Chapter 32

1. (a) Since the field lines of a bar magnet point towards its South pole, then the $\vec{B}$ arrows in one's sketch should point generally towards the left and also towards the central axis.

   (b) The sign of $\vec{B}\cdot d\vec{A}$ for every $d\vec{A}$ on the side of the paper cylinder is negative.

   (c) No, because Gauss' law for magnetism applies to an *enclosed* surface only. In fact, if we include the top and bottom of the cylinder to form an enclosed surface S then $\oint_s \vec{B}\cdot d\vec{A} = 0$ will be valid, as the flux through the open end of the cylinder near the magnet is positive.

2. We use $\sum_{n=1}^{6} \Phi_{Bn} = 0$ to obtain

$$\Phi_{B6} = -\sum_{n=1}^{5} \Phi_{Bn} = -(-1\,\text{Wb} + 2\,\text{Wb} - 3\,\text{Wb} + 4\,\text{Wb} - 5\,\text{Wb}) = +3\ \text{Wb}\ .$$

3. We use Gauss' law for magnetism: $\oint \vec{B}\cdot d\vec{A} = 0$. Now, $\oint \vec{B}\cdot d\vec{A} = \Phi_1 + \Phi_2 + \Phi_C$, where Φ_1 is the magnetic flux through the first end mentioned, Φ_2 is the magnetic flux through the second end mentioned, and Φ_C is the magnetic flux through the curved surface. Over the first end the magnetic field is inward, so the flux is $\Phi_1 = -25.0\,\mu\text{Wb}$. Over the second end the magnetic field is uniform, normal to the surface, and outward, so the flux is $\Phi_2 = AB = \pi r^2 B$, where A is the area of the end and r is the radius of the cylinder. It value is

$$\Phi_2 = \pi(0.120\,\text{m})^2(1.60\times10^{-3}\,\text{T}) = +7.24\times10^{-5}\,\text{Wb} = +72.4\,\mu\text{Wb}\ .$$

   Since the three fluxes must sum to zero,

$$\Phi_C = -\Phi_1 - \Phi_2 = 25.0\,\mu\text{Wb} - 72.4\,\mu\text{Wb} = -47.4\,\mu\text{Wb}\ .$$

   The minus sign indicates that the flux is inward through the curved surface.

4. The flux through Arizona is

$$\Phi = -B_r A = -(43\times10^{-6}\,\text{T})(295{,}000\,\text{km}^2)(10^3\,\text{m/km})^2 = -1.3\times10^7\ \text{Wb}\ ,$$

   inward. By Gauss' law this is equal to the negative value of the flux Φ' through the rest of the surface of the Earth. So $\Phi' = 1.3\times10^7\,\text{Wb}$, outward.

5. The horizontal component of the Earth's magnetic field is given by $B_h = B\cos\phi_i$, where B is the magnitude of the field and ϕ_i is the inclination angle. Thus

$$B = \frac{B_h}{\cos\phi_i} = \frac{16\,\mu\text{T}}{\cos 73^\circ} = 55\,\mu\text{T}\ .$$

6. (a) The Pythagorean theorem leads to

$$\begin{aligned} B &= \sqrt{B_h^2 + B_v^2} = \sqrt{\left(\frac{\mu_0\mu}{4\pi r^3}\cos\lambda_m\right)^2 + \left(\frac{\mu_0\mu}{2\pi r^3}\sin\lambda_m\right)^2} \\ &= \frac{\mu_0\mu}{4\pi r^3}\sqrt{\cos^2\lambda_m + 4\sin^2\lambda_m} = \frac{\mu_0\mu}{4\pi r^3}\sqrt{1+3\sin^2\lambda_m}\ , \end{aligned}$$

where $\cos^2\lambda_m + \sin^2\lambda_m = 1$ was used.

(b) We use Eq. 3-6:

$$\tan\phi_i = \frac{B_v}{B_h} = \frac{(\mu_0\mu/2\pi r^3)\sin\lambda_m}{(\mu_0\mu/4\pi r^3)\cos\lambda_m} = 2\tan\lambda_m\ .$$

7. (a) At the magnetic equator ($\lambda_m = 0$), the field is

$$B = \frac{\mu_0\mu}{4\pi r^3} = \frac{\left(4\pi\times10^{-7}\,\mathrm{T\cdot m/A}\right)\left(8.00\times10^{22}\,\mathrm{A\cdot m^2}\right)}{4\pi(6.37\times10^6\,\mathrm{m})^3} = 3.10\times10^{-5}\,\mathrm{T}\ ,$$

and $\phi_i = \tan^{-1}(2\tan\lambda_m) = \tan^{-1}(0) = 0$.

(b) At $\lambda_m = 60°$, we find

$$B = \frac{\mu_0\mu}{4\pi r^3}\sqrt{1+3\sin^2\lambda_m} = (3.10\times10^{-5})\sqrt{1+3\sin^2 60°} = 5.6\times10^{-5}\,\mathrm{T}\ ,$$

and $\phi_i = \tan^{-1}(2\tan 60°) = 74°$.

(c) At the north magnetic pole ($\lambda_m = 90.0°$), we obtain

$$B = \frac{\mu_0\mu}{4\pi r^3}\sqrt{1+3\sin^2\lambda_m} = (3.1\times10^{-5})\sqrt{1+3(1.00)^2} = 6.20\times10^{-5}\,\mathrm{T}\ ,$$

and $\phi_i = \tan^{-1}(2\tan 90°) = 90°$.

8. (a) At a distance r from the center of the Earth, the magnitude of the magnetic field is given by

$$B = \frac{\mu_0\mu}{4\pi r^3}\sqrt{1+3\sin^2\lambda_m}\ ,$$

where μ is the Earth's dipole moment and λ_m is the magnetic latitude. The ratio of the field magnitudes for two different distances at the same latitude is

$$\frac{B_2}{B_1} = \frac{r_1^3}{r_2^3}\ .$$

With B_1 being the value at the surface and B_2 being half of B_1, we set r_1 equal to the radius R_e of the Earth and r_2 equal to $R_e + h$, where h is altitude at which B is half its value at the surface. Thus,

$$\frac{1}{2} = \frac{R_e^3}{(R_e+h)^3}\ .$$

Taking the cube root of both sides and solving for h, we get

$$h = \left(2^{1/3}-1\right)R_e = \left(2^{1/3}-1\right)(6370\,\mathrm{km}) = 1660\ \mathrm{km}\ .$$

(b) We use the expression for B obtained in problem 6, part (a). For maximum B, we set $\sin\lambda_m = 1$. Also, $r = 6370\,\mathrm{km} - 2900\,\mathrm{km} = 3470\,\mathrm{km}$. Thus,

$$\begin{aligned} B_{\max} &= \frac{\mu_0\mu}{4\pi r^3}\sqrt{1+3\sin^2\lambda_m} \\ &= \frac{\left(4\pi\times10^{-7}\,\mathrm{T\cdot m/A}\right)\left(8.00\times10^{22}\,\mathrm{A\cdot m^2}\right)}{4\pi(3.47\times10^6\,\mathrm{m})^3}\sqrt{1+3(1)^2} = 3.83\times10^{-4}\ \mathrm{T}\ . \end{aligned}$$

(c) The angle between the magnetic axis and the rotational axis of the Earth is $11.5°$, so $\lambda_m = 90.0° - 11.5° = 78.5°$ at Earth's geographic north pole. Also $r = R_e = 6370\,\text{km}$. Thus,

$$\begin{aligned} B &= \frac{\mu_0\mu}{4\pi R_E^3}\sqrt{1+3\sin^2\lambda_m} \\ &= \frac{\left(4\pi\times 10^{-7}\,\text{T}\cdot\text{m/A}\right)(8.0\times 10^{22}\,\text{J/T})\sqrt{1+3\sin^2 78.5°}}{4\pi(6.37\times 10^6\,\text{m})^3} = 6.11\times 10^{-5}\ \text{T}\ , \end{aligned}$$

and, using the result of part (b) of problem 6,

$$\phi_i = \tan^{-1}(2\tan 78.5°) = 84.2°\ .$$

A plausible explanation to the discrepancy between the calculated and measured values of the Earth's magnetic field is that the formulas we obtained in problem 6 are based on dipole approximation, which does not accurately represent the Earth's actual magnetic field distribution on or near its surface. (Incidentally, the dipole approximation becomes more reliable when we calculate the Earth's magnetic field far from its center.)

