Chapter 12

1. The initial speed of the car is $v = (80.0)(1000/3600) = 22.2$ m/s. The tire radius is $R = 0.750/2 = 0.375$ m.

   (a) The initial speed of the car is the initial speed of the center of mass of the tire, so Eq. 12-2 leads to

   $$\omega_0 = \frac{v_{\text{com}\,0}}{R} = \frac{22.2}{0.375} = 59.3 \text{ rad/s} .$$

   (b) With $\theta = (30.0)(2\pi) = 188$ rad and $\omega = 0$, Eq. 11-14 leads to

   $$\omega^2 = \omega_0^2 + 2\alpha\theta \implies |\alpha| = \frac{59.3^2}{2(188)} = 9.31 \text{ rad/s}^2 .$$

   (c) Eq. 12-1 gives $R\theta = 70.7$ m for the distance traveled.

2. We define the direction of motion of the car as the $+x$ direction. The velocity of the car is a constant $\vec{v} = +(80)(1000/3600) = +22$ m/s, and the radius of the wheel is $r = 0.66/2 = 0.33$ m.

   (a) In the car's reference frame (where the lady perceives herself to be at rest) the road is moving towards the rear at $\vec{v}_{\text{road}} = -v = -22$ m/s, and the motion of the tire is purely rotational. In this frame, the center of the tire is "fixed" so $v_{\text{center}} = 0$.

   (b) This frame of reference is not accelerating, so "fixed" points within it have zero acceleration; thus, $a_{\text{center}} = 0$.

   (c) Since the tire's motion is only rotational (not translational) in this frame, Eq. 11-18 gives $\vec{v}_{\text{top}} = +v = +22$ m/s.

   (d) Not only is the motion purely rotational in this frame, but we also have $\omega =$ constant, which means the only acceleration for points on the rim is radial (centripetal). Therefore, the magnitude of the acceleration is

   $$a_{\text{top}} = \frac{v^2}{r} = \frac{22^2}{0.33} = 1.5 \times 10^3 \text{ m/s}^2 .$$

   (e) The bottom-most point of the tire is (momentarily) in firm contact with the road (not skidding) and has the same velocity as the road: $\vec{v}_{\text{bottom}} = -22$ m/s. This also follows from Eq. 11-18.

   (f) The magnitude of the acceleration is the same as in part (d): $a_{\text{bottom}} = 1.5 \times 10^3 \text{ m/s}^2$.

   (g) Now we examine the situation in the road's frame of reference (where the road is "fixed" and it is the car that appears to be moving). The center of the tire undergoes purely translational motion while points at the rim undergo a combination of translational and rotational motions. The velocity of the center of the tire is $\vec{v} = +v = +22$ m/s.

   (h) The translational motion of the center is constant; it does not accelerate.

(i) In part (c), we found $\vec{v}_{\text{top,car}} = +v$ and we use Eq. 4-39:

$$\begin{aligned}\vec{v}_{\text{top,ground}} &= \vec{v}_{\text{top,car}} + \vec{v}_{\text{car,ground}} \\ &= v + v\end{aligned}$$

which yields $2v = +44$ m/s. This is consistent with Fig. 12-3(c).

(j) Since we are transforming between constant-velocity frames of reference, the accelerations are unaffected. The answer is as it was in part (d): $1.5 \times 10^3\,\text{m/s}^2$.

(k) We can proceed as in part (i) or simply recall that the bottom-most point is in firm contact with the (zero-velocity) road. Either way – the answer is zero.

(l) As explained in part (j), $a = 1.5 \times 10^3\,\text{m/s}^2$.

3. By Eq. 11-44, the work required to stop the hoop is the negative of the initial kinetic energy of the hoop. The initial kinetic energy is $K = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$ (Eq. 12-5), where $I = mR^2$ is its rotational inertia about the center of mass, $m = 140$ kg, and $v = 0.150$ m/s is the speed of its center of mass. Eq. 12-2 relates the angular speed to the speed of the center of mass: $\omega = v/R$. Thus,

$$K = \frac{1}{2}mR^2\left(\frac{v^2}{R^2}\right) + \frac{1}{2}mv^2 = mv^2 = (140)(0.150)^2$$

which implies that the work required is -3.15 J.

4. The rotational kinetic energy is $K = \frac{1}{2}I\omega^2$, where $I = mR^2$ is its rotational inertia about the center of mass (Table 11-2(a)), $m = 140$ kg, and $\omega = v_{\text{com}}/R$ (Eq. 12-2). The asked-for ratio is

$$\frac{K_{\text{transl}}}{K_{\text{rot}}} = \frac{\frac{1}{2}mv_{\text{com}}^2}{\frac{1}{2}\left(mR^2\right)\left(v_{\text{com}}/R\right)^2} = 1\ .$$

5. Let M be the mass of the car (presumably including the mass of the wheels) and v be its speed. Let I be the rotational inertia of one wheel and ω be the angular speed of each wheel. The kinetic energy of rotation is

$$K_{\text{rot}} = 4\left(\frac{1}{2}I\omega^2\right)$$

where the factor 4 appears because there are four wheels. The total kinetic energy is given by $K = \frac{1}{2}Mv^2 + 4\left(\frac{1}{2}I\omega^2\right)$. The fraction of the total energy that is due to rotation is

$$\text{fraction} = \frac{K_{\text{rot}}}{K} = \frac{4I\omega^2}{Mv^2 + 4I\omega^2}\ .$$

For a uniform disk (relative to its center of mass) $I = \frac{1}{2}mR^2$ (Table 11-2(c)). Since the wheels roll without sliding $\omega = v/R$ (Eq. 12-2). Thus the numerator of our fraction is

$$4I\omega^2 = 4\left(\frac{1}{2}mR^2\right)\left(\frac{v}{R}\right)^2 = 2mv^2$$

and the fraction itself becomes

$$\text{fraction} = \frac{2mv^2}{Mv^2 + 2mv^2} = \frac{2m}{M + 2m} = \frac{2(10)}{1000} = \frac{1}{50}\ .$$

The wheel radius cancels from the equations and is not needed in the computation.

6. Interpreting h as the height increase for the center of mass of the body, then (using Eq. 12-5) mechanical energy conservation leads to

$$\begin{aligned} K_i &= U_f \\ \frac{1}{2}mv_{\text{com}}^2 + \frac{1}{2}I\omega^2 &= mgh \\ \frac{1}{2}mv^2 + \frac{1}{2}I\left(\frac{v}{R}\right)^2 &= mg\left(\frac{3v^2}{4g}\right) \end{aligned}$$

from which v cancels and we obtain $I = \frac{1}{2}mR^2$ (solid cylinder – Table 11-2(c)).

7. Rather than reproduce the analysis in §12-3, we simply use the results from that section.

   (a) We substitute $I = \frac{2}{5}MR^2$ (Table 11-2(f)) and $a = -0.10g$ into Eq. 12-10:

   $$-0.10g = -\frac{g\sin\theta}{1 + \left(\frac{2}{5}MR^2\right)/MR^2} = -\frac{g\sin\theta}{7/5}$$

   which yields $\theta = \sin^{-1}(0.14) = 8.0°$.

   (b) The acceleration would be more. We can look at this in terms of forces or in terms of energy. In terms of forces, the uphill static friction would then be absent so the downhill acceleration would be due only to the downhill gravitational pull. In terms of energy, the rotational term in Eq. 12-5 would be absent so that the potential energy it started with would simply become $\frac{1}{2}mv^2$ (without it being "shared" with another term) resulting in a greater speed (and, because of Eq. 2-16, greater acceleration).

8. We choose $+x$ rightward (so $\vec{F} = 10\,\hat{\imath}$ in Newtons) and apply Eq. 9-14 and Eq. 11-37.

   (a) Newton's second law in the x direction leads to

   $$F - f_s = ma \implies f_s = 10\,\text{N} - (10\,\text{kg})(0.60\,\text{m/s}^2)$$

   which yields $f_s = 4.0$ N. As assumed in setting up the equation, $\vec{f_s}$ points leftward.

   (b) With $R = 0.30$ m, we find the magnitude of the angular acceleration to be $|\alpha| = |a_{\text{com}}|/R = 2.0\,\text{rad/s}^2$, from Eq. 12-6. The only force not directed towards (or away from) the center of mass is $\vec{f_s}$, and the torque it produces is clockwise:

   $$\begin{aligned} |\tau| &= I\,|\alpha| \\ (0.30\,\text{m})(4.0\,\text{N}) &= I\left(2.0\,\text{rad/s}^2\right) \end{aligned}$$

   which yields the wheel's rotational inertia about its center of mass: $I = 0.60$ kg·m^2.

9. To find where the ball lands, we need to know its speed as it leaves the track (using conservation of energy). Its initial kinetic energy is $K_i = 0$ and its initial potential energy is $U_i = MgH$. Its final kinetic energy (as it leaves the track) is $K_f = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$ (Eq. 12-5) and its final potential energy is Mgh. Here we use v to denote the speed of its center of mass and ω is its angular speed – at the moment it leaves the track. Since (up to that moment) the ball rolls without sliding we can set $\omega = v/R$. Using $I = \frac{2}{5}MR^2$ (Table 11-2(f)), conservation of energy leads to

$$\begin{aligned} MgH &= \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 + Mgh \\ &= \frac{1}{2}Mv^2 + \frac{2}{10}Mv^2 + Mgh \\ &= \frac{7}{10}Mv^2 + Mgh\,. \end{aligned}$$

The mass M cancels from the equation, and we obtain

$$v = \sqrt{\frac{10}{7} g(H-h)} = \sqrt{\frac{10}{7}\left(9.8\,\mathrm{m/s^2}\right)(6.0\,\mathrm{m} - 2.0\,\mathrm{m})} = 7.48\,\mathrm{m/s}\ .$$

Now this becomes a projectile motion of the type examined in Chapter 4. We put the origin at the position of the center of mass when the ball leaves the track (the "initial" position for this part of the problem) and take $+x$ rightward and $+y$ downward. Then (since the initial velocity is purely horizontal) the projectile motion equations become

$$x = vt \quad \text{and} \quad y = -\frac{1}{2} g t^2\ .$$

Solving for x at the time when $y = h$, the second equation gives $t = \sqrt{2h/g}$. Then, substituting this into the first equation, we find

$$x = v\sqrt{\frac{2h}{g}} = (7.48)\sqrt{\frac{2(2.0)}{9.8}} = 4.8\ \mathrm{m}\ .$$

10. (a) When the small sphere is released at the edge of the large "bowl" (the hemisphere of radius R), its center of mass is at the same height at that edge, but when it is at the bottom of the "bowl" its center of mass is a distance r above the the bottom surface of the hemisphere. Since the small sphere descends by $R - r$, its loss in gravitational potential energy is $mg(R - r)$, which, by conservation of mechanical energy, is equal to its kinetic energy at the bottom of the track.

(b) Using Eq. 12-5 for K, the asked-for fraction becomes

$$\frac{K_{\mathrm{rot}}}{K} = \frac{\frac{1}{2} I\omega^2}{\frac{1}{2} I\omega^2 + \frac{1}{2} M v_{\mathrm{com}}^2} = \frac{1}{1 + \left(\frac{M}{I}\right)\left(\frac{v_{\mathrm{com}}}{\omega}\right)^2}\ .$$

Substituting $v_{\mathrm{com}} = R\omega$ (Eq. 12-2) and $I = \frac{2}{5} M R^2$ (Table 11-2(f)), we obtain

$$\frac{K_{\mathrm{rot}}}{K} = \frac{1}{1 + \left(\frac{5}{2R^2}\right) R^2} = \frac{2}{7}\ .$$

(c) The small sphere is executing circular motion so that when it reaches the bottom, it experiences a radial acceleration upward (in the direction of the normal force which the "bowl" exerts on it). From Newton's second law along the vertical axis, the normal force N satisfies $N - mg = ma_{\mathrm{com}}$ where $a_{\mathrm{com}} = v_{\mathrm{com}}^2/(R - r)$. Therefore,

$$N = mg + \frac{m v_{\mathrm{com}}^2}{R - r} = \frac{mg(R - r) + m v_{\mathrm{com}}^2}{R - r}\ .$$

But from part (a), $mg(R - r) = K$, and from Eq. 12-5, $\frac{1}{2} m v_{\mathrm{com}}^2 = K - K_{\mathrm{rot}}$. Thus,

$$N = \frac{K + 2\left(K - K_{\mathrm{rot}}\right)}{R - r} = 3\frac{K}{R - r} - 2\frac{K_{\mathrm{rot}}}{R - r}\ .$$

We now plug in $R - r = K/mg$ and use the result of part (b):

$$N = 3mg - 2mg\left(\frac{2}{7}\right) = \frac{17}{7} mg\ .$$

11. (a) We find its angular speed as it leaves the roof using conservation of energy. Its initial kinetic energy is $K_i = 0$ and its initial potential energy is $U_i = Mgh$ where $h = 6.0 \sin 30^\circ = 3.0$ m (we are using the edge of the roof as our reference level for computing U). Its final kinetic energy (as it leaves

the roof) is $K_f = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$ (Eq. 12-5). Here we use v to denote the speed of its center of mass and ω is its angular speed – at the moment it leaves the roof. Since (up to that moment) the ball rolls without sliding we can set $v = R\omega = v$ where $R = 0.10$ m. Using $I = \frac{1}{2}MR^2$ (Table 11-2(c)), conservation of energy leads to

$$\begin{aligned} Mgh &= \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 \\ &= \frac{1}{2}MR^2\omega^2 + \frac{1}{4}MR^2\omega^2 \\ &= \frac{3}{4}MR^2\omega^2 \ . \end{aligned}$$

The mass M cancels from the equation, and we obtain

$$\omega = \frac{1}{R}\sqrt{\frac{4}{3}gh} = \frac{1}{0.10\,\text{m}}\sqrt{\frac{4}{3}\left(9.8\,\text{m/s}^2\right)(3.0\,\text{m})} = 63\,\text{rad/s} \ .$$

(b) Now this becomes a projectile motion of the type examined in Chapter 4. We put the origin at the position of the center of mass when the ball leaves the track (the "initial" position for this part of the problem) and take $+x$ leftward and $+y$ downward. The result of part (a) implies $v_0 = R\omega = 6.3$ m/s, and we see from the figure that (with these positive direction choices) its components are

$$\begin{aligned} v_{0x} &= v_0 \cos 30^\circ = 5.4 \text{ m/s} \quad \text{and} \\ v_{0y} &= v_0 \sin 30^\circ = 3.1 \text{ m/s} \ . \end{aligned}$$

The projectile motion equations become

$$x = v_{0x}t \quad \text{and} \quad y = v_{0y}t + \frac{1}{2}gt^2 \ .$$

We first find the time when $y = 5.0$ m from the second equation (using the quadratic formula, choosing the positive root):

$$t = \frac{-v_{0y} + \sqrt{v_{0y}^2 + 2gy}}{g} = 0.74 \text{ s} \ .$$

Then we substitute this into the x equation and obtain

$$x = (5.4\,\text{m/s})(0.74\,\text{s}) = 4.0 \text{ m} \ .$$

12. Using the floor as the reference position for computing potential energy, mechanical energy conservation leads to

$$\begin{aligned} U_{\text{release}} &= K_{\text{top}} + U_{\text{top}} \\ mgh &= \frac{1}{2}mv_{\text{com}}^2 + \frac{1}{2}I\omega^2 + mg(2R) \ . \end{aligned}$$

Substituting $I = \frac{2}{5}mr^2$ (Table 11-2(f)) and $\omega = v_{\text{com}}/r$ (Eq. 12-2), we obtain

$$\begin{aligned} mgh &= \frac{1}{2}mv_{\text{com}}^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\frac{v_{\text{com}}}{r}\right)^2 + 2mgR \\ gh &= \frac{7}{10}v_{\text{com}}^2 + 2gR \end{aligned}$$

where we have canceled out mass m in that last step.

