Chapter 43

1. In order for the α particle to penetrate the gold nucleus, the separation between the centers of mass of the two particles must be no greater than $r = r_{\rm Cu} + r_\alpha = 6.23\,{\rm fm} + 1.80\,{\rm fm} = 8.03\,{\rm fm}$. Thus, the minimum energy K_α is given by

$$\begin{aligned} K_\alpha &= U = \frac{1}{4\pi\varepsilon_0}\frac{q_\alpha q_{\rm Au}}{r} = \frac{kq_\alpha q_{\rm Au}}{r} \\ &= \frac{(8.99\times10^9\,{\rm V\cdot m/C})(2e)(79)(1.60\times10^{-19}\,{\rm C})}{8.03\times10^{-15}\,{\rm m}} = 28.3\times10^6\ {\rm eV}\ . \end{aligned}$$

We note that the factor of e in $q_\alpha = 2e$ was not set equal to $1.60\times10^{-19}\,$C, but was instead carried through to become part of the final units.

2. Our calculation is similar to that shown in Sample Problem 43-1. We set $K = 5.30\,{\rm MeV} = U = (1/4\pi\varepsilon_0)(q_\alpha q_{\rm Cu}/r_{\rm min})$ and solve for the closest separation, $r_{\rm min}$:

$$\begin{aligned} r_{\rm min} &= \frac{q_\alpha q_{\rm Cu}}{4\pi\varepsilon_0 K} = \frac{kq_\alpha q_{\rm Cu}}{4\pi\varepsilon_0 K} \\ &= \frac{(2e)(29)(1.60\times10^{-19}\,{\rm C})(8.99\times10^9\,{\rm V\cdot m/C})}{5.30\times10^6\,{\rm eV}} \\ &= 1.58\times10^{-14}\,{\rm m} = 15.8\ {\rm fm}\ . \end{aligned}$$

We note that the factor of e in $q_\alpha = 2e$ was not set equal to $1.60\times10^{-19}\,$C, but was instead allowed to cancel the "e" in the non-SI energy unit, electronvolt.

3. The conservation laws of (classical kinetic) energy and (linear) momentum determine the outcome of the collision. The results are given in Chapter 10, Eqs. 10-30 and 10-31. The final speed of the α particle is

$$v_{\alpha f} = \frac{m_\alpha - m_{\rm Au}}{m_\alpha + m_{\rm Au}}\, v_{\alpha i}\ ,$$

and that of the recoiling gold nucleus is

$$v_{{\rm Au},f} = \frac{2m_\alpha}{m_\alpha + m_{\rm Au}}\, v_{\alpha i}\ .$$

(a) Therefore, the kinetic energy of the recoiling nucleus is

$$\begin{aligned} K_{{\rm Au},f} &= \frac{1}{2}m_{\rm Au}v_{{\rm Au},f}^2 \\ &= \frac{1}{2}m_{\rm Au}\left(\frac{2m_\alpha}{m_\alpha + m_{\rm Au}}\right)^2 v_{\alpha i}^2 = K_{\alpha i}\frac{4m_{\rm Au}m_\alpha}{(m_\alpha + m_{\rm Au})^2} \\ &= (5.00\,{\rm MeV})\frac{4(197\,{\rm u})(4.00\,{\rm u})}{(4.00\,{\rm u} + 197\,{\rm u})^2} \\ &= 0.390\ {\rm MeV}\ . \end{aligned}$$

(b) The final kinetic energy of the alpha particle is

$$\begin{aligned} K_{\alpha f} &= \frac{1}{2} m_\alpha v_{\alpha f}^2 \\ &= \frac{1}{2} m_\alpha \left(\frac{m_\alpha - m_{\rm Au}}{m_\alpha + m_{\rm Au}}\right)^2 v_{\alpha i}^2 = K_{\alpha i}\left(\frac{m_\alpha - m_{\rm Au}}{m_\alpha + m_{\rm Au}}\right)^2 \\ &= (5.00\,{\rm MeV})\left(\frac{4.00\,{\rm u} - 197\,{\rm u}}{4.00\,{\rm u} + 197\,{\rm u}}\right)^2 \\ &= 4.61 \text{ MeV} . \end{aligned}$$

We note that $K_{\alpha f} + K_{{\rm Au},f} = K_{\alpha i}$ is indeed satisfied.

4. We solve for A from Eq. 43-3:

$$A = \left(\frac{r}{r_0}\right)^3 = \left(\frac{3.6\,{\rm fm}}{1.2\,{\rm fm}}\right)^3 = 27 .$$

5. We locate a nuclide from Table 43-1 by finding the coordinate (N, Z) of the corresponding point in Fig. 43-4. It is clear that all the nuclides listed in Table 43-1 are stable except the last two, ^{227}Ac and ^{239}Pu.

6. We note that the mean density and mean radius for the Sun are given in Appendix C. Since $\rho = M/V$ where $V \propto r^3$, we get $r \propto \rho^{-1/3}$. Thus, the new radius would be

$$r = R_s\left(\frac{\rho_s}{\rho}\right)^{1/3} = (6.96\times 10^8\,{\rm m})\left(\frac{1410\,{\rm kg/m^3}}{2\times 10^{17}\,{\rm kg/m^3}}\right)^{1/3} = 1.3\times 10^4 \text{ m} .$$

7. (a) 6 protons, since $Z = 6$ for carbon (see Appendix F).

(b) 8 neutrons, since $A - Z = 14 - 6 = 8$ (see Eq. 43-1).

8. The problem with Web-based services is that there are no guarantees of accuracy or that the webpage addresses will not change from the time this solution is written to the time someone reads this. Still, it is worth mentioning that a very accessible website for a wide variety of periodic table and isotope-related information is http://www.webelements.com. Two websites aimed more towards the nuclear professional are http://nucleardata.nuclear.lu.se/nucleardata and http://www.nndc.bnl.gov/nndc/ensdf, which are where some of the information mentioned below was obtained.

(a) According to Appendix F, the atomic number 60 corresponds to the element Neodymium (Nd). The first website mentioned above gives ^{142}Nd, ^{143}Nd, ^{144}Nd, ^{145}Nd, ^{146}Nd, ^{148}Nd, and ^{150}Nd in its list of naturally occurring isotopes. Two of these, ^{144}Nd and ^{150}Nd, are not perfectly stable, but their half-lives are much longer than the age of the universe (detailed information on their half-lives, modes of decay, etc are available at the last two websites referred to, above).

(b) In this list, we are asked to put the nuclides which contain 60 neutrons and which are recognized to exist but not stable nuclei (this is why, for example, ^{108}Cd is not included here). Although the problem does not ask for it, we include the half-lives of the nuclides in our list, though it must be admitted that not all reference sources agree on those values (we picked the ones we regarded as "most reliable"). Thus, we have ^{97}Rb (0.2 s), ^{98}Sr (0.7 s), ^{99}Y (2 s), ^{100}Zr (7 s), ^{101}Nb (7 s), ^{102}Mo (11 minutes), ^{103}Tc (54 s), ^{105}Rh (35 hours), ^{109}In (4 hours), ^{110}Sn (4 hours), ^{111}Sb (75 s), ^{112}Te (2 minutes), ^{113}I (7 s), ^{114}Xe (10 s), ^{115}Cs (1.4 s), and ^{116}Ba (1.4 s).

(c) We would include in this list: ^{60}Zn, ^{60}Cu, ^{60}Ni, ^{60}Co, ^{60}Fe, ^{60}Mn, ^{60}Cr, and ^{60}V.

9. Although we haven't drawn the requested lines in the following table, we can indicate their slopes: lines of constant A would have $-45°$ slopes, and those of constant $N-Z$ would have 45°. As an example of the latter, the $N-Z = 20$ line (which is one of "eighteen-neutron excess") would pass through Cd-114 at the

lower left corner up through Te-122 at the upper right corner. The first column corresponds to $N = 66$, and the bottom row to $Z = 48$. The last column corresponds to $N = 70$, and the top row to $Z = 52$. Much of the information below (regarding values of $T_{1/2}$ particularly) was obtained from the websites http://nucleardata.nuclear.lu.se/nucleardata and http://www.nndc.bnl.gov/nndc/ensdf (we refer the reader to the remarks we made in the solution to problem 8).

^{118}Te 6.0 days	^{119}Te 16.0 h	^{120}Te 0.1%	^{121}Te 19.4 days	^{122}Te 2.6%
^{117}Sb 2.8 h	^{118}Sb 3.6 min	^{119}Sb 38.2 s	^{120}Sb 15.9 min	^{121}Sb 57.2%
^{116}Sn 14.5%	^{117}Sn 7.7%	^{118}Sn 24.2%	^{119}Sn 8.6%	^{120}Sn 32.6%
^{115}In 95.7%	^{116}In 14.1 s	^{117}In 43.2 min	^{118}In 5.0 s	^{119}In 2.4 min
^{114}Cd 28.7%	^{115}Cd 53.5 h	^{116}Cd 7.5%	^{117}Cd 2.5 h	^{118}Cd 50.3 min

10. (a) The atomic number $Z = 39$ corresponds to the element Yttrium (see Appendix F and/or Appendix G), and $Z = 53$ corresponds to Iodine.

    (b) A detailed listing of stable nuclides (such as the website http://nucleardata.nuclear.lu.se/nucleardata) shows that the stable isotope of Iodine has 74 neutrons, and that the stable isotope of Yttrium has 50 neutrons (this can also be inferred from the Molar Mass values listed in Appendix F).

    (c) The number of neutrons left over is $235 - 127 - 89 = 19$.

11. (a) For ^{239}Pu, $Q = 94e$ and $R = 6.64$ fm. Including a conversion factor for J → eV, we obtain

$$\begin{aligned} U &= \frac{3Q^2}{20\pi\varepsilon_0 r} = \frac{3[94(1.60\times10^{-19}\,\mathrm{C})]^2(8.99\times10^{9}\,\mathrm{N\cdot m^2/C^2})}{5(6.64\times10^{-15}\,\mathrm{m})}\left(\frac{1\,\mathrm{eV}}{1.60\times10^{-19}\,\mathrm{J}}\right) \\ &= 1.15\times10^{9}\,\mathrm{eV} = 1.15\ \mathrm{GeV}\ . \end{aligned}$$

    (b) Since $Z = 94$ and $A = 239$, the electrostatic potential per nucleon is 1.15 GeV/239 = 4.81 MeV/nucleon, and per proton is 1.15 GeV/94 = 12.2 MeV/proton. These are of the same order of magnitude as the binding energy per nucleon.

    (c) The binding energy is significantly reduced by the electrostatic repulsion among the protons.