9. We use Eq. 32-11: $\mu_{\text{orb},z} = -m_l\mu_B$.

(a) For $m_l = 1$, $\mu_{\text{orb},z} = -(1)\left(9.27\times 10^{-24}\,\text{J/T}\right) = -9.27\times 10^{-24}\,\text{J/T}$.

(b) For $m_l = -2$, $\mu_{\text{orb},z} = -(-2)\left(9.27\times 10^{-24}\,\text{J/T}\right) = 1.85\times 10^{-23}\,\text{J/T}$.

10. We use Eq. 32-7 to obtain $\Delta U = -\Delta(\mu_{s,z}B) = -B\Delta\mu_{s,z}$, where $\mu_{s,z} = \pm eh/4\pi m_e = \pm\mu_B$ (see Eqs. 32-4 and 32-5). Thus,

$$\Delta U = -B[\mu_B - (-\mu_B)] = 2\mu_B B = 2\left(9.27\times 10^{-24}\,\text{J/T}\right)(0.25\,\text{T}) = 4.6\times 10^{-24}\ \text{J}\ .$$

11. (a) Since $m_l = 0$, $L_{\text{orb},z} = m_l h/2\pi = 0$.

(b) Since $m_l = 0$, $\mu_{\text{orb},z} = -m_l\mu_B = 0$.

(c) Since $m_l = 0$, then from Eq. 32-12, $U = -\mu_{\text{orb},z}B_{\text{ext}} = -m_l\mu_B B_{\text{ext}} = 0$.

(d) Regardless of the value of m_l, we find for the spin part

$$U = -\mu_{s,z}B = \pm\mu_B B = \pm\left(9.27\times 10^{-24}\,\text{J/T}\right)(35\,\text{mT}) = \pm 3.2\times 10^{-25}\ \text{J}\ .$$

(e) Now $m_l = -3$, so

$$L_{\text{orb},z} = \frac{m_l h}{2\pi} = \frac{(-3)\left(6.63\times 10^{-27}\,\text{J}\cdot\text{s}\right)}{2\pi} = -3.16\times 10^{-34}\ \text{J}\cdot\text{s}$$

and

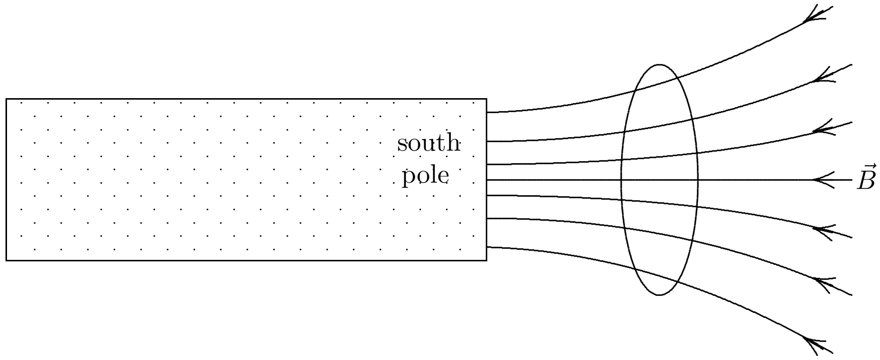
$$\mu_{\text{orb},z} = -m_l\mu_B = -(-3)\left(9.27\times 10^{-24}\,\text{J/T}\right) = 2.78\times 10^{-23}\ \text{J/T}\ .$$

The potential energy associated with the electron's orbital magnetic moment is now

$$U = -\mu_{\text{orb},z}B_{\text{ext}} = -(2.78\times 10^{-23}\,\text{J/T})(35\times 10^{-3}\,\text{T}) = -9.73\times 10^{-25}\ \text{J}\ ;$$

while the potential energy associated with the electron spin, being independent of m_l, remains the same: $\pm 3.2\times 10^{-25}$ J.

12. (a) A sketch of the field lines (due to the presence of the bar magnet) in the vicinity of the loop is shown below:



(b) The primary conclusion of §32-6 is two-fold: $\vec{\mu}$ is opposite to $\vec{B}$, and the effect of $\vec{F}$ is to move the material towards regions of smaller $|\vec{B}|$ values. The direction of the magnetic moment vector (of our loop) is toward the left in our sketch.

(c) See our comments in part (b). Since the size of $|\vec{B}|$ relates to the "crowdedness" of the field lines, we see that $\vec{F}$ is towards the right in our sketch.

13. An electric field with circular field lines is induced as the magnetic field is turned on. Suppose the magnetic field increases linearly from zero to B in time t. According to Eq. 31-27, the magnitude of the electric field at the orbit is given by

$$E = \left(\frac{r}{2}\right)\frac{dB}{dt} = \left(\frac{r}{2}\right)\frac{B}{t} \ ,$$

where r is the radius of the orbit. The induced electric field is tangent to the orbit and changes the speed of the electron, the change in speed being given by

$$\Delta v = at = \frac{eE}{m_e}t = \left(\frac{e}{m_e}\right)\left(\frac{r}{2}\right)\left(\frac{B}{t}\right)t = \frac{erB}{2m_e} \ .$$

The average current associated with the circulating electron is $i = ev/2\pi r$ and the dipole moment is

$$\mu = Ai = \left(\pi r^2\right)\left(\frac{ev}{2\pi r}\right) = \frac{1}{2}evr \ .$$

The change in the dipole moment is

$$\Delta\mu = \frac{1}{2}er\,\Delta v = \frac{1}{2}er\left(\frac{erB}{2m_e}\right) = \frac{e^2r^2B}{4m_e} \ .$$

14. Reviewing Sample Problem 32-1 before doing this exercise is helpful. Let

$$K = \frac{3}{2}kT = \left|\vec{\mu}\cdot\vec{B} - (-\vec{\mu}\cdot\vec{B})\right| = 2\mu B$$

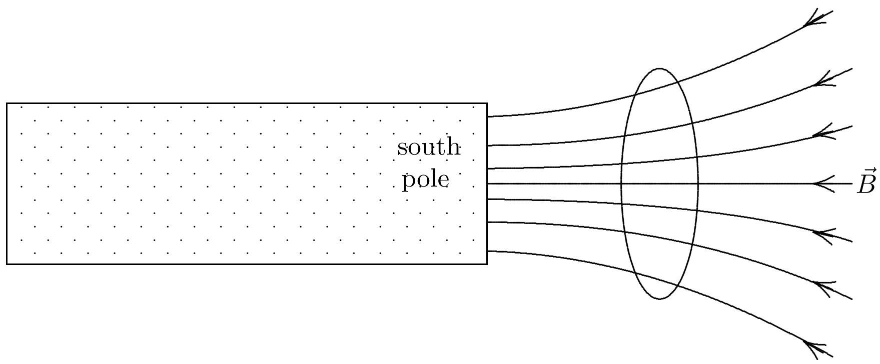
which leads to

$$T = \frac{4\mu B}{3k} = \frac{4(1.0\times10^{-23}\,\mathrm{J/T})(0.50\,\mathrm{T})}{3(1.38\times10^{-23}\,\mathrm{J/K})} = 0.48\ \mathrm{K} \ .$$

15. The magnetization is the dipole moment per unit volume, so the dipole moment is given by $\mu = M\mathcal{V}$, where M is the magnetization and $\mathcal{V}$ is the volume of the cylinder ($\mathcal{V} = \pi r^2 L$, where r is the radius of the cylinder and L is its length). Thus,

$$\mu = M\pi r^2 L = (5.30\times10^3\,\mathrm{A/m})\pi(0.500\times10^{-2}\,\mathrm{m})^2(5.00\times10^{-2}\,\mathrm{m}) = 2.08\times10^{-2}\ \mathrm{J/T} \ .$$

16. (a) A sketch of the field lines (due to the presence of the bar magnet) in the vicinity of the loop is shown below:



(b) The textbook, in §32-7, makes it clear that $\vec{\mu}$ is in the same direction as $\vec{B}$, and the effect of $\vec{F}$ is to move the material towards regions of larger $|\vec{B}|$ values. The direction of the magnetic moment vector (of our loop) is toward the right in our sketch.

