Chapter 15

1. The pressure increase is the applied force divided by the area: $\Delta p = F/A = F/\pi r^2$, where r is the radius of the piston. Thus $\Delta p = (42\,\text{N})/\pi(0.011\,\text{m})^2 = 1.1 \times 10^5\,\text{Pa}$. This is equivalent to 1.1 atm.

2. We note that the container is cylindrical, the important aspect of this being that it has a uniform cross-section (as viewed from above); this allows us to relate the pressure at the bottom simply to the total weight of the liquids. Using the fact that $1\,\text{L} = 1000\,\text{cm}^3$, we find the weight of the first liquid to be

$$\begin{aligned} W_1 &= m_1 g = \rho_1 V_1 g \\ &= (2.6\,\text{g/cm}^3)(0.50\,\text{L})(1000\,\text{cm}^3/\,\text{L})(980\,\text{cm/s}^2) = 1.27 \times 10^6\,\text{g}\cdot\text{cm/s}^2 = 12.7\ \text{N}\ . \end{aligned}$$

In the last step, we have converted grams to kilograms and centimeters to meters. Similarly, for the second and the third liquids, we have

$$W_2 = m_2 g = \rho_2 V_2 g = (1.0\,\text{g/cm}^3)(0.25\,\text{L})(1000\,\text{cm}^3/\,\text{L})(980\,\text{cm/s}^2) = 2.5\,\text{N}$$

and

$$W_3 = m_3 g = \rho_3 V_3 g = (0.80\,\text{g/cm}^3)(0.40\,\text{L})(1000\,\text{cm}^3/\text{L})(980\,\text{cm/s}^2) = 3.1\ \text{N}\ .$$

The total force on the bottom of the container is therefore $F = W_1 + W_2 + W_3 = 18\,\text{N}$.

3. The air inside pushes outward with a force given by $p_i A$, where p_i is the pressure inside the room and A is the area of the window. Similarly, the air on the outside pushes inward with a force given by $p_o A$, where p_o is the pressure outside. The magnitude of the net force is $F = (p_i - p_o)A$. Since $1\,\text{atm} = 1.013 \times 10^5\,\text{Pa}$,

$$F = (1.0\,\text{atm} - 0.96\,\text{atm})(1.013 \times 10^5\,\text{Pa/atm})(3.4\,\text{m})(2.1\,\text{m}) = 2.9 \times 10^4\ \text{N}\ .$$

4. Knowing the standard air pressure value in several units allows us to set up a variety of conversion factors:

   (a) $P = \left(28\,\text{lb/in.}^2\right)\left(\frac{1.01\times10^5\,\text{Pa}}{14.7\,\text{lb/in}^2}\right) = 190\ \text{kPa}$.

   (b) $(120\,\text{mmHg})\left(\frac{1.01\times10^5\,\text{Pa}}{760\,\text{mmHg}}\right) = 15.9\ \text{kPa}$,

   $(80\,\text{mmHg})\left(\frac{1.01\times10^5\,\text{Pa}}{760\,\text{mmHg}}\right) = 10.6\ \text{kPa}$.

5. Let the volume of the expanded air sacs be V_a and that of the fish with its air sacs collapsed be V. Then

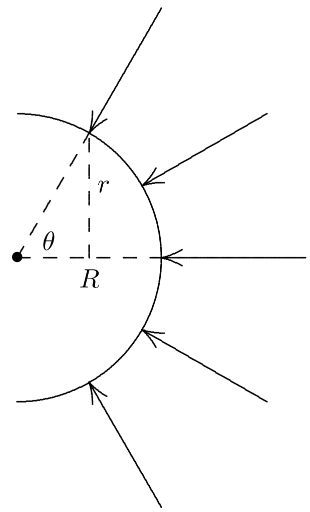
$$\rho_{\text{fish}} = \frac{m_{\text{fish}}}{V} = 1.08\,\text{g/cm}^3 \quad \text{and} \quad \rho_w = \frac{m_{\text{fish}}}{V + V_a} = 1.00\,\text{g/cm}^3\ .$$

where ρ_w is the density of the water. This implies $\rho_{\text{fish}} V = \rho_w (V + V_a)$ or $(V + V_a)/V = 1.08/1.00$, which gives $V_a/(V + V_a) = 7.4\%$.

6. The magnitude F of the force required to pull the lid off is $F = (p_o - p_i)A$, where p_o is the pressure outside the box, p_i is the pressure inside, and A is the area of the lid. Recalling that $1\,\text{N/m}^2 = 1\,\text{Pa}$, we obtain

$$p_i = p_o - \frac{F}{A} = 1.0 \times 10^5\,\text{Pa} - \frac{480\,\text{N}}{77 \times 10^{-4}\,\text{m}^2} = 3.8 \times 10^4 \text{ Pa} .$$

7. (a) The pressure difference results in forces applied as shown in the figure. We consider a team of horses pulling to the right. To pull the sphere apart, the team must exert a force at least as great as the horizontal component of the total force determined by "summing" (actually, integrating) these force vectors.



We consider a force vector at angle θ. Its leftward component is $\Delta p \cos\theta dA$, where dA is the area element for where the force is applied. We make use of the symmetry of the problem and let dA be that of a ring of constant θ on the surface. The radius of the ring is $r = R\sin\theta$, where R is the radius of the sphere. If the angular width of the ring is $d\theta$, in radians, then its width is $R\,d\theta$ and its area is $dA = 2\pi R^2 \sin\theta\, d\theta$. Thus the net horizontal component of the force of the air is given by

$$\begin{aligned} F_h &= 2\pi R^2\,\Delta p \int_0^{\pi/2} \sin\theta\cos\theta\, d\theta \\ &= \pi R^2\,\Delta p \sin^2\theta \,\Big|_0^{\pi/2} = \pi R^2\,\Delta p \ . \end{aligned}$$

(b) We use $1\,\text{atm} = 1.01 \times 10^5\,\text{Pa}$ to show that $\Delta p = 0.90\,\text{atm} = 9.09 \times 10^4\,\text{Pa}$. The sphere radius is $R = 0.30\,\text{m}$, so $F_h = \pi(0.30\,\text{m})^2(9.09 \times 10^4\,\text{Pa}) = 2.6 \times 10^4\,\text{N}$.

(c) One team of horses could be used if one half of the sphere is attached to a sturdy wall. The force of the wall on the sphere would balance the force of the horses.

8. We estimate the pressure difference (specifically due to hydrostatic effects) as follows:

$$\Delta p = \rho g h = (1.06 \times 10^3\,\text{kg/m}^3)\left(9.8\,\text{m/s}^2\right)(1.83\,\text{m}) = 1.90 \times 10^4 \text{ Pa} .$$

9. The pump must work against the hydrostatic pressure exerted by the column of sewage (of density ρ and height $\ell = 8.2\,\text{m} - 2.1\,\text{m} = 6.1\,\text{m}$). The (minimum) pressure difference that must be maintained by the pump is $\Delta p = \rho g \ell = (900\,\text{kg/m}^3)(9.8\,\text{m/s}^2)(6.1\,\text{m}) = 5.4 \times 10^4\,\text{Pa}$.

10. From the Figure, we see that the minimum pressure for diamond to form at 1000°C is $p_{\min} = 4.0\,\text{GPa}$. This pressure occurs at a minimum depth of $h_{\min}$ given by $p_{\min} = \rho g h_{\min}$. Thus,

$$h_{\min} = \frac{p_{\min}}{\rho g} = \frac{4.0 \times 10^9\,\text{Pa}}{\left(3.1 \times 10^3\,\text{kg/m}^3\right)\left(9.8\,\text{m/s}^2\right)} = 1.3 \times 10^5 \text{ m} .$$

11. (a) We note that the pool has uniform cross-section (as viewed from above); this allows us to relate the pressure at the bottom simply to the total weight of the liquid. Thus,

$$F_{\text{bottom}} = mg = \rho g V = \left(1000\,\text{kg/m}^3\right)\left(9.8\,\text{m/s}^2\right)\left(540\,\text{m}^3\right) = 5.3 \times 10^6 \text{ N} .$$

(b) The average pressure due to the water (that is, averaged over depth h) is

$$p_{\text{avg}} = \rho g \left(\frac{h}{2}\right)$$

where $h = 2.5$ m. Thus, the force on a short side (of area $A = 9.0 \times 2.5$ in SI units) is

$$F_{\text{short side}} = \rho g \left(\frac{h}{2}\right) A = 2.8 \times 10^5 \text{ N} .$$

(c) The area of a long side is $A' = 24 \times 2.5$ in SI units. Therefore, the force exerted by the water pressure on a long side is

$$F_{\text{long side}} = \rho g \left(\frac{h}{2}\right) A' = 7.4 \times 10^5 \text{ N} .$$

(d) If the pool is above ground, then it is clear that the air pressure outside the walls "cancels" any contribution of air pressure to the water pressure exerted by the liquid in the pool. If the pool is, as is often the case, surrounded by soil, then the situation may be more subtle, but our expectation is under normal circumstances the push from the soil certainly compensates for any atmospheric contribution to the water pressure (due to a "liberal interpretation" of Pascal's principle).

12. (a) The total weight is

$$W = \rho g h A = \left(1.00 \times 10^3 \, \text{kg/m}^3\right)\left(9.8 \, \text{m/s}^2\right)(200 \, \text{m})\left(3000 \, \text{m}^2\right) = 6.06 \times 10^9 \text{ N} .$$

(b) The water pressure is

$$p = \rho g h = \left(1.03 \times 10^3 \, \text{kg/m}^3\right)\left(9.8 \, \text{m/s}^2\right)(200 \, \text{m})\left(\frac{1 \, \text{atm}}{1.01 \times 10^5 \, \text{Pa}}\right) = 20 \text{ atm}$$

which is too much for anybody to endure without special equipment.

