Chapter 6

1. We do not consider the possibility that the bureau might tip, and treat this as a purely horizontal motion problem (with the person's push $\vec{F}$ in the $+x$ direction). Applying Newton's second law to the x and y axes, we obtain

$$\begin{aligned} F - f_{s,\text{max}} &= ma \\ N - mg &= 0 \end{aligned}$$

respectively. The second equation yields the normal force $N = mg$, whereupon the maximum static friction is found to be (from Eq. 6-1) $f_{s,\text{max}} = \mu_s mg$. Thus, the first equation becomes

$$F - \mu_s mg = ma = 0$$

where we have set $a = 0$ to be consistent with the fact that the static friction is still (just barely) able to prevent the bureau from moving.

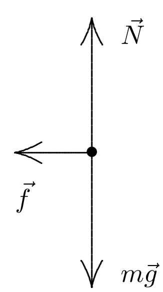
(a) With $\mu_s = 0.45$ and $m = 45$ kg, the equation above leads to $F = 198$ N. To bring the bureau into a state of motion, the person should push with any force greater than this value. Rounding to two significant figures, we can therefore say the minimum required push is $F = 2.0 \times 10^2$ N.

(b) Replacing $m = 45$ kg with $m = 28$ kg, the reasoning above leads to roughly $F = 1.2 \times 10^2$ N.

2. An excellent discussion and equation development related to this problem is given in Sample Problem 6-3. We merely quote (and apply) their main result (Eq. 6-13)

$$\theta = \tan^{-1} \mu_s = \tan^{-1} 0.04 \approx 2^\circ \ .$$

3. The free-body diagram for the player is shown below. $\vec{N}$ is the normal force of the ground on the player, $m\vec{g}$ is the force of gravity, and $\vec{f}$ is the force of friction. The force of friction is related to the normal force by $f = \mu_k N$. We use Newton's second law applied to the vertical axis to find the normal force. The vertical component of the acceleration is zero, so we obtain $N - mg = 0$; thus, $N = mg$. Consequently,



$$\begin{aligned} \mu_k &= \frac{f}{N} \\ &= \frac{470\,\text{N}}{(79\,\text{kg})\left(9.8\,\text{m/s}^2\right)} \\ &= 0.61 \ . \end{aligned}$$

4. To maintain the stone's motion, a horizontal force (in the $+x$ direction) is needed that cancels the retarding effect due to kinetic friction. Applying Newtons' second to the x and y axes, we obtain

$$\begin{aligned} F - f_k &= ma \\ N - mg &= 0 \end{aligned}$$

respectively. The second equation yields the normal force $N = mg$, so that (using Eq. 6-2) the kinetic friction becomes $f_k = \mu_k mg$. Thus, the first equation becomes

$$F - \mu_k mg = ma = 0$$

where we have set $a = 0$ to be consistent with the idea that the horizontal velocity of the stone should remain constant. With $m = 20$ kg and $\mu_k = 0.80$, we find $F = 1.6 \times 10^2$ N.

5. We denote $\vec{F}$ as the horizontal force of the person exerted on the crate (in the $+x$ direction), $\vec{f_k}$ is the force of kinetic friction (in the $-x$ direction), $\vec{N}$ is the vertical normal force exerted by the floor (in the $+y$ direction), and $m\vec{g}$ is the force of gravity. The magnitude of the force of friction is given by $f_k = \mu_k N$ (Eq. 6-2). Applying Newtons' second to the x and y axes, we obtain

$$\begin{aligned} F - f_k &= ma \\ N - mg &= 0 \end{aligned}$$

respectively.

   (a) The second equation yields the normal force $N = mg$, so that the friction is

$$f_k = \mu_k mg = (0.35)(55\,\text{kg})\left(9.8\,\text{m/s}^2\right) = 1.9 \times 10^2 \text{ N} .$$

   (b) The first equation becomes

$$F - \mu_k mg = ma$$

   which (with $F = 220$ N) we solve to find

$$a = \frac{F}{m} - \mu_k g = 0.56 \text{ m/s}^2 .$$

6. An excellent discussion and equation development related to this problem is given in Sample Problem 6-3. We merely quote (and apply) their main result (Eq. 6-13)

$$\theta = \tan^{-1}\mu_s = \tan^{-1} 0.5 = 27°$$

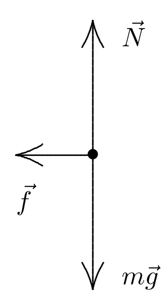
which implies that the angle through which the slope should be *reduced* is $\phi = 45° - 27° \approx 20°$.

7. The free-body diagram for the puck is shown below. $\vec{N}$ is the normal force of the ice on the puck, $\vec{f}$ is the force of friction (in the $-x$ direction), and $m\vec{g}$ is the force of gravity.

   (a) The horizontal component of Newton's second law gives $-f = ma$, and constant acceleration kinematics (Table 2-1) can be used to find the acceleration.

   Since the final velocity is zero, $v^2 = v_0^2 + 2ax$ leads to $a = -v_0^2/2x$. This is substituted into the Newton's law equation to obtain

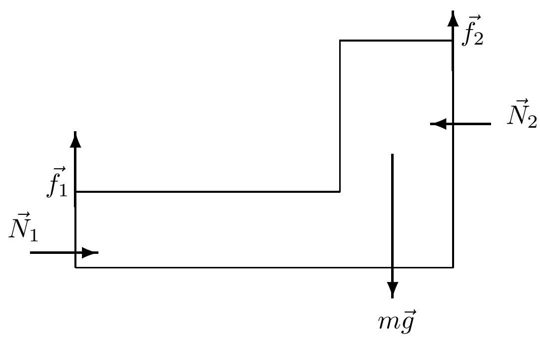
$$\begin{aligned} f &= \frac{mv_0^2}{2x} \\ &= \frac{(0.110\,\text{kg})(6.0\,\text{m/s})^2}{2(15\,\text{m})} \\ &= 0.13 \text{ N} . \end{aligned}$$



(b) The vertical component of Newton's second law gives $N - mg = 0$, so $N = mg$ which implies (using Eq. 6-2) $f = \mu_k mg$. We solve for the coefficient:

$$\mu_k = \frac{f}{mg} = \frac{0.13\,\text{N}}{(0.110\,\text{kg})\left(9.8\,\text{m/s}^2\right)} = 0.12\ .$$

8. (a) The free-body diagram for the person (shown as an L-shaped block) is shown below. The force that she exerts on the rock slabs is not directly shown (since the diagram should only show forces exerted on her), but it is related by Newton's third law) to the normal forces $\vec{N}_1$ and $\vec{N}_2$ exerted horizontally by the slabs onto her shoes and back, respectively. We will show in part (b) that $N_1 = N_2$ so that we there is no ambiguity in saying that the magnitude of her push is N_2. The total upward force due to (maximum) static friction is $\vec{f} = \vec{f}_1 + \vec{f}_2$ where (using Eq. 6-1) $f_1 = \mu_{s\,1} N_1$ and $f_2 = \mu_{s\,2} N_2$. The problem gives the values $\mu_{s\,1} = 1.2$ and $\mu_{s\,2} = 0.8$.



(b) We apply Newton's second law to the x and y axes (with $+x$ rightward and $+y$ upward and there is no acceleration in either direction).

$$\begin{aligned} N_1 - N_2 &= 0 \\ f_1 + f_2 - mg &= 0 \end{aligned}$$

The first equation tells us that the normal forces are equal $N_1 = N_2 = N$. Consequently, from Eq. 6-1

$$\begin{aligned} f_1 &= \mu_{s\,1} N \\ f_2 &= \mu_{s\,2} N \end{aligned}$$

we conclude that

$$f_1 = \left(\frac{\mu_{s\,1}}{\mu_{s\,2}}\right) f_2\ .$$

Therefore, $f_1 + f_2 - mg = 0$ leads to

$$\left(\frac{\mu_{s\,1}}{\mu_{s\,2}} + 1\right) f_2 = mg$$

which (with $m = 49$ kg) yields $f_2 = 192$ N. From this we find $N = f_2/\mu_{s\,2} = 240$ N. This is equal to the magnitude of the push exerted by the rock climber.

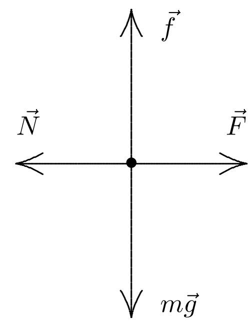
(c) From the above calculation, we find $f_1 = \mu_{s\,1} N = 288$ N which amounts to a fraction

$$\frac{f_1}{W} = \frac{288}{(49)(9.8)} = 0.60$$

or 60% of her weight.

9. (a) The free-body diagram for the block is shown below. $\vec{F}$ is the applied force, $\vec{N}$ is the normal force of the wall on the block, $\vec{f}$ is the force of friction, and $m\vec{g}$ is the force of gravity. To determine if the block falls, we find the magnitude f of the force of friction required to hold it without accelerating and also find the normal force of the wall on the block.

We compare f and $\mu_s N$. If $f < \mu_s N$, the block does not slide on the wall but if $f > \mu_s N$, the block does slide. The horizontal component of Newton's second law is $F - N = 0$, so $N = F = 12\,\text{N}$ and $\mu_s N = (0.60)(12\,\text{N}) = 7.2\,\text{N}$. The vertical component is $f - mg = 0$, so $f = mg = 5.0\,\text{N}$. Since $f < \mu_s N$ the block does not slide.



(b) Since the block does not move $f = 5.0\,\text{N}$ and $N = 12\,\text{N}$. The force of the wall on the block is

$$\vec{F}_w = -N\,\hat{\imath} + f\,\hat{\jmath} = -(12\,\text{N})\,\hat{\imath} + (5.0\,\text{N})\,\hat{\jmath}$$

where the axes are as shown on Fig. 6-21 of the text.

10. In addition to the forces already shown in Fig. 6-22, a free-body diagram would include an upward normal force $\vec{N}$ exerted by the floor on the block, a downward $m\vec{g}$ representing the gravitational pull exerted by Earth, and an assumed-leftward $\vec{f}$ for the kinetic or static friction. We choose $+x$ rightwards and $+y$ upwards. We apply Newton's second law to these axes:

$$\begin{aligned} (6.0\,\text{N}) - f &= ma \\ P + N - mg &= 0 \end{aligned}$$

where $m = 2.5$ kg is the mass of the block.

(a) In this case, $P = 8.0$ N leads to $N = (2.5)(9.8) - 8.0$ so that the normal force is $N = 16.5$ N. Using Eq. 6-1, this implies $f_{s,\max} = \mu_s N = 6.6$ N, which is larger than the 6.0 N rightward force – so the block (which was initially at rest) does not move. Putting $a = 0$ into the first of our equations above yields a static friction force of $f = P = 6.0$ N. Since its value is positive, then our assumption for the direction of $\vec{f}$ (leftward) is correct.

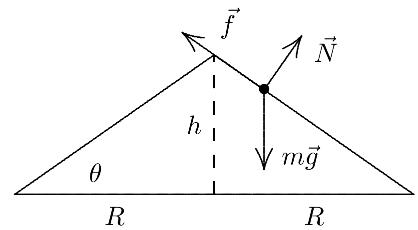
(b) In this case, $P = 10$ N leads to $N = (2.5)(9.8) - 10$ so that the normal force is $N = 14.5$ N. Using Eq. 6-1, this implies $f_{s,\max} = \mu_s N = 5.8$ N, which is less than the 6.0 N rightward force – so the block does move. Hence, we are dealing not with static but with kinetic friction, which Eq. 6-2 reveals to be $f_k = \mu_k N = 3.6$ N. Again, its value is positive, so our assumption for the direction of $\vec{f}$ (leftward) is correct.

(c) In this last case, $P = 12$ N leads to $N = 12.5$ N and thus to $f_{s,\max} = \mu_s N = 5.0$ N, which (as expected) is less than the 6.0 N rightward force – so the block moves. The kinetic friction force, then, is $f_k = \mu_k N = 3.1$ N. Once again, its value is positive, so our assumption for the direction of $\vec{f}$ (leftward) is correct.

11. A cross section of the cone of sand is shown below. To pile the most sand without extending the radius, sand is added to make the height h as great as possible. Eventually, however, the sides become so steep that sand at the surface begins to slide. The goal is to find the greatest height (corresponding to greatest slope) for which the sand does not slide. A grain of sand is shown on the diagram and the forces on it are labeled. $\vec{N}$ is the normal force of the surface, $m\vec{g}$ is the force of gravity, and $\vec{f}$ is the force of (static) friction. We take the x axis to be down the plane and the y axis to be in the direction of the normal

force. We assume the grain does not slide, so its acceleration is zero. Then the x component of Newton's second law is $mg\sin\theta - f = 0$ and the y component is $N - mg\cos\theta = 0$.

