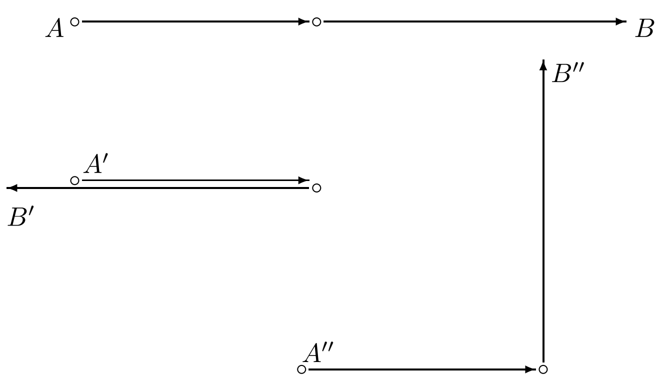
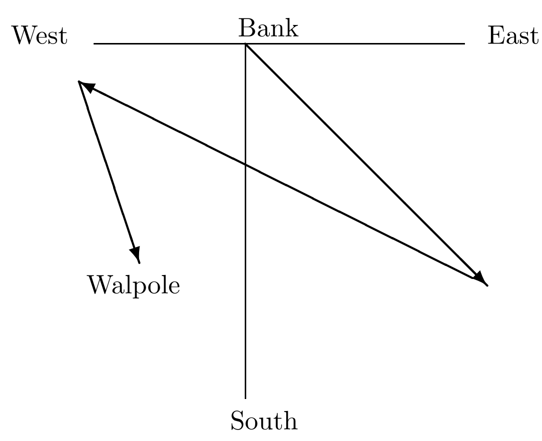
Chapter 3

1. The vectors should be parallel to achieve a resultant 7 m long (the unprimed case shown below), antiparallel (in opposite directions) to achieve a resultant 1 m long (primed case shown), and perpendicular to achieve a resultant $\sqrt{3^2 + 4^2} = 5$ m long (the double-primed case shown). In each sketch, the vectors are shown in a "head-to-tail" sketch but the resultant is not shown. The resultant would be a straight line drawn from beginning to end; the beginning is indicated by A (with or without primes, as the case may be) and the end is indicated by B.



2. A sketch of the displacements is shown. The resultant (not shown) would be a straight line from start (Bank) to finish (Walpole). With a careful drawing, one should find that the resultant vector has length 29.5 km at 35° west of south.



3. The x component of $\vec{a}$ is given by $a_x = 7.3 \cos 250° = -2.5$ and the y component is given by $a_y = 7.3 \sin 250° = -6.9$. In considering the variety of ways to compute these, we note that the vector is 70° below the $-x$ axis, so the components could also have been found from $a_x = -7.3 \cos 70°$ and $a_y = -7.3 \sin 70°$. In a similar vein, we note that the vector is 20° from the $-y$ axis, so one could use $a_x = -7.3 \sin 20°$ and $a_y = -7.3 \cos 20°$ to achieve the same results.

4. The angle described by a full circle is $360° = 2\pi$ rad, which is the basis of our conversion factor. Thus,

$$(20.0°)\,\frac{2\pi\,\text{rad}}{360°} = 0.349 \text{ rad}$$

and (similarly) $50.0° = 0.873$ rad and $100° = 1.75$ rad. Also,

$$(0.330\,\text{rad})\,\frac{360°}{2\pi\,\text{rad}} = 18.9°$$

and (similarly) $2.10\,\text{rad} = 120°$ and $7.70\,\text{rad} = 441°$.

5. The textbook's approach to this sort of problem is through the use of Eq. 3-6, and is illustrated in Sample Problem 3-3. However, most modern graphical calculators can produce the results quite efficiently using rectangular $\leftrightarrow$ polar "shortcuts."

   (a) $\sqrt{(-25)^2 + 40^2} = 47.2$ m

   (b) Recalling that $\tan(\theta) = \tan(\theta + 180°)$, we note that the two possibilities for $\tan^{-1}(40/-25)$ are $-58°$ and $122°$. Noting that the vector is in the third quadrant (by the signs of its x and y components) we see that $122°$ is the correct answer. The graphical calculator "shortcuts" mentioned above are designed to correctly choose the right possibility.

6. The x component of $\vec{r}$ is given by $15\cos 30° = 13$ m and the y component is given by $15\sin 30° = 7.5$ m.

7. The point P is displaced vertically by $2R$, where R is the radius of the wheel. It is displaced horizontally by half the circumference of the wheel, or πR. Since $R = 0.450$ m, the horizontal component of the displacement is 1.414 m and the vertical component of the displacement is 0.900 m. If the x axis is horizontal and the y axis is vertical, the vector displacement (in meters) is $\vec{r} = (1.414\,\hat{\imath} + 0.900\,\hat{\jmath})$. The displacement has a magnitude of

$$|\vec{r}| = \sqrt{(\pi R)^2 + (2R)^2} = R\sqrt{\pi^2 + 4} = 1.68 \text{ m}$$

and an angle of

$$\tan^{-1}\left(\frac{2R}{\pi R}\right) = \tan^{-1}\left(\frac{2}{\pi}\right) = 32.5°$$

above the floor. In physics there are no "exact" measurements, yet that angle computation seemed to yield something *exact*. However, there has to be some uncertainty in the observation that the wheel rolled half of a revolution, which introduces some indefiniteness in our result.

8. Although we think of this as a three-dimensional movement, it is rendered effectively two-dimensional by referring measurements to its well-defined plane of the fault.

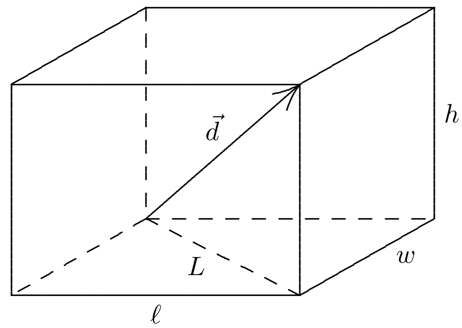
   (a) The magnitude of the net displacement is

$$|\vec{AB}| = \sqrt{|AD|^2 + |AC|^2} = \sqrt{17^2 + 22^2} = 27.8 \text{ m} .$$

   (b) The magnitude of the vertical component of $\vec{AB}$ is $|AD|\sin 52.0° = 13.4$ m.

9. The length unit meter is understood throughout the calculation.

   (a) We compute the distance from one corner to the diametrically opposite corner: $d = \sqrt{3.00^2 + 3.70^2 + 4.30^2} = 6.42$.

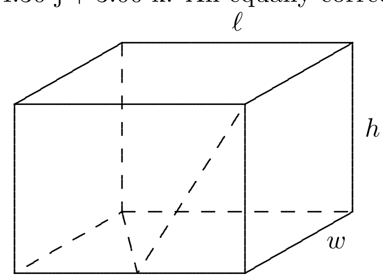


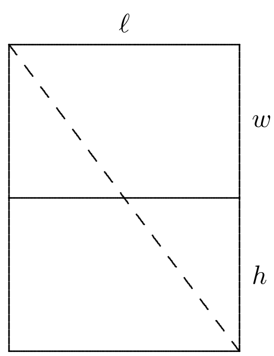
(b) The displacement vector is along the straight line from the beginning to the end point of the trip. Since a straight line is the shortest distance between two points, the length of the path cannot be less than the magnitude of the displacement.

(c) It can be greater, however. The fly might, for example, crawl along the edges of the room. Its displacement would be the same but the path length would be $\ell + w + h$.

(d) The path length is the same as the magnitude of the displacement if the fly flies along the displacement vector.

(e) We take the x axis to be out of the page, the y axis to be to the right, and the z axis to be upward. Then the x component of the displacement is $w = 3.70$, the y component of the displacement is 4.30, and the z component is 3.00. Thus $\vec{d} = 3.70\,\hat{\imath} + 4.30\,\hat{\jmath} + 3.00\,\hat{k}$. An equally correct answer is gotten by interchanging the length, width, and height.



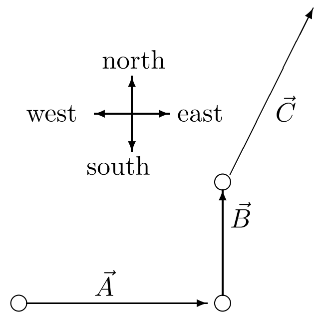


(f) Suppose the path of the fly is as shown by the dotted lines on the upper diagram. Pretend there is a hinge where the front wall of the room joins the floor and lay the wall down as shown on the lower diagram. The shortest walking distance between the lower left back of the room and the upper right front corner is the dotted straight line shown on the diagram. Its length is

$$L_{\text{min}} = \sqrt{(w+h)^2 + \ell^2} = \sqrt{(3.70 + 3.00)^2 + 4.30^2} = 7.96 \text{ m} .$$

10. We label the displacement vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ (and denote the result

of their vector sum as $\vec{r}$). We choose *east* as the $\hat{\imath}$ direction ($+x$ direction) and *north* as the $\hat{\jmath}$ direction ($+y$ direction). All distances are understood to be in kilometers. We note that the angle between $\vec{C}$ and the x axis is $60°$. Thus,



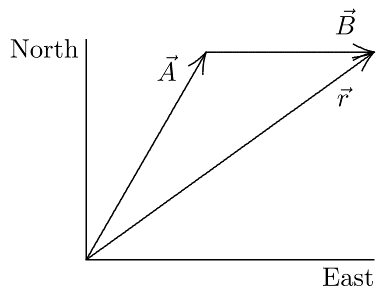
$$\begin{aligned}\vec{A} &= 50\,\hat{\imath} \\ \vec{B} &= 30\,\hat{\jmath} \\ \vec{C} &= 25\cos(60°)\,\hat{\imath} + 25\sin(60°)\,\hat{\jmath} \\ \vec{r} = \vec{A}+\vec{B}+\vec{C} &= 62.50\,\hat{\imath} + 51.65\,\hat{\jmath}\end{aligned}$$

which means that its magnitude is

$$|\vec{r}| = \sqrt{62.50^2 + 51.65^2} \approx 81 \text{ km} .$$

and its angle (counterclockwise from $+x$ axis) is $\tan^{-1}(51.65/62.50) \approx 40°$, which is to say that it points $40°$ *north of east*. Although the resultant $\vec{r}$ is shown in our sketch, it would be a direct line from the "tail" of $\vec{A}$ to the "head" of $\vec{C}$.

