Chapter 16

1. (a) During simple harmonic motion, the speed is (momentarily) zero when the object is at a "turning point" (that is, when $x = +x_m$ or $x = -x_m$). Consider that it starts at $x = +x_m$ and we are told that $t = 0.25$ second elapses until the object reaches $x = -x_m$. To execute a full cycle of the motion (which takes a period T to complete), the object which started at $x = +x_m$ must return to $x = +x_m$ (which, by symmetry, will occur 0.25 second *after* it was at $x = -x_m$). Thus, $T = 2t = 0.50$ s.

   (b) Frequency is simply the reciprocal of the period: $f = 1/T = 2.0$ Hz.

   (c) The 36 cm distance between $x = +x_m$ and $x = -x_m$ is $2x_m$. Thus, $x_m = 36/2 = 18$ cm.

2. (a) The problem describes the time taken to execute one cycle of the motion. The period is $T = 0.75$ s.

   (b) Frequency is simply the reciprocal of the period: $f = 1/T \approx 1.3$ Hz, where the SI unit abbreviation Hz stands for Hertz, which means a cycle-per-second.

   (c) Since 2π radians are equivalent to a cycle, the angular frequency ω (in radians-per-second) is related to frequency f by $\omega = 2\pi f$ so that $\omega \approx 8.4$ rad/s.

3. (a) The motion repeats every 0.500 s so the period must be $T = 0.500$ s.

   (b) The frequency is the reciprocal of the period: $f = 1/T = 1/(0.500\,\text{s}) = 2.00\,\text{Hz}$.

   (c) The angular frequency ω is $\omega = 2\pi f = 2\pi(2.00\,\text{Hz}) = 12.57\,\text{rad/s}$.

   (d) The angular frequency is related to the spring constant k and the mass m by $\omega = \sqrt{k/m}$. We solve for k: $k = m\omega^2 = (0.500\,\text{kg})(12.57\,\text{rad/s})^2 = 79.0\,\text{N/m}$.

   (e) Let x_m be the amplitude. The maximum speed is $v_m = \omega x_m = (12.57\,\text{rad/s})(0.350\,\text{m}) = 4.40\,\text{m/s}$.

   (f) The maximum force is exerted when the displacement is a maximum and its magnitude is given by $F_m = kx_m = (79.0\,\text{N/m})(0.350\,\text{m}) = 27.6\,\text{N}$.

4. The textbook notes (in the discussion immediately after Eq. 16-7) that the acceleration amplitude is $a_m = \omega^2 x_m$, where ω is the angular frequency ($\omega = 2\pi f$ since there are 2π radians in one cycle). Therefore, in this circumstance, we obtain

$$a_m = \left(2\pi(6.60\,\text{Hz})\right)^2 (0.0220\,\text{m}) = 37.8\ \text{m/s}^2 \ .$$

5. The magnitude of the maximum acceleration is given by $a_m = \omega^2 x_m$, where ω is the angular frequency and x_m is the amplitude. The angular frequency for which the maximum acceleration is g is given by $\omega = \sqrt{g/x_m}$, and the corresponding frequency is given by

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{g}{x_m}} = \frac{1}{2\pi}\sqrt{\frac{9.8\,\text{m/s}^2}{1.0 \times 10^{-6}\,\text{m}}} = 500\ \text{Hz} \ .$$

   For frequencies greater than 500 Hz, the acceleration exceeds g for some part of the motion.

6. (a) Hooke's law readily yields $k = (15\,\mathrm{kg})(9.8\,\mathrm{m/s^2})/(0.12\,\mathrm{m}) = 1225\,\mathrm{N/m}$. Rounding to three significant figures, the spring constant is therefore 1.23 kN/m.

   (b) We are told $f = 2.00\,\mathrm{Hz} = 2.00\,\mathrm{cycles/sec}$. Since a cycle is equivalent to 2π radians, we have $\omega = 2\pi(2.00) = 4\pi$ rad/s (understood to be valid to three significant figures). Using Eq. 16-12, we find

$$\omega = \sqrt{\frac{k}{m}} \implies m = \frac{1225\,\mathrm{N/m}}{(4\pi\,\mathrm{rad/s})^2} = 7.76\ \mathrm{kg}\ .$$

   Consequently, the weight of the package is $mg = 76$ N.

7. (a) The angular frequency ω is given by $\omega = 2\pi f = 2\pi/T$, where f is the frequency and T is the period. The relationship $f = 1/T$ was used to obtain the last form. Thus $\omega = 2\pi/(1.00 \times 10^{-5}\,\mathrm{s}) = 6.28 \times 10^5\,\mathrm{rad/s}$.

   (b) The maximum speed v_m and maximum displacement x_m are related by $v_m = \omega x_m$, so

$$x_m = \frac{v_m}{\omega} = \frac{1.00 \times 10^3\,\mathrm{m/s}}{6.28 \times 10^5\,\mathrm{rad/s}} = 1.59 \times 10^{-3}\,\mathrm{m}\ .$$

8. (a) The acceleration amplitude is related to the maximum force by Newton's second law: $F_{\max} = ma_m$. The textbook notes (in the discussion immediately after Eq. 16-7) that the acceleration amplitude is $a_m = \omega^2 x_m$, where ω is the angular frequency ($\omega = 2\pi f$ since there are 2π radians in one cycle). The frequency is the reciprocal of the period: $f = 1/T = 1/0.20 = 5.0$ Hz, so the angular frequency is $\omega = 10\pi$ (understood to be valid to two significant figures). Therefore,

$$F_{\max} = m\omega^2 x_m = (0.12\,\mathrm{kg})(10\pi\,\mathrm{rad/s})^2(0.085\,\mathrm{m}) = 10\ \mathrm{N}\ .$$

   (b) Using Eq. 16-12, we obtain

$$\omega = \sqrt{\frac{k}{m}} \implies k = (0.12\,\mathrm{kg})(10\pi\,\mathrm{rad/s})^2 = 1.2 \times 10^2\ \mathrm{N/m}\ .$$

9. (a) The amplitude is half the range of the displacement, or $x_m = 1.0\,\mathrm{mm}$.

   (b) The maximum speed v_m is related to the amplitude x_m by $v_m = \omega x_m$, where ω is the angular frequency. Since $\omega = 2\pi f$, where f is the frequency,

$$v_m = 2\pi f x_m = 2\pi(120\,\mathrm{Hz})\left(1.0 \times 10^{-3}\,\mathrm{m}\right) = 0.75\ \mathrm{m/s}\ .$$

   (c) The maximum acceleration is

$$a_m = \omega^2 x_m = (2\pi f)^2 x_m = (2\pi(120\,\mathrm{Hz}))^2\left(1.0 \times 10^{-3}\,\mathrm{m}\right) = 570\ \mathrm{m/s^2}\ .$$

10. (a) The problem gives the frequency $f = 440$ Hz, where the SI unit abbreviation Hz stands for Hertz, which means a cycle-per-second. The angular frequency ω is similar to frequency except that ω is in radians-per-second. Recalling that 2π radians are equivalent to a cycle, we have $\omega = 2\pi f \approx 2800$ rad/s.

   (b) In the discussion immediately after Eq. 16-6, the book introduces the velocity amplitude $v_m = \omega x_m$. With $x_m = 0.00075$ m and the above value for ω, this expression yields $v_m = 2.1$ m/s.

   (c) In the discussion immediately after Eq. 16-7, the book introduces the acceleration amplitude $a_m = \omega^2 x_m$, which (if the more precise value $\omega = 2765$ rad/s is used) yields $a_m = 5.7$ km/s.

11. (a) Since the problem gives the frequency $f = 3.00$ Hz, we have $\omega = 2\pi f = 6\pi$ rad/s (understood to be valid to three significant figures). Each spring is considered to support one fourth of the mass m_{car} so that Eq. 16-12 leads to

$$\omega = \sqrt{\frac{k}{\frac{1}{4}m_{\mathrm{car}}}} \implies k = \left(\frac{1}{4}(1450\,\mathrm{kg})\right)(6\pi\,\mathrm{rad/s})^2 = 1.29 \times 10^5\ \mathrm{N/m}\ .$$

(b) If the new mass being supported by the four springs is $m_{\text{total}} = 1450 + 5(73) = 1815$ kg, then Eq. 16-12 leads to

$$\omega_{\text{new}} = \sqrt{\frac{k}{\frac{1}{4}\,m_{\text{total}}}} \implies f_{\text{new}} = \frac{1}{2\pi}\sqrt{\frac{1.29\times 10^5}{1815/4}} = 2.68 \text{ Hz} .$$

12. (a) Making sure our calculator is in radians mode, we find

$$x = 6.0 \cos\left(3\pi(2.0) + \frac{\pi}{3}\right) = 3.0 \text{ m} .$$

(b) Differentiating with respect to time and evaluating at $t = 2.0$ s, we find

$$v = \frac{dx}{dt} = -3\pi(6.0)\sin\left(3\pi(2.0) + \frac{\pi}{3}\right) = -49 \text{ m/s} .$$

(c) Differentiating again, we obtain

$$a = \frac{dv}{dt} = -(3\pi)^2(6.0)\cos\left(3\pi(2.0) + \frac{\pi}{3}\right) = -2.7\times 10^2 \text{ m/s}^2 .$$

(d) In the second paragraph after Eq. 16-3, the textbook defines the phase of the motion. In this case (with $t = 2.0$ s) the phase is $3\pi(2.0) + \frac{\pi}{3} \approx 20$ rad.

(e) Comparing with Eq. 16-3, we see that $\omega = 3\pi$ rad/s. Therefore, $f = \omega/2\pi = 1.5$ Hz.

(f) The period is the reciprocal of the frequency: $T = 1/f \approx 0.67$ s.

13. We use $v_m = \omega x_m = 2\pi f x_m$, where the frequency is $180/(60\,\text{s}) = 3.0\,\text{Hz}$ and the amplitude is half the stroke, or $x_m = 0.38\,\text{m}$. Thus, $v_m = 2\pi(3.0\,\text{Hz})(0.38\,\text{m}) = 7.2\,\text{m/s}$.

14. (a) For a total mass of $m + M$, Eq. 16-12 becomes

$$\omega = \sqrt{\frac{k}{m+M}} \implies M = \frac{k}{\omega^2} - m .$$

Eq. 16-5 ($\omega = 2\pi/T$) is used to put this into its final form:

$$M = \frac{k}{(2\pi/T)^2} - m = \left(\frac{k}{4\pi^2}\right)T^2 - m .$$

(b) With $T = 0.90149$ s, $k = 605.6$ N/m and $M = 0$ in the above expression, we obtain $m = 12.47$ kg.

(c) With the same k and m, we plug $T = 2.08832$ s into the expression and obtain $M = 54.43$ kg.

15. From highest level to lowest level is twice the amplitude x_m of the motion. The period is related to the angular frequency by Eq. 16-5. Thus, $x_m = \frac{1}{2}d$ and $\omega = 0.503$ rad/h. The phase constant ϕ in Eq. 16-3 is zero since we start our clock when $x_o = x_m$ (at the highest point). We solve for t when x is one-fourth of the total distance from highest to lowest level, or (which is the same) half the distance from highest level to middle level (where we locate the origin of coordinates). Thus, we seek t when the ocean surface is at $x = \frac{1}{2}x_m = \frac{1}{4}d$.

$$\begin{aligned} x &= x_m \cos(\omega t + \phi) \\ \frac{1}{4}d &= \left(\frac{1}{2}d\right)\cos(0.503t + 0) \\ \frac{1}{2} &= \cos(0.503t) \end{aligned}$$

which has $t = 2.08$ h as the smallest positive root. The calculator is in radians mode during this calculation.

16. To be on the verge of slipping means that the force exerted on the smaller block (at the point of maximum acceleration) is $f_{\rm max} = \mu_s mg$. The textbook notes (in the discussion immediately after Eq. 16-7) that the acceleration amplitude is $a_m = \omega^2 x_m$, where ω is the angular frequency ($\omega = \sqrt{k/(m+M)}$ from Eq. 16-12). Therefore, using Newton's second law, we have

$$ma_m = \mu_s mg \implies \frac{k}{m+M}\, x_m = \mu_s\, g$$

which leads to $x_m = 0.22$ m.

17. The maximum force that can be exerted by the surface must be less than $\mu_s N$ or else the block will not follow the surface in its motion. Here, μ_s is the coefficient of static friction and N is the normal force exerted by the surface on the block. Since the block does not accelerate vertically, we know that $N = mg$, where m is the mass of the block. If the block follows the table and moves in simple harmonic motion, the magnitude of the maximum force exerted on it is given by $F = ma_m = m\omega^2 x_m = m(2\pi f)^2 x_m$, where a_m is the magnitude of the maximum acceleration, ω is the angular frequency, and f is the frequency. The relationship $\omega = 2\pi f$ was used to obtain the last form. We substitute $F = m(2\pi f)^2 x_m$ and $N = mg$ into $F < \mu_s N$ to obtain $m(2\pi f)^2 x_m < \mu_s mg$. The largest amplitude for which the block does not slip is

$$x_m = \frac{\mu_s g}{(2\pi f)^2} = \frac{(0.50)(9.8\,\mathrm{m/s^2})}{(2\pi \times 2.0\,\mathrm{Hz})^2} = 0.031\,\mathrm{m}\ .$$

A larger amplitude requires a larger force at the end points of the motion. The surface cannot supply the larger force and the block slips.

18. Both parts of this problem deal with the critical case when the maximum acceleration becomes equal to that of free fall. The textbook notes (in the discussion immediately after Eq. 16-7) that the acceleration amplitude is $a_m = \omega^2 x_m$, where ω is the angular frequency; this is the expression we set equal to $g = 9.8\,\mathrm{m/s^2}$.

