we find $v_B = 10$ m/s.

(b) The other unknown in the above set of equations is now easily found by plugging the result for v_B back in: $x_B = 100$ m.

(c) We can find a_1 a variety of ways, using the just-obtained results. We note that Eq. 2-11 is especially easy to use.

$$v = v_0 + a_1 t_1 \quad\Longrightarrow\quad 10 = 0 + a_1(20)$$

This leads to $a_1 = 0.50$ m/s^2.

(d) To find a_2 we proceed as just as we did in part (c), so that Eq. 2-11 for part 2 becomes $50 = 10 + a_2(40)$. Therefore, the acceleration is $a_2 = 1.0\,\text{m/s}^2$.