(c) See our comments in part (b). Since the size of $|\vec{B}|$ relates to the "crowdedness" of the field lines, we see that $\vec{F}$ is towards the left in our sketch.

17. For the measurements carried out, the largest ratio of the magnetic field to the temperature is $(0.50\,\text{T})/(10\,\text{K}) = 0.050\,\text{T/K}$. Look at Fig. 32-9 to see if this is in the region where the magnetization is a linear function of the ratio. It is quite close to the origin, so we conclude that the magnetization obeys Curie's law.

18. (a) From Fig. 32-9 we estimate a slope of $B/T = 0.50\,\text{T/K}$ when $M/M_{\text{max}} = 50\%$. So $B = 0.50\,\text{T} = (0.50\,\text{T/K})(300\,\text{K}) = 150\,\text{T}$.

(b) Similarly, now $B/T \approx 2$ so $B = (2)(300) = 600\,\text{T}$.

(c) Except for very short times and in very small volumes, these values are not attainable in the lab.

19. (a) A charge e traveling with uniform speed v around a circular path of radius r takes time $T = 2\pi r/v$ to complete one orbit, so the average current is

$$i = \frac{e}{T} = \frac{ev}{2\pi r} .$$

The magnitude of the dipole moment is this multiplied by the area of the orbit:

$$\mu = \frac{ev}{2\pi r}\,\pi r^2 = \frac{evr}{2} .$$

Since the magnetic force of with magnitude evB is centripetal, Newton's law yields $evB = m_e v^2/r$, so

$$r = \frac{m_e v}{eB} .$$

Thus,

$$\mu = \frac{1}{2}(ev)\left(\frac{m_e v}{eB}\right) = \left(\frac{1}{B}\right)\left(\frac{1}{2}m_e v^2\right) = \frac{K_e}{B} .$$

The magnetic force $-e\vec{v} \times \vec{B}$ must point toward the center of the circular path. If the magnetic field is directed into the page, for example, the electron will travel clockwise around the circle. Since the electron is negative, the current is in the opposite direction, counterclockwise and, by the right-hand rule for dipole moments, the dipole moment is out of the page. That is, the dipole moment is directed opposite to the magnetic field vector.

(b) We note that the charge canceled in the derivation of $\mu = K_e/B$. Thus, the relation $\mu = K_i/B$ holds for a positive ion. If the magnetic field is directed into the page, the ion travels counterclockwise around a circular orbit and the current is in the same direction. Therefore, the dipole moment is again out of the page, opposite to the magnetic field.

(c) The magnetization is given by $M = \mu_e n_e + \mu_i n_i$, where μ_e is the dipole moment of an electron, n_e is the electron concentration, μ_i is the dipole moment of an ion, and n_i is the ion concentration. Since $n_e = n_i$, we may write n for both concentrations. We substitute $\mu_e = K_e/B$ and $\mu_i = K_i/B$ to obtain

$$M = \frac{n}{B}(K_e + K_i) = \frac{5.3 \times 10^{21}\,\mathrm{m}^{-3}}{1.2\,\mathrm{T}}\left(6.2 \times 10^{-20}\,\mathrm{J} + 7.6 \times 10^{-21}\,\mathrm{J}\right) = 310\ \mathrm{A/m}\ .$$

20. The Curie temperature for iron is 770°C. If x is the depth at which the temperature has this value, then $10°\mathrm{C} + (30°\mathrm{C/km})x = 770°\mathrm{C}$. Therefore,

$$x = \frac{770°\mathrm{C} - 10°\mathrm{C}}{30°\mathrm{C/km}} = 25\ \mathrm{km}\ .$$

21. (a) The field of a dipole along its axis is given by Eq. 30-29:

$$B = \frac{\mu_0}{2\pi}\frac{\mu}{z^3}\ ,$$

where μ is the dipole moment and z is the distance from the dipole. Thus,

$$B = \frac{(4\pi \times 10^{-7}\,\mathrm{T{\cdot}m}/A)(1.5 \times 10^{-23}\,\mathrm{J/T})}{2\pi(10 \times 10^{-9}\,\mathrm{m})} = 3.0 \times 10^{-6}\ \mathrm{T}\ .$$

(b) The energy of a magnetic dipole $\vec{\mu}$ in a magnetic field $\vec{B}$ is given by $U = -\vec{\mu}\cdot\vec{B} = -\mu B\cos\phi$, where ϕ is the angle between the dipole moment and the field. The energy required to turn it end-for-end (from $\phi = 0°$ to $\phi = 180°$) is

$$\Delta U = 2\mu B = 2(1.5 \times 10^{-23}\,\mathrm{J/T})(3.0 \times 10^{-6}\,\mathrm{T}) = 9.0 \times 10^{-29}\ \mathrm{J} = 5.6 \times 10^{-10}\ \mathrm{eV}\ .$$

The mean kinetic energy of translation at room temperature is about 0.04 eV. Thus, if dipole-dipole interactions were responsible for aligning dipoles, collisions would easily randomize the directions of the moments and they would not remain aligned.

22. (a) The number of iron atoms in the iron bar is

$$N = \frac{\left(7.9\,\mathrm{g/cm^3}\right)(5.0\,\mathrm{cm})\left(1.0\,\mathrm{cm^2}\right)}{(55.847\,\mathrm{g/mol})\,/\,(6.022 \times 10^{23}/\mathrm{mol})} = 4.3 \times 10^{23}\ .$$

Thus the dipole moment of the iron bar is

$$\mu = \left(2.1 \times 10^{-23}\,\mathrm{J/T}\right)\left(4.3 \times 10^{23}\right) = 8.9\ \mathrm{A{\cdot}m^2}\ .$$

(b) $\tau = \mu B\sin 90° = (8.9\,\mathrm{A{\cdot}m^2})(1.57\,\mathrm{T}) = 13\,\mathrm{N{\cdot}m}$.

23. The saturation magnetization corresponds to complete alignment of all atomic dipoles and is given by $M_{\mathrm{sat}} = \mu n$, where n is the number of atoms per unit volume and μ is the magnetic dipole moment of an atom. The number of nickel atoms per unit volume is $n = \rho/m$, where ρ is the density of nickel. The mass of a single nickel atom is calculated using $m = M/N_A$, where M is the atomic mass of nickel and N_A is Avogadro's constant. Thus,

$$\begin{aligned} n &= \frac{\rho N_A}{M} = \frac{(8.90\,\mathrm{g/cm^3})(6.02 \times 10^{23}\,\mathrm{atoms/mol})}{58.71\,\mathrm{g/mol}} \\ &= 9.126 \times 10^{22}\,\mathrm{atoms/cm^3} = 9.126 \times 10^{28}\ \mathrm{atoms/m^3}\ . \end{aligned}$$

The dipole moment of a single atom of nickel is

$$\mu = \frac{M_{\rm sat}}{n} = \frac{4.70 \times 10^5\,{\rm A/m}}{9.126 \times 10^{28}\,{\rm m}^3} = 5.15 \times 10^{-24}\ {\rm A\cdot m^2}\ .$$

24. From the way the wire is wound it is clear that P_2 is the magnetic north pole while P_1 is the south pole.

(a) The deflection will be toward P_1 (away from the magnetic north pole).

(b) As the electromagnet is turned on, the magnetic flux Φ_B through the aluminum changes abruptly, causing a strong induced current which produces a magnetic field opposite to that of the electromagnet. As a result, the aluminum sample will be pushed toward P_1, away from the magnetic north pole of the bar magnet. As Φ_B reaches a constant value, however, the induced current disappears and the aluminum sample, being paramagnetic, will move slightly toward P_2, the magnetic north pole of the electromagnet.

