Chapter 29

1. (a) We use Eq. 29-3: $F_B = |q|vB\sin\phi = (+3.2\times10^{-19}\,\mathrm{C})(550\,\mathrm{m/s})(0.045\,\mathrm{T})(\sin 52^\circ) = 6.2\times10^{-18}\,\mathrm{N}$.
   (b) $a = F_B/m = (6.2\times10^{-18}\,\mathrm{N})/(6.6\times10^{-27}\,\mathrm{kg}) = 9.5\times10^{8}\,\mathrm{m/s^2}$.
   (c) Since it is perpendicular to $\vec{v}$, $\vec{F}_B$ does not do any work on the particle. Thus from the work-energy theorem both the kinetic energy and the speed of the particle remain unchanged.

2. (a) The largest value of force occurs if the velocity vector is perpendicular to the field. Using Eq. 29-3,

$$F_{B,\,\max} = |q|vB\sin(90^\circ) = evB = (1.60\times10^{-19}\,\mathrm{C})(7.20\times10^{6}\,\mathrm{m/s})(83.0\times10^{-3}\,\mathrm{T}) = 9.56\times10^{-14}\ \mathrm{N}\ .$$

   The smallest value occurs if they are parallel: $F_{B,\,\min} = |q|vB\sin(0) = 0$.
   (b) By Newton's second law, $a = F_B/m_e = |q|vB\sin\theta/m_e$, so the angle θ between $\vec{v}$ and $\vec{B}$ is

$$\theta = \sin^{-1}\left(\frac{m_e a}{|q|vB}\right) = \sin^{-1}\left[\frac{(9.11\times10^{-31}\,\mathrm{kg})(4.90\times10^{14}\,\mathrm{m/s^2})}{(1.60\times10^{-16}\,\mathrm{C})(7.20\times10^{6}\,\mathrm{m/s})(83.0\times10^{-3}\,\mathrm{T})}\right] = 0.267^\circ\ .$$

3. (a) Eq. 29-3 leads to

$$v = \frac{F_B}{eB\sin\phi} = \frac{6.50\times10^{-17}\,\mathrm{N}}{(1.60\times10^{-19}\,\mathrm{C})(2.60\times10^{-3}\,\mathrm{T})\sin 23.0^\circ} = 4.00\times10^{5}\ \mathrm{m/s}\ .$$

   (b) The kinetic energy of the proton is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(1.67\times10^{-27}\,\mathrm{kg})(4.00\times10^{5}\,\mathrm{m/s})^2 = 1.34\times10^{-16}\ \mathrm{J}\ .$$

   This is $(1.34\times10^{-16}\,\mathrm{J})/(1.60\times10^{-19}\,\mathrm{J/eV}) = 835$ eV.

4. (a) The force on the electron is

$$\begin{aligned}\vec{F}_B &= q\vec{v}\times\vec{B} = q(v_x\hat{\imath} + v_y\hat{\jmath})\times(B_x\hat{\imath} + B_y\vec{j}) \\ &= q(v_xB_y - v_yB_x)\hat{\mathrm{k}} \\ &= (-1.6\times10^{-19}\,\mathrm{C})[(2.0\times10^{6}\,\mathrm{m/s})(-0.15\,\mathrm{T}) - (3.0\times10^{6}\,\mathrm{m/s})(0.030\,\mathrm{T})] \\ &= (6.2\times10^{-14}\ \mathrm{N})\ \hat{\mathrm{k}}\ .\end{aligned}$$

   Thus, the magnitude of $\vec{F}_B$ is $6.2\times10^{14}\,\mathrm{N}$, and $\vec{F}_B$ points in the positive z direction.
   (b) This amounts to repeating the above computation with a change in the sign in the charge. Thus, $\vec{F}_B$ has the same magnitude but points in the negative z direction.

5. (a) The textbook uses "geomagnetic north" to refer to Earth's magnetic pole lying in the northern hemisphere. Thus, the electrons are traveling northward. The vertical component of the magnetic field is downward. The right-hand rule indicates that $\vec{v}\times\vec{B}$ is to the west, but since the electron is negatively charged (and $\vec{F} = q\vec{v}\times\vec{B}$), the magnetic force on it is to the east.

(b) We combine $F = m_e a$ with $F = evB\sin\phi$. Here, $B\sin\phi$ represents the downward component of Earth's field (given in the problem). Thus, $a = evB/m_e$. Now, the electron speed can be found from its kinetic energy. Since $K = \frac{1}{2}mv^2$,

$$v = \sqrt{\frac{2K}{m_e}} = \sqrt{\frac{2(12.0\times 10^3\,\mathrm{eV})(1.60\times 10^{-19}\ \mathrm{J/eV})}{9.11\times 10^{-31}\,\mathrm{kg}}} = 6.49\times 10^7\ \mathrm{m/s}\ .$$

Therefore,

$$a = \frac{evB}{m_e} = \frac{(1.60\times 10^{-19}\,\mathrm{C})(6.49\times 10^7\,\mathrm{m/s})(55.0\times 10^{-6}\,\mathrm{T})}{9.11\times 10^{-31}\,\mathrm{kg}} = 6.27\times 10^{14}\ \mathrm{m/s^2}\ .$$

(c) We ignore any vertical deflection of the beam which might arise due to the horizontal component of Earth's field. Technically, then, the electron should follow a circular arc. However, the deflection is so small that many of the technicalities of circular geometry may be ignored, and a calculation along the lines of projectile motion analysis (see Chapter 4) provides an adequate approximation:

$$\Delta x = vt \quad\Longrightarrow\quad t = \frac{\Delta x}{v} = \frac{0.200\,\mathrm{m}}{6.49\times 10^7\,\mathrm{m/s}}$$

which yields a time of $t = 3.08\times 10^{-9}$ s. Then, with our y axis oriented eastward,

$$\Delta y = \frac{1}{2}at^2 = \frac{1}{2}\left(6.27\times 10^{14}\right)\left(3.08\times 10^{-9}\right)^2 = 0.00298\ \mathrm{m}\ .$$

6. (a) The net force on the proton is given by

$$\begin{aligned}\vec{F} &= \vec{F}_E + \vec{F}_B = q\vec{E} + q\vec{v}\times\vec{B}\\ &= (1.6\times 10^{-19}\,\mathrm{C})[(4.0\,\mathrm{V/m})\hat{\mathrm{k}} + (2000\,\mathrm{m/s})\hat{\mathrm{j}}\times(-2.5\,\mathrm{mT})\hat{\mathrm{i}}]\\ &= (1.4\times 10^{-18}\ \mathrm{N})\ \hat{\mathrm{k}}\ .\end{aligned}$$

(b) In this case

$$\begin{aligned}\vec{F} &= \vec{F}_E + \vec{F}_B = q\vec{E} + q\vec{v}\times\vec{B}\\ &= (1.6\times 10^{-19}\,\mathrm{C})[(-4.0\,\mathrm{V/m})\hat{\mathrm{k}} + (2000\,\mathrm{m/s})\hat{\mathrm{j}}\times(-2.5\,\mathrm{mT})\hat{\mathrm{i}}]\\ &= (1.6\times 10^{-19}\ \mathrm{N})\ \hat{\mathrm{k}}\ .\end{aligned}$$

(c) In the final case,

$$\begin{aligned}\vec{F} &= \vec{F}_E + \vec{F}_B = q\vec{E} + q\vec{v}\times\vec{B}\\ &= (1.6\times 10^{-19}\,\mathrm{C})[(4.0\,\mathrm{V/m})\hat{\mathrm{i}} + (2000\,\mathrm{m/s})\hat{\mathrm{j}}\times(-2.5\,\mathrm{mT})\hat{\mathrm{i}}]\\ &= (6.4\times 10^{-19}\,\mathrm{N})\,\hat{\mathrm{i}} + (8.0\times 10^{-19}\ \mathrm{N})\,\hat{\mathrm{k}}\ .\end{aligned}$$

The magnitude of the force is now

$$\sqrt{F_x^2 + F_y^2 + F_z^2} = \sqrt{(6.4\times 10^{-19}\,\mathrm{N})^2 + 0 + (8.0\times 10^{-19}\,\mathrm{N})^2} = 1.0\times 10^{-18}\ \mathrm{N}\ .$$

7. (a) Equating the magnitude of the electric force ($F = eE$) with that of the magnetic force (Eq. 29-3), we obtain $B = E/v\sin\phi$. The field is smallest when the $\sin\phi$ factor is at its largest value; that is, when $\phi = 90°$. Now, we use $K = \frac{1}{2}mv^2$ to find the speed:

$$v = \sqrt{\frac{2K}{m_e}} = \sqrt{\frac{2(2.5\times 10^3\,\mathrm{eV})(1.60\times 10^{-19}\ \mathrm{J/eV})}{9.11\times 10^{-31}\,\mathrm{kg}}} = 2.96\times 10^7\ \mathrm{m/s}\ .$$

Thus,

$$B = \frac{E}{v} = \frac{10 \times 10^3 \text{V/m}}{2.96 \times 10^7 \text{ m/s}} = 3.4 \times 10^{-4} \text{ T} .$$

The magnetic field must be perpendicular to both the electric field and the velocity of the electron.

(b) A proton will pass undeflected if its velocity is the same as that of the electron. Both the electric and magnetic forces reverse direction, but they still cancel.