13. The pressure p at the depth d of the hatch cover is $p_0 + \rho g d$, where ρ is the density of ocean water and p_0 is atmospheric pressure. The downward force of the water on the hatch cover is $(p_0 + \rho g d)A$, where A is the area of the cover. If the air in the submarine is at atmospheric pressure then it exerts an upward force of $p_0 A$. The minimum force that must be applied by the crew to open the cover has magnitude $F = (p_0 + \rho g d)A - p_0 A = \rho g d A = (1025 \, \text{kg/m}^3)(9.8 \, \text{m/s}^2)(100 \, \text{m})(1.2 \, \text{m})(0.60 \, \text{m}) = 7.2 \times 10^5$ N.

14. Since the pressure (caused by liquid) at the bottom of the barrel is doubled due to the presence of the narrow tube, so is the hydrostatic force. The ratio is therefore equal to 2.0. The difference between the hydrostatic force and the weight is accounted for by the additional upward force exerted by water on the top of the barrel due to the increased pressure introduced by the water in the tube.

15. When the levels are the same the height of the liquid is $h = (h_1 + h_2)/2$, where h_1 and h_2 are the original heights. Suppose h_1 is greater than h_2. The final situation can then be achieved by taking liquid with volume $A(h_1 - h)$ and mass $\rho A(h_1 - h)$, in the first vessel, and lowering it a distance $h - h_2$. The work done by the force of gravity is $W = \rho A(h_1 - h)g(h - h_2)$. We substitute $h = (h_1 + h_2)/2$ to obtain $W = \frac{1}{4}\rho g A(h_1 - h_2)^2$.

16. Letting $p_a = p_b$, we find $\rho_c g(6.0 \, \text{km} + 32 \, \text{km} + D) + \rho_m(y - D) = \rho_c g(32 \, \text{km}) + \rho_m(y)$ and obtain

$$D = \frac{(6.0 \, \text{km})\rho_c}{\rho_m - \rho_c} = \frac{(6.0 \, \text{km})\left(2.9 \, \text{g/cm}^3\right)}{3.3 \, \text{g/cm}^3 - 2.9 \, \text{g/cm}^3} = 44 \text{ km} .$$

17. We assume that the pressure is the same at all points that are the distance $d = 20\,\text{km}$ below the surface. For points on the left side of Fig. 15-31, this pressure is given by $p = p_0 + \rho_o g d_o + \rho_c g d_c + \rho_m g d_m$, where p_0 is atmospheric pressure, ρ_o and d_o are the density and depth of the ocean, ρ_c and d_c are the density and thickness of the crust, and ρ_m and d_m are the density and thickness of the mantle (to a depth of 20 km). For points on the right side of the figure p is given by $p = p_0 + \rho_c g d$. We equate the two expressions for p and note that g cancels to obtain $\rho_c d = \rho_o d_o + \rho_c d_c + \rho_m d_m$. We substitute $d_m = d - d_o - d_c$ to obtain

$$\rho_c d = \rho_o d_o + \rho_c d_c + \rho_m d - \rho_m d_o - \rho_m d_c \ .$$

We solve for d_o:

$$\begin{aligned} d_o &= \frac{\rho_c d_c - \rho_c d + \rho_m d - \rho_m d_c}{\rho_m - \rho_o} = \frac{(\rho_m - \rho_c)(d - d_c)}{\rho_m - \rho_o} \\ &= \frac{\left(3.3\,\text{g/cm}^3 - 2.8\,\text{g/cm}^3\right)(20\,\text{km} - 12\,\text{km})}{3.3\,\text{g/cm}^3 - 1.0\,\text{g/cm}^3} = 1.7\ \text{km} \ . \end{aligned}$$

18. (a) The force on face A of area A_A is

$$\begin{aligned} F_A &= p_A A_A = \rho_w g h_A A_A = 2\rho_w g d^3 \\ &= 2\left(1.0 \times 10^3\,\text{kg/m}^3\right)\left(9.8\,\text{m/s}^2\right)(5.0\,\text{m})^3 = 2.5 \times 10^6\ \text{N} \ . \end{aligned}$$

(b) The force on face B is

$$\begin{aligned} F_B &= p_{\text{avg}B} A_B = \rho_w g \left(\frac{5d}{2}\right) d^2 = \frac{5}{2}\rho_w g d^3 \\ &= \frac{5}{2}\left(1.0 \times 10^3\,\text{kg/m}^3\right)\left(9.8\,\text{m/s}^2\right)(5.0\,\text{m})^3 = 3.1 \times 10^6\ \text{N} \ . \end{aligned}$$

Note that these figures are due to the water pressure only. If you add the contribution from the atmospheric pressure, then you need to add $F' = (1.0 \times 10^5\,\text{Pa})(5.0\,\text{m})^2 = 2.5 \times 10^6\,\text{N}$ to each of the figures above. The results would then be $5.0 \times 10^6\,\text{N}$ and $5.6 \times 10^6\,\text{N}$, respectively.

19. (a) At depth y the gauge pressure of the water is $p = \rho g y$, where ρ is the density of the water. We consider a horizontal strip of width W at depth y, with (vertical) thickness dy, across the dam. Its area is $dA = W\,dy$ and the force it exerts on the dam is $dF = p\,dA = \rho g y W\,dy$. The total force of the water on the dam is

$$F = \int_0^D \rho g y W\,dy = \frac{1}{2}\rho g W D^2 \ .$$

(b) Again we consider the strip of water at depth y. Its moment arm for the torque it exerts about O is $D - y$ so the torque it exerts is $d\tau = dF(D - y) = \rho g y W(D - y)dy$ and the total torque of the water is

$$\tau = \int_0^D \rho g y W(D - y)\,dy = \rho g W\left(\frac{1}{2}D^3 - \frac{1}{3}D^3\right) = \frac{1}{6}\rho g W D^3 \ .$$

(c) We write $\tau = rF$, where r is the effective moment arm. Then,

$$r = \frac{\tau}{F} = \frac{\frac{1}{6}\rho g W D^3}{\frac{1}{2}\rho g W D^2} = \frac{D}{3} \ .$$

20. The gauge pressure you can produce is

$$p = -\rho g h = -\frac{\left(1000\,\text{kg/m}^3\right)\left(9.8\,\text{m/s}^2\right)\left(4.0 \times 10^{-2}\,\text{m}\right)}{1.01 \times 10^5\ \text{Pa/atm}} = -3.9 \times 10^{-3}\ \text{atm}$$

where the minus sign indicates that the pressure inside your lung is less than the outside pressure.

21. (a) We use the expression for the variation of pressure with height in an incompressible fluid: $p_2 = p_1 - \rho g(y_2 - y_1)$. We take y_1 to be at the surface of Earth, where the pressure is $p_1 = 1.01 \times 10^5$ Pa, and y_2 to be at the top of the atmosphere, where the pressure is $p_2 = 0$. For this calculation, we take the density to be uniformly $1.3\,\mathrm{kg/m^3}$. Then,

$$y_2 - y_1 = \frac{p_1}{\rho g} = \frac{1.01 \times 10^5\,\mathrm{Pa}}{\left(1.3\,\mathrm{kg/m^3}\right)\left(9.8\,\mathrm{m/s^2}\right)} = 7.9 \times 10^3\,\mathrm{m} = 7.9\ \mathrm{km}\ .$$

(b) Let h be the height of the atmosphere. Now, since the density varies with altitude, we integrate

$$p_2 = p_1 - \int_0^h \rho g\, dy\ .$$

Assuming $\rho = \rho_0(1 - y/h)$, where ρ_0 is the density at Earth's surface and $g = 9.8\,\mathrm{m/s^2}$ for $0 \le y \le h$, the integral becomes

$$p_2 = p_1 - \int_0^h \rho_0 g \left(1 - \frac{y}{h}\right) dy = p_1 - \frac{1}{2}\rho_0 g h\ .$$

Since $p_2 = 0$, this implies

$$h = \frac{2p_1}{\rho_0 g} = \frac{2(1.01 \times 10^5\,\mathrm{Pa})}{(1.3\,\mathrm{kg/m^3})(9.8\,\mathrm{m/s^2})} = 16 \times 10^3\,\mathrm{m} = 16\ \mathrm{km}\ .$$

22. (a) According to Pascal's principle $F/A = f/a \ \rightarrow\ F = (A/a)f$.

(b) We obtain

$$f = \frac{a}{A}F = \frac{(3.80\,\mathrm{cm})^2}{(53.0\,\mathrm{cm})^2}(20.0 \times 10^3\,\mathrm{N}) = 103\ \mathrm{N}\ .$$

The ratio of the squares of diameters is equivalent to the ratio of the areas. We also note that the area units cancel.

23. We assume the fluid in the press is incompressible. Then, the work done by the output force is the same as the work done by the input force. If the large piston moves a distance D and the small piston moves a distance d, then $fd = FD$ and

$$D = \frac{fd}{F} = \frac{(103\,\mathrm{N})(0.85\,\mathrm{m})}{20.0 \times 10^3\,\mathrm{N}} = 4.4 \times 10^{-3}\,\mathrm{m} = 4.4\ \mathrm{mm}\ .$$

24. (a) Archimedes' principle makes it clear that a body, in order to float, displaces an amount of the liquid which corresponds to the weight of the body. The problem (indirectly) tells us that the weight of the boat is $W = 35.6$ kN. In salt water of density $\rho' = 1100\,\mathrm{kg/m^3}$, it must displace an amount of liquid having weight equal to 35.6 kN.

(b) The displaced volume of salt water is equal to

$$V' = \frac{W}{\rho' g} = \frac{35600}{(1100)(9.8)} = 3.30\ \mathrm{m^3}\ .$$

In freshwater, it displaces a volume of $V = W/\rho g = 3.63\,\mathrm{m^3}$, where $\rho = 1000\,\mathrm{kg/m^3}$. The difference is $V - V' = 0.33\,\mathrm{m^3}$.

