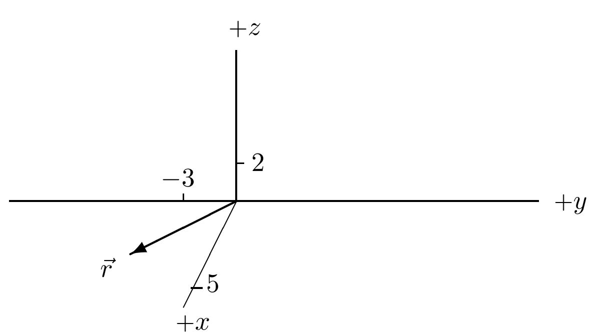
Chapter 4

1. Where the length unit is not specified (in this solution), the unit meter should be understood.
   (a) The position vector, according to Eq. 4-1, is $\vec{r} = -5.0\,\hat{\imath} + 8.0\,\hat{\jmath}$ (in meters).
   (b) The magnitude is $|\vec{r}| = \sqrt{x^2 + y^2 + z^2} = 9.4$ m.
   (c) Many calculators have polar $\leftrightarrow$ rectangular conversion capabilities which make this computation more efficient than what is shown below. Noting that the vector lies in the xy plane, we are using Eq. 3-6:
   $$\tan^{-1}\left(\frac{8.0}{-5.0}\right) = -58^\circ \quad \text{or} \quad 122^\circ$$
   where we choose the latter possibility (122° measured counterclockwise from the $+x$ direction) since the signs of the components imply the vector is in the second quadrant.
   (d) In the interest of saving space, we omit the sketch. The vector is 32° counterclockwise from the $+y$ direction, where the $+y$ direction is assumed to be (as is standard) $+90^\circ$ counterclockwise from $+x$, and the $+z$ direction would therefore be "out of the paper."
   (e) The displacement is $\Delta\vec{r} = \vec{r}\,' - \vec{r}$ where $\vec{r}$ is given in part (a) and $\vec{r}\,' = 3.0\,\hat{\imath}$. Therefore, $\Delta\vec{r} = 8.0\,\hat{\imath} - 8.0\,\hat{\jmath}$ (in meters).
   (f) The magnitude of the displacement is $|\Delta\vec{r}| = \sqrt{8^2 + (-8)^2} = 11$ m.
   (g) The angle for the displacement, using Eq. 3-6, is found from
   $$\tan^{-1}\left(\frac{8.0}{-8.0}\right) = -45^\circ \quad \text{or} \quad 135^\circ$$
   where we choose the former possibility (-45°, which means 45° measured clockwise from $+x$, or 315° counterclockwise from $+x$) since the signs of the components imply the vector is in the fourth quadrant.

2. (a) The magnitude of $\vec{r}$ is $\sqrt{5.0^2 + (-3.0)^2 + 2.0^2} = 6.2$ m.
   (b) A sketch is shown. The coordinate values are in meters.



3. Where the unit is not specified, the unit meter is understood. We use Eq. 4-2 and Eq. 4-3.
   (a) With the initial position vector as $\vec{r}_1$ and the later vector as $\vec{r}_2$, Eq. 4-3 yields
   $$\Delta r = ((-2) - 5)\,\hat{\imath} + (6 - (-6))\,\hat{\jmath} + (2 - 2)\,\hat{k} = -7.0\,\hat{\imath} + 12\,\hat{\jmath}$$
   for the displacement vector in unit-vector notation (in meters).
   (b) Since there is no z component (that is, the coefficient of $\hat{k}$ is zero), the displacement vector is in the xy plane.

4. We use a coordinate system with $+x$ eastward and $+y$ upward. We note that 123° is the angle between the initial position and later position vectors, so that the angle from $+x$ to the later position vector is $40° + 123° = 163°$. In unit-vector notation, the position vectors are
   $$\begin{aligned}\vec{r}_1 &= 360\cos(40°)\,\hat{\imath} + 360\sin(40°)\,\hat{\jmath} = 276\,\hat{\imath} + 231\,\hat{\jmath}\\ \vec{r}_2 &= 790\cos(163°)\,\hat{\imath} + 790\sin(163°)\,\hat{\jmath} = -755\,\hat{\imath} + 231\,\hat{\jmath}\end{aligned}$$
   respectively (in meters). Consequently, we plug into Eq. 4-3
   $$\Delta r = ((-755) - 276)\,\hat{\imath} + (231 - 231)\,\hat{\jmath}$$
   and find the displacement vector is horizontal (westward) with a length of 1.03 km. If unit-vector notation is not a priority in this problem, then the computation can be approached in a variety of ways – particularly in view of the fact that a number of vector capable calculators are on the market which reduce this problem to a very few keystrokes (using magnitude-angle notation throughout).

5. The average velocity is given by Eq. 4-8. The total displacement $\Delta\vec{r}$ is the sum of three displacements, each result of a (constant) velocity during a given time. We use a coordinate system with $+x$ East and $+y$ North. In unit-vector notation, the first displacement is given by
   $$\Delta\vec{r}_1 = \left(60\,\frac{\text{km}}{\text{h}}\right)\left(\frac{40\,\text{min}}{60\,\text{min/h}}\right)\hat{\imath} = 40\,\hat{\imath}$$
   in kilometers. The second displacement has a magnitude of $60\,\frac{\text{km}}{\text{h}} \cdot \frac{20\,\text{min}}{60\,\text{min/h}} = 20$ km, and its direction is 40° north of east. Therefore,
   $$\Delta\vec{r}_2 = 20\cos(40°)\,\hat{\imath} + 20\sin(40°)\,\hat{\jmath} = 15.3\,\hat{\imath} + 12.9\,\hat{\jmath}$$
   in kilometers. And the third displacement is
   $$\Delta\vec{r}_3 = -\left(60\,\frac{\text{km}}{\text{h}}\right)\left(\frac{50\,\text{min}}{60\,\text{min/h}}\right)\hat{\imath} = -50\,\hat{\imath}$$
   in kilometers. The total displacement is
   $$\begin{aligned}\Delta\vec{r} &= \Delta\vec{r}_1 + \Delta\vec{r}_2 + \Delta\vec{r}_3\\ &= 40\,\hat{\imath} + 15.3\,\hat{\imath} + 12.9\,\hat{\jmath} - 50\,\hat{\imath}\\ &= 5.3\,\hat{\imath} + 12.9\,\hat{\jmath}\quad(\text{km})\;.\end{aligned}$$
   The time for the trip is $40 + 20 + 50 = 110$ min, which is equivalent to 1.83 h. Eq. 4-8 then yields
   $$\vec{v}_{\text{avg}} = \left(\frac{5.3\,\text{km}}{1.83\,\text{h}}\right)\hat{\imath} + \left(\frac{12.9\,\text{km}}{1.83\,\text{h}}\right)\hat{\jmath} = 2.90\,\hat{\imath} + 7.01\,\hat{\jmath}$$
   in kilometers-per-hour. If it is desired to express this in magnitude-angle notation, then this is equivalent to a vector of magnitude $\sqrt{2.9^2 + 7.01^2} = 7.59$ km/h, which is inclined 67.5° north of east (or, 22.5° east of north). If unit-vector notation is not a priority in this problem, then the computation can be approached in a variety of ways – particularly in view of the fact that a number of vector capable calculators are on the market which reduce this problem to a very few keystrokes (using magnitude-angle notation throughout).

6. Using Eq. 4-3 and Eq. 4-8, we have

$$\begin{aligned}\vec{v}_{\text{avg}} &= \frac{(-2.0\,\hat{\imath}+8.0\,\hat{\jmath}-2.0\,\hat{k})-(5.0\,\hat{\imath}-6.0\,\hat{\jmath}+2.0\,\hat{k})}{10}\\ &= -0.70\,\hat{\imath}+1.40\,\hat{\jmath}-0.40\,\hat{k}\end{aligned}$$

in meters-per-second.

7. To emphasize the fact that the velocity is a function of time, we adopt the notation $v(t)$ for $\frac{dx}{dt}$.

   (a) Eq. 4-10 leads to
   $$v(t)=\frac{d}{dt}\left(3.00t\,\hat{\imath}-4.00t^2\,\hat{\jmath}+2.00\,\hat{k}\right)=3.00\,\hat{\imath}-8.00t\,\hat{\jmath}$$
   in meters-per-second.

   (b) Evaluating this result at $t = 2$ s produces $\vec{v} = 3.0\,\hat{\imath} - 16.0\,\hat{\jmath}$ m/s.

   (c) The speed at $t = 2$ s is $v = |\vec{v}| = \sqrt{3^2 + (-16)^2} = 16.3$ m/s.

   (d) And the angle of $\vec{v}$ at that moment is one of the possibilities
   $$\tan^{-1}\left(\frac{-16}{3}\right) = -79.4^\circ \quad\text{or}\quad 101^\circ$$
   where we choose the first possibility (79.4° measured clockwise from the $+x$ direction, or 281° counterclockwise from $+x$) since the signs of the components imply the vector is in the fourth quadrant.

8. On the one hand, we could perform the vector addition of the displacements with a vector capable calculator in polar mode $((75\ \angle\ 37^\circ) + (65\ \angle -90^\circ) = (63\ \angle -18^\circ))$, but in keeping with Eq. 3-5 and Eq. 3-6 we will show the details in unit-vector notation. We use a 'standard' coordinate system with $+x$ East and $+y$ North. Lengths are in kilometers and times are in hours.

   (a) We perform the vector addition of individual displacements to find the net displacement of the camel.
   $$\begin{aligned}\Delta\vec{r}_1 &= 75\cos(37^\circ)\,\hat{\imath}+75\sin(37^\circ)\,\hat{\jmath}\\ \Delta\vec{r}_2 &= -65\,\hat{\jmath}\\ \Delta\vec{r}_1+\Delta\vec{r}_2 &= 60\,\hat{\imath}-20\,\hat{\jmath}\quad\text{km}\ .\end{aligned}$$
   If it is desired to express this in magnitude-angle notation, then this is equivalent to a vector of length $\sqrt{60^2 + (-20)^2} = 63$ km, which is directed at 18° south of east.

   (b) We use the result from part (a) in Eq. 4-8 along with the fact that $\Delta t = 90$ h. In unit vector notation, we obtain
   $$\vec{v}_{\text{avg}}=\frac{60\,\hat{\imath}-20\,\hat{\jmath}}{90}=0.66\,\hat{\imath}-0.22\,\hat{\jmath}$$
   in kilometers-per-hour. This result in magnitude-angle notation is $\vec{v}_{\text{avg}} = 0.70$ km/h at 18° south of east.

   (c) Average speed is distinguished from the magnitude of average velocity in that it depends on the total distance as opposed to the net displacement. Since the camel travels 140 km, we obtain $140/90 = 1.56$ km/h.

   (d) The net displacement is required to be the 90 km East from A to B. The displacement from the resting place to B is denoted $\vec{r}_3$. Thus, we must have (in kilometers)
   $$\vec{r}_1+\vec{r}_2+\vec{r}_3=90\,\hat{\imath}$$

which produces $\vec{r}_3 = 30\,\hat{\imath} + 20\,\hat{\jmath}$ in unit-vector notation, or $(36 \angle\ 33^\circ)$ in magnitude-angle notation. Therefore, using Eq. 4-8 we obtain

$$|\vec{v}_{\text{avg}}| = \frac{36 \text{ km}}{120 - 90 \text{ h}} = 1.2 \text{ km/h}$$

and the direction of this vector is the same as $\vec{r}_3$ (that is, 33° north of east).

9. We apply Eq. 4-10 and Eq. 4-16.

   (a) Taking the derivative of the position vector with respect to time, we have

   $$\vec{v} = \frac{d}{dt}\left(\hat{\imath} + 4t^2\,\hat{\jmath} + t\,\hat{k}\right) = 8t\,\hat{\jmath} + \hat{k}$$

   in SI units (m/s).

   (b) Taking another derivative with respect to time leads to

   $$\vec{a} = \frac{d}{dt}\left(8t\,\hat{\jmath} + \hat{k}\right) = 8\,\hat{\jmath}$$

   in SI units (m/s^2).

10. We use Eq. 4-15 with $\vec{v}_1$ designating the initial velocity and $\vec{v}_2$ designating the later one.

    (a) The average acceleration during the $\Delta t = 4$ s interval is

    $$\vec{a}_{\text{avg}} = \frac{\left(-2\,\hat{\imath} - 2\,\hat{\jmath} + 5\,\hat{k}\right) - \left(4\,\hat{\imath} - 22\,\hat{\jmath} + 3\,\hat{k}\right)}{4} = -1.5\,\hat{\imath} + 0.5\,\hat{k}$$

    in SI units (m/s^2).

    (b) The magnitude of $\vec{a}_{\text{avg}}$ is $\sqrt{(-1.5)^2 + 0.5^2} = 1.6$ m/s^2. Its angle in the xz plane (measured from the $+x$ axis) is one of these possibilities:

    $$\tan^{-1}\left(\frac{0.5}{-1.5}\right) = -18^\circ \quad \text{or} \quad 162^\circ$$

    where we settle on the second choice since the signs of its components imply that it is in the second quadrant.

11. In parts (b) and (c), we use Eq. 4-10 and Eq. 4-16. For part (d), we find the direction of the velocity computed in part (b), since that represents the asked-for tangent line.

    (a) Plugging into the given expression, we obtain

    $$\vec{r}\,\Big|_{t=2} = (2(8) - 5(2))\,\hat{\imath} + (6 - 7(16))\,\hat{\jmath} = 6.00\,\hat{\imath} - 106\,\hat{\jmath}$$

    in meters.

    (b) Taking the derivative of the given expression produces

    $$\vec{v}(t) = \left(6.00t^2 - 5.00\right)\,\hat{\imath} + 28.0t^3\,\hat{\jmath}$$

    where we have written $v(t)$ to emphasize its dependence on time. This becomes, at $t = 2.00$ s, $\vec{v} = 19.0\,\hat{\imath} - 224\,\hat{\jmath}$ m/s.

    (c) Differentiating the $\vec{v}(t)$ found above, with respect to t produces $12.0t\,\hat{\imath} - 84.0t^2\,\hat{\jmath}$, which yields $\vec{a} = 24.0\,\hat{\imath} - 336\,\hat{\jmath}$ m/s^2 at $t = 2.00$ s.

(d) The angle of $\vec{v}$, measured from $+x$, is either

$$\tan^{-1}\left(\frac{-224}{19.0}\right) = -85.2^\circ \quad \text{or} \quad 94.8^\circ$$

where we settle on the first choice (-85.2°, which is equivalent to 275°) since the signs of its components imply that it is in the fourth quadrant.

12. Noting that $\vec{v}_2 = 0$, then, using Eq. 4-15, the average acceleration is

$$\vec{a}_{\text{avg}} = \frac{\Delta\vec{v}}{\Delta t} = \frac{0 - (6.30\,\hat{\imath} - 8.42\,\hat{\jmath})}{3} = -2.1\,\hat{\imath} + 2.8\,\hat{\jmath}$$

in SI units (m/s^2).

13. Constant acceleration in both directions (x and y) allows us to use Table 2-1 for the motion along each direction. This can be handled individually (for Δx and Δy) or together with the unit-vector notation (for Δr). Where units are not shown, SI units are to be understood.

(a) The velocity of the particle at any time t is given by $\vec{v} = \vec{v}_0 + \vec{a}t$, where $\vec{v}_0$ is the initial velocity and $\vec{a}$ is the (constant) acceleration. The x component is $v_x = v_{0x} + a_x t = 3.00 - 1.00t$, and the y component is $v_y = v_{0y} + a_y t = -0.500t$ since $v_{0y} = 0$. When the particle reaches its maximum x coordinate at $t = t_m$, we must have $v_x = 0$. Therefore, $3.00 - 1.00t_m = 0$ or $t_m = 3.00$ s. The y component of the velocity at this time is $v_y = 0 - 0.500(3.00) = -1.50$ m/s; this is the only nonzero component of $\vec{v}$ at t_m.

