Chapter 7

1. The kinetic energy is given by $K = \frac{1}{2}mv^2$, where m is the mass and v is the speed of the electron. The speed is therefore

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(6.7 \times 10^{-19}\,\text{J})}{9.11 \times 10^{-31}\,\text{kg}}} = 1.2 \times 10^6 \text{ m/s} .$$

2. (a) The change in kinetic energy for the meteorite would be

$$\begin{aligned} \Delta K &= K_f - K_i = -K_i = -\frac{1}{2}m_i v_i^2 \\ &= -\frac{1}{2}\left(4 \times 10^6\,\text{kg}\right)\left(15 \times 10^3\,\text{m/s}\right)^2 \\ &= -5 \times 10^{14} \text{ J} \end{aligned}$$

where the negative sign indicates that kinetic energy is lost.

(b) The energy loss in units of megatons of TNT would be

$$-\Delta K = \left(5 \times 10^{14}\,\text{J}\right)\left(\frac{1\,\text{megaton TNT}}{4.2 \times 10^{15}\,\text{J}}\right) = 0.1 \text{ megaton TNT} .$$

(c) The number of bombs N that the meteorite impact would correspond to is found by noting that megaton = 1000 kilotons and setting up the ratio:

$$N = \frac{0.1 \times 1000\,\text{kiloton TNT}}{13\,\text{kiloton TNT}} = 8 .$$

3. We convert to SI units (where necessary) and use $K = \frac{1}{2}mv^2$.

(a) $K = \frac{1}{2}(110)(8.1)^2 = 3.6 \times 10^3$ J.

(b) Since 1000 g = kg, we find

$$K = \frac{1}{2}\left(4.2 \times 10^{-3}\,\text{kg}\right)(950\,\text{m/s})^2 = 1.9 \times 10^3 \text{ J} .$$

(c) We note that the conversion from knots to m/s can be obtained from the information in Appendix D (knot = 1.688 ft/s where ft = 0.3048 m), which is also where the ton → kilogram conversion can be found. Therefore,

$$K = \frac{1}{2}\left((91400\,\text{tons})\,\frac{907.2\,\text{kg}}{\text{ton}}\right)\left((32\,\text{knots})\,\frac{0.515\,\text{m/s}}{\text{knot}}\right)^2 = 1.1 \times 10^{10} \text{ J} .$$

4. We denote the mass of the father as m and his initial speed v_i. The initial kinetic energy of the father is

$$K_i = \frac{1}{2} K_{\text{son}}$$

and his final kinetic energy (when his speed is $v_f = v_i + 1.0$ m/s) is

$$K_f = K_{\text{son}} \ .$$

We use these relations along with Eq. 7-1 in our solution.

(a) We see from the above that $K_i = \frac{1}{2} K_f$ which (with SI units understood) leads to

$$\frac{1}{2} m v_i^2 = \frac{1}{2} \left(\frac{1}{2} m (v_i + 1.0)^2 \right) \ .$$

The mass cancels and we find a second-degree equation for v_i :

$$\frac{1}{2} v_i^2 - v_i - \frac{1}{2} = 0 \ .$$

The positive root (from the quadratic formula) yields $v_i = 2.4$ m/s.

(b) From the first relation above ($K_i = \frac{1}{2} K_{\text{son}}$), we have

$$\frac{1}{2} m v_i^2 = \frac{1}{2} \left(\frac{1}{2} \left(\frac{m}{2} \right) v_{\text{son}}^2 \right)$$

and (after canceling m and one factor of 1/2) are led to $v_{\text{son}} = 2 v_i = 4.8$ m/s.

5. (a) From Table 2-1, we have $v^2 = v_0^2 + 2a\Delta x$. Thus,

$$v = \sqrt{v_0^2 + 2a\Delta x} = \sqrt{\left(2.4 \times 10^7\right)^2 + 2\left(3.6 \times 10^{15}\right)(0.035)} = 2.9 \times 10^7 \text{ m/s} \ .$$

(b) The initial kinetic energy is

$$K_i = \frac{1}{2} m v_0^2 = \frac{1}{2} \left(1.67 \times 10^{-27} \text{ kg}\right) \left(2.4 \times 10^7 \text{ m/s}\right)^2 = 4.8 \times 10^{-13} \text{ J} \ .$$

The final kinetic energy is

$$K_f = \frac{1}{2} m v^2 = \frac{1}{2} \left(1.67 \times 10^{-27} \text{ kg}\right) \left(2.9 \times 10^7 \text{ m/s}\right)^2 = 6.9 \times 10^{-13} \text{ J} \ .$$

The change in kinetic energy is $\Delta K = 6.9 \times 10^{-13} - 4.8 \times 10^{-13} = 2.1 \times 10^{-13}$ J.

6. Using Eq. 7-8 (and Eq. 3-23), we find the work done by the water on the ice block:

$$\begin{aligned} W &= \vec{F} \cdot \vec{d} \\ &= \left(210\,\hat{\imath} - 150\,\hat{\jmath}\right) \cdot \left(15\,\hat{\imath} - 12\,\hat{\jmath}\right) \\ &= (210)(15) + (-150)(-12) \\ &= 5.0 \times 10^3 \text{ J} \ . \end{aligned}$$

7. (a) The force of the worker on the crate is constant, so the work it does is given by $W_F = \vec{F} \cdot \vec{d} = F d \cos\phi$, where $\vec{F}$ is the force, $\vec{d}$ is the displacement of the crate, and ϕ is the angle between the force and the displacement. Here $F = 210$ N, $d = 3.0$ m, and $\phi = 20^\circ$. Thus $W_F = (210\text{ N})(3.0\text{ m})\cos 20^\circ = 590$ J.

(b) The force of gravity is downward, perpendicular to the displacement of the crate. The angle between this force and the displacement is 90° and $\cos 90° = 0$, so the work done by the force of gravity is zero.

(c) The normal force of the floor on the crate is also perpendicular to the displacement, so the work done by this force is also zero.

(d) These are the only forces acting on the crate, so the total work done on it is 590 J.