(c) A magnetic north pole will now be induced on the side of the sample closer to P_1, and a magnetic south pole will appear on the other side. If the magnitude of the field of the electromagnet is larger near P_1, then the sample will move toward P_1.

25. (a) If the magnetization of the sphere is saturated, the total dipole moment is $\mu_{\rm total} = N\mu$, where N is the number of iron atoms in the sphere and μ is the dipole moment of an iron atom. We wish to find the radius of an iron sphere with N iron atoms. The mass of such a sphere is Nm, where m is the mass of an iron atom. It is also given by $4\pi\rho R^3/3$, where ρ is the density of iron and R is the radius of the sphere. Thus $Nm = 4\pi\rho R^3/3$ and

$$N = \frac{4\pi\rho R^3}{3m}\ .$$

We substitute this into $\mu_{\rm total} = N\mu$ to obtain

$$\mu_{\rm total} = \frac{4\pi\rho R^3\mu}{3m}\ .$$

We solve for R and obtain

$$R = \left(\frac{3m\mu_{\rm total}}{4\pi\rho\mu}\right)^{1/3}\ .$$

The mass of an iron atom is

$$m = 56\,{\rm u} = (56\,{\rm u})(1.66 \times 10^{-27}\,{\rm kg/u}) = 9.30 \times 10^{-26}\ {\rm kg}\ .$$

Therefore,

$$R = \left[\frac{3(9.30 \times 10^{-26}\,{\rm kg})(8.0 \times 10^{22}\,{\rm J/T})}{4\pi(14 \times 10^3\,{\rm kg/m^3})(2.1 \times 10^{-23}\,{\rm J/T})}\right]^{1/3} = 1.8 \times 10^5\ {\rm m}\ .$$

(b) The volume of the sphere is

$$V_s = \frac{4\pi}{3}R^3 = \frac{4\pi}{3}(1.82 \times 10^5\,{\rm m})^3 = 2.53 \times 10^{16}\,{\rm m}^3$$

and the volume of the Earth is

$$V_e = \frac{4\pi}{3}(6.37 \times 10^6\,{\rm m})^3 = 1.08 \times 10^{21}\ {\rm m}^3\ ,$$

so the fraction of the Earth's volume that is occupied by the sphere is

$$\frac{2.53 \times 10^{16}\,{\rm m}^3}{1.08 \times 10^{21}\,{\rm m}^3} = 2.3 \times 10^{-5}\ .$$

26. Let R be the radius of a capacitor plate and r be the distance from axis of the capacitor. For points with $r \leq R$, the magnitude of the magnetic field is given by

$$B = \frac{\mu_0 \varepsilon_0 r}{2} \frac{dE}{dt} ,$$

and for $r \geq R$, it is

$$B = \frac{\mu_0 \varepsilon_0 R^2}{2r} \frac{dE}{dt} .$$

The maximum magnetic field occurs at points for which $r = R$, and its value is given by either of the formulas above:

$$B_{\max} = \frac{\mu_0 \varepsilon_0 R}{2} \frac{dE}{dt} .$$

There are two values of r for which $B = B_{\max}/2$: one less than R and one greater. To find the one that is less than R, we solve

$$\frac{\mu_0 \varepsilon_0 r}{2} \frac{dE}{dt} = \frac{\mu_0 \varepsilon_0 R}{4} \frac{dE}{dt}$$

for r. The result is $r = R/2 = (55.0\,\text{mm})/2 = 27.5\,\text{mm}$. To find the one that is greater than R, we solve

$$\frac{\mu_0 \varepsilon_0 R^2}{2r} \frac{dE}{dt} = \frac{\mu_0 \varepsilon_0 R}{4} \frac{dE}{dt}$$

for r. The result is $r = 2R = 2(55.0\,\text{mm}) = 110\,\text{mm}$.

27. We use the result of part (b) in Sample Problem 32-3:

$$B = \frac{\mu_0 \varepsilon_0 R^2}{2r} \frac{dE}{dt} \qquad\qquad \text{(for } r \geq R\text{)}$$

to solve for dE/dt:

$$\begin{aligned} \frac{dE}{dt} &= \frac{2Br}{\mu_0 \varepsilon_0 R^2} \\ &= \frac{2(2.0 \times 10^{-7}\ T)(6.0 \times 10^{-3}\,\text{m})}{(4\pi \times 10^{-7}\,\text{T}\cdot\text{m/A})\left(8.85 \times 10^{-12}\,\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)(3.0 \times 10^{-3}\,\text{m})^2} = 2.4 \times 10^{13}\,\frac{\text{V}}{\text{m}\cdot\text{s}} . \end{aligned}$$

28. (a) Noting that the magnitude of the electric field (assumed uniform) is given by $E = V/d$ (where $d = 5.0$ mm), we use the result of part (a) in Sample Problem 32-3

$$B = \frac{\mu_0 \varepsilon_0 r}{2} \frac{dE}{dt} = \frac{\mu_0 \varepsilon_0 r}{2d} \frac{dV}{dt} \qquad\qquad \text{(for } r \leq R\text{)} .$$

We also use the fact that the time derivative of $\sin(\omega t)$ (where $\omega = 2\pi f = 2\pi(60) \approx 377/\text{s}$ in this problem) is $\omega \cos(\omega t)$. Thus, we find the magnetic field as a function of r (for $r \leq R$; note that this neglects "fringing" and related effects at the edges):

$$B = \frac{\mu_0 \varepsilon_0 r}{2d} V_{\max} \omega \cos(\omega t) \implies B_{\max} = \frac{\mu_0 \varepsilon_0 r V_{\max} \omega}{2d}$$

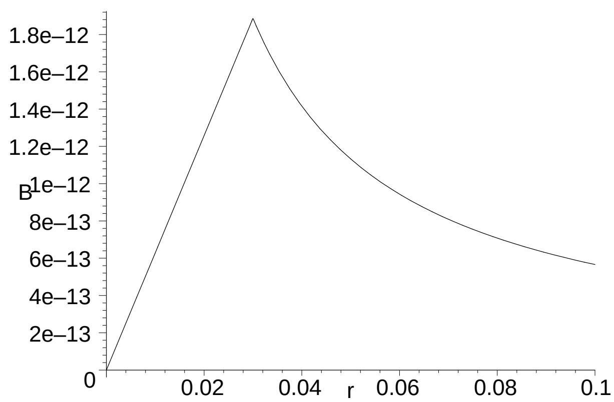
where $V_{\max} = 150$ V. This grows with r until reaching its highest value at $r = R = 30$ mm:

$$\begin{aligned} B_{\max}\Big|_{r=R} &= \frac{\mu_0 \varepsilon_0 R V_{\max} \omega}{2d} \\ &= \frac{\left(4\pi \times 10^{-7}\,\text{H/m}\right)\left(8.85 \times 10^{-12}\,\text{F/m}\right)\left(30 \times 10^{-3}\,\text{m}\right)(150\,\text{V})(377/\text{s})}{2(5.0 \times 10^{-3}\,\text{m})} \\ &= 1.9 \times 10^{-12}\ \text{T} . \end{aligned}$$

(b) For $r \le 0.03\,\mathrm{m}$, we use the $B_{\max} = \frac{\mu_0\varepsilon_0 r V_{\max}\omega}{2d}$ expression found in part (a) (note the $B \propto r$ dependence), and for $r \ge 0.03\,\mathrm{m}$ we perform a similar calculation starting with the result of part (b)in Sample Problem 32-3:

$$\begin{aligned} B_{\max} &= \left(\frac{\mu_0\varepsilon_0 R^2}{2r}\frac{dE}{dt}\right)_{\max} \\ &= \left(\frac{\mu_0\varepsilon_0 R^2}{2rd}\frac{dV}{dt}\right)_{\max} \\ &= \left(\frac{\mu_0\varepsilon_0 R^2}{2rd}V_{\max}\omega\cos(\omega t)\right)_{\max} \\ &= \frac{\mu_0\varepsilon_0 R^2 V_{\max}\omega}{2rd} \qquad \text{(for } r \ge R) \end{aligned}$$

(note the $B \propto r^{-1}$ dependence – See also Eqs. 32-40 and 32-41). The plot (with SI units understood) is shown below.



