Chapter 45

1. Using Table 45-1, the difference in mass between the muon and the pion is

$$\Delta m = \left(139.6\,\frac{\text{MeV}}{c^2} - 105.7\,\frac{\text{MeV}}{c^2}\right) = \frac{(33.9\,\text{MeV})(1.60\times10^{-13}\,\text{J/MeV})}{(2.998\times10^{8}\,\text{m/s})^2} = 6.03\times10^{-29}\ \text{kg}\ .$$

2. We establish a ratio, using Eq. 22-4 and Eq. 14-1:

$$\begin{aligned}\frac{F_{\text{gravity}}}{F_{\text{electric}}} &= \frac{Gm_e^2/r^2}{ke^2/r^2} = \frac{4\pi\varepsilon_0 G m_e^2}{e^2} \\ &= \frac{(6.67\times10^{-11}\,\text{N}\cdot\text{m}^2/\text{C}^2)(9.11\times10^{-31}\,\text{kg})^2}{(9.0\times10^{9}\,\text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\,\text{C})^2} \\ &= 2.4\times10^{-43}\ .\end{aligned}$$

Since $F_{\text{gravity}} \ll F_{\text{electric}}$, we can neglect the gravitational force acting between particles in a bubble chamber.

3. Conservation of momentum requires that the gamma ray particles move in opposite directions with momenta of the same magnitude. Since the magnitude p of the momentum of a gamma ray particle is related to its energy by $p = E/c$, the particles have the same energy E. Conservation of energy yields $m_\pi c^2 = 2E$, where m_π is the mass of a neutral pion. According to Table 45-4, the rest energy of a neutral pion is $m_\pi c^2 = 135.0\,\text{MeV}$. Hence, $E = (135.0\,\text{MeV})/2 = 67.5\,\text{MeV}$. We use the result of Exercise 3 of Chapter 39 to obtain the wavelength of the gamma rays:

$$\lambda = \frac{1240\,\text{eV}\cdot\text{nm}}{67.5\times10^{6}\,\text{eV}} = 1.84\times10^{-5}\,\text{nm} = 18.4\ \text{fm}\ .$$

4. By charge conservation, it is clear that reversing the sign of the pion means we must reverse the sign of the muon. In effect, we are replacing the charged particles by their antiparticles. Less obvious is the fact that we should now put a "bar" over the neutrino (something we should also have done for some of the reactions and decays discussed in the previous two chapters, except that we had not yet learned about antiparticles). To understand the "bar" we refer the reader to the discussion in §45-4. The decay of the negative pion is $\pi^- \to \mu^- + \bar{\nu}$. A subscript can be added to the antineutrino to clarify what "type" it is, as discussed in §45-4.

5. The energy released would be twice the rest energy of Earth, or $E = 2mc^2 = 2(5.98\times10^{24}\,\text{kg})(2.998\times10^{8}\,\text{m/s})^2 = 1.08\times10^{42}\,\text{J}$. The mass of Earth can be found in Appendix C.

6. Since the density of water is $\rho = 1000\,\text{kg/m}^3 = 1\,\text{kg/L}$, then the total mass of the pool is $\rho\mathcal{V} = 4.32\times10^{5}\,\text{kg}$, where $\mathcal{V}$ is the given volume. Now, the fraction of that mass made up by the protons is 10/18 (by counting the protons versus total nucleons in a water molecule). Consequently, if we ignore

the effects of neutron decay (neutrons can beta decay into protons) in the interest of making an order-of-magnitude calculation, then the number of particles susceptible to decay via this $T_{1/2} = 10^{32}$ y half-life is

$$N = \frac{\frac{10}{18} M_{\text{pool}}}{m_p} = \frac{\frac{10}{18}(4.32 \times 10^5\,\text{kg})}{1.67 \times 10^{-27}\,\text{kg}} = 1.44 \times 10^{32} .$$

Using Eq. 43-19, we obtain

$$R = \frac{N \ln 2}{T_{1/2}} = \frac{\left(1.44 \times 10^{32}\right) \ln 2}{10^{32}\,\text{y}} \approx 1 \text{ decay/y} .$$

7. From Eq. 38-45, the Lorentz factor would be

$$\gamma = \frac{E}{mc^2} = \frac{1.5 \times 10^6\,\text{eV}}{20\,\text{eV}} = 75000 .$$

Solving Eq. 38-8 for the speed, we find

$$\gamma = \frac{1}{\sqrt{1 - (v/c)^2}} \implies v = c\sqrt{1 - \frac{1}{\gamma^2}}$$

which implies that the difference between v and c is

$$c - v = c\left(1 - \sqrt{1 - \frac{1}{\gamma^2}}\right) \approx c\left(1 - \left(1 - \frac{1}{2\gamma^2} + \cdots\right)\right)$$

where we use the binomial expansion (see Appendix E) in the last step. Therefore,

$$c - v \approx c\left(\frac{1}{2\gamma^2}\right) = (299792458\,\text{m/s})\left(\frac{1}{2(75000)^2}\right) = 0.0266 \text{ m/s} .$$

8. From Eq. 38-49, the Lorentz factor is

$$\gamma = 1 + \frac{K}{mc^2} = 1 + \frac{80\,\text{MeV}}{135\,\text{MeV}} = 1.59 .$$

