Chapter 33

1. We find the capacitance from $U = \frac{1}{2}Q^2/C$:

$$C = \frac{Q^2}{2U} = \frac{(1.60 \times 10^{-6}\,\mathrm{C})^2}{2(140 \times 10^{-6}\,\mathrm{J})} = 9.14 \times 10^{-9}\ \mathrm{F}\ .$$

2. According to $U = \frac{1}{2}LI^2 = \frac{1}{2}Q^2/C$, the current amplitude is

$$I = \frac{Q}{\sqrt{LC}} = \frac{3.00 \times 10^{-6}\,\mathrm{C}}{\sqrt{(1.10 \times 10^{-3}\,\mathrm{H})(4.00 \times 10^{-6}\,\mathrm{F})}} = 4.52 \times 10^{-2}\ \mathrm{A}\ .$$

3. (a) All the energy in the circuit resides in the capacitor when it has its maximum charge. The current is then zero. If Q is the maximum charge on the capacitor, then the total energy is

$$U = \frac{Q^2}{2C} = \frac{(2.90 \times 10^{-6}\,\mathrm{C})^2}{2(3.60 \times 10^{-6}\,\mathrm{F})} = 1.17 \times 10^{-6}\ \mathrm{J}\ .$$

(b) When the capacitor is fully discharged, the current is a maximum and all the energy resides in the inductor. If I is the maximum current, then $U = LI^2/2$ leads to

$$I = \sqrt{\frac{2U}{L}} = \sqrt{\frac{2(1.168 \times 10^{-6}\,\mathrm{J})}{75 \times 10^{-3}\,\mathrm{H}}} = 5.58 \times 10^{-3}\ \mathrm{A}\ .$$

4. (a) The period is $T = 4(1.50\,\mu\mathrm{s}) = 6.00\,\mu\mathrm{s}$.

(b) The frequency is the reciprocal of the period:

$$f = \frac{1}{T} = \frac{1}{6.00\,\mu\mathrm{s}} = 1.67 \times 10^5\ \mathrm{Hz}\ .$$

(c) The magnetic energy does not depend on the direction of the current (since $U_B \propto i^2$), so this will occur after one-half of a period, or $3.00\,\mu\mathrm{s}$.

5. (a) We recall the fact that the period is the reciprocal of the frequency. It is helpful to refer also to Fig. 33-1. The values of t when plate A will again have maximum positive charge are multiples of the period:

$$t_A = nT = \frac{n}{f} = \frac{n}{2.00 \times 10^3\,\mathrm{Hz}} = n(5.00\,\mu\mathrm{s})\ ,$$

where $n = 1, 2, 3, 4, \cdots$.

(b) We note that it takes $t = \frac{1}{2}T$ for the charge on the other plate to reach its maximum positive value for the first time (compare steps a and e in Fig. 33-1). This is when plate A acquires its most negative charge. From that time onward, this situation will repeat once every period. Consequently,

$$t = \frac{1}{2}T + nT = \frac{1}{2}(2n+1)T = \frac{(2n+1)}{2f} = \frac{(2n+1)}{2(2 \times 10^3\,\mathrm{Hz})} = (2n+1)(2.50\,\mu\mathrm{s})\ ,$$

where $n = 0, 1, 2, 3, 4, \cdots$.

(c) At $t = \frac{1}{4}T$, the current and the magnetic field in the inductor reach maximum values for the first time (compare steps a and c in Fig. 33-1). Later this will repeat every half-period (compare steps c and g in Fig. 33-1). Therefore,

$$t_L = \frac{T}{4} + \frac{nT}{2} = (1 + 2n)\frac{T}{4} = (2n + 1)(1.25\,\mu\text{s}) ,$$

where $n = 0, 1, 2, 3, 4, \cdots$.

6. (a) The angular frequency is

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{F/x}{m}} = \sqrt{\frac{8.0\,\text{N}}{(2.0 \times 10^{-3}\,\text{m})(0.50\,\text{kg})}} = 89 \text{ rad/s} .$$

(b) The period is $1/f$ and $f = \omega/2\pi$. Therefore,

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{89\,\text{rad/s}} = 7.0 \times 10^{-2} \text{ s} .$$

(c) From $\omega = (LC)^{-1/2}$, we obtain

$$C = \frac{1}{\omega^2 L} = \frac{1}{(89\,\text{rad/s})^2(5.0\,\text{H})} = 2.5 \times 10^{-5} \text{ F} .$$

7. (a) The mass m corresponds to the inductance, so $m = 1.25\,\text{kg}$.

(b) The spring constant k corresponds to the reciprocal of the capacitance. Since the total energy is given by $U = Q^2/2C$, where Q is the maximum charge on the capacitor and C is the capacitance,

$$C = \frac{Q^2}{2U} = \frac{\left(175 \times 10^{-6}\,\text{C}\right)^2}{2(5.70 \times 10^{-6}\,\text{J})} = 2.69 \times 10^{-3}\,\text{F}$$

and

$$k = \frac{1}{2.69 \times 10^{-3}\,\text{m/N}} = 372 \text{ N/m} .$$

(c) The maximum displacement corresponds to the maximum charge, so $x_{\text{max}} = 175 \times 10^{-6}\,\text{m}$.

(d) The maximum speed v_{max} corresponds to the maximum current. The maximum current is

$$I = Q\omega = \frac{Q}{\sqrt{LC}} = \frac{175 \times 10^{-6}\,\text{C}}{\sqrt{(1.25\,\text{H})(2.69 \times 10^{-3}\,\text{F})}} = 3.02 \times 10^{-3} \text{ A} .$$

Consequently, $v_{\text{max}} = 3.02 \times 10^{-3}\,\text{m/s}$.

8. We find the inductance from $f = \omega/2\pi = (2\pi\sqrt{LC})^{-1}$.

$$L = \frac{1}{4\pi^2 f^2 C} = \frac{1}{4\pi^2(10 \times 10^3\,\text{Hz})^2(6.7 \times 10^{-6}\,\text{F})} = 3.8 \times 10^{-5} \text{ H} .$$

9. The time required is $t = T/4$, where the period is given by $T = 2\pi/\omega = 2\pi\sqrt{LC}$. Consequently,

$$t = \frac{T}{4} = \frac{2\pi\sqrt{LC}}{4} = \frac{2\pi\sqrt{(0.050\,\text{H})(4.0 \times 10^{-6}\,\text{F})}}{4} = 7.0 \times 10^{-4} \text{ s} .$$

10. We apply the loop rule to the entire circuit:

$$\begin{aligned}
\mathcal{E}_{\text{total}} &= \mathcal{E}_{L_1} + \mathcal{E}_{C_1} + \mathcal{E}_{R_1} + \cdots \\
&= \sum_j \left(\mathcal{E}_{L_j} + \mathcal{E}_{C_j} + \mathcal{E}_{R_j}\right) \\
&= \sum_j \left(L_j \frac{di}{dt} + \frac{q}{C_j} + iR_j\right) \\
&= L\frac{di}{dt} + \frac{q}{C} + iR \qquad \text{where } L = \sum_j L_j \ , \ \frac{1}{C} = \sum_j \frac{1}{C_j} \ , \ \ R = \sum_j R_j
\end{aligned}$$

where we require $\mathcal{E}_{\text{total}} = 0$. This is equivalent to the simple LRC circuit shown in Fig. 33-22(b).

11. (a) $Q = CV_{\text{max}} = (1.0 \times 10^{-9}\,\text{F})(3.0\,\text{V}) = 3.0 \times 10^{-9}\,\text{C}$.

(b) From $U = \frac{1}{2}LI^2 = \frac{1}{2}Q^2/C$ we get

$$I = \frac{Q}{\sqrt{LC}} = \frac{3.0 \times 10^{-9}\,\text{C}}{\sqrt{(3.0 \times 10^{-3}\,\text{H})(1.0 \times 10^{-9}\,\text{F})}} = 1.7 \times 10^{-3} \text{ A} \ .$$

(c) When the current is at a maximum, the magnetic field is at maximum:

$$U_{B,\text{max}} = \frac{1}{2}LI^2 = \frac{1}{2}(3.0 \times 10^{-3}\,\text{H})(1.7 \times 10^{-3}\,\text{A})^2 = 4.5 \times 10^{-9} \text{ J} \ .$$

12. (a) We use $U = \frac{1}{2}LI^2 = \frac{1}{2}Q^2/C$ to solve for L:

$$\begin{aligned}
L &= \frac{1}{C}\left(\frac{Q}{I}\right)^2 = \frac{1}{C}\left(\frac{CV_{\text{max}}}{I}\right)^2 \\
&= C\left(\frac{V_{\text{max}}}{I}\right)^2 \\
&= (4.00 \times 10^{-6}\,\text{F})\left(\frac{1.50\,\text{V}}{50.0 \times 10^{-3}\,\text{A}}\right)^2 \\
&= 3.60 \times 10^{-3} \text{ H} \ .
\end{aligned}$$

(b) Since $f = \omega/2\pi$, the frequency is

$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(3.60 \times 10^{-3}\,\text{H})(4.00 \times 10^{-6}\,\text{F})}} = 1.33 \times 10^3 \text{ Hz} \ .$$

(c) Referring to Fig. 33-1, we see that the required time is one-fourth of a period (where the period is the reciprocal of the frequency). Consequently,

$$t = \frac{1}{4}T = \frac{1}{4f} = \frac{1}{4(1.33 \times 10^3\,\text{Hz})} = 1.88 \times 10^{-4} \text{ s} \ .$$

13. (a) After the switch is thrown to position b the circuit is an LC circuit. The angular frequency of oscillation is $\omega = 1/\sqrt{LC}$. Consequently,

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(54.0 \times 10^{-3}\,\text{H})(6.20 \times 10^{-6}\,\text{F})}} = 275 \text{ Hz} \ .$$

(b) When the switch is thrown, the capacitor is charged to $V = 34.0\,\mathrm{V}$ and the current is zero. Thus, the maximum charge on the capacitor is $Q = VC = (34.0\,\mathrm{V})(6.20 \times 10^{-6}\,\mathrm{F}) = 2.11 \times 10^{-4}\,\mathrm{C}$. The current amplitude is

$$I = \omega Q = 2\pi f Q = 2\pi(275\,\mathrm{Hz})(2.11 \times 10^{-4}\,\mathrm{C}) = 0.365\ \mathrm{A}\ .$$

14. The capacitors C_1 and C_2 can be used in four different ways: (1) C_1 only; (2) C_2 only; (3) C_1 and C_2 in parallel; and (4) C_1 and C_2 in series. The corresponding oscillation frequencies are:

$$f_1 = \frac{1}{2\pi\sqrt{LC_1}} = \frac{1}{2\pi\sqrt{(1.0 \times 10^{-2}\,\mathrm{H})(5.0 \times 10^{-6}\,\mathrm{F})}} = 7.1 \times 10^2\ \mathrm{Hz}$$

$$f_2 = \frac{1}{2\pi\sqrt{LC_2}} = \frac{1}{2\pi\sqrt{(1.0 \times 10^{-2}\,\mathrm{H})(2.0 \times 10^{-6}\,\mathrm{F})}} = 1.1 \times 10^3\ \mathrm{Hz}$$

$$f_3 = \frac{1}{2\pi\sqrt{L(C_1 + C_2)}} = \frac{1}{2\pi\sqrt{(1.0 \times 10^{-2}\,\mathrm{H})(2.0 \times 10^{-6}\,\mathrm{F} + 5.0 \times 10^{-6}\,\mathrm{F})}} = 6.0 \times 10^2\ \mathrm{Hz}$$

$$\begin{aligned} f_4 &= \frac{1}{2\pi\sqrt{LC_1C_2/(C_1 + C_2)}} = \frac{1}{2\pi}\sqrt{\frac{2.0 \times 10^{-6}\,\mathrm{F} + 5.0 \times 10^{-6}\,\mathrm{F}}{(1.0 \times 10^{-2}\,\mathrm{H})(2.0 \times 10^{-6}\,\mathrm{F})(5.0 \times 10^{-6}\,\mathrm{F})}} \\ &= 1.3 \times 10^3\ \mathrm{Hz} \end{aligned}$$

15. (a) Since the frequency of oscillation f is related to the inductance L and capacitance C by $f = 1/2\pi\sqrt{LC}$, the smaller value of C gives the larger value of f. Consequently, $f_{\max} = 1/2\pi\sqrt{LC_{\min}}$, $f_{\min} = 1/2\pi\sqrt{LC_{\max}}$, and

$$\frac{f_{\max}}{f_{\min}} = \frac{\sqrt{C_{\max}}}{\sqrt{C_{\min}}} = \frac{\sqrt{365\,\mathrm{pF}}}{\sqrt{10\,\mathrm{pF}}} = 6.0\ .$$

(b) An additional capacitance C is chosen so the ratio of the frequencies is

$$r = \frac{1.60\,\mathrm{MHz}}{0.54\,\mathrm{MHz}} = 2.96\ .$$

Since the additional capacitor is in parallel with the tuning capacitor, its capacitance adds to that of the tuning capacitor. If C is in picofarads, then

$$\frac{\sqrt{C + 365\,\mathrm{pF}}}{\sqrt{C + 10\,\mathrm{pF}}} = 2.96\ .$$

The solution for C is

$$C = \frac{(365\,\mathrm{pF}) - (2.96)^2(10\,\mathrm{pF})}{(2.96)^2 - 1} = 36\ \mathrm{pF}\ .$$

We solve $f = 1/2\pi\sqrt{LC}$ for L. For the minimum frequency $C = 365\,\mathrm{pF} + 36\,\mathrm{pF} = 401\,\mathrm{pF}$ and $f = 0.54\,\mathrm{MHz}$. Thus

$$L = \frac{1}{(2\pi)^2 C f^2} = \frac{1}{(2\pi)^2(401 \times 10^{-12}\,\mathrm{F})(0.54 \times 10^6\,\mathrm{Hz})^2} = 2.2 \times 10^{-4}\ \mathrm{H}\ .$$

16. (a) Since the percentage of energy stored in the electric field of the capacitor is $(1 - 75.0\%) = 25.0\%$, then

$$\frac{U_E}{U} = \frac{q^2/2C}{Q^2/2C} = 25.0\%$$

which leads to $q = \sqrt{0.250}\,Q = 0.500Q$.

