Chapter 27

1. (a) The charge that passes through any cross section is the product of the current and time. Since $4.0\,\text{min} = (4.0\,\text{min})(60\,\text{s/min}) = 240\,\text{s}$, $q = it = (5.0\,\text{A})(240\,\text{s}) = 1200\,\text{C}$.

   (b) The number of electrons N is given by $q = Ne$, where e is the magnitude of the charge on an electron. Thus $N = q/e = (1200\,\text{C})/(1.60 \times 10^{-19}\,\text{C}) = 7.5 \times 10^{21}$.

2. We adapt the discussion in the text to a moving two-dimensional collection of charges. Using σ for the charge per unit area and w for the belt width, we can see that the transport of charge is expressed in the relationship $i = \sigma v w$, which leads to

$$\sigma = \frac{i}{vw} = \frac{100 \times 10^{-6}\,\text{A}}{(30\,\text{m/s})(50 \times 10^{-2}\,\text{m})} = 6.7 \times 10^{-6}\ \text{C/m}^2\ .$$

3. Suppose the charge on the sphere increases by Δq in time Δt. Then, in that time its potential increases by

$$\Delta V = \frac{\Delta q}{4\pi\varepsilon_0 r}\ ,$$

where r is the radius of the sphere. This means

$$\Delta q = 4\pi\varepsilon_0 r\,\Delta V\ .$$

Now, $\Delta q = (i_{\text{in}} - i_{\text{out}})\,\Delta t$, where i_{in} is the current entering the sphere and i_{out} is the current leaving. Thus,

$$\begin{aligned}\Delta t &= \frac{\Delta q}{i_{\text{in}} - i_{\text{out}}} = \frac{4\pi\varepsilon_0 r\,\Delta V}{i_{\text{in}} - i_{\text{out}}} \\ &= \frac{(0.10\,\text{m})(1000\,\text{V})}{(8.99 \times 10^9\,\text{F/m})(1.0000020\,\text{A} - 1.0000000\,\text{A})} = 5.6 \times 10^{-3}\ \text{s}\ .\end{aligned}$$

4. (a) The magnitude of the current density vector is

$$J = \frac{i}{A} = \frac{i}{\pi d^2/4} = \frac{4(1.2 \times 10^{-10}\,\text{A})}{\pi(2.5 \times 10^{-3}\,\text{m})^2} = 2.4 \times 10^{-5}\ \text{A/m}^2\ .$$

   (b) The drift speed of the current-carrying electrons is

$$v_d = \frac{J}{ne} = \frac{2.4 \times 10^{-5}\,\text{A/m}^2}{(8.47 \times 10^{28}/\text{m}^3)\,(1.60 \times 10^{-19}\,\text{C})} = 1.8 \times 10^{-15}\ \text{m/s}\ .$$

5. (a) The magnitude of the current density is given by $J = nqv_d$, where n is the number of particles per unit volume, q is the charge on each particle, and v_d is the drift speed of the particles. The particle

concentration is $n = 2.0 \times 10^8/\text{cm}^3 = 2.0 \times 10^{14}\,\text{m}^{-3}$, the charge is $q = 2e = 2(1.60 \times 10^{-19}\,\text{C}) = 3.20 \times 10^{-19}\,\text{C}$, and the drift speed is $1.0 \times 10^5\,\text{m/s}$. Thus,

$$J = (2 \times 10^{14}/\text{m})(3.2 \times 10^{-19}\,\text{C})(1.0 \times 10^5\,\text{m/s}) = 6.4\ \text{A/m}^2 .$$

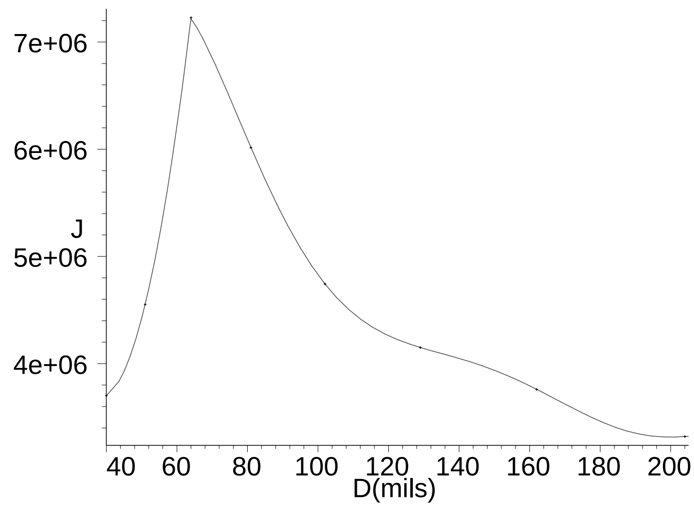
Since the particles are positively charged the current density is in the same direction as their motion, to the north.

(b) The current cannot be calculated unless the cross-sectional area of the beam is known. Then $i = JA$ can be used.

6. We express the magnitude of the current density vector in SI units by converting the diameter values in mils to inches (by dividing by 1000) and then converting to meters (by multiplying by 0.0254) and finally using

$$J = \frac{i}{A} = \frac{i}{\pi R^2} = \frac{4i}{\pi D^2} .$$

For example, the gauge 14 wire with $D = 64\,\text{mil} = 0.0016$ m is found to have a (maximum safe) current density of $J = 7.2 \times 10^6\ \text{A/m}^2$. In fact, this is the wire with the largest value of J allowed by the given data. The values of J in SI units are plotted below as a function of their diameters in mils.



7. The cross-sectional area of wire is given by $A = \pi r^2$, where r is its radius (half its thickness). The magnitude of the current density vector is $J = i/A = i/\pi r^2$, so

$$r = \sqrt{\frac{i}{\pi J}} = \sqrt{\frac{0.50\,\text{A}}{\pi(440 \times 10^4\,\text{A/m}^2)}} = 1.9 \times 10^{-4}\,\text{m} .$$

The diameter of the wire is therefore $d = 2r = 2(1.9 \times 10^{-4}\,\text{m}) = 3.8 \times 10^{-4}\,\text{m}$.

8. (a) Since $1\,\text{cm}^3 = 10^{-6}\,\text{m}^3$, the magnitude of the current density vector is

$$J = nev = \left(\frac{8.70}{10^{-6}\,\text{m}^3}\right)\left(1.60 \times 10^{-19}\,\text{C}\right)\left(470 \times 10^3\,\text{m/s}\right) = 6.54 \times 10^{-7}\ \text{A/m}^2 .$$

(b) Although the total surface area of Earth is $4\pi R_E^2$ (that of a sphere), the area to be used in a computation of how many protons in an approximately unidirectional beam (the solar wind) will be captured by Earth is its projected area. In other words, for the beam, the encounter is with a "target" of circular area πR_E^2. The rate of charge transport implied by the influx of protons is

$$i = AJ = \pi R_E^2 J = \pi\left(6.37 \times 10^6\,\text{m}\right)^2\left(6.54 \times 10^{-7}\,\text{A/m}^2\right) = 8.34 \times 10^7\ \text{A} .$$

9. (a) The charge that strikes the surface in time Δt is given by $\Delta q = i\,\Delta t$, where i is the current. Since each particle carries charge $2e$, the number of particles that strike the surface is

$$N = \frac{\Delta q}{2e} = \frac{i\,\Delta t}{2e} = \frac{(0.25\times 10^{-6}\,\mathrm{A})(3.0\,\mathrm{s})}{2(1.6\times 10^{-19}\,\mathrm{C})} = 2.3\times 10^{12}\ .$$