Solving Eq. 38-8 for the speed, we find

$$\gamma = \frac{1}{\sqrt{1 - (v/c)^2}} \implies v = c\sqrt{1 - \frac{1}{\gamma^2}}$$

which yields $v = 0.778c$ or $v = 2.33 \times 10^8$ m/s. Now, in the reference frame of the laboratory, the lifetime of the pion is not the given τ value but is "dilated." Using Eq. 38-9, the time in the lab is

$$t = \gamma\tau = (1.59)\left(8.3 \times 10^{-17}\,\text{s}\right) = 1.3 \times 10^{-16} \text{ s} .$$

Finally, using Eq. 38-10, we find the distance in the lab to be

$$x = vt = \left(2.33 \times 10^8\,\text{m/s}\right)\left(1.3 \times 10^{-16}\,\text{s}\right) = 3.1 \times 10^{-8} \text{ m} .$$

9. Table 45-4 gives the rest energy of each pion as 139.6 MeV. The magnitude of the momentum of each pion is $p_\pi = (358.3\,\text{MeV})/c$. We use the relativistic relationship between energy and momentum (Eq. 38-52) to find the total energy of each pion:

$$E_\pi = \sqrt{(p_\pi c)^2 + (m_\pi c^2)^2} = \sqrt{(358.3\,\text{MeV})^2 + (139.6\,\text{MeV})^2} = 384.5 \text{ MeV} .$$

Conservation of energy yields $m_\rho c^2 = 2E_\pi = 2(384.5\,\text{MeV}) = 769\,\text{MeV}$.

10. (a) In SI units, $K = (2200\,\text{MeV})(1.6 \times 10^{-13}\,\text{J/MeV}) = 3.52 \times 10^{-10}\,\text{J}$. Similarly, $mc^2 = 2.85 \times 10^{-10}\,\text{J}$ for the positive tau. Eq. 38-51 leads to the relativistic momentum:

$$p = \frac{1}{c}\sqrt{K^2 + 2Kmc^2} = \frac{1}{2.998 \times 10^8}\sqrt{\left(3.52 \times 10^{-10}\right)^2 + 2\left(3.52 \times 10^{-10}\right)\left(2.85 \times 10^{-10}\right)}$$

which yields $p = 1.90 \times 10^{-18}\,\text{kg}\cdot\text{m/s}$.

(b) According to problem 46 in Chapter 38, the radius should be calculated with the relativistic momentum:

$$r = \frac{\gamma mv}{|q|B} = \frac{p}{eB}$$

where we use the fact that the positive tau has charge $e = 1.6 \times 10^{-19}\,\text{C}$. With $B = 1.20\,\text{T}$, this yields $r = 9.9\,\text{m}$.

11. (a) Conservation of energy gives $Q = K_2 + K_3 = E_1 - E_2 - E_3$ where E refers here to the *rest* energies (mc^2) instead of the total energies of the particles. Writing this as $K_2 + E_2 - E_1 = -(K_3 + E_3)$ and squaring both sides yields

$$K_2^2 + 2K_2E_2 - 2K_2E_1 + (E_1 - E_2)^2 = K_3^2 + 2K_3E_3 + E_3^2 \ .$$

Next, conservation of linear momentum (in a reference frame where particle 1 was at rest) gives $|p_2| = |p_3|$ (which implies $(p_2c)^2 = (p_3c)^2$). Therefore, Eq. 38-51 leads to

$$K_2^2 + 2K_2E_2 = K_3^2 + 2K_3E_3$$

which we subtract from the above expression to obtain

$$-2K_2E_1 + (E_1 - E_2)^2 = E_3^2 \ .$$

This is now straightforward to solve for K_2 and yields the result stated in the problem.

(b) Setting $E_3 = 0$ in

$$K_2 = \frac{1}{2E_1}\left[(E_1 - E_2)^2 - E_3^2\right]$$

and using the rest energy values given in Table 45-1 readily gives the same result for K_μ as computed in Sample Problem 45-1.

12. (a) Eq. 45-14 conserves charge since both the proton and the positron have $q = +e$ (and the neutrino is uncharged).

(b) Energy conservation is not violated since $m_pc^2 > m_ec^2 + m_\nu c^2$.

(c) We are free to view the decay from the rest frame of the proton. Both the positron and the neutrino are able to carry momentum, and so long as they travel in opposite directions with appropriate values of p (so that $\sum \vec{p} = 0$) then linear momentum is conserved.

(d) If we examine the spin angular momenta, there does seem to be a violation of angular momentum conservation (Eq. 45-14 shows a spin-one-half particle decaying into two spin-one-half particles).

13. (a) The conservation laws considered so far are associated with energy, momentum, angular momentum, charge, baryon number, and the three lepton numbers. The rest energy of the muon is 105.7 MeV, the rest energy of the electron is 0.511 MeV, and the rest energy of the neutrino is zero. Thus, the total rest energy before the decay is greater than the total rest energy after. The excess energy can be carried away as the kinetic energies of the decay products and energy can be conserved. Momentum is conserved if the electron and neutrino move away from the decay in opposite directions with equal magnitudes of momenta. Since the orbital angular momentum is zero, we consider only spin angular momentum. All the particles have spin $\hbar/2$. The total angular momentum after the decay must be either $\hbar$ (if the spins are aligned) or zero (if the spins are antialigned). Since the spin before the

decay is $\hbar/2$, angular momentum cannot be conserved. The muon has charge $-e$, the electron has charge $-e$, and the neutrino has charge zero, so the total charge before the decay is $-e$ and the total charge after is $-e$. Charge is conserved. All particles have baryon number zero, so baryon number is conserved. The muon lepton number of the muon is $+1$, the muon lepton number of the muon neutrino is $+1$, and the muon lepton number of the electron is 0. Muon lepton number is conserved. The electron lepton numbers of the muon and muon neutrino are 0 and the electron lepton number of the electron is $+1$. Electron lepton number is not conserved. The laws of conservation of angular momentum and electron lepton number are not obeyed and this decay does not occur..

