Chapter 28

1. (a) The cost is $(100\,\mathrm{W}\cdot 8.0\,\mathrm{h}/2.0\,\mathrm{W}\cdot\mathrm{h})(\$0.80) = \$320$.
   (b) The cost is $(100\,\mathrm{W}\cdot 8.0\,\mathrm{h}/10^3\,\mathrm{W}\cdot\mathrm{h})(\$0.06) = \$0.048 = 4.8\,\mathrm{cents}$.

2. The chemical energy of the battery is reduced by $\Delta E = q\mathcal{E}$, where q is the charge that passes through in time $\Delta t = 6.0\,\mathrm{min}$, and $\mathcal{E}$ is the emf of the battery. If i is the current, then $q = i\,\Delta t$ and $\Delta E = i\mathcal{E}\,\Delta t = (5.0\,\mathrm{A})(6.0\,\mathrm{V})(6.0\,\mathrm{min})(60\,\mathrm{s/min}) = 1.1\times 10^4\,\mathrm{J}$. We note the conversion of time from minutes to seconds.

3. If P is the rate at which the battery delivers energy and Δt is the time, then $\Delta E = P\,\Delta t$ is the energy delivered in time Δt. If q is the charge that passes through the battery in time Δt and $\mathcal{E}$ is the emf of the battery, then $\Delta E = q\mathcal{E}$. Equating the two expressions for ΔE and solving for Δt, we obtain

$$\Delta t = \frac{q\mathcal{E}}{P} = \frac{(120\,\mathrm{A\cdot h})(12\,\mathrm{V})}{100\,\mathrm{W}} = 14.4\,\mathrm{h} = 14\,\mathrm{h}\ 24\,\mathrm{min}\ .$$

4. (a) Since $\mathcal{E}_1 > \mathcal{E}_2$ the current flows counterclockwise.
   (b) Battery 1, since the current flows through it from its negative terminal to the positive one.
   (c) Point B, since the current flows from B to A.

5. (a) Let i be the current in the circuit and take it to be positive if it is to the left in R_1. We use Kirchhoff's loop rule: $\mathcal{E}_1 - iR_2 - iR_1 - \mathcal{E}_2 = 0$. We solve for i:

$$i = \frac{\mathcal{E}_1 - \mathcal{E}_2}{R_1 + R_2} = \frac{12\,\mathrm{V} - 6.0\,\mathrm{V}}{4.0\,\Omega + 8.0\,\Omega} = 0.50\ \mathrm{A}\ .$$

   A positive value is obtained, so the current is counterclockwise around the circuit.
   (b) If i is the current in a resistor R, then the power dissipated by that resistor is given by $P = i^2R$. For R_1, $P_1 = (0.50\,\mathrm{A})^2(4.0\,\Omega) = 1.0\,\mathrm{W}$ and for R_2, $P_2 = (0.50\,\mathrm{A})^2(8.0\,\Omega) = 2.0\,\mathrm{W}$.
   (c) If i is the current in a battery with emf $\mathcal{E}$, then the battery supplies energy at the rate $P = i\mathcal{E}$ provided the current and emf are in the same direction. The battery absorbs energy at the rate $P = i\mathcal{E}$ if the current and emf are in opposite directions. For $\mathcal{E}_1$, $P_1 = (0.50\,\mathrm{A})(12\,\mathrm{V}) = 6.0\,\mathrm{W}$ and for $\mathcal{E}_2$, $P_2 = (0.50\,\mathrm{A})(6.0\,\mathrm{V}) = 3.0\,\mathrm{W}$. In battery 1 the current is in the same direction as the emf. Therefore, this battery supplies energy to the circuit; the battery is discharging. The current in battery 2 is opposite the direction of the emf, so this battery absorbs energy from the circuit. It is charging.

6. (a) The energy transferred is

$$U = Pt = \frac{\mathcal{E}^2 t}{r+R} = \frac{(2.0\,\mathrm{V})^2(2.0\,\mathrm{min})(60\,\mathrm{s/min})}{1.0\,\Omega + 5.0\,\Omega} = 80\ \mathrm{J}\ .$$

   (b) The amount of thermal energy generated is

$$U' = i^2Rt = \left(\frac{\mathcal{E}}{r+R}\right)^2 Rt = \left(\frac{2.0\,\mathrm{V}}{1.0\,\Omega + 5.0\,\Omega}\right)^2 (5.0\,\Omega)(2.0\,\mathrm{min})(60\,\mathrm{s/min}) = 67\ \mathrm{J}\ .$$

(c) The difference between U and U', which is equal to 13 J, is the thermal energy that is generated in the battery due to its internal resistance.

7. (a) The potential difference is $V = \mathcal{E} + ir = 12\,\text{V} + (0.040\,\Omega)(50\,\text{A}) = 14\,\text{V}$.

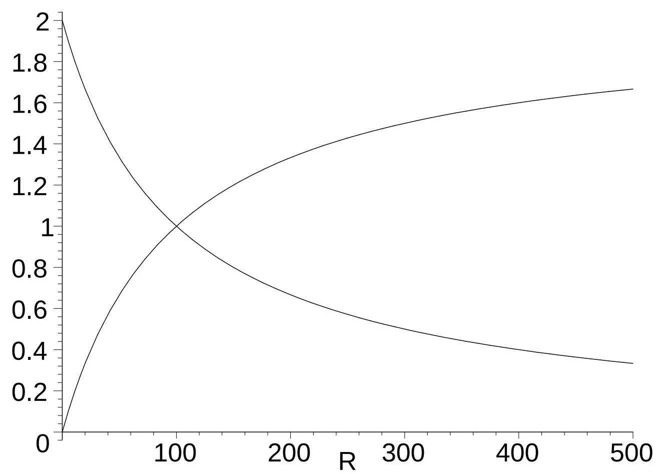
   (b) $P = i^2 r = (50\,\text{A})^2(0.040\,\Omega) = 100\,\text{W}$.

   (c) $P' = iV = (50\,\text{A})(12\,\text{V}) = 600\,\text{W}$.

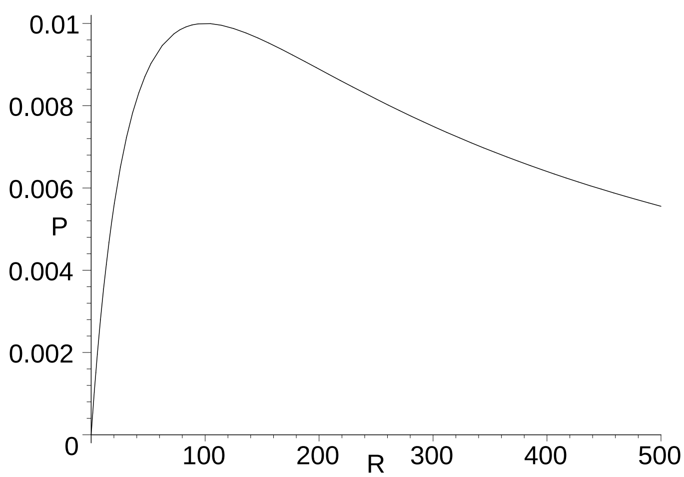
   (d) In this case $V = \mathcal{E} - ir = 12\,\text{V} - (0.040\,\Omega)(50\,\text{A}) = 10\,\text{V}$ and $P = i^2 r = 100\,\text{W}$.

8. (a) Below, we graph Eq. 28-4 (scaled by a factor of 100) for $\mathcal{E} = 2.0\,\text{V}$ and $r = 100\,\Omega$ over the range $0 \le R \le 500\,\Omega$. We multiplied the SI output of Eq. 28-4 by 100 so that this graph would not be vanishingly small with the other graph (see part (b)) when they are plotted together.

   (b) In the same graph, we show $V_R = iR$ over the same range. The graph of current i is the one that starts at 2 (which corresponds to 0.02 A in SI units) and the graph of voltage V_R is the one that starts at 0 (when $R = 0$). The value of V_R are in SI units (not scaled by any factor).



   (c) In our final graph, we show the dependence of power $P = iV_R$ (dissipated in resistor R) as a function of R. The units of the vertical axis are Watts. We note that it is maximum when $R = r$.



9. (a) If i is the current and ΔV is the potential difference, then the power absorbed is given by $P = i\,\Delta V$. Thus,

$$\Delta V = \frac{P}{i} = \frac{50\,\mathrm{W}}{1.0\,\mathrm{A}} = 50\ \mathrm{V}\ .$$

Since the energy of the charge decreases, point A is at a higher potential than point B; that is, $V_A - V_B = 50\,\mathrm{V}$.

(b) The end-to-end potential difference is given by $V_A - V_B = +iR + \mathcal{E}$, where $\mathcal{E}$ is the emf of element C and is taken to be positive if it is to the left in the diagram. Thus, $\mathcal{E} = V_A - V_B - iR = 50\,\mathrm{V} - (1.0\,\mathrm{A})(2.0\,\Omega) = 48\,\mathrm{V}$.

(c) A positive value was obtained for $\mathcal{E}$, so it is toward the left. The negative terminal is at B.

10. The current in the circuit is $i = (150\,\mathrm{V}-50\,\mathrm{V})/(3.0\,\Omega+2.0\,\Omega) = 20\,\mathrm{A}$. So from $V_Q+150\,\mathrm{V}-(2.0\,\Omega)i = V_P$, we get $V_Q = 100\,\mathrm{V} + (2.0\,\Omega)(20\,\mathrm{A}) - 150\,\mathrm{V} = -10\,\mathrm{V}$.

11. From $V_a - \mathcal{E}_1 = V_c - ir_1 - iR$ and $i = (\mathcal{E}_1 - \mathcal{E}_2)/(R + r_1 + r_2)$, we get

$$\begin{aligned} V_a - V_c &= \mathcal{E}_1 - i\,(r_1 + R) \\ &= \mathcal{E}_1 - \left(\frac{\mathcal{E}_1 - \mathcal{E}_2}{R + r_1 + r_2}\right)(r_1 + R) \\ &= 4.4\,\mathrm{V} - \left(\frac{4.4\,\mathrm{V} - 2.1\,\mathrm{V}}{5.5\,\Omega + 1.8\,\Omega + 2.3\,\Omega}\right)(2.3\,\Omega + 5.5\,\Omega) \\ &= 2.5\ \mathrm{V}\ . \end{aligned}$$

12. (a) We solve $i = (\mathcal{E}_2 - \mathcal{E}_1)/(r_1 + r_2 + R)$ for R:

$$R = \frac{\mathcal{E}_2 - \mathcal{E}_1}{i} - r_1 - r_2 = \frac{3.0\,\mathrm{V} - 2.0\,\mathrm{V}}{1.0 \times 10^{-3}\,\mathrm{A}} - 3.0\,\Omega - 3.0\,\Omega = 9.9 \times 10^2\,\Omega\ .$$

(b) $P = i^2R = (1.0 \times 10^{-3}\,\mathrm{A})^2(9.9 \times 10^2\,\Omega) = 9.9 \times 10^{-4}\,\mathrm{W}$.

13. Let the emf be V. Then $V = iR = i'(R + R')$, where $i = 5.0\,\mathrm{A}$, $i' = 4.0\,\mathrm{A}$ and $R' = 2.0\,\Omega$. We solve for R:

$$R = \frac{i'R'}{i - i'} = \frac{(4)(2)}{5 - 4} = 8.0\ \Omega\ .$$

14. The internal resistance of the battery is $r = (12\,\mathrm{V} - 11.4\,\mathrm{V})/50\,\mathrm{A} = 0.012\,\Omega < 0.020\,\Omega$, so the battery is OK. The resistance of the cable is $R = 3.0\,\mathrm{V}/50\,\mathrm{A} = 0.060\,\Omega > 0.040\,\Omega$, so the cable is defective.

15. To be as general as possible, we refer to the individual emf's as $\mathcal{E}_1$ and $\mathcal{E}_2$ and wait until the latter steps to equate them ($\mathcal{E}_1 = \mathcal{E}_2 = \mathcal{E}$). The batteries are placed in series in such a way that their voltages add; that is, they do not "oppose" each other. The total resistance in the circuit is therefore $R_{\text{total}} = R + r_1 + r_2$ (where the problem tells us $r_1 > r_2$), and the "net emf" in the circuit is $\mathcal{E}_1 + \mathcal{E}_2$. Since battery 1 has the higher internal resistance, it is the one capable of having a zero terminal voltage, as the computation in part (a) shows.