(b) From

$$\frac{U_B}{U} = \frac{Li^2/2}{LI^2/2} = 75.0\% \ ,$$

we find $i = \sqrt{0.750}\, I = 0.866I$.

17. (a) The total energy U is the sum of the energies in the inductor and capacitor:

$$\begin{aligned} U &= U_E + U_B = \frac{q^2}{2C} + \frac{i^2 L}{2} \\ &= \frac{(3.80 \times 10^{-6}\,\mathrm{C})^2}{2(7.80 \times 10^{-6}\,\mathrm{F})} + \frac{(9.20 \times 10^{-3}\,\mathrm{A})^2 (25.0 \times 10^{-3}\,\mathrm{H})}{2} = 1.98 \times 10^{-6}\ \mathrm{J}\ . \end{aligned}$$

(b) We solve $U = Q^2/2C$ for the maximum charge:

$$Q = \sqrt{2CU} = \sqrt{2(7.80 \times 10^{-6}\,\mathrm{F})(1.98 \times 10^{-6}\,\mathrm{J})} = 5.56 \times 10^{-6}\ \mathrm{C}\ .$$

(c) From $U = I^2 L/2$, we find the maximum current:

$$I = \sqrt{\frac{2U}{L}} = \sqrt{\frac{2(1.98 \times 10^{-6}\,\mathrm{J})}{25.0 \times 10^{-3}\,\mathrm{H}}} = 1.26 \times 10^{-2}\ \mathrm{A}\ .$$

(d) If q_0 is the charge on the capacitor at time $t = 0$, then $q_0 = Q\cos\phi$ and

$$\phi = \cos^{-1}\left(\frac{q}{Q}\right) = \cos^{-1}\left(\frac{3.80 \times 10^{-6}\,\mathrm{C}}{5.56 \times 10^{-6}\,\mathrm{C}}\right) = \pm 46.9^\circ\ .$$

For $\phi = +46.9^\circ$ the charge on the capacitor is decreasing, for $\phi = -46.9^\circ$ it is increasing. To check this, we calculate the derivative of q with respect to time, evaluated for $t = 0$. We obtain $-\omega Q \sin\phi$, which we wish to be positive. Since $\sin(+46.9^\circ)$ is positive and $\sin(-46.9^\circ)$ is negative, the correct value for increasing charge is $\phi = -46.9^\circ$.

(e) Now we want the derivative to be negative and $\sin\phi$ to be positive. Thus, we take $\phi = +46.9^\circ$.

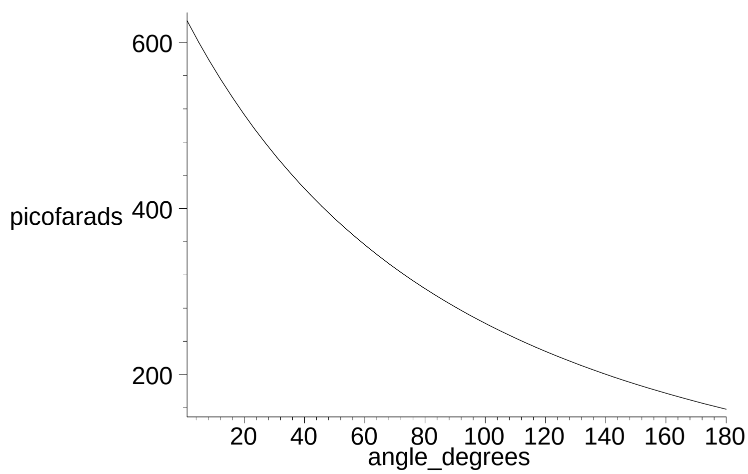
18. The linear relationship between θ (the knob angle in degrees) and frequency f is

$$f = f_0\left(1 + \frac{\theta}{180^\circ}\right) \implies \theta = 180^\circ\left(\frac{f}{f_0} - 1\right)$$

where $f_0 = 2 \times 10^5$ Hz. Since $f = \omega/2\pi = 1/2\pi\sqrt{LC}$, we are able to solve for C in terms of θ:

$$C = \frac{1}{4\pi^2 L f_0^2 \left(1 + \frac{\theta}{180^\circ}\right)^2} = \frac{81}{400000\pi^2(180^\circ + \theta)^2}$$

with SI units understood. After multiplying by 10^{12} (to convert to picofarads), this is plotted, below.



19. (a) The charge (as a function of time) is given by $q = Q\sin\omega t$, where Q is the maximum charge on the capacitor and ω is the angular frequency of oscillation. A sine function was chosen so that $q = 0$ at time $t = 0$. The current (as a function of time) is

$$i = \frac{dq}{dt} = \omega Q\cos\omega t\ ,$$

and at $t = 0$, it is $I = \omega Q$. Since $\omega = 1/\sqrt{LC}$,

$$Q = I\sqrt{LC} = (2.00\,\text{A})\sqrt{(3.00\times10^{-3}\,\text{H})(2.70\times10^{-6}\,\text{F})} = 1.80\times10^{-4}\ \text{C}\ .$$

(b) The energy stored in the capacitor is given by

$$U_E = \frac{q^2}{2C} = \frac{Q^2\sin^2\omega t}{2C}$$

and its rate of change is

$$\frac{dU_E}{dt} = \frac{Q^2\omega\sin\omega t\cos\omega t}{C}\ .$$

We use the trigonometric identity $\cos\omega t\sin\omega t = \frac{1}{2}\sin(2\omega t)$ to write this as

$$\frac{dU_E}{dt} = \frac{\omega Q^2}{2C}\sin(2\omega t)\ .$$

The greatest rate of change occurs when $\sin(2\omega t) = 1$ or $2\omega t = \pi/2\,\text{rad}$. This means

$$t = \frac{\pi}{4\omega} = \frac{\pi T}{4(2\pi)} = \frac{T}{8}$$

where T is the period of oscillation. The relationship $\omega = 2\pi/T$ was used.

(c) Substituting $\omega = 2\pi/T$ and $\sin(2\omega t) = 1$ into $dU_E/dt = (\omega Q^2/2C)\sin(2\omega t)$, we obtain

$$\left(\frac{dU_E}{dt}\right)_{\text{max}} = \frac{2\pi Q^2}{2TC} = \frac{\pi Q^2}{TC}\ .$$

Now $T = 2\pi\sqrt{LC} = 2\pi\sqrt{(3.00\times10^{-3}\,\text{H})(2.70\times10^{-6}\,\text{F})} = 5.655\times10^{-4}\,\text{s}$, so

$$\left(\frac{dU_E}{dt}\right)_{\text{max}} = \frac{\pi(1.80\times10^{-4}\,\text{C})^2}{(5.655\times10^{-4}\,\text{s})(2.70\times10^{-6}\,\text{F})} = 66.7\ \text{W}\ .$$

We note that this is a positive result, indicating that the energy in the capacitor is indeed increasing at $t = T/8$.

20. For the first circuit $\omega = (L_1C_1)^{-1/2}$, and for the second one $\omega = (L_2C_2)^{-1/2}$. When the two circuits are connected in series, the new frequency is

$$\begin{aligned}
\omega' &= \frac{1}{\sqrt{L_{\text{eq}}C_{\text{eq}}}} \\
&= \frac{1}{\sqrt{(L_1+L_2)C_1C_2/(C_1+C_2)}} = \frac{1}{\sqrt{(L_1C_1C_2+L_2C_2C_1)/(C_1+C_2)}} \\
&= \frac{1}{\sqrt{L_1C_1}}\,\frac{1}{\sqrt{(C_1+C_2)/(C_1+C_2)}} = \omega\ ,
\end{aligned}$$

where we use $\omega^{-1} = \sqrt{L_1C_1} = \sqrt{L_2C_2}$.

21. (a) We compare this expression for the current with $i = I\sin(\omega t + \phi_0)$. Setting $(\omega t + \phi) = 2500t + 0.680 = \pi/2$, we obtain $t = 3.56 \times 10^{-4}$ s.

    (b) Since $\omega = 2500\,\text{rad/s} = (LC)^{-1/2}$,

$$L = \frac{1}{\omega^2 C} = \frac{1}{(2500\,\text{rad/s})^2(64.0 \times 10^{-6}\,\text{F})} = 2.50 \times 10^{-3}\ \text{H}\ .$$

    (c) The energy is

$$U = \frac{1}{2}LI^2 = \frac{1}{2}(2.50 \times 10^{-3}\,\text{H})(1.60\,\text{A})^2 = 3.20 \times 10^{-3}\ \text{J}\ .$$

22. (a) The figure implies that the the instantaneous current through the leftmost inductor is the same as that through the rightmost one, which means there is no current through the middle inductor (at any instant). Applying the loop rule to the outer loop (including the rightmost and leftmost inductors), with the current suitably related to the rate of change of charge, we find

$$2L\frac{d^2q}{dt^2} + \frac{2}{C}q = 0 \implies \omega = \frac{1}{\sqrt{(2L)(C/2)}} = \frac{1}{\sqrt{LC}}\ .$$

    (b) In this case, we see that the middle inductor must have current $2i(t)$ flowing downward, and application of the loop rule to, say, the left loop leads to

$$L\frac{d^2q}{dt^2} + L\left(2\frac{d^2q}{dt^2}\right) + \frac{1}{C}q = 0 \implies \omega = \frac{1}{\sqrt{(3L)(C)}} = \frac{1}{\sqrt{3LC}}\ .$$

23. The energy needed to charge the 100 μF capacitor to 300 V is

$$\frac{1}{2}C_2V^2 = \frac{1}{2}(100 \times 10^{-6}\,\text{F})(300\,\text{V})^2 = 4.50\ \text{J}\ .$$

The energy initially in the 900 μF capacitor is

$$\frac{1}{2}C_1V^2 = \frac{1}{2}(900 \times 10^{-6}\,\text{F})(100\,\text{V})^2 = 4.50\ \text{J}\ .$$

All the energy originally in the 900 μF capacitor must be transferred to the 100 μF capacitor. The plan is to store it temporarily in the inductor. We do this by leaving switch S_1 open and closing switch S_2. We wait until the 900 μF capacitor is completely discharged and the current in the circuit is at maximum (this occurs at $t = T_1/4$, one quarter of the relevant period). Since

$$T_1 = 2\pi\sqrt{LC_1} = 2\pi\sqrt{(10.0\,\text{H})(900 \times 10^{-6}\,\text{F})} = 0.596\ \text{s}\ ,$$

we wait until $t = (0.596\,\text{s})/4 = 0.149\,\text{s}$. Now, we close switch S_1 while simultaneously opening switch S_2. Next, we wait for one-fourth of the T_2 period to elapse and open switch S_1. The 100 μF capacitor then has maximum charge, and all the energy resides in it. Since

$$T_2 = 2\pi\sqrt{LC_2} = 2\pi\sqrt{(10.0\,\text{H})(100 \times 10^{-6}\,\text{F})} = 0.199\ \text{s}\ ,$$

we must keep S_1 closed for $(0.199\,\text{s})/4 = 0.0497\,\text{s}$. It is helpful to refer to Figure 23-1 to appreciate the emphasis on "quarter-periods" in this solution.

24. (a) Since $T = 2\pi/\omega = 2\pi\sqrt{LC}$, we may rewrite the power on the exponential factor as

$$-\pi R\sqrt{\frac{C}{L}}\frac{t}{T} = -\pi R\sqrt{\frac{C}{L}}\frac{t}{2\pi\sqrt{LC}} = -\frac{Rt}{2L}\ .$$

    Thus $e^{-Rt/2L} = e^{-\pi R\sqrt{C/L}(t/T)}$.

(b) Since $-\pi R\sqrt{C/L}(t/T)$ must be unitless (as is t/T), $R\sqrt{C/L}$ must also be unitless. Thus, the SI unit of $\sqrt{C/L}$ must be Ω^{-1}. In other words, the SI unit for $\sqrt{L/C}$ is Ω.

(c) Since the amplitude of oscillation reduces by a factor of $e^{-\pi R\sqrt{C/L}(T/T)} = e^{-\pi R\sqrt{C/L}}$ after each cycle, the condition is equivalent to $\pi R\sqrt{C/L} \ll 1$, or $R \ll \sqrt{L/C}$.

