# Chapter 38

1. (a) The time an electron with a horizontal component of velocity $v$ takes to travel a horizontal distance $L$ is
$$t = \frac{L}{v} = \frac{20 \times 10^{-2}\,\mathrm{m}}{(0.992)(2.998 \times 10^{8}\,\mathrm{m/s})} = 6.72 \times 10^{-10}\ \mathrm{s}\ .$$

   (b) During this time, it falls a vertical distance
$$y = \frac{1}{2}gt^2 = \frac{1}{2}(9.8\,\mathrm{m/s^2})(6.72 \times 10^{-10}\,\mathrm{s})^2 = 2.2 \times 10^{-18}\ \mathrm{m}\ .$$

   This distance is much less than the radius of a proton. We can conclude that for particles traveling near the speed of light in a laboratory, Earth may be considered an approximately inertial frame.

2. (a) The speed parameter $\beta$ is $v/c$. Thus,
$$\beta = \frac{(3\,\mathrm{cm/y})(0.01\,\mathrm{m/cm})(1\,\mathrm{y}/3.15 \times 10^{7}\,\mathrm{s})}{3.0 \times 10^{8}\,\mathrm{m/s}} = 3 \times 10^{-18}\ .$$

   (b) For the highway speed limit, we find
$$\beta = \frac{(90\,\mathrm{km/h})(1000\,\mathrm{m/km})(1\,\mathrm{h}/3600\,\mathrm{s})}{3.0 \times 10^{8}\,\mathrm{m/s}} = 8.3 \times 10^{-8}\ .$$

   (c) Mach 2.5 corresponds to
$$\beta = \frac{(1200\,\mathrm{km/h})(1000\,\mathrm{m/km})(1\,\mathrm{h}/3600\,\mathrm{s})}{3.0 \times 10^{8}\,\mathrm{m/s}} = 1.1 \times 10^{-6}\ .$$

   (d) We refer to Table 14-2:
$$\beta = \frac{(11.2\,\mathrm{km/s})(1000\,\mathrm{m/km})}{3.0 \times 10^{8}\,\mathrm{m/s}} = 3.7 \times 10^{-5}\ .$$

   (e) For the quasar recession speed, we obtain
$$\beta = \frac{(3.0 \times 10^{4}\,\mathrm{km/s})(1000\,\mathrm{m/km})}{3.0 \times 10^{8}\,\mathrm{m/s}} = 0.10\ .$$

3. From the time dilation equation $\Delta t = \gamma \Delta t_0$ (where $\Delta t_0$ is the proper time interval, $\gamma = 1/\sqrt{1-\beta^2}$, and $\beta = v/c$), we obtain
$$\beta = \sqrt{1 - \left(\frac{\Delta t_0}{\Delta t}\right)^2}\ .$$

The proper time interval is measured by a clock at rest relative to the muon. Specifically, $\Delta t_0 = 2.2\,\mu\text{s}$. We are also told that Earth observers (measuring the decays of moving muons) find $\Delta t = 16\,\mu\text{s}$. Therefore,

$$\beta = \sqrt{1 - \left(\frac{2.2\,\mu\text{s}}{16\,\mu\text{s}}\right)^2} = 0.9905\ .$$

The muon speed is $v = \beta c = 0.9905(2.998 \times 10^8\,\text{m/s}) = 2.97 \times 10^8\,\text{m/s}$.

4. (a) We find $\beta$ from $\gamma = 1/\sqrt{1-\beta^2}$:

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}} = \sqrt{1 - \frac{1}{(1.01)^2}} = 0.140371 \ \approx\ 0.140\ .$$

(b) Similarly, $\beta = \sqrt{1-(10.0)^{-2}} = 0.994987 \approx 0.9950$.

(c) In this case, $\beta = \sqrt{1-(100)^{-2}} = 0.999\,950$.

(d) This last case might prove problematic for some calculators. The result is $\beta = \sqrt{1-(1000)^{-2}} = 0.999\,999\,50$. The discussion in Sample Problem 38-7 dealing with large $\gamma$ values may prove helpful for those whose calculators do not yield this answer.

5. In the laboratory, it travels a distance $d = 0.00105\,\text{m} = vt$, where $v = 0.992c$ and $t$ is the time measured on the laboratory clocks. We can use Eq. 38-7 to relate $t$ to the proper lifetime of the particle $t_0$:

$$t = \frac{t_0}{\sqrt{1-(v/c)^2}} \quad\Longrightarrow\quad t_0 = t\sqrt{1 - \left(\frac{v}{c}\right)^2} = \frac{d}{0.992c}\sqrt{1-0.992^2}$$

which yields $t_0 = 4.46 \times 10^{-13}$ s.

6. (a) The round-trip (discounting the time needed to "turn around") should be one year according to the clock you are carrying (this is your proper time interval $\Delta t_0$) and 1000 years according to the clocks on Earth which measure $\Delta t$. We solve Eq. 38-7 for $v$ and then plug in:

$$\begin{aligned} v &= c\sqrt{1 - \left(\frac{\Delta t_0}{\Delta t}\right)^2} \\ &= (299792458\,\text{m/s})\sqrt{1 - \left(\frac{1\,\text{y}}{1000\,\text{y}}\right)^2} \\ &= 299792308\ \text{m/s} \end{aligned}$$

which may also be expressed as $v = c\sqrt{1-(1000)^{-2}} = 0.999\,999\,50c$. The discussion in Sample Problem 38-7 dealing with these sorts of values may prove helpful for those whose calculators do not yield this answer.

(b) The equations do not show a dependence on acceleration (or on the direction of the velocity vector), which suggests that a circular journey (with its constant magnitude centripetal acceleration) would give the same result (if the speed is the same) as the one described in the problem. A more careful argument can be given to support this, but it should be admitted that this is a fairly subtle question which has occasionally precipitated debates among professional physicists.

7. The length $L$ of the rod, as measured in a frame in which it is moving with speed $v$ parallel to its length, is related to its rest length $L_0$ by $L = L_0/\gamma$, where $\gamma = 1/\sqrt{1-\beta^2}$ and $\beta = v/c$. Since $\gamma$ must be greater than 1, $L$ is less than $L_0$. For this problem, $L_0 = 1.70\,\text{m}$ and $\beta = 0.630$, so $L = (1.70\,\text{m})\sqrt{1-(0.630)^2} = 1.32\,\text{m}$.

8. The contracted length of the tube would be

$$L = L_0\sqrt{1-\beta^2} = (3.00\,\mathrm{m})\sqrt{1-0.999987^2} = 0.0153\ \mathrm{m}\ .$$

9. Only the "component" of the length in the $x$ direction contracts, so its $y$ component stays

$$\ell'_y = \ell_y = \ell\sin 30^\circ = 0.5000\ \mathrm{m}$$

while its $x$ component becomes

$$\ell'_x = \ell_x\sqrt{1-\beta^2} = \ell\cos 30^\circ\sqrt{1-0.90^2} = 0.3775\ \mathrm{m}\ .$$

Therefore, using the Pythagorean theorem, the length measured from $S'$ is

$$\ell' = \sqrt{\left(\ell'_x\right)^2 + \left(\ell'_y\right)^2} = 0.626\ \mathrm{m}\ .$$

10. (a) We solve Eq. 38-13 for $v$ and then plug in:

$$\begin{aligned} v &= c\sqrt{1-\left(\frac{L}{L_0}\right)^2} \\ &= (299792458\,\mathrm{m/s})\sqrt{1-\left(\frac{1}{2}\right)^2} \\ &= 259627884\ \mathrm{m/s} \end{aligned}$$

which may also be expressed as $v = 0.8660254c$.

(b) The Lorentz factor in this case is $\gamma = \frac{1}{\sqrt{1-(v/c)^2}} = 2$ "exactly."

11. (a) The rest length $L_0 = 130\,\mathrm{m}$ of the spaceship and its length $L$ as measured by the timing station are related by Eq. 38-13. Therefore, $L = (130\,\mathrm{m})\sqrt{1-(0.740)^2} = 87.4\,\mathrm{m}$.

(b) The time interval for the passage of the spaceship is

$$\Delta t = \frac{L}{v} = \frac{87.4\,\mathrm{m}}{(0.740)(3.00\times10^8\,\mathrm{m/s})} = 3.94\times10^{-7}\ \mathrm{s}\ .$$

12. (a) According solely to the principles of Special Relativity, yes. If the person moves fast enough, then the time dilation argument will allow for his proper travel time to be much less than that measured from the Earth. Stated differently, length contraction can make that travel distance seem much shorter to the traveler than to our Earth-based estimations. This does not include important considerations such as fuel requirements, stresses to the human body (due to the accelerations, primarily), and so on.

(b) Let $d = 23000\,\mathrm{ly} = 23000\,c\,\mathrm{y}$, which would give the distance in meters if we included a conversion factor for years $\rightarrow$ seconds. With $\Delta t_0 = 30\,\mathrm{y}$ and $\Delta t = d/v$ (see Eq. 38-10), we wish to solve for $v$ from Eq. 38-7. Our first step is as follows:

$$\begin{aligned} \Delta t &= \frac{\Delta t_0}{\sqrt{1-(v/c)^2}} \\ \frac{d}{v} &= \frac{\Delta t_0}{\sqrt{1-(v/c)^2}} \\ \frac{23000\,c\,\mathrm{y}}{v} &= \frac{30\,\mathrm{y}}{\sqrt{1-(v/c)^2}}\ , \end{aligned}$$

at which point we can cancel the unit year and manipulate the equation to solve for the speed. After a couple of algebraic steps, we obtain

$$\begin{aligned} v &= \frac{c}{\sqrt{1+\left(\frac{30}{23000}\right)^2}} \\ &= \frac{299792458\,\text{m/s}}{\sqrt{1+0.000017013}} \\ &= 299792203\ \text{m/s} \end{aligned}$$

which may also be expressed as $v = 0.999\,999\,15c$. The discussion in Sample Problem 38-7 dealing with these sorts of values may prove helpful for those whose calculators do not yield this answer.