12. (a) For ^{55}Mn the mass density is

$$\rho_m = \frac{M}{V} = \frac{0.055\,\mathrm{kg/mol}}{(4\pi/3)[(1.2\times10^{-15}\,\mathrm{m})(55)^{1/3}]^3(6.02\times10^{23}/\mathrm{mol})} = 2.3\times10^{17}\ \mathrm{kg/m^3}\ ,$$

and for ^{209}Bi

$$\rho_m = \frac{M}{V} = \frac{0.209\,\mathrm{kg/mol}}{(4\pi/3)[(1.2\times10^{-15}\,\mathrm{m})(209)^{1/3}]^3(6.02\times10^{23}/\mathrm{mol})} = 2.3\times10^{17}\ \mathrm{kg/m^3}\ .$$

(b) For ^{55}Mn the charge density is

$$\rho_q = \frac{Ze}{V} = \frac{(25)(1.6\times10^{-19}\,\mathrm{C})}{(4\pi/3)[(1.2\times10^{-15}\,\mathrm{m})(55)^{1/3}]^3} = 1.0\times10^{25}\ \mathrm{C/m^3}\ ,$$

and for ^{209}Bi

$$\rho_q = \frac{Ze}{V} = \frac{(83)(1.6\times10^{-19}\,\mathrm{C})}{(4\pi/3)[(1.2\times10^{-15}\,\mathrm{m})(209)^{1/3}]^3} = 8.8\times10^{24}\ \mathrm{C/m^3}\ .$$

(c) Since $V \propto r^3 = (r_0A^{1/3})^3 \propto A$, we expect $\rho_m \propto A/V \propto A/A \approx$ const. for all nuclides, while $\rho_q \propto Z/V \propto Z/A$ should gradually decrease since $A > 2Z$ for large nuclides.

13. The binding energy is given by $\Delta E_{\mathrm{be}} = [Zm_H + (A-Z)m_n - M_{\mathrm{Pu}}]\,c^2$, where Z is the atomic number (number of protons), A is the mass number (number of nucleons), m_H is the mass of a hydrogen atom, m_n is the mass of a neutron, and M_{Pu} is the mass of a $^{239}_{94}$Pu atom. In principle, nuclear masses should be used, but the mass of the Z electrons included in ZM_H is canceled by the mass of the Z electrons included in M_{Pu}, so the result is the same. First, we calculate the mass difference in atomic mass units: $\Delta m = (94)(1.00783\,\mathrm{u}) + (239-94)(1.00867\,\mathrm{u}) - (239.05216\,\mathrm{u}) = 1.94101\,\mathrm{u}$. Since 1 u is equivalent to 931.5 MeV, $\Delta E_{\mathrm{be}} = (1.94101\,\mathrm{u})(931.5\,\mathrm{MeV/u}) = 1808\,\mathrm{MeV}$. Since there are 239 nucleons, the binding energy per nucleon is $\Delta E_{\mathrm{ben}} = E/A = (1808\,\mathrm{MeV})/239 = 7.56\,\mathrm{MeV}$.

14. (a) The mass number A is the number of nucleons in an atomic nucleus. Since $m_p \approx m_n$ the mass of the nucleus is approximately Am_p. Also, the mass of the electrons is negligible since it is much less than that of the nucleus. So $M \approx Am_p$.

(b) For ^{1}H, the approximate formula gives $M \approx Am_p = (1)(1.007276\,\mathrm{u}) = 1.007276\,\mathrm{u}$. The actual mass is (see Table 47-1) 1.007825 u. The percent error committed is then $\delta = (1.007825\,\mathrm{u} - 1.007276\,\mathrm{u})/1.007825\,\mathrm{u} = 0.054\%$. Similarly, $\delta = 0.50\%$ for ^{7}Li, 0.81% for ^{31}P, 0.83% for ^{81}Br, 0.81% for ^{120}Sn, 0.78% for ^{157}Gd, 0.74% for ^{197}Au, 0.72% for ^{272}Ac, and 0.71% for ^{239}Pu.

(c) No. In a typical nucleus the binding energy per nucleon is several MeV, which is a bit less than 1% of the nucleon mass times c^2. This is comparable with the percent error calculated in part (b), so we need to use a more accurate method to calculate the nuclear mass.

15. (a) The de Broglie wavelength is given by $\lambda = h/p$, where p is the magnitude of the momentum. The kinetic energy K and momentum are related by Eq. 38-51, which yields

$$pc = \sqrt{K^2 + 2Kmc^2} = \sqrt{(200\,\mathrm{MeV})^2 + 2(200\,\mathrm{MeV})(0.511\,\mathrm{MeV})} = 200.5\,\mathrm{MeV}\ .$$

Thus,

$$\lambda = \frac{hc}{pc} = \frac{1240\,\mathrm{eV\cdot nm}}{200.5\times10^6\,\mathrm{eV}} = 6.18\times10^{-6}\,\mathrm{nm} = 6.18\ \mathrm{fm}\ .$$

(b) The diameter of a copper nucleus, for example, is about 8.6 fm, just a little larger than the de Broglie wavelength of a 200-MeV electron. To resolve detail, the wavelength should be smaller than the target, ideally a tenth of the diameter or less. 200-MeV electrons are perhaps at the lower limit in energy for useful probes.

16. We take the speed to be constant, and apply the classical kinetic energy formula:

$$\begin{aligned} t &= \frac{d}{v} = \frac{d}{\sqrt{2K/m}} = 2r\sqrt{\frac{m_n}{2K}} = \frac{r}{c}\sqrt{\frac{2mc^2}{K}} \\ &\approx \frac{(1.2\times10^{-15}\,\mathrm{m})(100)^{1/3}}{3.0\times10^8\,\mathrm{m/s}}\sqrt{\frac{2(938\,\mathrm{MeV})}{5\,\mathrm{MeV}}} \\ &\approx 10^{-22}\ \mathrm{s}\ . \end{aligned}$$

17. We note that $hc = 1240\,\text{MeV}\cdot\text{fm}$ (see problem 3 of Chapter 39), and that the classical kinetic energy $\frac{1}{2}mv^2$ can be written directly in terms of the classical momentum $p = mv$ (see below). Letting $p \simeq \Delta p \simeq h/\Delta x \simeq h/r$, we get

$$E = \frac{p^2}{2m} \simeq \frac{(hc)^2}{2(mc^2)r^2} = \frac{(1240\,\text{MeV}\cdot\text{fm})^2}{2(938\,\text{MeV})[(1.2\,\text{fm})(100)^{1/3}]^2} \simeq 30 \text{ MeV} .$$

18. (a) In terms of the original value of u, the newly defined u is greater by a factor of 1.007825. So the mass of ^{1}H would be 1.000000 u, the mass of ^{12}C would be $(12.000000/1.007825)\text{u} = 11.90683\,\text{u}$, and the mass of ^{238}U would be $(238.050785/1.007825)\text{u} = 236.2025\,\text{u}$.

    (b) Defining the mass of ^{1}H to be exactly 1 does not result in any overall simplification.

19. (a) Since the nuclear force has a short range, any nucleon interacts only with its nearest neighbors, not with more distant nucleons in the nucleus. Let N be the number of neighbors that interact with any nucleon. It is independent of the number A of nucleons in the nucleus. The number of interactions in a nucleus is approximately NA, so the energy associated with the strong nuclear force is proportional to NA and, therefore, proportional to A itself.

    (b) Each proton in a nucleus interacts electrically with every other proton. The number of pairs of protons is $Z(Z-1)/2$, where Z is the number of protons. The Coulomb energy is, therefore, proportional to $Z(Z-1)$.

    (c) As A increases, Z increases at a slightly slower rate but Z^2 increases at a faster rate than A and the energy associated with Coulomb interactions increases faster than the energy associated with strong nuclear interactions.

20. (a) The first step is to add energy to produce $^4\text{He} \to p + {}^3\text{H}$, which – to make the electrons "balance" – may be rewritten as $^4\text{He} \to {}^1\text{H} + {}^3\text{H}$. The energy needed is $\Delta E_1 = (m_{^3\text{H}} + m_{^1\text{H}} - m_{^4\text{He}})c^2 = (3.01605\,\text{u} + 1.00783\,\text{u} - 4.00260\,\text{u})(931.5\,\text{MeV/u}) = 19.8\,\text{MeV}$. The second step is to add energy to produce $^3\text{H} \to n + {}^2\text{H}$. The energy needed is $\Delta E_2 = (m_{^2\text{H}} + m_n - m_{^3\text{H}})c^2 = (2.01410\,\text{u} + 1.00867\,\text{u} - 3.01605\,\text{u})(931.5\,\text{MeV/u}) = 6.26\,\text{MeV}$. The third step: $^2\text{H} \to p + n$, which – to make the electrons "balance" – may be rewritten as $^2\text{H} \to {}^1\text{H} + n$. The work required is $\Delta E_3 = (m_{^1\text{H}} + m_n - m_{^2\text{H}})c^2 = (1.00783\,\text{u} + 1.00867\,\text{u} - 2.01410\,\text{u})(931.5\,\text{MeV/u}) = 2.23\,\text{MeV}$.

    (b) The total binding energy is $\Delta E_{\text{be}} = \Delta E_1 + \Delta E_2 + \Delta E_3 = 19.8\,\text{MeV} + 6.26\,\text{MeV} + 2.23\,\text{MeV} = 28.3\,\text{MeV}$.

    (c) The binding energy per nucleon is $\Delta E_{\text{ben}} = \Delta E_{\text{be}}/A = 28.3\,\text{MeV}/4 = 7.07\,\text{MeV}$.

21. Let f_{24} be the abundance of ^{24}Mg, let f_{25} be the abundance of ^{25}Mg, and let f_{26} be the abundance of ^{26}Mg. Then, the entry in the periodic table for Mg is $24.312 = 23.98504 f_{24} + 24.98584 f_{25} + 25.98259 f_{26}$. Since there are only three isotopes, $f_{24} + f_{25} + f_{26} = 1$. We solve for f_{25} and f_{26}. The second equation gives $f_{26} = 1 - f_{24} - f_{25}$. We substitute this expression and $f_{24} = 0.7899$ into the first equation to obtain $24.312 = (23.98504)(0.7899) + 24.98584 f_{25} + 25.98259 - (25.98259)(0.7899) - 25.98259 f_{25}$. The solution is $f_{25} = 0.09303$. Then, $f_{26} = 1 - 0.7899 - 0.09303 = 0.1171$. 78.99% of naturally occurring magnesium is ^{24}Mg, 9.30% is ^{25}Mg, and 11.71% is ^{26}Mg.

22. (a) Table 43-1 gives the atomic mass of ^{1}H as $m = 1.007825\,\text{u}$. Therefore, the *mass excess* for ^{1}H is $\Delta = (1.007825\,\text{u} - 1.000000\,\text{u})(931.5\,\text{MeV/u}) = +7.29\,\text{MeV}$.

    (b) The mass of the neutron is given in Sample Problem 43-3. Thus, for the neutron, $\Delta = (1.008665\,\text{u} - 1.000000\,\text{u})(931.5\,\text{MeV/u}) = +8.07\,\text{MeV}$.

    (c) Appealing again to Table 43-1, we obtain, for ^{120}Sn, $\Delta = (119.902199\,\text{u} - 120.000000\,\text{u})(931.5\,\text{MeV/u}) = -91.10\,\text{MeV}$.