(a) Using Eq. 16-5 and $T = 1.0$ s, we have

$$\left(\frac{2\pi}{T}\right)^2 x_m = g \implies x_m = \frac{gT^2}{4\pi^2} = 0.25\ \mathrm{m}\ .$$

(b) Since $\omega = 2\pi f$, and $x_m = 0.050$ m is given, we find

$$\left(2\pi f)^2\right) x_m = g \implies f = \frac{1}{2\pi}\sqrt{\frac{g}{x_m}} = 2.2\ \mathrm{Hz}\ .$$

19. (a) Eq. 16-8 leads to

$$a = -\omega^2 x \implies \omega = \sqrt{\frac{-a}{x}} = \sqrt{\frac{123}{0.100}}$$

which yields $\omega = 35.07$ rad/s. Therefore, $f = \omega/2\pi = 5.58$ Hz.

(b) Eq. 16-12 provides a relation between ω (found in the previous part) and the mass:

$$\omega = \sqrt{\frac{k}{m}} \implies m = \frac{400}{35.07^2} = 0.325\ \mathrm{kg}\ .$$

(c) By energy conservation, $\frac{1}{2}kx_m^2$ (the energy of the system at a turning point) is equal to the sum of kinetic and potential energies at the time t described in the problem.

$$\frac{1}{2}kx_m^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 \implies x_m = \frac{m}{k}v^2 + x^2\ .$$

Consequently, $x_m = \sqrt{(0.325/400)(13.6)^2 + 0.1^2} = 0.400$ m.

20. Eq. 16-12 gives the angular velocity:

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{100}{2.00}} = 7.07 \text{ rad/s} .$$

Energy methods (discussed in §16-4) provide one method of solution. Here, we use trigonometric techniques based on Eq. 16-3 and Eq. 16-6.

(a) Dividing Eq. 16-6 by Eq. 16-3, we obtain

$$\frac{v}{x} = -\omega \tan(\omega t + \phi)$$

so that the phase $(\omega t + \phi)$ is found from

$$\omega t + \phi = \tan^{-1}\left(\frac{-v}{\omega x}\right) = \tan^{-1}\left(\frac{-3.415}{(7.07)(0.129)}\right) .$$

With the calculator in radians mode, this gives the phase equal to -1.31 rad. Plugging this back into Eq. 16-3 leads to

$$0.129\,\text{m} = x_m \cos(-1.31) \quad \Longrightarrow \quad 0.500\,\text{m} = x_m .$$

(b) Since $\omega t + \phi = -1.31$ rad at $t = 1.00$ s. We can use the above value of ω to solve for the phase constant ϕ. We obtain $\phi = -8.38$ rad (though this, as well as the previous result, can have 2π or 4π (and so on) added to it without changing the physics of the situation). With this value of ϕ, we find $x_o = x_m \cos\phi = -0.251$ m.

(c) And we obtain $v_o = -x_m \omega \sin\phi = 3.06$ m/s.

21. (a) The object oscillates about its equilibrium point, where the downward force of gravity is balanced by the upward force of the spring. If ℓ is the elongation of the spring at equilibrium, then $k\ell = mg$, where k is the spring constant and m is the mass of the object. Thus $k/m = g/\ell$ and $f = \omega/2\pi = (1/2\pi)\sqrt{k/m} = (1/2\pi)\sqrt{g/\ell}$. Now the equilibrium point is halfway between the points where the object is momentarily at rest. One of these points is where the spring is unstretched and the other is the lowest point, 10 cm below. Thus $\ell = 5.0\,\text{cm} = 0.050\,\text{m}$ and

$$f = \frac{1}{2\pi}\sqrt{\frac{9.8\,\text{m/s}^2}{0.050\,\text{m}}} = 2.23\,\text{Hz} .$$

(b) Use conservation of energy. We take the zero of gravitational potential energy to be at the initial position of the object, where the spring is unstretched. Then both the initial potential and kinetic energies are zero. We take the y axis to be positive in the downward direction and let $y = 0.080\,\text{m}$. The potential energy when the object is at this point is $U = \frac{1}{2}ky^2 - mgy$. The energy equation becomes $0 = \frac{1}{2}ky^2 - mgy + \frac{1}{2}mv^2$. We solve for the speed.

$$\begin{aligned} v &= \sqrt{2gy - \frac{k}{m}y^2} = \sqrt{2gy - \frac{g}{\ell}y^2} \\ &= \sqrt{2(9.8\,\text{m/s}^2)(0.080\,\text{m}) - \left(\frac{9.8\,\text{m/s}^2}{0.050\,\text{m}}\right)(0.080\,\text{m})^2} = 0.56 \text{ m/s} \end{aligned}$$

(c) Let m be the original mass and Δm be the additional mass. The new angular frequency is $\omega' = \sqrt{k/(m + \Delta m)}$. This should be half the original angular frequency, or $\frac{1}{2}\sqrt{k/m}$. We solve $\sqrt{k/(m + \Delta m)} = \frac{1}{2}\sqrt{k/m}$ for m. Square both sides of the equation, then take the reciprocal to obtain $m + \Delta m = 4m$. This gives $m = \Delta m/3 = (300\,\text{g})/3 = 100\,\text{g}$.

(d) The equilibrium position is determined by the balancing of the gravitational and spring forces: $ky = (m + \Delta m)g$. Thus $y = (m + \Delta m)g/k$. We will need to find the value of the spring constant k. Use $k = m\omega^2 = m(2\pi f)^2$. Then

$$y = \frac{(m + \Delta m)g}{m(2\pi f)^2} = \frac{(0.10\,\mathrm{kg} + 0.30\,\mathrm{kg})\left(9.8\,\mathrm{m/s^2}\right)}{(0.10\,\mathrm{kg})(2\pi \times 2.24\,\mathrm{Hz})^2} = 0.20\,\mathrm{m}\ .$$

This is measured from the initial position.

22. They pass each other at time t, at $x_1 = x_2 = \frac{1}{2}x_m$ where

$$x_1 = x_m \cos(\omega t + \phi_1) \quad \text{and} \quad x_2 = x_m \cos(\omega t + \phi_2)\ .$$

From this, we conclude that $\cos(\omega t + \phi_1) = \cos(\omega t + \phi_2) = \frac{1}{2}$, and therefore that the phases (the arguments of the cosines) are either both equal to $\pi/3$ or one is $\pi/3$ while the other is $-\pi/3$. Also at this instant, we have $v_1 = -v_2 \neq 0$ where

$$v_1 = -x_m\omega \sin(\omega t + \phi_1) \quad \text{and} \quad v_2 = -x_m\omega \sin(\omega t + \phi_2)\ .$$

This leads to $\sin(\omega t + \phi_1) = -\sin(\omega t + \phi_2)$. This leads us to conclude that the phases have opposite sign. Thus, one phase is $\pi/3$ and the other phase is $-\pi/3$; the ωt term cancels if we take the phase difference, which is seen to be $\pi/3 - (-\pi/3) = 2\pi/3$.

23. (a) Let

$$x_1 = \frac{A}{2}\cos\left(\frac{2\pi t}{T}\right)$$

be the coordinate as a function of time for particle 1 and

$$x_2 = \frac{A}{2}\cos\left(\frac{2\pi t}{T} + \frac{\pi}{6}\right)$$

be the coordinate as a function of time for particle 2. Here T is the period. Note that since the range of the motion is A, the amplitudes are both $A/2$. The arguments of the cosine functions are in radians. Particle 1 is at one end of its path ($x_1 = A/2$) when $t = 0$. Particle 2 is at $A/2$ when $2\pi t/T + \pi/6 = 0$ or $t = -T/12$. That is, particle 1 lags particle 2 by one-twelfth a period. We want the coordinates of the particles 0.50 s later; that is, at $t = 0.50\,\mathrm{s}$,

$$x_1 = \frac{A}{2}\cos\left(\frac{2\pi \times 0.50\,\mathrm{s}}{1.5\,\mathrm{s}}\right) = -0.250A$$

and

$$x_2 = \frac{A}{2}\cos\left(\frac{2\pi \times 0.50\,\mathrm{s}}{1.5\,\mathrm{s}} + \frac{\pi}{6}\right) = -0.433A\ .$$

Their separation at that time is $x_1 - x_2 = -0.250A + 0.433A = 0.183A$.

(b) The velocities of the particles are given by

$$v_1 = \frac{\mathrm{d}x_1}{\mathrm{d}t} = \frac{\pi A}{T}\sin\left(\frac{2\pi t}{T}\right)$$

and

$$v_2 = \frac{\mathrm{d}x_2}{\mathrm{d}t} = \frac{\pi A}{T}\sin\left(\frac{2\pi t}{T} + \frac{\pi}{6}\right)\ .$$

We evaluate these expressions for $t = 0.50\,\mathrm{s}$ and find they are both negative-valued, indicating that the particles are moving in the same direction.

24. When displaced from equilibrium, the net force exerted by the springs is $-2kx$ acting in a direction so as to return the block to its equilibrium position ($x = 0$). Since the acceleration $a = d^2x/dt^2$, Newton's second law yields

$$m\,\frac{d^2x}{dt^2} = -2kx \ .$$

Substituting $x = x_m \cos(\omega t + \phi)$ and simplifying, we find

$$\omega^2 = \frac{2k}{m}$$

where ω is in radians per unit time. Since there are 2π radians in a cycle, and frequency f measures cycles per second, we obtain

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{2k}{m}} \ .$$

25. When displaced from equilibrium, the magnitude of the net force exerted by the springs is $|k_1x + k_2x|$ acting in a direction so as to return the block to its equilibrium position ($x = 0$). Since the acceleration $a = d^2x/dt^2$, Newton's second law yields

$$m\,\frac{d^2x}{dt^2} = -k_1x - k_2x \ .$$

Substituting $x = x_m \cos(\omega t + \phi)$ and simplifying, we find

$$\omega^2 = \frac{k_1 + k_2}{m}$$

where ω is in radians per unit time. Since there are 2π radians in a cycle, and frequency f measures cycles per second, we obtain

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k_1 + k_2}{m}} \ .$$

The single springs each acting alone would produce simple harmonic motions of frequency

$$f_1 = \frac{1}{2\pi}\sqrt{\frac{k_1}{m}} \quad \text{and} \quad f_2 = \frac{1}{2\pi}\sqrt{\frac{k_2}{m}} \ ,$$

respectively. Comparing these expressions, it is clear that $f = \sqrt{f_1^2 + f_2^2}$.

26. (a) The textbook notes (in the discussion immediately after Eq. 16-7) that the acceleration amplitude is $a_m = \omega^2 x_m$, where ω is the angular frequency ($\omega = 2\pi f$ since there are 2π radians in one cycle). Therefore, in this circumstance, we obtain

$$a_m = (2\pi(1000\,\mathrm{Hz}))^2\,(0.00040\,\mathrm{m}) = 1.6 \times 10^4 \ \mathrm{m/s^2} \ .$$

(b) Similarly, in the discussion after Eq. 16-6, we find $v_m = \omega x_m$ so that

$$v_m = (2\pi(1000\,\mathrm{Hz}))\,(0.00040\,\mathrm{m}) = 2.5 \ \mathrm{m/s} \ .$$

(c) From Eq. 16-8, we have (in absolute value)

$$|a| = (2\pi(1000\,\mathrm{Hz}))^2\,(0.00020\,\mathrm{m}) = 7.9 \times 10^3 \ \mathrm{m/s^2} \ .$$

(d) This can be approached with the energy methods of §16-4, but here we will use trigonometric relations along with Eq. 16-3 and Eq. 16-6. Thus, allowing for both roots stemming from the square root,

$$\begin{aligned} \sin(\omega t + \phi) &= \pm\sqrt{1 - \cos^2(\omega t + \phi)} \\ -\frac{v}{\omega x_m} &= \pm\sqrt{1 - \frac{x^2}{x_m^2}} \ . \end{aligned}$$

Taking absolute values and simplifying, we obtain

$$|v| = 2\pi f\sqrt{x_m^2 - x^2} = 2\pi(1000)\sqrt{0.00040^2 - 0.00020^2} = 2.2 \text{ m/s} .$$

27. We wish to find the effective spring constant for the combination of springs shown in Fig. 16–31. We do this by finding the magnitude F of the force exerted on the mass when the total elongation of the springs is Δx. Then $k_{\text{eff}} = F/\Delta x$. Suppose the left-hand spring is elongated by Δx_ℓ and the right-hand spring is elongated by Δx_r. The left-hand spring exerts a force of magnitude $k\,\Delta x_\ell$ on the right-hand spring and the right-hand spring exerts a force of magnitude $k\,\Delta x_r$ on the left-hand spring. By Newton's third law these must be equal, so $\Delta x_\ell = \Delta x_r$. The two elongations must be the same and the total elongation is twice the elongation of either spring: $\Delta x = 2\Delta x_\ell$. The left-hand spring exerts a force on the block and its magnitude is $F = k\,\Delta x_\ell$. Thus $k_{\text{eff}} = k\,\Delta x_\ell/2\Delta x_r = k/2$. The block behaves as if it were subject to the force of a single spring, with spring constant $k/2$. To find the frequency of its motion replace k_{eff} in $f = (1/2\pi)\sqrt{k_{\text{eff}}/m}$ with $k/2$ to obtain

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{2m}} .$$

28. (a) We interpret the problem as asking for the equilibrium position; that is, the block is gently lowered until forces balance (as opposed to being suddenly released and allowed to oscillate). If the amount the spring is stretched is x, then we examine force-components along the incline surface and find

$$kx = mg\sin\theta \implies x = \frac{14.0\sin 40.0°}{120} = 0.075 \text{ m}$$

at equilibrium. The calculator is in degrees mode in the above calculation. The distance from the top of the incline is therefore $0.450 + 0.75 = 0.525$ m.