11. The diagram shows the displacement vectors for the two segments of her walk, labeled $\vec{A}$ and $\vec{B}$, and the total ("final") displacement vector, labeled $\vec{r}$. We take east to be the $+x$ direction and north to be the $+y$ direction. We observe that the angle between $\vec{A}$ and the x axis is $60°$. Where the units are not explicitly shown, the distances are understood to be in meters. Thus, the components of $\vec{A}$ are $A_x = 250\cos 60° = 125$ and $A_y = 250\sin 30° = 216.5$. The components of $\vec{B}$ are $B_x = 175$ and $B_y = 0$. The components of the total displacement are $r_x = A_x + B_x = 125 + 175 = 300$ and $r_y = A_y + B_y = 216.5 + 0 = 216.5$.



(a) The magnitude of the resultant displacement is

$$|\vec{r}| = \sqrt{r_x^2 + r_y^2} = \sqrt{300^2 + 216.5^2} = 370 \text{ m} .$$

(b) The angle the resultant displacement makes with the $+x$ axis is

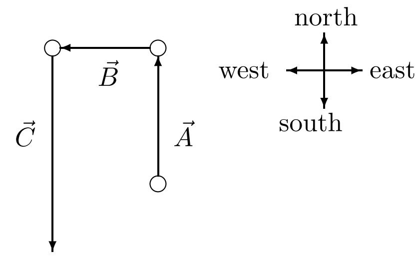
$$\tan^{-1}\left(\frac{r_y}{r_x}\right) = \tan^{-1}\left(\frac{216.5}{300}\right) = 36° .$$

(c) The total *distance* walked is $d = 250 + 175 = 425$ m.

(d) The total distance walked is greater than the magnitude of the resultant displacement. The diagram shows why: $\vec{A}$ and $\vec{B}$ are not collinear.

12. We label the displacement vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ (and denote the result

of their vector sum as $\vec{r}$). We choose *east* as the $\hat{\imath}$ direction ($+x$ direction) and *north* as the $\hat{\jmath}$ direction ($+y$ direction). All distances are understood to be in kilometers. Therefore,



$$\begin{aligned} \vec{A} &= 3.1\,\hat{\jmath} \\ \vec{B} &= -2.4\,\hat{\imath} \\ \vec{C} &= -5.2\,\hat{\jmath} \\ \vec{r} = \vec{A} + \vec{B} + \vec{C} &= -2.1\,\hat{\imath} - 2.4\,\hat{\jmath} \end{aligned}$$

which means that its magnitude is

$$|\vec{r}| = \sqrt{(-2.1)^2 + (-2.4)^2} \approx 3.2 \text{ km} .$$

and the two possibilities for its angle are

$$\tan^{-1}\left(\frac{-2.4}{-2.1}\right) = 41^\circ, \quad \text{or} \quad 221^\circ .$$

We choose the latter possibility since $\vec{r}$ is in the third quadrant. It should be noted that many graphical calculators have polar $\leftrightarrow$ rectangular "shortcuts" that automatically produce the correct answer for angle (measured counterclockwise from the $+x$ axis). We may phrase the angle, then, as 221° counterclockwise from East (a phrasing that sounds peculiar, at best) or as 41° south from west or 49° west from south. The resultant $\vec{r}$ is not shown in our sketch; it would be an arrow directed from the "tail" of $\vec{A}$ to the "head" of $\vec{C}$.

13. We write $\vec{r} = \vec{a} + \vec{b}$. When not explicitly displayed, the units here are assumed to be meters. Then $r_x = a_x + b_x = 4.0 - 13 = -9.0$ and $r_y = a_y + b_y = 3.0 + 7.0 = 10$. Thus $\vec{r} = (-9.0\,\text{m})\,\hat{\imath} + (10\,\text{m})\,\hat{\jmath}$. The magnitude of the resultant is

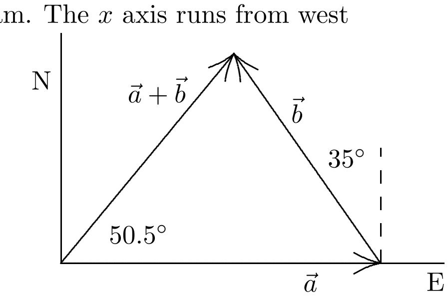
$$r = \sqrt{r_x^2 + r_y^2} = \sqrt{(-9.0)^2 + (10)^2} = 13 \text{ m} .$$

The angle between the resultant and the $+x$ axis is given by $\tan^{-1}(r_y/r_x) = \tan^{-1} 10/(-9.0)$ which is either -48° or 132°. Since the x component of the resultant is negative and the y component is positive, characteristic of the second quadrant, we find the angle is 132° (measured counterclockwise from $+x$ axis).

14. The x, y and z components (with meters understood) of $\vec{r}$ are

$$\begin{aligned} r_x = c_x + d_x &= 7.4 + 4.4 = 12 \\ r_y = c_y + d_y &= -3.8 - 2.0 = -5.8 \\ r_z = c_z + d_z &= -6.1 + 3.3 = -2.8 . \end{aligned}$$

15. The vectors are shown on the diagram. The x axis runs from west to east and the y axis run from south to north. Then $a_x = 5.0\,\text{m}$, $a_y = 0$, $b_x = -(4.0\,\text{m})\sin 35^\circ = -2.29\,\text{m}$, and $b_y = (4.0\,\text{m})\cos 35^\circ = 3.28\,\text{m}$.



(a) Let $\vec{c} = \vec{a} + \vec{b}$. Then $c_x = a_x + b_x = 5.0\,\text{m} - 2.29\,\text{m} = 2.71\,\text{m}$ and $c_y = a_y + b_y = 0 + 3.28\,\text{m} = 3.28\,\text{m}$. The magnitude of c is

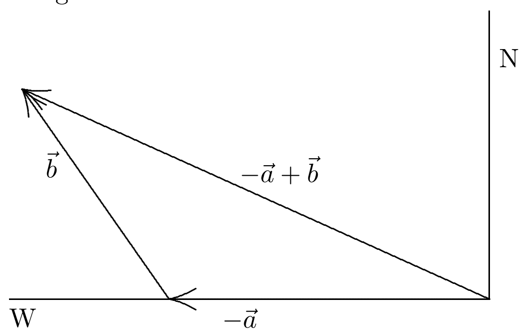
$$c = \sqrt{c_x^2 + c_y^2} = \sqrt{(2.71\,\text{m})^2 + (3.28\,\text{m})^2} = 4.3\ \text{m}\ .$$

(b) The angle θ that $\vec{c} = \vec{a} + \vec{b}$ makes with the $+x$ axis is

$$\theta = \tan^{-1}\frac{c_y}{c_x} = \tan^{-1}\frac{3.28\,\text{m}}{2.71\,\text{m}} = 50.4^\circ\ .$$

The second possibility ($\theta = 50.4^\circ + 180^\circ = 126^\circ$) is rejected because it would point in a direction opposite to $\vec{c}$.

(c) The vector $\vec{b} - \vec{a}$ is found by adding $-\vec{a}$ to $\vec{b}$. The result is shown on the diagram to the right. Let $\vec{c} = \vec{b} - \vec{a}$. Then $c_x = b_x - a_x = -2.29\,\text{m} - 5.0\,\text{m} = -7.29\,\text{m}$ and $c_y = b_y - a_y = 3.28\,\text{m}$. The magnitude of $\vec{c}$ is $c = \sqrt{c_x^2 + c_y^2} = 8.0\,\text{m}$.



(d) The tangent of the angle θ that $\vec{c}$ makes with the $+x$ axis (east) is

$$\tan\theta = \frac{c_y}{c_x} = \frac{3.28\,\text{m}}{-7.29\,\text{m}} = -4.50, .$$

There are two solutions: -24.2° and 155.8°. As the diagram shows, the second solution is correct. The vector $\vec{c} = -\vec{a} + \vec{b}$ is 24° north of west.

16. All distances in this solution are understood to be in meters.

(a) $\vec{a} + \vec{b} = (3.0\,\hat{\imath} + 4.0\,\hat{\jmath}) + (5.0\,\hat{\imath} - 2.0\,\hat{\jmath}) = 8.0\,\hat{\imath} + 2.0\,\hat{\jmath}$.