(a) The current in the circuit is

$$i = \frac{\mathcal{E}_1 + \mathcal{E}_2}{r_1 + r_2 + R}\ ,$$

and the requirement of zero terminal voltage leads to

$$\mathcal{E}_1 = ir_1 \quad\Longrightarrow\quad R = \frac{\mathcal{E}_2 r_1 - \mathcal{E}_1 r_2}{\mathcal{E}_1}$$

which reduces to $R = r_1 - r_2$ when we set $\mathcal{E}_1 = \mathcal{E}_2$.

(b) As mentioned above, this occurs in battery 1.

16. (a) Let the emf of the solar cell be $\mathcal{E}$ and the output voltage be V. Thus,

$$V = \mathcal{E} - ir = \mathcal{E} - \left(\frac{V}{R}\right) r$$

for both cases. Numerically, we get $0.10\,\text{V} = \mathcal{E} - (0.10\,\text{V}/500\,\Omega)r$ and $0.15\,\text{V} = \mathcal{E} - (0.15\,\text{V}/1000\,\Omega)r$. We solve for $\mathcal{E}$ and r: $\mathcal{E} = 0.30\,\text{V}$, $r = 1000\,\Omega$.

(b) The efficiency is

$$\frac{V^2/R}{P_{\text{received}}} = \frac{0.15\,\text{V}}{(1000\,\Omega)(5.0\,\text{cm}^2)(2.0\times10^{-3}\,\text{W/cm}^2)} = 2.3\times10^{-3}\ .$$

17. (a) Using Eq. 28-4, we take the derivative of the power $P = i^2R$ with respect to R and set the result equal to zero:

$$\frac{dP}{dR} = \frac{d}{dR}\left(\frac{\mathcal{E}^2 R}{(R+r)^2}\right) = \frac{\mathcal{E}^2(r-R)}{(R+r)^3} = 0$$

which clearly has the solution $R = r$.

(b) When $R = r$, the power dissipated in the external resistor equals

$$P_{\max} = \left.\frac{\mathcal{E}^2 R}{(R+r)^2}\right|_{R=r} = \frac{\mathcal{E}^2}{4r}\ .$$

18. Let the resistances of the two resistors be R_1 and R_2. Note that the smallest value of the possible R_{eq} must be the result of connecting R_1 and R_2 in parallel, while the largest one must be that of connecting them in series. Thus, $R_1R_2/(R_1+R_2) = 3.0\,\Omega$ and $R_1 + R_2 = 16\,\Omega$. So R_1 and R_2 must be $4.0\,\Omega$ and $12\,\Omega$, respectively.

19. The potential difference across each resistor is $V = 25.0\,\text{V}$. Since the resistors are identical, the current in each one is $i = V/R = (25.0\,\text{V})/(18.0\,\Omega) = 1.39\,\text{A}$. The total current through the battery is then $i_{\text{total}} = 4(1.39\,\text{A}) = 5.56\,\text{A}$. One might alternatively use the idea of equivalent resistance; for four identical resistors in parallel the equivalent resistance is given by

$$\frac{1}{R_{\text{eq}}} = \sum \frac{1}{R} = \frac{4}{R}\ .$$

When a potential difference of 25.0 V is applied to the equivalent resistor, the current through it is the same as the total current through the four resistors in parallel. Thus $i_{\text{total}} = V/R_{\text{eq}} = 4V/R = 4(25.0\,\text{V})/(18.0\,\Omega) = 5.56\,\text{A}$.

20. We note that two resistors in parallel, say R_1 and R_2, are equivalent to

$$R_{\text{parallel pair}} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{R_1R_2}{R_1+R_2}\ .$$

This situation (Figure 28-27) consists of a parallel pair which are then in series with a single $2.50\,\Omega$ resistor. Thus, the situation has an equivalent resistance of

$$R_{\text{eq}} = 2.50\,\Omega + \frac{(4.00\,\Omega)(4.00\,\Omega)}{4.00\,\Omega + 4.00\,\Omega} = 4.50\,\Omega\ .$$

21. Let i_1 be the current in R_1 and take it to be positive if it is to the right. Let i_2 be the current in R_2 and take it to be positive if it is upward. When the loop rule is applied to the lower loop, the result is

$$\mathcal{E}_2 - i_1 R_1 = 0 \ .$$

and when it is applied to the upper loop, the result is

$$\mathcal{E}_1 - \mathcal{E}_2 - \mathcal{E}_3 - i_2 R_2 = 0 \ .$$

The first equation yields

$$i_1 = \frac{\mathcal{E}_2}{R_1} = \frac{5.0\,\mathrm{V}}{100\,\Omega} = 0.050\ \mathrm{A}\ .$$

The second yields

$$i_2 = \frac{\mathcal{E}_1 - \mathcal{E}_2 - \mathcal{E}_3}{R_2} = \frac{6.0\,\mathrm{V} - 5.0\,\mathrm{V} - 4.0\,\mathrm{V}}{50\,\Omega} = -0.060\ \mathrm{A}\ .$$

The negative sign indicates that the current in R_2 is actually downward. If V_b is the potential at point b, then the potential at point a is $V_a = V_b + \mathcal{E}_3 + \mathcal{E}_2$, so $V_a - V_b = \mathcal{E}_3 + \mathcal{E}_2 = 4.0\,\mathrm{V} + 5.0\,\mathrm{V} = 9.0\,\mathrm{V}$.

22.
- S_1, S_2 and S_3 all open: $i_a = 0.00\,\mathrm{A}$.
- S_1 closed, S_2 and S_3 open: $i_a = \mathcal{E}/2R_1 = 120\,\mathrm{V}/40.0\,\Omega = 3.00\,\mathrm{A}$.
- S_2 closed, S_1 and S_3 open: $i_a = \mathcal{E}/(2R_1 + R_2) = 120\,\mathrm{V}/50.0\,\Omega = 2.40\,\mathrm{A}$.
- S_3 closed, S_1 and S_2 open: $i_a = \mathcal{E}/(2R_1 + R_2) = 120\,\mathrm{V}/60.0\,\Omega = 2.00\,\mathrm{A}$.
- S_1 open, S_2 and S_3 closed: $R_{\mathrm{eq}} = R_1 + R_2 + R_1(R_1 + R_2)/(2R_1 + R_2) = 20.0\,\Omega + 10.0\,\Omega + (20.0\,\Omega)(30.0\,\Omega)/(50.0\,\Omega) = 42.0\,\Omega$, so $i_a = \mathcal{E}/R_{\mathrm{eq}} = 120\,\mathrm{V}/42.0\,\Omega = 2.86\,\mathrm{A}$.
- S_2 open, S_1 and S_3 closed: $R_{\mathrm{eq}} = R_1 + R_1(R_1 + 2R_2)/(2R_1 + 2R_2) = 20.0\,\Omega + (20.0\,\Omega)\times (40.0\,\Omega)/(60.0\,\Omega) = 33.3\,\Omega$, so $i_a = \mathcal{E}/R_{\mathrm{eq}} = 120\,\mathrm{V}/33.3\,\Omega = 3.60\,\mathrm{A}$.
- S_3 open, S_1 and S_2 closed: $R_{\mathrm{eq}} = R_1 + R_1(R_1 + R_2)/(2R_1 + R_2) = 20.0\,\Omega + (20.0\,\Omega)\times (30.0\,\Omega)/(50.0\,\Omega) = 32.0\,\Omega$, so $i_a = \mathcal{E}/R_{\mathrm{eq}} = 120\,\mathrm{V}/32.0\,\Omega = 3.75\,\mathrm{A}$.
- S_1, S_2 and S_3 all closed: $R_{\mathrm{eq}} = R_1 + R_1R'/(R_1 + R')$ where $R' = R_2 + R_1(R_1 + R_2)/(2R_1 + R_2) = 22.0\,\Omega$, i.e., $R_{\mathrm{eq}} = 20.0\,\Omega + (20.0\,\Omega)(22.0\,\Omega)/(20.0\,\Omega + 22.0\,\Omega) = 30.5\,\Omega$, so $i_a = \mathcal{E}/R_{\mathrm{eq}} = 120\,\mathrm{V}/30.5\,\Omega = 3.94\,\mathrm{A}$.

23. (a) Let $\mathcal{E}$ be the emf of the battery. When the bulbs are connected in parallel, the potential difference across them is the same and is also the same as the emf of the battery. The power dissipated by bulb 1 is $P_1 = \mathcal{E}^2/R_1$, and the power dissipated by bulb 2 is $P_2 = \mathcal{E}^2/R_2$. Since R_1 is greater than R_2, bulb 2 dissipates energy at a greater rate than bulb 1 and is the brighter of the two.

(b) When the bulbs are connected in series the current in them is the same. The power dissipated by bulb 1 is now $P_1 = i^2R_1$ and the power dissipated by bulb 2 is $P_2 = i^2R_2$. Since R_1 is greater than R_2 greater power is dissipated by bulb 1 than by bulb 2 and bulb 1 is the brighter of the two.

24. The currents i_1, i_2 and i_3 are obtained from Eqs. 28-15 through 28-17:

$$\begin{aligned}
i_1 &= \frac{\mathcal{E}_1(R_2 + R_3) - \mathcal{E}_2R_3}{R_1R_2 + R_2R_3 + R_1R_3} = \frac{(4.0\,\mathrm{V})(10\,\Omega + 5.0\,\Omega) - (1.0\,\mathrm{V})(5.0\,\Omega)}{(10\,\Omega)(10\,\Omega) + (10\,\Omega)(5.0\,\Omega) + (10\,\Omega)(5.0\,\Omega)} \\
&= 0.275\ \mathrm{A}\ , \\
i_2 &= \frac{\mathcal{E}_1R_3 - \mathcal{E}_2(R_1 + R_2)}{R_1R_2 + R_2R_3 + R_1R_3} = \frac{(4.0\,\mathrm{V})(5.0\,\Omega) - (1.0\,\mathrm{V})(10\,\Omega + 5.0\,\Omega)}{(10\,\Omega)(10\,\Omega) + (10\,\Omega)(5.0\,\Omega) + (10\,\Omega)(5.0\,\Omega)} \\
&= 0.025\ \mathrm{A}\ , \\
i_3 &= i_2 - i_1 = 0.025\,\mathrm{A} - 0.275\,\mathrm{A} = -0.250\ \mathrm{A}\ .
\end{aligned}$$

$V_d - V_c$ can now be calculated by taking various paths. Two examples: from $V_d - i_2R_2 = V_c$ we get $V_d - V_c = i_2R_2 = (0.0250\,\mathrm{A})(10\,\Omega) = +0.25\,\mathrm{V}$; from $V_d + i_3R_3 + \mathcal{E}_2 = V_c$ we get $V_d - V_c = -i_3R_3 - \mathcal{E}_2 = -(-0.250\,\mathrm{A})(5.0\,\Omega) - 1.0\,\mathrm{V} = +0.25\,\mathrm{V}$.

25. Let r be the resistance of each of the narrow wires. Since they are in parallel the resistance R of the composite is given by

$$\frac{1}{R} = \frac{9}{r} ,$$

or $R = r/9$. Now $r = 4\rho\ell/\pi d^2$ and $R = 4\rho\ell/\pi D^2$, where ρ is the resistivity of copper. $A = \pi d^2/4$ was used for the cross-sectional area of a single wire, and a similar expression was used for the cross-sectional area of the thick wire. Since the single thick wire is to have the same resistance as the composite,

$$\frac{4\rho\ell}{\pi D^2} = \frac{4\rho\ell}{9\pi d^2} \implies D = 3d .$$

26. (a) $R_{\text{eq}}(FH) = (10.0\,\Omega)(10.0\,\Omega)(5.00\,\Omega)/[(10.0\,\Omega)(10.0\,\Omega) + 2(10.0\,\Omega)(5.00\,\Omega)] = 2.50\,\Omega$.
(b) $R_{\text{eq}}(FG) = (5.00\,\Omega)R/(R+5.00\,\Omega)$, where $R = 5.00\,\Omega + (5.00\,\Omega)(10.0\,\Omega)/(5.00\,\Omega+10.0\,\Omega) = 8.33\,\Omega$. So $R_{\text{eq}}(FG) = (5.00\,\Omega)(8.33\,\Omega)/(5.00\,\Omega + 8.33\,\Omega) = 3.13\,\Omega$.