8. Since this involves constant-acceleration motion, we can apply the equations of Table 2-1, such as $x = v_0t + \frac{1}{2}at^2$ (where $x_0 = 0$). We choose to analyze the third and fifth points, obtaining

$$\begin{aligned} 0.2\,\mathrm{m} &= v_0(1.0\,\mathrm{s}) + \frac{1}{2}a\,(1.0\,\mathrm{s})^2 \\ 0.8\,\mathrm{m} &= v_0(2.0\,\mathrm{s}) + \frac{1}{2}a\,(2.0\,\mathrm{s})^2 \end{aligned}$$

Simultaneous solution of the equations leads to $v_0 = 0$ and $a = 0.40\,\mathrm{m/s^2}$. We now have two ways to finish the problem. One is to compute force from $F = ma$ and then obtain the work from Eq. 7-7. The other is to find ΔK as a way of computing W (in accordance with Eq. 7-10). In this latter approach, we find the velocity at $t = 2.0$ s from $v = v_0 + at$ (so $v = 0.80$ m/s). Thus,

$$W = \Delta K = \frac{1}{2}(1.0\,\mathrm{kg})(0.80\,\mathrm{m/s})^2 = 0.32 \text{ J} .$$

9. We choose $+x$ as the direction of motion (so $\vec{a}$ and $\vec{F}$ are negative-valued).

(a) Newton's second law readily yields $\vec{F} = (85\,\mathrm{kg})(-2.0\,\mathrm{m/s^2})$ so that $F = |\vec{F}| = 170$ N.

(b) From Eq. 2-16 (with $v = 0$) we have

$$0 = v_0^2 + 2a\Delta x \quad \Longrightarrow \quad \Delta x = -\frac{(37\,\mathrm{m/s})^2}{2(-2.0\,\mathrm{m/s^2})}$$

which gives $\Delta x = 3.4 \times 10^2$ m. Alternatively, this can be worked using the work-energy theorem.

(c) Since $\vec{F}$ is opposite to the direction of motion (so the angle ϕ between $\vec{F}$ and $\vec{d} = \Delta x$ is 180°) then Eq. 7-7 gives the work done as $W = -F\Delta x = -5.8 \times 10^4$ J.

(d) In this case, Newton's second law yields $\vec{F} = (85\,\mathrm{kg})(-4.0\,\mathrm{m/s^2})$ so that $F = |\vec{F}| = 340$ N.

(e) From Eq. 2-16, we now have

$$\Delta x = -\frac{(37\,\mathrm{m/s})^2}{2(-4.0\,\mathrm{m/s^2})} = 1.7 \times 10^2 \text{ m} .$$

(f) The force $\vec{F}$ is again opposite to the direction of motion (so the angle ϕ is again 180°) so that Eq. 7-7 leads to $W = -F\Delta x = -5.8 \times 10^4$ J. The fact that this agrees with the result of part (c) provides insight into the concept of work.

10. We choose to work this using Eq. 7-10 (the work-kinetic energy theorem). To find the initial and final kinetic energies, we need the speeds, so

$$v = \frac{dx}{dt} = 3.0 - 8.0t + 3.0t^2$$

in SI units. Thus, the initial speed is $v_i = 3.0$ m/s and the speed at $t = 4$ s is $v_f = 19$ m/s. The change in kinetic energy for the object of mass $m = 3.0$ kg is therefore

$$\Delta K = \frac{1}{2}m\left(v_f^2 - v_i^2\right) = 528 \text{ J}$$

which we round off to two figures and (using the work-kinetic energy theorem) conclude that the work done is $W = 5.3 \times 10^2$ J.

11. (a) The forces are constant, so the work done by any one of them is given by $W = \vec{F} \cdot \vec{d}$, where $\vec{d}$ is the displacement. Force $\vec{F}_1$ is in the direction of the displacement, so

$$W_1 = F_1 d \cos\phi_1 = (5.00\,\text{N})(3.00\,\text{m})\cos 0^\circ = 15.0 \text{ J} .$$

Force $\vec{F}_2$ makes an angle of 120° with the displacement, so

$$W_2 = F_2 d \cos\phi_2 = (9.00\,\text{N})(3.00\,\text{m})\cos 120^\circ = -13.5 \text{ J} .$$

Force $\vec{F}_3$ is perpendicular to the displacement, so $W_3 = F_3 d \cos\phi_3 = 0$ since $\cos 90^\circ = 0$. The net work done by the three forces is

$$W = W_1 + W_2 + W_3 = 15.0\,\text{J} - 13.5\,\text{J} + 0 = +1.5 \text{ J} .$$

(b) If no other forces do work on the box, its kinetic energy increases by 1.5 J during the displacement.

12. By the work-kinetic energy theorem,

$$\begin{aligned} W &= \Delta K \\ &= \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 \\ &= \frac{1}{2}(2.0\,\text{kg})\left((6.0\,\text{m/s})^2 - (4.0\,\text{m/s})^2\right) \\ &= 20 \text{ J} . \end{aligned}$$

We note that the *directions* of $\vec{v}_f$ and $\vec{v}_i$ play no role in the calculation.

13. The forces are all constant, so the total work done by them is given by $W = F_{\text{net}}\,\Delta x$, where F_{net} is the magnitude of the net force and Δx is the magnitude of the displacement. We add the three vectors, finding the x and y components of the net force:

$$\begin{aligned} F_{\text{net}\,x} &= -F_1 - F_2 \sin 50^\circ + F_3 \cos 35^\circ \\ &= -3.00\,\text{N} - (4.00\,\text{N})\sin 35^\circ + (10.0\,\text{N})\cos 35^\circ \\ &= 2.127 \text{ N} \\ F_{\text{net}\,y} &= -F_2 \cos 50^\circ + F_3 \sin 35^\circ \\ &= -(4.00\,\text{N})\cos 50^\circ + (10.0\,\text{N})\sin 35^\circ \\ &= 3.165 \text{ N} . \end{aligned}$$

The magnitude of the net force is

$$F_{\text{net}} = \sqrt{F_{\text{net}\,x}^2 + F_{\text{net}\,y}^2} = \sqrt{2.127^2 + 3.165^2} = 3.813 \text{ N} .$$

The work done by the net force is

$$W = F_{\text{net}}\,d = (3.813\,\text{N})(4.00\,\text{m}) = 15.3 \text{ J}$$

where we have used the fact that $\vec{d} \parallel \vec{F}_{\text{net}}$ (which follows from the fact that the canister started from rest and moved horizontally under the action of horizontal forces – the resultant effect of which is expressed by $\vec{F}_{\text{net}}$).

