

Chapter 8

1. The potential energy stored by the spring is given by $U = \frac{1}{2}kx^2$, where k is the spring constant and x is the displacement of the end of the spring from its position when the spring is in equilibrium. Thus

$$k = \frac{2U}{x^2} = \frac{2(25\text{ J})}{(0.075\text{ m})^2} = 8.9 \times 10^3 \text{ N/m} .$$

2. (a) Noting that the vertical displacement is $10.0 - 1.5 = 8.5$ m downward (same direction as \vec{F}_g), Eq. 7-12 yields

$$W_g = mgd \cos \phi = (2.00)(9.8)(8.5) \cos 0^\circ = 167 \text{ J} .$$

- (b) One approach (which is fairly trivial) is to use Eq. 8-1, but we feel it is instructive to instead calculate this as ΔU where $U = mgy$ (with upwards understood to be the $+y$ direction).

$$\Delta U = mgy_f - mgy_i = (2.00)(9.8)(1.5) - (2.00)(9.8)(10.0) = -167 \text{ J} .$$

- (c) In part (b) we used the fact that $U_i = mgy_i = 196$ J.
- (d) In part (b), we also used the fact $U_f = mgy_f = 29$ J.
- (e) The computation of W_g does not use the new information (that $U = 100$ J at the ground), so we again obtain $W_g = 167$ J.
- (f) As a result of Eq. 8-1, we must again find $\Delta U = -W_g = -167$ J.
- (g) With this new information (that $U_0 = 100$ J where $y = 0$) we have $U_i = mgy_i + U_0 = 296$ J.
- (h) With this new information (that $U_0 = 100$ J where $y = 0$) we have $U_f = mgy_f + U_0 = 129$ J. We can check part (f) by subtracting the new U_i from this result.

3. (a) The force of gravity is constant, so the work it does is given by $W = \vec{F} \cdot \vec{d}$, where \vec{F} is the force and \vec{d} is the displacement. The force is vertically downward and has magnitude mg , where m is the mass of the flake, so this reduces to $W = mgh$, where h is the height from which the flake falls. This is equal to the radius r of the bowl. Thus

$$W = mgr = (2.00 \times 10^{-3} \text{ kg})(9.8 \text{ m/s}^2)(22.0 \times 10^{-2} \text{ m}) = 4.31 \times 10^{-3} \text{ J} .$$

- (b) The force of gravity is conservative, so the change in gravitational potential energy of the flake-Earth system is the negative of the work done: $\Delta U = -W = -4.31 \times 10^{-3}$ J.
- (c) The potential energy when the flake is at the top is greater than when it is at the bottom by $|\Delta U|$. If $U = 0$ at the bottom, then $U = +4.31 \times 10^{-3}$ J at the top.
- (d) If $U = 0$ at the top, then $U = -4.31 \times 10^{-3}$ J at the bottom.
- (e) All the answers are proportional to the mass of the flake. If the mass is doubled, all answers are doubled.

4. We use Eq. 7-12 for W_g and Eq. 8-9 for U .

- (a) The displacement between the initial point and A is horizontal, so $\phi = 90^\circ$ and $W_g = 0$ (since $\cos 90^\circ = 0$).
 - (b) The displacement between the initial point and B has a vertical component of $h/2$ downward (same direction as \vec{F}_g), so we obtain $W_g = \vec{F}_g \cdot \vec{d} = mgh/2$.
 - (c) The displacement between the initial point and C has a vertical component of h downward (same direction as \vec{F}_g), so we obtain $W_g = \vec{F}_g \cdot \vec{d} = mgh$.
 - (d) With the reference position at C , we obtain $U_B = mgh/2$.
 - (e) Similarly, we find $U_A = mgh$.
 - (f) All the answers are proportional to the mass of the object. If the mass is doubled, all answers are doubled.
- 5.
- (a) The only force that does work on the ball is the force of gravity; the force of the rod is perpendicular to the path of the ball and so does no work. In going from its initial position to the lowest point on its path, the ball moves vertically through a distance equal to the length L of the rod, so the work done by the force of gravity is $W = mgL$.
 - (b) In going from its initial position to the highest point on its path, the ball moves vertically through a distance equal to L , but this time the displacement is upward, opposite the direction of the force of gravity. The work done by the force of gravity is $W = -mgL$.
 - (c) The final position of the ball is at the same height as its initial position. The displacement is horizontal, perpendicular to the force of gravity. The force of gravity does no work during this displacement.
 - (d) The force of gravity is conservative. The change in the gravitational potential energy of the ball-Earth system is the negative of the work done by gravity: $\Delta U = -mgL$ as the ball goes to the lowest point
 - (e) Continuing this line of reasoning, we find $\Delta U = mgL$ as it goes to the highest point.
 - (f) Continuing this line of reasoning, we have $\Delta U = 0$ as it goes to the point at the same height.
 - (g) The change in the gravitational potential energy depends only on the initial and final positions of the ball, not on its speed anywhere. The change in the potential energy is the *same* since the initial and final positions are the same.
6. We use Eq. 7-12 for W_g and Eq. 8-9 for U .
- (a) The displacement between the initial point and Q has a vertical component of $h - R$ downward (same direction as \vec{F}_g), so (with $h = 5R$) we obtain $W_g = \vec{F}_g \cdot \vec{d} = 4mgR$.
 - (b) The displacement between the initial point and the top of the loop has a vertical component of $h - 2R$ downward (same direction as \vec{F}_g), so (with $h = 5R$) we obtain $W_g = \vec{F}_g \cdot \vec{d} = 3mgR$.
 - (c) With $y = h = 5R$, we find $U = 5mgR$ at P .
 - (d) With $y = R$, we find $U = mgR$ at Q .
 - (e) With $y = 2R$, we find $U = 2mgR$ at the top of the loop.
 - (f) The new information ($v_i \neq 0$) is not involved in any of the preceding computations; the above results are unchanged.
- 7.
- (a) The force of gravity is constant, so the work it does is given by $W = \vec{F} \cdot \vec{d}$, where \vec{F} is the force and \vec{d} is the displacement. The force is vertically downward and has magnitude mg , where m is the mass of the snowball. The expression for the work reduces to $W = mgh$, where h is the height through which the snowball drops. Thus

$$W = mgh = (1.50 \text{ kg})(9.8 \text{ m/s}^2)(12.5 \text{ m}) = 184 \text{ J} .$$

- (b) The force of gravity is conservative, so the change in the potential energy of the snowball-Earth system is the negative of the work it does: $\Delta U = -W = -184 \text{ J}$.
- (c) The potential energy when it reaches the ground is less than the potential energy when it is fired by $|\Delta U|$, so $U = -184 \text{ J}$ when the snowball hits the ground.
8. The main challenge for students in this type of problem seems to be working out the trigonometry in order to obtain the height of the ball (relative to the low point of the swing) $h = L - L \cos \theta$ (for angle θ measured from vertical as shown in Fig. 8-29). Once this relation (which we will not derive here since we have found this to be most easily illustrated at the blackboard) is established, then the principal results of this problem follow from Eq. 7-12 (for W_g) and Eq. 8-9 (for U).

- (a) The vertical component of the displacement vector is downward with magnitude h , so we obtain

$$W_g = \vec{F}_g \cdot \vec{d} = mgh = mgL(1 - \cos \theta) .$$

- (b) From Eq. 8-1, we have $\Delta U = -W_g = -mgL(1 - \cos \theta)$.
- (c) With $y = h$, Eq. 8-9 yields $U = mgL(1 - \cos \theta)$.
- (d) As the angle increases, we intuitively see that the height h increases (and, less obviously, from the mathematics, we see that $\cos \theta$ decreases so that $1 - \cos \theta$ increases), so the answers to parts (a) and (c) increase, and the absolute value of the answer to part (b) also increases.
9. (a) If K_i is the kinetic energy of the flake at the edge of the bowl, K_f is its kinetic energy at the bottom, U_i is the gravitational potential energy of the flake-Earth system with the flake at the top, and U_f is the gravitational potential energy with it at the bottom, then $K_f + U_f = K_i + U_i$. Taking the potential energy to be zero at the bottom of the bowl, then the potential energy at the top is $U_i = mgr$ where $r = 0.220 \text{ m}$ is the radius of the bowl and m is the mass of the flake. $K_i = 0$ since the flake starts from rest. Since the problem asks for the speed at the bottom, we write $\frac{1}{2}mv^2$ for K_f . Energy conservation leads to

$$mgr = \frac{1}{2}mv^2 \implies v = \sqrt{2gr} = \sqrt{2(9.8)(0.220)} = 2.08 \text{ m/s} .$$

- (b) We note that the expression for the speed ($v = \sqrt{2gr}$) does not contain the mass of the flake. The speed would be the same, 2.08 m/s , regardless of the mass of the flake.
- (c) The final kinetic energy is given by $K_f = K_i + U_i - U_f$. Since K_i is greater than before, K_f is greater. This means the final speed of the flake is greater.
10. We use Eq. 8-17, representing the conservation of mechanical energy (which neglects friction and other dissipative effects).
- (a) In the solution to exercise 2 (to which this problem refers), we found $U_i = mgy_i = 196 \text{ J}$ and $U_f = mgy_f = 29 \text{ J}$ (assuming the reference position is at the ground). Since $K_i = 0$ in this case, we have

$$\begin{aligned} K_i + U_i &= K_f + U_f \\ 0 + 196 &= K_f + 29 \end{aligned}$$

which gives $K_f = 167 \text{ J}$ and thus leads to

$$v = \sqrt{\frac{2K_f}{m}} = \sqrt{\frac{2(167)}{2.00}} = 12.9 \text{ m/s} .$$

- (b) If we proceed algebraically through the calculation in part (a), we find $K_f = -\Delta U = mgh$ where $h = y_i - y_f$ and is positive-valued. Thus,

$$v = \sqrt{\frac{2K_f}{m}} = \sqrt{2gh}$$

as we might also have derived from the equations of Table 2-1 (particularly Eq. 2-16). The fact that the answer is independent of mass means that the answer to part (b) is identical to that of part (a).

- (c) If $K_i \neq 0$, then we find $K_f = mgh + K_i$ (where K_i is necessarily positive-valued). This represents a larger value for K_f than in the previous parts, and thus leads to a larger value for v .
11. We use Eq. 8-18, representing the conservation of mechanical energy (which neglects friction and other dissipative effects).
- (a) In the solution to exercise 5 (to which this problem refers), we found $\Delta U = mgL$ as it goes to the highest point. Thus, we have

$$\begin{aligned}\Delta K + \Delta U &= 0 \\ K_{\text{top}} - K_0 + mgL &= 0\end{aligned}$$

which, upon requiring $K_{\text{top}} = 0$, gives $K_0 = mgL$ and thus leads to

$$v_0 = \sqrt{\frac{2K_0}{m}} = \sqrt{2gL}.$$

- (b) We also found in the solution to exercise 5 that the potential energy change is $\Delta U = -mgL$ in going from the initial point to the lowest point (the bottom). Thus,

$$\begin{aligned}\Delta K + \Delta U &= 0 \\ K_{\text{bottom}} - K_0 - mgL &= 0\end{aligned}$$

which, with $K_0 = mgL$, leads to $K_{\text{bottom}} = 2mgL$. Therefore,

$$v_{\text{bottom}} = \sqrt{\frac{2K_{\text{bottom}}}{m}} = \sqrt{4gL}$$

which simplifies to $2\sqrt{gL}$.

- (c) Since there is no change in height (going from initial point to the rightmost point), then $\Delta U = 0$, which implies $\Delta K = 0$. Consequently, the speed is the same as what it was initially ($\sqrt{2gL}$).
- (d) It is evident from the above manipulations that the results do not depend on mass. Thus, a different mass for the ball must lead to the same results.
12. We use Eq. 8-17, representing the conservation of mechanical energy (which neglects friction and other dissipative effects).

- (a) In the solution to exercise 4, we found $U_A = mgh$ (with the reference position at C). Referring again to Fig. 8-26, we see that this is the same as U_0 which implies that $K_A = K_0$ and thus that $v_A = v_0$.
- (b) In the solution to exercise 4, we also found $U_B = mgh/2$. In this case, we have

$$\begin{aligned}K_0 + U_0 &= K_B + U_B \\ \frac{1}{2}mv_0^2 + mgh &= \frac{1}{2}mv_B^2 + mg\left(\frac{h}{2}\right)\end{aligned}$$

which leads to $v_B = \sqrt{v_0^2 + gh}$.

(c) Similarly, $v_C = \sqrt{v_0^2 + 2gh}$.

(d) To find the “final” height, we set $K_f = 0$. In this case, we have

$$\begin{aligned} K_0 + U_0 &= K_f + U_f \\ \frac{1}{2}mv_0^2 + mgh &= 0 + mgh_f \end{aligned}$$

which leads to $h_f = h + v_0^2/2g$.

(e) It is evident that the above results do not depend on mass. Thus, a different mass for the coaster must lead to the same results.

13. We neglect any work done by friction. We work with SI units, so the speed is converted: $v = 130(1000/3600) = 36.1$ m/s.

(a) We use Eq. 8-17: $K_f + U_f = K_i + U_i$ with $U_i = 0$, $U_f = mgh$ and $K_f = 0$. Since $K_i = \frac{1}{2}mv^2$, where v is the initial speed of the truck, we obtain

$$\frac{1}{2}mv^2 = mgh \implies h = \frac{v^2}{2g} = \frac{36.1^2}{2(9.8)} = 66.5 \text{ m} .$$

If L is the length of the ramp, then $L \sin 15^\circ = 66.5$ m so that $L = 66.5 / \sin 15^\circ = 257$ m. Therefore, the ramp must be about 260 m long if friction is negligible.

(b) The answers do not depend on the mass of the truck. They remain the same if the mass is reduced.

(c) If the speed is decreased, h and L both decrease (note that h is proportional to the square of the speed and that L is proportional to h).

14. We use Eq. 8-18, representing the conservation of mechanical energy. The reference position for computing U is the lowest point of the swing; it is also regarded as the “final” position in our calculations.