(b) Since it started at the origin, the coordinates of the particle at any time t are given by $\vec{r} = \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$. At $t = t_m$ this becomes

$$(3.00\,\hat{\imath})(3.00) + \frac{1}{2}(-1.00\,\hat{\imath} - 0.50\,\hat{\jmath})(3.00)^2 = 4.50\,\hat{\imath} - 2.25\,\hat{\jmath}$$

in meters.

14. (a) Using Eq. 4-16, the acceleration as a function of time is

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left((6.0t - 4.0t^2)\hat{\imath} + 8.0\,\hat{\jmath}\right) = (6.0 - 8.0t)\hat{\imath}$$

in SI units. Specifically, we find the acceleration vector at $t = 3.0$ s to be $(6.0 - 8.0(3.0))\hat{\imath} = -18\,\hat{\imath}$ m/s^2.

(b) The equation is $\vec{a} = (6.0 - 8.0t)\hat{\imath} = 0$; we find $t = 0.75$ s.

(c) Since the y component of the velocity, $v_y = 8.0$ m/s, is never zero, the velocity cannot vanish.

(d) Since speed is the magnitude of the velocity, we have $v = |\vec{v}| = \sqrt{(6.0t - 4.0t^2)^2 + (8.0)^2} = 10$ in SI units (m/s). We solve for t as follows:

$$\begin{aligned}
\text{squaring} \quad (6t - 4t^2)^2 + 64 &= 100 \\
\text{rearranging} \quad (6t - 4t^2)^2 &= 36 \\
\text{taking square root} \quad 6t - 4t^2 &= \pm 6 \\
\text{rearranging} \quad 4t^2 - 6t \pm 6 &= 0 \\
\text{using quadratic formula} \quad t &= \frac{6 \pm \sqrt{36 - 4(4)(\pm 6)}}{2(8)}
\end{aligned}$$

where the requirement of a real positive result leads to the unique answer: $t = 2.2$ s.

15. Since the x and y components of the acceleration are constants, then we can use Table 2-1 for the motion along both axes. This can be handled individually (for Δx and Δy) or together with the unit-vector notation (for Δr). Where units are not shown, SI units are to be understood.

(a) Since $\vec{r}_0 = 0$, the position vector of the particle is (adapting Eq. 2-15)

$$\begin{aligned}\vec{r} &= \vec{v}_0 t + \frac{1}{2}\vec{a}t^2 \\ &= (8.0\,\hat{\jmath})t + \frac{1}{2}(4.0\,\hat{\imath} + 2.0\,\hat{\jmath})t^2 \\ &= (2.0t^2)\,\hat{\imath} + (8.0t + 1.0t^2)\,\hat{\jmath}\ .\end{aligned}$$

Therefore, we find when $x = 29$ m, by solving $2.0t^2 = 29$, which leads to $t = 3.8$ s. The y coordinate at that time is $y = 8.0(3.8) + 1.0(3.8)^2 = 45$ m.

(b) Adapting Eq. 2-11, the velocity of the particle is given by

$$\vec{v} = \vec{v}_0 + \vec{a}t\ .$$

Thus, at $t = 3.8$ s, the velocity is

$$\vec{v} = 8.0\,\hat{\jmath} + (4.0\,\hat{\imath} + 2.0\,\hat{\jmath})(3.8) = 15.2\,\hat{\imath} + 15.6\,\hat{\jmath}$$

which has a magnitude of

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{15.2^2 + 15.6^2} = 22\ \text{m/s}\ .$$

16. The acceleration is constant so that use of Table 2-1 (for both the x and y motions) is permitted. Where units are not shown, SI units are to be understood. Collision between particles A and B requires two things. First, the y motion of B must satisfy (using Eq. 2-15 and noting that θ is measured from the y axis)

$$y = \frac{1}{2}a_y t^2 \implies 30 = \frac{1}{2}\left(0.40\cos\theta\right)t^2\ .$$

Second, the x motions of A and B must coincide:

$$vt = \frac{1}{2}a_x t^2 \implies 3.0t = \frac{1}{2}\left(0.40\sin\theta\right)t^2\ .$$

We eliminate a factor of t in the last relationship and formally solve for time:

$$t = \frac{3}{0.2\sin\theta}\ .$$

This is then plugged into the previous equation to produce

$$30 = \frac{1}{2}\left(0.40\cos\theta\right)\left(\frac{3}{0.2\sin\theta}\right)^2$$

which, with the use of $\sin^2\theta = 1 - \cos^2\theta$, simplifies to

$$30 = \frac{9}{0.2}\,\frac{\cos\theta}{1-\cos^2\theta} \implies 1 - \cos^2\theta = \frac{9}{(0.2)(30)}\,\cos\theta\ .$$

We use the quadratic formula (choosing the positive root) to solve for $\cos\theta$:

$$\cos\theta = \frac{-1.5 + \sqrt{1.5^2 - 4(1)(-1)}}{2} = \frac{1}{2}$$

which yields

$$\theta = \cos^{-1}\left(\frac{1}{2}\right) = 60^\circ\ .$$

17. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable.

    (a) With the origin at the firing point, the y coordinate of the bullet is given by $y = -\frac{1}{2}gt^2$. If t is the time of flight and $y = -0.019$ m indicates where the bullet hits the target, then

    $$t = \sqrt{\frac{2(0.019)}{9.8}} = 6.2 \times 10^{-2} \text{ s} .$$

    (b) The muzzle velocity is the initial (horizontal) velocity of the bullet. Since $x = 30$ m is the horizontal position of the target, we have $x = v_0 t$. Thus,

    $$v_0 = \frac{x}{t} = \frac{30}{6.3 \times 10^{-2}} = 4.8 \times 10^2 \text{ m/s} .$$

18. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable.

    (a) With the origin at the initial point (edge of table), the y coordinate of the ball is given by $y = -\frac{1}{2}gt^2$. If t is the time of flight and $y = -1.20$ m indicates the level at which the ball hits the floor, then

    $$t = \sqrt{\frac{2(1.20)}{9.8}} = 0.495 \text{ s} .$$

    (b) The initial (horizontal) velocity of the ball is $\vec{v} = v_0\,\hat{\imath}$. Since $x = 1.52$ m is the horizontal position of its impact point with the floor, we have $x = v_0 t$. Thus,

    $$v_0 = \frac{x}{t} = \frac{1.52}{0.495} = 3.07 \text{ m/s} .$$

19. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The initial velocity is horizontal so that $v_{0y} = 0$ and $v_{0x} = v_0 = 161$ km/h. Converting to SI units, this is $v_0 = 44.7$ m/s.

    (a) With the origin at the initial point (where the ball leaves the pitcher's hand), the y coordinate of the ball is given by $y = -\frac{1}{2}gt^2$, and the x coordinate is given by $x = v_0 t$. From the latter equation, we have a simple proportionality between horizontal distance and time, which means the time to travel half the total distance is half the total time. Specifically, if $x = 18.3/2$ m, then $t = (18.3/2)/44.7 = 0.205$ s.

    (b) And the time to travel the next $18.3/2$ m must also be 0.205 s. It can be useful to write the horizontal equation as $\Delta x = v_0 \Delta t$ in order that this result can be seen more clearly.

    (c) From $y = -\frac{1}{2}gt^2$, we see that the ball has reached the height of $-\frac{1}{2}(9.8)(0.205)^2 = -0.205$ m at the moment the ball is halfway to the batter.

    (d) The ball's height when it reaches the batter is $-\frac{1}{2}(9.8)(0.409)^2 = -0.820$ m, which, when subtracted from the previous result, implies it has fallen another 0.615 m. Since the value of y is not simply proportional to t, we do not expect equal time-intervals to correspond to equal height-changes; in a physical sense, this is due to the fact that the initial y-velocity for the first half of the motion is not the same as the "initial" y-velocity for the second half of the motion.

20. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The initial velocity is horizontal so that $v_{0y} = 0$ and $v_{0x} = v_0 = 10$ m/s.

    (a) With the origin at the initial point (where the dart leaves the thrower's hand), the y coordinate of the dart is given by $y = -\frac{1}{2}gt^2$, so that with $y = -PQ$ we have $PQ = \frac{1}{2}(9.8)(0.19)^2 = 0.18$ m.

    (b) From $x = v_0 t$ we obtain $x = (10)(0.19) = 1.9$ m.

21. Since this problem involves constant downward acceleration of magnitude a, similar to the projectile motion situation, we use the equations of §4-6 as long as we substitute a for g. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The initial velocity is horizontal so that $v_{0y} = 0$ and $v_{0x} = v_0 = 1.0 \times 10^9$ cm/s.

   (a) If ℓ is the length of a plate and t is the time an electron is between the plates, then $\ell = v_0 t$, where v_0 is the initial speed. Thus

   $$t = \frac{\ell}{v_0} = \frac{2.0\,\text{cm}}{1.0 \times 10^9\,\text{cm/s}} = 2.0 \times 10^{-9}\ \text{s} .$$

   (b) The vertical displacement of the electron is

   $$y = -\frac{1}{2}at^2 = -\frac{1}{2}\left(1.0 \times 10^{17}\ \text{cm/s}^2\right)\left(2.0 \times 10^{-9}\,\text{s}\right)^2 = -0.20\ \text{cm} .$$

   (c) and (d) The x component of velocity does not change: $v_x = v_0 = 1.0 \times 10^9\,\text{cm/s}$, and the y component is

   $$v_y = a_y t = \left(1.0 \times 10^{17}\ \text{cm/s}^2\right)\left(2.0 \times 10^{-9}\,\text{s}\right) = 2.0 \times 10^8\,\text{cm/s} .$$

22. We use Eq. 4-26

   $$R_{\text{max}} = \left(\frac{v_0^2}{g}\sin 2\theta_0\right)_{\text{max}} = \frac{v_0^2}{g} = \frac{(9.5\,\text{m/s})^2}{9.80\,\text{m/s}^2} = 9.21\ \text{m}$$

   to compare with Powell's long jump; the difference from R_{max} is only $\Delta R = 9.21 - 8.95 = 0.26$ m.

23. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is throwing point (the stone's initial position). The x component of its initial velocity is given by $v_{0x} = v_0 \cos\theta_0$ and the y component is given by $v_{0y} = v_0 \sin\theta_0$, where $v_0 = 20$ m/s is the initial speed and $\theta_0 = 40.0°$ is the launch angle.

   (a) At $t = 1.10\,\text{s}$, its x coordinate is

   $$x = v_0 t\cos\theta_0 = (20.0\,\text{m/s})(1.10\,\text{s})\cos 40.0° = 16.9\,\text{m}$$

   (b) Its y coordinate at that instant is

   $$y = v_0 t\sin\theta_0 - \frac{1}{2}gt^2 = (20.0\,\text{m/s})(1.10\,\text{s})\sin 40° - \frac{1}{2}(9.80\,\text{m/s}^2)(1.10\,\text{s})^2 = 8.21\,\text{m}.$$

   (c) At $t' = 1.80\,\text{s}$, its x coordinate is

   $$x = (20.0\,\text{m/s})(1.80\,\text{s})\cos 40.0° = 27.6\,\text{m}$$

   (d) Its y coordinate at t' is

   $$y = (20.0\,\text{m/s})(1.80\,\text{s})\sin 40° - \frac{1}{2}\left(9.80\,\text{m/s}^2\right)(1.80\,\text{s})^2 = 7.26\ \text{m} .$$

   (e) and (f) The stone hits the ground earlier than $t = 5.0\,\text{s}$. To find the time when it hits the ground solve $y = v_0 t\sin\theta_0 - \frac{1}{2}gt^2 = 0$ for t. We find

   $$t = \frac{2v_0}{g}\sin\theta_0 = \frac{2(20.0\,\text{m/s})}{9.8\,\text{m/s}^2}\sin 40° = 2.62\ \text{s} .$$

   Its x coordinate on landing is

   $$x = v_0 t\cos\theta_0 = (20.0\,\text{m/s})(2.62\,\text{s})\cos 40° = 40.2\ \text{m}$$

   (or Eq. 4-26 can be used). Assuming it stays where it lands, its coordinates at $t = 5.00\,\text{s}$ are $x = 40.2\,\text{m}$ and $y = 0$.

24. In this projectile motion problem, we have $v_0 = v_x =$ constant, and what is plotted is $v = \sqrt{v_x^2 + v_y^2}$. We infer from the plot that at $t = 2.5$ s, the ball reaches its maximum height, where $v_y = 0$. Therefore, we infer from the graph that $v_x = 19$ m/s.

    (a) During $t = 5$ s, the horizontal motion is $x - x_0 = v_x t = 95$ m.

    (b) Since $\sqrt{19^2 + v_{0y}^2} = 31$ m/s (the first point on the graph), we find $v_{0y} = 24.5$ m/s. Thus, with $t = 2.5$ s, we can use $y_{\max} - y_0 = v_{0y}t - \frac{1}{2}gt^2$ or $v_y^2 = 0 = v_{0y}^2 - 2g(y_{\max} - y_0)$, or $y_{\max} - y_0 = \frac{1}{2}\left(v_y + v_{0y}\right)t$ to solve. Here we will use the latter:

    $$y_{\max} - y_0 = \frac{1}{2}\left(v_y + v_{0y}\right)t \implies y_{\max} = \frac{1}{2}(0 + 24.5)(2.5) = 31 \text{ m}$$

    where we have taken $y_0 = 0$ as the ground level.

25. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at the end of the rifle (the initial point for the bullet as it begins projectile motion in the sense of §4-5), and we let θ_0 be the firing angle. If the target is a distance d away, then its coordinates are $x = d$, $y = 0$. The projectile motion equations lead to $d = v_0 t\cos\theta_0$ and $0 = v_0 t\sin\theta_0 - \frac{1}{2}gt^2$. Eliminating t leads to $2v_0^2\sin\theta_0\cos\theta_0 - gd = 0$. Using $\sin\theta_0\cos\theta_0 = \frac{1}{2}\sin(2\theta_0)$, we obtain

    $$v_0^2\sin(2\theta_0) = gd \implies \sin(2\theta_0) = \frac{gd}{v_0^2} = \frac{(9.8)(45.7)}{460^2}$$

    which yields $\sin(2\theta_0) = 2.12 \times 10^{-3}$ and consequently $\theta_0 = 0.0606°$. If the gun is aimed at a point a distance ℓ above the target, then $\tan\theta_0 = \ell/d$ so that

    $$\ell = d\tan\theta_0 = 45.7\tan 0.0606° = 0.0484\,\text{m} = 4.84 \text{ cm} .$$

26. The figure offers many interesting points to analyze, and others are easily inferred (such as the point of maximum height). The focus here, to begin with, will be the final point shown (1.25 s after the ball is released) which is when the ball returns to its original height. In English units, $g = 32$ ft/s^2.

    (a) Using $x - x_0 = v_x t$ we obtain $v_x = (40 \text{ ft})/(1.25 \text{ s}) = 32$ ft/s. And $y - y_0 = 0 = v_{0y}t - \frac{1}{2}gt^2$ yields $v_{0y} = \frac{1}{2}(32)(1.25) = 20$ ft/s. Thus, the initial speed is

    $$v_0 = |\vec{v}_0| = \sqrt{32^2 + 20^2} = 38 \text{ ft/s} .$$

    (b) Since $v_y = 0$ at the maximum height and the horizontal velocity stays constant, then the speed at the top is the same as $v_x = 32$ ft/s.

    (c) We can infer from the figure (or compute from $v_y = 0 = v_{0y} - gt$) that the time to reach the top is 0.625 s. With this, we can use $y - y_0 = v_{0y}t - \frac{1}{2}gt^2$ to obtain 9.3 ft (where $y_0 = 3$ ft has been used). An alternative approach is to use $v_y^2 = v_{0y}^2 - 2g(y - y_0)$.