23. We first "separate" all the nucleons in one copper nucleus (which amounts to simply calculating the nuclear binding energy) and then figure the number of nuclei in the penny (so that we can multiply the

two numbers and obtain the result). To begin, we note that (using Eq. 43-1 with Appendix F and/or G) the copper-63 nucleus has 29 protons and 34 neutrons. We use the more accurate values given in Sample Problem 43-3:

$$\Delta E_{\rm be} = (29(1.007825\,{\rm u}) + 34(1.008665\,{\rm u}) - 62.92960\,{\rm u})\,(931.5\,{\rm MeV/u}) = 551.4\ {\rm MeV}\ .$$

To figure the number of nuclei (or, equivalently, the number of atoms), we adapt Eq. 43-20:

$$N_{\rm Cu} = \left(\frac{3.0\,{\rm g}}{62.92960\,{\rm g/mol}}\right)\left(6.02\times 10^{23}\,{\rm atoms/mol}\right) \approx 2.9\times 10^{22}\ {\rm atoms}\ .$$

Therefore, the total energy needed is

$$N_{\rm Cu}\Delta E_{\rm be} = (551.4\,{\rm MeV})\left(2.9\times 10^{22}\right) = 1.6\times 10^{25}\ {\rm MeV}\ .$$

24. It should be noted that when the problem statement says the "masses of the proton and the deuteron are ..." they are actually referring to the corresponding atomic masses (given to very high precision). That is, the given masses include the "orbital" electrons. As in many computations in this chapter, this circumstance (of implicitly including electron masses in what should be a purely nuclear calculation) does not cause extra difficulty in the calculation (see remarks in Sample Problems 43-4, 43-6, and 43-7). Setting the gamma ray energy equal to $\Delta E_{\rm be}$, we solve for the neutron mass (with each term understood to be in u units):

$$\begin{aligned} m_{\rm n} &= M_{\rm d} - m_{\rm H} + \frac{E_\gamma}{c^2} \\ &= 2.0141019 - 1.007825035 + \frac{2.2233}{931.502} \\ &= 1.0062769 + 0.0023868 \end{aligned}$$

which yields $m_{\rm n} = 1.0086637\,{\rm u}$, where the last digit (7) is uncertain to within roughly ± 2 (but this depends on what precisely the uncertainties are in the given data).

25. If a nucleus contains Z protons and N neutrons, its binding energy is $\Delta E_{\rm be} = (Zm_H + Nm_n - m)c^2$, where m_H is the mass of a hydrogen atom, m_n is the mass of a neutron, and m is the mass of the atom containing the nucleus of interest. If the masses are given in atomic mass units, then mass excesses are defined by $\Delta_H = (m_H - 1)c^2$, $\Delta_n = (m_n - 1)c^2$, and $\Delta = (m - A)c^2$. This means $m_H c^2 = \Delta_H + c^2$, $m_n c^2 = \Delta_n + c^2$, and $mc^2 = \Delta + Ac^2$. Thus $E = (Z\Delta_H + N\Delta_n - \Delta) + (Z + N - A)c^2 = Z\Delta_H + N\Delta_n - \Delta$, where $A = Z + N$ is used. For ${}^{197}_{79}{\rm Au}$, $Z = 79$ and $N = 197 - 79 = 118$. Hence,

$$\Delta E_{\rm be} = (79)(7.29\,{\rm MeV}) + (118)(8.07\,{\rm MeV}) - (-31.2\,{\rm MeV}) = 1560\,{\rm MeV}\ .$$

This means the binding energy per nucleon is $\Delta E_{\rm ben} = (1560\,{\rm MeV})/197 = 7.92\,{\rm MeV}$.

26. (a) Since $60\,{\rm y} = 2(30\,{\rm y}) = 2T_{1/2}$, the fraction left is $2^{-2} = 1/4$.
    (b) Since $90\,{\rm y} = 3(30\,{\rm y}) = 3T_{1/2}$, the fraction that remains is $2^{-3} = 1/8$.

27. By the definition of half-life, the same has reduced to $\frac{1}{2}$ its initial amount after 140 d. Thus, reducing it to $\frac{1}{4} = (\frac{1}{2})^2$ of its initial number requires that two half-lives have passed: $t = 2T_{1/2} = 280\,{\rm d}$.

28. We note that $t = 24\,{\rm h}$ is four times $T_{1/2} = 6.5\,{\rm h}$. Thus, it has reduced by half, four-fold:

$$\left(\frac{1}{2}\right)^4 \left(48\times 10^{19}\right) = 3\times 10^{19}\ .$$

29. (a) The decay rate is given by $R = \lambda N$, where λ is the disintegration constant and N is the number of undecayed nuclei. Initially, $R = R_0 = \lambda N_0$, where N_0 is the number of undecayed nuclei at that time. One must find values for both N_0 and λ. The disintegration constant is related to the half-life $T_{1/2}$ by $\lambda = (\ln 2)/T_{1/2} = (\ln 2)/(78\,\mathrm{h}) = 8.89 \times 10^{-3}\,\mathrm{h}^{-1}$. If M is the mass of the sample and m is the mass of a single atom of gallium, then $N_0 = M/m$. Now, $m = (67\,\mathrm{u})(1.661 \times 10^{-24}\,\mathrm{g/u}) = 1.113 \times 10^{-22}\,\mathrm{g}$ and $N_0 = (3.4\,\mathrm{g})/(1.113 \times 10^{-22}\,\mathrm{g}) = 3.05 \times 10^{22}$. Thus $R_0 = (8.89 \times 10^{-3}\,\mathrm{h}^{-1})(3.05 \times 10^{22}) = 2.71 \times 10^{20}\,\mathrm{h}^{-1} = 7.53 \times 10^{16}\,\mathrm{s}^{-1}$.

(b) The decay rate at any time t is given by

$$R = R_0\, e^{-\lambda t}$$

where R_0 is the decay rate at $t = 0$. At $t = 48\,\mathrm{h}$, $\lambda t = (8.89 \times 10^{-3}\,\mathrm{h}^{-1})(48\,\mathrm{h}) = 0.427$ and

$$R = (7.53 \times 10^{16}\,\mathrm{s}^{-1})\, e^{-0.427} = 4.91 \times 10^{16}\,\mathrm{s}^{-1} .$$

30. (a) Replacing differentials with deltas in Eq. 43-11, we use the fact that $\Delta N = -12$ during $\Delta t = 1.0$ s to obtain

$$\frac{\Delta N}{N} = -\lambda \Delta t \implies \lambda = 4.8 \times 10^{-18}/\mathrm{s}$$

where $N = 2.5 \times 10^{18}$, mentioned at the second paragraph of §43-3, is used.

(b) Eq. 43-17 yields $T_{1/2} = \ln 2/\lambda = 1.4 \times 10^{17}$ s, or about 4.6 billion years.

31. (a) The half-life $T_{1/2}$ and the disintegration constant are related by $T_{1/2} = (\ln 2)/\lambda$, so $T_{1/2} = (\ln 2)/(0.0108\,\mathrm{h}^{-1}) = 64.2\,\mathrm{h}$.

(b) At time t, the number of undecayed nuclei remaining is given by

$$N = N_0\, e^{-\lambda t} = N_0\, e^{-(\ln 2)t/T_{1/2}} .$$

We substitute $t = 3T_{1/2}$ to obtain

$$\frac{N}{N_0} = e^{-3 \ln 2} = 0.125 .$$

In each half-life, the number of undecayed nuclei is reduced by half. At the end of one half-life, $N = N_0/2$, at the end of two half-lives, $N = N_0/4$, and at the end of three half-lives, $N = N_0/8 = 0.125N_0$.

(c) We use

$$N = N_0\, e^{-\lambda t} .$$

$10.0\,d$ is 240 h, so $\lambda t = (0.0108\,\mathrm{h}^{-1})(240\,\mathrm{h}) = 2.592$ and

$$\frac{N}{N_0} = e^{-2.592} = 0.0749 .$$

32. (a) We adapt Eq. 43-20:

$$N_{\mathrm{Pu}} = \left(\frac{0.002\,\mathrm{g}}{239\,\mathrm{g/mol}}\right)\left(6.02 \times 10^{23}\,\mathrm{nuclei/mol}\right) \approx 5 \times 10^{18}\ \mathrm{nuclei} .$$

(b) Eq. 43-19 leads to

$$R = \frac{N \ln 2}{T_{1/2}} = \frac{5 \times 10^{18} \ln 2}{2.41 \times 10^4\,\mathrm{y}} = 1.4 \times 10^{14}/\mathrm{y}$$

which is equivalent to $4.6 \times 10^6/\mathrm{s} = 4.6 \times 10^6$ Bq (the unit becquerel is defined in §43-3).

33. The rate of decay is given by $R = \lambda N$, where λ is the disintegration constant and N is the number of undecayed nuclei. In terms of the half-life $T_{1/2}$, the disintegration constant is $\lambda = (\ln 2)/T_{1/2}$, so

$$\begin{aligned} N &= \frac{R}{\lambda} = \frac{RT_{1/2}}{\ln 2} = \frac{(6000\,\mathrm{Ci})(3.7\times 10^{10}\,\mathrm{s^{-1}/Ci})(5.27\,\mathrm{y})(3.16\times 10^{7}\,\mathrm{s/y})}{\ln 2} \\ &= 5.33\times 10^{22}\ \text{nuclei}\ . \end{aligned}$$

34. Using Eq. 43-14 and Eq. 43-17 (and the fact that mass is proportional to the number of atoms), the amount decayed is

$$\begin{aligned} |\Delta m| &= m|_{t_f=16.0\,\mathrm{h}} - m|_{t_i=14.0\,\mathrm{h}} \\ &= m_0(1 - e^{-t_i \ln 2/T_{1/2}}) - m_0(1 - e^{-t_f \ln 2/T_{1/2}}) \\ &= m_0(e^{-t_f \ln 2/T_{1/2}} - e^{-t_i \ln 2/T_{1/2}}) \\ &= (5.50\,\mathrm{g})\left[e^{-(16.0\,\mathrm{h}/12.7\,\mathrm{h})\ln 2} - e^{-(14.0\,\mathrm{h}/12.7\,\mathrm{h})\ln 2}\right] \\ &= 0.256\ \mathrm{g}\ . \end{aligned}$$

35. (a) We assume that the chlorine in the sample had the naturally occurring isotopic mixture, so the average mass number was 35.453, as given in Appendix F. Then, the mass of ^{226}Ra was

$$m = \frac{226}{226 + 2(35.453)}(0.10\,\mathrm{g}) = 76.1\times 10^{-3}\ \mathrm{g}\ .$$

The mass of a ^{226}Ra nucleus is $(226\,\mathrm{u})(1.661\times 10^{-24}\,\mathrm{g/u}) = 3.75\times 10^{-22}\,\mathrm{g}$, so the number of ^{226}Ra nuclei present was $N = (76.1\times 10^{-3}\,\mathrm{g})/(3.75\times 10^{-22}\,\mathrm{g}) = 2.03\times 10^{20}$.