(a) In the solution to problem 8 (to which this problem refers), we found $U = mgL(1 - \cos \theta)$ at the position shown in Fig. 8-29 (which we consider to be the initial position). Thus, we have

$$\begin{aligned} K_i + U_i &= K_f + U_f \\ 0 + mgL(1 - \cos \theta) &= \frac{1}{2}mv^2 + 0 \end{aligned}$$

which leads to

$$v = \sqrt{\frac{2mgL(1 - \cos \theta)}{m}} = \sqrt{2gL(1 - \cos \theta)} .$$

Plugging in $L = 2.00$ m and $\theta = 30.0^\circ$ we find $v = 2.29$ m/s.

(b) It is evident that the result for v does not depend on mass. Thus, a different mass for the ball must not change the result.

15. We use Eq. 8-18, representing the conservation of mechanical energy. We choose the reference position for computing U to be at the ground below the cliff; it is also regarded as the “final” position in our calculations.

(a) Using Eq. 8-9, the initial potential energy is $U_i = mgh$ where $h = 12.5$ m and $m = 1.50$ kg. Thus, we have

$$\begin{aligned} K_i + U_i &= K_f + U_f \\ \frac{1}{2}mv_i^2 + mgh &= \frac{1}{2}mv^2 + 0 \end{aligned}$$

which leads to the speed of the snowball at the instant before striking the ground:

$$v = \sqrt{\frac{2}{m} \left(\frac{1}{2} m v_i^2 + mgh \right)} = \sqrt{v_i^2 + 2gh}$$

where $v_i = 14.0$ m/s is the magnitude of its initial velocity (not just one component of it). Thus we find $v = 21.0$ m/s.

- (b) As noted above, v_i is the magnitude of its initial velocity and not just one component of it; therefore, there is no dependence on launch angle. The answer is again 21.0 m/s.
- (c) It is evident that the result for v in part (a) does not depend on mass. Thus, changing the mass of the snowball does not change the result for v .
16. We convert to SI units and choose upward as the $+y$ direction. Also, the relaxed position of the top end of the spring is the origin, so the initial compression of the spring (defining an equilibrium situation between the spring force and the force of gravity) is $y_0 = -0.100$ m and the additional compression brings it to the position $y_1 = -0.400$ m.

- (a) When the stone is in the equilibrium ($a = 0$) position, Newton's second law becomes

$$\begin{aligned} \vec{F}_{\text{net}} &= ma \\ F_{\text{spring}} - mg &= 0 \\ -k(-0.100) - (8.00)(9.8) &= 0 \end{aligned}$$

where Hooke's law (Eq. 7-21) has been used. This leads to a spring constant equal to $k = 784$ N/m.

- (b) With the additional compression (and release) the acceleration is no longer zero, and the stone will start moving upwards, turning some of its elastic potential energy (stored in the spring) into kinetic energy. The amount of elastic potential energy at the moment of release is, using Eq. 8-11,

$$U = \frac{1}{2} k y_1^2 = \frac{1}{2} (784) (-0.400)^2 = 62.7 \text{ J} .$$

- (c) Its maximum height y_2 is beyond the point that the stone separates from the spring (entering free-fall motion). As usual, it is characterized by having (momentarily) zero speed. If we choose the y_1 position as the reference position in computing the gravitational potential energy, then

$$\begin{aligned} K_1 + U_1 &= K_2 + U_2 \\ 0 + \frac{1}{2} k y_1^2 &= 0 + mgh \end{aligned}$$

where $h = y_2 - y_1$ is the height above the release point. Thus, mgh (the gravitational potential energy) is seen to be equal to the previous answer, 62.7 J, and we proceed with the solution in the next part.

- (d) We find $h = k y_1^2 / 2mg = 0.800$ m, or 80.0 cm.

17. We take the reference point for gravitational potential energy at the position of the marble when the spring is compressed.

- (a) The gravitational potential energy when the marble is at the top of its motion is $U_g = mgh$, where $h = 20$ m is the height of the highest point. Thus,

$$U_g = (5.0 \times 10^{-3} \text{ kg}) (9.8 \text{ m/s}^2) (20 \text{ m}) = 0.98 \text{ J} .$$

- (b) Since the kinetic energy is zero at the release point and at the highest point, then conservation of mechanical energy implies $\Delta U_g + \Delta U_s = 0$, where ΔU_s is the change in the spring's elastic potential energy. Therefore, $\Delta U_s = -\Delta U_g = -0.98$ J.

- (c) We take the spring potential energy to be zero when the spring is relaxed. Then, our result in the previous part implies that its initial potential energy is $U_s = 0.98 \text{ J}$. This must be $\frac{1}{2}kx^2$, where k is the spring constant and x is the initial compression. Consequently,

$$k = \frac{2U_s}{x^2} = \frac{2(0.98 \text{ J})}{(0.080 \text{ m})^2} = 3.1 \times 10^2 \text{ N/m} = 3.1 \text{ N/cm} .$$

18. We use Eq. 8-18, representing the conservation of mechanical energy (which neglects friction and other dissipative effects). The reference position for computing U (and height h) is the lowest point of the swing; it is also regarded as the “final” position in our calculations.

- (a) Careful examination of the figure leads to the trigonometric relation $h = L - L \cos \theta$ when the angle is measured from vertical as shown. Thus, the gravitational potential energy is $U = mgL(1 - \cos \theta_0)$ at the position shown in Fig. 8-32 (the initial position). Thus, we have

$$\begin{aligned} K_0 + U_0 &= K_f + U_f \\ \frac{1}{2}mv_0^2 + mgL(1 - \cos \theta_0) &= \frac{1}{2}mv^2 + 0 \end{aligned}$$

which leads to

$$v = \sqrt{\frac{2}{m} \left(\frac{1}{2}mv_0^2 + mgL(1 - \cos \theta_0) \right)} = \sqrt{v_0^2 + 2gL(1 - \cos \theta_0)} .$$

- (b) We look for the initial speed required to barely reach the horizontal position – described by $v_h = 0$ and $\theta = 90^\circ$ (or $\theta = -90^\circ$, if one prefers, but since $\cos(-\phi) = \cos \phi$, the sign of the angle is not a concern).

$$\begin{aligned} K_0 + U_0 &= K_h + U_h \\ \frac{1}{2}mv_0^2 + mgL(1 - \cos \theta_0) &= 0 + mgL \end{aligned}$$

which leads to $v_0 = \sqrt{2gL \cos \theta_0}$.

- (c) For the cord to remain straight, then the centripetal force (at the top) must be (at least) equal to gravitational force:

$$\frac{mv_t^2}{r} = mg \implies mv_t^2 = mgL$$

where we recognize that $r = L$. We plug this into the expression for the kinetic energy (at the top, where $\theta = 180^\circ$).

$$\begin{aligned} K_0 + U_0 &= K_t + U_t \\ \frac{1}{2}mv_0^2 + mgL(1 - \cos \theta_0) &= \frac{1}{2}mv_t^2 + mg(1 - \cos 180^\circ) \\ \frac{1}{2}mv_0^2 + mgL(1 - \cos \theta_0) &= \frac{1}{2}(mgL) + mg(2L) \end{aligned}$$

which leads to $v_0 = \sqrt{gL(3 + 2 \cos \theta_0)}$.

- (d) The more initial potential energy there is, the less initial kinetic energy there needs to be, in order to reach the positions described in parts (b) and (c). Increasing θ_0 amounts to increasing U_0 , so we see that a greater value of θ_0 leads to smaller results for v_0 in parts (b) and (c).
19. The reference point for the gravitational potential energy U_g (and height h) is at the block when the spring is maximally compressed. When the block is moving to its highest point, it is first accelerated by the spring; later, it separates from the spring and finally reaches a point where its speed v_f is (momentarily) zero. The x axis is along the incline, pointing uphill (so x_0 for the initial compression is negative-valued); its origin is at the relaxed position of the spring. We use SI units, so $k = 1960 \text{ N/m}$ and $x_0 = -0.200 \text{ m}$.

- (a) The elastic potential energy is $\frac{1}{2}kx_0^2 = 39.2$ J.
- (b) Since initially $U_g = 0$, the change in U_g is the same as its final value mgh where $m = 2.00$ kg. That this must equal the result in part (a) is made clear in the steps shown in the next part. Thus, $\Delta U_g = U_g = 39.2$ J.
- (c) The principle of mechanical energy conservation leads to

$$\begin{aligned} K_0 + U_0 &= K_f + U_f \\ 0 + \frac{1}{2}kx_0^2 &= 0 + mgh \end{aligned}$$

which yields $h = 2.00$ m. The problem asks for the distance *along the incline*, so we have $d = h/\sin 30^\circ = 4.00$ m.

20. (a) At Q the block (which is in circular motion at that point) experiences a centripetal acceleration v^2/R leftward. We find v^2 from energy conservation:

$$\begin{aligned} K_P + U_P &= K_Q + U_Q \\ 0 + mgh &= \frac{1}{2}mv^2 + mgR \end{aligned}$$

Using the fact (mentioned in problem 6) that $h = 5R$, we find $mv^2 = 8mgR$. Thus, the horizontal component of the net force on the block at Q is $mv^2/R = 8mg$ and points left (in the same direction as \vec{a}).

- (b) The downward component of the net force on the block at Q is the force of gravity mg downward.
- (c) To barely make the top of the loop, the centripetal force there must equal the force of gravity:

$$\frac{mv_t^2}{R} = mg \implies mv_t^2 = mgR$$

This requires a different value of h than was used above.

$$\begin{aligned} K_P + U_P &= K_t + U_t \\ 0 + mgh &= \frac{1}{2}mv_t^2 + mgh_t \\ mgh &= \frac{1}{2}(mgR) + mg(2R) \end{aligned}$$

Consequently, $h = 2.5R$.

- (d) The normal force N , for speeds v_t greater than \sqrt{gR} (which are the only possibilities for non-zero N – see the solution in the previous part), obeys

$$N = \frac{mv_t^2}{R} - mg$$

from Newton's second law. Since v_t^2 is related to h by energy conservation

$$K_P + U_P = K_t + U_t \implies gh = \frac{1}{2}v_t^2 + 2gR$$

then the normal force, as a function for h (so long as $h \geq 2.5R$ – see solution in previous part), becomes

$$N = \frac{2mg}{R}h - 5mg.$$

Thus, the graph for $h \geq 2.5R$ consists of a straight line of positive slope $2mg/R$ (which can be set to some convenient values for graphing purposes). For $h \leq 2.5R$, the normal force is zero. In the interest of saving space, we do not show the graph here.

21. We refer to its starting point as A , the point where it first comes into contact with the spring as B , and the point where the spring is compressed $|x| = 0.055 \text{ m}$ as C . Point C is our reference point for computing gravitational potential energy. Elastic potential energy (of the spring) is zero when the spring is relaxed. Information given in the second sentence allows us to compute the spring constant. From Hooke's law, we find

$$k = \frac{F}{x} = \frac{270 \text{ N}}{0.02 \text{ m}} = 1.35 \times 10^4 \text{ N/m} .$$

- (a) The distance between points A and B is ℓ , and we note that the total sliding distance $\ell + |x|$ is related to the initial height h of the block (measured relative to C) by

$$\frac{h}{\ell + |x|} = \sin \theta$$

where the incline angle θ is 30° . Mechanical energy conservation leads to

$$\begin{aligned} K_A + U_A &= K_C + U_C \\ 0 + mgh &= 0 + \frac{1}{2}kx^2 \end{aligned}$$

which yields

$$h = \frac{kx^2}{2mg} = \frac{(1.35 \times 10^4 \text{ N/m})(0.055 \text{ m})^2}{2(12 \text{ kg})(9.8 \text{ m/s}^2)} = 0.174 \text{ m} .$$

Therefore,

$$\ell + |x| = \frac{h}{\sin 30^\circ} = \frac{0.174 \text{ m}}{\sin 30^\circ} = 0.35 \text{ m} .$$

- (b) From this result, we find $\ell = 0.35 - 0.055 = 0.29 \text{ m}$, which means $\Delta y = -\ell \sin \theta = -0.15 \text{ m}$ in sliding from point A to point B . Thus, Eq. 8-18 gives

$$\begin{aligned} \Delta K + \Delta U &= 0 \\ \frac{1}{2}mv_B^2 + mg\Delta h &= 0 \end{aligned}$$

which yields

$$v_B = \sqrt{-2g\Delta h} = \sqrt{-(9.8)(-0.15)} = 1.7 \text{ m/s} .$$

22. Since time does not directly enter into the energy formulations, we return to Chapter 4 (or Table 2-1 in Chapter 2) to find the change of height during this $t = 6 \text{ s}$ flight.

$$\Delta y = v_{0y}t - \frac{1}{2}gt^2$$

This leads to $\Delta y = -32 \text{ m}$. Therefore $\Delta U = mg\Delta y = -318 \approx -320 \text{ J}$.

23. (a) As the string reaches its lowest point, its original potential energy $U = mgL$ (measured relative to the lowest point) is converted into kinetic energy. Thus,

$$mgL = \frac{1}{2}mv^2 \implies v = \sqrt{2gL} .$$

With $L = 1.20 \text{ m}$ we obtain $v = 4.85 \text{ m/s}$.

- (b) In this case, the total mechanical energy is shared between kinetic $\frac{1}{2}mv_b^2$ and potential $mg y_b$. We note that $y_b = 2r$ where $r = L - d = 0.450 \text{ m}$. Energy conservation leads to

$$mgL = \frac{1}{2}mv_b^2 + mg y_b$$

which yields

$$v_b = \sqrt{2gL - 2g(2r)} = 2.42 \text{ m/s} .$$

24. From Chapter 4, we know the height h of the skier's jump can be found from $v_y^2 = 0 = v_{0y}^2 - 2gh$ where $v_{0y} = v_0 \sin 28^\circ$ is the upward component of the skier's "launch velocity." To find v_0 we use energy conservation.

(a) The skier starts at rest $y = 20$ m above the point of "launch" so energy conservation leads to

$$mgy = \frac{1}{2}mv^2 \implies v = \sqrt{2gy} = 20 \text{ m/s}$$

which becomes the initial speed v_0 for the launch. Hence, the above equation relating h to v_0 yields

$$h = \frac{(v_0 \sin 28^\circ)^2}{2g} = 4.4 \text{ m}.$$

(b) We see that all reference to mass cancels from the above computations, so a new value for the mass will yield the same result as before.