13. (a) The speed of the traveler is $v = 0.99c$, which may be equivalently expressed as $0.99\,\text{ly/y}$. Let $d$ be the distance traveled. Then, the time for the trip as measured in the frame of Earth is $\Delta t = d/v = (26\,\text{ly})/(0.99\,\text{ly/y}) = 26.3\,\text{y}$.

(b) The signal, presumed to be a radio wave, travels with speed $c$ and so takes $26.0\,\text{y}$ to reach Earth. The total time elapsed, in the frame of Earth, is $26.3\,\text{y} + 26.0\,\text{y} = 52.3\,\text{y}$.

(c) The proper time interval is measured by a clock in the spaceship, so $\Delta t_0 = \Delta t/\gamma$. Now $\gamma = 1/\sqrt{1-\beta^2} = 1/\sqrt{1-(0.99)^2} = 7.09$. Thus, $\Delta t_0 = (26.3\,\text{y})/(7.09) = 3.7\,\text{y}$.

14. The "coincidence" of $x = x' = 0$ at $t = t' = 0$ is important for Eq. 38-20 to apply without additional terms. In part (a), we apply these equations directly with $v = +0.400c = 1.199 \times 10^8\,\text{m/s}$, and in part (b) we simply change $v \to -v$ and recalculate the primed values.

(a) The position coordinate measured in the $S'$ frame is

$$\begin{aligned} x' &= \gamma(x - vt) = \frac{x - vt}{\sqrt{1-\beta^2}} \\ &= \frac{3.00 \times 10^8\,\text{m} - (1.199 \times 10^8\,\text{m/s})(2.50\,\text{s})}{\sqrt{1-(0.400)^2}} \\ &= 2.7 \times 10^5\ \text{m/s} \quad \approx \quad 0\ , \end{aligned}$$

where we conclude that the numerical result ($2.7 \times 10^5$ or $2.3 \times 10^5$ depending on how precise a value of $v$ is used) is not meaningful (in the significant figures sense) and should be set equal to zero (that is, it is "consistent with zero" in view of the statistical uncertainties involved). The time coordinate measured in the $S'$ frame is

$$\begin{aligned} t' &= \gamma\left(t - \frac{vx}{c^2}\right) = \frac{t - \frac{\beta x}{c}}{\sqrt{1-\beta^2}} \\ &= \frac{2.50\,\text{s} - \frac{(0.400)(3.00\times 10^3\,\text{m})}{2.998\times 10^8\,\text{m/s}}}{\sqrt{1-(0.400)^2}} \\ &= 2.29\ \text{s}\ . \end{aligned}$$

(b) Now, we obtain

$$x' = \frac{x + vt}{\sqrt{1-\beta^2}} = \frac{3.00 \times 10^8\,\text{m} + (1.199 \times 10^8\,\text{m/s})(2.50\,\text{s})}{\sqrt{1-(0.400)^2}} = 6.54 \times 10^8\ \text{m}\ ,$$

and

$$t' = \gamma\left(t + \frac{vx}{c^2}\right) = \frac{2.50\,\text{s} + \frac{(0.400)(3.00\times 10^8\,\text{m})}{2.998\times 10^8\,\text{m/s}}}{\sqrt{1-(0.400)^2}} = 3.16\ \text{s}\ .$$

15. The proper time is not measured by clocks in either frame $S$ or frame $S'$ since a single clock at rest in either frame cannot be present at the origin and at the event. The full Lorentz transformation must be used:

$$x' = \gamma(x - vt) \qquad \text{and} \qquad t' = \gamma(t - \beta x/c)$$

where $\beta = v/c = 0.950$ and $\gamma = 1/\sqrt{1-\beta^2} = 1/\sqrt{1-(0.950)^2} = 3.20256$. Thus,

$$\begin{aligned} x' &= (3.20256)\left(100 \times 10^3\,\text{m} - (0.950)(2.998 \times 10^8\,\text{m/s})(200 \times 10^{-6}\,\text{s})\right) \\ &= 1.38 \times 10^5\,\text{m} = 138\ \text{km} \end{aligned}$$

and

$$t' = (3.20256)\left[200 \times 10^{-6}\,\text{s} - \frac{(0.950)(100 \times 10^3\,\text{m})}{2.998 \times 10^8\,\text{m/s}}\right] = -3.74 \times 10^{-4}\,\text{s} = -374\,\mu\text{s}\ .$$

16. The "coincidence" of $x = x' = 0$ at $t = t' = 0$ is important for Eq. 38-20 to apply without additional terms. We label the event coordinates with subscripts: $(x_1, t_1) = (0, 0)$ and $(x_2, t_2) = (3000, 4.0 \times 10^{-6})$ with SI units understood. Of course, we expect $(x'_1, t'_1) = (0, 0)$, and this may be verified using Eq. 38-20. We now compute $(x'_2, t'_2)$, assuming $v = +0.60c = +1.799 \times 10^8$ m/s (the sign of $v$ is not made clear in the problem statement, but the Figure referred to, Fig. 38-9, shows the motion in the positive $x$ direction).

$$\begin{aligned} x'_2 &= \frac{x - vt}{\sqrt{1-\beta^2}} = \frac{3000 - \left(1.799 \times 10^8\right)\left(4.0 \times 10^{-6}\right)}{\sqrt{1-(0.60)^2}} = 2.85 \times 10^3 \\ t'_2 &= \frac{t - \beta x/c}{\sqrt{1-\beta^2}} = \frac{4.0 \times 10^{-6} - (0.60)(3000)/(2.998 \times 10^8)}{\sqrt{1-(0.60)^2}} = -2.5 \times 10^{-6} \end{aligned}$$

The two events in frame $S$ occur in the order: first 1, then 2. However, in frame $S'$ where $t'_2 < 0$, they occur in the reverse order: first 2, then 1. We note that the distances $x_2 - x_1$ and $x'_2 - x'_1$ are larger than how far light can travel during the respective times ($c(t_2 - t_1) = 1.2$ km and $c|t'_2 - t'_1| \approx 750$ m), so that no inconsistencies arise as a result of the order reversal (that is, no signal from event 1 could arrive at event 2 or vice versa).

17. (a) We take the flashbulbs to be at rest in frame $S$, and let frame $S'$ be the rest frame of the second observer. Clocks in neither frame measure the proper time interval between the flashes, so the full Lorentz transformation (Eq. 38-20) must be used. Let $t_s$ be the time and $x_s$ be the coordinate of the small flash, as measured in frame $S$. Then, the time of the small flash, as measured in frame $S'$, is

$$t'_s = \gamma\left(t_s - \frac{\beta x_s}{c}\right)$$

where $\beta = v/c = 0.250$ and $\gamma = 1/\sqrt{1-\beta^2} = 1/\sqrt{1-(0.250)^2} = 1.0328$. Similarly, let $t_b$ be the time and $x_b$ be the coordinate of the big flash, as measured in frame $S$. Then, the time of the big flash, as measured in frame $S'$, is

$$t'_b = \gamma\left(t_b - \frac{\beta x_b}{c}\right)\ .$$

Subtracting the second Lorentz transformation equation from the first and recognizing that $t_s = t_b$ (since the flashes are simultaneous in $S$), we find

$$\Delta t' = -\frac{\gamma\beta(x_s - x_b)}{c} = -\frac{(1.0328)(0.250)(30 \times 10^3\,\text{m})}{3.00 \times 10^8\,\text{m/s}} = -2.58 \times 10^{-5}\ \text{s}$$

where $\Delta t' = t'_s - t'_b$.

(b) Since $\Delta t'$ is negative, $t'_b$ is greater than $t'_s$. The small flash occurs first in $S'$.

18. (a) In frame $S$, our coordinates are such that $x_1 = +1200$ m for the big flash, and $x_2 = 1200 - 720 = 480\,\text{m}$ for the small flash (which occurred later). Thus, $\Delta x = x_2 - x_1 = -720\,\text{m}$. If we set $\Delta x' = 0$ in Eq. 38-24, we find

$$0 = \gamma\,(\Delta x - v\Delta t) = \gamma\left(-720\,\text{m} - v(5.00 \times 10^{-6}\,\text{s})\right)$$

which yields $v = -1.44 \times 10^8$ m/s. Therefore, frame $S'$ must be moving in the $-x$ direction with a speed of $0.480c$.

(b) Eq. 38-27 leads to

$$\Delta t' = \gamma\left(\Delta t - \frac{v\Delta x}{c^2}\right) = \gamma\left(5.00 \times 10^{-6}\,\text{s} - \frac{(-1.44 \times 10^8\,\text{m/s})(-720\,\text{m})}{(2.998 \times 10^8\,\text{m/s})^2}\right)$$

which turns out to be positive (regardless of the specific value of $\gamma$). Thus, the order of the flashes is the same in the $S'$ frame as it is in the $S$ frame (where $\Delta t$ is also positive). Thus, the big flash occurs first, and the small flash occurs later.