27. Let the resistors be divided into groups of n resistors each, with all the resistors in the same group connected in series. Suppose there are m such groups that are connected in parallel with each other. Let R be the resistance of any one of the resistors. Then the equivalent resistance of any group is nR, and R_{eq}, the equivalent resistance of the whole array, satisfies

$$\frac{1}{R_{\text{eq}}} = \sum_{1}^{m} \frac{1}{nR} = \frac{m}{nR} .$$

Since the problem requires $R_{\text{eq}} = 10\,\Omega = R$, we must select $n = m$. Next we make use of Eq. 28-13. We note that the current is the same in every resistor and there are $n \cdot m = n^2$ resistors, so the maximum total power that can be dissipated is $P_{\text{total}} = n^2 P$, where $P = 1.0\,\text{W}$ is the maximum power that can be dissipated by any one of the resistors. The problem demands $P_{\text{total}} \geq 5.0P$, so n^2 must be at least as large as 5.0. Since n must be an integer, the smallest it can be is 3. The least number of resistors is $n^2 = 9$.

28. (a) R_2, R_3 and R_4 are in parallel. By finding a common denominator and simplifying, the equation $1/R = 1/R_2 + 1/R_3 + 1/R_4$ gives an equivalent resistance of

$$R = \frac{R_2R_3R_4}{R_2R_3 + R_2R_4 + R_3R_4} = \frac{(50\,\Omega)(50\,\Omega)(75\,\Omega)}{(50\,\Omega)(50\,\Omega) + (50\,\Omega)(75\,\Omega) + (50\,\Omega)(75\,\Omega)} = 19\,\Omega .$$

Thus, considering the series contribution of resistor R_1, the equivalent resistance for the network is $R_{\text{eq}} = R_1 + R = 100\,\Omega + 19\,\Omega = 1.2 \times 10^2\,\Omega$.
(b) $i_1 = \mathcal{E}/R_{\text{eq}} = 6.0\,\text{V}/(1.1875 \times 10^2\,\Omega) = 5.1 \times 10^{-2}\,\text{A}$; $i_2 = (\mathcal{E} - V_1)/R_2 = (\mathcal{E} - i_1R_1)/R_2 = [6.0\,\text{V} - (5.05 \times 10^{-2}\,\text{A})(100\,\Omega)]/50\,\Omega = 1.9 \times 10^{-2}\,\text{A}$; $i_3 = (\mathcal{E} - V_1)/R_3 = i_2R_2/R_3 = (1.9 \times 10^{-2}\,\text{A})(50\,\Omega/50\,\Omega) = 1.9\times10^{-2}\,\text{A}$; $i_4 = i_1 - i_2 - i_3 = 5.0\times10^{-2}\,\text{A} - 2(1.895\times10^{-2}\,\text{A}) = 1.2\times10^{-2}\,\text{A}$.

29. (a) The batteries are identical and, because they are connected in parallel, the potential differences across them are the same. This means the currents in them are the same. Let i be the current in either battery and take it to be positive to the left. According to the junction rule the current in R is $2i$ and it is positive to the right. The loop rule applied to either loop containing a battery and R yields

$$\mathcal{E} - ir - 2iR = 0 \implies i = \frac{\mathcal{E}}{r + 2R} .$$

The power dissipated in R is

$$P = (2i)^2 R = \frac{4\mathcal{E}^2 R}{(r + 2R)^2} .$$

We find the maximum by setting the derivative with respect to R equal to zero. The derivative is

$$\frac{dP}{dR} = \frac{4\mathcal{E}^2}{(r + 2R)^2} - \frac{16\mathcal{E}^2 R}{(r + 2R)^3} = \frac{4\mathcal{E}^2(r - 2R)}{(r + 2R)^3} .$$

The derivative vanishes (and P is a maximum) if $R = r/2$.

(b) We substitute $R = r/2$ into $P = 4\mathcal{E}^2 R/(r+2R)^2$ to obtain

$$P_{\max} = \frac{4\mathcal{E}^2(r/2)}{[r+2(r/2)]^2} = \frac{\mathcal{E}^2}{2r} \, .$$

30. (a) By symmetry, when the two batteries are connected in parallel the current i going through either one is the same. So from $\mathcal{E} = ir + (2i)R$ we get $i_R = 2i = 2\mathcal{E}/(r+2R)$. When connected in series $2\mathcal{E} - i_R r - i_R r - i_R R = 0$, or $i_R = 2\mathcal{E}/(2r+R)$.

(b) In series, since $R > r$.

(c) In parallel, since $R < r$.

31. (a) We first find the currents. Let i_1 be the current in R_1 and take it to be positive if it is upward. Let i_2 be the current in R_2 and take it to be positive if it is to the left. Let i_3 be the current in R_3 and take it to be positive if it is to the right. The junction rule produces

$$i_1 + i_2 + i_3 = 0 \, .$$

The loop rule applied to the left-hand loop produces

$$\mathcal{E}_1 - i_3 R_3 + i_1 R_1 = 0$$

and applied to the right-hand loop produces

$$\mathcal{E}_2 - i_2 R_2 + i_1 R_1 = 0 \, .$$

We substitute $i_1 = -i_2 - i_3$, from the first equation, into the other two to obtain

$$\mathcal{E}_1 - i_3 R_3 - i_2 R_1 - i_3 R_1 = 0$$

and

$$\mathcal{E}_2 - i_2 R_2 - i_2 R_1 - i_3 R_1 = 0 \, .$$

The first of these yields

$$i_3 = \frac{\mathcal{E}_1 - i_2 R_1}{R_1 + R_3} \, .$$

Substituting this into the second equation and solving for i_2, we obtain

$$\begin{aligned} i_2 &= \frac{\mathcal{E}_2(R_1+R_3) - \mathcal{E}_1 R_1}{R_1R_2 + R_1R_3 + R_2R_3} \\ &= \frac{(1.00\,\mathrm{V})(5.00\,\Omega + 4.00\,\Omega) - (3.00\,\mathrm{V})(5.00\,\Omega)}{(5.00\,\Omega)(2.00\,\Omega) + (5.00\,\Omega)(4.00\,\Omega) + (2.00\,\Omega)(4.00\,\Omega)} = -0.158\,\mathrm{A} \, . \end{aligned}$$

We substitute into the expression for i_3 to obtain

$$i_3 = \frac{\mathcal{E}_1 - i_2 R_1}{R_1 + R_3} = \frac{3.00\,\mathrm{V} - (-0.158\,\mathrm{A})(5.00\,\Omega)}{5.00\,\Omega + 4.00\,\Omega} = 0.421\,\mathrm{A} \, .$$

Finally,

$$i_1 = -i_2 - i_3 = -(-0.158\,\mathrm{A}) - (0.421\,\mathrm{A}) = -0.263\,\mathrm{A} \, .$$

Note that the current in R_1 is actually downward and the current in R_2 is to the right. The current in R_3 is also to the right. The power dissipated in R_1 is $P_1 = i_1^2 R_1 = (-0.263\,\mathrm{A})^2(5.00\,\Omega) = 0.346\,\mathrm{W}$.

(b) The power dissipated in R_2 is $P_2 = i_2^2 R_2 = (-0.158\,\mathrm{A})^2(2.00\,\Omega) = 0.0499\,\mathrm{W}$.

(c) The power dissipated in R_3 is $P_3 = i_3^2 R_3 = (0.421\,\mathrm{A})^2(4.00\,\Omega) = 0.709\,\mathrm{W}$.

(d) The power supplied by $\mathcal{E}_1$ is $i_3\mathcal{E}_1 = (0.421\,\mathrm{A})(3.00\,\mathrm{V}) = 1.26\,\mathrm{W}$.

(e) The power "supplied" by $\mathcal{E}_2$ is $i_2\mathcal{E}_2 = (-0.158\,\text{A})(1.00\,\text{V}) = -0.158\,\text{W}$. The negative sign indicates that $\mathcal{E}_2$ is actually absorbing energy from the circuit.

32. (a) We use $P = \mathcal{E}^2/R_{\text{eq}}$, where

$$R_{\text{eq}} = 7.00\,\Omega + \frac{(12.0\,\Omega)(4.00\,\Omega)R}{(12.0\,\Omega)(4.0\,\Omega) + (12.0\,\Omega)R + (4.00\,\Omega)R} \ .$$

Put $P = 60.0\,\text{W}$ and $\mathcal{E} = 24.0\,\text{V}$ and solve for R: $R = 19.5\,\Omega$.

(b) Since $P \propto R_{\text{eq}}$, we must minimize R_{eq}, which means $R = 0$.

(c) Now we must maximize R_{eq}, or set $R = \infty$.

(d) Since $R_{\text{eq, max}} = 7.00\,\Omega + (12.0\,\Omega)(4.00\,\Omega)/(12.0\,\Omega + 4.00\,\Omega) = 10.0\,\Omega$, $P_{\text{min}} = \mathcal{E}^2/R_{\text{eq, max}} = (24.0\,\text{V})^2/10.0\,\Omega = 57.6\,\text{W}$. Since $R_{\text{eq, min}} = 7.00\,\Omega$, $P_{\text{max}} = \mathcal{E}^2/R_{\text{eq, min}} = (24.0\,\text{V})^2/7.00\,\Omega = 82.3\,\text{W}$.

33. (a) We note that the R_1 resistors occur in series pairs, contributing net resistance $2R_1$ in each branch where they appear. Since $\mathcal{E}_2 = \mathcal{E}_3$ and $R_2 = 2R_1$, from symmetry we know that the currents through $\mathcal{E}_2$ and $\mathcal{E}_3$ are the same: $i_2 = i_3 = i$. Therefore, the current through $\mathcal{E}_1$ is $i_1 = 2i$. Then from $V_b - V_a = \mathcal{E}_2 - iR_2 = \mathcal{E}_1 + (2R_1)(2i)$ we get

$$i = \frac{\mathcal{E}_2 - \mathcal{E}_1}{4R_1 + R_2} = \frac{4.0\,\text{V} - 2.0\,\text{V}}{4(1.0\,\Omega) + 2.0\,\Omega} = 0.33\,\text{A} \ .$$

Therefore, the current through $\mathcal{E}_1$ is $i_1 = 2i = 0.67\,\text{A}$, flowing downward. The current through $\mathcal{E}_2$ is $0.33\,\text{A}$, flowing upward; the same holds for $\mathcal{E}_3$.

(b) $V_a - V_b = -iR_2 + \mathcal{E}_2 = -(0.333\,\text{A})(2.0\,\Omega) + 4.0\,\text{V} = 3.3\,\text{V}$.

34. The voltage difference across R is $V_R = \mathcal{E}R'/(R' + 2.00\,\Omega)$, where $R' = (5.00\,\Omega R)/(5.00\,\Omega + R)$. Thus,

$$\begin{aligned} P_R &= \frac{V_R^2}{R} = \frac{1}{R}\left(\frac{\mathcal{E}R'}{R' + 2.00\,\Omega}\right)^2 = \frac{1}{R}\left(\frac{\mathcal{E}}{1 + 2.00\,\Omega/R'}\right)^2 \\ &= \frac{\mathcal{E}^2}{R}\left[1 + \frac{(2.00\,\Omega)(5.00\,\Omega + R)}{(5.00\,\Omega)R}\right]^{-2} \equiv \frac{\mathcal{E}^2}{f(R)} \end{aligned}$$

where we use the equivalence symbol $\equiv$ to define the expression $f(R)$. To maximize P_R we need to minimize the expression $f(R)$. We set

$$\frac{df(R)}{dR} = -\frac{4.00\,\Omega^2}{R^2} + \frac{49}{25} = 0$$

to obtain $R = \sqrt{(4.00\,\Omega^2)(25)/49} = 1.43\,\Omega$.