8. (a) Letting $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) = 0$, we get $vB \sin\phi = E$. We note that (for given values of the fields) this gives a minimum value for speed whenever the $\sin\phi$ factor is at its maximum value (which is 1, corresponding to $\phi = 90°$). So $v_{\min} = E/B = (1.50 \times 10^3 \text{ V/m})/(0.400 \text{ T}) = 3.75 \times 10^3 \text{ m/s}$.

(b) Having noted already that $\vec{v} \perp \vec{B}$, we now point out that $\vec{v} \times \vec{B}$ (which direction is given by the right-hand rule) must be in the direction opposite to $\vec{E}$. Thus, we can use the *left* hand to indicate the arrangement of vectors: if one points the thumb, index finger, and middle finger on the left hand so that all three are mutually perpendicular, then the thumb represents $\vec{v}$, the index finger indicates $\vec{B}$, and the middle finger represents $\vec{E}$.

9. Straight line motion will result from zero net force acting on the system; we ignore gravity. Thus, $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) = 0$. Note that $\vec{v} \perp \vec{B}$ so $|\vec{v} \times \vec{B}| = vB$. Thus, obtaining the speed from the formula for kinetic energy, we obtain

$$\begin{aligned} B &= \frac{E}{v} = \frac{E}{\sqrt{2m_e K}} \\ &= \frac{100 \text{ V}/(20 \times 10^{-3} \text{ m})}{\sqrt{2(9.11 \times 10^{-31} \text{ kg})(1.0 \times 10^3 \text{ V})(1.60 \times 10^{-19} \text{ C})}} \\ &= 2.7 \times 10^{-4} \text{ T} . \end{aligned}$$

10. We apply $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) = m_e\vec{a}$ to solve for $\vec{E}$:

$$\begin{aligned} \vec{E} &= \frac{m_e \vec{a}}{q} + \vec{B} \times \vec{v} \\ &= \frac{(9.11 \times 10^{-31} \text{ kg})(2.00 \times 10^{12} \text{ m/s}^2)\hat{\imath}}{-1.60 \times 10^{-19} \text{ C}} + (400\,\mu\text{T})\hat{\imath} \times [(12.0 \text{ km/s})\hat{\jmath} + (15.0 \text{ km/s})\hat{k}] \\ &= (-11.4\hat{\imath} - 6.00\hat{\jmath} + 4.80\hat{k}) \text{ V/m} \quad . \end{aligned}$$

11. Since the total force given by $\vec{F} = e(\vec{E} + \vec{v} \times \vec{B})$ vanishes, the electric field $\vec{E}$ must be perpendicular to both the particle velocity $\vec{v}$ and the magnetic field $\vec{B}$. The magnetic field is perpendicular to the velocity, so $\vec{v} \times \vec{B}$ has magnitude vB and the magnitude of the electric field is given by $E = vB$. Since the particle has charge e and is accelerated through a potential difference V, $\frac{1}{2}mv^2 = eV$ and $v = \sqrt{2eV/m}$. Thus,

$$E = B\sqrt{\frac{2eV}{m}} = (1.2\text{T})\sqrt{\frac{2(1.60 \times 10^{-19} \text{ C})(10 \times 10^3 \text{ V})}{(6.0 \text{ u})(1.661 \times 10^{-27} \text{ kg/u})}} = 6.8 \times 10^5 \text{ V/m} .$$

12. We use Eq. 29-12 to solve for V:

$$V = \frac{iB}{nle} = \frac{(23 \text{ A})(0.65 \text{ T})}{(8.47 \times 10^{28}/\text{m}^3)(150\,\mu\text{m})(1.6 \times 10^{-19} \text{ C})} = 7.4 \times 10^{-6} \text{ V} .$$

13. (a) In Chapter 27, the electric field (called E_C in this problem) which "drives" the current through the resistive material is given by Eq. 27-11, which (in magnitude) reads $E_C = \rho J$. Combining this with Eq. 27-7, we obtain

$$E_C = \rho nev_d .$$

Now, regarding the Hall effect, we use Eq. 29-10 to write $E = v_d B$. Dividing one equation by the other, we get $E/E_c = B/ne\rho$.

(b) Using the value of copper's resistivity given in Chapter 27, we obtain

$$\frac{E}{E_c} = \frac{B}{ne\rho} = \frac{0.65\,\mathrm{T}}{(8.47 \times 10^{28}/\,\mathrm{m}^3)(1.60 \times 10^{-19}\,\mathrm{C})(1.69 \times 10^{-8}\,\Omega{\cdot}\mathrm{m})} = 2.84 \times 10^{-3} \ .$$

14. For a free charge q inside the metal strip with velocity $\vec{v}$ we have $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$. We set this force equal to zero and use the relation between (uniform) electric field and potential difference. Thus,

$$v = \frac{E}{B} = \frac{|V_x - V_y|/d_{xy}}{B} = \frac{(3.90 \times 10^{-9}\,\mathrm{V})}{(1.20 \times 10^{-3}\,\mathrm{T})(0.850 \times 10^{-2}\,\mathrm{m})} = 0.382 \mathrm{~m/s} \ .$$

15. From Eq. 29-16, we find

$$B = \frac{m_e v}{er} = \frac{(9.11 \times 10^{-31}\,\mathrm{kg})(1.3 \times 10^6\,\mathrm{m/s})}{(1.60 \times 10^{-19}\,\mathrm{C})(0.35\,\mathrm{m})} = 2.1 \times 10^{-5} \mathrm{~T} \ .$$

16. (a) The accelerating process may be seen as a conversion of potential energy eV into kinetic energy. Since it starts from rest, $\frac{1}{2}m_e v^2 = eV$ and

$$v = \sqrt{\frac{2eV}{m_e}} = \sqrt{\frac{2(1.60 \times 10^{-19}\,\mathrm{C})(350\,\mathrm{V})}{9.11 \times 10^{-31} \mathrm{~kg}}} = 1.11 \times 10^7 \mathrm{~m/s} \ .$$

(b) Eq. 29-16 gives

$$r = \frac{m_e v}{eB} = \frac{(9.11 \times 10^{-31} \mathrm{~kg})(1.11 \times 10^7 \mathrm{~m/s})}{(1.60 \times 10^{-19}\,\mathrm{C})(200 \times 10^{-3}\,\mathrm{T})} = 3.16 \times 10^{-4} \mathrm{~m} \ .$$

17. (a) From $K = \frac{1}{2}m_e v^2$ we get

$$v = \sqrt{\frac{2K}{m_e}} = \sqrt{\frac{2(1.20 \times 10^3\,\mathrm{eV})(1.60 \times 10^{-19}\,\mathrm{eV/J})}{9.11 \times 10^{-31}\,\mathrm{kg}}} = 2.05 \times 10^7 \mathrm{~m/s} \ .$$

(b) From $r = m_e v/qB$ we get

$$B = \frac{m_e v}{qr} = \frac{(9.11 \times 10^{-31}\,\mathrm{kg})(2.05 \times 10^7\,\mathrm{m/s})}{(1.60 \times 10^{-19}\,\mathrm{C})(25.0 \times 10^{-2}\,\mathrm{m})} = 4.67 \times 10^{-4} \mathrm{~T} \ .$$

(c) The "orbital" frequency is

$$f = \frac{v}{2\pi r} = \frac{2.07 \times 10^7\,\mathrm{m/s}}{2\pi(25.0 \times 10^{-2}\,\mathrm{m})} = 1.31 \times 10^7 \mathrm{~Hz} \ .$$

(d) $T = 1/f = (1.31 \times 10^7\,\mathrm{Hz})^{-1} = 7.63 \times 10^{-8}$ s.

18. The period of revolution for the iodine ion is $T = 2\pi r/v = 2\pi m/Bq$, which gives

$$m = \frac{BqT}{2\pi} = \frac{(45.0 \times 10^{-3}\,\mathrm{T})(1.60 \times 10^{-19}\,\mathrm{C})(1.29 \times 10^{-3}\,\mathrm{s})}{(7)(2\pi)(1.66 \times 10^{-27}\,\mathrm{kg/u})} = 127 \mathrm{~u} \ .$$

19. (a) The frequency of revolution is

$$f = \frac{Bq}{2\pi m_e} = \frac{(35.0 \times 10^{-6}\,\mathrm{T})(1.60 \times 10^{-19}\,\mathrm{C})}{2\pi(9.11 \times 10^{-31}\,\mathrm{kg})} = 9.78 \times 10^5\,\mathrm{Hz} \ .$$

(b) Using Eq. 29-16, we obtain

$$r = \frac{m_e v}{qB} = \frac{\sqrt{2m_eK}}{qB} = \frac{\sqrt{2(9.11\times10^{-31}\,\text{kg})(100\,\text{eV})(1.60\times10^{-19}\,\text{J/eV})}}{(1.60\times10^{-19}\,\text{C})(35.0\times10^{-6}\,\text{T})} = 0.964\,\text{m}\ .$$

20. (a) Using Eq. 29-16, we obtain

$$v = \frac{rqB}{m_\alpha} = \frac{2eB}{4.00\,\text{u}} = \frac{2(4.50\times10^{-2}\,\text{m})(1.60\times10^{-19}\,\text{C})(1.20\,\text{T})}{(4.00\,\text{u})(1.66\times10^{-27}\,\text{kg/u})} = 2.60\times10^{6}\ \text{m/s}\ .$$

(b) $T = 2\pi r/v = 2\pi(4.50\times10^{-2}\,\text{m})/(2.60\times10^{6}\,\text{m/s}) = 1.09\times10^{-7}\,\text{s}$.