27. Taking the y axis to be upward and placing the origin at the firing point, the y coordinate is given by $y = v_0 t\sin\theta_0 - \frac{1}{2}gt^2$ and the y component of the velocity is given by $v_y = v_0\sin\theta_0 - gt$. The maximum height occurs when $v_y = 0$. Thus, $t = (v_0/g)\sin\theta_0$ and

    $$y = v_0\left(\frac{v_0}{g}\right)\sin\theta_0\sin\theta_0 - \frac{1}{2}\frac{g(v_0\sin\theta_0)^2}{g^2} = \frac{(v_0\sin\theta_0)^2}{2g} .$$

28. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at the release point (the initial position for the ball as it begins projectile motion in the sense of §4-5), and we let θ_0 be the angle of throw (shown in the figure). Since the horizontal component of the velocity of the ball is $v_x = v_0\cos 40.0°$, the time it takes for the ball to hit the wall is

    $$t = \frac{\Delta x}{v_x} = \frac{22.0}{25.0\cos 40.0°} = 1.15 \text{ s} .$$

(a) The vertical distance is

$$\begin{aligned}\Delta y &= (v_0 \sin\theta_0)t - \frac{1}{2}gt^2 \\ &= (25.0\sin 40.0^\circ)(1.15) - \frac{1}{2}(9.8)(1.15)^2 = 12.0 \text{ m} .\end{aligned}$$

(b) The horizontal component of the velocity when it strikes the wall does not change from its initial value: $v_x = v_0 \cos 40.0^\circ = 19.2$ m/s, while the vertical component becomes (using Eq. 4-23)

$$v_y = v_0 \sin\theta_0 - gt = 25.0\sin 40.0^\circ - (9.8)(1.15) = 4.80 \text{ m/s} .$$

(c) Since $v_y > 0$ when the ball hits the wall, it has not reached the highest point yet.

29. We designate the given velocity $\vec{v} = 7.6\,\hat{\imath} + 6.1\,\hat{\jmath}$ (SI units understood) as $\vec{v}_1$ – as opposed to the velocity when it reaches the max height $\vec{v}_2$ or the velocity when it returns to the ground $\vec{v}_3$ – and take $\vec{v}_0$ as the launch velocity, as usual. The origin is at its launch point on the ground.

(a) Different approaches are available, but since it will be useful (for the rest of the problem) to first find the initial y velocity, that is how we will proceed. Using Eq. 2-16, we have

$$\begin{aligned}v_{1\,y}^2 &= v_{0\,y}^2 - 2g\Delta y \\ 6.1^2 &= v_{0\,y}^2 - 2(9.8)(9.1)\end{aligned}$$

which yields $v_{0\,y} = 14.7$ m/s. Knowing that $v_{2\,y}$ must equal 0, we use Eq. 2-16 again but now with $\Delta y = h$ for the maximum height:

$$\begin{aligned}v_{2\,y}^2 &= v_{0\,y}^2 - 2gh \\ 0 &= 14.7^2 - 2(9.8)h\end{aligned}$$

which yields $h = 11$ m.

(b) Recalling the derivation of Eq. 4-26, but using $v_{0\,y}$ for $v_0 \sin\theta_0$ and $v_{0\,x}$ for $v_0 \cos\theta_0$, we have

$$\begin{aligned}0 &= v_{0\,y}t - \frac{1}{2}gt^2 \\ R &= v_{0\,x}t\end{aligned}$$

which leads to $R = \frac{2\,v_{0\,x}\,v_{0\,y}}{g}$. Noting that $v_{0\,x} = v_{1\,x} = 7.6$ m/s, we plug in values and obtain $R = 2(7.6)(14.7)/9.8 = 23$ m.

(c) Since $v_{3\,x} = v_{1\,x} = 7.6$ m/s and $v_{3\,y} = -v_{0\,y} = -14.7$ m/s, we have

$$v_3 = \sqrt{v_{3\,x}^2 + v_{3\,y}^2} = \sqrt{(-14.7)^2 + 7.6^2} = 17 \text{ m/s} .$$

(d) The angle (measured from horizontal) for $\vec{v}_3$ is one of these possibilities:

$$\tan^{-1}\left(\frac{-14.7}{7.6}\right) = -63^\circ \quad \text{or} \quad 117^\circ$$

where we settle on the first choice (-63°, which is equivalent to 297°) since the signs of its components imply that it is in the fourth quadrant.

30. We apply Eq. 4-21, Eq. 4-22 and Eq. 4-23.

(a) From $\Delta x = v_{0x}t$, we find $v_{0x} = 40/2 = 20$ m/s.

(b) From $\Delta y = v_{0y}t - \frac{1}{2}gt^2$, we find $v_{0y} = (53 + \frac{1}{2}(9.8)(2)^2)/2 = 36$ m/s.

(c) From $v_y = v_{0y} - gt'$ with $v_y = 0$ as the condition for maximum height, we obtain $t' = 36/9.8 = 3.7$ s. During that time the x-motion is constant, so $x' - x_0 = (20)(3.7) = 74$ m.

31. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at the the initial position for the football as it begins projectile motion in the sense of §4-5), and we let θ_0 be the angle of its initial velocity measured from the $+x$ axis.

(a) $x = 46\,\text{m}$ and $y = -1.5\,\text{m}$ are the coordinates for the landing point; it lands at time $t = 4.5\,\text{s}$. Since $x = v_{0x}t$,

$$v_{0\,x} = \frac{x}{t} = \frac{46\,\text{m}}{4.5\,\text{s}} = 10.2\ \text{m/s}\ .$$

Since $y = v_{0y}t - \frac{1}{2}gt^2$,

$$v_{0\,y} = \frac{y + \frac{1}{2}gt^2}{t} = \frac{(-1.5\,\text{m}) + \frac{1}{2}(9.8\,\text{m/s}^2)(4.5\,\text{s})^2}{4.5\,\text{s}} = 21.7\ \text{m/s}\ .$$

The magnitude of the initial velocity is

$$v_0 = \sqrt{v_{0\,x}^2 + v_{0\,y}^2} = \sqrt{(10.2\,\text{m/s})^2 + (21.7\,\text{m/s})^2} = 24\ \text{m/s}\ .$$

(b) The initial angle satisfies $\tan\theta_0 = v_{0\,y}/v_{0\,x}$. Thus, $\theta_0 = \tan^{-1}(21.7/10.2) = 64.8°$.

32. The initial velocity has no vertical component – only an x component equal to $+2.00$ m/s. Also, $y_0 = +10.0$ m if the water surface is established as $y = 0$.

(a) $x - x_0 = v_x t$ readily yields $x - x_0 = 1.60$ m.

(b) Using $y - y_0 = v_{0y}t - \frac{1}{2}gt^2$, we obtain $y = 6.86$ m when $t = 0.800$ s.

(c) With t unknown and $y = 0$, the equation $y - y_0 = v_{0y}t - \frac{1}{2}gt^2$ leads to $t = \sqrt{2(10)/9.8} = 1.43$ s. During this time, the x-displacement of the diver is $x - x_0 = (2.00\ \text{m/s})(1.43\ \text{s}) = 2.86$ m.

33. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at ground level directly below the release point. We write $\theta_0 = -30°$ since the angle shown in the figure is measured clockwise from horizontal. We note that the initial speed of the decoy is the plane's speed at the moment of release: $v_0 = 290$ km/h, which we convert to SI units: $(290)(1000/3600) = 80.6$ m/s.

(a) We use Eq. 4-12 to solve for the time:

$$\Delta x = (v_0\cos\theta_0)\,t \quad\Longrightarrow\quad t = \frac{700}{(80.6)\cos -30°} = 10.0\ \text{s}\ .$$

(b) And we use Eq. 4-22 to solve for the initial height y_0:

$$\begin{aligned} y - y_0 &= (v_0\sin\theta_0)\,t - \frac{1}{2}gt^2 \\ 0 - y_0 &= (-40.3)(10.0) - \frac{1}{2}(9.8)(10.0)^2 \end{aligned}$$

which yields $y_0 = 897$ m.

34. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at its initial position (where it is launched). At maximum height, we observe $v_y = 0$ and denote $v_x = v$ (which is also equal to $v_{0\,x}$). In this notation, we have

$$v_0 = 5v\ .$$

Next, we observe $v_0 \cos\theta_0 = v_{0\,x} = v$, so that we arrive at an equation (where $v \neq 0$ cancels) which can be solved for θ_0:

$$(5v)\cos\theta_0 = v \implies \theta_0 = \cos^{-1}\frac{1}{5} = 78^\circ \ .$$

35. We denote h as the height of a step and w as the width. To hit step n, the ball must fall a distance nh and travel horizontally a distance between $(n-1)w$ and nw. We take the origin of a coordinate system to be at the point where the ball leaves the top of the stairway, and we choose the y axis to be positive in the upward direction. The coordinates of the ball at time t are given by $x = v_{0\,x}t$ and $y = -\frac{1}{2}gt^2$ (since $v_{0\,y} = 0$). We equate y to $-nh$ and solve for the time to reach the level of step n:

$$t = \sqrt{\frac{2nh}{g}} \, .$$

The x coordinate then is

$$x = v_{0\,x}\sqrt{\frac{2nh}{g}} = (1.52\,\mathrm{m/s})\sqrt{\frac{2n(0.203\,\mathrm{m})}{9.8\,\mathrm{m/s^2}}} = (0.309\,\mathrm{m})\sqrt{n} \ .$$

The method is to try values of n until we find one for which x/w is less than n but greater than $n-1$. For $n = 1$, $x = 0.309\,\mathrm{m}$ and $x/w = 1.52$, which is greater than n. For $n = 2$, $x = 0.437\,\mathrm{m}$ and $x/w = 2.15$, which is also greater than n. For $n = 3$, $x = 0.535\,\mathrm{m}$ and $x/w = 2.64$. Now, this is less than n and greater than $n - 1$, so the ball hits the third step.

36. Although we could use Eq. 4-26 to find where it lands, we choose instead to work with Eq. 4-21 and Eq. 4-22 (for the soccer ball) since these will give information about where *and when* and these are also considered more fundamental than Eq. 4-26. With $\Delta y = 0$, we have

$$\Delta y = (v_0 \sin\theta_0)\,t - \frac{1}{2}gt^2 \implies t = \frac{(19.5)\sin 45^\circ}{\frac{1}{2}(9.8)} = 2.81 \text{ s} \ .$$

Then Eq. 4-21 yields $\Delta x = (v_0\cos\theta_0)\,t = 38.3$ m. Thus, using Eq. 4-8 and SI units, the player must have an average velocity of

$$\vec{v}_{\text{avg}} = \frac{\Delta\vec{r}}{\Delta t} = \frac{38.3\,\hat{\imath} - 5\,\hat{\imath}}{2.81} = -5.8\,\hat{\imath}$$

which means his average speed (assuming he ran in only one direction) is 5.8 m/s.

37. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at ground level directly below the release point. We write $\theta_0 = -37^\circ$ for the angle measured from $+x$, since the angle given in the problem is measured from the $-y$ direction. We note that the initial speed of the projectile is the plane's speed at the moment of release.

(a) We use Eq. 4-22 to find v_0 (SI units are understood).

$$\begin{aligned} y - y_0 &= (v_0\sin\theta_0)\,t - \frac{1}{2}gt^2 \\ 0 - 730 &= v_0\sin(-37^\circ)\,(5.00) - \frac{1}{2}(9.8)(5.00)^2 \end{aligned}$$

which yields $v_0 = 202$ m/s.

(b) The horizontal distance traveled is $x = v_0 t\cos\theta_0 = (202)(5.00)\cos -37.0^\circ = 806$ m.

(c) The x component of the velocity (just before impact) is $v_x = v_0\cos\theta_0 = (202)\cos -37.0^\circ = 161$ m/s.

(d) The y component of the velocity (just before impact) is $v_y = v_0\sin\theta_0 - gt = (202)\sin(-37^\circ) - (9.80)(5.00) = -171$ m/s.

38. We assume the ball's initial velocity is perpendicular to the plane of the net. We choose coordinates so that $(x_0, y_0) = (0, 3.0)$ m, and $v_x > 0$ (note that $v_{0y} = 0$).

(a) To (barely) clear the net, we have

$$y - y_0 = v_{0y}t - \frac{1}{2}gt^2 \quad \Longrightarrow \quad 3.0 - 2.24 = 0 - \frac{1}{2}(9.8)t^2$$

which gives $t = 0.39$ s for the time it is passing over the net. This is plugged into the x-equation to yield the (minimum) initial velocity $v_x = (8.0 \text{ m})/(0.39 \text{ s}) = 20.3$ m/s.

(b) We require $y = 0$ and find t from $y - y_0 = v_{0y}t - \frac{1}{2}gt^2$. This value ($t = \sqrt{2(3.0)/9.8} = 0.78$ s) is plugged into the x-equation to yield the (maximum) initial velocity $v_x = (17.0 \text{ m})/(0.78 \text{ s}) = 21.7$ m/s.

39. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at ground level directly below impact point between bat and ball. The *Hint* given in the problem is important, since it provides us with enough information to find v_0 directly from Eq. 4-26.

(a) We want to know how high the ball is from the ground when it is at $x = 97.5$ m, which requires knowing the initial velocity. Using the range information and $\theta_0 = 45°$, we use Eq. 4-26 to solve for v_0:

$$v_0 = \sqrt{\frac{g\,R}{\sin 2\theta_0}} = \sqrt{\frac{(9.8)(107)}{1}} = 32.4 \text{ m/s} .$$

Thus, Eq. 4-21 tells us the time it is over the fence:

$$t = \frac{x}{v_0 \cos\theta_0} = \frac{97.5}{(32.4)\cos 45°} = 4.26 \text{ s} .$$

At this moment, the ball is at a height (above the ground) of

$$y = y_0 + (v_0 \sin\theta_0)\,t - \frac{1}{2}gt^2 = 9.88 \text{ m}$$

which implies it does indeed clear the 7.32 m high fence.

(b) At $t = 4.26$ s, the center of the ball is $9.88 - 7.32 = 2.56$ m above the fence.

40. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at ground level directly below the point where the ball was hit by the racquet.

(a) We want to know how high the ball is above the court when it is at $x = 12$ m. First, Eq. 4-21 tells us the time it is over the fence:

$$t = \frac{x}{v_0 \cos\theta_0} = \frac{12}{(23.6)\cos 0°} = 0.508 \text{ s} .$$

At this moment, the ball is at a height (above the court) of

$$y = y_0 + (v_0 \sin\theta_0)\,t - \frac{1}{2}gt^2 = 1.103 \text{ m}$$

which implies it does indeed clear the 0.90 m high fence.

(b) At $t = 0.508$ s, the center of the ball is $1.103 - 0.90 = 0.20$ m above the net.

(c) Repeating the computation in part (a) with $\theta_0 = -5°$ results in $t = 0.510$ s and $y = 0.04$ m, which clearly indicates that it cannot clear the net.

(d) In the situation discussed in part (c), the distance between the top of the net and the center of the ball at $t = 0.510$ s is $0.90 - 0.04 = 0.86$ m.

41. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at the point where the ball is kicked. Where units are not displayed, SI units are understood. We use x and y to denote the coordinates of ball at the goalpost, and try to find the kicking angle(s) θ_0 so that $y = 3.44$ m when $x = 50$ m. Writing the kinematic equations for projectile motion: $x = v_0 t \cos\theta_0$ and $y = v_0 t \sin\theta_0 - \frac{1}{2}gt^2$, we see the first equation gives $t = x/v_0 \cos\theta_0$, and when this is substituted into the second the result is

$$y = x\tan\theta_0 - \frac{gx^2}{2v_0^2\cos^2\theta_0} .$$

One may solve this by trial and error: systematically trying values of θ_0 until you find the two that satisfy the equation. A little manipulation, however, will give an algebraic solution:
Using the trigonometric identity $1/\cos^2\theta_0 = 1 + \tan^2\theta_0$, we obtain

$$\frac{1}{2}\frac{gx^2}{v_0^2}\tan^2\theta_0 - x\tan\theta_0 + y + \frac{1}{2}\frac{gx^2}{v_0^2} = 0$$

which is a second-order equation for $\tan\theta_0$. To simplify writing the solution, we denote $c = \frac{1}{2}gx^2/v_0^2 = \frac{1}{2}(9.80)(50)^2/(25)^2 = 19.6$ m. Then the second-order equation becomes $c\tan^2\theta_0 - x\tan\theta_0 + y + c = 0$. Using the quadratic formula, we obtain its solution(s).

$$\begin{aligned}\tan\theta_0 &= \frac{x \pm \sqrt{x^2 + 4(y+c)c}}{2c} \\ &= \frac{50 \pm \sqrt{50^2 - 4(3.44 + 19.6)(19.6)}}{2(19.6)} .\end{aligned}$$

The two solutions are given by $\tan\theta_0 = 1.95$ and $\tan\theta_0 = 0.605$. The corresponding (first-quadrant) angles are $\theta_0 = 63°$ and $\theta_0 = 31°$. If kicked at any angle between these two, the ball will travel above the cross bar on the goalposts.