The first equation gives $f = mg\sin\theta$ and the second gives $N = mg\cos\theta$. If the grain does not slide, the condition $f < \mu_s N$ must hold. This means $mg\sin\theta < \mu_s mg\cos\theta$ or $\tan\theta < \mu_s$. The surface of the cone has the greatest slope (and the height of the cone is the greatest) if $\tan\theta = \mu_s$.



Since R and h are two sides of a right triangle, $h = R\tan\theta$. Replacing $\tan\theta$ with μ_s we obtain $h = \mu_s R$. We substitute this into the volume equation $V = \pi R^2 h/3$ to obtain the result $V = \pi\mu_s R^3/3$.

12. We denote the magnitude of 110 N force exerted by the worker on the crate as F. The magnitude of the static frictional force can vary between zero and $f_{s,\,\max} = \mu_s N$.

   (a) In this case, application of Newton's second law in the vertical direction yields $N = mg$. Thus,

$$\begin{aligned} f_{s,\,\max} &= \mu_s N = \mu_s mg \\ &= (0.37)(35\,\text{kg})\left(9.8\,\text{m/s}^2\right) = 126.9\text{ N} \end{aligned}$$

   which is greater than F. The block, which is initially at rest, stays at rest. This implies, by applying Newton's second law to the horizontal direction, that the magnitude of the frictional force exerted on the crate is $f_s = F = 110\,\text{N}$.

   (b) As calculated in part (a), $f_{s,\,\max} = 1.3 \times 10^2\,\text{N}$.

   (c) As remarked above, the crate does not move (since $F < f_{s,\,\max}$).

   (d) Denoting the upward force exerted by the second worker as F_2, then application of Newton's second law in the vertical direction yields $N = mg - F_2$. Therefore, in this case, $f_{s,\,\max} = \mu_s N = \mu_s(mg - F_2)$. In order to move the crate, F must satisfy $F > f_{s,\,\max} = \mu_s(mg - F_2)$, i.e.,

$$110\,\text{N} > (0.37)\left((35\,\text{kg})\left(9.8\,\text{m/s}^2\right) - F_2\right).$$

   The minimum value of F_2 that satisfies this inequality is a value slightly bigger than 45.7 N, so we express our answer as $F_{2,\,\min} = 46\,\text{N}$.

   (e) In this final case, moving the crate requires a greater horizontal push from the worker than static friction (as computed in part (a)) can resist. Thus, Newton's law in the horizontal direction leads to

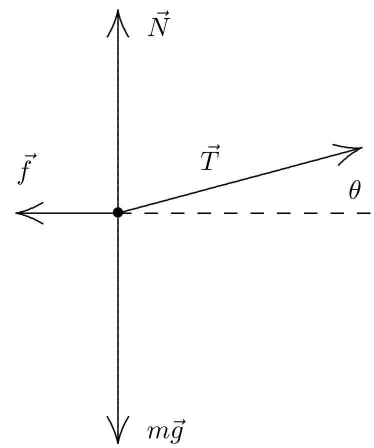
$$\begin{aligned} F + F_2 &> f_{s,\,\max} \\ 110\,\text{N} + F_2 &> 126.9\,\text{N} \end{aligned}$$

   which leads (after appropriate rounding) to $F_{2,\,\min} = 17\,\text{N}$.

13. (a) The free-body diagram for the crate is shown below. $\vec{T}$ is the tension force of the rope on the crate, $\vec{N}$ is the normal force of the floor on the crate, $m\vec{g}$ is the force of gravity, and $\vec{f}$ is the force of friction. We take the $+x$ direction to be horizontal to the right and the $+y$ direction to be up. We assume the crate is motionless. The x component of Newton's second law leads to $T\cos\theta - f = 0$ and the y component becomes $T\sin\theta + N - mg = 0$, where $\theta = 15°$ is the angle between the rope and the horizontal.

The first equation gives $f = T\cos\theta$ and the second gives $N = mg - T\sin\theta$. If the crate is to remain at rest, f must be less than $\mu_s N$, or $T\cos\theta < \mu_s(mg - T\sin\theta)$. When the tension force is sufficient to just start the crate moving, we must have $T\cos\theta = \mu_s(mg - T\sin\theta)$. We solve for the tension:

$$\begin{aligned} T &= \frac{\mu_s mg}{\cos\theta + \mu_s \sin\theta} \\ &= \frac{(0.50)(68)(9.8)}{\cos 15^\circ + 0.50\sin 15^\circ} \\ &= 304 \approx 300 \text{ N} . \end{aligned}$$



(b) The second law equations for the moving crate are $T\cos\theta - f = ma$ and $N + T\sin\theta - mg = 0$. Now $f = \mu_k N$. The second equation gives $N = mg - T\sin\theta$, as before, so $f = \mu_k(mg - T\sin\theta)$. This expression is substituted for f in the first equation to obtain $T\cos\theta - \mu_k(mg - T\sin\theta) = ma$, so the acceleration is

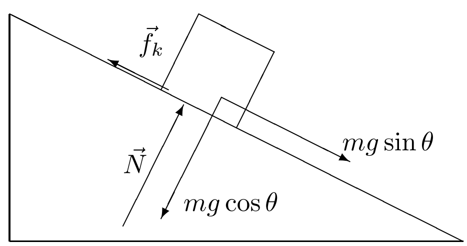
$$a = \frac{T(\cos\theta + \mu_k \sin\theta)}{m} - \mu_k g$$

which we evaluate:

$$a = \frac{(304\,\text{N})(\cos 15^\circ + 0.35\sin 15^\circ)}{68\,\text{kg}} - (0.35)(9.8\,\text{m/s}^2) = 1.3\,\text{m/s}^2 .$$

14. We first analyze the forces on the pig of mass m. The incline angle is θ.

The $+x$ direction is "downhill."



Application of Newton's second law to the x and y axes leads to

$$\begin{aligned} mg\sin\theta - f_k &= ma \\ N - mg\cos\theta &= 0 . \end{aligned}$$

Solving these along with Eq. 6-2 ($f_k = \mu_k N$) produces the following result for the pig's downhill acceleration:

$$a = g\left(\sin\theta - \mu_k \cos\theta\right) .$$

To compute the time to slide from rest through a downhill distance ℓ, we use Eq. 2-15:

$$\ell = v_0 t + \frac{1}{2}at^2 \implies t = \sqrt{\frac{2\ell}{a}} .$$

We denote the frictionless ($\mu_k = 0$) case with a prime and set up a ratio:

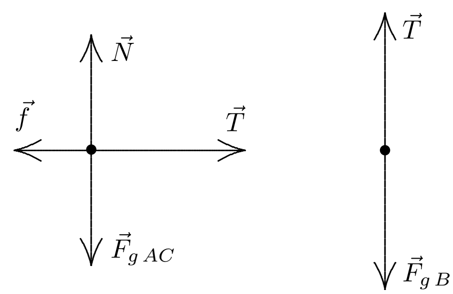
$$\frac{t}{t'} = \frac{\sqrt{2\ell/a}}{\sqrt{2\ell/a'}} = \sqrt{\frac{a'}{a}}$$

which leads us to conclude that if $t/t' = 2$ then $a' = 4a$. Putting in what we found out above about the accelerations, we have

$$g\sin\theta = 4g\left(\sin\theta - \mu_k\cos\theta\right) \ .$$

Using $\theta = 35°$, we obtain $\mu_k = 0.53$.

15. (a) Free-body diagrams for the blocks A and C, considered as a single object, and for the block B are shown below. T is the magnitude of the tension force of the rope, N is the magnitude of the normal force of the table on block A, f is the magnitude of the force of friction, W_{AC} is the combined weight of blocks A and C (the magnitude of force $\vec{F}_{g\,AC}$ shown in the figure), and W_B is the weight of block B (the magnitude of force $\vec{F}_{g\,B}$ shown). Assume the blocks are not moving. For the blocks on the table we take the x axis to be to the right and the y axis to be upward. The x component of Newton's second law is then $T - f = 0$ and the y component is $N - W_{AC} = 0$. For block B take the downward direction to be positive. Then Newton's second law for that block is $W_B - T = 0$. The third equation gives $T = W_B$ and the first gives $f = T = W_B$. The second equation gives $N = W_{AC}$. If sliding is not to occur, f must be less than $\mu_s N$, or $W_B < \mu_s W_{AC}$. The smallest that W_{AC} can be with the blocks still at rest is $W_{AC} = W_B/\mu_s = (22\,\text{N})/(0.20) = 110\,\text{N}$. Since the weight of block A is 44 N, the least weight for C is $110 - 44 = 66\,\text{N}$.



(b) The second law equations become $T - f = (W_A/g)a$, $N - W_A = 0$, and $W_B - T = (W_B/g)a$. In addition, $f = \mu_k N$. The second equation gives $N = W_A$, so $f = \mu_k W_A$. The third gives $T = W_B - (W_B/g)a$. Substituting these two expressions into the first equation, we obtain $W_B - (W_B/g)a - \mu_k W_A = (W_A/g)a$. Therefore,

$$a = \frac{g(W_B - \mu_k W_A)}{W_A + W_B} = \frac{(9.8\,\text{m/s}^2)\,(22\,\text{N} - (0.15)(44\,\text{N}))}{44\,\text{N} + 22\,\text{N}} = 2.3\,\text{m/s}^2 \ .$$

16. We choose $+x$ horizontally rightwards and $+y$ upwards and observe that the 15 N force has components $F_x = F\cos\theta$ and $F_y = -F\sin\theta$.

(a) We apply Newton's second law to the y axis:

$$N - F\sin\theta - mg = 0 \implies N = (15)\sin 40° + (3.5)(9.8) = 44$$

in SI units. With $\mu_k = 0.25$, Eq. 6-2 leads to $f_k = 11$ N.

(b) We apply Newton's second law to the x axis:

$$F\cos\theta - f_k = ma \implies a = \frac{(15)\cos 40° - 11}{3.5} = 0.14$$

in SI units (m/s^2). Since the result is positive-valued, then the block is accelerating in the $+x$ (rightward) direction.

17. (a) Although details in Fig. 6-27 might suggest otherwise, we assume (as the problem states) that only static friction holds block B in place. An excellent discussion and equation development related to this topic is given in Sample Problem 6-3. We merely quote (and apply) their main result (Eq. 6-13) for the maximum angle for which static friction applies (in the absence of additional forces such as the $\vec{F}$ of part (b) of this problem).

$$\theta_{\max} = \tan^{-1}\mu_s = \tan^{-1} 0.63 \approx 32^\circ \ .$$

This is greater than the dip angle in the problem, so the block does not slide.

(b) We analyze forces in a manner similar to that shown in Sample Problem 6-3, but with the addition of a downhill force F.

$$\begin{aligned} F + mg\sin\theta - f_{s,\max} &= ma = 0 \\ N - mg\cos\theta &= 0 \ . \end{aligned}$$

Along with Eq. 6-1 ($f_{s,\max} = \mu_s N$) we have enough information to solve for F. With $\theta = 24^\circ$ and $m = 1.8 \times 10^7$ kg, we find

$$F = mg\left(\mu_s\cos\theta - \sin\theta\right) = 3.0 \times 10^7 \text{ N} \ .$$

18. We use coordinates and weight-components as indicated in Fig. 5-18 (see Sample Problem 5-7 from the previous chapter).

(a) In this situation, we take $\vec{f_s}$ to point uphill and to be equal to its maximum value, in which case $f_{s,\max} = \mu_s N$ applies, where $\mu_s = 0.25$. Applying Newton's second law to the block of mass $m = W/g = 8.2$ kg, in the x and y directions, produces

$$\begin{aligned} F_{\min 1} - mg\sin\theta + f_{s,\max} &= ma = 0 \\ N - mg\cos\theta &= 0 \end{aligned}$$

which (with $\theta = 20^\circ$) leads to

$$F_{\min 1} = mg\left(\sin\theta - \mu_s\cos\theta\right) = 8.6 \text{ N} \ .$$

(b) Now we take $\vec{f_s}$ to point downhill and to be equal to its maximum value, in which case $f_{s,\max} = \mu_s N$ applies, where $\mu_s = 0.25$. Applying Newton's second law to the block of mass $m = W/g = 8.2$ kg, in the x and y directions, produces

$$\begin{aligned} F_{\min 2} - mg\sin\theta - f_{s,\max} &= ma = 0 \\ N - mg\cos\theta &= 0 \end{aligned}$$

which (with $\theta = 20^\circ$) leads to

$$F_{\min 2} = mg\left(\sin\theta + \mu_s\cos\theta\right) = 46 \text{ N} \ .$$

A value slightly larger than the "exact" result of this calculation is required to make it accelerate up hill, but since we quote our results here to two significant figures, 46 N is a "good enough" answer.