(c) The kinetic energy of the alpha particle is

$$K = \frac{1}{2}m_\alpha v^2 = \frac{(4.00\,\text{u})(1.66\times10^{-27}\,\text{kg/u})(2.60\times10^{6}\,\text{m/s})^2}{2(1.60\times10^{-19}\,\text{J/eV})} = 1.40\times10^{5}\ \text{eV}\ .$$

(d) $\Delta V = K/q = 1.40\times10^{5}\,\text{eV}/2e = 7.00\times10^{4}\,\text{V}$.

21. So that the magnetic field has an effect on the moving electrons, we need a non-negligible component of $\vec{B}$ to be perpendicular to $\vec{v}$ (the electron velocity). It is most efficient, therefore, to orient the magnetic field so it is perpendicular to the plane of the page. The magnetic force on an electron has magnitude $F_B = evB$, and the acceleration of the electron has magnitude $a = v^2/r$. Newton's second law yields $evB = m_e v^2/r$, so the radius of the circle is given by $r = m_e v/eB$ in agreement with Eq. 29-16. The kinetic energy of the electron is $K = \frac{1}{2}m_e v^2$, so $v = \sqrt{2K/m_e}$. Thus,

$$r = \frac{m_e}{eB}\sqrt{\frac{2K}{m_e}} = \sqrt{\frac{2m_eK}{e^2B^2}}\ .$$

This must be less than d, so

$$\sqrt{\frac{2m_eK}{e^2B^2}} \le d$$

or

$$B \ge \sqrt{\frac{2m_eK}{e^2d^2}}\ .$$

If the electrons are to travel as shown in Fig. 29-33, the magnetic field must be out of the page. Then the magnetic force is toward the center of the circular path, as it must be (in order to make the circular motion possible).

22. Let $v_\parallel = v\cos\theta$. The electron will proceed with a uniform speed $v_\parallel$ in the direction of $\vec{B}$ while undergoing uniform circular motion with frequency f in the direction perpendicular to B: $f = eB/2\pi m_e$. The distance d is then

$$\begin{aligned} d &= v_\parallel T = \frac{v_\parallel}{f} = \frac{(v\cos\theta)2\pi m_e}{eB} \\ &= \frac{2\pi(1.5\times10^{7}\,\text{m/s})(9.11\times10^{-31}\,\text{kg})(\cos 10^\circ)}{(1.60\times10^{-19}\,\text{C})(1.0\times10^{-3}\,\text{T})} = 0.53\,\text{m}\ . \end{aligned}$$

23. Referring to the solution of problem 19 part (b), we see that $r = \sqrt{2mK}/qB$ implies $K = (rqB)^2/2m \propto q^2m^{-1}$. Thus,

(a) $K_\alpha = (q_\alpha/q_p)^2(m_p/m_\alpha)K_p = (2)^2(1/4)K_p = K_p = 1.0\,\text{MeV}$;

(b) $K_d = (q_d/q_p)^2(m_p/m_d)K_p = (1)^2(1/2)K_p = 1.0\,\text{MeV}/2 = 0.50\,\text{MeV}$.

24. Referring to the solution of problem 19 part (b), we see that $r = \sqrt{2mK}/qB$ implies the proportionality: $r \propto \sqrt{mK}/qB$. Thus,

$$\begin{aligned} r_\alpha &= \sqrt{\frac{m_\alpha K_\alpha}{m_p K_p}} \frac{q_p}{q_\alpha} r_p = \sqrt{\frac{4.0\,\mathrm{u}}{1.0\,\mathrm{u}}} \frac{e r_p}{2e} = r_p\,; \\ r_d &= \sqrt{\frac{m_d K_d}{m_p K_p}} \frac{q_p}{q_d} r_d = \sqrt{\frac{2.0\,\mathrm{u}}{1.0\,\mathrm{u}}} \frac{e r_d}{e} = \sqrt{2} r_p\,. \end{aligned}$$

25. (a) We solve for B from $m = B^2 q x^2/8V$ (see Sample Problem 29-3):

$$B = \sqrt{\frac{8Vm}{qx^2}}\,.$$

We evaluate this expression using $x = 2.00\,\mathrm{m}$:

$$B = \sqrt{\frac{8(100 \times 10^3\,\mathrm{V})(3.92 \times 10^{-25}\,\mathrm{kg})}{(3.20 \times 10^{-19}\,\mathrm{C})(2.00\,\mathrm{m})^2}} = 0.495\ \mathrm{T}\,.$$

(b) Let N be the number of ions that are separated by the machine per unit time. The current is $i = qN$ and the mass that is separated per unit time is $M = mN$, where m is the mass of a single ion. M has the value

$$M = \frac{100 \times 10^{-6}\,\mathrm{kg}}{3600\,\mathrm{s}} = 2.78 \times 10^{-8}\,\mathrm{kg/s}\,.$$

Since $N = M/m$ we have

$$i = \frac{qM}{m} = \frac{(3.20 \times 10^{-19}\,\mathrm{C})(2.78 \times 10^{-8}\ \mathrm{kg/s})}{3.92 \times 10^{-25}\,\mathrm{kg}} = 2.27 \times 10^{-2}\ \mathrm{A}\,.$$

(c) Each ion deposits energy qV in the cup, so the energy deposited in time Δt is given by

$$E = NqV\,\Delta t = \frac{iqV}{q}\,\Delta t = iV\,\Delta t\,.$$

For $\Delta t = 1.0\mathrm{h}$,

$$E = (2.27 \times 10^{-2}\,\mathrm{A})(100 \times 10^3\,\mathrm{V})(3600\,\mathrm{s}) = 8.17 \times 10^6\ \mathrm{J}\,.$$

To obtain the second expression, i/q is substituted for N.

26. The equation of motion for the proton is

$$\begin{aligned} \vec{F} &= q\vec{v} \times \vec{B} = q(v_x \hat{\mathrm{i}} + v_y \hat{\mathrm{j}} + v_z \hat{\mathrm{k}}) \times B\hat{\mathrm{i}} = qB(v_z \hat{\mathrm{j}} - v_y \hat{\mathrm{k}}) \\ &= m_p \vec{a} = m_p \left[\left(\frac{dv_x}{dt}\right) \hat{\mathrm{i}} + \left(\frac{dv_y}{dt}\right) \hat{\mathrm{j}} + \left(\frac{dv_z}{dt}\right) \hat{\mathrm{k}} \right]\,. \end{aligned}$$

Thus,

$$\begin{aligned} \frac{dv_x}{dt} &= 0 \\ \frac{dv_y}{dt} &= \omega v_z \\ \frac{dv_z}{dt} &= -\omega v_y\,, \end{aligned}$$

where $\omega = eB/m_p$. The solution is $v_x = v_{0x}$, $v_y = v_{0y} \cos \omega t$ and $v_z = -v_{0y} \sin \omega t$. In summary, we have $\vec{v}(t) = v_{0x}\hat{\mathrm{i}} + v_{0y} \cos(\omega t)\hat{\mathrm{j}} - v_{0y}(\sin \omega t)\hat{\mathrm{k}}$.

27. (a) If v is the speed of the positron then $v \sin\phi$ is the component of its velocity in the plane that is perpendicular to the magnetic field. Here ϕ is the angle between the velocity and the field (89°). Newton's second law yields $eBv\sin\phi = m_e(v\sin\phi)^2/r$, where r is the radius of the orbit. Thus $r = (m_e v/eB)\sin\phi$. The period is given by

$$T = \frac{2\pi r}{v\sin\phi} = \frac{2\pi m_e}{eB} = \frac{2\pi(9.11\times10^{-31}\,\text{kg})}{(1.60\times10^{-19}\,\text{C})(0.10\,\text{T})} = 3.6\times10^{-10}\ \text{s}\ .$$

The equation for r is substituted to obtain the second expression for T.

(b) The pitch is the distance traveled along the line of the magnetic field in a time interval of one period. Thus $p = vT\cos\phi$. We use the kinetic energy to find the speed: $K = \frac{1}{2}m_e v^2$ means

$$v = \sqrt{\frac{2K}{m_e}} = \sqrt{\frac{2(2.0\times10^3\,\text{eV})(1.60\times10^{-19}\ \text{J/eV})}{9.11\times10^{-31}\,\text{kg}}} = 2.651\times10^7\ \text{m/s}\ .$$

Thus

$$p = (2.651\times10^7\,\text{m/s})(3.58\times10^{-10}\,\text{s})\cos 89° = 1.7\times10^{-4}\ \text{m}\ .$$

(c) The orbit radius is

$$R = \frac{m_e v\sin\phi}{eB} = \frac{(9.11\times10^{-31}\,\text{kg})\left(2.651\times10^7\,\text{m/s}\right)\sin 89°}{(1.60\times10^{-19}\,\text{C})\,(0.10\text{T})} = 1.5\times10^{-3}\ \text{m}\ .$$

28. We consider the point at which it enters the field-filled region, velocity vector pointing downward. The field points out of the page so that $\vec{v}\times\vec{B}$ points leftward, which indeed seems to be the direction it is "pushed"; therefore, $q > 0$ (it is a proton).

(a) Eq. 29-17 becomes

$$\begin{aligned} T &= \frac{2\pi m_\text{p}}{e\,|\vec{B}|} \\ 2\left(130\times10^{-9}\right) &= \frac{2\pi\left(1.67\times10^{-27}\right)}{(1.60\times10^{-19})\,|\vec{B}|} \end{aligned}$$

which yields $|\vec{B}| = 0.252$ T.