(b) Now let N be the number of particles in a length L of the beam. They will all pass through the beam cross section at one end in time $t = L/v$, where v is the particle speed. The current is the charge that moves through the cross section per unit time. That is, $i = 2eN/t = 2eNv/L$. Thus $N = iL/2ev$. To find the particle speed, we note the kinetic energy of a particle is

$$K = 20\,\mathrm{MeV} = (20\times 10^{6}\,\mathrm{eV})(1.60\times 10^{-19}\,\mathrm{J/eV}) = 3.2\times 10^{-12}\ \mathrm{J}\ .$$

Since $K = \frac{1}{2}mv^2$, then the speed is $v = \sqrt{2K/m}$. The mass of an alpha particle is (very nearly) 4 times the mass of a proton, or $m = 4(1.67\times 10^{-27}\,\mathrm{kg}) = 6.68\times 10^{-27}\,\mathrm{kg}$, so

$$v = \sqrt{\frac{2(3.2\times 10^{-12}\,\mathrm{J})}{6.68\times 10^{-27}\,\mathrm{kg}}} = 3.1\times 10^{7}\,\mathrm{m/s}$$

and

$$N = \frac{iL}{2ev} = \frac{(0.25\times 10^{-6})(20\times 10^{-2}\,\mathrm{m})}{2(1.60\times 10^{-19}\,\mathrm{C})(3.1\times 10^{7}\,\mathrm{m/s})} = 5.0\times 10^{3}\ .$$

(c) We use conservation of energy, where the initial kinetic energy is zero and the final kinetic energy is $20\,\mathrm{MeV} = 3.2\times 10^{-12}\,\mathrm{J}$. We note, too, that the initial potential energy is $U_i = qV = 2eV$, and the final potential energy is zero. Here V is the electric potential through which the particles are accelerated. Consequently,

$$K_f = U_i = 2eV \implies V = \frac{K_f}{2e} = \frac{3.2\times 10^{-12}\,\mathrm{J}}{2(1.60\times 10^{-19}\mathrm{C})} = 10\times 10^{6}\ \mathrm{V}\ .$$

10. (a) The current resulting from this non-uniform current density is

$$i = \int_{\text{cylinder}} J\,dA = \int_0^R J_0\left(1 - \frac{r}{R}\right) 2\pi r dr = \frac{1}{3}\pi R^2 J_0 = \frac{1}{3} A J_0\ .$$

(b) In this case,

$$i = \int_{\text{cylinder}} J\,dA = \frac{J_0}{R}\int_0^R r\cdot 2\pi r dr = \frac{2}{3}\pi R^2 J_0 = \frac{2}{3} A J_0\ .$$

The result is different from that in part (a) because the current density in part (b) is lower near the center of the cylinder (where the area is smaller for the same radial interval) and higher outward, resulting in a higher average current density over the cross section and consequently a greater current than that in part (a).

11. We use $v_d = J/ne = i/Ane$. Thus,

$$\begin{aligned} t &= \frac{L}{v_d} = \frac{L}{i/Ane} = \frac{LAne}{i} \\ &= \frac{(0.85\,\mathrm{m})(0.21\times 10^{-4}\,\mathrm{m}^2)(8.47\times 10^{28}/\,\mathrm{m}^3)(1.60\times 10^{-19}\,\mathrm{C})}{300\,\mathrm{A}} \\ &= 8.1\times 10^{2}\,\mathrm{s} = 13\ \mathrm{min}\ . \end{aligned}$$

12. We find the conductivity of Nichrome (the reciprocal of its resistivity) as follows:

$$\sigma = \frac{1}{\rho} = \frac{L}{RA} = \frac{L}{(V/i)A} = \frac{Li}{VA} = \frac{(1.0\,\mathrm{m})(4.0\,\mathrm{A})}{(2.0\,\mathrm{V})(1.0\times 10^{-6}\,\mathrm{m}^2)} = 2.0\times 10^{6}/\Omega{\cdot}\mathrm{m}\ .$$

13. The resistance of the wire is given by $R = \rho L/A$, where ρ is the resistivity of the material, L is the length of the wire, and A is its cross-sectional area. In this case,

$$A = \pi r^2 = \pi(0.50 \times 10^{-3}\,\mathrm{m})^2 = 7.85 \times 10^{-7}\ \mathrm{m}^2 \ .$$

Thus,

$$\rho = \frac{RA}{L} = \frac{(50 \times 10^{-3}\,\Omega)(7.85 \times 10^{-7}\,\mathrm{m}^2)}{2.0\,\mathrm{m}} = 2.0 \times 10^{-8}\ \Omega{\cdot}\mathrm{m} \ .$$

14. Since $100\,\mathrm{cm} = 1\,\mathrm{m}$, then $10^4\,\mathrm{cm}^2 = 1\,\mathrm{m}^2$. Thus,

$$R = \frac{\rho L}{A} = \frac{(3.00 \times 10^{-7}\,\Omega{\cdot}\mathrm{m})(10.0 \times 10^3\,\mathrm{m})}{56.0 \times 10^{-4}\,\mathrm{m}^2} = 0.536\ \Omega \ .$$

15. Since the potential difference V and current i are related by $V = iR$, where R is the resistance of the electrician, the fatal voltage is $V = (50 \times 10^{-3}\,\mathrm{A})(2000\,\Omega) = 100\,\mathrm{V}$.

16. (a) $i = V/R = 23.0\,\mathrm{V}/15.0 \times 10^{-3}\,\Omega = 1.53 \times 10^3\,\mathrm{A}$.

(b) The cross-sectional area is $A = \pi r^2 = \frac{1}{4}\pi D^2$. Thus, the magnitude of the current density vector is

$$J = \frac{i}{A} = \frac{4i}{\pi D^2} = \frac{4(1.53 \times 10^{-3}\,\mathrm{A})}{\pi(6.00 \times 10^{-3}\,\mathrm{m})^2} = 5.41 \times 10^7\ \mathrm{A/m}^2 \ .$$

(c) The resistivity is $\rho = RA/L = (15.0 \times 10^{-3}\,\Omega)(\pi)(6.00 \times 10^{-3}\,\mathrm{m})^2/[4(4.00\,\mathrm{m})] = 10.6 \times 10^{-8}\,\Omega{\cdot}\mathrm{m}$. The material is platinum.

17. The resistance of the coil is given by $R = \rho L/A$, where L is the length of the wire, ρ is the resistivity of copper, and A is the cross-sectional area of the wire. Since each turn of wire has length $2\pi r$, where r is the radius of the coil, then $L = (250)2\pi r = (250)(2\pi)(0.12\,\mathrm{m}) = 188.5\,\mathrm{m}$. If r_w is the radius of the wire itself, then its cross-sectional area is $A = \pi r_w^2 = \pi(0.65 \times 10^{-3}\,\mathrm{m})^2 = 1.33 \times 10^{-6}\,\mathrm{m}^2$. According to Table 27-1, the resistivity of copper is $1.69 \times 10^{-8}\,\Omega{\cdot}\mathrm{m}$. Thus,

$$R = \frac{\rho L}{A} = \frac{(1.69 \times 10^{-8}\,\Omega{\cdot}\mathrm{m})(188.5\,\mathrm{m})}{1.33 \times 10^{-6}\,\mathrm{m}^2} = 2.4\ \Omega \ .$$

18. In Eq. 27-17, we let $\rho = 2\rho_0$ where ρ_0 is the resistivity at $T_0 = 20°\mathrm{C}$:

$$\rho - \rho_0 = 2\rho_0 - \rho_0 = \rho_0 \alpha\,(T - T_0) \ ,$$

and solve for the temperature T:

$$T = T_0 + \frac{1}{\alpha} = 20°\mathrm{C} + \frac{1}{4.3 \times 10^{-3}/\mathrm{K}} \approx 250°\mathrm{C} \ .$$

Since a change in Celsius is equivalent to a change on the Kelvin temperature scale, the value of α used in this calculation is not inconsistent with the other units involved. It is worth nothing that this agrees well with Fig. 27-10.