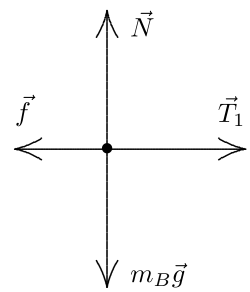
(c) Finally, we are dealing with kinetic friction (pointing downhill), so that

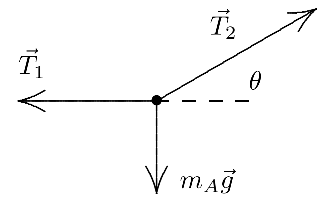
$$\begin{aligned} F - mg\sin\theta - f_k &= ma = 0 \\ N - mg\cos\theta &= 0 \end{aligned}$$

along with $f_k = \mu_k N$ (where $\mu_k = 0.15$) brings us to

$$F = mg\left(\sin\theta + \mu_k\cos\theta\right) = 39 \text{ N} \ .$$

19. The free-body diagrams for block B and for the knot just above block A are shown below. $\vec{T}_1$ is the tension force of the rope pulling on block B or pulling on the knot (as the case may be), $\vec{T}_2$ is the tension force exerted by the second rope (at angle $\theta = 30°$) on the knot, $\vec{f}$ is the force of static friction exerted by the horizontal surface on block B, $\vec{N}$ is normal force exerted by the surface on block B, W_A is the weight of block A (W_A is the magnitude of $m_A\vec{g}$), and W_B is the weight of block B ($W_B = 711$ N is the magnitude of $m_B\vec{g}$).





For each object we take $+x$ horizontally rightward and $+y$ upward. Applying Newton's second law in the x and y directions for block B and then doing the same for the knot results in four equations:

$$\begin{aligned} T_1 - f_{s,\max} &= 0 \\ N - W_B &= 0 \\ T_2 \cos\theta - T_1 &= 0 \\ T_2 \sin\theta - W_A &= 0 \end{aligned}$$

where we assume the static friction to be at its maximum value (permitting us to use Eq. 6-1). Solving these equations with $\mu_s = 0.25$, we obtain $W_A = 103 \approx 100$ N.

20. If the block is sliding then we compute the kinetic friction from Eq. 6-2; if it is not sliding, then we determine the extent of static friction from applying Newton's law, with zero acceleration, to the x axis (which is parallel to the incline surface). The question of whether or not it is sliding is therefore crucial, and depends on the maximum static friction force, as calculated from Eq. 6-1. The forces are resolved in the incline plane coordinate system in Figure 6-5 in the textbook. The acceleration, if there is any, is along the x axis, and we are taking uphill as $+x$. The net force along the y axis, then, is certainly zero, which provides the following relationship:

$$\sum \vec{F}_y = 0 \quad \Longrightarrow \quad N = W\cos\theta$$

where $W = 45$ N is the weight of the block, and $\theta = 15°$ is the incline angle. Thus, $N = 43.5$ N, which implies that the maximum static friction force should be $f_{s,\max} = (0.50)(43.5) = 21.7$ N.

(a) For $\vec{P} = 5.0$ N downhill, Newton's second law, applied to the x axis becomes

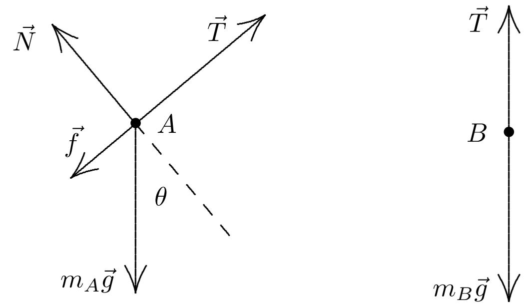
$$f - P - W\sin\theta = ma \quad \text{where} \quad m = \frac{W}{g} \; .$$

Here we are assuming $\vec{f}$ is pointing uphill, as shown in Figure 6-5, and if it turns out that it points downhill (which *is* a possibility), then the result for f_s will be negative. If $f = f_s$ then $a = 0$, we obtain $f_s = 17$ N, which is clearly allowed since it is less than $f_{s,\max}$.

(b) For $\vec{P} = 8.0$ N downhill, we obtain (from the same equation) $f_s = 20$ N, which is still allowed since it is less than $f_{s,\max}$.

(c) But for $\vec{P} = 15$ N downhill, we obtain (from the same equation) $f_s = 27$ N, which is not allowed since it is larger than $f_{s,\max}$. Thus, we conclude that it is the kinetic friction, not the static friction, that is relevant in this case. We compute the result $f_k = (0.34)(43.5) = 15$ N. Here, as in the other parts of this problem, the friction is directed uphill.

21. First, we check to see if the bodies start to move. We assume they remain at rest and compute the force of (static) friction which holds them there, and compare its magnitude with the maximum value $\mu_s N$. The free-body diagrams are shown below. T is the magnitude of the tension force of the string, f is the magnitude of the force of friction on body A, N is the magnitude of the normal force of the plane on body A, $m_A\vec{g}$ is the force of gravity on body A (with magnitude $W_A = 102$ N), and $m_B\vec{g}$ is the force of gravity on body B (with magnitude $W_B = 32$ N). $\theta = 40^\circ$ is the angle of incline. We are not told the direction of $\vec{f}$ but we assume it is downhill. If we obtain a negative result for f, then we know the force is actually up the plane.



(a) For A we take the $+x$ to be uphill and $+y$ to be in the direction of the normal force. The x and y components of Newton's second law become

$$\begin{aligned} T - f - W_A \sin\theta &= 0 \\ N - W_A \cos\theta &= 0 . \end{aligned}$$

Taking the positive direction to be *downward* for body B, Newton's second law leads to

$$W_B - T = 0 .$$

Solving these three equations leads to

$$f = W_B - W_A \sin\theta = 32 - 102 \sin 40^\circ = -34 \text{ N}$$

(indicating that the force of friction is *uphill*) and to

$$N = W_A \cos\theta = 102 \cos 40^\circ = 78 \text{ N}$$

which means that $f_{s,\max} = \mu_s N = (0.56)(78) = 44$ N. Since the magnitude f of the force of friction that holds the bodies motionless is less than $f_{s,\max}$ the bodies remain at rest. The acceleration is zero.

(b) Since A is moving up the incline, the force of friction is downhill with magnitude $f_k = \mu_k N$. Newton's second law, using the same coordinates as in part (a), leads to

$$\begin{aligned} T - f_k - W_A \sin\theta &= m_A\, a \\ N - W_A \cos\theta &= 0 \\ W_B - T &= m_B\, a \end{aligned}$$

for the two bodies. We solve for the acceleration:

$$\begin{aligned} a &= \frac{W_B - W_A \sin\theta - \mu_k W_A \cos\theta}{m_B + m_A} \\ &= \frac{32\,\text{N} - (102\,\text{N}) \sin 40^\circ - (0.25)(102\,\text{N}) \cos 40^\circ}{(32\,\text{N} + 102\,\text{N}) \,/ \left(9.8\,\text{m/s}^2\right)} \\ &= -3.9 \text{ m/s}^2 . \end{aligned}$$

The acceleration is down the plane, which is to say (since the initial velocity was uphill) that the objects are slowing down. We note that $m = W/g$ has been used to calculate the masses in the calculation above.

(c) Now body A is initially moving down the plane, so the force of friction is uphill with magnitude $f_k = \mu_k N$. The force equations become

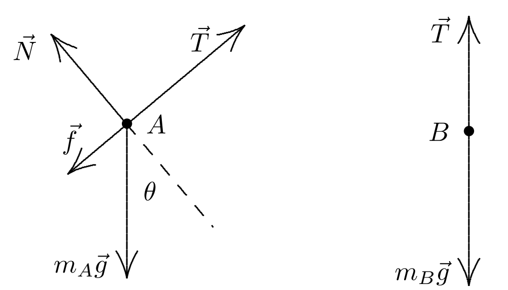
$$\begin{aligned} T + f_k - W_A \sin\theta &= m_A\, a \\ N - W_A \cos\theta &= 0 \\ W_B - T &= m_B\, a \end{aligned}$$

which we solve to obtain

$$\begin{aligned} a &= \frac{W_B - W_A \sin\theta + \mu_k W_A \cos\theta}{m_B + m_A} \\ &= \frac{32\,\mathrm{N} - (102\,\mathrm{N})\sin 40° + (0.25)(102\,\mathrm{N})\cos 40°}{(32\,\mathrm{N} + 102\,\mathrm{N})\,/\left(9.8\,\mathrm{m/s^2}\right)} \\ &= -1.0\ \mathrm{m/s^2}\ . \end{aligned}$$

The acceleration is again downhill the plane. In this case, the objects are speeding up.

22. The free-body diagrams are shown below. T is the magnitude of the tension force of the string, f is the magnitude of the force of friction on block A, N is the magnitude of the normal force of the plane on block A, $m_A\vec{g}$ is the force of gravity on body A (where $m_A = 10$ kg), and $m_B\vec{g}$ is the force of gravity on block B. $\theta = 30°$ is the angle of incline. For A we take the $+x$ to be uphill and $+y$ to be in the direction of the normal force; the positive direction is chosen *downward* for block B.



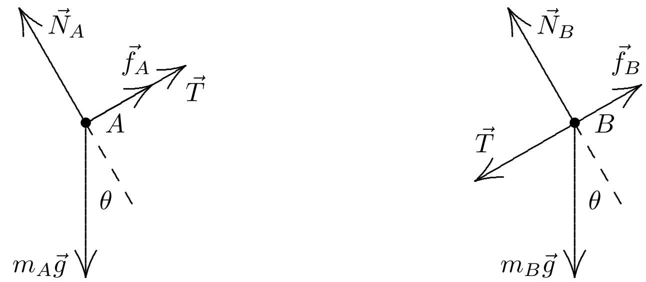
Since A is moving down the incline, the force of friction is uphill with magnitude $f_k = \mu_k N$ (where $\mu_k = 0.20$). Newton's second law leads to

$$\begin{aligned} T - f_k + m_A g \sin\theta &= m_A\, a = 0 \\ N - m_A g \cos\theta &= 0 \\ m_B g - T &= m_B\, a = 0 \end{aligned}$$

for the two bodies (where $a = 0$ is a consequence of the velocity being constant). We solve these for the mass of block B.

$$m_B = m_A\,(\sin\theta - \mu_k \cos\theta) = 3.3\ \mathrm{kg}\ .$$

23. The free-body diagrams for the two blocks are shown below. T is the magnitude of the tension force of the string, $\vec{N}_A$ is the normal force on block A (the leading block), $\vec{N}_B$ is the the normal force on block B, $\vec{f}_A$ is kinetic friction force on block A, $\vec{f}_B$ is kinetic friction force on block B. Also, m_A is the mass of block A (where $m_A = W_A/g$ and $W_A = 3.6$ N), and m_B is the mass of block B (where $m_B = W_B/g$ and $W_B = 7.2$ N). The angle of the incline is $\theta = 30°$.



For each block we take $+x$ downhill (which is toward the lower-left in these diagrams) and $+y$ in the direction of the normal force. Applying Newton's second law to the x and y directions of first block A and next block B, we arrive at four equations:

$$\begin{aligned} W_A \sin\theta - f_A - T &= m_A\, a \\ N_A - W_A \cos\theta &= 0 \\ W_B \sin\theta - f_B + T &= m_B\, a \\ N_B - W_B \cos\theta &= 0\ . \end{aligned}$$

which, when combined with Eq. 6-2 ($f_A = \mu_{k\,A} N_A$ where $\mu_{k\,A} = 0.10$ and $f_B = \mu_{k\,B} N_B$ where $\mu_{k\,B} = 0.20$), fully describe the dynamics of the system so long as the blocks have the same acceleration and $T > 0$.

(a) These equations lead to an acceleration equal to

$$a = g\left(\sin\theta - \left(\frac{\mu_{k\,A} W_A + \mu_{k\,B} W_B}{W_A + W_B}\right)\cos\theta\right) = 3.5\ \mathrm{m/s^2}\ .$$

(b) We solve the above equations for the tension and obtain

$$T = \left(\frac{W_A\, W_B}{W_A + W_B}\right)(\mu_{k\,B} - \mu_{k\,A})\cos\theta = 0.21\ \mathrm{N}\ .$$

Simply returning the value for a found in part (a) into one of the above equations is certainly fine, and probably easier than solving for T algebraically as we have done, but the algebraic form does illustrate the $\mu_{k\,B} - \mu_{k\,A}$ factor which aids in the understanding of the next part.