25. We denote m as the mass of the block, $h = 0.40$ m as the height from which it dropped (measured from the relaxed position of the spring), and x the compression of the spring (measured downward so that it yields a positive value). Our reference point for the gravitational potential energy is the initial position of the block. The block drops a total distance $h + x$, and the final gravitational potential energy is $-mg(h + x)$. The spring potential energy is $\frac{1}{2}kx^2$ in the final situation, and the kinetic energy is zero both at the beginning and end. Since energy is conserved

$$\begin{aligned} K_i + U_i &= K_f + U_f \\ 0 &= -mg(h + x) + \frac{1}{2}kx^2 \end{aligned}$$

which is a second degree equation in x . Using the quadratic formula, its solution is

$$x = \frac{mg \pm \sqrt{(mg)^2 + 2mghk}}{k}.$$

Now $mg = 19.6$ N, $h = 0.40$ m, and $k = 1960$ N/m, and we choose the positive root so that $x > 0$.

$$x = \frac{19.6 + \sqrt{19.6^2 + 2(19.6)(0.40)(1960)}}{1960} = 0.10 \text{ m}.$$

26. To find out whether or not the vine breaks, it is sufficient to examine it at the moment Tarzan swings through the lowest point, which is when the vine – if it didn't break – would have the greatest tension. Choosing upward positive, Newton's second law leads to

$$T - mg = m\frac{v^2}{r}$$

where $r = 18$ m and $m = W/g = 688/9.8 = 70.2$ kg. We find the v^2 from energy conservation (where the reference position for the potential energy is at the lowest point).

$$mgh = \frac{1}{2}mv^2 \implies v^2 = 2gh$$

where $h = 3.2$ m. Combining these results, we have

$$T = mg + m\frac{2gh}{r} = mg \left(1 + \frac{2h}{r} \right)$$

which yields 933 N. Thus, the vine does not break. And rounding to an appropriate number of significant figures, we see the maximum tension is roughly 930 N.

27. The distance the marble travels is determined by its initial speed (and the methods of Chapter 4), and the initial speed is determined (using energy conservation) by the original compression of the spring. We denote h as the height of the table, and x as the horizontal distance to the point where the marble lands. Then $x = v_0 t$ and $h = \frac{1}{2}gt^2$ (since the vertical component of the marble's "launch velocity" is zero). From these we find $x = v_0 \sqrt{2h/g}$. We note from this that the distance to the landing point is directly proportional to the initial speed. We denote v_{01} be the initial speed of the first shot and $x_1 = 1.93$ m be the horizontal distance to its landing point; similarly, v_{02} is the initial speed of the second shot and $x_2 = 2.20$ m is the horizontal distance to its landing spot. Then

$$\frac{v_{02}}{v_{01}} = \frac{x_2}{x_1} \implies v_{02} = \frac{x_2}{x_1} v_{01} .$$

When the spring is compressed an amount ℓ , the elastic potential energy is $\frac{1}{2}k\ell^2$. When the marble leaves the spring its kinetic energy is $\frac{1}{2}mv_0^2$. Mechanical energy is conserved: $\frac{1}{2}mv_0^2 = \frac{1}{2}k\ell^2$, and we see that the initial speed of the marble is directly proportional to the original compression of the spring. If ℓ_1 is the compression for the first shot and ℓ_2 is the compression for the second, then $v_{02} = (\ell_2/\ell_1)v_{01}$. Relating this to the previous result, we obtain

$$\ell_2 = \frac{x_2}{x_1} \ell_1 = \left(\frac{2.20 \text{ m}}{1.93 \text{ m}} \right) (1.10 \text{ cm}) = 1.25 \text{ cm} .$$

28. We place the reference position for evaluating gravitational potential energy at the relaxed position of the spring. We use x for the spring's compression, measured positively downwards (so $x > 0$ means it is compressed).
- (a) With $x = 0.190$ m, Eq. 7-26 gives $W_s = -\frac{1}{2}kx^2 = -7.22$ J for the work done by the spring force. Using Newton's third law, we conclude the work done on the spring is 7.22 J.
- (b) As noted above, $W_s = -7.22$ J.
- (c) Energy conservation leads to

$$\begin{aligned} K_i + U_i &= K_f + U_f \\ mgh_0 &= -mgx + \frac{1}{2}kx^2 \end{aligned}$$

which (with $m = 0.700$ kg) yields $h_0 = 0.862$ m.

- (d) With a new value for the height $h'_0 = 2h_0 = 1.72$ m, we solve for a new value of x using the quadratic formula (taking its positive root so that $x > 0$).

$$mgh'_0 = -mgx + \frac{1}{2}kx^2 \implies x = \frac{mg + \sqrt{(mg)^2 + 2mgkh'_0}}{k}$$

which yields $x = 0.261$ m.

29. We use conservation of mechanical energy: the mechanical energy must be the same at the top of the swing as it is initially. Newton's second law is used to find the speed, and hence the kinetic energy, at the top. There the tension force T of the string and the force of gravity are both downward, toward the center of the circle. We notice that the radius of the circle is $r = L - d$, so the law can be written $T + mg = mv^2/(L - d)$, where v is the speed and m is the mass of the ball. When the ball passes the highest point with the least possible speed, the tension is zero. Then

$$mg = m \frac{v^2}{L - d} \implies v = \sqrt{g(L - d)} .$$

We take the gravitational potential energy of the ball-Earth system to be zero when the ball is at the bottom of its swing. Then the initial potential energy is mgL . The initial kinetic energy is zero since

the ball starts from rest. The final potential energy, at the top of the swing, is $2mg(L - d)$ and the final kinetic energy is $\frac{1}{2}mv^2 = \frac{1}{2}mg(L - d)$ using the above result for v . Conservation of energy yields

$$mgL = 2mg(L - d) + \frac{1}{2}mg(L - d) \implies d = 3L/5.$$

If d is greater than this value, so the highest point is lower, then the speed of the ball is greater as it reaches that point and the ball passes the point. If d is less, the ball cannot go around. Thus the value we found for d is a lower limit.

30. The connection between angle θ (measured from vertical – see Fig. 8-29) and height h (measured from the lowest point, which is our choice of reference position in computing the gravitational potential energy) is given by $h = L(1 - \cos \theta)$ where L is the length of the pendulum.

(a) We use energy conservation in the form of Eq. 8-17.

$$\begin{aligned} K_1 + U_1 &= K_2 + U_2 \\ 0 + mgL(1 - \cos \theta_1) &= \frac{1}{2}mv_2^2 + mgL(1 - \cos \theta_2) \end{aligned}$$

This leads to

$$v_2 = \sqrt{2gL(\cos \theta_2 - \cos \theta_1)} = 1.4 \text{ m/s}$$

since $L = 1.4 \text{ m}$, $\theta_1 = 30^\circ$, and $\theta_2 = 20^\circ$.

- (b) The maximum speed v_3 is at the lowest point. Our formula for h gives $h_3 = 0$ when $\theta_3 = 0^\circ$, as expected.

$$\begin{aligned} K_1 + U_1 &= K_3 + U_3 \\ 0 + mgL(1 - \cos \theta_1) &= \frac{1}{2}mv_3^2 + 0 \end{aligned}$$

This yields $v_3 = 1.9 \text{ m/s}$.

- (c) We look for an angle θ_4 such that the speed there is $v_4 = v_3/3$. To be as accurate as possible, we proceed algebraically (substituting $v_3^2 = 2gL(1 - \cos \theta_1)$ at the appropriate place) and plug numbers in at the end. Energy conservation leads to

$$\begin{aligned} K_1 + U_1 &= K_4 + U_4 \\ 0 + mgL(1 - \cos \theta_1) &= \frac{1}{2}mv_4^2 + mgL(1 - \cos \theta_4) \\ mgL(1 - \cos \theta_1) &= \frac{1}{2}m\frac{v_3^2}{9} + mgL(1 - \cos \theta_4) \\ -gL\cos \theta_1 &= \frac{1}{2}\frac{2gL(1 - \cos \theta_1)}{9} - gL\cos \theta_4 \end{aligned}$$

where in the last step we have subtracted out mgL and then divided by m . Thus, we obtain

$$\theta_4 = \cos^{-1} \left(\frac{1}{9} + \frac{8}{9} \cos \theta_1 \right) = 28.2^\circ$$

where we have quoted the answer to three significant figures since the problem gives θ_1 to three figures.

31. The connection between angle θ (measured from vertical – see Fig. 8-29) and height h (measured from the lowest point, which is our choice of reference position in computing the gravitational potential energy mgh) is given by $h = L(1 - \cos \theta)$ where L is the length of the pendulum.

- (a) Using this formula (or simply using intuition) we see the initial height is $h_1 = 2L$, and of course $h_2 = 0$. We use energy conservation in the form of Eq. 8-17.

$$\begin{aligned} K_1 + U_1 &= K_2 + U_2 \\ 0 + mg(2L) &= \frac{1}{2}mv^2 + 0 \end{aligned}$$

This leads to $v = 2\sqrt{gL}$.

- (b) The ball is in circular motion with the center of the circle above it, so $\vec{a} = v^2/r$ upward, where $r = L$. Newton's second law leads to

$$T - mg = m \frac{v^2}{r} \implies T = m \left(g + \frac{4gL}{L} \right) = 5mg .$$

- (c) The pendulum is now started (with zero speed) at $\theta_i = 90^\circ$ (that is, $h_i = L$), and we look for an angle θ such that $T = mg$. When the ball is moving through a point at angle θ , then Newton's second law applied to the axis along the rod yields

$$T - mg \cos \theta = m \frac{v^2}{r}$$

which (since $r = L$) implies $v^2 = gL(1 - \cos \theta)$ at the position we are looking for. Energy conservation leads to

$$\begin{aligned} K_i + U_i &= K + U \\ 0 + mgL &= \frac{1}{2}mv^2 + mgL(1 - \cos \theta) \\ gL &= \frac{1}{2}(gL(1 - \cos \theta)) + gL(1 - \cos \theta) \end{aligned}$$

where we have divided by mass in the last step. Simplifying, we obtain

$$\theta = \cos^{-1} \left(\frac{1}{3} \right) = 70.5^\circ .$$

32. All heights h are measured from the lower end of the incline (which is our reference position for computing gravitational potential energy mgh). Our x axis is along the incline, with $+x$ being uphill (so spring compression corresponds to $x > 0$) and its origin being at the relaxed end of the spring. The 1.00 m distance indicated in Fig. 8-40 will be referred to as ℓ , and the 37.0° angle will be referred to as θ . Thus, the height that corresponds to the canister's initial position (with spring compressed amount $x = 0.200$ m) is given by $h_1 = (\ell + x) \sin \theta$.

- (a) Energy conservation leads to

$$\begin{aligned} K_1 + U_1 &= K_2 + U_2 \\ 0 + mg(\ell + x) \sin \theta + \frac{1}{2}kx^2 &= \frac{1}{2}mv_2^2 + mg\ell \sin \theta \end{aligned}$$

which yields $v_2 = \sqrt{2gx \sin \theta + kx^2/m} = 2.40$ m/s using the data $m = 2.00$ kg and $k = 170$ N/m.

- (b) In this case, energy conservation leads to

$$\begin{aligned} K_1 + U_1 &= K_3 + U_3 \\ 0 + mg(\ell + x) \sin \theta + \frac{1}{2}kx^2 &= \frac{1}{2}mv_3^2 + 0 \end{aligned}$$

which yields $v_3 = \sqrt{2g(\ell + x) \sin \theta + kx^2/m} = 4.19$ m/s.

33. The work required is the change in the gravitational potential energy as a result of the chain being pulled onto the table. Dividing the hanging chain into a large number of infinitesimal segments, each of length dy , we note that the mass of a segment is $(m/L) dy$ and the change in potential energy of a segment when it is a distance $|y|$ below the table top is $dU = (m/L)g|y| dy = -(m/L)gy dy$ since y is negative-valued (we have $+y$ upward and the origin is at the tabletop). The total potential energy change is

$$\Delta U = -\frac{mg}{L} \int_{-L/4}^0 y dy = \frac{1}{2} \frac{mg}{L} (L/4)^2 = mgL/32.$$

The work required to pull the chain onto the table is therefore $W = \Delta U = mgL/32$.

34. If the description of the scenario seems confusing, reference to Figure 8-31 in the textbook is helpful. We note that the block being unattached means that for $y > 0.25$ m, the elastic potential energy vanishes. With $k = 400$ N/m, $m = 40.0/9.8 = 4.08$ kg and length in meters, the energy equation is

$$E = \begin{cases} \frac{1}{2}k\left(\frac{1}{4}\right)^2 & y = 0 \\ K + mgy + \frac{1}{2}k\left(\frac{1}{4} - y\right)^2 & 0 \leq y \leq \frac{1}{4} \\ K + mgy & \frac{1}{4} \leq y \end{cases}$$

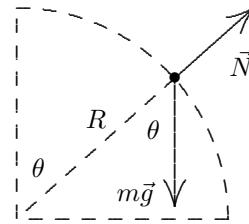
In this way, the kinetic energy K for each region is related to E – which by conservation of energy is always equal to the value 12.5 J that it had at $y = 0$. We arrange our results in a table (with energies in Joules) where it is clear that the sum of each column (of energies) is 12.5 J:

part	(a)	(b)	(c)	(d)	(e)	(f)	(g)
position y	0	0.05	0.10	0.15	0.20	0.25	0.30
U_g	0	2.0	4.0	6.0	8.0	10.0	12.0
U_e	12.5	8.0	4.5	2.0	0.5	0	0
K	0	2.5	4.0	4.5	4.0	2.5	0.5

Finally (for part (h)), where $y \geq 0.25$ m, we have $K = E - mgy$, so that $K = 0$ occurs when $y = (12.5 \text{ J})/(40 \text{ N}) = 0.313$ m.

35. The free-body diagram for the boy is shown below. \vec{N} is the normal force of the ice on him and m is his mass. The net inward force is $mg \cos \theta - N$ and, according to Newton's second law, this must be equal to mv^2/R , where v is the speed of the boy. At the point where the boy leaves the ice $N = 0$, so $g \cos \theta = v^2/R$. We wish to find his speed. If the gravitational potential energy is taken to be zero when he is at the top of the ice mound, then his potential energy at the time shown is $U = -mgR(1 - \cos \theta)$.