(b) Doubling the kinetic energy implies multiplying the speed by $\sqrt{2}$. Since the period T does not depend on speed, then it remains the same (even though the radius increases by a factor of $\sqrt{2}$). Thus, $t = T/2 = 130$ ns, again.

29. (a) $-q$, from conservation of charges.

(b) Each of the two particles will move in the same circular path, initially going in the opposite direction. After traveling half of the circular path they will collide. So the time is given by $t = T/2 = \pi m/Bq$ (where Eq. 29-17 has been used).

30. (a) Using Eq. 29-23 and Eq. 29-18, we find

$$f_\text{osc} = \frac{qB}{2\pi m_p} = \frac{(1.60\times10^{-19}\,\text{C})(1.2\,\text{T})}{2\pi(1.67\times10^{-27}\,\text{kg})} = 1.8\times10^7\ \text{Hz}\ .$$

(b) From $r = m_p v/qB = \sqrt{2m_pK}/qB$ we have

$$K = \frac{(rqB)^2}{2m_p} = \frac{[(0.50\,\text{m})(1.60\times10^{-19}\,\text{C})(1.2\,\text{T})]^2}{2(1.67\times10^{-27}\,\text{kg})(1.60\times10^{-19}\,\text{J/eV})} = 1.7\times10^7\ \text{eV}\ .$$

31. We approximate the total distance by the number of revolutions times the circumference of the orbit corresponding to the average energy. This should be a good approximation since the deuteron receives the same energy each revolution and its period does not depend on its energy. The deuteron accelerates twice in each cycle, and each time it receives an energy of $qV = 80 \times 10^3$ eV. Since its final energy is 16.6 MeV, the number of revolutions it makes is

$$n = \frac{16.6 \times 10^6\,\text{eV}}{2(80 \times 10^3\,\text{eV})} = 104 .$$

Its average energy during the accelerating process is 8.3 MeV. The radius of the orbit is given by $r = mv/qB$, where v is the deuteron's speed. Since this is given by $v = \sqrt{2K/m}$, the radius is

$$r = \frac{m}{qB}\sqrt{\frac{2K}{m}} = \frac{1}{qB}\sqrt{2Km} .$$

For the average energy

$$r = \frac{\sqrt{2(8.3 \times 10^6\,\text{eV})(1.60 \times 10^{-19}\,\text{J/eV})(3.34 \times 10^{-27}\,\text{kg})}}{(1.60 \times 10^{-19}\,\text{C})(1.57\,\text{T})} = 0.375\ \text{m} .$$

The total distance traveled is about $n2\pi r = (104)(2\pi)(0.375) = 2.4 \times 10^2$ m.

32. (a) Since $K = \frac{1}{2}mv^2 = \frac{1}{2}m(2\pi R f_{\text{osc}})^2 \propto m$,

$$K_p = \left(\frac{m_p}{m_d}\right) K_d = \frac{1}{2}K_d = \frac{1}{2}(17\,\text{MeV}) = 8.5\ \text{MeV} .$$

(b) We require a magnetic field of strength

$$B_p = \frac{1}{2}B_d = \frac{1}{2}(1.6\,\text{T}) = 0.80\ \text{T} .$$

(c) Since $K \propto B^2/m$,

$$K_p' = \left(\frac{m_d}{m_p}\right) K_d = 2K_d = 2(17\,\text{MeV}) = 34\ \text{MeV} .$$

(d) Since $f_{\text{osc}} = Bq/(2\pi m) \propto m^{-1}$,

$$f_{\text{osc},\,d} = \left(\frac{m_d}{m_P}\right) f_{\text{osc},\,p} = 2(12 \times 10^6\,\text{s}^{-1}) = 2.4 \times 10^7\ \text{Hz} .$$

(e) Now,

$$\begin{aligned} K_\alpha &= \left(\frac{m_\alpha}{m_d}\right) K_d = 2K_d = 2(17\,\text{MeV}) = 34\,\text{MeV} , \\ B_\alpha &= \left(\frac{m_\alpha}{m_d}\right)\left(\frac{q_d}{q_\alpha}\right) B_d = 2\left(\frac{1}{2}\right)(1.6\,\text{T}) = 1.6\,\text{T} , \\ K_\alpha' &= K_\alpha = 34\,\text{MeV} \qquad (\text{Since } B_\alpha = B_d = 1.6\,\text{T}) , \end{aligned}$$

and

$$f_{\text{osc},\,\alpha} = \left(\frac{q_\alpha}{a_d}\right)\left(\frac{m_d}{m_\alpha}\right) f_{\text{osc, d}} = 2\left(\frac{2}{4}\right)\left(12 \times 10^6\,\text{s}^{-1}\right) = 1.2 \times 10^7\ \text{Hz} .$$

33. The magnitude of the magnetic force on the wire is given by $F_B = iLB\sin\phi$, where i is the current in the wire, L is the length of the wire, B is the magnitude of the magnetic field, and ϕ is the angle between the current and the field. In this case $\phi = 70°$. Thus,

$$F_B = (5000\,\text{A})(100\,\text{m})(60.0 \times 10^{-6}\,\text{T})\sin 70° = 28.2\ \text{N} .$$

We apply the right-hand rule to the vector product $\vec{F}_B = i\vec{L} \times \vec{B}$ to show that the force is to the west.

34. The magnetic force on the (straight) wire is

$$F_B = iBL\sin\theta = (13.0\,\mathrm{A})(1.50\,\mathrm{T})(1.80\,\mathrm{m})(\sin 35.0^\circ) = 20.1\,\mathrm{N}\ .$$

35. The magnetic force on the wire must be upward and have a magnitude equal to the gravitational force mg on the wire. Applying the right-hand rule reveals that the current must be from left to right. Since the field and the current are perpendicular to each other the magnitude of the magnetic force is given by $F_B = iLB$, where L is the length of the wire. Thus,

$$iLB = mg \implies i = \frac{mg}{LB} = \frac{(0.0130\,\mathrm{kg})(9.8\,\mathrm{m/s^2})}{(0.620\,\mathrm{m})(0.440\,\mathrm{T})} = 0.467\ \mathrm{A}\ .$$

36. The magnetic force on the wire is

$$\begin{aligned}\vec{F}_B &= i\vec{L}\times\vec{B} = iL\hat{\imath}\times(B_y\hat{\jmath}+B_z\hat{\mathrm{k}}) = iL(-B_z\hat{\jmath}+B_y\hat{\mathrm{k}})\\ &= (0.50\,\mathrm{A})(0.50\,\mathrm{m})[-(0.010\,\mathrm{T})\hat{\jmath}+(0.0030\,\mathrm{T})\hat{\mathrm{k}}]\\ &= (-2.5\times10^{-3}\hat{\jmath}+0.75\times10^{-3}\hat{\mathrm{k}})\,\mathrm{N}\ .\end{aligned}$$

37. The magnetic force must push horizontally on the rod to overcome the force of friction, but it can be oriented so that it also pulls up on the rod and thereby reduces both the normal force and the force of friction. The forces acting on the rod are: $\vec{F}$, the force of the magnetic field; mg, the magnitude of the (downward) force of gravity; $\vec{N}$, the normal force exerted by the stationary rails upward on the rod; and $\vec{f}$, the (horizontal) force of friction. For definiteness, we assume the rod is on the verge of moving eastward, which means that $\vec{f}$ points westward (and is equal to its maximum possible value $\mu_s N$). Thus, $\vec{F}$ has an eastward component F_x and an upward component F_y, which can be related to the components of the magnetic field once we assume a direction for the current in the rod. Thus, again for definiteness, we assume the current flows northward. Then, by the righthand rule, a downward component (B_d) of $\vec{B}$ will produce the eastward F_x, and a westward component (B_w) will produce the upward F_y. Specifically,

$$F_x = iLB_d \qquad \text{and} \qquad F_y = iLB_w\ .$$

Considering forces along a vertical axis, we find

$$N = mg - F_y = mg - iLB_w$$

so that

$$f = f_{s,\max} = \mu_s\,(mg - iLB_w)\ .$$

It is on the verge of motion, so we set the horizontal acceleration to zero:

$$F_x - f = 0 \implies iLB_d = \mu_s\,(mg - iLB_w)\ .$$

The angle of the field components is adjustable, and we can minimize with respect to it. Defining the angle by $B_w = B\sin\theta$ and $B_d = B\cos\theta$ (which means θ is being measured from a vertical axis) and writing the above expression in these terms, we obtain

$$iLB\cos\theta = \mu_s\,(mg - iLB\sin\theta) \implies B = \frac{\mu_s mg}{iL(\cos\theta + \mu_s\sin\theta)}$$

which we differentiate (with respect to θ) and set the result equal to zero. This provides a determination of the angle:

$$\theta = \tan^{-1}(\mu_s) = \tan^{-1}(0.60) = 31^\circ\ .$$

Consequently,

$$B_{\min} = \frac{0.60(1.0\,\mathrm{kg})(9.8\,\mathrm{m/s^2})}{(50\,\mathrm{A})(1.0\,\mathrm{m})(\cos 31^\circ + 0.60\sin 31^\circ)} = 0.10\ \mathrm{T}\ .$$

38. (a) From $F_B = iLB$ we get

$$i = \frac{F_B}{LB} = \frac{10 \times 10^3\,\text{N}}{(3.0\,\text{m})(10 \times 10^{-6}\,\text{T})} = 3.3 \times 10^8 \text{ A} .$$

(b) $P = i^2 R = (3.3 \times 10^8\,\text{A})^2(1.0\,\Omega) = 1.0 \times 10^{17}\,\text{W}$.

