# Chapter 26

1. The minimum charge measurable is

$$q_{\min} = CV_{\min} = (50\,\text{pF})(0.15\,\text{V}) = 7.5\ \text{pC}\ .$$

2. (a) The capacitance of the system is

$$C = \frac{q}{\Delta V} = \frac{70\,\text{pC}}{20\,\text{V}} = 3.5\ \text{pF}\ .$$

(b) The capacitance is independent of $q$; it is still 3.5 pF.

(c) The potential difference becomes

$$\Delta V = \frac{q}{C} = \frac{200\,\text{pC}}{3.5\,\text{pF}} = 57\ \text{V}\ .$$

3. Charge flows until the potential difference across the capacitor is the same as the potential difference across the battery. The charge on the capacitor is then $q = CV$, and this is the same as the total charge that has passed through the battery. Thus, $q = (25 \times 10^{-6}\,\text{F})(120\,\text{V}) = 3.0 \times 10^{-3}\,\text{C}$.

4. We verify the units relationship as follows:

$$[\varepsilon_0] = \frac{\text{F}}{\text{m}} = \frac{\text{C}}{\text{V}\cdot\text{m}} = \frac{\text{C}}{(\text{N}\cdot\text{m}/\text{C})\,\text{m}} = \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\ .$$

5. (a) The capacitance of a parallel-plate capacitor is given by $C = \varepsilon_0 A/d$, where $A$ is the area of each plate and $d$ is the plate separation. Since the plates are circular, the plate area is $A = \pi R^2$, where $R$ is the radius of a plate. Thus,

$$C = \frac{\varepsilon_0 \pi R^2}{d} = \frac{(8.85 \times 10^{-12}\,\text{F/m})\pi(8.2 \times 10^{-2}\,\text{m})^2}{1.3 \times 10^{-3}\,\text{m}} = 1.4 \times 10^{-10}\,\text{F} = 140\ \text{pF}\ .$$

(b) The charge on the positive plate is given by $q = CV$, where $V$ is the potential difference across the plates. Thus, $q = (1.4 \times 10^{-10}\,\text{F})(120\,\text{V}) = 1.7 \times 10^{-8}\,\text{C} = 17\,\text{nC}$.

6. We use $C = A\varepsilon_0/d$. Thus

$$d = \frac{A\varepsilon_0}{C} = \frac{(1.00\,\text{m}^2)\left(8.85 \times 10^{-12}\,\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)}{1.00\,\text{F}} = 8.85 \times 10^{-12}\ \text{m}\ .$$

Since $d$ is much less than the size of an atom ($\sim 10^{-10}\,\text{m}$), this capacitor cannot be constructed.

7. Assuming conservation of volume, we find the radius of the combined spheres, then use $C = 4\pi\varepsilon_0 R$ to find the capacitance. When the drops combine, the volume is doubled. It is then $V = 2(4\pi/3)R^3$. The new radius $R'$ is given by

$$\frac{4\pi}{3}(R')^3 = 2\frac{4\pi}{3}R^3 ,$$

so

$$R' = 2^{1/3}R .$$

The new capacitance is

$$C' = 4\pi\varepsilon_0 R' = 4\pi\varepsilon_0 2^{1/3} R = 5.04\pi\varepsilon_0 .$$

8. (a) We use Eq. 26-17:

$$C = 4\pi\varepsilon_0 \frac{ab}{b-a} = \frac{(40.0\,\text{mm})(38.0\,\text{mm})}{\left(8.99\times 10^9\,\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(40.0\,\text{mm} - 38.0\,\text{mm})} = 84.5\ \text{pF} .$$

(b) Let the area required be $A$. Then $C = \varepsilon_0 A/(b-a)$, or

$$A = \frac{C(b-a)}{\varepsilon_0} = \frac{(84.5\,\text{pF})(40.0\,\text{mm} - 38.0\,\text{mm})}{\left(8.85\times 10^{-12}\,\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)} = 191\ \text{cm}^2 .$$

9. According to Eq. 26-17 the capacitance of a spherical capacitor is given by

$$C = 4\pi\varepsilon_0 \frac{ab}{b-a} ,$$

where $a$ and $b$ are the radii of the spheres. If $a$ and $b$ are nearly the same then $4\pi ab$ is nearly the surface area of either sphere. Replace $4\pi ab$ with $A$ and $b - a$ with $d$ to obtain

$$C \approx \frac{\varepsilon_0 A}{d} .$$

10. The equivalent capacitance is

$$C_{\text{eq}} = C_3 + \frac{C_1 C_2}{C_1 + C_2} = 4.00\,\mu\text{F} + \frac{(10.0\,\mu\text{F})(5.00\,\mu\text{F})}{10.0\,\mu\text{F} + 5.00\,\mu\text{F}} = 7.33\,\mu\text{F} .$$

11. The equivalent capacitance is given by $C_{\text{eq}} = q/V$, where $q$ is the total charge on all the capacitors and $V$ is the potential difference across any one of them. For $N$ identical capacitors in parallel, $C_{\text{eq}} = NC$, where $C$ is the capacitance of one of them. Thus, $NC = q/V$ and

$$N = \frac{q}{VC} = \frac{1.00\,\text{C}}{(110\,\text{V})(1.00\times 10^{-6}\,\text{F})} = 9090 .$$

12. The charge that passes through meter $A$ is

$$q = C_{\text{eq}} V = 3CV = 3(25.0\,\mu\text{F})(4200\,\text{V}) = 0.315\ \text{C} .$$

13. The equivalent capacitance is

$$C_{\text{eq}} = \frac{(C_1 + C_2)C_3}{C_1 + C_2 + C_3} = \frac{(10.0\,\mu\text{F} + 5.00\,\mu\text{F})(4.00\,\mu\text{F})}{10.0\,\mu\text{F} + 5.00\,\mu\text{F} + 4.00\,\mu\text{F}} = 3.16\,\mu\text{F} .$$

14. (a) and (b) The original potential difference $V_1$ across $C_1$ is

$$V_1 = \frac{C_{\text{eq}} V}{C_1 + C_2} = \frac{(3.16\,\mu\text{F})(100\,\text{V})}{10.0\,\mu\text{F} + 5.00\,\mu\text{F}} = 21.1\ \text{V} .$$

Thus $\Delta V_1 = 100\,\text{V} - 21.1\,\text{V} = 79\,\text{V}$ and $\Delta q_1 = C_1 \Delta V_1 = (10.0\,\mu\text{F})(79\,\text{V}) = 7.9\times 10^{-4}\,\text{C}$.

15. Let $x$ be the separation of the plates in the lower capacitor. Then the plate separation in the upper capacitor is $a-b-x$. The capacitance of the lower capacitor is $C_\ell = \varepsilon_0 A/x$ and the capacitance of the upper capacitor is $C_u = \varepsilon_0 A/(a-b-x)$, where $A$ is the plate area. Since the two capacitors are in series, the equivalent capacitance is determined from

$$\frac{1}{C_{\rm eq}} = \frac{1}{C_\ell} + \frac{1}{C_u} = \frac{x}{\varepsilon_0 A} + \frac{a-b-x}{\varepsilon_0 A} = \frac{a-b}{\varepsilon_0 A} \ .$$

Thus, the equivalent capacitance is given by $C_{\rm eq} = \varepsilon_0 A/(a-b)$ and is independent of $x$.

16. (a) The potential difference across $C_1$ is $V_1 = 10\,{\rm V}$. Thus, $q_1 = C_1V_1 = (10\,\mu{\rm F})(10\,{\rm V}) = 1.0\times10^{-4}\,{\rm C}$.

(b) Let $C = 10\,\mu{\rm F}$. We first consider the three-capacitor combination consisting of $C_2$ and its two closest neighbors, each of capacitance $C$. The equivalent capacitance of this combination is

$$C_{\rm eq} = C + \frac{C_2C}{C+C_2} = 1.5C \ .$$

Also, the voltage drop across this combination is

$$V = \frac{CV_1}{C+C_{eq}} = \frac{CV_1}{C+1.5C} = \frac{2}{5}V_1 \ .$$

Since this voltage difference is divided equally between $C_2$ and the one connected in series with it, the voltage difference across $C_2$ satisfies $V_2 = V/2 = V_1/5$. Thus

$$q_2 = C_2V_2 = (10\,\mu{\rm F})\left(\frac{10\,{\rm V}}{5}\right) = 2.0\times10^{-5}\ {\rm V} \ .$$

17. The charge initially on the charged capacitor is given by $q = C_1V_0$, where $C_1 = 100\,{\rm pF}$ is the capacitance and $V_0 = 50\,{\rm V}$ is the initial potential difference. After the battery is disconnected and the second capacitor wired in parallel to the first, the charge on the first capacitor is $q_1 = C_1V$, where $v = 35\,{\rm V}$ is the new potential difference. Since charge is conserved in the process, the charge on the second capacitor is $q_2 = q - q_1$, where $C_2$ is the capacitance of the second capacitor. Substituting $C_1V_0$ for $q$ and $C_1V$ for $q_1$, we obtain $q_2 = C_1(V_0 - V)$. The potential difference across the second capacitor is also $V$, so the capacitance is

$$C_2 = \frac{q_2}{V} = \frac{V_0 - V}{V}C_1 = \frac{50\,{\rm V} - 35\,{\rm V}}{35\,{\rm V}}(100\,{\rm pF}) = 3\ {\rm pF} \ .$$

18. (a) First, the equivalent capacitance of the two $4.0\,\mu{\rm F}$ capacitors connected in series is given by $4.0\,\mu{\rm F}/2 = 2.0\,\mu{\rm F}$. This combination is then connected in parallel with two other 2.0-$\mu$F capacitors (one on each side), resulting in an equivalent capacitance $C = 3(2.0\,\mu{\rm F}) = 6.0\,\mu{\rm F}$. This is now seen to be in series with another combination, which consists of the two 3.0-$\mu$F capacitors connected in parallel (which are themselves equivalent to $C' = 2(3.0\,\mu{\rm F}) = 6.0\,\mu{\rm F}$). Thus, the equivalent capacitance of the circuit is

$$C_{\rm eq} = \frac{CC'}{C+C'} = \frac{(6.0\,\mu{\rm F})(6.0\,\mu{\rm F})}{6.0\,\mu{\rm F}+6.0\,\mu{\rm F}} = 3.0\,\mu{\rm F} \ .$$

(b) Let $V = 20\,{\rm V}$ be the potential difference supplied by the battery. Then $q = C_{\rm eq}V = (3.0\,\mu{\rm F})(20\,{\rm V}) = 6.0\times10^{-5}\,{\rm C}$.