He starts from rest and his kinetic energy at the time shown is $\frac{1}{2}mv^2$. Thus conservation of energy gives $0 = \frac{1}{2}mv^2 - mgR(1 - \cos \theta)$, or $v^2 = 2gR(1 - \cos \theta)$. We substitute this expression into the equation developed from the second law to obtain $g \cos \theta = 2g(1 - \cos \theta)$. This gives $\cos \theta = 2/3$. The height of the boy above the bottom of the mound is $R \cos \theta = 2R/3$.



36. We use Eq. 8-20 and various observations made in §8-5.

(a) The force at $x = 2.0$ m is

$$F = -\frac{dU}{dx} \approx -\frac{(-17.5) - (-2.8)}{4.0 - 1.0} = 4.9 \text{ N}$$

in the $+x$ direction (but there is some uncertainty in reading the graph which makes the last digit not very significant).

- (b) The total mechanical energy at $x = 2.0$ m is

$$E = \frac{1}{2}mv^2 + U \approx \frac{1}{2}(2.0)(-1.5)^2 - 7.7 = -5.5$$

in SI units (Joules). Again, there is some uncertainty in reading the graph which makes the last digit not very significant. At that level (-5.5 J) on the graph, we find two points where the potential energy curve has that value – at $x \approx 1.5$ m and $x \approx 13.5$ m. Therefore, the particle remains in the region $1.5 < x < 13.5$ m.

- (c) At $x = 7.0$ m, we read $U \approx -17.5$ J. Thus, if its total energy (calculated in the previous part) is $E \approx -5.5$ J, then we find

$$\frac{1}{2}mv^2 = E - U \approx 12 \text{ J} \implies v = \sqrt{\frac{2}{m}(E - U)} \approx 3.5 \text{ m/s}$$

where there is certainly room for disagreement on that last digit for the reasons cited above.

37. We use Eq. 8-20 and various observations made in §8-5.

- (a) The force at the equilibrium position $r = r_{\text{eq}}$ is

$$\begin{aligned} F = -\frac{dU}{dr} \Big|_{r=r_{\text{eq}}} &= 0 \\ -\frac{12A}{r_{\text{eq}}^{13}} + \frac{6B}{r_{\text{eq}}^7} &= 0 \end{aligned}$$

which leads to the result

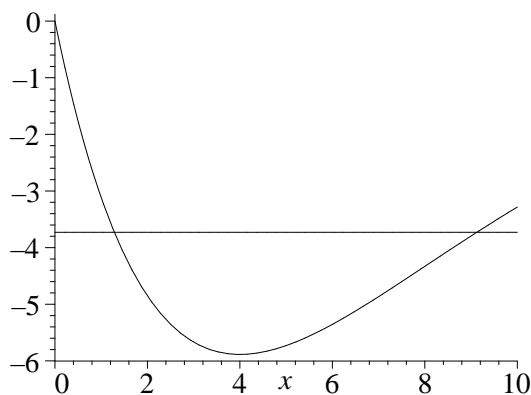
$$r_{\text{eq}} = \left(\frac{2A}{B}\right)^{\frac{1}{6}} = 1.12 \left(\frac{A}{B}\right)^{\frac{1}{6}}.$$

- (b) This defines a minimum in the potential energy curve (as can be verified either by a graph or by taking another derivative and verifying that it is concave upward at this point), which means that for values of r slightly smaller than r_{eq} the slope of the curve is negative (so the force is positive, repulsive).

- (c) And for values of r slightly larger than r_{eq} the slope of the curve must be positive (so the force is negative, attractive).

38. (a) The energy at $x = 5.0$ m is $E = K + U = 2.0 - 5.7 = -3.7$ J.

- (b) A plot of the potential energy curve (SI units understood) and the energy E (the horizontal line) is shown for $0 \leq x \leq 10$ m.



- (c) The problem asks for a graphical determination of the turning points, which are the points on the curve corresponding to the total energy computed in part (a). The result for the smallest turning point (determined, to be honest, by more careful means) is $x = 1.29$ m.
- (d) And the result for the largest turning point is $x = 9.12$ m.
- (e) Since $K = E - U$, then maximizing K involves finding the minimum of U . A graphical determination suggests that this occurs at $x = 4.0$ m, which plugs into the expression $E - U = -3.7 - (-4xe^{-x/4})$ to give $K = 2.16$ J. Alternatively, one can measure from the graph from the minimum of the U curve up to the level representing the total energy E and thereby obtain an estimate of K at that point.
- (f) As mentioned in the previous part, the minimum of the U curve occurs at $x = 4.0$ m.
- (g) The force (understood to be in Newtons) follows from the potential energy, using Eq. 8-20 (and Appendix E if students are unfamiliar with such derivatives).

$$F = \frac{dU}{dx} = (4 - x)e^{-x/4}$$

- (h) This revisits the considerations of parts (d) and (e) (since we are returning to the minimum of $U(x)$) – but now with the advantage of having the analytic result of part (g). We see that the location which produces $F = 0$ is exactly $x = 4$ m.
39. (a) Using Eq. 7-8, we have

$$W_{\text{applied}} = (8.0 \text{ N})(0.70 \text{ m}) = 5.6 \text{ J} .$$

- (b) Using Eq. 8-29, the thermal energy generated is

$$\Delta E_{\text{th}} = f_k d = (5.0 \text{ N})(0.70 \text{ m}) = 3.5 \text{ J} .$$

40. Since the speed is constant $\Delta K = 0$ and Eq. 8-31 (an application of the energy conservation concept) implies

$$W_{\text{applied}} = \Delta E_{\text{th}} = \Delta E_{\text{th (cube)}} + \Delta E_{\text{th (floor)}} .$$

Thus, if $W_{\text{applied}} = (15)(3.0) = 45$ J, and we are told that $\Delta E_{\text{th (cube)}} = 20$ J, then we conclude that $\Delta E_{\text{th (floor)}} = 25$ J.

41. (a) The work done on the block by the force in the rope is, using Eq. 7-7,

$$W = Fd \cos \theta = (7.68 \text{ N})(4.06 \text{ m}) \cos 15.0^\circ = 30.1 \text{ J} .$$

- (b) Using f for the magnitude of the kinetic friction force, Eq. 8-29 reveals that the increase in thermal energy is

$$\Delta E_{\text{th}} = fd = (7.42 \text{ N})(4.06 \text{ m}) = 30.1 \text{ J} .$$

- (c) We can use Newton's second law of motion to obtain the frictional and normal forces, then use $\mu_k = f/N$ to obtain the coefficient of friction. Place the x axis along the path of the block and the y axis normal to the floor. The x component of Newton's second law is $F \cos \theta - f = 0$ and the y component is $N + F \sin \theta - mg = 0$, where m is the mass of the block, F is the force exerted by the rope, and θ is the angle between that force and the horizontal. The first equation gives $f = F \cos \theta = (7.68) \cos 15.0^\circ = 7.42$ N and the second gives $N = mg - F \sin \theta = (3.57)(9.8) - (7.68) \sin 15.0^\circ = 33.0$ N. Thus

$$\mu_k = \frac{f}{N} = \frac{7.42 \text{ N}}{33.0 \text{ N}} = 0.22 .$$

42. Since the velocity is constant, $\vec{a} = 0$ and the horizontal component of the worker's push $F \cos \theta$ (where $\theta = 32^\circ$) must equal the friction force magnitude $f_k = \mu_k N$. Also, the vertical forces must cancel, implying

$$N = mg + F \sin \theta \implies F \cos \theta = \mu_k (mg + F \sin \theta)$$

which is solved to find $F = 71$ N.

- (a) The work done on the block by the worker is, using Eq. 7-7,

$$W = Fd \cos \theta = (71 \text{ N})(9.2 \text{ m}) \cos 32^\circ = 5.6 \times 10^2 \text{ J} .$$

- (b) Since $f_k = \mu_k (mg + F \sin \theta)$, we find

$$\Delta E_{\text{th}} = f_k d = (60 \text{ N})(9.2 \text{ m}) = 5.6 \times 10^2 \text{ J} .$$

43. (a) We take the initial gravitational potential energy to be $U_i = 0$. Then the final gravitational potential energy is $U_f = -mgL$, where L is the length of the tree. The change is

$$U_f - U_i = -mgL = -(25 \text{ kg}) (9.8 \text{ m/s}^2) (12 \text{ m}) = -2.9 \times 10^3 \text{ J} .$$

- (b) The kinetic energy is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(25 \text{ kg})(5.6 \text{ m/s})^2 = 3.9 \times 10^2 \text{ J} .$$

- (c) The changes in the mechanical and thermal energies must sum to zero. The change in thermal energy is $\Delta E_{\text{th}} = fL$, where f is the magnitude of the average frictional force; therefore,

$$f = -\frac{\Delta K + \Delta U}{L} = -\frac{3.92 \times 10^2 \text{ J} - 2.94 \times 10^3 \text{ J}}{12 \text{ m}} = 210 \text{ N} .$$

44. We use SI units so $m = 0.030$ kg and $d = 0.12$ m.

- (a) Since there is no change in height (and we assume no changes in elastic potential energy), then $\Delta U = 0$ and we have

$$\Delta E_{\text{mech}} = \Delta K = -\frac{1}{2}mv_0^2 = -3.8 \times 10^3 \text{ J}$$

where $v_0 = 500$ m/s and the final speed is zero.

- (b) By Eq. 8-31 (with $W = 0$) we have $\Delta E_{\text{th}} = 3.8 \times 10^3 \text{ J}$, which implies

$$f = \frac{\Delta E_{\text{th}}}{d} = 3.1 \times 10^4 \text{ N}$$

using Eq. 8-29 with f_k replaced by f (effectively generalizing that equation to include a greater variety of dissipative forces than just those obeying Eq. 6-2).

45. Equation 8-31 provides $\Delta E_{\text{th}} = -\Delta E_{\text{mec}}$ for the energy “lost” in the sense of this problem. Thus,

$$\begin{aligned} \Delta E_{\text{th}} &= \frac{1}{2}m(v_i^2 - v_f^2) + mg(y_i - y_f) \\ &= \frac{1}{2}(60)(24^2 - 22^2) + (60)(9.8)(14) \\ &= 1.1 \times 10^4 \text{ J} . \end{aligned}$$

That the angle of 25° is nowhere used in this calculation is indicative of the fact that energy is a scalar quantity.

46. We use SI units so $m = 0.075$ kg. Equation 8-30 provides $\Delta E_{\text{th}} = -\Delta E_{\text{mec}}$ for the energy “lost” in the sense of this problem. Thus,

$$\begin{aligned}\Delta E_{\text{th}} &= \frac{1}{2}m(v_i^2 - v_f^2) + mg(y_i - y_f) \\ &= \frac{1}{2}(0.075)(12^2 - 10.5^2) + (0.075)(9.8)(1.1 - 2.1) \\ &= 0.53 \text{ J} .\end{aligned}$$

47. We work this using the English units (with $g = 32$ ft/s), but for consistency we convert the weight to pounds

$$mg = (9.0 \text{ oz}) \left(\frac{1 \text{ lb}}{16 \text{ oz}} \right) = 0.56 \text{ lb}$$

which implies $m = 0.018 \text{ lb}\cdot\text{s}^2/\text{ft}$ (which can be phrased as 0.018 slug as explained in Appendix D). And we convert the initial speed to feet-per-second

$$v_i = (81.8 \text{ mi/h}) \left(\frac{5280 \text{ ft/mi}}{3600 \text{ s/h}} \right) = 120 \text{ ft/s}$$

or a more “direct” conversion from Appendix D can be used. Equation 8-30 provides $\Delta E_{\text{th}} = -\Delta E_{\text{mec}}$ for the energy “lost” in the sense of this problem. Thus,

$$\begin{aligned}\Delta E_{\text{th}} &= \frac{1}{2}m(v_i^2 - v_f^2) + mg(y_i - y_f) \\ &= \frac{1}{2}(0.018)(120^2 - 110^2) + 0 \\ &= 20 \text{ ft} \cdot \text{lb} .\end{aligned}$$

48. (a) During one second, the decrease in potential energy is

$$-\Delta U = mg(-\Delta y) = (5.5 \times 10^6 \text{ kg}) (9.8 \text{ m/s}^2) (50 \text{ m}) = 2.7 \times 10^9 \text{ J}$$

where $+y$ is upward and $\Delta y = y_f - y_i$.

- (b) The information relating mass to volume is not needed in the computation. By Eq. 8-36 (and the SI relation $W = J/s$), the result follows: $(2.7 \times 10^9 \text{ J})/(1 \text{ s}) = 2.7 \times 10^9 \text{ W}$.
(c) One year is equivalent to $24 \times 365.25 = 8766$ h which we write as 8.77 kh. Thus, the energy supply rate multiplied by the cost and by the time is

$$(2.7 \times 10^9 \text{ W}) (8.77 \text{ kh}) \left(\frac{1 \text{ cent}}{1 \text{ kWh}} \right) = 2.4 \times 10^{10} \text{ cents}$$

which equals $\$2.4 \times 10^8$.

49. (a) The initial potential energy is

$$U_i = mgy_i = (520 \text{ kg}) (9.8 \text{ m/s}^2) (300 \text{ m}) = 1.53 \times 10^6 \text{ J}$$

where $+y$ is upward and $y = 0$ at the bottom (so that $U_f = 0$).

- (b) Since $f_k = \mu_k N = \mu_k mg \cos \theta$ we have

$$\Delta E_{\text{th}} = f_k d = \mu_k mg d \cos \theta$$

from Eq. 8-29. Now, the hillside surface (of length $d = 500$ m) is treated as an hypotenuse of a 3-4-5 triangle, so $\cos \theta = x/d$ where $x = 400$ m. Therefore,

$$\Delta E_{\text{th}} = \mu_k mg d \frac{x}{d} = \mu_k mg x = (0.25)(520)(9.8)(400) = 5.1 \times 10^5 \text{ J} .$$

(c) Using Eq. 8-31 (with $W = 0$) we find

$$\begin{aligned} K_f &= K_i + U_i - U_f - \Delta E_{\text{th}} \\ &= 0 + 1.53 \times 10^6 - 0 - 5.1 \times 10^5 \\ &= 0 + 1.02 \times 10^6 \text{ J} . \end{aligned}$$

(d) From $K_f = \frac{1}{2}mv^2$ we obtain $v = 62.6 \text{ m/s}$.