25. (a) The anchor is completely submerged in water of density ρ_w. Its effective weight is $W_{\text{eff}} = W - \rho_w g V$, where W is its actual weight (mg). Thus,

$$V = \frac{W - W_{\text{eff}}}{\rho_w g} = \frac{200\,\mathrm{N}}{\left(1000\,\mathrm{kg/m^3}\right)\left(9.8\,\mathrm{m/s^2}\right)} = 2.04 \times 10^{-2}\ \mathrm{m^3}\ .$$

(b) The mass of the anchor is $m = \rho V$, where ρ is the density of iron (found in Table 15-1). Its weight in air is

$$W = mg = \rho V g = \left(7870\,\mathrm{kg/m^3}\right)\left(2.04 \times 10^{-2}\,\mathrm{m^3}\right)\left(9.8\,\mathrm{m/s^2}\right) = 1.6 \times 10^3\ \mathrm{N}\ .$$

26. (a) The pressure (including the contribution from the atmosphere) at a depth of $h_{\mathrm{top}} = L/2$ (corresponding to the top of the block) is

$$p_{\mathrm{top}} = p_{\mathrm{atm}} + \rho g h_{\mathrm{top}} = 1.01 \times 10^5 + (1030)(9.8)(0.300) = 1.04 \times 10^5\ \mathrm{Pa}$$

where the unit Pa (Pascal) is equivalent to $\mathrm{N/m^2}$. The force on the top surface (of area $A = L^2 = 0.36\,\mathrm{m^2}$) is $F_{\mathrm{top}} = p_{\mathrm{top}} A = 3.75 \times 10^4\,\mathrm{N}$.

(b) The pressure at a depth of $h_{\mathrm{bot}} = 3L/2$ (that of the bottom of the block) is

$$p_{\mathrm{bot}} = p_{\mathrm{atm}} + \rho g h_{\mathrm{bot}} = 1.01 \times 10^5 + (1030)(9.8)(0.900) = 1.10 \times 10^5\ \mathrm{Pa}$$

where we recall that the unit Pa (Pascal) is equivalent to $\mathrm{N/m^2}$. The force on the bottom surface is $F_{\mathrm{bot}} = p_{\mathrm{bot}} A = 3.96 \times 10^4\,\mathrm{N}$.

(c) Taking the difference $F_{\mathrm{bot}} - F_{\mathrm{top}}$ cancels the contribution from the atmosphere (including any numerical uncertainties associated with that value) and leads to

$$F_{\mathrm{bot}} - F_{\mathrm{top}} = \rho g \left(h_{\mathrm{bot}} - h_{\mathrm{top}}\right) A = \rho g L^3 = 2180\ \mathrm{N}$$

which is to be expected on the basis of Archimedes' principle. Two other forces act on the block: an upward tension T and a downward pull of gravity mg. To remain stationary, the tension must be

$$T = mg - (F_{\mathrm{bot}} - F_{\mathrm{top}}) = (450)(9.8) - 2180 = 2230\ \mathrm{N}\ .$$

(d) This has already been noted in the previous part: $F_b = 2180\,\mathrm{N}$, and $T + F_b = mg$.

27. (a) Let V be the volume of the block. Then, the submerged volume is $V_s = 2V/3$. Since the block is floating, the weight of the displaced water is equal to the weight of the block, so $\rho_w V_s = \rho_b V$, where ρ_w is the density of water, and ρ_b is the density of the block. We substitute $V_s = 2V/3$ to obtain $\rho_b = 2\rho_w/3 = 2(1000\,\mathrm{kg/m^3})/3 \approx 670\,\mathrm{kg/m^3}$.

(b) If ρ_o is the density of the oil, then Archimedes' principle yields $\rho_o V_s = \rho_b V$. We substitute $V_s = 0.90V$ to obtain $\rho_o = \rho_b/0.90 = 740\,\mathrm{kg/m^3}$.

28. The weight of the additional cargo ΔW the blimp could carry is equal to the difference between the weight of the helium and that of the hydrogen gas inside the blimp:

$$\begin{aligned}\Delta W &= W_{\mathrm{He}} - W_{\mathrm{H_2}} = (\rho_{\mathrm{He}} - \rho_{\mathrm{H_2}})gV \\ &= \left(0.16\,\mathrm{kg/m^3} - 0.081\,\mathrm{kg/m^3}\right)\left(9.8\,\mathrm{m/s^2}\right)\left(5000\,\mathrm{m^3}\right) \\ &= 3.9 \times 10^3\ \mathrm{N}\end{aligned}$$

which corresponds to about 400 kg of mass. The reason why helium is used is because it is safer (non-flammable).

29. (a) The downward force of gravity mg is balanced by the upward buoyant force of the liquid: $mg = \rho g V_s$. Here m is the mass of the sphere, ρ is the density of the liquid, and V_s is the submerged volume. Thus $m = \rho V_s$. The submerged volume is half the total volume of the sphere, so $V_s = \frac{1}{2}(4\pi/3)r_o^3$, where r_o is the outer radius. Therefore,

$$m = \frac{2\pi}{3}\rho r_o^3 = \left(\frac{2\pi}{3}\right)\left(800\,\mathrm{kg/m^3}\right)(0.090\,\mathrm{m})^3 = 1.22\ \mathrm{kg}\ .$$

(b) The density ρ_m of the material, assumed to be uniform, is given by $\rho_m = m/V$, where m is the mass of the sphere and V is its volume. If r_i is the inner radius, the volume is

$$V = \frac{4\pi}{3}\left(r_o^3 - r_i^3\right) = \frac{4\pi}{3}\left((0.090\,\mathrm{m})^3 - (0.080\,\mathrm{m})^3\right) = 9.09 \times 10^{-4}\ \mathrm{m}^3 \ .$$

The density is

$$\rho_m = \frac{1.22\,\mathrm{kg}}{9.09 \times 10^{-4}\,\mathrm{m}^3} = 1.3 \times 10^3\ \mathrm{kg/m}^3 \ .$$

30. Equilibrium of forces (on the floating body) is expressed as

$$F_b = m_{\mathrm{body}}g \implies \rho_{\mathrm{liquid}}gV_{\mathrm{submerged}} = \rho_{\mathrm{body}}gV_{\mathrm{total}}$$

which leads to

$$\frac{V_{\mathrm{submerged}}}{V_{\mathrm{total}}} = \frac{\rho_{\mathrm{body}}}{\rho_{\mathrm{liquid}}} \ .$$

We are told (indirectly) that two-thirds of the body is below the surface, so the fraction above is 2/3. Thus, with $\rho_{\mathrm{body}} = 0.98\,\mathrm{g/cm}^3$, we find $\rho_{\mathrm{liquid}} \approx 1.5\,\mathrm{g/cm}^3$ – certainly much more dense than normal seawater (the Dead Sea is about seven times saltier than the ocean due to the high evaporation rate and low rainfall in that region).

31. For our estimate of $V_{\mathrm{submerged}}$ we interpret "almost completely submerged" to mean

$$V_{\mathrm{submerged}} \approx \frac{4}{3}\pi r_{\mathrm{o}}^3 \qquad \text{where} \quad r_{\mathrm{o}} = 60\ \mathrm{cm} \ .$$

Thus, equilibrium of forces (on the iron sphere) leads to

$$F_b = m_{\mathrm{iron}}g \implies \rho_{\mathrm{water}}gV_{\mathrm{submerged}} = \rho_{\mathrm{iron}}g\left(\frac{4}{3}\pi r_{\mathrm{o}}^3 - \frac{4}{3}\pi r_{\mathrm{i}}^3\right)$$

where r_{i} is the inner radius (half the inner diameter). Plugging in our estimate for $V_{\mathrm{submerged}}$ as well as the densities of water ($1.0\,\mathrm{g/cm}^3$) and iron ($7.87\,\mathrm{g/cm}^3$), we obtain the inner diameter:

$$2r_{\mathrm{i}} = 2r_{\mathrm{o}}\left(1 - \frac{1}{7.87}\right)^{1/3} = 57.3\ \mathrm{cm} \ .$$

32. (a) Since the lead is not displacing any water (of density ρ_w), the lead's volume is not contributing to the buoyant force F_b. If the immersed volume of wood is V_i, then

$$F_b = \rho_w V_i g = 0.90\rho_w V_{\mathrm{wood}} g = 0.90\rho_w g\left(\frac{m_{\mathrm{wood}}}{\rho_{\mathrm{wood}}}\right) ,$$

which, when floating, equals the weights of the wood and lead:

$$F_b = 0.90\rho_w g\left(\frac{m_{\mathrm{wood}}}{\rho_{\mathrm{wood}}}\right) = (m_{\mathrm{wood}} + m_{\mathrm{lead}})g \ .$$

Thus,

$$\begin{aligned} m_{\mathrm{lead}} &= 0.90\rho_w\left(\frac{m_{\mathrm{wood}}}{\rho_{\mathrm{wood}}}\right) - m_{\mathrm{wood}} \\ &= \frac{(0.90)(1000\,\mathrm{kg/m}^3)(3.67\,\mathrm{kg})}{600\,\mathrm{kg/m}^3} - 3.67\,\mathrm{kg} = 1.84\,\mathrm{kg} \approx 1.8\ \mathrm{kg} \ . \end{aligned}$$

(b) In this case, the volume $V_{\text{lead}} = m_{\text{lead}}/\rho_{\text{lead}}$ also contributes to F_b. Consequently,

$$F_b = 0.90\rho_w g\left(\frac{m_{\text{wood}}}{\rho_{\text{wood}}}\right) + \left(\frac{\rho_w}{\rho_{\text{lead}}}\right) m_{\text{lead}}g = (m_{\text{wood}} + m_{\text{lead}})g\,,$$

which leads to

$$\begin{aligned} m_{\text{lead}} &= \frac{0.90(\rho_w/\rho_{\text{wood}})m_{\text{wood}} - m_{\text{wood}}}{1 - \rho_w/\rho_{\text{lead}}} \\ &= \frac{1.84\,\text{kg}}{1 - \left(1.00 \times 10^3\,\text{kg/m}^3/1.13 \times 10^4\,\text{kg/m}^3\right)} = 2.0\ \text{kg}\,. \end{aligned}$$