35. (a) The copper wire and the aluminum sheath are connected in parallel, so the potential difference is the same for them. Since the potential difference is the product of the current and the resistance, $i_C R_C = i_A R_A$, where i_C is the current in the copper, i_A is the current in the aluminum, R_C is the resistance of the copper, and R_A is the resistance of the aluminum. The resistance of either component is given by $R = \rho L/A$, where ρ is the resistivity, L is the length, and A is the cross-sectional area. The resistance of the copper wire is $R_C = \rho_C L/\pi a^2$, and the resistance of the aluminum sheath is $R_A = \rho_A L/\pi(b^2 - a^2)$. We substitute these expressions into $i_C R_C = i_A R_A$, and cancel the common factors L and π to obtain

$$\frac{i_C \rho_C}{a^2} = \frac{i_A \rho_A}{b^2 - a^2} \ .$$

We solve this equation simultaneously with $i = i_C + i_A$, where i is the total current. We find

$$i_C = \frac{r_C^2 \rho_C i}{(r_A^2 - r_C^2)\rho_C + r_C^2 \rho_A}$$

and

$$i_A = \frac{(r_A^2 - r_C^2)\rho_C i}{(r_A^2 - r_C^2)\rho_C + r_C^2 \rho_A} \ .$$

The denominators are the same and each has the value

$$\begin{aligned}(b^2 - a^2)\rho_C + a^2\rho_A &= \left[(0.380 \times 10^{-3}\,\mathrm{m})^2 - (0.250 \times 10^{-3}\,\mathrm{m})^2\right](1.69 \times 10^{-8}\,\Omega{\cdot}\mathrm{m}) \\ &\qquad +(0.250 \times 10^{-3}\,\mathrm{m})^2(2.75 \times 10^{-8}\,\Omega{\cdot}\mathrm{m}) \\ &= 3.10 \times 10^{-15}\,\Omega{\cdot}\mathrm{m}^3 \ .\end{aligned}$$

Thus,

$$i_C = \frac{(0.250 \times 10^{-3}\,\mathrm{m})^2(2.75 \times 10^{-8}\Omega{\cdot}\mathrm{m})(2.00\,\mathrm{A})}{3.10 \times 10^{-15}\,\Omega{\cdot}\mathrm{m}^3} = 1.11 \ \mathrm{A}$$

and

$$\begin{aligned}i_A &= \frac{\left[(0.380 \times 10^{-3}\,\mathrm{m})^2 - (0.250 \times 10^{-3}\,\mathrm{m})^2\right](1.69 \times 10^{-8}\,\Omega{\cdot}\mathrm{m})(2.00\,\mathrm{A})}{3.10 \times 10^{-15}\,\Omega{\cdot}\mathrm{m}^3} \\ &= 0.893\,\mathrm{A} \ .\end{aligned}$$

(b) Consider the copper wire. If V is the potential difference, then the current is given by $V = i_C R_C = i_C \rho_C L/\pi a^2$, so

$$L = \frac{\pi a^2 V}{i_C \rho_C} = \frac{(\pi)(0.250 \times 10^{-3}\,\mathrm{m})^2(12.0\,\mathrm{V})}{(1.11\,\mathrm{A})(1.69 \times 10^{-8}\,\Omega{\cdot}\mathrm{m})} = 126\,\mathrm{m} \ .$$

36. (a) Since $i = \mathcal{E}/(r + R_{\text{ext}})$ and $i_{\max} = \mathcal{E}/r$, we have $R_{\text{ext}} = R(i_{\max}/i - 1)$ where $r = 1.50\,\mathrm{V}/1.00\,\mathrm{mA} = 1.50 \times 10^3\,\Omega$. Thus, $R_{\text{ext}} = (1.5 \times 10^3\,\Omega)(1/0.10 - 1) = 1.35 \times 10^4\,\Omega$;

(b) $R_{\text{ext}} = (1.5 \times 10^3\,\Omega)(1/0.50 - 1) = 1.50 \times 10^3\,\Omega$;

(c) $R_{\text{ext}} = (1.5 \times 10^3\,\Omega)(1/0.90 - 1) = 167\,\Omega$.

(d) Since $r = 20.0\,\Omega + R$, $R = 1.50 \times 10^3\,\Omega - 20.0\,\Omega = 1.48 \times 10^3\,\Omega$.

37. (a) The current in R_1 is given by

$$i_1 = \frac{\mathcal{E}}{R_1 + R_2R_3/(R_2 + R_3)} = \frac{5.0\,\mathrm{V}}{2.0\,\Omega + (4.0\,\Omega)(6.0\,\Omega)/(4.0\,\Omega + 6.0\,\Omega)} = 1.14\,\mathrm{A} \ .$$

Thus

$$i_3 = \frac{\mathcal{E} - V_1}{R_3} = \frac{\mathcal{E} - i_1R_1}{R_3} = \frac{5.0\,\mathrm{V} - (1.14\,\mathrm{A})(2.0\,\Omega)}{6.0\,\Omega} = 0.45\,\mathrm{A} \ .$$

(b) We simply interchange subscripts 1 and 3 in the equation above. Now

$$\begin{aligned}i_3 &= \frac{\mathcal{E}}{R_3 + (R_2R_1/(R_2 + R_1))} \\ &= \frac{5.0\,\mathrm{V}}{6.0\,\Omega + ((2.0\,\Omega)(4.0\,\Omega)/(2.0\,\Omega + 4.0\,\Omega))} \\ &= 0.6818 \ \mathrm{A}\end{aligned}$$

and

$$i_1 = \frac{5.0\,\mathrm{V} - (0.6818\,\mathrm{A})(6.0\,\Omega)}{2.0\,\Omega} = 0.45 \ \mathrm{A} \ ,$$

the same as before.

38. (a) $\mathcal{E} = V + ir = 12\,\mathrm{V} + (10\,\mathrm{A})(0.050\,\Omega) = 12.5\,\mathrm{V}$.

(b) Now $\mathcal{E} = V' + (i_{\text{motor}} + 8.0\,\text{A})r$, where $V' = i'_A R_{\text{light}} = (8.0\,\text{A})(12\,\text{V}/10\,\text{A}) = 9.6\,\text{V}$. Therefore,

$$i_{\text{motor}} = \frac{\mathcal{E} - V'}{r} - 8.0\,\text{A} = \frac{12.5\,\text{V} - 9.6\,\text{V}}{0.050\,\Omega} - 8.0\,\text{A} = 50\ \text{A}\ .$$

39. The current in R_2 is i. Let i_1 be the current in R_1 and take it to be downward. According to the junction rule the current in the voltmeter is $i - i_1$ and it is downward. We apply the loop rule to the left-hand loop to obtain

$$\mathcal{E} - iR_2 - i_1R_1 - ir = 0\ .$$

We apply the loop rule to the right-hand loop to obtain

$$i_1R_1 - (i - i_1)R_V = 0\ .$$

The second equation yields

$$i = \frac{R_1 + R_V}{R_V}\, i_1\ .$$

We substitute this into the first equation to obtain

$$\mathcal{E} - \frac{(R_2 + r)(R_1 + R_V)}{R_V}\, i_1 + R_1 i_1 = 0\ .$$

This has the solution

$$i_1 = \frac{\mathcal{E}R_V}{(R_2 + r)(R_1 + R_V) + R_1R_V}\ .$$

The reading on the voltmeter is

$$\begin{aligned} i_1R_1 &= \frac{\mathcal{E}R_VR_1}{(R_2 + r)(R_1 + R_V) + R_1R_V} \\ &= \frac{(3.0\,\text{V})(5.0 \times 10^3\,\Omega)(250\,\Omega)}{(300\,\Omega + 100\,\Omega)(250\,\Omega + 5.0 \times 10^3\,\Omega) + (250\,\Omega)(5.0 \times 10^3\,\Omega)} = 1.12\,\text{V}\ . \end{aligned}$$

The current in the absence of the voltmeter can be obtained by taking the limit as R_V becomes infinitely large. Then

$$i_1R_1 = \frac{\mathcal{E}R_1}{R_1 + R_2 + r} = \frac{(3.0\,\text{V})(250\,\Omega)}{250\,\Omega + 300\,\Omega + 100\,\Omega} = 1.15\,\text{V}\ .$$

The fractional error is $(1.12 - 1.15)/(1.15) = -0.030$, or -3.0%.

40. The currents in R and R_V are i and $i' - i$, respectively. Since $V = iR = (i' - i)R_V$ we have, by dividing both sides by V, $1 = (i'/V - i/V)R_V = (1/R' - 1/R)R_V$. Thus,

$$\frac{1}{R} = \frac{1}{R'} - \frac{1}{R_V}\ .$$

41. Let the current in the ammeter be i'. We have $V = i'(R + R_A)$, or $R = V/i' - R_A = R' - R_A$, where $R' = V/i'$ is the apparent reading of the resistance.

42. (a) In the first case

$$\begin{aligned} i' &= \frac{\mathcal{E}}{R_{\text{eq}}} = \frac{\mathcal{E}}{R_A + R_0 + R_VR/(R + R_V)} \\ &= \frac{12.0\,\text{V}}{3.00\,\Omega + 100\,\Omega + (300\,\Omega)(85.0\,\Omega)/(300\,\Omega + 85.0\,\Omega)} \\ &= 7.09 \times 10^{-2}\ \text{A}\ , \end{aligned}$$

and $V = \mathcal{E} - i'(R_A + R_0) = 12.0\,\mathrm{V} - (0.0709\,\mathrm{A})(103.00\,\Omega) = 4.70\,\mathrm{V}$. In the second case $V = \mathcal{E}R'/(R' + R_0)$, where

$$R' = \frac{R_V\,(R + R_A)}{R_V + R + R_A} = \frac{(300\,\Omega)(300\,\Omega + 85.0\,\Omega)}{300\,\Omega + 85.0\,\Omega + 3.00\,\Omega} = 68.0\,\Omega\ .$$

So $V = (12.0\,\mathrm{V})(68.0\,\Omega)/(68.0\,\Omega{+}100\,\Omega) = 4.86\,\mathrm{V}$, and $i' = V(R{+}R_A) = 4.86\,\mathrm{V}/(300\,\Omega{+}85.0\,\Omega) = 5.52 \times 10^{-2}\,\mathrm{A}$.

(b) In the first case $R' = V/i' = 4.70\,\mathrm{V}/(7.09 \times 10^{-2}\,\mathrm{A}) = 66.3\,\Omega$. In the second case $R' = V/i' = 4.86\,\mathrm{V}/(5.52 \times 10^{-2}\,\mathrm{A}) = 88.0\,\Omega$.

43. Let i_1 be the current in R_1 and R_2, and take it to be positive if it is toward point a in R_1. Let i_2 be the current in R_s and R_x, and take it to be positive if it is toward b in R_s. The loop rule yields $(R_1 + R_2)i_1 - (R_x + R_s)i_2 = 0$. Since points a and b are at the same potential, $i_1R_1 = i_2R_s$. The second equation gives $i_2 = i_1R_1/R_s$, which is substituted into the first equation to obtain

$$(R_1 + R_2)i_1 = (R_x + R_s)\,\frac{R_1}{R_s}\,i_1 \quad\Longrightarrow\quad R_x = \frac{R_2R_s}{R_1}\ .$$

44. (a) We use $q = q_0e^{-t/\tau}$, or $t = \tau\ln(q_0/q)$, where $\tau = RC$ is the capacitive time constant. Thus, $t_{1/3} = \tau\ln[q_0/(2q_0/3)] = \tau\ln(3/2) = 0.41\tau$.

(b) $t_{2/3} = \tau\ln[q_0/(q_0/3)] = \tau\ln 3 = 1.1\tau$.

45. During charging, the charge on the positive plate of the capacitor is given by

$$q = C\mathcal{E}(1 - e^{-t/\tau})\ ,$$

where C is the capacitance, $\mathcal{E}$ is applied emf, and $\tau = RC$ is the capacitive time constant. The equilibrium charge is $q_{\mathrm{eq}} = C\mathcal{E}$. We require $q = 0.99q_{\mathrm{eq}} = 0.99C\mathcal{E}$, so

$$0.99 = 1 - e^{-t/\tau}\ .$$

Thus,

$$e^{-t/\tau} = 0.01\ .$$

Taking the natural logarithm of both sides, we obtain $t/\tau = -\ln 0.01 = 4.6$ and $t = 4.6\tau$.