14. In both cases, there is no acceleration, so the lifting force is equal to the weight of the object.

(a) Eq. 7-8 leads to $W = \vec{F} \cdot \vec{d} = (360\,\text{kN})(0.10\,\text{m}) = 36$ kJ.

(b) In this case, we find $W = (4000\,\text{N})(0.050\,\text{m}) = 200$ J.

15. There is no acceleration, so the lifting force is equal to the weight of the object. We note that the person's pull $\vec{F}$ is equal (in magnitude) to the tension in the cord.

    (a) As indicated in the *hint*, tension contributes twice to the lifting of the canister: $2T = mg$. Since, $|\vec{F}| = T$, we find $|\vec{F}| = 98$ N.

    (b) To rise 0.020 m, two segments of the cord (see Fig. 7-28) must shorten by that amount. Thus, the amount of string pulled down at the left end (this is the magnitude of $\vec{d}$, the downward displacement of the hand) is $d = 0.040$ m.

    (c) Since (at the left end) both $\vec{F}$ and $\vec{d}$ are downward, then Eq. 7-7 leads to $W = \vec{F} \cdot \vec{d} = (98)(0.040) = 3.9$ J.

    (d) Since the force of gravity $\vec{F}_g$ (with magnitude mg) is opposite to the displacement $\vec{d}_c = 0.020$ m (up) of the canister, Eq. 7-7 leads to $W = \vec{F}_g \cdot \vec{d}_c = -(196)(0.020) = -3.9$ J. This is consistent with Eq. 7-15 since there is no change in kinetic energy.

16. (a) The component of the force of gravity exerted on the ice block (of mass m) along the incline is $mg \sin\theta$, where $\theta = \sin^{-1}(0.91/1.5)$ gives the angle of inclination for the inclined plane. Since the ice block slides down with uniform velocity, the worker must exert a force $\vec{F}$ "uphill" with a magnitude equal to $mg \sin\theta$. Consequently,

$$F = mg \sin\theta = (45\,\mathrm{kg})\left(9.8\,\mathrm{m/s^2}\right)\left(\frac{0.91\,\mathrm{m}}{1.5\,\mathrm{m}}\right) = 2.7 \times 10^2 \text{ N} .$$

    (b) Since the "downhill" displacement is opposite to $\vec{F}$, the work done by the worker is

$$W_1 = -\left(2.7 \times 10^2\,\mathrm{N}\right)(1.5\,\mathrm{m}) = -4.0 \times 10^2 \text{ J} .$$

    (c) Since the displacement has a vertically downward component of magnitude 0.91 m (in the same direction as the force of gravity), we find the work done by gravity to be

$$W_2 = (45\,\mathrm{kg})\left(9.8\,\mathrm{m/s^2}\right)(0.91\,\mathrm{m}) = 4.0 \times 10^2 \text{ J} .$$

    (d) Since $\vec{N}$ is perpendicular to the direction of motion of the block, and $\cos 90° = 0$, work done by the normal force is $W_3 = 0$ by Eq. 7-7.

    (e) The resultant force $\vec{F}_{\text{net}}$ is zero since there is no acceleration. Thus, it's work is zero, as can be checked by adding the above results $W_1 + W_2 + W_3 = 0$.

17. (a) We use $\vec{F}$ to denote the upward force exerted by the cable on the astronaut. The force of the cable is upward and the force of gravity is mg downward. Furthermore, the acceleration of the astronaut is $g/10$ upward. According to Newton's second law, $F - mg = mg/10$, so $F = 11mg/10$. Since the force $\vec{F}$ and the displacement $\vec{d}$ are in the same direction, the work done by $\vec{F}$ is

$$W_F = Fd = \frac{11mgd}{10} = \frac{11(72\,\mathrm{kg})\left(9.8\,\mathrm{m/s^2}\right)(15\,\mathrm{m})}{10} = 1.164 \times 10^4 \text{ J}$$

    which (with respect to significant figures) should be quoted as 1.2×10^4 J.

    (b) The force of gravity has magnitude mg and is opposite in direction to the displacement. Thus, using Eq. 7-7, the work done by gravity is

$$W_g = -mgd = -(72\,\mathrm{kg})\left(9.8\,\mathrm{m/s^2}\right)(15\,\mathrm{m}) = -1.058 \times 10^4 \text{ J}$$

    which should be quoted as -1.1×10^4 J.

(c) The total work done is $W = 1.164\times10^4\,\mathrm{J} - 1.058\times10^4\,\mathrm{J} = 1.06\times10^3\,\mathrm{J}$. Since the astronaut started from rest, the work-kinetic energy theorem tells us that this (which we round to 1.1×10^3 J) is her final kinetic energy.

(d) Since $K = \frac{1}{2}mv^2$, her final speed is

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(1.06\times10^3\,\mathrm{J})}{72\,\mathrm{kg}}} = 5.4\ \mathrm{m/s}\ .$$

18. We use d to denote the magnitude of the spelunker's displacement during each stage. The mass of the spelunker is $m = 80.0$ kg. The work done by the lifting force is denoted W_i where $i = 1, 2, 3$ for the three stages. We apply the work-energy theorem, Eq. 17-15.

(a) For stage 1, $W_1 - mgd = \Delta K_1 = \frac{1}{2}mv_1^2$, where $v_1 = 5.00\,\mathrm{m/s}$. This gives

$$W_1 = mgd + \frac{1}{2}mv_1^2 = (80.0)(9.8)(10.0) + \frac{1}{2}(80.0)(5.00)^2 = 8.84\times10^3\ \mathrm{J}\ .$$

(b) For stage 2, $W_2 - mgd = \Delta K_2 = 0$, which leads to

$$W_2 = mgd = (80.0\,\mathrm{kg})\left(9.8\,\mathrm{m/s^2}\right)(10.0\,\mathrm{m}) = 7.84\times10^3\ \mathrm{J}\ .$$

(c) For stage 3, $W_3 - mgd = \Delta K_3 = -\frac{1}{2}mv_1^2$. We obtain

$$W_3 = mgd - \frac{1}{2}mv_1^2 = (80.0)(9.8)(10.0) - \frac{1}{2}(80.0)(5.00)^2 = 6.84\times10^3\ \mathrm{J}\ .$$

19. (a) We use F to denote the magnitude of the force of the cord on the block. This force is upward, opposite to the force of gravity (which has magnitude Mg). The acceleration is $\vec{a} = g/4$ downward. Taking the downward direction to be positive, then Newton's second law yields

$$\vec{F}_{\mathrm{net}} = m\vec{a} \quad\Longrightarrow\quad Mg - F = M\left(\frac{g}{4}\right)$$

so $F = 3Mg/4$. The displacement is downward, so the work done by the cord's force is $W_F = -Fd = -3Mgd/4$, using Eq. 7-7.