(b) We analyze the decay in the same way. We find that only charge is not conserved.

(c) Here we find that energy and muon lepton number cannot be conserved.

14. (a) Noting that there are two positive pions created (so, in effect, its decay products are doubled), then we count up the electrons, positrons and neutrinos: $2\mathrm{e}^+ + \mathrm{e}^- + 5\nu + 4\bar{\nu}$.

(b) The final products are all leptons, so the baryon number of A_2^+ is zero. Both the pion and rho meson have integer-valued spins, so A_2^+ is a meson (and a boson).

15. For purposes of deducing the properties of the antineutron, one may cancel a proton from each side of the reaction and write the equivalent reaction as

$$\pi^+ \to \mathrm{p} + \overline{\mathrm{n}} \ .$$

Particle properties can be found in Tables 45-3 and 45-4. The pion and proton each have charge $+e$, so the antineutron must be neutral. The pion has baryon number zero (it is a meson) and the proton has baryon number $+1$, so the baryon number of the antineutron must be -1. The pion and the proton each have strangeness zero, so the strangeness of the antineutron must also be zero. In summary, $q = 0$, $B = -1$, and $S = 0$ for the antineutron.

16. (a) Referring to Tables 45-3 and 45-4, we find the strangeness of K^0 is $+1$, while it is zero for both π^+ and π^-. Consequently, strangeness is not conserved in this decay; $\mathrm{K}^0 \to \pi^+ + \pi^-$ does not proceed via the strong interaction.

(b) The strangeness of each side is -1, which implies that the decay is governed by the strong interaction.

(c) The strangeness or Λ^0 is -1 while that of $\mathrm{p}+\pi^-$ is zero, so the decay is not via the strong interaction.

(d) The strangeness of each side is -1; it proceeds via the strong interaction.

17. (a) See the solution to Exercise 13 for the quantities to be considered, adding strangeness to the list. The lambda has a rest energy of 1115.6 MeV, the proton has a rest energy of 938.3 MeV, and the kaon has a rest energy of 493.7 MeV. The rest energy before the decay is less than the total rest energy after, so energy cannot be conserved. Momentum can be conserved. The lambda and proton each have spin $\hbar/2$ and the kaon has spin zero, so angular momentum can be conserved. The lambda has charge zero, the proton has charge $+e$, and the kaon has charge $-e$, so charge is conserved. The lambda and proton each have baryon number $+1$, and the kaon has baryon number zero, so baryon number is conserved. The lambda and kaon each have strangeness -1 and the proton has strangeness zero, so strangeness is conserved. Only energy cannot be conserved.

(b) The omega has a rest energy of 1680 MeV, the sigma has a rest energy of 1197.3 MeV, and the pion has a rest energy of 135 MeV. The rest energy before the decay is greater than the total rest energy after, so energy can be conserved. Momentum can be conserved. The omega and sigma each have spin $\hbar/2$ and the pion has spin zero, so angular momentum can be conserved. The omega has charge $-e$, the sigma has charge $-e$, and the pion has charge zero, so charge is conserved. The omega and sigma have baryon number $+1$ and the pion has baryon number 0, so baryon number is conserved. The omega has strangeness -3, the sigma has strangeness -1, and the pion has strangeness zero, so strangeness is not conserved.

(c) The kaon and proton can bring kinetic energy to the reaction, so energy can be conserved even though the total rest energy after the collision is greater than the total rest energy before. Momentum can be conserved. The proton and lambda each have spin $\hbar/2$ and the kaon and pion each have spin zero, so angular momentum can be conserved. The kaon has charge $-e$, the proton has charge $+e$, the lambda has charge zero, and the pion has charge $+e$, so charge is not conserved. The proton and lambda each have baryon number $+1$, and the kaon and pion each have baryon number zero; baryon number is conserved. The kaon has strangeness -1, the proton and pion each have strangeness zero, and the lambda has strangeness -1, so strangeness is conserved. Only charge is not conserved.

18. (a) From Eq. 38-47,

$$\begin{aligned} Q &= -\Delta m\, c^2 = (m_{\Sigma^+} + m_{K^+} - m_{\pi^+} - m_p)c^2 \\ &= 1189.4\,\text{MeV} + 493.7\,\text{MeV} - 139.6\,\text{MeV} - 938.3\,\text{MeV} \\ &= 605\ \text{MeV}\ . \end{aligned}$$

(b) Similarly,

$$\begin{aligned} Q &= -\Delta m\, c^2 = (m_{\Lambda^0} + m_{\pi^0} - m_{K^-} - m_p)c^2 \\ &= 1115.6\,\text{MeV} + 135.0\,\text{MeV} - 493.7\,\text{MeV} - 938.3\,\text{MeV} \\ &= -181\ \text{MeV}\ . \end{aligned}$$

19. Conservation of energy (see Eq. 38-44) leads to

$$\begin{aligned} K_f &= -\Delta m\, c^2 + K_i = (m_{\Sigma^-} - m_{\pi^-} - m_n)c^2 + K_i \\ &= 1197.3\,\text{MeV} - 139.6\,\text{MeV} - 939.6\,\text{MeV} + 220\,\text{MeV} \\ &= 338\ \text{MeV}\ . \end{aligned}$$

20. The formula for T_z as it is usually written to include strange baryons is $T_z = q - (S + B)/2$. Also, we interpret the symbol q in the T_z formula in terms of elementary charge units; this is how q is listed in Table 45-3. In terms of charge q as we have used it in previous chapters, the formula is $T_z = \frac{q}{e} - \frac{1}{2}(B+S)$. For instance, $T_z = +\frac{1}{2}$ for the proton (and the neutral Xi) and $T_z = -\frac{1}{2}$ for the neutron (and the negative Xi). The baryon number B is $+1$ for all the particles in Fig. 45-4(a). Rather than use a sloping axis as in Fig. 45-4 (there it is done for the q values), one reproduces (if one uses the "corrected" formula for T_z mentioned above) exactly the same pattern using regular rectangular axes (T_z values along the horizontal axis and Y values along the vertical) with the neutral lambda and sigma particles situated at the origin.