19. Since the mass and density of the material do not change, the volume remains the same. If L_0 is the original length, L is the new length, A_0 is the original cross-sectional area, and A is the new cross-sectional area, then $L_0 A_0 = LA$ and $A = L_0 A_0/L = L_0 A_0/3L_0 = A_0/3$. The new resistance is

$$R = \frac{\rho L}{A} = \frac{\rho 3 L_0}{A_0/3} = 9\frac{\rho L_0}{A_0} = 9R_0 \ ,$$

where R_0 is the original resistance. Thus, $R = 9(6.0\,\Omega) = 54\,\Omega$.

20. The thickness (diameter) of the wire is denoted by D. We use $R \propto L/A$ (Eq. 27-16) and note that $A = \frac{1}{4}\pi D^2 \propto D^2$. The resistance of the second wire is given by

$$R_2 = R\left(\frac{A_1}{A_2}\right)\left(\frac{L_2}{L_1}\right) = R\left(\frac{D_1}{D_2}\right)^2\left(\frac{L_2}{L_1}\right) = R(2)^2\left(\frac{1}{2}\right) = 2R\ .$$

21. The resistance of conductor A is given by

$$R_A = \frac{\rho L}{\pi r_A^2}\ ,$$

where r_A is the radius of the conductor. If r_o is the outside diameter of conductor B and r_i is its inside diameter, then its cross-sectional area is $\pi(r_o^2 - r_i^2)$, and its resistance is

$$R_B = \frac{\rho L}{\pi\left(r_o^2 - r_i^2\right)}\ .$$

The ratio is

$$\frac{R_A}{R_B} = \frac{r_o^2 - r_i^2}{r_A^2} = \frac{(1.0\,\text{mm})^2 - (0.50\,\text{mm})^2}{(0.50\,\text{mm})^2} = 3.0\ .$$

22. (a) The current in each strand is $i = 0.750\,\text{A}/125 = 6.00 \times 10^{-3}\,\text{A}$.
    (b) The potential difference is $V = iR = (6.00 \times 10^{-3}\,\text{A})(2.65 \times 10^{-6}\,\Omega) = 1.59 \times 10^{-8}\,\text{V}$.
    (c) The resistance is $R_{\text{total}} = 2.65 \times 10^{-6}\,\Omega/125 = 2.12 \times 10^{-8}\,\Omega$.

23. We use $J = E/\rho$, where E is the magnitude of the (uniform) electric field in the wire, J is the magnitude of the current density, and ρ is the resistivity of the material. The electric field is given by $E = V/L$, where V is the potential difference along the wire and L is the length of the wire. Thus $J = V/L\rho$ and

$$\rho = \frac{V}{LJ} = \frac{115\,\text{V}}{(10\,\text{m})\left(1.4 \times 10^4\,\text{A/m}^2\right)} = 8.2 \times 10^{-4}\,\Omega{\cdot}\text{m}\ .$$

24. (a) $i = V/R = 35.8\,\text{V}/935\,\Omega = 3.83 \times 10^{-2}\,\text{A}$.
    (b) $J = i/A = (3.83 \times 10^{-2}\,\text{A})/(3.50 \times 10^{-4}\,\text{m}^2) = 109\,\text{A/m}^2$.
    (c) $v_d = J/ne = (109\,\text{A/m}^2)/[(5.33 \times 10^{22}/\text{m}^3)(1.60 \times 10^{-19}\,\text{C})] = 1.28 \times 10^{-2}\,\text{m/s}$.
    (d) $E = V/L = 35.8\,\text{V}/0.158\,\text{m} = 227\,\text{V/m}$.

25. The resistance at operating temperature T is $R = V/i = 2.9\,\text{V}/0.30\,\text{A} = 9.67\,\Omega$. Thus, from $R - R_0 = R_0\alpha(T - T_0)$, we find

$$\begin{aligned} T &= T_0 + \frac{1}{\alpha}\left(\frac{R}{R_0} - 1\right) \\ &= 20°\text{C} + \left(\frac{1}{4.5 \times 10^{-3}/\text{K}}\right)\left(\frac{9.67\,\Omega}{1.1\,\Omega} - 1\right) \end{aligned}$$

which yields approximately 1900°C. Since a change in Celsius is equivalent to a change on the Kelvin temperature scale, the value of α used in this calculation is not inconsistent with the other units involved. Table 27-1 has been used.

26. We use $J = \sigma E = (n_+ + n_-)ev_d$, which combines Eq. 27-13 and Eq. 27-7.

    (a) The drift velocity is

$$v_d = \frac{\sigma E}{(n_+ + n_-)e} = \frac{(2.70 \times 10^{-14}/\Omega{\cdot}\text{m})(120\,\text{V/m})}{[(620 + 550)/\text{cm}^3](1.60 \times 10^{-19}\,\text{C})} = 1.73\ \text{cm/s}\ .$$

(b) $J = \sigma E = (2.70 \times 10^{-14}/\,\Omega\cdot\text{m})(120\,\text{V/m}) = 3.24 \times 10^{-12}\,\text{A/m}^2$.

27. (a) Let ΔT be the change in temperature and κ be the coefficient of linear expansion for copper. Then $\Delta L = \kappa L\,\Delta T$ and

$$\frac{\Delta L}{L} = \kappa\,\Delta T = (1.7 \times 10^{-5}/\text{K})(1.0°\text{C}) = 1.7 \times 10^{-5}\ .$$

This is equivalent to 0.0017%. Since a change in Celsius is equivalent to a change on the Kelvin temperature scale, the value of κ used in this calculation is not inconsistent with the other units involved. Incorporating a factor of 2 for the two-dimensional nature of A, the fractional change in area is

$$\frac{\Delta A}{A} = 2\kappa\,\Delta T = 2(1.7 \times 10^{-5}/\text{K})(1.0°\text{C}) = 3.4 \times 10^{-5}$$

which is 0.0034%. For small changes in the resistivity ρ, length L, and area A of a wire, the change in the resistance is given by

$$\Delta R = \frac{\partial R}{\partial \rho}\,\Delta\rho + \frac{\partial R}{\partial L}\,\Delta L + \frac{\partial R}{\partial A}\,\Delta A\ .$$

Since $R = \rho L/A$, $\partial R/\partial\rho = L/A = R/\rho$, $\partial R/\partial L = \rho/A = R/L$, and $\partial R/\partial A = -\rho L/A^2 = -R/A$. Furthermore, $\Delta\rho/\rho = \alpha\,\Delta T$, where α is the temperature coefficient of resistivity for copper ($4.3 \times 10^{-3}/\text{K} = 4.3 \times 10^{-3}/\text{C}°$, according to Table 27-1). Thus,

$$\begin{aligned}
\frac{\Delta R}{R} &= \frac{\Delta\rho}{\rho} + \frac{\Delta L}{L} - \frac{\Delta A}{A} \\
&= (\alpha + \kappa - 2\kappa)\,\Delta T = (\alpha - \kappa)\,\Delta T \\
&= (4.3 \times 10^{-3}/\text{C}° - 1.7 \times 10^{-5}/\text{C}°)\,(1.0\,\text{C}°) = 4.3 \times 10^{-3}\ .
\end{aligned}$$

This is 0.43%, which we note (for the purposes of the next part) is primarily determined by the $\Delta\rho/\rho$ term in the above calculation.