50. Energy conservation, as expressed by Eq. 8-31 (with $W = 0$) leads to

$$\begin{aligned} \Delta E_{\text{th}} &= K_i - K_f + U_i - U_f \\ f_k d &= 0 - 0 + \frac{1}{2}kx^2 - 0 \\ \mu_k mgd &= \frac{1}{2}(200 \text{ N/m})(0.15 \text{ m})^2 \\ \mu_k(2.0 \text{ kg})(9.8 \text{ m/s}^2)(0.75 \text{ m}) &= 2.25 \text{ J} \end{aligned}$$

which yields $\mu_k = 0.15$ as the coefficient of kinetic friction.

51. (a) The vertical forces acting on the block are the normal force, upward, and the force of gravity, downward. Since the vertical component of the block's acceleration is zero, Newton's second law requires $N = mg$, where m is the mass of the block. Thus $f = \mu_k N = \mu_k mg$. The increase in thermal energy is given by $\Delta E_{\text{th}} = fd = \mu_k mgd$, where d is the distance the block moves before coming to rest. Using Eq. 8-29, we have

$$\Delta E_{\text{th}} = (0.25)(3.5 \text{ kg})(9.8 \text{ m/s}^2)(7.8 \text{ m}) = 67 \text{ J} .$$

(b) The block has its maximum kinetic energy K_{max} just as it leaves the spring and enters the region where friction acts. Therefore, the maximum kinetic energy equals the thermal energy generated in bringing the block back to rest, 67 J.

(c) The energy that appears as kinetic energy is originally in the form of potential energy in the compressed spring. Thus $K_{\text{max}} = U_i = \frac{1}{2}kx^2$, where k is the spring constant and x is the compression. Thus,

$$x = \sqrt{\frac{2K_{\text{max}}}{k}} = \sqrt{\frac{2(67 \text{ J})}{640 \text{ N/m}}} = 0.46 \text{ m} .$$

52. We use Eq. 8-29

$$\Delta E_{\text{th}} = f_k d = (10 \text{ N})(5.0 \text{ m}) = 50 \text{ J}$$

and Eq. 7-8

$$W = Fd = (2.0 \text{ N})(5.0 \text{ m}) = 10 \text{ J}$$

and Eq. 8-31

$$\begin{aligned} W &= \Delta K + \Delta U + \Delta E_{\text{th}} \\ 10 &= 35 + \Delta U + 50 \end{aligned}$$

which yields $\Delta U = -75 \text{ J}$. By Eq. 8-1, then, the work done by gravity is $W = -\Delta U = 75 \text{ J}$.

53. (a) To stretch the spring an external force, equal in magnitude to the force of the spring but opposite to its direction, is applied. Since a spring stretched in the positive x direction exerts a force in the negative x direction, the applied force must be $F = 52.8x + 38.4x^2$, in the $+x$ direction. The work it does is

$$W = \int_{0.50}^{1.00} (52.8x + 38.4x^2) dx = \left[\frac{52.8}{2}x^2 + \frac{38.4}{3}x^3 \right]_{0.50}^{1.00} = 31.0 \text{ J} .$$

- (b) The spring does 31.0 J of work and this must be the increase in the kinetic energy of the particle. Its speed is then

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(31.0 \text{ J})}{2.17 \text{ kg}}} = 5.35 \text{ m/s} .$$

- (c) The force is conservative since the work it does as the particle goes from any point x_1 to any other point x_2 depends only on x_1 and x_2 , not on details of the motion between x_1 and x_2 .
54. We look for the distance along the incline d which is related to the height ascended by $\Delta h = d \sin \theta$. By a force analysis of the style done in Ch. 6, we find the normal force has magnitude $N = mg \cos \theta$ which means $f_k = \mu_k mg \cos \theta$. Thus, Eq. 8-31 (with $W = 0$) leads to

$$\begin{aligned} 0 &= K_f - K_i + \Delta U + \Delta E_{\text{th}} \\ &= 0 - K_i + mgd \sin \theta + \mu_k mgd \cos \theta \end{aligned}$$

which leads to

$$d = \frac{K_i}{mg(\sin \theta + \mu_k \cos \theta)} = \frac{128}{(4.0)(9.8)(\sin 30^\circ + 0.30 \cos 30^\circ)} = 4.3 \text{ m} .$$

55. (a) We take the gravitational potential energy of the skier-Earth system to be zero when the skier is at the bottom of the peaks. The initial potential energy is $U_i = mgh_i$, where m is the mass of the skier, and h_i is the height of the higher peak. The final potential energy is $U_f = mgh_f$, where h_f is the height of the lower peak. The skier initially has a kinetic energy of $K_i = 0$, and the final kinetic energy is $K_f = \frac{1}{2}mv^2$, where v is the speed of the skier at the top of the lower peak. The normal force of the slope on the skier does no work and friction is negligible, so mechanical energy is conserved.

$$\begin{aligned} U_i + K_i &= U_f + K_f \\ mgh_i &= mgh_f + \frac{1}{2}mv^2 \end{aligned}$$

Thus,

$$v = \sqrt{2g(h_i - h_f)} = \sqrt{2(9.8)(850 - 750)} = 44 \text{ m/s} .$$

- (b) We recall from analyzing objects sliding down inclined planes that the normal force of the slope on the skier is given by $N = mg \cos \theta$, where θ is the angle of the slope from the horizontal, 30° for each of the slopes shown. The magnitude of the force of friction is given by $f = \mu_k N = \mu_k mg \cos \theta$. The thermal energy generated by the force of friction is $f d = \mu_k mgd \cos \theta$, where d is the total distance along the path. Since the skier gets to the top of the lower peak with no kinetic energy, the increase in thermal energy is equal to the decrease in potential energy. That is, $\mu_k mgd \cos \theta = mg(h_i - h_f)$. Consequently,

$$\mu_k = \frac{(h_i - h_f)}{d \cos \theta} = \frac{(850 - 750)}{(3.2 \times 10^3) \cos 30^\circ} = 0.036 .$$

56. (a) By a force analysis in the style of Chapter 6, we find the normal force $N = mg \cos \theta$ (where $mg = 267 \text{ N}$) which means $f_k = \mu_k mg \cos \theta$. Thus, Eq. 8-29 yields

$$\Delta E_{\text{th}} = f_k d = \mu_k mgd \cos \theta = (0.10)(267)(6.1) \cos 20^\circ = 1.5 \times 10^2 \text{ J} .$$

- (b) The potential energy change is $\Delta U = mg(-d \sin \theta) = (267)(-6.1 \sin 20^\circ) = -5.6 \times 10^2 \text{ J}$. The initial kinetic energy is

$$K_i = \frac{1}{2}mv_i^2 = \frac{1}{2} \left(\frac{267 \text{ N}}{9.8 \text{ m/s}^2} \right) (0.457 \text{ m/s})^2 = 2.8 \text{ J} .$$

Therefore, using Eq. 8-31 (with $W = 0$), the final kinetic energy is

$$K_f = K_i - \Delta U - \Delta E_{\text{th}} = 2.8 - (-5.6 \times 10^2) - 1.5 \times 10^2 = 4.1 \times 10^2 \text{ J} .$$

Consequently, the final speed is $v_f = \sqrt{2K_f/m} = 5.5 \text{ m/s}$

57. (a) With $x = 0.075 \text{ m}$ and $k = 320 \text{ N/m}$, Eq. 7-26 yields $W_s = -\frac{1}{2}kx^2 = -0.90 \text{ J}$. For later reference, this is equal to the negative of ΔU .

- (b) Analyzing forces, we find $N = mg$ which means $f_k = \mu_k mg$. With $d = x$, Eq. 8-29 yields

$$\Delta E_{\text{th}} = f_k d = \mu_k mgx = (0.25)(2.5)(9.8)(0.075) = 0.46 \text{ J} .$$

- (c) Eq. 8-31 (with $W = 0$) indicates that the initial kinetic energy is

$$K_i = \Delta U + \Delta E_{\text{th}} = 0.90 + 0.46 = 1.36 \text{ J}$$

which leads to $v_i = \sqrt{2K_i/m} = 1.0 \text{ m/s}$.

58. This can be worked entirely by the methods of Chapters 2-6, but we will use energy methods in as many steps as possible.

- (a) By a force analysis in the style of Chapter 6, we find the normal force has magnitude $N = mg \cos \theta$ (where $\theta = 39^\circ$) which means $f_k = \mu_k mg \cos \theta$ where $\mu_k = 0.28$. Thus, Eq. 8-29 yields $\Delta E_{\text{th}} = f_k d = \mu_k mgd \cos \theta$. Also, elementary trigonometry leads us to conclude that $\Delta U = -mgd \sin \theta$ where $d = 3.7 \text{ m}$. Since $K_i = 0$, Eq. 8-31 (with $W = 0$) indicates that the final kinetic energy is

$$K_f = -\Delta U - \Delta E_{\text{th}} = mgd (\sin \theta - \mu_k \cos \theta)$$

which leads to the speed at the bottom of the ramp

$$v = \sqrt{\frac{2K_f}{m}} = \sqrt{2gd (\sin \theta - \mu_k \cos \theta)} = 5.5 \text{ m/s} .$$

- (b) This speed begins its horizontal motion, where $f_k = \mu_k mg$ and $\Delta U = 0$. It slides a distance d' before it stops. According to Eq. 8-31 (with $W = 0$),

$$\begin{aligned} 0 &= \Delta K + \Delta U + \Delta E_{\text{th}} \\ &= 0 - \frac{1}{2}mv^2 + 0 + \mu_k mgd' \\ &= -\frac{1}{2}(2gd (\sin \theta - \mu_k \cos \theta)) + \mu_k gd' \end{aligned}$$

where we have divided by mass and substituted from part (a) in the last step. Therefore,

$$d' = \frac{d (\sin \theta - \mu_k \cos \theta)}{\mu_k} = 5.4 \text{ m} .$$

- (c) We see from the algebraic form of the results, above, that the answers do not depend on mass. A 90 kg crate should have the same speed at the bottom and sliding distance across the floor, to the extent that the friction relations in Ch. 6 are accurate. Interestingly, since g does not appear in the relation for d' , the sliding distance would seem to be the same if the experiment were performed on Mars!

59. Since the valley is frictionless, the only reason for the speed being less when it reaches the higher level is the gain in potential energy $\Delta U = mgh$ where $h = 1.1 \text{ m}$. Sliding along the rough surface of the higher level, the block finally stops since its remaining kinetic energy has turned to thermal energy

$$\Delta E_{\text{th}} = f_k d = \mu mgd$$

where $\mu = 0.60$. Thus, Eq. 8-31 (with $W = 0$) provides us with an equation to solve for the distance d :

$$K_i = \Delta U + \Delta E_{\text{th}} = mg(h + \mu d)$$

where $K_i = \frac{1}{2}mv_i^2$ and $v_i = 6.0$ m/s. Dividing by mass and rearranging, we obtain

$$d = \frac{v_i^2}{2\mu g} - \frac{h}{\mu} = 1.2 \text{ m} .$$

60. This can be worked entirely by the methods of Chapters 2-6, but we will use energy methods in as many steps as possible.

- (a) By a force analysis of the style done in Ch. 6, we find the normal force has magnitude $N = mg \cos \theta$ (where $\theta = 40^\circ$) which means $f_k = \mu_k mg \cos \theta$ where $\mu_k = 0.15$. Thus, Eq. 8-29 yields $\Delta E_{\text{th}} = f_k d = \mu_k mgd \cos \theta$. Also, elementary trigonometry leads us to conclude that $\Delta U = mgd \sin \theta$. Eq. 8-31 (with $W = 0$ and $K_f = 0$) provides an equation for determining d :

$$\begin{aligned} K_i &= \Delta U + \Delta E_{\text{th}} \\ \frac{1}{2}mv_i^2 &= mgd(\sin \theta + \mu_k \cos \theta) \end{aligned}$$

where $v_i = 1.4$ m/s. Dividing by mass and rearranging, we obtain

$$d = \frac{v_i^2}{2g(\sin \theta + \mu_k \cos \theta)} = 0.13 \text{ m} .$$

- (b) Now that we know where on the incline it stops ($d' = 0.13 + 0.55 = 0.68$ m from the bottom), we can use Eq. 8-31 again (with $W = 0$ and now with $K_i = 0$), to describe the final kinetic energy (at the bottom):

$$\begin{aligned} K_f &= -\Delta U - \Delta E_{\text{th}} \\ \frac{1}{2}mv^2 &= mgd'(\sin \theta - \mu_k \cos \theta) \end{aligned}$$

which – after dividing by the mass and rearranging – yields

$$v = \sqrt{2gd'(\sin \theta - \mu_k \cos \theta)} = 2.7 \text{ m/s} .$$

- (c) In part (a) it is clear that d increases if μ_k decreases – both mathematically (since it is a positive term in the denominator) and intuitively (less friction – less energy “lost”). In part (b), there are two terms in the expression for v which imply that it should increase if μ_k were smaller: the increased value of $d' = d_0 + d$ and that last factor $\sin \theta - \mu_k \cos \theta$ which indicates that less is being subtracted from $\sin \theta$ when μ_k is less (so the factor itself increases in value).
61. (a) The maximum height reached is h . The thermal energy generated by air resistance as the stone rises to this height is $\Delta E_{\text{th}} = fh$ by Eq. 8-29. We use energy conservation in the form of Eq. 8-31 (with $W = 0$):

$$K_f + U_f + \Delta E_{\text{th}} = K_i + U_i$$

and we take the potential energy to be zero at the throwing point (ground level). The initial kinetic energy is $K_i = \frac{1}{2}mv_0^2$, the initial potential energy is $U_i = 0$, the final kinetic energy is $K_f = 0$, and the final potential energy is $U_f = wh$. Thus $wh + fh = \frac{1}{2}mv_0^2$, and we solve for the height:

$$h = \frac{mv_0^2}{2(w + f)} = \frac{wv_0^2}{2g(w + f)} = \frac{v_0^2}{2g(1 + f/w)} .$$

- (b) We notice that the force of the air is downward on the trip up and upward on the trip down, since it is opposite to the direction of motion. Over the entire trip the increase in thermal energy is $\Delta E_{\text{th}} = 2fh$. The final kinetic energy is $K_f = \frac{1}{2}mv^2$, where v is the speed of the stone just before it hits the ground. The final potential energy is $U_f = 0$. Thus, using Eq. 8-31 (with $W = 0$), we find

$$\frac{1}{2}mv^2 + 2fh = \frac{1}{2}mv_0^2.$$

We substitute the expression found for h to obtain

$$-\frac{2fv_0^2}{2g(1+f/w)} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$$

which leads to

$$v^2 = v_0^2 - \frac{2fv_0^2}{mg(1+f/w)} = v_0^2 - \frac{2fv_0^2}{w(1+f/w)} = v_0^2 \left(1 - \frac{2f}{w+f}\right) = v_0^2 \frac{w-f}{w+f}$$

where w was substituted for mg and some algebraic manipulations were carried out. Therefore,

$$v = v_0 \sqrt{\frac{w-f}{w+f}}.$$

62. (a) The assumption is that the slope of the bottom of the slide is horizontal, like the ground. A useful analogy is that of the pendulum of length $R = 12$ m that is pulled leftward to an angle θ (corresponding to being at the top of the slide at height $h = 4.0$ m) and released so that the pendulum swings to the lowest point (zero height) gaining speed $v = 6.2$ m/s. Exactly as we would analyze the trigonometric relations in the pendulum problem, we find

$$h = R(1 - \cos \theta) \implies \theta = \cos^{-1} \left(1 - \frac{h}{R}\right) = 48^\circ$$

or 0.84 radians. The slide, representing a circular arc of length $s = R\theta$, is therefore $(12)(0.84) = 10$ m long.