(c) The potential difference across $C_1$ is given by

$$V_1 = \frac{CV}{C+C'} = \frac{(6.0\,\mu{\rm F})(20\,{\rm V})}{6.0\,\mu{\rm F}+6.0\,\mu{\rm F}} = 10\,{\rm V} \ ,$$

and the charge carried by $C_1$ is $q_1 = C_1V_1 = (3.0\,\mu{\rm F})(10\,{\rm V}) = 3.0\times10^{-5}\ {\rm C}$.

(d) The potential difference across $C_2$ is given by $V_2 = V - V_1 = 20\,\text{V} - 10\,\text{V} = 10\,\text{V}$. Consequently, the charge carried by $C_2$ is $q_2 = C_2V_2 = (2.0\,\mu\text{F})(10\,\text{V}) = 2.0 \times 10^{-5}$ C.

(e) Since this voltage difference $V_2$ is divided equally between $C_3$ and the other 4.0-$\mu$F capacitors connected in series with it, the voltage difference across $C_3$ is given by $V_3 = V_2/2 = 10\,\text{V}/2 = 5.0\,\text{V}$. Thus, $q_3 = C_3V_3 = (4.0\,\mu\text{F})(5.0\,\text{V}) = 2.0 \times 10^{-5}$ C.

19. (a) After the switches are closed, the potential differences across the capacitors are the same and the two capacitors are in parallel. The potential difference from $a$ to $b$ is given by $V_{ab} = Q/C_{\text{eq}}$, where $Q$ is the net charge on the combination and $C_{\text{eq}}$ is the equivalent capacitance. The equivalent capacitance is $C_{\text{eq}} = C_1 + C_2 = 4.0 \times 10^{-6}\,\text{F}$. The total charge on the combination is the net charge on either pair of connected plates. The charge on capacitor 1 is

$$q_1 = C_1V = (1.0 \times 10^{-6}\,\text{F})(100\,\text{V}) = 1.0 \times 10^{-4}\,\text{C}$$

and the charge on capacitor 2 is

$$q_2 = C_2V = (3.0 \times 10^{-6}\,\text{F})(100\,\text{V}) = 3.0 \times 10^{-4}\,\text{C}\ ,$$

so the net charge on the combination is $3.0 \times 10^{-4}\,\text{C} - 1.0 \times 10^{-4}\,\text{C} = 2.0 \times 10^{-4}\,\text{C}$. The potential difference is

$$V_{ab} = \frac{2.0 \times 10^{-4}\,\text{C}}{4.0 \times 10^{-6}\,\text{F}} = 50\,\text{V}\ .$$

(b) The charge on capacitor 1 is now $q_1 = C_1V_{ab} = (1.0 \times 10^{-6}\,\text{F})(50\,\text{V}) = 5.0 \times 10^{-5}\,\text{C}$.

(c) The charge on capacitor 2 is now $q_2 = C_2V_{ab} = (3.0 \times 10^{-6}\,\text{F})(50\,\text{V}) = 1.5 \times 10^{-4}\,\text{C}$.

20. (a) In this situation, capacitors 1 and 3 are in series, which means their charges are necessarily the same:

$$q_1 = q_3 = \frac{C_1C_3V}{C_1 + C_3} = \frac{(1.0\,\mu\text{F})(3.0\,\mu\text{F})(12\,\text{V})}{1.0\,\mu\text{F} + 3.0\,\mu\text{F}} = 9.0\,\mu\text{C}\ .$$

Also, capacitors 2 and 4 are in series:

$$q_2 = q_4 = \frac{C_2C_4V}{C_2 + C_4} = \frac{(2.0\,\mu\text{F})(4.0\,\mu\text{F})(12\,\text{V})}{2.0\,\mu\text{F} + 4.0\,\mu\text{F}} = 16\,\mu\text{C}\ .$$

(b) With switch 2 also closed, the potential difference $V_1$ across $C_1$ must equal the potential difference across $C_2$ and is

$$V_1 = \frac{C_3 + C_4}{C_1 + C_2 + C_3 + C_4}V = \frac{(3.0\,\mu\text{F} + 4.0\,\mu\text{F})(12\,\text{V})}{1.0\,\mu\text{F} + 2.0\,\mu\text{F} + 3.0\,\mu\text{F} + 4.0\,\mu\text{F}} = 8.4\ \text{V}\ .$$

Thus, $q_1 = C_1V_1 = (1.0\,\mu\text{F})(8.4\,\text{V}) = 8.4\,\mu\text{C}$, $q_2 = C_2V_1 = (2.0\,\mu\text{F})(8.4\,\text{V}) = 17\,\mu\text{C}$, $q_3 = C_3(V - V_1) = (3.0\,\mu\text{F})(12\,\text{V} - 8.4\,\text{V}) = 11\,\mu\text{C}$, and $q_4 = C_4(V - V_1) = (4.0\,\mu\text{F})(12\,\text{V} - 8.4\,\text{V}) = 14\,\mu\text{C}$.

21. The charges on capacitors 2 and 3 are the same, so these capacitors may be replaced by an equivalent capacitance determined from

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_2} + \frac{1}{C_3} = \frac{C_2 + C_3}{C_2C_3}\ .$$

Thus, $C_{\text{eq}} = C_2C_3/(C_2 + C_3)$. The charge on the equivalent capacitor is the same as the charge on either of the two capacitors in the combination and the potential difference across the equivalent capacitor is given by $q_2/C_{\text{eq}}$. The potential difference across capacitor 1 is $q_1/C_1$, where $q_1$ is the charge on this capacitor. The potential difference across the combination of capacitors 2 and 3 must be the same as the potential difference across capacitor 1, so $q_1/C_1 = q_2/C_{\text{eq}}$. Now some of the charge originally on capacitor 1 flows to the combination of 2 and 3. If $q_0$ is the original charge, conservation of charge yields

$q_1 + q_2 = q_0 = C_1 V_0$, where $V_0$ is the original potential difference across capacitor 1. Solving the two equations

$$\frac{q_1}{C_1} = \frac{q_2}{C_{\text{eq}}} \quad \text{and} \quad q_1 + q_2 = C_1 V_0$$

for $q_1$ and $q_2$, we find

$$q_2 = C_1 V_0 - q_1 \qquad \text{and} \quad q_1 = \frac{C_1^2 V_0}{C_{\text{eq}} + C_1} = \frac{C_1^2 V_0}{\dfrac{C_2 C_3}{C_2 + C_3} + C_1} = \frac{C_1^2 (C_2 + C_3) V_0}{C_1 C_2 + C_1 C_3 + C_2 C_3} \ .$$

The charges on capacitors 2 and 3 are

$$q_2 = q_3 = C_1 V_0 - q_1 = C_1 V_0 - \frac{C_1^2 (C_2 + C_3) V_0}{C_1 C_2 + C_1 C_3 + C_2 C_3} = \frac{C_1 C_2 C_3 V_0}{C_1 C_2 + C_1 C_3 + C_2 C_3} \ .$$

22. Let $\mathcal{V} = 1.00\,\text{m}^3$. Using Eq. 26-23, the energy stored is

$$\begin{aligned} U &= u\mathcal{V} = \frac{1}{2}\varepsilon_0 E^2 \mathcal{V} \\ &= \frac{1}{2}\left(8.85 \times 10^{-12}\,\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)(150\,\text{V/m})^2 (1.00\,\text{m}^3) \\ &= 9.96 \times 10^{-8}\ \text{J} \ . \end{aligned}$$

23. The energy stored by a capacitor is given by $U = \frac{1}{2}CV^2$, where $V$ is the potential difference across its plates. We convert the given value of the energy to Joules. Since a Joule is a watt·second, we multiply by $(10^3\,\text{W/kW})(3600\,\text{s/h})$ to obtain $10\,\text{kW}\cdot\text{h} = 3.6 \times 10^7\,\text{J}$. Thus,

$$C = \frac{2U}{V^2} = \frac{2(3.6 \times 10^7\,\text{J})}{(1000\,\text{V})^2} = 72\ \text{F} \ .$$

24. (a) The capacitance is

$$C = \frac{\varepsilon_0 A}{d} = \frac{\left(8.85 \times 10^{-12}\,\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)(40 \times 10^{-4}\,\text{m}^2)}{1.0 \times 10^{-3}\,\text{m}} = 3.5 \times 10^{-11}\,\text{F} = 35\ \text{pF} \ .$$

(b) $q = CV = (35\,\text{pF})(600\,\text{V}) = 2.1 \times 10^{-8}\,\text{C} = 21\,\text{nC}$.

(c) $U = \frac{1}{2}CV^2 = \frac{1}{2}(35\,\text{pF})(21\,\text{nC})^2 = 6.3 \times 10^{-6}\,\text{J} = 6.3\,\mu\text{J}$.

(d) $E = V/d = 600\,\text{V}/1.0 \times 10^{-3}\,\text{m} = 6.0 \times 10^5\,\text{V/m}$.