(c) Reversing the blocks is equivalent to switching the labels (so A is now the block of weight 7.2 N and $\mu_{k\,A}$ is now the 0.20 value). We see from our algebraic result in part (b) that this gives a negative value for T, which is impossible. We conclude that the above set of four equations are not valid in this circumstance (specifically, a for one block is not equal to a for the other block). The blocks move independently of each other.

24. Treating the two boxes as a single system of total mass $1.0 + 3.0 = 4.0$ kg, subject to a total (leftward) friction of magnitude $2.0 + 4.0 = 6.0$ N, we apply Newton's second law (with $+x$ rightward):

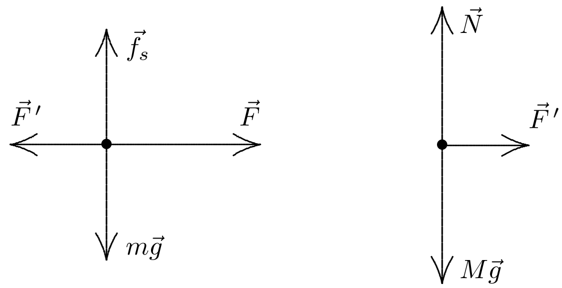
$$\begin{aligned} F - f_{\text{total}} &= m_{\text{total}}\, a \\ 12.0 - 6.0 &= (4.0)a \end{aligned}$$

which yields the acceleration $a = 1.5\ \mathrm{m/s^2}$. We have treated F as if it were known to the nearest tenth of a Newton so that our acceleration is "good" to two significant figures. Turning our attention to the larger box (the Wheaties box of mass 3.0 kg) we apply Newton's second law to find the contact force F' exerted by the smaller box on it.

$$\begin{aligned} F' - f_{\mathrm{W}} &= m_{\mathrm{W}}\, a \\ F' - 4.0 &= (3.0)(1.5) \end{aligned}$$

This yields the contact force $F' = 8.5$ N.

25. The free-body diagrams for the two blocks, treated individually, are shown below (first m and then M). F' is the contact force between the two blocks, and the static friction force $\vec{f}_s$ is at its maximum value (so Eq. 6-1 leads to $f_s = f_{s,\max} = \mu_s F'$ where $\mu_s = 0.38$).



Treating the two blocks together as a single system (sliding across a frictionless floor), we apply Newton's second law (with $+x$ rightward) to find an expression for the acceleration.

$$F = m_{\text{total}}\, a \quad \Longrightarrow a = \frac{F}{m+M}$$

This is equivalent to having analyzed the two blocks individually and then combined their equations. Now, when we analyze the small block individually, we apply Newton's second law to the x and y axes, substitute in the above expression for a, and use Eq. 6-1.

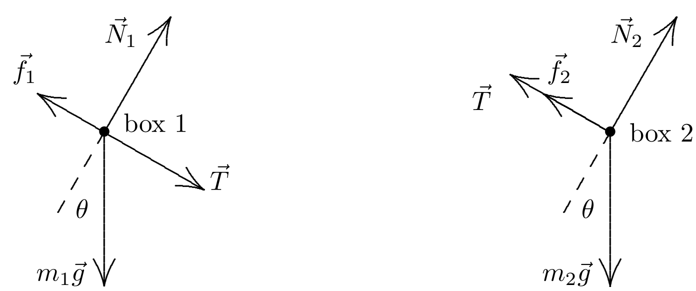
$$\begin{aligned} F - F' = ma \quad &\Longrightarrow \quad F' = F - m\left(\frac{F}{m+M}\right) \\ f_s - mg = 0 \quad &\Longrightarrow \quad \mu_s F' - mg = 0 \end{aligned}$$

These expressions are combined (to eliminate F') and we arrive at

$$F = \frac{mg}{\mu_s\left(1 - \frac{m}{m+M}\right)}$$

which we find to be $F = 4.9 \times 10^2$ N.

26. The free-body diagrams for the two boxes are shown below. T is the magnitude of the force in the rod (when $T > 0$ the rod is said to be in tension and when $T < 0$ the rod is under compression), $\vec{N}_2$ is the normal force on box 2 (the uncle box), $\vec{N}_1$ is the the normal force on the aunt box (box 1), $\vec{f}_1$ is kinetic friction force on the aunt box, and $\vec{f}_2$ is kinetic friction force on the uncle box. Also, $m_1 = 1.65$ kg is the mass of the aunt box and $m_2 = 3.30$ kg is the mass of the uncle box (which is a lot of ants!).



For each block we take $+x$ downhill (which is toward the lower-right in these diagrams) and $+y$ in the direction of the normal force. Applying Newton's second law to the x and y directions of first box 2 and next box 1, we arrive at four equations:

$$\begin{aligned} m_2 g \sin\theta - f_2 - T &= m_2\, a \\ N_2 - m_2 g \cos\theta &= 0 \\ m_1 g \sin\theta - f_1 + T &= m_1\, a \\ N_1 - m_1 g \cos\theta &= 0\ . \end{aligned}$$

which, when combined with Eq. 6-2 ($f_1 = \mu_1 N_1$ where $\mu_1 = 0.226$ and $f_2 = \mu_2 N_2$ where $\mu_2 = 0.113$), fully describe the dynamics of the system.

(a) We solve the above equations for the tension and obtain

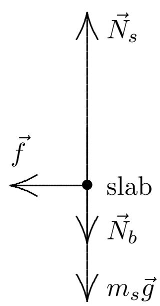
$$T = \left(\frac{m_2\, m_1\, g}{m_2 + m_1}\right)(\mu_1 - \mu_2)\cos\theta = 1.05 \text{ N} .$$

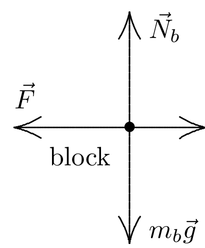
(b) These equations lead to an acceleration equal to

$$a = g\left(\sin\theta - \left(\frac{\mu_2 m_2 + \mu_1 m_1}{m_2 + m_1}\right)\cos\theta\right) = 3.62 \text{ m/s}^2 .$$

(c) Reversing the blocks is equivalent to switching the labels. We see from our algebraic result in part (a) that this gives a negative value for T (equal in magnitude to the result we got before). Thus, the situation is as it was before except that the rod is now in a state of compression.

27. The free-body diagrams for the slab and block are shown below. $\vec{F}$ is the 100 N force applied to the block, $\vec{N}_s$ is the normal force of the floor on the slab, N_b is the magnitude of the normal force between the slab and the block, $\vec{f}$ is the force of friction between the slab and the block, m_s is the mass of the slab, and m_b is the mass of the block. For both objects, we take the $+x$ direction to be to the left and the $+y$ direction to be up.





Applying Newton's second law for the x and y axes for (first) the slab and (second) the block results in four equations:

$$\begin{aligned} f &= m_s a_s \\ N_s - N_b - m_s g &= 0 \\ F - f &= m_b a_b \\ N_b - m_b g &= 0 \end{aligned}$$

from which we note that the maximum possible static friction magnitude would be

$$\mu_s N_b = \mu_s m_b g = (0.60)(10\,\text{kg})(9.8\,\text{m/s}^2) = 59 \text{ N} .$$

We check to see if the block slides on the slab. Assuming it does not, then $a_s = a_b$ (which we denote simply as a) and we solve for f:

$$f = \frac{m_s F}{m_s + m_b} = \frac{(40\,\text{kg})(100\,\text{N})}{40\,\text{kg} + 10\,\text{kg}} = 80 \text{ N}$$

which is greater than $f_{s,\max}$ so that we conclude the block is sliding across the slab (their accelerations are different).

(a) Using $f = \mu_k N_b$ the above equations yield

$$a_b = \frac{F - \mu_k m_b g}{m_b} = \frac{100\,\text{N} - (0.40)(10\,\text{kg})(9.8\,\text{m/s}^2)}{10\,\text{kg}} = 6.1 \text{ m/s}^2 .$$

The result is positive which means (recalling our choice of $+x$ direction) that it accelerates leftward.

(b) We also obtain

$$a_s = \frac{\mu_k m_b g}{m_s} = \frac{(0.40)(10\,\text{kg})(9.8\,\text{m/s}^2)}{40\,\text{kg}} = 0.98\ \text{m/s}^2\ .$$

As mentioned above, this means it accelerates to the left.

28. We may treat all 25 cars as a single object of mass $m = 25 \times 5.0 \times 10^4$ kg and (when the speed is 30 km/h = 8.3 m/s) subject to a friction force equal to $f = 25 \times 250 \times 8.3 = 5.2 \times 10^4$ N.

(a) Along the level track, this object experiences a "forward" force T exerted by the locomotive, so that Newton's second law leads to

$$T - f = ma \quad\Longrightarrow\quad T = 5.2 \times 10^4 + \left(1.25 \times 10^6\right)(0.20)$$

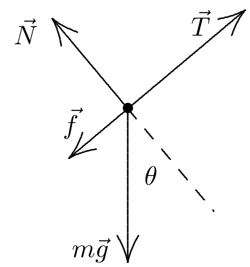
which yields $T = 3.0 \times 10^5$ N.

(b) The free-body diagram is shown below, with θ as the angle of the incline. The $+x$ direction (which is the only direction to which we will be applying Newton's second law) is uphill (to the upper right in our sketch).

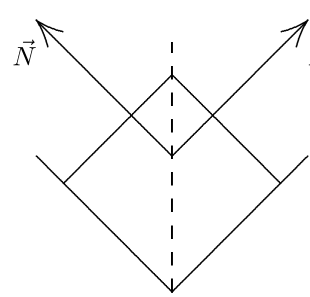
Thus, we obtain

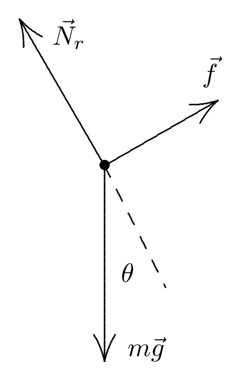
$$T - f - mg\sin\theta = ma$$

where we set $a = 0$ (implied by the problem statement) and solve for the angle. We obtain $\theta = 1.2°$.



29. Each side of the trough exerts a normal force on the crate. The first diagram shows the view looking in toward a cross section. The net force is along the dashed line. Since each of the normal forces makes an angle of 45° with the dashed line, the magnitude of the resultant normal force is given by $N_r = 2N\cos 45° = \sqrt{2}N$. The second diagram is the free-body diagram for the crate (from a "side" view, similar to that shown in the first picture in Fig. 6-36). The force of gravity has magnitude mg, where m is the mass of the crate, and the magnitude of the force of friction is denoted by f. We take the $+x$ direction to be down the incline and $+y$ to be in the direction of $\vec{N}_r$. Then the x component of Newton's second law is $mg\sin\theta - f = ma$ and the y component is $N_r - mg\cos\theta = 0$. Since the crate is moving, each side of the trough exerts a force of kinetic friction, so the total frictional force has magnitude $f = 2\mu_k N = 2\mu_k N_r/\sqrt{2} = \sqrt{2}\mu_k N_r$. Combining this expression with $N_r = mg\cos\theta$ and substituting into the x component equation, we obtain $mg\sin\theta - \sqrt{2}mg\cos\theta = ma$. Therefore $a = g(\sin\theta - \sqrt{2}\mu_k\cos\theta)$.





30. Fig. 6-4 in the textbook shows a similar situation (using ϕ for the unknown angle) along with a free-body diagram. We use the same coordinate system as in that figure.

(a) Thus, Newton's second law leads to

$$\begin{aligned} T\cos\phi - f &= ma \qquad \text{along } x \text{ axis} \\ T\sin\phi + N - mg &= 0 \qquad \text{along } y \text{ axis} \end{aligned}$$

Setting $a = 0$ and $f = f_{s,\max} = \mu_s N$, we solve for the mass of the box-and-sand (as a function of angle):

$$m = \frac{T}{g}\left(\sin\phi + \frac{\cos\phi}{\mu_s}\right)$$

which we will solve with calculus techniques (to find the angle ϕ_m corresponding to the maximum mass that can be pulled).

$$\frac{dm}{dt} = \frac{T}{g}\left(\cos\phi_m - \frac{\sin\phi_m}{\mu_s}\right) = 0$$

This leads to $\tan\phi_m = \mu_s$ which (for $\mu_s = 0.35$) yields $\phi_m = 19°$.