(a) To be on the verge of losing contact with the loop (at the top) means the normal force is vanishingly small. In this case, Newton's second law along the vertical direction ($+y$ downward) leads to

$$mg = ma_r \implies g = \frac{v_{\text{com}}^2}{R-r}$$

where we have used Eq. 11-23 for the radial (centripetal) acceleration (of the center of mass, which at this moment is a distance $R-r$ from the center of the loop). Plugging the result $v_{\text{com}}^2 = g(R-r)$ into the previous expression stemming from energy considerations gives

$$gh = \frac{7}{10}(g)(R-r) + 2gR$$

which leads to

$$h = 2.7R - 0.7r \quad \approx \quad 2.7R \ .$$

(b) The energy considerations shown above (now with $h = 6R$) can be applied to point Q (which, however, is only at a height of R) yielding the condition

$$g(6R) = \frac{7}{10}v_{\text{com}}^2 + gR$$

which gives us $v_{\text{com}}^2 = 50gR/7$. Recalling previous remarks about the radial acceleration, Newton's second law applied to the horizontal axis at Q ($+x$ leftward) leads to

$$\begin{aligned} N &= m\,\frac{v_{\text{com}}^2}{R-r} \\ &= m\,\frac{50gR}{7(R-r)} \end{aligned}$$

which (for $R \gg r$) gives $N \approx 50mg/7$.

13. From $I = \frac{2}{3}MR^2$ (Table 11-2(g)) we find

$$M = \frac{3I}{2R^2} = \frac{3(0.040)}{2(0.15)^2} = 2.7 \text{ kg} \ .$$

It also follows from the rotational inertia expression that $\frac{1}{2}I\omega^2 = \frac{1}{3}MR^2\omega^2$. Furthermore, it rolls without slipping, $v_{\text{com}} = R\omega$, and we find

$$\frac{K_{\text{rot}}}{K_{\text{com}} + K_{\text{rot}}} = \frac{\frac{1}{3}MR^2\omega^2}{\frac{1}{2}mR^2\omega^2 + \frac{1}{3}MR^2\omega^2} \ .$$

(a) Simplifying the above ratio, we find $K_{\text{rot}}/K = 0.4$. Thus, 40% of the kinetic energy is rotational, or $K_{\text{rot}} = (0.4)(20) = 8.0$ J.

(b) From $K_{\text{rot}} = \frac{1}{3}MR^2\omega^2 = 8.0$ J (and using the above result for M) we find

$$\omega = \frac{1}{0.15\,\text{m}}\sqrt{\frac{3(8.0\,\text{J})}{2.7\,\text{kg}}} = 20 \text{ rad/s}$$

which leads to $v_{\text{com}} = (0.15)(20) = 3.0$ m/s.

(c) We note that the inclined distance of 1.0 m corresponds to a height $h = 1.0\sin 30° = 0.50$ m. Mechanical energy conservation leads to

$$\begin{aligned} K_i &= K_f + U_f \\ 20\,\text{J} &= K_f + Mgh \end{aligned}$$

which yields (using the values of M and h found above) $K_f = 6.9$ J.

(d) We found in part (a) that 40% of this must be rotational, so

$$\frac{1}{3}MR^2\omega_f^2 = (0.40)K_f \implies \omega_f = \frac{1}{0.15}\sqrt{\frac{3(0.40)(6.9)}{2.7}}$$

which yields $\omega_f = 12$ rad/s and leads to

$$v_{\text{com}f} = R\omega_f = (0.15)(12) = 1.8 \text{ m/s} .$$

14. (a) We choose clockwise as the negative rotational sense and rightwards as the positive translational direction. Thus, since this is the moment when it begins to roll smoothly, Eq. 12-2 becomes

$$v_{\text{com}} = -R\omega = (-0.11\,\text{m})\omega .$$

This velocity is positive-valued (rightward) since ω is negative-valued (clockwise) as shown in Fig. 12-34.

(b) The force of friction exerted on the ball of mass m is $-\mu_k mg$ (negative since it points left), and setting this equal to ma_{com} leads to

$$a_{\text{com}} = -\mu g = -(0.21)\left(9.8\,\text{m/s}^2\right) = -2.1 \text{ m/s}^2$$

where the minus sign indicates that the center of mass acceleration points left, opposite to its velocity, so that the ball is decelerating.

(c) Measured about the center of mass, the torque exerted on the ball due to the frictional force is given by $\tau = -\mu mgR$. Using Table 11-2(f) for the rotational inertia, the angular acceleration becomes (using Eq. 11-37)

$$\alpha = \frac{\tau}{I} = \frac{-\mu mgR}{2mR^2/5} = \frac{-5\mu g}{2R} = \frac{-5(0.21)(9.8)}{2(0.11)} = -47 \text{ rad/s}^2$$

where the minus sign indicates that the angular acceleration is clockwise, the same direction as ω (so its angular motion is "speeding up").

(d) The center-of-mass of the sliding ball decelerates from $v_{\text{com},0}$ to v_{com} during time t according to Eq. 2-11:

$$v_{\text{com}} = v_{\text{com},0} - \mu gt .$$

During this time, the angular speed of the ball increases (in magnitude) from zero to $|\omega|$ according to Eq. 11-12:

$$|\omega| = |\alpha|\, t = \frac{5\mu gt}{2R} = \frac{v_{\text{com}}}{R}$$

where we have made use of our part (a) result in the last equality. We have two equations involving v_{com}, so we eliminate that variable and find

$$t = \frac{2v_{\text{com},0}}{7\mu g} = \frac{2(8.5)}{7(0.21)(9.8)} = 1.2 \text{ s} .$$

(e) The skid length of the ball is (using Eq. 2-15)

$$\Delta x = v_{\text{com},0}t - \frac{1}{2}(\mu g)t^2 = (8.5)(1.2) - \frac{1}{2}(0.21)(9.8)(1.2)^2 = 8.6 \text{ m} .$$

(f) The center of mass velocity at the time found in part (d) is

$$v_{\text{com}} = v_{\text{com},0} - \mu gt = 8.5 - (0.21)(9.8)(1.2) = 6.1 \text{ m/s} .$$

15. (a) The derivation of the acceleration is found in §12-4; Eq. 12-13 gives

$$a_{\rm com} = -\frac{g}{1 + I_{\rm com}/MR_0^2}$$

where the positive direction is upward. We use $I_{\rm com} = 950\,{\rm g\cdot cm^2}$, $M = 120\,{\rm g}$, $R_0 = 0.32\,{\rm cm}$ and $g = 980\,{\rm cm/s^2}$ and obtain

$$|a_{\rm com}| = \frac{980}{1 + (950)/(120)(0.32)^2} = 12.5 \text{ cm/s}^2 .$$

(b) Taking the coordinate origin at the initial position, Eq. 2-15 leads to $y_{\rm com} = \frac{1}{2}a_{\rm com}t^2$. Thus, we set $y_{\rm com} = -120\,{\rm cm}$, and find

$$t = \sqrt{\frac{2y_{\rm com}}{a_{\rm com}}} = \sqrt{\frac{2(-120\,{\rm cm})}{-12.5\,{\rm cm/s^2}}} = 4.38 \text{ s} .$$

(c) As it reaches the end of the string, its center of mass velocity is given by Eq. 2-11: $v_{\rm com} = a_{\rm com}t = (-12.5\,{\rm cm/s^2})(4.38\,{\rm s}) = -54.8\,{\rm cm/s}$, so its linear speed then is approximately 55 cm/s.

(d) The translational kinetic energy is $\frac{1}{2}mv_{\rm com}^2 = \frac{1}{2}(0.120\,{\rm kg})(0.548\,{\rm m/s})^2 = 1.8 \times 10^{-2}\,{\rm J}$.

(e) The angular velocity is given by $\omega = -v_{\rm com}/R_0$ and the rotational kinetic energy is

$$\frac{1}{2}I_{\rm com}\omega^2 = \frac{1}{2}I_{\rm com}\frac{v_{\rm com}^2}{R_0^2} = \frac{1}{2}\frac{(9.50 \times 10^{-5}\,{\rm kg\cdot m^2})(0.548\,{\rm m/s})^2}{(3.2 \times 10^{-3}\,{\rm m})^2}$$

which yields $K_{\rm rot} = 1.4$ J.

(f) The angular speed is $\omega = |v_{\rm com}|/R_0 = (0.548\,{\rm m/s})/(3.2 \times 10^{-3}\,{\rm m}) = 1.7 \times 10^2\,{\rm rad/s} = 27\,{\rm rev/s}$.

16. (a) The acceleration is given by Eq. 12-13:

$$a_{\rm com} = -\frac{g}{1 + I_{\rm com}/MR_0^2}$$

where upward is the positive translational direction. Taking the coordinate origin at the initial position, Eq. 2-15 leads to

$$y_{\rm com} = v_{\rm com,0}\,t + \frac{1}{2}a_{\rm com}t^2 = v_{\rm com,0}\,t - \frac{\frac{1}{2}gt^2}{1 + I_{\rm com}/MR_0^2}$$

where $y_{\rm com} = -1.2\,{\rm m}$ and $v_{\rm com,0} = -1.3$ m/s. Substituting $I_{\rm com} = 0.000095\,{\rm kg\cdot m^2}$, $M = 0.12\,{\rm kg}$, $R_0 = 0.0032\,{\rm m}$ and $g = 9.8\,{\rm m/s^2}$, we use the quadratic formula and find

$$\begin{aligned} t &= \frac{\left(1 + \frac{I_{\rm com}}{MR_0^2}\right)\left(v_{\rm com,0} \mp \sqrt{v_{\rm com,0}^2 - \frac{2gy_{\rm com}}{1+I_{\rm com}/MR_0^2}}\right)}{g} \\ &= \frac{\left(1 + \frac{0.000095}{(0.12)(0.0032)^2}\right)\left(-1.3 \mp \sqrt{1.3^2 - \frac{2(9.8)(-1.2)}{1+0.000095/(0.12)(0.0032)^2}}\right)}{9.8} \\ &= -21.7 \quad \text{or} \quad 0.885 \end{aligned}$$

where we choose $t = 0.89$ s as the answer.

(b) We note that the initial potential energy is $U_i = Mgh$ and $h = 1.2$ m (using the bottom as the reference level for computing U). The initial kinetic energy is as shown in Eq. 12-5, where the initial angular and linear speeds are related by Eq. 12-2. Energy conservation leads to

$$\begin{aligned} K_f &= K_i + U_i \\ &= \frac{1}{2}mv_{\text{com},0}^2 + \frac{1}{2}I\left(\frac{v_{\text{com},0}}{R_0}\right)^2 + Mgh \\ &= \frac{1}{2}(0.12)(1.3)^2 + \frac{1}{2}\left(9.5\times10^{-5}\right)\left(\frac{1.3}{0.0032}\right)^2 + (0.12)(9.8)(1.2) \\ &= 9.4\ \text{J} . \end{aligned}$$

(c) As it reaches the end of the string, its center of mass velocity is given by Eq. 2-11:

$$v_{\text{com}} = v_{\text{com},0} + a_{\text{com}}t = v_{\text{com},0} - \frac{gt}{1 + I_{\text{com}}/MR_0^2} \quad .$$

Thus, we obtain

$$v_{\text{com}} = -1.3 - \frac{(9.8)(0.885)}{1 + \frac{0.000095}{(0.12)(0.0032)^2}} = -1.41\ \text{m/s}$$

so its linear speed at that moment is approximately 1.4 m/s.

(d) The translational kinetic energy is $\frac{1}{2}mv_{\text{com}}^2 = \frac{1}{2}(0.12)(1.41)^2 = 0.12\,\text{J}$.

(e) The angular velocity at that moment is given by

$$\omega = -\frac{v_{\text{com}}}{R_0} = -\frac{-1.41}{0.0032} = 441$$

or approximately 440 rad/s.

(f) And the rotational kinetic energy is

$$\frac{1}{2}I_{\text{com}}\omega^2 = \frac{1}{2}\left(9.50\times10^{-5}\,\text{kg}\cdot\text{m}^2\right)(441\,\text{rad/s})^2 = 9.2\ \text{J} .$$

17. One method is to show that $\vec{r}\cdot\left(\vec{r}\times\vec{F}\right) = \vec{F}\cdot\left(\vec{r}\times\vec{F}\right) = 0$, but we choose here a more pedestrian approach: without loss of generality we take $\vec{r}$ and $\vec{F}$ to be in the xy plane – and will show that $\vec{\tau}$ has no x and y components (that it is parallel to the $\hat{\text{k}}$ direction). We proceed as follows: in the general expression $\vec{r} = x\hat{\text{i}} + y\hat{\text{j}} + z\hat{\text{k}}$, we will set $z = 0$ to constrain $\vec{r}$ to the xy plane, and similarly for $\vec{F}$. Using Eq. 3-30, we find $\vec{r}\times\vec{F}$ is equal to

$$(yF_z - zF_y)\,\hat{\text{i}} + (zF_x - xF_z)\,\hat{\text{j}} + (xF_y - yF_x)\,\hat{\text{k}}$$

and once we set $z = 0$ and $F_z = 0$ we obtain

$$\vec{\tau} = \vec{r}\times\vec{F} = (xF_y - yF_x)\,\hat{\text{k}}$$

which demonstrates that $\vec{\tau}$ has no component in the xy plane.

18. If we write $\vec{r} = x\hat{\text{i}} + y\hat{\text{j}} + z\hat{\text{k}}$, then (using Eq. 3-30) we find $\vec{r}\times\vec{F}$ is equal to

$$(yF_z - zF_y)\,\hat{\text{i}} + (zF_x - xF_z)\,\hat{\text{j}} + (xF_y - yF_x)\,\hat{\text{k}} .$$

(a) In the above expression, we set (with SI units understood) $x = -2$, $y = 0$, $z = 4$, $F_x = 6$, $F_y = 0$ and $F_z = 0$. Then we obtain $\vec{\tau} = \vec{r}\times\vec{F} = 24\,\hat{\text{j}}$ N·m.