42. The magnitude of the acceleration is

$$a = \frac{v^2}{r} = \frac{(10\,\mathrm{m/s})^2}{25\,\mathrm{m}} = 4.0\,\mathrm{m/s}^2 .$$

43. We apply Eq. 4-33 to solve for speed v and Eq. 4-32 to find acceleration a.

(a) Since the radius of Earth is 6.37×10^6 m, the radius of the satellite orbit is $6.37\times10^6\,\mathrm{m} + 640\times10^3\,\mathrm{m} = 7.01\times10^6$ m. Therefore, the speed of the satellite is

$$v = \frac{2\pi r}{T} = \frac{2\pi(7.01\times10^6\,\mathrm{m})}{(98.0\,\mathrm{min})(60\,\mathrm{s/min})} = 7.49\times10^3\,\mathrm{m/s} .$$

(b) The magnitude of the acceleration is

$$a = \frac{v^2}{r} = \frac{(7.49\times10^3\,\mathrm{m/s})^2}{7.01\times10^6\,\mathrm{m}} = 8.00\,\mathrm{m/s}^2 .$$

44. We note that the period of revolution is $(1200\,\mathrm{rev/min})^{-1} = 8.3\times10^{-4}$ min which becomes, in SI units, $T = 0.050$ s.

(a) The circumference is $c = 2\pi r = 2\pi(0.15) = 0.94$ m.

(b) The speed is $v = c/T = (0.94)/(0.050) = 19$ m/s. This is equivalent to using Eq. 4-33.

(c) The magnitude of the acceleration is $a = v^2/r = 19^2/0.15 = 2.4 \times 10^3$ m/s^2.

(d) As noted above, $T = 50$ ms.

45. We apply Eq. 4-32 to solve for speed v and Eq. 4-33 to find the period T.

(a) We obtain

$$v = \sqrt{ra} = \sqrt{(5.0\,\text{m})(7.0)(9.8\,\text{m/s}^2)} = 19 \text{ m/s} .$$

(b) The time to go around once (the period) is $T = 2\pi r/v = 1.7$ s. Therefore, in one minute ($t = 60$ s), the astronaut executes

$$\frac{t}{T} = \frac{60}{1.7} = 35$$

revolutions. Thus, 35 rev/min is needed to produce a centripetal acceleration of $7g$ when the radius is 5.0 m.

(c) As noted above, $T = 1.7$ s.

46. The magnitude of centripetal acceleration ($a = v^2/r$) and its direction (towards the center of the circle) form the basis of this problem.

(a) If a passenger at this location experiences $\vec{a} = 1.83$ m/s^2 east, then the center of the circle is *east* of this location. And the distance is $r = v^2/a = (3.66^2)/(1.83) = 7.32$ m. Thus, relative to the center, the passenger at that moment is located 7.32 m toward the west.

(b) We see the distance is the same, but now the direction of $\vec{a}$ experienced by the passenger is *south* – indicating that the center of the merry-go-round is south of him. Therefore, relative to the center, the passenger at that moment located 7.32 m toward the north.

47. The radius of Earth may be found in Appendix C.

(a) The speed of a person at Earth's equator is $v = 2\pi R/T$, where R is the radius of Earth (6.37×10^6 m) and T is the length of a day (8.64×10^4 s): $v = 2\pi(6.37 \times 10^6\,\text{m})/(8.64 \times 10^4\,\text{s}) = 463\,\text{m/s}$. The magnitude of the acceleration is given by

$$a = \frac{v^2}{R} = \frac{(463\,\text{m/s})^2}{6.37 \times 10^6\,\text{m}} = 0.034 \text{ m/s}^2 .$$

(b) If T is the period, then $v = 2\pi R/T$ is the speed and $a = v^2/R = 4\pi^2 R^2/T^2 R = 4\pi^2 R/T^2$ is the magnitude of the acceleration. Thus

$$T = 2\pi\sqrt{\frac{R}{a}} = 2\pi\sqrt{\frac{6.37 \times 10^6\,\text{m}}{9.8\,\text{m/s}^2}} = 5.1 \times 10^3\,\text{s} = 84 \text{ min} .$$

48. Eq. 4-32 describes an inverse proportionality between r and a, so that a large acceleration results from a small radius. Thus, an upper limit for a corresponds to a lower limit for r.

(a) The minimum turning radius of the train is given by

$$r_{\text{min}} = \frac{v^2}{a_{\text{max}}} = \frac{(216\,\text{km/h})^2}{(0.050)(9.8\,\text{m/s}^2)} = 7.3 \times 10^3\,\text{m} .$$

(b) The speed of the train must be reduced to no more than

$$v = \sqrt{a_{\text{max}}\, r} = \sqrt{0.050(9.8)(1.00 \times 10^3)} = 22 \text{ m/s}$$

which is roughly 80 km/h.

49. (a) Since the wheel completes 5 turns each minute, its period is one-fifth of a minute, or 12 s.

 (b) The magnitude of the centripetal acceleration is given by $a = v^2/R$, where R is the radius of the wheel, and v is the speed of the passenger. Since the passenger goes a distance $2\pi R$ for each revolution, his speed is

$$v = \frac{2\pi(15\,\text{m})}{12\,\text{s}} = 7.85\,\text{m/s}$$

 and his centripetal acceleration is

$$a = \frac{(7.85\,\text{m/s})^2}{15\,\text{m}} = 4.1\,\text{m/s}^2 \ .$$

 When the passenger is at the highest point, his centripetal acceleration is downward, toward the center of the orbit.

 (c) At the lowest point, the centripetal acceleration vector points up, toward the center of the orbit. It has the same magnitude as in part (b).

50. We apply Eq. 4-33 to solve for speed v and Eq. 4-32 to find centripetal acceleration a.

 (a) $v = 2\pi r/T = 2\pi(20\,\text{km})/1.0\,\text{s} = 1.3 \times 10^5\,\text{km/s}$.

 (b)

$$a = \frac{v^2}{r} = \frac{(126\,\text{km/s})^2}{20\,\text{km}} = 7.9 \times 10^5\,\text{m/s}^2 \ .$$

 (c) Clearly, both v and a will increase if T is reduced.

51. To calculate the centripetal acceleration of the stone, we need to know its speed during its circular motion (this is also its initial speed when it flies off). We use the kinematic equations of projectile motion (discussed in §4-6) to find that speed. Taking the $+y$ direction to be upward and placing the origin at the point where the stone leaves its circular orbit, then the coordinates of the stone during its motion as a projectile are given by $x = v_0 t$ and $y = -\frac{1}{2}gt^2$ (since $v_{0\,y} = 0$). It hits the ground at $x = 10\,\text{m}$ and $y = -2.0\,\text{m}$. Formally solving the second equation for the time, we obtain $t = \sqrt{-2y/g}$, which we substitute into the first equation:

$$v_0 = x\sqrt{-\frac{g}{2y}} = (10\,\text{m})\sqrt{-\frac{9.8\,\text{m/s}^2}{2(-2.0\,\text{m})}} = 15.7\ \text{m/s} \ .$$

Therefore, the magnitude of the centripetal acceleration is

$$a = \frac{v^2}{r} = \frac{(15.7\,\text{m/s})^2}{1.5\,\text{m}} = 160\ \text{m/s}^2 \ .$$

52. We write our magnitude-angle results in the form $(R \angle \theta)$ with SI units for the magnitude understood (m for distances, m/s for speeds, m/s^2 for accelerations). All angles θ are measured counterclockwise from $+x$, but we will occasionally refer to angles ϕ which are measured counterclockwise from the vertical line between the circle-center and the coordinate origin and the line drawn from the circle-center to the particle location (see r in the figure). We note that the speed of the particle is $v = 2\pi r/T$ where $r = 3.00$ m and $T = 20.0$ s; thus, $v = 0.942$ m/s. The particle is moving counterclockwise in Fig. 4-37.

 (a) At $t = 5.00$ s, the particle has traveled a fraction of

$$\frac{t}{T} = \frac{5.00}{20.0} = \frac{1}{4}$$

 of a full revolution around the circle (starting at the origin). Thus, relative to the circle-center, the particle is at

$$\phi = \frac{1}{4}\,(360°) = 90°$$

measured from vertical (as explained above). Referring to Fig. 4-37, we see that this position (which is the "3 o'clock" position on the circle) corresponds to $x = 3.00$ m and $y = 3.00$ m relative to the coordinate origin. In our magnitude-angle notation, this is expressed as $(R \angle \theta) = (4.24 \angle 45°)$. Although this position is easy to analyze without resorting to trigonometric relations, it is useful (for the computations below) to note that these values of x and y relative to coordinate origin can be gotten from the angle ϕ from the relations $x = r\sin\phi$ and $y = r - r\cos\phi$. Of course, $R = \sqrt{x^2 + y^2}$ and θ comes from choosing the appropriate possibility from $\tan^{-1}(y/x)$ (or by using particular functions of vector capable calculators).

(b) At $t = 7.50$ s, the particle has traveled a fraction of $7.50/20.0 = 3/8$ of a revolution around the circle (starting at the origin). Relative to the circle-center, the particle is therefore at $\phi = 3/8\,(360°) = 135°$ measured from vertical in the manner discussed above. Referring to Fig. 4-37, we compute that this position corresponds to $x = 3.00\sin 135° = 2.12$ m and $y = 3.00 - 3.00\cos 135° = 5.12$ m relative to the coordinate origin. In our magnitude-angle notation, this is expressed as $(R \angle \theta) = (5.54 \angle 67.5°)$.

(c) At $t = 10.0$ s, the particle has traveled a fraction of $10.0/20.0 = 1/2$ of a revolution around the circle. Relative to the circle-center, the particle is at $\phi = 180°$ measured from vertical (see explanation, above). Referring to Fig. 4-37, we see that this position corresponds to $x = 0$ and $y = 6.00$ m relative to the coordinate origin. In our magnitude-angle notation, this is expressed as $(R \angle \theta) = (6.00 \angle 90.0°)$.

(d) We subtract the position vector in part (a) from the position vector in part (c): $(6.00 \angle 90.0°) - (4.24 \angle 45°) = (4.24 \angle 135°)$ using magnitude-angle notation (convenient when using vector capable calculators). If we wish instead to use unit-vector notation, we write

$$\Delta\vec{R} = (0 - 3)\,\hat{\imath} + (6 - 3)\,\hat{\jmath} = -3\,\hat{\imath} + 3\,\hat{\jmath}$$

which leads to $|\Delta\vec{R}| = 4.24$ m and $\theta = 135°$.

(e) From Eq. 4-8, we have

$$\vec{v}_{\text{avg}} = \frac{\Delta\vec{R}}{\Delta t} \quad \text{where} \quad \Delta t = 5.00 \text{ s}$$

which produces $-0.6\,\hat{\imath} + 0.6\,\hat{\jmath}$ m/s in unit-vector notation or $(0.849 \angle 135°)$ in magnitude-angle notation.

(f) The speed has already been noted ($v = 0.942$ m/s), but its direction is best seen by referring again to Fig. 4-37. The velocity vector is tangent to the circle at its "3 o'clock position" (see part (a)), which means $\vec{v}$ is vertical. Thus, our result is $(0.942 \angle 90°)$.

(g) Again, the speed has been noted above ($v = 0.942$ m/s), but its direction is best seen by referring to Fig. 4-37. The velocity vector is tangent to the circle at its "12 o'clock position" (see part (c)), which means $\vec{v}$ is horizontal. Thus, our result is $(0.942 \angle 180°)$.

(h) The acceleration has magnitude $v^2/r = 0.296$ m/s^2, and at this instant (see part (a)) it is horizontal (towards the center of the circle). Thus, our result is $(0.296 \angle 180°)$.

(i) Again, $a = v^2/r = 0.296$ m/s^2, but at this instant (see part (c)) it is vertical (towards the center of the circle). Thus, our result is $(0.296 \angle 270°)$.

53. We use Eq. 4-15 first using velocities relative to the truck (subscript t) and then using velocities relative to the ground (subscript g). We work with SI units, so 20 km/h $\to$ 5.6 m/s, 30 km/h $\to$ 8.3 m/s, and 45 km/h $\to$ 12.5 m/s. We choose east as the $+\,\hat{\imath}$ direction.

(a) The velocity of the cheetah (subscript c) at the end of the 2.0 s interval is (from Eq. 4-42)

$$\vec{v}_{\text{c t}} = \vec{v}_{\text{c g}} - \vec{v}_{\text{t g}} = 12.5\,\hat{\imath} - (-5.6\,\hat{\imath}) = 18.1\,\hat{\imath} \text{ m/s}$$

relative to the truck. The (average) acceleration vector relative to the cameraman (in the truck) is

$$\vec{a}_{\text{avg}} = \frac{18.1\,\hat{\imath} - (-8.3\,\hat{\imath})}{2.0} = 13\,\hat{\imath} \text{ m/s}^2 .$$

(b) The velocity of the cheetah at the start of the 2.0 s interval is (from Eq. 4-42)

$$\vec{v}_{0\,\mathrm{cg}} = \vec{v}_{0\,\mathrm{ct}} + \vec{v}_{0\,\mathrm{tg}} = (-8.3\,\hat{\imath}) + (-5.6\,\hat{\imath}) = -13.9\,\hat{\imath}\ \mathrm{m/s}$$

relative to the ground. The (average) acceleration vector relative to the crew member (on the ground) is

$$\vec{a}_{\mathrm{avg}} = \frac{12.5\,\hat{\imath} - (-13.9\,\hat{\imath})}{2.0} = 13\,\hat{\imath}\ \mathrm{m/s^2}$$

identical to the result of part (a).

54. We choose upstream as the $+\hat{\imath}$ direction, and use Eq. 4-42.

(a) The subscript b is for the boat, w is for the water, and g is for the ground.

$$\vec{v}_{\mathrm{b\,g}} = \vec{v}_{\mathrm{b\,w}} + \vec{v}_{\mathrm{w\,g}} = (14\,\mathrm{km/h})\,\hat{\imath} + (-9\,\mathrm{km/h})\,\hat{\imath} = (5\,\mathrm{km/h})\,\hat{\imath}$$

(b) And we use the subscript c for the child.

$$\vec{v}_{\mathrm{c\,g}} = \vec{v}_{\mathrm{c\,b}} + \vec{v}_{\mathrm{b\,g}} = (-6\,\mathrm{km/h})\,\hat{\imath} + (5\,\mathrm{km/h})\,\hat{\imath} = (-1\,\mathrm{km/h})\,\hat{\imath}$$

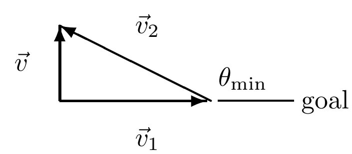
55. When the escalator is stalled the speed of the person is $v_p = \ell/t$, where ℓ is the length of the escalator and t is the time the person takes to walk up it. This is $v_p = (15\,\mathrm{m})/(90\,\mathrm{s}) = 0.167\,\mathrm{m/s}$. The escalator moves at $v_e = (15\,\mathrm{m})/(60\,\mathrm{s}) = 0.250\,\mathrm{m/s}$. The speed of the person walking up the moving escalator is $v = v_p + v_e = 0.167\,\mathrm{m/s} + 0.250\,\mathrm{m/s} = 0.417\,\mathrm{m/s}$ and the time taken to move the length of the escalator is

$$t = \ell/v = (15\,\mathrm{m})/(0.417\,\mathrm{m/s}) = 36\,\mathrm{s}\ .$$

If the various times given are independent of the escalator length, then the answer does not depend on that length either. In terms of ℓ (in meters) the speed (in meters per second) of the person walking on the stalled escalator is $\ell/90$, the speed of the moving escalator is $\ell/60$, and the speed of the person walking on the moving escalator is $v = (\ell/90) + (\ell/60) = 0.0278\ell$. The time taken is $t = \ell/v = \ell/0.0278\ell = 36\,\mathrm{s}$ and is independent of ℓ.