25. Since $\omega \approx \omega'$, we may write $T = 2\pi/\omega$ as the period and $\omega = 1/\sqrt{LC}$ as the angular frequency. The time required for 50 cycles (with 3 significant figures understood) is

$$\begin{aligned} t &= 50T = 50\left(\frac{2\pi}{\omega}\right) = 50\left(2\pi\sqrt{LC}\right) \\ &= 50\left(2\pi\sqrt{(220\times10^{-3}\,\mathrm{H})\,(12.0\times10^{-6}\,\mathrm{F})}\right) = 0.5104\ \mathrm{s}\ . \end{aligned}$$

The maximum charge on the capacitor decays according to

$$q_{\max} = Qe^{-Rt/2L}$$

(this is called the *exponentially decaying amplitude* in §33-5), where Q is the charge at time $t = 0$ (if we take $\phi = 0$ in Eq. 33-25). Dividing by Q and taking the natural logarithm of both sides, we obtain

$$\ln\left(\frac{q_{\max}}{Q}\right) = -\frac{Rt}{2L}$$

which leads to

$$R = -\frac{2L}{t}\ln\left(\frac{q_{\max}}{Q}\right) = -\frac{2(220\times10^{-3}\,\mathrm{H})}{0.5104\,\mathrm{s}}\ln(0.99) = 8.66\times10^{-3}\ \Omega\ .$$

26. The charge q after N cycles is obtained by substituting $t = NT = 2\pi N/\omega'$ into Eq. 33-25:

$$\begin{aligned} q &= Qe^{-Rt/2L}\cos(\omega' t + \phi) = Qe^{-RNT/2L}\cos(\omega'(2\pi N/\omega') + \phi) \\ &= Qe^{-RN(2\pi\sqrt{L/C})/2L}\cos(2\pi N + \phi) \\ &= Qe^{-N\pi R\sqrt{C/L}}\cos(\phi). \end{aligned}$$

We note that the initial charge (setting $N = 0$ in the above expression) is $q_0 = Q\cos\phi$, where $q_0 = 6.2\,\mu\mathrm{C}$ is given (with 3 significant figures understood). Consequently, we write the above result as $q_N = q_0 e^{-N\pi R\sqrt{C/L}}$ and obtain

$$\begin{aligned} q_5 &= (6.2\,\mu\mathrm{C})e^{-5\pi(7.2\,\Omega)\sqrt{0.0000032\,\mathrm{F}/12\,\mathrm{H}}} = 5.85\,\mu\mathrm{C} \\ q_{10} &= (6.2\,\mu\mathrm{C})e^{-10\pi(7.2\,\Omega)\sqrt{0.0000032\,\mathrm{F}/12\,\mathrm{H}}} = 5.52\,\mu\mathrm{C} \\ q_{100} &= (6.2\,\mu\mathrm{C})e^{-100\pi(7.2\,\Omega)\sqrt{0.0000032\,\mathrm{F}/12\,\mathrm{H}}} = 1.93\,\mu\mathrm{C}\ . \end{aligned}$$

27. The assumption stated at the end of the problem is equivalent to setting $\phi = 0$ in Eq. 33-25. Since the maximum energy in the capacitor (each cycle) is given by $q_{\max}^2/2C$, where $q_{\max}$ is the maximum charge (during a given cycle), then we seek the time for which

$$\frac{q_{\max}^2}{2C} = \frac{1}{2}\frac{Q^2}{2C} \quad\Longrightarrow\quad q_{\max} = \frac{Q}{\sqrt{2}}\ .$$

Now $q_{\max}$ (referred to as the *exponentially decaying amplitude* in §33-5) is related to Q (and the other parameters of the circuit) by

$$q_{\max} = Qe^{-Rt/2L} \quad\Longrightarrow\quad \ln\left(\frac{q_{\max}}{Q}\right) = -\frac{Rt}{2L}\ .$$

Setting $q_{\max} = Q/\sqrt{2}$, we solve for t:

$$t = -\frac{2L}{R}\ln\left(\frac{q_{\max}}{Q}\right) = -\frac{2L}{R}\ln\left(\frac{1}{\sqrt{2}}\right) = \frac{L}{R}\ln 2\ .$$

The identities $\ln(1/\sqrt{2}) = -\ln\sqrt{2} = -\frac{1}{2}\ln 2$ were used to obtain the final form of the result.

28. (a) In Eq. 33-25, we set $q = 0$ and $t = 0$ to obtain $0 = Q\cos\phi$. This gives $\phi = \pm\pi/2$ (assuming $Q \neq 0$). It should be noted that other roots are possible (for instance, $\cos(3\pi/2) = 0$) but the $\pm\pi/2$ choices for the phase constant are in some sense the "simplest." We choose $\phi = -\pi/2$ to make the manipulation of signs in the expressions below easier to follow. To simplify the work in part (b), we note that $\cos(\omega' t - \pi/2) = \sin(\omega' t)$.

(b) First, we calculate the time-dependent current $i(t)$ from Eq. 33-25:

$$\begin{aligned} i(t) &= \frac{dq}{dt} = \frac{d}{dt}\left(Qe^{-Rt/2L}\sin(\omega' t)\right) \\ &= -\frac{QR}{2L}e^{-Rt/2L}\sin(\omega' t) + Q\omega' e^{-Rt/2L}\cos(\omega' t) \\ &= Qe^{-Rt/2L}\left(-\frac{R\sin(\omega' t)}{2L} + \omega'\cos(\omega' t)\right)\ , \end{aligned}$$

which we evaluate at $t = 0$: $i(0) = Q\omega'$. If we denote $i(0) = I$ as suggested in the problem, then $Q = I/\omega'$. Returning this to Eq. 33-25 leads to

$$q = Qe^{-Rt/2L}\cos(\omega' t + \phi) = \left(\frac{I}{\omega'}\right)e^{-Rt/2L}\cos\left(\omega' t - \frac{\pi}{2}\right) = Ie^{-Rt/2L}\,\frac{\sin(\omega' t)}{\omega'}$$

which answers the question if we interpret "current amplitude" as I. If one, instead, interprets an (exponentially decaying) "current amplitude" to be more appropriately defined as $i_{\max} = i(t)/\cos(\cdots)$ (that is, the current after dividing out its oscillatory behavior), then another step is needed in the $i(t)$ manipulations, above. Using the identity $x\cos\alpha - y\sin\alpha = r\cos(\alpha + \beta)$ where $r = \sqrt{x^2 + y^2}$ and $\tan\beta = y/x$, we can write the current as

$$i(t) = Qe^{-Rt/2L}\left(-\frac{R\sin(\omega' t)}{2L} + \omega'\cos(\omega' t)\right) = Q\sqrt{\omega'^2 + \left(\frac{R}{2L}\right)^2}\,e^{-Rt/2L}\cos(\omega' t + \theta)$$

where $\theta = \tan^{-1}(R/2L\omega')$. Thus, the current amplitude defined in this second way becomes (using Eq. 33-26 for ω')

$$i_{\max} = Q\sqrt{\omega'^2 + \left(\frac{R}{2L}\right)^2}\,e^{-Rt/2L} = Q\omega e^{-Rt/2L}\ .$$

In terms of $i_{\max}$ the expression for charge becomes

$$q = Qe^{-Rt/2L}\sin(\omega' t) = \left(\frac{i_{\max}}{\omega}\right)\sin(\omega' t)$$

which is remarkably similar to our previous "result" in terms of I, except for the fact that ω' in the denominator has now been replaced with ω (and, of course, the exponential has been absorbed into the definition of $i_{\max}$).

29. Let t be a time at which the capacitor is fully charged in some cycle and let $q_{\max 1}$ be the charge on the capacitor then. The energy in the capacitor at that time is

$$U(t) = \frac{q_{\max 1}^2}{2C} = \frac{Q^2}{2C}\,e^{-Rt/L}$$

where

$$q_{\max 1} = Q\, e^{-Rt/2L}$$

(see the discussion of the *exponentially decaying amplitude* in §33-5). One period later the charge on the fully charged capacitor is

$$q_{\max 2} = Q\, e^{-R(t+T)/2L} \qquad \text{where} \quad T = \frac{2\pi}{\omega'} \ ,$$

and the energy is

$$U(t+T) = \frac{q_{\max 2}^2}{2C} = \frac{Q^2}{2C}\, e^{-R(t+T)/L} \ .$$

The fractional loss in energy is

$$\frac{|\Delta U|}{U} = \frac{U(t) - U(t+T)}{U(t)} = \frac{e^{-Rt/L} - e^{-R(t+T)/L}}{e^{-Rt/L}} = 1 - e^{-RT/L} \ .$$

Assuming that RT/L is very small compared to 1 (which would be the case if the resistance is small), we expand the exponential (see Appendix E). The first few terms are:

$$e^{-RT/L} \quad \approx \quad 1 - \frac{RT}{L} + \frac{R^2T^2}{2L^2} + \cdots \ .$$

If we approximate $\omega \approx \omega'$, then we can write T as $2\pi/\omega$. As a result, we obtain

$$\frac{|\Delta U|}{U} \quad \approx \quad 1 - \left(1 - \frac{RT}{L} + \cdots\right) \quad \approx \quad \frac{RT}{L} = \frac{2\pi R}{\omega L} \ .$$

30. (a) We use $I = \mathcal{E}/X_c = \omega_d C\mathcal{E}$:

$$I = \omega_d C\mathcal{E}_m = 2\pi f_d C\mathcal{E}_m = 2\pi(1.00 \times 10^3\,\text{Hz})(1.50 \times 10^{-6}\,\text{F})(30.0\,\text{V}) = 0.283\ \text{A} \ .$$

(b) $I = 2\pi(8.00 \times 10^3\,\text{Hz})(1.50 \times 10^{-6}\,\text{F})(30.0\,\text{V}) = 2.26\,\text{A}$.

31. (a) The current amplitude I is given by $I = V_L/X_L$, where $X_L = \omega_d L = 2\pi f_d L$. Since the circuit contains only the inductor and a sinusoidal generator, $V_L = \mathcal{E}_m$. Therefore,

$$I = \frac{V_L}{X_L} = \frac{\mathcal{E}_m}{2\pi f_d L} = \frac{30.0\,\text{V}}{2\pi(1.00 \times 10^3\,\text{Hz})(50.0 \times 10^{-3}\,\text{H})} = 0.0955\ \text{A} \ .$$

(b) The frequency is now eight times larger than in part (a), so the inductive reactance X_L is eight times larger and the current is one-eighth as much. The current is now $(0.0955\,\text{A})/8 = 0.0119\,\text{A}$.

32. (a) and (b) Regardless of the frequency of the generator, the current through the resistor is

$$I = \frac{\mathcal{E}_m}{R} = \frac{30.0\,\text{V}}{50\,\Omega} = 0.60\ \text{A} \ .$$

33. (a) The inductive reactance for angular frequency ω_d is given by $X_L = \omega_d L$, and the capacitive reactance is given by $X_C = 1/\omega_d C$. The two reactances are equal if $\omega_d L = 1/\omega_d C$, or $\omega_d = 1/\sqrt{LC}$. The frequency is

$$f_d = \frac{\omega_d}{2\pi} = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(6.0 \times 10^{-3}\,\text{H})(10 \times 10^{-6}\,\text{F})}} = 650\ \text{Hz} \ .$$

(b) The inductive reactance is $X_L = \omega_d L = 2\pi f_d L = 2\pi(650\,\text{Hz})(6.0 \times 10^{-3}\,\text{H}) = 24\,\Omega$. The capacitive reactance has the same value at this frequency.

(c) The natural frequency for free LC oscillations is $f = \omega/2\pi = 1/2\pi\sqrt{LC}$, the same as we found in part (a).

34. (a) The circuit consists of one generator across one inductor; therefore, $\mathcal{E}_m = V_L$. The current amplitude is

$$I = \frac{\mathcal{E}_m}{X_L} = \frac{\mathcal{E}_m}{\omega_d L} = \frac{25.0\,\text{V}}{(377\,\text{rad/s})(12.7\,\text{H})} = 5.22 \times 10^{-3}\ \text{A} .$$

(b) When the current is at a maximum, its derivative is zero. Thus, Eq. 31-37 gives $\mathcal{E}_L = 0$ at that instant. Stated another way, since $\mathcal{E}(t)$ and $i(t)$ have a 90° phase difference, then $\mathcal{E}(t)$ must be zero when $i(t) = I$. The fact that $\phi = 90° = \pi/2$ rad is used in part (c).

(c) Consider Eq. 32-28 with $\mathcal{E} = -\frac{1}{2}\mathcal{E}_m$. In order to satisfy this equation, we require $\sin(\omega_d t) = -1/2$. Now we note that the problem states that $\mathcal{E}$ is increasing *in magnitude*, which (since it is already negative) means that it is becoming more negative. Thus, differentiating Eq. 32-28 with respect to time (and demanding the result be negative) we must also require $\cos(\omega_d t) < 0$. These conditions imply that ωt must equal $(2n\pi - 5\pi/6)$ [$n =$ integer]. Consequently, Eq. 33-29 yields (for all values of n)

$$i = I\,\sin\left(2n\pi - \frac{5\pi}{6} - \frac{\pi}{2}\right) = \left(5.22 \times 10^{-3}\,\text{A}\right)\left(\frac{\sqrt{3}}{2}\right) = 4.51 \times 10^{-3}\ \text{A} .$$

35. (a) The generator emf is a maximum when $\sin(\omega_d t - \pi/4) = 1$ or $\omega_d t - \pi/4 = (\pi/2) \pm 2n\pi$ [$n =$ integer]. The first time this occurs after $t = 0$ is when $\omega_d t - \pi/4 = \pi/2$ (that is, $n = 0$). Therefore,

$$t = \frac{3\pi}{4\omega_d} = \frac{3\pi}{4(350\,\text{rad/s})} = 6.73 \times 10^{-3}\ \text{s} .$$

(b) The current is a maximum when $\sin(\omega_d t - 3\pi/4) = 1$, or $\omega_d t - 3\pi/4 = (\pi/2) \pm 2n\pi$ [$n =$ integer]. The first time this occurs after $t = 0$ is when $\omega_d t - 3\pi/4 = \pi/2$ (as in part (a), $n = 0$). Therefore,

$$t = \frac{5\pi}{4\omega_d} = \frac{5\pi}{4(350\,\text{rad/s})} = 1.12 \times 10^{-2}\ \text{s} .$$

(c) The current lags the emf by $+\frac{\pi}{2}$ rad, so the circuit element must be an inductor.