(b) The values are just as in part (a) with the exception that now $F_x = -6$. We find $\vec{\tau} = \vec{r}\times\vec{F} = -24\,\hat{\text{j}}$ N·m.

(c) In the above expression, we set $x = -2$, $y = 0$, $z = 4$, $F_x = 0$, $F_y = 0$ and $F_z = 6$. We get $\vec{\tau} = \vec{r} \times \vec{F} = 12\,\hat{\jmath}$ N·m.

(d) The values are just as in part (c) with the exception that now $F_z = -6$. We find $\vec{\tau} = \vec{r} \times \vec{F} = -12\,\hat{\jmath}$ N·m.

19. If we write $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$, then (using Eq. 3-30) we find $\vec{r} \times \vec{F}$ is equal to

$$(yF_z - zF_y)\,\hat{\imath} + (zF_x - xF_z)\,\hat{\jmath} + (xF_y - yF_x)\,\hat{k}\ .$$

(a) In the above expression, we set (with SI units understood) $x = 0$, $y = -4$, $z = 3$, $F_x = 2$, $F_y = 0$ and $F_z = 0$. Then we obtain $\vec{\tau} = \vec{r} \times \vec{F} = \left(6\,\hat{\jmath} + 8\,\hat{k}\right)$ N·m. This has magnitude $\sqrt{6^2 + 8^2} = 10$ N·m and is seen to be parallel to the yz plane. Its angle (measured counterclockwise from the $+y$ direction) is $\tan^{-1}(8/6) = 53°$.

(b) In the above expression, we set $x = 0$, $y = -4$, $z = 3$, $F_x = 0$, $F_y = 2$ and $F_z = 4$. Then we obtain $\vec{\tau} = \vec{r} \times \vec{F} = -22\,\hat{\imath}$ N·m. This has magnitude 22 N·m and points in the $-x$ direction.

20. We use the notation $\vec{r}\,'$ to indicate the vector pointing from the axis of rotation directly to the position of the particle. If we write $\vec{r}\,' = x'\,\hat{\imath} + y'\,\hat{\jmath} + z'\,\hat{k}$, then (using Eq. 3-30) we find $\vec{r}\,' \times \vec{F}$ is equal to

$$(y'F_z - z'F_y)\,\hat{\imath} + (z'F_x - x'F_z)\,\hat{\jmath} + (x'F_y - y'F_x)\,\hat{k}\ .$$

(a) Here, $\vec{r}\,' = \vec{r}$. Dropping the primes in the above expression, we set (with SI units understood) $x = 0$, $y = 0.5$, $z = -2.0$, $F_x = 2$, $F_y = 0$ and $F_z = -3$. Then we obtain $\vec{\tau} = \vec{r} \times \vec{F} = \left(-1.5\,\hat{\imath} - 4\,\hat{\jmath} - \hat{k}\right)$ N·m.

(b) Now $\vec{r}\,' = \vec{r} - \vec{r}_o$ where $\vec{r}_o = 2\,\hat{\imath} - 3\,\hat{k}$. Therefore, in the above expression, we set $x' = -2.0$, $y' = 0.5$, $z' = 1.0$, $F_x = 2$, $F_y = 0$ and $F_z = -3$. Thus, we obtain $\vec{\tau} = \vec{r}\,' \times \vec{F} = \left(-1.5\,\hat{\imath} - 4\,\hat{\jmath} - \hat{k}\right)$ N·m.

21. If we write $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$, then (using Eq. 3-30) we find $\vec{r} \times \vec{F}$ is equal to

$$(yF_z - zF_y)\,\hat{\imath} + (zF_x - xF_z)\,\hat{\jmath} + (xF_y - yF_x)\,\hat{k}\ .$$

(a) Plugging in, we find

$$\vec{\tau} = ((3.0\,\text{m})(6.0\,\text{N}) - (4.0\,\text{m})(-8.0\,\text{N}))\,\hat{k} = 50\,\hat{k}\ \text{N·m}\ .$$

(b) We use Eq. 3-27, $|\vec{r} \times \vec{F}| = rF\sin\phi$, where ϕ is the angle between $\vec{r}$ and $\vec{F}$. Now $r = \sqrt{x^2 + y^2} = 5.0\,\text{m}$ and $F = \sqrt{F_x^2 + F_y^2} = 10\,\text{N}$. Thus $rF = (5.0\,\text{m})(10\,\text{N}) = 50\,\text{N}\cdot\text{m}$, the same as the magnitude of the vector product calculated in part (a). This implies $\sin\phi = 1$ and $\phi = 90°$.

22. If we write $\vec{r}\,' = x'\,\hat{\imath} + y'\,\hat{\jmath} + z'\,\hat{k}$, then (using Eq. 3-30) we find $\vec{r}\,' \times \vec{F}$ is equal to

$$(y'F_z - z'F_y)\,\hat{\imath} + (z'F_x - x'F_z)\,\hat{\jmath} + (x'F_y - y'F_x)\,\hat{k}\ .$$

(a) Here, $\vec{r}\,' = \vec{r}$ where $\vec{r} = 3\,\hat{\imath} - 2\,\hat{\jmath} + 4\,\hat{k}$, and $\vec{F} = \vec{F}_1$. Thus, dropping the primes in the above expression, we set (with SI units understood) $x = 3$, $y = -2$, $z = 4$, $F_x = 3$, $F_y = -4$ and $F_z = 5$. Then we obtain $\vec{\tau} = \vec{r} \times \vec{F}_1 = \left(6.0\,\hat{\imath} - 3.0\,\hat{\jmath} - 6.0\,\hat{k}\right)$ N·m.

(b) This is like part (a) but with $\vec{F} = \vec{F}_2$. We plug in $F_x = -3$, $F_y = -4$ and $F_z = -5$ and obtain $\vec{\tau} = \vec{r} \times \vec{F}_2 = \left(26\,\hat{\imath} + 3.0\,\hat{\jmath} - 18\,\hat{k}\right)$ N·m.

(c) We can proceed in either of two ways. We can add (vectorially) the answers from parts (a) and (b), or we can first add the two force vectors and then compute $\vec{\tau} = \vec{r} \times \left(\vec{F}_1 + \vec{F}_2\right)$ (these total force components are computed in the next part). The result is $\left(32\,\hat{\imath} - 24\,\hat{k}\right)$ N·m.

(d) Now $\vec{r}' = \vec{r} - \vec{r}_o$ where $\vec{r}_o = 3\,\hat{\imath} + 2\,\hat{\jmath} + 4\,\hat{k}$. Therefore, in the above expression, we set $x' = 0$, $y' = -4$, $z' = 0$, $F_x = 3 - 3 = 0$, $F_y = -4 - 4 = -8$ and $F_z = 5 - 5 = 0$. We get $\vec{\tau} = \vec{r}' \times \left(\vec{F}_1 + \vec{F}_2\right) = 0$.

23. We could proceed formally by setting up an xyz coordinate system and using Eq. 3-30 for the vector cross product, or we can approach this less formally in the style of Sample Problem 12-4 (which is our choice). For the 3.1 kg particle, Eq. 12-21 yields

$$\ell_1 = r_{\perp\,1}\, m v_1 = (2.8)(3.1)(3.6) = 31.2 \text{ kg}\cdot\text{m}^2/\text{s} .$$

Using the right-hand rule for vector products, we find this $(\vec{r}_1 \times \vec{p}_1)$ is out of the page, perpendicular to the plane of Fig. 12-35. And for the 6.5 kg particle, we find

$$\ell_2 = r_{\perp\,2}\, m v_2 = (1.5)(6.5)(2.2) = 21.4 \text{ kg}\cdot\text{m}^2/\text{s} .$$

And we use the right-hand rule again, finding that this $(\vec{r}_2 \times \vec{p}_2)$ is into the page. Consequently, the two angular momentum vectors are in opposite directions, so their vector sum is the *difference* of their magnitudes:

$$L = \ell_1 - \ell_2 = 9.8 \text{ kg}\cdot\text{m}^2/\text{s} .$$

24. We note that the component of $\vec{v}$ perpendicular to $\vec{r}$ has magnitude $v \sin\phi$ where $\phi = 30°$. A similar observation applies to $\vec{F}$.

(a) Eq. 12-20 leads to

$$\ell = r m v_\perp = (3.0)(2.0)(4.0)\sin 30° = 12 \text{ kg}\cdot\text{m}^2/\text{s} .$$

Using the right-hand rule for vector products, we find $\vec{r} \times \vec{p}$ points out of the page, perpendicular to the plane of Fig. 12-36.

(b) Eq. 11-31 (which is the same as Eq. 12-15) leads to

$$\tau = rF\sin\phi = (3.0)(2.0)\sin 30° = 3.0 \text{ N}\cdot\text{m} .$$

Using the right-hand rule for vector products, we find $\vec{r} \times \vec{F}$ is also out of the page.

25. (a) We use $\vec{\ell} = m\vec{r} \times \vec{v}$, where $\vec{r}$ is the position vector of the object, $\vec{v}$ is its velocity vector, and m is its mass. Only the x and z components of the position and velocity vectors are nonzero, so Eq. 3-30 leads to $\vec{r} \times \vec{v} = (-xv_z + zv_x)\,\hat{\jmath}$. Therefore,

$$\begin{aligned} \vec{\ell} &= m\,(-xv_z + zv_x)\,\hat{\jmath} \\ &= (0.25\,\text{kg})\,(-(2.0\,\text{m})(5.0\,\text{m/s}) + (-2.0\,\text{m})(-5.0\,\text{m/s}))\,\hat{\jmath} \\ &= 0 . \end{aligned}$$

(b) If we write $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$, then (using Eq. 3-30) we find $\vec{r} \times \vec{F}$ is equal to

$$(yF_z - zF_y)\,\hat{\imath} + (zF_x - xF_z)\,\hat{\jmath} + (xF_y - yF_x)\,\hat{k} .$$

With $x = 2.0$, $z = -2.0$, $F_y = 4.0$ and all other components zero (and SI units understood) the expression above yields $\vec{\tau} = \vec{r} \times \vec{F} = \left(8.0\,\hat{\imath} + 8.0\,\hat{k}\right)$ N·m.

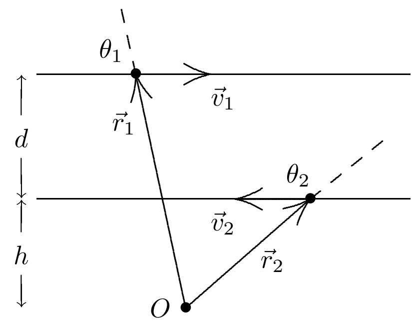
26. If we write $\vec{r}' = x'\,\hat{\imath} + y'\,\hat{\jmath} + z'\,\hat{k}$, then (using Eq. 3-30) we find $\vec{r}' \times \vec{v}$ is equal to

$$(y'v_z - z'v_y)\,\hat{\imath} + (z'v_x - x'v_z)\,\hat{\jmath} + (x'v_y - y'v_x)\,\hat{k} .$$

(a) Here, $\vec{r}' = \vec{r}$ where $\vec{r} = 3\,\hat{\imath} - 4\,\hat{\jmath}$. Thus, dropping the primes in the above expression, we set (with SI units understood) $x = 3$, $y = -4$, $z = 0$, $v_x = 30$, $v_y = 60$ and $v_z = 0$. Then (with $m = 2.0$ kg) we obtain $\vec{\ell} = m\,(\vec{r} \times \vec{v}) = 600\,\hat{k}$ kg·m²/s.

(b) Now $\vec{r}\,' = \vec{r} - \vec{r}_o$ where $\vec{r}_o = -2\,\hat{\imath} - 2\,\hat{\jmath}$. Therefore, in the above expression, we set $x' = 5$, $y' = -2$, $z' = 0$, $v_x = 30$, $v_y = 60$ and $v_z = 0$. We get $\vec{\ell} = m\,(\vec{r}\,' \times \vec{v}) = 720\,\hat{k}$ kg·m^2/s.

27. (a) The diagram below shows the particles and their lines of motion. The origin is marked O and may be anywhere. The angular momentum of particle 1 has magnitude $\ell_1 = mvr_1 \sin\theta_1 = mv(d+h)$ and it is into the page. The angular momentum of particle 2 has magnitude $\ell_2 = mvr_2 \sin\theta_2 = mvh$ and it is out of the page. The net angular momentum has magnitude $L = mv(d+h) - mvh = mvd$ and is into the page. This result is independent of the location of the origin.



(b) As indicated above, the expression does not change.

(c) Suppose particle 2 is traveling to the right. Then $L = mv(d+h) + mvh = mv(d+2h)$. This result depends on h, the distance from the origin to one of the lines of motion. If the origin is midway between the lines of motion, then $h = -d/2$ and $L = 0$.

28. (a) With $\vec{p} = m\vec{v} = -16\,\hat{\jmath}$ kg · m/s, we take the vector cross product (using either Eq. 3-30 or, more simply, Eq. 12-20 and the right-hand rule):

$$\vec{\ell} = \vec{r} \times \vec{p} = -32\,\hat{k}\ \text{kg}\cdot\text{m}^2/\text{s}\ .$$

(b) Now the axis passes through the point $\vec{R} = 4.0\,\hat{\jmath}$ m, parallel with the z axis. With $\vec{r}\,' = \vec{r} - \vec{R} = 2.0\,\hat{\imath}$ m, we again take the cross product and arrive at the same result as before:

$$\vec{\ell}\,' = \vec{r}\,' \times \vec{p} = -32\,\hat{k}\ \text{kg}\cdot\text{m}^2/\text{s}\ .$$

(c) Torque is defined in Eq. 12-14: $\vec{\tau} = \vec{r} \times \vec{F} = 12\,\hat{k}$ N·m.

(d) Using the notation from part (b),

$$\vec{\tau}\,' = \vec{r}\,' \times \vec{F} = 0\ \ .$$

29. If we write (for the general case) $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$, then (using Eq. 3-30) we find $\vec{r} \times \vec{v}$ is equal to

$$(yv_z - zv_y)\,\hat{\imath} + (zv_x - xv_z)\,\hat{\jmath} + (xv_y - yv_x)\,\hat{k}\ .$$

(a) The angular momentum is given by the vector product $\vec{\ell} = m\vec{r} \times \vec{v}$, where $\vec{r}$ is the position vector of the particle, $\vec{v}$ is its velocity, and $m = 3.0$ kg is its mass. Substituting (with SI units understood) $x = 3$, $y = 8$, $z = 0$, $v_x = 5$, $v_y = -6$ and $v_z = 0$ into the above expression, we obtain

$$\vec{\ell} = (3.0)\,((3)(-6) - (8.0)(5.0))\,\hat{k} = -1.7 \times 10^2\,\hat{k}\ \text{kg}\cdot\text{m}^2/\text{s}\ .$$

(b) The torque is given by Eq. 12-14, $\vec{\tau} = \vec{r} \times \vec{F}$. We write $\vec{r} = x\hat{\imath} + y\hat{\jmath}$ and $\vec{F} = F_x\,\hat{\imath}$ and obtain

$$\vec{\tau} = \left(x\,\hat{\imath} + y\,\hat{\jmath}\right) \times (F_x\,\hat{\imath}) = -yF_x\,\hat{k}$$

since $\hat{\imath} \times \hat{\imath} = 0$ and $\hat{\jmath} \times \hat{\imath} = -\hat{k}$. Thus, we find $\vec{\tau} = -(8.0\,\text{m})(-7.0\,\text{N})\,\hat{k} = 56\,\hat{k}$ N·m.