29. The displacement current is given by

$$i_d = \varepsilon_0 A\frac{dE}{dt} ,$$

where A is the area of a plate and E is the magnitude of the electric field between the plates. The field between the plates is uniform, so $E = V/d$, where V is the potential difference across the plates and d is the plate separation. Thus

$$i_d = \frac{\varepsilon_0 A}{d}\frac{dV}{dt} .$$

Now, $\varepsilon_0 A/d$ is the capacitance C of a parallel-plate capacitor (not filled with a dielectric), so

$$i_d = C\frac{dV}{dt} .$$

30. Let the area plate be A and the plate separation be d. We use Eq. 32-34:

$$i_d = \varepsilon_0\frac{d\Phi_E}{dt} = \varepsilon_0\frac{d}{dt}(AE) = \varepsilon_0 A\frac{d}{dt}\left(\frac{V}{d}\right) = \frac{\varepsilon_0 A}{d}\left(\frac{dV}{dt}\right) ,$$

or

$$\frac{dV}{dt} = \frac{i_d d}{\varepsilon_0 A} = \frac{i_d}{C} = \frac{1.5\ \mathrm{A}}{2.0\times10^{-6}\ \mathrm{F}} = 7.5\times10^5\ \mathrm{V/s} .$$

Therefore, we need to change the voltage difference across the capacitor at the rate of $7.5\times10^5\,\mathrm{V/s}$.

31. Consider an area A, normal to a uniform electric field $\vec{E}$. The displacement current density is uniform and normal to the area. Its magnitude is given by $J_d = i_d/A$. For this situation

$$i_d = \varepsilon_0 A \frac{dE}{dt} ,$$

so

$$J_d = \frac{1}{A}\varepsilon_0 A \frac{dE}{dt} = \varepsilon_0 \frac{dE}{dt} .$$

32. We use Eq. 32-38:

$$i_d = \varepsilon_0 A \frac{dE}{dt} .$$

Note that, in this situation, A is the area over which a changing electric field is present. In this case $r > R$, so $A = \pi R^2$. Thus,

$$\frac{dE}{dt} = \frac{i_d}{\varepsilon_0 A} = \frac{i_d}{\varepsilon_0 \pi R^2} = \frac{2.0\,\mathrm{A}}{\pi\left(8.85\times 10^{-12}\,\frac{\mathrm{C}^2}{\mathrm{N\cdot m^2}}\right)(0.10\,\mathrm{m})^2} = 7.2\times 10^{12}\,\frac{\mathrm{V}}{\mathrm{m\cdot s}} .$$

33. (a) We use $\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{\mathrm{enclosed}}$ to find

$$\begin{aligned} B &= \frac{\mu_0 I_{\mathrm{enclosed}}}{2\pi r} = \frac{\mu_0 (J_d \pi r^2)}{2\pi r} = \frac{1}{2}\mu_0 J_d r \\ &= \frac{1}{2}(1.26\times 10^{-6}\,\mathrm{H/m})(20\,\mathrm{A/m^2})(50\times 10^{-3}\,\mathrm{m}) = 6.3\times 10^{-7}\ \mathrm{T} . \end{aligned}$$

(b) From

$$i_d = J_d \pi r^2 = \varepsilon_0 \frac{d\Phi_E}{dt} = \varepsilon_0 \pi r^2 \frac{dE}{dt}$$

we get

$$\frac{dE}{dt} = \frac{J_d}{\varepsilon_0} = \frac{20\,\mathrm{A/m^2}}{8.85\times 10^{-12}\,\mathrm{F/m}} = 2.3\times 10^{12}\,\frac{\mathrm{V}}{\mathrm{m\cdot s}} .$$

34. (a) From Eq. 32-34,

$$\begin{aligned} i_d &= \varepsilon_0 \frac{d\Phi_E}{dt} = \varepsilon_0 A \frac{dE}{dt} = \varepsilon_0 A \frac{d}{dt}\left[(4.0\times 10^5) - (6.0\times 10^4 t)\right] \\ &= -\varepsilon_0 A (6.0\times 10^4\,\mathrm{V/m\cdot s}) \\ &= -\left(8.85\times 10^{-12}\,\frac{\mathrm{C}^2}{\mathrm{N\cdot m^2}}\right)(4.0\times 10^{-2}\,\mathrm{m^2})(6.0\times 10^4\,\mathrm{V/m\cdot s}) \\ &= -2.1\times 10^{-8}\,\mathrm{A} . \end{aligned}$$

(b) If one draws a counterclockwise circular loop s around the plates, then according to Eq. 32-42

$$\oint_s \vec{B}\cdot d\vec{s} = \mu_0 i_d < 0 ,$$

which means that $\vec{B}\cdot d\vec{s} < 0$. Thus $\vec{B}$ must be clockwise.

35. (a) In region a of the graph,

$$\begin{aligned} |i_d| &= \varepsilon_0 \left|\frac{d\Phi_E}{dt}\right| = \varepsilon_0 A \left|\frac{dE}{dt}\right| \\ &= (8.85\times 10^{-12}\,\mathrm{F/m})(1.6\,\mathrm{m^2})\left|\frac{4.5\times 10^5\,\mathrm{N/C} - 6.0\times 10^5\,\mathrm{N/C}}{4.0\times 10^{-6}\,\mathrm{s}}\right| = 0.71\ \mathrm{A}. \end{aligned}$$

(b) $i_d \propto dE/dt = 0$.

(c) In region c of the graph,

$$|i_d| = \varepsilon_0 A \left|\frac{dE}{dt}\right| = (8.85 \times 10^{-12}\,\mathrm{F/m})(1.6\,\mathrm{m}^2)\left|\frac{-4.0 \times 10^5\,\mathrm{N/C}}{15 \times 10^{-6}\,\mathrm{s} - 10 \times 10^{-6}\,\mathrm{s}}\right| = 1.1 \text{ A} .$$

36. Using Eq. 32-38, we have

$$\frac{d\left|\vec{E}\right|}{dt} = \frac{i_d}{\varepsilon_0\, A} = 7.2 \times 10^{12}$$

where $A = \pi(0.10)^2$ (fringing is being neglected in §32-10) and SI units are understood.

37. (a) At any instant the displacement current i_d in the gap between the plates equals the conduction current i in the wires. Thus $i_d = i = 2.0\,\mathrm{A}$.

(b) The rate of change of the electric field is

$$\frac{dE}{dt} = \frac{1}{\varepsilon_0 A}\left(\varepsilon_0 \frac{d\Phi_E}{dt}\right) = \frac{i_d}{\varepsilon_0 A} = \frac{2.0\,\mathrm{A}}{(8.85 \times 10^{-12}\,\mathrm{F/m})(1.0\,\mathrm{m})^2} = 2.3 \times 10^{11}\ \frac{\mathrm{V}}{\mathrm{m{\cdot}s}} .$$

(c) The displacement current through the indicated path is

$$i'_d = i_d \times \left(\frac{\text{area enclosed by the path}}{\text{area of each plate}}\right) = (2.0\,\mathrm{A})\left(\frac{0.50\,\mathrm{m}}{1.0\,\mathrm{m}}\right)^2 = 0.50 \text{ A} .$$

(d) The integral of the field around the indicated path is

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 i'_d = (1.26 \times 10^{-6}\,\mathrm{H/m})(0.50\,\mathrm{A}) = 6.3 \times 10^{-7}\ \mathrm{T{\cdot}m} .$$

38. (a) Since $i = i_d$ (Eq. 32-39) then the portion of displacement current enclosed is

$$i_{d,\mathrm{enc}} = i\,\frac{\pi\left(\frac{R}{3}\right)^2}{\pi R^2} = i\,\frac{1}{9} = 1.33 \text{ A} .$$

(b) We see from Sample Problems 32-3 and 32-4 that the maximum field is at $r = R$ and that (in the interior) the field is simply proportional to r. Therefore,

$$\frac{B}{B_{\max}} = \frac{3.00 \text{ mT}}{12.0 \text{ mT}} = \frac{r}{R}$$

which yields $r = R/4$ as a solution. We now look for a solution in the exterior region, where the field is inversely proportional to r (by Eq. 32-41):

$$\frac{B}{B_{\max}} = \frac{3.00 \text{ mT}}{12.0 \text{ mT}} = \frac{R}{r}$$

which yields $r = 4R$ as a solution.