(c) Finishing the computation begun in part (b), we obtain

$$\Delta t' = \frac{5.00 \times 10^{-6}\,\text{s} - \frac{(-1.44\times10^8\,\text{m/s})(-720\,\text{m})}{(2.998\times10^8\,\text{m/s})^2}}{\sqrt{1 - 0.480^2}} = 4.39 \times 10^{-6}\ \text{s}\ .$$

19. (a) The Lorentz factor is

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-(0.600)^2}} = 1.25\ .$$

(b) In the unprimed frame, the time for the clock to travel from the origin to $x = 180\,\text{m}$ is

$$t = \frac{x}{v} = \frac{180\,\text{m}}{(0.600)(3.00 \times 10^8\,\text{m/s})} = 1.00 \times 10^{-6}\ \text{s}\ .$$

The proper time interval between the two events (at the origin and at $x = 180\,\text{m}$) is measured by the clock itself. The reading on the clock at the beginning of the interval is zero, so the reading at the end is

$$t' = \frac{t}{\gamma} = \frac{1.00 \times 10^{-6}\,\text{s}}{1.25} = 8.00 \times 10^{-7}\ \text{s}\ .$$

20. We refer to the solution of problem 18. We wish to adjust $\Delta t$ so that

$$\Delta x' = 0 = \gamma\,(-720\,\text{m} - v\Delta t)$$

in the limiting case of $|v| \to c$. Thus,

$$\Delta t = \frac{720\,\text{m}}{2.998 \times 10^8\,\text{m/s}} = 2.40 \times 10^{-6}\ \text{s}\ .$$

21. We assume $S'$ is moving in the $+x$ direction. With $u' = +0.40c$ and $v = +0.60c$, Eq. 38-28 yields

$$u = \frac{u' + v}{1 + u'v/c^2} = \frac{0.40c + 0.60c}{1 + (0.40c)(+0.60c)/c^2} = 0.81c\ .$$

22. (a) We use Eq. 38-28:

$$v = \frac{v' + u}{1 + uv'/c^2} = \frac{0.47c + 0.62c}{1 + (0.47)(0.62)} = 0.84c\ ,$$

in the direction of increasing $x$ (since $v > 0$). The classical theory predicts that $v = 0.47c + 0.62c = 1.1c > c$.

(b) Now $v' = -0.47c$ so

$$v = \frac{v' + u}{1 + uv'/c^2} = \frac{-0.47c + 0.62c}{1 + (-0.47)(0.62)} = 0.21c \ ,$$

again in the direction of increasing $x$. By contrast, the classical prediction is $v = 0.62c - 0.47c = 0.15c$.

23. (a) One thing Einstein's relativity has in common with the more familiar (Galilean) relativity is the reciprocity of relative velocity. If Joe sees Fred moving at 20 m/s eastward away from him (Joe), then Fred should see Joe moving at 20 m/s westward away from him (Fred). Similarly, if we see Galaxy A moving away from us at $0.35c$ then an observer in Galaxy A should see our galaxy move away from him at $0.35c$.

(b) We take the positive axis to be in the direction of motion of Galaxy A, as seen by us. Using the notation of Eq. 38-28, the problem indicates $v = +0.35c$ (velocity of Galaxy A relative to Earth) and $u = -0.35c$ (velocity of Galaxy B relative to Earth). We solve for the velocity of B relative to A:

$$u' = \frac{u - v}{1 - uv/c^2} = \frac{(-0.35c) - 0.35c}{1 - (-0.35)(0.35)} = -0.62c$$

or $u' = -1.87 \times 10^8$ m/s.

24. Using the notation of Eq. 38-28 and taking "away" (from us) as the positive direction, the problem indicates $v = +0.4c$ and $u = +0.8c$ (with 3 significant figures understood). We solve for the velocity of $Q_2$ relative to $Q_1$:

$$u' = \frac{u - v}{1 - uv/c^2} = \frac{0.8c - 0.4c}{1 - (0.8)(0.4)} = 0.588c$$

or $u' = 1.76 \times 10^8$ m/s in a direction away from Earth.

25. Using the notation of Eq. 38-28 and taking the micrometeorite motion as the positive direction, the problem indicates $v = -0.82c$ (spaceship velocity) and $u = +0.82c$ (micrometeorite velocity). We solve for the velocity of the micrometeorite relative to the spaceship:

$$u' = \frac{u - v}{1 - uv/c^2} = \frac{0.82c - (-0.82c)}{1 - (0.82)(-0.82)} = 0.98c$$

or $2.94 \times 10^8$ m/s. Using Eq. 38-10, we conclude that observers on the ship measure a transit time for the micrometeorite (as it passes along the length of the ship) equal to

$$\Delta t = \frac{d}{u'} = \frac{350\,\text{m}}{2.94 \times 10^8\,\text{m/s}} = 1.2 \times 10^{-6}\ \text{s} \ .$$

26. (a) In the messenger's rest system (called $S_m$), the velocity of the armada is

$$v' = \frac{v - v_m}{1 - vv_m/c^2} = \frac{0.80c - 0.95c}{1 - (0.80c)(0.95c)/c^2} = -0.625c \ .$$

The length of the armada as measured in $S_m$ is

$$L_1 = \frac{L_0}{\gamma_{v'}} = (1.0\,\text{ly})\sqrt{1 - (-0.625)^2} = 0.781\ \text{ly} \ .$$

Thus, the length of the trip is

$$t' = \frac{L'}{|v'|} = \frac{0.781\,\text{ly}}{0.625c} = 1.25\ \text{y} \ .$$

(b) In the armada's rest frame (called $S_a$), the velocity of the messenger is

$$v' = \frac{v - v_a}{1 - vv_a/c^2} = \frac{0.95c - 0.80c}{1 - (0.95c)(0.80c)/c^2} = 0.625c \ .$$

Now, the length of the trip is

$$t' = \frac{L_0}{v'} = \frac{1.0\,\text{ly}}{0.625c} = 1.6 \text{ y} \ .$$

(c) Measured in system $S$, the length of the armada is

$$L = \frac{L_0}{\gamma} = 1.0\,\text{ly}\sqrt{1 - (0.80)^2} = 0.60 \text{ ly} \ ,$$

so the length of the trip is

$$t = \frac{L}{v_m - v_a} = \frac{0.60\,\text{ly}}{0.95c - 0.80c} = 4.0 \text{ y} \ .$$

27. The spaceship is moving away from Earth, so the frequency received is given directly by Eq. 38-30. Thus,

$$f = f_0\sqrt{\frac{1-\beta}{1+\beta}} = (100\,\text{MHz})\sqrt{\frac{1-0.9000}{1+0.9000}} = 22.9 \text{ MHz} \ .$$

28. (a) Eq. 38-33 leads to

$$v = \frac{\Delta\lambda}{\lambda}c = \frac{12\,\text{nm}}{513\,\text{nm}}\left(2.998 \times 10^8\,\text{m/s}\right) = 7.0 \times 10^6 \text{ m/s} \ .$$

(b) The line is shifted to a larger wavelength, which means shorter frequency. Recalling Eq. 38-30 and the discussion that follows it, this means galaxy NGC is moving away from Earth.

29. Eq. 38-33 leads to a recessional speed of

$$v = \frac{\Delta\lambda}{\lambda}c = (0.004)\left(3.0 \times 10^8\,\text{m/s}\right) = 1 \times 10^6 \text{ m/s} \ .$$

30. We obtain

$$v = \frac{\Delta\lambda}{\lambda}c = \left(\frac{620 - 540}{620}\right)c = 0.13c = 3.9 \times 10^6 \text{ m/s} \ .$$

31. The frequency received is given by

$$\begin{aligned} f &= f_0\sqrt{\frac{1-\beta}{1+\beta}} \\ \frac{c}{\lambda} &= \frac{c}{\lambda_0}\sqrt{\frac{1-0.20}{1+0.20}} \end{aligned}$$

which implies

$$\lambda = (450\,\text{nm})\sqrt{\frac{1+0.20}{1-0.20}} = 550 \text{ nm} \ .$$

This is in the yellow-green portion of the visible spectrum.

32. (a) The work-kinetic energy theorem applies as well to Einsteinian physics as to Newtonian; the only difference is the specific formula for kinetic energy. Thus, we use $W = \Delta K = m_e c^2(\gamma - 1)$ (Eq. 38-49) and $m_e c^2 = 511\,\text{keV} = 0.511\,\text{MeV}$ (Table 38-3), and obtain

$$W = m_e c^2\left(\frac{1}{\sqrt{1-\beta^2}} - 1\right) = (511\,\text{keV})\left[\frac{1}{\sqrt{1-(0.50)^2}} - 1\right] = 79 \text{ keV} \ .$$

(b)

$$W = (0.511\,\text{MeV})\left(\frac{1}{\sqrt{1-(0.990)^2}} - 1\right) = 3.11\text{ MeV} .$$

(c)

$$W = (0.511\,\text{MeV})\left(\frac{1}{\sqrt{1-(0.9990)^2}} - 1\right) = 10.9\text{ MeV} .$$

33. (a) Using $K = m_e c^2(\gamma - 1)$ (Eq. 38-49) and $m_e c^2 = 511\,\text{keV} = 0.511\,\text{MeV}$ (Table 38-3), we obtain

$$\gamma = \frac{K}{m_e c^2} + 1 = \frac{1.00\,\text{keV}}{511\,\text{keV}} + 1 = 1.00196 .$$

Therefore, the speed parameter is

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}} = \sqrt{1 - \frac{1}{1.00196^2}} = 0.0625 .$$

(b) We could first find $\beta$ and then find $\gamma$, as illustrated here: With $K = 1.00\,\text{MeV}$, we find

$$\beta = \sqrt{1 - \left(\frac{1.00\,\text{MeV}}{0.511\,\text{MeV}} + 1\right)^{-2}} = 0.941$$

and $\gamma = 1/\sqrt{1-\beta^2} = 2.96$.