56. We denote the velocity of the player with $\vec{v}_1$ and the relative velocity between the player and the ball be $\vec{v}_2$. Then the velocity $\vec{v}$ of the ball relative to the field is given by $\vec{v} = \vec{v}_1 + \vec{v}_2$. The smallest angle θ_{min} corresponds to the case when $\vec{v} \perp \vec{v}_1$. Hence,

$$\begin{aligned}\theta_{\mathrm{min}} &= 180^\circ - \cos^{-1}\left(\frac{|\vec{v}_1|}{|\vec{v}_2|}\right) \\ &= 180^\circ - \cos^{-1}\left(\frac{4.0\,\mathrm{m/s}}{6.0\,\mathrm{m/s}}\right) \\ &\approx 130^\circ\ .\end{aligned}$$



57. Relative to the car the velocity of the snowflakes has a vertical component of $8.0\,\mathrm{m/s}$ and a horizontal component of $50\,\mathrm{km/h} = 13.9\,\mathrm{m/s}$. The angle θ from the vertical is found from

$$\tan\theta = v_h/v_v = (13.9\,\mathrm{m/s})/(8.0\,\mathrm{m/s}) = 1.74$$

which yields $\theta = 60^\circ$.

58. We denote the police and the motorist with subscripts p and m, respectively. The coordinate system is indicated in Fig. 4-38.

(a) The velocity of the motorist with respect to the police car is

$$\vec{v}_{m\,p} = \vec{v}_m - \vec{v}_p = -60\,\hat{\jmath} - (-80\,\hat{\imath}) = 80\,\hat{\imath} - 60\,\hat{\jmath}\ \text{(km/h)}\ .$$

(b) $\vec{v}_{m\,p}$ does happen to be along the line of sight. Referring to Fig. 4-38, we find the vector pointing from car to another is $\vec{r} = 800\,\hat{\imath} - 600\,\hat{\jmath}$ m (from M to P). Since the ratio of components in $\vec{r}$ is the same as in $\vec{v}_{m\,p}$, they must point the same direction.

(c) No, they remain unchanged.

59. Since the raindrops fall vertically relative to the train, the horizontal component of the velocity of a raindrop is $v_h = 30\,\text{m/s}$, the same as the speed of the train. If v_v is the vertical component of the velocity and θ is the angle between the direction of motion and the vertical, then $\tan\theta = v_h/v_v$. Thus $v_v = v_h/\tan\theta = (30\,\text{m/s})/\tan 70^\circ = 10.9\,\text{m/s}$. The speed of a raindrop is $v = \sqrt{v_h^2 + v_v^2} = \sqrt{(30\,\text{m/s})^2 + (10.9\,\text{m/s})^2} = 32\,\text{m/s}$.

60. Here, the subscript W refers to the water. Our coordinates are chosen with $+x$ being *east* and $+y$ being *north*. In these terms, the angle specifying *east* would be 0° and the angle specifying *south* would be -90° or 270°. Where the length unit is not displayed, km is to be understood.

(a) We have $\vec{v}_{A\,W} = \vec{v}_{A\,B} + \vec{v}_{B\,W}$, so that $\vec{v}_{A\,B} = (22\ \angle -90^\circ) - (40\ \angle\ 37^\circ) = (56\ \angle -125^\circ)$ in the magnitude-angle notation (conveniently done with a vector capable calculator in polar mode). Converting to rectangular components, we obtain

$$\vec{v}_{A\,B} = -32\,\hat{\imath} - 46\,\hat{\jmath}\ \ \text{km/h}\ .$$

Of course, this could have been done in unit-vector notation from the outset.

(b) Since the velocity-components are constant, integrating them to obtain the position is straightforward ($\vec{r} - \vec{r}_0 = \int \vec{v}\,dt$)

$$\vec{r} = (2.5 - 32t)\,\hat{\imath} + (4.0 - 46t)\,\hat{\jmath}$$

with lengths in kilometers and time in hours.

(c) The magnitude of this $\vec{r}$ is

$$r = \sqrt{(2.5 - 32t)^2 + (4.0 - 46t)^2}$$

We minimize this by taking a derivative and requiring it to equal zero – which leaves us with an equation for t

$$\frac{dr}{dt} = \frac{1}{2}\frac{6286t - 528}{\sqrt{(2.5 - 32t)^2 + (4.0 - 46t)^2}} = 0$$

which yields $t = 0.084$ h.

(d) Plugging this value of t back into the expression for the distance between the ships (r), we obtain $r = 0.2$ km. Of course, the calculator offers more digits ($r = 0.225...$), but they are not significant; in fact, the uncertainties implicit in the given data, here, should make the ship captains worry.

61. The velocity vector (relative to the shore) for ships A and B are given by

$$\vec{v}_A = -\left(v_A \cos 45^\circ\right)\,\hat{\imath} + \left(v_A \sin 45^\circ\right)\,\hat{\jmath}$$

and

$$\vec{v}_B = -\left(v_B \sin 40^\circ\right)\,\hat{\imath} - \left(v_B \cos 40^\circ\right)\,\hat{\jmath}$$

respectively (where $v_A = 24$ knots and $v_B = 28$ knots). We are taking East as $+\,\hat{\imath}$ and North as $\hat{\jmath}$.

(a) Their relative velocity is

$$\vec{v}_{A\,B} = \vec{v}_A - \vec{v}_B = (v_B \sin 40^\circ - v_A \cos 45^\circ)\,\hat{\imath} + (v_B \cos 40^\circ + v_A \sin 45^\circ)\,\hat{\jmath}$$

the magnitude of which is $|\vec{v}_{A\,B}| = \sqrt{1.0^2 + 38.4^2} \approx 38$ knots. The angle θ which $\vec{v}_{A\,B}$ makes with North is given by

$$\theta = \tan^{-1}\left(\frac{v_{AB,x}}{v_{AB,y}}\right) = \tan^{-1}\left(\frac{1.0}{38.4}\right) = 1.5^\circ$$

which is to say that $\vec{v}_{A\,B}$ points 1.5° east of north.

(b) Since they started at the same time, their relative velocity describes at what rate the distance between them is increasing. Because the rate is steady, we have

$$t = \frac{|\Delta r_{A\,B}|}{|\vec{v}_{A\,B}|} = \frac{160}{38} = 4.2\ \text{h}\ .$$

(c) The velocity $\vec{v}_{A\,B}$ does not change with time in this problem, and $\vec{r}_{A\,B}$ is in the same direction as $\vec{v}_{A\,B}$ since they started at the same time. Reversing the points of view, we have $\vec{v}_{A\,B} = -\vec{v}_{B\,A}$ so that $\vec{r}_{A\,B} = -\vec{r}_{B\,A}$ (i.e., they are 180° opposite to each other). Hence, we conclude that B stays at a bearing of 1.5° west of south relative to A during the journey (neglecting the curvature of Earth).

62. The (box)car has velocity $\vec{v}_{c\,g} = v_1\,\hat{\imath}$ relative to the ground, and the bullet has velocity

$$\vec{v}_{0\,b\,g} = v_2 \cos\theta\,\hat{\imath} + v_2 \sin\theta\,\hat{\jmath}$$

relative to the ground before entering the car (we are neglecting the effects of gravity on the bullet). While in the car, its velocity relative to the outside ground is $\vec{v}_{b\,g} = 0.8 v_2 \cos\theta\,\hat{\imath} + 0.8 v_2 \sin\theta\,\hat{\jmath}$ (due to the 20% reduction mentioned in the problem). The problem indicates that the velocity of the bullet in the car *relative to the car* is (with v_3 unspecified) $\vec{v}_{b\,c} = v_3\,\hat{\jmath}$. Now, Eq. 4-42 provides the condition

$$\begin{aligned} \vec{v}_{b\,g} &= \vec{v}_{b\,c} + \vec{v}_{c\,g} \\ 0.8 v_2 \cos\theta\,\hat{\imath} + 0.8 v_2 \sin\theta\,\hat{\jmath} &= v_3\,\hat{\jmath} + v_1\,\hat{\imath} \end{aligned}$$

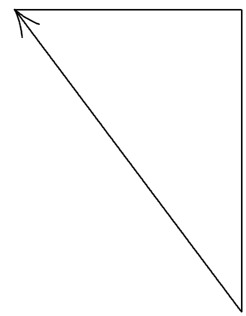
so that equating x components allows us to find θ. If one wished to find v_3 one could also equate the y components, and from this, if the car width were given, one could find the time spent by the bullet in the car, but this information is not asked for (which is why the width is irrelevant). Therefore, examining the x components in SI units leads to

$$\theta = \cos^{-1}\left(\frac{v_1}{0.8 v_2}\right) = \cos^{-1}\left(\frac{85\left(\frac{1000}{3600}\right)}{0.8(650)}\right)$$

which yields 87° for the direction of $\vec{v}_{b\,g}$ (measured from $\hat{\imath}$, which is the direction of motion of the car). The problem asks, "from what direction was it fired?" – which means the answer is not 87° but rather its supplement 93° (measured from the direction of motion). Stating this more carefully, in the coordinate system we have adopted in our solution, the bullet velocity vector is in the first quadrant, at 87° measured counterclockwise from the $+x$ direction (the direction of train motion), which means that the direction from which the bullet came (where the sniper is) is in the third quadrant, at -93° (that is, 93° measured clockwise from $+x$).

63. We construct a right triangle starting from the clearing on the south

bank, drawing a line (200 m long) due north (*upward* in our sketch) across the river, and then a line due west (upstream, leftward in our sketch) along the north bank for a distance $(82\,\text{m}) + (1.1\,\text{m/s})t$, where the t-dependent contribution is the distance that the river will carry the boat downstream during time t.



The hypotenuse of this right triangle (the arrow in our sketch) also depends on t and on the boat's speed (relative to the water), and we set it equal to the Pythagorean "sum" of the triangle's sides:

$$(4.0)t = \sqrt{200^2 + (82 + 1.1t)^2}$$

which leads to a quadratic equation for t

$$46724 + 180.4t - 14.8t^2 = 0 \ .$$

We solve this and find a positive value: $t = 62.6$ s. The angle between the northward (200 m) leg of the triangle and the hypotenuse (which is measured "west of north") is then given by

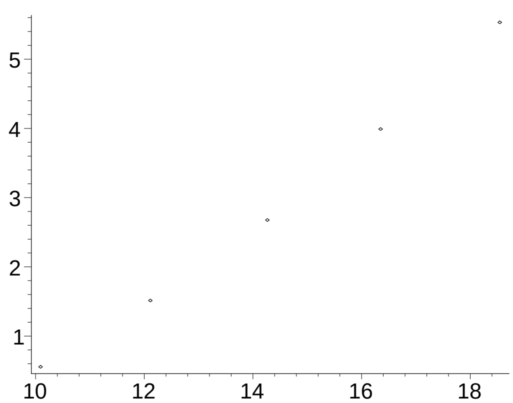
$$\theta = \tan^{-1}\left(\frac{82 + 1.1t}{200}\right) = \tan^{-1}\left(\frac{151}{200}\right) = 37^\circ \ .$$

64. (a) We compute the coordinate pairs (x, y) from $x = v_0 \cos\theta t$ and $x = v_0 \sin\theta t - \frac{1}{2}gt^2$ for $t = 20$ s and the speeds and angles given in the problem. We obtain (in kilometers)

$$(\,x_A\,,\,y_A\,) = (10.1, 0.56) \qquad (\,x_B\,,\,y_B\,) = (12.1, 1.51)$$
$$(\,x_C\,,\,y_C\,) = (14.3, 2.68) \qquad (\,x_D\,,\,y_D\,) = (16.4, 3.99)$$

and $(\,x_E\,,\,y_E\,) = (18.5, 5.53)$ which we plot in the next part.

(b) The vertical (y) and horizontal (x) axes are in kilometers. The graph does not start at the origin. The curve to "fit" the data is not shown, but is easily imagined (forming the "curtain of death").

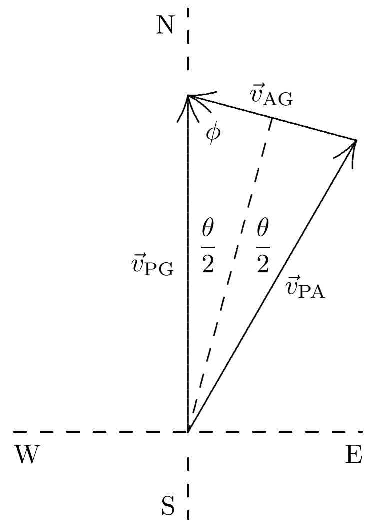


65. We denote $\vec{v}_{\text{PG}}$ as the velocity of the plane relative to the ground, $\vec{v}_{\text{AG}}$ as the velocity of the air relative to the ground, and $\vec{v}_{\text{PA}}$ be the velocity of the plane relative to the air.

(a) The vector diagram is shown below. $\vec{v}_{\text{PG}} = \vec{v}_{\text{PA}} + \vec{v}_{\text{AG}}$. Since the magnitudes v_{PG} and v_{PA} are equal the triangle is isosceles, with two sides of equal length. Consider either of the right triangles formed when the bisector of θ is drawn (the dashed line). It bisects $\vec{v}_{\rm AG}$, so

$$\sin(\theta/2) = \frac{v_{\rm AG}}{2v_{\rm PG}} = \frac{70.0\,\text{mi/h}}{2(135\,\text{mi/h})}$$

which leads to $\theta = 30.1°$. Now $\vec{v}_{\rm AG}$ makes the same angle with the E-W line as the dashed line does with the N-S line. The wind is blowing in the direction 15° north of west. Thus, it is blowing *from* 75° east of south.



(b) The plane is headed along $\vec{v}_{\rm PA}$, in the direction 30° east of north. There is another solution, with the plane headed 30° west of north and the wind blowing 15° north of east (that is, from 75° west of south).

66. (a) The ball must increase in height by $\Delta y = 0.193$ m and cover a horizontal distance $\Delta x = 0.910$ m during a very short time $t_0 = 1.65 \times 10^{-2}$ s. The statement that the "initial curvature of the ball's path can be neglected" is essentially the same as saying the average velocity for $0 \le t \le t_0$ may be taken equal to the instantaneous initial velocity $\vec{v}_0$. Thus, using Eq. 4-8 to figure its two components, we have

$$\tan\theta_0 = \frac{v_{0y}}{v_{0x}} = \frac{\frac{\Delta y}{t_0}}{\frac{\Delta x}{t_0}} = \frac{\Delta y}{\Delta x}$$

so that $\theta_0 = \tan^{-1}(0.193/0.910) = 12°$.

(b) The magnitude of $\vec{v}_0$ is

$$\sqrt{v_{0x}^2 + v_{0y}^2} = \sqrt{\left(\frac{\Delta x}{t_0}\right)^2 + \left(\frac{\Delta y}{t_0}\right)^2} = \frac{\sqrt{\Delta x^2 + \Delta y^2}}{t_0}$$

which yields $v_0 = 56.4$ m/s.

(c) The range is given by Eq. 4-26:

$$R = \frac{v_0^2}{g}\sin 2\theta_0 = 132 \text{ m} .$$

(d) Partly because of its dimpled surface (but other air-flow related effects are important here) the golf ball travels farther than one would expect based on the simple projectile-motion analysis done here.