(c) According to Newton's second law $\vec{\tau} = d\vec{\ell}/dt$, so the rate of change of the angular momentum is $56\,\text{kg}\cdot\text{m}^2/\text{s}^2$, in the positive z direction.

30. The rate of change of the angular momentum is

$$\frac{d\vec{\ell}}{dt} = \vec{\tau}_1 + \vec{\tau}_2 = 2.0\hat{\imath} - 4.0\hat{\jmath}\ \mathrm{N{\cdot}m}\ .$$

Consequently, the vector $d\vec{\ell}/dt$ has a magnitude $\sqrt{2.0^2 + (-4.0)^2} = 4.5\,\mathrm{N\cdot m}$ and is at an angle θ (in the xy plane, or a plane parallel to it) measured from the positive x axis, where $\theta = \tan^{-1}\left(\frac{-4.0}{2.0}\right) = -63°$, the negative sign indicating that the angle is measured clockwise as viewed "from above" (by a person on the $+z$ axis).

31. We use a right-handed coordinate system with $+\hat{\mathrm{k}}$ directed out of the xy plane so as to be consistent with counterclockwise rotation (and the right-hand rule). Thus, all the angular momenta being considered are along the $-\hat{\mathrm{k}}$ direction; for example, in part (b) $\vec{\ell} = -4.0t^2\,\hat{\mathrm{k}}$ in SI units. We use Eq. 12-23.

(a) The angular momentum is constant so its derivative is zero. There is no torque in this instance.

(b) Taking the derivative with respect to time, we obtain the torque:

$$\vec{\tau} = \frac{d\vec{\ell}}{dt} = (-4.0\,\hat{\mathrm{k}})\,\frac{dt^2}{dt} = -8.0t\,\hat{\mathrm{k}}$$

in SI units (N·m). This vector points in the $-\hat{\mathrm{k}}$ direction (causing the clockwise motion to speed up) for all $t > 0$.

(c) With $\vec{\ell} = -4.0\sqrt{t}\,\hat{\mathrm{k}}$ in SI units, the torque is

$$\vec{\tau} = \left(-4.0\hat{\mathrm{k}}\right)\frac{d\sqrt{t}}{dt} = \left(-4.0\hat{\mathrm{k}}\right)\left(\frac{1}{2\sqrt{t}}\right)$$

which yields $\vec{\tau} = -2.0/\sqrt{t}\,\hat{\mathrm{k}}$ in SI units. This vector points in the $-\hat{\mathrm{k}}$ direction (causing the clockwise motion to speed up) for all $t > 0$ (and it is undefined for $t < 0$).

(d) Finally, we have

$$\vec{\tau} = \left(-4.0\hat{\mathrm{k}}\right)\frac{dt^{-2}}{dt} = \left(-4.0\hat{\mathrm{k}}\right)\left(\frac{-2}{t^3}\right)$$

which yields $\vec{\tau} = 8.0/t^3\,\hat{\mathrm{k}}$ in SI units. This vector points in the $+\hat{\mathrm{k}}$ direction (causing the initially clockwise motion to slow down) for all $t > 0$.

32. Both $\vec{r}$ and $\vec{v}$ lie in the xy plane. The position vector $\vec{r}$ has an x component that is a function of time (being the integral of the x component of velocity, which is itself time-dependent) and a y component that is constant ($y = -2.0$ m). In the cross product $\vec{r}\times\vec{v}$, all that matters is the y component of $\vec{r}$ since $v_x \neq 0$ but $v_y = 0$:

$$\vec{r}\times\vec{v} = -yv_x\,\hat{\mathrm{k}}\ .$$

(a) The angular momentum is $\vec{\ell} = m\,(\vec{r}\times\vec{v})$ where the mass is $m = 2.0$ kg in this case. With SI units understood and using the above cross-product expression, we have

$$\vec{\ell} = (2.0)\left(-(-2.0)\left(-6.0t^2\right)\right)\hat{\mathrm{k}} = -24t^2\,\hat{\mathrm{k}}$$

in kg·m^2/s. This implies the particle is moving clockwise (as observed by someone on the $+z$ axis) for $t > 0$.

(b) The torque is caused by the (net) force $\vec{F} = m\vec{a}$ where

$$\vec{a} = \frac{d\vec{v}}{dt} = -12t\,\hat{\imath}\ \mathrm{m/s^2}\ .$$

The remark above that only the y component of $\vec{r}$ still applies, since $a_y = 0$. We use $\vec{\tau} = \vec{r} \times \vec{F} = m\,(\vec{r} \times \vec{a})$ and obtain

$$\vec{\tau} = (2.0)\,(-(-2.0)(-12t))\,\hat{\mathrm{k}} = -48t\,\hat{\mathrm{k}}$$

in N·m. The torque on the particle (as observed by someone on the $+z$ axis) is clockwise, causing the particle motion (which was clockwise to begin with) to increase.

(c) We replace $\vec{r}$ with $\vec{r}\,'$ (measured relative to the new reference point) and note (again) that only its y component matters in these calculations. Thus, with $y' = -2.0 - (-3.0) = 1.0$ m, we find

$$\vec{\ell}\,' = (2.0)\left(-(1.0)\left(-6.0t^2\right)\right)\hat{\mathrm{k}} = 12t^2\,\hat{\mathrm{k}}$$

in kg·m^2/s. The fact that this is positive implies that the particle is moving counterclockwise relative to the new reference point.

(d) Using $\vec{\tau}\,' = \vec{r}\,' \times \vec{F} = m\,(\vec{r}\,' \times \vec{a})$, we obtain

$$\vec{\tau} = (2.0)\,(-(1.0)\,(-12t))\,\hat{\mathrm{k}} = 24t\,\hat{\mathrm{k}}$$

in N·m. The torque on the particle (as observed by someone on the $+z$ axis) is counterclockwise, relative to the new reference point.

33. (a) Since $\tau = dL/dt$, the average torque acting during any interval Δt is given by $\tau_{\text{avg}} = (L_f - L_i)\,/\Delta t$, where L_i is the initial angular momentum and L_f is the final angular momentum. Thus

$$\tau_{\text{avg}} = \frac{0.800\,\mathrm{kg\cdot m^2/s} - 3.00\,\mathrm{kg\cdot m^2/s}}{1.50\,\mathrm{s}}$$

which yields $\tau_{\text{avg}} = -1.467 \approx= -1.47\,\mathrm{N\cdot m}$. In this case the negative sign indicates that the direction of the torque is opposite the direction of the initial angular momentum, implicitly taken to be positive.

(b) The angle turned is $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$. If the angular acceleration α is uniform, then so is the torque and $\alpha = \tau/I$. Furthermore, $\omega_0 = L_i/I$, and we obtain

$$\begin{aligned}\theta &= \frac{L_i t + \frac{1}{2}\tau t^2}{I} \\ &= \frac{(3.00\,\mathrm{kg\cdot m^2/s})(1.50\,\mathrm{s}) + \frac{1}{2}(-1.467\,\mathrm{N\cdot m})(1.50\,\mathrm{s})^2}{0.140\,\mathrm{kg\cdot m^2}} \\ &= 20.4\ \text{rad}\ .\end{aligned}$$

(c) The work done on the wheel is

$$W = \tau\theta = (-1.47\,\mathrm{N\cdot m})(20.4\,\mathrm{rad}) = -29.9\ \mathrm{J}$$

where more precise values are used in the calculation than what is shown here. An equally good method for finding W is Eq. 11-44, which, if desired, can be rewritten as $W = (L_f^2 - L_i^2)/2I$.

(d) The average power is the work done by the flywheel (the negative of the work done on the flywheel) divided by the time interval:

$$P_{\text{avg}} = -\frac{W}{\Delta t} = -\frac{-29.8\,\mathrm{J}}{1.50\,\mathrm{s}} = 19.9\ \mathrm{W}\ .$$

34. (a) Eq. 11-27 gives $\alpha = \tau/I$ and Eq. 11-12 leads to $\omega = \alpha t = \tau t/I$. Therefore, the angular momentum at $t = 0.033$ s is

$$I\omega = \tau t = (16\,\mathrm{N\cdot m})(0.033\,\mathrm{s}) = 0.53\ \mathrm{kg\cdot m^2/s}$$

where this is essentially a derivation of the angular version of the impulse-momentum theorem.

(b) We find

$$\omega = \frac{\tau t}{I} = \frac{(16)(0.033)}{1.2 \times 10^{-3}} = 440 \text{ rad}$$

which we convert as follows: $\omega = (440)(60/2\pi) \approx 4200$ rev/min.

35. (a) A particle contributes mr^2 to the rotational inertia. Here r is the distance from the origin O to the particle. The total rotational inertia is

$$I = m(3d)^2 + m(2d)^2 + m(d)^2 = 14md^2 \ .$$

(b) The angular momentum of the middle particle is given by $L_m = I_m\omega$, where $I_m = 4md^2$ is its rotational inertia. Thus $L_m = 4md^2\omega$.

(c) The total angular momentum is $I\omega = 14md^2\omega$.

36. We integrate Eq. 12-29 (for a single torque) over the time interval (where the angular speed at the beginning is ω_i and at the end is ω_f)

$$\int \tau \, dt = \int \frac{dL}{dt} \, dt = L_f - L_i = I\left(\omega_f - \omega_i\right)$$

and if we use the calculus-based notion of the average of a function f

$$f_{\text{avg}} = \frac{1}{\Delta t} \int f \, dt$$

then (using Eq. 12-16) we obtain

$$\int \tau \, dt = \tau_{\text{avg}} \Delta t = F_{\text{avg}} R \Delta t \ .$$

Inserting this into the top line proves the relationship shown in the problem.

37. Suppose cylinder 1 exerts a uniform force of magnitude F on cylinder 2, tangent to the cylinder's surface at the point of contact. The torque applied to cylinder 2 is $\tau_2 = R_2F$ and the angular acceleration of that cylinder is $\alpha_2 = \tau_2/I_2 = R_2F/I_2$. As a function of time its angular velocity is

$$\omega_2 = \alpha_2 t = \frac{R_2Ft}{I_2} \ .$$

The forces of the cylinders on each other obey Newton's third law, so the magnitude of the force of cylinder 2 on cylinder 1 is also F. The torque exerted by cylinder 2 on cylinder 1 is $\tau_1 = R_1F$ and the angular acceleration of cylinder 1 is $\alpha_1 = \tau_1/I_1 = R_1F/I_1$. This torque slows the cylinder. As a function of time, its angular velocity is $\omega_1 = \omega_0 - R_1Ft/I_1$. The force ceases and the cylinders continue rotating with constant angular speeds when the speeds of points on their rims are the same ($R_1\omega_1 = R_2\omega_2$). Thus,

$$R_1\omega_0 - \frac{R_1^2Ft}{I_1} = \frac{R_2^2Ft}{I_2} \ .$$

When this equation is solved for the product of force and time, the result is

$$Ft = \frac{R_1I_1I_2}{I_1R_2^2 + I_2R_1^2} \, \omega_0 \ .$$

Substituting this expression for Ft in the ω_2 equation above, we obtain

$$\omega_2 = \frac{R_1R_2I_1}{I_1R_2^2 + I_2R_1^2} \, \omega_0 \ .$$

38. (a) For the hoop, we use Table 11-2(h) and the parallel-axis theorem to obtain

$$I_1 = I_{\text{com}} + mh^2 = \frac{1}{2}mR^2 + mR^2 = \frac{3}{2}mR^2 \ .$$

Of the thin bars (in the form of a square), the member along the rotation axis has (approximately) no rotational inertia about that axis (since it is thin), and the member farthest from it is very much like it (by being parallel to it) except that it is displaced by a distance h; it has rotational inertia given by the parallel axis theorem:

$$I_2 = I_{\text{com}} + mh^2 = 0 + mR^2 = mR^2 \ .$$

Now the two members of the square perpendicular to the axis have the same rotational inertia (that is, $I_3 = I_4$). We find I_3 using Table 11-2(e) and the parallel-axis theorem:

$$I_3 = I_{\text{com}} + mh^2 = \frac{1}{12}mR^2 + m\left(\frac{R}{2}\right)^2 = \frac{1}{3}mR^2 \ .$$

Therefore, the total rotational inertia is

$$I_1 + I_2 + I_3 + I_4 = \frac{19}{6}mR^2 = 1.6 \text{ kg}\cdot\text{m}^2 \ .$$

(b) The angular speed is constant:

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{2.5} = 2.5 \text{ rad/s} \ .$$

Thus, $L = I_{\text{total}}\omega = 4.0$ kg·m^2/s.

39. (a) No external torques act on the system consisting of the man, bricks, and platform, so the total angular momentum of the system is conserved. Let I_i be the initial rotational inertia of the system and let I_f be the final rotational inertia. Then $I_i\omega_i = I_f\omega_f$ and

$$\begin{aligned}\omega_f &= \left(\frac{I_i}{I_f}\right)\omega_i \\ &= \left(\frac{6.0\,\text{kg}\cdot\text{m}^2}{2.0\,\text{kg}\cdot\text{m}^2}\right)(1.2\,\text{rev/s}) \\ &= 3.6 \text{ rev/s} \ .\end{aligned}$$

(b) The initial kinetic energy is $K_i = \frac{1}{2}I_i\omega_i^2$, the final kinetic energy is $K_f = \frac{1}{2}I_f\omega_f^2$, and their ratio is

$$\frac{K_f}{K_i} = \frac{I_f\omega_f^2}{I_i\omega_i^2} = \frac{(2.0\,\text{kg}\cdot\text{m}^2)(3.6\,\text{rev/s})^2}{(6.0\,\text{kg}\cdot\text{m}^2)(1.2\,\text{rev/s})^2} = 3.0 \ .$$

(c) The man did work in decreasing the rotational inertia by pulling the bricks closer to his body. This energy came from the man's store of internal energy.

40. We use conservation of angular momentum: $I_m\omega_m = I_p\omega_p$. The respective angles θ_m and θ_p by which the motor and probe rotate are therefore related by

$$\int I_m\omega_m dt = I_m\theta_m = \int I_p\omega_p dt = I_p\theta_p$$

which gives

$$\theta_m = \frac{I_p\theta_p}{I_m} = \frac{\left(12\,\text{kg}\cdot\text{m}^2\right)(30^\circ)}{2.0\times10^{-3}\,\text{kg}\cdot\text{m}^2} = 180000^\circ \ .$$

The number of revolutions for the rotor is then $1.8\times10^5/360 = 500\,\text{rev}$.