(c) Finally, $K = 1000\,\text{MeV}$, so

$$\beta = \sqrt{1 - \left(\frac{1000\,\text{MeV}}{0.511\,\text{MeV}} + 1\right)^{-2}} = 0.999\,999\,87$$

and $\gamma = 1000\,\text{MeV}/0.511\,\text{MeV} + 1 = 1.96 \times 10^3$. The discussion in Sample Problem 38-7 dealing with these sorts of values may prove helpful for those whose calculators do not yield these answers.

34. From Eq. 38-49, $\gamma = (K/mc^2) + 1$, and from Eq. 38-8, the speed parameter is $\beta = \sqrt{1-(1/\gamma)^2}$.

(a) Table 38-3 gives $m_e c^2 = 511\,\text{keV} = 0.511\,\text{MeV}$, so the Lorentz factor is

$$\gamma = \frac{10.0\,\text{MeV}}{0.511\,\text{MeV}} + 1 = 20.57 ,$$

and the speed parameter is

$$\beta = \sqrt{1 - \frac{1}{(20.57)^2}} = 0.9988 .$$

(b) Table 38-3 gives $m_p c^2 = 938\,\text{MeV}$, so the Lorentz factor is $\gamma = 1 + 10.0\,\text{MeV}/938\,\text{MeV} = 1.01$, and the speed parameter is

$$\beta = \sqrt{1 - \frac{1}{1.01^2}} = 0.145 .$$

(c) If we refer to the data shown in problem 36, we find $m_\alpha = 4.0026$ u, which (using Eq. 38-43) implies $m_\alpha c^2 = 3728$ MeV. This leads to $\gamma = 10/3728 + 1 = 1.0027$. And, being careful not to do any unnecessary rounding off in the intermediate steps, we find $\beta = 0.073$. We remark that the mass value used in our solution is not exactly the alpha particle mass (it's the helium-4 atomic mass), but this slight difference does not introduce significant error in this computation.

35. From Eq. 38-49, $\gamma = (K/mc^2) + 1$, and from Eq. 38-8, the speed parameter is $\beta = \sqrt{1 - (1/\gamma)^2}$. Table 38-3 gives $m_e c^2 = 511\,\text{keV} = 0.511\,\text{MeV}$, so the Lorentz factor is

$$\gamma = \frac{100\,\text{MeV}}{0.511\,\text{MeV}} + 1 = 197 ,$$

and the speed parameter is

$$\beta = \sqrt{1 - \frac{1}{(197)^2}} = 0.999987 .$$

Thus, the speed of the electron is $0.999987c$, or 99.9987% of the speed of light. The discussion in Sample Problem 38-7 dealing with these sorts of values may prove helpful for those whose calculators do not yield this answer.

36. The mass change is

$$\Delta M = (4.002603\,\text{u} + 15.994915\,\text{u}) - (1.007825\,\text{u} + 18.998405\,\text{u}) = -0.008712\ \text{u} .$$

Using Eq. 38-47 and Eq. 38-43, this leads to

$$Q = -\Delta M\,c^2 = -(-0.008712\,\text{u})(931.5\,\text{MeV/u}) = 8.12\ \text{MeV} .$$

37. Since the rest energy $E_0$ and the mass $m$ of the quasar are related by $E_0 = mc^2$, the rate $P$ of energy radiation and the rate of mass loss are related by $P = dE_0/dt = (dm/dt)c^2$. Thus,

$$\frac{dm}{dt} = \frac{P}{c^2} = \frac{1 \times 10^{41}\,\text{W}}{(2.998 \times 10^8\,\text{m/s})^2} = 1.11 \times 10^{24}\ \text{kg/s} .$$

Since a solar mass is $2.0 \times 10^{30}\,\text{kg}$ and a year is $3.156 \times 10^7\,\text{s}$,

$$\frac{dm}{dt} = (1.11 \times 10^{24}\,\text{kg/s}) \left( \frac{3.156 \times 10^7\,\text{s/y}}{2.0 \times 10^{30}\,\text{kg/smu}} \right) \approx 18\ \text{smu/y} .$$

38. (a) The work-kinetic energy theorem applies as well to Einsteinian physics as to Newtonian; the only difference is the specific formula for kinetic energy. Thus, we use $W = \Delta K$ where $K = m_e c^2(\gamma - 1)$ (Eq. 38-49), and $m_e c^2 = 511\,\text{keV} = 0.511\,\text{MeV}$ (Table 38-3). Noting that $\Delta K = m_e c^2(\gamma_f - \gamma_i)$, we obtain

$$W = m_e c^2 \left( \frac{1}{\sqrt{1 - \beta_f^2}} - \frac{1}{\sqrt{1 - \beta_i^2}} \right) = (511\,\text{keV}) \left( \frac{1}{\sqrt{1 - (0.19)^2}} - \frac{1}{\sqrt{1 - (0.18)^2}} \right) = 0.996\ \text{keV} .$$

(b) Similarly,

$$W = (511\,\text{keV}) \left( \frac{1}{\sqrt{1 - (0.99)^2}} - \frac{1}{\sqrt{1 - (0.98)^2}} \right) = 1055\ \text{keV} .$$

We see the dramatic increase in difficulty in trying to accelerate a particle when its initial speed is very close to the speed of light.

39. (a) We set Eq. 38-38 equal to $mc$, as required by the problem, and solve for the speed. Thus,

$$\frac{mv}{\sqrt{1 - v^2/c^2}} = mc$$

leads to $v = c/\sqrt{2} = 0.707c$.

(b) Substituting $v = \sqrt{2}c$ into the definition of $\gamma$, we obtain

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{1}{\sqrt{1 - (1/2)}} = \sqrt{2} \approx 1.41 \; .$$

(c) The kinetic energy is

$$K = (\gamma - 1)mc^2 = (\sqrt{2} - 1)mc^2 = 0.414mc^2 \; .$$

40. (a) We set Eq. 38-49 equal to $2mc^2$, as required by the problem, and solve for the speed. Thus,

$$mc^2 \left( \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} - 1 \right) = 2mc^2$$

leads to $v = \frac{2\sqrt{2}}{3}\,c \approx 0.943c$.

(b) We now set Eq. 38-45 equal to $2mc^2$ and solve for the speed. In this case,

$$\frac{mc^2}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} = 2mc^2$$

leads to $v = \frac{\sqrt{3}}{2}\,c \approx 0.866c$.

41. We set Eq. 38-52 equal to $(3mc^2)^2$, as required by the problem, and solve for the speed. Thus,

$$(pc)^2 + (mc^2)^2 = 9(mc^2)^2$$

leads to $p = mc\sqrt{8}$.

42. (a) Squaring Eq. 38-44 gives

$$E^2 = \left(mc^2\right)^2 + 2mc^2K + K^2$$

which we set equal to Eq. 38-52. Thus,

$$\left(mc^2\right)^2 + 2mc^2K + K^2 = (pc)^2 + \left(mc^2\right)^2 \implies m = \frac{(pc)^2 - K^2}{2Kc^2} \; .$$

(b) At low speeds, the pre-Einsteinian expressions $p = mv$ and $K = \frac{1}{2}mv^2$ apply. We note that $pc \gg K$ at low speeds since $c \gg v$ in this regime. Thus,

$$m \rightarrow \frac{(mvc)^2 - \left(\frac{1}{2}mv^2\right)^2}{2\left(\frac{1}{2}mv^2\right)c^2} \approx \frac{(mvc)^2}{2\left(\frac{1}{2}mv^2\right)c^2} = m \; .$$

(c) Here, $pc = 121\,\text{MeV}$, so

$$m = \frac{121^2 - 55^2}{2(55)c^2} = 105.6 \text{ MeV/c}^2 \; .$$

Now, the mass of the electron (see Table 38-3) is $m_e = 0.511\,\text{MeV/c}^2$, so our result is roughly 207 times bigger than an electron mass.

43. The energy equivalent of one tablet is $mc^2 = (320 \times 10^{-6}\,\text{kg})(3.00 \times 10^8\,\text{m/s})^2 = 2.88 \times 10^{13}\,\text{J}$. This provides the same energy as $(2.88 \times 10^{13}\,\text{J})/(3.65 \times 10^7\,\text{J/L}) = 7.89 \times 10^5\,\text{L}$ of gasoline. The distance the car can go is $d = (7.89 \times 10^5\,\text{L})(12.75\,\text{km/L}) = 1.01 \times 10^7\,\text{km}$. This is roughly 250 times larger than the circumference of Earth (see Appendix C).

44. (a) The proper lifetime $\Delta t_0$ is $2.20\,\mu$s, and the lifetime measured by clocks in the laboratory (through which the muon is moving at high speed) is $\Delta t = 6.90\,\mu$s. We use Eq. 38-7 to solve for the speed:

$$v = c\sqrt{1 - \left(\frac{\Delta t_0}{\Delta t}\right)^2} = 0.9478c$$

or $v = 2.84 \times 10^8$ m/s.