21. (a) As far as the conservation laws are concerned, we may cancel a proton from each side of the reaction equation and write the reaction as $\text{p} \to \Lambda^0 + x$. Since the proton and the lambda each have a spin angular momentum of $\hbar/2$, the spin angular momentum of x must be either zero or $\hbar$. Since the proton has charge $+e$ and the lambda is neutral, x must have charge $+e$. Since the proton and the lambda each have a baryon number of $+1$, the baryon number of x is zero. Since the strangeness of the proton is zero and the strangeness of the lambda is -1, the strangeness of x is $+1$. We take the unknown particle to be a spin zero meson with a charge of $+e$ and a strangeness of $+1$. Look at Table 45-4 to identify it as a K^+ particle.

(b) Similar analysis tells us that x is a spin-$\frac{1}{2}$ antibaryon ($B = -1$) with charge and strangeness both zero. Inspection of Table 45-3 reveals it is an antineutron.

(c) Here x is a spin-0 (or spin-1) meson with charge zero and strangeness -1. According to Table 45-4, it could be a $\overline{K}^0$ particle.

22. (a) From Eq. 38-47,

$$\begin{aligned} Q &= -\Delta m\, c^2 = (m_{\Lambda^0} - m_p - m_{\pi^-})c^2 \\ &= 1115.6\,\text{MeV} - 938.3\,\text{MeV} - 139.6\,\text{MeV} = 37.7\ \text{MeV}\ . \end{aligned}$$

(b) We use the formula obtained in problem 11 (where it should be emphasized that E is used to mean the rest energy, not the total energy):

$$\begin{aligned} K_p &= \frac{1}{2E_\Lambda}\left[(E_\Lambda - E_p)^2 - E_\pi^2\right] \\ &= \frac{(1115.6\,\text{MeV} - 938.3\,\text{MeV})^2 - (139.6\,\text{MeV})^2}{2(1115.6\,\text{MeV})} = 5.35\ \text{MeV}\ . \end{aligned}$$

(c) By conservation of energy,

$$K_{\pi^-} = Q - K_p = 37.7\,\text{MeV} - 5.35\,\text{MeV} = 32.4\ \text{MeV}\ .$$

23. (a) We indicate the antiparticle nature of each quark with a “bar” over it. Thus, $\bar{\text{u}}\bar{\text{u}}\bar{\text{d}}$ represents an antiproton.

(b) Similarly, $\bar{\text{u}}\bar{\text{d}}\bar{\text{d}}$ represents an antineutron.

24. (a) The combination ddu has a total charge of $(-\frac{1}{3} - \frac{1}{3} + \frac{2}{3}) = 0$, and a total strangeness of zero. From Table 45-3, we find it to be a neutron (n).

(b) For the combination uus, we have $Q = +\frac{2}{3} + \frac{2}{3} - \frac{1}{3} = 1$ and $S = 0 + 0 - 1 = 1$. This is the Σ^+ particle.

(c) For the quark composition ssd, we have $Q = -\frac{1}{3} - \frac{1}{3} - \frac{1}{3} = -1$ and $S = -1 - 1 + 0 = -2$. This is a Ξ^-.

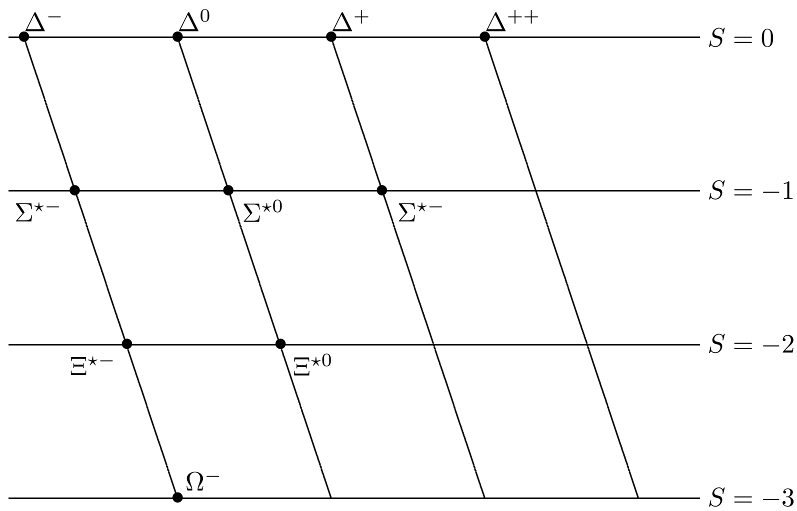
25. (a) Looking at the first three lines of Table 45-5, since the particle is a baryon, we determine that it must consist of three quarks. To obtain a strangeness of -2, two of them must be s quarks. Each of these has a charge of $-e/3$, so the sum of their charges is $-2e/3$. To obtain a total charge of e, the charge on the third quark must be $5e/3$. There is no quark with this charge, so the particle cannot be constructed. In fact, such a particle has never been observed.