33. The volume V_{cav} of the cavities is the difference between the volume V_{cast} of the casting as a whole and the volume V_{iron} contained: $V_{\text{cav}} = V_{\text{cast}} - V_{\text{iron}}$. The volume of the iron is given by $V_{\text{iron}} = W/g\rho_{\text{iron}}$, where W is the weight of the casting and ρ_{iron} is the density of iron. The effective weight in water (of density ρ_w) is $W_{\text{eff}} = W - g\rho_w V_{\text{cast}}$. Thus, $V_{\text{cast}} = (W - W_{\text{eff}})/g\rho_w$ and

$$\begin{aligned} V_{\text{cav}} &= \frac{W - W_{\text{eff}}}{g\rho_w} - \frac{W}{g\rho_{\text{iron}}} \\ &= \frac{6000\,\text{N} - 4000\,\text{N}}{(9.8\,\text{m/s}^2)(1000\,\text{kg/m}^3)} - \frac{6000\,\text{N}}{(9.8\,\text{m/s}^2)(7.87 \times 10^3\,\text{kg/m}^3)} \\ &= 0.126\ \text{m}^3\,. \end{aligned}$$

34. Let F_o be the buoyant force of air exerted on the object (of mass m and volume V), and F_{brass} be the buoyant force on the brass weights (of total mass m_{brass} and volume V_{brass}). Then we have

$$F_o = \rho_{\text{air}} V g = \rho_{\text{air}}\left(\frac{mg}{\rho}\right)$$

and

$$F_{\text{brass}} = \rho_{\text{air}} V_{\text{brass}} g = \rho_{\text{air}}\left(\frac{m_{\text{brass}}}{\rho_{\text{brass}}}\right)\,.$$

For the two arms of the balance to be in mechanical equilibrium, we require $mg - F_o = m_{\text{brass}}g - F_{\text{brass}}$, or

$$mg - mg\left(\frac{\rho_{\text{air}}}{\rho}\right) = m_{\text{brass}}g - m_{\text{brass}}g\left(\frac{\rho_{\text{air}}}{\rho_{\text{brass}}}\right),$$

which leads to

$$m_{\text{brass}} = \left(\frac{1 - \rho_{\text{air}}/\rho}{1 - \rho_{\text{air}}/\rho_{\text{brass}}}\right)m\,.$$

Therefore, the percent error in the measurement of m is

$$\begin{aligned} \frac{\Delta m}{m} &= \frac{m - m_{\text{brass}}}{m} = 1 - \frac{1 - \rho_{\text{air}}/\rho}{1 - \rho_{\text{air}}/\rho_{\text{brass}}} = \frac{\rho_{\text{air}}(1/\rho - 1/\rho_{\text{brass}})}{1 - \rho_{\text{air}}/\rho_{\text{brass}}} \\ &= \frac{0.0012(1/\rho - 1/8.0)}{1 - 0.0012/8.0} \approx 0.0012\left(\frac{1}{\rho} - \frac{1}{8.0}\right)\,, \end{aligned}$$

where ρ is in g/cm^3. Stating this as a *percent* error, our result is 0.12% multiplied by $\left(\frac{1}{\rho} - \frac{1}{8.0}\right)$.

35. (a) We assume that the top surface of the slab is at the surface of the water and that the automobile is at the center of the ice surface. Let M be the mass of the automobile, ρ_i be the density of ice, and ρ_w be the density of water. Suppose the ice slab has area A and thickness h. Since the volume of

ice is Ah, the downward force of gravity on the automobile and ice is $(M + \rho_i Ah)g$. The buoyant force of the water is $\rho_w Ahg$, so the condition of equilibrium is $(M + \rho_i Ah)g - \rho_w Ahg = 0$ and

$$A = \frac{M}{(\rho_w - \rho_i)h} = \frac{1100\,\text{kg}}{(998\,\text{kg/m}^3 - 917\,\text{kg/m}^3)(0.30\,\text{m})} = 45\ \text{m}^2\ .$$

These density values are found in Table 15-1 of the text.

(b) It does matter where the car is placed since the ice tilts if the automobile is not at the center of its surface.

36. The problem intends for the children to be completely above water. The total downward pull of gravity on the system is

$$3(356\,\text{N}) + N\rho_{\text{wood}}gV$$

where N is the (minimum) number of logs needed to keep them afloat and V is the volume of each log: $V = \pi(0.15\,\text{m})^2(1.80\,\text{m}) = 0.13\,\text{m}^3$. The buoyant force is $F_b = \rho_{\text{water}}gV_{\text{submerged}}$ where we require $V_{\text{submerged}} \le NV$. The density of water is $1000\,\text{kg/m}^3$. To obtain the minimum value of N we set $V_{\text{submerged}} = NV$ and then round our "answer" for N up to the nearest integer:

$$3(356\,\text{N}) + N\rho_{\text{wood}}gV = \rho_{\text{water}}gNV \implies N = \frac{3(356\,\text{N})}{gV\,(\rho_{\text{water}} - \rho_{\text{wood}})}$$

which yields $N = 4.28 \rightarrow 5$ logs.

37. (a) We assume the center of mass is closer to the right end of the rod, so the distance from the left end to the center of mass is $\ell = 0.60$ m. Four forces act on the rod: the upward force of the left rope T_L, the upward force of the right rope T_R, the downward force of gravity mg, and the upward buoyant force F_b. The force of gravity (effectively) acts at the center of mass, and the buoyant force acts at the geometric center of the rod (which has length $L = 0.80$ m). Computing torques about the left end of the rod, we find

$$T_R L + F_b\left(\frac{L}{2}\right) - mg\ell = 0 \implies T_R = \frac{mg\ell - F_b L/2}{L}\ .$$

Now, the buoyant force is equal to the weight of the displaced water (where the volume of displacement is $V = AL$). Thus,

$$F_b = \rho_w gAL = \left(1000\,\text{kg/m}^3\right)\left(9.8\,\text{m/s}^2\right)\left(6.0\times10^{-4}\,\text{m}^2\right)(0.80\,\text{m}) = 4.7\ \text{N}\ .$$

Consequently, the tension in the right rope is

$$T_R = \frac{(1.6\,\text{kg})\left(9.8\,\text{m/s}^2\right)(0.60\,\text{m}) - (4.7\,\text{N})(0.40\,\text{m})}{0.80\,\text{m}} = 9.4\ \text{N}\ .$$

(b) Newton's second law (for the case of zero acceleration) leads to

$$T_L + T_R + F_B - mg = 0 \implies T_L = mg - F_B - T_R = (1.6\,\text{kg})\left(9.8\,\text{m/s}^2\right) - 4.69\,\text{N} - 9.4\,\text{N} = 1.6\ \text{N}\ .$$

38. (a) If the volume of the car below water is V_1 then $F_b = \rho_w V_1 g = W_{\text{car}}$, which leads to

$$V_1 = \frac{W_{\text{car}}}{\rho_w g} = \frac{(1800\,\text{kg})\left(9.8\,\text{m/s}^2\right)}{\left(1000\,\text{kg/m}^3\right)\left(9.8\,\text{m/s}^2\right)} = 1.80\ \text{m}^3\ .$$

(b) We denote the total volume of the car as V and that of the water in it as V_2. Then

$$F_b = \rho_w V g = W_{\rm car} + \rho_w V_2 g$$

which gives

$$\begin{aligned} V_2 &= V - \frac{W_{\rm car}}{\rho_w g} \\ &= \left(0.750\,{\rm m}^3 + 5.00\,{\rm m}^3 + 0.800\,{\rm m}^3\right) - \frac{1800\,{\rm kg}}{1000\,{\rm kg/m}^3} \\ &= 4.75\ {\rm m}^3\ . \end{aligned}$$

39. We use the equation of continuity. Let v_1 be the speed of the water in the hose and v_2 be its speed as it leaves one of the holes. $A_1 = \pi R^2$ is the cross-sectional area of the hose. If there are N holes and A_2 is the area of a single hole, then the equation of continuity becomes

$$v_1 A_1 = v_2\,(N A_2) \implies v_2 = \frac{A_1}{N A_2}\,v_1 = \frac{R^2}{N r^2}\,v_1$$

where R is the radius of the hose and r is the radius of a hole. Noting that $R/r = D/d$ (the ratio of diameters) we find

$$v_2 = \frac{D^2}{N d^2} v_1 = \frac{(1.9\,{\rm cm})^2}{24(0.13\,{\rm cm})^2}(0.91\,{\rm m/s}) = 8.1\ {\rm m/s}\ .$$

40. We use the equation of continuity and denote the depth of the river as h. Then,

$$(8.2\,{\rm m})(3.4\,{\rm m})(2.3\,{\rm m/s}) + (6.8\,{\rm m})(3.2{\rm m})(2.6\,{\rm m/s}) = h(10.5\,{\rm m})(2.9\,{\rm m/s})$$

which leads to $h = 4.0\,{\rm m}$.

41. Suppose that a mass Δm of water is pumped in time Δt. The pump increases the potential energy of the water by $\Delta m g h$, where h is the vertical distance through which it is lifted, and increases its kinetic energy by $\frac{1}{2}\Delta m v^2$, where v is its final speed. The work it does is $\Delta W = \Delta m g h + \frac{1}{2}\Delta m v^2$ and its power is

$$P = \frac{\Delta W}{\Delta t} = \frac{\Delta m}{\Delta t}\left(gh + \frac{1}{2}v^2\right)\ .$$

Now the rate of mass flow is $\Delta m/\Delta t = \rho_w A v$, where ρ_w is the density of water and A is the area of the hose. The area of the hose is $A = \pi r^2 = \pi(0.010\,{\rm m})^2 = 3.14\times10^{-4}\,{\rm m}^2$ and $\rho_w A v = (1000\,{\rm kg/m}^3)(3.14\times 10^{-4}\,{\rm m}^2)(5.0\,{\rm m/s}) = 1.57\,{\rm kg/s}$. Thus,

$$\begin{aligned} P &= \rho A v\left(gh + \frac{1}{2}v^2\right) \\ &= (1.57\,{\rm kg/s})\left((9.8\,{\rm m/s}^2)(3.0\,{\rm m}) + \frac{(5.0\,{\rm m/s})^2}{2}\right) = 66\ {\rm W}\ . \end{aligned}$$

42. (a) The equation of continuity provides $26 + 19 + 11 = 56$ L/min for the flow rate in the main (1.9 cm diameter) pipe.