67. (a) Since the performer returns to the original level, Eq. 4-26 applies. With $R = 4.0$ m and $\theta_0 = 30°$, the initial speed (for the projectile motion) is consequently

$$v_0 = \sqrt{\frac{gR}{\sin 2\theta_0}} = 6.7 \text{ m/s} .$$

This is, of course, the final speed v for the Air Ramp's acceleration process (for which the initial speed is taken to be zero) Then, for that process, Eq. 2-11 leads to

$$a = \frac{v}{t} = \frac{6.7}{0.25} = 27 \text{ m/s}^2 .$$

We express this as a multiple of g by setting up a ratio: $a = (27/9.8)g = 2.7g$.

(b) Repeating the above steps for $R = 12$ m, $t = 0.29$ s and $\theta_0 = 45°$ gives $a = 3.8g$.

68. The initial position vector $\vec{r}_o$ satisfies $\vec{r} - \vec{r}_o = \Delta\vec{r}$, which results in

$$\vec{r}_o = \vec{r} - \Delta\vec{r} = (3\,\hat{j} - 4\,\hat{k}) - (2\,\hat{i} - 3\,\hat{j} + 6\,\hat{k}) = -2.0\,\hat{i} + 6.0\,\hat{j} - 10\,\hat{k}$$

where the understood unit is meters.

69. We adopt a coordinate system with $\hat{i}$ pointed east and $\hat{j}$ pointed north; the coordinate origin is the flagpole. With SI units understood, we "translate" the given information into unit-vector notation as follows:

$$\vec{r}_o = 40\,\hat{i} \quad \text{and} \quad \vec{v}_o = -10\,\hat{j}$$
$$\vec{r} = 40\,\hat{j} \quad \text{and} \quad \vec{v} = 10\,\hat{i}\ .$$

(a) Using Eq. 4-2, the displacement $\Delta\vec{r}$ is

$$\vec{r} - \vec{r}_o = (56.6 \angle\ 135°)$$

where we have expressed the result in magnitude-angle notation. The displacement has magnitude $40\sqrt{2} = 56.6$ m and points due Northwest.

(b) Eq. 4-8 shows that $\vec{v}_{avg}$ points in the same direction as $\Delta\vec{r}$, and that its magnitude is simply the magnitude of the displacement divided by the time ($\Delta t = 30$ s). Thus, the average velocity has magnitude $56.6/30 = 1.89$ m/s and points due Northwest.

(c) Using Eq. 4-15, we have

$$\vec{a}_{avg} = \frac{\vec{v} - \vec{v}_o}{\Delta t} = 0.333\,\hat{i} + 0.333\,\hat{j}$$

in SI units. The magnitude of the average acceleration vector is therefore $0.333\sqrt{2} = 0.471\ \text{m/s}^2$, and it points due Northeast.

70. The velocity of Larry is v_1 and that of Curly is v_2. Also, we denote the length of the corridor by L. Now, Larry's time of passage is $t_1 = 150$ s (which must equal L/v_1), and Curly's time of passage is $t_2 = 70$ s (which must equal L/v_2). The time Moe takes is therefore

$$t = \frac{L}{v_1 + v_2} = \frac{1}{v_1/L + v_2/L} = \frac{1}{\frac{1}{150} + \frac{1}{70}} = 48\ \text{s}\ .$$

71. We choose horizontal x and vertical y axes such that both components of $\vec{v}_0$ are positive. Positive angles are counterclockwise from $+x$ and negative angles are clockwise from it. In unit-vector notation, the velocity at each instant during the projectile motion is

$$\vec{v} = v_0 \cos\theta_0\,\hat{i} + (v_0 \sin\theta_0 - gt)\,\hat{j}\ .$$

(a) With $v_0 = 30$ m/s and $\theta_0 = 60°$, we obtain $\vec{v} = 15\,\hat{i} + \hat{j}$ in m/s, for $t = 2.0$ s. Converting to magnitude-angle notation, this is $\vec{v} = (16 \angle\ 23°)$ with the magnitude in m/s.

(b) Now with $t = 5.0$ s, we find $\vec{v} = (27 \angle -57°)$.

72. (a) The helicopter's speed is $v' = 6.2$ m/s. From the discussions in §4-9 we see that the speed of the package is $v_0 = 12 - v' = 5.8$ m/s, relative to the ground.

(b) Letting $+x$ be in the direction of $\vec{v}_0$ for the package and $+y$ be downward, we have (for the motion of the package)

$$\Delta x = v_0 t \quad \text{and} \quad \Delta y = \frac{1}{2}gt^2$$

where $\Delta y = 9.5$ m. From these, we find $t = 1.39$ s and $\Delta x = 8.08$ m for the package, while $\Delta x'$ (for the helicopter, which is moving in the opposite direction) is $-v'\,t = -8.63$ m. Thus, the horizontal separation between them is $8.08 - (-8.63) = 16.7$ m.

(c) The components of $\vec{v}$ at the moment of impact are $(v_x, v_y) = (5.8, 13.6)$ in SI units. The vertical component has been computed using Eq. 2-11. The angle (which is below horizontal) for this vector is $\tan^{-1}(13.6/5.8) = 67°$.

73. (a) By symmetry, $y = H$ occurs at $x = R/2$ (taking the coordinate origin to be at the launch point). Substituting this into Eq. 4-25 gives

$$H = \frac{R}{2}\tan\theta_0 - \frac{gR^2/4}{2v_0^2\cos^2\theta_0}$$

which leads immediately to

$$\frac{H}{R} = \frac{1}{2}\left(\tan\theta_0 - \frac{gR}{4v_0^2\cos^2\theta_0}\right) .$$

In the far right term, we substitute from Eq. 4-26 for the range:

$$\frac{H}{R} = \frac{1}{2}\left(\tan\theta_0 - \frac{g\left(v_0^2\sin(2\theta_0)/g\right)}{4v_0^2\cos^2\theta_0}\right)$$

which, upon setting $\sin 2\theta_0 = 2\sin\theta_0\cos\theta_0$ and simplifying that last term, yields

$$\frac{H}{R} = \frac{1}{2}\left(\tan\theta_0 - \frac{\sin\theta_0}{2\cos\theta_0}\right)$$

which clearly leads to the relation we wish to prove.

(b) Setting $H/R = 1$ in that relation, we have $\theta_0 = \tan^{-1}(4) = 76°$.

74. (a) The tangent of the angle ϕ is found from the ratio of y to x coordinates of the highest point (taking the coordinate origin to be at the launch point). Using the same notation as in problem 73, we have

$$\phi = \tan^{-1}\left(\frac{H}{\frac{1}{2}R}\right)\tan^{-1}\left(2\frac{H}{R}\right) .$$

Substituting $H/R = \frac{1}{4}\tan\theta_0$ from problem 73, we obtain the relation

$$\tan^{-1}\left(\frac{1}{2}\tan\theta_0\right) .$$

(b) Since $\tan 45° = 1$, then $\phi = \tan^{-1}\left(\frac{1}{2}\right) = 27°$.

75. The initial velocity has magnitude v_0 and because it is horizontal, it is equal to v_x the horizontal component of velocity at impact. Thus, the speed at impact is

$$\sqrt{v_0^2 + v_y^2} = 3v_0 \quad \text{where} \quad v_y = \sqrt{2gh}$$

where we use Eq. 2-16 with Δx replaced with the $h = 20$ m to obtain that second equality. Squaring both sides of the first equality and substituting from the second, we find

$$v_0^2 + 2gh = (3v_0)^2$$

which leads to $gh = 4v_0^2$ and therefore to $v_0 = \sqrt{(9.8)(20)}/2 = 7.0$ m/s.

76. (a) The magnitude of the displacement vector $\Delta\vec{r}$ is given by

$$|\Delta\vec{r}| = \sqrt{21.5^2 + 9.7^2 + 2.88^2} = 23.8 \text{ km} .$$

Thus,

$$|\vec{v}_{\text{avg}}| = \frac{|\Delta\vec{r}|}{\Delta t} = \frac{23.8}{3.50} = 6.79 \text{ km/h} .$$

(b) The angle θ in question is given by

$$\theta = \tan^{-1}\left(\frac{2.88}{\sqrt{21.5^2 + 9.7^2}}\right) = 6.96^\circ \ .$$

77. With no acceleration in the x direction yet a constant acceleration of 1.4 m/s^2 in the y direction, the position (in meters) as a function of time (in seconds) must be

$$\vec{r} = (6.0t)\hat{\imath} + \left(\frac{1}{2}(1.4)t^2\right)\hat{\jmath}$$

and $\vec{v}$ is its derivative with respect to t.

(a) At $t = 3.0$ s, therefore, $\vec{v} = 6.0\hat{\imath} + 4.2\hat{\jmath}$ m/s.

(b) At $t = 3.0$ s, the position is $\vec{r} = 18\,\hat{\imath} + 6.3\,\hat{\jmath}$ m.

78. We choose a coordinate system with origin at the clock center and $+x$ rightward (towards the "3:00" position) and $+y$ upward (towards "12:00").

(a) In unit-vector notation, we have (in centimeters) $\vec{r}_1 = 10\,\hat{\imath}$ and $\vec{r}_2 = -10\,\hat{\jmath}$. Thus, Eq. 4-2 gives

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = -10\,\hat{\imath} - 10\,\hat{\jmath} \ \longrightarrow \ (14 \ \angle -135^\circ)$$

where we have switched to magnitude-angle notation in the last step.

(b) In this case, $\vec{r}_1 = -10\,\hat{\jmath}$ and $\vec{r}_2 = 10\,\hat{\jmath}$, and $\Delta\vec{r} = 20\,\hat{\jmath}$ cm.

(c) In a full-hour sweep, the hand returns to its starting position, and the displacement is zero.

79. We let g_p denote the magnitude of the gravitational acceleration on the planet. A number of the points on the graph (including some "inferred" points – such as the max height point at $x = 12.5$ m and $t = 1.25$ s) can be analyzed profitably; for future reference, we label (with subscripts) the first ($(x_0, y_0) = (0, 2)$ at $t_0 = 0$) and last ("final") points ($(x_f, y_f) = (25, 2)$ at $t_f = 2.5$), with lengths in meters and time in seconds.

(a) The x-component of the initial velocity is found from $x_f - x_0 = v_{0x} t_f$. Therefore, $v_{0x} = 25/2.5 =$ 10 m/s. And we try to obtain the y-component from $y_f - y_0 = 0 = v_{0y} t_f - \frac{1}{2} g_p t_f^2$. This gives us $v_{0y} = 1.25 g_p$, and we see we need another equation (by analyzing another point, say, the next-to-last one) $y - y_0 = v_{0y} t - \frac{1}{2} g_p t^2$ with $y = 6$ and $t = 2$; this produces our second equation $v_{0y} = 2 + g_p$. Simultaneous solution of these two equations produces results for v_{0y} and g_p (relevant to part (b)). Thus, our complete answer for the initial velocity is $\vec{v} = 10\hat{\imath} + 10\hat{\jmath}$ m/s.

(b) As a by-product of the part (a) computations, we have $g_p = 8.0\,\text{m/s}^2$.

(c) Solving for t_g (the time to reach the ground) in $y_g = 0 = y_0 + v_{0y} t_g - \frac{1}{2} g_p t_g^2$ leads to a positive answer: $t_g = 2.7$ s.

(d) With $g = 9.8\,\text{m/s}^2$, the method employed in part (c) would produce the quadratic equation $-4.9t_g^2 + 10t_g + 2 = 0$ and then the positive result $t_g = 2.2$ s.

80. At maximum height, the y-component of a projectile's velocity vanishes, so the given 10 m/s is the (constant) x-component of velocity.

(a) Using v_{0y} to denote the y-velocity 1.0 s before reaching the maximum height, then (with $v_y = 0$) the equation $v_y = v_{0y} - gt$ leads to $v_{0y} = 9.8$ m/s. The magnitude of the velocity vector at that moment (also known as the *speed*) is therefore

$$\sqrt{v_x^2 + {v_{0y}}^2} = \sqrt{10^2 + 9.8^2} = 14 \text{ m/s} \ .$$

(b) It is clear from the symmetry of the problem that the speed is the same 1.0 s after reaching the top, as it was 1.0 s before (14 m/s again). This may be verified by using $v_y = v_{0y} - gt$ again but now "starting the clock" at the highest point so that $v_{0y} = 0$ (and $t = 1.0$ s). This leads to $v_y = -9.8$ m/s and ultimately to $\sqrt{10^2 + (-18)^2} = 14$ m/s.

(c) With v_{0y} denoting the y-component of velocity one second before the top of the trajectory – as in part (a) – then we have $y = 0 = y_0 + v_{0y}t - \frac{1}{2}gt^2$ where $t = 1.0$ s. This yields $y_0 = -4.9$ m. Alternatively, Eq. 2-18 could have been used, with $v_y = 0$ to the same end. The x_0 value more simply results from $x = 0 = x_0 + (10\,\text{m/s})(1.0\,\text{s})$. Thus, the coordinates (in meters) of the projectile one second before reaching maximum height is $(-10, -4.9)$.

(d) It is clear from symmetry that the coordinate one second after the maximum height is reached is $(10, -4.9)$ (in meters). But this can be verified by considering $t = 0$ at the top and using $y - y_0 = v_{0y}t - \frac{1}{2}gt^2$ where $y_0 = v_{0y} = 0$ and $t = 1$ s. And by using $x - x_0 = (10\,\text{m/s})(1.0\,\text{s})$ where $x_0 = 0$. Thus, $x = 10$ m and $y = -4.9$ m is obtained.

81. With $g_B = 9.8128$ m/s^2 and $g_M = 9.7999$ m/s^2, we apply Eq. 4-26:

$$R_M - R_B = \frac{v_0^2 \sin 2\theta_0}{g_M} - \frac{v_0^2 \sin 2\theta_0}{g_B} = \frac{v_0^2 \sin 2\theta_0}{g_B}\left(\frac{g_B}{g_M} - 1\right)$$

which becomes

$$R_M - R_B = R_B\left(\frac{9.8128}{9.7999} - 1\right)$$

and yields (upon substituting $R_B = 8.09$ m) $R_M - R_B = 0.01$ m.

82. (a) Using the same coordinate system assumed in Eq. 4-25, we rearrange that equation to solve for the initial speed:

$$v_0 = \frac{x}{\cos\theta_0}\sqrt{\frac{g}{2\,(x\tan\theta_0 - y)}}$$

which yields $v_0 = 255.5 \approx 2.6 \times 10^2$ m/s for $x = 9400$ m, $y = -3300$ m, and $\theta_0 = 35°$.

(b) From Eq. 4-21, we obtain the time of flight:

$$t = \frac{x}{v_0\cos\theta_0} = \frac{9400}{255.5\cos 35°} = 45 \text{ s} .$$

(c) We expect the air to provide resistance but no appreciable lift to the rock, so we would need a greater launching speed to reach the same target.

83. (a) Using the same coordinate system assumed in Eq. 4-22, we solve for $y = h$:

$$h = y_0 + v_0 \sin\theta_0 - \frac{1}{2}gt^2$$

which yields $h = 51.8$ m for $y_0 = 0$, $v_0 = 42$ m/s, $\theta_0 = 60°$ and $t = 5.5$ s.

(b) The horizontal motion is steady, so $v_x = v_{0x} = v_0\cos\theta_0$, but the vertical component of velocity varies according to Eq. 4-23. Thus, the speed at impact is

$$v = \sqrt{(v_0\cos\theta_0)^2 + (v_0\sin\theta_0 - gt)^2} = 27 \text{ m/s} .$$

(c) We use Eq. 4-24 with $v_y = 0$ and $y = H$:

$$H = \frac{(v_0\sin\theta_0)^2}{2g} = 67.5 \text{ m} .$$

84. (a) Using the same coordinate system assumed in Eq. 4-25, we find

$$y = x\tan\theta_0 - \frac{gx^2}{2\left(v_0\cos\theta_0\right)^2} = -\frac{gx^2}{2v_0^2} \quad \text{if } \theta_0 = 0 \ .$$

Thus, with $v_0 = 3 \times 10^6$ m/s and $x = 1$ m, we obtain $y = -5.4 \times 10^{-13}$ m which is not practical to measure (and suggests why gravitational processes play such a small role in the fields of atomic and subatomic physics).