(b) Just as with a vertical spring, the effect of gravity (or one of its components) is simply to shift the equilibrium position; it does not change the characteristics (such as the period) of simple harmonic motion. Thus, Eq. 16-13 applies, and we obtain

$$T = 2\pi\sqrt{\frac{14.0/9.8}{120}} = 0.686 \text{ s} .$$

29. (a) First consider a single spring with spring constant k and unstretched length L. One end is attached to a wall and the other is attached to an object. If it is elongated by Δx the magnitude of the force it exerts on the object is $F = k\,\Delta x$. Now consider it to be two springs, with spring constants k_1 and k_2, arranged so spring 1 is attached to the object. If spring 1 is elongated by Δx_1 then the magnitude of the force exerted on the object is $F = k_1\,\Delta x_1$. This must be the same as the force of the single spring, so $k\,\Delta x = k_1\,\Delta x_1$. We must determine the relationship between Δx and Δx_1. The springs are uniform so equal unstretched lengths are elongated by the same amount and the elongation of any portion of the spring is proportional to its unstretched length. This means spring 1 is elongated by $\Delta x_1 = CL_1$ and spring 2 is elongated by $\Delta x_2 = CL_2$, where C is a constant of proportionality. The total elongation is $\Delta x = \Delta x_1 + \Delta x_2 = C(L_1 + L_2) = CL_2(n+1)$, where $L_1 = nL_2$ was used to obtain the last form. Since $L_2 = L_1/n$, this can also be written $\Delta x = CL_1(n+1)/n$. We substitute $\Delta x_1 = CL_1$ and $\Delta x = CL_1(n+1)/n$ into $k\,\Delta x = k_1\,\Delta x_1$ and solve for k_1. The result is $k_1 = k(n+1)/n$.

(b) Now suppose the object is placed at the other end of the composite spring, so spring 2 exerts a force on it. Now $k\,\Delta x = k_2\,\Delta x_2$. We use $\Delta x_2 = CL_2$ and $\Delta x = CL_2(n+1)$, then solve for k_2. The result is $k_2 = k(n+1)$.

(c) To find the frequency when spring 1 is attached to mass m, we replace k in $(1/2\pi)\sqrt{k/m}$ with $k(n+1)/n$ to obtain

$$f_1 = \frac{1}{2\pi}\sqrt{\frac{(n+1)k}{nm}} = \sqrt{\frac{n+1}{n}}f$$

where the substitution $f = (1/2\pi)\sqrt{k/m}$ was made.

(d) To find the frequency when spring 2 is attached to the mass, we replace k with $k(n+1)$ to obtain

$$f_2 = \frac{1}{2\pi}\sqrt{\frac{(n+1)k}{m}} = \sqrt{n+1}f$$

where the same substitution was made.

30. The magnitude of the downhill component of the gravitational force acting on each ore car is

$$w_x = (10000\,\text{kg})\left(9.8\,\text{m/s}^2\right)\sin\theta$$

where $\theta = 30°$ (and it is important to have the calculator in degrees mode during this problem). We are told that a downhill pull of $3w_x$ causes the cable to stretch $x = 0.15\,\text{m}$. Since the cable is expected to obey Hooke's law, its spring constant is

$$k = \frac{3w_x}{x} = 9.8 \times 10^5 \text{ N/m} .$$

(a) Noting that the oscillating mass is that of *two* of the cars, we apply Eq. 16-12 (divided by 2π).

$$f = \frac{1}{2\pi}\sqrt{\frac{9.8 \times 10^5\,\text{N/m}}{20000\,\text{kg}}} = 1.1 \text{ Hz} .$$

(b) The difference between the equilibrium positions of the end of the cable when supporting two as opposed to three cars is

$$\Delta x = \frac{3w_x - 2w_x}{k} = 0.050 \text{ m} .$$

31. When the block is at the end of its path and is momentarily stopped, its displacement is equal to the amplitude and all the energy is potential in nature. If the spring potential energy is taken to be zero when the block is at its equilibrium position, then

$$E = \frac{1}{2}kx_m^2 = \frac{1}{2}(1.3 \times 10^2\,\text{N/m})(0.024\,\text{m})^2 = 3.7 \times 10^{-2} \text{ J} .$$

32. (a) The energy at the turning point is all potential energy: $E = \frac{1}{2}kx_m^2$ where $E = 1.00$ J and $x_m = 0.100$ m. Thus,

$$k = \frac{2E}{x_m^2} = 200 \text{ N/m} .$$

(b) The energy as the block passes through the equilibrium position (with speed $v_m = 1.20$ m/s)is purely kinetic:

$$E = \frac{1}{2}mv_m^2 \implies m = \frac{2E}{v_m^2} = 1.39 \text{ kg} .$$

(c) Eq. 16-12 (divided by 2π) yields

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = 1.91 \text{ Hz} .$$

33. (a) Eq. 16-12 (divided by 2π) yields

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{1000\,\text{N/m}}{5.00\,\text{kg}}} = 2.25 \text{ Hz} .$$

(b) With $x_o = 0.500$ m, we have $U_o = \frac{1}{2}kx_o^2 = 125$ J.

(c) With $v_o = 10.0$ m/s, the initial kinetic energy is $K_o = \frac{1}{2}mv_o^2 = 250$ J.

(d) Since the total energy $E = K_o + U_o = 375$ J is conserved, then consideration of the energy at the turning point leads to

$$E = \frac{1}{2}kx_m^2 \implies x_m = \sqrt{\frac{2E}{k}} = 0.866 \text{ m} .$$

34. (a) We require

$$\frac{1}{2}kx_m^2 = \frac{1}{2}mv_m^2 \implies k = m\left(\frac{v_m}{x_m}\right)^2$$

where $m = 0.130$ kg, $v_m = 11200$ m/s and $x_m = 1.50$ m. This yields $k = 7.25 \times 10^6$ N/m.

(b) The force required to produce an elongation x_m if the spring constant is k is $kx_m = 1.087 \times 10^7$ N. Dividing this among N persons, each one exerting a force of 220 N, requires $N = 1.087 \times 10^7/220 \approx 49400$.

35. (a) The spring stretches until the magnitude of its upward force on the block equals the magnitude of the downward force of gravity: $ky = mg$, where $y = 0.096$ m is the elongation of the spring at equilibrium, k is the spring constant, and $m = 1.3$ kg is the mass of the block. Thus $k = mg/y = (1.3)(9.8)/0.096 = 133\,\text{N/m}$.

(b) The period is given by $T = 1/f = 2\pi/\omega = 2\pi\sqrt{m/k} = 2\pi\sqrt{1.3/133} = 0.62\,\text{s}$.

(c) The frequency is $f = 1/T = 1/0.62\,\text{s} = 1.6\,\text{Hz}$.

(d) The block oscillates in simple harmonic motion about the equilibrium point determined by the forces of the spring and gravity. It is started from rest 5.0 cm below the equilibrium point so the amplitude is 5.0 cm.

(e) The block has maximum speed as it passes the equilibrium point. At the initial position, the block is not moving but it has potential energy

$$U_i = -mgy_i + \frac{1}{2}ky_i^2 = -(1.3)(9.8)(0.146) + \frac{1}{2}(133)(0.146)^2 = -0.44 \text{ J} .$$

When the block is at the equilibrium point, the elongation of the spring is $y = 9.6$ cm and the potential energy is

$$U_f = -mgy + \frac{1}{2}ky^2 = -(1.3)(9.8)(0.096) + \frac{1}{2}(133)(0.096)^2 = -0.61 \text{ J} .$$

We write the equation for conservation of energy as $U_i = U_f + \frac{1}{2}mv^2$ and solve for v:

$$v = \sqrt{\frac{2(U_i - U_f)}{m}} = \sqrt{\frac{2(-0.44\,\text{J} + 0.61\,\text{J})}{1.3\,\text{kg}}} = 0.51 \text{ m/s} .$$

36. The problem consists of two distinct parts: the completely inelastic collision (which is assumed to occur instantaneously, the bullet embedding itself in the block before the block moves through significant distance) followed by simple harmonic motion (of mass $m + M$ attached to a spring of spring constant k).

(a) Momentum conservation readily yields $v' = mv/(m + M)$.

(b) Since v' occurs at the equilibrium position, then $v' = v_m$ for the simple harmonic motion. The relation $v_m = \omega x_m$ can be used to solve for x_m, or we can pursue the alternate (though related) approach of energy conservation. Here we choose the latter:

$$\begin{aligned} \frac{1}{2}(m+M)\,(v')^2 &= \frac{1}{2}kx_m^2 \\ \frac{1}{2}(m+M)\frac{m^2v^2}{(m+M)^2} &= \frac{1}{2}kx_m^2 \end{aligned}$$

which simplifies to

$$x_m = \frac{mv}{\sqrt{k(m+M)}} \ .$$

37. The total energy is given by $E = \frac{1}{2}kx_m^2$, where k is the spring constant and x_m is the amplitude. We use the answer from part (b) to do part (a), so it is best to look at the solution for part (b) first.

   (a) The fraction of the energy that is kinetic is

   $$\frac{K}{E} = \frac{E-U}{E} = 1 - \frac{U}{E} = 1 - \frac{1}{4} = \frac{3}{4}$$

   where the result from part (b) has been used.

   (b) When $x = \frac{1}{2}x_m$ the potential energy is $U = \frac{1}{2}kx^2 = \frac{1}{8}kx_m^2$. The ratio is

   $$\frac{U}{E} = \frac{\frac{1}{8}kx_m^2}{\frac{1}{2}kx_m^2} = \frac{1}{4} \ .$$

   (c) Since $E = \frac{1}{2}kx_m^2$ and $U = \frac{1}{2}kx^2$, $U/E = x^2/x_m^2$. We solve $x^2/x_m^2 = 1/2$ for x. We should get $x = x_m/\sqrt{2}$.

38. The textbook notes (in the discussion immediately after Eq. 16-7) that the acceleration amplitude is $a_m = \omega^2 x_m$, where ω is the angular frequency and $x_m = 0.0020$ m is the amplitude. Thus, $a_m = 8000\,\text{m/s}^2$ leads to $\omega = 2000$ rad/s.

   (a) Using Newton's second law with $m = 0.010$ kg, we have

   $$F = ma = m\left(-a_m \cos(\omega t + \phi)\right) = -(80\,\text{N})\cos\left(2000t - \frac{\pi}{3}\right)$$

   where t is understood to be in seconds.

   (b) Eq. 16-5 gives $T = 2\pi/\omega = 3.1 \times 10^{-3}$ s.

   (c) The relation $v_m = \omega x_m$ can be used to solve for v_m, or we can pursue the alternate (though related) approach of energy conservation. Here we choose the latter. By Eq. 16-12, the spring constant is $k = \omega^2 m = 40000$ N/m. Then, energy conservation leads to

   $$\frac{1}{2}kx_m^2 = \frac{1}{2}mv_m^2 \implies v_m = x_m\sqrt{\frac{k}{m}} = 4.0 \text{ m/s} \ .$$

   (d) The total energy is $\frac{1}{2}kx_m^2 = \frac{1}{2}mv_m^2 = 0.080$ J.

39. (a) Assume the bullet becomes embedded and moves with the block before the block moves a significant distance. Then the momentum of the bullet-block system is conserved during the collision. Let m be the mass of the bullet, M be the mass of the block, v_0 be the initial speed of the bullet, and v be the final speed of the block and bullet. Conservation of momentum yields $mv_0 = (m+M)v$, so

   $$v = \frac{mv_0}{m+M} = \frac{(0.050\,\text{kg})(150\,\text{m/s})}{0.050\,\text{kg} + 4.0\,\text{kg}} = 1.85\,\text{m/s} \ .$$

   When the block is in its initial position the spring and gravitational forces balance, so the spring is elongated by Mg/k. After the collision, however, the block oscillates with simple harmonic motion about the point where the spring and gravitational forces balance with the bullet embedded. At this point the spring is elongated a distance $\ell = (M+m)g/k$, somewhat different from the initial elongation. Mechanical energy is conserved during the oscillation. At the initial position, just after the bullet is embedded, the kinetic energy is $\frac{1}{2}(M+m)v^2$ and the elastic potential energy is $\frac{1}{2}k(Mg/k)^2$. We take the gravitational potential energy to be zero at this point. When the block and bullet reach the highest point in their motion the kinetic energy is zero. The block is then a distance y_m above the position where the spring and gravitational forces balance. Note that y_m is the amplitude of the motion. The spring is compressed by $y_m - \ell$, so the elastic potential energy is $\frac{1}{2}k(y_m - \ell)^2$. The gravitational potential energy is $(M+m)gy_m$. Conservation of mechanical energy yields

$$\frac{1}{2}(M+m)v^2 + \frac{1}{2}k\left(\frac{Mg}{k}\right)^2 = \frac{1}{2}k(y_m - \ell)^2 + (M+m)gy_m \ .$$

We substitute $\ell = (M+m)g/k$. Algebraic manipulation leads to

$$\begin{aligned} y_m &= \sqrt{\frac{(m+M)v^2}{k} - \frac{mg^2}{k^2}(2M+m)} \\ &= \sqrt{\frac{(0.050\,\text{kg} + 4.0\,\text{kg})(1.85\,\text{m/s})^2}{500\,\text{N/m}} - \frac{(0.050\,\text{kg})(9.8\,\text{m/s}^2)^2}{(500\,\text{N/m})^2}\,[2(4.0\,\text{kg}) + 0.050\,\text{kg}]} \\ &= 0.166\ \text{m} \ . \end{aligned}$$

(b) The original energy of the bullet is $E_0 = \frac{1}{2}mv_0^2 = \frac{1}{2}(0.050\,\text{kg})(150\,\text{m/s})^2 = 563\,\text{J}$. The kinetic energy of the bullet-block system just after the collision is

$$E = \frac{1}{2}(m+M)v^2 = \frac{1}{2}(0.050\,\text{kg} + 4.0\,\text{kg})(1.85\,\text{m/s})^2 = 6.94\ \text{J} \ .$$

Since the block does not move significantly during the collision, the elastic and gravitational potential energies do not change. Thus, E is the energy that is transferred. The ratio is $E/E_0 = (6.94\,\text{J})/(563\,\text{J}) = 0.0123$ or 1.23%.