(b) The magnitude of $\vec{a} + \vec{b}$ is

$$|\vec{a} + \vec{b}| = \sqrt{8.0^2 + 2.0^2} = 8.2\ \text{m}\ .$$

(c) The angle between this vector and the $+x$ axis is $\tan^{-1}(2.0/8.0) = 14^\circ$.

(d) $\vec{b} - \vec{a} = (5.0\,\hat{\imath} - 2.0\,\hat{\jmath}) - (3.0\,\hat{\imath} + 4.0\,\hat{\jmath}) = 2.0\,\hat{\imath} - 6.0\,\hat{\jmath}$.

(e) The magnitude of the difference vector $\vec{b} - \vec{a}$ is

$$|\vec{b} - \vec{a}| = \sqrt{2.0^2 + (-6.0)^2} = 6.3\ \text{m}\ .$$

(f) The angle between this vector and the $+x$ axis is $\tan^{-1}(-6.0/2.0) = -72^\circ$. The vector is 72° *clockwise* from the axis defined by $\hat{\imath}$.

17. All distances in this solution are understood to be in meters.

(a) $\vec{a} + \vec{b} = (4.0 + (-1.0))\,\hat{\imath} + ((-3.0) + 1.0)\,\hat{\jmath} + (1.0 + 4.0)\,\hat{k} = 3.0\,\hat{\imath} - 2.0\,\hat{\jmath} + 5.0\,\hat{k}$.

(b) $\vec{a} - \vec{b} = (4.0 - (-1.0))\,\hat{\imath} + ((-3.0) - 1.0)\,\hat{\jmath} + (1.0 - 4.0)\,\hat{k} = 5.0\,\hat{\imath} - 4.0\,\hat{\jmath} - 3.0\,\hat{k}$.

(c) The requirement $\vec{a} - \vec{b} + \vec{c} = 0$ leads to $\vec{c} = \vec{b} - \vec{a}$, which we note is the opposite of what we found in part (b). Thus, $\vec{c} = -5.0\,\hat{\imath} + 4.0\,\hat{\jmath} + 3.0\,\hat{k}$.

18. Many of the operations are done efficiently on most modern graphical calculators using their built-in vector manipulation and rectangular $\leftrightarrow$ polar "shortcuts." In this solution, we employ the "traditional" methods (such as Eq. 3-6).

   (a) The magnitude of $\vec{a}$ is $\sqrt{4^2+(-3)^2} = 5.0$ m.

   (b) The angle between $\vec{a}$ and the $+x$ axis is $\tan^{-1}(-3/4) = -37°$. The vector is 37° *clockwise* from the axis defined by $\hat{\imath}$.

   (c) The magnitude of $\vec{b}$ is $\sqrt{6^2+8^2} = 10$ m.

   (d) The angle between $\vec{b}$ and the $+x$ axis is $\tan^{-1}(8/6) = 53°$.

   (e) $\vec{a}+\vec{b} = (4+6)\,\hat{\imath} + ((-3)+8)\,\hat{\jmath} = 10\,\hat{\imath} + 5\,\hat{\jmath}$, with the unit meter understood. The magnitude of this vector is $\sqrt{10^2+5^2} = 11$ m; we rounding to two significant figures in our results.

   (f) The angle between the vector described in part (e) and the $+x$ axis is $\tan^{-1}(5/10) = 27°$.

   (g) $\vec{b}-\vec{a} = (6-4)\,\hat{\imath} + (8-(-3))\,\hat{\jmath} = 2\,\hat{\imath} + 11\,\hat{\jmath}$, with the unit meter understood. The magnitude of this vector is $\sqrt{2^2+11^2} = 11$ m, which is, interestingly, the same result as in part (e) (exactly, not just to 2 significant figures) (this curious coincidence is made possible by the fact that $\vec{a} \perp \vec{b}$).

   (h) The angle between the vector described in part (g) and the $+x$ axis is $\tan^{-1}(11/2) = 80°$.

   (i) $\vec{a}-\vec{b} = (4-6)\,\hat{\imath} + ((-3)-8)\,\hat{\jmath} = -2\,\hat{\imath} - 11\,\hat{\jmath}$, with the unit meter understood. The magnitude of this vector is $\sqrt{(-2)^2+(-11)^2} = 11$ m.

   (j) The two possibilities presented by a simple calculation for the angle between the vector described in part (i) and the $+x$ direction are $\tan^{-1}(11/2) = 80°$, and $180° + 80° = 260°$. The latter possibility is the correct answer (see part (k) for a further observation related to this result).

   (k) Since $\vec{a}-\vec{b} = (-1)(\vec{b}-\vec{a})$, they point in opposite (antiparallel) directions; the angle between them is 180°.

19. Many of the operations are done efficiently on most modern graphical calculators using their built-in vector manipulation and rectangular $\leftrightarrow$ polar "shortcuts." In this solution, we employ the "traditional" methods (such as Eq. 3-6). Where the length unit is not displayed, the unit meter should be understood.

   (a) Using unit-vector notation,

$$\begin{aligned}
\vec{a} &= 50\cos(30°)\,\hat{\imath} + 50\sin(30°)\,\hat{\jmath} \\
\vec{b} &= 50\cos(195°)\,\hat{\imath} + 50\sin(195°)\,\hat{\jmath} \\
\vec{c} &= 50\cos(315°)\,\hat{\imath} + 50\sin(315°)\,\hat{\jmath} \\
\vec{a}+\vec{b}+\vec{c} &= 30.4\,\hat{\imath} - 23.3\,\hat{\jmath}\ .
\end{aligned}$$

   The magnitude of this result is $\sqrt{30.4^2+(-23.3)^2} = 38$ m.

   (b) The two possibilities presented by a simple calculation for the angle between the vector described in part (a) and the $+x$ direction are $\tan^{-1}(-23.2/30.4) = -37.5°$, and $180° + (-37.5°) = 142.5°$. The former possibility is the correct answer since the vector is in the fourth quadrant (indicated by the signs of its components). Thus, the angle is $-37.5°$, which is to say that it is roughly 38° *clockwise* from the $+x$ axis. This is equivalent to 322.5° counterclockwise from $+x$.

   (c) We find $\vec{a}-\vec{b}+\vec{c} = (43.3-(-48.3)+35.4)\,\hat{\imath} - (25-(-12.9)+(-35.4))\,\hat{\jmath} = 127\,\hat{\imath} + 2.6\,\hat{\jmath}$ in unit-vector notation. The magnitude of this result is $\sqrt{127^2+2.6^2} \approx 1.3\times 10^2$ m.

   (d) The angle between the vector described in part (c) and the $+x$ axis is $\tan^{-1}(2.6/127) \approx 1°$.

   (e) Using unit-vector notation, $\vec{d}$ is given by

$$\begin{aligned}
\vec{d} &= \vec{a}+\vec{b}-\vec{c} \\
&= -40.4\,\hat{\imath} + 47.4\,\hat{\jmath}
\end{aligned}$$

   which has a magnitude of $\sqrt{(-40.4)^2+47.4^2} = 62$ m.

(f) The two possibilities presented by a simple calculation for the angle between the vector described in part (e) and the $+x$ axis are $\tan^{-1}(47.4/(-40.4)) = -50°$, and $180° + (-50°) = 130°$. We choose the latter possibility as the correct one since it indicates that $\vec{d}$ is in the second quadrant (indicated by the signs of its components).

20. Angles are given in 'standard' fashion, so Eq. 3-5 applies directly. We use this to write the vectors in unit-vector notation before adding them. However, a very different-looking approach using the special capabilities of most graphical calculators can be imagined. Where the length unit is not displayed in the solution below, the unit meter should be understood.

(a) Allowing for the different angle units used in the problem statement, we arrive at

$$\begin{aligned} \vec{E} &= 3.73\,\hat{\imath} + 4.70\,\hat{\jmath} \\ \vec{F} &= 1.29\,\hat{\imath} - 4.83\,\hat{\jmath} \\ \vec{G} &= 1.45\,\hat{\imath} + 3.73\,\hat{\jmath} \\ \vec{H} &= -5.20\,\hat{\imath} + 3.00\,\hat{\jmath} \\ \vec{E} + \vec{F} + \vec{G} + \vec{H} &= 1.28\,\hat{\imath} + 6.60\,\hat{\jmath}\ . \end{aligned}$$

(b) The magnitude of the vector sum found in part (a) is $\sqrt{1.28^2 + 6.60^2} = 6.72$ m. Its angle measured counterclockwise from the $+x$ axis is $\tan^{-1}(6.6/1.28) = 79° = 1.38$ rad.

21. It should be mentioned that an efficient way to work this vector addition problem is with the cosine law for general triangles (and since $\vec{a}$, $\vec{b}$ and $\vec{r}$ form an isosceles triangle, the angles are easy to figure). However, in the interest of reinforcing the usual systematic approach to vector addition, we note that the angle $\vec{b}$ makes with the $+x$ axis is 135° and apply Eq. 3-5 and Eq. 3-6 where appropriate.

(a) The x component of $\vec{r}$ is $10\cos 30° + 10\cos 135° = 1.59$ m.

(b) The y component of $\vec{r}$ is $10\sin 30° + 10\sin 135° = 12.1$ m.