39. (a) Using Eq. 27-10, we find

$$E = \rho J = \frac{\rho i}{A} = \frac{(1.62 \times 10^{-8}\,\Omega{\cdot}\mathrm{m})(100\,\mathrm{A})}{5.00 \times 10^{-6}\,\mathrm{m}^2} = 0.324 \text{ V/m} .$$

(b) The displacement current is

$$\begin{aligned} i_d &= \varepsilon_0 \frac{d\Phi_E}{dt} = \varepsilon_0 A \frac{dE}{dt} = \varepsilon_0 A \frac{d}{dt}\left(\frac{\rho i}{A}\right) = \varepsilon_0 \rho \frac{di}{dt} \\ &= (8.85 \times 10^{-12}\,\mathrm{F})(1.62 \times 10^{-8}\,\Omega)(2000\,\mathrm{A/s}) = 2.87 \times 10^{-16} \text{ A} . \end{aligned}$$

(c) The ratio of fields is

$$\frac{B(\text{ due to } i_d)}{B(\text{ due to } i)} = \frac{\mu_0 i_d/2\pi r}{\mu_0 i/2\pi r} = \frac{i_d}{i} = \frac{2.87\times10^{-16}\,\text{A}}{100\,\text{A}} = 2.87\times10^{-18}\ .$$

40. (a) From Sample Problem 32-3 we know that $B \propto r$ for $r \le R$ and $B \propto r^{-1}$ for $r \ge R$. So the maximum value of B occurs at $r = R$, and there are two possible values of r at which the magnetic field is 75% of B_{max}. We denote these two values as r_1 and r_2, where $r_1 < R$ and $r_2 > R$. Then $0.75B_{\text{max}}/B_{\text{max}} = r_1/R$, or $r_1 = 0.75R$; and $0.75B_{\text{max}}/B_{\text{max}} = (r_2/R)^{-1}$, or $r_2 = R/0.75 = 1.3R$.

(b) From Eqs. 32-39 and 32-41,

$$B_{\text{max}} = \frac{\mu_0 i_d}{2\pi R} = \frac{\mu_0 i}{2\pi R} = \frac{\left(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}\right)(6.0\,\text{A})}{2\pi(0.040\,\text{m})} = 3.0\times10^{-5}\ \text{T}\ .$$

41. (a) At any instant the displacement current i_d in the gap between the plates equals the conduction current i in the wires. Thus $i_{\text{max}} = i_{d\ \text{max}} = 7.60\,\mu\text{A}$.

(b) Since $i_d = \varepsilon_0\,(d\Phi_E/dt)$,

$$\left(\frac{d\Phi_E}{dt}\right)_{\text{max}} = \frac{i_{d\ \text{max}}}{\varepsilon_0} = \frac{7.60\times10^{-6}\,\text{A}}{8.85\times10^{-12}\ \text{F/m}} = 8.59\times10^{5}\text{V}\cdot\text{m/s}\ .$$

(c) According to problem 29,

$$i_d = C\frac{dV}{dt} = \frac{\varepsilon_0 A}{d}\frac{dV}{dt}\ .$$

Now the potential difference across the capacitor is the same in magnitude as the emf of the generator, so $V = \mathcal{E}_{\text{m}}\sin\omega t$ and $dV/dt = \omega\mathcal{E}_{\text{m}}\cos\omega t$. Thus,

$$i_d = \frac{\varepsilon_0 A\omega\mathcal{E}_{\text{m}}}{d}\cos\omega t$$

and

$$i_{d\ \text{max}} = \frac{\varepsilon_0 A\omega\mathcal{E}_{\text{m}}}{d}\ .$$

This means

$$\begin{aligned} d &= \frac{\varepsilon_0 A\omega\mathcal{E}_{\text{m}}}{i_{d\,\text{max}}} = \frac{(8.85\times10^{-12}\,\text{F/m})\pi(0.180\text{m})^2(130\,\text{rad/s})(220\text{V})}{7.60\times10^{-6}\,\text{A}} \\ &= 3.39\times10^{-3}\,\text{m}\ , \end{aligned}$$

where $A = \pi R^2$ was used.

(d) We use the Ampere-Maxwell law in the form $\oint \vec{B}\cdot d\vec{s} = \mu_0 I_d$, where the path of integration is a circle of radius r between the plates and parallel to them. I_d is the displacement current through the area bounded by the path of integration. Since the displacement current density is uniform between the plates $I_d = (r^2/R^2)i_d$, where i_d is the total displacement current between the plates and R is the plate radius. The field lines are circles centered on the axis of the plates, so $\vec{B}$ is parallel to $d\vec{s}$. The field has constant magnitude around the circular path, so $\oint \vec{B}\cdot d\vec{s} = 2\pi rB$. Thus,

$$2\pi rB = \mu_0\left(\frac{r^2}{R^2}\right)i_d$$

and

$$B = \frac{\mu_0 i_d r}{2\pi R^2}\ .$$

The maximum magnetic field is given by

$$B_{\text{max}} = \frac{\mu_0 i_{d\ \text{max}}\,r}{2\pi R^2} = \frac{(4\pi\times10^{-7}\text{T}\cdot\text{m/A})(7.6\times10^{-6}\,\text{A})(0.110\text{m})}{2\pi(0.180\text{m})^2} = 5.16\times10^{-12}\ \text{T}\ .$$

42. From Gauss' law for magnetism, the flux through S_1 is equal to that through S_2, the portion of the xz plane that lies within the cylinder. Here the normal direction of S_2 is $+y$. Therefore,

$$\begin{aligned}\Phi_B(S_1) &= \Phi_B(S_2) = \int_{-r}^{r} B(x)L\,dx \\ &= 2\int_{-r}^{r} B_{\text{left}}(x)L\,dx \\ &= 2\int_{-r}^{r} \frac{\mu_0 i}{2\pi}\frac{1}{2r-x}L\,dx \quad = \quad \frac{\mu_0 iL}{\pi}\ln 3\ .\end{aligned}$$

43. (a) Again from Fig. 32-9, for $M/M_{\text{max}} = 50\%$ we have $B/T = 0.50$. So $T = B/0.50 = 2/0.50 = 4\,\text{K}$.

(b) Now $B/T = 2.0$, so $T = 2/2.0 = 1\,\text{K}$.

44. (a) For a given value of l, m_l varies from $-l$ to $+l$. Thus, in our case $l = 3$, and the number of different m_l's is $2l + 1 = 2(3) + 1 = 7$. Thus, since $L_{\text{orb},z} \propto m_l$, there are a total of seven different values of $L_{\text{orb},z}$.

(b) Similarly, since $\mu_{\text{orb},z} \propto m_l$, there are also a total of seven different values of $\mu_{\text{orb},z}$.

(c) Since $L_{\text{orb},z} = m_l h/2\pi$, the greatest allowed value of $L_{\text{orb},z}$ is given by $|m_l|_{\text{max}}h/2\pi = 3h/2\pi$; while the least allowed value is given by $|m_l|_{\text{min}}h/2\pi = 0$.

(d) Similar to part (c), since $\mu_{\text{orb},z} = -m_l\mu_B$, the greatest allowed value of $\mu_{\text{orb},z}$ is given by $|m_l|_{\text{max}}\mu_B = 3eh/4\pi m_e$; while the least allowed value is given by $|m_l|_{\text{min}}\mu_B = 0$.