(c) It is totally unrealistic because of the huge current and the accompanying high power loss.

39. The applied field has two components: $B_x > 0$ and $B_z > 0$. Considering each straight-segment of the rectangular coil, we note that Eq. 29-26 produces a non-zero force only for the component of $\vec{B}$ which is perpendicular to that segment; we also note that the equation is effectively multiplied by $N = 20$ due to the fact that this is a 20-turn coil. Since we wish to compute the torque about the hinge line, we can ignore the force acting on the straight-segment of the coil which lies along the y axis (forces acting at the axis of rotation produce no torque about that axis). The top and bottom straight-segments experience forces due to Eq. 29-26 (caused by the B_z component), but these forces are (by the right-hand rule) in the $\pm y$ directions and are thus unable to produce a torque about the y axis. Consequently, the torque derives completely from the force exerted on the straight-segment located at $x = 0.050$ m, which has length $L = 0.10$ m and is shown in Figure 29-36 carrying current in the $-y$ direction. Now, the B_z component will produce a force on this straight-segment which points in the $-x$ direction (back towards the hinge) and thus will exert no torque about the hinge. However, the B_x component (which is equal to $B\cos\theta$ where $B = 0.50$ T and $\theta = 30°$) produces a force equal to $NiLB_x$ which points (by the right-hand rule) in the $+z$ direction. Since the action of this force is perpendicular to the plane of the coil, and is located a distance x away from the hinge, then the torque has magnitude

$$\tau = (NiLB_x)\,(x) = NiLxB\cos\theta = (20)(0.10)(0.10)(0.050)(0.50)\cos 30° = 0.0043$$

in SI units (N·m). Since $\vec{\tau} = \vec{r} \times \vec{F}$, the direction of the torque is $-y$. An alternative way to do this problem is through the use of Eq. 29-37. We do not show those details here, but note that the magnetic moment vector (a necessary part of Eq. 29-37) has magnitude

$$|\vec{\mu}| = NiA = (20)(0.10\,\text{A})(0.0050\,\text{m}^2)$$

and points in the $-z$ direction. At this point, Eq. 3-30 may be used to obtain the result for the torque vector.

40. We establish coordinates such that the two sides of the right triangle meet at the origin, and the $\ell_y = 50$ cm side runs along the $+y$ axis, while the $\ell_x = 120$ cm side runs along the $+x$ axis. The angle made by the hypotenuse (of length 130 cm) is $\theta = \tan^{-1}(50/120) = 22.6°$, relative to the 120 cm side. If one measures the angle counterclockwise from the $+x$ direction, then the angle for the hypotenuse is $180° - 22.6° = +157°$. Since we are only asked to find the magnitudes of the forces, we have the freedom to assume the current is flowing, say, counterclockwise in the triangular loop (as viewed by an observer on the $+z$ axis. We take $\vec{B}$ to be in the same direction as that of the current flow in the hypotenuse. Then, with $B = |\vec{B}| = 0.0750$ T,

$$B_x = -B\cos\theta = -0.0692 \text{ T} \qquad \text{and} \qquad B_y = B\sin\theta = 0.0288 \text{ T} .$$

(a) Eq. 29-26 produces zero force when $\vec{L} \parallel \vec{B}$ so there is no force exerted on the hypotenuse. On the 50 cm side, the B_x component produces a force $i\ell_y B_x \hat{\text{k}}$, and there is no contribution from the B_y component. Using SI units, the magnitude of the force on the ℓ_y side is therefore

$$(4.00\,\text{A})(0.500\,\text{m})(0.0692\,\text{T}) = 0.138 \text{ N} .$$

On the 120 cm side, the B_y component produces a force $i\ell_x B_y \hat{\text{k}}$, and there is no contribution from the B_x component. Using SI units, the magnitude of the force on the ℓ_x side is also

$$(4.00\,\text{A})(1.20\,\text{m})(0.0288\,\text{T}) = 0.138 \text{ N} .$$

(b) The net force is

$$i\ell_y B_x \hat{\mathrm{k}} + i\ell_x B_y \hat{\mathrm{k}} = 0 \ ,$$

keeping in mind that $B_x < 0$ due to our initial assumptions. If we had instead assumed $\vec{B}$ went the opposite direction of the current flow in the hypotenuse, then $B_x > 0$ but $B_y < 0$ and a zero net force would still be the result.

41. If N closed loops are formed from the wire of length L, the circumference of each loop is L/N, the radius of each loop is $R = L/2\pi N$, and the area of each loop is $A = \pi R^2 = \pi(L/2\pi N)^2 = L^2/4\pi N^2$. For maximum torque, we orient the plane of the loops parallel to the magnetic field, so the dipole moment is perpendicular to the field. The magnitude of the torque is then

$$\tau = NiAB = (Ni)\left(\frac{L^2}{4\pi N^2}\right) B = \frac{iL^2 B}{4\pi N} \ .$$

To maximize the torque, we take N to have the smallest possible value, 1. Then $\tau = iL^2B/4\pi$.

42. We replace the current loop of arbitrary shape with an assembly of small adjacent rectangular loops filling the same area which was enclosed by the original loop (as nearly as possible). Each rectangular loop carries a current i flowing in the same sense as the original loop. As the sizes of these rectangles shrink to infinitesimally small values, the assembly gives a current distribution equivalent to that of the original loop. The magnitude of the torque $\Delta\vec{\tau}$ exerted by $\vec{B}$ on the nth rectangular loop of area ΔA_n is given by $\Delta\tau_n = NiB\sin\theta\Delta A_n$. Thus, for the whole assembly

$$\tau = \sum_n \Delta\tau_n = NiB\sum_n \Delta A_n = NiAB\sin\theta \ .$$

43. Consider an infinitesimal segment of the loop, of length ds. The magnetic field is perpendicular to the segment, so the magnetic force on it is has magnitude $dF = iB\,ds$. The horizontal component of the force has magnitude $dF_h = (iB\cos\theta)\,ds$ and points inward toward the center of the loop. The vertical component has magnitude $dF_v = (iB\sin\theta)\,ds$ and points upward. Now, we sum the forces on all the segments of the loop. The horizontal component of the total force vanishes, since each segment of wire can be paired with another, diametrically opposite, segment. The horizontal components of these forces are both toward the center of the loop and thus in opposite directions. The vertical component of the total force is

$$F_v = iB\sin\theta\int ds = (iB\sin\theta)2\pi a \ .$$

We note the i, B, and θ have the same value for every segment and so can be factored from the integral.

44. The total magnetic force on the loop L is

$$\vec{F}_B = i\oint_L (d\vec{L}\times\vec{B}) = i(\oint_L d\vec{L})\times\vec{B} = 0 \ .$$

We note that $\oint_L d\vec{L} = 0$. If $\vec{B}$ is not a constant, however, then the equality

$$\oint_L (d\vec{L}\times\vec{B}) = (\oint_L d\vec{L})\times\vec{B}$$

is not necessarily valid, so $\vec{F}_B$ is not always zero.

45. (a) The current in the galvanometer should be 1.62 mA when the potential difference across the resistor-galvanometer combination is 1.00 V. The potential difference across the galvanometer alone is $iR_g = (1.62\times10^{-3}\,\mathrm{A})(75.3\,\Omega) = 0.122\,\mathrm{V}$, so the resistor must be in series with the galvanometer and the potential difference across it must be $1.00\,\mathrm{V} - 0.122\,\mathrm{V} = 0.878\mathrm{V}$. The resistance should be $R = (0.878\,\mathrm{V})/(1.62\times10^{-3}\,\mathrm{A}) = 542\,\Omega$.

(b) The current in the galvanometer should be 1.62 mA when the total current in the resistor and galvanometer combination is 50.0 mA. The resistor should be in parallel with the galvanometer, and the current through it should be $50.0\,\text{mA} - 1.62\,\text{mA} = 48.38\,\text{mA}$. The potential difference across the resistor is the same as that across the galvanometer, 0.122 V, so the resistance should be $R = (0.122\,\text{V})/(48.38 \times 10^{-3}\,\text{A}) = 2.52\,\Omega$.