(b) Again the particle consists of three quarks (and no antiquarks). To obtain a strangeness of zero, none of them may be s quarks. We must find a combination of three u and d quarks with a total charge of $2e$. The only such combination consists of three u quarks.

26. (a) Using Table 45-3, we find $q = 0$ and $S = -1$ for this particle (also, $B = 1$, since that is true for all particles in that table). From Table 45-5, we see it must therefore contain a strange quark (which has charge $-1/3$), so the other two quarks must have charges to add to zero. Assuming the others are among the lighter quarks (none of them being an antiquark, since $B = 1$), then the quark composition is $\bar{\text{u}}\bar{\text{s}}\bar{\text{d}}$.

(b) The reasoning is very similar to that of part (a). The main difference is that this particle must have two strange quarks. Its quark combination turns out to be $\bar{\text{u}}\bar{\text{s}}\bar{\text{s}}$.

27. If we were to use regular rectangular axes, then this would appear as a right triangle. Using the sloping q axis as the problem suggests, it is similar to an “upside down” equilateral triangle as we show below. The leftmost slanted line is for the -1 charge, and the rightmost slanted line is for the $+2$ charge.



28. Since only the strange quark (s) has non-zero strangeness, in order to obtain $S = -1$ we need to combine s with some non-strange antiquark (which would have the negative of the quantum numbers listed in Table 45-5). The difficulty is that the charge of the strange quark is $-1/3$, which means that (to obtain a total charge of $+1$) the antiquark would have to have a charge of $+\frac{4}{3}$. Clearly, there are no such antiquarks in our list. Thus, a meson with $S = -1$ and $q = +1$ cannot be formed with the quarks/antiquarks of Table 45-5. Similarly, one can show that, since no quark has $q = -\frac{4}{3}$, there cannot be a meson with $S = +1$ and $q = -1$.

29. From $\gamma = 1 + K/mc^2$ (see Eq. 38-49) and $v = \beta c = c\sqrt{1-\gamma^{-2}}$ (see Eq. 38-8), we get

$$v = c\sqrt{1 - \left(1 + \frac{K}{mc^2}\right)^{-2}} \ .$$

Therefore, for the Σ^{*0} particle,

$$v = (2.9979 \times 10^8\,\text{m/s})\sqrt{1 - \left(1 + \frac{1000\,\text{MeV}}{1385\,\text{MeV}}\right)^{-2}} = 2.4406 \times 10^8 \text{ m/s} \ ,$$

and for Σ^0,

$$v' = (2.9979 \times 10^8\,\text{m/s})\sqrt{1 - \left(1 + \frac{1000\,\text{MeV}}{1192.5\,\text{MeV}}\right)^{-2}} = 2.5157 \times 10^8 \text{ m/s} \ .$$

Thus Σ^0 moves faster than Σ^{*0} by

$$\Delta v = v' - v = (2.5157 - 2.4406)(10^8\,\text{m/s}) = 7.51 \times 10^6 \text{ m/s} \ .$$

30. Letting $v = Hr = c$, we obtain

$$r = \frac{c}{H} = \frac{3.0 \times 10^8\,\text{m/s}}{0.0193\,\text{m/s}\cdot\text{ly}} = 1.6 \times 10^{10} \text{ ly} \ .$$

31. We apply Eq. 38-33 for the Doppler shift in wavelength:

$$\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$$

where v is the recessional speed of the galaxy. We use Hubble's law to find the recessional speed: $v = Hr$, where r is the distance to the galaxy and H is the Hubble constant $(19.3 \times 10^{-3} \frac{\text{m}}{\text{s·ly}})$. Thus, $v = [19.3 \times 10^{-3} \frac{\text{m}}{\text{s·ly}}](2.40 \times 10^8 \text{ ly}) = 4.63 \times 10^6 \text{ m/s}$ and

$$\Delta\lambda = \frac{v}{c}\lambda = \left(\frac{4.63 \times 10^6 \text{ m/s}}{3.00 \times 10^8 \text{ m/s}}\right)(656.3 \text{ nm}) = 10.1 \text{ nm} .$$

Since the galaxy is receding, the observed wavelength is longer than the wavelength in the rest frame of the galaxy. Its value is $656.3 \text{ nm} + 10.1 \text{ nm} = 666.4 \text{ nm}$.

32. First, we find the speed of the receding galaxy from Eq. 38-30:

$$\begin{aligned}\beta &= \frac{1-(f/f_0)^2}{1+(f/f_0)^2} = \frac{1-(\lambda_0/\lambda)^2}{1+(\lambda_0/\lambda)^2} \\ &= \frac{1-(590.0 \text{ nm}/602.0 \text{ nm})^2}{1+(590.0 \text{ nm}/602.0 \text{ nm})^2} = 0.02013\end{aligned}$$

where we use $f = c/\lambda$ and $f_0 = c/\lambda_0$. Then from Eq. 45-19,

$$r = \frac{v}{H} = \frac{\beta c}{H} = \frac{(0.02013)(2.998 \times 10^8 \text{ m/s})}{19.3 \text{ mm/s·ly}} = 3.13 \times 10^8 \text{ ly} .$$

(Note: if one uses the classical Doppler shift formula instead of the relativistic version in Eq. 38-30, one obtains $r = 31.7 \times 10^8$ ly, which is reasonably close to the value we obtained above. This is to be expected since $\beta \approx 0.02 \ll 1$.)