(b) The fractional change in resistivity is much larger than the fractional change in length and area. Changes in length and area affect the resistance much less than changes in resistivity.

28. We use $R \propto L/A$. The diameter of a 22-gauge wire is 1/4 that of a 10-gauge wire. Thus from $R = \rho L/A$ we find the resistance of 25 ft of 22-gauge copper wire to be $R = (1.00\,\Omega)(25\,\text{ft}/1000\,\text{ft})(4)^2 = 0.40\,\Omega$.

29. (a) The current i is shown in Fig. 27-22 entering the truncated cone at the left end and leaving at the right. This is our choice of positive x direction. We make the assumption that the current density J at each value of x may be found by taking the ratio i/A where $A = \pi r^2$ is the cone's cross-section area at that particular value of x. The direction of $\vec{J}$ is identical to that shown in the figure for i (our $+x$ direction). Using Eq. 27-11, we then find an expression for the electric field at each value of x, and next find the potential difference V by integrating the field along the x axis, in accordance with the ideas of Chapter 25. Finally, the resistance of the cone is given by $R = V/i$. Thus,

$$J = \frac{i}{\pi r^2} = \frac{E}{\rho}$$

where we must deduce how r depends on x in order to proceed. We note that the radius increases linearly with x, so (with c_1 and c_2 to be determined later) we may write

$$r = c_1 + c_2\,x\ .$$

Choosing the origin at the left end of the truncated cone, the coefficient c_1 is chosen so that $r = a$ (when $x = 0$); therefore, $c_1 = a$. Also, the coefficient c_2 must be chosen so that (at the right end of the truncated cone) we have $r = b$ (when $x = L$); therefore, $c_2 = (b - a)/L$. Our expression, then, becomes

$$r = a + \left(\frac{b - a}{L}\right)x\ .$$

Substituting this into our previous statement and solving for the field, we find

$$E = \frac{i\rho}{\pi}\left(a + \frac{b-a}{L}x\right)^{-2} .$$

Consequently, the potential difference between the faces of the cone is

$$\begin{aligned} V &= -\int_0^L E\,dx = -\frac{i\rho}{\pi}\int_0^L \left(a + \frac{b-a}{L}x\right)^{-2} dx \\ &= \frac{i\rho}{\pi}\frac{L}{b-a}\left(a + \frac{b-a}{L}x\right)^{-1}\Bigg|_0^L = \frac{i\rho}{\pi}\frac{L}{b-a}\left(\frac{1}{a} - \frac{1}{b}\right) \\ &= \frac{i\rho}{\pi}\frac{L}{b-a}\frac{b-a}{ab} = \frac{i\rho L}{\pi ab} . \end{aligned}$$

The resistance is therefore

$$R = \frac{V}{i} = \frac{\rho L}{\pi ab} .$$

(b) If $b = a$, then $R = \rho L/\pi a^2 = \rho L/A$, where $A = \pi a^2$ is the cross-sectional area of the cylinder.

30. From Eq. 27-20, $\rho \propto \tau^{-1} \propto v_{\text{eff}}$. The connection with v_{eff} is indicated in part (b) of Sample Problem 27-5, which contains useful insight regarding the problem we are working now. According to Chapter 20, $v_{\text{eff}} \propto \sqrt{T}$. Thus, we may conclude that $\rho \propto \sqrt{T}$.

31. The power dissipated is given by the product of the current and the potential difference:

$$P = iV = (7.0 \times 10^{-3}\,\text{A})(80 \times 10^3\,\text{V}) = 560\text{W} .$$

32. Since $P = iV$, $q = it = Pt/V = (7.0\,\text{W})(5.0\,\text{h})(3600\,\text{s/h})/9.0\,\text{V} = 1.4 \times 10^4\,\text{C}$.

33. (a) Electrical energy is converted to heat at a rate given by

$$P = \frac{V^2}{R} ,$$

where V is the potential difference across the heater and R is the resistance of the heater. Thus,

$$P = \frac{(120\,\text{V})^2}{14\,\Omega} = 1.0 \times 10^3\,\text{W} = 1.0\ \text{kW} .$$

(b) The cost is given by

$$(1.0\,\text{kW})(5.0\,\text{h})(5.0\,\text{cents/kW}\cdot\text{h}) = 25\ \text{cents} .$$

34. The resistance is $R = P/i^2 = (100\,\text{W})/(3.00\,\text{A})^2 = 11.1\,\Omega$.

35. The relation $P = V^2/R$ implies $P \propto V^2$. Consequently, the power dissipated in the second case is

$$P = \left(\frac{1.50\,\text{V}}{3.00\,\text{V}}\right)^2 (0.540\,\text{W}) = 0.135\ \text{W} .$$

36. (a) From $P = V^2/R$ we find $R = V^2/P = (120\,\text{V})^2/500\,\text{W} = 28.8\,\Omega$.

(b) Since $i = P/V$, the rate of electron transport is

$$\frac{i}{e} = \frac{P}{eV} = \frac{500\,\text{W}}{(1.60 \times 10^{-19}\,\text{C})(120\,\text{V})} = 2.60 \times 10^{19}/\text{s} .$$

37. (a) The power dissipated, the current in the heater, and the potential difference across the heater are related by $P = iV$. Therefore,

$$i = \frac{P}{V} = \frac{1250\,\text{W}}{115\,\text{V}} = 10.9\ \text{A}\ .$$

(b) Ohm's law states $V = iR$, where R is the resistance of the heater. Thus,

$$R = \frac{V}{i} = \frac{115\,\text{V}}{10.9\,\text{A}} = 10.6\ \Omega\ .$$

(c) The thermal energy E generated by the heater in time $t = 1.0\,\text{h} = 3600\,\text{s}$ is

$$E = Pt = (1250\,\text{W})(3600\,\text{s}) = 4.5 \times 10^6\ \text{J}\ .$$

38. (a) From $P = V^2/R = AV^2/\rho L$, we solve for the length:

$$L = \frac{AV^2}{\rho P} = \frac{(2.60 \times 10^{-6}\,\text{m}^2)(75.0\,\text{V})^2}{(5.00 \times 10^{-7}\,\Omega\cdot\text{m})(5000\,\text{W})} = 5.85\ \text{m}\ .$$

(b) Since $L \propto V^2$ the new length should be

$$L' = L\left(\frac{V'}{V}\right)^2 = (5.85\,\text{m})\left(\frac{100\,\text{V}}{75.0\,\text{V}}\right)^2 = 10.4\ \text{m}\ .$$

39. Let R_H be the resistance at the higher temperature (800°C) and let R_L be the resistance at the lower temperature (200°C). Since the potential difference is the same for the two temperatures, the power dissipated at the lower temperature is $P_L = V^2/R_L$, and the power dissipated at the higher temperature is $P_H = V^2/R_H$, so $P_L = (R_H/R_L)P_H$. Now $R_L = R_H + \alpha R_H\,\Delta T$, where ΔT is the temperature difference $T_L - T_H = -600\,\text{C}^\circ = -600\,\text{K}$. Thus,

$$P_L = \frac{R_H}{R_H + \alpha R_H\,\Delta T}\,P_H = \frac{P_H}{1 + \alpha\,\Delta T} = \frac{500\,\text{W}}{1 + (4.0 \times 10^{-4}/\text{K})(-600\,\text{K})} = 660\ \text{W}\ .$$

40. (a) The monthly cost is $(100\,\text{W})(24\,\text{h/day})(31\,\text{day/month})(6\,\text{cents/kW}\cdot\text{h}) = 446\,\text{cents} = \$4.46$, assuming a 31-day month.