(b) From the answer to part (a), we find $\gamma = 3.136$. Thus, with $m_\mu c^2 = 207 m_e c^2 = 105.8\,\text{MeV}$ (see Table 38-3), Eq. 38-49 yields

$$K = m_\mu c^2(\gamma - 1) = 226 \text{ MeV} .$$

(c) We write $m_\mu c = 105.8\,\text{MeV}/c$ and apply Eq. 38-38:

$$p = \gamma m_\mu v = \gamma m_\mu c\beta = (3.136)(105.8\,\text{MeV}/c)(0.9478) = 314 \text{ MeV}/c$$

which can also be expressed in SI units ($p = 1.7 \times 10^{-19}\,\text{kg}\cdot\text{m/s}$).

45. The distance traveled by the pion in the frame of Earth is (using Eq. 38-12) $d = v\,\Delta t$. The proper lifetime $\Delta t_0$ is related to $\Delta t$ by the time-dilation formula: $\Delta t = \gamma \Delta t_0$. To use this equation, we must first find the Lorentz factor $\gamma$ (using Eq. 38-45). Since the total energy of the pion is given by $E = 1.35 \times 10^5$ MeV and its $mc^2$ value is 139.6 MeV, then

$$\gamma = \frac{E}{mc^2} = \frac{1.35 \times 10^5\,\text{MeV}}{139.6\,\text{MeV}} = 967.05 .$$

Therefore, the lifetime of the moving pion as measured by Earth observers is

$$\Delta t = \gamma \Delta t_0 = (967.1)(35.0 \times 10^{-9}\,\text{s}) = 3.385 \times 10^{-5} \text{ s} ,$$

and the distance it travels is

$$d \approx c\Delta t = (2.998 \times 10^8\,\text{m/s})(3.385 \times 10^{-5}\,\text{s}) = 1.015 \times 10^4\,\text{m} = 10.15 \text{ km}$$

where we have approximated its speed as $c$ (note: its speed can be found by solving Eq. 38-8, which gives $v = 0.9999995c$; this more precise value for $v$ would not significantly alter our final result). Thus, the altitude at which the pion decays is $120\,\text{km} - 10.15\,\text{km} = 110\,\text{km}$.

46. The $q$ in the denominator is to be interpreted as $|q|$ (so that the orbital radius $r$ is a positive number). We interpret the given 10.0 MeV to be the kinetic energy of the electron. In order to make use of the $mc^2$ value for the electron given in Table 38-3 (511 keV = 0.511 MeV) we write the classical kinetic energy formula as

$$K_{\text{classical}} = \frac{1}{2}mv^2 = \frac{1}{2}\left(mc^2\right)\left(\frac{v^2}{c^2}\right) = \frac{1}{2}\left(mc^2\right)\beta^2 .$$

(a) If $K_{\text{classical}} = 10.0\,\text{MeV}$, then

$$\beta = \sqrt{\frac{2K_{\text{classical}}}{mc^2}} = \sqrt{\frac{2(10.0\,\text{MeV})}{0.511\,\text{MeV}}} = 6.256 ,$$

which, of course, is impossible (see the Ultimate Speed subsection of §38-2). If we use this value anyway, then the classical orbital radius formula yields

$$\begin{aligned} r &= \frac{mv}{|q|B} = \frac{m\beta c}{eB} \\ &= \frac{\left(9.11 \times 10^{-31}\,\text{kg}\right)(6.256)\left(2.998 \times 10^8\,\text{m/s}\right)}{(1.6 \times 10^{-19}\,\text{C})\,(2.20\,\text{T})} \\ &= 4.85 \times 10^{-3} \text{ m} . \end{aligned}$$

If, however, we use the correct value for $\beta$ (calculated in the next part) then the classical radius formula would give about 0.77 mm.

(b) Before using the relativistically correct orbital radius formula, we must compute $\beta$ in a relativistically correct way:

$$K = mc^2(\gamma - 1) \implies \gamma = \frac{10.0\,\text{MeV}}{0.511\,\text{MeV}} + 1 = 20.57$$

which implies (from Eq. 38-8)

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}} = 0.99882 \ .$$

Therefore,

$$\begin{aligned} r &= \frac{\gamma m v}{|q|B} = \frac{\gamma m \beta c}{eB} \\ &= \frac{(20.57)\left(9.11 \times 10^{-31}\,\text{kg}\right)(0.99882)\left(2.998 \times 10^{8}\,\text{m/s}\right)}{(1.6 \times 10^{-19}\,\text{C})\,(2.20\,\text{T})} \\ &= 1.59 \times 10^{-2}\ \text{m} \ . \end{aligned}$$

(c) The period is

$$T = \frac{2\pi r}{\beta c} = \frac{2\pi(0.0159\,\text{m})}{(0.99882)\,(2.998 \times 10^{8}\,\text{m/s})} = 3.34 \times 10^{-10}\ \text{s} \ .$$

Whereas the purely classical result gives a period which is independent of speed, this is no longer true in the relativistic case (due to the $\gamma$ factor in the equation).

47. The radius $r$ of the path is given in problem 46 as $r = \gamma mvqB$. Thus,

$$\begin{aligned} m &= \frac{qBr\sqrt{1-\beta^2}}{v} \\ &= \frac{2(1.60 \times 10^{-19}\,\text{C})(1.00\,\text{T})(6.28\,\text{m})\sqrt{1-(0.710)^2}}{(0.710)(3.00 \times 10^{8}\,\text{m/s})} \\ &= 6.64 \times 10^{-27}\ \text{kg} \ . \end{aligned}$$

Since $1.00\,\text{u} = 1.66 \times 10^{-27}\,\text{kg}$, the mass is $m = 4.00\,\text{u}$. The nuclear particle contains four nucleons. Since there must be two protons to provide the charge $2e$, the nuclear particle is a helium nucleus (usually referred to as an alpha particle) with two protons and two neutrons.

48. We interpret the given $10\,\text{GeV} = 10000\,\text{MeV}$ to be the kinetic energy of the proton. Using Table 38-3 and Eq. 38-49, we find

$$\gamma = \frac{K}{m_p c^2} + 1 = \frac{10000\,\text{MeV}}{938\,\text{MeV}} + 1 = 11.66 \ ,$$

and (from Eq. 38-8)

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}} = 0.9963 \ .$$

Therefore, using the equation introduced in problem 46, we obtain

$$\begin{aligned} r &= \frac{\gamma m v}{qB} = \frac{\gamma m_p \beta c}{eB} \\ &= \frac{(11.66)\left(1.67 \times 10^{-27}\,\text{kg}\right)(0.9963)\left(2.998 \times 10^{8}\,\text{m/s}\right)}{(1.6 \times 10^{-19}\,\text{C})\,(55 \times 10^{-6}\,\text{T})} \\ &= 6.6 \times 10^{5}\ \text{m} \ . \end{aligned}$$

49. We interpret the given $2.50\,\text{MeV} = 2500\,\text{keV}$ to be the kinetic energy of the electron. Using Table 38-3 and Eq. 38-49, we find

$$\gamma = \frac{K}{m_e c^2} + 1 = \frac{2500\,\text{keV}}{511\,\text{keV}} + 1 = 5.892 \ ,$$

and (from Eq. 38-8)

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}} = 0.9855 \ .$$

Therefore, using the equation introduced in problem 46 (with "$q$" interpreted as $|q|$), we obtain

$$\begin{aligned} B &= \frac{\gamma\, m_e\, v}{|q|\, r} = \frac{\gamma m_e \beta c}{er} \\ &= \frac{(5.892)\left(9.11\times 10^{-31}\,\text{kg}\right)(0.9855)\left(2.998\times 10^{8}\,\text{m/s}\right)}{(1.6\times 10^{-19}\,\text{C})\,(0.030\,\text{m})} \\ &= 0.33\ \text{T} \ . \end{aligned}$$

50. (a) Using Table 38-3 and Eq. 38-49 (or, to be more precise, the value given at the end of the problem statement), we find

$$\gamma = \frac{K}{m_p c^2} + 1 = \frac{500\times 10^3\,\text{MeV}}{938.3\,\text{MeV}} + 1 = 533.88 \ .$$

(b) From Eq. 38-8, we obtain

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}} = 0.99999825 \ .$$

The discussion in Sample Problem 38-7 dealing with large $\gamma$ values may prove helpful for those whose calculators do not yield this answer.

(c) To make use of the precise $m_p c^2$ value given here, we rewrite the expression introduced in problem 46 (as applied to the proton) as follows:

$$r = \frac{\gamma m v}{qB} = \frac{\gamma\left(mc^2\right)\left(\frac{v}{c^2}\right)}{eB} = \frac{\gamma\left(mc^2\right)\beta}{ecB} \ .$$

Therefore, the magnitude of the magnetic field is

$$\begin{aligned} B &= \frac{\gamma\left(mc^2\right)\beta}{ecr} \\ &= \frac{(533.88)(938.3\,\text{MeV})(0.99999825)}{ec(750\,\text{m})} \\ &= \frac{667.92\times 10^6\,\text{V/m}}{c} \end{aligned}$$

where we note the cancellation of the "e" in MeV with the $e$ in the denominator. After substituting $c = 2.998\times 10^8$ m/s, we obtain $B = 2.23$ T.