46. We use $\tau_{\text{max}} = |\vec{\mu} \times \vec{B}|_{\text{max}} = \mu B = i\pi a^2 B$, and note that $i = qf = qv/2\pi a$. So

$$\tau_{\text{max}} = \left(\frac{qv}{2\pi a}\right) \pi a^2 B = \frac{1}{2} qvaB \ .$$

47. We use Eq. 29-37 where $\vec{\mu}$ is the magnetic dipole moment of the wire loop and $\vec{B}$ is the magnetic field, as well as Newton's second law. Since the plane of the loop is parallel to the incline the dipole moment is normal to the incline. The forces acting on the cylinder are the force of gravity mg, acting downward from the center of mass, the normal force of the incline N, acting perpendicularly to the incline through the center of mass, and the force of friction f, acting up the incline at the point of contact. We take the x axis to be positive down the incline. Then the x component of Newton's second law for the center of mass yields

$$mg \sin\theta - f = ma \ .$$

For purposes of calculating the torque, we take the axis of the cylinder to be the axis of rotation. The magnetic field produces a torque with magnitude $\mu B \sin\theta$, and the force of friction produces a torque with magnitude fr, where r is the radius of the cylinder. The first tends to produce an angular acceleration in the counterclockwise direction, and the second tends to produce an angular acceleration in the clockwise direction. Newton's second law for rotation about the center of the cylinder, $\tau = I\alpha$, gives

$$fr - \mu B \sin\theta = I\alpha \ .$$

Since we want the current that holds the cylinder in place, we set $a = 0$ and $\alpha = 0$, and use one equation to eliminate f from the other. The result is $mgr = \mu B$. The loop is rectangular with two sides of length L and two of length $2r$, so its area is $A = 2rL$ and the dipole moment is $\mu = NiA = 2NirL$. Thus, $mgr = 2NirLB$ and

$$i = \frac{mg}{2NLB} = \frac{(0.250\,\text{kg})(9.8\,\text{m/s}^2)}{2(10.0)(0.100\,\text{m})(0.500\,\text{T})} = 2.45\ \text{A} \ .$$

48. From $\mu = NiA = i\pi r^2$ we get

$$i = \frac{\mu}{\pi r^2} = \frac{8.00 \times 10^{22}\,\text{J/T}}{\pi(3500 \times 10^3\,\text{m})^2} = 2.08 \times 10^9\ \text{A} \ .$$

49. (a) The magnitude of the magnetic dipole moment is given by $\mu = NiA$, where N is the number of turns, i is the current in each turn, and A is the area of a loop. In this case the loops are circular, so $A = \pi r^2$, where r is the radius of a turn. Thus

$$i = \frac{\mu}{N\pi r^2} = \frac{2.30\,\text{A}\cdot\text{m}^2}{(160)(\pi)(0.0190\,\text{m})^2} = 12.7\ \text{A} \ .$$

(b) The maximum torque occurs when the dipole moment is perpendicular to the field (or the plane of the loop is parallel to the field). It is given by

$$\tau_{\text{max}} = \mu B = \left(2.30\,\text{A}\cdot\text{m}^2\right)\left(35.0 \times 10^{-3}\,\text{T}\right) = 8.05 \times 10^{-2}\ \text{N}\cdot\text{m} \ .$$

50. (a) $\mu = NAi = \pi r^2 i = \pi(0.150\,\text{m})^2(2.60\,\text{A}) = 0.184\,\text{A}\cdot\text{m}^2$.

(b) The torque is

$$\tau = |\vec{\mu} \times \vec{B}| = \mu B \sin\theta = \left(0.184\,\text{A}\cdot\text{m}^2\right)(12.0\,\text{T}) \sin 41.0^\circ = 1.45\ \text{N}\cdot\text{m} \ .$$

51. (a) The area of the loop is $A = \frac{1}{2}(30\,\text{cm})(40\,\text{cm}) = 6.0 \times 10^2\,\text{cm}^2$, so

$$\mu = iA = (5.0\,\text{A})\left(6.0 \times 10^{-2}\,\text{m}^2\right) = 0.30\ \text{A}\cdot\text{m}^2\ .$$

(b) The torque on the loop is

$$\tau = \mu B \sin\theta = \left(0.30\,\text{A}\cdot\text{m}^2\right)\left(80 \times 10^3\,\text{T}\right)\sin 90^\circ = 2.4 \times 10^{-2}\ \text{N}\cdot\text{m}\ .$$

52. (a) We use $\vec{\tau} = \vec{\mu} \times \vec{B}$, where $\vec{\mu}$ points into the wall (since the current goes clockwise around the clock). Since $\vec{B}$ points towards the one-hour (or "5-minute") mark, and (by the properties of vector cross products) $\vec{\tau}$ must be perpendicular to it, then (using the right-hand rule) we find $\vec{\tau}$ points at the 20-minute mark. So the time interval is 20 min.

(b) The torque is given by

$$\begin{aligned}\tau &= \left|\vec{\mu} \times \vec{B}\right| = \mu B \sin 90^\circ \\ &= NiAB = \pi N i r^2 B \\ &= 6\pi(2.0\,\text{A})(0.15\,\text{m})^2(70 \times 10^{-3}\,\text{T}) \\ &= 5.9 \times 10^{-2}\ \text{N}\cdot\text{m}\ .\end{aligned}$$

53. (a) The magnitude of the magnetic moment vector is

$$\mu = \sum_n i_n A_n = \pi r_1^2 i_1 + \pi r_2^2 i_2 = \pi(7.00\,\text{A})[(0.300\,\text{m})^2 + (0.200\,\text{m})^2] = 2.86\ \text{A}\cdot\text{m}^2\ .$$

(b) Now,

$$\mu = \pi r_1^2 i_1 - \pi r_2^2 i_2 = \pi(7.00\,\text{A})[(0.300\,\text{m})^2 - (0.200\,\text{m})^2] = 1.10\ \text{A}\cdot\text{m}^2\ .$$

54. Let $a = 30.0\,\text{cm}$, $b = 20.0\,\text{cm}$, and $c = 10.0\,\text{cm}$. From the given hint, we write

$$\begin{aligned}\vec{\mu} &= \vec{\mu}_1 + \vec{\mu}_2 = iab(-\hat{\text{k}}) + iac(\hat{\text{j}}) \\ &= ia(c\hat{\text{j}} - b\hat{\text{k}}) \\ &= (5.00\,\text{A})(0.300\,\text{m})[(0.100\,\text{m})\hat{\text{j}} - (0.200\,\text{m})\hat{\text{k}}] \\ &= (0.150\hat{\text{j}} - 0.300\hat{\text{k}})\,\text{A}\cdot\text{m}^2.\end{aligned}$$

Thus, using the Pythagorean theorem,

$$\mu = \sqrt{(0.150)^2 + (0.300)^2} = 0.335\ \text{A}\cdot\text{m}^2\ ,$$

and $\vec{\mu}$ is in the yz plane at angle θ to the $+y$ direction, where

$$\theta = \tan^{-1}\left(\frac{\mu_y}{\mu_x}\right) = \tan^{-1}\left(\frac{-0.300}{0.150}\right) = -63.4^\circ\ .$$

55. The magnetic dipole moment is $\vec{\mu} = \mu(0.60\,\hat{\text{i}} - 0.80\,\hat{\text{j}})$, where $\mu = NiA = Ni\pi r^2 = 1(0.20\text{A})\pi(0.080\text{m})^2 = 4.02 \times 10^{-4}\,\text{A}\cdot\text{m}^2$. Here i is the current in the loop, N is the number of turns, A is the area of the loop, and r is its radius.

(a) The torque is

$$\begin{aligned}\vec{\tau} &= \vec{\mu} \times \vec{B} = \mu(0.60\,\hat{\text{i}} - 0.80\,\hat{\text{j}}) \times (0.25\,\hat{\text{i}} + 0.30\,\hat{\text{k}}) \\ &= \mu\left[(0.60)(0.30)(\,\hat{\text{i}} \times \hat{\text{k}}) - (0.80)(0.25)(\,\hat{\text{j}} \times \hat{\text{i}}) - (0.80)(0.30)(\,\hat{\text{j}} \times \hat{\text{k}})\right] \\ &= \mu[-0.18\,\hat{\text{j}} + 0.20\,\hat{\text{k}} - 0.24\,\hat{\text{i}}]\ .\end{aligned}$$

Here $\hat{\imath} \times \hat{k} = -\hat{\jmath}$, $\hat{\jmath} \times \hat{\imath} = -\hat{k}$, and $\hat{\jmath} \times \hat{k} = \hat{\imath}$ are used. We also use $\hat{\imath} \times \hat{\imath} = 0$. Now, we substitute the value for μ to obtain

$$\vec{\tau} = \left(-0.97 \times 10^{-4}\,\hat{\imath} - 7.2 \times 10^{-4}\,\hat{\jmath} + 8.0 \times 10^{-4}\,\hat{k}\right) \text{N}\cdot\text{m} .$$

(b) The potential energy of the dipole is given by

$$\begin{aligned} U &= -\vec{\mu}\cdot\vec{B} = -\mu(0.60\,\hat{\imath} - 0.80\,\hat{\jmath})\cdot(0.25\,\hat{\imath} + 0.30\,\hat{k}) \\ &= -\mu(0.60)(0.25) = -0.15\mu = -6.0 \times 10^{-4}\text{ J} . \end{aligned}$$

Here $\hat{\imath}\cdot\hat{\imath} = 1$, $\hat{\imath}\cdot\hat{k} = 0$, $\hat{\jmath}\cdot\hat{\imath} = 0$, and $\hat{\jmath}\cdot\hat{k} = 0$ are used.

56. The unit vector associated with the current element (of magnitude $d\ell$) is $-\hat{\jmath}$. The (infinitesimal) force on this element is

$$d\vec{F} = i\,d\ell(-\hat{\jmath}) \times (0.3y\hat{\imath} + 0.4y\hat{\jmath})$$

with SI units (and 3 significant figures) understood.