40. (a) The rotational inertia is $I = \frac{1}{2}MR^2 = \frac{1}{2}(3.00\,\text{kg})(0.700\,\text{m})^2 = 0.735\,\text{kg}\cdot\text{m}^2$.

(b) Using Eq. 16-22 (in absolute value), we find

$$\kappa = \frac{\tau}{\theta} = \frac{0.0600\,\text{N}\cdot\text{m}}{2.5\,\text{rad}} = 0.024\ \text{N}\cdot\text{m} \ .$$

(c) Using Eq. 16-5, Eq. 16-23 leads to

$$\omega = \sqrt{\frac{\kappa}{I}} = \sqrt{\frac{0.024\,\text{N}\cdot\text{m}}{0.735\,\text{kg}\cdot\text{m}^2}} 0.181\ \text{rad/s} \ .$$

41. (a) We take the angular displacement of the wheel to be $\theta = \theta_m \cos(2\pi t/T)$, where θ_m is the amplitude and T is the period. We differentiate with respect to time to find the angular velocity: $\Omega = -(2\pi/T)\theta_m \sin(2\pi t/T)$. The symbol Ω is used for the angular velocity of the wheel so it is not confused with the angular frequency. The maximum angular velocity is

$$\Omega_m = \frac{2\pi\theta_m}{T} = \frac{(2\pi)(\pi\,\text{rad})}{0.500\,\text{s}} = 39.5\ \text{rad/s} \ .$$

(b) When $\theta = \pi/2$, then $\theta/\theta_m = 1/2$, $\cos(2\pi t/T) = 1/2$, and

$$\sin(2\pi t/T) = \sqrt{1 - \cos^2(2\pi t/T)} = \sqrt{1 - (1/2)^2} = \sqrt{3}/2$$

where the trigonometric identity $\cos^2\theta + \sin^2\theta = 1$ is used. Thus,

$$\Omega = -\frac{2\pi}{T}\,\theta_m \sin\left(\frac{2\pi t}{T}\right) = -\left(\frac{2\pi}{0.500\,\text{s}}\right)(\pi\,\text{rad})\left(\frac{\sqrt{3}}{2}\right) = -34.2\ \text{rad/s} \ .$$

During another portion of the cycle its angular speed is $+34.2\,\text{rad/s}$ when its angular displacement is $\pi/2\,\text{rad}$.

(c) The angular acceleration is

$$\alpha = \frac{d^2\theta}{dt^2} = -\left(\frac{2\pi}{T}\right)^2 \theta_m \cos(2\pi t/T) = -\left(\frac{2\pi}{T}\right)^2 \theta \ .$$

When $\theta = \pi/4$,

$$\alpha = -\left(\frac{2\pi}{0.500\,\mathrm{s}}\right)^2 \left(\frac{\pi}{4}\right) = -124 \ \mathrm{rad/s}^2 \ .$$

42. (a) Eq. 16-28 gives

$$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{17\,\mathrm{m}}{9.8\,\mathrm{m/s^2}}} = 8.3 \ \mathrm{s} \ .$$

(b) Plugging $I = mL^2$ into Eq. 16-25, we see that the mass m cancels out. Thus, the characteristics (such as the period) of the periodic motion do not depend on the mass.

43. The period of a simple pendulum is given by $T = 2\pi\sqrt{L/g}$, where L is its length. Thus,

$$L = \frac{T^2 g}{4\pi^2} = \frac{(2.0\,\mathrm{s})^2(9.8\,\mathrm{m/s^2})}{4\pi^2} = 0.99 \ \mathrm{m} \ .$$

44. From Eq. 16-28, we find the length of the pendulum when the period is $T = 8.85$ s:

$$L = \frac{gT^2}{4\pi^2} \ .$$

The new length is $L' = L - d$ where $d = 0.350\,\mathrm{m}$. The new period is

$$T' = 2\pi\sqrt{\frac{L'}{g}} = 2\pi\sqrt{\frac{L}{g} - \frac{d}{g}} = 2\pi\sqrt{\frac{T^2}{4\pi^2} - \frac{d}{g}}$$

which yields $T' = 8.77$ s.

45. We use Eq. 16-29 and the parallel-axis theorem $I = I_{\mathrm{cm}} + mh^2$ where $h = d$, the unknown. For a meter stick of mass m, the rotational inertia about its center of mass is $I_{\mathrm{cm}} = mL^2/12$ where $L = 1.0$ m. Thus, for $T = 2.5$ s, we obtain

$$T = 2\pi\sqrt{\frac{mL^2/12 + md^2}{mgd}} = 2\pi\sqrt{\frac{L^2}{12gd} + \frac{d}{g}} \ .$$

Squaring both sides and solving for d leads to the quadratic formula:

$$d = \frac{g\,(T/2\pi)^2 \pm \sqrt{d^2\,(T/2\pi)^4 - L^2/3}}{2} \ .$$

Choosing the plus sign leads to an impossible value for d ($d = 1.5 > L$). If we choose the minus sign, we obtain a physically meaningful result: $d = 0.056$ m.

46. We use Eq. 16-29 and the parallel-axis theorem $I = I_{\mathrm{cm}} + mh^2$ where $h = d$. For a solid disk of mass m, the rotational inertia about its center of mass is $I_{\mathrm{cm}} = mR^2/2$. Therefore,

$$T = 2\pi\sqrt{\frac{mR^2/2 + md^2}{mgd}} = 2\pi\sqrt{\frac{R^2 + 2d^2}{2gd}} \ .$$

47. (a) The period of the pendulum is given by $T = 2\pi\sqrt{I/mgd}$, where I is its rotational inertia, m is its mass, and d is the distance from the center of mass to the pivot point. The rotational inertia of a rod pivoted at its center is $mL^2/12$ and, according to the parallel-axis theorem, its rotational inertia when it is pivoted a distance d from the center is $I = mL^2/12 + md^2$. Thus

$$T = 2\pi\sqrt{\frac{m(L^2/12 + d^2)}{mgd}} = 2\pi\sqrt{\frac{L^2 + 12d^2}{12gd}} \ .$$

(b) $(L^2 + 12d^2)/12gd$, considered as a function of d, has a minimum at $d = L/\sqrt{12}$, so the period increases as d decreases if $d < L/\sqrt{12}$ and decreases as d decreases if $d > L/\sqrt{12}$.

(c) L occurs only in the numerator of the expression for the period, so T increases as L increases.

(d) The period does not depend on the mass of the pendulum, so T does not change when m increases.

48. (a) We use Eq. 16-29 and the parallel-axis theorem $I = I_{\text{cm}} + mh^2$ where $h = R = 0.125$ m. For a solid disk of mass m, the rotational inertia about its center of mass is $I_{\text{cm}} = mR^2/2$. Therefore,

$$T = 2\pi\sqrt{\frac{mR^2/2 + mR^2}{mgR}} = 2\pi\sqrt{\frac{3R}{2g}} = 0.869 \text{ s} \ .$$

(b) We seek a value of $r \neq R$ such that

$$2\pi\sqrt{\frac{R^2 + 2r^2}{2gr}} = 2\pi\sqrt{\frac{3R}{2g}}$$

and are led to the quadratic formula:

$$r = \frac{3R \pm \sqrt{(3R)^2 - 8R^2}}{4} = R \quad \text{or} \quad \frac{R}{2} \ .$$

Thus, our result is $r = 0.125/2 = 0.0625$ m.

49. (a) A uniform disk pivoted at its center has a rotational inertia of $\frac{1}{2}MR^2$, where M is its mass and R is its radius. The disk of this problem rotates about a point that is displaced from its center by $R + L$, where L is the length of the rod, so, according to the parallel-axis theorem, its rotational inertia is $\frac{1}{2}MR^2 + M(L+R)^2$. The rod is pivoted at one end and has a rotational inertia of $mL^2/3$, where m is its mass. The total rotational inertia of the disk and rod is

$$\begin{aligned} I &= \frac{1}{2}MR^2 + M(L+R)^2 + \frac{1}{3}mL^2 \\ &= \frac{1}{2}(0.500\,\text{kg})(0.100\,\text{m})^2 + (0.500\,\text{kg})(0.500\,\text{m} + 0.100\,\text{m})^2 + \frac{1}{3}(0.270\,\text{kg})(0.500\,\text{m})^2 \\ &= 0.205 \text{ kg}\cdot\text{m}^2 \ . \end{aligned}$$

(b) We put the origin at the pivot. The center of mass of the disk is

$$\ell_d = L + R = 0.500\,\text{m} + 0.100\,\text{m} = 0.600 \text{ m}$$

away and the center of mass of the rod is $\ell_r = L/2 = (0.500\,\text{m})/2 = 0.250\,\text{m}$ away, on the same line. The distance from the pivot point to the center of mass of the disk-rod system is

$$d = \frac{M\ell_d + m\ell_r}{M + m} = \frac{(0.500\,\text{kg})(0.600\,\text{m}) + (0.270\,\text{kg})(0.250\,\text{m})}{0.500\,\text{kg} + 0.270\,\text{kg}} = 0.477 \text{ m} \ .$$

(c) The period of oscillation is

$$T = 2\pi\sqrt{\frac{I}{(M+m)gd}} = 2\pi\sqrt{\frac{0.205\,\mathrm{kg\cdot m^2}}{(0.500\,\mathrm{kg}+0.270\,\mathrm{kg})(9.8\,\mathrm{m/s^2})(0.447\,\mathrm{m})}} = 1.50\ \mathrm{s}\ .$$

50. (a) Referring to Sample Problem 16-5, we see that the distance between P and C is $h = \frac{2}{3}L - \frac{1}{2}L = \frac{1}{6}L$. The parallel axis theorem (see Eq. 16-30) leads to

$$I = \frac{1}{12}mL^2 + mh^2 = \left(\frac{1}{12} + \frac{1}{36}\right)mL^2 = \frac{1}{9}mL^2\ .$$

And Eq. 16-29 gives

$$T = 2\pi\sqrt{\frac{I}{mgh}} = 2\pi\sqrt{\frac{L^2/9}{gL/6}} = 2\pi\sqrt{\frac{2L}{3g}}$$

which yields $T = 1.64$ s for $L = 1.00$ m.

(b) Comparing with Eq. 16-32, we note that this T is identical to that computed in Sample Problem 16-5. As far as the characteristics of the periodic motion are concerned, the center of oscillation provides a pivot which is equivalent to that chosen in the Sample Problem (pivot at the edge of the stick).

51. We require

$$T = 2\pi\sqrt{\frac{L_o}{g}} = 2\pi\sqrt{\frac{I}{mgh}}$$

similar to the approach taken in part (b) of Sample Problem 16-5, but treating in our case a more general possibility for I. Canceling 2π, squaring both sides, and canceling g leads directly to the result; $L_o = I/mh$.

52. (a) This is similar to the situation treated in Sample Problem 16-5, except that O is no longer at the end of the stick. Referring to the center of mass as C (assumed to be the geometric center of the stick), we see that the distance between O and C is $h = x$. The parallel axis theorem (see Eq. 16-30) leads to

$$I = \frac{1}{12}mL^2 + mh^2 = m\left(\frac{L^2}{12} + x^2\right)\ .$$

And Eq. 16-29 gives

$$T = 2\pi\sqrt{\frac{I}{mgh}} = 2\pi\sqrt{\frac{\left(\frac{L^2}{12} + x^2\right)}{gx}} = 2\pi\sqrt{\frac{(L^2 + 12x^2)}{12gx}}\ .$$

(b) Minimizing T by graphing (or special calculator functions) is straightforward, but the standard calculus method (setting the derivative equal to zero and solving) is somewhat awkward. We pursue the calculus method but choose to work with $12gT^2/2\pi$ instead of T (it should be clear that $12gT^2/2\pi$ is a minimum whenever T is a minimum).

$$\frac{d\left(\frac{12gT^2}{2\pi}\right)}{dx} = 0 = \frac{d\left(\frac{L^2}{x} + 12x\right)}{dx} = -\frac{L^2}{x^2} + 12$$

which yields $x = L/\sqrt{12}$ as the value of x which should produce the smallest possible value of T. Stated as a ratio, this means $x/L = 0.289$.

(c) With $L = 1.00$ m and $x = 0.289$ m, we obtain $T = 1.53$ s from the expression derived in part (a).