51. (a) Before looking at our solution to part (a) (which uses momentum conservation), it might be advisable to look at our solution (and accompanying remarks) for part (b) (where a very different approach is used). Since momentum is a vector, its conservation involves two equations (along the original direction of alpha particle motion, the $x$ direction, as well as along the final proton direction of motion, the $y$ direction). The problem states that all speeds are much less than the speed of light, which allows us to use the classical formulas for kinetic energy and momentum ($K = \frac{1}{2}mv^2$ and $\vec{p} = m\vec{v}$, respectively). Along the $x$ and $y$ axes, momentum conservation gives (for the components of $\vec{v}_{\text{oxy}}$):

$$\begin{aligned} m_\alpha v_\alpha &= m_{\text{oxy}} v_{\text{oxy},x} \implies v_{\text{oxy},x} = \frac{m_\alpha}{m_{\text{oxy}}}\, v_\alpha \approx \frac{4}{17}\, v_\alpha \\ 0 &= m_{\text{oxy}} v_{\text{oxy},y} + m_p v_p \implies v_{\text{oxy},y} = -\frac{m_p}{m_{\text{oxy}}}\, v_p \approx -\frac{1}{17}\, v_p \ . \end{aligned}$$

To complete these determinations, we need values (inferred from the kinetic energies given in the problem) for the initial speed of the alpha particle ($v_\alpha$) and the final speed of the proton ($v_p$). One way to do this is to rewrite the classical kinetic energy expression as $K = \frac{1}{2}(mc^2)\beta^2$ and solve for $\beta$ (using Table 38-3 and/or Eq. 38-43). Thus, for the proton, we obtain

$$\beta_p = \sqrt{\frac{2K_p}{m_p c^2}} = \sqrt{\frac{2(4.44\,\text{MeV})}{938\,\text{MeV}}} = 0.0973 \ .$$

This is almost 10% the speed of light, so one might worry that the relativistic expression (Eq. 38-49) should be used. If one does so, one finds $\beta_p = 0.969$, which is reasonably close to our previous result based on the classical formula. For the alpha particle, we write $m_\alpha c^2 = (4.0026\,\text{u})(931.5\,\text{MeV/u}) =$ 3728 MeV (which is actually an overestimate due to the use of the "atomic mass" value in our calculation, but this does not cause significant error in our result), and obtain

$$\beta_\alpha = \sqrt{\frac{2K_\alpha}{m_\alpha c^2}} = \sqrt{\frac{2(7.70\,\text{MeV})}{3728\,\text{MeV}}} = 0.064 \ .$$

Returning to our oxygen nucleus velocity components, we are now able to conclude:

$$\begin{aligned} v_{\text{oxy},x} &\approx \frac{4}{17}\, v_\alpha & \implies \quad & \beta_{\text{oxy},x} \approx \frac{4}{17}\,\beta_\alpha = \frac{4}{17}(0.064) = 0.015 \\ |v_{\text{oxy},y}| &\approx \frac{1}{17}\, v_p & \implies \quad & \beta_{\text{oxy},y} \approx \frac{1}{17}\,\beta_p = \frac{1}{17}(0.097) = 0.0057 \end{aligned}$$

Consequently, with $m_{\text{oxy}}c^2 \approx (17\,\text{u})(931.5\,\text{MeV/u}) = 1.58 \times 10^4$ MeV, we obtain

$$K_{\text{oxy}} = \frac{1}{2}\left(m_{\text{oxy}}c^2\right)\left(\beta^2_{\text{oxy},x} + \beta^2_{\text{oxy},y}\right) = \frac{1}{2}\left(1.58 \times 10^4\,\text{MeV}\right)\left(0.015^2 + 0.0057^2\right) \approx 2.0 \text{ MeV} \ .$$

(b) Using Eq. 38-47 and Eq. 38-43,

$$Q = -(1.007825\,\text{u} + 16.99914\,\text{u} - 4.00260\,\text{u} - 14.00307\,\text{u})c^2 = -(0.001295\,\text{u})(931.5\,\text{MeV/u})$$

which yields $Q = -1.206$ MeV. Incidentally, this provides an alternate way to obtain the answer (and a more accurate one at that!) to part (a). Eq. 38-46 leads to

$$K_{\text{oxy}} = K_\alpha + Q - K_p = 7.70\,\text{MeV} - 1.206\,\text{MeV} - 4.44\,\text{MeV} = 2.05 \text{ MeV} \ .$$

This approach to finding $K_{\text{oxy}}$ avoids the many computational steps and approximations made in part (a).

52. (a) From the length contraction equation, the length $L'_c$ of the car according to Garageman is

$$L'_c = \frac{L_c}{\gamma} = L_c\sqrt{1-\beta^2} = (30.5\,\text{m})\,\sqrt{1-(0.9980)^2} = 1.93 \text{ m} \ .$$

(b) Since the $x_g$ axis is fixed to the garage $x_{g2} = L_g = 6.00\,\text{m}$. As for $t_{g2}$, note from Fig. 38-21(b) that, at $t_g = t_{g1} = 0$ the coordinate of the front bumper of the limo in the $x_g$ frame is $L'_c$, meaning that the front of the limo is still a distance $L_g - L'_c$ from the back door of the garage. Since the limo travels at a speed $v$, the time it takes for the front of the limo to reach the back door of the garage is given by

$$\Delta t_g = t_{g2} - t_{g1} = \frac{L_g - L'_c}{v} = \frac{6.00\,\text{m} - 1.93\,\text{m}}{0.9980(2.998 \times 10^8\,\text{m/s})} = 1.36 \times 10^{-8} \text{ s} \ .$$

Thus $t_{g2} = t_{g1} + \Delta t_g = 0 + 1.36 \times 10^{-8}\,\text{s} = 1.36 \times 10^{-8}\,\text{s}$.

(c) The limo is inside the garage between times $t_{g1}$ and $t_{g2}$, so the time duration is $t_{g2} - t_{g1} = 1.36 \times 10^{-8}$ s.

(d) Again from Eq. 38-13, the length $L'_g$ of the garage according to Carman is

$$L'_g = \frac{L_g}{\gamma} = L_g\sqrt{1-\beta^2} = (6.00\,\text{m})\,\sqrt{1-(0.9980)^2} = 0.379 \text{ m} .$$

(e) Again, since the $x_c$ axis is fixed to the limo $x_{c2} = L_c = 30.5\,\text{m}$. Now, from the two diagrams described in part (h) below, we know that at $t_c = t_{c2}$ (when event 2 takes place), the distance between the rear bumper of the limo and the back door of the garage is given by $L_c - L'_g$ . Since the garage travels at a speed $v$, the front door of the garage will reach the rear bumper of the limo a time $\Delta t_c$ later, where $\Delta t_c$ satisfies

$$\Delta t_c = t_{c1} - t_{c2} = \frac{L_c - L'_g}{v} = \frac{30.5\,\text{m} - 0.379\,\text{m}}{0.9980(2.998 \times 10^8\,\text{m/s})} = 1.01 \times 10^{-7} \text{ s} .$$

Thus $t_{c2} = t_{c1} - \Delta t_c = 0 - 1.01 \times 10^{-7}\,\text{s} = -1.01 \times 10^{-7}\,\text{s}$.

(f) From Carman's point of view, the answer is clearly no.

(g) Event 2 occurs first according to Carman, since $t_{c2} < t_{c1}$.

(h) We describe the essential features of the two pictures. For event 2, the front of the limo coincides with the back door, and the garage itself seems very short (perhaps failing to reach as far as the front window of the limo). For event 1, the rear of the car coincides with the front door and the front of the limo has traveled a significant distance beyond the back door. In this picture, as in the other, the garage seems very short compared to the limo.

(i) Both Carman and Garageman are correct in their respective reference frames. But, in a sense, Carman should lose the bet since he dropped his physics course before reaching the Theory of Special Relativity!

53. (a) The spatial separation between the two bursts is $vt$. We project this length onto the direction perpendicular to the light rays headed to Earth and obtain $D_{\text{app}} = vt\sin\theta$.

(b) Burst 1 is emitted a time $t$ ahead of burst 2. Also, burst 1 has to travel an extra distance $L$ more than burst 2 before reaching the Earth, where $L = vt\cos\theta$ (see Fig. 38-22); this requires an additional time $t' = L/c$. Thus, the apparent time is given by

$$T_{\text{app}} = t - t' = t - \frac{vt\cos\theta}{c} = t\left[1 - \left(\frac{v}{c}\right)\cos\theta\right] .$$

(c) We obtain

$$V_{\text{app}} = \frac{D_{\text{app}}}{T_{\text{app}}} = \left[\frac{(v/c)\sin\theta}{1-(v/c)\cos\theta}\right]c = \left[\frac{(0.980)\sin 30.0^\circ}{1-(0.980)\cos 30.0^\circ}\right]c = 3.24\,c .$$

54. (a) The strategy is to find the $\gamma$ factor from $E = 14.24 \times 10^{-9}$ J and $m_pc^2 = 1.5033 \times 10^{-10}$ J and from that find the contracted length. From the energy relation (Eq. 38-45), we obtain

$$\gamma = \frac{E}{mc^2} = 94.73 .$$

Consequently, Eq. 38-13 yields

$$L = \frac{L_0}{\gamma} = 0.222\,\text{cm} = 2.22 \times 10^{-3} \text{ m} .$$

(b) and (c) From the $\gamma$ factor, we find the speed:

$$v = c\sqrt{1 - \left(\frac{1}{\gamma}\right)^2} = 0.99994c \; .$$

Therefore, the trip (according to the proton) took $\Delta t_0 = 2.22 \times 10^{-3}/0.99994c = 7.40 \times 10^{-12}$ s. Finally, the time dilation formula (Eq. 38-7) leads to

$$\Delta t = \gamma \Delta t_0 = 7.01 \times 10^{-10} \text{ s}$$

which can be checked using $\Delta t = L_0/v$ in our frame of reference.