(b) Plugging our value for ϕ_m into the equation we found for the mass of the box-and-sand yields $m = 340$ kg. This corresponds to a weight of $mg = 3.3 \times 10^3$ N.

31. We denote the magnitude of the frictional force αv, where $\alpha = 70\,\mathrm{N\cdot s/m}$. We take the direction of the boat's motion to be positive. Newton's second law gives

$$-\alpha v = m\,\frac{dv}{dt}\ .$$

Thus,

$$\int_{v_0}^{v} \frac{dv}{v} = -\frac{\alpha}{m}\int_0^t dt$$

where v_0 is the velocity at time zero and v is the velocity at time t. The integrals are evaluated with the result

$$\ln\frac{v}{v_0} = -\frac{\alpha t}{m}\ .$$

We take $v = v_0/2$ and solve for time:

$$t = \frac{m}{\alpha}\ln 2 = \frac{1000\,\mathrm{kg}}{70\,\mathrm{N\cdot s/m}}\ln 2 = 9.9\ \mathrm{s}\ .$$

32. In the solution to exercise 4, we found that the force provided by the wind needed to equal $F = 157$ N (where that last figure is not "significant").

(a) Setting $F = D$ (for Drag force) we use Eq. 6-14 to find the wind speed V along the ground (which actually is relative to the moving stone, but we assume the stone is moving slowly enough that this does not invalidate the result):

$$V = \sqrt{\frac{2F}{C\rho A}} = \sqrt{\frac{2(157)}{(0.80)(1.21)(0.040)}}$$

which yields $V = 90$ m/s which converts to $V = 3.2 \times 10^2$ km/h.

(b) Doubling our previous result, we find the reported speed to be 6.5×10^2 km/h, which is not reasonable for a terrestrial storm. (A category 5 hurricane has speeds on the order of 2.6×10^2 m/s.)

33. We use Eq. 6-14, $D = \frac{1}{2}C\rho A v^2$, where ρ is the air density, A is the cross-sectional area of the missile, v is the speed of the missile, and C is the drag coefficient. The area is given by $A = \pi R^2$, where $R = 0.265\,\mathrm{m}$ is the radius of the missile. Thus

$$D = \frac{1}{2}(0.75)(1.2\,\mathrm{kg/m^3})\pi(0.265\,\mathrm{m})^2(250\,\mathrm{m/s})^2 = 6.2 \times 10^3\ \mathrm{N}\ .$$

34. Using Eq. 6-16, we solve for the area

$$A = \frac{2\,m\,g}{C\,\rho\,v_t^2}$$

which illustrates the inverse proportionality between the area and the speed-squared. Thus, when we set up a ratio of areas – of the slower case to the faster case – we obtain

$$\frac{A_{\text{slow}}}{A_{\text{fast}}} = \left(\frac{310\,\text{km/h}}{160\,\text{km/h}}\right)^2 = 3.75\ .$$

35. For the passenger jet $D_j = \frac{1}{2}C\rho_1 A v_j^2$, and for the prop-driven transport $D_t = \frac{1}{2}C\rho_2 A v_t^2$, where ρ_1 and ρ_2 represent the air density at 10 km and 5.0 km, respectively. Thus the ratio in question is

$$\frac{D_j}{D_t} = \frac{\rho_1 v_j^2}{\rho_2 v_t^2} = \frac{(0.38\,\text{kg/m}^3)(1000\,\text{km/h})^2}{(0.67\,\text{kg/m}^3)(500\,\text{km/h})^2} = 2.3\ .$$

36. With $v = 96.6$ km/h $= 26.8$ m/s, Eq. 6-17 readily yields

$$a = \frac{v^2}{R} = \frac{26.8^2}{7.6} = 94.7\ \text{m/s}^2$$

which we express as a multiple of g:

$$a = \left(\frac{a}{g}\right) g = \left(\frac{94.7}{9.8}\right) g = 9.7g\ .$$

37. The magnitude of the acceleration of the car as it rounds the curve is given by v^2/R, where v is the speed of the car and R is the radius of the curve. Since the road is horizontal, only the frictional force of the road on the tires makes this acceleration possible. The horizontal component of Newton's second law is $f = mv^2/R$. If N is the normal force of the road on the car and m is the mass of the car, the vertical component of Newton's second law leads to $N = mg$. Thus, using Eq. 6-1, the maximum value of static friction is $f_{s,\max} = \mu_s N = \mu_s mg$. If the car does not slip, $f \le \mu_s mg$. This means

$$\frac{v^2}{R} \le \mu_s g \implies v \le \sqrt{\mu_s R g}\ .$$

Consequently, the maximum speed with which the car can round the curve without slipping is

$$v_{\max} = \sqrt{\mu_s R g} = \sqrt{(0.60)(30.5)(9.8)} = 13\ \text{m/s}\ .$$

38. We will start by assuming that the normal force (on the car from the rail) points up. Note that gravity points down, and the y axis is chosen positive upwards. Also, the direction to the center of the circle (the direction of centripetal acceleration) is down. Thus, Newton's second law leads to

$$N - mg = m\left(-\frac{v^2}{r}\right)\ .$$

(a) When $v = 11$ m/s, we obtain $N = 3.7 \times 10^3$ N. The fact that this answer is positive means that $\vec{N}$ does indeed point upward as we had assumed.

(b) When $v = 14$ m/s, we obtain $N = -1.3 \times 10^3$ N. The fact that this answer is negative means that $\vec{N}$ points opposite to what we had assumed. Thus, the magnitude of $\vec{N}$ is 1.3 kN and its direction is *down*.

39. The magnitude of the acceleration of the cyclist as it rounds the curve is given by v^2/R, where v is the speed of the cyclist and R is the radius of the curve. Since the road is horizontal, only the frictional force of the road on the tires makes this acceleration possible. The horizontal component of Newton's second law is $f = mv^2/R$. If N is the normal force of the road on the bicycle and m is the mass of the bicycle and rider, the vertical component of Newton's second law leads to $N = mg$. Thus, using Eq. 6-1, the maximum value of static friction is $f_{s,\max} = \mu_s N = \mu_s mg$. If the bicycle does not slip, $f \le \mu_s mg$. This means
$$\frac{v^2}{R} \le \mu_s g \implies R \ge \frac{v^2}{\mu_s\, g} \, .$$
Consequently, the minimum radius with which a cyclist moving at 29 km/h = 8.1 m/s can round the curve without slipping is
$$R_{\min} = \frac{v^2}{\mu_s\, g} = \frac{8.1^2}{(0.32)(9.8)} = 21 \text{ m} \, .$$

40. The situation is somewhat similar to that shown in the "loop-the-loop" example done in the textbook (see Figure 6-10) except that, instead of a downward normal force, we are dealing with the force of the boom $\vec{F}_B$ on the car – which is capable of pointing any direction. We will assume it to be upward as we apply Newton's second law to the car (of total weight 5000 N):
$$F_B - W = ma \quad \text{where} \quad m = \frac{W}{g} \;, \quad \text{and } a = -\frac{v^2}{r}$$
Note that the centripetal acceleration is downward (our choice for negative direction) for a body at the top of its circular trajectory.
   (a) If $r = 10$ m and $v = 5.0$ m/s, we obtain $F_B = 3.7 \times 10^3$ N = 3.7 kN (up).
   (b) If $r = 10$ m and $v = 12$ m/s, we obtain $F_B = -2.3 \times 10^3$ N = -2.3 kN where the minus sign indicates that $\vec{F}_B$ points downward.

41. For the puck to remain at rest the magnitude of the tension force T of the cord must equal the gravitational force Mg on the cylinder. The tension force supplies the centripetal force that keeps the puck in its circular orbit, so $T = mv^2/r$. Thus $Mg = mv^2/r$. We solve for the speed: $v = \sqrt{Mgr/m}$.

42. The magnitude of the acceleration of the cyclist as it moves along the horizontal circular path is given by v^2/R, where v is the speed of the cyclist and R is the radius of the curve.
   (a) The horizontal component of Newton's second law is $f = mv^2/R$, where f is the static friction exerted horizontally by the ground on the tires. Thus,
$$f = \frac{(85.0)(9.00)^2}{25.0} = 275 \text{ N} \, .$$
   (b) If N is the vertical force of the ground on the bicycle and m is the mass of the bicycle and rider, the vertical component of Newton's second law leads to $N = mg = 833$ N. The magnitude of the force exerted by the ground on the bicycle is therefore
$$\sqrt{f^2 + N^2} = \sqrt{275^2 + 833^2} = 877 \text{ N} \, .$$

43. (a) At the top (the highest point in the circular motion) the seat pushes up on the student with a force of magnitude $N = 556$ N. Earth pulls down with a force of magnitude $W = 667$ N. The seat is pushing up with a force that is smaller than the student's weight, and we say the student experiences a decrease in his "apparent weight" at the highest point.
   (b) When the student is at the highest point, the net force toward the center of the circular orbit is $W - F_t$ (note that we are choosing downward as the positive direction). According to Newton's second law, this must equal mv^2/R, where v is the speed of the student and R is the radius of the orbit. Thus
$$mv^2/R = W - N = 667\,\text{N} - 556\,\text{N} = 111 \text{ N} \, .$$

(c) Now N is the magnitude of the upward force exerted by the seat when the student is at the lowest point. The net force toward the center of the circle is $F_b - W = mv^2/R$ (note that we are now choosing upward as the positive direction). The Ferris wheel is "steadily rotating" so the value mv^2/R is the same as in part (a). Thus,

$$N = \frac{mv^2}{R} + W = 111\,\text{N} + 667\,\text{N} = 778\ \text{N}\ .$$

(d) If the speed is doubled, mv^2/R increases by a factor of 4, to 444 N. Therefore, at the highest point we have $W - N = mv^2/R$, which leads to

$$N = 667\,\text{N} - 444\,\text{N} = 223\ \text{N}\ .$$

Similarly, the normal force at the lowest point is now found to be $N = 667 + 444 \approx 1.1$ kN.

44. The free-body diagram (for the hand straps of mass m) is the view that a passenger might see if she was looking forward and the streetcar was curving towards the right (so $\vec{a}$ points rightwards in the figure) We note that $|\vec{a}| = v^2/R$ where $v = 16$ km/h $= 4.4$ m/s.

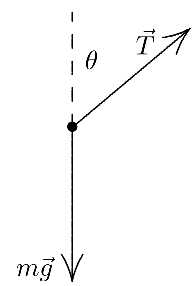
Applying Newton's law to the axes of the problem ($+x$ is rightward and $+y$ is upward) we obtain

$$\begin{aligned} T\sin\theta &= m\frac{v^2}{R} \\ T\cos\theta &= mg\ . \end{aligned}$$

We solve these equations for the angle:

$$\theta = \tan^{-1}\left(\frac{v^2}{Rg}\right)$$

which yields $\theta = 12°$.



45. The free-body diagram (for the airplane of mass m) is shown below. We note that $\vec{F_\ell}$ is the force of aerodynamic lift and $\vec{a}$ points rightwards in the figure. We also note that $|\vec{a}| = v^2/R$ where $v = 480$ km/h $= 133$ m/s.

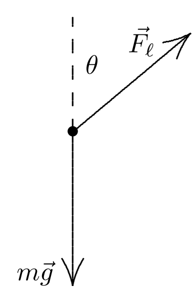
Applying Newton's law to the axes of the problem ($+x$ rightward and $+y$ upward) we obtain

$$\begin{aligned} \vec{F_\ell}\sin\theta &= m\frac{v^2}{R} \\ \vec{F_\ell}\cos\theta &= mg \end{aligned}$$

where $\theta = 40°$. Eliminating mass from these equations leads to

$$\tan\theta = \frac{v^2}{gR}$$

which yields $R = v^2/g\tan\theta = 2.2 \times 10^3$ m.

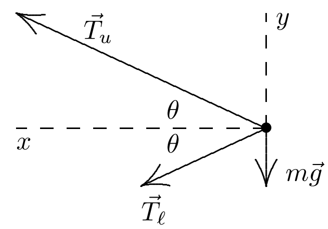


46. (a) The upward force exerted by the car on the passenger is equal to the downward force of gravity ($W = 500$ N) on the passenger. So the *net* force does not have a vertical contribution; it only has the contribution from the horizontal force (which is necessary for maintaining the circular motion). Thus $\left|\vec{F}_{\text{net}}\right| = F = 210$ N.

(b) Using Eq. 6-18, we have

$$v = \sqrt{\frac{FR}{m}} = \sqrt{\frac{(210)(470)}{51.0}} = 44.0 \text{ m/s} .$$

47. (a) The free-body diagram for the ball is shown below. $\vec{T}_u$ is the tension exerted by the upper string on the ball, $\vec{T}_\ell$ is the tension force of the lower string, and m is the mass of the ball. Note that the tension in the upper string is greater than the tension in the lower string. It must balance the downward pull of gravity and the force of the lower string.