53. If the torque exerted by the spring on the rod is proportional to the angle of rotation of the rod and if the torque tends to pull the rod toward its equilibrium orientation, then the rod will oscillate in simple harmonic motion. If $\tau = -C\theta$, where τ is the torque, θ is the angle of rotation, and C is a constant of proportionality, then the angular frequency of oscillation is $\omega = \sqrt{C/I}$ and the period is $T = 2\pi/\omega = 2\pi\sqrt{I/C}$, where I is the rotational inertia of the rod. The plan is to find the torque as a function of θ and identify the constant C in terms of given quantities. This immediately gives the period in terms of given quantities. Let ℓ_0 be the distance from the pivot point to the wall. This is also the equilibrium length of the spring. Suppose the rod turns through the angle θ, with the left end moving away from the wall. This end is now $(L/2)\sin\theta$ further from the wall and has moved $(L/2)(1-\cos\theta)$ to the right. The length of the spring is now $\sqrt{(L/2)^2(1-\cos\theta)^2 + [\ell_0 + (L/2)\sin\theta]^2}$. If the angle θ is small we may approximate $\cos\theta$ with 1 and $\sin\theta$ with θ in radians. Then the length of the spring is given by $\ell_0 + L\theta/2$ and its elongation is $\Delta x = L\theta/2$. The force it exerts on the rod has magnitude $F = k\,\Delta x = kL\theta/2$. Since θ is small we may approximate the torque exerted by the spring on the rod by $\tau = -FL/2$, where the pivot point was taken as the origin. Thus $\tau = -(kL^2/4)\theta$. The constant of proportionality C that relates the torque and angle of rotation is $C = kL^2/4$. The rotational inertia for a rod pivoted at its center is $I = mL^2/12$, where m is its mass. See Table 11–2. Thus the period of oscillation is

$$T = 2\pi\sqrt{\frac{I}{C}} = 2\pi\sqrt{\frac{mL^2/12}{kL^2/4}} = 2\pi\sqrt{\frac{m}{3k}}\ .$$

54. Since the centripetal acceleration is horizontal and Earth's gravitational $\vec{g}$ is downward, we can define the magnitude of an "effective" gravitational acceleration using the Pythagorean theorem:

$$g_{\text{eff}} = \sqrt{g^2 + \left(\frac{v^2}{R}\right)^2}\ .$$

Then, since frequency is the reciprocal of the period, Eq. 16-28 leads to

$$f = \frac{1}{2\pi}\sqrt{\frac{g_{\text{eff}}}{L}} = \frac{1}{2\pi}\sqrt{\frac{\sqrt{g^2 + v^4/R^2}}{L}}\ .$$

55. (a) The frequency for small amplitude oscillations is $f = (1/2\pi)\sqrt{g/L}$, where L is the length of the pendulum. This gives $f = (1/2\pi)\sqrt{(9.80\,\text{m/s}^2)/(2.0\,\text{m})} = 0.35\,\text{Hz}$.

(b) The forces acting on the pendulum are the tension force $\vec{T}$ of the rod and the force of gravity $m\vec{g}$. Newton's second law yields $\vec{T} + m\vec{g} = m\vec{a}$, where m is the mass and $\vec{a}$ is the acceleration of the pendulum. Let $\vec{a} = \vec{a}_e + \vec{a}\,'$, where $\vec{a}_e$ is the acceleration of the elevator and $\vec{a}\,'$ is the acceleration of the pendulum relative to the elevator. Newton's second law can then be written $m(\vec{g}-\vec{a}_e)+\vec{T} = m\vec{a}\,'$. Relative to the elevator the motion is exactly the same as it would be in an inertial frame where the acceleration due to gravity is $\vec{g} - \vec{a}_e$. Since $\vec{g}$ and $\vec{a}_e$ are along the same line and in opposite directions we can find the frequency for small amplitude oscillations by replacing g with $g + a_e$ in the expression $f = (1/2\pi)\sqrt{g/L}$. Thus

$$f = \frac{1}{2\pi}\sqrt{\frac{g + a_e}{L}} = \frac{1}{2\pi}\sqrt{\frac{9.8\,\text{m/s}^2 + 2.0\,\text{m/s}^2}{2.0\,\text{m}}} = 0.39\ \text{Hz}\ .$$

(c) Now the acceleration due to gravity and the acceleration of the elevator are in the same direction and have the same magnitude. That is, $\vec{g} - \vec{a}_e = 0$. To find the frequency for small amplitude oscillations, replace g with zero in $f = (1/2\pi)\sqrt{g/L}$. The result is zero. The pendulum does not oscillate.

56. For simple harmonic motion, Eq. 16-24 must reduce to

$$\tau = -L\,(F_g \sin\theta) \ \longrightarrow\ -L\,(F_g\theta)$$

where θ is in radians. We take the percent difference (in absolute value)

$$\left|\frac{(-LF_g \sin\theta) - (-LF_g\theta)}{-LF_g \sin\theta}\right| = \left|1 - \frac{\theta}{\sin\theta}\right|$$

and set this equal to 0.010 (corresponding to 1.0%). In order to solve for θ (since this is not possible "in closed form"), several approaches are available. Some calculators have built-in numerical routines to facilitate this, and most math software packages have this capability. Alternatively, we could expand $\sin\theta \approx \theta - \theta^3/6$ (valid for small θ) and thereby find an approximate solution (which, in turn, might provide a seed value for a numerical search). Here we show the latter approach:

$$\left|1 - \frac{\theta}{\theta - \theta^3/6}\right| \approx 0.010 \quad\Longrightarrow\quad \frac{1}{1-\theta^2/6} \approx 1.010$$

which leads to $\theta \approx \sqrt{6(0.01/1.01)} = 0.24\,\text{rad} = 14°$. A more accurate value (found numerically) for the θ value which results in a 1.0% deviation is 13.986°.

57. Careful consideration of how the angle θ relates to height h (measured from the lowest position) gives $h = R(1 - \cos\theta)$. The energy at the amplitude point is equal to the energy as it swings through the lowest position:

$$\begin{aligned} mgh &= \frac{1}{2}mv^2 \\ gR\,(1-\cos\theta_m) &= \frac{1}{2}v^2 \end{aligned}$$

where the mass has been canceled in the last step. The tension (acting upward on the bob when it swings through the lowest position) is related to the bob's weight mg and the centripetal acceleration using Newton's second law:

$$T - mg = m\,\frac{v^2}{R}\ .$$

From the above, we substitute for v^2:

$$T - mg = m\,\frac{2gR\,(1-\cos\theta_m)}{R} = 2mg\,(1-\cos\theta_m)\ .$$

(a) This provides an "exact" answer for the tension, but the problem directs us to examine the small angle behavior: $\cos\theta \approx 1-\theta^2/2$ (where θ is in radians). Solving for T and using this approximation, we find

$$T \approx mg + 2mg\left(\frac{\theta_m^2}{2}\right) = mg\left(1+\theta_m^2\right)\ .$$

(b) At other values of θ (other than the lowest position, where $\theta = 0$), Newton's second law yields

$$T' - mg\cos\theta = m\,\frac{v^2}{R} \quad \text{or} \quad T' - mg\left(1 - \frac{\theta^2}{2}\right) \approx m\,\frac{v^2}{R}\ .$$

Making the same substitutions as before, we obtain

$$T' \approx mg\left(1+\theta_m^2-\theta^2\right)$$

which is clearly smaller than the result of part (a).

58. (a) The rotational inertia of a hoop is $I = mR^2$, and the energy of the system becomes

$$E = \frac{1}{2}I\omega^2 + \frac{1}{2}kx^2$$

and θ is in radians. We note that $r\omega = v$ (where $v = \frac{dx}{dt}$). Thus, the energy becomes

$$E = \frac{1}{2}\left(\frac{m\,R^2}{r^2}\right)v^2 + \frac{1}{2}kx^2$$

which looks like the energy of the simple harmonic oscillator discussed in §16-4 *if* we identify the mass m in that section with the term mR^2/r^2 appearing in this problem. Making this identification, Eq. 16-12 yields

$$\omega = \sqrt{\frac{k}{mR^2/r^2}} = \frac{r}{R}\sqrt{\frac{k}{m}} \ .$$

(b) If $r = R$ the result of part (a) reduces to $\omega = \sqrt{k/m}$.

(c) And if $r = 0$ then $\omega = 0$ (the spring exerts no restoring torque on the wheel so that it is not brought back towards its equilibrium position).

59. Referring to the numbers in Sample Problem 16-7, we have $m = 0.25$ kg, $b = 0.070$ kg/s and $T = 0.34$ s. Thus, when $t = 20T$, the damping factor becomes

$$e^{-bt/2m} = e^{-(0.070)(20)(0.34)/2(0.25)} = 0.39 \ .$$

60. Since the energy is proportional to the amplitude squared (see Eq. 16-21), we find the fractional change (assumed small) is

$$\frac{E' - E}{E} \approx \frac{dE}{E} = \frac{dx_m^2}{x_m^2} = \frac{2x_m\,dx_m}{x_m^2} = 2\frac{dx_m}{x_m} \ .$$

Thus, if we approximate the fractional change in x_m as dx_m/x_m, then the above calculation shows that multiplying this by 2 should give the fractional energy change. Therefore, if x_m decreases by 3%, then E must decrease by 6%.

61. (a) We want to solve $e^{-bt/2m} = 1/3$ for t. We take the natural logarithm of both sides to obtain $-bt/2m = \ln(1/3)$. Therefore, $t = -(2m/b)\ln(1/3) = (2m/b)\ln 3$. Thus,

$$t = \frac{2(1.50\,\text{kg})}{0.230\,\text{kg/s}}\ln 3 = 14.3 \text{ s} \ .$$

(b) The angular frequency is

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}} = \sqrt{\frac{8.00\,\text{N/m}}{1.50\,\text{kg}} - \frac{(0.230\,\text{kg/s})^2}{4(1.50\,\text{kg})^2}} = 2.31 \text{ rad/s} \ .$$

The period is $T = 2\pi/\omega' = (2\pi)/(2.31\,\text{rad/s}) = 2.72\,\text{s}$ and the number of oscillations is $t/T = (14.3\,\text{s})/(2.72\,\text{s}) = 5.27$.

62. (a) From Hooke's law, we have

$$k = \frac{(500\,\text{kg})\left(9.8\,\text{m/s}^2\right)}{10\,\text{cm}} = 490 \text{ N/cm} \ .$$

(b) The amplitude decreasing by 50% during one period of the motion implies

$$e^{-bT/2m} = \frac{1}{2} \quad \text{where} \quad T = \frac{2\pi}{\omega'} \ .$$

Since the problem asks us to estimate, we let $\omega' \approx \omega = \sqrt{k/m}$. That is, we let

$$\omega' \approx \sqrt{\frac{49000\,\text{N/m}}{500\,\text{kg}}} \approx 9.9 \text{ rad/s} \ ,$$

so that $T \approx 0.63$ s. Taking the (natural) log of both sides of the above equation, and rearranging, we find

$$b = \frac{2m}{T} \ln 2 \approx \frac{2(500)}{0.63}(0.69) = 1.1 \times 10^3 \text{ kg/s} .$$

Note: if one worries about the $\omega' \approx \omega$ approximation, it is quite possible (though messy) to use Eq. 16-41 in its full form and solve for b. The result would be (quoting more figures than are significant)

$$b = \frac{2 \ln 2 \sqrt{mk}}{\sqrt{(\ln 2)^2 + 4\pi^2}} = 1086 \text{ kg/s}$$

which is in good agreement with the value gotten "the easy way" above.

63. (a) We set $\omega = \omega_d$ and find that the given expression reduces to $x_m = F_m/b\omega$ at resonance.

(b) In the discussion immediately after Eq. 16-6, the book introduces the velocity amplitude $v_m = \omega x_m$. Thus, at resonance, we have $v_m = \omega F_m/b\omega = F_m/b$.

64. With $M = 1000$ kg and $m = 82$ kg, we adapt Eq. 16-12 to this situation by writing

$$\omega = \sqrt{\frac{k}{M + 4m}} \quad \text{where} \quad \omega = \frac{2\pi}{T} .$$

If $d = 4.0$ m is the distance traveled (at constant car speed v) between impulses, then we may write $T = v/d$, in which case the above equation may be solved for the spring constant:

$$\frac{2\pi v}{d} = \sqrt{\frac{k}{M + 4m}} \implies k = (M + 4m)\left(\frac{2\pi v}{d}\right)^2 .$$

Before the people got out, the equilibrium compression is $x_i = (M + 4m)g/k$, and afterward it is $x_f = Mg/k$. Therefore, with $v = 16000/3600 = 4.44$ m/s, we find the rise of the car body on its suspension is

$$x_i - x_f = \frac{4mg}{k} = \frac{4mg}{M + 4m}\left(\frac{d}{2\pi v}\right)^2 = 0.050 \text{ m} .$$

65. The rotational inertia for an axis through A is $I_{\text{cm}} + mh_A^2$ and that for an axis through B is $I_{\text{cm}} + mh_B^2$. Using Eq. 16-29, we require

$$2\pi\sqrt{\frac{I_{\text{cm}} + mh_A^2}{mgh_A}} = 2\pi\sqrt{\frac{I_{\text{cm}} + mh_B^2}{mgh_B}}$$

which (after canceling 2π and squaring both sides) becomes

$$\frac{I_{\text{cm}} + mh_A^2}{mgh_A} = \frac{I_{\text{cm}} + mh_B^2}{mgh_B} .$$

Cross-multiplying and rearranging, we obtain

$$I_{\text{cm}}(h_B - h_A) = m\left(h_A h_B^2 - h_B h_A^2\right) = mh_A h_B (h_B - h_A)$$

which simplifies to $I_{\text{cm}} = mh_A h_B$. We plug this back into the first period formula above and obtain

$$T = 2\pi\sqrt{\frac{mh_A h_B + mh_A^2}{mgh_A}} = 2\pi\sqrt{\frac{h_B + h_A}{g}} .$$

From the figure, we see that $h_B + h_A = L$, and (after squaring both sides) we can solve the above equation for the gravitational acceleration:

$$g = \left(\frac{2\pi}{T}\right)^2 L = \frac{4\pi^2 L}{T^2} .$$

66. (a) The net horizontal force is F since the batter is assumed to exert no horizontal force on the bat. Thus, the horizontal acceleration (which applies as long as F acts on the bat) is $a = F/m$.