- (b) To find the magnitude f of the frictional force, we use Eq. 8-31 (with $W = 0$):

$$\begin{aligned} 0 &= \Delta K + \Delta U + \Delta E_{\text{th}} \\ &= \frac{1}{2}mv^2 - mgh + fs \end{aligned}$$

so that (with $m = 25$ kg) we obtain $f = 49$ N.

- (c) The assumption is no longer that the slope of the bottom of the slide is horizontal, but rather that the slope of the top of the slide is vertical (and 12 m to the left of the center of curvature). Returning to the pendulum analogy, this corresponds to releasing the pendulum from horizontal (at $\theta_1 = 90^\circ$ measured from vertical) and taking a snapshot of its motion a few moments later when it is at angle θ_2 with speed $v = 6.2$ m/s. The difference in height between these two positions is (just as we would figure for the pendulum of length R)

$$\Delta h = R(1 - \cos \theta_2) - R(1 - \cos \theta_1) = -R \cos \theta_2$$

where we have used the fact that $\cos \theta_1 = 0$. Thus, with $\Delta h = -4.0$ m, we obtain $\theta_2 = 70.5^\circ$ which means the arc subtends an angle of $|\Delta \theta| = 19.5^\circ$ or 0.34 radians. Multiplying this by the radius gives a slide length of $s' = 4.1$ m.

- (d) We again find the magnitude f' of the frictional force by using Eq. 8-31 (with $W = 0$):

$$\begin{aligned} 0 &= \Delta K + \Delta U + \Delta E_{\text{th}} \\ &= \frac{1}{2}mv^2 - mgh + f's' \end{aligned}$$

so that we obtain $f' = 1.2 \times 10^2$ N.

63. The initial and final kinetic energies are zero, and we set up energy conservation in the form of Eq. 8-31 (with $W = 0$) according to our assumptions. Certainly, it can only come to a permanent stop somewhere in the flat part, but the question is whether this occurs during its first pass through (going rightward) or its second pass through (going leftward) or its third pass through (going rightward again), and so on. If it occurs during its first pass through, then the thermal energy generated is $\Delta E_{\text{th}} = f_k d$ where $d \leq L$ and $f_k = \mu_k mg$. If it occurs during its second pass through, then the total thermal energy is $\Delta E_{\text{th}} = \mu_k mg(L + d)$ where we again use the symbol d for how far through the level area it goes during that last pass (so $0 \leq d \leq L$). Generalizing to the n^{th} pass through, we see that $\Delta E_{\text{th}} = \mu_k mg((n-1)L + d)$. In this way, Eq. 8-39 leads to

$$mgh = \mu_k mg((n-1)L + d)$$

which simplifies (when $h = L/2$ is inserted) to

$$\frac{d}{L} = 1 + \frac{1}{2\mu_k} - n.$$

The first two terms give $1 + 1/2\mu_k = 3.5$, so that the requirement $0 \leq d/L \leq 1$ demands that $n = 3$. We arrive at the conclusion that $d/L = \frac{1}{2}$ and that this occurs on its third pass through the flat region.

64. We observe that the last line of the problem indicates that static friction is not to be considered a factor in this problem. The friction force of magnitude $f = 4400$ N mentioned in the problem is kinetic friction and (as mentioned) is constant (and directed upward), and the thermal energy change associated with it is $\Delta E_{\text{th}} = fd$ (Eq. 8-29) where $d = 3.7$ m in part (a) (but will be replaced by x , the spring compression, in part (b)).

- (a) With $W = 0$ and the reference level for computing $U = mgy$ set at the top of the (relaxed) spring, Eq. 8-31 leads to

$$U_i = K + \Delta E_{\text{th}} \implies v = \sqrt{2d\left(g - \frac{f}{m}\right)}$$

which yields $v = 7.4$ m/s for $m = 1800$ kg.

- (b) We again utilize Eq. 8-31 (with $W = 0$), now relating its kinetic energy at the moment it makes contact with the spring to the system energy at the bottom-most point. Using the same reference level for computing $U = mgy$ as we did in part (a), we end up with gravitational potential energy equal to $mg(-x)$ at that bottom-most point, where the spring (with spring constant $k = 1.5 \times 10^5$ N/m) is fully compressed.

$$K = mg(-x) + \frac{1}{2}kx^2 + fx$$

where $K = \frac{1}{2}mv^2 = 4.9 \times 10^4$ J using the speed found in part (a). Using the abbreviation $\xi = mg - f = 1.3 \times 10^4$ N, the quadratic formula yields

$$x = \frac{\xi \pm \sqrt{\xi^2 + 2kK}}{k} = 0.90 \text{ m}$$

where we have taken the positive root.

- (c) We relate the energy at the bottom-most point to that of the highest point of rebound (a distance d' above the relaxed position of the spring). We assume $d' > x$. We now use the bottom-most point as the reference level for computing gravitational potential energy.

$$\frac{1}{2}kx^2 = mgd' + fd' \implies d' = \frac{kx^2}{2(mg + d)} = 2.8 \text{ m}.$$

- (d) The non-conservative force (§8-1) is friction, and the energy term associated with it is the one that keeps track of the total distance traveled (whereas the potential energy terms, coming as they do from conservative forces, depend on positions – but not on the paths that led to them). We assume the elevator comes to final rest at the equilibrium position of the spring, with the spring compressed an amount d_{eq} given by

$$mg = kd_{\text{eq}} \implies d_{\text{eq}} = \frac{mg}{k} = 0.12 \text{ m} .$$

In this part, we use that final-rest point as the reference level for computing gravitational potential energy, so the original $U = mgy$ becomes $mg(d_{\text{eq}} + d)$. In that final position, then, the gravitational energy is zero and the spring energy is $\frac{1}{2}kd_{\text{eq}}^2$. Thus, Eq. 8-31 becomes

$$\begin{aligned} mg(d_{\text{eq}} + d) &= \frac{1}{2}kd_{\text{eq}}^2 + fd_{\text{total}} \\ (1800)(9.8)(0.12 + 3.7) &= \frac{1}{2}(1.5 \times 10^5)(0.12)^2 + (4400)d_{\text{total}} \end{aligned}$$

which yields $d_{\text{total}} = 15 \text{ m}$.

65. (a) Since the speed of the crate of mass m increases from 0 to 1.20 m/s relative to the factory ground, the kinetic energy supplied to it is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(300 \text{ kg})(1.20 \text{ m/s})^2 = 216 \text{ J} .$$

- (b) The magnitude of the kinetic frictional force is

$$f = \mu N = \mu mg = (0.400)(300 \text{ kg})(9.8 \text{ m/s}^2) = 1.18 \times 10^3 \text{ N} .$$

- (c) and (d) The energy supplied by the motor is the work W it does on the system, and must be greater than the kinetic energy gained by the crate computed in part (b). This is due to the fact that part of the energy supplied by the motor is being used to compensate for the energy dissipated ΔE_{th} while it was slipping. Let the distance the crate moved relative to the conveyor belt before it stops slipping be d , then from Eq. 2-16 ($v^2 = 2ad = 2(f/m)d$) we find

$$\Delta E_{\text{th}} = fd = \frac{1}{2}mv^2 = K .$$

Thus, the total energy that must be supplied by the motor is

$$W = K + \Delta E_{\text{th}} = 2K = (2)(216 \text{ J}) = 432 \text{ J} .$$

66. (a) The compression is “spring-like” so the maximum force relates to the distance x by Hooke’s law:

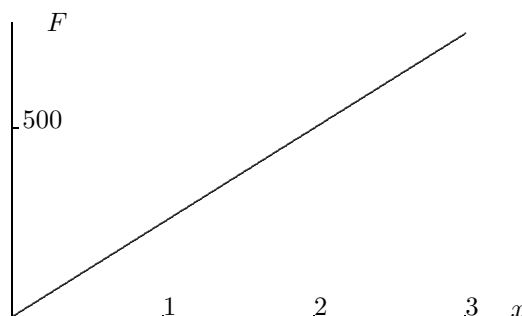
$$F_x = kx \implies x = \frac{750}{2.5 \times 10^5} = 0.0030 \text{ m} .$$

- (b) The work is what produces the “spring-like” potential energy associated with the compression. Thus, using Eq. 8-11,

$$W = \frac{1}{2}kx^2 = \frac{1}{2}(2.5 \times 10^5)(0.0030)^2 = 1.1 \text{ J} .$$

- (c) By Newton’s third law, the force

F exerted by the tooth is equal and opposite to the “spring-like” force exerted by the licorice, so the graph of F is a straight line of slope k . We plot F (in Newtons) versus x (in millimeters); both are taken as positive.



- (d) As mentioned in part (b), the spring potential energy expression is relevant. Now, whether or not we can ignore dissipative processes is a deeper question. In other words, it seems unlikely that – if the tooth at any moment were to reverse its motion – that the licorice could “spring back” to its original shape. Still, to the extent that $U = \frac{1}{2}kx^2$ applies, the graph is a parabola (not shown here) which has its vertex at the origin and is either concave upward or concave downward depending on how one wishes to define the sign of F (the connection being $F = -dU/dx$).
- (e) As a crude estimate, the area under the curve is roughly half the area of the entire plotting-area (8000 N by 12 mm). This leads to an approximate work of $\frac{1}{2}(8000)(0.012) \approx 50$ J. Estimates in the range $40 \leq W \leq 50$ J are acceptable.
- (f) Certainly dissipative effects dominate this process, and we cannot assign it a meaningful potential energy.
67. (a) The drawings in the Figure (especially pictures (b) and (c)) show this geometric relationship very clearly. But we can work out the details, if need be. If ℓ is the length we are to compute (that of the still moving upper section) and ℓ' is the length of the lower (motionless) section, then clearly $\ell + \ell' = L$. Also, (as is especially easy to see in picture (c)) $x + \ell$ must equal ℓ' . These two equations, then, lead to the conclusion $\ell = \frac{1}{2}(L - x)$.
- (b) The mass of the still moving upper section is

$$m = \rho\ell = \frac{\rho}{2}(L - x) .$$

- (c) The assumptions stated in the problem lead to

$$\frac{1}{2}(\rho L + m_f)v_0^2 = \frac{1}{2}\left(\frac{\rho}{2}(L - x) + m_f\right)v^2$$

which yields the speed of the still moving upper section:

$$v = v_0 \sqrt{\frac{\rho L + m_f}{\rho(L - x)/2 + m_f}} .$$

- (d) As x approaches L , we obtain

$$v_f = v_0 \sqrt{\frac{\rho L + m_f}{m_f}} = (6.0) \sqrt{\frac{(1.3)(20) + 0.8}{0.8}}$$

which yields $v_f = 35$ m/s.

68. (a) The effect of a (sliding) friction is described in terms of energy dissipated as shown in Eq. 8-29. We have

$$\Delta E = K + \frac{1}{2}k(0.08)^2 - \frac{1}{2}k(0.10)^2 = -f_k(0.02)$$

where distances are in meters and energies are in Joules. With $k = 4000$ N/m and $f_k = 80$ N, we obtain $K = 5.6$ J.

- (b) In this case, we have $d = 0.10$ m. Thus,

$$\Delta E = K + 0 - \frac{1}{2}k(0.10)^2 = -f_k(0.10)$$

which leads to $K = 12$ J.

- (c) We can approach this two ways. One way is to examine the dependence of energy on the variable d :

$$\Delta E = K + \frac{1}{2}k(d_0 - d)^2 - \frac{1}{2}kd_0^2 = -f_k d$$

where $d_0 = 0.10$ m, and solving for K as a function of d :

$$K = -\frac{1}{2}kd^2 + (kd_0)d - f_k d .$$

In this first approach, we could work through the $\frac{dK}{dd} = 0$ condition (or with the special capabilities of a graphing calculator) to obtain the answer $K_{\max} = \frac{1}{2k}(kd_0 - f_k)^2$. In the second (and perhaps easier) approach, we note that K is maximum where v is maximum – which is where $a = 0 \implies$ equilibrium of forces. Thus, the second approach simply solves for the equilibrium position

$$|F_{\text{spring}}| = f_k \implies kx = 80 .$$

Thus, with $k = 4000$ N/m we obtain $x = 0.02$ m. But $x = d_0 - d$ so this corresponds to $d = 0.08$ m. Then the methods of part (a) lead to the answer $K_{\max} = 12.8 \approx 13$ J.