(d) The current amplitude I is related to the voltage amplitude V_L by $V_L = IX_L$, where X_L is the inductive reactance, given by $X_L = \omega_d L$. Furthermore, since there is only one element in the circuit, the amplitude of the potential difference across the element must be the same as the amplitude of the generator emf: $V_L = \mathcal{E}_m$. Thus, $\mathcal{E}_m = I\omega_d L$ and

$$L = \frac{\mathcal{E}_m}{I\omega_d} = \frac{30.0\,\text{V}}{(620 \times 10^{-3}\,\text{A})(350\,\text{rad/s})} = 0.138\ \text{H} .$$

36. (a) The circuit consists of one generator across one capacitor; therefore, $\mathcal{E}_m = V_C$. Consequently, the current amplitude is

$$I = \frac{\mathcal{E}_m}{X_C} = \omega C\mathcal{E}_m = (377\,\text{rad/s})(4.15 \times 10^{-6}\,\text{F})(25.0\,\text{V}) = 3.91 \times 10^{-2}\,\text{A} .$$

(b) When the current is at a maximum, the charge on the capacitor is changing at its largest rate. This happens not when it is fully charged ($\pm q_{\text{max}}$), but rather as it passes through the (momentary) states of being uncharged ($q = 0$). Since $q = CV$, then the voltage across the capacitor (and at the generator, by the loop rule) is zero when the current is at a maximum. Stated more precisely, the time-dependent emf $\mathcal{E}(t)$ and current $i(t)$ have a $\phi = -90°$ phase relation, implying $\mathcal{E}(t) = 0$ when $i(t) = I$. The fact that $\phi = -90° = -\pi/2$ rad is used in part (c).

(c) Consider Eq. 32-28 with $\mathcal{E} = -\frac{1}{2}\mathcal{E}_m$. In order to satisfy this equation, we require $\sin(\omega_d t) = -1/2$. Now we note that the problem states that $\mathcal{E}$ is increasing *in magnitude*, which (since it is already negative) means that it is becoming more negative. Thus, differentiating Eq. 32-28 with respect to time (and demanding the result be negative) we must also require $\cos(\omega_d t) < 0$. These conditions imply that ωt must equal $(2n\pi - 5\pi/6)$ [$n =$ integer]. Consequently, Eq. 33-29 yields (for all values of n)

$$i = I \sin\left(2n\pi - \frac{5\pi}{6} + \frac{\pi}{2}\right) = \left(3.91 \times 10^{-3}\,\text{A}\right)\left(-\frac{\sqrt{3}}{2}\right) = -3.38 \times 10^{-2}\ \text{A}\ .$$

37. (a) Now $X_C = 0$, while $R = 160\,\Omega$ and $X_L = 86.7\,\Omega$ remain unchanged. Therefore, the impedance is

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{(160\,\Omega)^2 + (86.7\,\Omega)^2} = 182\ \Omega\ .$$

The current amplitude is now found to be

$$I = \frac{\mathcal{E}_m}{Z} = \frac{36.0\,\text{V}}{182\,\Omega} = 0.198\ \text{A}\ .$$

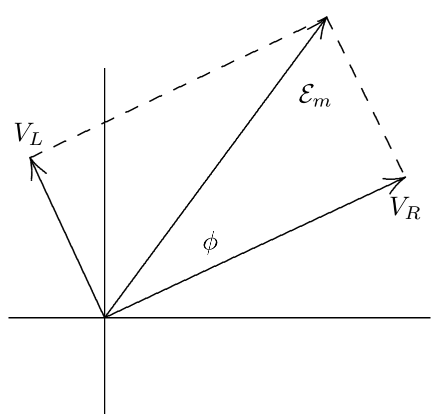
The phase angle is, from Eq. 33-65,

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{86.7\,\Omega - 0}{160\,\Omega}\right) = 28.5^\circ\ .$$

(b) We first find the voltage amplitudes across the circuit elements:

$$\begin{aligned} V_R &= IR = (0.198\,\text{A})(160\,\Omega) \approx 32\ \text{V} \\ V_L &= IX_L = (0.216\,\text{A})(86.7\,\Omega) \approx 17\ \text{V} \end{aligned}$$

This is an inductive circuit, so $\mathcal{E}_m$ leads I. The phasor diagram is drawn to scale below.



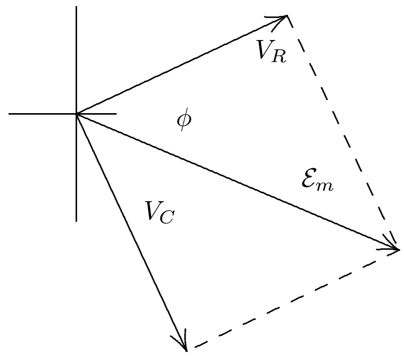
38. (a) Now $X_L = 0$, while $R = 160\,\Omega$ and $X_C = 177\,\Omega$ remain as shown in the Sample Problem. Therefore, the impedance, current amplitude and phase angle are

$$\begin{aligned} Z &= \sqrt{R^2 + X_C^2} = \sqrt{(160\,\Omega)^2 + (177\,\Omega)^2} = 239\ \Omega\ , \\ I &= \frac{\mathcal{E}_m}{Z} = \frac{36.0\,\text{V}}{239\,\Omega} = 0.151\ \text{A}\ , \\ \phi &= \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{0 - 177\,\Omega}{160\,\Omega}\right) = -47.9^\circ\ . \end{aligned}$$

(b) We first find the voltage amplitudes across the circuit elements:

$$\begin{aligned} V_R &= IR = (0.151\,\mathrm{A})(160\,\Omega) &\approx& \quad 24\ \mathrm{V} \\ V_C &= IX_C = (0.151\,\mathrm{A})(177\,\Omega) &\approx& \quad 27\ \mathrm{V} \end{aligned}$$

The circuit is capacitive, so I leads $\mathcal{E}_m$. The phasor diagram is drawn to scale below.



39. (a) The capacitive reactance is

$$X_C = \frac{1}{\omega_d C} = \frac{1}{2\pi f_d C} = \frac{1}{2\pi(60.0\,\mathrm{Hz})(70.0\times10^{-6}\,\mathrm{F})} = 37.9\ \Omega\ .$$

The inductive reactance $86.7\,\Omega$ is unchanged. The new impedance is

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(160\,\Omega)^2 + (37.9\,\Omega - 86.7\,\Omega)^2} = 167\ \Omega\ .$$

The current amplitude is

$$I = \frac{\mathcal{E}_m}{Z} = \frac{36.0\,\mathrm{V}}{167\,\Omega} = 0.216\ \mathrm{A}\ .$$

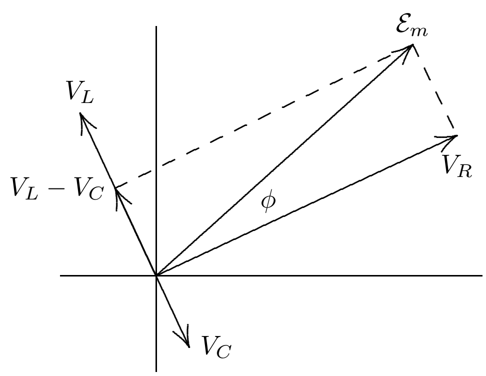
The phase angle is

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{86.7\,\Omega - 37.9\,\Omega}{160\,\Omega}\right) = 17.0^\circ\ .$$

(b) We first find the voltage amplitudes across the circuit elements:

$$\begin{aligned} V_R &= IR = (0.216\,\mathrm{A})(160\,\Omega) = 34.6\ \mathrm{V} \\ V_L &= IX_L = (0.216\,\mathrm{A})(86.7\,\Omega) = 18.7\ \mathrm{V} \\ V_C &= IX_C = (0.216\,\mathrm{A})(37.9\,\Omega) = 8.19\ \mathrm{V} \end{aligned}$$

Note that $X_L > X_C$, so that $\mathcal{E}_m$ leads I. The phasor diagram is drawn to scale below.



40. (a) The resonance frequency f_0 of the circuit is about $(1.50\,\text{kHz} + 1.30\,\text{kHz})/2 = 1.40\,\text{kHz}$. Thus, from $2\pi f_0 = (LC)^{-1/2}$ we get

$$L = \frac{1}{4\pi^2 f_0^2 C} = \frac{1}{4\pi^2(1.40\times10^3\,\text{Hz})^2(5.50\times10^{-6}\,\text{F})} = 2.35\times10^{-3}\ \text{H}\ .$$

(b) From the resonance curves shown in the textbook, we see that as R increases the resonance curve gets more spread out, so the two frequencies at which the amplitude is at half-maximum level will move away from each other.

41. The amplitude of the voltage across the inductor in an RLC series circuit is given by $V_L = IX_L = I\omega_d L$. At resonance, the driving angular frequency equals the natural angular frequency: $\omega_d = \omega = 1/\sqrt{LC}$. For the given circuit

$$X_L = \frac{L}{\sqrt{LC}} = \frac{1.0\,\text{H}}{\sqrt{(1.0\,\text{H})(1.0\times10^{-6}\,\text{F})}} = 1000\ \Omega\ .$$

At resonance the capacitive reactance has this same value, and the impedance reduces simply: $Z = R$. Consequently,

$$I = \left.\frac{\mathcal{E}_m}{Z}\right|_{\text{resonance}} = \frac{\mathcal{E}_m}{R} = \frac{10\,\text{V}}{10\,\Omega} = 1.0\ \text{A}\ .$$

The voltage amplitude across the inductor is therefore

$$V_L = IX_L = (1.0\,\text{A})(1000\,\Omega) = 1000\ \text{V}$$

which is much larger than the amplitude of the generator emf.

42. (a) We note that we obtain the maximum value in Eq. 33-28 when we set

$$t = \frac{\pi}{2\omega_d} = \frac{1}{4f} = \frac{1}{4(60)} = 0.00417\ \text{s}$$

or 4.17 ms. The result is $\mathcal{E}_m \sin(\pi/2) = \mathcal{E}_m \sin(90^\circ) = 36.0\,\text{V}$. We note, for reference in the subsequent parts, that at $t = 4.17\,\text{ms}$, the current is

$$i = I\sin(\omega_d t - \phi) = I\sin(90^\circ - (-29.4^\circ)) = (0.196\,\text{A})\cos(29.4^\circ) = 0.171\ \text{A}$$

using Eq. 33-29 and the results of the Sample Problem.

(b) At $t = 4.17$ ms, Ohm's law directly gives

$$v_R = iR = (I\cos(29.4^\circ))\,R(0.171\,\text{A})(160\,\Omega) = 27.3\ \text{V}\ .$$

(c) The capacitor voltage phasor is 90° less than that of the current. Thus, at $t = 4.17$ ms, we obtain

$$v_C = I\sin(90^\circ - (-29.4^\circ) - 90^\circ)X_C = IX_C\sin(29.4^\circ) = (0.196\,\text{A})(177\,\Omega)\sin(29.4^\circ) = 17.0\ \text{V}\ .$$

(d) The inductor voltage phasor is 90° more than that of the current. Therefore, at $t = 4.17$ ms, we find

$$v_L = I\sin(90^\circ - (-29.4^\circ) + 90^\circ)X_L = -IX_L\sin(29.4^\circ) = -(0.196\,\text{A})(86.7\,\Omega)\sin(29.4^\circ) = -8.3\ \text{V}\ .$$

(e) Our results for parts (b), (c) and (d) add to give 36.0 V, the same as the answer for part (a).

43. The resistance of the coil is related to the reactances and the phase constant by Eq. 33-65. Thus,

$$\frac{X_L - X_C}{R} = \frac{\omega_d L - 1/\omega_d C}{R} = \tan\phi\ ,$$

which we solve for R:

$$\begin{aligned} R &= \frac{1}{\tan\phi}\left(\omega_d L - \frac{1}{\omega_d C}\right) \\ &= \frac{1}{\tan 75^\circ}\left[(2\pi)(930\,\text{Hz})(8.8\times 10^{-2}\,\text{H}) - \frac{1}{(2\pi)(930\,\text{Hz})(0.94\times 10^{-6}\,\text{F})}\right] \\ &= 89\ \Omega\ . \end{aligned}$$

44. (a) The capacitive reactance is

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(400\,\text{Hz})(24.0\times 10^{-6}\,\text{F})} = 16.6\ \Omega\ .$$

(b) The impedance is

$$\begin{aligned} Z &= \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + (2\pi f L - X_C)^2} \\ &= \sqrt{(220\,\Omega)^2 + [2\pi(400\,\text{Hz})(150\times 10^{-3}\,\text{H}) - 16.6\,\Omega]^2} = 422\ \Omega\ . \end{aligned}$$

(c) The current amplitude is

$$I = \frac{\mathcal{E}_m}{Z} = \frac{220\,\text{V}}{422\,\Omega} = 0.521\ \text{A}\ .$$

(d) Now $X_C \propto C_{\text{eq}}^{-1}$. Thus, X_C increases as C_{eq} decreases.

(e) Now $C_{\text{eq}} = C/2$, and the new impedance is

$$Z = \sqrt{(220\,\Omega)^2 + [2\pi(400\,\text{Hz})(150\times 10^{-3}\,\text{H}) - 2(16.6\,\Omega)]^2} = 408\,\Omega < 422\,\Omega\ .$$

Therefore, the impedance decreases.

(f) Since $I \propto Z^{-1}$, it increases.