(b) The only torque on the system is that due to F, which is exerted at P, at a distance $L_o - \frac{1}{2}L$ from C. Since $L_o = 2L/3$ (see Sample Problem 16-5), then the distance from C to P is $\frac{2}{3}L - \frac{1}{2}L = \frac{1}{6}L$. Since the net torque is equal to the rotational inertia ($I = \frac{1}{12}mL^2$ about the center of mass) multiplied by the angular acceleration, we obtain

$$\alpha = \frac{\tau}{I} = \frac{F\left(\frac{1}{6}L\right)}{\frac{1}{12}mL^2} = \frac{2F}{mL} \; .$$

(c) The distance from C to O is $r = L/2$, so the contribution to the acceleration at O stemming from the angular acceleration (in the counterclockwise direction of Fig. 16-11) is $\alpha r = \frac{1}{2}\alpha L$ (leftward in that figure). Also, the contribution to the acceleration at O due to the result of part (a) is F/m (rightward in that figure). Thus, if we choose rightward as positive, then the net acceleration of O is

$$a_O = \frac{F}{m} - \frac{1}{2}\alpha L = \frac{F}{m} - \frac{1}{2}\left(\frac{2F}{mL}\right)L = 0 \; .$$

(d) Point O stays relatively stationary in the batting process, and that might be possible due to a force exerted by the batter or due to a finely tuned cancellation such as we have shown here. We assumed that the batter exerted no force, and our first expectation is that the impulse delivered by the impact would make all points on the bat go into motion, but for this particular choice of impact point, we have seen that the point being held by the batter is naturally stationary and exerts no force on the batter's hands which would otherwise have to "fight" to keep a good hold of it.

67. Since $\omega = 2\pi f$ where $f = 2.2$ Hz, we find that the angular frequency is $\omega = 13.8$ rad/s. Thus, with $x = 0.010$ m, the acceleration amplitude is $a_m = x_m\omega^2 = 1.91\,\text{m/s}^2$. We set up a ratio:

$$a_m = \left(\frac{a_m}{g}\right)g = \left(\frac{1.91}{9.8}\right)g = 0.19g \; .$$

68. We adjust the phase constant ϕ in Eq. 16-3 so that $x = -x_m$ when $t = 0$.

$$-x_m = x_m \cos\phi \implies \phi = \pi \text{ rad} \; .$$

We also note that $\omega = 2\pi/T = 5\pi$ rad/s.

(a) With this information, Eq. 16-3 becomes

$$x = 0.10\cos(5\pi t + \pi)$$

where t is in seconds and x is in meters.

(b) By taking the derivative of the previous expression (or by plugging into Eq. 16-6) we have

$$v = -0.50\pi\sin(5\pi t + \pi)$$

with SI units again understood. Both of these expression can be simplified using standard trig identities.

69. (a) We are told

$$e^{-bt/2m} = \frac{3}{4} \quad \text{where} \quad t = 4T$$

where $T = 2\pi/\omega' \approx 2\pi\sqrt{m/k}$ (neglecting the second term in Eq. 16.41). Thus,

$$T \approx 2\pi\sqrt{(2.00\,\text{kg})/(10.0\,\text{N/m})} = 2.81\,\text{s}$$

and we find

$$\frac{b(4T)}{2m} = \ln\left(\frac{4}{3}\right) = 0.288 \implies b = \frac{2(2.00)(0.288)}{4(2.81)} = 0.102 \text{ kg/s} \; .$$

(b) Initially, the energy is $E_o = \frac{1}{2}kx_{m\,o}^2 = \frac{1}{2}(10.0)(0.250)^2 = 0.313$ J. At $t = 4T$, $E = \frac{1}{2}k(\frac{3}{4}x_{m\,o})^2 = 0.176$ J. Therefore, $E_o - E = 0.137$ J.

70. (a) Sample Problem 16-7 gives $b = 0.070$ kg/s, $m = 0.25$ kg and $k = 85$ N/m, and notes that $b \ll \sqrt{km}$ which implies $\omega' \approx \omega = \sqrt{k/m}$ (and, as will be important below, $b/m \ll \omega$). Thus, from Eq. 16-40, we find

$$v = \frac{dx}{dt} = x_m \left(\frac{-b}{2m}\right) e^{-bt/2m} \cos(\omega t + \phi) - \omega x_m e^{-bt/2m} \sin(\omega t + \phi)$$

where the first term is considered negligible ($b/2m \ll \omega$) and we write

$$v \approx -\omega x_m e^{-bt/2m} \sin(\omega t + \phi) \quad \Longrightarrow \quad v_m = \omega x_m e^{-bt/2m} \ .$$

Thus, the ratio of maximum values of forces is

$$\frac{F_{m\,\text{damp}}}{F_{m\,\text{spring}}} = \frac{bv_m}{kx_m e^{-bt/2m}} \approx \frac{b\omega}{k} = \frac{b}{\sqrt{km}} = 0.015 \ .$$

(b) The ratio of force amplitudes found in part (a) displays no time dependence, implying there is no (or, since approximations were made, approximately no) change in the ratio as the system undergoes further oscillations.

71. We take derivatives and let $dg \approx \Delta g$ and $dT \approx \Delta T$. The derivative of Eq. 16-28 is

$$\frac{dT}{dg} = 2\pi \left(\frac{1}{2}\right) \frac{-L/g^2}{\sqrt{L/g}}$$

which (after dividing the left side by T and the right side by $2\pi\sqrt{L/g}$) can be written

$$\frac{\Delta T}{T} = -\frac{1}{2}\frac{\Delta g}{g}$$

where both sides have also been multiplied by $dg \to \Delta g$. To make the units consistent, we write

$$\frac{\Delta T}{T} = \frac{2.5\,\text{min}}{1\,\text{day}} = \frac{2.5\,\text{min}}{1440\,\text{min}} = 0.00174 \ .$$

Therefore, with $g = 9.81$ m/s^2, we obtain

$$0.00174 = -\frac{1}{2}\frac{\Delta g}{9.81\,\text{m/s}^2} \quad \Longrightarrow \quad \Delta g = -0.034\ \text{m/s}^2$$

which yields $g' = g + \Delta g = 9.78\,\text{m/s}^2$.

72. The speed of the submarine going eastward is

$$v_{\text{east}} = v_{\text{equator}} + v_{\text{sub}}$$

where $v_{\text{sub}} = 16000/3600 = 4.44$ m/s. The term v_{equator} is the speed that any point at the equator (at radius $R = 6.37 \times 10^6$ m) would have in order to keep up with the spinning earth. With $T = 1\,\text{day} = 86400\,\text{s}$, we note that $v_{\text{equator}} = R\omega = R2\pi/T = 463$ m/s and is much larger than v_{sub}. Similarly, when it travels westward, its speed is

$$v_{\text{west}} = v_{\text{equator}} - v_{\text{sub}} \ .$$

The effective gravity g_e (or apparent gravity) combines the gravitational pull of the earth g (which cancels when we take the difference) and the effect of the centripetal acceleration v^2/R. Considering the two motions of the submarine, the difference is therefore

$$\begin{aligned}
\Delta g_e = g'_e - g_e &= \frac{v_{\text{east}}^2}{R} - \frac{v_{\text{west}}^2}{R} \\
&= \frac{1}{R}\left((v_{\text{equator}} + v_{\text{sub}})^2 - (v_{\text{equator}} - v_{\text{sub}})^2\right) \\
&= \frac{4v_{\text{equator}}v_{\text{sub}}}{R} = \frac{8\pi v_{\text{sub}}}{T}
\end{aligned}$$

where in the last step we have used $v_{\text{equator}} = R2\pi/T$. Consequently, we find

$$\frac{\Delta g_e}{g} = \frac{8\pi v_{\text{sub}}}{gT} = \frac{8\pi(4.44)}{(9.8)(86400)} = 1.3 \times 10^{-4} \ .$$

The problem asks for $\Delta g/g$ for *either* travel direction, and since our computation examines eastward travel as opposed to westward travel, then we infer that either-way travel versus no-travel should be half of our result. Thus, the answer to the problem is $\frac{1}{2}(1.3 \times 10^{-4}) = 6.6 \times 10^{-5}$.

73. (a) The graph makes it clear that the period is $T = 0.20$ s.

(b) Eq. 16-13 states

$$T = 2\pi\sqrt{\frac{m}{k}} \ .$$

Thus, using the result from part (a) with $k = 200$ N/m, we obtain $m = 0.203 \approx 0.20$ kg.

(c) The graph indicates that the speed is (momentarily) zero at $t = 0$, which implies that the block is at $x_0 = \pm x_m$. From the graph we also note that the slope of the velocity curve (hence, the acceleration) is positive at $t = 0$, which implies (from $ma = -kx$) that the value of x is negative. Therefore, with $x_m = 0.20$ m, we obtain $x_0 = -0.20$ m.

(d) We note from the graph that $v = 0$ at $t = 0.10$ s, which implied $a = \pm a_m = \pm\omega^2 x_m$. Since acceleration is the instantaneous slope of the velocity graph, then (looking again at the graph) we choose the negative sign. Recalling $\omega^2 = k/m$ we obtain $a = -197 \approx -200$ m/s^2.

(e) The graph shows $v_m = 6.28$ m/s, so

$$K_m = \frac{1}{2}mv_m^2 = 4.0 \text{ J} \ .$$

74. (a) The Hooke's law force (of magnitude $(100)(0.30) = 30$ N) is directed upward and the weight (20 N) is downward. Thus, the net force is 10 N upward.

(b) The equilibrium position is where the upward Hooke's law force balances the weight, which corresponds to the spring being stretched (from unstretched length) by 20 N/100 N/m = 0.20 m. Thus, relative to the equilibrium position, the block (at the instant described in part (a)) is at what one might call *the bottom turning point* (since $v = 0$) at $x = -x_m$ where the amplitude is $x_m = 0.30 - 0.20 = 0.10$ m.

(c) Using Eq. 16-13 with $m = W/g \approx 2.0$ kg, we have

$$T = 2\pi\sqrt{\frac{m}{k}} = 0.90 \text{ s} \ .$$

(d) The maximum kinetic energy is equal to the maximum potential energy $\frac{1}{2}kx_m^2$. Thus,

$$K_m = U_m = \frac{1}{2}(100 \text{ N/m})(0.10 \text{ m})^2 = 0.50 \text{ J} \ .$$

75. (a) Comparing with Eq. 16-3, we see $\omega = 10$ rad/s in this problem. Thus, $f = \omega/2\pi = 1.6$ Hz.

(b) Since $v_m = \omega x_m$ and $x_m = 10$ cm (see Eq. 16-3), then $v_m = (10\,\text{rad/s})(10\,\text{cm}) = 100$ cm/s or 1.0 m/s.

(c) Since $a_m = \omega^2 x_m$ then $v_m = (10\,\text{rad/s})^2(10\,\text{cm}) = 1000$ cm/s^2 or 10 m/s^2.

(d) The acceleration extremes occur at the displacement extremes: $x = \pm x_m$ or $x = \pm 10$ cm.

(e) Using Eq. 16-12, we find

$$\omega = \sqrt{\frac{k}{m}} \implies k = (0.10\,\text{kg})(10\,\text{rad/s})^2 = 10 \text{ N/m} \ .$$

Thus, Hooke's law gives $F = -kx = -10x$ in SI units.

76. (a) We take the x axis along the tunnel, with $x = 0$ at the middle. At any instant during the train's motion, it is a distance r from the center of Earth, and we can think of this as a vector $\vec{r}$ pointing from the train to the Earth's center. We neglect any effects associated with the spinning of Earth (which has mass M and radius R). Based on the theory of Ch. 14, we know that the magnitude of gravitational force on the train of mass m_o at any instant is

$$|F_g| = \frac{Gm_oM\left(r^3/R^3\right)}{r^2} = \frac{Gm_oMr}{R^3} .$$

It is only the horizontal component of this force which leads to acceleration/deceleration of the train, so a $\cos\theta$ factor (with θ giving the angle of $\vec{r}$ measured from the x axis) must be included, and we can relate $\cos\theta = x/r$ and obtain

$$m_oa = F_x = -\frac{Gm_oMr}{R^3}\frac{x}{r}$$

where the minus sign is necessary because the force pulls towards the $x = 0$ position, so when the train is, say, at a large negative value of x the force is in the positive x direction (towards the origin of the x axis). The above expression simplifies to exactly the form (Eq. 16-8) required for simple harmonic motion:

$$a = -\omega^2 x \quad \text{where} \quad \omega = \sqrt{\frac{GM}{R^3}} .$$

Since a full cycle of the motion would return the train to its starting point, then a half cycle is required to travel from the departure city to the destination city. Therefore, $t_{\text{travel}} = \frac{1}{2}T$.

(b) Since $T = 2\pi/\omega$, we obtain

$$t_{\text{travel}} = \pi\sqrt{\frac{R^3}{GM}} = \pi\sqrt{\frac{\left(6.37\times10^6\right)^3}{\left(6.67\times10^{-11}\right)\left(5.98\times10^{24}\right)}}$$

which yields 2530 s or 42 min.