(b) We take the $+x$ direction to be leftward (toward the center of the circular orbit) and $+y$ upward. Since the magnitude of the acceleration is $a = v^2/R$, the x component of Newton's second law is

$$T_u \cos\theta + T_\ell \cos\theta = \frac{mv^2}{R} ,$$

where v is the speed of the ball and R is the radius of its orbit. The y component is

$$T_u \sin\theta - T_\ell \sin\theta - mg = 0 .$$

The second equation gives the tension in the lower string: $T_\ell = T_u - mg/\sin\theta$. Since the triangle is equilateral $\theta = 30°$. Thus

$$T_\ell = 35 - \frac{(1.34)(9.8)}{\sin 30°} = 8.74 \text{ N} .$$

(c) The net force is leftward ("radially inward") and has magnitude

$$F_{\text{net}} = (T_u + T_\ell)\cos\theta = (35 + 8.74)\cos 30° = 37.9 \text{ N} .$$

(d) The radius of the path is $[(1.70\,\text{m})/2)]\tan 30° = 1.47\,\text{m}$. Using $F_{\text{net}} = mv^2/R$, we find that the speed of the ball is

$$v = \sqrt{\frac{RF_{\text{net}}}{m}} = \sqrt{\frac{(1.47\,\text{m})(37.9\,\text{N})}{1.34\,\text{kg}}} = 6.45 \text{ m/s} .$$

48. In the solution to exercise 4, we found that the force provided by the wind needed to equal $F = \mu_k mg$. In this situation, we have a much smaller value of μ_k (0.10) and a much larger mass (one hundred stones and the layer of ice). The layer of ice has a mass of

$$m_{\text{ice}} = \left(917\,\text{kg/m}^3\right)(400\,\text{m} \times 500\,\text{m} \times 0.0040\,\text{m})$$

which yields $m_{\text{ice}} = 7.34 \times 10^5$ kg. This added to the mass of the hundred stones (at 20 kg each) comes to $m = 7.36 \times 10^5$ kg.

(a) Setting $F = D$ (for Drag force) we use Eq. 6-14 to find the wind speed v along the ground (which actually is relative to the moving stone, but we assume the stone is moving slowly enough that this does not invalidate the result):

$$v = \sqrt{\frac{\mu_k mg}{4C_{\text{ice}}\rho A_{\text{ice}}}} = \sqrt{\frac{(0.10)\,(7.36 \times 10^5)\,(9.8)}{4(0.002)(1.21)(400 \times 500)}}$$

which yields $v = 19$ m/s which converts to $v = 69$ km/h.

(b) and (c) Doubling our previous result, we find the reported speed to be 139 km/h, which is a reasonable for a storm winds. (A category 5 hurricane has speeds on the order of 2.6×10^2 m/s.)

49. (a) The distance traveled by the coin in 3.14 s is $3(2\pi r) = 6\pi(0.050) = 0.94$ m. Thus, its speed is $v = 0.94/3.14 = 0.30$ m/s.

(b) The acceleration vector (at any instant) is horizontal and points from the coin towards the center of the turntable. This centripetal acceleration is given by Eq. 6-17:

$$a = \frac{v^2}{r} = \frac{0.30^2}{0.050} = 1.8 \text{ m/s}^2 .$$

(c) The only horizontal force acting on the coin is static friction f_s and must be large enough to supply the acceleration of part (b) for the $m = 0.0020$ kg coin. Using Newton's second law,

$$f_s = ma = (0.0020)(1.8) = 3.6 \times 10^{-3} \text{ N}$$

which must point in the same direction as the acceleration (towards the center of the turntable).

(d) We note that the normal force exerted upward on the coin by the turntable must equal the coin's weight (since there is no vertical acceleration in the problem). We also note that if we repeat the computations in parts (a) and (b) for $r' = 0.10$ m, then we obtain $v' = 0.60$ m/s and $a' = 3.6$ m/s^2. Now, if friction is at its maximum at $r = r'$, then, by Eq. 6-1, we obtain

$$\mu_s = \frac{f_{s,\text{max}}}{mg} = \frac{ma'}{mg} = 0.37 .$$

50. (a) The angle made by the cord with the vertical axis is given by $\theta = \cos^{-1}(18/30) = 53°$. This means the radius of the plane's circular path is $r = 30 \sin\theta = 24$ m (we also could have arrived at this using the Pythagorean theorem). The speed of the plane is

$$v = \frac{4.4(2\pi r)}{1\,\text{min}} = \frac{8.8\pi(24\,\text{m})}{60\,\text{s}}$$

which yields $v = 11$ m/s. Eq. 6-17 then gives the acceleration (which at any instant is horizontally directed from the plane to the center of its circular path)

$$a = \frac{v^2}{r} = \frac{11^2}{24} = 5.1 \text{ m/s}^2 .$$

(b) The only horizontal force on the airplane is that component of tension, so Newton's second law gives

$$T \sin\theta = \frac{mv^2}{r} \quad \Longrightarrow \quad T = \frac{(0.75)(11)^2}{24 \sin 53°}$$

which yields $T = 4.8$ N.

(c) The net vertical force on the airplane is zero (since its only acceleration is horizontal), so

$$F_{\text{lift}} = T \cos\theta + mg = 4.8 \cos 53° + (0.75)(9.8) = 10 \text{ N} .$$

51. (a) The centripetal force is given by Eq. 6-18:

$$F = \frac{mv^2}{R} = \frac{(1)(465)^2}{6.4 \times 10^6} = 0.034 \text{ N} .$$

(b) Calling downward (towards the center of Earth) the positive direction, Newton's second law leads to

$$mg - T = ma$$

where $mg = 9.80$ N and $ma = 0.034$ N, calculated in part (a). Thus, the tension in the cord by which the body hangs from the balance is $T = 9.80 - 0.03 = 9.77$ N. Thus, this is the reading for a standard kilogram mass, of the scale at the equator of the spinning Earth.

52. There is no acceleration, so the (upward) static friction forces (there are four of them, one for each thumb and one for each set of opposing fingers) equals the magnitude of the (downward) pull of gravity. Using Eq. 6-1, we have

$$4\mu_s N = mg = (79\,\text{kg})\left(9.8\,\text{m/s}^2\right)$$

which, with $\mu_s = 0.70$, yields $N = 2.8 \times 10^2$ N.

53. (a) From Table 6-1 and Eq. 6-16, we have

$$v_t = \sqrt{\frac{2F_g}{C\rho A}} \implies C\rho A = 2\frac{mg}{v_t^2}$$

where $v_t = 60$ m/s. We estimate the pilot's mass at about $m = 70$ kg. Now, we convert $v = 1300(1000/3600) \approx 360$ m/s and plug into Eq. 6-14:

$$D = \frac{1}{2}C\rho A v^2 = \frac{1}{2}\left(2\frac{mg}{v_t^2}\right)v^2 = mg\left(\frac{v}{v_t}\right)^2$$

which yields $D = (690)(360/60)^2 \approx 2 \times 10^4$ N.

(b) We assume the mass of the ejection seat is roughly equal to the mass of the pilot. Thus, Newton's second law (in the horizontal direction) applied to this system of mass $2m$ gives the magnitude of acceleration:

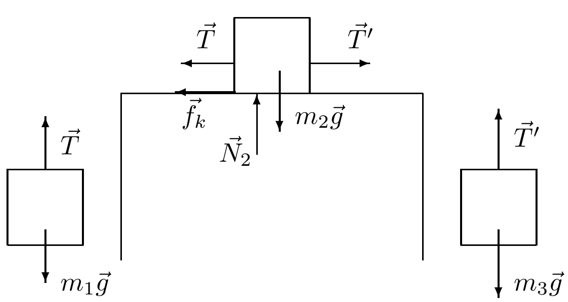
$$|a| = \frac{D}{2m} = \frac{g}{2}\left(\frac{v}{v_t}\right)^2 = 18g\ .$$

54. Although the object in question is a sphere, the area A in Eq. 6-16 is the cross sectional area presented by the object as it moves through the air (the cross section is perpendicular to $\vec{v}$). Thus, A is that of a circle: $A = \pi R^2$. We also note that 16 lb equates to an SI weight of 71 N. Thus,

$$v_t = \sqrt{\frac{2F_g}{C\rho\pi R^2}} \quad\implies\quad R = \frac{1}{145}\sqrt{\frac{2(71)}{(0.49)(1.2)\pi}}$$

which yields a diameter of $2R = 0.12$ m.

55. In the following sketch, T and T' are the tensions in the left and right strings, respectively. Also, $m_1 = M = 2.0$ kg, $m_2 = 2M = 4.0$ kg, and $m_3 = 2M = 4.0$ kg. Since it does, in fact, slide (presumably rightward), the type of friction that is acting upon m_2 is *kinetic* friction.



We use the familiar axes with $+x$ rightward and $+y$ upward for each block. This has the consequence that m_1 and m_2 accelerate with the same sign, but the acceleration of m_3 has the opposite sign. We take this into account as we apply Newton's second law to the three blocks.

$$\begin{aligned} T - m_1 g &= m_1(+a) \\ T' - T - f_k &= m_2(+a) \\ T' - m_3 g &= m_3(-a) \end{aligned}$$

Adding the first two equations, and subtracting the last, we obtain

$$(m_3 - m_1)\,g - f_k = (m_1 + m_2 + m_3)\,a$$

or (using M as in the problem statement)

$$Mg - f_k = 5Ma\ .$$

With $a = 1.5\ \text{m/s}^2$, we find $f_k = 4.6$ N.

56. (a) The component of the weight along the incline (with downhill understood as the positive direction) is $mg\sin\theta$ where $m = 630$ kg and $\theta = 10.2°$. With $f = 62.0$ N, Newton's second law leads to

$$mg\sin\theta - f = ma$$

which yields $a = 1.64\ \text{m/s}^2$. Using Eq. 2-15, we have

$$80.0\ \text{m} \;=\; \left(6.20\ \frac{\text{m}}{\text{s}}\right)t + \frac{1}{2}\left(1.64\ \frac{\text{m}}{\text{s}^2}\right)t^2\ .$$

This is solved using the quadratic formula. The positive root is $t = 6.80$ s.

(b) Running through the calculation of part (a) with $f = 42.0$ N instead of $f = 62$ N results in $t = 6.76$ s.

57. We convert to SI units: $v = 94(1000/3600) = 26$ m/s. Eq. 6-18 yields

$$F = \frac{mv^2}{R} = \frac{(85)(26)^2}{220} = 263\ \text{N}$$

for the horizontal force exerted on the passenger by the seat. But the seat also exerts an upward force equal to $mg = 833$ N. The magnitude of force is therefore $\sqrt{263^2 + 833^2} = 874$ N.

58. (a) Comparing the $t = 2.0$ s photo with the $t = 0$ photo, we see that the distance traveled by the box is

$$d = \sqrt{4.0^2 + 2.0^2} = 4.5\ \text{m}\ .$$

Thus (from Table 2-1, with *downhill* positive) $d = v_0 t + \frac{1}{2}at^2$, we obtain $a = 2.2\ \text{m/s}^2$; note that the boxes are assumed to start from rest.

(b) For the axis along the incline surface, we have

$$mg\sin\theta - f_k = ma\ .$$

We compute mass m from the weight $m = 240/9.8 = 24$ kg, and θ is figured from the absolute value of the slope of the graph: $\theta = \tan^{-1} 2.5/5.0 = 27°$. Therefore, we find $f_k = 53$ N.

59. (a) If the skier covers a distance L during time t with zero initial speed and a constant acceleration a, then $L = at^2/2$, which gives the acceleration a_1 for the first (old) pair of skis:

$$a_1 = \frac{2L}{t_1^2} = \frac{2(200\,\text{m})}{(61\,\text{s})^2} = 0.11\,\text{m/s}^2$$

and the acceleration a_2 for the second (new) pair:

$$a_2 = \frac{2L}{t_2^2} = \frac{2(200\,\text{m})}{(42\,\text{s})^2} = 0.23\,\text{m/s}^2\ .$$

(b) The net force along the slope acting on the skier of mass m is

$$F_{\text{net}} = mg\sin\theta - f_k = mg(\sin\theta - \mu_k\cos\theta) = ma$$

which we solve for μ_{k1} for the first pair of skis:

$$\mu_{k1} = \tan\theta - \frac{a_1}{g\cos\theta} = \tan 3.0^\circ - \frac{0.11}{9.8\cos 3.0^\circ} = 0.041$$

and μ_{k2} for the second pair:

$$\mu_{k2} = \tan\theta - \frac{a_2}{g\cos\theta} = \tan 3.0^\circ - \frac{0.23}{9.8\cos 3.0^\circ} = 0.029 \ .$$

60. (a) The box doesn't move until $t = 2.8$ s, which is when the applied force $\vec{F}$ reaches a magnitude of $F = (1.8)(2.8) = 5.0$ N, implying therefore that $f_{s,\,\max} = 5.0$ N. Analysis of the vertical forces on the block leads to the observation that the normal force magnitude equals the weight $N = mg = 15$ N. Thus, $\mu_s = f_{s,\,\max}/N = 0.34$.