55. Since it has two protons, its kinetic energy is 600 MeV. With the given value $mc^2 = 3727$ MeV, we use Eq. 38-37:

$$pc = \sqrt{K^2 + 2Kmc^2} = \sqrt{600^2 + 2(600)(3727)}$$

which yields $p = 2198$ MeV/$c$.

56. For the purposes of using Eq. 38-28, we choose our frame to be the primed frame and note that, as a consequence, $v = -0.800c\,\hat{\imath}$ for the velocity of us relative to Bullwinkle.

$$u = \frac{u' + v}{1 + u'v/c^2} = \frac{0.990c\hat{\imath} - 0.800c\hat{\imath}}{1 - (0.990)(0.800)} = 0.913c\,\hat{\imath} \; .$$

57. (a) We compute

$$\gamma = \frac{1}{\sqrt{1 - (0.9990)^2}} = 22.4$$

Now, the length contraction formula (Eq. 38-13) yields

$$L = \frac{2.50 \text{ m}}{\gamma} = 0.112 \text{ m} \; .$$

(b) (c) and (d) We assume our spacetime coordinate origins coincide and use the Lorentz transformations (Eq. 38-20, but with primes and non-primes swapped, and $v \to -v$). Lengths are in meters and time is in nanoseconds (so that $c = 0.2998$ in these units).

$$\begin{aligned}
x_\alpha &= \gamma\left(4.0 + (0.9990c)(40)\right) = 357 \\
t_\alpha &= \gamma\left(40 + (0.9990c)(4.0)/c^2\right) = 1193 \\
x_\beta &= \gamma\left(-4.0 + (0.9990c)(80)\right) = 446 \\
t_\beta &= \gamma\left(80 + (0.9990c)(-4.0)/c^2\right) = 1491
\end{aligned}$$

Thus, our reckoning of the distance between events is $x_\beta - x_\alpha = 89.0$ m. We note that event alpha took place first (smallest value of $t$) and that the time-separation is $t_\alpha - t_\beta = 298$ ns.

58. Using Eq. 38-10,

$$v = \frac{d}{t} = \frac{6.0\,\text{ly}}{2.0\,\text{y} + 6.0\,\text{y}} = \frac{(6.0c)(1.0\,\text{y})}{2.0\,\text{y} + 6.0\,\text{y}} = 0.75c.$$

59. To illustrate the technique, we derive Eq. 1′ from Eqs. 1 and 2 (in Table 38-2). We multiply Eq. 2 by speed $v$ and subtract it from Eq. 1:

$$\Delta x - v\Delta t = \gamma\left(\Delta x' + v\Delta t'\right) - v\gamma\left(\Delta t' + \frac{v\Delta x'}{c^2}\right) = \gamma\Delta x'\left(1 - \frac{v^2}{c^2}\right)$$

We note that $\gamma\left(1 - v^2/c^2\right) = 1/\gamma$ (using Eq. 38-8), so that if we multiply the above equation by $\gamma$ we obtain Eq. 1′:

$$\gamma\left(\Delta x - v\Delta t\right) = \gamma\left(\gamma\Delta x'\left(1 - \frac{v^2}{c^2}\right)\right) = \Delta x'$$

60. (a) $v_r = 2v = 2(27000\,\text{km/h}) = 54000\,\text{km/h}$.

(b) We can express $c$ in these units by multiplying by 3.6: $c = 1.08 \times 10^9$ km/h. The correct formula for $v_r$ is $v_r = 2v/(1 + v^2/c^2)$, so the fractional error is

$$1 - \frac{1}{1 + v^2/c^2} = 1 - \frac{1}{1 + [(27000\,\text{km/h})/(1.08 \times 10^9\,\text{km/h})]^2} = 6.3 \times 10^{-10} \ .$$

The discussion in Sample Problem 38-7 dealing with numerical considerations may prove helpful for those whose calculators do not yield this answer.

61. (a) We assume the electron starts from rest. The classical formula for kinetic energy is Eq. 38-48, so if $v = c$ then this (for an electron) would be $\frac{1}{2}mc^2 = \frac{1}{2}(511\,\text{keV}) = 255.5$ keV (using Table 38-3). Setting this equal to the potential energy loss (which is responsible for its acceleration), we find (using Eq. 25-7)

$$V = \frac{255.5\,\text{keV}}{|q|} = \frac{255\,\text{keV}}{e} = 255.5 \text{ kV} \ .$$

(b) Setting this amount of potential energy loss ($|\Delta U| = 255.5$ keV) equal to the correct relativistic kinetic energy, we obtain (using Eq. 38-49)

$$mc^2\left(\frac{1}{\sqrt{1-(v/c)^2}} - 1\right) = |\Delta U| \implies v = c\sqrt{1 + \left(\frac{1}{1 - \Delta U/mc^2}\right)^2}$$

which yields $v = 0.745c = 2.23 \times 10^8$ m/s.

62. (a) $\Delta E = \Delta mc^2 = (3.0\,\text{kg})(0.0010)(2.998 \times 10^8\,\text{m/s})^2 = 2.7 \times 10^{14}\,\text{J}$.

(b) The mass of TNT is

$$m_{\text{TNT}} = \frac{(2.7 \times 10^{14}\,\text{J})(0.227\,\text{kg/mol})}{3.4 \times 10^6\,\text{J}} = 1.8 \times 10^7 \text{ kg} \ .$$

(c) The fraction of mass converted in the TNT case is

$$\frac{\Delta m_{\text{TNT}}}{m_{\text{TNT}}} = \frac{(3.0\,\text{kg})(0.0010)}{1.8 \times 10^7\,\text{kg}} = 1.6 \times 10^{-9} \ ,$$

Therefore, the fraction is $0.0010/1.6 \times 10^{-9} = 6.0 \times 10^6$.

63. (a) Eq. 38-33 yields

$$v = \frac{\Delta\lambda}{\lambda}c = \left(\frac{462 - 434}{434}\right)c = 0.065c$$

or $v = 1.93 \times 10^7$ m/s.

(b) Since it is shifted "towards the red" (towards longer wavelengths) then the galaxy is moving away from us (receding).

64. When $\beta = 0.9860$, we have $\gamma = 5.9972$, and when $\beta = 0.9850$, we have $\gamma = 5.7953$. Thus, $\Delta\gamma = 0.202$ and the change in kinetic energy (equal to the work) becomes (using Eq. 38-49)

$$W = \Delta K = mc^2\Delta\gamma = 189 \text{ MeV}$$

where $mc^2 = 938$ MeV has been used (see Table 38-3).

65. Using $m_p = 1.672623 \times 10^{-27}$ kg in Eq. 38-45 yields

$$\gamma = \frac{E}{m_p c^2} = \frac{14.242 \times 10^{-9}\,\mathrm{J}}{1.50328 \times 10^{-10}\,\mathrm{J}} = 94.740 \ .$$

Solving for the speed , we obtain

$$v = c\sqrt{1 - \left(\frac{1}{\gamma}\right)^2} = 0.99994c \ .$$

66. (a) According to ship observers, the duration of proton flight is $\Delta t' = (760\,\mathrm{m})/0.980c = 2.59\ \mu\mathrm{s}$ (assuming it travels the entire length of the ship).

(b) To transform to our point of view, we use Eq. 2 in Table 38-2. Thus, with $\Delta x' = -750$ m, we have

$$\Delta t = \gamma\left(\Delta t' + (0.950c)\Delta x'/c^2\right) = 0.57\ \mu\mathrm{s} \ .$$

(c) and (d) For the ship observers, firing the proton from back to front makes no difference, and $\Delta t' = 2.59\ \mu$s as before. For us, the fact that now $\Delta x' = +750$ m is a significant change.

$$\Delta t = \gamma\left(\Delta t' + (0.950c)\Delta x'/c^2\right) = 16.0\ \mu\mathrm{s} \ .$$

67. (a) Our lab-based measurement of its lifetime is figured simply from $t = L/v = 7.99 \times 10^{-13}$ s. Use of the time-dilation relation (Eq. 38-7) leads to

$$\Delta t_0 = \left(7.99 \times 10^{-13}\,\mathrm{s}\right)\sqrt{1 - (0.960)^2} = 2.24 \times 10^{-13}\,\mathrm{s} \ .$$

(b) The length contraction formula can be used, or we can use the simple speed-distance relation (from the point of view of the particle, who watches the lab and all its meter sticks rushing past him at $0.960c$ until he expires): $L = v\Delta t_0 = 6.44 \times 10^{-5}$ m.

68. Using Appendix C, we find that the contraction is

$$\begin{aligned} |\Delta L| &= L_0 - L = L_0\left(1 - \frac{1}{\gamma}\right) = L_0(1 - \sqrt{1-\beta^2}) \\ &= 2(6.370 \times 10^6\,\mathrm{m})\left(1 - \sqrt{1 - \left(\frac{3.0 \times 10^4\,\mathrm{m/s}}{2.998 \times 10^8\,\mathrm{m/s}}\right)^2}\right) \\ &= 0.064\ \mathrm{m} \ . \end{aligned}$$

The discussion in Sample Problem 38-7 dealing with numerical considerations may prove helpful for those whose calculators do not yield this answer.