(e) The energy density (energy per unit volume) is

$$u = \frac{U}{Ad} = \frac{6.3 \times 10^{-6}\,\text{J}}{(40 \times 10^{-4}\,\text{m}^2)(1.0 \times 10^{-3}\,\text{m})} = 1.6\ \text{J/m}^3 \ .$$

25. The total energy is the sum of the energies stored in the individual capacitors. Since they are connected in parallel, the potential difference $V$ across the capacitors is the same and the total energy is

$$U = \frac{1}{2}\left(C_1 + C_2\right)V^2 = \frac{1}{2}\left(2.0 \times 10^{-6}\,\text{F} + 4.0 \times 10^{-6}\,\text{F}\right)(300\,\text{V})^2 = 0.27\ \text{J} \ .$$

26. The total energy stored in the capacitor bank is

$$U = \frac{1}{2}C_{\text{total}}V^2 = \frac{1}{2}(2000)(5.00 \times 10^{-6}\,\text{F})(50000\,\text{V})^2 = 1.3 \times 10^7\ \text{J} \ .$$

Thus, the cost is

$$\frac{(1.3 \times 10^7\,\text{J})(3.0\,\text{cent}/\,\text{kW}\cdot\text{h})}{3.6 \times 10^6\,\text{J}/\,\text{kW}\cdot\text{h}} = 10\,\text{cents} \ .$$

27. (a) In the first case $U = q^2/2C$, and in the second case $U = 2(q/2)^2/2C = q^2/4C$. So the energy is now $4.0\,\text{J}/2 = 2.0\,\text{J}$.

    (b) It becomes the thermal energy generated in the wire connecting the capacitors during the discharging process (although a small fraction of it is probably radiated away in the form of radio waves).

28. (a) The potential difference across $C_1$ (the same as across $C_2$) is given by

$$V_1 = V_2 = \frac{C_3 V}{C_1 + C_2 + C_3} = \frac{(4.00\,\mu\text{F})(100\,\text{V})}{10.0\,\mu\text{F} + 5.00\,\mu\text{F} + 4.00\,\mu\text{F}} = 21.1\ \text{V} .$$

Also, $V_3 = V - V_1 = V - V_2 = 100\,\text{V} - 21.1\,\text{V} = 78.9\,\text{V}$. Thus,

$$\begin{aligned} q_1 &= C_1 V_1 = (10.0\,\mu\text{F})(21.1\,\text{V}) = 2.11 \times 10^{-4}\ \text{C} \\ q_2 &= C_2 V_2 = (5.00\,\mu\text{F})(21.1\,\text{V}) \ = 1.05 \times 10^{-4}\ \text{C} \\ q_3 &= q_1 + q_2 = 2.11 \times 10^{-4}\,\text{C} + 1.05 \times 10^{-4}\,\text{C} = 3.16 \times 10^{-4}\ \text{C} . \end{aligned}$$

    (b) The potential differences were found in the course of solving for the charges in part (a).

    (c) The stored energies are as follows:

$$\begin{aligned} U_1 &= \frac{1}{2} C_1 V_1^2 = \frac{1}{2}(10.0\,\mu\text{F})(21.1\,\text{V})^2 = 2.22 \times 10^{-3}\ \text{J} , \\ U_2 &= \frac{1}{2} C_2 V_2^2 = \frac{1}{2}(5.00\,\mu\text{F})(21.1\,\text{V})^2 = 1.11 \times 10^{-3}\ \text{J} , \\ U_3 &= \frac{1}{2} C_3 V_3^2 = \frac{1}{2}(4.00\,\mu\text{F})(78.9\,\text{V})^2 = 1.25 \times 10^{-2}\ \text{J} . \end{aligned}$$

29. (a) Let $q$ be the charge on the positive plate. Since the capacitance of a parallel-plate capacitor is given by $\varepsilon_0 A/d$, the charge is $q = CV = \varepsilon_0 AV/d$. After the plates are pulled apart, their separation is $2d$ and the potential difference is $V'$. Then $q = \varepsilon_0 AV'/2d$ and

$$V' = \frac{2d}{\varepsilon_0 A}\, q = \frac{2d}{\varepsilon_0 A}\, \frac{\varepsilon_0 A}{d}\, V = 2V .$$

    (b) The initial energy stored in the capacitor is

$$U_i = \frac{1}{2} CV^2 = \frac{\varepsilon_0 A V^2}{2d}$$

and the final energy stored is

$$U_f = \frac{1}{2}\,\frac{\varepsilon_0 A}{2d}\,(V')^2 = \frac{1}{2}\,\frac{\varepsilon_0 A}{2d}\, 4V^2 = \frac{\varepsilon_0 A V^2}{d} .$$

This is twice the initial energy.

    (c) The work done to pull the plates apart is the difference in the energy: $W = U_f - U_i = \varepsilon_0 AV^2/2d$.

30. (a) The charge in the Figure is

$$q_3 = C_3 V = (4.00\,\mu\text{F})(100\,\text{V}) = 4.00 \times 10^{-4}\,\text{mC} ,$$

$$q_1 = q_2 = \frac{C_1 C_2 V}{C_1 + C_2} = \frac{(10.0\,\mu\text{F})(5.00\,\mu\text{F})(100\,\text{V})}{10.0\,\mu\text{F} + 5.00\,\mu\text{F}} = 3.33 \times 10^{-4}\,\text{C} .$$

    (b) $V_1 = q_1/C_1 = 3.33 \times 10^{-4}\,\text{C}/10.0\,\mu\text{F} = 33.3\,\text{V}$, $V_2 = V - V_1 = 100\,\text{V} - 33.3\,\text{V} = 66.7\,\text{V}$, and $V_3 = V = 100\,\text{V}$.

(c) We use $U_i = \frac{1}{2}C_iV_i^2$, where $i = 1$, 2, 3. The answers are $U_1 = 5.6\,\mathrm{mJ}$, $U_1 = 11\,\mathrm{mJ}$, and $U_1 = 20\,\mathrm{mJ}$.

31. We first need to find an expression for the energy stored in a cylinder of radius $R$ and length $L$, whose surface lies between the inner and outer cylinders of the capacitor ($a < R < b$). The energy density at any point is given by $u = \frac{1}{2}\varepsilon_0 E^2$, where $E$ is the magnitude of the electric field at that point. If $q$ is the charge on the surface of the inner cylinder, then the magnitude of the electric field at a point a distance $r$ from the cylinder axis is given by

$$E = \frac{q}{2\pi\varepsilon_0 Lr}$$

(see Eq. 26-12), and the energy density at that point is given by

$$u = \frac{1}{2}\varepsilon_0 E^2 = \frac{q^2}{8\pi^2\varepsilon_0 L^2 r^2} \ .$$

The energy in the cylinder is the volume integral

$$U_R = \int u\, d\mathcal{V} \ .$$

Now, $d\mathcal{V} = 2\pi r L\, dr$, so

$$U_R = \int_a^R \frac{q^2}{8\pi^2\varepsilon_0 L^2 r^2}\, 2\pi r L\, dr = \frac{q^2}{4\pi\varepsilon_0 L}\int_a^R \frac{dr}{r} = \frac{q^2}{4\pi\varepsilon_0 L}\ln\frac{R}{a} \ .$$

To find an expression for the total energy stored in the capacitor, we replace $R$ with $b$:

$$U_b = \frac{q^2}{4\pi\varepsilon_0 L}\ln\frac{b}{a} \ .$$

We want the ratio $U_R/U_b$ to be 1/2, so

$$\ln\frac{R}{a} = \frac{1}{2}\ln\frac{b}{a}$$

or, since $\frac{1}{2}\ln(b/a) = \ln(\sqrt{b/a})$, $\ln(R/a) = \ln(\sqrt{b/a})$. This means $R/a = \sqrt{b/a}$ or $R = \sqrt{ab}$.

32. We use $E = q/4\pi\varepsilon_0 R^2 = V/R$. Thus

$$u = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0\left(\frac{V}{R}\right)^2 = \frac{1}{2}\left(8.85\times10^{-12}\,\frac{\mathrm{C}^2}{\mathrm{N}\cdot\mathrm{m}^2}\right)\left(\frac{8000\,\mathrm{V}}{0.050\,\mathrm{m}}\right)^2 = 0.11\ \mathrm{J/m^3}.$$

33. The charge is held constant while the plates are being separated, so we write the expression for the stored energy as $U = q^2/2C$, where $q$ is the charge and $C$ is the capacitance. The capacitance of a parallel-plate capacitor is given by $C = \varepsilon_0 A/x$, where $A$ is the plate area and $x$ is the plate separation, so

$$U = \frac{q^2 x}{2\varepsilon_0 A} \ .$$

If the plate separation increases by $dx$, the energy increases by $dU = (q^2/2\varepsilon_0 A)\,dx$. Suppose the agent pulling the plate apart exerts force $F$. Then the agent does work $F\,dx$ and if the plates begin and end at rest, this must equal the increase in stored energy. Thus,

$$F\,dx = \left(\frac{q^2}{2\varepsilon_0 A}\right)dx$$

and

$$F = \frac{q^2}{2\varepsilon_0 A} \ .$$

The net force on a plate is zero, so this must also be the magnitude of the force one plate exerts on the other. The force can also be computed as the product of the charge $q$ on one plate and the electric field $E_1$ due to the charge on the other plate. Recall that the field produced by a uniform plane surface of charge is $E_1 = q/2\varepsilon_0 A$. Thus, $F = q^2/2\varepsilon_0 A$.

34. If the original capacitance is given by $C = \varepsilon_0 A/d$, then the new capacitance is $C' = \varepsilon_0 \kappa A/2d$. Thus $C'/C = \kappa/2$ or $\kappa = 2C'/C = 2(2.6\,\text{pF}/1.3\,\text{pF}) = 4.0$.