77. Using $\Delta m = 2.0$ kg, $T_1 = 2.0$ s and $T_2 = 3.0$ s, we write

$$T_1 = 2\pi\sqrt{\frac{m}{k}} \quad \text{and} \quad T_2 = 2\pi\sqrt{\frac{m+\Delta m}{k}} .$$

Dividing one relation by the other, we obtain

$$\frac{T_2}{T_1} = \sqrt{\frac{m+\Delta m}{m}}$$

which (after squaring both sides) simplifies to

$$m = \frac{\Delta m}{\left(\frac{T_2}{T_1}\right)^2 - 1} = 1.6 \text{ kg} .$$

78. (a) Hooke's law readily yields $(0.300\,\text{kg})(9.8\,\text{m/s}^2)/(0.0200\,\text{m}) = 147$ N/m.

(b) With $m = 2.00$ kg, the period is

$$T = 2\pi\sqrt{\frac{m}{k}} = 0.733 \text{ s} .$$

79. Since $T = 0.500$ s, we note that $\omega = 2\pi/T = 4\pi$ rad/s. We work with SI units, so $m = 0.0500$ kg and $v_m = 0.150$ m/s.

(a) Since $\omega = \sqrt{k/m}$, the spring constant is

$$k = \omega^2 m = (4\pi)^2(0.0500) = 7.90 \text{ N/m} .$$

(b) We use the relation $v_m = x_m\omega$ and obtain

$$x_m = \frac{v_m}{\omega} = \frac{0.150}{4\pi} = 0.0119 \text{ m} .$$

(c) The frequency is $f = \omega/2\pi = 2.00$ Hz (which is equivalent to $f = 1/T$).

80. (a) Hooke's law provides the spring constant: $k = (20\,\text{N})/(0.20\,\text{m}) = 100$ N/m.
(b) The attached mass is $m = (5.0\,\text{N})/(9.8\,\text{m/s}^2) = 0.51$ kg. Consequently, Eq. 16-13 leads to

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.51}{100}} = 0.45 \text{ s} .$$

81. Since a mole of silver atoms has a mass of 0.108 kg, then the mass of one atom is

$$m = \frac{0.108\,\text{kg}}{6.02 \times 10^{23}} = 1.8 \times 10^{-25} \text{ kg} .$$

Using Eq. 16-12 and the fact that $f = \omega/2\pi$, we have

$$1 \times 10^{13}\,\text{Hz} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \implies k = \left(2\pi \times 10^{13}\right)^2 \left(1.8 \times 10^{-25}\right) \approx 7.1 \times 10^2 \text{ N/m} .$$

82. (a) Hooke's law provides the spring constant: $k = (4.00\,\text{kg})(9.8\,\text{m/s}^2)/(0.160\,\text{m}) = 245$ N/m.
(b) The attached mass is $m = 0.500$ kg. Consequently, Eq. 16-13 leads to

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.500}{245}} = 0.284 \text{ s} .$$

83. (a) By Eq. 16-13, the mass of the block is

$$m_b = \frac{kT_0^2}{4\pi^2} = 2.43 \text{ kg} .$$

Therefore, with $m_p = 0.50$ kg, the new period is

$$T = 2\pi\sqrt{\frac{m_p + m_b}{k}} = 0.44 \text{ s} .$$

(b) The speed before the collision (since it is at its maximum, passing through equilibrium) is $v_0 = x_m\omega_0$ where $\omega_0 = 2\pi/T_0$; thus, $v_0 = 3.14$ m/s. Using momentum conservation (along the horizontal direction) we find the speed after the collision.

$$V = v_0\,\frac{m_b}{m_p + m_b} = 2.61 \text{ m/s} .$$

The equilibrium position has not changed, so (for the new system of greater mass) this represents the maximum speed value for the subsequent harmonic motion: $V = x'_m\omega$ where $\omega = 2\pi/T = 14.3$ rad/s. Therefore, $x'_m = 0.18$ m.

84. The period is the time for one oscillation: $T = 180/72 = 2.5$ s. Thus, by Eq. 16-28, we have

$$T = 2\pi\sqrt{\frac{L}{g}} \implies g = L\left(\frac{2\pi}{T}\right)^2 = 9.47 \text{ m/s}^2 .$$

85. Using Eq. 16-12, we find $\omega = \sqrt{k/m} = 10$ rad/s. We also use $v_m = x_m\omega$ and $a_m = x_m\omega^2$.

(a) The amplitude (meaning "displacement amplitude") is $x_m = v_m/\omega = 3/10 = 0.30$ m.

(b) The acceleration-amplitude is $a_m = (0.30)(10)^2 = 30\,\mathrm{m/s^2}$.

(c) One interpretation of this question is "what is the most negative value of the acceleration?" in which case the answer is $-a_m = -30\,\mathrm{m/s^2}$. Another interpretation is "what is the smallest value of the absolute-value of the acceleration?" in which case the answer is zero.

(d) Since the period is $T = 2\pi/\omega = 0.628$ s. Therefore, seven cycles of the motion requires $t = 7T = 4.4$ s.

86. We find that the spring constant is $k = mg/h$. Thus, Eq. 16-13 becomes

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{(mg/h)}}2\pi\sqrt{\frac{h}{g}}$$

which we recognize as the period formula for a simple pendulum of length h (see Eq. 16-28).

87. Using Eq. 16-28, we obtain

$$L = g\left(\frac{T}{2\pi}\right)^2 = (9.75)\left(\frac{1.50}{2\pi}\right)^2 = 0.556 \text{ m} .$$

88. Using Eq. 16-29 and the parallel-axis formula for rotational inertia, we have

$$I = 2\pi\sqrt{\frac{I_{\mathrm{cm}} + mh^2}{mgh}} = 2\pi\sqrt{\frac{L^2}{12gh} + \frac{h}{g}}$$

where we have used the fact (from Ch. 11) that $I_{\mathrm{cm}} = mL^2/12$ for a uniform rod. We wish to minimize by taking the derivative and setting equal to zero, but we observe that this is done more easily if we consider I^2 (the square of the above expression) instead of I. Thus,

$$\frac{dI^2}{dh} = 0 = 4\pi^2\left(-\frac{L^2}{12gh^2} + \frac{1}{g}\right)$$

which leads to

$$0 = -\frac{L^2}{12h^2} + 1 \implies h = \frac{L}{\sqrt{12}} \approx 0.29L .$$

89. To use Eq. 16-29 we need to locate the center of mass and we need to compute the rotational inertia about A. The center of mass of the stick shown horizontal in the figure is at A, and the center of mass of the other stick is 0.50 m below A. The two sticks are of equal mass so the center of mass of the system is $h = \frac{1}{2}(0.50) = 0.25$ m below A, as shown in the figure. Now, the rotational inertia of the system is the sum of the rotational inertia I_1 of the stick shown horizontal in the figure and the rotational inertia I_2 of the stick shown vertical. Thus, we have

$$I = I_1 + I_2 = \frac{1}{12}ML^2 + \frac{1}{3}ML^2 = \frac{5}{12}ML^2$$

where $L = 1.00$ m and M is the mass of a meter stick (which cancels in the next step). Now, with $m = 2M$ (the total mass), Eq. 16-29 yields

$$T = 2\pi\sqrt{\frac{\frac{5}{12}ML^2}{2Mgh}} = 2\pi\sqrt{\frac{5L}{6g}}$$

where $h = L/4$ was used. Thus, $T = 1.83$ s.

90. The period formula, Eq. 16-29, requires knowing the distance h from the axis of rotation and the center of mass of the system. We also need the rotational inertia I about the axis of rotation. From Figure 16-53, we see $h = L + R$ where $R = 0.15$ m. Using the parallel-axis theorem, we find

$$I = \frac{1}{2}MR^2 + M(L+R)^2 \quad \text{where } M = 1.0 \text{ kg} .$$

Thus, Eq. 16-29, with $T = 2.0$ s, leads to

$$2.0 = 2\pi\sqrt{\frac{\frac{1}{2}MR^2 + M(L+R)^2}{Mg(L+R)}}$$

which leads to $L = 0.8315$ m.

91. (a) From Eq. 16-12, $T = 2\pi\sqrt{m/k} = 0.45$ s.

(b) For a vertical spring, the distance between the unstretched length and the equilibrium length (with a mass m attached) is mg/k, where in this problem $mg = 10$ N and $k = 200$ N/m (so that the distance is 0.05 m). During simple harmonic motion, the convention is to establish $x = 0$ at the equilibrium length (the middle level for the oscillation) and to write the total energy without any gravity term; i.e.,

$$E = K + U \qquad \text{where } U = \frac{1}{2}kx^2 \quad .$$

Thus, as the block passes through the unstretched position, the energy is $E = 2.0 + \frac{1}{2}k(0.05)^2 = 2.25$ J. At its topmost and bottommost points of oscillation, the energy (using this convention) is all elastic potential: $\frac{1}{2}kx_m^2$. Therefore, by energy conservation,

$$2.25 = \frac{1}{2}kx_m^2 \implies x_m = \pm 0.15 \text{ m} .$$

This gives the amplitude of oscillation as 0.15 m, but how far are these points from the *unstretched* position? We add (or subtract) the 0.05 m value found above and obtain 0.10 m for the top-most position and 0.20 m for the bottom-most position.

(c) As noted in part (b), $x_m = \pm 0.15$ m.

(d) The maximum kinetic energy equals the maximum potential energy (found in part (b)) and is equal to 2.25 J.

92. (a) Eq. 16-21 leads to

$$E = \frac{1}{2}kx_m^2 \implies x_m = \sqrt{\frac{2E}{k}} = \sqrt{\frac{2(4.0)}{200}} = 0.020 \text{ m} .$$

(b) Since $T = 2\pi\sqrt{m/k} = 2\pi\sqrt{0.80/200} \approx 0.4$ s, then the block completes $10/0.4 = 25$ cycles during the specified interval.

(c) The maximum kinetic energy is the total energy, 4.0 J.

(d) This can be approached more than one way; we choose to use energy conservation:

$$E = K + U \implies 4.0 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 .$$

Therefore, when $x = 0.15$ m, we find $v = 2.1$ m/s.

93. (a) The rotational inertia of a uniform rod with pivot point at its end is $I = mL^2/12 + mL^2 = \frac{1}{3}ML^2$. Therefore, Eq. 16-29 leads to

$$T = 2\pi\sqrt{\frac{\frac{1}{3}ML^2}{Mg(L/2)}} \implies L = \frac{3gT^2}{8\pi^2}$$

so that $L = 0.84$ m.

(b) By energy conservation

$$\begin{aligned} E_{\text{bottom of swing}} &= E_{\text{end of swing}} \\ K_m &= U_m \end{aligned}$$

where $U = Mg\ell\,(1-\cos\theta)$ with ℓ being the distance from the axis of rotation to the center of mass. If we use the small angle approximation ($\cos\theta \approx 1 - \frac{1}{2}\theta^2$ with θ in radians (Appendix E)), we obtain

$$U_m = (0.5)(9.8)\left(\frac{L}{2}\right)\left(\frac{1}{2}\theta_m^2\right)$$

where $\theta_m = 0.17$ rad. Thus, $K_m = U_m = 0.031$ J. If we calculate $(1-\cos\theta)$ straightforwardly (without using the small angle approximation) then we obtain within 0.3% of the same answer.

94. From Eq. 16-23 (in absolute value) we find the torsion constant:

$$\kappa = \left|\frac{\tau}{\theta}\right| = \frac{0.20}{0.85} = 0.235$$

in SI units. With $I = 2mR^2/5$ (the rotational inertia for a solid sphere – from Chapter 11), Eq. 16-23 leads to

$$T = 2\pi\sqrt{\frac{\frac{2}{5}\,m\,R^2}{\kappa}} = 2\pi\sqrt{\frac{\frac{2}{5}(95)(0.15)^2}{0.235}} = 12 \text{ s} .$$

95. The time for one cycle is $T = (50\,\text{s})/20 = 2.5$ s. Thus, from Eq. 16-23, we find

$$I = \kappa\left(\frac{T}{2\pi}\right)^2 = (0.50)\left(\frac{2.5}{2\pi}\right)^2 = 0.079 \text{ kg}\cdot\text{m}^2 .$$

96. The distance from the relaxed position of the bottom end of the spring to its equilibrium position when the body is attached is given by Hooke's law: $\Delta x = F/k = (0.20\,\text{kg})(9.8\,\text{m/s}^2)/(19\,\text{N/m}) = 0.103$ m.

(a) The body, once released, will not only fall through the Δx distance but continue through the equilibrium position to a "turning point" equally far on the other side. Thus, the total descent of the body is $2\Delta x = 0.21$ m.

(b) Since $f = \omega/2\pi$, Eq. 16-12 leads to

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = 1.6 \text{ Hz} .$$

(c) The maximum distance from the equilibrium position is the amplitude: $x_m = \Delta x = 0.10$ m.

97. The rotational inertia of a uniform rod with pivot point at its end is $I = mL^2/12 + mL^2 = \frac{1}{3}ML^2$. Therefore, Eq. 16-29 leads to

$$T_0 = 2\pi\sqrt{\frac{\frac{1}{3}ML^2}{Mg(L/2)}} = 2\pi\sqrt{\frac{2L}{3g}} .$$

If we replace L with $L/2$ (for the case where half has been cut off) then the new period is $T = 2\pi\sqrt{L/3g}$. Since frequency is the reciprocal of the period, then $T_0/T = f/f_0$ which leads to

$$\frac{f}{f_0} = \frac{2\pi\sqrt{2L/3g}}{2\pi\sqrt{L/3g}} \implies f = f_0\sqrt{2} .$$

98. We note that for a horizontal spring, the relaxed position is the equilibrium position (in a regular simple harmonic motion setting); thus, we infer that the given $v = 5.2$ m/s at $x = 0$ is the maximum value v_m (which equals ωx_m where $\omega = \sqrt{k/m} = 20$ rad/s).

(a) Since $\omega = 2\pi f$, we find $f = 3.2$ Hz.