69. The speed of the spaceship after the first increment is $v_1 = 0.5c$. After the second one, it becomes

$$v_2 = \frac{v' + v_1}{1 + v'v_1/c^2} = \frac{0.50c + 0.50c}{1 + (0.50c)^2/c^2} = 0.80c \ ,$$

and after the third one, the speed is

$$v_3 = \frac{v' + v_2}{1 + v'v_2/c^2} = \frac{0.50c + 0.50c}{1 + (0.50c)(0.80c)/c^2} = 0.929c \ .$$

Continuing with this process, we get $v_4 = 0.976c$, $v_5 = 0.992c$, $v_6 = 0.997c$ and $v_7 = 0.999c$. Thus, seven increments are needed.

70. We use the transverse Doppler shift formula, Eq. 38-34: $f = f_0\sqrt{1-\beta^2}$, or

$$\frac{1}{\lambda} = \frac{1}{\lambda_0}\sqrt{1-\beta^2} \ .$$

We solve for $\lambda - \lambda_0$:

$$\lambda - \lambda_0 = \lambda_0\left(\frac{1}{\sqrt{1-\beta^2}} - 1\right) = (589.00\,\text{mm})\left[\frac{1}{\sqrt{1-(0.100)^2}} - 1\right] = +2.97 \text{ nm} \ .$$

71. The mean lifetime of a pion measured by observers on the Earth is $\Delta t = \gamma\Delta t_0$, so the distance it can travel (using Eq. 38-12) is

$$d = v\Delta t = \gamma v\Delta t_0 = \frac{(0.99)(2.998\times10^8\,\text{m/s})(26\times10^{-9}\,\text{s})}{\sqrt{1-(0.99)^2}} = 55 \text{ m} \ .$$

72. (a) For a proton (using Table 38-3), our results are:

$$E = \gamma m_p c^2 = \frac{938\,\text{MeV}}{\sqrt{1-(0.990)^2}} = 6.65 \text{ GeV}$$

$$K = E - m_p c^2 = 6.65\,\text{GeV} - 938\,\text{MeV} = 5.71 \text{ GeV}$$

$$p = \gamma m_p v = \gamma(m_p c^2)\beta/c = \frac{(938\,\text{MeV})(0.990)/c}{\sqrt{1-(0.990)^2}} = 6.59 \text{ GeV}/c$$

(b) For an electron:

$$E = \gamma m_e c^2 = \frac{0.511\,\text{MeV}}{\sqrt{1-(0.990)^2}} = 3.62 \text{ MeV}$$

$$K = E - m_e c^2 = 3.625\,\text{MeV} - 0.511\,\text{MeV} = 3.11 \text{ MeV}$$

$$p = \gamma m_e v = \gamma(m_e c^2)\beta/c = \frac{(0.511\,\text{MeV})(0.990)/c}{\sqrt{1-(0.990)^2}} = 3.59 \text{ MeV}/c$$

73. The strategy is to find the speed from $E = 1533$ MeV and $mc^2 = 0.511$ MeV (see Table 38-3) and from that find the time. From the energy relation (Eq. 38-45), we obtain

$$v = c\sqrt{1-\left(\frac{mc^2}{E}\right)^2} = 0.99999994c \ \approx \ c$$

so that we conclude it took the electron 26 y to reach us. In order to transform to its own "clock" it's useful to compute $\gamma$ directly from Eq. 38-45:

$$\gamma = \frac{E}{mc^2} = 3000$$

though if one is careful one can also get this result from $\gamma = 1/\sqrt{1-(v/c)^2}$. Then, Eq. 38-7 leads to

$$\Delta t_0 = \frac{26 \text{ y}}{\gamma} = 0.0087\,\text{y}$$

so that the electron "concludes" the distance he traveled is 0.0087 light-years (stated differently, the Earth, which is rushing towards him at very nearly the speed of light, seemed to start its journey from a distance of 0.0087 light-years away).

74. (a) Using Eq. 38-7, we expect the dilated time intervals to be

$$\tau = \gamma\tau_0 = \frac{\tau_0}{\sqrt{1-(v/c)^2}} .$$

(b) We rewrite Eq. 38-30 using the fact that period is the reciprocal of frequency ($f_R = \tau_R^{-1}$ and $f_0 = \tau_0^{-1}$):

$$\tau_R = \frac{1}{f_R} = \left(f_0\sqrt{\frac{1-\beta}{1+\beta}}\right)^{-1} = \tau_0\sqrt{\frac{1+\beta}{1-\beta}} = \tau_0\sqrt{\frac{c+v}{c-v}} .$$

(c) The Doppler shift combines two physical effects: the time dilation of the moving source *and* the travel-time differences involved in periodic emission (like a sine wave or a series of pulses) from a traveling source to a "stationary" receiver). To isolate the purely time-dilation effect, it's useful to consider "local" measurements (say, comparing the readings on a moving clock to those of two of your clocks, spaced some distance apart, such that the moving clock and each of your clocks can make a close-comparison of readings at the moment of passage).

75. We use the relative velocity formula (Eq. 38-28) with the primed measurements being those of the scout ship. We note that $v = -0.900c$ since the velocity of the scout ship relative to the cruiser is opposite to that of the cruiser relative to the scout ship.

$$u = \frac{u' + v}{1 + u'v/c^2} = \frac{0.980c - 0.900c}{1 - (0.980)(0.900)} = 0.678c .$$

76. We solve the time dilation equation for the time elapsed (as measured by Earth observers):

$$\Delta t = \frac{\Delta t_0}{\sqrt{1-(0.9990)^2}}$$

where $\Delta t_0 = 120$ y. This yields $\Delta t = 2684$ y.

77. (a) The relative contraction is

$$\begin{aligned}
\frac{|\Delta L|}{L_0} &= \frac{L_0(1-\gamma^{-1})}{L_0} = 1 - \sqrt{1-\beta^2} \\
&\approx 1 - \left(1 - \frac{1}{2}\beta^2\right) = \frac{1}{2}\beta^2 \\
&= \frac{1}{2}\left(\frac{630\,\mathrm{m/s}}{3.00\times10^8\,\mathrm{m/s}}\right)^2 \\
&= 2.21\times10^{-12} .
\end{aligned}$$

(b) Letting $|\Delta t - \Delta t_0| = \Delta t_0(\gamma - 1) = \tau = 1.00\,\mu$s, we solve for $\Delta t_0$:

$$\begin{aligned}
\Delta t_0 &= \frac{\tau}{\gamma - 1} = \frac{\tau}{(1-\beta^2)^{-1/2} - 1} \approx \frac{\tau}{1 + \frac{1}{2}\beta^2 - 1} = \frac{2\tau}{\beta^2} \\
&= \frac{2(1.00\times10^{-6}\,\mathrm{s})(1\,\mathrm{d}/86400\,\mathrm{s})}{[(630\,\mathrm{m/s})/(2.998\times10^8\,\mathrm{m/s})]^2} \\
&= 5.25\ \mathrm{d} .
\end{aligned}$$

78. Let the reference frame be $S$ in which the particle (approaching the South Pole) is at rest, and let the frame that is fixed on Earth be $S'$. Then $v = 0.60c$ and $u' = 0.80c$ (calling "downwards" [in the sense of Fig. 38-31] positive). The relative speed is now the speed of the other particle as measured in $S$:

$$u = \frac{u' + v}{1 + u'v/c^2} = \frac{0.80c + 0.60c}{1 + (0.80c)(0.60c)/c^2} = 0.95c .$$

79. We refer to the particle in the first sentence of the problem statement as particle 2. Since the total momentum of the two particles is zero in $S'$, it must be that the velocities of these two particles are equal in magnitude and opposite in direction in $S'$. Letting the velocity of the $S'$ frame be $v$ relative to $S$, then the particle which is at rest in $S$ must have a velocity of $u'_1 = -v$ as measured in $S'$, while the velocity of the other particle is given by solving Eq. 38-28 for $u'$:

$$u'_2 = \frac{u_2 - v}{1 - u_2 v/c^2} = \frac{\left(\frac{c}{2}\right) - v}{1 - \left(\frac{c}{2}\right)\left(\frac{v}{c^2}\right)} .$$

Letting $u'_2 = -u'_1 = v$, we obtain

$$\frac{\left(\frac{c}{2}\right) - v}{1 - \left(\frac{c}{2}\right)\left(\frac{v}{c^2}\right)} = v \implies v = c(2 \pm \sqrt{3}) \approx 0.27c$$

where the quadratic formula has been used (with the smaller of the two roots chosen so that $v \le c$).

80. From Eq. 28-37, we have

$$Q = -\Delta M c^2 = -\left(3(4.00151\,\mathrm{u}) - 11.99671\,\mathrm{u}\right)c^2 = -(0.00782\,\mathrm{u})(931.5\,\mathrm{MeV/u}) = -7.28 \text{ MeV} .$$

Thus, it takes a minimum of 7.28 MeV supplied to the system to cause this reaction. We note that the masses given in this problem are strictly for the nuclei involved; they are not the "atomic" masses which are quoted in several of the other problems in this chapter.

81. We use Eq. 38-51 with $mc^2 = 0.511$ MeV (see Table 38-3):

$$pc = \sqrt{K^2 + 2Kmc^2} = \sqrt{(2.00)^2 + 2(2.00)(0.511)}$$

This readily yields $p = 2.46$ MeV$/c$.