(c) The magnitude of $\vec{r}$ is $\sqrt{1.59^2 + 12.1^2} = 12.2$ m.

(d) The angle between $\vec{r}$ and the $+x$ direction is $\tan^{-1}(12.1/1.59) = 82.5°$.

22. If we wish to use Eq. 3-5 in an unmodified fashion, we should note that the angle between $\vec{C}$ and the $+x$ axis is $180° + 20° = 200°$.

(a) The x component of $\vec{B}$ is given by $C_x - A_x = 15\cos 200° - 12\cos 40° = -23.3$ m, and the y component of $\vec{B}$ is given by $C_y - A_y = 15\sin 200° - 12\sin 40° = -12.8$ m. Consequently, its magnitude is $\sqrt{(-23.3)^2 + (-12.8)^2} = 26.6$ m.

(b) The two possibilities presented by a simple calculation for the angle between $\vec{B}$ and the $+x$ axis are $\tan^{-1}((-12.8)/(-23.3)) = 28.9°$, and $180° + 28.9° = 209°$. We choose the latter possibility as the correct one since it indicates that $\vec{B}$ is in the third quadrant (indicated by the signs of its components). We note, too, that the answer can be equivalently stated as $-151°$.

23. We consider $\vec{A}$ with (x, y) components given by $(A\cos\alpha, A\sin\alpha)$. Similarly, $\vec{B} \to (B\cos\beta, B\sin\beta)$. The angle (measured from the $+x$ direction) for their vector sum must have a slope given by

$$\tan\theta = \frac{A\sin\alpha + B\sin\beta}{A\cos\alpha + B\cos\beta}\ .$$

The problem requires that we now consider the orthogonal direction, where $\tan\theta + 90° = -\cot\theta$. If this (the negative reciprocal of the above expression) is to equal the slope for their vector *difference*, then we must have

$$-\frac{A\cos\alpha + B\cos\beta}{A\sin\alpha + B\sin\beta} = \frac{A\sin\alpha - B\sin\beta}{A\cos\alpha - B\cos\beta}\ .$$

Multiplying both sides by $A\sin\alpha + B\sin\beta$ and doing the same with $A\cos\alpha - B\cos\beta$ yields

$$A^2\,\cos^2\alpha - B^2\,\cos^2\beta = A^2\,\sin^2\alpha - B^2\,\sin^2\beta\ .$$

Rearranging, using the $\cos^2\phi + \sin^2\phi = 1$ identity, we obtain

$$A^2 = B^2 \quad\Longrightarrow\quad A = B\ .$$

In a *later* section, the scalar (dot) product of vectors is presented and this result can be revisited with a more compact derivation.

24. If we wish to use Eq. 3-5 directly, we should note that the angles for $\vec{Q}$,$\vec{R}$ and $\vec{S}$ are 100°, 250° and 310°, respectively, if they are measured counterclockwise from the $+x$ axis.

    (a) Using unit-vector notation, with the unit meter understood, we have

$$\begin{aligned}\vec{P} &= 10\cos(25^\circ)\,\hat{\imath} + 10\sin(25^\circ)\,\hat{\jmath}\\ \vec{Q} &= 12\cos(100^\circ)\,\hat{\imath} + 12\sin(100^\circ)\,\hat{\jmath}\\ \vec{R} &= 8\cos(250^\circ)\,\hat{\imath} + 8\sin(250^\circ)\,\hat{\jmath}\\ \vec{S} &= 9\cos(310^\circ)\,\hat{\imath} + 9\sin(310^\circ)\,\hat{\jmath}\\ \vec{P}+\vec{Q}+\vec{R}+\vec{S} &= 10.0\,\hat{\imath} + 1.6\,\hat{\jmath}\end{aligned}$$

    (b) The magnitude of the vector sum is $\sqrt{10^2 + 1.6^2} = 10.2$ m and its angle is $\tan^{-1}(1.6/10) \approx 9.2^\circ$ measured counterclockwise from the $+x$ axis. The appearance of this solution would be quite different using the vector manipulation capabilities of most modern graphical calculators, although the principle would be basically the same.

25. Without loss of generality, we assume $\vec{a}$ points along the $+x$ axis, and that $\vec{b}$ is at θ measured counterclockwise from $\vec{a}$. We wish to verify that

$$r^2 = a^2 + b^2 + 2ab\cos\theta$$

where $a = |\vec{a}| = a_x$ (we'll call it a for simplicity) and $b = |\vec{b}| = \sqrt{b_x^2 + b_y^2}$. Since $\vec{r} = \vec{a} + \vec{b}$ then $r = |\vec{r}| = \sqrt{(a+b_x)^2 + b_y^2}$. Thus, the above expression becomes

$$\begin{aligned}\left(\sqrt{(a+b_x)^2 + b_y^2}\right)^2 &= a^2 + \left(\sqrt{b_x^2 + b_y^2}\right)^2 + 2ab\cos\theta\\ a^2 + 2ab_x + b_x^2 + b_y^2 &= a^2 + b_x^2 + b_y^2 + 2ab\cos\theta\end{aligned}$$

which makes a valid equality since (the last term) $2ab\cos\theta$ is indeed the same as $2ab_x$ (on the left-hand side). In a *later* section, the scalar (dot) product of vectors is presented and this result can be revisited with a somewhat different-looking derivation.

26. The vector equation is $\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D}$. Expressing $\vec{B}$ and $\vec{D}$ in unit-vector notation, we have $1.69\,\hat{\imath} + 3.63\,\hat{\jmath}$ and $-2.87\,\hat{\imath} + 4.10\,\hat{\jmath}$, respectively. Where the length unit is not displayed in the solution below, the unit meter should be understood.

    (a) Adding corresponding components, we obtain $\vec{R} = -3.18\,\hat{\imath} + 4.72\,\hat{\jmath}$.

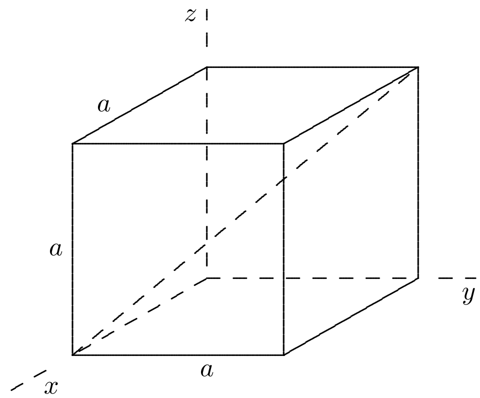
    (b) and (c) Converting this result to polar coordinates (using Eq. 3-6 or functions on a vector-capable calculator), we obtain

$$(-3.18, 4.72) \;\longrightarrow\; (5.69\ \angle\ 124^\circ)$$

    which tells us the magnitude is 5.69 m and the angle (measured counterclockwise from $+x$ axis) is 124°.

27. (a) There are 4 such lines, one from each of the corners on the lower face to the diametrically opposite corner on the upper face. One is shown on the diagram. Using an xyz coordinate system as shown (with the origin at the back lower left corner) The position vector of the "starting point" of the diagonal shown is $a\,\hat{\imath}$ and the position vector of the ending point is $a\,\hat{\jmath} + a\,\hat{k}$, so the vector along the line is the difference $a\,\hat{\jmath} + a\,\hat{k} - a\,\hat{\imath}$.

The point diametrically opposite the origin has position vector $a\,\hat{\imath} + a\,\hat{\jmath} + a\,\hat{k}$ and this is the vector along the diagonal. Another corner of the bottom face is at $a\,\hat{\imath} + a\,\hat{\jmath}$ and the diametrically opposite corner is at $a\,\hat{k}$, so another cube diagonal is $a\,\hat{k} - a\,\hat{\imath} - a\,\hat{\jmath}$. The fourth diagonal runs from $a\,\hat{\jmath}$ to $a\,\hat{\imath} + a\,\hat{k}$, so the vector along the diagonal is $a\,\hat{\imath} + a\,\hat{k} - a\,\hat{\jmath}$.



(b) Consider the vector from the back lower left corner to the front upper right corner. It is $a\,\hat{\imath} + a\,\hat{\jmath} + a\,\hat{k}$. We may think of it as the sum of the vector $a\,\hat{\imath}$ parallel to the x axis and the vector $a\,\hat{\jmath} + a\,\hat{k}$ perpendicular to the x axis. The tangent of the angle between the vector and the x axis is the perpendicular component divided by the parallel component. Since the magnitude of the perpendicular component is $\sqrt{a^2 + a^2} = a\sqrt{2}$ and the magnitude of the parallel component is a, $\tan\theta = \left(a\sqrt{2}\right)/a = \sqrt{2}$. Thus $\theta = 54.7°$. The angle between the vector and each of the other two adjacent sides (the y and z axes) is the same as is the angle between any of the other diagonal vectors and any of the cube sides adjacent to them.

(c) The length of any of the diagonals is given by $\sqrt{a^2 + a^2 + a^2} = a\sqrt{3}$.