(b) Using $v = R/A$ and $A = \pi d^2/4$, we set up ratios:

$$\frac{v_{56}}{v_{26}} = \frac{\frac{56}{\pi(1.9)^2/4}}{\frac{26}{\pi(1.3)^2/4}} \approx 1\ .$$

43. (a) We use the equation of continuity: $A_1 v_1 = A_2 v_2$. Here A_1 is the area of the pipe at the top and v_1 is the speed of the water there; A_2 is the area of the pipe at the bottom and v_2 is the speed of the water there. Thus $v_2 = (A_1/A_2)v_1 = \left((4.0\,{\rm cm}^2)/(8.0\,{\rm cm}^2)\right)(5.0\,{\rm m/s}) = 2.5\,{\rm m/s}$.

(b) We use the Bernoulli equation: $p_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$, where ρ is the density of water, h_1 is its initial altitude, and h_2 is its final altitude. Thus

$$\begin{aligned} p_2 &= p_1 + \frac{1}{2}\rho(v_1^2 - v_2^2) + \rho g(h_1 - h_2) \\ &= 1.5 \times 10^5\,\text{Pa} + \frac{1}{2}(1000\,\text{kg/m}^3)\left((5.0\,\text{m/s})^2 - (2.5\,\text{m/s})^2\right) + (1000\,\text{kg/m}^3)(9.8\,\text{m/s}^2)(10\,\text{m}) \\ &= 2.6 \times 10^5 \text{ Pa}. \end{aligned}$$

44. (a) We use $Av =$ const. The speed of water is

$$v = \frac{(25.0\,\text{cm})^2 - (5.00\,\text{cm})^2}{(25.0\,\text{cm})^2}(2.50\,\text{m/s}) = 2.40 \text{ m/s}.$$

(b) Since $p + \frac{1}{2}\rho v^2 =$ const., the pressure difference is

$$\Delta p = \frac{1}{2}\rho\Delta v^2 = \frac{1}{2}(1000\,\text{kg/m}^3)[(2.50\,\text{m/s})^2 - (2.40\,\text{m/s})^2] = 245 \text{ Pa}.$$

45. (a) The equation of continuity leads to

$$v_2 A_2 = v_1 A_1 \implies v_2 = v_1\left(\frac{r_1^2}{r_2^2}\right)$$

which gives $v_2 = 3.9\,\text{m/s}$.

(b) With $h = 7.6\,\text{m}$ and $p_1 = 1.7 \times 10^5\,\text{Pa}$, Bernoulli's equation reduces to

$$p_2 = p_1 - \rho g h + \frac{1}{2}\rho\left(v_1^2 - v_2^2\right) = 8.8 \times 10^4 \text{ Pa}.$$

46. We use Bernoulli's equation:

$$p_2 - p_i = \rho g h + \frac{1}{2}\rho\left(v_1^2 - v_2^2\right)$$

where $\rho = 1000\,\text{kg/m}^3$, $h = 180\,\text{m}$, $v_1 = 0.40\,\text{m/s}$ and $v_2 = 9.5\,\text{m/s}$. Therefore, we find $\Delta p = 1.7 \times 10^6\,\text{Pa}$, or 1.7 MPa. The SI unit for pressure is the Pascal (Pa) and is equivalent to N/m^2.

47. (a) We use the Bernoulli equation: $p_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$, where h_1 is the height of the water in the tank, p_1 is the pressure there, and v_1 is the speed of the water there; h_2 is the altitude of the hole, p_2 is the pressure there, and v_2 is the speed of the water there. ρ is the density of water. The pressure at the top of the tank and at the hole is atmospheric, so $p_1 = p_2$. Since the tank is large we may neglect the water speed at the top; it is much smaller than the speed at the hole. The Bernoulli equation then becomes $\rho g h_1 = \frac{1}{2}\rho v_2^2 + \rho g h_2$ and

$$v_2 = \sqrt{2g(h_1 - h_2)} = \sqrt{2(9.8\,\text{m/s}^2)(0.30\,\text{m})} = 2.42 \text{ m/s}.$$

The flow rate is $A_2 v_2 = (6.5 \times 10^{-4}\,\text{m}^2)(2.42\,\text{m/s}) = 1.6 \times 10^{-3} \text{ m}^3/\text{s}$.

(b) We use the equation of continuity: $A_2 v_2 = A_3 v_3$, where $A_3 = \frac{1}{2}A_2$ and v_3 is the water speed where the area of the stream is half its area at the hole. Thus $v_3 = (A_2/A_3)v_2 = 2v_2 = 4.84\,\text{m/s}$. The water is in free fall and we wish to know how far it has fallen when its speed is doubled to 4.84 m/s. Since the pressure is the same throughout the fall, $\frac{1}{2}\rho v_2^2 + \rho g h_2 = \frac{1}{2}\rho v_3^2 + \rho g h_3$. Thus

$$h_2 - h_3 = \frac{v_3^2 - v_2^2}{2g} = \frac{(4.84\,\text{m/s})^2 - (2.42\,\text{m/s})^2}{2(9.8\,\text{m/s}^2)} = 0.90 \text{ m}.$$

48. The lift force follows from the pressure difference (large pressure on the bottom surface than on the top) and the fact that the pressure difference is related to force through the relation $\Delta p = F/A$ where we are asked to use L for F. From Bernoulli's equation, we have

$$p_u - p_t = \frac{1}{2}\rho v_t^2 - \frac{1}{2}\rho v_u^2 + \rho g \Delta z$$

where Δz is the thickness of the wing. The last term makes a negligible contribution (we will return to this point in a moment) and can be ignored. We then have

$$\Delta p = \frac{1}{2}\rho\left(v_t^2 - v_u^2\right) \implies L = \frac{1}{2}\rho A\left(v_t^2 - v_u^2\right)$$

as desired. The contribution of the "potential" term would have been $\rho g A \Delta z$ which we can estimate as follows: let $\rho \approx 1$ kg/m^3, $A \approx 100$ m^2, and $\Delta z \approx 1$ m. Then $\rho g A \Delta z \approx 1000$ N which perhaps corresponds to the weight of a couple of adults, and is at least an order of magnitude less than the weight of an airplane with wings (the size of which are as estimated above) and equipment and crew.

49. We use the Bernoulli equation: $p_\ell + \frac{1}{2}\rho v_\ell^2 = p_u + \frac{1}{2}\rho v_u^2$, where p_ℓ is the pressure at the lower surface, p_u is the pressure at the upper surface, v_ℓ is the air speed at the lower surface, v_u is the air speed at the upper surface, and ρ is the density of air. The two tubes of flow are essentially at the same altitude. We want to solve for v_u such that $p_\ell - p_u = 900\,\text{Pa}$. That is,

$$v_u = \sqrt{\frac{2(p_\ell - p_u)}{\rho} + v_\ell^2} = \sqrt{\frac{2(900\,\text{Pa})}{1.30\,\text{kg/m}^3} + (110\,\text{m/s})^2} = 116\text{ m/s} .$$

50. (a) The speed v of the fluid flowing out of the hole satisfies $\frac{1}{2}\rho v^2 = \rho g h$ or $v = \sqrt{2gh}$. Thus, $\rho_1 v_1 A_1 = \rho_2 v_2 A_2$, which leads to

$$\rho_1 \sqrt{2gh} A_1 = \rho_2 \sqrt{2gh} A_2 \implies \frac{\rho_1}{\rho_2} = \frac{A_2}{A_1} = 2 .$$

(b) The ratio of volume flow is

$$\frac{R_1}{R_2} = \frac{v_1 A_1}{v_2 A_2} = \frac{A_1}{A_2} = \frac{1}{2} .$$

(c) Letting $R_1/R_2 = 1$, we obtain $v_1/v_2 = A_2/A_1 = 2 = \sqrt{h_1/h_2}$. Thus $h_2 = h_1/4$.

51. (a) The volume of water (during 10 minutes) is

$$V = (v_1 t)A_1 = (15\,\text{m/s})(10\,\text{min})(60\,\text{s/min})\left(\frac{\pi}{4}\right)(0.03\,\text{m})^2 = 6.4\text{ m}^3 .$$

(b) The speed in the left section of pipe is

$$v_2 = v_1\left(\frac{A_1}{A_2}\right) = v_1\left(\frac{d_1}{d_2}\right)^2 = (15\,\text{m/s})\left(\frac{3.0\,\text{cm}}{5.0\,\text{cm}}\right)^2 = 5.4\text{ m/s} .$$

(c) Since $p_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$ and $h_1 = h_2$, $p_1 = p_0$ (= atmospheric pressure),

$$\begin{aligned} p_2 &= p_0 + \frac{1}{2}\rho(v_1^2 - v_2^2) \\ &= 1.01 \times 10^5\,\text{Pa} + \frac{1}{2}(1.0 \times 10^3\,\text{kg/m}^3)[(15\,\text{m/s})^2 - (5.4\,\text{m/s})^2] \\ &= 1.99 \times 10^5\,\text{Pa} = 1.97\text{ atm} . \end{aligned}$$

Thus the gauge pressure is $1.97\,\text{atm} - 1.00\,\text{atm} = 0.97\,\text{atm} = 9.8 \times 10^4\,\text{Pa}$.