35. The capacitance with the dielectric in place is given by $C = \kappa C_0$, where $C_0$ is the capacitance before the dielectric is inserted. The energy stored is given by $U = \frac{1}{2}CV^2 = \frac{1}{2}\kappa C_0 V^2$, so

$$\kappa = \frac{2U}{C_0 V^2} = \frac{2(7.4 \times 10^{-6}\,\text{J})}{(7.4 \times 10^{-12}\,\text{F})(652\,\text{V})^2} = 4.7\ .$$

According to Table 26-1, you should use Pyrex.

36. (a) We use $C = \varepsilon_0 A/d$ to solve for $d$:

$$d = \frac{\varepsilon_0 A}{C} = \frac{\left(8.85 \times 10^{-12}\,\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)(0.35\,\text{m}^2)}{50 \times 10^{-12}\,\text{F}} = 6.2 \times 10^{-2}\,\text{m}\ .$$

(b) We use $C \propto \kappa$. The new capacitance is $C' = C(\kappa/\kappa_{\text{air}}) = (50\,\text{pf})(5.6/1.0) = 280\,\text{pF}$.

37. The capacitance of a cylindrical capacitor is given by

$$C = \kappa C_0 = \frac{2\pi\kappa\varepsilon_0 L}{\ln(b/a)}\ ,$$

where $C_0$ is the capacitance without the dielectric, $\kappa$ is the dielectric constant, $L$ is the length, $a$ is the inner radius, and $b$ is the outer radius. The capacitance per unit length of the cable is

$$\frac{C}{L} = \frac{2\pi\kappa\varepsilon_0}{\ln(b/a)} = \frac{2\pi(2.6)(8.85 \times 10^{-12}\,\text{F/m})}{\ln\left[(0.60\,\text{mm})/(0.10\,\text{mm})\right]} = 8.1 \times 10^{-11}\,\text{F/m} = 81\ \text{pF/m}\ .$$

38. (a) We use Eq. 26-14:

$$C = 2\pi\varepsilon_0\kappa\frac{L}{\ln(b/a)} = \frac{(4.7)(0.15\,\text{m})}{2\left(8.99 \times 10^9\,\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\ln(3.8\,\text{cm}/3.6\,\text{cm})} = 0.73\ \text{nF}\ .$$

(b) The breakdown potential is $(14\,\text{kV/mm})(3.8\,\text{cm} - 3.6\,\text{cm}) = 28\,\text{kV}$.

39. The capacitance is given by $C = \kappa C_0 = \kappa\varepsilon_0 A/d$, where $C_0$ is the capacitance without the dielectric, $\kappa$ is the dielectric constant, $A$ is the plate area, and $d$ is the plate separation. The electric field between the plates is given by $E = V/d$, where $V$ is the potential difference between the plates. Thus, $d = V/E$ and $C = \kappa\varepsilon_0 AE/V$. Thus,

$$A = \frac{CV}{\kappa\varepsilon_0 E}\ .$$

For the area to be a minimum, the electric field must be the greatest it can be without breakdown occurring. That is,

$$A = \frac{(7.0 \times 10^{-8}\,\text{F})(4.0 \times 10^3\,\text{V})}{2.8(8.85 \times 10^{-12}\,\text{F/m})(18 \times 10^6\,\text{V/m})} = 0.63\ \text{m}^2\ .$$

40. The capacitor can be viewed as two capacitors $C_1$ and $C_2$ in parallel, each with surface area $A/2$ and plate separation $d$, filled with dielectric materials with dielectric constants $\kappa_1$ and $\kappa_2$, respectively. Thus

$$C = C_1 + C_2 = \frac{\varepsilon_0(A/2)\kappa_1}{d} + \frac{\varepsilon_0(A/2)\kappa_2}{d} = \frac{\varepsilon_0 A}{d}\left(\frac{\kappa_1 + \kappa_2}{2}\right)\ .$$

41. We assume there is charge $q$ on one plate and charge $-q$ on the other. The electric field in the lower half of the region between the plates is

$$E_1 = \frac{q}{\kappa_1 \varepsilon_0 A} ,$$

where $A$ is the plate area. The electric field in the upper half is

$$E_2 = \frac{q}{\kappa_2 \varepsilon_0 A} .$$

Let $d/2$ be the thickness of each dielectric. Since the field is uniform in each region, the potential difference between the plates is

$$V = \frac{E_1 d}{2} + \frac{E_2 d}{2} = \frac{qd}{2\varepsilon_0 A}\left[\frac{1}{\kappa_1} + \frac{1}{\kappa_2}\right] = \frac{qd}{2\varepsilon_0 A}\,\frac{\kappa_1+\kappa_2}{\kappa_1\kappa_2} ,$$

so

$$C = \frac{q}{V} = \frac{2\varepsilon_0 A}{d}\,\frac{\kappa_1\kappa_2}{\kappa_1+\kappa_2} .$$

This expression is exactly the same as the that for $C_{\text{eq}}$ of two capacitors in series, one with dielectric constant $\kappa_1$ and the other with dielectric constant $\kappa_2$. Each has plate area $A$ and plate separation $d/2$. Also we note that if $\kappa_1 = \kappa_2$, the expression reduces to $C = \kappa_1\varepsilon_0 A/d$, the correct result for a parallel-plate capacitor with plate area $A$, plate separation $d$, and dielectric constant $\kappa_1$.

42. Let $C_1 = \varepsilon_0(A/2)\kappa_1/2d = \varepsilon_0 A\kappa_1/4d$, $C_2 = \varepsilon_0(A/2)\kappa_2/d = \varepsilon_0 A\kappa_2/2d$, and $C_3 = \varepsilon_0 A\kappa_3/2d$. Note that $C_2$ and $C_3$ are effectively connected in series, while $C_1$ is effectively connected in parallel with the $C_2$-$C_3$ combination. Thus,

$$\begin{aligned} C &= C_1 + \frac{C_2 C_3}{C_2 + C_3} = \frac{\varepsilon_0 A\kappa_1}{4d} + \frac{(\varepsilon_0 A/d)(\kappa_2/2)(\kappa_3/2)}{\kappa_2/2+\kappa_3/2} \\ &= \frac{\varepsilon_0 A}{4d}\left(\kappa_1 + \frac{2\kappa_2\kappa_3}{\kappa_2+\kappa_3}\right) . \end{aligned}$$

43. (a) The electric field in the region between the plates is given by $E = V/d$, where $V$ is the potential difference between the plates and $d$ is the plate separation. The capacitance is given by $C = \kappa\varepsilon_0 A/d$, where $A$ is the plate area and $\kappa$ is the dielectric constant, so $d = \kappa\varepsilon_0 A/C$ and

$$E = \frac{VC}{\kappa\varepsilon_0 A} = \frac{(50\,\text{V})(100\times10^{-12}\,\text{F})}{5.4(8.85\times10^{-12}\,\text{F/m})(100\times10^{-4}\,\text{m}^2)} = 1.0\times10^4\,\text{V/m} .$$

(b) The free charge on the plates is $q_f = CV = (100\times10^{-12}\,\text{F})(50\,\text{V}) = 5.0\times10^{-9}\,\text{C}$.

(c) The electric field is produced by both the free and induced charge. Since the field of a large uniform layer of charge is $q/2\varepsilon_0 A$, the field between the plates is

$$E = \frac{q_f}{2\varepsilon_0 A} + \frac{q_f}{2\varepsilon_0 A} - \frac{q_i}{2\varepsilon_0 A} - \frac{q_i}{2\varepsilon_0 A} ,$$

where the first term is due to the positive free charge on one plate, the second is due to the negative free charge on the other plate, the third is due to the positive induced charge on one dielectric surface, and the fourth is due to the negative induced charge on the other dielectric surface. Note that the field due to the induced charge is opposite the field due to the free charge, so they tend to cancel. The induced charge is therefore

$$\begin{aligned} q_i &= q_f - \varepsilon_0 AE \\ &= 5.0\times10^{-9}\,\text{C} - (8.85\times10^{-12}\,\text{F/m})(100\times10^{-4}\,\text{m}^2)(1.0\times10^4\,\text{V/m}) \\ &= 4.1\times10^{-9}\,\text{C} = 4.1\,\text{nC} . \end{aligned}$$

44. (a) The electric field $E_1$ in the free space between the two plates is $E_1 = q/\varepsilon_0 A$ while that inside the slab is $E_2 = E_1/\kappa = q/\kappa\varepsilon_0 A$. Thus,

$$V_0 = E_1(d-b) + E_2 b = \left(\frac{q}{\varepsilon_0 A}\right)\left(d - b + \frac{b}{\kappa}\right) ,$$

and the capacitance is

$$\begin{aligned} C &= \frac{q}{V_0} = \frac{\varepsilon_0 A\kappa}{\kappa(d-b)+b} \\ &= \frac{\left(8.85\times10^{-12}\,\frac{\mathrm{C}^2}{\mathrm{N\cdot m^2}}\right)(115\times10^{-4}\,\mathrm{m}^2)(2.61)}{(2.61)(0.0124\,\mathrm{m} - 0.00780\,\mathrm{m}) + (0.00780\,\mathrm{m})} \\ &= 13.4\ \mathrm{pF}\ . \end{aligned}$$

(b) $q = CV = (13.4\times10^{-12}\,\mathrm{F})(85.5\,\mathrm{V}) = 1.15\,\mathrm{nC}$.