28. Reference to Figure 3-18 (and the accompanying material in that section) is helpful. If we convert $\vec{B}$ to the magnitude-angle notation (as $\vec{A}$ already is) we have $\vec{B} = (14.4 \angle\ 33.7°)$ (appropriate notation especially if we are using a vector capable calculator in polar mode). Where the length unit is not displayed in the solution, the unit meter should be understood. In the magnitude-angle notation, rotating the axis by $+20°$ amounts to subtracting that angle from the angles previously specified. Thus, $\vec{A} = (12.0\ \angle 40.0°)'$ and $\vec{B} = (14.4\ \angle\ 13.7°)'$, where the 'prime' notation indicates that the description is in terms of the new coordinates. Converting these results to (x, y) representations, we obtain

$$\begin{aligned}\vec{A} &= 9.19\,\hat{\imath}' + 7.71\,\hat{\jmath}' \\ \vec{B} &= 14.0\,\hat{\imath}' + 3.41\,\hat{\jmath}'\end{aligned}$$

with the unit meter understood, as already mentioned.

29. We apply Eq. 3-20 and Eq. 3-27.

(a) The scalar (dot) product of the two vectors is $\vec{a} \cdot \vec{b} = ab\cos\phi = (10)(6.0)\cos 60° = 30$.

(b) The magnitude of the vector (cross) product of the two vectors is $|\vec{a} \times \vec{b}| = ab\sin\phi = (10)(6.0)\sin 60° = 52$.

30. We consider all possible products and then simplify using relations such as $\hat{\imath} \cdot \hat{k} = 0$ and $\hat{\imath} \cdot \hat{\imath} = 1$. Thus,

$$\begin{aligned}\vec{a} \cdot \vec{b} &= \left(a_x\,\hat{\imath} + a_y\,\hat{\jmath} + a_z\,\hat{k}\right) \cdot \left(b_x\,\hat{\imath} + b_y\,\hat{\jmath} + b_z\,\hat{k}\right) \\ &= a_xb_x\,\hat{\imath} \cdot \hat{\imath} + a_xb_y\,\hat{\imath} \cdot \hat{\jmath} + a_xb_z\,\hat{\imath} \cdot \hat{k} + a_yb_x\,\hat{\jmath} \cdot \hat{\imath} + a_yb_j\,\hat{\jmath} \cdot \hat{\jmath} + \cdots \\ &= a_xb_x(1) + a_xb_y(0) + a_xb_z(0) + a_yb_x(0) + a_yb_j(1) + \cdots\end{aligned}$$

which is seen to reduce to the desired result (one might wish to show this in two dimensions before tackling the additional tedium of working with these three-component vectors).

31. Since $ab\cos\phi = a_x b_x + a_y b_y + a_z b_z$,

$$\cos\phi = \frac{a_x b_x + a_y b_y + a_z b_z}{ab} .$$

The magnitudes of the vectors given in the problem are

$$\begin{aligned} a = |\vec{a}| &= \sqrt{(3.0)^2 + (3.0)^2 + (3.0)^2} = 5.2 \\ b = |\vec{b}| &= \sqrt{(2.0)^2 + (1.0)^2 + (3.0)^2} = 3.7 . \end{aligned}$$

The angle between them is found from

$$\cos\phi = \frac{(3.0)(2.0) + (3.0)(1.0) + (3.0)(3.0)}{(5.2)(3.7)} = 0.926 .$$

The angle is $\phi = 22°$.

32. We consider all possible products and then simplify using relations such as $\hat{\imath} \times \hat{\imath} = 0$ and the important fundamental products

$$\begin{aligned} \hat{\imath} \times \hat{\jmath} = -\hat{\jmath} \times \hat{\imath} &= \hat{k} \\ \hat{\jmath} \times \hat{k} = -\hat{k} \times \hat{\jmath} &= \hat{\imath} \\ \hat{k} \times \hat{\imath} = -\hat{\imath} \times \hat{k} &= \hat{\jmath} . \end{aligned}$$

Thus,

$$\begin{aligned} \vec{a} \times \vec{b} &= \left(a_x\,\hat{\imath} + a_y\,\hat{\jmath} + a_z\,\hat{k}\right) \times \left(b_x\,\hat{\imath} + b_y\,\hat{\jmath} + b_z\,\hat{k}\right) \\ &= a_x b_x\,\hat{\imath} \times \hat{\imath} + a_x b_y\,\hat{\imath} \times \hat{\jmath} + a_x b_z\,\hat{\imath} \times \hat{k} + a_y b_x\,\hat{\jmath} \times \hat{\imath} + a_y b_j\,\hat{\jmath} \times \hat{\jmath} + \cdots \\ &= a_x b_x(0) + a_x b_y(\hat{k}) + a_x b_z(-\hat{\jmath}) + a_y b_x(-\hat{k}) + a_y b_j(0) + \cdots \end{aligned}$$

which is seen to simplify to the desired result.

33. The area of a triangle is half the product of its base and altitude. The base is the side formed by vector $\vec{a}$. Then the altitude is $b\sin\phi$ and the area is $A = \frac{1}{2}ab\sin\phi = \frac{1}{2}|\vec{a} \times \vec{b}|$.

34. Applying Eq. 3-23, $\vec{F} = q\vec{v} \times \vec{B}$ (where q is a scalar) becomes

$$F_x\,\hat{\imath} + F_y\,\hat{\jmath} + F_z\,\hat{k} = q\,(v_y B_z - v_z B_y)\,\hat{\imath} + q\,(v_z B_x - v_x B_z)\,\hat{\jmath} + q\,(v_x B_y - v_y B_x)\,\hat{k}$$

which – plugging in values – leads to three equalities:

$$\begin{aligned} 4.0 &= 2\,(4.0B_z - 6.0B_y) \\ -20 &= 2\,(6.0B_x - 2.0B_z) \\ 12 &= 2\,(2.0B_y - 4.0B_x) \end{aligned}$$

Since we are told that $B_x = B_y$, the third equation leads to $B_y = -3.0$. Inserting this value into the first equation, we find $B_z = -4.0$. Thus, our answer is

$$\vec{B} = -3.0\,\hat{\imath} - 3.0\,\hat{\jmath} - 4.0\,\hat{k} .$$

35. Both proofs shown below utilize the fact that the vector (cross) product of $\vec{a}$ and $\vec{b}$ is perpendicular to both $\vec{a}$ and $\vec{b}$. This is mentioned in the book, and is fundamental to its discussion of the right-hand rule.

(a) $(\vec{b} \times \vec{a})$ is a vector that is perpendicular to $\vec{a}$, so the scalar product of $\vec{a}$ with this vector is zero. This can also be verified by using Eq. 3-30, and then (with suitable notation changes) Eq. 3-23.

(b) Let $\vec{c} = \vec{b} \times \vec{a}$. Then the magnitude of $\vec{c}$ is $c = ab\sin\phi$. Since $\vec{c}$ is perpendicular to $\vec{a}$ the magnitude of $\vec{a} \times \vec{c}$ is ac. The magnitude of $\vec{a} \times (\vec{b} \times \vec{a})$ is consequently $|\vec{a} \times (\vec{b} \times \vec{a})| = ac = a^2 b \sin\phi$. This too can be verified by repeated application of Eq. 3-30, although it must be admitted that this is much less intimidating if one is using a math software package such as MAPLE or Mathematica.

36. If a vector capable calculator is used, this makes a good exercise for getting familiar with those features. Here we briefly sketch the method. Eq. 3-30 leads to

$$2\vec{A} \times \vec{B} = 2\left(2\,\hat{\imath} + 3\,\hat{\jmath} - 4\,\hat{k}\right) \times \left(-3\,\hat{\imath} + 4\,\hat{\jmath} + 2\,\hat{k}\right) = 44\,\hat{\imath} + 16\,\hat{\jmath} + 34\,\hat{k} \ .$$

We now apply Eq. 3-23 to evaluate $3\vec{C} \cdot \left(2\vec{A} \times \vec{B}\right)$:

$$3\left(7\,\hat{\imath} - 8\,\hat{\jmath}\right) \cdot \left(44\,\hat{\imath} + 16\,\hat{\jmath} + 34\,\hat{k}\right) = 3\left((7)(44) + (-8)(16) + (0)(34)\right) = 540 \ .$$

37. From the figure, we note that $\vec{c} \perp \vec{b}$, which implies that the angle between $\vec{c}$ and the $+x$ axis is 120°.

(a) Direct application of Eq. 3-5 yields the answers for this and the next few parts. $a_x = a\cos 0° = a = 3.00$ m.

(b) $a_y = a\sin 0° = 0$.

(c) $b_x = b\cos 30° = (4.00\,\text{m})\cos 30° = 3.46$ m.

(d) $b_y = b\sin 30° = (4.00\,\text{m})\sin 30° = 2.00$ m.

(e) $c_x = c\cos 120° = (10.0\,\text{m})\cos 120° = -5.00$ m.

(f) $c_y = c\sin 30° = (10.0\,\text{m})\sin 120° = 8.66$ m.

(g) In terms of components (first x and then y), we must have

$$\begin{aligned} -5.00\,\text{m} &= p(3.00\,\text{m}) + q(3.46\,\text{m}) \\ 8.66\,\text{m} &= p(0) + q(2.00\,\text{m}) \ . \end{aligned}$$

Solving these equations, we find $p = -6.67$

(h) and $q = 4.33$ (note that it's easiest to solve for q first). The numbers p and q have no units.