52. (a) We denote a point at the top surface of the liquid A and a point at the opening B. Point A is a vertical distance $h = 0.50$ m above B. Bernoulli's equation yields $p_A = p_B + \frac{1}{2}\rho v_B^2 - \rho g h$. Noting that $p_A = p_B$ we obtain

$$\begin{aligned} v_B &= \sqrt{2gh + \frac{2}{\rho}(p_A - p_B)} \\ &= \sqrt{2(9.8\,\mathrm{m/s^2})(0.50\,\mathrm{m})} = 3.1\ \mathrm{m/s}\ . \end{aligned}$$

(b)

$$\begin{aligned} v_B &= \sqrt{2gh + \frac{2}{\rho}(p_A - p_B)} \\ &= \sqrt{2(9.8\,\mathrm{m/s^2})(0.50\,\mathrm{m}) + \frac{2(1.40\,\mathrm{atm} - 1.00\,\mathrm{atm})}{1.0\times 10^3\,\mathrm{kg/m^3}}} = 9.5\ \mathrm{m/s}\ . \end{aligned}$$

53. (a) The friction force is

$$\begin{aligned} f &= A\Delta p = \rho_w g h A \\ &= \left(1.0\times 10^3\,\mathrm{kg/m^3}\right)\left(9.8\,\mathrm{m/s^2}\right)(6.0\mathrm{m})\left(\frac{\pi}{4}\right)(0.040\,\mathrm{m})^2 = 74\ \mathrm{N}\ . \end{aligned}$$

(b) The speed of water flowing out of the hole is $v = \sqrt{2gh}$. Thus, the volume of water flowing out of the pipe in $t = 3.0\,\mathrm{h}$ is

$$\begin{aligned} V &= Avt = \frac{\pi d v t}{4} \\ &= \frac{\pi^2}{4}(0.040\,\mathrm{m})^2\sqrt{2\,(9.8\,\mathrm{m/s^2})\,(6.0\,\mathrm{m})}(3.0\,\mathrm{h})(3600\,\mathrm{s/h}) \\ &= 1.5\times 10^2\ \mathrm{m^3}\ . \end{aligned}$$

54. (a) Since Sample Problem 15-9 deals with a similar situation, we use the final equation (labeled "Answer") from it:

$$v = \sqrt{2gh} \quad\Longrightarrow\quad v = v_\mathrm{o} \text{ for the projectile motion.}$$

The stream of water emerges horizontally ($\theta_\mathrm{o} = 0°$ in the notation of Chapter 4), and setting $y - y_\mathrm{o} = -(H - h)$ in Eq. 4-22, we obtain the "time-of-flight"

$$t = \sqrt{\frac{-2(H-h)}{-g}} = \sqrt{\frac{2}{g}(H-h)}\ .$$

Using this in Eq. 4-21, where $x_\mathrm{o} = 0$ by choice of coordinate origin, we find

$$x = v_\mathrm{o} t = \sqrt{2gh}\sqrt{\frac{2}{g}(H-h)} = 2\sqrt{h(H-h)}\ .$$

(b) The result of part (a) (which, when squared, reads $x^2 = 4h(H-h)$) is a quadratic equation for h once x and H are specified. Two solutions for h are therefore mathematically possible, but are they both physically possible? For instance, are both solutions positive and less than H? We employ the quadratic formula:

$$h^2 - Hh + \frac{x^2}{4} = 0 \quad\Longrightarrow\quad h = \frac{H \pm \sqrt{H^2 - x^2}}{2}$$

which permits us to see that both roots are physically possible, so long as $x < H$. Labeling the larger root h_1 (where the plus sign is chosen) and the smaller root as h_2 (where the minus sign is chosen), then we note that their sum is simply

$$h_1 + h_2 = \frac{H + \sqrt{H^2 - x^2}}{2} + \frac{H - \sqrt{H^2 - x^2}}{2} = H \ .$$

Thus, one root is related to the other (generically labeled h' and h) by $h' = H - h$.

(c) We wish to maximize the function $f = x^2 = 4h(H - h)$. We differentiate with respect to h and set equal to zero to obtain

$$\frac{df}{dh} = 4H - 8h = 0 \implies h = \frac{H}{2}$$

as the depth from which an emerging stream of water will travel the maximum horizontal distance.

55. (a) The continuity equation yields $Av = aV$, and Bernoulli's equation yields $\Delta p + \frac{1}{2}\rho v^2 = \frac{1}{2}\rho V^2$, where $\Delta p = p_1 - p_2$. The first equation gives $V = (A/a)v$. We use this to substitute for V in the second equation, and obtain $\Delta p + \frac{1}{2}\rho v^2 = \frac{1}{2}\rho (A/a)^2 v^2$. We solve for v. The result is

$$v = \sqrt{\frac{2\,\Delta p}{\rho\left(\frac{A^2}{a^2} - 1\right)}} = \sqrt{\frac{2a^2\,\Delta p}{\rho(A^2 - a^2)}} \ .$$

(b) We substitute values to obtain

$$v = \sqrt{\frac{2(32 \times 10^{-4}\,\mathrm{m^2})^2(55 \times 10^3\,\mathrm{Pa} - 41 \times 10^3\,\mathrm{Pa})}{(1000\,\mathrm{kg/m^3})\left((64 \times 10^{-4}\,\mathrm{m^2})^2 - (32 \times 10^{-4}\,\mathrm{m^2})^2\right)}} = 3.06\ \mathrm{m/s}\ .$$

Consequently, the flow rate is

$$Av = \left(64 \times 10^{-4}\,\mathrm{m^2}\right)(3.06\,\mathrm{m/s}) = 2.0 \times 10^{-2}\ \mathrm{m^3/s}\ .$$

56. We use the result of part (a) in the previous problem.

(a) In this case, we have $\Delta p = p_1 = 2.0\,\mathrm{atm}$. Consequently,

$$v = \sqrt{\frac{2\Delta p}{\rho\left((A/a)^2 - 1\right)}} = \sqrt{\frac{4(1.01 \times 10^5\,\mathrm{Pa})}{(1000\,\mathrm{kg/m^3})\left((5a/a)^2 - 1\right)}} = 4.1\ \mathrm{m/s}\ .$$

(b) And the equation of continuity yields $V = (A/a)v = (5a/a)v = 5v = 21\,\mathrm{m/s}$.

(c) The flow rate is given by

$$Av = \frac{\pi}{4}\left(5.0 \times 10^{-4}\,\mathrm{m^2}\right)(4.1\,\mathrm{m/s}) = 8.0 \times 10^{-3}\ \mathrm{m^3/s}\ .$$

57. (a) Bernoulli's equation gives $p_A = p_B + \frac{1}{2}\rho_{\mathrm{air}} v^2$. But $\Delta p = p_A - p_B = \rho g h$ in order to balance the pressure in the two arms of the U-tube. Thus $\rho g h = \frac{1}{2}\rho_{\mathrm{air}} v^2$, or

$$v = \sqrt{\frac{2\rho g h}{\rho_{\mathrm{air}}}}\ .$$

(b) The plane's speed relative to the air is

$$v = \sqrt{\frac{2\rho g h}{\rho_{\mathrm{air}}}} = \sqrt{\frac{2\left(810\,\mathrm{kg/m^3}\right)(9.8\,\mathrm{m/s^2})(0.260\,\mathrm{m})}{1.03\,\mathrm{kg/m^3}}} = 63.3\ \mathrm{m/s}\ .$$

58. We use the formula for v obtained in the previous problem:

$$v = \sqrt{\frac{2\Delta p}{\rho_{\rm air}}} = \sqrt{\frac{2(180\,{\rm Pa})}{0.031\,{\rm kg/m^3}}} = 1.1 \times 10^2 \text{ m/s} .$$

59. (a) To avoid confusing weight with work, we write out the word instead of using the symbol W. Thus,

$$\text{weight} = mg = \left(1.85 \times 10^4\,\text{kg}\right)\left(9.8\,\text{m/s}^2\right) \approx 1.8 \times 10^5 \text{ N} .$$

(b) The buoyant force is $F_b = \rho_w g V_w$ where $\rho_w = 1000\,\text{kg/m}^3$ is the density of water and V_w is the volume of water displaced by the dinosaur. If we use f for the fraction of the dinosaur's total volume V which is submerged, then $V_w = fV$. We can further relate V to the dinosaur's mass using the assumption that the density of the dinosaur is 90% that of water: $V = m/(0.9\rho_w)$. Therefore, the apparent weight of the dinosaur is

$$\text{weight}_{\text{app}} = \text{weight} - \rho_w g \left(f \frac{m}{0.9\rho_w}\right) = \text{weight} - gf\frac{m}{0.9} .$$

If $f = 0.50$, this yields 81 kN for the apparent weight.

(c) If $f = 0.80$, our formula yields 20 kN for the apparent weight.

(d) If $f = 0.90$, we find the apparent weight is zero (it floats).

(e) Eq. 15-8 indicates that the water pressure at that depth is greater than standard air pressure (the assumed pressure at the surface) by $\rho_w gh = (1000)(9.8)(8) = 7.8 \times 10^4$ Pa. If we assume the pressure of air in the dinosaur's lungs is approximately standard air pressure, then this value represents the pressure difference which the lung muscles would have to work against.

(f) Assuming the maximum pressure difference the muscles can work with is 8 kPa, then our previous result (78 kPa) spells doom to the wading Diplodocus hypothesis.

60. The volume rate of flow is $R = vA$ where $A = \pi r^2$ and $r = d/2$. Solving for speed, we obtain

$$v = \frac{R}{A} = \frac{R}{\pi(d/2)^2} = \frac{4R}{\pi d^2} .$$

(a) With $R = 7.0 \times 10^{-3}\,\text{m}^3/\text{s}$ and $d = 14 \times 10^{-3}$ m, our formula yields $v = 45$ m/s, which is about 13% of the speed of sound (which we establish by setting up a ratio: v/v_s where $v_s = 343$ m/s).

(b) With the contracted trachea ($d = 5.2 \times 10^{-3}$ m) we obtain $v = 330$ m/s, or 96% of the speed of sound.