(a) Since $\hat{\jmath} \times \hat{\imath} = -\hat{k}$ and $\hat{\jmath} \times \hat{\jmath} = 0$, we obtain

$$d\vec{F} = 0.3iy\,d\ell\,\hat{k} = \left(6.00 \times 10^{-4}\,\text{N/m}^2\right) y\,d\ell\,\hat{k} .$$

(b) We integrate the force element found in part (a), using the symbol ξ to stand for the coefficient $6.00 \times 10^{-4}\,\text{N/m}^2$, and obtain

$$\vec{F} = \int d\vec{F} = \xi\hat{k} \int_0^{0.25} y\,dy = \xi\hat{k}\left(\frac{0.25^2}{2}\right) = 1.88 \times 10^{-5}\,\text{N}\,\hat{k} .$$

57. Since the velocity is constant, the net force on the proton vanishes. Using Eq. 29-2 and Eq. 23-28, we obtain the requirement (Eq. 29-7) for the proton's speed in terms of the crossed fields:

$$v = \frac{E}{B} \implies E = (50\,\text{m/s})(0.0020\,\text{T}) = 0.10\text{ V/m} .$$

By the right-hand rule, the magnetic force points in the $\hat{k}$ direction. To cancel this, the electric force must be in the $-\hat{k}$ direction. Since $q > 0$ for the proton, we conclude $\vec{E} = -0.10\,\text{V/m}\,\hat{k}$.

58. (a) The kinetic energy gained is due to the potential energy decrease as the dipole swings from a position specified by angle θ to that of being aligned (zero angle) with the field. Thus,

$$K = U_i - U_f = -\mu B\cos\theta - (-\mu B\cos 0^\circ) .$$

Therefore, using SI units, the angle is

$$\theta = \cos^{-1}\left(1 - \frac{K}{\mu B}\right) = \cos^{-1}\left(1 - \frac{0.00080}{(0.020)(0.052)}\right) = 77^\circ .$$

(b) Since we are making the assumption that no energy is dissipated in this process, then the dipole will continue its rotation (similar to a pendulum) until it reaches an angle $\theta = 77^\circ$ on the other side of the alignment axis.

59. Using Eq. 29-2 and Eq. 3-30, we obtain

$$\vec{F} = q\left(v_x B_y - v_y B_x\right)\hat{k} = q\left(v_x(3B_x) - v_y B_x\right)\hat{k}$$

where we use the fact that $B_y = 3B_x$. Since the force (at the instant considered) is $F_z\,\hat{k}$ where $F_z = 6.4 \times 10^{-19}\,\text{N}$, then we are led to the condition

$$q\left(3v_x - v_y\right)B_x = F_z \implies B_x = \frac{F_z}{q\left(3v_x - v_y\right)} .$$

Substituting $V_x = 2.0\,\text{m/s}$, $v_y = 4.0\,\text{m/s}$ and $q = -1.6 \times 10^{-19}\,\text{C}$, we obtain $B_x = -2.0$ T.

60. The current is in the $+\hat{\imath}$ direction. Thus, the $\hat{\imath}$ component of $\vec{B}$ has no effect, and (with x in meters) we evaluate

$$\begin{aligned}\vec{F} &= (3.00\ \mathrm{A})\int_0^1 \left(-0.600\ \mathrm{T/m^2}\right) x^2\,dx\left(\hat{\imath}\times\hat{\jmath}\right)\\ &= -1.80\hat{\mathrm{k}}\left(\frac{1^3}{3}\right)\ \mathrm{A{\cdot}T{\cdot}m}\\ &= -0.600\,\mathrm{N}\ \hat{\mathrm{k}}\ .\end{aligned}$$

61. (a) We seek the electrostatic field established by the separation of charges (brought on by the magnetic force). We use the ideas discussed in §29-4; especially, see SAMPLE PROBLEM 29-2. With Eq. 29-10, we define the magnitude of the electric field as $|\vec{E}| = v|\vec{B}| = (20)(0.03) = 0.6$ V/m. Its direction may be inferred from Figure 29-8; its direction is opposite to that defined by $\vec{v}\times\vec{B}$. In summary,

$$\vec{E} = -0.600\,\mathrm{V/m}\ \hat{\mathrm{k}}$$

which insures that $\vec{F} = q(\vec{E}+\vec{v}\times\vec{B})$ vanishes.

(b) Eq. 29-9 yields $V = (0.6\ \mathrm{V/m})(2\ \mathrm{m}) = 1.20$ V.

62. With the $\vec{B}$ pointing "out of the page," we evaluate the force (using the right-hand rule) at, say, the dot shown on the left edge of the particle's path, where its velocity is down. If the particle were positively charged, then the force at the dot would be toward the left, which is at odds with the figure (showing it being bent towards the right). Therefore, the particle is negatively charged; it is an electron.

(a) Using Eq. 29-3 (with angle ϕ equal to 90°), we obtain

$$v = \frac{|\vec{F}|}{e\,|\vec{B}|} = 4.99\times10^6\ \mathrm{m/s}\ .$$

(b) Using either Eq. 29-14 or Eq. 29-16, we find $r = 0.00710$ m.

(c) Using Eq. 29-17 (in either its first or last form) readily yields $T = 8.93\times10^{-9}$ s.

63. (a) We are given $\vec{B} = B_x\hat{\imath} = 6\times10^{-5}\,\hat{\imath}$ T, so that $\vec{v}\times\vec{B} = -v_yB_x\hat{\mathrm{k}}$ where $v_y = 4\times10^4$ m/s. We note that the magnetic force on the electron is $(-e)(-v_yB_x\hat{\mathrm{k}})$ and therefore points in the $+\hat{\mathrm{k}}$ direction, at the instant the electron enters the field-filled region. In these terms, Eq. 29-16 becomes

$$r = \frac{m_e\,v_y}{e\,B_x} = 0.0038\ \mathrm{m}\ .$$

(b) One revolution takes $T = 2\pi r/v_y = 0.60\ \mu$s, and during that time the "drift" of the electron in the x direction (which is the *pitch* of the helix) is $\Delta x = v_xT = 0.019$ m where $v_x = 32\times10^3$ m/s.

(c) Returning to our observation of force direction made in part (a), we consider how this is perceived by an observer at some point on the $-x$ axis. As the electron moves away from him, he sees it enter the region with positive v_y (which he might call "upward") but "pushed" in the $+z$ direction (to his right). Hence, he describes the electron's spiral as clockwise.

64. The force associated with the magnetic field must point in the $\hat{\jmath}$ direction in order to cancel the force of gravity in the $-\hat{\jmath}$ direction. By the right-hand rule, $\vec{B}$ points in the $-\hat{\mathrm{k}}$ direction (since $\hat{\imath}\times(-\hat{\mathrm{k}}) = \hat{\jmath}$). Note that the charge is positive; also note that we need to assume $B_y = 0$. The magnitude $|B_z|$ is given by Eq. 29-3 (with $\phi = 90°$). Therefore, with $m = 10\times10^{-3}$ kg, $v = 2.0\times10^4$ m/s and $q = 80\times10^{-6}$ C, we find

$$\vec{B} = B_z\hat{\mathrm{k}} = -\left(\frac{mg}{qv}\right)\hat{\mathrm{k}} = -0.061\hat{\mathrm{k}}$$

in SI units (Tesla).

65. By the right-hand rule, we see that $\vec{v} \times \vec{B}$ points along $-\hat{\mathrm{k}}$. From Eq. 29-2 ($\vec{F} = q\vec{v} \times \vec{B}$), we find that for the force to point along $+\hat{\mathrm{k}}$, we must have $q < 0$. Now, examining the magnitudes (in SI units) in Eq. 29-3, we find

$$\begin{aligned} |\vec{F}| &= |q|\, v\, |\vec{B}| \sin\phi \\ 0.48 &= |q|(4000)(0.0050)\sin 35^\circ \end{aligned}$$

which yields $|q| = 0.040$ C. In summary, then, $q = -40\,\mathrm{mC}$.

66. (a) Since $K = qV$ we have $K_p = K_d = \frac{1}{2}K_\alpha$ (as $q_\alpha = 2K_d = 2K_p$).

(b) and (c) Since $r = \sqrt{2mK}/qB \propto \sqrt{mK}/q$, we have

$$\begin{aligned} r_d &= \sqrt{\frac{m_d K_d}{m_p K_p}}\,\frac{q_p r_p}{q_d} = \sqrt{\frac{(2.00\,\mathrm{u})K_p}{(1.00\,\mathrm{u})K_p}}\, r_p = 10\sqrt{2}\,\mathrm{cm} = 14 \text{ cm} , \\ r_\alpha &= \sqrt{\frac{m_\alpha K_\alpha}{m_p K_p}}\,\frac{q_p r_p}{q_\alpha} = \sqrt{\frac{(4.00\,\mathrm{u})K_\alpha}{(1.00\,\mathrm{u})(K_\alpha/2)}}\,\frac{e r_p}{2e} = 10\sqrt{2}\,\mathrm{cm} = 14 \text{ cm} . \end{aligned}$$

67. (a) The radius of the cyclotron dees should be

$$r = \frac{m_p v}{qB} = \frac{(1.67\times10^{-27}\,\mathrm{kg})(3.00\times10^8\,\mathrm{m/s})/10}{(1.60\times10^{-19}\,\mathrm{C})(1.4\,\mathrm{T})} = 0.22 \text{ m} .$$

(b) The frequency should be

$$f_{\mathrm{osc}} = \frac{v}{2\pi r} = \frac{3.00\times10^7\,\mathrm{m/s}}{2\pi(0.22\,\mathrm{m})} = 2.1\times10^7 \text{ Hz} .$$

68. The magnetic force on the wire is $F_B = idB$, pointing to the left. Thus $v = at = (F_B/m)t = idBt/m$, to the left (away from the generator).