(e) From Eqs. 32-3 and 32-9 the z component of the net angular momentum of the electron is given by

$$L_{\text{net},z} = L_{\text{orb},z} + L_{s,z} = \frac{m_l h}{2\pi} + \frac{m_s h}{2\pi}\ .$$

For the maximum value of $L_{\text{net},z}$ let $m_l = [m_l]_{\text{max}} = 3$ and $m_s = \frac{1}{2}$. Thus

$$[L_{\text{net},z}]_{\text{max}} = \left(3 + \frac{1}{2}\right)\frac{h}{2\pi} = \frac{3.5h}{2\pi}\ .$$

(f) Since the maximum value of $L_{\text{net},z}$ is given by $[m_J]_{\text{max}}h/2\pi$ with $[m_J]_{\text{max}} = 3.5$ (see the last part above), the number of allowed values for the z component of $L_{\text{net},z}$ is given by $2[m_J]_{\text{max}} + 1 = 2(3.5) + 1 = 8$.

45. (a) We use the result of part (a) in Sample Problem 32-3:

$$B = \frac{\mu_0\varepsilon_0 r}{2}\frac{dE}{dt} \qquad\qquad (\text{for } r \le R)\ ,$$

where $r = 0.80R$ and

$$\frac{dE}{dt} = \frac{d}{dt}\left(\frac{V}{d}\right) = \frac{1}{d}\frac{d}{dt}\left(V_0 e^{-t/\tau}\right) = -\frac{V_0}{\tau d}e^{-t/\tau}\ .$$

Here $V_0 = 100\,\text{V}$. Thus

$$\begin{aligned}B(t) &= \left(\frac{\mu_0\varepsilon_0 r}{2}\right)\left(-\frac{V_0}{\tau d}e^{-t/\tau}\right) = -\frac{\mu_0\varepsilon_0 V_0 r}{2\tau d}\,e^{-t/\tau} \\ &= -\frac{\left(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}\right)\left(8.85\times10^{-12}\,\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)(100\,\text{V})(0.80)(16\,\text{mm})}{2(12\times10^{-3}\,\text{s})(5.0\,\text{mm})}\,e^{-t/12\,\text{ms}} \\ &= -(1.2\times10^{-13}\,\text{T})\,e^{-t/12\,\text{ms}}\ .\end{aligned}$$

The minus sign here is insignificant.

(b) At time $t = 3\tau$, $B(t) = -(1.2 \times 10^{-13}\,\mathrm{T})e^{-3\tau/\tau} = -5.9 \times 10^{-15}\,\mathrm{T}$.

46. The given value 7.0 mW should be 7.0 mWb. From Eq. 32-1, we have

$$\begin{aligned}(\Phi_B)_{\rm in} &= (\Phi_B)_{\rm out} \\ &= 0.0070\,\mathrm{Wb} + (0.40\,\mathrm{T})(\pi r^2) \\ &= 9.2 \times 10^{-3}\ \mathrm{Wb}\ .\end{aligned}$$

Thus, the magnetic flux at the sides is inward with absolute-value equal to 9.2 mWb.

47. The definition of displacement current is Eq. 32-34, and the formula of greatest convenience here is Eq. 32-41:

$$i_d = \frac{2\pi\, r\, B}{\mu_0} = \frac{2\pi(0.0300\,\mathrm{m})\left(2.00 \times 10^{-6}\,\mathrm{T}\right)}{4\pi \times 10^{-7}\,\mathrm{T}\cdot\mathrm{m/A}} = 0.30\ \mathrm{A}\ .$$

48. Ignoring points where the determination of the slope is problematic, we find the interval of largest $\Delta|\vec{E}|/\Delta t$ is $6\,\mu\mathrm{s} < t < 7\,\mu\mathrm{s}$. During that time, we have, from Eq. 32-38,

$$i_d = \varepsilon_0\, A\, \frac{\Delta|\vec{E}|}{\Delta t} = \varepsilon_0 \left(2.0\,\mathrm{m}^2\right)\left(2.0 \times 10^6\,\mathrm{V/m}\right)$$

which yields $i_d = 3.5 \times 10^{-5}$ A.

49. (a) We use the notation $P(\mu)$ for the probability of a dipole being parallel to $\vec{B}$, and $P(-\mu)$ for the probability of a dipole being antiparallel to the field. The magnetization may be thought of as a "weighted average" in terms of these probabilities:

$$M = \frac{N\mu P(\mu) - N\mu P(-\mu)}{P(\mu) + P(-\mu)} = \frac{N\mu\left(e^{\mu B/KT} - e^{-\mu B/KT}\right)}{e^{\mu B/KT} + e^{-\mu B/KT}} = N\mu \tanh\left(\frac{\mu B}{kT}\right)\ .$$

(b) For $\mu B \ll kT$ (that is, $\mu B/kT \ll 1$) we have $e^{\pm \mu B/kT} \approx 1 \pm \mu B/kT$, so

$$M = N\mu \tanh\left(\frac{\mu B}{kT}\right) \approx \frac{N\mu[(1+\mu B/kT) - (1-\mu B/kT)]}{(1+\mu B/kT) + (1-\mu B/kT)} = \frac{N\mu^2 B}{kT}\ .$$

(c) For $\mu B \gg kT$ we have $\tanh(\mu B/kT) \approx 1$, so

$$M = N\mu \tanh\left(\frac{\mu B}{kT}\right) \approx N\mu\ .$$

(d) One can easily plot the tanh function using, for instance, a graphical calculator. One can then note the resemblance between such a plot and Fig. 32-9. By adjusting the parameters used in one's plot, the curve in Fig. 32-9 can reliably be fit with a tanh function.

50. (a) From Eq. 22-3,

$$E = \frac{e}{4\pi\varepsilon_0 r^2} = \frac{(1.60 \times 10^{-19}\,\mathrm{C})(8.99 \times 10^9\,\mathrm{N\cdot m^2/\,C^2})}{(5.2 \times 10^{-11}\,\mathrm{m})^2} = 5.3 \times 10^{11}\ \mathrm{N/C}\ .$$

(b) We use Eq. 30-28:

$$B = \frac{\mu_0}{2\pi}\frac{\mu_p}{r^3} = \frac{\left(4\pi \times 10^{-7}\,\mathrm{T\cdot m/A}\right)(1.4 \times 10^{-26}\,\mathrm{J/T})}{2\pi(5.2 \times 10^{-11}\,\mathrm{m})^3} = 2.0 \times 10^{-2}\ \mathrm{T}\ .$$

(c) From Eq. 32-10,

$$\frac{\mu_{\rm orb}}{\mu_p} = \frac{eh/4\pi m_e}{\mu_p} = \frac{\mu_B}{\mu_p} = \frac{9.27 \times 10^{-24}\,\mathrm{J/T}}{1.4 \times 10^{-26}\,\mathrm{J/T}} = 6.6 \times 10^2\ .$$

51. The interacting potential energy between the magnetic dipole of the compass and the Earth's magnetic field is $U = -\vec{\mu}\cdot\vec{B}_e = -\mu B_e \cos\theta$, where θ is the angle between $\vec{\mu}$ and $\vec{B}_e$. For small angle θ

$$U(\theta) = -\mu B_e \cos\theta \quad \approx \quad -\mu B_e \left(1 - \frac{\theta^2}{2}\right) = \frac{1}{2}\kappa\theta^2 - \mu B_e$$

where $\kappa = \mu B_e$. Conservation of energy for the compass then gives

$$\frac{1}{2} I \left(\frac{d\theta}{dt}\right)^2 + \frac{1}{2}\kappa\theta^2 = \text{const.} \ .$$

This is to be compared with the following expression for the mechanical energy of a spring-mass system:

$$\frac{1}{2} m \left(\frac{dx}{dt}\right)^2 + \frac{1}{2}kx^2 = \text{const.} \ ,$$

which yields $\omega = \sqrt{k/m}$. So by analogy, in our case

$$\omega = \sqrt{\frac{\kappa}{I}} = \sqrt{\frac{\mu B_e}{I}} = \sqrt{\frac{\mu B_e}{ml^2/12}} \ ,$$

which leads to

$$\mu = \frac{ml^2\omega^2}{12B_e} = \frac{(0.050\,\text{kg})(4.0\times10^{-2}\,\text{m})^2(45\,\text{rad/s})^2}{12(16\times10^{-6}\,\text{T})} = 8.4\times10^2 \ \text{J/T} \ .$$

52. Let the area of each circular plate be A and that of the central circular section be a, then

$$\frac{A}{a} = \frac{\pi R^2}{\pi (R/2)^2} = 4 \ .$$

Thus, from Eqs. 32-38 and 32-39 the total discharge current is given by $i = i_d = 4(2.0\,\text{A}) = 8.0\,\text{A}$.