(b) $R = V^2/P = (120\,\text{V})^2/100\,\text{W} = 144\,\Omega$.

(c) $i = P/V = 100\,\text{W}/120\,\text{V} = 0.833\,\text{A}$.

41. (a) The charge q that flows past any cross section of the beam in time Δt is given by $q = i\,\Delta t$, and the number of electrons is $N = q/e = (i/e)\,\Delta t$. This is the number of electrons that are accelerated. Thus

$$N = \frac{(0.50\,\text{A})(0.10 \times 10^{-6}\,\text{s})}{1.60 \times 10^{-19}\,\text{C}} = 3.1 \times 10^{11}\ .$$

(b) Over a long time t the total charge is $Q = nqt$, where n is the number of pulses per unit time and q is the charge in one pulse. The average current is given by $i_{\text{avg}} = Q/t = nq$. Now $q = i\,\Delta t = (0.50\,\text{A})(0.10 \times 10^{-6}\,\text{s}) = 5.0 \times 10^{-8}\,\text{C}$, so

$$i_{\text{avg}} = (500/\text{s})(5.0 \times 10^{-8}\,\text{C}) = 2.5 \times 10^{-5}\ \text{A}\ .$$

(c) The accelerating potential difference is $V = K/e$, where K is the final kinetic energy of an electron. Since $K = 50\,\text{MeV}$, the accelerating potential is $V = 50\,\text{kV} = 5.0 \times 10^7\,\text{V}$. During a pulse the power output is

$$P = iV = (0.50\,\text{A})(5.0 \times 10^7\,\text{V}) = 2.5 \times 10^7\ \text{W}\ .$$

This is the peak power. The average power is

$$P_{\text{avg}} = i_{\text{avg}}V = (2.5 \times 10^{-5}\,\text{A})(5.0 \times 10^7\,\text{V}) = 1.3 \times 10^3\ \text{W}\ .$$

42. (a) Since $P = i^2R = J^2A^2R$, the current density is

$$\begin{aligned} J &= \frac{1}{A}\sqrt{\frac{P}{R}} = \frac{1}{A}\sqrt{\frac{P}{\rho L/A}} = \sqrt{\frac{P}{\rho L A}} \\ &= \sqrt{\frac{1.0\,\mathrm{W}}{\pi(3.5\times10^{-5}\,\Omega\cdot\mathrm{m})(2.0\times10^{-2}\,\mathrm{m})(5.0\times10^{-3}\,\mathrm{m})^2}} = 1.3\times10^5\ \mathrm{A/m^2}\ . \end{aligned}$$

(b) From $P = iV = JAV$ we get

$$V = \frac{P}{AJ} = \frac{P}{\pi r^2 J} = \frac{1.0\,\mathrm{W}}{\pi(5.0\times10^{-3}\,\mathrm{m})^2(1.3\times10^5\,\mathrm{A/m^2})} = 9.4\times10^{-2}\ \mathrm{V}\ .$$

43. (a) Using Table 27-1 and Eq. 27-10 (or Eq. 27-11), we have

$$\left|\vec{E}\right| = \rho\left|\vec{J}\right| = \left(1.69\times10^{-8}\,\Omega\cdot\mathrm{m}\right)\left(\frac{2.0\,\mathrm{A}}{2.0\times10^{-6}\,\mathrm{m^2}}\right) = 1.7\times10^{-2}\ \mathrm{V/m}\ .$$

(b) Using $L = 4.0$ m, the resistance is found from Eq. 27-16: $R = \rho L/A = 0.034\,\Omega$. The rate of thermal energy generation is found from Eq. 27-22: $P = i^2R = 0.14$ W. Assuming a steady rate, the thermal energy generated in 30 minutes is $(0.14\,\mathrm{J/s})(30\times60\,\mathrm{s}) = 2.4\times10^2$ J.

44. (a) Current is the transport of charge; here it is being transported "in bulk" due to the volume rate of flow of the powder. From Chapter 15, we recall that the volume rate of flow is the product of the cross-sectional area (of the stream) and the (average) stream velocity. Thus, $i = \rho A v$ where ρ is the charge per unit volume. If the cross-section is that of a circle, then $i = \rho\pi R^2 v$.

(b) Recalling that a Coulomb per second is an Ampere, we obtain

$$i = \left(1.1\times10^{-3}\,\mathrm{C/m^3}\right)\pi(0.050\,\mathrm{m})^2(2.0\,\mathrm{m/s}) = 1.7\times10^{-5}\ \mathrm{A}\ .$$

(c) The motion of charge is not in the same direction as the potential difference computed in problem 57 of Chapter 25. It might be useful to think of (by analogy) Eq. 7-48; there, the scalar (dot) product in $P = \vec{F}\cdot\vec{v}$ makes it clear that $P = 0$ if $\vec{F}\perp\vec{v}$. This suggests that a radial potential difference and an axial flow of charge will not together produce the needed transfer of energy (into the form of a spark).

(d) With the assumption that there is (at least) a voltage equal to that computed in problem 57 of Chapter 25, in the proper direction to enable the transference of energy (into a spark), then we use our result from that problem in Eq. 27-21:

$$P = iV = \left(1.7\times10^{-5}\,\mathrm{A}\right)\left(7.8\times10^4\,\mathrm{V}\right) = 1.3\ \mathrm{W}\ .$$

(e) Recalling that a Joule per second is a Watt, we obtain $(1.3\,\mathrm{W})(0.20\,\mathrm{s}) = 0.27\,\mathrm{J}$ for the energy that can be transferred at the exit of the pipe.

(f) This result is greater than the 0.15 J needed for a spark, so we conclude that the spark was likely to have occurred at the exit of the pipe, going into the silo.

45. (a) Since the area of a hemisphere is $2\pi r^2$ then the magnitude of the current density vector is

$$\left|\vec{J}\right| = \frac{i}{A} = \frac{I}{2\pi r^2}\ .$$

(b) Eq. 27-11 yields $\left|\vec{E}\right| = \rho\left|\vec{J}\right| = \rho I/2\pi r^2$.

(c) Eq. 25-18 leads to

$$\Delta V = V_r - V_b = -\int_b^r \vec{E} \cdot d\vec{r} = -\int_b^r \left(\frac{\rho I}{2\pi r^2}\right) dr = \frac{\rho I}{2\pi}\left(\frac{1}{r} - \frac{1}{b}\right) .$$

(d) Using the given values, we obtain $\left|\vec{J}\right| = \frac{100}{2\pi(10)^2} = 0.16\ \mathrm{A/m^2}$.

(e) Also, $\left|\vec{E}\right| = 16$ V/m (or 16 N/C).

(f) With $b = 0.010$ m, the voltage is $\Delta V = -1.6 \times 10^5$ V.