38. We apply Eq. 3-20 with Eq. 3-23. Where the length unit is not displayed, the unit meter is understood.

(a) We first note that $a = |\vec{a}| = \sqrt{3.2^2 + 1.6^2} = 3.58$ m and $b = |\vec{b}| = \sqrt{0.5^2 + 4.5^2} = 4.53$ m. Now,

$$\begin{aligned} \vec{a} \cdot \vec{b} = a_x b_x + a_y b_y &= ab\cos\phi \\ (3.2)(0.5) + (1.6)(4.5) &= (3.58)(4.53)\cos\phi \end{aligned}$$

which leads to $\phi = 57°$ (the inverse cosine is double-valued as is the inverse tangent, but we know this is the right solution since both vectors are in the same quadrant).

(b) Since the angle (measured from $+x$) for $\vec{a}$ is $\tan^{-1}(1.6/3.2) = 26.6°$, we know the angle for $\vec{c}$ is $26.6° - 90° = -63.4°$ (the other possibility, $26.6° + 90°$ would lead to a $c_x < 0$). Therefore, $c_x = c\cos -63.4° = (5.0)(0.45) = 2.2$ m.

(c) Also, $c_y = c\sin -63.4° = (5.0)(-0.89) = -4.5$ m.

(d) And we know the angle for $\vec{d}$ to be $26.6° + 90° = 116.6°$, which leads to $d_x = d\cos 116.6° = (5.0)(-0.45) = -2.2$ m.

(e) Finally, $d_y = d\sin 116.6° = (5.0)(0.89) = 4.5$ m.

39. The solution to problem 27 showed that each diagonal has a length given by $a\sqrt{3}$, where a is the length of a cube edge. Vectors along two diagonals are $\vec{b} = a\,\hat{\imath} + a\,\hat{\jmath} + a\,\hat{k}$ and $\vec{c} = -a\,\hat{\imath} + a\,\hat{\jmath} + a\,\hat{k}$. Using Eq. 3-20 with Eq. 3-23, we find the angle between them:

$$\cos\phi = \frac{b_x c_x + b_y c_y + b_z c_z}{bc} = \frac{-a^2 + a^2 + a^2}{3a^2} = \frac{1}{3}\,.$$

The angle is $\phi = \cos^{-1}(1/3) = 70.5°$.

40. (a) The vector equation $\vec{r} = \vec{a} - \vec{b} - \vec{v}$ is computed as follows: $(5.0 - (-2.0) + 4.0)\hat{\imath} + (4.0 - 2.0 + 3.0)\hat{\jmath} + ((-6.0) - 3.0 + 2.0)\hat{k}$. This leads to $\vec{r} = 11\,\hat{\imath} + 5.0\,\hat{\jmath} - 7.0\,\hat{k}$.

(b) We find the angle from $+z$ by "dotting" (taking the scalar product) $\vec{r}$ with $\hat{k}$. Noting that $r = |\vec{r}| = \sqrt{11^2 + 5^2 + (-7)^2} = 14$, Eq. 3-20 with Eq. 3-23 leads to

$$\vec{r}\cdot\hat{k} = -7.0 = (14)(1)\cos\phi \implies \phi = 120°\,.$$

(c) To find the component of a vector in a certain direction, it is efficient to "dot" it (take the scalar product of it) with a unit-vector in that direction. In this case, we make the desired unit-vector by

$$\hat{b} = \frac{\vec{b}}{|\vec{b}|} = \frac{-2\,\hat{\imath} + 2\,\hat{\jmath} + 3\,\hat{k}}{\sqrt{(-2)^2 + 2^2 + 3^2}}\,.$$

We therefore obtain

$$a_b = \vec{a}\cdot\hat{b} = \frac{(5)(-2) + (4)(2) + (-6)(3)}{\sqrt{(-2)^2 + 2^2 + 3^2}} = -4.9\,.$$

(d) One approach (if we all we require is the magnitude) is to use the vector cross product, as the problem suggests; another (which supplies more information) is to subtract the result in part (c) (multiplied by $\hat{b}$) from $\vec{a}$. We briefly illustrate both methods. We note that if $a\cos\theta$ (where θ is the angle between $\vec{a}$ and $\vec{b}$) gives a_b (the component along $\hat{b}$) then we expect $a\sin\theta$ to yield the orthogonal component:

$$a\sin\theta = \frac{|\vec{a}\times\vec{b}|}{b} = 7.3$$

(alternatively, one might compute θ form part (c) and proceed more directly). The second method proceeds as follows:

$$\begin{aligned}\vec{a} - a_b\,\hat{b} &= (5.0 - 2.35)\hat{\imath} + (4.0 - (-2.35))\hat{\jmath} + ((-6.0) - (-3.53))\hat{k} \\ &= 2.65\,\hat{\imath} + 6.35\,\hat{\jmath} - 2.47\,\hat{k}\end{aligned}$$

This describes the perpendicular part of $\vec{a}$ completely. To find the magnitude of this part, we compute

$$\sqrt{2.65^2 + 6.35^2 + (-2.47)^2} = 7.3$$

which agrees with the first method.

41. The volume of a parallelepiped is equal to the product of its altitude and the area of its base. Take the base to be the parallelogram formed by the vectors $\vec{b}$ and $\vec{c}$. Its area is $bc\sin\phi$, where ϕ is the angle between $\vec{b}$ and $\vec{c}$. This is just the magnitude of the vector (cross) product $\vec{b}\times\vec{c}$. The altitude of the parallelepiped is $a\cos\theta$, where θ is the angle between $\vec{a}$ and the normal to the plane of $\vec{b}$ and $\vec{c}$. Since $\vec{b}\times\vec{c}$ is normal to that plane, θ is the angle between $\vec{a}$ and $\vec{b}\times\vec{c}$. Thus, the volume of the parallelepiped is $V = a|\vec{b}\times\vec{c}|\cos\theta = \vec{a}\cdot(\vec{b}\times\vec{c})$.

42. We apply Eq. 3-30 and Eq. 3-23.

(a) $\vec{a}\times\vec{b} = (a_x b_y - a_y b_x)\,\hat{k}$ since all other terms vanish, due to the fact that neither $\vec{a}$ nor $\vec{b}$ have any z components. Consequently, we obtain $((3.0)(4.0) - (5.0)(2.0))\,\hat{k} = 2.0\,\hat{k}$.

(b) $\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y$ yields $(3)(2) + (5)(4) = 26$.

(c) $\vec{a} + \vec{b} = (3+2)\,\hat{\imath} + (5+4)\,\hat{\jmath}$, so that $\left(\vec{a} + \vec{b}\right) \cdot \vec{b} = (5)(2) + (9)(4) = 46$.

(d) Several approaches are available. In this solution, we will construct a $\hat{b}$ unit-vector and "dot" it (take the scalar product of it) with $\vec{a}$. In this case, we make the desired unit-vector by

$$\hat{b} = \frac{\vec{b}}{|\vec{b}|} = \frac{2\,\hat{\imath} + 4\,\hat{\jmath}}{\sqrt{2^2 + 4^2}} \ .$$

We therefore obtain

$$a_b = \vec{a} \cdot \hat{b} = \frac{(3)(2) + (5)(4)}{\sqrt{2^2 + 4^2}} = 5.8 \ .$$

43. We apply Eq. 3-30 and Eq.3-23. If a vector capable calculator is used, this makes a good exercise for getting familiar with those features. Here we briefly sketch the method.

(a) We note that $\vec{b} \times \vec{c} = -8\,\hat{\imath} + 5\,\hat{\jmath} + 6\,\hat{k}$. Thus, $\vec{a} \cdot \left(\vec{b} \times \vec{c}\right) = (3)(-8) + (3)(5) + (-2)(6) = -21$.

(b) We note that $\vec{b} + \vec{c} = 1\,\hat{\imath} - 2\,\hat{\jmath} + 3\,\hat{k}$. Thus, $\vec{a} \cdot \left(\vec{b} + \vec{c}\right) = (3)(1) + (3)(-2) + (-2)(3) = -9$.

(c) Finally, $\vec{a} \times \left(\vec{b} + \vec{c}\right) = ((3)(3) - (-2)(-2))\,\hat{\imath} + ((-2)(1) - (3)(3))\,\hat{\jmath} + ((3)(-2) - (3)(1))\,\hat{k} = 5\,\hat{\imath} - 11\,\hat{\jmath} - 9\,\hat{k}$.

44. The components of $\vec{a}$ are $a_x = 0$, $a_y = 3.20\cos 63° = 1.45$, and $a_z = 3.20\sin 63° = 2.85$. The components of $\vec{b}$ are $b_x = 1.40\cos 48° = 0.937$, $b_y = 0$, and $b_z = 1.40\sin 48° = 1.04$.