33. (a) Letting $v(r) = Hr \le v_e = \sqrt{2GM/r}$, we get $M/r^3 \ge H^2/2G$. Thus,

$$\rho = \frac{M}{4\pi r^3/3} = \frac{3}{4\pi}\frac{M}{r^3} \ge \frac{3H^2}{8\pi G} .$$

(b) The density being expressed in H-atoms/m^3 is equivalent to expressing it in terms of $\rho_0 = m_{\text{H}}/\text{m}^3 = 1.67 \times 10^{-27} \text{ kg/m}^3$. Thus,

$$\begin{aligned}\rho &= \frac{3H^2}{8\pi G\rho_0}\left(\text{H atoms/m}^3\right) = \frac{3(0.0193 \text{ m/s·ly})^2(1.00 \text{ ly}/9.460 \times 10^{15} \text{ m})^2(\text{H atoms/m}^3)}{8\pi(6.67 \times 10^{-11} \text{ m}^3/\text{kg·s}^2)(1.67 \times 10^{-27} \text{ kg/m}^3)} \\ &= 4.5 \text{ H atoms/m}^3 .\end{aligned}$$

34. (a) From $f = c/\lambda$ and Eq. 38-30, we get

$$\lambda_0 = \lambda\sqrt{\frac{1-\beta}{1+\beta}} = (\lambda_0 + \Delta\lambda)\sqrt{\frac{1-\beta}{1+\beta}} .$$

Dividing both sides by λ_0 leads to

$$1 = (1+z)\sqrt{\frac{1-\beta}{1+\beta}} .$$

We solve for β:

$$\beta = \frac{(1+z)^2 - 1}{(1+z)^2 + 1} = \frac{z^2 + 2z}{z^2 + 2z + 2} .$$

(b) Now $z = 4.43$, so

$$\beta = \frac{(4.43)^2 + 2(4.43)}{(4.43)^2 + 2(4.43) + 2} = 0.934 .$$

(c) From Eq. 45-19,

$$r = \frac{v}{H} = \frac{\beta c}{H} = \frac{(0.943)(3.0 \times 10^8\,\text{m/s})}{0.0193\,\text{m/s}\cdot\text{ly}} = 1.5 \times 10^{10}\ \text{ly}\ .$$

35. (a) From Eq. 41-29, we know that $N_2/N_1 = e^{-\Delta E/kT}$. We solve for ΔE:

$$\begin{aligned}\Delta E &= kT\,\ln\frac{N_1}{N_2} = (8.62 \times 10^{-5}\,\text{eV/K})(2.7\,\text{K})\,\ln\left(\frac{1-0.25}{0.25}\right) \\ &= 2.56 \times 10^{-4}\,\text{eV} = 256\,\mu\text{eV}\ .\end{aligned}$$

(b) Using the result of problem 3 in Chapter 39,

$$\lambda = \frac{hc}{\Delta E} = \frac{1240\,\text{eV}\cdot\text{nm}}{2.56 \times 10^{-4}\,\text{eV}} = 4.84 \times 10^6\,\text{nm} = 4.84\ \text{mm}\ .$$

36. From $F_{\text{grav}} = GMm/r^2 = mv^2/r$ we find $M \propto v^2$. Thus, the mass of the Sun would be

$$M_s' = \left(\frac{v_{\text{Mercury}}}{v_{\text{Pluto}}}\right)^2 M_s = \left(\frac{47.9\,\text{km/s}}{4.74\,\text{km/s}}\right)^2 M_s = 102 M_s\ .$$

37. (a) The mass M within Earth's orbit is used to calculate the gravitational force on Earth. If r is the radius of the orbit, R is the radius of the new Sun, and M_S is the mass of the Sun, then

$$M = \left(\frac{r}{R}\right)^3 M_S = \left(\frac{1.50 \times 10^{11}\,\text{m}}{5.90 \times 10^{12}\,\text{m}}\right)^3 (1.99 \times 10^{30}\,\text{kg}) = 3.27 \times 10^{25}\,\text{kg}\ .$$

The gravitational force on Earth is given by GMm/r^2, where m is the mass of Earth and G is the universal gravitational constant. Since the centripetal acceleration is given by v^2/r, where v is the speed of Earth, $GMm/r^2 = mv^2/r$ and

$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{(6.67 \times 10^{-11}\,\text{m}^3/\text{s}^2\cdot\text{kg})(3.27 \times 10^{25}\,\text{kg})}{1.50 \times 10^{11}\,\text{m}}} = 1.21 \times 10^2\ \text{m/s}\ .$$

(b) The period of revolution is

$$T = \frac{2\pi r}{v} = \frac{2\pi(1.50 \times 10^{11}\,\text{m})}{1.21 \times 10^2\,\text{m/s}} = 7.82 \times 10^9\,\text{s} = 248\ \text{y}\ .$$

38. (a) The mass of the portion of the galaxy within the radius r from its center is given by $M' = (r/R)^3 M$. Thus, from $GM'm/r^2 = mv^2/r$ (where m is the mass of the star) we get

$$v = \sqrt{\frac{GM'}{r}} = \sqrt{\frac{GM}{r}\left(\frac{r}{R}\right)^3} = r\sqrt{\frac{GM}{R^3}}\ .$$

(b) In the case where $M' = M$, we have

$$T = \frac{2\pi r}{v} = 2\pi r\sqrt{\frac{r}{GM}} = \frac{2\pi r^{3/2}}{\sqrt{GM}}\ .$$