(b) The decay rate is given by $R = N\lambda = (N \ln 2)/T_{1/2}$, where λ is the disintegration constant, $T_{1/2}$ is the half-life, and N is the number of nuclei. The relationship $\lambda = (\ln 2)/T_{1/2}$ is used. Thus,

$$R = \frac{(2.03\times 10^{20})\ln 2}{(1600\,\mathrm{y})(3.156\times 10^{7}\,\mathrm{s/y})} = 2.79\times 10^{9}\ \mathrm{s^{-1}}\ .$$

36. (a) We use $R = R_0 e^{-\lambda t}$ to find t:

$$t = \frac{1}{\lambda}\ln\frac{R_0}{R} = \frac{T_{1/2}}{\ln 2}\ln\frac{R_0}{R} = \frac{14.28\,\mathrm{d}}{\ln 2}\ln\frac{3050}{170} = 59.5\ \mathrm{d}\ .$$

(b) The required factor is

$$\frac{R_0}{R} = e^{\lambda t} = e^{t\ln 2/T_{1/2}} = e^{(3.48\,\mathrm{d}/14.28\,\mathrm{d})\ln 2} = 1.18\ .$$

37. We label the two isotopes with subscripts 1 (for ^{32}P) and 2 (for ^{33}P). Initially, 10% of the decays come from ^{33}P, which implies that the initial rate $R_{02} = 9R_{01}$. Using Eq. 43-16, this means

$$R_{01} = \lambda_1 N_{01} = \frac{1}{9}R_{02} = \frac{1}{9}\lambda_2 N_{02}\ .$$

At time t, we have $R_1 = R_{01}e^{-\lambda_1 t}$ and $R_2 = R_{02}e^{-\lambda_2 t}$. We seek the value of t for which $R_1 = 9R_2$ (which means 90% of the decays arise from ^{33}P). We divide equations to obtain $(R_{01}/R_{02})e^{-(\lambda_1-\lambda_2)t} = 9$, and solve for t:

$$\begin{aligned} t &= \frac{1}{\lambda_1 - \lambda_2}\ln\left(\frac{R_{01}}{9R_{02}}\right) = \frac{\ln(R_{01}/9R_{02})}{\ln 2/T_{1/2\,1} - \ln 2/T_{1/2\,2}} \\ &= \frac{\ln[(1/9)^2]}{\ln 2[(14.3\,\mathrm{d})^{-1} - (25.3\,\mathrm{d})^{-1}]} = 209\ \mathrm{d}\ . \end{aligned}$$

38. We have one alpha particle (helium nucleus) produced for every plutonium nucleus that decays. To find the number that have decayed, we use Eq. 43-14, Eq. 43-17, and adapt Eq. 43-20:

$$N_0 - N = N_0\left(1 - e^{-t\ln 2/T_{1/2}}\right) = N_A \frac{12.0\,\mathrm{g/mol}}{239\,\mathrm{g/mol}}\left(1 - e^{-20000\ln 2/24100}\right)$$

where N_A is the Avogadro constant. This yields 1.32×10^{22} alpha particles produced. In terms of the amount of helium gas produced (assuming the α particles slow down and capture the appropriate number of electrons), this corresponds to

$$m_{\mathrm{He}} = \left(\frac{1.32 \times 10^{22}}{6.02 \times 10^{23}/\mathrm{mol}}\right)(4.0\,\mathrm{g/mol}) = 87.9 \times 10^{-3}\ \mathrm{g}\ .$$

39. The number N of undecayed nuclei present at any time and the rate of decay R at that time are related by $R = \lambda N$, where λ is the disintegration constant. The disintegration constant is related to the half-life $T_{1/2}$ by $\lambda = (\ln 2)/T_{1/2}$, so $R = (N\ln 2)/T_{1/2}$ and $T_{1/2} = (N\ln 2)/R$. Since 15.0% by mass of the sample is ^{147}Sm, the number of ^{147}Sm nuclei present in the sample is

$$N = \frac{(0.150)(1.00\,\mathrm{g})}{(147\,\mathrm{u})(1.661 \times 10^{-24}\,\mathrm{g/u})} = 6.143 \times 10^{20}\ .$$

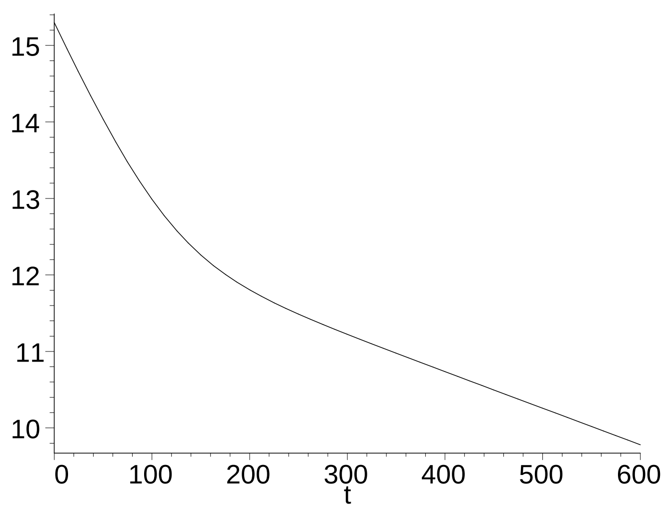
Thus

$$T_{1/2} = \frac{(6.143 \times 10^{20})\ln 2}{120\,\mathrm{s}^{-1}} = 3.55 \times 10^{18}\,\mathrm{s} = 1.12 \times 10^{11}\ \mathrm{y}\ .$$

40. We note that $2.42\,\mathrm{min} = 145.2\,\mathrm{s}$. We are asked to plot (with SI units understood)

$$\ln R = \ln\left(R_0 e^{-\lambda t} + R_0' e^{-\lambda' t}\right)$$

where $R_0 = 3.1 \times 10^5$, $R_0' = 4.1 \times 10^6$, $\lambda = \ln 2/145.2$ and $\lambda' = \ln 2/24.6$. Our plot is shown below.



We note that the magnitude of the slope for small t is λ' (the disintegration constant for ^{110}Ag), and for large t is λ (the disintegration constant for ^{108}Ag).

41. If N is the number of undecayed nuclei present at time t, then

$$\frac{dN}{dt} = R - \lambda N$$

where R is the rate of production by the cyclotron and λ is the disintegration constant. The second term gives the rate of decay. Rearrange the equation slightly and integrate:

$$\int_{N_0}^{N} \frac{dN}{R-\lambda N} = \int_0^t dt$$

where N_0 is the number of undecayed nuclei present at time $t = 0$. This yields

$$-\frac{1}{\lambda}\ln\frac{R-\lambda N}{R-\lambda N_0} = t\ .$$

We solve for N:

$$N = \frac{R}{\lambda} + \left(N_0 - \frac{R}{\lambda}\right) e^{-\lambda t}\ .$$

After many half-lives, the exponential is small and the second term can be neglected. Then, $N = R/\lambda$, regardless of the initial value N_0. At times that are long compared to the half-life, the rate of production equals the rate of decay and N is a constant.

42. Combining Eqs. 43-19 and 43-20, we obtain

$$M_{\text{sam}} = N\frac{M_{\text{K}}}{N_{\text{A}}} = \left(\frac{RT_{1/2}}{\ln 2}\right)\left(\frac{40\,\text{g/mol}}{6.02\times 10^{23}/\text{mol}}\right)$$

which gives 0.66 g for the mass of the sample once we plug in 1.7×10^5/s for the decay rate and $1.28\times 10^9\,\text{y} = 4.04\times 10^{16}\,\text{s}$ for the half-life.

43. (a) The sample is in secular equilibrium with the source and the decay rate equals the production rate. Let R be the rate of production of ^{56}Mn and let λ be the disintegration constant. According to the result of problem 41, $R = \lambda N$ after a long time has passed. Now, $\lambda N = 8.88\times 10^{10}\,\text{s}^{-1}$, so $R = 8.88\times 10^{10}\,\text{s}^{-1}$.

(b) They decay at the same rate as they are produced, $8.88\times 10^{10}\,\text{s}^{-1}$.

(c) We use $N = R/\lambda$. If $T_{1/2}$ is the half-life, then the disintegration constant is $\lambda = (\ln 2)/T_{1/2} = (\ln 2)/(2.58\,\text{h}) = 0.269\,\text{h}^{-1} = 7.46\times 10^{-5}\,\text{s}^{-1}$, so $N = (8.88\times 10^{10}\,\text{s}^{-1})/(7.46\times 10^{-5}\,\text{s}^{-1}) = 1.19\times 10^{15}$.

(d) The mass of a ^{56}Mn nucleus is $(56\,\text{u})(1.661\times 10^{-24}\,\text{g/u}) = 9.30\times 10^{-23}\,\text{g}$ and the total mass of ^{56}Mn in the sample at the end of the bombardment is $Nm = (1.19\times 10^{15})(9.30\times 10^{-23}\,\text{g}) = 1.11\times 10^{-7}\,\text{g}$.

44. (a) The rate at which Radium-226 is decaying is

$$R = \lambda N = \left(\frac{\ln 2}{T_{1/2}}\right)\left(\frac{M}{m}\right) = \frac{(\ln 2)(1.00\,\text{mg})(6.02\times 10^{23}/\text{mol})}{(1600\,\text{y})(3.15\times 10^7\,\text{s/y})(226\,\text{g/mol})} = 3.66\times 10^7\ \text{s}^{-1}\ .$$

(b) Since $1600\,\text{y} \gg 3.82\,\text{d}$ the time required is $t \gg 3.82\,\text{d}$.

(c) It is decaying at the same rate as it is produced, or $R = 3.66\times 10^7\,\text{s}^{-1}$.

(d) From $R_{\text{Ra}} = R_{\text{Rn}}$ and $R = \lambda N = (\ln 2/T_{1/2})(M/m)$, we get

$$\begin{aligned} M_{\text{Rn}} &= \left(\frac{T_{1/2\text{Rn}}}{T_{1/2\text{Ra}}}\right)\left(\frac{m_{\text{Rn}}}{m_{\text{Ra}}}\right) M_{\text{Ra}} \\ &= \frac{(3.82\,\text{d})(1.00\times 10^{-3}\,\text{g})(222\,\text{u})}{(1600\,\text{y})(365\,\text{d/y})(226\,\text{u})} \\ &= 6.42\times 10^{-9}\ \text{g}\ . \end{aligned}$$

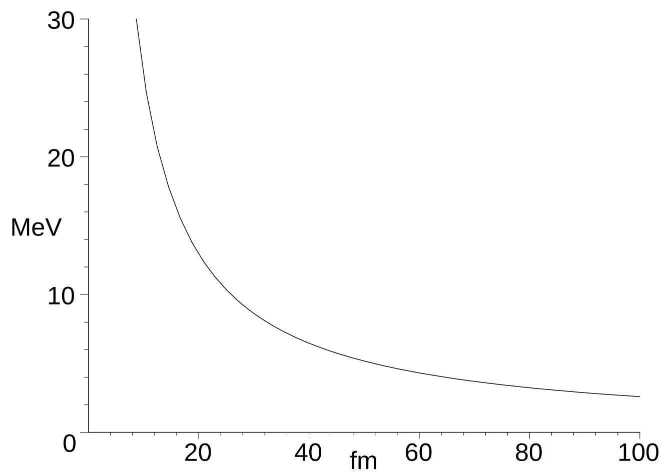
45. Since the spreading is assumed uniform, the count rate $R = 74,000/\text{s}$ is given by $R = \lambda N = \lambda(M/m)(a/A)$, where $M = 400\,\text{g}$, m is the mass of the ^{90}Sr nucleus, $A = 2000\,\text{km}^2$, and a is the area in question. We solve for a:

$$\begin{aligned} a &= A\left(\frac{m}{M}\right)\left(\frac{R}{\lambda}\right) = \frac{AmRT_{1/2}}{M\ln 2} \\ &= \frac{(2000\times 10^6\,\text{m}^2)(90\,\text{g/mol})(29\,\text{y})(3.15\times 10^7\,\text{s/y})(74,000/\text{s})}{(400\,\text{g})(6.02\times 10^{23}/\text{mol})(\ln 2)} \\ &= 7.3\times 10^{-2}\,\text{m}^2 = 730\ \text{cm}^2\ . \end{aligned}$$

46. Eq. 25-43 gives the electrostatic potential energy between two uniformly charged spherical charges (in this case $q_1 = 2e$ and $q_2 = 90e$) with r being the distance between their centers. Assuming the "uniformly charged spheres" condition is met in this instance, we write the equation in such a way that we can make use of $k = 1/4\pi\varepsilon_0$ and the electronvolt unit:

$$U = k\frac{(2e)(90e)}{r} = \left(8.99\times 10^9\,\frac{\text{V}\cdot\text{m}}{\text{C}}\right)\frac{\left(3.2\times 10^{-19}\,\text{C}\right)(90e)}{r} = \frac{2.59\times 10^{-7}}{r}\ \text{eV}$$

with r understood to be in meters. It is convenient to write this for r in femtometers, in which case $U = 259/r$ MeV. This is shown plotted below.