45. (a) For a given amplitude $(E)_m$ of the generator emf, the current amplitude is given by

$$I = \frac{(E)_m}{Z} = \frac{(E)_m}{\sqrt{R^2 + (\omega_d L - 1/\omega_d C)^2}}\ .$$

We find the maximum by setting the derivative with respect to ω_d equal to zero:

$$\frac{dI}{d\omega_d} = -(E)_m\left[R^2 + (\omega_d L - 1/\omega_d C)^2\right]^{-3/2}\left[\omega_d L - \frac{1}{\omega_d C}\right]\left[L + \frac{1}{\omega_d^2 C}\right]\ .$$

The only factor that can equal zero is $\omega_d L - (1/\omega_d C)$; it does so for $\omega_d = 1/\sqrt{LC} = \omega$. For this circuit,

$$\omega_d = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(1.00\,\text{H})(20.0\times 10^{-6}\,\text{F})}} = 224\ \text{rad/s}\ .$$

(b) When $\omega_d = \omega$, the impedance is $Z = R$, and the current amplitude is

$$I = \frac{(E)_m}{R} = \frac{30.0\,\text{V}}{5.00\,\Omega} = 6.00\ \text{A}\ .$$

(c) We want to find the (positive) values of ω_d for which $I = \frac{(E)_m}{2R}$:

$$\frac{(E)_m}{\sqrt{R^2 + (\omega_d L - 1/\omega_d C)^2}} = \frac{(E)_m}{2R}\ .$$

This may be rearranged to yield

$$\left(\omega_d L - \frac{1}{\omega_d C}\right)^2 = 3R^2 \ .$$

Taking the square root of both sides (acknowledging the two $\pm$ roots) and multiplying by $\omega_d C$, we obtain

$$\omega_d^2 (LC) \pm \omega_d \left(\sqrt{3}CR\right) - 1 = 0 \ .$$

Using the quadratic formula, we find the smallest positive solution

$$\begin{aligned}
\omega_2 &= \frac{-\sqrt{3}CR + \sqrt{3C^2R^2 + 4LC}}{2LC} \\
&= \frac{-\sqrt{3}(20.0 \times 10^{-6}\,\mathrm{F})(5.00\,\Omega)}{2(1.00\,\mathrm{H})(20.0 \times 10^{-6}\,\mathrm{F})} \\
&\qquad + \frac{\sqrt{3(20.0 \times 10^{-6}\,\mathrm{F})^2(5.00\,\Omega)^2 + 4(1.00\,\mathrm{H})(20.0 \times 10^{-6}\,\mathrm{F})}}{2(1.00\,\mathrm{H})(20.0 \times 10^{-6}\,\mathrm{F})} \\
&= 219 \text{ rad/s} \ ,
\end{aligned}$$

and the largest positive solution

$$\begin{aligned}
\omega_1 &= \frac{+\sqrt{3}CR + \sqrt{3C^2R^2 + 4LC}}{2LC} \\
&= \frac{+\sqrt{3}(20.0 \times 10^{-6}\,\mathrm{F})(5.00\,\Omega)}{2(1.00\,\mathrm{H})(20.0 \times 10^{-6}\,\mathrm{F})} \\
&\qquad + \frac{\sqrt{3(20.0 \times 10^{-6}\,\mathrm{F})^2(5.00\,\Omega)^2 + 4(1.00\,\mathrm{H})(20.0 \times 10^{-6}\,\mathrm{F})}}{2(1.00\,\mathrm{H})(20.0 \times 10^{-6}\,\mathrm{F})} \\
&= 228 \text{ rad/s} \ .
\end{aligned}$$

(d) The fractional width is

$$\frac{\omega_1 - \omega_2}{\omega_0} = \frac{228\,\mathrm{rad/s} - 219\,\mathrm{rad/s}}{224\,\mathrm{rad/s}} = 0.04 \ .$$

46. Four possibilities exist: (1) $C_1 = 4.00\,\mu\mathrm{F}$ is used alone; (2) $C_2 = 6.00\,\mu\mathrm{F}$ is used alone; (3) C_1 and C_2 are connected in series; and (4) C_1 and C_2 are connected in parallel. The corresponding resonant frequencies are

$$\begin{aligned}
f_1 &= \frac{1}{2\pi\sqrt{LC_1}} = \frac{1}{2\pi\sqrt{(2.00 \times 10^{-3}\,\mathrm{H})(4.00 \times 10^{-6}\,\mathrm{F})}} = 1.78 \times 10^3 \text{ Hz} \\
f_2 &= \frac{1}{2\pi\sqrt{LC_2}} = \frac{1}{2\pi\sqrt{(2.00 \times 10^{-3}\,\mathrm{H})(6.00 \times 10^{-6}\,\mathrm{F})}} = 1.45 \times 10^3 \text{ Hz} \\
f_3 &= \frac{1}{2\pi\sqrt{LC_1C_2/(C_1 + C_2)}} = 2.30 \times 10^3 \text{ Hz} \\
f_4 &= \frac{1}{2\pi\sqrt{L(C_1 + C_2)}} = 1.13 \times 10^3 \text{ Hz} \ .
\end{aligned}$$

47. We use the expressions found in Problem 45:

$$\begin{aligned}
\omega_1 &= \frac{+\sqrt{3}CR + \sqrt{3C^2R^2 + 4LC}}{2LC} \\
\omega_2 &= \frac{-\sqrt{3}CR + \sqrt{3C^2R^2 + 4LC}}{2LC} \ .
\end{aligned}$$

We also use Eq. 33-4. Thus,

$$\frac{\Delta\omega_d}{\omega} = \frac{\omega_1 - \omega_2}{\omega} = \frac{2\sqrt{3}CR\sqrt{LC}}{2LC} = R\sqrt{\frac{3C}{L}} \ .$$

For the data of Problem 45,

$$\frac{\Delta\omega_d}{\omega} = (5.00\,\Omega)\sqrt{\frac{3(20.0\times10^{-6}\,\text{F})}{1.00\,\text{H}}} = 3.87\times10^{-2} \ .$$

This is in agreement with the result of Problem 45. The method of Problem 45, however, gives only one significant figure since two numbers close in value are subtracted ($\omega_1 - \omega_2$). Here the subtraction is done algebraically, and three significant figures are obtained.

48. (a) Since $L_{\text{eq}} = L_1 + L_2$ and $C_{\text{eq}} = C_1 + C_2 + C_3$ for the circuit, the resonant frequency is

$$\begin{aligned}\omega &= \frac{1}{2\pi\sqrt{L_{\text{eq}}C_{\text{eq}}}} = \frac{1}{2\pi\sqrt{(L_1+L_2)(C_1+C_2+C_3)}} \\ &= \frac{1}{2\pi\sqrt{(1.70\times10^{-3}\,\text{H}+2.30\times10^{-3}\,\text{H})(4.00\times10^{-6}\,\text{F}+2.50\times10^{-6}\,\text{F}+3.50\times10^{-6}\,\text{F})}} \\ &= 796\ \text{Hz} \ .\end{aligned}$$

(b) The resonant frequency does not depend on R so it will not change as R increases.

(c) Since $\omega \propto (L_1 + L_2)^{-1/2}$, it will decrease as L_1 increases.

(d) Since $\omega \propto C_{\text{eq}}^{-1/2}$ and C_{eq} decreases as C_3 is removed, ω will increase.

49. The average power dissipated in resistance R when the current is alternating is given by $P_{\text{avg}} = I_{\text{rms}}^2 R$, where I_{rms} is the root-mean-square current. Since $I_{\text{rms}} = I/\sqrt{2}$, where I is the current amplitude, this can be written $P_{\text{avg}} = I^2R/2$. The power dissipated in the same resistor when the current i_d is direct is given by $P = i_d^2 R$. Setting the two powers equal to each other and solving, we obtain

$$i_d = \frac{I}{\sqrt{2}} = \frac{2.60\ \text{A}}{\sqrt{2}} = 1.84\ \text{A} \ .$$

50. Since the impedance of the voltmeter is large, it will not affect the impedance of the circuit when connected in parallel with the circuit. So the reading will be 100 V in all three cases.

51. The amplitude (peak) value is

$$V_{\max} = \sqrt{2}\,V_{\text{rms}} = \sqrt{2}(100\,\text{V}) = 141\ \text{V} \ .$$

52. (a) We refer to problem 34, part (c). The power delivered by the generator at this instant is $P = \mathcal{E}(t)i(t) = \mathcal{E}_m \sin(2n\pi - \pi/6) I \sin(\pi/3) = -\mathcal{E}_m I \sin(\pi/6)\sin(\pi/3)$. This is less than zero, so it is taking energy from the rest of the circuit.

(b) We refer to problem 36, part (c). The power delivered by the generator at this instant is $P = \mathcal{E}(t)i(t) = \mathcal{E}_m \sin(2n\pi - \pi/6) I \sin(-2\pi/3) = \mathcal{E}_m I \sin(\pi/6)\sin(2\pi/3)$. Since this is positive, it is supplying energy to the rest of the system.

53. We use $P_{\text{avg}} = I_{\text{rms}}^2 R = \frac{1}{2}I^2R$.

- $P_{\text{avg}} = 0$, since $R = 0$.
- $P_{\text{avg}} = \frac{1}{2}I^2R = \frac{1}{2}(0.600\,\text{A})^2(50\,\Omega) = 9.0\,\text{W}$.
- $P_{\text{avg}} = \frac{1}{2}I^2R = \frac{1}{2}(0.198\,\text{A})^2(160\,\Omega) = 3.14\,\text{W}$.
- $P_{\text{avg}} = \frac{1}{2}I^2R = \frac{1}{2}(0.151\,\text{A})^2(160\,\Omega) = 1.82\,\text{W}$.

54. We start with Eq. 33-76:

$$P_{\text{avg}} = \mathcal{E}_{\text{rms}} I_{\text{rms}} \cos\phi = \mathcal{E}_{\text{rms}} \left(\frac{\mathcal{E}_{\text{rms}}}{Z}\right)\left(\frac{R}{Z}\right) = \frac{\mathcal{E}_{\text{rms}}^2 R}{Z^2} \ .$$

For a purely resistive circuit, $Z = R$, and this result reduces to Eq. 27-23 (with V replaced with $\mathcal{E}_{\text{rms}}$). This is also the case for a series RLC circuit at resonance. The average rate for dissipating energy is, of course, zero if $R = 0$, as would be the case for a purely inductive circuit.

55. (a) Using Eq. 33-61, the impedance is

$$Z = \sqrt{(12.0\,\Omega)^2 + (1.30\,\Omega - 0)^2} = 12.1\ \Omega \ .$$

(b) We use the result of problem 54:

$$P_{\text{avg}} = \frac{\mathcal{E}_{\text{rms}}^2 R}{Z^2} = \frac{(120\,\text{V})^2(12.0\,\Omega)}{(12.1\,\Omega)^2} = 1.18 \times 10^3\ \text{W} \ .$$

56. The current in the circuit satisfies $i(t) = I \sin(\omega_d t - \phi)$, where

$$\begin{aligned} I &= \frac{\mathcal{E}_m}{Z} = \frac{\mathcal{E}_m}{\sqrt{R^2 + (\omega_d L - 1/\omega_d C)^2}} \\ &= \frac{45.0\,\text{V}}{\sqrt{(16.0\,\Omega)^2 + \{(3000\,\text{rad/s})(9.20\,\text{mH}) - 1/[(3000\,\text{rad/s})(31.2\,\mu\text{F})]\}^2}} \\ &= 1.93\ \text{A} \end{aligned}$$

and

$$\begin{aligned} \phi &= \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{\omega_d L - 1/\omega_d C}{R}\right) \\ &= \tan^{-1}\left[\frac{(3000\,\text{rad/s})(9.20\,\text{mH})}{16.0\,\Omega} - \frac{1}{(3000\,\text{rad/s})(16.0\,\Omega)(31.2\,\mu\text{F})}\right] \\ &= 46.5^\circ \ . \end{aligned}$$

(a) The power supplied by the generator is

$$\begin{aligned} P_g &= i(t)\mathcal{E}(t) = I \sin(\omega_d t - \phi)\mathcal{E}_m \sin\omega_d t \\ &= (1.93\,\text{A})(45.0\,\text{V}) \sin[(3000\,\text{rad/s})(0.442\,\text{ms})] \sin[(3000\,\text{rad/s})(0.442\,\text{ms}) - 46.5^\circ] \\ &= 41.4\ \text{W} \ . \end{aligned}$$

(b) The rate at which the energy in the capacitor changes is

$$\begin{aligned} P_c &= -\frac{d}{dt}\left(\frac{q^2}{2C}\right) = -i\frac{q}{C} = -iV_c \\ &= -I \sin(\omega_d t - \phi)\left(\frac{I}{\omega_d C}\right)\cos(\omega_d t - \phi) = -\frac{I^2}{2\omega_d C}\sin[2(\omega_d t - \phi)] \\ &= -\frac{(1.93\,\text{A})^2}{2(3000\,\text{rad/s})(31.2 \times 10^{-6}\,\text{F})}\sin[2(3000\,\text{rad/s})(0.442\,\text{ms}) - 2(46.5^\circ)] \\ &= -17.0\ \text{W} \ . \end{aligned}$$

(c) The rate at which the energy in the inductor changes is

$$\begin{aligned}
P_i &= \frac{d}{dt}\left(\frac{1}{2}Li^2\right) = Li\frac{di}{dt} = LI\sin(\omega_d t - \phi)\frac{d}{dt}[I\sin(\omega_d t - \phi)] \\
&= \frac{1}{2}\omega_d LI^2 \sin[2(\omega_d t - \phi)] \\
&= \frac{1}{2}(3000\,\text{rad/s})(1.93\,\text{A})^2(9.20\,\text{mH})\sin[2(3000\,\text{rad/s})(0.442\,\text{ms}) - 2(46.5°)] \\
&= 44.1\ \text{W}\ .
\end{aligned}$$

(d) The rate at which energy is being dissipated by the resistor is

$$\begin{aligned}
P_r &= i^2R = I^2R\sin^2(\omega_d t - \phi) \\
&= (1.93\,\text{A})^2(16.0\,\Omega)\sin^2\left[(3000\,\text{rad/s})(0.442\,\text{ms}) - 46.5°\right] \\
&= 14.4\,\text{W}\ .
\end{aligned}$$

(e) The negative result for P_i means that energy is being taken away from the inductor at this particular time.