69. (a) We use Eq. 29-10: $v_d = E/B = (10\times10^{-6}\,\mathrm{V}/1.0\times10^{-2}\,\mathrm{m})/(1.5\,\mathrm{T}) = 6.7\times10^{-4}\,\mathrm{m/s}$.

(b) We rewrite Eq. 29-12 in terms of the electric field:

$$n = \frac{Bi}{V\ell e} = \frac{Bi}{(Ed)\ell e} = \frac{Bi}{EAe}$$

which we use $A = \ell d$. In this experiment, $A = (0.010\,\mathrm{m})(10\times10^{-6}\,\mathrm{m}) = 1.0\times10^{-7}\,\mathrm{m}^2$. By Eq. 29-10, v_d equals the ratio of the fields (as noted in part (a)), so we are led to

$$\begin{aligned} n &= \frac{Bi}{EAe} = \frac{i}{v_d A e} \\ &= \frac{3.0\,\mathrm{A}}{(6.7\times10^{-4}\,\mathrm{m/s})(1.0\times10^{-7}\,\mathrm{m}^2)(1.6\times10^{-19}\,\mathrm{C})} \\ &= 2.8\times10^{29}/\mathrm{m}^3 . \end{aligned}$$

(c) Since a drawing of an inherently 3-D situation can be misleading, we describe it in terms of horizontal *north, south, east, west* and vertical *up* and *down* directions. We assume $\vec{B}$ points up and the conductor's width of 0.010 m is along an east-west line. We take the current going northward. The conduction electrons experience a westward magnetic force (by the right-hand rule), which results in the west side of the conductor being negative and the east side being positive (with reference to the Hall voltage which becomes established).

70. The fact that the fields are uniform, with the feature that the charge moves in a straight line, implies the speed is constant (if it were not, then the magnetic *force* would vary while the electric force could not – causing it to deviate from straight-line motion). This is then the situation leading to Eq. 29-7, and we find

$$|\vec{E}| = v|\vec{B}| = 500 \text{ V/m} .$$

Its direction (so that $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$ vanishes) is downward (in "page" coordinates).

71. (a) We use Eq. 29-2 and Eq. 3-30:

$$\begin{aligned}
\vec{F} &= q\vec{v} \times \vec{B} \\
&= (+e)\left((v_yB_z - v_zB_y)\,\hat{\imath} + (v_zB_x - v_xB_z)\,\hat{\jmath} + (v_xB_y - v_yB_x)\,\hat{k}\right) \\
&= \left(1.60 \times 10^{-19}\right)\Big(((4)(0.008) - (-6)(-0.004))\,\hat{\imath} + \\
&\qquad ((-6)(0.002) - (-2)(0.008))\,\hat{\jmath} + ((-2)(-0.004) - (4)(0.002))\,\hat{k}\Big) \\
&= \left(1.28 \times 10^{-21}\right)\hat{\imath} + \left(6.41 \times 10^{-22}\right)\hat{\jmath}
\end{aligned}$$

with SI units understood.

(b) By definition of the cross product, $\vec{v} \perp \vec{F}$. This is easily verified by taking the dot (scalar) product of $\vec{v}$ with the result of part (a), yielding zero, provided care is taken not to introduce any round-off error.

(c) There are several ways to proceed. It may be worthwhile to note, first, that if B_z were 6.00 mT instead of 8.00 mT then the two vectors would be exactly antiparallel. Hence, the angle θ between $\vec{B}$ and $\vec{v}$ is presumably "close" to 180°. Here, we use Eq. 3-20:

$$\theta = \cos^{-1}\frac{\vec{v}\cdot\vec{B}}{|\vec{v}|\,|\vec{B}|} = \cos^{-1}\frac{-68}{\sqrt{56}\,\sqrt{84}} = 173^\circ \ .$$

72. (a) From symmetry, we conclude that any x-component of force will vanish (evaluated over the entirety of the bent wire as shown). By the right-hand rule, a field in the $\hat{k}$ direction produces on each part of the bent wire a y-component of force pointing in the $-\hat{\jmath}$ direction; each of these components has magnitude

$$|F_y| = i\,\ell\,|\vec{B}|\sin 30^\circ = 8 \text{ N} .$$

Therefore, the the force (in Newtons) on the wire shown in the figure is $-16\,\hat{\jmath}$.

(b) The force exerted on the left half of the bent wire points in the $-\hat{k}$ direction, by the right-hand rule, and the force exerted on the right half of the wire points in the $+\hat{k}$ direction. It is clear that the magnitude of each force is equal, so that the force (evaluated over the entirety of the bent wire as shown) must necessarily vanish.

73. The contribution to the force by the magnetic field ($\vec{B} = B_x\hat{\imath} = -0.020\hat{\imath}$ T) is given by Eq. 29-2:

$$\begin{aligned}
\vec{F}_B &= q\vec{v} \times \vec{B} \\
&= q\left((17000\hat{\imath} \times B_x\hat{\imath}) + \left(-11000\hat{\jmath} \times B_x\hat{\imath}\right) + \left(7000\hat{k} \times B_x\hat{\imath}\right)\right) \\
&= q\left(-220\hat{k} - 140\hat{\jmath}\right)
\end{aligned}$$

in SI units. And the contribution to the force by the electric field ($\vec{E} = E_y\hat{\jmath} = 300\hat{\jmath}$ V/m) is given by Eq. 23-1: $\vec{F}_E = qE_y\hat{\jmath}$. Using $q = 5.0 \times 10^{-6}$ C, the net force (with the unit newton understood) on the particle is

$$\vec{F} = 0.0008\hat{\jmath} - 0.0011\hat{k} \ .$$

74. Letting $B_x = B_y = B_1$ and $B_z = B_2$ and using Eq. 29-2 and Eq. 3-30, we obtain (with SI units understood)

$$\begin{aligned}\vec{F} &= q\vec{v}\times\vec{B}\\ 4\hat{\imath} - 20\hat{\jmath} + 12\hat{k} &= 2\left((4B_2 - 6B_1)\,\hat{\imath} + (6B_1 - 2B_2)\,\hat{\jmath} + (2B_1 - 4B_1)\,\hat{k}\right)\ .\end{aligned}$$

Equating like components, we find $B_1 = -3$ and $B_2 = -4$. In summary (with the unit Tesla understood), $\vec{B} = -3.0\hat{\imath} - 3.0\hat{\jmath} - 4.0\hat{k}$.

75. (a) We use Eq. 29-16 to calculate r:

$$r = \frac{m_e v}{qB} = \frac{(9.11\times10^{-31}\,\text{kg})(0.10)(3.00\times10^{8}\,\text{m/s})}{(1.60\times10^{-19}\,\text{C})(0.50\,\text{T})} = 3.4\times10^{-4}\ \text{m}\ .$$

(b) The kinetic energy, computed using the formula from Chapter 7, is

$$K = \frac{1}{2}m_e v^2 = \frac{(9.11\times10^{-31}\,\text{kg})(3.0\times10^{7}\,\text{m/s})^2}{2(1.6\times10^{-19}\,\text{J/eV})} = 2.6\times10^{3}\ \text{eV}\ .$$

76. (a) From $m = B^2qx^2/8V$ we have $\Delta m = (B^2q/8V)(2x\Delta x)$. Here $x = \sqrt{8Vm/B^2q}$, which we substitute into the expression for Δm to obtain

$$\Delta m = \left(\frac{B^2q}{8V}\right)2\sqrt{\frac{8mV}{B^2q}}\Delta x = B\sqrt{\frac{mq}{2V}}\,\Delta x\ .$$

(b) The distance between the spots made on the photographic plate is

$$\begin{aligned}\Delta x &= \frac{\Delta m}{B}\sqrt{\frac{2V}{mq}}\\ &= \frac{(37\,\text{u} - 35\,\text{u})(1.66\times10^{-27}\,\text{kg/u})}{0.50\,\text{T}}\sqrt{\frac{2(7.3\times10^{3}\,\text{V})}{(36\,\text{u})(1.66\times10^{-27}\,\text{kg/u})(1.60\times10^{-19}\,\text{C})}}\\ &= 8.2\times10^{-3}\ \text{m}\ .\end{aligned}$$

77. (a) Since $\vec{B}$ is uniform,

$$\vec{F}_B = \int_{\text{wire}} i d\vec{L}\times\vec{B} = i\left(\int_{\text{wire}} d\vec{L}\right)\times\vec{B} = i\vec{L}_{ab}\times\vec{B}\ ,$$

where we note that $\int_{\text{wire}} d\vec{L} = \vec{L}_{ab}$, with $\vec{L}_{ab}$ being the displacement vector from a to b.

(b) Now $\vec{L}_{ab} = 0$, so $\vec{F}_B = i\vec{L}_{ab}\times\vec{B} = 0$.

78. We use $d\vec{F}_B = id\vec{L}\times\vec{B}$, where $d\vec{L} = dx\hat{\imath}$ and $\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath}$. Thus,

$$\begin{aligned}\vec{F}_B &= \int i d\vec{L}\times\vec{B}\\ &= \int_{x_i}^{x_f} i\,dx\,\hat{\imath}\times(B_x\hat{\imath} + B_y\hat{\jmath}) = i\int_{x_i}^{x_f} B_y\,dx\hat{k}\\ &= (-5.0\,\text{A})\left(\int_{1.0}^{3.0}(8.0x^2\,dx)\,(\text{m}\cdot\text{mT})\right)\hat{k}\\ &= -0.35\ \text{N}\ \hat{k}\ .\end{aligned}$$