69. Let the amount of stretch of the spring be x . For the object to be in equilibrium

$$kx - mg = 0 \implies x = mg/k .$$

Thus the gain in elastic potential energy for the spring is

$$\Delta U_e = \frac{1}{2}kx^2 = \frac{1}{2}k \left(\frac{mg}{k} \right)^2 = \frac{m^2 g^2}{2k}$$

while the loss in the gravitational potential energy of the system is

$$-\Delta U_g = mgx = mg \left(\frac{mg}{k} \right) = \frac{m^2 g^2}{k}$$

which we see (by comparing with the previous expression) is equal to $2\Delta U_e$. The reason why $|\Delta U_g| \neq \Delta U_e$ is that, since the object is slowly lowered, an upward external force (e.g., due to the hand) must have been exerted on the object during the lowering process, preventing it from accelerating downward. This force does *negative* work on the object, reducing the total mechanical energy of the system.

70. (a) The rate of change of the gravitational potential energy is

$$\frac{dU}{dt} = mg \frac{dy}{dt} = -mg|v| = -(68)(9.8)(59) = -3.9 \times 10^4 \text{ J/s} .$$

Thus, the gravitational energy is being reduced at the rate of 3.9×10^4 W.

- (b) Since the velocity is constant, the rate of change of the kinetic energy is zero. Thus the rate at which the mechanical energy is being dissipated is the same as that of the gravitational potential energy (3.9×10^4 W).

71. The power generation (assumed constant, so average power is the same as instantaneous power) is

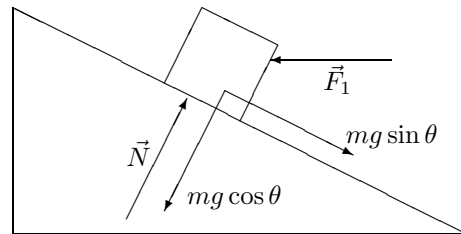
$$P = \frac{mgh}{t} = \frac{(3/4)(1200 \text{ m}^3) (10^3 \text{ kg/m}^3) (9.8 \text{ m/s}^2) (100 \text{ m})}{1.0 \text{ s}}$$

which yields $P = 8.8 \times 10^8$ W.

72. The free-body diagram for the trunk is shown.

The x and y applications of Newton's second law provide two equations:

$$\begin{aligned} F_1 \cos \theta - f_k - mg \sin \theta &= ma \\ N - F_1 \sin \theta - mg \cos \theta &= 0 . \end{aligned}$$



- (a) The trunk is moving up the incline at constant velocity, so $a = 0$. Using $f_k = \mu_k N$, we solve for the push-force F_1 and obtain

$$F_1 = \frac{mg(\sin \theta + \mu_k \cos \theta)}{\cos \theta - \mu_k \sin \theta} .$$

The work done by the push-force \vec{F}_1 as the trunk is pushed through a distance ℓ up the inclined plane is therefore

$$\begin{aligned} W_1 &= F_1 \ell \cos \theta = \frac{(mg \ell \cos \theta)(\sin \theta + \mu_k \cos \theta)}{\cos \theta - \mu_k \sin \theta} \\ &= \frac{(50 \text{ kg})(9.8 \text{ m/s}^2)(6.0 \text{ m})(\cos 30^\circ)(\sin 30^\circ + (0.20) \cos 30^\circ)}{\cos 30^\circ - (0.20) \sin 30^\circ} \\ &= 2.2 \times 10^3 \text{ J} . \end{aligned}$$

- (b) The increase in the gravitational potential energy of the trunk is

$$\Delta U = mg \ell \sin \theta = (50 \text{ kg})(9.8 \text{ m/s}^2)(6.0 \text{ m}) \sin 30^\circ = 1.5 \times 10^3 \text{ J} .$$

Since the speed (and, therefore, the kinetic energy) of the trunk is unchanged, Eq. 8-31 leads to

$$W_1 = \Delta U + \Delta E_{\text{th}} .$$

Thus, using more precise numbers than are shown above, the increase in thermal energy (generated by the kinetic friction) is $2.24 \times 10^3 - 1.47 \times 10^3 = 7.7 \times 10^2 \text{ J}$. An alternate way to this result is to use $\Delta E_{\text{th}} = f_k \ell$ (Eq. 8-29).

73. The style of reasoning used here is presented in §8-5.

- The horizontal line representing E_1 intersects the potential energy curve at a value of $r \approx 0.07 \text{ nm}$ and seems not to intersect the curve at larger r (though this is somewhat unclear since $U(r)$ is graphed only up to $r = 0.4 \text{ nm}$). Thus, if m were propelled towards M from large r with energy E_1 it would “turn around” at 0.07 nm and head back in the direction from which it came.
- The line representing E_2 has two intersections points $r_1 \approx 0.16 \text{ nm}$ and $r_2 \approx 0.28 \text{ nm}$ with the $U(r)$ plot. Thus, if m starts in the region $r_1 < r < r_2$ with energy E_2 it will bounce back and forth between these two points, presumably forever.
- At $r = 0.3 \text{ nm}$, the potential energy is roughly $U = -1.1 \times 10^{-19} \text{ J}$.
- With $M \gg m$, the kinetic energy is essentially just that of m . Since $E = 1 \times 10^{-19} \text{ J}$, its kinetic energy is $K = E - U \approx 2.1 \times 10^{-19} \text{ J}$.
- Since force is related to the slope of the curve, we must (crudely) estimate $|F| \approx 1 \times 10^{-9} \text{ N}$ at this point. The sign of the slope is positive, so by Eq. 8-20, the force is negative-valued. This is interpreted to mean that the atoms are attracted to each other.
- Recalling our remarks in the previous part, we see that the sign of F is positive (meaning it’s repulsive) for $r < 0.2 \text{ nm}$.
- And the sign of F is negative (attractive) for $r > 0.2 \text{ nm}$.
- At $r = 0.2 \text{ nm}$, the slope (hence, F) vanishes.

74. We take her original elevation to be the $y = 0$ reference level and observe that the top of the hill must consequently have $y_A = R(1 - \cos 20^\circ) = 1.2 \text{ m}$, where R is the radius of the hill. The mass of the skier is $600/9.8 = 61 \text{ kg}$.

- (a) Applying energy conservation, Eq. 8-17, we have

$$K_B + U_B = K_A + U_A \implies K_B + 0 = K_A + mgy_A$$

Using $K_B = \frac{1}{2}(61 \text{ kg})(8.0 \text{ m/s})^2$, we obtain $K_A = 1.2 \times 10^3 \text{ J}$. Thus, we find the speed at the hilltop is $v = \sqrt{2K/m} = 6.4 \text{ m/s}$. (Note: one might wish to check that the skier stays in contact with the hill – which is indeed the case, here. For instance, at A we find $v^2/r \approx 2 \text{ m/s}^2$ which is considerably less than g .)

- (b) With $K_A = 0$, we have

$$K_B + U_B = K_A + U_A \implies K_B + 0 = 0 + mgy_A$$

which yields $K_B = 724 \text{ J}$, and the corresponding speed is $v = \sqrt{2K/m} = 4.9 \text{ m/s}$.

- (c) Expressed in terms of mass, we have

$$\begin{aligned} K_B + U_B &= K_A + U_A \implies \\ \frac{1}{2}mv_B^2 + mgy_B &= \frac{1}{2}mv_A^2 + mgy_A . \end{aligned}$$

Thus, the mass m cancels, and we observe that solving for speed does not depend on the value of mass (or weight).

75. The spring is relaxed at $y = 0$, so the elastic potential energy (Eq. 8-11) is $U_{el} = \frac{1}{2}ky^2$. The total energy is conserved, and is zero (determined by evaluating it at its initial position). We note that U is the same as ΔU in these manipulations. Thus, we have

$$0 = K + U_g + U_e \implies K = -U_g - U_e$$

where $U_g = mgy = (20 \text{ N})y$ with y in meters (so that the energies are in Joules). We arrange the results

in a table:

position y	-0.05	-0.10	-0.15	-0.20
U_g	-1.0	-2.0	-3.0	-4.0
U_e	0.25	1.0	2.25	4.0
K	0.75	1.0	0.75	0

76. From Eq. 8-6, we find (with SI units understood)

$$U(\xi) = -\int_0^\xi (-3x - 5x^2) dx = \frac{3}{2}\xi^2 + \frac{5}{3}\xi^3 .$$

- (a) Using the above formula, we obtain $U(2) \approx 19 \text{ J}$.
 (b) When its speed is $v = 4 \text{ m/s}$, its mechanical energy is $\frac{1}{2}mv^2 + U(5)$. This must equal the energy at the origin:

$$\frac{1}{2}mv^2 + U(5) = \frac{1}{2}mv_o^2 + U(0)$$

so that the speed at the origin is

$$v_o = \sqrt{v^2 + \frac{2}{m} (U(5) - U(0))} .$$

Thus, with $U(5) = 246 \text{ J}$, $U(0) = 0$ and $m = 20 \text{ kg}$, we obtain $v_o = 6.4 \text{ m/s}$.

- (c) Our original formula for U is changed to $U(x) = -8 + \frac{3}{2}x^2 + \frac{5}{3}x^3$ in this case. Therefore, $U(2) = 11 \text{ J}$. But we still have $v_o = 6.4 \text{ m/s}$ since that calculation only depended on the difference of potential energy values (specifically, $U(5) - U(0)$).
77. (a) At the top of its flight, the vertical component of the velocity vanishes, and the horizontal component (neglecting air friction) is the same as it was when it was thrown. Thus,

$$K_{\text{top}} = \frac{1}{2}mv_x^2 = \frac{1}{2}(0.050 \text{ kg}) ((8.0 \text{ m/s}) \cos 30^\circ)^2$$

which yields $K_{\text{top}} = 1.2 \text{ J}$.

- (b) We choose the point 3.0 m below the window as the reference level for computing the potential energy. Thus, equating the mechanical energy when it was thrown to when it is at this reference level, we have (with SI units understood)

$$\begin{aligned} mgy_0 + K_0 &= K \\ m(9.8)(3.0) + \frac{1}{2}m(8.0)^2 &= \frac{1}{2}mv^2 \end{aligned}$$

which yields (after canceling m and simplifying) $v = 11$ m/s.

- (c) As mentioned, m cancels – and is therefore not relevant to that computation.
 (d) The v in the kinetic energy formula is the magnitude of the velocity vector; it does not depend on the direction.

78. From the slope of the graph, we find the spring constant

$$k = \frac{\Delta F}{\Delta x} = 0.10 \text{ N/cm} = 10 \text{ N/m} .$$

- (a) Equating the potential energy of the compressed spring to the kinetic energy of the cork at the moment of release, we have

$$\frac{1}{2}kx^2 = \frac{1}{2}mv^2 \implies v = x\sqrt{\frac{k}{m}}$$

which yields $v = 2.8$ m/s for $m = 0.0038$ kg and $x = 0.055$ m.

- (b) The new scenario involves some potential energy at the moment of release. With $d = 0.015$ m, energy conservation becomes

$$\frac{1}{2}kx^2 = \frac{1}{2}mv^2 + \frac{1}{2}kd^2 \implies v = \sqrt{\frac{k}{m}(x^2 - d^2)}$$

which yields $v = 2.7$ m/s.

79. We assume his initial kinetic energy (when he jumps) is negligible. Then, his initial gravitational potential energy measured relative to where he momentarily stops is what becomes the elastic potential energy of the stretched net (neglecting air friction). Thus,

$$U_{\text{net}} = U_{\text{grav}} = mgh$$

where $h = 11.0 + 1.5 = 12.5$ m. Consequently, with $m = 70$ kg, we obtain $U_{\text{net}} = 8.6 \times 10^3$ J.

80. The work done by \vec{F} is the negative of its potential energy change (see Eq. 8-6), so $U_B = U_A - 25 = 15$ J.

81. (a) During the final $d = 12$ m of motion, we use

$$K_1 + U_1 = K_2 + U_2 + f_k d \implies \frac{1}{2}mv^2 + 0 = 0 + 0 + f_k d$$

where $v = 4.2$ m/s. This gives $f_k = 0.31$ N. Therefore, the thermal energy change is $f_k d = 3.7$ J.

- (b) Using $f_k = 0.31$ N we obtain $f_k d_{\text{total}} = 4.3$ J for the thermal energy generated by friction; here, $d_{\text{total}} = 14$ m.

- (c) During the initial $d' = 2$ m of motion, we have

$$K_0 + U_0 + W_{\text{app}} = K_1 + U_1 + f_k d' \implies 0 + 0 + W_{\text{app}} = \frac{1}{2}mv^2 + 0 + f_k d'$$

which essentially combines Eq. 8-31 and Eq. 8-29. This leads to the result $W_{\text{app}} = 4.3$ J, and – reasonably enough – is the same as our answer in part (b).

82. (a) This part is essentially a free-fall problem, which can be easily done with Chapter 2 methods. Instead, choosing energy methods, we take $y = 0$ to be the ground level.

$$K_i + U_i = K + U \implies 0 + mgy_i = \frac{1}{2}mv^2 + 0$$

Therefore $v = \sqrt{2gy_i} = 9.2 \text{ m/s}$, where $y_i = 4.3 \text{ m}$.

- (b) Eq. 8-29 provides $\Delta E_{\text{th}} = f_k d$ for thermal energy generated by the kinetic friction force. We apply Eq. 8-31:

$$K_i + U_i = K + U \implies 0 + mgy_i = \frac{1}{2}mv^2 + 0 + f_k d$$

With $d = y_i$, $m = 70 \text{ kg}$ and $f_k = 500 \text{ N}$, this yields $v = 4.8 \text{ m/s}$.