(a) The scalar (dot) product is therefore

$$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z = (0)(0.937) + (1.45)(0) + (2.85)(1.04) = 2.97 \ .$$

(b) The vector (cross) product is

$$\begin{aligned} \vec{a} \times \vec{b} &= (a_y b_z - a_z b_y)\,\hat{\imath} + (a_z b_x - a_x b_z)\,\hat{\jmath} + (a_x b_y - a_y b_x)\,\hat{k} \\ &= ((1.45)(1.04) - 0))\,\hat{\imath} + ((2.85)(0.937) - 0))\,\hat{\jmath} + (0 - (1.45)(0.94))\,\hat{k} \\ &= 1.51\,\hat{\imath} + 2.67\hat{\jmath} - 1.36\,\hat{k} \ . \end{aligned}$$

(c) The angle θ between $\vec{a}$ and $\vec{b}$ is given by

$$\theta = \cos^{-1}\left(\frac{\vec{a} \cdot \vec{b}}{ab}\right) = \cos^{-1}\left(\frac{2.96}{(3.30)(1.40)}\right) = 48° \ .$$

45. We observe that $|\hat{\imath} \times \hat{\imath}| = |\hat{\imath}|\,|\hat{\imath}| \sin 0°$ vanishes because $\sin 0° = 0$. Similarly, $\hat{\jmath} \times \hat{\jmath} = \hat{k} \times \hat{k} = 0$. When the unit vectors are perpendicular, we have to do a little more work to show the cross product results. First, the magnitude of the vector $\hat{\imath} \times \hat{\jmath}$ is

$$\left|\hat{\imath} \times \hat{\jmath}\right| = |\hat{\imath}|\,\left|\hat{\jmath}\right| \sin 90°$$

which equals 1 because $\sin 90° = 1$ and these are all units vectors (each has magnitude equal to 1). This is consistent with the claim that $\hat{\imath} \times \hat{\jmath} = \hat{k}$ since the magnitude of $\hat{k}$ is certainly 1. Now, we use the right-hand rule to show that $\hat{\imath} \times \hat{\jmath}$ is in the positive z direction. Thus $\hat{\imath} \times \hat{\jmath}$ has the same magnitude and direction as $\hat{k}$, so it is equal to $\hat{k}$. Similarly, $\hat{k} \times \hat{\imath} = \hat{\jmath}$ and $\hat{\jmath} \times \hat{k} = \hat{\imath}$. If, however, the coordinate system is left-handed, we replace $\hat{k} \to -\hat{k}$ in the work we have shown above and get

$$\hat{\imath} \times \hat{\imath} = \hat{\jmath} \times \hat{\jmath} = \hat{k} \times \hat{k} = 0 \ .$$

just as before. But the relations that are different are

$$\hat{\imath} \times \hat{\jmath} = -\hat{k} \quad \hat{k} \times \hat{\imath} = -\hat{\jmath} \quad \hat{\jmath} \times \hat{k} = -\hat{\imath} \ .$$

46. (a) By the right-hand rule, $\vec{A} \times \vec{B}$ points upward if $\vec{A}$ points north and $\vec{B}$ points west. If $\vec{A}$ and $\vec{B}$ have magnitude$=1$ then, by Eq. 3-27, the result also has magnitude equal to 1.
    (b) Since $\cos 90° = 0$, the scalar dot product between perpendicular vectors is zero. Thus, $\vec{A} \cdot \vec{B} = 0$ is $\vec{A}$ points down and $\vec{B}$ points south.
    (c) By the right-hand rule, $\vec{A} \times \vec{B}$ points south if $\vec{A}$ points east and $\vec{B}$ points up. If $\vec{A}$ and $\vec{B}$ have unit magnitude then, by Eq. 3-27, the result also has unit magnitude.
    (d) Since $\cos 0° = 1$, then $\vec{A} \cdot \vec{B} = AB$ (where A is the magnitude of $\vec{A}$ and B is the magnitude of $\vec{B}$). If, additionally, we have $A = B = 1$, then the result is 1.
    (e) Since $\sin 0° = 0$, $\vec{A} \times \vec{B} = 0$ if both $\vec{A}$ and $\vec{B}$ point south.

47. Let A denote the magnitude of $\vec{A}$; similarly for the other vectors. The vector equation is $\vec{A} + \vec{B} = \vec{C}$ where $B = 8.0$ m and $C = 2A$. We are also told that the angle (measured in the 'standard' sense) for $\vec{A}$ is 0° and the angle for $\vec{C}$ is 90°, which makes this a right triangle (when drawn in a "head-to-tail" fashion) where B is the size of the hypotenuse. Using the Pythagorean theorem,

$$B = \sqrt{A^2 + C^2} \quad \Longrightarrow \quad 8.0 = \sqrt{A^2 + 4A^2}$$

which leads to $A = 8/\sqrt{5} = 3.6$ m.

48. We choose $+x$ east and $+y$ north and measure all angles in the "standard" way (positive ones are counterclockwise from $+x$). Thus, vector $\vec{d_1}$ has magnitude $d_1 = 4$ (with the unit meter and three significant figures assumed) and direction $\theta_1 = 225°$. Also, $\vec{d_2}$ has magnitude $d_2 = 5$ and direction $\theta_2 = 0°$, and vector $\vec{d_3}$ has magnitude $d_3 = 6$ and direction $\theta_3 = 60°$.
    (a) The x-component of $\vec{d_1}$ is $d_1 \cos\theta_1 = -2.83$ m.
    (b) The y-component of $\vec{d_1}$ is $d_1 \sin\theta_1 = -2.83$ m.
    (c) The x-component of $\vec{d_2}$ is $d_2 \cos\theta_2 = 5.00$ m.
    (d) The y-component of $\vec{d_2}$ is $d_2 \sin\theta_2 = 0$.
    (e) The x-component of $\vec{d_3}$ is $d_3 \cos\theta_3 = 3.00$ m.
    (f) The y-component of $\vec{d_3}$ is $d_3 \sin\theta_3 = 5.20$ m.
    (g) The sum of x-components is $-2.83 + 5.00 + 3.00 = 5.17$ m.
    (h) The sum of y-components is $-2.83 + 0 + 5.20 = 2.37$ m.
    (i) The magnitude of the resultant displacement is $\sqrt{5.17^2 + 2.37^2} = 5.69$ m.
    (j) And its angle is $\theta = \tan^{-1}(2.37/5.17) = 24.6°$ which (recalling our coordinate choices) means it points at about 25° north of east.
    (k) and (ℓ) This new displacement (the direct line home) when vectorially added to the previous (net) displacement must give zero. Thus, the new displacement is the negative, or opposite, of the previous (net) displacement. That is, it has the same magnitude (5.69 m) but points in the opposite direction (25° south of west).

49. Reading carefully, we see that the (x, y) specifications for each "dart" are to be interpreted as $(\Delta x, \Delta y)$ descriptions of the corresponding displacement vectors. We combine the different parts of this problem into a single exposition. Thus, along the x axis, we have (with the centimeter unit understood)

$$30.0 + b_x - 20.0 - 80.0 = -140.0 \quad ,$$

and along y axis we have

$$40.0 - 70.0 + c_y - 70.0 = -20.0 \quad .$$

Hence, we find $b_x = -70.0$ cm and $c_y = 80.0$ cm. And we convert the final location $(-140, -20)$ into polar coordinates and obtain $(141 \angle -172°)$, an operation quickly done using a vector capable calculator in polar mode. Thus, the ant is 141 cm from where it started at an angle of $-172°$, which means 172° clockwise from the $+x$ axis or 188° counterclockwise from the $+x$ axis.

50. We find the components and then add them (as scalars, not vectors). With $d = 3.40$ km and $\theta = 35.0°$ we find $d\cos\theta + d\sin\theta = 4.74$ km.

51. We choose $+x$ east and $+y$ north and measure all angles in the "standard" way (positive ones counterclockwise from $+x$, negative ones clockwise). Thus, vector $\vec{d}_1$ has magnitude $d_1 = 3.66$ (with the unit meter and three significant figures assumed) and direction $\theta_1 = 90°$. Also, $\vec{d}_2$ has magnitude $d_2 = 1.83$ and direction $\theta_2 = -45°$, and vector $\vec{d}_3$ has magnitude $d_3 = 0.91$ and direction $\theta_3 = -135°$. We add the x and y components, respectively:

$$\begin{array}{ll} x: & d_1\cos\theta_1 + d_2\cos\theta_2 + d_3\cos\theta_3 = 0.651 \text{ m} \\ y: & d_1\sin\theta_1 + d_2\sin\theta_2 + d_3\sin\theta_3 = 1.723 \text{ m} . \end{array}$$

(a) The magnitude of the direct displacement (the vector sum $\vec{d}_1+\vec{d}_2+\vec{d}_3$) is $\sqrt{0.651^2 + 1.723^2} = 1.84$ m.

(b) The angle (understood in the sense described above) is $\tan^{-1}(1.723/0.651) = 69°$. That is, the first putt must aim in the direction 69° north of east.

52. (a) We write $\vec{b} = b\,\hat{\jmath}$ where $b > 0$. We are asked to consider

$$\frac{\vec{b}}{d} = \left(\frac{b}{d}\right)\hat{\jmath}$$

in the case $d > 0$. Since the coefficient of $\hat{\jmath}$ is positive, then the vector points in the $+y$ direction.

(b) If, however, $d < 0$, then the coefficient is negative and the vector points in the $-y$ direction.

(c) Since $\cos 90° = 0$, then $\vec{a}\cdot\vec{b} = 0$, using Eq. 3-20.

(d) Since $\vec{b}/d$ is along the y axis, then (by the same reasoning as in the previous part) $\vec{a}\cdot(\vec{b}/d) = 0$.

(e) By the right-hand rule, $\vec{a}\times\vec{b}$ points in the $+z$ direction.

(f) By the same rule, $\vec{b}\times\vec{a}$ points in the $-z$ direction. We note that $\vec{b}\times\vec{a} = -\vec{a}\times\vec{b}$ is true in this case and quite generally.