(b) The force of gravity is in the same direction as the displacement, so it does work $W_g = Mgd$.

(c) The total work done on the block is $-3Mgd/4 + Mgd = Mgd/4$. Since the block starts from rest, we use Eq. 7-15 to conclude that this $(Mgd/4)$ is the block's kinetic energy K at the moment it has descended the distance d.

(d) Since $K = \frac{1}{2}Mv^2$, the speed is

$$v = \sqrt{\frac{2K}{M}} = \sqrt{\frac{2\,(Mgd/4)}{M}} = \sqrt{\frac{gd}{2}}$$

at the moment the block has descended the distance d.

20. The spring constant is $k = 100$ N/m and the maximum elongation is $x_i = 5.00$ m. Using Eq. 7-25 with $x_f = 0$, the work is found to be

$$W = \frac{1}{2}kx_i^2 = \frac{1}{2}(100)(5.00)^2 = 1.25\times10^3\ \mathrm{J}\ .$$

21. (a) The spring constant is $k = 1500$ N/m and the elongation is $x = 0.0076$ m. Our $+x$ direction is rightward. Using Eq. 7-26, the work is found to be

$$W = -\frac{1}{2}kx^2 = -\frac{1}{2}(1500)(0.0076)^2 = -0.043\ \mathrm{J}\ .$$

(b) We use Eq. 7-25 with $x_i = x = 0.0076$ m and $x_f = 2x = 0.0152$ m to find the additional work:

$$\begin{aligned} W &= \frac{1}{2}k\left(x_i^2 - x_f^2\right) \\ &= \frac{1}{2}k\left(x^2 - 4x^2\right) \\ &= -\frac{3}{2}kx^2 \\ &= -\frac{3}{2}(1500)(0.0076)^2 = -0.13 \text{ J} . \end{aligned}$$

We note that this is greater (in magnitude) than the work done in the first interval (even though the displacements have the same magnitude), due to the fact that the force is larger throughout the second interval.

22. (a) The compression of the spring is $d = 0.12$ m. The work done by the force of gravity (acting on the block) is, by Eq. 7-12,

$$W_1 = mgd = (0.25\,\text{kg})\left(9.8\,\text{m/s}^2\right)(0.12\,\text{m}) = 0.29 \text{ J} .$$

(b) The work done by the spring is, by Eq. 7-26,

$$W_2 = -\frac{1}{2}kd^2 = -\frac{1}{2}(250\,\text{N/m})(0.12\,\text{m})^2 = -1.8 \text{ J} .$$

(c) The speed v_i of the block just before it hits the spring is found from the work-kinetic energy theorem (Eq. 7-15).

$$\Delta K = 0 - \frac{1}{2}mv_i^2 = W_1 + W_2$$

which yields

$$v_i = \sqrt{\frac{(-2)(W_1 + W_2)}{m}} = \sqrt{\frac{(-2)(0.29 - 1.8)}{0.25}} = 3.5 \text{ m/s} .$$

(d) If we instead had $v_i' = 7$ m/s, we reverse the above steps and solve for d'. Recalling the theorem used in part (c), we have

$$\begin{aligned} 0 - \frac{1}{2}mv_i'^{\,2} &= W_1' + W_2' \\ &= mgd' - \frac{1}{2}kd'^{\,2} \end{aligned}$$

which (choosing the positive root) leads to

$$d' = \frac{mg + \sqrt{m^2g^2 + mkv_i'^{\,2}}}{k}$$

which yields $d' = 0.23$ m. In order to obtain this, we have used more digits in our intermediate results than are shown above (so $v_i = \sqrt{12.048} = 3.471$ m/s and $v_i' = 6.942$ m/s).

23. (a) As the body moves along the x axis from $x_i = 3.0\,\text{m}$ to $x_f = 4.0\,\text{m}$ the work done by the force is

$$\begin{aligned} W &= \int_{x_i}^{x_f} F_x\,dx \\ &= \int_{x_i}^{x_f} -6x\,dx = -3(x_f^2 - x_i^2) \\ &= -3\left(4.0^2 - 3.0^2\right) = -21 \text{ J} . \end{aligned}$$

According to the work-kinetic energy theorem, this gives the change in the kinetic energy:

$$W = \Delta K = \frac{1}{2}m\left(v_f^2 - v_i^2\right)$$

where v_i is the initial velocity (at x_i) and v_f is the final velocity (at x_f). The theorem yields

$$v_f = \sqrt{\frac{2W}{m} + v_i^2} = \sqrt{\frac{2(-21)}{2.0} + 8.0^2} = 6.6 \text{ m/s} .$$

(b) The velocity of the particle is $v_f = 5.0\,\text{m/s}$ when it is at $x = x_f$. The work-kinetic energy theorem is used to solve for x_f. The net work done on the particle is $W = -3(x_f^2 - x_i^2)$, so the theorem leads to

$$-3\left(x_f^2 - x_i^2\right) = \frac{1}{2}m\left(v_f^2 - v_i^2\right) .$$

Thus,

$$\begin{aligned} x_f &= \sqrt{-\frac{m}{6}\left(v_f^2 - v_i^2\right) + x_i^2} \\ &= \sqrt{-\frac{2.0\,\text{kg}}{6\,\text{N/m}}\left((5.0\,\text{m/s})^2 - (8.0\,\text{m/s})^2\right) + (3.0\,\text{m})^2} \\ &= 4.7 \text{ m} . \end{aligned}$$

24. From Eq. 7-32, we see that the "area" in the graph is equivalent to the work done. Finding that area (in terms of rectangular [length×width] and triangular [$\frac{1}{2}$base×height] areas) we obtain

$$\begin{aligned} W &= W_{0<x<2} + W_{2<x<4} + W_{4<x<6} + W_{6<x<8} \\ &= 20 + 10 + 0 - 5 = 25 \text{ J} . \end{aligned}$$