(f) $P_i + P_r + P_c = 44.1\text{W} - 17.0\,\text{W} + 14.4\,\text{W} = 41.5\,\text{W} = P_g$.

57. (a) The power factor is $\cos\phi$, where ϕ is the phase constant defined by the expression $i = I\sin(\omega t - \phi)$. Thus, $\phi = -42.0°$ and $\cos\phi = \cos(-42.0°) = 0.743$.

(b) Since $\phi < 0$, $\omega t - \phi > \omega t$. The current leads the emf.

(c) The phase constant is related to the reactance difference by $\tan\phi = (X_L - X_C)/R$. We have $\tan\phi = \tan(-42.0°) = -0.900$, a negative number. Therefore, $X_L - X_C$ is negative, which leads to $X_C > X_L$. The circuit in the box is predominantly capacitive.

(d) If the circuit were in resonance X_L would be the same as X_C, $\tan\phi$ would be zero, and ϕ would be zero. Since ϕ is not zero, we conclude the circuit is not in resonance.

(e) Since $\tan\phi$ is negative and finite, neither the capacitive reactance nor the resistance are zero. This means the box must contain a capacitor and a resistor. The inductive reactance may be zero, so there need not be an inductor. If there is an inductor its reactance must be less than that of the capacitor at the operating frequency.

(f) The average power is

$$P_{\text{avg}} = \frac{1}{2}\mathcal{E}_m I\cos\phi = \frac{1}{2}(75.0\,\text{V})(1.20\,\text{A})(0.743) = 33.4\ \text{W}\ .$$

(g) The answers above depend on the frequency only through the phase constant ϕ, which is given. If values were given for R, L and C then the value of the frequency would also be needed to compute the power factor.

58. This circuit contains no reactances, so $\mathcal{E}_{\text{rms}} = I_{\text{rms}}R_{\text{total}}$. Using Eq. 33-71, we find the average dissipated power in resistor R is

$$P_R = I_{\text{rms}}^2 R = \left(\frac{\mathcal{E}_m}{r+R}\right)^2 R\ .$$

In order to maximize P_R we set the derivative equal to zero:

$$\frac{dP_R}{dR} = \frac{\mathcal{E}_m^2[(r+R)^2 - 2(r+R)R]}{(r+R)^4} = \frac{\mathcal{E}_{\text{m}}^2(r-R)}{(r+R)^3} = 0 \implies R = r$$

59. We use the result of problem 54:

$$P_{\text{avg}} = \frac{(E)_m^2 R}{2Z^2} = \frac{(E)_m^2 R}{2\left[R^2 + (\omega_d L - 1/\omega_d C)^2\right]} \ .$$

We use the expression $Z = \sqrt{R^2 + (\omega_d L - 1/\omega_d C)^2}$ for the impedance in terms of the angular frequency.

(a) Considered as a function of C, P_{avg} has its largest value when the factor $R^2 + (\omega_d L - 1/\omega_d C)^2$ has the smallest possible value. This occurs for $\omega_d L = 1/\omega_d C$, or

$$C = \frac{1}{\omega_d^2 L} = \frac{1}{(2\pi)^2 (60.0\,\text{Hz})^2 (60.0 \times 10^{-3}\,\text{H})} = 1.17 \times 10^{-4} \text{ F} \ .$$

The circuit is then at resonance.

(b) In this case, we want Z^2 to be as large as possible. The impedance becomes large without bound as C becomes very small. Thus, the smallest average power occurs for $C = 0$ (which is not very different from a simple open switch).

(c) When $\omega_d L = 1/\omega_d C$, the expression for the average power becomes

$$P_{\text{avg}} = \frac{(E)_m^2}{2R} \ ,$$

so the maximum average power is in the resonant case and is equal to

$$P_{\text{avg}} = \frac{(30.0\,\text{V})^2}{2(5.00\,\Omega)} = 90.0 \text{ W} \ .$$

On the other hand, the minimum average power is $P_{\text{avg}} = 0$ (as it would be for an open switch).

(d) At maximum power, the reactances are equal: $X_L = X_C$. The phase angle ϕ in this case may be found from

$$\tan\phi = \frac{X_L - X_C}{R} = 0 \ ,$$

which implies $\phi = 0$. On the other hand, at minimum power $X_C \propto 1/C$ is infinite, which leads us to set $\tan\phi = -\infty$. In this case, we conclude that $\phi = -90^\circ$.

(e) At maximum power, the power factor is $\cos\phi = \cos 0^\circ = 1$, and at minimum power, it is $\cos\phi = \cos(-90^\circ) = 0$.

60. (a) The power consumed by the light bulb is $P = I^2R/2$. So we must let $P_{\max}/P_{\min} = (I/I_{\min})^2 = 5$, or

$$\left(\frac{I}{I_{\min}}\right)^2 = \left(\frac{\mathcal{E}_m/Z_{\min}}{\mathcal{E}_m/Z_{\max}}\right)^2 = \left(\frac{Z_{\max}}{Z_{\min}}\right)^2 = \left(\frac{\sqrt{R^2 + (\omega L_{\max})^2}}{R}\right)^2 = 5 \ .$$

We solve for $L_{\max}$:

$$L_{\max} = \frac{2R}{\omega} = \frac{2(120\,\text{V})^2/1000\,\text{W}}{2\pi(60.0\,\text{Hz})} = 7.64 \times 10^{-2} \text{ H} \ .$$

(b) Now we must let

$$\left(\frac{R_{\max} + R_{\text{bulb}}}{R_{\text{bulb}}}\right)^2 = 5 \ ,$$

or

$$R_{\max} = (\sqrt{5} - 1)R_{\text{bulb}} = (\sqrt{5} - 1)\frac{(120\,\text{V})^2}{1000\,\text{W}} = 17.8 \ \Omega \ .$$

This is not done because the resistors would consume, rather than temporarily store, electromagnetic energy.

61. (a) The rms current is

$$\begin{aligned} I_{\text{rms}} &= \frac{\mathcal{E}_{\text{rms}}}{Z} = \frac{\mathcal{E}_{\text{rms}}}{\sqrt{R^2 + (2\pi f L - 1/2\pi f C)^2}} \\ &= \frac{75.0\,\text{V}}{\sqrt{(15.0\,\Omega)^2 + \{2\pi(550\,\text{Hz})(25.0\,\text{mH}) - 1/[2\pi(550\,\text{Hz})(4.70\mu\text{F})]\}^2}} \\ &= 2.59\ \text{A}\ . \end{aligned}$$

(b) The various rms voltages are:

$$\begin{aligned} V_{ab} &= I_{\text{rms}} R = (2.59\,\text{A})(15.0\,\Omega) = 38.8\ \text{V} \\ V_{bc} &= I_{\text{rms}} X_C = \frac{I_{\text{rms}}}{2\pi f C} = \frac{2.59\,\text{A}}{2\pi(550\,\text{Hz})(4.70\,\mu\text{F})} = 159\ \text{V} \\ V_{cd} &= I_{\text{rms}} X_L = 2\pi I_{\text{rms}} f L = 2\pi(2.59\,\text{A})(550\,\text{Hz})(25.0\,\text{mH}) = 224\ \text{V} \\ V_{bd} &= |V_{bc} - V_{cd}| = |159.5\,\text{V} - 223.7\,\text{V}| = 64.2\ \text{V} \\ V_{ad} &= \sqrt{V_{ab}^2 + V_{bd}^2} = \sqrt{(38.8\,\text{V})^2 + (64.2\,\text{V})^2} = 75.0\ \text{V} \end{aligned}$$

(c) For L and C, the rate is zero since they do not dissipate energy. For R,

$$P_R = \frac{V_{ab}^2}{R} = \frac{(38.8\,\text{V})^2}{15.0\,\Omega} = 100\ \text{W}\ .$$

62. We use Eq. 33-79 to find

$$V_s = V_p\left(\frac{N_s}{N_p}\right) = (100\,\text{V})\left(\frac{500}{50}\right) = 1.00 \times 10^3\ \text{V}\ .$$

63. (a) The stepped-down voltage is

$$V_s = V_p\left(\frac{N_s}{N_p}\right) = (120\,\text{V})\left(\frac{10}{500}\right) = 2.4\ \text{V}\ .$$

(b) By Ohm's law, the current in the secondary is

$$I_s = \frac{V_s}{R_s} = \frac{2.4\,\text{V}}{15\,\Omega} = 0.16\ \text{A}\ .$$

We find the primary current from Eq. 33-80:

$$I_p = I_s\left(\frac{N_s}{N_p}\right) = (0.16\,\text{A})\left(\frac{10}{500}\right) = 3.2 \times 10^{-3}\ \text{A}\ .$$

64. Step up:

- We use T_1T_2 as primary and T_1T_3 as secondary coil: $V_{13}/V_{12} = (800 + 200)/200 = 5.00$.
- We use T_1T_2 as primary and T_2T_3 as secondary coil: $V_{23}/V_{13} = 800/200 = 4.00$.
- We use T_2T_3 as primary and T_1T_3 as secondary coil: $V_{13}/V_{23} = (800 + 200)/800 = 1.25$.

Step down: By exchanging the primary and secondary coils in each of the three cases above we get the following possible ratios:

- $1/5.00 = 0.200$
- $1/4.00 = 0.250$

- $1/1.25 = 0.800$

65. The amplifier is connected across the primary windings of a transformer and the resistor R is connected across the secondary windings. If I_s is the rms current in the secondary coil then the average power delivered to R is $P_{\rm avg} = I_s^2 R$. Using $I_s = (N_p/N_s)I_p$, we obtain

$$P_{\rm avg} = \left(\frac{I_p N_p}{N_s}\right)^2 R \, .$$

Next, we find the current in the primary circuit. This is effectively a circuit consisting of a generator and two resistors in series. One resistance is that of the amplifier (r), and the other is the equivalent resistance $R_{\rm eq}$ of the secondary circuit. Therefore,

$$I_p = \frac{\mathcal{E}_{\rm rms}}{r + R_{\rm eq}} = \frac{\mathcal{E}_{\rm rms}}{r + (N_p/N_s)^2 R}$$

where Eq. 33-82 is used for $R_{\rm eq}$. Consequently,

$$P_{\rm avg} = \frac{\mathcal{E}^2 (N_p/N_s)^2 R}{\left[r + (N_p/N_s)^2 R\right]^2} \, .$$

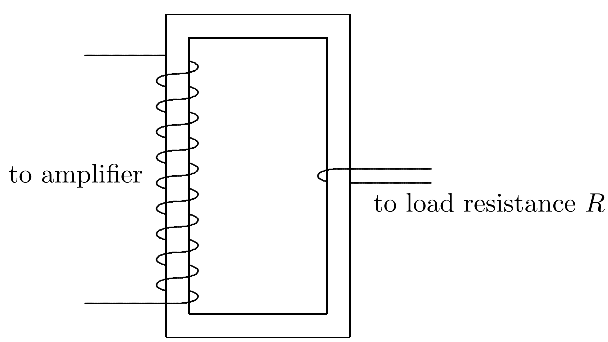
Now, we wish to find the value of N_p/N_s such that $P_{\rm avg}$ is a maximum. For brevity, let $x = (N_p/N_s)^2$. Then

$$P_{\rm avg} = \frac{\mathcal{E}^2 R x}{(r + xR)^2} \, ,$$

and the derivative with respect to x is

$$\frac{dP_{\rm avg}}{dx} = \frac{\mathcal{E}^2 R (r - xR)}{(r + xR)^3} \, .$$

This is zero for $x = r/R = (1000\,\Omega)/(10\,\Omega) = 100$. We note that for small x, $P_{\rm avg}$ increases linearly with x, and for large x it decreases in proportion to $1/x$. Thus $x = r/R$ is indeed a maximum, not a minimum. Recalling $x = (N_p/N_s)^2$, we conclude that the maximum power is achieved for $N_p/N_s = \sqrt{x} = 10$. The diagram below is a schematic of a transformer with a ten to one turns ratio. An actual transformer would have many more turns in both the primary and secondary coils.



66. The effective resistance $R_{\rm eff}$ satisfies $I_{\rm rms}^2 R_{\rm eff} = P_{\rm mechanical}$, or

$$R_{\rm eff} = \frac{P_{\rm mechanical}}{I_{\rm rms}^2} = \frac{(0.100\,{\rm hp})(746\,{\rm W/hp})}{(0.650\,{\rm A})^2} = 177\ \Omega \, .$$

This is not the same as the resistance R of its coils, but just the effective resistance for power transfer from electrical to mechanical form. In fact $I_{\rm rms}^2 R$ would not give $P_{\rm mechanical}$ but rather the rate of energy loss due to thermal dissipation.