46. (a) $\tau = RC = (1.40 \times 10^6\,\Omega)(1.80 \times 10^{-6}\,\mathrm{F}) = 2.52\,\mathrm{s}$.

(b) $q_o = \mathcal{E}C = (12.0\,\mathrm{V})(1.80\,\mu\,\mathrm{F}) = 21.6\,\mu\mathrm{C}$.

(c) The time t satisfies $q = q_0(1 - e^{-t/RC})$, or

$$t = RC\ln\left(\frac{q_0}{q_0 - q}\right) = (2.52\,\mathrm{s})\ln\left(\frac{21.6\,\mu\mathrm{C}}{21.6\,\mu\mathrm{C} - 16.0\,\mu\mathrm{C}}\right) = 3.40\ \mathrm{s}\ .$$

47. (a) The voltage difference V across the capacitor varies with time as $V(t) = \mathcal{E}(1 - e^{-t/RC})$. At $t = 1.30\,\mu\,\mathrm{s}$ we have $V(t) = 5.00\,\mathrm{V}$, so $5.00\,\mathrm{V} = (12.0\,\mathrm{V})(1 - e^{-1.30\,\mu\mathrm{s}/RC})$, which gives $\tau = (1.30\,\mu\,\mathrm{s})/\ln(12/7) = 2.41\,\mu\mathrm{s}$.

(b) $C = \tau/R = 2.41\,\mu\mathrm{s}/15.0\,\mathrm{k}\Omega = 161\,\mathrm{pF}$.

48. The potential difference across the capacitor varies as a function of time t as $V(t) = V_0e^{-t/RC}$. Using $V = V_0/4$ at $t = 2.0\,\mathrm{s}$, we find

$$R = \frac{t}{C\ln(V_0/V)} = \frac{2.0\,\mathrm{s}}{(2.0 \times 10^{-6}\,\mathrm{F})\ln 4} = 7.2 \times 10^5\,\Omega\ .$$

49. (a) The charge on the positive plate of the capacitor is given by

$$q = C\mathcal{E}\left(1 - e^{-t/\tau}\right),$$

where $\mathcal{E}$ is the emf of the battery, C is the capacitance, and τ is the time constant. The value of τ is $\tau = RC = (3.00 \times 10^6\,\Omega)(1.00 \times 10^{-6}\,\mathrm{F}) = 3.00\,\mathrm{s}$. At $t = 1.00\,\mathrm{s}$, $t/\tau = (1.00\,\mathrm{s})/(3.00\,\mathrm{s}) = 0.333$ and the rate at which the charge is increasing is

$$\frac{dq}{dt} = \frac{C\mathcal{E}}{\tau}\,e^{-t/\tau} = \frac{(1.00 \times 10^{-6})(4.00\,\mathrm{V})}{3.00\,\mathrm{s}}\,e^{-0.333} = 9.55 \times 10^{-7}\,\mathrm{C/s}\,.$$

(b) The energy stored in the capacitor is given by

$$U_C = \frac{q^2}{2C}\,.$$

and its rate of change is

$$\frac{dU_C}{dt} = \frac{q}{C}\,\frac{dq}{dt}\,.$$

Now

$$q = C\mathcal{E}\left(1 - e^{-t/\tau}\right) = (1.00 \times 10^{-6})(4.00\,\mathrm{V})(1 - e^{-0.333}) = 1.13 \times 10^{-6}\,\mathrm{C}\,,$$

so

$$\frac{dU_C}{dt} = \left(\frac{1.13 \times 10^{-6}\,\mathrm{C}}{1.00 \times 10^{-6}\,\mathrm{F}}\right)(9.55 \times 10^{-7}\,\mathrm{C/s}) = 1.08 \times 10^{-6}\ \mathrm{W}\,.$$

(c) The rate at which energy is being dissipated in the resistor is given by $P = i^2R$. The current is $9.55 \times 10^{-7}\mathrm{A}$, so

$$P = (9.55 \times 10^{-7}\mathrm{A})^2(3.00 \times 10^6\,\Omega) = 2.74 \times 10^{-6}\ \mathrm{W}\,.$$

(d) The rate at which energy is delivered by the battery is

$$i\mathcal{E} = (9.55 \times 10^{-7}\mathrm{A})(4.00\,\mathrm{V}) = 3.82 \times 10^{-6}\,\mathrm{W}\,.$$

The energy delivered by the battery is either stored in the capacitor or dissipated in the resistor. Conservation of energy requires that $i\mathcal{E} = (q/C)\,(dq/dt) + i^2R$. Except for some round-off error the numerical results support the conservation principle.

50. (a) The charge q on the capacitor as a function of time is $q(t) = (\mathcal{E}C)(1 - e^{-t/RC})$, so the charging current is $i(t) = dq/dt = (\mathcal{E}/R)e^{-t/RC}$. The energy supplied by the emf is then

$$U = \int_0^\infty \mathcal{E}i\,dt = \frac{\mathcal{E}^2}{R}\int_0^\infty e^{-t/RC}dt = C\mathcal{E}^2 = 2U_C$$

where $U_C = \frac{1}{2}C\mathcal{E}^2$ is the energy stored in the capacitor.

(b) By directly integrating i^2R we obtain

$$U_R = \int_0^\infty i^2Rdt = \frac{\mathcal{E}^2}{R}\int_0^\infty e^{-2t/RC}\,dt = \frac{1}{2}C\mathcal{E}^2\,.$$

51. (a) The potential difference V across the plates of a capacitor is related to the charge q on the positive plate by $V = q/C$, where C is capacitance. Since the charge on a discharging capacitor is given by $q = q_0\,e^{-t/\tau}$, this means $V = V_0\,e^{-t/\tau}$ where V_0 is the initial potential difference. We solve for the time constant τ by dividing by V_0 and taking the natural logarithm:

$$\tau = -\frac{t}{\ln\left(V/V_0\right)} = -\frac{10.0\,\mathrm{s}}{\ln\left[(1.00\,\mathrm{V})/(100\,\mathrm{V})\right]} = 2.17\ \mathrm{s}\,.$$

(b) At $t = 17.0\,\mathrm{s}$, $t/\tau = (17.0\,\mathrm{s})/(2.17\,\mathrm{s}) = 7.83$, so

$$V = V_0\, e^{-t/\tau} = (100\,\mathrm{V})\, e^{-7.83} = 3.96 \times 10^{-2}\ \mathrm{V}\ .$$

52. The time it takes for the voltage difference across the capacitor to reach V_L is given by $V_L = \mathcal{E}(1 - e^{-t/RC})$. We solve for R:

$$R = \frac{t}{C \ln[\mathcal{E}/(\mathcal{E} - V_L)]} = \frac{0.500\,\mathrm{s}}{(0.150 \times 10^{-6}\,\mathrm{F}) \ln[95.0\,\mathrm{V}/(95.0\,\mathrm{V} - 72.0\,\mathrm{V})]} = 2.35 \times 10^{6}\,\Omega$$

where we used $t = 0.500\,\mathrm{s}$ given (implicitly) in the problem.

53. (a) The initial energy stored in a capacitor is given by

$$U_C = \frac{q_0^2}{2C}\ ,$$

where C is the capacitance and q_0 is the initial charge on one plate. Thus

$$q_0 = \sqrt{2CU_C} = \sqrt{2(1.0 \times 10^{-6}\,\mathrm{F})(0.50\,\mathrm{J})} = 1.0 \times 10^{-3}\ \mathrm{C}\ .$$

(b) The charge as a function of time is given by $q = q_0\, e^{-t/\tau}$, where τ is the capacitive time constant. The current is the derivative of the charge

$$i = -\frac{dq}{dt} = \frac{q_0}{\tau}\, e^{-t/\tau}\ ,$$

and the initial current is $i_0 = q_0/\tau$. The time constant is $\tau = RC = (1.0 \times 10^{-6}\,\mathrm{F})(1.0 \times 10^{6}\,\Omega) = 1.0\,\mathrm{s}$. Thus $i_0 = (1.0 \times 10^{-3}\,\mathrm{C})/(1.0\,\mathrm{s}) = 1.0 \times 10^{-3}\,\mathrm{A}$.

(c) We substitute $q = q_0\, e^{-t/\tau}$ into $V_C = q/C$ to obtain

$$V_C = \frac{q_0}{C}\, e^{-t/\tau} = \left(\frac{1.0 \times 10^{-3}\,\mathrm{C}}{1.0 \times 10^{-6}\,\mathrm{F}}\right) e^{-t/1.0\,\mathrm{s}} = (1.0 \times 10^{3}\,\mathrm{V})\, e^{-1.0t}\ ,$$

where t is measured in seconds. We substitute $i = (q_0/\tau)\, e^{-t/\tau}$ into $V_R = iR$ to obtain

$$V_R = \frac{q_0 R}{\tau}\, e^{-t/\tau} = \frac{(1.0 \times 10^{-3}\,\mathrm{C})(1.0 \times 10^{6}\,\Omega)}{1.0\,\mathrm{s}}\, e^{-t/1.0\,\mathrm{s}} = (1.0 \times 10^{3}\,\mathrm{V})\, e^{-1.0t}\ ,$$

where t is measured in seconds.

(d) We substitute $i = (q_0/\tau)\, e^{-t/\tau}$ into $P = i^2 R$ to obtain

$$P = \frac{q_0^2 R}{\tau^2}\, e^{-2t/\tau} = \frac{(1.0 \times 10^{-3}\,\mathrm{C})^2 (1.0 \times 10^{6}\,\Omega)}{(1.0\,\mathrm{s})^2}\, e^{-2t/1.0\,\mathrm{s}} = (1.0\,\mathrm{W})\, e^{-2.0t}\ ,$$

where t is again measured in seconds.

54. We use the result of problem 48: $R = t/[C \ln(V_0/V)]$. Then, for $t_{\min} = 10.0\,\mu\mathrm{s}$

$$R_{\min} = \frac{10.0\,\mu\mathrm{s}}{(0.220\,\mu\mathrm{F}) \ln(5.00/0.800)} = 24.8\,\Omega.$$

For $t_{\max} = 6.00\,\mathrm{ms}$,

$$R_{\max} = \left(\frac{6.00\,\mathrm{ms}}{10.0\,\mu\mathrm{s}}\right)(24.8\,\Omega) = 1.49 \times 10^{4}\,\Omega\ ,$$

where in the last equation we used $\tau = RC$.

55. (a) At $t = 0$ the capacitor is completely uncharged and the current in the capacitor branch is as it would be if the capacitor were replaced by a wire. Let i_1 be the current in R_1 and take it to be positive if it is to the right. Let i_2 be the current in R_2 and take it to be positive if it is downward. Let i_3 be the current in R_3 and take it to be positive if it is downward. The junction rule produces

$$i_1 = i_2 + i_3 ,$$

the loop rule applied to the left-hand loop produces

$$\mathcal{E} - i_1R_1 - i_2R_2 = 0 ,$$

and the loop rule applied to the right-hand loop produces

$$i_2R_2 - i_3R_3 = 0 .$$

Since the resistances are all the same we can simplify the mathematics by replacing R_1, R_2, and R_3 with R. The solution to the three simultaneous equations is

$$i_1 = \frac{2\mathcal{E}}{3R} = \frac{2(1.2 \times 10^3\,\mathrm{V})}{3(0.73 \times 10^6\,\Omega)} = 1.1 \times 10^{-3}\,\mathrm{A}$$

and

$$i_2 = i_3 = \frac{\mathcal{E}}{3R} = \frac{1.2 \times 10^3\,\mathrm{V}}{3(0.73 \times 10^6\,\Omega)} = 5.5 \times 10^{-4}\,\mathrm{A} .$$

At $t = \infty$ the capacitor is fully charged and the current in the capacitor branch is 0. Thus, $i_1 = i_2$, and the loop rule yields

$$\mathcal{E} - i_1R_1 - i_1R_2 = 0 .$$

The solution is

$$i_1 = i_2 = \frac{\mathcal{E}}{2R} = \frac{1.2 \times 10^3\,\mathrm{V}}{2(0.73 \times 10^6\,\Omega)} = 8.2 \times 10^{-4}\,\mathrm{A} .$$