25. According to the graph the acceleration a varies linearly with the coordinate x. We may write $a = \alpha x$, where α is the slope of the graph. Numerically,

$$\alpha = \frac{20\,\text{m/s}^2}{8.0\,\text{m}} = 2.5 \text{ s}^{-2} .$$

The force on the brick is in the positive x direction and, according to Newton's second law, its magnitude is given by $F = a/m = (\alpha/m)x$. If x_f is the final coordinate, the work done by the force is

$$W = \int_0^{x_f} F\,dx = \frac{\alpha}{m}\int_0^{x_f} x\,dx = \frac{\alpha}{2m}x_f^2 = \frac{2.5}{2(10)}(8.0)^2 = 800 \text{ J} .$$

26. From Eq. 7-32, we see that the"area" in the graph is equivalent to the work done. We find the area in terms of rectangular [length×width] and triangular [$\frac{1}{2}$base×height] areas and use the work-kinetic energy theorem appropriately. The initial point is taken to be $x = 0$, where $v_0 = 4.0$ m/s.

(a) With $K_i = \frac{1}{2}mv_0^2 = 16$ J, we have

$$K_3 - K_0 = W_{0<x<1} + W_{1<x<2} + W_{2<x<3} = -4 \text{ J}$$

so that K_3 (the kinetic energy when $x = 3.0$ m) is found to equal 12 J.

(b) With SI units understood, we write $W_{3<x<x_f}$ as $F_x\Delta x = (-4)(x_f - 3.0)$ and apply the work-kinetic energy theorem:

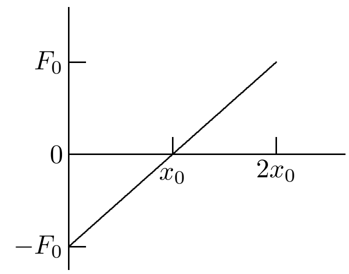
$$\begin{aligned} K_{x_f} - K_3 &= W_{3<x<x_f} \\ K_{x_f} - 12 &= (-4)(x_f - 3.0) \end{aligned}$$

so that the requirement $K_{x_f} = 8$ J leads to $x_f = 4.0$ m.

(c) As long as the work is positive, the kinetic energy grows. The graph shows this situation to hold until $x = 1.0$ m. At that location, the kinetic energy is

$$\begin{aligned} K_1 &= K_0 + W_{0<x<1} \\ &= 16 + 2 \;=\; 18 \text{ J} . \end{aligned}$$

27. (a) The graph shows F as a function of x assuming x_0 is positive. The work is negative as the object moves from $x = 0$ to $x = x_0$ and positive as it moves from $x = x_0$ to $x = 2x_0$. Since the area of a triangle is $\frac{1}{2}$(base)(altitude), the work done from $x = 0$ to $x = x_0$ is $-\frac{1}{2}(x_0)(F_0)$ and the work done from $x = x_0$ to $x = 2x_0$ is $\frac{1}{2}(2x_0 - x_0)(F_0) = \frac{1}{2}(x_0)(F_0)$. The total work is the sum, which is zero.



(b) The integral for the work is

$$W = \int_0^{2x_0} F_0 \left(\frac{x}{x_0} - 1\right) dx = F_0 \left(\frac{x^2}{2x_0} - x\right)\Bigg|_0^{2x_0} = 0 .$$

28. (a) Using the work-kinetic energy theorem

$$K_f = K_i + \int_0^2 \left(2.5 - x^2\right) dx = 0 + (2.5)(2) - \frac{1}{3}(2)^3$$

we obtain $K_f = 2.3$ J.

(b) For a variable end-point, we have K_f as a function of x, which could be differentiated to find the extremum value, but we recognize that this is equivalent to solving $F = 0$ for x:

$$F = 0 \implies 2.5 - x^2 = 0$$

Thus, K is extremized at $x = \sqrt{2.5}$ and we compute

$$K_f = K_i + \int_0^{\sqrt{2.5}} \left(2.5 - x^2\right) dx = 0 + (2.5)(\sqrt{2.5}) - \frac{1}{3}(\sqrt{2.5})^3 .$$

Therefore, $K = 2.6$ J at $x = \sqrt{2.5} = 1.6$ m. Recalling our answer for part (a), it is clear that this extreme value is a maximum.

29. One approach is to assume a "path" from $\vec{r}_i$ to $\vec{r}_f$ and do the line-integral accordingly. Another approach is to simply use Eq. 7-36, which we demonstrate:

$$\begin{aligned} W &= \int_{x_i}^{x_f} F_x\, dx + \int_{y_i}^{y_f} F_y\, dy \\ &= \int_2^{-4} (2x)\, dx + \int_3^{-3} (3)\, dy \end{aligned}$$

with SI units understood. Thus, we obtain $W = 12 - 18 = -6$ J.

30. Recognizing that the force in the cable must equal the total weight (since there is no acceleration), we employ Eq. 7-47:

$$P = Fv\cos\theta = mg\left(\frac{\Delta x}{\Delta t}\right)$$

where we have used the fact that $\theta = 0°$ (both the force of the cable and the elevator's motion are upward). Thus,

$$P = \left(3.0 \times 10^3\,\text{kg}\right)\left(9.8\,\text{m/s}^2\right)\left(\frac{210\,\text{m}}{23\,\text{s}}\right) = 2.7 \times 10^5\ \text{W}\ .$$

31. The power associated with force $\vec{F}$ is given by $P = \vec{F} \cdot \vec{v}$, where $\vec{v}$ is the velocity of the object on which the force acts. Thus,

$$P = \vec{F} \cdot \vec{v} = Fv\cos\phi = (122\,\text{N})(5.0\,\text{m/s})\cos 37° = 490\ \text{W}\ .$$

32. (a) Using Eq.7-48 and Eq. 3-23, we obtain

$$P = \vec{F} \cdot \vec{v} = (4.0\,\text{N})(-2.0\,\text{m/s}) + (9.0\,\text{N})(4.0\,\text{m/s}) = 28\ \text{W}\ .$$

(b) We again use Eq.7-48 and Eq. 3-23, but with a one-component velocity: $\vec{v} = v\hat{\jmath}$.

$$\begin{aligned} P &= \vec{F} \cdot \vec{v} \\ -12\,\text{W} &= (-2.0\,\text{N})v \end{aligned}$$

which yields $v = 6$ m/s.