(b) We have $v_m = 5.2 = (20)x_m$, which leads to $x_m = 0.26$ m.

(c) With meters, seconds and radians understood,

$$\begin{aligned} x &= 0.26\cos(20t + \phi) \\ v &= -5.2\sin(20t + \phi) \end{aligned}$$

The requirement that $x = 0$ at $t = 0$ implies (from the first equation above) that either $\phi = +\pi/2$ or $\phi = -\pi/2$. Only one of these choices meets the further requirement that $v > 0$ when $t = 0$; that choice is $\phi = -\pi/2$. Therefore,

$$x = 0.26\cos\left(20t - \frac{\pi}{2}\right) = 0.26\sin(20t) \ .$$

99. (a) The potential energy at the turning point is equal (in the absence of friction) to the total kinetic energy (translational plus rotational) as it passes through the equilibrium position:

$$\begin{aligned} \frac{1}{2}kx_m^2 &= \frac{1}{2}Mv_{\rm cm}^2 + \frac{1}{2}I_{\rm cm}\omega^2 \\ &= \frac{1}{2}Mv_{\rm cm}^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{v_{\rm cm}}{R}\right)^2 \\ &= \frac{1}{2}Mv_{\rm cm}^2 + \frac{1}{4}Mv_{\rm cm}^2 = \frac{3}{4}Mv_{\rm cm}^2 \ . \end{aligned}$$

which leads to $Mv_{\rm cm}^2 = 2kx_m^2/3 = 0.125$ J. The translational kinetic energy is therefore $\frac{1}{2}Mv_{\rm cm}^2 = kx_m^2/3 = 0.0625$ J.

(b) And the rotational kinetic energy is $\frac{1}{4}Mv_{\rm cm}^2 = kx_m^2/6 = 0.03125$ J.

(c) In this part, we use $v_{\rm cm}$ to denote the speed at any instant (and not just the maximum speed as we had done in the previous parts). Since the energy is constant, then

$$\begin{aligned} \frac{dE}{dt} &= 0 \\ \frac{d}{dt}\left(\frac{3}{4}Mv_{\rm cm}^2\right)\frac{d}{dt}\left(\frac{1}{2}kx^2\right) &= 0 \\ \frac{3}{2}Mv_{\rm cm}a_{\rm cm} + kxv_{\rm cm} &= 0 \end{aligned}$$

which leads to

$$a_{\rm cm} = -\left(\frac{2k}{3M}\right)x \ .$$

Comparing with Eq. 16-8, we see that $\omega = \sqrt{2k/3M}$ for this system. Since $\omega = 2\pi/T$, we obtain the desired result: $T = 2\pi\sqrt{3M/2k}$.

100. Eq. 16-28 gives $T = 2\pi\sqrt{L/g}$. Replacing L by $L/2$ gives the new period $T' = 2\pi\sqrt{L/2g}$. The ratio is

$$\frac{T'}{T} = \frac{2\pi\sqrt{L/2g}}{2\pi\sqrt{L/g}} = \frac{1}{\sqrt{2}} \ .$$

Therefore, we conclude that $T' = T/\sqrt{2}$.

101. (a) We require $U = \frac{1}{2}E$ at some value of x. Using Eq. 16-21, this becomes

$$\frac{1}{2}kx^2 = \frac{1}{2}\left(\frac{1}{2}kx_m^2\right) \implies x = \frac{x_m}{\sqrt{2}} \ .$$

We compare the given expression x as a function of t with Eq. 16-3 and find $x_m = 5.0$ m. Thus, the value of x we seek is $x = 5.0/\sqrt{2} \approx 3.5$ m.

(b) We solve the given expression (with $x = 5.0/\sqrt{2}$), making sure our calculator is in radians mode:

$$t = \frac{\pi}{4} + \frac{3}{\pi}\cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = 1.54 \text{ s} .$$

Since we are asked for the interval $t_{\text{eq}} - t$ where t_{eq} specifies the instant the particle passes through the equilibrium position, then we set $x = 0$ and find

$$t_{\text{eq}} = \frac{\pi}{4} + \frac{3}{\pi}\cos^{-1}(0) = 2.29 \text{ s} .$$

Consequently, the time interval is $t_{\text{eq}} - t = 0.75$ s.

102. (a) From the graph, it is clear that $x_m = 0.30$ m.

(b) With $F = -kx$, we see k is the (negative) slope of the graph – which is $75/0.30 = 250$ N/m. Plugging this into Eq. 16-13 yields

$$T = 2\pi\sqrt{\frac{m}{k}} = 0.28 \text{ s} \quad .$$

(c) As discussed in §16-2, the maximum acceleration is

$$a_m = \omega^2 x_m = \frac{k}{m}\, x_m = 150 \text{ m/s}^2 \ .$$

Alternatively, we could arrive at this result using $a_m = \left(\frac{2\pi}{T}\right)^2 x_m$.

(d) Also in §16-2 is $v_m = \omega x_m$ so that the maximum kinetic energy is

$$K_m = \frac{1}{2}mv_m^2 = \frac{1}{2}m\omega^2 x_m^2 = \frac{1}{2}kx_m^2$$

which yields $11.3 \approx 11$ J. We note that the above manipulation reproduces the notion of energy conservation for this system (maximum kinetic energy being equal to the maximum potential energy).

103. Since the particle has zero speed (momentarily) at $x \neq 0$, then it must be at its turning point; thus, $x_\text{o} = x_m = 0.37$ cm. It is straightforward to infer from this that the phase constant ϕ in Eq. 16-2 is zero. Also, $f = 0.25$ Hz is given, so we have $\omega = 2\pi f = \pi/2$ rad/s. The variable t is understood to take values in seconds.

(a) The period is $T = 1/f = 4.0$ s.

(b) As noted above, $\omega = \frac{\pi}{2}$ rad/s.

(c) The amplitude, as observed above, is 0.37 cm.

(d) Eq. 16-3 becomes $x = (0.37)\cos(\pi t/2)$ in centimeters.

(e) The derivative of x is $v = -(0.37)(\pi/2)\sin(\pi t/2) \approx (-0.58)\sin(\pi t/2)$ in centimeters-per-second.

(f) From the previous part, we conclude $v_m = 0.58$ cm/s.

(g) The acceleration-amplitude is $a_m = \omega^2 x_m = 0.91$ cm/s^2.

(h) Making sure our calculator is in radians mode, we find $x = (0.37)\cos(\pi(3.0)/2) = 0$. It is important to avoid rounding off the value of π in order to get precisely zero, here.

(i) With our calculator still in radians mode, we obtain $v = -(0.58)\sin(\pi(3.0)/2) = 0.58$ cm/s.

104. (a) Since no torque is being applied to the system, the angular momentum is constant.

(b) The maximum ω occurs when the maximum speed v occurs (as it passes through vertical: $\theta = 0$). The angular momentum of the "particle" may be written as $mvr = mr^2\omega$ so that conservation of momentum (applied to the $\theta = 0$ position) leads to

$$mr^2\omega_{\rm max} = mr_0^2\omega_{0,\rm max} \implies \omega_{\rm max} = \left(\frac{r_0}{r}\right)^2 \omega_{0,\rm max}$$

which becomes (with $r_0 = 0.80$ m and $\omega_{0,\rm max} = 1.30$ rad/s) $\omega_{\rm max} = 0.832/r^2$ in SI units.

(c) The maximum kinetic energy occurs at this same position: $K_{\rm max} = \frac{1}{2}mv_{\rm max}^2$ which we write as

$$K_{\rm max} = \frac{1}{2}mr^2\omega_{\rm max}^2 = \frac{1}{2}mr^2\left(\left(\frac{r_0}{r}\right)^2\omega_{0,\rm max}\right)^2 = \frac{mr_0^4\omega_{0,\rm max}^2}{2\,r^2} \ .$$

(d) We note from the previous result that $K_{\rm max}$ depends *inversely* on r^2, so it decreases as r increases.

(e) Measuring height h from the low point of the swing, consideration of the geometry leads to the relation $h = r(1-\cos\theta)$. The maximum height is therefore related to the maximum angle (measured from vertical) by

$$h_{\rm max} = r\,(1 - \cos\theta_{\rm max}\,)$$

which means the maximum potential energy (which must equal the same numerical value as the maximum kinetic energy if we assume mechanical energy conservation) is

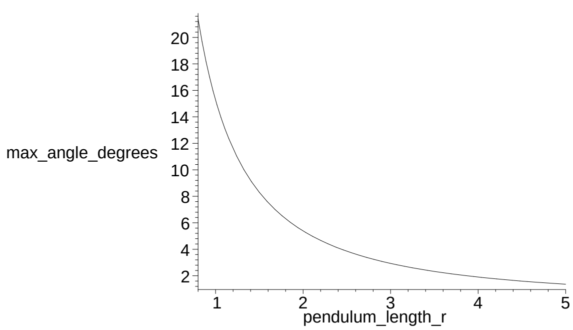
$$U_{\rm max} = K_{\rm max} = mgh_{\rm max} = mgr\,(1 - \cos\theta_{\rm max}\,) \ .$$

(f) Combining the results of part (c) and part (e), we obtain

$$\frac{mr_0^4\omega_{0,\rm max}^2}{2\,r^2} = mgr\,(1 - \cos\theta_{\rm max}\,) \implies \theta_{\rm max} = \cos^{-1}\left(1 - \frac{r_0^4\omega_{0,\rm max}^2}{2gr^3}\right)$$

which evaluates to be $\theta_{\rm max} = \cos^{-1}(1 - 0.0353/r^3\,)$ in SI units.

(g) As can be seen in the graph below, the angle of the pendulum "turning point" decreases as the pendulum lengthens (note that r is in meters).



(h) The original value of $\theta_{\rm max}$ is $\cos^{-1}(1 - 0.0353/r_0^3)$ where $r_0 = 0.80$ m. This gives 21.4° as the initial "turning point" angle. The question, then, asks us to solve for r in the case that $\theta_{\rm max} = \frac{1}{2}(21.4°) = 10.7°$. We know to look for half the initial value (as opposed to one twice as big) because the previous part shows $\theta_{\rm max}$ decreases with r. This value of the turning point angle occurs for

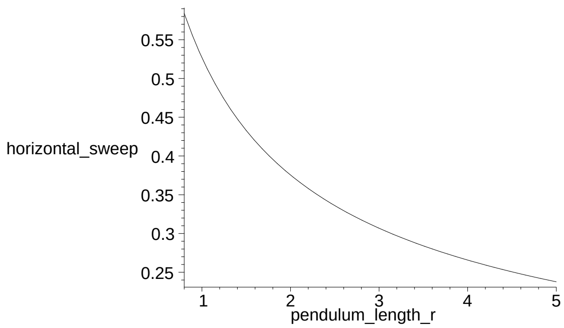
$$r = \left(\frac{0.0353}{1 - \cos 10.7°}\right)^{1/3} = 1.27 \text{ m} \ .$$

(i) The angle $\theta_{\rm max}$ is measured from vertical, so the horizontal sweep involves $\sin\theta_{\rm max}$. From one turning point to the opposite one covers a horizontal distance of $\Delta x = 2r\sin\theta_{\rm max}$.

(j) Plugging in from part (f), we find

$$\Delta x = 2r\sin\left(\cos^{-1}\left(1 - \frac{r_0^4\omega_{0,\max}^2}{2gr^3}\right)\right) = \frac{r_0^2\omega_{0,\max}}{gr^2}\sqrt{4gr^3 - r_0^4\omega_{0,\max}^2}$$

where that last equality is (depending on one's viewpoint) a simplification and should not viewed as a necessary step. With $r_0 = 0.80$ m and $\omega_{0,\max} = 1.30$ rad/s, we plot this expression (with r and Δx (the horizontal sweep) in meters) and see that it is a decreasing function.

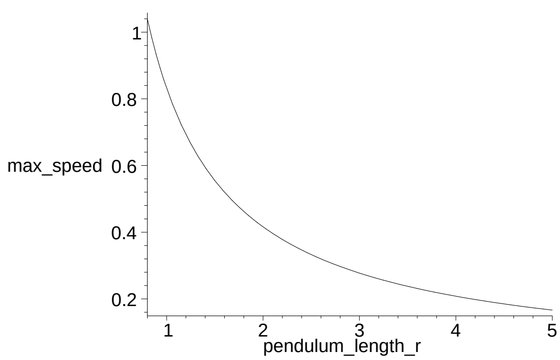


(k) When $r = r_0$ we find $\Delta x_0 = 0.584$ m. Any later value must be smaller (according to the above graph), so we seek a value of r that gives half of Δx_0 (that is, $\Delta x = 0.292$ m). If we numerically solve the expression in the previous part for r in the range $0.8\,\text{m} \le r \le 5\,\text{m}$, we obtain $r = 3.31$ m.

(l) Returning to part (b) with $v_{\max} = r\omega_{\max}$ we obtain

$$v_{\max} = \left(\frac{r_0^2}{r}\right)\omega_{0,\max} = \frac{0.832\,\text{m}^2/\text{s}}{r} \; .$$

(m) This result is again a decreasing function of r. We graph $v_{\max}$ versus r (with SI units understood) below.



(n) When $r = r_0$ we find $v_{\max} = 1.04$ m/s. Any later value must be smaller (according to the above graph), so we seek a value of r that gives $v_{\max} = 0.520$ m. This can be solved for algebraically:

$$r = \frac{0.832}{0.520} = 1.60 \text{ m} \; .$$

(o) If we do not examine changes in perspective (the fact that the blade is getting closer to the observer), then Poe's description must be considered misleading. We have found that the angular swing, the horizontal sweep and the maximum speed should decrease as r increases, which is contrary to the description given in Poe's story.