61. To be as general as possible, we denote the ratio of body density to water density as f (so that $f = \rho/\rho_w = 0.95$ in this problem). Floating involves an equilibrium of vertical forces acting on the body (Earth's gravity pulls down and the buoyant force pushes up). Thus,

$$F_b = F_g \implies \rho_w g V_w = \rho g V$$

where V is the total volume of the body and V_w is the portion of it which is submerged.

(a) We rearrange the above equation to yield

$$\frac{V_w}{V} = \frac{\rho}{\rho_w} = f$$

which means that 95% of the body is submerged and therefore 5% is above the water surface.

(b) We replace ρ_w with $1.6\rho_w$ in the above equilibrium of forces relationship, and find

$$\frac{V_w}{V} = \frac{\rho}{1.6\rho_w} = \frac{f}{1.6}$$

which means that 59% of the body is submerged and thus 41% is above the quicksand surface.

(c) The answer to part (b) suggests that a person in that situation is able to breathe.

(d) The thixotropic property is warning that slow motions are best. Reasonable steps are: lay back on the surface, slowly pull your legs free, and then roll to the shore.

62. (a) The volume rate of flow is related to speed by $R = vA$. Thus,

$$v_1 = \frac{R_1}{\pi r_{\text{stream}}^2} = \frac{7.9\,\text{cm}^3/\text{s}}{\pi(0.13\,\text{cm})^2} = 148.8\,\text{cm/s} = 1.5\ \text{m/s}\ .$$

(b) The depth d of spreading water becomes smaller as r (the distance from the impact point) increases due to the equation of continuity (and the assumption that the water speed remains equal to v_1 in this region). The water that has reached radius r (with perimeter $2\pi r$) is crossing an area of $2\pi rd$. Thus, the equation of continuity gives

$$R_1 = v_1 2\pi rd \implies d = \frac{R}{2\pi rv_1}\ .$$

(c) As noted above, d is a decreasing function of r.

(d) At $r = r_J$ we apply the formula from part (b):

$$d_J = \frac{R_1}{2\pi r_J v_1} = \frac{7.9\,\text{cm}^3/\text{s}}{2\pi(2.0\,\text{cm})(148.8\,\text{cm/s})} = 0.0042\ \text{cm}\ .$$

(e) We are told "the depth just after the jump is 2.0 mm" which means $d_2 = 0.20$ cm, and we are asked to find v_2. We use the equation of continuity:

$$R_1 = R_2 \implies 2\pi r_J\, v_1 d_J = 2\pi r'_J\, v_2 d_2$$

where r'_J is some very small amount greater than r_J (and for calculation purposes is taken to be the same numerical value, 2.0 cm). This yields

$$v_2 = v_1\left(\frac{d_1}{d_2}\right) = (148.8\,\text{cm/s})\left(\frac{0.0042\,\text{cm}}{0.20\,\text{cm}}\right) = 3.1\ \text{cm/s}\ .$$

(f) The kinetic energy per unit volume at $r = r_J$ with $v = v_1$ is

$$\frac{1}{2}\rho_w v_1^2 = \frac{1}{2}\left(1000\,\text{kg/m}^3\right)(1.488\,\text{m/s})^2 = 1.1\times 10^3\ \text{J/m}^3\ .$$

(g) The kinetic energy per unit volume at $r = r'_J$ with $v = v_2$ is

$$\frac{1}{2}\rho_w v_2^2 = \frac{1}{2}\left(1000\,\text{kg/m}^3\right)(0.031\,\text{m/s})^2 = 0.49\ \text{J/m}^3\ .$$

(h) The hydrostatic pressure change is due to the change in depth:

$$\Delta p = \rho_w g\,(d_2 - d_1) = \left(1000\,\text{kg/m}^3\right)\left(9.8\,\text{m/s}^2\right)(0.0020\,\text{m} - 0.000042\,\text{m}) = 19\ \text{Pa}\ .$$

(i) Certainly, $\frac{1}{2}\rho_w v_1^2 + \rho_w g d_1 + p_1$ is greater than $\frac{1}{2}\rho_w v_2^2 + \rho_w g d_2 + p_2$ which is not unusual with "shock-like" fluids structures such as this hydraulic jump. Not only does Bernoulli's equation not apply but the very concept of a streamline becomes difficult to define in this circumstance.

63. (a) We rewrite the formula for work W (when the force is constant in a direction parallel to the displacement d) in terms of pressure:

$$W = Fd = \left(\frac{F}{A}\right)(Ad) = pV$$

where V is the volume of the chocolate cylinder. On a per unit mass basis (utilizing the equation for density $\rho = m/V$) we have

$$\frac{W}{m} = p\left(\frac{V}{m}\right) = \frac{p}{\rho} \ .$$

(b) If $p = 5.5 \times 10^6$ Pa and $\rho = 1200\,\text{kg/m}^3$, we obtain $W/m = p/\rho = 4.6 \times 10^3$ J/kg.

64. (a) When the model is suspended (in air) the reading is F_g (its true weight, neglecting any buoyant effects caused by the air). When the model is submerged in water, the reading is lessened because of the buoyant force: $F_g - F_b$. We denote the difference in readings as Δm. Thus,

$$(F_g) - (F_g - F_b) = \Delta m g$$

which leads to $F_b = \Delta m g$. Since $F_b = \rho_w g V_m$ (the weight of water displaced by the model) we obtain

$$V_m = \frac{\Delta m}{\rho_w} = \frac{0.63776\,\text{kg}}{1000\,\text{kg/m}^3} = 6.3776 \times 10^{-4}\ \text{m}^3 \ .$$

(b) The $\frac{1}{20}$ scaling factor is discussed in the problem (and for purposes of significant figures is treated as exact). The actual volume of the dinosaur is

$$V_{\text{dino}} = 20^3\, V_m = 5.1021\ \text{m}^3 \ .$$

(c) Using $\rho \approx \rho_w = 1000\,\text{kg/m}^3$, we find

$$\rho = \frac{m_{\text{dino}}}{V_{\text{dino}}} \implies m_{\text{dino}} = \left(1000\,\text{kg/m}^3\right)\left(5.1021\,\text{m}^3\right)$$

which yields 5.1×10^3 kg for the *T. Rex* mass.

(d) We estimate the mass range for college students as $50 \le m \le 115$ kg. Dividing these values into the previous result leads to ratios r in the range of roughly $100 \ge r \ge 45$.

65. We apply Bernoulli's equation to the central streamline:

$$p_1 + \frac{1}{2}\rho_{\text{air}} v_1^2 = p_\text{o} + \frac{1}{2}\rho_{\text{air}} v_\text{o}^2 \implies p_1 - p_\text{o} = \frac{1}{2}\rho_{\text{air}}\left(v_\text{o}^2 - v_1^2\right)$$

where $v_\text{o} = 65\,\text{m/s}$, $v_1 = 2\,\text{m/s}$ and the density of air is $\rho_{\text{air}} = 1.2\,\text{kg/m}^3$ (see Table 15-1). Thus, we obtain $p_1 - p_\text{o} \approx 2500$ Pa.

66. The pressure (relative to standard air pressure) is given by Eq. 15-8:

$$\rho g h = \left(1024\,\text{kg/m}^3\right)\left(9.8\,\text{m/s}^2\right)\left(6.0 \times 10^3\,\text{m}\right) = 6.02 \times 10^7\ \text{Pa} \ .$$

67. Recalling that $1\,\text{atm} = 1.01 \times 10^5\,\text{atm}$, Eq. 15-8 leads to

$$\rho g h = \left(1024\,\text{kg/m}^3\right)\left(9.8\,\text{m/s}^2\right)\left(10.9 \times 10^3\,\text{m}\right)\left(\frac{1\,\text{atm}}{1.01 \times 10^5\,\text{atm}}\right) \approx 1080\ \text{atm} \ .$$

68. (a) We consider a point D on the surface of the liquid in the container, in the same tube of flow with points A, B and C. Applying Bernoulli's equation to points D and C, we obtain

$$p_D + \frac{1}{2}\rho v_D^2 + \rho g h_D = p_C + \frac{1}{2}\rho v_C^2 + \rho g h_C$$

which leads to

$$v_C = \sqrt{\frac{2(p_D - p_C)}{\rho} + 2g(h_D - h_C) + v_D^2} \approx \sqrt{2g\,(d + h_2)}$$

where in the last step we set $p_D = p_C = p_{\text{air}}$ and $v_D/v_C \approx 0$.

(b) We now consider points B and C:

$$p_B + \frac{1}{2}\rho v_B^2 + \rho g h_B = p_C + \frac{1}{2}\rho v_C^2 + \rho g h_C \ .$$

Since $v_B = v_C$ by equation of continuity, and $p_C = p_{\text{air}}$, Bernoulli's equation becomes

$$p_B = p_C + \rho g(h_C - h_B) = p_{\text{air}} - \rho g(h_1 + h_2 + d) \ .$$

(c) Since $p_B \geq 0$, we must let $p_{\text{air}} - \rho g(h_1 + d + h_2) \geq 0$, which yields

$$h_1 \leq h_{1,\max} = \frac{p_{\text{air}}}{\rho} - d - h_2 \leq \frac{p_{\text{air}}}{\rho} = 10.3 \text{ m} \ .$$

69. An object of mass $m = \rho V$ floating in a liquid of density ρ_{liquid} is able to float if the downward pull of gravity mg is equal to the upward buoyant force $F_b = \rho_{\text{liquid}} g V_{\text{sub}}$ where V_{sub} is the portion of the object which is submerged. This readily leads to the relation:

$$\frac{\rho}{\rho_{\text{liquid}}} = \frac{V_{\text{sub}}}{V}$$

for the fraction of volume submerged of a floating object. When the liquid is water, as described in this problem, this relation leads to

$$\frac{\rho}{\rho_w} = 1$$

since the object "floats fully submerged" in water (thus, the object has the same density as water). We assume the block maintains an "upright" orientation in each case (which is not necessarily realistic).