41. (a) No external torques act on the system consisting of the two wheels, so its total angular momentum is conserved. Let I_1 be the rotational inertia of the wheel that is originally spinning (at ω_i) and I_2 be the rotational inertia of the wheel that is initially at rest. Then $I_1\omega_i = (I_1 + I_2)\omega_f$ and

$$\omega_f = \frac{I_1}{I_1 + I_2}\,\omega_i$$

where ω_f is the common final angular velocity of the wheels. Substituting $I_2 = 2I_1$ and $\omega_i = 800\,\text{rev/min}$, we obtain $\omega_f = 267\,\text{rev/min}$.

(b) The initial kinetic energy is $K_i = \frac{1}{2}I_1\omega_i^2$ and the final kinetic energy is $K_f = \frac{1}{2}(I_1 + I_2)\omega_f^2$. We rewrite this as

$$K_f = \frac{1}{2}(I_1 + 2I_1)\left(\frac{I_1\omega_i}{I_1 + 2I_1}\right)^2 = \frac{1}{6}I\omega_i^2 \ .$$

Therefore, the fraction lost, $(K_i - K_f)/K_i$, is

$$1 - \frac{K_f}{K_i} = 1 - \frac{\frac{1}{6}I\omega_i^2}{\frac{1}{2}I\omega_i^2} = \frac{2}{3} \ .$$

42. (a) We apply conservation of angular momentum: $I_1\omega_1 + I_2\omega_2 = (I_1 + I_2)\omega$. The angular speed after coupling is therefore

$$\omega = \frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2} = \frac{\left(3.3\,\text{kg}\cdot\text{m}^2\right)(450\,\text{rev/min}) + \left(6.6\,\text{kg}\cdot\text{m}^2\right)(900\,\text{rev/min})}{3.3\,\text{kg}\cdot\text{m}^2 + 6.6\,\text{kg}\cdot\text{m}^2} = 750\ \text{rev/min} \ .$$

(b) In this case, we obtain

$$\omega = \frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2} = \frac{(3.3)(450) + (6.6)(-900)}{3.3 + 6.6} = -450\,\text{rev/min}$$

where the minus sign indicates that $\vec{\omega}$ is in the direction of the second disk's initial angular velocity.

43. (a) In terms of the radius of gyration k, the rotational inertia of the merry-go-round is $I = Mk^2$. We obtain $I = (180\,\text{kg})(0.910\,\text{m})^2 = 149\,\text{kg}\cdot\text{m}^2$.

(b) An object moving along a straight line has angular momentum about any point that is not on the line. The magnitude of the angular momentum of the child about the center of the merry-go-round is given by Eq. 12-21, mvR, where R is the radius of the merry-go-round. Therefore,

$$\left|\vec{L}_{\text{child}}\right| = (44.0\,\text{kg})(3.00\,\text{m/s})(1.20\,\text{m}) = 158\,\text{kg}\cdot\text{m}^2/\text{s} \ .$$

(c) No external torques act on the system consisting of the child and the merry-go-round, so the total angular momentum of the system is conserved. The initial angular momentum is given by mvR; the final angular momentum is given by $\left(I + mR^2\right)\omega$, where ω is the final common angular velocity of the merry-go-round and child. Thus $mvR = (I + mR^2)\omega$ and

$$\omega = \frac{mvR}{I + mR^2} = \frac{158\,\text{kg}\cdot\text{m}^2/\text{s}}{149\,\text{kg}\cdot\text{m}^2 + (44.0\,\text{kg})(1.20\,\text{m})^2} = 0.744\ \text{rad/s} \ .$$

44. Angular momentum conservation $I_i\omega_i = I_f\omega_f$ leads to

$$\frac{\omega_f}{\omega_i} = \frac{I_i}{I_f}\omega_i = 3$$

which implies

$$\frac{K_f}{K_i} = \frac{\frac{1}{2}I_f\omega_f^2}{\frac{1}{2}I_i\omega_i^2} = \frac{I_f}{I_i}\left(\frac{\omega_f}{\omega_i}\right)^2 = 3 \ .$$

45. No external torques act on the system consisting of the train and wheel, so the total angular momentum of the system (which is initially zero) remains zero. Let $I = MR^2$ be the rotational inertia of the wheel. Its final angular momentum is $= I\omega\hat{\mathrm{k}} = -MR^2|\omega|\hat{\mathrm{k}}$, where $\hat{\mathrm{k}}$ is *up* in Fig. 12-40 and that last step (with the minus sign) is done in recognition that the wheel's clockwise rotation implies a negative value for ω. The linear speed of a point on the track is ωR and the speed of the train (going counterclockwise in Fig. 12-40 with speed v' relative to an outside observer) is therefore $v' = v - |\omega|R$ where v is its speed relative to the tracks. Consequently, the angular momentum of the train is $m(v - |\omega|R)R\,\hat{\mathrm{k}}$. Conservation of angular momentum yields

$$0 = -MR^2|\omega|\,\hat{\mathrm{k}} + m\,(v - |\omega|R)\,R\,\hat{\mathrm{k}}\ .$$

When this equation is solved for the angular speed, the result is

$$|\omega| = \frac{mvR}{(M+m)R^2} = \frac{mv}{(M+m)R}\ .$$

46. We assume that from the moment of grabbing the stick onward, they maintain rigid postures so that the system can be analyzed as a symmetrical rigid body with center of mass midway between the skaters.

(a) The total linear momentum is zero (the skaters have the same mass and equal-and-opposite velocities). Thus, their center of mass (the middle of the 3.0 m long stick) remains fixed and they execute circular motion (of radius $r = 1.5$ m) about it. Using Eq. 11-18, their angular velocity (counterclockwise as seen in Fig. 12-41) is

$$\omega = \frac{v}{r} = \frac{1.4}{1.5} = 0.93 \text{ rad/s}\ .$$

(b) Their rotational inertia is that of two particles in circular motion at $r = 1.5$ m, so Eq. 11-26 yields

$$I = \sum mr^2 = 2(50)(1.5)^2 = 225 \text{ kg}\cdot\text{m}^2\ .$$

Therefore, Eq. 11-27 leads to

$$K = \frac{1}{2}I\omega^2 = \frac{1}{2}(225)(0.93)^2 = 98 \text{ J}\ .$$

(c) Angular momentum is conserved in this process. If we label the angular velocity found in part (a) ω_i and the rotational inertia of part (b) as I_i, we have

$$I_i\omega_i = (225)(0.93) = I_f\omega_f\ .$$

The final rotational inertia is $\sum mr_f^2$ where $r_f = 0.5$ m so $I_f = 25$ kg$\cdot$m^2. Using this value, the above expression gives $\omega_f = 8.4$ rad/s.

(d) We find

$$K_f = \frac{1}{2}I_f\omega_f^2 = \frac{1}{2}(25)(8.4)^2 = 8.8\times 10^2 \text{ J}\ .$$

(e) We account for the large increase in kinetic energy (part (d) minus part (b)) by noting that the skaters do a great deal of work (converting their internal energy into mechanical energy) as they pull themselves closer – "fighting" what appears to them to be large "centrifugal forces" trying to keep them apart.

47. So that we don't get confused about $\pm$ signs, we write the angular *speed* of the lazy Susan as $|\omega|$ and reserve the ω symbol for the angular velocity (which, using a common convention, is negative-valued when the rotation is clockwise). When the roach "stops" we recognize that it comes to rest relative to the lazy Susan (not relative to the ground).

(a) Angular momentum conservation leads to

$$mvR + I\omega_0 = \left(mR^2 + I\right)\omega_f$$

which we can write (recalling our discussion about angular speed versus angular velocity) as

$$mvR - I\,|\omega_0| = -\left(mR^2 + I\right)|\omega_f| \ .$$

We solve for the final angular speed of the system:

$$|\omega_f| = \frac{mvR - I\,|\omega_0|}{mR^2 + I} \ .$$

(b) No, $K_f \neq K_i$ and – if desired – we can solve for the difference:

$$K_i - K_f = \frac{m\,I}{2}\,\frac{v^2 + \omega_0^2R^2 + 2Rv\,|\omega_0|}{mR^2 + I}$$

which is clearly positive. Thus, some of the initial kinetic energy is "lost" – that is, transferred to another form. And the culprit is the roach, who must find it difficult to stop (and "internalize" that energy).

48. The initial angular momentum of the system is zero. The final angular momentum of the girl-plus-merry-go-round is $\left(I + MR^2\right)\omega$ which we will take to be positive. The final angular momentum we associate with the thrown rock is negative: $-mRv$, where v is the speed (positive, by definition) of the rock relative to the ground.

(a) Angular momentum conservation leads to

$$0 = \left(I + MR^2\right)\omega - mRv \implies \omega = \frac{mRv}{I + MR^2} \ .$$

(b) The girl's linear speed is given by Eq. 11-18:

$$R\omega = \frac{mvR^2}{I + MR^2} \ .$$

49. For simplicity, we assume the record is turning freely, without any work being done by its motor (and without any friction at the bearings or at the stylus trying to slow it down). Before the collision, the angular momentum of the system (presumed positive) is $I_i\omega_i$ where $I_i = 5.0 \times 10^{-4}$ kg·m^2 and $\omega_i = 4.7$ rad/s. The rotational inertia afterwards is $I_f = I_i + mR^2$ where $m = 0.020$ kg and $R = 0.10$ m. The mass of the record (0.10 kg), although given in the problem, is not used in the solution. Angular momentum conservation leads to

$$I_i\omega_i = I_f\omega_f \implies \omega_f = \frac{I_i\omega_i}{I_i + mR^2} = 3.4 \text{ rad/s} \ .$$

50. The axis of rotation is in the middle of the rod, $r = 0.25$ m from either end. By Eq. 12-19, the initial angular momentum of the system (which is just that of the bullet, before impact) is $rmv\sin\phi$ where $m = 0.003$ kg and $\phi = 60°$. Relative to the axis, this is counterclockwise and thus (by the common convention) positive. After the collision, the moment of inertia of the system is $I = I_{\text{rod}} + mr^2$ where $I_{\text{rod}} = ML^2/12$ by Table 11-2(e), with $M = 4.0$ kg and $L = 0.5$ m. Angular momentum conservation leads to

$$rmv\sin\phi = \left(\frac{1}{12}ML^2 + mr^2\right)\omega \ .$$

Thus, with $\omega = 10$ rad/s, we obtain

$$v = \frac{\left(\frac{1}{12}(4.0)(0.5)^2 + (0.003)(0.25)^2\right)(10)}{(0.25)(0.003)\sin 60°} = 1.3 \times 10^3 \text{ m/s} \ .$$

51. (a) If we consider a short time interval from just before the wad hits to just after it hits and sticks, we may use the principle of conservation of angular momentum. The initial angular momentum is the angular momentum of the falling putty wad. The wad initially moves along a line that is $d/2$ distant from the axis of rotation, where $d = 0.500\,\text{m}$ is the length of the rod. The angular momentum of the wad is $mvd/2$ where $m = 0.0500\,\text{kg}$ and $v = 3.00$ m/s are the mass and initial speed of the wad. After the wad sticks, the rod has angular velocity ω and angular momentum $I\omega$, where I is the rotational inertia of the system consisting of the rod with the two balls and the wad at its end. Conservation of angular momentum yields $mvd/2 = I\omega$ where $I = (2M + m)(d/2)^2$ and $M = 2.00$ kg is the mass of each of the balls. We solve $mvd/2 = (2M + m)(d/2)^2\omega$ for the angular speed:

$$\omega = \frac{2mv}{(2M + m)d} = \frac{2(0.0500)(3.00)}{(2(2.00) + 0.0500)(0.500)} = 0.148 \text{ rad/s} .$$

(b) The initial kinetic energy is $K_i = \frac{1}{2}mv^2$, the final kinetic energy is $K_f = \frac{1}{2}I\omega^2$, and their ratio is $K_f/K_i = I\omega^2/mv^2$. When $I = (2M + m)d^2/4$ and $\omega = 2mv/(2M + m)d$ are substituted, this becomes

$$\frac{K_f}{K_i} = \frac{m}{2M + m} = \frac{0.0500}{2(2.00) + 0.0500} = 0.0123 .$$

(c) As the rod rotates, the sum of its kinetic and potential energies is conserved. If one of the balls is lowered a distance h, the other is raised the same distance and the sum of the potential energies of the balls does not change. We need consider only the potential energy of the putty wad. It moves through a $90°$ arc to reach the lowest point on its path, gaining kinetic energy and losing gravitational potential energy as it goes. It then swings up through an angle θ, losing kinetic energy and gaining potential energy, until it momentarily comes to rest. Take the lowest point on the path to be the zero of potential energy. It starts a distance $d/2$ above this point, so its initial potential energy is $U_i = mgd/2$. If it swings up to the angular position θ, as measured from its lowest point, then its final height is $(d/2)(1 - \cos\theta)$ above the lowest point and its final potential energy is $U_f = mg(d/2)(1 - \cos\theta)$. The initial kinetic energy is the sum of that of the balls and wad: $K_i = \frac{1}{2}I\omega^2 = \frac{1}{2}(2M + m)(d/2)^2\omega^2$. At its final position, we have $K_f = 0$. Conservation of energy provides the relation:

$$mg\,\frac{d}{2} + \frac{1}{2}(2M + m)\left(\frac{d}{2}\right)^2\omega^2 = mg\,\frac{d}{2}\,(1 - \cos\theta) .$$

When this equation is solved for $\cos\theta$, the result is

$$\begin{aligned}\cos\theta &= -\frac{1}{2}\left(\frac{2M + m}{mg}\right)\left(\frac{d}{2}\right)\omega^2 \\ &= -\frac{1}{2}\left(\frac{2(2.00\,\text{kg}) + 0.0500\,\text{kg}}{(0.0500\,\text{kg})(9.8\,\text{m/s}^2)}\right)\left(\frac{0.500\,\text{m}}{2}\right)(0.148\,\text{rad/s})^2 \\ &= -0.0226 .\end{aligned}$$

Consequently, the result for θ is $91.3°$. The total angle through which it has swung is $90° + 91.3° = 181°$.