(b) We take the upper plate of the capacitor to be positive. This is consistent with current flowing into that plate. The junction equation is $i_1 = i_2 + i_3$, and the loop equations are

$$\mathcal{E} - i_1R - i_2R = 0 \qquad \text{and} \qquad -\frac{q}{C} - i_3R + i_2R = 0 .$$

We use the first equation to substitute for i_1 in the second and obtain $\mathcal{E} - 2i_2R - i_3R = 0$. Thus $i_2 = (\mathcal{E} - i_3R)/2R$. We substitute this expression into the third equation above to obtain $-(q/C) - (i_3R) + (\mathcal{E}/2) - (i_3R/2) = 0$. Now we replace i_3 with dq/dt to obtain

$$\frac{3R}{2}\frac{dq}{dt} + \frac{q}{C} = \frac{\mathcal{E}}{2} .$$

This is just like the equation for an RC series circuit, except that the time constant is $\tau = 3RC/2$ and the impressed potential difference is $\mathcal{E}/2$. The solution is

$$q = \frac{C\mathcal{E}}{2}\left(1 - e^{-2t/3RC}\right) .$$

The current in the capacitor branch is

$$i_3 = \frac{dq}{dt} = \frac{\mathcal{E}}{3R}\,e^{-2t/3RC} .$$

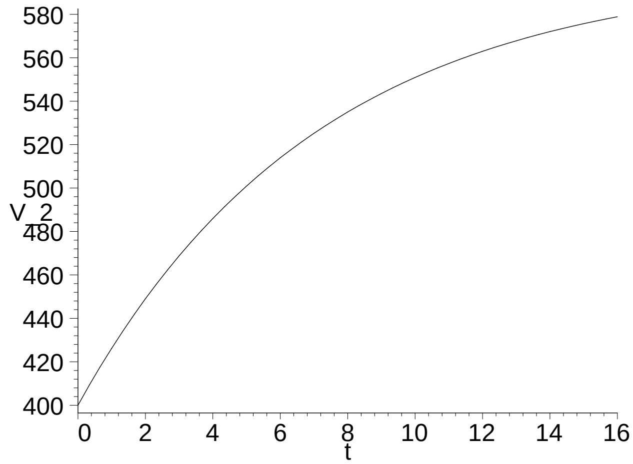
The current in the center branch is

$$\begin{aligned} i_2 &= \frac{\mathcal{E}}{2R} - \frac{i_3}{2} = \frac{\mathcal{E}}{2R} - \frac{\mathcal{E}}{6R}\,e^{-2t/3RC} \\ &= \frac{\mathcal{E}}{6R}\left(3 - e^{-2t/3RC}\right) \end{aligned}$$

and the potential difference across R_2 is

$$V_2 = i_2 R = \frac{\mathcal{E}}{6}\left(3 - e^{-2t/3RC}\right) .$$

This is shown in the following graph.



(c) For $t = 0$, $e^{-2t/3RC}$ is 1 and $V_R = \mathcal{E}/3 = (1.2 \times 10^3\,\mathrm{V})/3 = 400\,\mathrm{V}$. For $t = \infty$, $e^{-2t/3RC}$ is 0 and $V_R = \mathcal{E}/2 = (1.2 \times 20^3\,\mathrm{V})/2 = 600\,\mathrm{V}$.

(d) After "a long time" means after several time constants. Then, the current in the capacitor branch is very small and can be approximated by 0.

56. (a) We found in part (e) of problem 45 in Chapter 27 that the magnitude of the electric field is $E = 16$ V/m. Taking this to be roughly constant over the small distance ($\ell = 0.50$ m) involved here, then we approximate the potential difference between the man's feet as

$$\Delta V \approx E\ell = 8\ \mathrm{V} .$$

(b) The voltage found in part (a) drives a current i through the two feet (each represented by $R_f = 300\,\Omega$) and the torso (represented by $R_t = 1000\,\Omega$). Thus,

$$i = \frac{\Delta V}{2R_f + R_t} = \frac{8\,\mathrm{V}}{2(300\,\Omega) + 1000\,\Omega}$$

which yields $i \approx 5$ mA.

(c) Our value for i is far less than the stated 100 mA minimum required to put the heart into fibrillation.

57. (a) The four tires act as resistors in parallel, with an equivalent value given by

$$\frac{1}{R_{\mathrm{eq}}} = \sum_{n=1}^{4} \frac{1}{R_{\mathrm{tire}}} = \frac{4}{R_{\mathrm{tire}}} \implies R_{\mathrm{eq}} = \frac{R_{\mathrm{tire}}}{4} .$$

Using the stated values ($C = 5.0 \times 10^{-10}\,\mathrm{F}$ and $10^8\,\Omega < R_{\mathrm{tire}} < 10^{11}\,\Omega$) we find the capacitive time constant $\tau = R_{\mathrm{eq}}C$ in the range $0.012\,\mathrm{s} < \tau < 13\,\mathrm{s}$.

(b) Eq. 26-22 leads to

$$U_0 = \frac{1}{2}CV^2 = \frac{1}{2}\left(5.00 \times 10^{-10}\,\mathrm{F}\right)\left(30.0 \times 10^3\,\mathrm{V}\right)^2 = 0.225\ \mathrm{J} .$$

(c) As demonstrated in Sample Problem 28-5, the energy "decays" exponentially according to

$$U = U_0 e^{-2t/\tau} .$$

Solving for the time which gives $U = 0.050$ J, we find

$$t = \frac{\tau}{2} \ln\left(\frac{U_0}{U}\right) = \frac{\tau}{2} \ln\left(\frac{0.225}{0.050}\right)$$

which yields, for the range of time constants found in part (a), values of t in the range $0.094\,\text{s} < t < 9.4\,\text{s}$. To obtain these particular values, we used 3-figure versions of the part (a) results ($0.0125\,\text{s} < \tau < 12.5\,\text{s}$).

(d) The lower range of resistance leads to the smaller times to discharge, which is the more desirable situation. Based on this criterion, low resistance tires are favored.

(e) There are a variety of ways to safely and quickly ground a large charged object. A large metal cable connected to, say, the (metal) building frame and held at the end of, say, a long lucite rod might be used (to touch a part of the car that does not have much paint or grease on it) to make the car safe to handle.

58. (a) In the process described in the problem, no charge is gained or lost. Thus, $q =$ constant. Hence,

$$q = C_1 V_1 = C_2 V_2 \implies V_2 = V_1 \frac{C_1}{C_2} = (200)\left(\frac{150}{10}\right) = 3000 \text{ V} .$$

(b) Eq. 28-36, with $\tau = RC$, describes not only the discharging of q but also of V. Thus,

$$V = V_0 e^{-t/\tau} \implies t = RC \ln\left(\frac{V_0}{V}\right) = \left(300 \times 10^9\,\Omega\right)\left(10 \times 10^{-12}\,\text{F}\right) \ln\left(\frac{3000}{100}\right)$$

which yields $t = 10$ s. This is a longer time than most people are inclined to wait before going on to their next task (such as handling the sensitive electronic equipment).

(c) We solve $V = V_0 e^{-t/RC}$ for R with the new values $V_0 = 1400$ V and $t = 0.30$ s. Thus,

$$R = \frac{t}{C \ln(V_0/V)} = \frac{0.30\,\text{s}}{(10 \times 10^{-12}\,\text{F}) \ln(1400/100)} = 1.1 \times 10^{10}\ \Omega .$$

59. (a) Since $R_{\text{tank}} = 140\,\Omega$, $i = 12\,\text{V}/(10\,\Omega + 140\,\Omega) = 8.0 \times 10^{-2}\,\text{A}$.

(b) Now, $R_{\text{tank}} = (140\,\Omega + 20\,\Omega)/2 = 80\,\Omega$, so $i = 12\,\text{V}/(10\,\Omega + 80\,\Omega) = 0.13\,\text{A}$.

(c) When full, $R_{\text{tank}} = 20\,\Omega$ so $i = 12\,\text{V}/(10\,\Omega + 20\,\Omega) = 0.40\,\text{A}$.

60. (a) The magnitude of the current density vector is

$$\begin{aligned} J_A = J_B &= \frac{i}{A} = \frac{V}{(R_1 + R_2)A} = \frac{4\,\text{V}}{(R_1 + R_2)\pi D^2} \\ &= \frac{4(60.0\,\text{V})}{\pi(0.127\,\Omega + 0.729\,\Omega)(2.60 \times 10^{-3}\,\text{m})^2} \\ &= 1.32 \times 10^7\ \text{A/m}^2 . \end{aligned}$$

(b) $V_A = V R_1/(R_1 + R_2) = (60.0\,\text{V})(0.127\,\Omega)/(0.127\,\Omega + 0.729\,\Omega) = 8.90\,\text{V}$, and $V_B = V - V_A = 60.0\,\text{V} - 8.9\,\text{V} = 51.1\,\text{V}$.

(c) Calculate the resistivity from $R = \rho L/A$ for both materials: $\rho_A = R_A A/L_A = \pi R_A D^2/4L_A = \pi(0.127\,\Omega)(2.60 \times 10^{-3}\,\text{m})^2/[4(40.0\,\text{m})] = 1.69 \times 10^{-8}\,\Omega\cdot\text{m}$. So A is made of copper. Similarly we find $\rho_B = 9.68 \times 10^{-8}\,\Omega\cdot\text{m}$, so B is made of iron.

61. We denote silicon with subscript s and iron with i. Let $T_0 = 20°$. If

$$\begin{aligned} R(T) &= R_s(T) + R_i(T) = R_s(T_0)[1 + \alpha(T - T_0)] + R_i(T_0)[1 + \alpha_i(T - T_0)] \\ &= (R_s(T)_0\alpha_s + R_i(T_0)\alpha_i) + (\text{temperature independentterms}) \end{aligned}$$

is to be temperature-independent, we must require that $R_s(T_0)\alpha_s + R_i(T_0)\alpha_i = 0$. Also note that $R_s(T_0) + R_i(T_0) = R = 1000\,\Omega$. We solve for $R_s(T_0)$ and $R_i(T_0)$ to obtain

$$R_s(T_0) = \frac{R\alpha_i}{\alpha_i - \alpha_s} = \frac{(1000\,\Omega)(6.5 \times 10^{-3})}{6.5 \times 10^{-3} + 70 \times 10^{-3}} = 85.0\ \Omega\ ,$$

and $R_i(T_0) = 1000\,\Omega - 85.0\,\Omega = 915\,\Omega$.

62. The potential difference across R_2 is

$$V_2 = iR_2 = \frac{\mathcal{E}\,R_2}{R_1 + R_2 + R_3} = \frac{(12\,\text{V})(4.0\,\Omega)}{3.0\,\Omega + 4.0\,\Omega + 5.0\,\Omega} = 4.0\ \text{V}\ .$$

63. Since $R_{\text{eq}} < R$, the two resistors ($R = 12.0\,\Omega$ and R_x) must be connected in parallel:

$$R_{\text{eq}} = 3.00\,\Omega = \frac{R_x R}{R + R_x} = \frac{R_x(12.0\,\Omega)}{12.0\,\Omega + R_x}\ .$$

We solve for R_x: $R_x = R_{\text{eq}}R/(R - R_{\text{eq}}) = (3.00\,\Omega)(12.0\,\Omega)/(12.0\,\Omega - 3.00\,\Omega) = 4.00\,\Omega$.

64. Consider the lowest branch with the two resistors $R_1 = 3.0\,\Omega$ and $R_2 = 5.0\,\Omega$. The voltage difference across the $5.0\,\Omega$ resistor is

$$V = i_2R_2 = \frac{\mathcal{E}\,R_2}{R_1 + R_2} = \frac{(120\,\text{V})(5.0\,\Omega)}{3.0\,\Omega + 5.0\,\Omega} = 7.5\ \text{V}\ .$$

65. When all the batteries are connected in parallel, each supplies a current i; thus, $i_R = Ni$. Then from $\mathcal{E} = ir + i_RR = ir + Nir$, we get $i_R = N\mathcal{E}/[(N+1)r]$. When all the batteries are connected in series, $i_r = i_R$ and $\mathcal{E}_{\text{total}} = N\mathcal{E} = Ni_rr + i_RR = Ni_Rr + i_Rr$, so $i_R = N\mathcal{E}/[(N+1)r]$.