39. (a) We substitute $\lambda = (2898\,\mu\text{m}\cdot\text{K})/T$ into the result of Exercise 3 of Chapter 39: $E = (1240\,\text{eV}\cdot\text{nm})/\lambda$. First, we convert units: $2898\,\mu\text{m}\cdot\text{K} = 2.898 \times 10^6\,\text{nm}\cdot\text{K}$ and $1240\,\text{eV}\cdot\text{nm} = 1.240 \times 10^{-3}\,\text{MeV}\cdot\text{nm}$. Hence,

$$E = \frac{(1.240 \times 10^{-3}\,\text{MeV}\cdot\text{nm})T}{2.898 \times 10^6\,\text{nm}\cdot\text{K}} = (4.28 \times 10^{-10}\ \text{MeV/K})T\ .$$

(b) The minimum energy required to create an electron-positron pair is twice the rest energy of an electron, or $2(0.511\,\text{MeV}) = 1.022\,\text{MeV}$. Hence,

$$T = \frac{E}{4.28 \times 10^{-10}\,\text{MeV/K}} = \frac{1.022\,\text{MeV}}{4.28 \times 10^{-10}\,\text{MeV/K}} = 2.39 \times 10^9\ \text{K}\ .$$

40. (a) For the universal microwave background, Wien's law leads to

$$T = \frac{2898\,\mu\text{m}\cdot\text{K}}{\lambda_{\text{max}}} = \frac{2.898\,\text{mm}\cdot\text{K}}{1.1\,\text{mm}} = 2.6\ \text{K}\ .$$

(b) At "decoupling" (when the universe became approximately "transparent"),

$$\lambda_{\text{max}} = \frac{2898\,\mu\text{m}\cdot\text{K}}{T} = \frac{2898\,\mu\text{m}\cdot\text{K}}{10^5\,\text{K}} = 29\ \text{nm}\ .$$

41. (a) We use the relativistic relationship between speed and momentum:

$$p = \gamma m v = \frac{mv}{\sqrt{1-(v/c)^2}}\ ,$$

which we solve for the speed v:

$$\frac{v}{c} = \sqrt{1 - \frac{1}{\left(\frac{pc}{mc^2}\right)^2 + 1}}\ .$$

For an antiproton $mc^2 = 938.3\,\text{MeV}$ and $pc = 1.19\,\text{GeV} = 1190\,\text{MeV}$, so

$$v = c\sqrt{1 - \frac{1}{(1190\,\text{MeV}/938.3\,\text{MeV})^2 + 1}} = 0.785c\ .$$

For the negative pion $mc^2 = 193.6\,\text{MeV}$, and pc is the same. Therefore,

$$v = c\sqrt{1 - \frac{1}{(1190\,\text{MeV}/193.6\,\text{MeV})^2 + 1}} = 0.993c\ .$$

(b) See part (a).

(c) Since the speed of the antiprotons is about $0.78c$ but not over $0.79c$, an antiproton will trigger C1.

(d) Since the speed of the negative pions exceeds $0.79c$, a negative pion will trigger C2.

(e) and (f) We use $\Delta t = d/v$, where $d = 12\,\text{m}$. For an antiproton

$$\Delta t = \frac{12\,\text{m}}{0.785(2.998 \times 10^8\,\text{m/s})} = 5.1 \times 10^{-8}\,\text{s} = 51\ \text{ns}\ ,$$

and for a negative pion

$$\Delta t = \frac{12\,\text{m}}{0.993(2.998 \times 10^8\,\text{m/s})} = 4.0 \times 10^{-8}\,\text{s} = 40\ \text{ns}\ .$$

42. We note from track 1, and the quantum numbers of the original particle (A), that positively charged particles move in counterclockwise curved paths, and – by inference – negatively charged ones move along clockwise arcs. This immediately shows that tracks 1, 2, 4, 6, and 7 belong to positively charged particles, and tracks 5, 8 and 9 belong to negatively charged ones. Looking at the fictitious particles

in the table (and noting that each appears in the cloud chamber once [or not at all]), we see that this observation (about charged particle motion) greatly narrows the possibilities:

$$\begin{aligned} \text{tracks } 2,4,6,7 &\quad\leftrightarrow\quad \text{particles } C, F, H, J \\ \text{tracks } 5,8,9 &\quad\leftrightarrow\quad \text{particles } D, E, G \end{aligned}$$

This tells us, too, that the particle that does not appear at all is either B or I (since only one neutral particle "appears"). By charge conservation, tracks 2, 4 and 6 are made by particles with a single unit of positive charge (note that track 5 is made by one with a single unit of negative charge), which implies (by elimination) that track 7 is made by particle H. This is confirmed by examining charge conservation at the end-point of track 6. Having exhausted the charge-related information, we turn now to the fictitious quantum numbers. Consider the vertex where tracks 2, 3 and 4 meet (the Whimsy number is listed here as a subscript):

$$\begin{aligned} \text{tracks } 2,4 &\quad\leftrightarrow\quad \text{particles } C_2, F_0, J_{-6} \\ \text{tracks } 3 &\quad\leftrightarrow\quad \text{particle } B_4 \text{ or } I_6 \end{aligned}$$

The requirement that the Whimsy quantum number of the particle making track 4 must equal the sum of the Whimsy values for the particles making tracks 2 and 3 places a powerful constraint (see the subscripts above). A fairly quick trial and error procedure leads to the assignments: particle F makes track 4, and particles J and I make tracks 2 and 3, respectively. Particle B, then, is irrelevant to this set of events. By elimination, the particle making track 6 (the only positively charged particle not yet assigned) must be C. At the vertex defined by

$$A \;\rightarrow\; F + C + (\text{track } 5)_{-} \quad ,$$

where the charge of that particle is indicated by the subscript, we see that Cuteness number conservation requires that the particle making track 5 has Cuteness $= -1$, so this must be particle G. We have only one decision remaining:

$$\text{tracks } 8,9 \quad\leftrightarrow\quad \text{particles } D, E$$

Re-reading the problem, one finds that the particle making track 8 must be particle D since it is the one with seriousness $= 0$. Consequently, the particle making track 9 must be E.