(g) Since $\sin 90° = 1$, Eq. 3-27 gives $|\vec{a}\times\vec{b}| = ab$ where a is the magnitude of $\vec{a}$. Also, $|\vec{a}\times\vec{b}| = |\vec{b}\times\vec{a}|$ so both results have the same magnitude.

(h) and (i) With $d > 0$, we find that $\vec{a}\times(\vec{b}/d)$ has magnitude ab/d and is pointed in the $+z$ direction.

53. (a) With $a = 17.0$ m and $\theta = 56.0°$ we find $a_x = a\cos\theta = 9.51$ m.

(b) And $a_y = a\sin\theta = 14.1$ m.

(c) The angle relative to the new coordinate system is $\theta' = 56 - 18 = 38°$. Thus, $a'_x = a\cos\theta' = 13.4$ m.

(d) And $a'_y = a\sin\theta' = 10.5$ m.

54. Since $\cos 0° = 1$ and $\sin 0° = 0$, these follows immediately from Eq. 3-20 and Eq. 3-27.

55. (a) The magnitude of the vector $\vec{a} = 4.0\vec{d}$ is $(4.0)(2.5) = 10$ m.

(b) The direction of the vector $\vec{a} = 4.0\vec{d}$ is the same as the direction of $\vec{d}$ (north).

(c) The magnitude of the vector $\vec{c} = -3.0\vec{d}$ is $(3.0)(2.5) = 7.5$ m.

(d) The direction of the vector $\vec{c} = -3.0\vec{d}$ is the opposite of the direction of $\vec{d}$. Thus, the direction of $\vec{c}$ is south.

56. The vector sum of the displacements $\vec{d}_{\text{storm}}$ and $\vec{d}_{\text{new}}$ must give the same result as its originally intended displacement $\vec{d}_{\text{o}} = 120\,\hat{\jmath}$ where east is $\hat{\imath}$, north is $\hat{\jmath}$, and the assumed length unit is km. Thus, we write

$$\vec{d}_{\text{storm}} = 100\,\hat{\imath} \quad \text{and} \quad \vec{d}_{\text{new}} = A\,\hat{\imath} + B\,\hat{\jmath} .$$

(a) The equation $\vec{d}_{\rm storm} + \vec{d}_{\rm new} = \vec{d}_{\rm o}$ readily yields $A = -100$ km and $B = 120$ km. The magnitude of $\vec{d}_{\rm new}$ is therefore $\sqrt{A^2 + B^2} = 156$ km.

(b) And its direction is $\tan^{-1}(B/A) = -50.2°$ or $180° + (-50.2°) = 129.8°$. We choose the latter value since it indicates a vector pointing in the second quadrant, which is what we expect here. The answer can be phrased several equivalent ways: 129.8° counterclockwise from east, or 39.8° west from north, or 50.2° north from west.

57. (a) The height is $h = d \sin\theta$, where $d = 12.5$ m and $\theta = 20.0°$. Therefore, $h = 4.28$ m.

(b) The horizontal distance is $d \cos\theta = 11.7$ m.

58. (a) We orient $\hat{\imath}$ eastward, $\hat{\jmath}$ northward, and $\hat{k}$ upward. The displacement in meters is consequently $1000\,\hat{\imath} + 2000\,\hat{\jmath} - 500\,\hat{k}$.

(b) The net displacement is zero since his final position matches his initial position.

59. (a) If we add the equations, we obtain $2\vec{a} = 6\vec{c}$, which leads to $\vec{a} = 3\vec{c} = 9\,\hat{\imath} + 12\,\hat{\jmath}$.

(b) Plugging this result back in, we find $\vec{b} = \vec{c} = 3\,\hat{\imath} + 4\,\hat{\jmath}$.

60. (First problem in **Cluster 1**)
The given angle $\theta = 130°$ is assumed to be measured counterclockwise from the $+x$ axis. Angles (if positive) in our results follow the same convention (but if negative are clockwise from $+x$).

(a) With $A = 4.00$, the x-component of $\vec{A}$ is $A \cos\theta = -2.57$.

(b) The y-component of $\vec{A}$ is $A \sin\theta = 3.06$.

(c) Adding $\vec{A}$ and $\vec{B}$ produces a vector we call R with components $R_x = -6.43$ and $R_y = -1.54$. Using Eq. 3-6 (or special functions on a calculator) we present this in magnitude-angle notation: $\vec{R} = (6.61\ \angle - 167°)$.

(d) From the discussion in the previous part, it is clear that $\vec{R} = -6.43\,\hat{\imath} - 1.54\,\hat{\jmath}$.

(e) The vector $\vec{C}$ is the difference of $\vec{A}$ and $\vec{B}$. In unit-vector notation, this becomes

$$\vec{C} = \vec{A} - \vec{B} = \left(-2.57\,\hat{\imath} - 3.06\,\hat{\jmath}\right) - \left(-3.86\,\hat{\imath} - 4.60\,\hat{\jmath}\right)$$

which yields $\vec{C} = 1.29\,\hat{\imath} + 7.66\,\hat{\jmath}$.

(f) Using Eq. 3-6 (or special functions on a calculator) we present this in magnitude-angle notation: $\vec{C} = (7.77\ \angle\ 80.5°)$.

(g) We note that $\vec{C}$ is the "constant" in all six pictures. Remembering that the negative of a vector simply reverses it, then we see that in form or another, all six pictures express the relation $\vec{C} = \vec{A} - \vec{B}$.

61. (Second problem in **Cluster 1**)

(a) The dot (scalar) product of $3\vec{A}$ and $\vec{B}$ is found using Eq. 3-23:

$$3\vec{A} \cdot \vec{B} = 3\,(4.00 \cos 130°)\,(-3.86) + 3\,(4.00 \sin 130°)\,(-4.60) = -12.5\ .$$

(b) We call the result $\vec{D}$ and combine the scalars ((3)(4) = 12). Thus, $\vec{D} = (4\vec{A}) \times (3\vec{B})$ becomes, using Eq. 3-30,

$$12\vec{A} \times \vec{B} = 12\,((4.00 \cos 130°)\,(-4.60) - (4.00 \sin 130°)\,(-3.86))\,\hat{k}$$

which yields $\vec{D} = 284\,\hat{k}$.

(c) Since $\vec{D}$ has magnitude 284 and points in the $+z$ direction, it has radial coordinate 284 and angle-measured-from-z-axis equal to 0°. The angle measured in the xy plane does not have a well-defined value (since this vector does not have a component in that plane)

(d) Since $\vec{A}$ is in the xy plane, then it is clear that $\vec{A} \perp \vec{D}$. The angle between them is $90°$.

(e) Calling this new result $\vec{G}$ we have

$$\vec{G} = (4.00 \cos 130°)\,\hat{\imath} + (4.00 \sin 130°)\,\hat{\jmath} + (3.00)\hat{k}$$

which yields $\vec{G} = -2.57\,\hat{\imath} + 3.06\hat{\jmath} + 3.00\,\hat{k}$.

(f) It is straightforward using a vector-capable calculator to convert the above into spherical coordinates. We, however, proceed "the hard way", using the notation in Fig. 3-44 (where θ is in the xy plane and ϕ is measured from the z axis):

$$\begin{aligned} |\vec{G}| = r &= \sqrt{(-2.57)^2 + 3.06^2 + 3.00^2} = 5.00 \\ \phi &= \tan^{-1}(4.00/3.00) = 53.1° \\ \theta &= 130° \quad \text{given in problem 60} . \end{aligned}$$

62. (Third problem in **Cluster 1**)

(a) Looking at the xy plane in Fig. 3-44, it is clear that the angle to $\vec{A}$ (which is the vector lying *in* the plane, not the one rising out of it, which we called $\vec{G}$ in the previous problem) measured counterclockwise from the $-y$ axis is $90° + 130° = 220°$. Had we measured this *clockwise* we would obtain (in absolute value) $360° - 220° = 140°$.

(b) We found in part (b) of the previous problem that $\vec{A} \times \vec{B}$ points along the z axis, so it is perpendicular to the $-y$ direction.

(c) Let $\vec{u} = -\hat{\jmath}$ represent the $-y$ direction, and $\vec{w} = 3\,\hat{k}$ is the vector being added to $\vec{B}$ in this problem. The vector being examined in this problem (we'll call it $\vec{Q}$) is, using Eq. 3-30 (or a vector-capable calculator),

$$\vec{Q} = \vec{A} \times \left(\vec{B} + \vec{w}\right) = 9.19\,\hat{\imath} + 7.71\,\hat{\jmath} + 23.7\,\hat{k}$$

and is clearly in the first octant (since all components are positive); using Pythagorean theorem, its magnitude is $Q = 26.52$. From Eq. 3-23, we immediately find $\vec{u} \cdot \vec{Q} = -7.71$. Since $\vec{u}$ has unit magnitude, Eq. 3-20 leads to

$$\cos^{-1}\left(\frac{\vec{u} \cdot \vec{Q}}{Q}\right) = \cos^{-1}\left(\frac{-7.71}{26.52}\right)$$

which yields a choice of angles $107°$ or $-107°$. Since we have already observed that $\vec{Q}$ is in the first octant, the the angle measured counterclockwise (as observed by someone high up on the $+z$ axis) from the $-y$ axis to $\vec{Q}$ is $107°$.