(c) The magnitude of the electric field in the gap is

$$E_1 = \frac{q}{\varepsilon_0 A} = \frac{1.15\times10^{-9}\,\mathrm{C}}{\left(8.85\times10^{-12}\,\frac{\mathrm{C}^2}{\mathrm{N\cdot m^2}}\right)(115\times10^{-4}\,\mathrm{m}^2)} = 1.13\times10^4\ \mathrm{N/C}\ .$$

(d) Using Eq. 26-32, we obtain

$$E_2 = \frac{E_1}{\kappa} = \frac{1.13\times10^4\,\mathrm{N/C}}{2.61} = 4.33\times10^3\ \mathrm{N/C}\ .$$

45. (a) According to Eq. 26-17 the capacitance of an air-filled spherical capacitor is given by

$$C_0 = 4\pi\varepsilon_0\frac{ab}{b-a}\ .$$

When the dielectric is inserted between the plates the capacitance is greater by a factor of the dielectric constant $\kappa$. Consequently, the new capacitance is

$$C = 4\pi\kappa\varepsilon_0\frac{ab}{b-a}\ .$$

(b) The charge on the positive plate is

$$q = CV = 4\pi\kappa\varepsilon_0\frac{ab}{b-a}V\ .$$

(c) Let the charge on the inner conductor to be $-q$. Immediately adjacent to it is the induced charge $q'$. Since the electric field is less by a factor $1/\kappa$ than the field when no dielectric is present, then $-q + q' = -q/\kappa$. Thus,

$$q' = \frac{\kappa-1}{\kappa}q = 4\pi(\kappa-1)\varepsilon_0\frac{ab}{b-a}V\ .$$

46. (a) We apply Gauss's law with dielectric: $q/\varepsilon_0 = \kappa EA$, and solve for $\kappa$:

$$\kappa = \frac{q}{\varepsilon_0 EA} = \frac{8.9\times10^{-7}\,\mathrm{C}}{\left(8.85\times10^{-12}\,\frac{\mathrm{C}^2}{\mathrm{N\cdot m^2}}\right)(1.4\times10^{-6}\,\mathrm{V/m})(100\times10^{-4}\,\mathrm{m}^2)} = 7.2\ .$$

(b) The charge induced is

$$q' = q\left(1-\frac{1}{\kappa}\right) = (8.9\times10^{-7}\,\mathrm{C})\left(1-\frac{1}{7.2}\right) = 7.7\times10^{-7}\ \mathrm{C}\ .$$

47. Assuming the charge on one plate is $+q$ and the charge on the other plate is $-q$, we find an expression for the electric field in each region, in terms of $q$, then use the result to find an expression for the potential difference $V$ between the plates. The capacitance is

$$C = \frac{q}{V} .$$

The electric field in the dielectric is $E_d = q/\kappa\varepsilon_0 A$, where $\kappa$ is the dielectric constant and $A$ is the plate area. Outside the dielectric (but still between the capacitor plates) the field is $E = q/\varepsilon_0 A$. The field is uniform in each region so the potential difference across the plates is

$$V = E_d b + E(d-b) = \frac{qb}{\kappa\varepsilon_0 A} + \frac{q(d-b)}{\varepsilon_0 A} = \frac{q}{\varepsilon_0 A}\,\frac{b+\kappa(d-b)}{\kappa} .$$

The capacitance is

$$C = \frac{q}{V} = \frac{\kappa\varepsilon_0 A}{\kappa(d-b)+b} = \frac{\kappa\varepsilon_0 A}{\kappa d - b(\kappa-1)} .$$

The result does not depend on where the dielectric is located between the plates; it might be touching one plate or it might have a vacuum gap on each side.

For the capacitor of Sample Problem 26-8, $\kappa = 2.61$, $A = 115\,\text{cm}^2 = 115\times 10^{-4}\,\text{m}^2$, $d = 1.24\,\text{cm} = 1.24\times 10^{-2}\,\text{m}$, and $b = 0.78\,\text{cm} = 0.78\times 10^{-2}\,\text{m}$, so

$$\begin{aligned} C &= \frac{2.61(8.85\times 10^{-12}\,\text{F/m})(115\times 10^{-4}\,\text{m}^2)}{2.61(1.24\times 10^{-2}\,\text{m}) - (0.780\times 10^{-2}\,\text{m})(2.61-1)} \\ &= 1.34\times 10^{-11}\,\text{F} = 13.4\ \text{pF} \end{aligned}$$

in agreement with the result found in the sample problem. If $b = 0$ and $\kappa = 1$, then the expression derived above yields $C = \varepsilon_0 A/d$, the correct expression for a parallel-plate capacitor with no dielectric. If $b = d$, then the derived expression yields $C = \kappa\varepsilon_0 A/d$, the correct expression for a parallel-plate capacitor completely filled with a dielectric.

48. (a) Eq. 26-22 yields

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\left(200\times 10^{-12}\,\text{F}\right)\left(7.0\times 10^3\,\text{V}\right)^2 = 4.9\times 10^{-3}\ \text{J} .$$

(b) Our result from part (a) is much less than the required 150 mJ, so such a spark should not have set off an explosion.

49. (a) With the potential difference equal to 600 V, a capacitance of $2.5\times 10^{-10}\,\text{F}$ can only store energy equal to $U = \frac{1}{2}CV^2 = 4.5\times 10^{-5}\,\text{J}$.

(b) No, our result from part (a) is only about 20% of that needed to produce a spark.

(c) Considering the charge as a constant, then voltage should be inversely proportional to the capacitance. Therefore, if the capacitance drops by a factor of ten, then we expect the voltage to increase by that same factor: $V_f = 6000$ V.

(d) Now the energy stored is $U' = \frac{1}{2}C_f V_f^2 = 4.5\times 10^{-4}\,\text{J}$, a factor of ten greater than the value we obtained in part (a).

(e) Yes, this new value of energy is nearly double that needed for a spark.

50. (a) We calculate the charged surface area of the cylindrical volume as follows:

$$A = 2\pi rh + \pi r^2 = 2\pi(0.20\,\text{m})(0.10\,\text{m}) + \pi(0.20\,\text{m})^2 = 0.25\ \text{m}^2$$

where we note from the figure that although the bottom is charged, the top is not. Therefore, the charge is $q = \sigma A = -0.50\,\mu\text{C}$ on the exterior surface, and consequently (according to the assumptions in the problem) that same charge $q$ is induced in the interior of the fluid.

(b) By Eq. 26-21, the energy stored is

$$U = \frac{q^2}{2C} = \frac{\left(5.0\times10^{-7}\,\mathrm{C}\right)^2}{2\left(35\times10^{-12}\,\mathrm{F}\right)} = 3.6\times10^{-3}\ \mathrm{J}\ .$$

(c) Our result is within a factor of three of that needed to cause a spark. Our conclusion is that it will probably not cause a spark; however, there is not enough of a safety factor to be sure.

51. (a) We know from Eq. 26-7 that the magnitude of the electric field is directly proportional to the surface charge density:

$$E = \frac{\sigma}{\varepsilon_0} = \frac{15\times10^{-6}\,\mathrm{C/m^2}}{8.85\times10^{-12}\,\mathrm{C^2/N{\cdot}m^2}} = 1.7\times10^6\ \mathrm{V/m}\ .$$

Regarding the units, it is worth noting that a Volt is equivalent to a N·m/C.

(b) Eq. 26-23 yields

$$u = \frac{1}{2}\varepsilon_0 E^2 = 13\ \mathrm{J/m^3}\ .$$

(c) The energy $U$ is the energy-per-unit-volume multiplied by the (variable) volume of the region between the layers of plastic food wrap. Since the distance between the layers is $x$, and we use $A$ for the area over which the (say, positive) charge is spread, then that volume is $Ax$. Thus,

$$U = uAx \qquad \text{where} \qquad u = 13\ \mathrm{J/m^3}\ .$$

(d) The magnitude of force is

$$\left|\vec{F}\right| = \frac{dU}{dx} = uA\ .$$

(e) The force per unit area is

$$\frac{\left|\vec{F}\right|}{A} = u = 13\ \mathrm{N/m^2}\ .$$

Regarding units, it is worth noting that a Joule is equivalent to a N·m, which explains how J/m$^3$ may be set equal to N/m$^2$ in the above manipulation. We note, too, that the pressure unit N/m$^2$ is generally known as a Pascal (Pa).

(f) Combining our steps in parts (a) through (e), we have

$$\begin{aligned} \frac{\left|\vec{F}\right|}{A} &= u = \frac{1}{2}\varepsilon_0 E^2 \\ 6.0\ \mathrm{N/m^2} &= \frac{1}{2}\varepsilon_0\left(\frac{\sigma}{\varepsilon_0}\right)^2 = \frac{\sigma^2}{2\varepsilon_0} \end{aligned}$$

which leads to $\sigma = \sqrt{2(8.85\times10^{-12})(6.0)} = 1.0\times10^{-5}\ \mathrm{C/m^2}$.

52. (a) We do not employ energy conservation since, in reaching equilibrium, some energy is dissipated either as heat or radio waves. Charge is conserved; therefore, if $Q = C_1 V_{\mathrm{bat}} = 40\ \mu\mathrm{C}$, and $q_1$ and $q_2$ are the charges on $C_1$ and $C_2$ after the switch is thrown to the right and equilibrium is reached, then

$$Q = q_1 + q_2\ .$$

Reducing the right portion of the circuit (the $C_3$, $C_4$ parallel pair which are in series with $C_2$) we have an equivalent capacitance of $C' = 8.0\ \mu\mathrm{F}$ which has charge $q' = q_2$ and potential difference equal to that of $C_1$. Thus,

$$\begin{aligned} V_1 &= V' \\ \frac{q_1}{C_1} &= \frac{q_2}{C'} \end{aligned}$$

which yields $4q_1 = q_2$. Therefore,

$$Q = q_1 + 4q_1$$

leads to $q_1 = 8.0\ \mu\text{C}$ and consequently to $q_2 = 32\,\mu\text{C}$.

(b) From Eq. 26-1, we have $V_2 = (32\,\mu\text{C})(16\,\mu\text{F}) = 2.0$ V.