(b) We apply Newton's second law to the horizontal x axis (positive in the direction of motion).

$$F - f_k = ma \quad\Longrightarrow\quad 1.8t - f_k = (1.5)(1.2t - 2.4)$$

Thus, we find $f_k = 3.6$ N. Therefore, $\mu_k = f_k/N = 0.24$.

61. In both cases (highest point and lowest point), the normal force (on the child from the seat) points up, gravity points down, and the y axis is chosen positive upwards. At the high point, the direction to the center of the circle (the direction of centripetal acceleration) is down, and at the low point that direction is up.

(a) Newton's second law (using Eq. 6-17 for the magnitude of the acceleration) leads to

$$N - mg = m\left(-\frac{v^2}{R}\right) \ .$$

With $m = 26$ kg, $v = 5.5$ m/s and $R = 12$ m, this yields $N = 189$ N which we round off to $N \approx 190$ N.

(b) Now, Newton's second law leads to

$$N - mg = m\left(\frac{v^2}{r}\right)$$

which yields $N = 320$ N. As already mentioned, the direction of $\vec{N}$ is *up* in both cases.

62. The mass of the car is $m = 10700/9.8 = 1.09 \times 10^3$ kg. We choose "inward" (horizontally towards the center of the circular path) as the positive direction.

(a) With $v = 13.4$ m/s and $R = 61$ m, Newton's second law (using Eq. 6-18) leads to

$$f_s = \frac{mv^2}{R} = 3.21 \times 10^3 \text{ N} \ .$$

(b) Noting that $N = mg$ in this situation, the maximum possible static friction is found to be

$$f_{s,\max} = \mu_s mg = (0.35)(10700) = 3.75 \times 10^3 \text{ N}$$

using Eq. 6-1. We see that the static friction found in part (a) is less than this, so the car rolls (no skidding) and successfully negotiates the curve.

63. (a) The distance traveled in one revolution is $2\pi R = 2\pi(4.6) = 29$ m. The (constant) speed is consequently $v = 29/30 = 0.96$ m/s.

(b) Newton's second law (using Eq. 6-17 for the magnitude of the acceleration) leads to

$$f_s = m\left(\frac{v^2}{R}\right) = m(0.20)$$

in SI units. Noting that $N = mg$ in this situation, the maximum possible static friction is $f_{s,\max} = \mu_s mg$ using Eq. 6-1. Equating this with $f_s = m(0.20)$ we find the mass m cancels and we obtain $\mu_s = 0.20/9.8 = 0.021$.

64. At the top of the hill the vertical forces on the car are the upward normal force exerted by the ground and the downward pull of gravity. Designating $+y$ downward, we have

$$mg - N = \frac{mv^2}{R}$$

from Newton's second law. To find the greatest speed without leaving the hill, we set $N = 0$ and solve for v:

$$v = \sqrt{gR} = \sqrt{(9.8)(250)} = 49.5 \text{ m/s}$$

which converts to $49.5(3600/1000) = 178$ km/h.

65. For simplicity, we denote the 70° angle as θ and the magnitude of the push (80 N) as P. The vertical forces on the block are the downward normal force exerted on it by the ceiling, the downward pull of gravity (of magnitude mg) and the vertical component of $\vec{P}$ (which is upward with magnitude $P\sin\theta$). Since there is no acceleration in the vertical direction, we must have

$$N = P\sin\theta - mg$$

in which case the leftward-pointed kinetic friction has magnitude

$$f_k = \mu_k\left(P\sin\theta - mg\right) \ .$$

Choosing $+x$ rightward, Newton's second law leads to

$$P\cos\theta - f_k = ma \quad\Longrightarrow\quad a = \frac{P\cos\theta - \mu_k\left(P\sin\theta - mg\right)}{m}$$

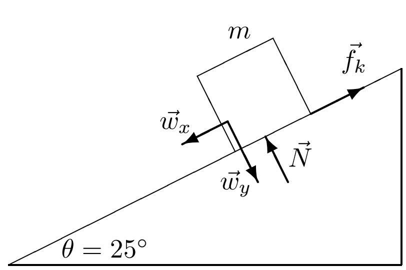
which yields $a = 3.4$ m/s^2 when $\mu_k = 0.40$ and $m = 5.0$ kg.

66. Probably the most appropriate picture in the textbook to represent the situation in this problem is in the previous chapter: Fig. 5-9. We adopt the familiar axes with $+x$ rightward and $+y$ upward, and refer to the 85 N horizontal push of the worker as P (and assume it to be rightward). Applying Newton's second law to the x axis and y axis, respectively, produces

$$\begin{aligned} P - f_k &= ma \\ N - mg &= 0 \ . \end{aligned}$$

Using $v^2 = v_0^2 + 2a\Delta x$ we find $a = 0.36$ m/s^2. Consequently, we obtain $f_k = 71$ N and $N = 392$ N. Therefore, $\mu_k = f_k/N = 0.18$.

67. In the figure below, $m = 140/9.8 = 14.3$ kg is the mass of the child. We use $\vec{w}_x$ and $\vec{w}_y$ as the components of the gravitational pull of Earth on the block; their magnitudes are $w_x = mg\sin\theta$ and $w_y = mg\cos\theta$.



(a) With the x axis directed up along the incline (so that $a = -0.86\ \mathrm{m/s^2}$), Newton's second law leads to

$$f_k - 140\sin 25° = m\,(-0.86)$$

which yields $f_k = 47$ N. We also apply Newton's second law to the y axis (perpendicular to the incline surface), where the acceleration-component is zero:

$$N - 140\cos 25° = 0 \implies N = 127\ \mathrm{N}\ .$$

Therefore, $\mu_k = f_k/N = 0.37$.

(b) Returning to our first equation in part (a), we see that if the downhill component of the weight force were insufficient to overcome static friction, the child would not slide at all. Therefore, we require $140\sin 25° > f_{s,\,\max} = \mu_s N$, which leads to $\tan 25° = 0.47 > \mu_s$. The minimum value of μ_s equals μ_k and is more subtle; reference to §6-1 is recommended. If μ_k exceeded μ_s then when static friction were overcome (as the incline is raised) then it should start to move – which is impossible if f_k is large enough to cause deceleration! The bounds on μ_s are therefore given by $\tan 25° > \mu_s > \mu_k$.

68. (a) The intuitive conclusion, that the tension is greatest at the bottom of the swing, is certainly supported by application of Newton's second law there:

$$T - mg = \frac{mv^2}{R} \implies T = m\left(g + \frac{v^2}{R}\right)$$

where Eq. 6-18 has been used. Increasing the speed eventually leads to the tension at the bottom of the circle reaching that breaking value of 40 N.

(b) Solving the above equation for the speed, we find

$$v = \sqrt{R\left(\frac{T}{m} - g\right)} = \sqrt{(0.91)\left(\frac{40}{0.37} - 9.8\right)}$$

which yields $v = 9.5$ m/s.

69. (a) We denote the apparent weight of the crew member of mass m on the spaceship as $W_a = 300\,\mathrm{N}$, his weight on Earth as $W_e = mg = 600\,\mathrm{N}$, and the radius of the spaceship as $R = 500\,\mathrm{m}$. Since $mv_s^2/R = W_a$, we get

$$v_s = \sqrt{\frac{W_a R}{m}} = \sqrt{\left(\frac{W_a}{W_e}\right) gR}$$

where we substituted $m = W_e/g$. Thus,

$$v_s = \sqrt{\left(\frac{300\,\mathrm{N}}{600\,\mathrm{N}}\right)(9.8\,\mathrm{m/s^2})\,(500\,\mathrm{m})} = 49.5\ \mathrm{m/s}\ .$$

(b) For any object of mass m on the spaceship $W_a = mv^2/R \propto v^2$, where v is the speed of the circular motion of the object relative to the center of the circle. In the previous case $v = v_s = 49.5\,\mathrm{m/s}$, and in the present case $v = 10\,\mathrm{m/s} + 49.5\,\mathrm{m/s} = 59.5\,\mathrm{m/s} \equiv v'$. Thus the apparent weight of the running crew member is

$$W_a' = W_a\left(\frac{v'}{v}\right)^2 = (300\,\mathrm{N})\left(\frac{59.5\,\mathrm{m/s}}{49.5\,\mathrm{m/s}}\right)^2 = 4.3\times 10^2\ \mathrm{N}\ .$$

70. We refer the reader to Sample Problem 6-11, and use the result Eq. 6-29:

$$\theta = \tan^{-1}\left(\frac{v^2}{gR}\right)$$

with $v = 60(1000/3600) = 17$ m/s and $R = 200$ m. The banking angle is therefore $\theta = 8.1°$. Now we consider a vehicle taking this banked curve at $v' = 40(1000/3600) = 11$ m/s. Its (horizontal) acceleration is $a' = v'^2/R$, which has components parallel the incline and perpendicular to it.

$$a_{\|} = a' \cos\theta = \frac{v'^2 \cos\theta}{R} \quad \text{and} \quad a_{\perp} = a' \sin\theta = \frac{v'^2 \sin\theta}{R}$$

These enter Newton's second law as follows (choosing downhill as the $+x$ direction and away-from-incline as $+y$):

$$mg\sin\theta - f_s = ma_{\|} \quad \text{and} \quad N - mg\cos\theta = ma_{\perp}$$

and we are led to

$$\frac{f_s}{N} = \frac{mg\sin\theta - mv'^2\cos\theta/R}{mg\cos\theta + mv'^2\sin\theta/R} \; .$$

We cancel the mass and plug in, obtaining $f_s/N = 0.078$. The problem implies we should set $f_s = f_{s,\max}$ so that, by Eq. 6-1, we have $\mu_s = 0.078$.

71. (a) The force which provides the horizontal acceleration v^2/R necessary for the circular motion of radius $R = 0.25$ m is $T\sin\theta$, where T in the tension in the $L = 1.2$ m string and θ is the angle of the string measured from vertical. The other component of tension must equal the bob's weight so that there is no vertical acceleration: $T\cos\theta = mg$. Combining these observations leads to

$$\frac{v^2}{R} = g\tan\theta \quad \text{where} \quad \sin\theta = \frac{R}{L}$$

so that $\theta = \sin^{-1}(0.25/1.2) = 12°$ and $v = \sqrt{gR\tan\theta} = 0.72\,\text{m/s}$. It should be mentioned that Sample Problem 6-11 discusses the conical pendulum.

(b) Thus, $a = v^2/R = 2.1$ m/s^2.

(c) The tension is

$$T = \frac{mg}{\cos\theta} = \frac{(0.050)(9.8)}{\cos 12°} = 0.50 \text{ N} \; .$$

72. (a) Our $+x$ direction is horizontal and is chosen (as we also do with $+y$) so that the components of the 100 N force $\vec{F}$ are non-negative. Thus, $F_x = F\cos\theta = 100$ N, which the textbook denotes F_h in this problem.

(b) Since there is no vertical acceleration, application of Newton's second law in the y direction gives

$$N + F_y = mg \quad \Longrightarrow N = mg - F\sin\theta$$

where $m = 25$ kg. This yields $N = 245$ N in this case ($\theta = 0°$).

(c) Now, $F_x = F_h = F\cos\theta = 86.6$ N for $\theta = 30°$.

(d) And $N = mg - F\sin\theta = 195$ N.

(e) We find $F_x = F_h = F\cos\theta = 50$ N for $\theta = 60°$.

(f) And $N = mg - F\sin\theta = 158$ N.

(g) The condition for the chair to slide is

$$F_x > f_{s,\max} = \mu_s N \quad \text{where} \quad \mu_s = 0.42 \; .$$

For $\theta = 0°$, we have

$$F_x = 100\,\text{N} < f_{s,\max} = (0.42)(245) = 103\,\text{N}$$

so the crate remains at rest.

(h) For $\theta = 30.0°$, we find

$$F_x = 86.6\,\mathrm{N} > f_{s,\,\max} = (0.42)(195) = 81.9\,\mathrm{N}$$

so the crate slides.