47. The fraction of undecayed nuclei remaining after time t is given by

$$\frac{N}{N_0} = e^{-\lambda t} = e^{-(\ln 2)t/T_{1/2}}$$

where λ is the disintegration constant and $T_{1/2}$ $(= (\ln 2)/\lambda)$ is the half-life. The time for half the original ^{238}U nuclei to decay is 4.5×10^9 y. For ^{244}Pu at that time,

$$\frac{(\ln 2)t}{T_{1/2}} = \frac{(\ln 2)(4.5\times 10^9\,\text{y})}{8.2\times 10^7\,\text{y}} = 38.0$$

and

$$\frac{N}{N_0} = e^{-38.0} = 3.1\times 10^{-17}\ .$$

For ^{248}Cm at that time,

$$\frac{(\ln 2)t}{T_{1/2}} = \frac{(\ln 2)(4.5\times 10^9\,\text{y})}{3.4\times 10^5\,\text{y}} = 9170$$

and

$$\frac{N}{N_0} = e^{-9170} = 3.31 \times 10^{-3983} \ .$$

For any reasonably sized sample this is less than one nucleus and may be taken to be zero. A standard calculator probably cannot evaluate e^{-9170} directly. Our recommendation is to treat it as $(e^{-91.70})^{100}$.

48. (a) The nuclear reaction is written as $^{238}\text{U} \rightarrow {}^{234}\text{Th} + {}^{4}\text{He}$. The energy released is

$$\begin{aligned} \Delta E_1 &= (m_\text{U} - m_\text{He} - m_\text{Th})c^2 \\ &= (238.05079\,\text{u} - 4.00260\,\text{u} - 234.04363\,\text{u})(931.5\,\text{MeV/u}) \\ &= 4.25 \text{ MeV} \ . \end{aligned}$$

(b) The reaction series consists of $^{238}\text{U} \rightarrow {}^{237}\text{U} + n$, followed by

$$\begin{aligned} {}^{237}\text{U} \rightarrow \ & {}^{236}\text{Pa} + p \\ & {}^{236}\text{Pa} \rightarrow {}^{235}\text{Pa} + n \\ & \qquad {}^{235}\text{Pa} \rightarrow {}^{234}\text{Th} + p \end{aligned}$$

The net energy released is then

$$\begin{aligned} \Delta E_2 &= (m_{^{238}\text{U}} - m_{^{237}\text{U}} - m_n)c^2 + (m_{^{237}\text{U}} - m_{^{236}\text{Pa}} - m_p)c^2 \\ &\qquad + (m_{^{236}\text{Pa}} - m_{^{235}\text{Pa}} - m_n)c^2 + (m_{^{235}\text{Pa}} - m_{^{234}\text{Th}} - m_p)c^2 \\ &= (m_{^{238}\text{U}} - 2m_n - 2m_p - m_{^{234}\text{Th}})c^2 \\ &= [238.05079\,\text{u} - 2(1.00867\,\text{u}) - 2(1.00783\,\text{u}) - 234.04363\,\text{u}](931.5\,\text{MeV/u}) \\ &= -24.1 \text{ MeV} \ . \end{aligned}$$

(c) This leads us to conclude that the binding energy of the α particle is

$$\left|(2m_n + 2m_p - m_\text{He})c^2\right| = |-24.1\,\text{MeV} - 4.25\,\text{MeV}| = 28.3 \text{ MeV} \ .$$

49. Energy and momentum are conserved. We assume the residual thorium nucleus is in its ground state. Let K_α be the kinetic energy of the alpha particle and K_Th be the kinetic energy of the thorium nucleus. Then, $Q = K_\alpha + K_\text{Th}$. We assume the uranium nucleus is initially at rest. Then, conservation of momentum yields $0 = p_\alpha + p_\text{Th}$, where p_α is the momentum of the alpha particle and p_Th is the momentum of the thorium nucleus. Both particles travel slowly enough that the classical relationship between momentum and energy can be used. Thus $K_\text{Th} = p_\text{Th}^2/2m_\text{Th}$, where m_Th is the mass of the thorium nucleus. We substitute $p_\text{Th} = -p_\alpha$ and use $K_\alpha = p_\alpha^2/2m_\alpha$ to obtain $K_\text{Th} = (m_\alpha/m_\text{Th})K_\alpha$. Consequently,

$$Q = K_\alpha + \frac{m_\alpha}{m_\text{Th}} K_\alpha = \left(1 + \frac{m_\alpha}{m_\text{Th}}\right) K_\alpha = \left(1 + \frac{4.00\,\text{u}}{234\,\text{u}}\right)(4.196\,\text{MeV}) = 4.27 \text{ MeV} \ .$$

50. (a) The disintegration energy for uranium-235 "decaying" into thorium-232 is

$$\begin{aligned} Q_3 &= \left(m_{^{235}\text{U}} - m_{^{232}\text{Th}} - m_{^{3}\text{He}}\right)c^2 \\ &= (235.0439\,\text{u} - 232.0381\,\text{u} - 3.0160\,\text{u})(931.5\,\text{MeV/u}) \\ &= -9.50 \text{ MeV} \ . \end{aligned}$$

(b) Similarly, the disintegration energy for uranium-235 decaying into thorium-231 is

$$\begin{aligned} Q_4 &= \left(m_{^{235}\text{U}} - m_{^{231}\text{Th}} - m_{^{4}\text{He}}\right)c^2 \\ &= (235.0439\,\text{u} - 231.0363\,\text{u} - 4.0026\,\text{u})(931.5\,\text{MeV/u}) \\ &= 4.66 \text{ MeV} \ . \end{aligned}$$

(c) Finally, the considered transmutation of uranium-235 into thorium-230 has a Q-value of

$$\begin{aligned} Q_5 &= \left(m_{^{235}\mathrm{U}} - m_{^{230}\mathrm{Th}} - m_{^{5}\mathrm{He}}\right)c^2 \\ &= (235.0439\,\mathrm{u} - 230.0331\,\mathrm{u} - 5.0122\,\mathrm{u})(931.5\,\mathrm{MeV/u}) \\ &= -1.30\ \mathrm{MeV}\ . \end{aligned}$$

Only the second decay process (the α decay) is spontaneous, as it releases energy.

51. (a) For the first reaction

$$\begin{aligned} Q_1 &= (m_{\mathrm{Ra}} - m_{\mathrm{Pb}} - m_{\mathrm{C}})c^2 \\ &= (223.01850\,\mathrm{u} - 208.98107\,\mathrm{u} - 14.00324\,\mathrm{u})(931.5\,\mathrm{MeV/u}) \\ &= 31.8\ \mathrm{MeV}\ , \end{aligned}$$

and for the second one

$$\begin{aligned} Q_2 &= (m_{\mathrm{Ra}} - m_{\mathrm{Rn}} - m_{\mathrm{He}})c^2 \\ &= (223.01850\,\mathrm{u} - 219.00948\,\mathrm{u} - 4.00260\,\mathrm{u})(931.5\,\mathrm{MeV/u}) \\ &= 5.98\ \mathrm{MeV}\ . \end{aligned}$$

(b) From $U \propto q_1 q_2/r$, we get

$$U_1 \approx U_2\left(\frac{q_{\mathrm{Pb}}\,q_C}{q_{\mathrm{Rn}}\,q_{\mathrm{He}}}\right) = (30.0\,\mathrm{MeV})\frac{(82e)(6.0e)}{(86e)(2.0e)} = 86\ \mathrm{MeV}\ .$$

52. (a) The mass number A of a radionuclide changes by 4 in an α decay and is unchanged in a β decay. If the mass numbers of two radionuclides are given by $4n+k$ and $4n'+k$ (where $k = 0, 1, 2, 3$), then the heavier one can decay into the lighter one by a series of α (and β) decays, as their mass numbers differ by only an integer times 4. If $A = 4n+k$, then after α-decaying for m times, its mass number becomes $A = 4n+k-4m = 4(n-m)+k$, still in the same chain.

(b) $235 = 58\times 4+3 = 4n_1+3,\ 236 = 59\times 4 = 4n_2,\ 238 = 59\times 4+2 = 4n_2+2,\ 239 = 59\times 4+3 = 4n_2+3,\ 240 = 60\times 4 = 4n_3,\ 245 = 61\times 4+1 = 4n_4+1,\ 246 = 61\times 4+2 = 4n_4+2,\ 249 = 62\times 4+1 = 4n_5+1,\ 253 = 63\times 4+1 = 4n_6+1.$

53. Let ${}^A_Z X$ represent the unknown nuclide. The reaction equation is

$${}^A_Z X + {}^1_0\mathrm{n} \to {}^{\ 0}_{-1}\mathrm{e} + 2\,{}^4_2\mathrm{He}\ .$$

Conservation of charge yields $Z+0 = -1+4$ or $Z = 3$. Conservation of mass number yields $A+1 = 0+8$ or $A = 7$. According to the periodic table in Appendix G (also see Appendix F), lithium has atomic number 3, so the nuclide must be ${}^7_3\mathrm{Li}$.

54. (a) We recall that $mc^2 = 0.511$ MeV from Table 38-3, and note that the result of problem 3 in Chapter 39 can be written as $hc = 1240\,\mathrm{MeV\cdot fm}$. Using Eq. 38-51 and Eq. 39-13, we obtain

$$\begin{aligned} \lambda &= \frac{h}{p} = \frac{hc}{\sqrt{K^2 + 2Kmc^2}} \\ &= \frac{1240\,\mathrm{MeV\cdot fm}}{\sqrt{(1.0\,\mathrm{MeV})^2 + 2(1.0\,\mathrm{MeV})(0.511\,\mathrm{MeV})}} = 9.0\times 10^2\ \mathrm{fm}\ . \end{aligned}$$

(b) $r = r_0 A^{1/3} = (1.2\,\mathrm{fm})(150)^{1/3} = 6.4\,\mathrm{fm}$.