66. (a) They are in parallel and the portions of A and B between the load and their respective sliding contacts have the same potential difference. It is clearly important not to "short" the system (particularly if the load turns out to have very little resistance) by having the sliding contacts too close to the load-ends of A and B to start with. Thus, we suggest putting the contacts roughly in the middle of each. Since $R_1 > R_2$, larger currents generally go through B (depending on the position of the sliding contact) than through A. Therefore, B is analogous to a "coarse" control, as A is to a "fine control." Hence, we recommend adjusting the current roughly with B, and then making fine adjustments with A.

    (b) Relatively large percentage changes in A cause only small percentage charges in the resistance of the parallel combination, thus permitting fine adjustment; any change in A causes half as much change in this combination.

67. When connected in series, the rate at which electric energy dissipates is $P_s = \mathcal{E}^2/(R_1 + R_2)$. When connected in parallel, the corresponding rate is $P_p = \mathcal{E}^2(R_1 + R_2)/R_1R_2$. Letting $P_p/P_s = 5$, we get $(R_1 + R_2)^2/R_1R_2 = 5$, where $R_1 = 100\,\Omega$. We solve for R_2: $R_2 = 38\,\Omega$ or $260\,\Omega$.

68. (a) Placing a wire (of resistance r) with current i running directly from point a to point b in Fig. 28-41 divides the top of the picture into a left and a right triangle. If we label the currents through each resistor with the corresponding subscripts (for instance, i_s goes toward the lower right through R_s and i_x goes toward the upper right through R_x), then the currents must be related as follows:

$$\begin{aligned} i_0 &= i_1 + i_s \qquad &&\text{and} \qquad i_1 = i + i_2 \\ i_s + i &= i_x \qquad &&\text{and} \qquad i_2 + i_x = i_0 \end{aligned}$$

where the last relation is not independent of the previous three. The loop equations for the two triangles and also for the bottom loop (containing the battery and point b) lead to

$$\begin{aligned} i_s R_s - i_1 R_1 - ir &= 0 \\ i_2 R_2 - i_x R_x - ir &= 0 \\ \mathcal{E} - i_0 R_0 - i_s R_s - i_x R_x &= 0 \ . \end{aligned}$$

We incorporate the current relations from above into these loop equations in order to obtain three well-posed "simultaneous" equations, for three unknown currents (i_s, i_1 and i):

$$\begin{aligned} i_s R_s - i_1 R_1 - ir &= 0 \\ i_1 R_2 - i_s R_x - i\,(r + R_x + R_2) &= 0 \\ \mathcal{E} - i_s\,(R_0 + R_s + R_x) - i_1 R_0 - iR_x &= 0 \end{aligned}$$

The problem statement further specifies $R_1 = R_2 = R$ and $R_0 = 0$, which causes our solution for i to simplify significantly. It becomes

$$i = \frac{\mathcal{E}\,(R_s - R_x)}{2rR_s + 2R_x R_s + R_s R + 2rR_x + R_x R}$$

which is equivalent to the result shown in the problem statement.

(b) Examining the numerator of our final result in part (a), we see that the condition for $i = 0$ is $R_s = R_x$. Since $R_1 = R_2 = R$, this is equivalent to $R_x = R_s R_2 / R_1$, consistent with the result of Problem 43.

69. The voltage across the rightmost resistors is $V_{12} = (1.4 \text{ A})(8.0\ \Omega + 4.0\ \Omega) = 16.8$ V, which is equal to V_{16} (the voltage across the 16 Ω resistor, which has current equal to $V_{16}/(16\ \Omega) = 1.05$ A). By the junction rule, the current in the rightmost 2.0 Ω resistor is $1.05 + 1.4 = 2.45$ A, so its voltage is $V_2 = (2.0\ \Omega)(2.45 \text{ A}) = 4.9$ V. The loop rule tells us the voltage across the 2.0 Ω resistor (the one going "downward" in the figure) is $V_2' = V_2 + V_{16} = 21.7$ V (implying that the current through it is $i_2' = V_2'/(2.0\ \Omega) = 10.85$ A). The junction rule now gives the current in the leftmost 2.0 Ω resistor as $10.85 + 2.45 = 13.3$ A, implying that the voltage across it is $V_2'' = (13.3 \text{ A})(2.0\ \Omega) = 26.6$ V. Therefore, by the loop rule, $\mathcal{E} = V_2'' + V_2' = 48.3$ V.

70. In the steady state situation, the capacitor voltage will equal the voltage across the 15 kΩ resistor:

$$V_0 = (15 \text{ k}\Omega)\left(\frac{20 \text{ V}}{10 \text{ k}\Omega + 15 \text{ k}\Omega}\right) = 12 \text{ V} \ .$$

Now, multiplying Eq. 28-36 by the capacitance leads to $V = V_0 e^{-t/RC}$ describing the voltage across the capacitor (and across the $R = 15$ kΩ resistor) after the switch is opened (at $t = 0$). Thus, with $t = 0.00400$ s, we obtain

$$V = (12)e^{-0.004/(15000)(0.4\times10^{-6})} = 6.16 \text{ V} \ .$$

Therefore, using Ohm's law, the current through the 15 kΩ resistor is $6.16/15000 = 4.11 \times 10^{-4}$ A.

71. (a) By symmetry, we see that i_1 is half the current that goes through the battery. The battery current is found by dividing $\mathcal{E}$ by the equivalent resistance of the circuit, which is easily found to be 6.0 Ω. Thus,

$$i_1 = \frac{1}{2}\,i_{\text{bat}} = \frac{1}{2}\,\frac{12 \text{ V}}{6.0\ \Omega} = 1.0 \text{ A}$$

and is clearly downward (in the figure).

(b) We use Eq. 28-14: $P = i_{\text{bat}}\mathcal{E} = 24$ W.

72. The series pair of 2.0 Ω resistors on the right reduce to $R' = 4.0\ \Omega$, and the parallel pair of identical 4.0 Ω resistors on the left reduce to $R'' = 2.0\ \Omega$. The voltage across R' must equal that across R''; thus,

$$\begin{aligned} V' &= V'' \\ i'R' &= i''R'' \\ i' &= \frac{1}{2}i'' \end{aligned}$$

where in the last step we divide by R' and simplify. This relation, plus the junction rule condition $6.0\ \text{A} = i' + i''$ leads to the solution $i'' = 4.0$ A. It is clear by symmetry that $i = \frac{1}{2}i''$, so we conclude $i = 2.0$ A.

73. (a) We reduce the parallel pair of identical 2.0 Ω resistors (on the right side) to $R' = 1.0\ \Omega$, and we reduce the series pair of identical 2.0 Ω resistors (on the upper left side) to $R'' = 4.0\ \Omega$. With R denoting the 2.0 Ω resistor at the bottom (between V_2 and V_1), we now have three resistors in series which are equivalent to

$$R + R' + R'' = 7.0\ \Omega$$

across which the voltage is 7.0 V (by the loop rule, this is 12 V − 5.0 V), implying that the current is 1.0 A (clockwise). Thus, the voltage across R' is (1.0 A)(1.0 Ω) = 1.0 V, which means that (examining the right side of the circuit) the voltage difference between *ground* and V_1 is $12 - 1 = 11$ V. Noting the orientation of the battery, we conclude $V_1 = -11$ V.

(b) The voltage across R'' is (1.0 A)(4.0 Ω) = 4.0 V, which means that (examining the left side of the circuit) the voltage difference between *ground* and V_2 is $5.0 + 4.0 = 9.0$ V. Noting the orientation of the battery, we conclude $V_2 = -9.0$ V. This can be verified by considering the voltage across R and the value we obtained for V_1.

74. (a) From symmetry we see that the current through the top set of batteries (i) is the same as the current through the second set. This implies that the current through the $R = 4.0\,\Omega$ resistor at the bottom is $i_R = 2i$. Thus, with r denoting the internal resistance of each battery (equal to 4.0 Ω) and $\mathcal{E}$ denoting the 20 V emf, we consider one loop equation (the outer loop), proceeding counterclockwise:

$$3\,(\mathcal{E} - ir) - (2i)R = 0\ .$$

This yields $i = 3.0$ A. Consequently, $i_R = 6.0$ A.

(b) The terminal voltage of each battery is $\mathcal{E} - ir = 8.0$ V.

(c) Using Eq. 28-14, we obtain $P = i\mathcal{E} = (3)(20) = 60$ W.

(d) Using Eq. 27-22, we have $P = i^2 r = 36$ W.

75. (a) The work done by the battery relates to the potential energy change:

$$q\Delta V = eV = e(12\,\text{V}) = 12\,\text{eV} = (12\,\text{eV})(1.6\times10^{-19}\,\text{J/eV}) = 1.9\times10^{-18}\ \text{J}\ .$$

(b) $P = iV = neV = (3.4\times10^{18}/\text{s})(1.6\times10^{-19}\,\text{C})(12\,\text{V}) = 6.5\,\text{W}$.

76. (a) We denote $L = 10\,\text{km}$ and $\alpha = 13\,\Omega/\text{km}$. Measured from the east end we have $R_1 = 100\,\Omega = 2\alpha(L - x) + R$, and measured from the west end $R_2 = 200\,\Omega = 2\alpha x + R$. Thus,

$$x = \frac{R_2 - R_1}{4\alpha} + \frac{L}{2} = \frac{200\,\Omega - 100\,\Omega}{4(13\,\Omega/\text{km})} + \frac{10\,\text{km}}{2} = 6.9\ \text{km}\ .$$

(b) Also, we obtain

$$R = \frac{R_1 + R_2}{2} - \alpha L = \frac{100\,\Omega + 200\,\Omega}{2} - (13\,\Omega/\text{km})(10\,\text{km}) = 20\ \Omega\ .$$

77. (a) From $P = V^2/R$ we find $V = \sqrt{PR} = \sqrt{(10\,\text{W})(0.10\,\Omega)} = 1.0\,\text{V}$.

(b) From $i = V/R = (\mathcal{E} - V)/r$ we find

$$r = R\left(\frac{\mathcal{E} - V}{V}\right) = (0.10\,\Omega)\left(\frac{1.5\,\text{V} - 1.0\,\text{V}}{1.0\,\text{V}}\right) = 0.050\ \Omega\ .$$

78. (a) The power delivered by the motor is $P = (2.00\,\text{V})(0.500\,\text{m/s}) = 1.00\,\text{W}$. From $P = i^2 R_{\text{motor}}$ and $\mathcal{E} = i(r + R_{\text{motor}})$ we then find $i^2 r - i\mathcal{E} + P = 0$ (which also follows directly from the conservation of energy principle). We solve for i:

$$i = \frac{\mathcal{E} \pm \sqrt{\mathcal{E}^2 - 4rP}}{2r} = \frac{2.00\,\text{V} \pm \sqrt{(2.00\,\text{V})^2 - 4(0.500\,\Omega)(1.00\,\text{W})}}{2(0.500\,\Omega)}\ .$$

The answer is either $3.41\,\text{A}$ or $0.586\,\text{A}$.

(b) We use $V = \mathcal{E} - ir = 2.00\,\text{V} - i(0.500\,\Omega)$. We substitute the two values of i obtained in part (a) into the above formula to get $V = 0.293\,\text{V}$ or $1.71\,\text{V}$.

(c) The power P delivered by the motor is the same for either solution. Since $P = iV$ we may have a lower i and higher V or, alternatively, a lower V and higher i. One can check that the two sets of solutions for i and V above do yield the same power $P = iV$.

79. Let the power supplied be P_s and that dissipated be P_d. Since $P_d = i^2 R$ and $i = P_s/\mathcal{E}$, we have $P_d = P_s^2/\mathcal{E}^2 R \propto \mathcal{E}^{-2}$. The ratio is then

$$\frac{P_d(\mathcal{E} = 110,000\,\text{V})}{P_d(\mathcal{E} = 110\,\text{V})} = \left(\frac{110\,\text{V}}{110,000\,\text{V}}\right)^2 = 1.0 \times 10^{-6}\ .$$

80. (a) $R_{\text{eq}}(AB) = 20.0\,\Omega/3 = 6.67\,\Omega$ (three $20.0\,\Omega$ resistors in parallel).

(b) $R_{\text{eq}}(AC) = 20.0\,\Omega/3 = 6.67\,\Omega$ (three $20.0\,\Omega$ resistors in parallel).