(i) For $\theta = 60°$, we get

$$F_x = 50.0\,\mathrm{N} < f_{s,\,\max} = (0.42)(158) = 66.4\,\mathrm{N}$$

which means the crate must remain at rest.

73. We note that $N = mg$ in this situation, so $f_k = \mu_k mg = (0.32)(220) = 70.4\,\mathrm{N}$ and $f_{s,\max} = \mu_s mg = (0.41)(220) = 90.2\,\mathrm{N}$.

(a) The person needs to push at least as hard as the static friction maximum if he hopes to start it moving. Denoting his force as P, this means a value of P slightly larger than 90.2 N is sufficient. Rounding to two figures, we obtain $P = 90$ N.

(b) Constant velocity (zero acceleration) implies the push equals the kinetic friction, so $P = 70$ N.

(c) Applying Newton's second law, we have

$$P - f_k = ma \implies a = \frac{\mu_s mg - \mu_k mg}{m}$$

which simplifies to $a = g(\mu_s - \mu_k) = 0.88\ \mathrm{m/s^2}$.

74. Except for replacing f_s with f_k, Fig. 6-5 in the textbook is appropriate. With that figure in mind, we choose uphill as the $+x$ direction. Applying Newton's second law to the x axis, we have

$$f_k - W\sin\theta = ma \quad \text{where} \quad m = \frac{W}{g}\ ,$$

and where $W = 40$ N, $a = +0.80\ \mathrm{m/s^2}$ and $\theta = 25°$. Thus, we find $f_k = 20$ N. Along the y axis, we have

$$\sum \vec{F}_y = 0 \implies N = W\cos\theta$$

so that $\mu_k = f_k/N = 0.56$.

75. We use the familiar horizontal and vertical axes for x and y directions, with rightward and upward positive, respectively. The rope is assumed massless so that the force exerted by the child $\vec{F}$ is identical to the tension uniformly through the rope. The x and y components of $\vec{F}$ are $F\cos\theta$ and $F\sin\theta$, respectively. The static friction force points leftward.

(a) Newton's Law applied to the y axis, where there is presumed to be no acceleration, leads to

$$N + F\sin\theta - mg = 0$$

which implies that the maximum static friction is $\mu_s(mg - F\sin\theta)$. If $f_s = f_{s,\,\max}$ is assumed, then Newton's second law applied to the x axis (which also has $a = 0$ even though it is "verging" on moving) yields

$$\begin{aligned} F\cos\theta - f_s &= ma\ , \quad \text{or} \\ F\cos\theta - \mu_s(mg - F\sin\theta) &= 0 \end{aligned}$$

which we solve, for $\theta = 42°$ and $\mu_s = 0.42$, to obtain $F = 74$ N.

(b) Solving the above equation algebraically for F, with W denoting the weight, we obtain

$$F = \frac{\mu_s W}{\cos\theta + \mu_s\sin\theta}\ .$$

(c) We minimize the above expression for F by working through the $\frac{dF}{d\theta} = 0$ condition:

$$\frac{dF}{d\theta} = \frac{\mu_s W (\sin\theta - \mu_s \cos\theta)}{(\cos\theta + \mu_s \sin\theta)^2} = 0$$

which leads to the result $\theta = \tan^{-1}\mu_s = 23°$.

(d) Plugging $\theta = 23°$ into the above result for F, with $\mu_s = 0.42$ and $W = 180$ N, yields $F = 70$ N.

76. (a) We note that $N = mg$ in this situation, so $f_{s,\max} = \mu_s mg = (0.52)(11)(9.8) = 56\,\text{N}$. Consequently, the horizontal force $\vec{F}$ needed to initiate motion must be (at minimum) slightly more than 56 N.

(b) Analyzing vertical forces when $\vec{F}$ is at nonzero θ yields

$$F\sin\theta + N = mg \implies f_{s,\max} = \mu_s (mg - F\sin\theta) \ .$$

Now, the horizontal component of $\vec{F}$ needed to initiate motion must be (at minimum) slightly more than this, so

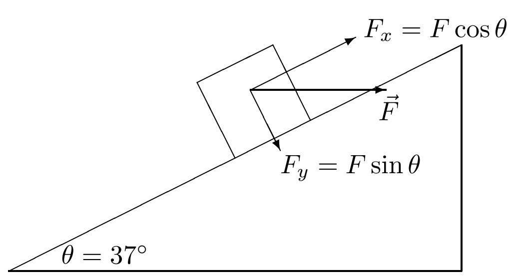
$$F\cos\theta = \mu_s (mg - F\sin\theta) \implies F = \frac{\mu_s mg}{\cos\theta + \mu_s \sin\theta}$$

which yields $F = 59$ N when $\theta = 60°$.

(c) We now set $\theta = -60°$ and obtain

$$F = \frac{(0.52)(11)(9.8)}{\cos(-60°) + (0.52)\sin(-60°)} = 1.1 \times 10^3\,\text{N} \ .$$

77. The coordinate system we wish to use is shown in Fig. 5-18 in the textbook, so we resolve this horizontal force into appropriate components.



(a) Applying Newton's second law to the x (directed uphill) and y (directed away from the incline surface) axes, we obtain

$$\begin{aligned} F\cos\theta - f_k - mg\sin\theta &= ma \\ N - F\sin\theta - mg\cos\theta &= 0 \ . \end{aligned}$$

Using $f_k = \mu_k N$, these equations lead to

$$a = \frac{F}{m}(\cos\theta - \mu_k \sin\theta) - g(\sin\theta + \mu_k \cos\theta)$$

which yields $a = -2.1$ m/s^2 for $\mu_k = 0.30$, $F = 50$ N and $m = 5.0$ kg.

(b) With $v_0 = +4.0$ m/s and $v = 0$, Eq. 2-16 gives

$$\Delta x = -\frac{4.0^2}{2(-2.1)} = 3.9 \text{ m} \ .$$

(c) We expect $\mu_s \geq \mu_k$; otherwise, an object started into motion would immediately start decelerating (before it gained any speed)! In the minimal expectation case, where $\mu_s = 0.30$, the maximum possible (downhill) static friction is, using Eq. 6-1,

$$f_{s,\max} = \mu_s N = \mu_s (F \sin\theta + mg\cos\theta)$$

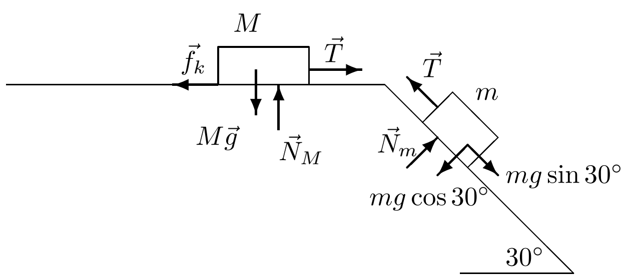
which turns out to be 21 N. But in order to have no acceleration along the x axis, we must have

$$f_s = F\cos\theta - mg\sin\theta = 10 \text{ N}$$

(the fact that this is positive reinforces our suspicion that $\vec{f_s}$ points downhill). Since the f_s needed to remain at rest is less than $f_{s,\max}$ then it stays at that location.

78. Since the problem is allowing for student creativity and research here, we only present a problem and solution for part (a).

(a) We show below two blocks M and m, the first on a horizontal surface with $\mu_k = 0.25$ and the second on a frictionless incline. They are connected by a rope (not shown) in which the tension is T. The goal is to find T given $M = 2.0$ kg and $m = 3.0$ kg. We assume f_s is not relevant to this computation.



Solution: We apply Newton's second law to each block's x axis, which for M is positive rightward and for m is positive downhill:

$$\begin{aligned} T - f_k &= Ma \\ mg\sin 30° - T &= ma \end{aligned}$$

Adding the equations, we obtain the acceleration.

$$a = \frac{mg\sin 30° - f_k}{m + M}$$

For $f_k = \mu_k N_M = \mu_k Mg$, we obtain $a = 1.96$ m/s^2. Returning this value to either of the above equations, we find $T = 8.8$ N.

79. (First problem in **Cluster 1**)
Since the block remains stationary, then $\sum \vec{F} = 0$, and we have (along the horizontal x axis) $f_s = 25$ N, where $\vec{f_s}$ points left.

80. (Second problem in **Cluster 1**)
To keep the block stationary, we require $\sum \vec{F} = 0$ (equilibrium of forces), which leads (along the horizontal x axis) to $f_s = 50$ N. Now, we take $f_s = f_{s,\max} = \mu_s N$ and find that N must equal $50/0.4 = 125$ N. Equilibrium of forces along the y axis implies $N - mg - F = 0$, so that (with $mg = 98$ N) we must have $F = 27$ N.

81. (Third problem in **Cluster 1**)
A useful diagram (where some of these forces are analyzed) is Fig. 6-5 in the textbook. Using that figure for this problem, W is the weight (equal to $mg = 98$ N), and $\theta = 25°$.

   (a) The maximum static friction is given by Eq. 6-1:
$$f_{s,\,\max} = \mu_s N = (0.60)W\cos\theta = 53 \text{ N} \quad .$$

   (b) $W\sin\theta = 41$ N.

   (c) If there is *no* motion, then $\sum\vec{F} = 0$ along the incline, so $f_s - W\sin\theta - F = 0$ (if uphill is positive). And if the system verges on motion, then $f_s = f_{s,\,\max}$. Therefore, in that case we find $F = 53 - 41 = 12$ N.

   (d) With the block sliding, with no applied force F, then Newton's second law yields $f_k - W\sin\theta = ma$ (if uphill is positive) where $f_k = \mu_k N = (0.20)W\cos\theta = 18$ N. We thus obtain $a = -2.4$ m/s^2. Therefore, the magnitude of $\vec{a}$ is 2.4 m/s^2 and the direction is downhill.

82. (Fourth problem in **Cluster 1**)
A useful diagram (where some of these forces are analyzed) is Fig. 6-5 in the textbook; however, since the block is about to move uphill, one must imagine $\vec{f_s}$ turned around (so that it points downhill). Using that figure for this problem, W is the weight (equal to $mg = 98$ N), and $\theta = 25°$.

   (a) If there is *no* motion, then $\sum\vec{F} = 0$ along the incline, so $F - f_s - W\sin\theta = 0$ (if uphill is positive). And if the system verges on motion, then $f_s = f_{s,\,\max} = \mu_s W\cos\theta = 53$ N. Therefore, in that case we find $F = 95$ N.

   (b) With the block sliding, and the applied force F still equal to the value found in part (a), then Newton's second law yields $F - f_k - W\sin\theta = ma$ (if uphill is positive) where $f_k = \mu_k N = (0.20)W\cos\theta = 18$ N. We thus obtain $a = 3.6$ m/s^2. Therefore, the magnitude of $\vec{a}$ is 3.6 m/s^2 and the direction is uphill.

   (c) With the block sliding uphill, but with no applied force F, then Newton's second law yields $-f_k - W\sin\theta = ma$ (if uphill is positive) where $f_k = 18$ N. We thus obtain $a = -5.9$ m/s^2. Therefore, the magnitude of $\vec{a}$ is 5.9 m/s^2 and the direction is downhill. It is decelerating and will ultimately come to a stop and remain at there at equilibrium.

83. (Fifth problem in **Cluster 1**)
A useful diagram (where these forces are analyzed) is Fig. 6-5 in the textbook. In that figure, W is the weight (equal to $mg = 98$ N).

   (a) Since there is no motion, then $\sum\vec{F} = 0$ along the incline, so $f_s - W\sin\theta = 0$ (if uphill is positive, which is the direction assumed for $\vec{f_s}$). We therefore obtain $f_s = 25$ N. Our result is positive, so it indeed points uphill as we had assumed. One can check that this value of f_s does not exceed the maximum possible value $f_{s,\,\max}$ (see next part).

   (b) As in part (a), we have $f_s - W\sin\theta = 0$, but since the system is on the verge of motion we also have $f_s = f_{s,\,\max} = \mu_s W\cos\theta$. Therefore,
$$\mu_s W\cos\theta - W\sin\theta = 0 \implies \mu_s = \tan\theta$$
which leads to $\theta_s = \tan^{-1}\mu_s = 31°$ (this is often called "the angle of repose").

   (c) If the block slides with no acceleration then we have $f_k - W\sin\theta = 0$ from Newton's second law applied along the incline surface. With $f_k = \mu_k W\cos\theta$ we are led to $\theta_k = \tan^{-1}\mu_k$ as the condition for this constant velocity sliding downhill. Since $\mu_k < \mu_s$ then we see that $\theta_k < \theta_s$ from part (b).

   (d) We find $\theta_k = \tan^{-1}\mu_k = 11°$.