33. (a) The power is given by $P = Fv$ and the work done by $\vec{F}$ from time t_1 to time t_2 is given by

$$W = \int_{t_1}^{t_2} P\,\text{dt} = \int_{t_1}^{t_2} Fv\,\text{dt}\,.$$

Since $\vec{F}$ is the net force, the magnitude of the acceleration is $a = F/m$, and, since the initial velocity is $v_0 = 0$, the velocity as a function of time is given by $v = v_0 + at = (F/m)t$. Thus

$$W = \int_{t_1}^{t_2} (F^2/m)t\,\text{dt} = \frac{1}{2}(F^2/m)(t_2^2 - t_1^2)\,.$$

For $t_1 = 0$ and $t_2 = 1.0\,\text{s}$,

$$W = \frac{1}{2}\left(\frac{(5.0\,\text{N})^2}{15\,\text{kg}}\right)(1.0\,\text{s})^2 = 0.83\,\text{J}\ .$$

(b) For $t_1 = 1.0\,\text{s}$ and $t_2 = 2.0\,\text{s}$,

$$W = \frac{1}{2}\left(\frac{(5.0\,\text{N})^2}{15\,\text{kg}}\right)\left((2.0\,\text{s})^2 - (1.0\,\text{s})^2\right) = 2.5\ \text{J}\ .$$

(c) For $t_1 = 2.0\,\text{s}$ and $t_2 = 3.0\,\text{s}$,

$$W = \frac{1}{2}\left(\frac{(5.0\,\text{N})^2}{15\,\text{kg}}\right)\left((3.0\,\text{s})^2 - (2.0\,\text{s})^2\right) = 4.2\ \text{J}\ .$$

(d) Substituting $v = (F/m)t$ into $P = Fv$ we obtain $P = F^2t/m$ for the power at any time t. At the end of the third second

$$P = \frac{(5.0\,\text{N})^2(3.0\,\text{s})}{15\,\text{kg}} = 5.0\ \text{W}\ .$$

34. (a) Since constant speed implies $\Delta K = 0$, we require $W_a = -W_g$, by Eq. 7-15. Since W_g is the same in both cases (same weight and same path), then $W_a = 900$ J just as it was in the first case.

(b) Since the speed of 1.0 m/s is constant, then 8.0 meters is traveled in 8.0 seconds. Using Eq. 7-42, and noting that average power is *the* power when the work is being done at a steady rate, we have

$$P = \frac{W}{\Delta t} = \frac{900\ \text{J}}{8.0\ \text{s}}$$

which results in $P = 113$ W.

(c) Since the speed of 2.0 m/s is constant, 8.0 meters is traveled in 4.0 seconds. Using Eq. 7-42, with *average power* replaced by *power*, we have

$$P = \frac{W}{\Delta t} = \frac{900\ \text{J}}{4.0\ \text{s}}$$

from which we obtain $P = 225$ W.

35. The total work is the sum of the work done by gravity on the elevator, the work done by gravity on the counterweight, and the work done by the motor on the system: $W_T = W_e + W_c + W_s$. Since the elevator moves at constant velocity, its kinetic energy does not change and according to the work-kinetic energy theorem the total work done is zero. This means $W_e + W_c + W_s = 0$. The elevator moves upward through 54 m, so the work done by gravity on it is

$$W_e = -m_e g d = -(1200\,\text{kg})(9.8\,\text{m/s}^2)(54\,\text{m}) = -6.35 \times 10^5\,\text{J}\,.$$

The counterweight moves downward the same distance, so the work done by gravity on it is

$$W_c = m_c g d = (950\,\text{kg})\left(9.8\,\text{m/s}^2\right)(54\,\text{m}) = 5.03 \times 10^5\ \text{J}\ .$$

Since $W_T = 0$, the work done by the motor on the system is

$$W_s = -W_e - W_c = 6.35 \times 10^5\,\text{J} - 5.03 \times 10^5\,\text{J} = 1.32 \times 10^5\ \text{J}\ .$$

This work is done in a time interval of $\Delta t = 3.0\,\text{min} = 180\,\text{s}$, so the power supplied by the motor to lift the elevator is

$$P = \frac{W_s}{\Delta t} = \frac{1.32 \times 10^5\,\text{J}}{180\,\text{s}} = 7.4 \times 10^2\ \text{W}\ .$$

36. (a) Since the force exerted by the spring on the mass is zero when the mass passes through the equilibrium position of the spring, the rate at which the spring is doing work on the mass at this instant is also zero.

(b) The rate is given by $P = \vec{F} \cdot \vec{v} = -Fv$, where the minus sign corresponds to the fact that $\vec{F}$ and $\vec{v}$ are antiparallel to each other. The magnitude of the force is given by $F = kx = (500\,\text{N/m})(0.10\,\text{m}) = 50\,\text{N}$, while v is obtained from conservation of energy for the spring-mass system:

$$E = K + U = 10\,\text{J} = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}(0.30\,\text{kg})v^2 + \frac{1}{2}(500\,\text{N/m})(0.10\,\text{m})^2$$

which gives $v = 7.1\,\text{m/s}$. Thus

$$P = -Fv = -(50\,\text{N})(7.1\,\text{m/s}) = -3.5\ \times 10^2\,\text{W}\ .$$

37. We write the force as $F = \alpha v$, where v is the speed and α is a constant of proportionality. The power required is $P = Fv = \alpha v^2$. Let P_1 be the power required for speed v_1 and P_2 be the power required for speed v_2. Dividing $P_2 = \alpha v_2^2$ by $P_1 = \alpha v_1^2$, we find

$$P_2 = \left(\frac{v_2}{v_1}\right)^2 P_1\ .$$

Since $P_1 = 7.5\,\text{kW}$ and $v_2 = 3v_1$,

$$P_2 = (3)^2(7.5\,\text{kW}) = 68\ \text{kW}\ .$$

38. (a) The force $\vec{F}$ of the incline is a combination of normal and friction force which is serving to "cancel" the tendency of the box to fall downward (due to its 19.6 N weight). Thus, $\vec{F} = mg$ upward. In this part of the problem, the angle ϕ between the belt and $\vec{F}$ is 80°. From Eq. 7-47, we have

$$P = Fv\cos\phi = (19.6)(0.50)\cos 80°$$

which leads to $P = 1.7$ W.