46. (a) Using Eq. 27-11 and Eq. 25-42, we obtain

$$\left|\vec{J}_A\right| = \frac{\left|\vec{E}_A\right|}{\rho} = \frac{|\Delta V_A|}{\rho L} = \frac{40 \times 10^{-6}\,\mathrm{V}}{(100\,\Omega\cdot\mathrm{m})(20\,\mathrm{m})} = 2.0 \times 10^{-8}\ \mathrm{A/m^2} .$$

(b) Similarly, in region B we find

$$\left|\vec{J}_B\right| = \frac{|\Delta V_B|}{\rho L} = \frac{60 \times 10^{-6}\,\mathrm{V}}{(100\,\Omega\cdot\mathrm{m})(20\,\mathrm{m})} = 3.0 \times 10^{-8}\ \mathrm{A/m^2} .$$

(c) With $w = 1.0$ m and $d_A = 3.8$ m (so that the cross-section area is $d_A w$) we have (using Eq. 27-5)

$$i_A = \left|\vec{J}_A\right| d_A w = \left(2.0 \times 10^{-8}\,\mathrm{A/m^2}\right)(1.0\,\mathrm{m})(3.8\,\mathrm{m}) = 7.6 \times 10^{-8}\ \mathrm{A} .$$

(d) Assuming $i_A = i_B$ we obtain

$$d_B = \frac{i_B}{\left|\vec{J}_B\right| w} = \frac{7.6 \times 10^{-8}\,\mathrm{A}}{(3.0 \times 10^{-8}\,\mathrm{A/m^2})\,(1.0\,\mathrm{m})} = 2.5\ \mathrm{m} .$$

(e) We do not show the graph-and-figure here, but describe it briefly. To be meaningful (as a function of x) we would plot $V(x)$ measured relative to $V(0)$ (the voltage at, say, the left edge of the figure, which we are effectively setting equal to 0). From the problem statement, we note that $V(x)$ would grow linearly in region A, increasing by $40\,\mu$V for each 20 m distance. Once we reach the transition region (between A and B) we might assume a parabolic shape for $V(x)$ as it changes from the $40\,\mu$V-per-20 m slope to a $60\,\mu$V-per-20 m slope (which becomes its constant slope once we are into region B, where the function is again linear). The figure goes further than region B, so as we leave region B, we might assume again a parabolic shape for the function as it tends back down toward some lower slope value.

47. (a) It is useful to read the whole problem before considering the sketch here in part (a) (which we do not show, but briefly describe). We find in part (d) and part (f), below, that $J_A > J_B$ which suggests that the streamlines should be closer together in region A than in B (at least for portions of those regions which lie close to the pipe). Associated with this (see part (g)) the sketch of the streamlines should reflect that fact that some of the conduction charge-carriers are entering the pipe walls during the transition from region A to region B.

(b) Eq. 27-16 yields

$$\rho_{\mathrm{pipe}} = R\frac{A}{L} = (6.0\,\Omega)\left(\frac{0.010\,\mathrm{m^2}}{1.0 \times 10^6\,\mathrm{m}}\right) = 6.0 \times 10^{-8}\ \Omega\cdot\mathrm{m} .$$

(c) If the resistance of 1000 km of pipe is $6.0\,\Omega$ then the resistance of $L = 1.0$ km of pipe is $R = 6.0\,\mathrm{m}\Omega$. Thus in region A, Ohm's law leads to

$$i_{\mathrm{pipe}} = \frac{V_{ab}}{R} = \frac{8.0\,\mathrm{mV}}{6.0\,\mathrm{m}\Omega} = 1.3\ \mathrm{A} .$$

(d) Using Eq. 27-11 and Eq. 25-42 (in absolute value), we find the magnitude of the current density vector in region A:

$$\left|\vec{J}_{\text{ground}}\right| = \frac{V_{ab}}{\rho_{\text{ground}} L} = \frac{0.0080\,\text{V}}{(500\,\Omega\cdot\text{m})(1000\,\text{m})} = 1.6 \times 10^{-8}\ \text{A/m}^2 \ .$$

(e) Similarly, in region B we obtain

$$i_{\text{pipe}} = \frac{V_{cd}}{R} = \frac{9.5\,\text{mV}}{6.0\,\text{m}\Omega} = 1.6\ \text{A} \ ,$$

(f) and

$$\left|\vec{J}_{\text{ground}}\right| = \frac{V_{cd}}{\rho_{\text{ground}} L} = \frac{0.0095\,\text{V}}{(1000\,\Omega\cdot\text{m})(1000\,\text{m})} = 9.5 \times 10^{-9}\ \text{A/m}^2 \ .$$

(g) These results suggest that the pipe walls, in leaving region A and entering region B, have "absorbed" some of the current, leaving the current density in the nearby ground somewhat "depleted" of the telluric flows.

(h) We assume the transition $B \to A$ is the reverse of that discussed in part (g). Here, some current leaves the pipe walls and joins in the ground-supported telluric flows.

(i) There is no current here, because there is no potential difference along this section of pipe. The reason $V_{gh} = 0$ is best seen using Eq. 27-11 and Eq. 25-18 (and remembering that the scalar dot product gives zero for perpendicular vectors). The arrows shown in the figure for current actually refer, in the technical sense, to the direction of $\vec{J}$. We refer to this as the x direction. The pipe section gh is oriented in what we will refer to as the y direction. Eq. 27-11 implies that $\vec{J}$ and $\vec{E}$ must be in the same direction (x). But a nonzero voltage difference here would require (by Eq. 25-18) $\int \vec{E} \cdot d\vec{s} \neq 0$. But since $d\vec{s} = dy$ for this section of pipe, then $\vec{E} \cdot d\vec{s}$ vanishes identically.

(j) Our discussion in part (j) serves also to motivate the fact that the current in section fg is less than that in section ef by a factor of $\cos 45° = 1/\sqrt{2}$. To see this, one may consider the component of the electric field which would "drive" the current (in the sense of Eq. 27-11) along section fg; it is less than the field responsible for the current in section ef by exactly the factor just mentioned. Thus,

$$i_{fg} = i_{ef} \cos 45° = \frac{1.0\,\text{A}}{\sqrt{2}} = 0.71\ \text{A} \ .$$

(k) The answers to the previous parts indicate that current leaves the pipe at point f and

(l) at point g.

48. (a) We use Eq. 27-16. The new area is $A' = AL/L' = A/2$.

(b) The new resistance is $R' = R(A/A')(L'/L) = 4R$.

49. We use $P = i^2R = i^2\rho L/A$, or $L/A = P/i^2\rho$. So the new values of L and A satisfy

$$\left(\frac{L}{A}\right)_{\text{new}} = \left(\frac{P}{i^2\rho}\right)_{\text{new}} = \frac{30}{4^2}\left(\frac{P}{i^2\rho}\right)_{\text{old}} = \frac{30}{16}\left(\frac{L}{A}\right)_{\text{old}} \ .$$

Consequently, $(L/A)_{\text{new}} = 1.875(L/A)_{\text{old}}$. Note, too, that $(LA)_{\text{new}} = (LA)_{\text{old}}$. We solve the above two equations for L_{new} and A_{new}:

$$\begin{aligned} L_{\text{new}} &= \sqrt{1.875}L_{\text{old}} = 1.369L_{\text{old}} \\ A_{\text{new}} &= \sqrt{1/1.875}A_{\text{old}} = 0.730A_{\text{old}} \ . \end{aligned}$$

50. (a) We denote the copper wire with subscript c and the aluminum wire with subscript a.

$$R = \rho_a \frac{L}{A} = \frac{(2.75 \times 10^{-8}\,\Omega\cdot\text{m})(1.3\,\text{m})}{(5.2 \times 10^{-3}\,\text{m})^2} = 1.3 \times 10^{-3}\ \Omega \ .$$

(b) Let $R = \rho_c L/(\pi d^2/4)$ and solve for the diameter d of the copper wire:

$$d = \sqrt{\frac{4\rho_c L}{\pi R}} = \sqrt{\frac{4(1.69 \times 10^{-8}\,\Omega{\cdot}\mathrm{m})(1.3\,\mathrm{m})}{\pi(1.3 \times 10^{-3}\,\Omega)}} = 4.6 \times 10^{-3}\ \mathrm{m}\ .$$

51. We use Eq. 27-17: $\rho - \rho_0 = \rho\alpha(T - T_0)$, and solve for T:

$$T = T_0 + \frac{1}{\alpha}\left(\frac{\rho}{\rho_0} - 1\right) = 20^\circ\mathrm{C} + \frac{1}{4.3 \times 10^{-3}/\mathrm{K}}\left(\frac{58\,\Omega}{50\,\Omega} - 1\right) = 57^\circ\mathrm{C}\ .$$

We are assuming that $\rho/\rho_0 = R/R_0$.