(c) Since $\lambda \gg r$ the electron cannot be confined in the nuclide. We recall from Chapters 40 and 41, that at least $\lambda/2$ was needed in any particular direction, to support a standing wave in an "infinite well." A finite well is able to support *slightly* less than $\lambda/2$ (as one can infer from the ground state wavefunction in Fig. 40-8), but in the present case λ/r is far too big to be supported.

(d) A strong case can be made on the basis of the remarks in part (c), above.

55. Let M_{Cs} be the mass of one atom of ${}^{137}_{55}\text{Cs}$ and M_{Ba} be the mass of one atom of ${}^{137}_{56}\text{Ba}$. To obtain the nuclear masses, we must subtract the mass of 55 electrons from M_{Cs} and the mass of 56 electrons from M_{Ba}. The energy released is $Q = [(M_{\text{Cs}} - 55m) - (M_{\text{Ba}} - 56m) - m]\, c^2$, where m is the mass of an electron. Once cancellations have been made, $Q = (M_{\text{Cs}} - M_{\text{Ba}})c^2$ is obtained. Therefore,

$$Q = [136.9071\,\text{u} - 136.9058\,\text{u}]\, c^2 = (0.0013\,\text{u})c^2 = (0.0013\,\text{u})(931.5\,\text{MeV/u}) = 1.21\ \text{MeV}\ .$$

56. Assuming the neutrino has negligible mass, then

$$\Delta m\, c^2 = (\mathbf{m}_{\text{Ti}} - \mathbf{m}_{\text{V}} - m_e)\, c^2\ .$$

Now, since Vanadium has 23 electrons (see Appendix F and/or G) and Titanium has 22 electrons, we can add and subtract $22m_e$ to the above expression and obtain

$$\Delta m\, c^2 = (\mathbf{m}_{\text{Ti}} + 22m_e - \mathbf{m}_{\text{V}} - 23m_e)\, c^2 = (m_{\text{Ti}} - m_{\text{V}}\,)\, c^2\ .$$

We note that our final expression for Δmc^2 involves the *atomic* masses, and that this assumes (due to the way they are usually tabulated) the atoms are in the ground states (which is certainly not the case here, as we discuss below). The question now is: do we set $Q = -\Delta mc^2$ as in Sample Problem 43-7? The answer is "no." The atom is left in an excited (high energy) state due to the fact that an electron was captured from the lowest shell (where the absolute value of the energy, E_K, is quite large for large Z – see Eq. 41-25). To a very good approximation, the energy of the K-shell electron in Vanadium is equal to that in Titanium (where there is now a "vacancy" that must be filled by a readjustment of the whole electron cloud), and we write $Q = -\Delta mc^2 - E_K$ so that Eq. 43-27 still holds. Thus,

$$Q = (m_{\text{V}} - m_{\text{Ti}}\,)\, c^2 - E_K\ .$$

57. The decay scheme is $\text{n} \to \text{p} + \text{e}^- + \nu$. The electron kinetic energy is a maximum if no neutrino is emitted. Then, $K_{\max} = (m_n - m_p - m_e)c^2$, where m_n is the mass of a neutron, m_p is the mass of a proton, and m_e is the mass of an electron. Since $m_p + m_e = m_H$, where m_H is the mass of a hydrogen atom, this can be written $K_{\max} = (m_n - m_H)c^2$. Hence, $K_{\max} = (840 \times 10^{-6}\,\text{u})c^2 = (840 \times 10^{-6}\,\text{u})(931.5\,\text{MeV/u}) = 0.783\,\text{MeV}$.

58. We obtain

$$\begin{aligned} Q &= (m_{\text{V}} - m_{\text{Ti}}\,)\, c^2 - E_K \\ &= (48.94852\,\text{u} - 48.94787\,\text{u})\,(931.5\,\text{MeV/u}) - 0.00547\,\text{MeV} \\ &= 0.600\ \text{MeV}\ . \end{aligned}$$

59. (a) Since the positron has the same mass as an electron, and the neutrino has negligible mass, then

$$\Delta m\, c^2 = (\mathbf{m}_{\text{B}} + m_e - \mathbf{m}_{\text{C}})\, c^2\ .$$

Now, since Carbon has 6 electrons (see Appendix F and/or G) and Boron has 5 electrons, we can add and subtract $6m_e$ to the above expression and obtain

$$\Delta m\, c^2 = (\mathbf{m}_{\text{B}} + 7m_e - \mathbf{m}_{\text{C}} - 6m_e)\, c^2 = (m_{\text{B}} + 2m_e - m_{\text{C}}\,)\, c^2\ .$$

We note that our final expression for Δmc^2 involves the *atomic* masses, as well an "extra" term corresponding to two electron masses. From Eq. 38-47 and Table 38-3, we obtain

$$Q = (m_{\text{C}} - m_{\text{B}} - 2m_e)\, c^2 = (m_{\text{C}} - m_{\text{B}})\, c^2 - 2(0.511\,\text{MeV})\ .$$

(b) The disintegration energy for the positron decay of Carbon-11 is

$$Q = (11.011434\,\mathrm{u} - 11.009305\,\mathrm{u})\,(931.5\,\mathrm{MeV/u}) - 1.022\,\mathrm{MeV} = 0.961\ \mathrm{MeV}\ .$$

60. (a) The rate of heat production is

$$\begin{aligned}\frac{dE}{dt} &= \sum_{i=1}^{3} R_iQ_i = \sum_{i=1}^{3}\lambda_1 N_i Q_i = \sum_{i=1}^{3}\left(\frac{\ln 2}{T_{1/2_i}}\right)\frac{(1.00\,\mathrm{kg})f_i}{m_i}Q_i \\ &= \frac{(1.00\,\mathrm{kg})(\ln 2)(1.60\times 10^{-13}\,\mathrm{J/MeV})}{(3.15\times 10^{7}\,\mathrm{s/y})(1.661\times 10^{-27}\,\mathrm{kg/u})}\left[\frac{(4\times 10^{-6})(51.7\,\mathrm{MeV})}{(238\,\mathrm{u})(4.47\times 10^{9}\,\mathrm{y})}\right. \\ &\qquad \left. + \frac{(13\times 10^{-6})(42.7\,\mathrm{MeV})}{(232\,\mathrm{u})(1.41\times 10^{10}\,\mathrm{y})} + \frac{(4\times 10^{-6})(1.31\,\mathrm{MeV})}{(40\,\mathrm{u})(1.28\times 10^{9}\,\mathrm{y})}\right] \\ &= 1.0\times 10^{-9}\ \mathrm{W}\ .\end{aligned}$$

(b) The contribution to heating, due to radioactivity, is $P = (2.7\times 10^{22}\,\mathrm{kg})(1.0\times 10^{-9}\,\mathrm{W/kg}) = 2.7\times 10^{13}\,\mathrm{W}$, which is very small compared to what is received from the Sun.

61. Since the electron has the maximum possible kinetic energy, no neutrino is emitted. Since momentum is conserved, the momentum of the electron and the momentum of the residual sulfur nucleus are equal in magnitude and opposite in direction. If p_e is the momentum of the electron and p_S is the momentum of the sulfur nucleus, then $p_S = -p_e$. The kinetic energy K_S of the sulfur nucleus is $K_S = p_S^2/2M_S = p_e^2/2M_S$, where M_S is the mass of the sulfur nucleus. Now, the electron's kinetic energy K_e is related to its momentum by the relativistic equation $(p_ec)^2 = K_e^2 + 2K_emc^2$, where m is the mass of an electron. See Eq. 38-51. Thus,

$$\begin{aligned}K_S &= \frac{(p_ec)^2}{2M_Sc^2} = \frac{K_e^2 + 2K_emc^2}{2M_Sc^2} = \frac{(1.71\,\mathrm{MeV})^2 + 2(1.71\,\mathrm{MeV})(0.511\,\mathrm{MeV})}{2(32\,\mathrm{u})(931.5\,\mathrm{MeV/u})} \\ &= 7.83\times 10^{-5}\,\mathrm{MeV} = 78.3\ \mathrm{eV}\end{aligned}$$

where $mc^2 = 0.511\,\mathrm{MeV}$ is used (see Table 38-3).

62. We solve for t from $R = R_0e^{-\lambda t}$:

$$t = \frac{1}{\lambda}\ln\frac{R_0}{R} = \left(\frac{5730\,\mathrm{y}}{\ln 2}\right)\ln\left[\left(\frac{15.3}{63.0}\right)\left(\frac{5.00}{1.00}\right)\right] = 1.61\times 10^{3}\ \mathrm{y}\ .$$

63. (a) The mass of a ^{238}U atom is $(238\,\mathrm{u})(1.661\times 10^{-24}\,\mathrm{g/u}) = 3.95\times 10^{-22}\,\mathrm{g}$, so the number of uranium atoms in the rock is $N_U = (4.20\times 10^{-3}\,\mathrm{g})/(3.95\times 10^{-22}\,\mathrm{g}) = 1.06\times 10^{19}$. The mass of a ^{206}Pb atom is $(206\,\mathrm{u})(1.661\times 10^{-24}\,\mathrm{g}) = 3.42\times 10^{-22}\,\mathrm{g}$, so the number of lead atoms in the rock is $N_{\mathrm{Pb}} = (2.135\times 10^{-3}\,\mathrm{g})/(3.42\times 10^{-22}\,\mathrm{g}) = 6.24\times 10^{18}$.

(b) If no lead was lost, there was originally one uranium atom for each lead atom formed by decay, in addition to the uranium atoms that did not yet decay. Thus, the original number of uranium atoms was $N_{U0} = N_U + N_{\mathrm{Pb}} = 1.06\times 10^{19} + 6.24\times 10^{18} = 1.68\times 10^{19}$.