(b) Now the angle between the belt and $\vec{F}$ is 90°, so that $P = 0$.

(c) In this part, the angle between the belt and $\vec{F}$ is 100°, so that $P = (19.6)(0.50)\cos 100° = -1.7$ W.

39. (a) In 10 min the cart moves

$$\left(6.0\,\frac{\text{mi}}{\text{h}}\right)\left(\frac{5280\,\text{ft/mi}}{60\,\text{min/h}}\right)(10\,\text{min}) = 5280\text{ ft}$$

so that Eq. 7-7 yields

$$W = Fd\cos\phi = (40\,\text{lb})(5280\,\text{ft})\cos 30° = 1.8\times10^5\ \text{ft}\cdot\text{lb}\ .$$

(b) The average power is given by Eq. 7-42, and the conversion to horsepower (hp) can be found on the inside back cover. We note that 10 min is equivalent to 600 s.

$$P_{\text{avg}} = \frac{1.8\times10^5\ \text{ft}\cdot\text{lb}}{600\,\text{s}} = 305\ \text{ft}\cdot\text{lb/s}$$

which (upon dividing by 550) converts to $P_{\text{avg}} = 0.55$ hp.

40. The acceleration is constant, so we may use the equations in Table 2-1. We choose the direction of motion as $+x$ and note that the displacement is the same as the distance traveled, in this problem. We designate the force (assumed singular) along the x direction acting on the $m = 2.0$ kg object as F.

(a) With $v_0 = 0$, Eq. 2-11 leads to $a = v/t$. And Eq. 2-17 gives $\Delta x = \frac{1}{2}vt$ Newton's second law yields the force $F = ma$. Eq. 7-8, then, gives the work:

$$W = F\Delta x = m\left(\frac{v}{t}\right)\left(\frac{1}{2}vt\right) = \frac{1}{2}mv^2$$

as we expect from the work-kinetic energy theorem. With $v = 10$ m/s, this yields $W = 100$ J.

(b) Instantaneous power is defined in Eq. 7-48. With $t = 3.0$ s, we find

$$P = Fv = m\left(\frac{v}{t}\right)v = 67\ \text{W}\ .$$

(c) The velocity at $t' = 1.5$ s is $v' = at' = 5.0$ m/s. Thus,

$$P' = Fv' = 33\ \text{W}\ .$$

41. The total weight is $(100)(660) = 6.6\times10^4$ N, and the words "raises ... at constant speed" imply zero acceleration, so the lift-force is equal to the total weight. Thus $P = Fv = (6.6\times10^4)(150/60) = 1.65\times10^5$ W.

42. Using Eq. 7-32, we find

$$W = \int_{0.25}^{1.25} e^{-4x^2}\,dx = 0.21\text{ J}$$

where the result has been obtained numerically. Many modern calculators have that capability, as well as most math software packages that a great many students have access to.

43. (a) and (b) Hooke's law and the work done by a spring is discussed in the chapter. We apply Work-kinetic energy theorem, in the form of $\Delta K = W_a + W_s$, to the points in Figure 7-48 at $x = 1.0$ m and $x = 2.0$ m, respectively. The "applied" work W_a is that due to the constant force $\vec{P}$.

$$\begin{aligned} 4 &= P(1.0) - \frac{1}{2}k(1.0)^2 \\ 0 &= P(2.0) - \frac{1}{2}k(2.0)^2 \end{aligned}$$

Simultaneous solution leads to $P = 8.0$ N and $k = 8.0$ N/m.

44. Using Eq. 7-8, we find

$$W = \vec{F} \cdot \vec{d} = \left(F\cos\theta\,\hat{\imath} + F\sin\theta\,\hat{\jmath}\right) \cdot \left(x\hat{\imath} + y\hat{\jmath}\right) = Fx\cos\theta + Fy\sin\theta$$

where $x = 2.0$ m, $y = -4.0$ m, $F = 10$ N, and $\theta = 150°$. Thus, we obtain $W = -37$ J. Note that the given mass value (2.0 kg) is not used in the computation.

45. (a) Estimating the initial speed from the slope of the graph near the origin is somewhat difficult, and it may be simpler to determine it from the constant-acceleration equations from chapter 2: $v = v_0 + at$ and $x = v_0 + \frac{1}{2}at^2$, where $x_0 = 0$ has been used. Applying these to the last point on the graph (where the slope is apparently zero) or applying just the x equation to any two points on the graph, leads to a pair of simultaneous equations from which $a = -2$ m/s^2 and $v_0 = 10$ m/s can be found. Then,

$$K_0 = \frac{1}{2}mv_0^2 = 2.5 \times 10^3 \text{ J} = 2.5 \text{ kJ} .$$

(b) The speed at $t = 3.0$ s is obtained by

$$v = v_0 + at = 10 + (-2)(3) = 4 \text{ m/s}$$

or by estimating the slope from the graph (not recommended). Then the work-kinetic energy theorem yields

$$W = \Delta K = \frac{1}{2}(50 \text{ kg})(4 \text{ m/s})^2 - 2.5 \times 10^3 \text{ J} = -2.1 \text{ kJ} .$$

46. (a) Using Eq. 7-8 and SI units, we find

$$W = \vec{F} \cdot \vec{d} = \left(2\,\hat{\imath} - 4\,\hat{\jmath}\right) \cdot \left(8\hat{\imath} + c\hat{\jmath}\right) = 16 - 4c$$

which, if equal zero, implies $c = 16/4 = 4$ m.

(b) If $W > 0$ then $16 > 4c$, which implies $c < 4$ m.

(c) If $W < 0$ then $16 < 4c$, which implies $c > 4$ m.

47. With speed $v = 11200$ m/s, we find

$$K = \frac{1}{2}mv^2 = \frac{1}{2}\left(2.9 \times 10^5\right)(11200)^2 = 1.8 \times 10^{13} \text{ J} .$$

48. (a) Hooke's law and the work done by a spring is discussed in the chapter. Taking absolute values, and writing that law in terms of differences ΔF and Δx, we analyze the first two pictures as follows:

$$\begin{aligned} |\Delta F| &= k\,|\Delta x| \\ 240\,\text{N} - 110\,\text{N} &= k(60\,\text{mm} - 40\,\text{mm}) \end{aligned}$$

which yields $k = 6.5$ N/mm. Designating the relaxed position (as read by that scale) as x_o we look again at the first picture:

$$110\,\text{N} = k\,(40\,\text{mm} - x_o)$$

which (upon using the above result for k) yields $x_o = 23$ mm.