52. We denote the cockroach with subscript 1 and the disk with subscript 2.

(a) Initially the angular momentum of the system consisting of the cockroach and the disk is

$$L_i = m_1 v_{1i} r_{1i} + I_2\omega_{2i} = m_1\omega_0 R^2 + \frac{1}{2}m_2\omega_0 R^2 .$$

After the cockroach has completed its walk, its position (relative to the axis) is $r_{1f} = R/2$ so the final angular momentum of the system is

$$L_f = m_1\omega_f\left(\frac{R}{2}\right)^2 + \frac{1}{2}m_2\omega_f R^2 .$$

Then from $L_f = L_i$ we obtain

$$\omega_f \left(\frac{1}{4} m_1 R^2 + \frac{1}{2} m_2 R \right) = \omega_0 \left(m_1 R^2 + \frac{1}{2} m_2 R^2 \right) .$$

Thus,

$$\begin{aligned} \omega_f - \omega_0 &= \omega_0 \left(\frac{m_1 R^2 + m_2 R^2/2}{m_1 R^2/4 + m_2 R^2/2} \right) - \omega_0 \\ &= \omega_0 \left(\frac{m + 10m/2}{m/4 + 10m/2} - 1 \right) \\ &= \omega_0 (1.14 - 1) \end{aligned}$$

which yields $\Delta\omega = 0.14\omega_0$. For later use, we note that $\omega_f/\omega_i = 1.14$.

(b) We substitute $I = L/\omega$ into $K = \frac{1}{2} I\omega^2$ and obtain $K = \frac{1}{2} L\omega$. Since we have $L_i = L_f$, the the kinetic energy ratio becomes

$$\frac{K}{K_0} = \frac{\frac{1}{2} L_f \omega_f}{\frac{1}{2} L_i \omega_i} = \frac{\omega_f}{\omega_i} = 1.14 .$$

(c) The cockroach does positive work while walking toward the center of the disk, increasing the total kinetic energy of the system.

53. If the polar cap melts, the resulting body of water will effectively increase the equatorial radius of the Earth from R_e to $R'_e = R_e + \Delta R$, thereby increasing the moment of inertia of the Earth and slowing its rotation (by conservation of angular momentum), causing the duration T of a day to increase by ΔT. We note that (in rad/s) $\omega = 2\pi/T$ so

$$\frac{\omega'}{\omega} = \frac{2\pi/T'}{2\pi/T} = \frac{T}{T'}$$

from which it follows that

$$\frac{\Delta\omega}{\omega} = \frac{\omega'}{\omega} - 1 = \frac{T}{T'} - 1 = -\frac{\Delta T}{T'} .$$

We can approximate that last denominator as T so that we end up with the simple relationship $|\Delta\omega|/\omega = \Delta T/T$. Now, conservation of angular momentum gives us

$$\Delta L = 0 = \Delta(I\omega) \approx I(\Delta\omega) + \omega(\Delta I)$$

so that $|\Delta\omega|/\omega = \Delta I/I$. Thus, using our expectation that rotational inertia is proportional to the equatorial radius squared (supported by Table 11-2(f) for a perfect uniform sphere, but then this isn't a perfect uniform sphere) we have

$$\begin{aligned} \frac{\Delta T}{T} &= \frac{\Delta I}{I} \\ &= \frac{\Delta(R_e^2)}{R_e^2} \approx \frac{2\Delta R_e}{R_e} \\ &= \frac{2(30\,\mathrm{m})}{6.37 \times 10^6\,\mathrm{m}} \end{aligned}$$

so with $T = 86400\,\mathrm{s}$ we find (approximately) that $\Delta T = 0.8$ s. The radius of the earth can be found in Appendix C or on the inside front cover of the textbook.

54. The initial rotational inertia of the system is $I_i = I_{\text{disk}} + I_{\text{student}}$ where $I_{\text{disk}} = 300\ \mathrm{kg{\cdot}m^2}$ (which, incidentally, does agree with Table 11-2(c)) and $I_{\text{student}} = mR^2$ where $m = 60$ kg and $R = 2.0$ m. The rotational inertia when the student reaches $r = 0.5$ m is $I_f = I_{\text{disk}} + mr^2$. Angular momentum conservation leads to

$$I_i\omega_i = I_f\omega_f \implies \omega_f = \omega_i \frac{I_{\text{disk}} + mR^2}{I_{\text{disk}} + mr^2}$$

which yields, for $\omega_i = 1.5$ rad/s, a final angular velocity of $\omega_f = 2.6$ rad/s.

55. Their angular velocities, when they are stuck to each other, are equal, regardless of whether they share the same central axis. The initial rotational inertia of the system is

$$I_0 = I_{\text{big disk}} + I_{\text{small disk}} \qquad \text{where} \quad I_{\text{big disk}} = \frac{1}{2}MR^2$$

using Table 11-2(c). Similarly, since the small disk is initially concentric with the big one, $I_{\text{small disk}} = \frac{1}{2}mr^2$. After it slides, the rotational inertia of the small disk is found from the parallel axis theorem (using $h = R - r$). Thus, the new rotational inertia of the system is

$$I = \frac{1}{2}MR^2 + \frac{1}{2}mr^2 + m(R-r)^2 \ .$$

(a) Angular momentum conservation, $I_0\omega_0 = I\omega$, leads to the new angular velocity:

$$\omega = \omega_0 \, \frac{\frac{1}{2}MR^2 + \frac{1}{2}mr^2}{\frac{1}{2}MR^2 + \frac{1}{2}mr^2 + m(R-r)^2} \ .$$

Substituting $M = 10m$ and $R = 3r$, this becomes $\omega = \omega_0(91/99)$. Thus, with $\omega_0 = 20$ rad/s, we find $\omega = 18$ rad/s.

(b) From the previous part, we know that

$$\frac{I_0}{I} = \frac{91}{99} \quad \text{and} \quad \frac{\omega}{\omega_0} = \frac{91}{99} \ .$$

Plugging these into the ratio of kinetic energies, we have

$$\frac{\frac{1}{2}I\omega^2}{\frac{1}{2}I_0\omega_0^2} = \frac{I}{I_0}\left(\frac{\omega}{\omega_0}\right)^2 = \frac{99}{91}\left(\frac{91}{99}\right)^2$$

which yields $K/K_0 = 0.92$.

56. This is a completely inelastic collision which we analyze using angular momentum conservation. Let m and v_0 be the mass and initial speed of the ball and R the radius of the merry-go-round. The initial angular momentum is

$$\vec{\ell}_0 = \vec{r}_0 \times \vec{p}_0 \quad \Longrightarrow \quad \ell_0 = R\,(mv_0)\sin 53^\circ$$

where 53° is the angle between the radius vector pointing to the child and the direction of $\vec{v}_0$. Thus, $\ell_0 = 19\,\text{kg}\cdot\text{m}^2/\text{s}$. Now, with SI units understood,

$$\begin{aligned} \ell_0 &= L_f \\ 19 &= I\omega \\ &= \left(150 + (30)R^2 + (1.0)R^2\right)\omega \end{aligned}$$

so that $\omega = 0.070$ rad/s.

57. (a) With $r = 0.60$ m, we obtain $I = 0.060 + (0.501)r^2 = 0.24\ \text{kg}\cdot\text{m}^2$.

(b) Invoking angular momentum conservation, with SI units understood,

$$\begin{aligned} \ell_0 &= L_f \\ mv_0r &= = I\omega \\ (0.001)v_0(0.60) &= (0.24)(4.5) \end{aligned}$$

which leads to $v_0 = 1.8 \times 10^3$ m/s.

58. We make the unconventional choice of *clockwise* sense as positive, so that the angular velocities in this problem are positive. With $r = 0.60$ m and $I_0 = 0.12\ \text{kg}\cdot\text{m}^2$, the rotational inertia of the putty-rod system (after the collision) is $I = I_0 + (0.20)r^2 = 0.19\ \text{kg}\cdot\text{m}^2$. Invoking angular momentum conservation, with SI units understood, we have

$$\begin{aligned} L_0 &= L_f \\ I_0\omega_0 &= I\omega \\ (0.12)(2.4) &= (0.19)\omega \end{aligned}$$

which yields $\omega = 1.5$ rad/s.

59. We make the unconventional choice of *clockwise* sense as positive, so that the angular velocities (and angles) in this problem are positive. Mechanical energy conservation applied to the particle (before impact) leads to

$$mgh = \frac{1}{2}mv^2 \implies v = \sqrt{2gh}$$

for its speed right before undergoing the completely inelastic collision with the rod. The collision is described by angular momentum conservation:

$$mvd = \left(I_{\text{rod}} + md^2\right)\omega$$

where I_{rod} is found using Table 11-2(e) and the parallel axis theorem:

$$I_{\text{rod}} = \frac{1}{12}Md^2 + M\left(\frac{d}{2}\right)^2 = \frac{1}{3}Md^2 \ .$$

Thus, we obtain the angular velocity of the system immediately after the collision:

$$\omega = \frac{md\sqrt{2gh}}{\frac{1}{3}Md^2 + md^2}$$

which means the system has kinetic energy $\frac{1}{2}(I_{\text{rod}} + md^2)\omega^2$ which will turn into potential energy in the final position, where the block has reached a height H (relative to the lowest point) and the center of mass of the stick has increased its height by $H/2$. From trigonometric considerations, we note that $H = d(1 - \cos\theta)$, so we have

$$\begin{aligned} \frac{1}{2}\left(I_{\text{rod}} + md^2\right)\omega^2 &= mgH + Mg\frac{H}{2} \\ \frac{1}{2}\frac{m^2d^2(2gh)}{\frac{1}{3}Md^2 + md^2} &= \left(m + \frac{M}{2}\right)gd\,(1 - \cos\theta) \end{aligned}$$

from which we obtain

$$\theta = \cos^{-1}\left(1 - \frac{m^2h}{\left(m + \frac{1}{2}M\right)\left(m + \frac{1}{3}M\right)}\right) \ .$$

60. (a) Since the motorcycle is going leftward across our field of view, then when its wheels are rolling they must be going counterclockwise (which we take as the positive sense of rotation, which is the usual convention).

(b) Just before the rear wheel spins up to ω_{wf} it has the angular velocity necessary for rolling $\omega_{wR} = v/R$ where $v = 32$ m/s and $R = 0.30$ m. Since $\omega_{wf} > \omega_{wR}$ the system would seem to have suddenly acquired an increase in (positive) angular momentum − without the action of external torques! Since this is not possible, then the other constituents of the system (the man and the motorcycle body, which the problem just refers to as "the motorcycle") must have acquired some (negative) angular momentum. Thus, the motorcycle rotated clockwise.

(c) Assuming the system's (translational) projectile motion is symmetrical (as in Fig. 4-34 in the textbook) then (with $+y$ upward) it starts with $v_{0y} = v \sin 15°$ and returns with $v_y = -v \sin 15°$. Substituting these into Eq. 2-11 (with $a = -g$) leads to

$$-v \sin 15° = v \sin 15° - gt \implies t = \frac{2v \sin 15°}{g} = 1.7 \text{ s} .$$

(d) As noted in our solution of part (b), $\omega_{wR} = v/R$ which yields the value $\omega_{wR} = 32/0.30 = 106.7$ rad/s. In keeping with the significant figures rules, we round this to 1.1×10^2 rad/s.

(e) We have $L_w = I_w \omega_{wR} = (0.40)(106.7) = 43$ kg·m^2/s.

(f) Recalling our discussion in part (b), we apply angular momentum conservation:

$$I_w \omega_{wR} = I_w \omega_{wf} + I_c \omega_c \implies \omega_c = -\frac{I_w (\omega_{wf} - \omega_{wR})}{I_c}$$

which yields $\omega_c = -1.067$ rad/s or $|\omega_c| \approx 1.1$ rad/s.

(g) The problem states that the spin up occurs immediately – the moment this becomes a projectile motion problem (for the center of mass). We assume the motorcycle turns at the (constant) rate $|\omega_c|$ for the duration of the motion. Using the more precise values from our previous results, we are led to

$$\theta = \omega_c t = -1.80 \text{ rad}$$

which we convert (multiplying by $180/\pi$) to $-103°$. Rounding off, we find $|\theta| \approx 100°$.

61. (a) The derivation of the acceleration is found in §12-4; Eq. 12-13 gives

$$a_{\text{com}} = -\frac{g}{1 + I_{\text{com}}/MR_0^2}$$

where the positive direction is upward. We use $I_{\text{com}} = \frac{1}{2}MR^2$ where the radius is $R = 0.32$ m and $M = 116$ kg is the *total* mass (thus including the fact that there are two disks) and obtain

$$a = -\frac{g}{1 + \frac{1}{2}MR^2/MR_0^2} = \frac{g}{1 + \frac{1}{2}\left(\frac{R}{R_0}\right)^2}$$

which yields $a = -g/51$ upon plugging in $R_0 = R/10 = 0.032$ m. Thus, the magnitude of the center of mass acceleration is 0.19 m/s^2 and the direction of that vector is down.

(b) As observed in §12-4, our result in part (a) applies to both the descending and the rising yoyo motions.

(c) The external forces on the center of mass consist of the cord tension (upward) and the pull of gravity (downward). Newton's second law leads to

$$T - Mg = ma \implies T = M\left(g - \frac{g}{51}\right)$$

which yields $T = 1.1 \times 10^3$ N.

(d) Our result in part (c) indicates that the tension is well below the ultimate limit for the cord.

(e) As we saw in our acceleration computation, all that mattered was the ratio R/R_0 (and, of course, g). So if it's a scaled-up version, then such ratios are unchanged and we obtain the same result.

(f) Since the tension also depends on mass, then the larger yoyo will involve a larger cord tension.

62. We denote the wheel with subscript 1 and the whole system with subscript 2. We take clockwise as the negative sense for rotation (as is the usual convention). Conservation of angular momentum gives $L = I_1\omega_1 = I_2\omega_2$, where $I_1 = m_1R_1^2$. Thus

$$\omega_2 = \omega_1\,\frac{I_1}{I_2} = (-57.7\,\text{rad/s})\frac{\left(37\,\text{N}/9.8\,\text{m/s}^2\right)(0.35\,\text{m})^2}{2.1\,\text{kg}\cdot\text{m}^2}$$

which yields $\omega_2 = -12.7\,\text{rad/s}$. The system therefore rotates clockwise (as seen from above) at the rate of 12.7 rad/s.

63. We use $L = I\omega$ and $K = \frac{1}{2}I\omega^2$ and observe that the speed of points on the rim (corresponding to the speed of points on the belt) of wheels A and B must be the same (so $\omega_A R_A = \omega_B r_B$).