67. The rms current in the motor is

$$I_{\rm rms} = \frac{\mathcal{E}_{\rm rms}}{Z} = \frac{\mathcal{E}_{\rm rms}}{\sqrt{R^2 + X_L^2}} = \frac{420\,{\rm V}}{\sqrt{(45.0\,\Omega)^2 + (32.0\,\Omega)^2}} = 7.61\ {\rm A}\ .$$

68. We use $nT/2$ to represent the integer number of half-periods specified in the problem. Note that $T = 2\pi/\omega$. We use the calculus-based definition of an average of a function:

$$\begin{aligned}\left[\sin^2(\omega t - \phi)\right]_{\rm avg} &= \frac{1}{nT/2}\int_0^{\frac{nT}{2}} \sin^2(\omega t - \phi)\,dt \\ &= \frac{2}{nT}\int_0^{\frac{nT}{2}} \frac{1 - \cos(2\omega t - 2\phi)}{2}\,dt \\ &= \frac{2}{nT}\left[\frac{t}{2} - \frac{1}{4\omega}\sin(2\omega t - 2\phi)\right]\Bigg|_0^{\frac{nT}{2}} \\ &= \frac{1}{2} - \frac{1}{2nT\omega}\left[\sin(n\omega T - 2\phi) + \sin 2\phi\right]\ .\end{aligned}$$

Since $n\omega T = n\omega(2\pi/\omega) = 2n\pi$, we have $\sin(n\omega T - 2\phi) = \sin(2n\pi - 2\phi) = -\sin 2\phi$ so $[\sin(n\omega T - 2\phi) + \sin 2\phi] = 0$. Thus,

$$\left[\sin^2(\omega t - \phi)\right]_{\rm avg} = \frac{1}{2}\ .$$

69. (a) The energy stored in the capacitor is given by $U_E = q^2/2C$. Since q is a periodic function of t with period T, so must be U_E. Consequently, U_E will not be changed over one complete cycle. Actually, U_E has period $T/2$, which does not alter our conclusion.

(b) Similarly, the energy stored in the inductor is $U_B = \frac{1}{2}i^2L$. Since i is a periodic function of t with period T, so must be U_B.

(c) The energy supplied by the generator is

$$P_{\rm avg}T = (I_{\rm rms}\mathcal{E}_{\rm rms}\cos\phi)T = \left(\frac{1}{2}T\right)\mathcal{E}_m I\cos\phi$$

where we substitute $I_{\rm rms} = I/\sqrt{2}$ and $\mathcal{E}_{\rm rms} = \mathcal{E}_m/\sqrt{2}$.

(d) The energy dissipated by the resistor is

$$P_{\rm avg,resistor}\,T = (I_{\rm rms}V_R)T = I_{\rm rms}(I_{\rm rms}R)T = \left(\frac{1}{2}T\right)I^2R\ .$$

(e) Since $\mathcal{E}_m I\cos\phi = \mathcal{E}_m I(V_R/\mathcal{E}_m) = \mathcal{E}_m I(IR/\mathcal{E}_m) = I^2R$, the two quantities are indeed the same.

70. (a) The rms current in the cable is $I_{\rm rms} = P/V_t = 250\times10^3\,{\rm W}/(80\times10^3\,{\rm V}) = 3.125\,{\rm A}$. The rms voltage drop is then $\Delta V = I_{\rm rms}R = (3.125\,{\rm A})(2)(0.30\,\Omega) = 1.9\,{\rm V}$, and the rate of energy dissipation is $P_d = I_{\rm rms}^2R = (3.125\,{\rm A})(2)(0.60\,\Omega) = 5.9\,{\rm W}$.

(b) Now $I_{\rm rms} = 250\times10^3\,{\rm W}/(8.0\times10^3\,{\rm V}) = 31.25\,{\rm A}$, so $\Delta V = (31.25\,{\rm A})(0.60\,\Omega) = 19\,{\rm V}$ and $P_d = (3.125\,{\rm A})^2(0.60\,\Omega) = 5.9\times10^2\,{\rm W}$.

(c) Now $I_{\rm rms} = 250\times10^3\,{\rm W}/(0.80\times10^3\,{\rm V}) = 312.5\,{\rm A}$, so $\Delta V = (312.5\,{\rm A})(0.60\,\Omega) = 1.9\times10^2\,{\rm V}$ and $P_d = (312.5\,{\rm A})^2(0.60\,\Omega) = 5.9\times10^4\,{\rm W}$. Both the rate of energy dissipation and the voltage drop increase as V_t decreases. Therefore, to minimize these effects the best choice among the three V_t's above is $V_t = 80\,{\rm kV}$.

71. (a) The impedance is

$$Z = \frac{\mathcal{E}_{\rm m}}{I} = \frac{125\,{\rm V}}{3.20\,{\rm A}} = 39.1\ \Omega\ .$$

(b) From $V_R = IR = \mathcal{E}_m \cos\phi$, we get

$$R = \frac{\mathcal{E}_m \cos\phi}{I} = \frac{(125\,\text{V})\cos(0.982\,\text{rad})}{3.20\,\text{A}} = 21.7\ \Omega\ .$$

(c) Since $X_L - X_C \propto \sin\phi = \sin(-0.982\,\text{rad})$, we conclude that $X_L < X_C$. The circuit is predominantly capacitive.

72. (a) The phase constant is given by

$$\phi = \tan^{-1}\left(\frac{V_L - V_C}{R}\right) = \tan^{-1}\left(\frac{V_L - V_L/2.00}{V_L/2.00}\right) = \tan^{-1}(1.00) = 45.0^\circ\ .$$

(b) We solve R from $\mathcal{E}_m \cos\phi = IR$:

$$R = \frac{\mathcal{E}_m \cos\phi}{I} = \frac{(30.0\,\text{V})(\cos 45^\circ)}{300 \times 10^{-3}\,\text{A}} = 70.7\ \Omega\ .$$

73. (a) We solve L from Eq. 33-4, using the fact that $\omega = 2\pi f$:

$$L = \frac{1}{4\pi^2 f^2 C} = \frac{1}{4\pi^2(10.4 \times 10^3\,\text{Hz})^2(340 \times 10^{-6}\,\text{F})} = 6.89 \times 10^{-7}\ \text{H}\ .$$

(b) The total energy may be figured from the inductor (when the current is at maximum):

$$U = \frac{1}{2}LI^2 = \frac{1}{2}(6.89 \times 10^{-7}\,\text{H})(7.20 \times 10^{-3}\,\text{A})^2 = 1.79 \times 10^{-11}\ \text{J}\ .$$

(c) We solve for Q from $U = \frac{1}{2}Q^2/C$:

$$Q = \sqrt{2CU} = \sqrt{2(340 \times 10^{-6}\,\text{F})(1.79 \times 10^{-11}\,\text{J})} = 1.10 \times 10^{-7}\ \text{C}\ .$$

74. (a) Let $\omega t - \pi/4 = \pi/2$ to obtain $t = 3\pi/4\omega = 3\pi/[4(350\,\text{rad/s})] = 6.73 \times 10^{-3}\,\text{s}$.

(b) Let $\omega t + \pi/4 = \pi/2$ to obtain $t = \pi/4\omega = \pi/[4(350\,\text{rad/s})] = 2.24 \times 10^{-3}\,\text{s}$.

(c) Since i leads $\mathcal{E}$ in phase by $\pi/2$, the element must be a capacitor.

(d) We solve C from $X_C = (\omega C)^{-1} = \mathcal{E}_m/I$:

$$C = \frac{I}{\mathcal{E}_m \omega} = \frac{6.20 \times 10^{-3}\,\text{A}}{(30.0\,\text{V})(350\,\text{rad/s})} = 5.90 \times 10^{-5}\ \text{F}\ .$$

75. From the problem statement $2\pi f_0 = (LC)^{-1/2} = 6000\,\text{Hz}$, $Z = \sqrt{R^2 + (2\pi f_1 L - 1/2\pi f_1 C)^2} = 1000\,\Omega$ where $f_1 = 8000\,\text{Hz}$, and $\cos\phi = R/Z = \cos 45^\circ$. We solve these equations for the unknowns.

(a) $R = Z\cos\phi = (1000\,\Omega)\cos 45^\circ = 707\,\Omega$

(b) The self-inductance is

$$L = \frac{\sqrt{Z^2 - R^2}}{2\pi(f_1 - f_0^2/f_1)} = \frac{\sqrt{(1000\,\Omega)^2 - (707\,\Omega)^2}}{2\pi[8000\,\text{Hz} - (6000\,\text{Hz})^2/8000\,\text{Hz}]} = 3.22 \times 10^{-2}\ \text{H}\ .$$

(c) The capacitance is

$$C = \frac{1}{4\pi^2 f_0^2 L} = \frac{1}{4\pi^2(6000\,\text{Hz})^2(3.22 \times 10^{-2}\,\text{H})} = 2.19 \times 10^{-8}\ \text{F}\ .$$

76. (a) From Eq. 33-65, we have

$$\phi = \tan^{-1}\left(\frac{V_L - V_C}{V_R}\right) = \tan^{-1}\left(\frac{V_L - (V_L/1.50)}{(V_L/2.00)}\right)$$

which becomes $\tan^{-1} 2/3 = 33.7°$ or 0.588 rad.

(b) Since $\phi > 0$, it is inductive ($X_L > X_C$).

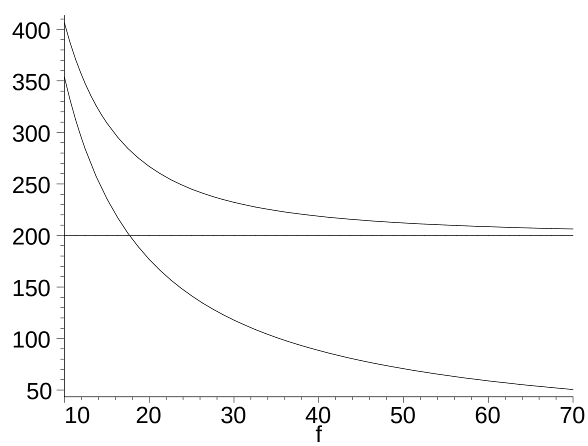
(c) We have $V_R = IR = 9.98$ V, so that $V_L = 2.00V_R = 20.0$ V and $V_C = V_L/1.50 = 13.3$ V. Therefore, from Eq. 33-60,

$$\mathcal{E}_m = \sqrt{V_R^2 + (V_L - V_C)^2}$$

we find $\mathcal{E}_m = 12.0$ V.

77. (a) With f understood to be in Hertz, the capacitive reactance is $X_C = \left[(2\pi)(45 \times 10^{-6}\,\mathrm{F})f\right]^{-1}$.

(b) The resistance, reactance and impedance are plotted over the range $10 \le f \le 70$ Hz. The horizontal line is R, and the curve that crosses that line is X_C. SI units are understood.

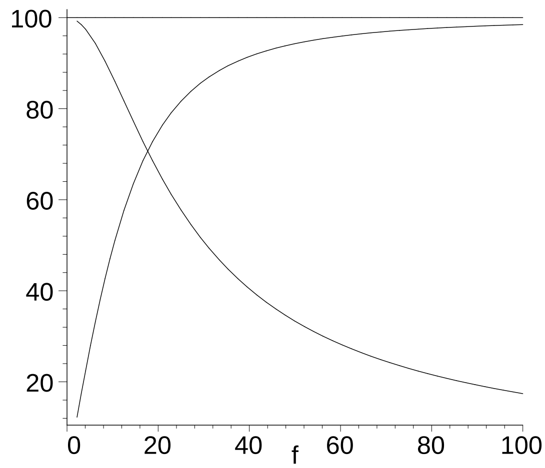


(c) From the graph, we estimate the crossing point to be at about 18 Hz. More careful considerations lead to $f = 17.7$ Hz as the frequency where $X_C = R$.

78. (a) The voltage amplitude for the source is $V_s = 100\,\mathrm{V} = IZ = I\sqrt{R^2 + X_C^2}$, from which we can determine the current at each frequency (the explicit dependence of X_C on frequency is stated in the solution to part (a) of problem 77). This leads to the voltage amplitude across the resistor $V_R = IR$ and the voltage amplitude across the capacitor

$$V_C = IX_C = \left(\frac{V_s}{\sqrt{R^2 + X_C^2}}\right) X_C \quad \text{where} \quad X_C = \frac{1}{2\pi C f}$$

using the values $R = 200\,\Omega$ and $C = 45 \times 10^{-6}$ F given in problem 77. We show, below, the graphs of V_s, V_R and V_C over the range $0 < f < 100$ Hz. The falling curve is V_C and the rising curve is V_R.



(b) The graph indicates that V_C and V_R are equal at roughly 18 Hz. More careful considerations lead to $f = 17.7$ Hz as the frequency for which $V_C = V_R$.

79. When switch S_1 is closed and the others are open, the inductor is essentially out of the circuit and what remains is an RC circuit. The time constant is $\tau_C = RC$. When switch S_2 is closed and the others are open, the capacitor is essentially out of the circuit. In this case, what we have is an LR circuit with time constant $\tau_L = L/R$. Finally, when switch S_3 is closed and the others are open, the resistor is essentially out of the circuit and what remains is an LC circuit that oscillates with period $T = 2\pi\sqrt{LC}$. Substituting $L = R\tau_L$ and $C = \tau_C/R$, we obtain $T = 2\pi\sqrt{\tau_C\tau_L}$.