53. Using Eq. 26-27, with $\sigma = q/A$, we have

$$\left|\vec{E}\right| = \frac{q}{\kappa\varepsilon_0 A} = 200 \times 10^3 \text{ N/C}$$

which yields $q = 3.3 \times 10^{-7}$ C. Eq. 26-21 and Eq. 26-25 therefore lead to

$$U = \frac{q^2}{2C} = \frac{q^2 d}{2\kappa\varepsilon_0 A} = 6.6 \times 10^{-5} \text{ J} .$$

54. (a) The potential across capacitor 1 is 10 V, so the charge on it is

$$q_1 = C_1 V_1 = (10\ \mu\text{F})(10\ \text{V}) \ = \ 100\ \mu\text{C} .$$

(b) Reducing the right portion of the circuit produces an equivalence equal to $6.0\,\mu$F, with 10 V across it. Thus, a charge of $60\,\mu$C is on it – and consequently also on the bottom right capacitor. The bottom right capacitor has, as a result, a potential across it equal to

$$V = \frac{q}{C} = \frac{60\,\mu\text{C}}{10\,\mu\text{F}} \ = \ 6.0 \text{ V} ,$$

which leaves $10 - 6 = 4.0$ V across the group of capacitors in the upper right portion of the circuit. Inspection of the arrangement (and capacitance values) of that group reveals that this 4.0 V must be equally divided by $C_2$ and the capacitor directly below it (in series with it). Therefore, with 2.0 V across capacitor 2, we find

$$q_2 = C_2 V_2 = (10\,\mu\text{F})(2.0\ \text{V}) = 20\,\mu\text{C} .$$

55. (a) We use $q = CV = \varepsilon_0 AV/d$ to solve for $A$:

$$A = \frac{Cd}{\varepsilon_0} = \frac{(10 \times 10^{-12}\,\text{F})(1.0 \times 10^{-3}\,\text{m})}{\left(8.85 \times 10^{-12}\,\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)} = 1.1 \times 10^{-3}\,\text{m}^2 .$$

(b) Now,

$$C' = C\left(\frac{d}{d'}\right) = (10\,\text{pF})\left(\frac{1.0\,\text{mm}}{0.9\,\text{mm}}\right) = 11\,\text{pF} .$$

(c) The new potential difference is $V' = q/C' = CV/C'$. Thus,

$$\Delta V = V' - V = \frac{(10\,\text{pF})(12\,\text{V})}{11\,\text{pF}} - 12\,\text{V} = 1.2\ \text{V} .$$

In a microphone, mechanical pressure applied to the aluminum foil as a result of sound can cause the capacitance of the foil to change, thereby inducing a variable $\Delta V$ in response to the sound signal.

56. (a) Here $D$ is not attached to anything, so that the $6C$ and $4C$ capacitors are in series (equivalent to $2.4C$). This is then in parallel with the $2C$ capacitor, which produces an equivalence of $4.4C$. Finally the $4.4C$ is in series with $C$ and we obtain

$$C_{\text{eq}} = \frac{(C)\,(4.4C)}{C + 4.4C} = 0.82C = 41\,\mu\text{F}$$

where we have used the fact that $C = 50\,\mu$F.

(b) Now, $B$ is the point which is not attached to anything, so that the $6C$ and $2C$ capacitors are now in series (equivalent to $1.5C$), which is then in parallel with the $4C$ capacitor (and thus equivalent to $5.5C$). The $5.5C$ is then in series with the $C$ capacitor; consequently,

$$C_{\text{eq}} = \frac{(C)\,(5.5C)}{C + 5.5C} = 0.85C = 42\,\mu\text{F} \ .$$

57. In the first case the two capacitors are effectively connected in series, so the output potential difference is $V_{\text{out}} = CV_{\text{in}}/2C = V_{\text{in}}/2 = 50.0\,\text{V}$. In the second case the lower diode acts as a wire so $V_{\text{out}} = 0$.

58. For maximum capacitance the two groups of plates must face each other with maximum area. In this case the whole capacitor consists of $(n-1)$ identical single capacitors connected in parallel. Each capacitor has surface area $A$ and plate separation $d$ so its capacitance is given by $C_0 = \varepsilon_0 A/d$. Thus, the total capacitance of the combination is

$$C = (n-1)C_0 = \frac{(n-1)\varepsilon_0 A}{d} \ .$$

59. The voltage across capacitor 1 is

$$V_1 = \frac{q_1}{C_1} = \frac{30\,\mu\text{C}}{10\,\mu\text{F}} = 3.0 \text{ V} \ .$$

Since $V_1 = V_2$, the total charge on capacitor 2 is

$$q_2 = C_2 V_2 = (20\,\mu\text{F})\,(2 \text{ V}) = 60 \ \mu\text{C} \ ,$$

which means a total of $90\,\mu$C of charge is on the pair of capacitors $C_1$ and $C_2$. This implies there is a total of $90\,\mu$C of charge also on the $C_3$ and $C_4$ pair. Since $C_3 = C_4$, the charge divides equally between them, so $q_3 = q_4 = 45\,\mu$C. Thus, the voltage across capacitor 3 is

$$V_3 = \frac{q_3}{C_3} = \frac{45\,\mu\text{C}}{20\,\mu\text{F}} = 2.3 \text{ V} \ .$$

Therefore, $|V_A - V_B| = V_1 + V_3 \ = \ 5.3$ V.

60. (a) The equivalent capacitance is

$$C_{\text{eq}} = \frac{C_1 C_2}{C_1 + C_2} = \frac{(6.00\,\mu\text{F})(4.00\,\mu\text{F})}{6.00\,\mu\text{F} + 4.00\,\mu\text{F}} = 2.40\,\mu\text{F} \ .$$

(b) $q = C_{\text{eq}} V = (2.40\,\mu\text{F})(200\,\text{V}) = 4.80 \times 10^4\,\text{C}$.

(c) $V_1 = q/C_1 = 4.80 \times 10^4\,\text{C}/2.40\,\mu\text{F} = 120\,\text{V}$, and $V_2 = V - V_1 = 200\,\text{V} - 120\,\text{V} = 80\,\text{V}$.

61. (a) Now $C_{\text{eq}} = C_1 + C_2 = 6.00\,\mu\text{F} + 4.00\,\mu\text{F} = 10.0\,\mu\text{F}$.

(b) $q_1 = C_1 V = (6.00\,\mu\text{F})(200\,\text{V}) = 1.20 \times 10^{-3}\,\text{C}$, $q_2 = C_2 V = (4.00\,\mu\text{F})(200\,\text{V}) = 8.00 \times 10^{-4}\,\text{C}$.

(c) $V_1 = V_2 = 200\,\text{V}$.

62. We cannot expect simple energy conservation to hold since energy is presumably dissipated either as heat in the hookup wires or as radio waves while the charge oscillates in the course of the system "settling down" to its final state (of having 40 V across the parallel pair of capacitors $C$ and 60 $\mu$F). We do expect charge to be conserved. Thus, if $Q$ is the charge originally stored on $C$ and $q_1$, $q_2$ are the charges on the parallel pair after "setting down," then

$$\begin{aligned} Q &= q_1 + q_2 \\ C(100 \text{ V}) &= C(40 \text{ V}) + (60\,\mu\text{F})\,(40 \text{ V}) \end{aligned}$$

which leads to the solution $C = 40\,\mu$F.

63. (a) Put five such capacitors in series. Then, the equivalent capacitance is $2.0\,\mu\text{F}/5 = 0.40\,\mu\text{F}$. With each capacitor taking a 200-V potential difference, the equivalent capacitor can withstand 1000 V.

(b) As one possibility, you can take three identical arrays of capacitors, each array being a five-capacitor combination described in part (a) above, and hook up the arrays in parallel. The equivalent capacitance is now $C_{\text{eq}} = 3(0.40\,\mu\text{F}) = 1.2\,\mu\text{F}$. With each capacitor taking a 200-V potential difference the equivalent capacitor can withstand 1000 V.

64. (a) The energy per unit volume is

$$u = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0 \left(\frac{e}{4\pi\varepsilon_0 r^2}\right)^2 = \frac{e^2}{32\pi^2\varepsilon_0 r^4} .$$

(b) From the expression above $u \propto r^{-4}$. So for $r \to 0 \quad u \to \infty$.

65. (a) They each store the same charge, so the maximum voltage is across the smallest capacitor. With 100 V across 10 $\mu$F, then the voltage across the 20 $\mu$F capacitor is 50 V and the voltage across the 25 $\mu$F capacitor is 40 V. Therefore, the voltage across the arrangement is 190 V.

(b) Using Eq. 26-21 or Eq. 26-22, we sum the energies on the capacitors and obtain $U_{\text{total}} = 0.095$ J.

66. (a) Since the field is constant and the capacitors are in parallel (each with 600 V across them) with identical distances ($d = 0.00300$ m) between the plates, then the field in $A$ is equal to the field in $B$:

$$\left|\vec{E}\right| = \frac{V}{d} = 2.00 \times 10^5 \text{ V/m} .$$

(b) See the note in part (a).

(c) For the air-filled capacitor, Eq. 26-4 leads to

$$\sigma = \frac{q}{A} = \varepsilon_0 \left|\vec{E}\right| = 1.77 \times 10^{-6} \text{ C/m}^2 .$$

(d) For the dielectric-filled capacitor, we use Eq. 26-27:

$$\sigma = \kappa\varepsilon_0 \left|\vec{E}\right| = 4.60 \times 10^{-6} \text{ C/m}^2 .$$

(e) Although the discussion in the textbook (§26-8) is in terms of the charge being held fixed (while a dielectric is inserted), it is readily adapted to this situation (where comparison is made of two capacitors which have the same *voltage* and are identical except for the fact that one has a dielectric). The fact that capacitor $B$ has a relatively large charge but only produces the field that $A$ produces (with its smaller charge) is in line with the point being made (in the text) with Eq. 26-32 and in the material that follows. Adapting Eq. 26-33 to this problem, we see that the difference in charge densities between parts (c) and (d) is due, in part, to the (negative) layer of charge at the top surface of the dielectric; consequently,

$$\sigma' = \left(1.77 \times 10^{-6}\right) - \left(4.60 \times 10^{-6}\right) = -2.83 \times 10^{-6} \text{ C/m}^2 .$$

67. (a) The equivalent capacitance is $C_{\text{eq}} = C_1C_2/(C_1 + C_2)$. Thus the charge $q$ on each capacitor is

$$q = C_{\text{eq}}V = \frac{C_1C_2V}{C_1 + C_2} = \frac{(2.0\,\mu\text{F})(8.0\,\mu\text{F})(300\text{ V})}{2.0\,\mu\text{F} + 8.0\,\mu\text{F}} = 4.8 \times 10^{-4} \text{ C} .$$

The potential differences are: $V_1 = q/C_1 = 4.8 \times 10^{-4}\,\text{C}/2.0\,\mu\text{F} = 240\,\text{V}$, $V_2 = V - V_1 = 300\,\text{V} - 240\,\text{V} = 60\,\text{V}$.