(c) We use

$$N_U = N_{U0}\,e^{-\lambda t}$$

where λ is the disintegration constant for the decay. It is related to the half-life $T_{1/2}$ by $\lambda = (\ln 2)/T_{1/2}$. Thus

$$t = -\frac{1}{\lambda}\,\ln\left(\frac{N_U}{N_{U0}}\right) = -\frac{T_{1/2}}{\ln 2}\,\ln\left(\frac{N_U}{N_{U0}}\right) = -\frac{4.47\times 10^{9}\,\mathrm{y}}{\ln 2}\,\ln\left(\frac{1.06\times 10^{19}}{1.68\times 10^{19}}\right) = 2.97\times 10^{9}\ \mathrm{y}\ .$$

64. The original amount of ^{238}U the rock contains is given by

$$m_0 = me^{\lambda t} = (3.70\,\text{mg})\, e^{(\ln 2)(260\times 10^6\,\text{y})/(4.47\times 10^9\,\text{y})} = 3.85 \text{ mg} .$$

Thus, the amount of lead produced is

$$m' = (m_0 - m)\left(\frac{m_{206}}{m_{238}}\right) = (3.85\,\text{mg} - 3.70\,\text{mg})\left(\frac{206}{238}\right) = 0.132 \text{ mg} .$$

65. We can find the age t of the rock from the masses of ^{238}U and ^{206}Pb. The initial mass of ^{238}U is

$$m_{\text{U}_0} = m_\text{U} + \frac{238}{206} m_\text{Pb} .$$

Therefore, $m_\text{U} = m_{\text{U}_0} e^{-\lambda_\text{U} t} = (m_\text{U} + m_{^{238}\text{Pb}}/206)e^{-(t\ln 2)/T_{1/2\text{U}}}$. We solve for t:

$$\begin{aligned} t &= \frac{T_{1/2\text{U}}}{\ln 2}\ln\left(\frac{m_\text{U} + (238/206)m_\text{Pb}}{m_\text{U}}\right) \\ &= \frac{4.47\times 10^9\text{ y}}{\ln 2}\ln\left[1 + \left(\frac{238}{206}\right)\left(\frac{0.15\,\text{mg}}{0.86\,\text{mg}}\right)\right] \\ &= 1.18\times 10^9 \text{ y} . \end{aligned}$$

For the β decay of ^{40}K, the initial mass of ^{40}K is

$$m_{\text{K}_0} = m_\text{K} + (40/40)m_\text{Ar} = m_\text{K} + m_\text{Ar} ,$$

so

$$m_\text{K} = m_{\text{K}_0} e^{-\lambda_\text{K} t} = (m_\text{K} + m_\text{Ar})e^{-\lambda_\text{K} t} .$$

We solve for m_K:

$$\begin{aligned} m_\text{K} &= \frac{m_\text{Ar} e^{-\lambda_\text{K} t}}{1 - e^{-\lambda_\text{K} t}} = \frac{m_\text{Ar}}{e^{\lambda_\text{K} t} - 1} \\ &= \frac{1.6\,\text{mg}}{e^{(\ln 2)(1.18\times 10^9\,\text{y})/(1.25\times 10^9\,\text{y})} - 1} = 1.7 \text{ mg} . \end{aligned}$$

66. The becquerel (Bq) and curie (Ci) are defined in §43-3. Thus, $R = 8700/60 = 145$ Bq, and

$$R = \frac{145\,\text{Bq}}{3.7\times 10^{10}\,\text{Bq/Ci}} = 3.92\times 10^{-9} \text{ Ci} .$$

67. The decay rate R is related to the number of nuclei N by $R = \lambda N$, where λ is the disintegration constant. The disintegration constant is related to the half-life $T_{1/2}$ by $\lambda = (\ln 2)/T_{1/2}$, so $N = R/\lambda = RT_{1/2}/\ln 2$. Since $1\,\text{Ci} = 3.7\times 10^{10}$ disintegrations/s,

$$N = \frac{(250\,\text{Ci})(3.7\times 10^{10}\,\text{s}^{-1}/\text{Ci})(2.7\,\text{d})(8.64\times 10^4\,\text{s/d})}{\ln 2} = 3.11\times 10^{18} .$$

The mass of a ^{198}Au atom is $M = (198\,\text{u})(1.661\times 10^{-24}\,\text{g/u}) = 3.29\times 10^{-22}\,\text{g}$, so the mass required is $NM = (3.11\times 10^{18})(3.29\times 10^{-22}\,\text{g}) = 1.02\times 10^{-3}\,\text{g} = 1.02\,\text{mg}$.

68. The annual dose equivalent is $(20\,\text{h})(52\,\text{week/y})(7.0\,\mu\text{Sv/h}) = 7.3\,\text{mSv}$.

69. The dose equivalent is the product of the absorbed dose and the RBE factor, so the absorbed dose is (dose equivalent)/(RBE) $= (250\times 10^{-6}\,\text{Sv})/(0.85) = 2.94\times 10^{-4}\,\text{Gy}$. But $1\,\text{Gy} = 1\,\text{J/kg}$, so the absorbed dose is

$$\left(2.94\times 10^{-4}\,\text{Gy}\right)\left(1\,\frac{\text{J}}{\text{kg}\cdot\text{Gy}}\right) = 2.94\times 10^{-4} \text{ J/kg} .$$

To obtain the total energy received, we multiply this by the mass receiving the energy: $E = (2.94\times 10^{-4}\,\text{J/kg})(44\,\text{kg}) = 1.29\times 10^{-2}\,\text{J}$.

70. (a) Using Eq. 43-31, the energy absorbed is

$$(2.4\times 10^{-4}\,\text{Gy})(75\,\text{kg}) = 18\,\text{mJ} .$$

(b) The dose equivalent is

$$(2.4\times 10^{-4}\,\text{Gy})(12) = 2.9\times 10^{-3}\,\text{Sv} = 0.29\ \text{rem}$$

where Eq. 43-32 is used in the last step.

71. (a) Adapting Eq. 43-20, we find

$$N_0 = \frac{(2.5\times 10^{-3}\,\text{g})(6.02\times 10^{23}/\text{mol})}{239\,\text{g/mol}} = 6.3\times 10^{18} .$$

(b) From Eq. 43-14 and Eq. 43-17,

$$\begin{aligned} |\Delta N| &= N_0\left[1 - e^{-t\ln 2/T_{1/2}}\right] \\ &= (6.3\times 10^{18})\left[1 - e^{-(12\,\text{h})\ln 2/(24{,}100\,\text{y})(8760\,\text{h/y})}\right] \\ &= 2.5\times 10^{11} . \end{aligned}$$

(c) The energy absorbed by the body is

$$(0.95)E_\alpha|\Delta N| = (0.95)\,(5.2\,\text{MeV})\left(2.5\times 10^{11}\right)\left(1.6\times 10^{-13}\,\text{J/MeV}\right) = 0.20\ \text{J} .$$

(d) On a per unit mass basis, the previous result becomes (according to Eq. 43-31)

$$\frac{0.20\,\text{mJ}}{85\,\text{kg}} = 2.3\times 10^{-3}\,\text{J/kg} = 2.3\ \text{mGy} .$$

(e) Using Eq. 43-32, $(2.3\,\text{mGy})(13) = 30\,\text{mSv}$.

72. From Eq. 20-24, we obtain

$$T = \frac{2}{3}\left(\frac{K_{\text{avg}}}{k}\right) = \frac{2}{3}\left(\frac{5.00\times 10^6\,\text{eV}}{8.62\times 10^{-5}\,\text{eV/K}}\right) = 3.9\times 10^{10}\ \text{K} .$$

73. (a) Following Sample Problem 43-10, we compute

$$\Delta E \approx \frac{\hbar}{t_{\text{avg}}} = \frac{(4.14\times 10^{-15}\,\text{eV}\cdot\text{fs})/2\pi}{1.0\times 10^{-22}\,\text{s}} = 6.6\times 10^6\ \text{eV} .$$

(b) In order to fully distribute the energy in a fairly large nucleus, and create a "compound nucleus" equilibrium configuration, about 10^{-15} s is typically required. A reaction state that exists no more than about 10^{-22} s does not qualify as a compound nucleus.

74. (a) We compare both the proton numbers (atomic numbers, which can be found in Appendix F and/or G) and the neutron numbers (see Eq. 43-1) with the magic nucleon numbers (special values of either Z or N) listed in §43-8. We find that ^{18}O, ^{60}Ni, ^{92}Mo, ^{144}Sm, and ^{207}Pb each have a filled shell for either the protons or the neutrons (two of these, ^{18}O and ^{92}Mo, are explicitly discussed in that section).

(b) Consider ^{40}K, which has $Z = 19$ protons (which is one less than the magic number 20). It has $N = 21$ neutrons, so it has one neutron outside a closed shell for neutrons, and thus qualifies for this list. Others in this list include ^{91}Zr, ^{121}Sb, and ^{143}Nd.

(c) Consider ^{13}C, which has $Z = 6$ and $N = 13 - 6 = 7$ neutrons. Since 8 is a magic number, then ^{13}C has a vacancy in an otherwise filled shell for neutrons. Similar arguments lead to inclusion of ^{40}K, ^{49}Ti, ^{205}Tl, and ^{207}Pb in this list.

75. A generalized formation reaction can be written $\mathrm{X} + \mathrm{x} \to \mathrm{Y}$, where X is the target nucleus, x is the incident light particle, and Y is the excited compound nucleus (^{20}Ne). We assume X is initially at rest. Then, conservation of energy yields

$$m_X c^2 + m_x c^2 + K_x = m_Y c^2 + K_Y + E_Y$$

where m_X, m_x, and m_Y are masses, K_x and K_Y are kinetic energies, and E_Y is the excitation energy of Y. Conservation of momentum yields

$$p_x = p_Y \ .$$

Now, $K_Y = p_Y^2/2m_Y = p_x^2/2m_Y = (m_x/m_Y)K_x$, so

$$m_X c^2 + m_x c^2 + K_x = m_Y c^2 + (m_x/m_Y)K_x + E_Y$$

and

$$K_x = \frac{m_Y}{m_Y - m_x}\left[(m_Y - m_X - m_x)c^2 + E_Y\right] \ .$$

(a) Let x represent the alpha particle and X represent the ^{16}O nucleus. Then, $(m_Y - m_X - m_x)c^2 = (19.99244\,\mathrm{u} - 15.99491\,\mathrm{u} - 4.00260\,\mathrm{u})(931.5\,\mathrm{MeV/u}) = -4.722\,\mathrm{MeV}$ and

$$K_\alpha = \frac{19.99244\,\mathrm{u}}{19.99244\,\mathrm{u} - 4.00260\,\mathrm{u}}(-4.722\,\mathrm{MeV} + 25.0\,\mathrm{MeV}) = 25.35\ \mathrm{MeV} \ .$$

(b) Let x represent the proton and X represent the ^{19}F nucleus. Then, $(m_Y - m_X - m_x)c^2 = (19.99244\,\mathrm{u} - 18.99841\,\mathrm{u} - 1.00783\,\mathrm{u})(931.5\,\mathrm{MeV/u}) = -12.85\,\mathrm{MeV}$ and

$$K_\alpha = \frac{19.99244\,\mathrm{u}}{19.99244\,\mathrm{u} - 1.00783\,\mathrm{u}}(-12.85\,\mathrm{MeV} + 25.0\,\mathrm{MeV}) = 12.80\ \mathrm{MeV} \ .$$

(c) Let x represent the photon and X represent the ^{20}Ne nucleus. Since the mass of the photon is zero, we must rewrite the conservation of energy equation: if E_γ is the energy of the photon, then $E_\gamma + m_X c^2 = m_Y c^2 + K_Y + E_Y$. Since $m_X = m_Y$, this equation becomes $E_\gamma = K_Y + E_Y$. Since the momentum and energy of a photon are related by $p_\gamma = E_\gamma/c$, the conservation of momentum equation becomes $E_\gamma/c = p_Y$. The kinetic energy of the compound nucleus is $K_Y = p_Y^2/2m_Y = E_\gamma^2/2m_Y c^2$. We substitute this result into the conservation of energy equation to obtain

$$E_\gamma = \frac{E_\gamma^2}{2m_Y c^2} + E_Y \ .$$

This quadratic equation has the solutions

$$E_\gamma = m_Y c^2 \pm \sqrt{(m_Y c^2)^2 - 2m_Y c^2 E_Y} \ .$$

If the problem is solved using the relativistic relationship between the energy and momentum of the compound nucleus, only one solution would be obtained, the one corresponding to the negative sign above. Since $m_Y c^2 = (19.99244\,\mathrm{u})(931.5\,\mathrm{MeV/u}) = 1.862 \times 10^4\,\mathrm{MeV}$,

$$\begin{aligned} E_\gamma &= (1.862 \times 10^4\,\mathrm{MeV}) - \sqrt{(1.862 \times 10^4\,\mathrm{MeV})^2 - 2(1.862 \times 10^4\,\mathrm{MeV})(25.0\,\mathrm{MeV})} \\ &= 25.0\ \mathrm{MeV} \ . \end{aligned}$$

The kinetic energy of the compound nucleus is very small; essentially all of the photon energy goes to excite the nucleus.