(b) It is clear from the above expression that $|y|$ decreases as v_0 is reduced.

85. (a) Using the same coordinate system assumed in Eq. 4-21, we obtain the time of flight

$$t = \frac{\Delta x}{v_0\cos\theta_0} = \frac{20}{15\cos 35^\circ} = 1.63 \text{ s} \ .$$

(b) At that moment, its height of above the ground (taking $y_0 = 0$) is

$$y = \left(v_0\sin\theta_0\right)t - \frac{1}{2}gt^2 = 1.02 \text{ m} \ .$$

Thus, the ball is 18 cm below the center of the circle; since the circle radius is 15 cm, we see that it misses it altogether.

(c) The horizontal component of velocity (at $t = 1.63$ s) is the same as initially:

$$v_x = v_{0x} = v_0\cos\theta_0 = 15\cos 35^\circ = 12.3 \text{ m/s} \ .$$

The vertical component is given by Eq. 4-23:

$$v_y = v_0\sin\theta_0 - gt = 15\sin 35^\circ - (9.8)(1.63) = -7.3 \text{ m/s} \ .$$

Thus, the magnitude of its speed at impact is $\sqrt{v_x^2 + v_y^2} = 14.3$ m/s.

(d) As we saw in the previous part, the sign of v_y is negative, implying that it is now heading down (after reaching its max height).

86. (a) From Eq. 4-22 (with $\theta_0 = 0$), the time of flight is

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2(45)}{9.8}} = 3.03 \text{ s} \ .$$

(b) The horizontal distance traveled is given by Eq. 4-21:

$$\Delta x = v_0 t = (250)(3.03) = 758 \text{ m} \ .$$

(c) And from Eq. 4-23, we find

$$|v_y| = gt = (9.80)(3.03) = 29.7 \text{ m/s} \ .$$

87. Using the same coordinate system assumed in Eq. 4-25, we find x for the elevated cannon from

$$y = x\tan\theta_0 - \frac{gx^2}{2\left(v_0\cos\theta_0\right)^2} \quad \text{where } y = -30 \text{ m.}$$

Using the quadratic formula (choosing the positive root), we find

$$x = v_0\cos\theta_0\left(\frac{v_0\sin\theta_0 + \sqrt{\left(v_0\sin\theta_0\right)^2 - 2gy}}{g}\right)$$

which yields $x = 715$ m for $v_0 = 82$ m/s (from Sample Problem 4-7) and $\theta_0 = 45^\circ$. This is 29 m longer than the 686 m found in that Sample Problem. The "9" in 29 m is not reliable, considering the low level of precision in the given data.

88. (a) With $r = 0.15$ m and $a = 3.0 \times 10614$ m/s^2, Eq. 4-32 gives

$$v = \sqrt{ra} = 6.7 \times 10^6 \text{ m/s} .$$

(b) The period is given by Eq. 4-33:

$$T = \frac{2\pi r}{v} = 1.4 \times 10^{-7} \text{ s} .$$

89. The type of acceleration involved in steady-speed circular motion is the centripetal acceleration $a = v^2/r$ which is at each moment directed towards the center of the circle. The radius of the circle is $r = 12^2/3 = 48$ m. Thus, the car is at the present moment 48 m west of the center of its circular path; this is equally true in part (a) and part (b).

90. (a) With $v = c/10 = 3 \times 10^7$ m/s and $a = 20g = 196$ m/s^2, Eq. 4-32 gives

$$r = \frac{v^2}{a} = 4.6 \times 10^{12} \text{ m} .$$

(b) The period is given by Eq. 4-33:

$$T = \frac{2\pi r}{v} = 9.6 \times 10^5 \text{ s} .$$

Thus, the time to make a quarter-turn is $T/4 = 2.4 \times 10^5$ s or about 2.8 days.

91. (a) Using the same coordinate system assumed in Eq. 4-21 and Eq. 4-22 (so that $\theta_0 = -20.0°$), we use $v_0 = 15.0$ m/s and find the horizontal displacement of the ball at $t = 2.30$ s:

$$\Delta x = (v_0 \cos\theta_0)\, t = 32.4 \text{ m} .$$

(b) And we find the vertical displacement:

$$\Delta y = (v_0 \sin\theta_0)\, t - \frac{1}{2} g t^2 = -37.7 \text{ m} .$$

92. This is a classic problem involving two-dimensional relative motion; see §4-9. The steps in Sample Problem 4-11 in the textbook are similar to those used here. We align our coordinates so that *east* corresponds to $+x$ and *north* corresponds to $+y$. We write the vector addition equation as $\vec{v}_{BG} = \vec{v}_{BW} + \vec{v}_{WG}$. We have $\vec{v}_{WG} = (2.0 \angle 0°)$ in the magnitude-angle notation (with the unit m/s understood), or $\vec{v}_{WG} = 2.0\,\hat{\imath}$ in unit-vector notation. We also have $\vec{v}_{BW} = (8.0 \angle 120°)$ where we have been careful to phrase the angle in the 'standard' way (measured counterclockwise from the $+x$ axis), or $\vec{v}_{BW} = -4.0\,\hat{\imath} + 6.9\,\hat{\jmath}$.

(a) We can solve the vector addition equation for $\vec{v}_{BG}$:

$$\vec{v}_{BG} = \vec{v}_{BW} + \vec{v}_{WG} = (2.0 \angle 0°) + (8.0 \angle 120°) = (7.2 \angle 106°)$$

which is very efficiently done using a vector capable calculator in polar mode. Thus $|\vec{v}_{BG}| = 7.2$ m/s, and its direction is 16° west of north, or 74° north of west.

(b) The velocity is constant, and we apply $y - y_0 = v_y t$ in a reference frame. Thus, in the *ground* reference frame, we have $200 = 7.2\sin(106°)t \;\rightarrow\; t = 29$ s. Note: if a student obtains "28 s", then the student has probably neglected to take the y component properly (a common mistake).

93. The topic of relative motion (with constant velocity motion) in a two-dimensional setting is covered in §4-9. We note that

$$\vec{v}_{PG} = \vec{v}_{PA} + \vec{v}_{AG}$$

describes a right triangle, with one leg being $\vec{v}_{PG}$ (east), another leg being $\vec{v}_{AG}$ (magnitude = 20, direction = south), and the hypotenuse being $\vec{v}_{PA}$ (magnitude = 70). Lengths are in kilometers and time is in hours. Using the Pythagorean theorem, we have

$$|\vec{v}_{PA}| = \sqrt{|\vec{v}_{PG}|^2 + |\vec{v}_{AG}|^2} \quad \Longrightarrow \quad 70 = \sqrt{|\vec{v}_{PG}|^2 + 20^2}$$

which is easily solved for the ground speed: $|\vec{v}_{PG}| = 67$ km/h.

94. Our coordinate system has $\hat{\imath}$ pointed east and $\hat{\jmath}$ pointed north. All distances are in kilometers, times in hours, and speeds in km/h. The first displacement is $\vec{r}_{AB} = 483\,\hat{\imath}$ and the second is $\vec{r}_{BC} = -966\,\hat{\jmath}$.

(a) The net displacement is

$$\vec{r}_{AC} = \vec{r}_{AB} + \vec{r}_{BC} = 483\,\hat{\imath} - 966\,\hat{\jmath} \longrightarrow (1080 \angle -63.4^\circ)$$

where we have expressed the result in magnitude-angle notation in the last step. We observe that the angle can be alternatively expressed as 63.4° south of east, or 26.6° east of south.

(b) Dividing the magnitude of $\vec{r}_{AC}$ by the total time (2.25 h) gives the magnitude of $\vec{v}_{\rm avg}$ and its direction is the same as in part (a). Thus, $\vec{v}_{\rm avg} = (480 \angle -63.4^\circ)$ in magnitude-angle notation (with km/h understood).

(c) Assuming the AB trip was a straight one, and similarly for the BC trip, then $|\vec{r}_{AB}|$ is the distance traveled during the AB trip, and $|\vec{r}_{BC}|$ is the distance traveled during the BC trip. Since the average speed is the total distance divided by the total time, it equals

$$\frac{483 + 966}{2.25} = 644 \text{ km/h} .$$

95. We take the initial (x, y) specification to be $(0.000, 0.762)$ m, and the positive x direction to be towards the "green monster." The components of the initial velocity are $(33.53 \angle 55^\circ) \rightarrow (19.23, 27.47)$ m/s.

(a) With $t = 5.00$ s, we have $x = x_0 + v_x t = 96.2$ m.

(b) At that time, $y = y_0 + v_{0y}t - \frac{1}{2}gt^2 = 15.59$ m, which is 4.31 m above the wall.

(c) The moment in question is specified by $t = 4.50$ s. At that time, $x - x_0 = (19.23)(4.5) = 86.5$ m, and $y = y_0 + v_{0y}t - \frac{1}{2}gt^2 = 25.1$ m.

96. The displacement of the one-way trip is the same as the displacement, which has magnitude $D = 4350$ km for the flight (we are in a frame of reference that rotates with the earth). The velocity of the flight relative to the earth is

$$\vec{v}_{fe} = \vec{va} + \vec{ae}$$

where $\vec{ae}$ is the velocity of the (eastward) jet stream (with magnitude $v > 0$), and $\vec{ae}$ is the velocity of the plane relative to the air (with magnitude $u = 966$ m/s). And the magnitudes of the eastward flight velocity (relative to earth) and of the westward flight velocity (primed) are, respectively,

$$|\vec{v}_{fe}| = \frac{D}{t} \quad \text{and} \quad \left|\vec{v}\,'_{fe}\right| = \frac{D}{t'} .$$

The time difference (5/6 of an hour) is therefore

$$\begin{aligned} t' - t &= \frac{D}{\left|\vec{v}\,'_{fe}\right|} - \frac{D}{|\vec{v}_{fe}|} \\ \Delta t &= \frac{D}{u - v} - \frac{D}{u + v} . \end{aligned}$$

Using the quadratic formula to solve for v, we obtain

$$v = \frac{-D + \sqrt{D^2 + u^2(\Delta t)^2}}{\Delta t} = 89 \text{ km/h} .$$

97. Using the same coordinate system assumed in Eq. 4-25, we rearrange that equation to solve for the initial speed:

$$v_0 = \frac{x}{\cos\theta_0}\sqrt{\frac{g}{2\,(x\tan\theta_0 - y)}}$$

which yields $v_0 = 23$ ft/s for $g = 32$ ft/s^2, $x = 13$ ft, $y = 3$ ft and $\theta_0 = 55°$.

98. We establish coordinates with $\hat{\imath}$ pointing to the far side of the river (perpendicular to the current) and $\hat{\jmath}$ pointing in the direction of the current. We are told that the magnitude (presumed constant) of the velocity of the boat relative to the water is $|\vec{v}_{bw}| = u = 6.4$ km/h. Its angle, relative to the x axis is θ. With km and h as the understood units, the velocity of the water (relative to the ground) is $\vec{v}_{wg} = 3.2\,\hat{\jmath}$.

(a) To reach a point "directly opposite" means that the velocity of her boat relative to ground must be $\vec{bg} = v\,\hat{\imath}$ where $v > 0$ is unknown. Thus, all $\hat{\jmath}$ components must cancel in the vector sum

$$\vec{v}_{bw} + \vec{v}_{wg} = \vec{v}_{bg}$$

which means the $u\sin\theta = -3.2$, so $\theta = \sin^{-1}(-3.2/6.4) = -30°$.

(b) Using the result from part (a), we find $v = u\cos\theta = 5.5$ km/h. Thus, traveling a distance of $\ell = 6.4$ km requires a time of $6.4/5.5 = 1.15$ h or 69 min.

(c) If her motion is completely along the y axis (as the problem implies) then with $v_w = 3.2$ km/h (the water speed) we have

$$t_{\text{total}} = \frac{D}{u + v_w} + \frac{D}{u - v_w} = 1.33 \text{ h}$$

where $D = 3.2$ km. This is equivalent to 80 min.

(d) Since

$$\frac{D}{u + v_w} + \frac{D}{u - v_w} = \frac{D}{u - v_w} + \frac{D}{u + v_w}$$

the answer is the same as in the previous part.

(e) The case of general θ leads to

$$\vec{v}_{bg} = \vec{v}_{bw} + \vec{v}_{wg} = u\cos\theta\,\hat{\imath} + (u\sin\theta + v_w)\,\hat{\jmath}$$

where the x component of $\vec{v}_{bg}$ must equal ell/t. Thus,

$$t = \frac{\ell}{u\cos\theta}$$

which can be minimized using $dt/d\theta = 0$ (though, of course, an easier way is to appeal to either physical or mathematical intuition – concluding that the shortest-time path should have $\theta = 0$). Then $t = 6.4/6.4 = 1.0$ h, or 60 min.

99. With $v_0 = 30$ m/s and $R = 20$ m, Eq. 4-26 gives

$$\sin 2\theta_0 = \frac{gR}{v_0^2} = 0.218 \ .$$

Because $\sin(\phi) = \sin(180° - \phi)$, there are two roots of the above equation:

$$2\theta_0 = \sin^{-1}(0.218) = 12.6° \quad \text{and} \quad 167.4° \ .$$

Therefore, the two possible launch angles that will hit the target (in the absence of air friction and related effects) are $\theta_0 = 6.3°$ and $\theta_0 = 83.7°$. An alternative approach to this problem in terms of Eq. 4-25 (with $y = 0$ and $1/\cos^2 = 1 + \tan^2$) is possible – and leads to a quadratic equation for $\tan\theta_0$ with the roots providing these two possible θ_0 values.

100. (a) The time available before the train arrives at the impact spot is

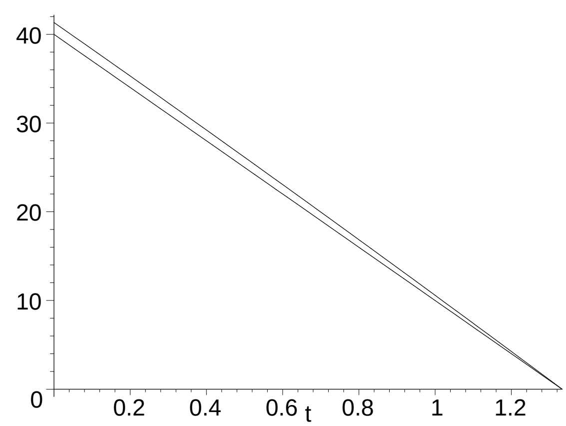
$$t_{\text{train}} = \frac{40\,\text{m}}{30\,\text{m/s}} = 1.33\text{ s}$$

(the train does not reduce its speed). We interpret the phrase "distance between the car and the center of the crossing" to refer to the distance from the front bumper of the car to that point. In which case, the car needs to travel a total distance of $\Delta x = 40 + 5 + 1.5 = 46.5$ m in order for its rear bumper and the edge of the train not to collide (the distance from the center of the train to either edge of the train is 1.5 m). With a starting velocity of $v_0 = 30$ m/s and an acceleration of $a = 1.5$ m/s^2, Eq. 2-15 leads to

$$\Delta x = v_0 t + \frac{1}{2}at^2 \implies t = \frac{-v_0 \pm \sqrt{v_0^2 + 2a\Delta x}}{a}$$

which yields (upon taking the positive root) a time $t_{\text{car}} = 1.49$ s needed for the car to make it. Recalling our result for t_{train} we see the car doesn't have enough time available to make it across.

(b) The difference is $t_{\text{car}} - t_{\text{train}} = 0.16$ s. We note that at $t = t_{\text{train}}$ the front bumper of the car is $v_0 t + \frac{1}{2}at^2 = 41.33$ m from where it started, which means it is 1.33 m past the center of the track (but the edge of the track is 1.5 m from the center). If the car was coming from the south, then the point P on the car impacted by the southern-most corner of the front of the train is 2.83 m behind the front bumper (or 2.17 m in front of the rear bumper). The motion of P is what is plotted below (the top graph – looking like a line instead of a parabola because the final speed of the car is not much different than its initial speed). Since the position of the train is on an entirely different axis than that of the car, we plot the distance (in meters) from P to "south" rail of the tracks (the top curve shown), and the distance of the "south" front corner of the train to the line-of-motion of the car (the bottom line shown).