(c) $R_{\text{eq}}(BC) = 0$ (as B and C are connected by a conducting wire).

81. The maximum power output is $(120\,\text{V})(15\,\text{A}) = 1800\,\text{W}$. Since $1800\,\text{W}/500\,\text{W} = 3.6$, the maximum number of 500 W lamps allowed is 3.

82. The part of R_0 connected in parallel with R is given by $R_1 = R_0 x/L$, where $L = 10\,\text{cm}$. The voltage difference across R is then $V_R = \mathcal{E}R'/R_{\text{eq}}$, where $R' = RR_1/(R + R_1)$ and $R_{\text{eq}} = R_0(1 - x/L) + R'$. Thus

$$P_R = \frac{V_R^2}{R} = \frac{1}{R}\left(\frac{\mathcal{E}RR_1/(R + R_1)}{R_0(1 - x/L) + RR_1/(R + R_1)}\right)^2\ .$$

Algebraic manipulation then leads to

$$P_R = \frac{100R(\mathcal{E}x/R_0)^2}{(100R/R_0 + 10x - x^2)^2}$$

where x is measured in cm.

83. (a) Since $P = \mathcal{E}^2/R_{\text{eq}}$, the higher the power rating the smaller the value of R_{eq}. To achieve this, we can let the low position connect to the larger resistance (R_1), middle position connect to the smaller resistance (R_2), and the high position connect to both of them in parallel.

(b) For $P = 100\,\text{W}$, $R_{\text{eq}} = R_1 = \mathcal{E}^2/P = (120\,\text{V})^2/100\,\text{W} = 144\,\Omega$; for $P = 300\,\text{W}$, $R_{\text{eq}} = R_1R_2/(R_1 + R_2) = (144\,\Omega)R_2/(144\,\Omega + R_2) = (120\,\text{V})^2/300\,\text{W}$. We obtain $R_2 = 72\,\Omega$.

84. Note that there is no voltage drop across the ammeter. Thus, the currents in the bottom resistors are the same, which we call i (so the current through the battery is $2i$ and the voltage drop across each of the bottom resistors is iR). The resistor network can be reduced to an equivalence of

$$R_{\text{eq}} = \frac{(2R)(R)}{2R+R} + \frac{(R)(R)}{R+R} = \frac{7}{6}R$$

which means that we can determine the current through the battery (and also through each of the bottom resistors):

$$2i = \frac{\mathcal{E}}{R_{\text{eq}}} \implies i = \frac{3\mathcal{E}}{7R} .$$

By the loop rule (going around the left loop, which includes the battery, resistor $2R$ and one of the bottom resistors), we have

$$\mathcal{E} - i_{2R}(2R) - iR = 0 \implies i_{2R} = \frac{\mathcal{E} - iR}{2R} .$$

Substituting $i = 3\mathcal{E}/7R$, this gives $i_{2R} = 2\mathcal{E}/7R$. The difference between i_{2R} and i is the current through the ammeter. Thus,

$$i_{\text{ammeter}} = i - i_{2R} = \frac{3\mathcal{E}}{7R} - \frac{2\mathcal{E}}{7R} = \frac{\mathcal{E}}{7R} .$$

85. The current in the ammeter is given by $i_A = \mathcal{E}/(r + R_1 + R_2 + R_A)$. The current in R_1 and R_2 without the ammeter is $i = \mathcal{E}/(r + R_1 + R_2)$. The percent error is then

$$\begin{aligned} \frac{\Delta i}{i} &= \frac{i - i_A}{i} = 1 - \frac{r + R_1 + R_2}{r + R_1 + R_2 + R_A} = \frac{R_A}{r + R_1 + R_2 + R_A} \\ &= \frac{0.10\,\Omega}{2.0\,\Omega + 5.0\,\Omega + 4.0\,\Omega + 0.10\,\Omega} = 0.90\% . \end{aligned}$$

86. When S is open for a long time, the charge on C is $q_i = \mathcal{E}_2 C$. When S is closed for a long time, the current i in R_1 and R_2 is $i = (\mathcal{E}_2 - \mathcal{E}_1)/(R_1 + R_2) = (3.0\,\text{V} - 1.0\,\text{V})/(0.20\,\Omega + 0.40\,\Omega) = 3.33\,\text{A}$. The voltage difference V across the capacitor is then $V = \mathcal{E}_2 - iR_2 = 3.0\,\text{V} - (3.33\,\text{A})(0.40\,\Omega) = 1.67\,\text{V}$. Thus the final charge on C is $q_f = VC$. So the change in the charge on the capacitor is $\Delta q = q_f - q_i = (V - \mathcal{E}_2)C = (1.67\,\text{V} - 3.0\,\text{V})(10\,\mu\text{F}) = -13\,\mu\text{C}$.

87. Requiring no current through the $10.0\,\Omega$ resistor means that 20.0 V will be across R (which has current i_R). The current through the $20.0\,\Omega$ resistor is also i_R, so the loop rule leads to

$$50.0\,\text{V} - 20.0\,\text{V} - i_R\,(20.0\,\Omega) = 0$$

which yields $i_R = 1.5$ A. Therefore,

$$R = \frac{20.0\,\text{V}}{i_R} = 13.3\ \Omega .$$

88. (a) The capacitor is *initially* uncharged, which implies (by the loop rule) that there is zero voltage (at $t = 0$) across the 10 kΩ resistor, and that 30 V is across the 20 kΩ resistor. Therefore, by Ohm's law, $i_{10} = 0$,

(b) and $i_{20} = (30\,\text{V})/(20\ \text{k}\Omega) = 1.5 \times 10^{-3}$ A.

(c) As $t \to \infty$ the current to the capacitor reduces to zero and the 20 kΩ and 10 kΩ resistors behave more like a series pair (having the same current), equivalent to 30 kΩ. The current through them, then, at long times, is $i = (30\,\text{V})/(30\ \text{k}\Omega) = 1.0 \times 10^{-3}$ A.

89. (a) The six resistors to the left of $\mathcal{E}_1 = 16$ V battery can be reduced to a single resistor $R = 8.0\ \Omega$, through which the current must be $i_R = \mathcal{E}_1/R = 2.0$ A. Now, by the loop rule, the current through the 3.0 Ω and 1.0 Ω resistors at the upper right corner is

$$i' = \frac{16.0\,\text{V} - 8.0\,\text{V}}{3.0\ \Omega + 1.0\ \Omega} = 2.0\ \text{A}$$

in a direction that is "backward" relative to the $\mathcal{E}_2 = 8.0$ V battery. Thus, by the junction rule,

$$i_1 = i_R + i' = 4.0\ \text{A}$$

and is upward (that is, in the "forward" direction relative to $\mathcal{E}_1$).

(b) The current i_2 derives from a succession of symmetric splittings of i_R (reversing the procedure of reducing those six resistors to find R in part (a)). We find

$$i_2 = \frac{1}{2}\left(\frac{1}{2}\, i_R\right) = 0.50\ \text{A}$$

and is clearly downward.

(c) Using our conclusions from part (a) in Eq. 28-14, we obtain $P = i_1\mathcal{E}_1 = (4)(16) = 64$ W supplied.

(d) Using results calculated in part (a) in Eq. 28-14, we obtain $P = i'\mathcal{E}_2 = (2)(8) = 16$ W absorbed.

90. We reduce the parallel pair of identical 4.0 Ω resistors to $R' = 2.0\ \Omega$, which has current $i = 2i_1$ going through it. It is in series with a 2.0 Ω resistor, which leads to an equivalence of $R = 4.0\ \Omega$ with current i. We find a path (for use with the loop rule) that goes through this R, the 4.0 V battery, and the 20 V battery, and proceed counterclockwise (assuming i goes rightward through R):

$$20\,\text{V} + 4.0\,\text{V} - iR = 0$$

which leads to $i = 6.0$ A. Consequently, $i_1 = \frac{1}{2}i = 3.0$ A going rightward.

91. With the unit Ω understood, the equivalent resistance for this circuit is

$$R_{\text{eq}} = \frac{20R + 100}{R + 10}\ .$$

Therefore, the power supplied by the battery (equal to the power dissipated in the resistors) is

$$P = \frac{V^2}{R} = V^2\,\frac{R + 10}{20R + 100}$$

where $V = 12$ V. We attempt to extremize the expression by working through the $dP/dR = 0$ condition and do not find a value of R that satisfies it. We note, then, that the function is a monotonically decreasing function of R, with $R = 0$ giving the maximum possible value (since $R < 0$ values are not being allowed). With the value $R = 0$, we obtain $P = 14.4$ W.

92. The resistor by the letter i is above three other resistors; together, these four resistors are equivalent to a resistor $R = 10\ \Omega$ (with current i). As if we were presented with a maze, we find a path through R that passes through any number of batteries (10, it turns out) but no other resistors, which – as in any good maze – winds "all over the place." Some of the ten batteries are opposing each other (particularly the ones along the outside), so that their net emf is only $\mathcal{E} = 40$ V. The current through R is then $i = \mathcal{E}/R = 4.0$ A, and is directed upward in the figure.

93. (First problem of **Cluster**)

(a) R_2 and R_3 are in parallel; their equivalence is in series with R_1. Therefore,

$$R_{\text{eq}} = R_1 + \frac{R_2\,R_3}{R_2 + R_3} = 300\ \Omega\ .$$

(b) The current through the battery is $\mathcal{E}/R_{\text{eq}} = 0.0200$ A, which is also the current through R_1. Hence, the voltage across R_1 is $V_1 = (0.0200 \text{ A})(100\ \Omega) = 2.00$ V.

(c) From the loop rule,

$$\mathcal{E} - V_1 - i_3 R_3 = 0$$

which yields $i_3 = 6.67 \times 10^{-3}$ A.

94. (Second problem of **Cluster**)

(a) The loop rule (proceeding counterclockwise around the right loop) leads to $\mathcal{E}_2 - i_1 R_1 = 0$ (where i_1 was assumed downward). This yields $i_1 = 0.060$ A (downward).

(b) The loop rule (counterclockwise around the left loop) gives

$$(+\mathcal{E}_1) + (+i_1 R_1) + (-i_3 R_3) = 0$$

where i_3 has been assumed leftward. This yields $i_3 = 0.180$ A (leftward).

(c) The junction rule tells us that the current through the 12 V battery is $0.180 + 0.060 = 0.240$ A upward.

95. (Third problem of **Cluster**)

(a) Using the junction rule ($i_1 = i_2 + i_3$) we write two loop rule equations:

$$\begin{aligned} \mathcal{E}_1 - i_2 R_2 - (i_2 + i_3) R_1 &= 0 \\ \mathcal{E}_2 - i_3 R_3 - (i_2 + i_3) R_1 &= 0 \ . \end{aligned}$$

Solving, we find $i_2 = 0.0109$ A (rightward, as was assumed in writing the equations as we did), $i_3 = 0.0273$ A (leftward), and $i_1 = i_2 + i_3 = 0.0382$ A (downward).

(b) See the results in part (a).

(c) See the results in part (a).

(d) The voltage across R_1 equals V_A: $(0.0382 \text{ A})(100\ \Omega) = +3.82$ V.

96. (Fourth problem of **Cluster**)

(a) The symmetry of the problem allows us to use i_2 as the current in *both* of the R_2 resistors and i_1 for the R_1 resistors. We see from the junction rule that $i_3 = i_1 - i_2$. There are only two independent loop rule equations:

$$\begin{aligned} \mathcal{E} - i_2 R_2 - i_1 R_1 &= 0 \\ \mathcal{E} - 2 i_1 R_1 - (i_1 - i_2) R_3 &= 0 \ . \end{aligned}$$

where in the latter equation, a zigzag path through the bridge has been taken. Solving, we find $i_1 = 0.002625$ A , $i_2 = 0.00225$ A and $i_3 = i_1 - i_2 = 0.000375$ A. Therefore, $V_A - V_B = i_1 R_1 = 5.25$ V.

(b) It follows also that $V_B - V_C = i_3 R_3 = 1.50$ V.

(c) We find $V_C - V_D = i_1 R_1 = 5.25$ V.

(d) Finally, $V_A - V_C = i_2 R_2 = 6.75$ V.