52. Since values from the referred-to graph can only be crudely estimated, we do not present a graph here, but rather indicate a few values. Since $R = V/i$ then we see $R = \infty$ when $i = 0$ (which the graph seems to show throughout the range $-\infty < V < 2$ V) and $V \neq 0$. For voltages values larger than 2 V, the resistance changes rapidly according to the ratio V/i. For instance, $R \approx 3.1/0.002 = 1550\,\Omega$ when $V = 3.1$ V, and $R \approx 3.8/0.006 = 633\,\Omega$ when $V = 3.8$ V

53. (a)

$$V = iR = i\rho\frac{L}{A} = \frac{(12\,\mathrm{A})(1.69 \times 10^{-8}\,\Omega{\cdot}\mathrm{m})(4.0 \times 10^{-2}\,\mathrm{m})}{\pi(5.2 \times 10^{-3}\,\mathrm{m}/2)^2} = 3.8 \times 10^{-4}\,\mathrm{V}\ .$$

(b) Since it moves in the direction of the electron drift which is against the direction of the current, its tail is negative compared to its head.

(c) The time of travel relates to the drift speed:

$$\begin{aligned} t &= \frac{L}{v_d} = \frac{lAne}{i} = \frac{\pi L d^2 ne}{4i} \\ &= \frac{\pi(1.0 \times 10^{-2}\,\mathrm{m})(5.2 \times 10^{-3}\,\mathrm{m})^2(8.47 \times 10^{28}/\mathrm{m}^3)(1.60 \times 10^{-19}\,\mathrm{C})}{4(12\,\mathrm{A})} \\ &= 238\,\mathrm{s} = 3\,\mathrm{min}\,58\,\mathrm{s}\ . \end{aligned}$$

54. Using Eq. 7-48 and Eq. 27-22, the rate of change of mechanical energy of the piston-Earth system, mgv, must be equal to the rate at which heat is generated from the coil: $mgv = i^2R$. Thus

$$v = \frac{i^2R}{mg} = \frac{(0.240\,\mathrm{A})^2(550\,\Omega)}{(12\,\mathrm{kg})\,(9.8\,\mathrm{m/s^2})} = 0.27\ \mathrm{m/s}\ .$$

55. Eq. 27-21 gives the rate of thermal energy production:

$$P = iV = (10\ \mathrm{A})(120\ \mathrm{V}) = 1.2\ \mathrm{kW}\ .$$

Dividing this into the 180 kJ necessary to cook the three hot-dogs leads to the result $t = 150$ s.

56. We find the drift speed from Eq. 27-7:

$$v_d = \frac{|\vec{J}|}{ne} = 1.5 \times 10^{-4}\,\mathrm{m/s}\ .$$

At this (average) rate, the time required to travel $L = 5.0$ m is

$$t = \frac{L}{v_d} = 3.4 \times 10^4\,\mathrm{s}\ .$$

57. (a) $i = (n_h + n_e)e = (2.25 \times 10^{15}/\mathrm{s} + 3.50 \times 10^{15}/\mathrm{s})(1.60 \times 10^{-19}\,\mathrm{C}) = 9.20 \times 10^{-4}\,\mathrm{A}$.

(b) The magnitude of the current density vector is

$$\left|\vec{J}\right| = \frac{i}{A} = \frac{9.20 \times 10^{-4}\,\mathrm{A}}{\pi(0.165 \times 10^{-3}\,\mathrm{m})^2} = 1.08 \times 10^4\ \mathrm{A/m^2}\ .$$

58. (a) Since $\rho = RA/L = \pi R d^2/4L = \pi(1.09 \times 10^{-3}\,\Omega)(5.50 \times 10^{-3}\,\mathrm{m})^2/[4(1.60\,\mathrm{m})] = 1.62 \times 10^{-8}\,\Omega{\cdot}\mathrm{m}$, the material is silver.

(b) The resistance of the round disk is

$$R = \rho\frac{L}{A} = \frac{4\rho L}{\pi d^2} = \frac{4(1.62 \times 10^{-8}\,\Omega{\cdot}\mathrm{m})(1.00 \times 10^{-3}\,\mathrm{m})}{\pi(2.00 \times 10^{-2}\,\mathrm{m})^2} = 5.16 \times 10^{-8}\ \Omega\ .$$

59. The horsepower required is

$$P = \frac{iV}{0.80} = \frac{(10\,\mathrm{A})(12\,\mathrm{V})}{(0.80)(746\,\mathrm{W/hp})} = 0.20\ \mathrm{hp}\ .$$

60. (a) The current is

$$\begin{aligned} i &= \frac{V}{R} = \frac{V}{\rho L/A} = \frac{\pi V d^2}{4\rho L} \\ &= \frac{\pi(1.20\,\mathrm{V})[(0.0400\,\mathrm{in.})(2.54 \times 10^{-2}\,\mathrm{m/in.})]^2}{4(1.69 \times 10^{-8}\,\Omega{\cdot}\mathrm{m})(33.0\,\mathrm{m})} = 1.74\,\mathrm{A}\ . \end{aligned}$$

(b) The magnitude of the current density vector is

$$\left|\vec{J}\right| = \frac{i}{A} = \frac{4i}{\pi d^2} = \frac{4(1.74\,\mathrm{A})}{\pi[(0.0400\,\mathrm{in.})(2.54 \times 10^{-2}\,\mathrm{m/in.})]^2} = 2.15 \times 10^6\ \mathrm{A/m^2}.$$

(c) $E = V/L = 1.20\,\mathrm{V}/33.0\,\mathrm{m} = 3.63 \times 10^{-2}\,\mathrm{V/m}$.

(d) $P = Vi = (1.20\,\mathrm{V})(1.74\,\mathrm{A}) = 2.09\,\mathrm{W}$.

61. We use $R/L = \rho/A = 0.150\,\Omega/\mathrm{km}$.

(a) For copper $J = i/A = (60.0\,\mathrm{A})(0.150\,\Omega/\mathrm{km})/(1.69 \times 10^{-8}\,\Omega\cdot\mathrm{m}) = 5.32 \times 10^5\,\mathrm{A/m^2}$; and for aluminum $J = (60.0\,\mathrm{A})(0.150\,\Omega/\mathrm{km})/(2.75 \times 10^{-8}\,\Omega{\cdot}\mathrm{m}) = 3.27 \times 10^5\,\mathrm{A/m^2}$.

(b) We denote the mass densities as ρ_m. For copper $(m/L)_c = (\rho_m A)_c = (8960\,\mathrm{kg/m^3})\,(1.69 \times 10^{-8}\,\Omega{\cdot}\mathrm{m})/(0.150\,\Omega/\mathrm{km}) = 1.01\,\mathrm{kg/m}$; and for aluminum $(m/L)_a = (\rho_m A)_a = (2700\,\mathrm{kg/m^3})(2.75 \times 10^{-8}\,\Omega{\cdot}\mathrm{m})/(0.150\,\Omega/\mathrm{km}) = 0.495\,\mathrm{kg/m}$.