(a) If $L_A = L_B$ (call it L) then the ratio of rotational inertias is

$$\frac{I_A}{I_B} = \frac{L/\omega_A}{L/\omega_B} = \frac{\omega_A}{\omega_B} = \frac{R_A}{R_B} = \frac{1}{3}\ .$$

(b) If we have $K_A = K_B$ (call it K) then the ratio of rotational inertias becomes

$$\frac{I_A}{I_B} = \frac{2K/\omega_A^2}{2K/\omega_B^2} = \left(\frac{\omega_B}{\omega_A}\right)^2 = \left(\frac{R_A}{R_B}\right)^2 = \frac{1}{9}\ .$$

64. Since we will be taking the vector cross product in the course of our calculations, below, we note first that when the two vectors in a cross product $\vec{A}\times\vec{B}$ are in the xy plane, we have $\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath}$ and $\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath}$, and Eq. 3-30 leads to

$$\vec{A}\times\vec{B} = (A_xB_y - A_yB_x)\,\hat{k}\ .$$

(a) We set up a coordinate system with its origin at the firing point, the positive x axis in the horizontal direction of motion of the projectile and the positive y axis vertically upward. The projectile moves in the xy plane, and if $+x$ is to our right then the "rotation" sense will be clockwise. Thus, we expect our answer to be negative. The position vector for the projectile (as a function of time) is given by

$$\vec{r} = (v_{0x}t)\hat{\imath} + \left(v_{0y}t - \frac{1}{2}gt^2\right)\hat{\jmath} = (v_0\cos\theta_0 t)\hat{\imath} + (v_0\sin\theta_0 - gt)\hat{\jmath}$$

and the velocity vector is

$$\vec{v} = v_x\hat{\imath} + v_y\hat{\jmath} = (v_0\cos\theta_0)\,\hat{\imath} + (v_0\sin\theta_0 - gt)\hat{\jmath}\ .$$

Thus (using the above observation about the cross product of vectors in the xy plane) the angular momentum of the projectile as a function of time is

$$\vec{\ell} = m\vec{r}\times\vec{v} = -\frac{1}{2}mv_0\cos\theta_0 gt^2\hat{k}\,.$$

(b) We take the derivative of our result in part (a): $\frac{d\vec{\ell}}{dt} = -v_0mgt\cos\theta_0\hat{k}$.

(c) Again using the above observation about the cross product of vectors in the xy plane, we find

$$\vec{r}\times\vec{F} = \left((v_0\cos\theta_0 t)\hat{\imath} + r_y\hat{\jmath}\right)\times(-mg\hat{\jmath}) = -v_0mgt\cos\theta_0\,\hat{k}$$

which is the same as the result in part (b).

(d) They are the same because $d\vec{\ell}/dt = \tau = \vec{r}\times\vec{F}$.

65. The problem asks that we put the origin of coordinates at point O but compute all the angular momenta and torques relative to point A. This requires some care in defining $\vec{r}$ (which occurs in the angular momentum and torque formulas). If $\vec{r}_O$ locates the point (where the block is) in the prescribed coordinates, and $\vec{r}_{OA} = -1.2\,\hat{\jmath}$ points from O to A, then $\vec{r} = \vec{r}_O - \vec{r}_{OA}$ gives the position of the block relative to point A. SI units are used throughout this problem.

(a) Here, the momentum is $\vec{p}_0 = m\vec{v}_0 = 1.5\,\hat{\imath}$ and $\vec{r}_0 = 1.2\,\hat{\jmath}$, so that

$$\vec{\ell}_0 = \vec{r}_0 \times \vec{p}_0 = -1.8\,\hat{\mathrm{k}}\ \mathrm{kg{\cdot}m^2/s}\ .$$

(b) The horizontal component of momentum doesn't change in projectile motion (without friction), and its vertical component depends on how far its fallen. From either the free-fall equations of Ch. 2 or the energy techniques of Ch. 8, we find the vertical momentum component after falling a distance h to be $-m\sqrt{2gh}$. Thus, with $m = 0.50$ and $h = 1.2$, the momentum just before the block hits the floor is $\vec{p} = 1.5\,\hat{\imath} - 2.4\,\hat{\jmath}$. Now, $\vec{r} = R\,\hat{\imath}$ where R is figured from the projectile motion equations of Ch. 4 to be $R = v_0\sqrt{\frac{2h}{g}} = 1.5$ m. Consequently,

$$\vec{\ell} = \vec{r} \times \vec{p} = -3.6\,\hat{\mathrm{k}}\ \mathrm{kg{\cdot}m^2/s}\ .$$

(c) and (d) The only force on the object is its weight $m\vec{g} = -4.9\,\hat{\jmath}$. Thus,

$$\begin{aligned} \vec{\tau}_0 &= \vec{r}_0 \times \vec{F} = 0 \\ \vec{\tau} &= \vec{r} \times \vec{F} = -7.3\,\hat{\mathrm{k}}\ \mathrm{N{\cdot}m}\ . \end{aligned}$$

66. Since we will be taking the vector cross product in the course of our calculations, below, we note first that when the two vectors in a cross product $\vec{A} \times \vec{B}$ are in the xy plane, we have $\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath}$ and $\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath}$, and Eq. 3-30 leads to

$$\vec{A} \times \vec{B} = (A_xB_y - A_yB_x)\,\hat{\mathrm{k}}\ .$$

Now, we choose coordinates centered on point O, with $+x$ rightwards and $+y$ upwards. In unit-vector notation, the initial position of the particle, then, is $\vec{r}_0 = s\,\hat{\imath}$ and its later position (halfway to the ground) is $\vec{r} = s\,\hat{\imath} - \frac{1}{2}h\,\hat{\jmath}$. Using either the free-fall equations of Ch. 2 or the energy techniques of Ch. 8, we find the speed at its later position to be $v = \sqrt{2g|\Delta y|} = \sqrt{gh}$. Its momentum there is $\vec{p} = -M\sqrt{gh}\,\hat{\jmath}$. We find the angular momentum using Eq. 12-18 and our observation, above, about the cross product of two vectors in the xy plane.

$$\vec{\ell} = \vec{r} \times \vec{p} = -sM\sqrt{gh}\ \hat{\mathrm{k}}$$

Therefore, its magnitude is $|\vec{\ell}| = sM\sqrt{gh}$.

67. We may approximate the planets and their motions as particles in circular orbits, and use Eq. 12-26

$$L = \sum_{i=1}^{9} \ell_i = \sum_{i=1}^{9} m_i r_i^2 \omega_i$$

to compute the total angular momentum. Since we assume the angular speed of each one is constant, we have (in rad/s) $\omega_i = 2\pi/T_i$ where T_i is the time for that planet to go around the Sun (this and related information is found in Appendix C but there, the T_i are expressed in years and we'll need to convert with 3.156×10^7 s/y, and the M_i are expressed as multiples of M_{earth} which we'll convert by multiplying by 5.98×10^{24} kg).

(a) Using SI units, we find (with $i = 1$ designating Mercury)

$$\begin{aligned} L &= \sum_{i=1}^{9} m_i r_i^2 \left(\frac{2\pi}{T_i}\right) \\ &= 2\pi \frac{3.34 \times 10^{23}}{7.61 \times 10^{6}} \left(57.9 \times 10^{9}\right)^2 + 2\pi \frac{4.87 \times 10^{24}}{19.4 \times 10^{7}} \left(108 \times 10^{9}\right)^2 + \\ &\quad 2\pi \frac{5.98 \times 10^{24}}{3.156 \times 10^{7}} \left(150 \times 10^{9}\right)^2 + 2\pi \frac{6.40 \times 10^{23}}{5.93 \times 10^{7}} \left(228 \times 10^{9}\right)^2 + \\ &\quad 2\pi \frac{1.9 \times 10^{27}}{3.76 \times 10^{8}} \left(778 \times 10^{9}\right)^2 + 2\pi \frac{5.69 \times 10^{26}}{9.31 \times 10^{8}} \left(1430 \times 10^{9}\right)^2 + \\ &\quad 2\pi \frac{8.67 \times 10^{25}}{2.65 \times 10^{9}} \left(2870 \times 10^{9}\right)^2 + 2\pi \frac{1.03 \times 10^{26}}{5.21 \times 10^{9}} \left(4500 \times 10^{9}\right)^2 + \\ &\quad 2\pi \frac{1.2 \times 10^{22}}{7.83 \times 10^{9}} \left(5900 \times 10^{9}\right)^2 \\ &= 3.14 \times 10^{43}\ \mathrm{kg{\cdot}m^2/s}\ . \end{aligned}$$

(b) The fractional contribution of Jupiter is

$$\frac{\ell_5}{L} = \frac{2\pi \frac{1.9 \times 10^{27}}{3.76 \times 10^{8}} \left(778 \times 10^{9}\right)^2}{3.14 \times 10^{43}} = 0.61\ .$$

68. If we write $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$, then (using Eq. 3-30) we find $\vec{r} \times \vec{F}$ is equal to

$$(yF_z - zF_y)\,\hat{\imath} + (zF_x - xF_z)\,\hat{\jmath} + (xF_y - yF_x)\,\hat{k}\ .$$

With (using SI units) $x = 0$, $y = -4.0$, $z = 5.0$, $F_x = 0$, $F_y = -2.0$ and $F_z = 3.0$ (these latter terms being the individual forces that contribute to the net force), the expression above yields

$$\vec{\tau} = \vec{r} \times \vec{F} = -2.0\,\hat{\imath}\ \mathrm{N{\cdot}m}\ .$$

69. We make the unconventional choice of *clockwise* sense as positive, so that the angular acceleration are positive (as is the linear acceleration of the center of mass, since we take rightwards as positive).

(a) We approach this in the manner of Eq. 12-3 (*pure rotation* about point P) but use torques instead of energy:

$$\tau = I_P\,\alpha \quad \text{where } I_P = \frac{1}{2}MR^2 + MR^2$$

where the parallel-axis theorem and Table 11-2(c) has been used. The torque (relative to point P) is due to the $F = 12$ N force and is $\tau = F(2R)$. In this way, we find

$$\alpha = \frac{(12)(0.20)}{0.05 + 0.10} = 16\ \mathrm{rad/s^2}\ .$$

Hence, $a_{\mathrm{com}} = R\alpha = 1.6\ \mathrm{m/s^2}$.

(b) As shown above, $\alpha = 16\ \mathrm{rad/s^2}$.

(c) Applying Newton's second law in its linear form yields

$$(12\ \mathrm{N}) - f = Ma_{\mathrm{com}}\ .$$

Therefore, $f = -4.0$ N. Contradicting what we assumed in setting up our force equation, the friction force is found to point *rightward* (with magnitude 4.0 N).

70. The speed of the center of mass of the car is $v = (40)(1000/3600) = 11$ m/s. The angular speed of the wheels is given by Eq. 12-2: $\omega = v/R$ where the wheel radius R is not given (but will be seen to cancel in these calculations).

   (a) For one wheel of mass $M = 32$ kg, Eq. 11-27 gives (using Table 11-2(c))

   $$K_{\text{rot}} = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{v}{R}\right)^2 = \frac{1}{4}Mv^2$$

   which yields $K_{\text{rot}} = 9.9 \times 10^2$ J. The time given in the problem (10 s) is not used in the solution.

   (b) Adding the above to the wheel's translational kinetic energy, $\frac{1}{2}Mv^2$, leads to

   $$K_{\text{wheel}} = \frac{1}{2}Mv^2 + \frac{1}{4}Mv^2 = \frac{3}{4}(32)(11)^2 = 3.0 \times 10^3 \text{ J} .$$

   (c) With $M_{\text{car}} = 1700$ kg and the fact that there are four wheels, we have

   $$\frac{1}{2}M_{\text{car}}v^2 + 4\left(\frac{3}{4}Mv^2\right) = 1.2 \times 10^5 \text{ J} .$$

71. Information relevant to this calculation can be found in Appendix C. We apply angular momentum conservation using Table 11-2(f):

   $$I_i\omega_i = I_f\omega_f \quad \Longrightarrow \quad \frac{\omega_i}{\omega_f} = \frac{I_f}{I_i} = \frac{\frac{2}{5}MR_f^2}{\frac{2}{5}MR_i^2}$$

   and we note that $\omega = 2\pi/T$ in rad/min if T is the period in minutes. Plugging this into to our expression above (and simplifying) yields

   $$\frac{T_f}{T_i} = \left(\frac{R_f}{R_i}\right)^2 .$$

   Substituting $T_i = 25(24)(60) = 36000$ min, $R_f = 6.37 \times 10^6$ m and $R_i = 6.96 \times 10^8$ m into this relation, we obtain $T_f = 3.0$ min.

72. (a) We use Table 11-2(e) and the parallel-axis theorem to obtain the rod's rotational inertia about an axis through one end:

   $$I = I_{\text{com}} + Mh^2 = \frac{1}{12}ML^2 + M\left(\frac{L}{2}\right)^2 = \frac{1}{3}ML^2$$

   where $L = 6.00$ m and $M = 10.0/9.8 = 1.02$ kg. Thus, $I = 12.2$ kg$\cdot$m^2.

   (b) Using $\omega = (240)(2\pi/60) = 25.1$ rad/s, Eq. 12-31 gives the magnitude of the angular momentum as $I\omega = (12.2)(25.1) = 308$ kg$\cdot$m^2/s. Since it is rotating clockwise as viewed from above, then the right-hand rule indicates that its direction is down.

73. This problem involves the vector cross product of vectors lying in the xy plane. For such vectors, if we write $\vec{r}\,' = x'\,\hat{\imath} + y'\,\hat{\jmath}$, then (using Eq. 3-30) we find

   $$\vec{r}\,' \times \vec{v} = (x'v_y - y'v_x)\,\hat{k} .$$

   (a) Here, $\vec{r}\,'$ points in either the $+\hat{\imath}$ or the $-\hat{\imath}$ direction (since the particle moves along the x axis). It has no y' or z' components, and neither does $\vec{v}$, so it is clear from the above expression (or, more simply, from the fact that $\hat{\imath} \times \hat{\imath} = 0$) that $\vec{\ell} = m(\vec{r}\,' \times \vec{v}) = 0$ in this case.

   (b) The net force is in the $-\hat{\imath}$ direction (as one finds from differentiating the velocity expression, yielding the acceleration), so, similar to what we found in part (a), we obtain $\tau = \vec{r}\,' \times \vec{F} = 0$.

(c) Now, $\vec{r}\,' = \vec{r} - \vec{r}_o$ where $\vec{r}_o = 2.0\,\hat{\imath} + 5.0\,\hat{\jmath}$ (with SI units understood) and points from $(2.0, 5.0, 0)$ to the instantaneous position of the car (indicated by $\vec{r}$ which points in either the $+x$ or $-x$ directions, or nowhere (if the car is passing through the origin)). Since $\vec{r} \times \vec{v} = 0$ we have (plugging into our general expression above)

$$\vec{\ell} = m\,(\vec{r}\,' \times \vec{v}) = -m\,(\vec{r}_o \times \vec{v}) = -(3.0)\left((2.0)(0) - (5.0)\left(-2.0t^3\right)\right)\hat{\mathrm{k}}$$

which yields $\vec{\ell} = -30t^3\,\hat{\mathrm{k}}$ in SI units (kg·m^2/s).

(d) The acceleration vector is given by $\vec{a} = \frac{d\vec{v}}{dt} = -6.0t^2\,\hat{\imath}$ in SI units, and the net force on the car is $m\vec{a}$. In a similar argument to that given in the previous part, we have

$$\vec{\tau} = m\,(\vec{r}\,' \times \vec{a}) = -m\,(\vec{r}_o \times \vec{a}) = -(3.0)\left((2.0)(0) - (5.0)\left(-6.0t^2\right)\right)\hat{\mathrm{k}}$$

which yields $\vec{\tau} = -90t^2\,\hat{\mathrm{k}}$ in SI units (N·m).