(b) Using the results from part (a) to analyze that last picture, we find

$$W = k\,(30\,\text{mm} - x_\text{o}\,) = 45\ \text{N}\ .$$

49. (a) To hold the crate at equilibrium in the final situation, $\vec{F}$ must have the same magnitude as the horizontal component of the rope's tension $T\sin\theta$, where θ is the angle between the rope (in the final position) and vertical:

$$\theta = \sin^{-1}\left(\frac{4.00}{12.0}\right) = 19.5^\circ\ .$$

But the vertical component of the tension supports against the weight: $T\cos\theta = mg$. Thus, the tension is $T = (230)(9.8)/\cos 19.5^\circ = 2391\,\text{N}$ and $F = (2391)\sin 19.5^\circ = 797\,\text{N}$. An alternative approach based on drawing a vector triangle (of forces) in the final situation provides a quick solution.

(b) Since there is no change in kinetic energy, the net work on it is zero.

(c) The work done by gravity is $W_g = \vec{F}_g \cdot \vec{d} = -mgh$, where $h = L(1-\cos\theta)$ is the vertical component of the displacement. With $L = 12.0$ m, we obtain $W_g = -1547$ J which should be rounded to three figures: -1.55 kJ.

(d) The tension vector is everywhere perpendicular to the direction of motion, so its work is zero (since $\cos 90^\circ = 0$).

(e) The implication of the previous three parts is that the work due to $\vec{F}$ is $-W_g$ (so the net work turns out to be zero). Thus, $W_F = -W_g = 1.55$ kJ.

(f) Since $\vec{F}$ does not have constant magnitude, we cannot expect Eq. 7-8 to apply.

50. (a) In the work-kinetic energy theorem, we include both the work due to an applied force W_a and work done by gravity W_g in order to find the latter quantity.

$$\Delta K = W_a + W_g \quad \Longrightarrow \quad 30 = (100)(1.8)\cos 180^\circ + W_g$$

leading to $W_g = 210$ J.

(b) The value of W_g obtained in part (a) still applies since the weight and the path of the child remain the same, so $\Delta K = W_g = 210$ J.

51. Using Eq. 7-7, we have $W = Fd\cos\phi = 1504$ J. Then, by the work-kinetic energy theorem, we find the kinetic energy $K_f = K_i + W = 0 + 1504$ J. The answer is therefore 1.5 kJ.

52. (a) Before the cord is cut, each spring (which might be described as being "in series" in this case) is stretched by the force $F = 100$ N. Thus, each spring is stretched by $x = 100/500 = 0.20\,\text{m}$ in the initial configuration. Since the relaxed length of each spring is 0.50 m, then the full length of each spring in the initial configuration is $0.20 + 0.50 = 0.70$ m. Therefore (including that 0.10 m length of string) the distance from the box to the ceiling is $2(0.70) + 0.10 = 1.50$ m, before the string is cut. In the moments after the short string is cut, there is some "transient motion" that is difficult to analyze, but after it has settled down again (in its new equilibrium position) the springs (which now might be described as being "in parallel") are sharing half the weight, so the force stretching each one is $F/2 = 50$ N. This means the elongation of each is $x/2 = 0.10$ m. The total distance (recalling that the longer cords are each of length 0.85 m) of the box to the ceiling is now $0.85 + 0.10 + 0.50 = 1.45$ m. Thus, the box is closer to the ceiling now than it was before. It has moved up.

(b) The distance moved up by the box is $d = 1.50 - 1.45 = 0.05$ m.

(c) To avoid worrying about friction-related (dissipative) processes which are involved in making the "transient motion" ultimately disappear, we consider that the person who cut the cord (and has predicted the new equilibrium position) very carefully and gradually moves it up to that new position, in which case the work being done on the system is due to the person. In this variation of the problem, it is easy to see that the work done by the person *against gravity* is $-W_g = mgd = 5.0$ J (though this is not the full work done by the person, since Eq. 7-25 hasn't been used). Returning to the problem in its original form, we can say that the work done on the block in raising it the distance d is 5.0 J, regardless of the agent doing the work (and in its original form, that agent is the pair of springs, and this represents part of the full work they do).

53. (a) We set up the ratio

$$\frac{50\,\text{km}}{1\,\text{km}} = \left(\frac{E}{1\,\text{megaton}}\right)^{1/3}$$

and find $E = 50^3 \approx 1 \times 10^5$ megatons of TNT.

(b) We note that 15 kilotons is equivalent to 0.015 megatons. Dividing the result from part (a) by 0.013 yields about ten million bombs.

54. (a) With SI units (and three significant figures) understood, the object's displacement is

$$\vec{d} = \vec{d_f} - \vec{d_i} = -8\,\hat{\text{i}} + 6\,\hat{\text{j}} + 2\,\hat{\text{k}}\ .$$

Thus, Eq. 7-8 gives

$$W = \vec{F} \cdot \vec{d} = (3)(-8) + (7)(6) + (7)(2) = 32.0 \text{ J}\ .$$

(b) The average power is given by Eq. 7-42:

$$P_{\text{avg}} = \frac{W}{t} = \frac{32}{4} = 8.00 \text{ W}\ .$$

(c) The distance from the coordinate origin to the initial position is $d_i = \sqrt{3^2 + (-2)^2 + 5^2} = 6.16$ m, and the magnitude of the distance from the coordinate origin to the final position is $d_f = \sqrt{(-5)^2 + 4^2 + 7^2} =$ 9.49 m. Their scalar (dot) product is

$$\vec{d_i} \cdot \vec{d_f} = (3)(-5) + (-2)(4) + (5)(7) = 12.0 \text{ m}^2\ .$$

Thus, the angle between the two vectors is

$$\phi = \cos^{-1}\left(\frac{\vec{d_i} \cdot \vec{d_f}}{d_i\, d_f}\right) = \cos^{-1}\left(\frac{12.0}{(6.16)(9.49)}\right)$$

which yields $\phi = 78°$.