(b) Now we have $q_1'/C_1 = q_2'/C_2 = V'$ ($V'$ being the new potential difference across each capacitor) and $q_1' + q_2' = 2q$. We solve for $q_1'$, $q_2'$ and $V$:

$$\begin{aligned} q_1' &= \frac{2C_1 q}{C_1 + C_2} = \frac{2(2.0\,\mu\text{F})(4.8 \times 10^{-4}\,\text{C})}{2.0\,\mu\text{F} + 8.0\,\mu\text{F}} = 1.9 \times 10^{-4}\ \text{C}\ , \\ q_2' &= 2q - q_1 = 7.7 \times 10^{-4}\ \text{C}\ , \\ V' &= \frac{q_1'}{C_1} = \frac{1.92 \times 10^{-4}\,\text{C}}{2.0\,\mu\text{F}} = 96\ \text{V}\ . \end{aligned}$$

(c) In this circumstance, the capacitors will simply discharge themselves, leaving $q_1 = q_2 = 0$ and $V_1 = V_2 = 0$.

68. We use $U = \frac{1}{2}CV^2$. As $V$ is increased by $\Delta V$, the energy stored in the capacitor increases correspondingly from $U$ to $U + \Delta U$: $U + \Delta U = \frac{1}{2}C(V + \Delta V)^2$. Thus, $(1 + \Delta V/V)^2 = 1 + \Delta U/U$, or

$$\frac{\Delta V}{V} = \sqrt{1 + \frac{\Delta U}{U}} - 1 = \sqrt{1 + 10\%} - 1 = 4.9\%\ .$$

69. (a) The voltage across $C_1$ is 12 V, so the charge is

$$q_1 = C_1 V_1 = 24\,\mu\text{C}\ \ .$$

(b) We reduce the circuit, starting with $C_4$ and $C_3$ (in parallel) which are equivalent to $4\,\mu$F. This is then in series with $C_2$, resulting in an equivalence equal to $\frac{4}{3}\,\mu$F which would have 12 V across it. The charge on this $\frac{4}{3}\,\mu$F capacitor (and therefore on $C_2$) is $(\frac{4}{3}\,\mu\text{F})(12\,\text{V}) = 16\,\mu\text{C}$. Consequently, the voltage across $C_2$ is

$$V_2 = \frac{q_2}{C_2} = \frac{16\,\mu\text{C}}{2\,\mu\text{F}} = 8\ \text{V}\ .$$

This leaves $12 - 8 = 4$ V across $C_4$ (similarly for $C_3$).

70. (a) The energy stored is

$$U = \frac{1}{2}CV^2 = \frac{1}{2}(130 \times 10^{-12}\,\text{F})(56.0\,\text{V})^2 = 2.04 \times 10^{-7}\,\text{J}\ .$$

(b) No, because we don't know the volume of the space inside the capacitor where the electric field is present.

71. We reduce the circuit, starting with $C_1$ and $C_2$ (in series) which are equivalent to $4\,\mu$F. This is then parallel to $C_3$ and results in a total of $8\,\mu$F, which is now in series with $C_4$ and can be further reduced. However, the final step in the reduction is not necessary, as we observe that the $8\,\mu$F equivalence from the top 3 capacitors has the same capacitance as $C_4$ and therefore the same voltage; since they are in series, that voltage is then $12/2 = 6$ V.

72. We use $C = \varepsilon_0 \kappa A/d \propto \kappa/d$. To maximize $C$ we need to choose the material with the greatest value of $\kappa/d$. It follows that the mica sheet should be chosen.

73. (a) After reducing the pair of $4\,\mu$F capacitors to a series equivalence of $2\,\mu$F, we have three 2 $\mu$F capacitors in the upper right portion of the circuit all in parallel – and thus equivalent to $6\,\mu$F. In the lower right portion of the circuit are two $3\,\mu$F capacitors in parallel, equivalent also to 6 $\mu$F. These two $6\,\mu$F equivalent-capacitors are then in series, so that the full reduction leads to an equivalence of $3.0\,\mu$F.

(b) With 20 V across the result of part (a), we have a charge equal to $q = CV = (3.0\,\mu\text{F})(20\text{ V}) = 60\,\mu\text{C}$.

74. (a) The length $d$ is effectively shortened by $b$ so $C' = \varepsilon_0 A/(d - b)$.

(b) The energy before, divided by the energy after inserting the slab is

$$\frac{U}{U'} = \frac{q^2/2C}{q^2/2C'} = \frac{C'}{C} = \frac{\varepsilon_0 A/(d-b)}{\varepsilon_0 A/d} = \frac{d}{d-b} \; .$$

(c) The work done is

$$W = \Delta U = U' - U = \frac{q^2}{2}\left(\frac{1}{C'} - \frac{1}{C}\right) = \frac{q^2}{2\varepsilon_0 A}(d-b-d) = -\frac{q^2 b}{2\varepsilon_0 A} \; .$$

Since $W < 0$ the slab is sucked in.

75. (a) $C' = \varepsilon_0 A/(d-b)$, the same as part (a) in problem 74.

(b) Now,

$$\frac{U}{U'} = \frac{\frac{1}{2}CV^2}{\frac{1}{2}C'V^2} = \frac{C}{C'} = \frac{\varepsilon_0 A/d}{\varepsilon_0 A/(d-b)} = \frac{d-b}{d} \; .$$

(c) The work done is

$$W = \Delta U = U' - U = \frac{1}{2}(C' - C)V^2 = \frac{\varepsilon_0 A}{2}\left(\frac{1}{d-b} - \frac{1}{d}\right)V^2 = \frac{\varepsilon_0 A b V^2}{2d(d-b)} \; .$$

Since $W > 0$ the slab must be pushed in.

76. We do not employ energy conservation since, in reaching equilibrium, some energy is dissipated either as heat or radio waves. Charge is conserved; therefore, if $Q = 48\,\mu\text{C}$, and $q_1$ and $q_3$ are the charges on $C_1$ and $C_3$ after the switch is thrown to the right (and equilibrium is reached), then

$$Q = q_1 + q_3 \quad .$$

We note that $V_{1 \text{ and } 2} = V_3$ because of the parallel arrangement, and $V_1 = \frac{1}{2}V_{1 \text{ and } 2}$ since they are identical capacitors. This leads to

$$\begin{aligned} 2V_1 &= V_3 \\ 2\frac{q_1}{C_1} &= \frac{q_3}{C_3} \\ 2q_1 &= q_3 \end{aligned}$$

where the last step follows from multiplying both sides by $2.00\,\mu\text{F}$. Therefore,

$$Q = q_1 + (2q_1)$$

which yields $q_1 = 16\,\mu\text{C}$ and $q_3 = 32\,\mu\text{C}$.

77. (a) Since $u = \frac{1}{2}\kappa\varepsilon_0 E^2$, we select the material with the greatest value of $\kappa E_{\text{max}}^2$, where $E_{\text{max}}$ is its dielectric strength. We therefore choose strontium titanate, with the corresponding minimum volume

$$\mathcal{V}_{\text{min}} = \frac{U}{U_{\text{max}}} = \frac{2U}{\kappa\varepsilon_0 E_{\text{max}}^2} = \frac{2(250\,\text{kJ})}{(310)\left(8.85 \times 10^{-12}\,\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)(8\,\text{kV/mm})^2} = 2.85\ \text{m}^3 \; .$$

(b) We solve for $\kappa'$ from $U = \frac{1}{2}\kappa'\varepsilon_0 E_{\text{max}}^2 \mathcal{V}'_{\text{min}}$:

$$\kappa' = \frac{2U}{\varepsilon_0 \mathcal{V}' E_{\text{max}}^2} = \frac{2(250\,\text{kJ})}{\left(8.85 \times 10^{-12}\,\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)(0.0870\,\text{m}^3)(8\,\text{kV/mm})^2} = 1.01 \times 10^4 \; .$$

78. (a) Initially, the capacitance is

$$C_0 = \frac{\varepsilon_0 A}{d} = \frac{\left(8.85 \times 10^{-12}\,\frac{\mathrm{C}^2}{\mathrm{N \cdot m^2}}\right)(0.12\,\mathrm{m}^2)}{1.2 \times 10^{-2}\,\mathrm{m}} = 89\ \mathrm{pF}\ .$$

(b) Working through Sample Problem 26-6 algebraically, we find:

$$C = \frac{\varepsilon_0 A \kappa}{\kappa(d-b)+b} = \frac{\left(8.85 \times 10^{-12}\,\frac{\mathrm{C}^2}{\mathrm{N \cdot m^2}}\right)(0.12\,\mathrm{m}^2)(4.8)}{(4.8)(1.2-0.40)(10^{-2}\,\mathrm{m}) + (4.0 \times 10^{-3}\,\mathrm{m})} = 120\ \mathrm{pF}\ .$$

(c) Before the insertion, $q = C_0 V (89\,\mathrm{pF})(120\,\mathrm{V}) = 11\,\mathrm{nC}$. Since the battery is disconnected, $q$ will remain the same after the insertion of the slab.