62. (a) We use $P = V^2/R \propto V^2$, which gives $\Delta P \propto \Delta V^2 \approx 2V\Delta V$. The percentage change is roughly $\Delta P/P = 2\Delta V/V = 2(110 - 115)/115 = -8.6\%$.

(b) A drop in V causes a drop in P, which in turn lowers the temperature of the resistor in the coil. At a lower temperature R is also decreased. Since $P \propto R^{-1}$ a decrease in R will result in an increase in P, which partially offsets the decrease in P due to the drop in V. Thus, the actual drop in P will be smaller when the temperature dependency of the resistance is taken into consideration.

63. Using $A = \pi r^2$ with $r = 5 \times 10^{-4}$ m with Eq. 27-5 yields

$$|\vec{J}| = \frac{i}{A} = 2.5 \times 10^6\,\mathrm{A/m^2}\ .$$

Then, with $|\vec{E}| = 5.3$ V/m, Eq. 27-10 leads to

$$\rho = \frac{5.3\,\mathrm{V/m}}{2.5 \times 10^6\,\mathrm{A/m^2}} = 2.1 \times 10^{-6}\ \Omega{\cdot}\mathrm{m}\ .$$

64. A least squares fit of the data gives $R = \frac{537}{5} + \frac{1111}{1750}T$ with T in degrees Celsius.

   (a) At $T = 20°\text{C}$, our expression gives $R = \frac{21017}{175} \approx 120\,\Omega$.

   (b) At $T = 0°\text{C}$, our expression gives $R = \frac{537}{5} \approx 107\,\Omega$.

   (c) Defining α_R by

$$\alpha_R = \frac{R - R_{20}}{R_{20}\,(T - 20°\text{C})}$$

   then we are effectively requiring $\alpha_R R_{20}$ to equal the $\frac{1111}{1750}$ factor in our least squares fit. This implies that $\alpha_R = 1111/210170 = 0.00529/\text{C}°$ if $R_{20} = \frac{21017}{175} \approx 120\,\Omega$ is used as the reference.

   (d) Now we define α_R by

$$\alpha_R = \frac{R - R_0}{R_0\,(T - 0°\text{C})}\ ,$$

   which means we require $\alpha_R R_0$ to equal the $\frac{1111}{1750}$ factor in our least squares fit. In this case, $\alpha_R = 1111/187950 = 0.00591/\text{C}°$ if $R_0 = \frac{537}{5} \approx 107\,\Omega$ is used as the reference.

   (e) Our least squares fit expression predicts $R = 96473/350 \approx 276\,\Omega$ at $T = 265°\text{C}$.

65. The electric field points towards lower values of potential (see Eq. 25-40) so $\vec{E}$ is directed towards point B (which we take to be the $\hat{\imath}$ direction in our calculation). Since the field is considered to be uniform inside the wire, then its magnitude is, by Eq. 25-42,

$$|\vec{E}| = \frac{|\Delta V|}{L} = \frac{50}{200} = 0.25\ \text{V/m}\ .$$

Using Eq. 27-11, with $\rho = 1.7 \times 10^{-8}\ \Omega{\cdot}\text{m}$, we obtain

$$\vec{E} = \rho\vec{J} \implies \vec{J} = 1.5 \times 10^7\ \hat{\imath}$$

in SI units (A/m^2).

66. Assuming $\vec{J}$ is directed along the wire (with no radial flow) we integrate, starting with Eq. 27-4,

$$i = \int \left|\vec{J}\right|\,dA = \int_{R/2}^{R} kr\,2\pi r\,dr = \frac{2}{3}\,k\pi\left(R^3 - \frac{R^3}{8}\right)$$

where $k = 3.0 \times 10^8$ and SI units understood. Therefore, if $R = 0.00200$ m, we obtain $i = 4.40$ A.

67. (First problem of **Cluster**)

   (a) We are told that $r_B = \frac{1}{2}r_A$ and $L_B = 2L_A$. Thus, using Eq. 27-16,

$$R_B = \rho\,\frac{L_B}{\pi r_B^2} = \rho\,\frac{2L_A}{\frac{1}{4}\,\pi r_A^2} = 8R_A = 64\ \Omega\ .$$

   (b) The current-densities are assumed uniform.

$$\frac{J_A}{J_B} = \frac{\frac{i}{\pi r_A^2}}{\frac{i}{\pi r_B^2}} = \frac{\frac{i}{\pi r_A^2}}{\frac{i}{\frac{1}{4}\,\pi r_A^2}} = \frac{1}{4}\ .$$

68. (Second problem of **Cluster**)

(a) We use Eq. 27-16 to compute the resistances in SI units:

$$\begin{aligned} R_C &= \rho_C \frac{L_C}{\pi r_C^2} = \left(2 \times 10^{-6}\right) \frac{1}{\pi (0.0005)^2} = 2.5\ \Omega \\ R_D &= \rho_D \frac{L_D}{\pi r_D^2} = \left(1 \times 10^{-6}\right) \frac{1}{\pi (0.00025)^2} = 5.1\ \Omega\ . \end{aligned}$$

The voltages follow from Ohm's law:

$$\begin{aligned} |V_1 - V_2| = V_C &= iR_C = 5.1\ \text{V} \\ |V_2 - V_3| = V_D &= iR_D = 10\ \text{V}\ . \end{aligned}$$

(b) See solution for part (a).

(c) and (d) The power is calculated from Eq. 27-22:

$$P = i^2 R = \begin{cases} 10\ \text{W} & \text{for } R = R_C \\ 20\ \text{W} & \text{for } R = R_D \end{cases}$$

69. (Third problem of **Cluster**)

(a) We use Eq. 27-17 with $\rho = \frac{10}{8}\rho_0$ (we are neglecting any thermal expansion of the material) and $T - T_0 = 100$ K in order to obtain $\alpha = 2.5 \times 10^{-3}$/K. Now with this value of α but $T = 600$ K (so $T - T_0 = 300$ K) we find $\rho = 1.75\rho_0 \rightarrow R = 1.75(8.0\,\Omega) = 14\,\Omega$.

(b) We are assuming the wires have unknown but equal length (not the lengths shown in Figure 27-33). With $\alpha_D = 5.0 \times 10^{-3}$/K, we find $\rho = 2.5\rho_0$ for $T - T_0 = 300$ K. With the same assumptions as in part (a), this implies $R = 2.5R_0$ where $R_0 = 16\,\Omega$ (that the resistance of D is twice that of C at 300 K is evident in part (a) of the *previous* solution. Therefore, $R = 2.5(16\,\Omega) = 40\,\Omega$ for wire D at $T = 600$ K.

70. (Fourth problem of **Cluster**)
From Eq. 27-23, we obtain the resistance at temperature T:

$$R = \frac{V^2}{P} = \frac{12^2}{10} = 14.4\ \Omega\ .$$

Thus, the ratio R/R_0 with R_0 representing the resistance at 300 K is 7.2, which we take to equal the ratio of resistivities (ignoring any thermal expansion of the filament). Eq. 27-17, then, leads to

$$\frac{\rho}{\rho_0} = 7.2 = 1 + \alpha\,(T - 300)\quad .$$

Using Table 27-1 ($\alpha = 4.5 \times 10^{-3}$/K) we find $T = 1.7 \times 10^3$ K.