(e) In this situation, $\vec{r}\,' = \vec{r} - \vec{r}_o$ where $\vec{r}_o = 2.0\,\hat{\imath} - 5.0\,\hat{\jmath}$ (with SI units understood) and points from $(2.0, -5.0, 0)$ to the instantaneous position of the car (indicated by $\vec{r}$ which points in either the $+x$ or $-x$ directions, or nowhere (if the car is passing through the origin)). Since $\vec{r} \times \vec{v} = 0$ we have (plugging into our general expression above)

$$\vec{\ell} = m\,(\vec{r}\,' \times \vec{v}) = -m\,(\vec{r}_o \times \vec{v}) = -(3.0)\left((2.0)(0) - (-5.0)\left(-2.0t^3\right)\right)\hat{\mathrm{k}}$$

which yields $\vec{\ell} = 30t^3\,\hat{\mathrm{k}}$ in SI units (kg·m^2/s).

(f) Again, the acceleration vector is given by $\vec{a} = -6.0t^2\,\hat{\imath}$ in SI units, and the net force on the car is $m\vec{a}$. In a similar argument to that given in the previous part, we have

$$\vec{\tau} = m\,(\vec{r}\,' \times \vec{a}) = -m\,(\vec{r}_o \times \vec{a}) = -(3.0)\left((2.0)(0) - (-5.0)\left(-6.0t^2\right)\right)\hat{\mathrm{k}}$$

which yields $\vec{\tau} = 90t^2\,\hat{\mathrm{k}}$ in SI units (N·m).

74. This problem involves the vector cross product of vectors lying in the xy plane. For such vectors, if we write $\vec{r} = x\,\hat{\imath} + y\,\hat{\jmath}$, then (using Eq. 3-30) we find

$$\vec{r} \times \vec{p} = (\Delta x p_y - \Delta y p_x)\,\hat{\mathrm{k}}\ .$$

The momentum components are $p_x = p\cos\theta$ and $p_y = p\sin\theta$ where $p = 2.4$ (SI units understood) and $\theta = 115°$. The mass (0.80 kg) given in the problem is not used in the solution. Thus, with $x = 2.0$, $y = 3.0$ and the momentum components described above, we obtain

$$\vec{\ell} = \vec{r} \times \vec{p} = 7.4\,\hat{\mathrm{k}}\ \mathrm{kg\cdot m^2/s}\ .$$

75. Information relevant to this calculation can be found in Appendix C or on the inside front cover of the textbook. The angular speed is constant so

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{86400} = 7.3 \times 10^{-5}\ \mathrm{rad/s}\ .$$

Thus, with $m = 84$ kg and $R = 6.37 \times 10^6$ m, we find $\ell = mR^2\omega = 2.5 \times 10^{11}$ kg·m^2/s.

76. With $r_\perp = 1300$ m, Eq. 12-21 gives

$$\ell = r_\perp m v = (1300)(1200)(80) = 1.2 \times 10^8\ \mathrm{kg\cdot m^2/s}\ .$$

77. The result follows immediately from Eq. 3-30. If what is desired to show here is basically a derivation of Eq. 3-30, then (with the slight change to position and force notation) that is shown in some detail in our solution to problem 32 of Chapter 3.

78. (a) Using Eq. 2-16 for the translational (center-of-mass) motion, we find

$$v^2 = v_0^2 + 2a\Delta x \implies a = -\frac{v_0^2}{2\Delta x}$$

which yields $a = -4.11$ for $v_0 = 43$ and $\Delta x = 225$ (SI units understood). The magnitude of the linear acceleration of the center of mass is therefore 4.11 m/s^2.

(b) With $R = 0.250$ m, Eq. 12-6 gives $|\alpha| = |a|/R = 16.4$ rad/s^2. If the wheel is going rightward, it is rotating in a clockwise sense. Since it is slowing down, this angular acceleration is counterclockwise (opposite to ω) so (with the usual convention that counterclockwise is positive) there is no need for the absolute value signs for α.

(c) Eq. 12-8 applies with Rf_s representing the magnitude of the frictional torque. Thus, $Rf_s = I\alpha = (0.155)(16.4) = 2.55\,\text{N}\cdot\text{m}$.

79. We note that its mass is $M = 36/9.8 = 3.67$ kg and its rotational inertia is $I_{\text{com}} = \frac{2}{5}MR^2$ (Table 11-2(f)).

(a) Using Eq. 12-2, Eq. 12-5 becomes

$$\begin{aligned} K &= \frac{1}{2}I_{\text{com}}\omega^2 + \frac{1}{2}Mv_{\text{com}}^2 \\ &= \frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{v_{\text{com}}}{R}\right)^2 + \frac{1}{2}Mv_{\text{com}}^2 \\ &= \frac{7}{10}Mv_{\text{com}}^2 \end{aligned}$$

which yields $K = 61.7$ J for $v_{\text{com}} = 4.9$ m/s.

(b) This kinetic energy turns into potential energy Mgh at some height $h = d\sin\theta$ where the sphere comes to rest. Therefore, we find the distance traveled up the $\theta = 30°$ incline from energy conservation:

$$\frac{7}{10}Mv_{\text{com}}^2 = Mgd\sin\theta \implies d = \frac{7v_{\text{com}}^2}{10g\sin\theta} = 3.43 \text{ m} .$$

(c) As shown in the previous part, M cancels in the calculation for d. Since the answer is independent of mass, then, it is also independent of the sphere's weight.

80. Although we will not be "working" this problem, we do – briefly – share a few thoughts about it.

(a) A figure in the textbook that may be referred to is Fig. 8-16. The idea, crudely stated, is to show that although all bodies will return to the same height they're released from (in the absence of dissipative effects), the one with the least rotational inertia (say, a sphere) will get there the fastest because its speed is greatest at every point inbetween.

(b) Several people might be pulling on ropes attached to a merry-go-round to set it into motion. The ropes should be at different angles (measured relative to tangent lines at the appropriate points). The idea is to calculate the net torque using Eq. 12-15 and then to find the angular acceleration (using Eq. 11-37) of the merry-go-round.

(c) This might require particular care in the wording, especially regarding a clown "falling off." If he falls off in what might described as the "natural way" (simply letting go and pursuing a straight-line trajectory tangent to the merry-go-round) then there is no change in the angular momentum. It's easier to see that there'd be a change in angular momentum in the case of a clown (initially at rest) stepping onto the moving merry-go-round.

(d) This is an important astrophysical application of the angular momentum concept (angular momentum is conserved in gravitational-dominated situations such as binary star systems). When the masses of the stars are similar and the mass transfer is relatively steady, they are often known as Algol binaries, and realistic numerical values can be found in many astronomy textbooks (and, probably, on the Web).

81. (First problem in **Cluster 1**)

(a) Applying Newton's second law in its linear form yields

$$(200 \text{ N}) - f = M_{\text{cart}}\, a \quad .$$

Therefore, $f = 200 - (50.0)(3.00) = 50$ N.

(b) The torque associated with the friction is $\tau_f = fR = (50)(0.200) = 10$ N·m. (We make the unconventional choice of the clockwise sense as positive, so that the frictional torque and this angular acceleration are positive.)

(c) Applying the rotational form of Newton's second law (relative to the axle) yields

$$\tau_f = I\alpha \qquad\qquad \text{where } \alpha = \frac{a}{R} = 15.0 \text{ rad/s}^2 \ .$$

Therefore, $I = 0.667$ kg·m^2.

82. (Second problem in **Cluster 1**)

(a) If we interpret this "one-wheel cart" which has a wheel that is a "long cylinder" as simply the cylinder itself, then an appropriate picture for this problem is Fig. 12-30 in the textbook. We make the unconventional choice of *clockwise* sense as positive, so that the angular velocity in this problem is positive; we choose *downhill* positive for the x axis (which is parallel to the incline surface) so that $a_{\text{com}} = R\alpha$ holds. We can combine the rotational (about the center of mass) and linear forms of Newton's second law, or we can more simply adopt the view of pure rotation (see, for example, Eq. 12-3) and examine torques about the bottom-most point P:

$$MgR\sin\theta = I_P\,\alpha = I_P\,\frac{a_{\text{com}}}{R}$$

We have assumed that the center of mass of the cart-wheel system is at the center of the wheel (the axle), although this is not stated in the problem. Now, $\theta = 30.0°$, $R = 0.200$ m, $M = 50.0$ kg, and $I_P = 0.667 \text{ kg}\cdot\text{m}^2 + MR^2 = 2.67 \text{ kg}\cdot\text{m}^2$ (using the parallel-axis theorem and the result of the previous problem). Thus, we find $a_{\text{com}} = 3.68\,\text{m/s}^2$.

(b) If we apply the linear form of Newton's law, we have

$$\begin{aligned}\sum F_x &= Mg\sin\theta - f_{s,\,\max} = Ma_{\text{com}}\\ \sum F_y &= N - Mg\sin\theta = 0\end{aligned}$$

Solving for $f_{s,\,\max}$ and N and dividing, we obtain

$$\mu_s = \frac{f_{s,\,\max}}{N} = 0.14 \quad .$$

83. (Third problem in **Cluster 1**)
An appropriate picture for this problem is Fig. 12-7 in the textbook. We make the unconventional choice of *clockwise* sense as positive, so that the angular velocity in this problem is positive; we choose *downhill* positive for the x axis (which is parallel to the incline surface) so that $a_{\text{com}} = R\alpha$ holds. For simplicity, we refer to a_{com} as a. We examine the rotational (about the center of mass) and linear forms of Newton's second law:

$$\begin{aligned}\sum \tau_z &= f_sR = I\alpha = I\frac{a}{R}\\ \sum F_x &= Mg\sin\theta - f_s = Ma\\ \sum F_y &= N - Mg\cos\theta = 0\end{aligned}$$

Combining the first two of these equations, we obtain

$$f_s = \frac{Mg\sin\theta}{1 + \frac{MR^2}{I}} \quad .$$

We now let $f_s = f_{s,\,\max} = \mu_s N$ and combine this with the third equation above:

$$\mu_s Mg\cos\theta = \frac{Mg\sin\theta}{1 + \frac{MR^2}{I}} \implies \theta = \tan^{-1}\left(\mu_s + \frac{MR^2\mu_s}{I}\right) \, .$$

84. (Fourth problem in **Cluster 1**)

(a) We take the tangential acceleration of the bottom-most point on the (positively) accelerating disk to equal $R\alpha + a_{\text{com}}$. This in turn must equal the (forward) acceleration of the truck $a_{\text{truck}} = a > 0$. Since the disk is rolling toward the back of the truck, $a_{\text{com}} < a$ which implies that α is positive. If the forward direction is *rightward*, then this makes it consistent to choose counterclockwise as the positive rotational sense, which is the usual convention. Thus, $\sum \tau = I\alpha$ becomes

$$f_s R = I\alpha \qquad\qquad \text{where } I = \frac{1}{2}MR^2$$

and $\sum F_x = Ma_{\text{com}}$ becomes

$$f_s = M\left(a - R\alpha\right) \quad .$$

Combining these two equations, we find $R\alpha = \frac{2}{3}\,a$. From the previous discussion, we see acceleration of the disk relative to the truck bed is $a_{\text{com}} - a = -R\alpha$, so this has a magnitude of $\frac{2}{3}$ and is directed leftward.

(b) Returning to $R\alpha + a_{\text{com}} = a$ with our result that $R\alpha = \frac{2}{3}\,a$, we find $a_{\text{com}} = \frac{1}{3}\,a$. This is positive, hence rightward.

85. (First problem in **Cluster 2**)
The last line of the problem indicates our choice of positive directions: up for m_2, down for m_1 and counterclockwise for the two-pulley device. This allows us to write $R_2\alpha = a_2$ and $R_1\alpha = a_1$ with all terms positive. We apply Newton's second law to the elements of this system:

$$\begin{aligned} T_2 - m_2 g &= m_2 a_2 = m_2 R_2 \alpha \\ m_1 g - T_1 &= m_1 a_1 = m_1 R_1 \alpha \\ T_1 R_1 - T_2 R_2 &= I\alpha \end{aligned}$$

Multiplying the first equation by R_2, the second by R_1 and adding the equations leads to

$$\alpha = \frac{m_1 g R_1 - m_2 g R_2}{I + m_1 R_1^2 + m_2 R_2^2} \quad .$$

(a) Therefore, again using $R_1\alpha = a_1$, we obtain

$$a_1 = \frac{m_1 g R_1^2 - m_2 g R_1 R_2}{I + m_1 R_1^2 + m_2 R_2^2} \quad .$$

(b) Once more, we use $R_2\alpha = a_2$ and find

$$a_2 = \frac{m_1 g R_1 R_2 - m_2 g R_2^2}{I + m_1 R_1^2 + m_2 R_2^2} \quad .$$

86. (Second problem in **Cluster 2**)
This system is extensively discussed in §12-4. Rather than repeat those steps here, we refer to their conclusion, Eq. 12-13.

(a) The magnitude of the result in Eq. 12-13 is

$$|a| = g\,\frac{1}{1+\frac{I}{MR^2}} \quad .$$

(b) The relation $a = a_{\rm com} = -R\alpha$ used in §12-3 must now be modified to read $a_f - a_{\rm com} = R\alpha$ where a_f is the acceleration of the finger. With this in mind, the linear and angular versions of Newton's second law become

$$\begin{aligned} T - Mg &= Ma_{\rm com} \\ TR &= I\alpha \qquad \text{where } \alpha = \frac{a_f - a_{\rm com}}{R} \end{aligned}$$

If we require $a_{\rm com} = 0$ then these equations yield

$$a_f = g\,\frac{MR^2}{I} \quad .$$

87. (Third problem in **Cluster 2**)
Our analysis of spool 2 is exactly as in the solution of part (b) of the previous problem, but with a_f replaced with $-a_s$. The negative sign is due to the wording of the problem (which refers to a "downward acceleration a_s"):

$$\begin{aligned} T - Mg &= Ma_1 \\ TR_1 &= I_1\alpha_1 = I_1\left(\frac{-a_s - a_1}{R_1}\right) \end{aligned}$$

In our analysis of spool 1, we pay close attention to signs: positive (downward) a_s corresponds to clockwise (conventionally taken to be negative) rotation of spool 1; hence, $R_2\alpha_2 = -a_s$. For spool 1, we therefore have

$$\sum \tau_z = -TR_2 = I_2\alpha_1 = I_2\left(\frac{-a_s}{R_2}\right) \quad .$$

(a) Simultaneous solution (certainly non-trivial) of these three equations yields

$$a_1 = -\frac{g}{1+\dfrac{1}{\dfrac{MR_1^2}{I_1}+\dfrac{MR_2^2}{I_2}}} \quad .$$

The problem asks for the magnitude of this (which eliminates the negative sign).

(b) This amounts to eliminating the $\frac{MR_2^2}{I_2}$ term in the expression for a_1.