80. (a) From Eq. 33-25,

$$\frac{dq}{dt} = \frac{d}{dt}\left[Qe^{-Rt/2L}\cos(\omega' t + \phi)\right] = -\frac{RQ}{2L}e^{-Rt/2L}\cos(\omega' t + \phi) - \omega' Q e^{-Rt/2L}\sin(\omega' t + \phi)$$

and

$$\begin{aligned}\frac{d^2q}{dt^2} &= \left(\frac{R}{2L}\right)e^{-Rt/2L}\left[\left(\frac{RQ}{2L}\right)\cos(\omega' t + \phi) - \omega' Q\sin(\omega' t + \phi)\right] \\ &\quad + e^{-Rt/2L}\left[\frac{RQ\omega'}{2L}\sin(\omega' t + \phi) - \omega'^2 Q\cos(\omega' t + \phi)\right].\end{aligned}$$

Substituting these expressions, and Eq. 33-25 itself, into Eq. 33-24, we obtain

$$Qe^{-Rt/2L}\left[-\omega'^2 L - \left(\frac{R}{2L}\right)^2 + \frac{1}{c}\right]\cos(\omega' t + \phi) = 0 .$$

Since this equation is valid at any time t, we must have

$$-\omega'^2 L - \left(\frac{R}{2L}\right)^2 + \frac{1}{C} = 0 \implies \omega' = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2} = \sqrt{\omega^2 - \left(\frac{R}{2L}\right)^2} .$$

(b) The fractional shift in frequency is

$$\begin{aligned}\frac{\Delta f}{f} = \frac{\Delta\omega}{\omega} &= \frac{\omega - \omega'}{\omega} = 1 - \frac{\sqrt{(1/LC) - (R/2L)^2}}{\sqrt{1/LC}} = 1 - \sqrt{1 - \frac{R^2C}{4L}} \\ &= 1 - \sqrt{1 - \frac{(100\,\Omega)^2(7.30\times10^{-6}\,\mathrm{F})}{4(4.40\,\mathrm{H})}} = 0.00210 = 0.210\% .\end{aligned}$$

81. (a) We find L from $X_L = \omega L = 2\pi f L$:

$$f = \frac{X_L}{2\pi L} = \frac{1.30 \times 10^3 \Omega}{2\pi(45.0 \times 10^{-3}\,\text{H})} = 4.60 \times 10^3 \text{ Hz} .$$

(b) The capacitance is found from $X_C = (\omega C)^{-1} = (2\pi f C)^{-1}$:

$$C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi(4.60 \times 10^3\,\text{Hz})(1.30 \times 10^3\,\Omega)} = 2.66 \times 10^{-8} \text{ F} .$$

(c) Noting that $X_L \propto f$ and $X_C \propto f^{-1}$, we conclude that when f is doubled, X_L doubles and X_C reduces by half. Thus, $X_L = 2(1.30 \times 10^3\,\Omega) = 2.60 \times 10^3\,\Omega$ and $X_C = 1.30 \times 10^3\,\Omega/2 = 6.50 \times 10^2\,\Omega$.

82. (a) We consider the following combinations: $\Delta V_{12} = V_1 - V_2$, $\Delta V_{13} = V_1 - V_3$, and $\Delta V_{23} = V_2 - V_3$. For ΔV_{12},

$$\Delta V_{12} = A\sin(\omega_d t) - A\sin(\omega_d t - 120°) = 2A\sin\left(\frac{120°}{2}\right)\cos\left(\frac{2\omega_d t - 120°}{2}\right) = \sqrt{3}\,A\cos(\omega_d t - 60°)$$

where we use $\sin\alpha - \sin\beta = 2\sin[(\alpha-\beta)/2]\cos[(\alpha+\beta)/2]$ and $\sin 60° = \sqrt{3}/2$. Similarly,

$$\Delta V_{13} = A\sin(\omega_d t) - A\sin(\omega_d t - 240°) = 2A\sin\left(\frac{240°}{2}\right)\cos\left(\frac{2\omega_d t - 240°}{2}\right) = \sqrt{3}\,A\cos(\omega_d t - 120°)$$

and

$$\Delta V_{23} = A\sin(\omega_d t - 120°) - A\sin(\omega_d t - 240°) = 2A\sin\left(\frac{120°}{2}\right)\cos\left(\frac{2\omega_d t - 360°}{2}\right) = \sqrt{3}\,A\cos(\omega_d t - 180°) .$$

All three expressions are sinusoidal functions of t with angular frequency ω_d.

(b) We note that each of the above expressions has an amplitude of $\sqrt{3}A$.

83. When the switch is open, we have a series LRC circuit involving just the one capacitor near the upper right corner. Eq. 33-65 leads to

$$\frac{\omega_d L - \frac{1}{\omega_d C}}{R} = \tan\phi_o = \tan(-20°) \;=\; -\tan 20° .$$

Now, when the switch is in position 1, the equivalent capacitance in the circuit is $2C$. In this case, we have

$$\frac{\omega_d L - \frac{1}{2\omega_d C}}{R} = \tan\phi_1 \;=\; \tan 10.0° .$$

Finally, with the switch in position 2, the circuit is simply an LC circuit with current amplitude

$$I_2 = \frac{\mathcal{E}_m}{Z_{LC}} = \frac{\mathcal{E}_m}{\sqrt{\left(\omega_d L - \frac{1}{\omega_d C}\right)^2}} = \frac{\mathcal{E}_m}{\frac{1}{\omega_d C} - \omega_d L}$$

where we use the fact that $\frac{1}{\omega_d C} > \omega_d L$ in simplifying the square root (this fact is evident from the description of the first situation, when the switch was open). We solve for L, R and C from the three equations above:

$$\begin{aligned}
R &= \frac{-\mathcal{E}_m}{I_2\tan\phi_o} = \frac{120\,\text{V}}{(2.00\,\text{A})\tan 20.0°} = 165\ \Omega \\
C &= \frac{I_2}{2\omega_d\mathcal{E}_m\left(1 - \frac{\tan\phi_1}{\tan\phi_o}\right)} = \frac{2.00\,\text{A}}{2(2\pi)(60.0\,\text{Hz})(120\,\text{V})\left(1 + \frac{\tan 10.0°}{\tan 20.0°}\right)} = 1.49 \times 10^{-5} \text{ F} \\
L &= \frac{\mathcal{E}_m}{\omega_d I_2}\left(1 - 2\frac{\tan\phi_1}{\tan\phi_o}\right) = \frac{120\,\text{V}}{2\pi(60.0\,\text{Hz})(2.00\,\text{A})}\left(1 + 2\frac{\tan 10.0°}{\tan 20.0°}\right) 0.313 \text{ H}
\end{aligned}$$

84. (a) Using $X_C = 1/\omega C$ and $V_C = I_C X_C$, we find

$$\omega = \frac{I_C}{CV_C} = 5.77 \times 10^5 \text{ rad/s} .$$

This value is used in the subsequent parts. The period is $T = 2\pi/\omega = 1.09 \times 10^{-5}$ s.

(b) Adapting Eq. 26-22 to the notation of this chapter,

$$U_{E,\max} = \frac{1}{2} C V_C^2 = 4.5 \times 10{-}9 \text{ J} .$$

(c) The discussion in §33-4 shows that $U_{E,\max} = U_{B,\max}$.

(d) We return to Eq. 31-37 (though other, equivalent, approaches could be explored):

$$\frac{di}{dt} = \frac{-\mathcal{E}_L}{L}$$

By the loop rule, $\mathcal{E}_L$ is at its most negative value when the capacitor voltage is at its most positive (V_C). Using this plus the frequency relationship between L and C (Eq. 33-4) leads to

$$\left|\frac{di}{dt}\right|_{\max} = \omega^2 C V_C = 998 \text{ A/s} .$$

(e) Differentiating Eq. 31-51, we have

$$\frac{dU_B}{dt} = Li\frac{di}{dt} .$$

As in the previous part, we use $L = 1/\omega^2 C$. We also use a simple sinusoidal form for the current, $i = I \sin \omega t$:

$$\frac{dU_B}{dt} = \frac{1}{\omega^2 C} I^2 \omega \sin \omega t \cos \omega t$$

where this I is equivalent to the I_C used in part (a). Using a well-known trig identity, we obtain

$$\left(\frac{dU_B}{dt}\right)_{\max} = \frac{I^2}{2\omega^2 C} (\sin 2\omega t)_{\max} = \frac{I^2}{2\omega^2 C}$$

which yields a (maximum) time rate of change (for U_B) equal to 2.60×10^{-3} J/s.

85. (a) At any time, the total energy U in the circuit is the sum of the energy U_E in the capacitor and the energy U_B in the inductor. When $U_E = 0.500 U_B$ (at time t), then $U_B = 2.00 U_E$ and $U = U_E + U_B = 3.00 U_E$. Now, U_E is given by $q^2/2C$, where q is the charge on the capacitor at time t. The total energy U is given by $Q^2/2C$, where Q is the maximum charge on the capacitor. Thus, $Q^2/2C = 3.00 q^2/2C$ or $q = Q/\sqrt{3.00} = 0.577Q$.

(b) If the capacitor is fully charged at time $t = 0$, then the time-dependent charge on the capacitor is given by $q = Q \cos \omega t$. This implies that the condition $q = 0.577Q$ is satisfied when $\cos \omega t = 0.557$, or $\omega t = 0.955$ rad. Since $\omega = 2\pi/T$ (where T is the period of oscillation), $t = 0.955T/2\pi = 0.152T$.

86. (a) Eqs. 33-4 and 33-14 lead to

$$Q = \frac{I}{\omega} = I\sqrt{LC} = 1.27 \times 10^{-6} \text{ C} .$$

(b) We choose the phase constant in Eq. 33-12 to be $\phi = -\pi/2$, so that $i_0 = I$ in Eq. 33-15). Thus, the energy in the capacitor is

$$U_E = \frac{q^2}{2C} = \frac{Q^2}{2C} (\sin \omega t)^2 .$$

Differentiating and using the fact that $2\sin\theta\,\cos\theta = \sin 2\theta$, we obtain

$$\frac{dU_E}{dt} = \frac{Q^2}{2C}\,\omega\,\sin 2\omega t\ .$$

We find the maximum value occurs whenever $\sin 2\omega t = 1$, which leads (with $n =$ *odd* integer) to

$$t = \frac{1}{2\omega}\,\frac{n\,\pi}{2} = \frac{n\,\pi}{4\,\omega} = \frac{n\,\pi}{4}\sqrt{LC} = 8.31\times10^{-5}\ \text{s},\ 2.49\times10^{-4}\ \text{s},\ \ldots\quad .$$

(c) Returning to the above expression for dU_E/dt with the requirement that $\sin 2\omega t = 1$, we obtain

$$\left(\frac{dU_E}{dt}\right)_{\text{max}} = \frac{Q^2}{2C}\,\omega = \frac{\left(I\sqrt{LC}\right)^2}{2C}\,\frac{1}{\sqrt{LC}} = \frac{I^2}{2}\sqrt{\frac{L}{C}} = 5.44\times10^{-3}\ \text{J/s}\ .$$

87. (a) We observe that $\omega = 6597$ rad/s, and, consequently, $X_L = 594\ \Omega$ and $X_C = 303\ \Omega$. Since $X_L > X_C$, the phase angle is positive: $\phi = +60°$.

(b) From Eq. 33-65, we obtain

$$R = \frac{X_L - X_C}{\tan\phi} = 168\ \Omega\ \ .$$

(c) Since we are already on the "high side" of resonance, increasing f will only decrease the current further, but *decreasing* f brings us closer to resonance and, consequently, large values of I.

(d) Increasing L increases X_L, but we already have $X_L > X_C$. Thus, if we wish to move closer to resonance (where X_L must equal X_C), we need to *decrease* the value of L.

(e) To change the present condition of $X_C < X_L$ to something closer to $X_C = X_L$ (resonance, large current), we can increase X_C. Since X_C depends inversely on C, this means *decreasing* C.

88. (a) We observe that $\omega_d = 12566$ rad/s. Consequently, $X_L = 754\ \Omega$ and $X_C = 199\ \Omega$. Hence, Eq. 33-65 gives

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = 1.22\ \text{rad}\ .$$

(b) We find the current amplitude from Eq. 33-60:

$$I = \frac{\mathcal{E}_m}{\sqrt{R^2 + (X_L - X_C)^2}} = 0.288\ \text{A}\ .$$

89. From Eq. 33-60, we have

$$\left(\frac{220\ \text{V}}{3.00\ \text{A}}\right)^2 = R^2 + X_L^2 \quad\Longrightarrow\quad X_L = 69.3\ \Omega\ .$$

90. (a) We observe that $\omega = 7540$ rad/s, and, consequently, $X_L = 377\ \Omega$ and $X_C = 15.3\ \Omega$. Therefore, Eq. 33-64 leads to

$$I_{\text{rms}} = \frac{112\ \text{V}}{\sqrt{(35\,\Omega)^2 + (377\,\Omega - 15\,\Omega)^2}} = 0.308\ \text{A}\ .$$

(b) (c) (d) (e) (f) and (g) For the individual elements, we have:

$$\begin{aligned} V_{R,\text{rms}} &= I_{\text{rms}}R = 10.8\ \text{V} \\ V_{C,\text{rms}} &= I_{\text{rms}}X_C = 4.73\ \text{V} \\ V_{L,\text{rms}} &= I_{\text{rms}}X_L = 116\ \text{V} \end{aligned}$$

The capacitor and inductor are not dissipative elements; the only power dissipated (by definition) is in the resistor. If a coil, perhaps referred to as an inductor in building a circuit, is found to have an internal resistance, then the coil (for purposes of analysis) is taken to be an inductor plus a resistor. The power dissipated in the resistive element is $P_{\text{avg}} = (0.308\ \text{A})^2(35\ \Omega) = 3.33$ W.

91. From Eq. 33-4, with $\omega = 2\pi f = 4.49 \times 10^3$ rad/s, we obtain

$$L = \frac{1}{\omega^2 C} = 7.08 \times 10^{-3} \text{ H} .$$

92. (a) From Eq. 33-4, with $\omega = 2\pi f$, we have

$$f = \frac{1}{2\pi\sqrt{LC}} = 7.08 \times 10^{-3} \text{ H} .$$

(b) The maximum current in the oscillator is

$$i_{\text{max}} = I_C = \frac{V_C}{X_C} = \omega C v_{\text{max}} = 4.00 \times 10^{-3} \text{ A} .$$

(c) Using Eq. 31-51, we find the maximum magnetic energy:

$$U_{B,\text{max}} = \frac{1}{2} L i_{\text{max}}^2 = 1.6 \times 10^{-8} \text{ J} .$$

(d) Adapting Eq. 31-37 to the notation of this chapter,

$$v_{\text{max}} = L \left| \frac{di}{dt} \right|_{\text{max}}$$

which yields a (maximum) time rate of change (for i) equal to 2000 A/s.