76. (a) From the decay series, we know that N_{210}, the amount of ^{210}Pb nuclei, changes because of two decays: the decay from ^{226}Ra into ^{210}Pb at the rate $R_{226} = \lambda_{226} N_{226}$, and the decay from ^{210}Pb into ^{206}Pb at the rate $R_{210} = \lambda_{210} N_{210}$. The first of these decays causes N_{210} to increase while the second one causes it to decrease. Thus,

$$\frac{dN_{210}}{dt} = R_{226} - R_{210} = \lambda_{226} N_{226} - \lambda_{210} N_{210} \ .$$

(b) We set $dN_{210}/dt = R_{226} - R_{210} = 0$ to obtain $R_{226}/R_{210} = 1$.

(c) From $R_{226} = \lambda_{226} N_{226} = R_{210} = \lambda_{210} N_{210}$, we obtain

$$\frac{N_{226}}{N_{210}} = \frac{\lambda_{210}}{\lambda_{226}} = \frac{T_{1/2_{226}}}{T_{1/2_{210}}} = \frac{1.60 \times 10^3\,\mathrm{y}}{22.6\,\mathrm{y}} = 70.8 \ .$$

(d) Since only 1.00% of the ^{226}Ra remains, the ratio R_{226}/R_{210} is 0.00100 of that of the equilibrium state computed in part (b). Thus the ratio is $(0.0100)(1) = 0.0100$.

(e) This is similar to part (d) above. Since only 1.00% of the ^{226}Ra remains, the ratio N_{226}/N_{210} is 1.00% of that of the equilibrium state computed in part (c), or $(0.0100)(70.8) = 0.708$.

(f) Since the actual value of N_{226}/N_{210} is 0.09, which much closer to 0.0100 than to 1, the sample of the lead pigment cannot be 300 years old. So *Emmaus* is not a *Vermeer*.

77. Using Eq. 43-14 with Eq. 43-17, we find the fraction remaining:

$$\frac{N}{N_0} = e^{-t\ln 2/T_{1/2}} = e^{-30\ln 2/29} \ = \ 0.49 \ .$$

78. Using Eq. 43-15 with Eq. 43-17, we find the initial activity:

$$R_0 = Re^{t\ln 2/T_{1/2}} = \left(7.4 \times 10^8\,\mathrm{Bq}\right) e^{24\ln 2/83.61} \ = \ 9.0 \times 10^8\ \mathrm{Bq} \ .$$

79. (a) Molybdenum beta decays into Technetium:

$$^{99}_{42}\mathrm{Mo} \ \rightarrow \ ^{99}_{43}\mathrm{Tc} + e^- + \nu$$

(b) Each decay corresponds to a photon produced when the Technetium nucleus de-excites [note that the de-excitation half-life is much less than the beta decay half-life]. Thus, the gamma rate is the same as the decay rate: 8.2×10^7/s.

(c) Eq. 43-19 leads to

$$N = \frac{RT_{1/2}}{\ln 2} = \frac{(38/\mathrm{s})(6.0\,\mathrm{h})(3600\,\mathrm{s/h})}{\ln 2} = 1.2 \times 10^6 \ .$$

80. (a) Assuming a “target” area of one square meter, we establish a ratio:

$$\frac{\text{rate through you}}{\text{total rate upward}} = \frac{1\,\mathrm{m}^2}{(2.6 \times 10^5\,\mathrm{km}^2)(1000\,\mathrm{m/km})^2} = 3.8 \times 10^{-12} \ .$$

The SI unit becquerel is equivalent to a disintegration per second. With half the beta-decay electrons moving upward, we find

$$\text{rate through you} = \frac{1}{2}\left(1 \times 10^{16}/\mathrm{s}\right)\left(3.8 \times 10^{-12}\right) = 1.9 \times 10^4/\mathrm{s}$$

which implies (converting s $\rightarrow$ h) the rate of electrons you would intercept is $R_0 = 7 \times 10^7$/h.

(b) Let D indicate the current year (2000, 2001, etc) Combining Eq. 43-15 and Eq. 43-17, we find

$$R = R_0 e^{-t\ln 2/T_{1/2}} = \left(7 \times 10^7/\mathrm{h}\right) e^{-(D-1996)\ln 2/(30.2\,\mathrm{y})} \ .$$

81. Eq. 43-19 leads to

$$\begin{aligned} R &= \frac{\ln 2}{T_{1/2}} N \\ &= \frac{\ln 2}{30.2\,\mathrm{y}} \left(\frac{M_{\mathrm{sam}}}{m_{\mathrm{atom}}} \right) \\ &= \frac{\ln 2}{9.53 \times 10^8\,\mathrm{s}} \left(\frac{0.0010\,\mathrm{kg}}{137 \times 1.661 \times 10^{-27}\,\mathrm{kg}} \right) \\ &= 3.2 \times 10^{12}\ \mathrm{Bq}\ =\ 86\ \mathrm{Ci}\ . \end{aligned}$$

82. The lines that lead toward the lower left are alpha decays, involving an atomic number change of $\Delta Z_\alpha = -2$ and a mass number change of $\Delta A_\alpha = -4$. The short horizontal lines toward the right are beta decays (involving electrons, not positrons) in which case A stays the same but the change in atomic number is $\Delta Z_\beta = +1$. Fig. 43-16 shows three alpha decays and two beta decays; thus,

$$Z_f = Z_i + 3\Delta Z_\alpha + 2\Delta Z_\beta \qquad \text{and} \qquad A_f = A_i + 3\Delta A_\alpha \ \ .$$

Referring to Appendix F or G, we find $Z_i = 93$ for Neptunium, so $Z_f = 93 + 3(-2) + 2(1) = 89$, which indicates the element Actinium. We are given $A_i = 237$, so $A_f = 237 + 3(-4) = 225$. Therefore, the final isotope is $^{225}\mathrm{Ac}$.

83. We note that every Calcium-40 atom and Krypton-40 atom found now in the sample was once one of the original number of Potassium atoms. Thus, using Eq. 43-13 and Eq. 43-17, we find

$$\begin{aligned} \ln\left(\frac{N_{\mathrm{K}}}{N_{\mathrm{K}} + N_{\mathrm{Ar}} + N_{\mathrm{Ca}}} \right) &= -\lambda t \\ \ln\left(\frac{1}{1 + 1 + 8.54} \right) &= -\frac{\ln 2}{T_{1/2}}\, t \end{aligned}$$

which (with $T_{1/2} = 1.26 \times 10^9\,\mathrm{y}$) yields $t = 4.3 \times 10^9\,\mathrm{y}$.

84. We note that 3.82 days is 330048 s, and that a becquerel is a disintegration per second (see §43-3). From Eq. 34-19, we have

$$\frac{N}{\mathcal{V}} = \frac{R}{\mathcal{V}} \frac{T_{1/2}}{\ln 2} = \left(1.55 \times 10^5\ \frac{\mathrm{Bq}}{\mathrm{m}^3} \right) \frac{330048\,\mathrm{s}}{\ln 2} = 7.4 \times 10^{10}\ \frac{\mathrm{atoms}}{\mathrm{m}^3}$$

where we have divided by volume $\mathcal{V}$. We estimate $\mathcal{V}$ (the volume breathed in $48\,\mathrm{h} = 2880\,\mathrm{min}$) as follows:

$$\left(2\,\frac{\mathrm{Liters}}{\mathrm{breath}} \right) \left(\frac{1\,\mathrm{m}^3}{1000\,\mathrm{L}} \right) \left(40\,\frac{\mathrm{breaths}}{\mathrm{min}} \right) (2880\,\mathrm{min})$$

which yields $\mathcal{V} \approx 200\,\mathrm{m}^3$. Thus, the order of magnitude of N is

$$\left(\frac{N}{\mathcal{V}} \right) (\mathcal{V}) \approx \left(7 \times 10^{10}\ \frac{\mathrm{atoms}}{\mathrm{m}^3} \right) (200\,\mathrm{m}^3)\ \approx\ 10^{13}\ \mathrm{atoms}\ .$$

85. Kinetic energy (we use the classical formula since v is much less than c) is converted into potential energy (see Eq. 25-43). From Appendix F or G, we find $Z = 3$ for Lithium and $Z = 90$ for Thorium; the charges on those nuclei are therefore $3e$ and $90e$, respectively. We manipulate the terms so that one of the factors of e cancels the "e" in the kinetic energy unit MeV, and the other factor of e is set equal to its SI value $1.6 \times 10^{-19}\,\mathrm{C}$. We note that $k = 1/4\pi\varepsilon_0$ can be written as $8.99 \times 10^9\,\mathrm{V{\cdot}m/C}$. Thus, from energy conservation, we have

$$K = U \quad \Longrightarrow \quad r = \frac{kq_1q_2}{K} = \frac{\left(8.99 \times 10^9\ \frac{\mathrm{V\cdot m}}{\mathrm{C}}\right)\left(3 \times 1.6 \times 10^{-19}\,\mathrm{C}\right)(90e)}{3.00 \times 10^6\,\mathrm{eV}}$$

which yields $r = 1.3 \times 10^{-13}\,\mathrm{m}$ (or about 130 fm).

86. Form Appendix F and/or G, we find $Z = 107$ for Bohrium, so this isotope has $N = A - Z = 262 - 107 = 155$ neutrons. Thus,

$$\Delta E_{\text{ben}} = \frac{(Zm_{\text{H}} + Nm_n - m_{\text{Bh}})\, c^2}{A} = \frac{((107)(1.007825\,\text{u}) + (155)(1.008665\,\text{u}) - 262.1231\,\text{u})\,(931.5\,\text{MeV/u})}{262}$$

which yields 7.3 MeV per nucleon.

87. Since R is proportional to N (see Eq. 43-16) then $N/N_0 = R/R_0$. Combining Eq. 43-13 and Eq.43-17 leads to

$$t = -\frac{T_{1/2}}{\ln 2} \ln\left(\frac{R}{R_0}\right) = -\frac{5730\,\text{y}}{\ln 2} \ln(0.020) = 3.2 \times 10^4 \text{ y} .$$

88. Adapting Eq. 43-20, we have

$$N_{\text{Kr}} = \frac{M_{\text{sam}}}{M_{\text{Kr}}} N_A = \left(\frac{20 \times 10^{-9}\,\text{g}}{92\,\text{g/mol}}\right)\left(6.02 \times 10^{23}\,\text{atoms/mol}\right) = 1.3 \times 10^{14} \text{ atoms} .$$

Consequently, Eq. 43-19 leads to

$$R = \frac{N \ln 2}{T_{1/2}} = \frac{\left(1.3 \times 10^{14}\right) \ln 2}{1.84\,\text{s}} = 4.9 \times 10^{13} \text{ Bq} .$$