101. (a) With $v_0 = 6.3$ m/s and $R = 0.40$ m, Eq. 4-26 gives

$$\sin 2\theta_0 = \frac{gR}{v_0^2} = 0.0988\ .$$

Because $\sin(\phi) = \sin(180° - \phi)$, there are two roots of the above equation:

$$2\theta_0 = \sin^{-1}(0.0988) = 5.7° \quad \text{and} \quad 174.3°\ .$$

Therefore, the two possible launch angles that will hit the target (in the absence of air friction and related effects) are $\theta_0 = 2.8°$ and $\theta_0 = 87.1°$. But the juggler is trying to achieve a visual effect by having a relatively high trajectory for the balls, so $\theta_0 = 87.1°$ is the result he should choose.

(b) We do not show the graph here. It would be very much like the higher parabola shown in Fig. 4-51.

(c) , (d) and (e) The problem requests that the student work with his graphs, here, but we – for doublechecking purposes – use Eq. 4-26 to calculate $R - 0.40$ m for $\theta_0 - 87.1° = -2°, -1°, 1°$, and $2°$. We obtain the respective values (in meters) $0.28, 0.14, -0.14$, and -0.28.

102. (First problem in **Cluster 1**)
Using the coordinate system employed in §4-5 and §4-6, we have $v_{0x} = v_x > 0$ and $v_{0y} = 0$. Also, $y_0 = h > 0$, $x_0 = 0$, $y = 0$ (when it hits the ground at $t = 3.00$)), and $x = 150$, with lengths in meters and time in seconds.

 (a) The equation $y - y_0 = v_{0y}t - \frac{1}{2}gt^2$ becomes $-h = -\frac{1}{2}(9.8)(3.00)^2$, so that $h = 44.1$ m.

 (b) The equation $v_y = v_{0y} - gt$ gives the y-component of the "final" velocity as $v_y = -(9.8)(3.00) = 29.4$ m/s. The x-component of velocity (which is constant) is computed from $v_x = (x - x_0)/t = 150/3.00 = 50.0$ m/s. Therefore,

$$|\vec{v}| = \sqrt{v_x^2 + v_y^2} = \sqrt{50^2 + 29.4^2} = 58.0 \text{ m/s} .$$

103. (Second problem in **Cluster 1**)
Using the coordinate system employed in §4-5 and §4-6, we have $v_{0x} = v_0 \cos 30° > 0$ and $v_{0y} = v_0 \sin 30° > 0$. Also, $y_0 = 0$ (corresponding to the dashed line in the figure), $x_0 = 0$, $y = h > 0$ (where it lands at $t = 3.00$), and $x = 100$, with lengths in meters and time in seconds.

 (a) The x-equation determines v_0

$$x - x_0 = v_0 \cos(30)t \implies 100 = v_0(0.866)(3.00)$$

 which leads to $v_0 = 38.5$ m/s. The y-equation $y - y_0 = v_{0y}t - \frac{1}{2}gt^2$ becomes $h = (38.5)(\sin 30)(3.00) - \frac{1}{2}(9.8)(3.00)^2 = 13.6$ m.

 (b) As a byproduct of part (a)'s computation, we found $v_0 = 38.5$ m/s.

 (c) Although a somewhat easier method will be found in the energy chapter (especially Chapter 8), we will find the "final" velocity components with the methods of §4-6. We have $v_x = v_{0x} = 38.5 \cos 30 = 33.3$ m/s. And $v_y = v_{0y} - gt = 38.5 \sin 30 - (9.8)(3.00) = -10.2$ m/s. Therefore,

$$|\vec{v}| = \sqrt{v_x^2 + v_y^2} = \sqrt{(33.3)^2 + (-10.2)^2} = 34.8 \text{ m/s} .$$

104. (Third problem in **Cluster 1**)
Following the hint, we have the time-reversed problem with the ball thrown from the roof, towards the left, at 60° measured clockwise from a leftward axis. We see in this time-reversed situation that it is convenient to take $+x$ as *leftward* with positive angles measured clockwise. Lengths are in meters and time is in seconds.

 (a) With $y_0 = 20.0$, and $y = 0$ at $t = 4.00$, we have $y - y_0 = v_{0y}t - \frac{1}{2}gt^2$ where $v_{0y} = v_0 \sin 60°$. This leads to $v_0 = 16.9$ m/s. This plugs into the x-equation (with $x_0 = 0$ and $x = d$) to produce $d = (16.9 \cos 60°)(4.00) = 33.7$ m.

 (b) Although a somewhat easier method will be found in the energy chapter (especially Chapter 8), we will find the "final" velocity components with the methods of §4-6. Note that we're still working the time-reversed problem; this "final" $\vec{v}$ is actually the velocity with which it was thrown. We have $v_x = v_{0x} = 16.9 \cos 60° = 8.43$ m/s. And $v_y = v_{0y} - gt = 16.9 \sin 60° - (9.8)(4.00) = -24.6$ m/s. We convert from rectangular components to polar (that is, magnitude-angle) representation:

$$\vec{v} = (8.43, -24.6) \longrightarrow (26.0 \angle -71.1°) .$$

 and we now interpret our result ("undoing" the time reversal) as an initial velocity of magnitude 26 m/s with angle (up from rightward) of 71°.

105. (Fourth problem in **Cluster 1**)
Following the hint, we have the time-reversed problem with the ball thrown from the ground, towards the right, at 60° measured counterclockwise from a rightward axis. We see in this time-reversed situation that it is convenient to use the familiar coordinate system with $+x$ as *rightward* and with positive angles measured counterclockwise. Lengths are in meters and time is in seconds.

(a) The x-equation (with $x_0 = 0$ and $x = 25.0$) leads to $25 = (v_0 \cos 60°)(1.50)$, so that $v_0 = 33.3$ m/s. And with $y_0 = 0$, and $y = h > 0$ at $t = 1.50$, we have $y - y_0 = v_{0y}t - \frac{1}{2}gt^2$ where $v_{0y} = v_0 \sin 60°$. This leads to $h = 32.3$ m.

(b) Although a somewhat easier method will be found in the energy chapter (especially Chapter 8), we will find the "final" velocity components with the methods of §4-6. Note that we're still working the time-reversed problem; this "final" $\vec{v}$ is actually the velocity with which it was thrown. We have $v_x = v_{0x} = 33.3 \cos 60° = 16.7$ m/s. And $v_y = v_{0y} - gt = 33.3 \sin 60° - (9.8)(1.50) = 14.2$ m/s. We convert from rectangular to polar in terms of the magnitude-angle notation:

$$\vec{v} = (16.7, 14.2) \longrightarrow (21.9 \angle 40.4°) .$$

We now interpret this result ("undoing" the time reversal) as an initial velocity (from the edge of the building) of magnitude 22 m/s with angle (down from leftward) of 40°.

106. (Fifth problem in **Cluster 1**)

Let $y_0 = 1.0$ m at $x_0 = 0$ when the ball is hit. Let $y_1 = h$ (the height of the wall) and x_1 describe the point where it first rises above the wall one second after being hit; similarly, $y_2 = h$ and x_2 describe the point where it passes back down behind the wall four seconds later. And $y_f = 1.0$ m at $x_f = R$ is where it is caught. Lengths are in meters and time is in seconds.

(a) Keeping in mind that v_x is constant, we have $x_2 - x_1 = 50.0 = v_{1x}(4.00)$, which leads to $v_{1x} = 12.5$ m/s. Thus, applied to the full six seconds of motion: $x_f - x_0 = R = v_x(6.00) = 75.0$ m.

(b) We apply $y - y_0 = v_{0y}t - \frac{1}{2}gt^2$ to the motion above the wall.

$$y_2 - y_1 = 0 = v_{1y}(4.00) - \frac{1}{2}g(4.00)^2$$

leads to $v_{1y} = 19.6$ m/s. One second earlier, using $v_{1y} = v_{0y} - g(1.00)$, we find $v_{0y} = 29.4$ m/s. We convert from (x, y) to magnitude-angle (polar) representation:

$$\vec{v}_0 = (16.7, 14.2) \longrightarrow (31.9 \angle 66.9°) .$$

We interpret this result as a velocity of magnitude 32 m/s, with angle (up from rightward) of 67°.

(c) During the first 1.00 s of motion, $y = y_0 + v_{0y}t - \frac{1}{2}gt^2$ yields $h = 1.0 + (29.4)(1.00) - \frac{1}{2}(9.8)(1.00)^2 = 25.5$ m.

107. (First problem in **Cluster 2**)

(a) Since $v_y^2 = v_{0y}^2 - 2g\Delta y$, and $v_y = 0$ at the target, we obtain $v_{0y} = \sqrt{2(9.8)(5.00)} = 9.90$ m/s. Since $v_0 \sin \theta_0 = v_{0y}$, with $v_0 = 12$ m/s, we find $\theta_0 = 55.6°$.

(b) Now, $v_y = v_{0y} - gt$ gives $t = 9.90/9.8 = 1.01$ s. Thus, $\Delta x = (v_0 \cos \theta_0)t = 6.85$ m.

(c) The velocity at the target has only the v_x component, which is equal to $v_{0x} = v_0 \cos \theta_0 = 6.78$ m/s.

108. (Second problem in **Cluster 2**)

(a) The magnitudes of the components are equal at point A, but in terms of the coordinate system usually employed in projectile motion problems, we have $v_x > 0$ and $v_y = -v_x$. The problem gives v_0 which is related to its components by $v_0^2 = v_{0x}^2 + v_{0y}^2$ which suggests that we look at the pair of equations

$$\begin{aligned} v_y^2 &= v_{0y}^2 - 2g\Delta y \\ v_x^2 &= v_{0x}^2 \end{aligned}$$

which we can add to obtain $2v_x^2 = v_0^2 - 2g\Delta y$ (this is closely related to the type of reasoning that will be employed in some Chapter 8 problems). Therefore, we find $v_x = -v_y = 6.53$ m/s. Therefore, $\Delta y = v_y t + \frac{1}{2} g t^2$ (Eq. 2-16) can be used to find t.

$$3.00 = (-6.53)t + \frac{1}{2}(9.8)t^2 \quad \Longrightarrow \quad t = 1.69 \text{ or } -0.36$$

from the quadratic formula or or with a polynomial solver available with some calculators. We choose the positive root: $t = 1.69$ s. Finally, we obtain

$$\Delta x = v_x t = 11.1 \text{ m} \quad .$$

(b) The speed is $v = \sqrt{v_x^2 + v_y^2} = 9.23$ m/s.

109. (Third problem in **Cluster 2**)

(a) Eq. 4-25, which assumes $(x_0, y_0) = (0, 0)$, gives

$$y = 5.00 = (\tan\theta_0)x - \frac{gx^2}{2\,(v_0\cos\theta_0)^2}$$

where $x = 30.0$ (lengths are in meters and time is in seconds). Using the trig identity suggested in the problem and letting u stand for $\tan\theta_0$, we have a second-degree equation for u (its two roots leading to the values $\theta_{0\,\mathrm{min}}$, and $\theta_{0\,\mathrm{max}}$) parameterized by the initial speed v_0.

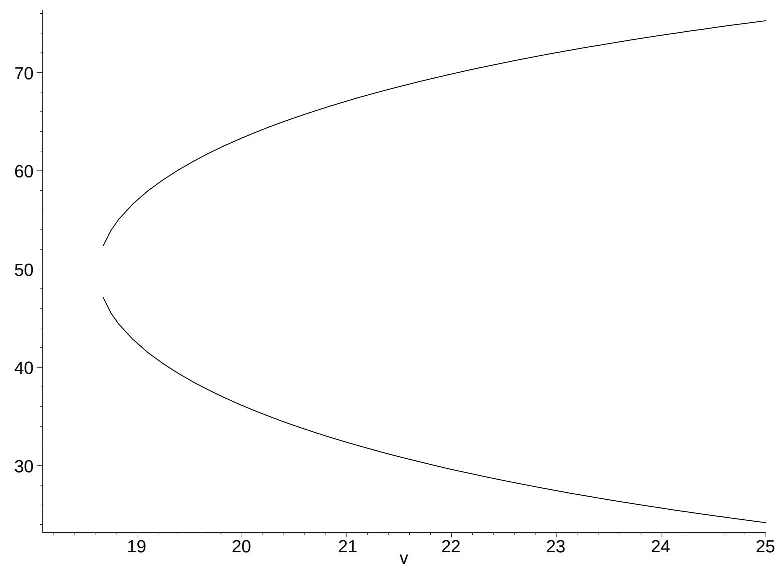
$$\frac{4410}{v_0^2}\,u^2 - 30.0u + \left(\frac{4410}{v_0^2} + 5.00\right) = 0$$

where numerical simplifications have already been made. To see these steps written with the *variables* x, y, v_0 and g made explicit, see the solution to problem 111, below. Now, we solve for u using the quadratic formula, and then find the angles:

$$\theta_0 = \tan^{-1}\left(\frac{1}{294}\,v_0^2 \pm \frac{1}{294}\sqrt{v_0^4 - 98\,v_0^2 - 86436}\right)$$

where the plus is chosen for $\theta_{0\mathrm{max}}$ and the negative is chosen for $\theta_{0\mathrm{min}}$.

(b) These angles are plotted (in degrees) versus v_0 (in m/s) as follows. There are no (real) solutions of the above equations for $18.0 \leq v_0 \leq 18.6$ m/s (this is further discussed in the next problem).



110. (Fourth problem in **Cluster 2**)
Following the hint in the problem (regarding *analytic* solution), we equate the square root expression, above, to zero:

$$\sqrt{v_0^4 - 98\,v_0^2 - 86436} = 0 \quad\Longrightarrow\quad v_0 = 18.6 \text{ m/s} \ .$$

That solution can be obtained either with the quadratic formula (by writing the equation, first, in terms of $w = v_0^2$) or with a polynomial solver built into many calculators; in the latter approach, this is straightforwardly handled as a fourth degree polynomial. Note that the other root ($v_0 = 15.8$ m/s) is dismissed since we are finding where the *real* solutions for angle disappear as one decreases the initial speed from roughly 20 m/s. In case this problem was assigned without assigning Problem 109 first, then this (the choice of root) might be a confusing point. Plugging $v_0 = 18.6$ m/s into

$$\theta_0 = \tan^{-1}\left(\frac{1}{294}\,v_0^2 \pm \frac{1}{294}\sqrt{v_0^4 - 98\,v_0^2 - 86436}\right)$$

(which is unambiguous since the square root factor is zero) provides the launch angle: $\theta_0 = 49.7°$ in this "critical" case.

111. (Fifth problem in **Cluster 2**)

(a) This builds directly on the solutions of the previous two problems. If we return to the solution of problem 109 without plugging in the data for x, y, and g, we obtain the following expression for the θ_0 roots.

$$\theta_0 = \tan^{-1}\left(\frac{v_0^2}{gx}\left(1 \pm \sqrt{1 - \frac{g}{v_0^2}\left(2y + \frac{gx^2}{v_0^2}\right)}\right)\right)$$

And for the "critical case" of maximum distance for a given launch-speed, we set the square root expression to zero (as in the previous problem) and solve for $x_{\max}$.

$$x_{\max} = \frac{v_0^2}{g}\sqrt{1 - \frac{2gy}{v_0^2}}$$

which one might wish to check for the "straight-up" case (where $x = 0$, and the familiar result $y_{\text{max}} = \frac{1}{2}v_0^2/g$ is obtained) and for the "range" case (where $y = 0$ and this then agrees with Eq. 4-26 where $\theta_0 = 45°$). In the problem at hand, we have $y = 5.00$ m, and $v_0 = 15.0$ m/s. This leads to $x_{\text{max}} = 17.2$ m.

(b) When the square root term vanishes, the expression for θ_0 becomes

$$\theta_0 = \tan^{-1}\left(\frac{v_0^2}{gx}\right) = 53.1°$$

using $x = x_{\text{max}}$ from part (a).