(d) $E = q/\varepsilon_0 A = 11 \times 10^{-9}\,\mathrm{C}/\left(8.85 \times 10^{-12}\,\frac{\mathrm{C}^2}{\mathrm{N \cdot m^2}}\right)(0.12\,\mathrm{m}^2) = 10\,\mathrm{kV/m}$.

(e) $E' = E/\kappa = (10\,\mathrm{kV/m})/4.8 = 2.1\,\mathrm{kV/m}$.

(f) $V = E(d-b) + E'b = (10\,\mathrm{kV/m})(0.012\,\mathrm{m} - 0.0040\,\mathrm{m}) + (2.1\,\mathrm{kV/m})(0.40 \times 10^{-3}\,\mathrm{m}) = 88\,\mathrm{V}$.

(g) The work done is

$$\begin{aligned} W_{\mathrm{ext}} &= \Delta U = \frac{q^2}{2}\left(\frac{1}{C} - \frac{1}{C_0}\right) \\ &= \frac{(11 \times 10^{-9}\,\mathrm{C})^2}{2}\left(\frac{1}{89 \times 10^{-12}\,\mathrm{F}} - \frac{1}{120 \times 10^{-12}\,\mathrm{F}}\right) \\ &= -1.7 \times 10^{-7}\ \mathrm{J}\ . \end{aligned}$$

79. (a) Since $u = \frac{1}{2}\kappa\varepsilon_0 E^2$, $E_{\mathrm{slab}} = E_{\mathrm{air}}/\kappa_{\mathrm{slab}}$, and $U = u\mathcal{V}$ (where $\mathcal{V} =$ volume), then the fraction of energy stored in the air gaps is

$$\begin{aligned} \frac{U_{\mathrm{air}}}{U_{\mathrm{total}}} &= \frac{E_{\mathrm{air}}^2 A(d-b)}{E_{\mathrm{air}}^2 A(d-b) + \kappa_{\mathrm{slab}} E_{\mathrm{slab}}^2 Ab} = \frac{1}{1 + \kappa_{\mathrm{slab}}(E_{\mathrm{slab}}/E_{\mathrm{air}})^2[b/(d-b)]} \\ &= \frac{1}{1 + (2.61)(1/2.61)^2[0.780/(1.24-0.780)]} = 0.606\ . \end{aligned}$$

(b) The fraction of energy stored in the slab is $1 - 0.606 = 0.394$.

80. (a) The equivalent capacitance of the three capacitors connected in parallel is $C_{\mathrm{eq}} = 3C = 3\varepsilon_0 A/d = \varepsilon_0 A/(d/3)$. Thus, the required spacing is $d/3$.

(b) Now, $C_{\mathrm{eq}} = C/3 = \varepsilon_0 A/3d$, so the spacing should be $3d$.

81. We do not employ energy conservation since, in reaching equilibrium, some energy is dissipated either as heat or radio waves. Charge is conserved; therefore, if $Q = C_1 V_{\mathrm{bat}} = 24\,\mu\mathrm{C}$, and $q_1$ and $q_3$ are the charges on $C_1$ and $C_3$ after the switch is thrown to the right (and equilibrium is reached), then

$$Q = q_1 + q_3\ .$$

We reduce the series pair $C_2$ and $C_3$ to $C' = 4/3\ \mu\mathrm{F}$ which has charge $q' = q_3$ and the same voltage that we find across $C_1$. Therefore,

$$\begin{aligned} V_1 &= V' \\ \frac{q_1}{C_1} &= \frac{q_3}{C'} \end{aligned}$$

which leads to $q_1 = 1.5 q_3$. Hence,

$$Q = (1.5 q_3) + q_3$$

leads to $q_3 = 9.6\,\mu\mathrm{C}$.

82. (First problem of **Cluster**)

(a) We do not employ energy conservation since, in reaching equilibrium, some energy is dissipated either as heat or radio waves. Charge is conserved; therefore, if $Q = C_1 V_{\rm bat} = 400\,\mu$C, and $q_1$ and $q_2$ are the charges on $C_1$ and $C_2$ after the switch S is closed (and equilibrium is reached), then

$$Q = q_1 + q_2 \quad .$$

After switch S is closed, the capacitor voltages are equal, so that

$$\begin{aligned} V_1 &= V_2 \\ \frac{q_1}{C_1} &= \frac{q_2}{C_2} \end{aligned}$$

which yields $\frac{3}{4}q_1 = q_2$. Therefore,

$$Q = q_1 + \left(\frac{3}{4}\,q_1\right)$$

which gives the result $q_1 = 229\,\mu$C.

(b) The relation $\frac{3}{4}q_1 = q_2$ gives the result $q_2 = 171\,\mu$C.

(c) We apply Eq. 27-1: $V_1 = q_1/C_1 = 5.71$ V.

(d) Similarly, $V_2 = q_2/C_2 = 5.71$ V (which is equal to $V_1$, of course – since that fact was used in the solution to part (a)).

(e) When $C_1$ had charge $Q$ and was connected to the battery, the energy stored was $\frac{1}{2}C_1 V_{\rm bat}^2 = 2.00 \times 10^{-3}$ J. The energy stored after S is closed is $\frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2 = 1.14 \times 10^{-3}$ J. The *decrease* is therefore $8.6 \times 10^{-4}$ J.

83. (Second problem of **Cluster**)

(a) The change (from the previous problem) is that the initial charge (before switch S is closed) is $Q + Q'$ where $Q$ is as before but $Q' = C_2(10\text{ V}) = 600\,\mu$C. We assume the polarities of these capacitor charges are the same. With this modification, we follow steps similar to those in the previous solution:

$$\begin{aligned} Q + Q' &= q_1 + q_2 \\ &= q_1 + \left(\frac{3}{4}\,q_1\right) \end{aligned}$$

which yields $q_1 = 571\,\mu$C.

(b) The relation $\frac{3}{4}q_1 = q_2$ gives the result $q_2 = 429\,\mu$C.

(c) We apply Eq. 27-1: $V_1 = q_1/C_1 = 14.3$ V.

(d) Similarly, $V_2 = q_2/C_2 = 14.3$ V.

(e) The initial energy now includes $\frac{1}{2}C_2(20\text{ V})^2$ in addition to the $\frac{1}{2}C_1V_{\rm bat}^2$ computed in the previous case. Thus, the total initial energy is $8.00\times10^{-3}$ J. And the final stored energy is $\frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2 = 7.14 \times 10^{-3}$ J. The *decrease* is therefore $8.6 \times 10^{-4}$ J, as it was in the previous problem.

84. (Third problem of **Cluster**)

(a) With the series pair $C_2$ and $C_3$ reduced to a single $C' = 10\,\mu$F capacitor, this becomes very similar to problem 82. Noting for later use that $q' = q_2 = q_3$, and using notation similar to that used in the solution to problem 82, we have

$$Q = q_1 + q'$$

where $Q = C_1 V_{\rm bat} = 400\,\mu$C. Also, after switch S is closed,

$$\begin{aligned} V_1 &= V' \\ \frac{q_1}{C_1} &= \frac{q'}{C'} \end{aligned}$$

which yields $\frac{1}{4}q_1 = q'$. Therefore,

$$Q = q_1 + \left(\frac{1}{4}\,q_1\right)$$

which gives the result $q_1 = 320\,\mu$C.

(b) We use $q_2 = q_3 = \frac{1}{4}q_1$ to obtain the result $80\,\mu$C.

(c) See part (b).

(d) (e) and (f) Eq. 26-1 yields

$$V = \frac{q}{C} = \begin{cases} 8.0\text{ V} & \text{for } C_1 \\ 5.3\text{ V} & \text{for } C_2 \\ 2.7\text{ V} & \text{for } C_3 \end{cases}$$

85. (Fourth problem of **Cluster**)

(a) With the parallel pair $C_2$ and $C_3$ reduced to a single $C' = 45\,\mu$F capacitor, this becomes very similar to problem 82. Using notation similar to that used in the solution to 82, we have

$$Q = q_1 + q'$$

where $Q = C_1 V_{\rm bat} = 400\,\mu$C. Also, after switch S is closed,

$$\begin{aligned} V_1 &= V' \\ \frac{q_1}{C_1} &= \frac{q'}{C'} \end{aligned}$$

which yields $\frac{9}{8}q_1 = q'$. Therefore,

$$Q = q_1 + \left(\frac{9}{8}\,q_1\right)$$

which gives the result $q_1 = 188\,\mu$C.

(b) We find the voltage across capacitor 1 from $q_1/C_1$ (see below) and (since the capacitors are in parallel) use the fact that $V_1 = V_2 = V_3$ with $q = CV$ to obtain the charges: $q_2 = 71\,\mu$C and $q_3 = 141\,\mu$C.

(c) See part (b).

(d) (e) and (f) The capacitors all have the same voltage. $V = q_1/C_1 = 4.7$ V.

86. (Fifth problem of **Cluster**)

(a) To begin with, the charge on capacitor 1 is $Q_1 = C_1 V_{\rm bat} = 400\,\mu$C, and the charge on capacitor 2 is $Q_2 = C_2 V_{\rm bat} = 150\,\mu$C. After the rearrangement and closing of the switch, the total charge in the upper portion of the circuit is $Q_1 - Q_2 = Q = 250\,\mu$C. With notation similar to that in the previous problems,

$$\begin{aligned} Q &= q_1 + q_2 \\ &= C_1 V + C_2 V \end{aligned}$$

which yields $V = 4.55$ V, which, in turn implies $q_1 = C_1 V = 182\,\mu$C and $q_2 = C_2 V = 68\,\mu$C. To achieve this distribution (with $+182\,\mu$C on one upper plate and $+68\,\mu$C on the other upper plate) from the arrangement right before closing the switch (with $+400\,\mu$C on one upper plate and $-150\,\mu$C on the other upper plate), it is necessary for $218\,\mu$C to flow through the switch.

(b) As shown above, $V = 4.55$ V $= V_1 = V_2$.