43. (a) During the time interval Δt, the light emitted from galaxy A has traveled a distance $c\Delta t$. Meanwhile, the distance between Earth and the galaxy has expanded from r to $r' = r + r\alpha\Delta t$. Let $c\Delta t = r' = r + r\alpha\Delta t$, which leads to

$$\Delta t = \frac{r}{c - r\alpha} \ .$$

(b) The detected wavelength λ' is longer then λ by $\lambda\alpha\Delta t$ due to the expansion of the universe: $\lambda' = \lambda + \lambda\alpha\Delta t$. Thus,

$$\frac{\Delta\lambda}{\lambda} = \frac{\lambda' - \lambda}{\lambda} = \alpha\Delta t = \frac{\alpha r}{c - \alpha r} \ .$$

(c) We use the binomial expansion formula (see Appendix E):

$$(1 \pm x)^n = 1 \pm \frac{nx}{1!} + \frac{n(n-1)x^2}{2!} + \cdots \qquad (x^2 < 1)$$

to obtain

$$\begin{aligned} \frac{\Delta\lambda}{\lambda} &= \frac{\alpha r}{c - \alpha r} = \frac{\alpha r}{c}\left(1 - \frac{\alpha r}{c}\right)^{-1} \\ &= \frac{\alpha r}{c}\left[1 + \frac{-1}{1!}\left(-\frac{\alpha r}{c}\right) + \frac{(-1)(-2)}{2!}\left(-\frac{\alpha r}{c}\right)^2 + \cdots\right] \\ &\approx \frac{\alpha r}{c} + \left(\frac{\alpha r}{c}\right)^2 + \left(\frac{\alpha r}{c}\right)^3 \ . \end{aligned}$$

(d) When only the first term in the expansion for $\Delta\lambda/\lambda$ is retained we have

$$\frac{\Delta\lambda}{\lambda} \approx \frac{\alpha r}{c} .$$

(e) We set

$$\frac{\Delta\lambda}{\lambda} = \frac{v}{c} = \frac{Hr}{c}$$

and compare with the result of part (d) to obtain $\alpha = H$.

(f) We use the formula $\Delta\lambda/\lambda = \alpha r/(c - \alpha r)$ to solve for r:

$$r = \frac{c(\Delta\lambda/\lambda)}{\alpha(1+\Delta\lambda/\lambda)} = \frac{(2.998\times10^8\,\mathrm{m/s})(0.050)}{(0.0193\,\mathrm{m/s\cdot ly})(1+0.050)} = 7.4\times10^8 \text{ ly} .$$

(g) From the result of part (a),

$$\Delta t = \frac{r}{c-\alpha r} = \frac{(7.4\times10^8\,\mathrm{ly})(9.46\times10^{15}\,\mathrm{m/ly})}{2.998\times10^8\,\mathrm{m/s} - (0.0193\,\mathrm{m/s\cdot ly})(7.4\times10^8\,\mathrm{ly})} = 2.5\times10^{16} \text{ s} ,$$

which is equivalent to 7.8×10^8 y.

(h) Letting $r = c\Delta t$, we solve for Δt:

$$\Delta t = \frac{r}{c} = \frac{7.4\times10^8\,\mathrm{ly}}{c} = 7.4\times10^8 \text{ y} .$$

(i) The distance is given by

$$r = c\Delta t = c(7.8\times10^8\,\mathrm{y}) = 7.8\times10^8 \text{ ly} .$$

(j) From the result of part (f),

$$r_\mathrm{B} = \frac{c(\Delta\lambda/\lambda)}{\alpha(1+\Delta\lambda/\lambda)} = \frac{(2.998\times10^8\,\mathrm{m/s})(0.080)}{(0.0193\,\mathrm{mm/s\cdot ly})(1+0.080)} = 1.15\times10^9 \text{ ly} .$$

(k) From the formula obtained in part (a),

$$\Delta t_\mathrm{B} = \frac{r_\mathrm{B}}{c - r_\mathrm{B}\alpha} = \frac{(1.15\times10^9\,\mathrm{ly})(9.46\times10^{15}\,\mathrm{m/ly})}{2.998\times10^8\,\mathrm{m/s} - (1.15\times10^9\,\mathrm{ly})(0.0193\,\mathrm{m/s\cdot ly})} = 3.9\times10^{16} \text{ s} ,$$

which is equivalent to 1.2×10^9 y.

(l) At the present time, the separation between the two galaxies A and B is given by $r_\mathrm{now} = c\Delta t_\mathrm{B} - c\Delta t_\mathrm{A}$. Since $r_\mathrm{now} = r_\mathrm{then} + r_\mathrm{then}\alpha\Delta t$, we get

$$r_\mathrm{then} = \frac{r_\mathrm{now}}{1+\alpha\Delta t} = 4.4\times10^8 \text{ ly} .$$

44. Assuming the line passes through the origin, its slope is $0.40c/(5.3\times10^9\,\mathrm{ly})$. Then,

$$T = \frac{1}{H} = \frac{1}{\text{slope}} = \frac{5.3\times10^9\,\mathrm{ly}}{0.40c} = \frac{5.3\times10^9\,\mathrm{y}}{0.40} \approx 13\times10^9 \text{ y} .$$