53. (a) Using Eq. 32-11, we find $\mu_{\text{orb},z} = -3\mu_B = -2.78\times10^{-23}$ J/T (that these are acceptable units for magnetic moment is seen from Eq. 32-12 or Eq. 32-7; they are equivalent to A·m^2).

(b) Similarly, for $m_\ell = -4$ we obtain $\mu_{\text{orb},z} = 3.71\times10^{-23}$ J/T.

54. (a) Since the field is decreasing, the displacement current (by Eq. 32-38) is downward, which produces (by the right-hand rule) a clockwise sense for the induced magnetic field.

(b) See the solution for part (a).

(c) and (d) We write $\vec{E} = E_z\hat{\text{k}} = (E_0 - \xi t)\hat{\text{k}}$ where $\xi = 60000$(V/m)/s. From Eq. 32-36 (treated in absolute value)

$$i_d = \varepsilon_0 \, A \left|\frac{dE_z}{dt}\right| = \varepsilon_0 \, A \, \xi$$

which yields $i_d = 2.1\times10^{-8}$ A for all values of t.

55. (a) From $\mu = iA = i\pi R_e^2$ we get

$$i = \frac{\mu}{\pi R_e^2} = \frac{8.0\times10^{22}\,\text{J/T}}{\pi(6.37\times10^6\,\text{m})^2} = 6.3\times10^8 \ \text{A} \ .$$

(b) Yes, because far away from the Earth the fields of both the Earth itself and the current loop are dipole fields. If these two dipoles cancel each other out, then the net field will be zero.

(c) No, because the field of the current loop is not that of a magnetic dipole in the region close to the loop.

56. (a) The period of rotation is $T = 2\pi/\omega$ and in this time all the charge passes any fixed point near the ring. The average current is $i = q/T = q\omega/2\pi$ and the magnitude of the magnetic dipole moment is

$$\mu = iA = \frac{q\omega}{2\pi}\,\pi r^2 = \frac{1}{2}q\omega r^2 \ .$$

(b) We curl the fingers of our right hand in the direction of rotation. Since the charge is positive, the thumb points in the direction of the dipole moment. It is the same as the direction of the angular momentum vector of the ring.

57. (a) The potential energy of the atom in association with the presence an external magnetic field $\vec{B}_{\text{ext}}$ is given by Eqs. 32-11 and 32-12:

$$U = -\mu_{\text{orb}}\cdot\vec{B}_{\text{ext}} = -\mu_{\text{orb},z}B_{\text{ext}} = -m_l\mu_B B_{\text{ext}} \ .$$

For level E_1 there is no change in energy as a result of the introduction of $\vec{B}_{\text{ext}}$, so $U \propto m_l = 0$, meaning that $m_l = 0$ for this level. For level E_2 the single level splits into a triplet (i.e., three separate ones) in the presence of $\vec{B}_{\text{ext}}$, meaning that there are three different values of m_l. The middle one in the triplet is unshifted from the original value of E_2 so its m_l must be equal to 0.

(b) The other two in the triplet then correspond to $m_l = -1$ and $m_1 = +1$, respectively.

(c) For any pair of adjacent levels in the triplet $|\Delta m_l| = 1$. Thus, the spacing is given by

$$\begin{aligned}\Delta U &= |\Delta(-m_l\mu_B B)| = |\Delta m_l|\mu_B B = \mu_B B \\ &= \left(9.27\times10^{-24}\,\text{J/T}\right)(0.50\,\text{T}) = 4.6\times10^{-24}\ \text{J}\end{aligned}$$

which is equivalent to $2.9\times10^{-5}\,\text{eV}$.

58. (a) The magnitude of the toroidal field is given by $B_0 = \mu_0 n i_p$, where n is the number of turns per unit length of toroid and i_p is the current required to produce the field (in the absence of the ferromagnetic material). We use the average radius ($r_{\text{avg}} = 5.5$cm) to calculate n:

$$n = \frac{N}{2\pi r_{\text{avg}}} = \frac{400\,\text{turns}}{2\pi(5.5\times10^{-2}\,\text{m})} = 1.16\times10^3\,\text{turns/m} \ .$$

Thus,

$$i_p = \frac{B_0}{\mu_0 n} = \frac{0.20\times10^{-3}\,\text{T}}{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(1.16\times10^3/\text{m})} = 0.14\ \text{A} \ .$$

(b) If Φ is the magnetic flux through the secondary coil, then the magnitude of the emf induced in that coil is $\mathcal{E} = N(d\Phi/dt)$ and the current in the secondary is $i_s = \mathcal{E}/R$, where R is the resistance of the coil. Thus

$$i_s = \left(\frac{N}{R}\right)\frac{d\Phi}{dt} \ .$$

The charge that passes through the secondary when the primary current is turned on is

$$q = \int i_s\,dt = \frac{N}{R}\int\frac{d\Phi}{dt}\,dt = \frac{N}{R}\int_0^{\Phi} d\Phi = \frac{N\Phi}{R} \ .$$

The magnetic field through the secondary coil has magnitude $B = B_0 + B_M = 801B_0$, where B_M is the field of the magnetic dipoles in the magnetic material. The total field is perpendicular to the plane of the secondary coil, so the magnetic flux is $\Phi = AB$, where A is the area of the Rowland ring (the field is inside the ring, not in the region between the ring and coil). If r is the radius of the ring's cross section, then $A = \pi r^2$. Thus

$$\Phi = 801\pi r^2 B_0 \ .$$

The radius r is $(6.0\,\text{cm} - 5.0\,\text{cm})/2 = 0.50\,\text{cm}$ and

$$\Phi = 801\pi(0.50 \times 10^{-2}\,\text{m})^2(0.20 \times 10^{-3}\,\text{T}) = 1.26 \times 10^{-5}\,\text{Wb} \ .$$

Consequently,

$$q = \frac{50(1.26 \times 10^{-5}\,\text{Wb})}{8.0\,\Omega} = 7.9 \times 10^{-5}\ \text{C} \ .$$

59. Combining Eq. 32-7 with Eq. 32-2 and Eq. Eq. 32-3, we obtain

$$\Delta U = 2\,\mu_B\,B$$

where μ_B is the Bohr magneton (evaluated in Eq. 32-5). Thus, with $\Delta U = 6.0 \times 10^{-25}$ J, we find $B = |\vec{B}| = 0.032$ T.

60. (a) Using Eq. 32-37 but noting that the capacitor is being *discharged*, we have

$$\frac{d|\vec{E}|}{dt} = -\frac{i}{\varepsilon_0\,A} = -8.8 \times 10^{15}$$

where $A = (0.0080)^2$ and SI units are understood.

(b) Assuming a perfectly uniform field, even so near to an edge (which is consistent with the fact that fringing is neglected in §32-10), we follow part (a) of Sample Problem 32-4 and relate the (absolute value of the) line integral to the portion of displacement current enclosed.

$$\begin{aligned} \left| \oint \vec{B} \cdot d\vec{s} \right| &= \mu_0\, i_{d,\text{enc}} \\ &= \mu_0\, \frac{W\,H}{L^2}\, i \\ &= 5.9 \times 10^{-7}\ \text{Wb/m} \ . \end{aligned}$$