83. We want to convert (at least in theory) the water that falls through $h = 500 \text{ m}$ into electrical energy. The problem indicates that in one year, a volume of water equal to $A\Delta z$ lands in the form of rain on the country, where $A = 8 \times 10^{12} \text{ m}^2$ and $\Delta z = 0.75 \text{ m}$. Multiplying this volume by the density $\rho = 1000 \text{ kg/m}^3$ leads to

$$m_{\text{total}} = \rho A \Delta z = (1000) (8 \times 10^{12}) (0.75) = 6 \times 10^{15} \text{ kg}$$

for the mass of rainwater. One-third of this “falls” to the ocean, so it is $m = 2 \times 10^{15} \text{ kg}$ that we want to use in computing the gravitational potential energy mgh (which will turn into electrical energy during the year). Since a year is equivalent to $3.2 \times 10^7 \text{ s}$, we obtain

$$P_{\text{avg}} = \frac{(2 \times 10^{15}) (9.8) (500)}{3.2 \times 10^7} = 3.1 \times 10^{11} \text{ W} .$$

84. With the potential energy reference level set at the point of throwing, we have (with SI units understood)

$$\Delta E = mgh - \frac{1}{2}mv_0^2 = m \left((9.8)(8.1) - \frac{1}{2}(14)^2 \right)$$

which yields $\Delta E = -12 \text{ J}$ for $m = 0.63 \text{ kg}$. This “loss” of mechanical energy is presumably due to air friction.

85. We note that in one second, the block slides $d = 1.34 \text{ m}$ up the incline, which means its height increase is $h = d \sin \theta$ where

$$\theta = \tan^{-1} \left(\frac{30}{40} \right) = 37^\circ .$$

We also note that the force of kinetic friction in this inclined plane problem is $f_k = \mu_k mg \cos \theta$ where $\mu_k = 0.40$ and $m = 1400 \text{ kg}$. Thus, using Eq. 8-31 and Eq. 8-29, we find

$$W = mgh + f_k d = mgd (\sin \theta + \mu_k \cos \theta)$$

or $W = 1.69 \times 10^4 \text{ J}$ for this one-second interval. Thus, the power associated with this is

$$P = \frac{1.69 \times 10^4 \text{ J}}{1 \text{ s}} = 1.69 \times 10^4 \text{ W} .$$

86. We take the original height of the box to be the $y = 0$ reference level and observe that, in general, the height of the box (when the box has moved a distance d downhill) is $y = -d \sin 40^\circ$.

- (a) Using the conservation of energy, we have

$$K_i + U_i = K + U \implies 0 + 0 = \frac{1}{2}mv^2 + mgy + \frac{1}{2}kd^2 .$$

Therefore, with $d = 0.10 \text{ m}$, we obtain $v = 0.81 \text{ m/s}$.

- (b) We look for a value of $d \neq 0$ such that $K = 0$.

$$K_i + U_i = K + U \implies 0 + 0 = 0 + mgy + \frac{1}{2}kd^2$$

Thus, we obtain $mgd \sin 40^\circ = \frac{1}{2}kd^2$ and find $d = 0.21$ m.

- (c) The uphill force is caused by the spring (Hooke's law) and has magnitude $kd = 25.2$ N. The downhill force is the component of gravity $mg \sin 40^\circ = 12.6$ N. Thus, the net force on the box is $25.2 - 12.6 = 12.6$ N uphill, and the acceleration is uphill with magnitude $12.6/2 = 6.3$ m/s².
87. Equating the mechanical energy at his initial position (as he emerges from the canon, where we set the reference level for computing potential energy) to his energy as he lands, we obtain

$$\begin{aligned} K_i &= K_f + U_f \\ \frac{1}{2}(60 \text{ kg})(16 \text{ m/s})^2 &= K_f + (60 \text{ kg})(9.8 \text{ m/s}^2)(3.9 \text{ m}) \end{aligned}$$

which leads to $K_f = 5.4 \times 10^3$ J.

88. (a) The initial kinetic energy is $K_i = \frac{1}{2}(1.5)(20)^2 = 300$ J.
- (b) At the point of maximum height, the vertical component of velocity vanishes but the horizontal component remains what it was when it was "shot" (if we neglect air friction). Its kinetic energy at that moment is

$$K = \frac{1}{2}(1.5)(20 \cos 34^\circ)^2 = 206 \text{ J}.$$

Thus, $\Delta U = K_i - K = 300 - 206 = 94$ J.

- (c) Since $\Delta U = mg\Delta y$, we obtain

$$\Delta y = \frac{94 \text{ J}}{(1.5 \text{ kg})(9.8 \text{ m/s}^2)} = 6.4 \text{ m}.$$

89. We note that if the larger mass ($M = 2$ kg) falls $d = 0.25$ m, then the smaller mass ($m = 1$ kg) must increase its height by $h = d \sin 30^\circ$. Thus, by mechanical energy conservation, the kinetic energy of the system is

$$K_{\text{total}} = Mgd - mgh = 3.7 \text{ J}.$$

90. (a) At the point of maximum height, where $y = 140$ m, the vertical component of velocity vanishes but the horizontal component remains what it was when it was launched (if we neglect air friction). Its kinetic energy at that moment is

$$K = \frac{1}{2}(0.55 \text{ kg})v_x^2.$$

Also, its potential energy (with the reference level chosen at the level of the cliff edge) at that moment is $U = mgy = 755$ J. Thus, by mechanical energy conservation,

$$K = K_i - U = 1550 - 755 \implies v_x = \sqrt{\frac{2(1550 - 755)}{0.55}}$$

which yields $v_x = 54$ m/s.

- (b) As mentioned $v_x = v_{ix}$ so that the initial kinetic energy

$$K_i = \frac{1}{2}m(v_{ix}^2 + v_{iy}^2)$$

can be used to find v_{iy} . We obtain $v_{iy} = 52$ m/s.

(c) Applying Eq. 2-16 to the vertical direction (with $+y$ upward), we have

$$\begin{aligned} v_y^2 &= v_{iy}^2 - 2g\Delta y \\ 65^2 &= 52^2 - 2(9.8)\Delta y \end{aligned}$$

which yields $\Delta y = -76$ m. The minus sign tells us it is below its launch point.

91. (a) The initial kinetic energy is $K_i = \frac{1}{2}(1.5)(3)^2 = 6.75$ J.
 (b) The work of gravity is the negative of its change in potential energy. At the highest point, all of K_i has converted into U (if we neglect air friction) so we conclude the work of gravity is -6.75 J.
 (c) And we conclude that $\Delta U = 6.75$ J.
 (d) The potential energy there is $U_f = U_i + \Delta U = 6.75$ J.
 (e) If $U_f = 0$, then $U_i = U_f - \Delta U = -6.75$ J.
 (f) Since $mg\Delta y = \Delta U$, we obtain $\Delta y = 0.46$ m.
92. (a) With energy in Joules and length in meters, we have

$$\Delta U = U(x) - U(0) = - \int_0^x (6x' - 12) dx' .$$

Therefore, with $U(0) = 27$ J, we obtain $U(x)$ (written simply as U) by integrating and rearranging:

$$U = 27 + 12x - 3x^2 .$$

(b) We can maximize the above function by working through the $\frac{dU}{dx} = 0$ condition, or we can treat this as a force equilibrium situation – which is the approach we show.

$$F = 0 \implies 6x_{eq} - 12 = 0$$

Thus, $x_{eq} = 2.0$ m, and the above expression for the potential energy becomes $U = 39$ J.

(c) Using the quadratic formula or using the polynomial solver on an appropriate calculator, we find the values of x for which $U = 0$ to be 5.6 m and -1.6 m.

93. Since the aim of this problem is to invite student creativity (and possibly some research), we “invent a problem” (and give its solution) somewhat along the lines of part (b) (in fact, the student might consider running our example “in reverse”). Consider a block of mass M that falls from rest a distance H to a vertical spring of spring constant k . The spring compresses by x_c in order to halt the block, but on the rebound (due to the fact that the block is stuck on the end of the spring) the spring stretches (relative to its original relaxed length) an amount x_s before the block is momentarily at rest again. Take both values of x to be positive. Find x_c and x_s and their difference.

Solution: The height to which the spring reaches when it is relaxed is our $y = 0$ reference level. We relate the initial situation (when the block is dropped) to the situation of maximum compression using energy conservation.

$$K_0 + U_0 = K_c + U_c \implies 0 + MgH = 0 + Mg(-x_c) + \frac{1}{2}kx_c^2$$

The positive root stemming from a quadratic formula solution for x_c yields

$$x_c = \frac{Mg}{k} \left(1 + \sqrt{1 + \frac{2kH}{Mg}} \right) .$$

Next, we relate the initial situation to the final situation (of maximal stretch) using energy conservation.

$$K_0 + U_0 = K_s + U_s \implies 0 + MgH = 0 + Mgx_s + \frac{1}{2}kx_s^2$$

The positive root stemming from a quadratic formula solution for x_s yields

$$x_s = \frac{Mg}{k} \left(-1 + \sqrt{1 + \frac{2kH}{Mg}} \right) .$$

Finally, we note that $x_c > x_s$ with the difference being $x_c - x_s = 2Mg/k$.

94. (First problem in **Cluster 1**)

We take the bottom of the incline to be the $y = 0$ reference level. The incline angle is $\theta = 30^\circ$. The distance along the incline d (measured from the bottom) is related to height y by the relation $y = d \sin \theta$.

(a) Using the conservation of energy, we have

$$K_0 + U_0 = K_{\text{top}} + U_{\text{top}} \implies \frac{1}{2}mv_0^2 + 0 = 0 + mgy$$

with $v_0 = 5.0$ m/s. This yields $y = 1.3$ m, from which we obtain $d = 2.6$ m.

(b) An analysis of forces in the manner of Chapter 6 reveals that the magnitude of the friction force is $f_k = \mu_k mg \cos \theta$. Now, we write Eq. 8-31 as

$$\begin{aligned} K_0 + U_0 &= K_{\text{top}} + U_{\text{top}} + f_k d \\ \frac{1}{2}mv_0^2 + 0 &= 0 + mgy + f_k d \\ \frac{1}{2}mv_0^2 &= mgd \sin \theta + \mu_k mgd \cos \theta \end{aligned}$$

which – upon cancelling the mass and rearranging – provides the result for d :

$$d = \frac{v_0^2}{2g(\mu_k \cos \theta + \sin \theta)} = 1.5 \text{ m} .$$

(c) The thermal energy generated by friction is $f_k d = \mu_k mgd \cos \theta = 26$ J.

(d) The slide back down, from the height $y = 1.5 \sin 30^\circ$ is also described by Eq. 8-31. With ΔE_{th} again equal to 26 J, we have

$$K_{\text{top}} + U_{\text{top}} = K_{\text{bot}} + U_{\text{bot}} + f_k d \implies 0 + mgy = \frac{1}{2}mv_{\text{bot}}^2 + 0 + 26$$

from which we find $v_{\text{bot}} = 2.1$ m/s.

95. (Second problem in **Cluster 1**)

Converting to SI units, $v_0 = 8.3$ m/s and $v = 11.1$ m/s. The incline angle is $\theta = 5.0^\circ$. The height difference between the car's highest and lowest points is $(50 \text{ m}) \sin \theta = 4.4$ m. We take the lowest point (the car's final reported location) to correspond to the $y = 0$ reference level.

(a) Using Eq. 8-31 and Eq. 8-29, we find

$$f_k d = -\Delta K - \Delta U \implies f_k d = \frac{1}{2}m(v_0^2 - v^2) + mgy_0 .$$

Therefore, the mechanical energy reduction (due to friction) is $f_k d = 2.4 \times 10^4$ J.

(b) With $d = 50$ m, we solve for f_k and obtain 471 N, which can be rounded to 470 N.

96. (Third problem in **Cluster 1**)

(a) When there is no change in potential energy, Eq. 8-24 leads to

$$W_{\text{app}} = \Delta K = \frac{1}{2}m(v^2 - v_0^2) .$$

Therefore, $\Delta E = 6.0 \times 10^3$ J.

- (b) From the above manipulation, we see $W_{\text{app}} = 6.0 \times 10^3 \text{ J}$. Also, from Chapter 2, we know that $\Delta t = \Delta v/a = 10 \text{ s}$. Thus, using Eq. 7-42,

$$P_{\text{avg}} = \frac{W}{\Delta t} = \frac{6.0 \times 10^3}{10} = 600 \text{ W} .$$

- (c) and (d) The constant applied force is $ma = 30 \text{ N}$ and clearly in the direction of motion, so Eq. 7-48 provides the results for instantaneous power

$$P = \vec{F} \cdot \vec{v} = \begin{cases} 300 \text{ W} & \text{for } v = 10 \text{ m/s} \\ 900 \text{ W} & \text{for } v = 30 \text{ m/s} \end{cases}$$

We note that the average of these two values agrees with the result in part (b).

97. (Fourth problem in **Cluster 1**)

The distance traveled up the incline can be figured with Chapter 2 techniques: $v^2 = v_0^2 + 2a\Delta x \longrightarrow \Delta x = 200 \text{ m}$. This corresponds to an increase in height equal to $y = 200 \sin \theta = 17 \text{ m}$, where $\theta = 5.0^\circ$. We take its initial height to be $y = 0$.

- (a) Eq. 8-24 leads to

$$W_{\text{app}} = \Delta E = \frac{1}{2}m(v^2 - v_0^2) + mgy .$$

Therefore, $\Delta E = 8.6 \times 10^3 \text{ J}$.

- (b) From the above manipulation, we see $W_{\text{app}} = 8.6 \times 10^3 \text{ J}$. Also, from Chapter 2, we know that $\Delta t = \Delta v/a = 10 \text{ s}$. Thus, using Eq. 7-42,

$$P_{\text{avg}} = \frac{W}{\Delta t} = \frac{8.6 \times 10^3}{10} = 860 \text{ W}$$

where the answer has been rounded off (from the 856 value that is provided by the calculator).

- (c) and (d) Taking into account the component of gravity along the incline surface, the applied force is $ma + mg \sin \theta = 43 \text{ N}$ and clearly in the direction of motion, so Eq. 7-48 provides the results for instantaneous power

$$P = \vec{F} \cdot \vec{v} = \begin{cases} 430 \text{ W} & \text{for } v = 10 \text{ m/s} \\ 1300 \text{ W} & \text{for } v = 30 \text{ m/s} \end{cases}$$

where these answers have been rounded off (from 428 and 1284, respectively). We note that the average of these two values agrees with the result in part (b).