(a) For liquid A,

$$\frac{\rho}{\rho_A} = \frac{1}{2}$$

so that, in view of the fact that $\rho = \rho_w$, we obtain $\rho_A/\rho_w = 2$.

(b) For liquid B, noting that two-thirds *above* means one-third *below*,

$$\frac{\rho}{\rho_B} = \frac{1}{3}$$

so that $\rho_B/\rho_w = 2$.

(c) For liquid C, noting that one-fourth *above* means three-fourths *below*,

$$\frac{\rho}{\rho_C} = \frac{3}{4}$$

so that $\rho_C/\rho_w = 4/3$.

70. In this case, Bernoulli's equation, reduces to Eq. 15-10. Thus,

$$p_g = \rho g(-h) = -(1800)(9.8)(1.5) = -2.6 \times 10^4 \text{ Pa} .$$

71. The downward force on the balloon is mg and the upward force is $F_b = \rho_{\text{out}} V g$. Newton's second law (with $m = \rho_{\text{in}} V$) leads to

$$\rho_{\text{out}} V g - \rho_{\text{in}} V g = \rho_{\text{in}} V a \implies \left(\frac{\rho_{\text{out}}}{\rho_{\text{in}}} - 1\right) g = a .$$

The problem specifies $\rho_{\text{out}}/\rho_{\text{in}} = 1.39$ (the outside air is cooler and thus more dense than the hot air inside the balloon). Thus, the upward acceleration is $(1.39 - 1)(9.8) = 3.8\,\text{m/s}^2$.

72. We rewrite the formula for work W (when the force is constant in a direction parallel to the displacement d) in terms of pressure:

$$W = Fd = \left(\frac{F}{A}\right)(Ad) = pV$$

where V is the volume of the water being forced through, and p is to be interpreted as the pressure difference between the two ends of the pipe. Thus,

$$W = \left(1.01 \times 10^5\,\text{Pa}\right)\left(1.4\,\text{m}^3\right) = 1.5 \times 10^5 \text{ J} .$$

73. (a) Using Eq. 15-10, we have $p_g = \rho g h = 1.21 \times 10^7$ Pa.

(b) By definition, $p = p_g + p_{\text{atm}} = 1.22 \times 10^7$ Pa.

(c) We interpret the question as asking for the total force *compressing* the sphere's surface, and we multiply the pressure by total area:

$$p\left(4\pi r^2\right) = 3.82 \times 10^5 \text{ N} .$$

(d) The (upward) buoyant force exerted on the sphere by the seawater is

$$F_b = \rho_w g V \qquad\qquad \text{where } V = \frac{4}{3}\pi r^3 \quad .$$

Therefore, $F_b = 5.26$ N.

(e) Newton's second law applied to the sphere (of mass $m = 7.0$ kg) yields

$$F_b - mg = ma$$

which results in $a = -9.04$, which means the acceleration vector has a magnitude of 9.04 m/s^2 and is directed downward.

74. Neglecting the buoyant force caused by air, then the 30 N value is interpreted as the true weight W of the object. The buoyant force of the water on the object is therefore $30 - 20 = 10$ N, which means

$$F_b = \rho_w V g \implies V = \frac{10\text{N}}{\left(1000\,\text{kg/m}^3\right)\left(9.8\,\text{m/s}^2\right)} = 1.02 \times 10^{-3} \text{ m}^3$$

is the volume of the object. When the object is in the second liquid, the buoyant force is $30 - 24 = 6$ N, which implies

$$\rho_2 = \frac{6\,\text{N}}{\left(9.8\,\text{m/s}^2\right)\left(1.02 \times 10^{-3}\,\text{m}^3\right)} = 600 \text{ kg/m}^3 .$$

75. The beaker is indicated by the subscript b. The volume of the glass of which the beaker walls and base are made is $V_b = m_b/\rho_b$. We consider the case where the beaker is slightly more than half full (which, for calculation purposes, will be simply set equal to half-volume) and thus remains on the bottom of the sink – as the water around it reaches its rim. At this point, the force of buoyancy exerted on it is given by $F = (V_b + V)\rho_w g$, where V is the interior volume of the beaker. Thus $F = (V_b + V)\rho_w g = \rho_w g(V/2) + m_b$, which we solve for ρ_b:

$$\rho_b = \frac{2m_b\rho_w}{2m_b - \rho_w V} = \frac{2(390\,\mathrm{g})(1.00\,\mathrm{g/cm^3})}{2(390\,\mathrm{g}) - \left(1.00\,\mathrm{g/cm^3}\right)(500\,\mathrm{cm^3})} = 2.79\ \mathrm{g/cm^3}\ .$$

76. If the mercury level in one arm of the tube is lowered by an amount x, it will rise by x in the other arm. Thus, the net difference in mercury level between the two arms is $2x$, causing a pressure difference of $\Delta p = 2\rho_{\mathrm{Hg}} g x$, which should be compensated for by the water pressure $p_w = \rho_w g h$, where $h = 11.2\,\mathrm{cm}$. In these units, $\rho_w = 1\,\mathrm{g/cm^3}$ and $\rho_{\mathrm{Hg}} = 13.6\,\mathrm{g/cm^3}$ (see Table 15-1). We obtain

$$x = \frac{\rho_w g h}{2\rho_{\mathrm{Hg}} g} = \frac{\left(1.00\,\mathrm{g/cm^3}\right)(11.2\,\mathrm{cm})}{2\,(13.6\,\mathrm{g/cm^3})} = 0.412\ \mathrm{cm}\ .$$

77. (a) Since the pressure (due to the water) increases linearly with depth, we use its average (multiplied by the dam area) to compute the force exerts on the face of the dam, its average being simply half the pressure value near the bottom (at depth $d = 48$ m). The maximum static friction will be μN where the normal force N (exerted upward by the portion of the bedrock directly underneath the concrete) is equal to the weight mg of the dam. Since $m = \rho_c V$ with ρ_c being the density of the concrete and V being the volume (thickness times width times height: $\ell w h$), we write $N = \rho_c \ell w h g$. Thus, the safety factor is

$$\frac{\mu\rho_c\ell w h g}{\frac{1}{2}\rho_w g d A_{\mathrm{face}}} = \frac{2\mu\rho_c\ell w h}{\rho_w d(wd)} = \frac{2\mu\rho_c\ell h}{\rho_w d^2}$$

which (since $\rho_w = 1\,\mathrm{g/cm^3}$) yields $2(.47)(3.2)(24)(71)/48^2 = 2.2$.

(b) To compute the torque due to the water pressure, we will need to integrate Eq. 15-7 (multiplied by $(d - y)$ and the dam width w) as shown below. The countertorque due to the weight of the concrete is the weight multiplied by half the thickness ℓ, since we take the center of mass of the dam is at its geometric center and the axis of rotation at A. Thus, the safety factor relative to rotation is

$$\frac{mg\frac{\ell}{2}}{\int_{\mathrm{o}}^{d}\rho_w g y(d-y) w\,dy} = \frac{\rho_c\ell w h g\frac{\ell}{2}}{\frac{1}{6}\rho_w g w d^3} = \frac{3\rho_c\ell^2 h}{\rho_w d^3}$$

which yields $3(3.2)(24)^2(71)/(48)^3 = 3.55$.

78. We use $p = p_{\mathrm{air}} = \rho g h$ to obtain

$$h = \frac{p_{\mathrm{air}}}{\rho g} = \frac{1.01\times10^5\ \mathrm{Pa}}{\left(1000\,\mathrm{kg/m^3}\right)\left(9.8\,\mathrm{m/s^2}\right)} = 10.3\ \mathrm{m}\ .$$

79. We consider the can with nearly its total volume submerged, and just the rim above water. For calculation purposes, we take its submerged volume to be $V = 1200\,\mathrm{cm^3}$. To float, the total downward force of gravity (acting on the tin mass m_t and the lead mass m_ℓ) must be equal to the buoyant force upward:

$$(m_t + m_\ell)\,g = \rho_w V g \quad\Longrightarrow\quad m_\ell = \left(1\,\mathrm{g/cm^3}\right)\left(1200\,\mathrm{cm^3}\right) - 130\,\mathrm{g}$$

which yields 1070 g for the (maximum) mass of the lead (for which the can still floats). The given density of lead is not used in the solution.

80. The force f that is required to tether the airship of volume V and weight W is given by

$$\begin{aligned} f &= F_b - W = \rho_{\text{air}} g V - \rho_{\text{gas}} g V \\ &= \left(1.21\,\text{kg/m}^3 - 0.80\,\text{kg/m}^3\right)\left(9.8\,\text{m/s}^2\right)\left(1.0\times 10^6\,\text{m}^3\right) \\ &= 4.0\times 10^6 \text{ N} . \end{aligned}$$

81. The weight of the air inside the balloon of volume V is $W = \rho_{\text{gas}} V g$, and the buoyant force exerted on it is given by $F_b = \rho_{\text{air}} V g$. Thus, we have $F_b = W + mg$, where m is the mass of the payload. we have $\rho_{\text{air}} V g = \rho_{\text{gas}} V g + mg$, which gives

$$V = \frac{m}{\rho_{\text{air}} - \rho_{\text{gas}}} = \frac{40\,\text{kg} + 15\,\text{kg}}{0.035\,\text{kg/m}^3 - 0.0051\,\text{kg/m}^3} = 1.8\times 10^3 \text{ m}^3 .$$

82. (a) We consider a thin slab of water with bottom area A and infinitesimal thickness dh. We apply Newton's second law to the slab:

$$\begin{aligned} dF_{\text{net}} &= (p+dp)A - pA \\ &= dp\cdot A - dm\cdot g \\ &= Adp - \rho g A dh \\ &= dm\cdot a = \rho a A dh \end{aligned}$$

which gives

$$\frac{dp}{dh} = \rho(g+a) .$$

Integrating over the range $(0, h)$, we get

$$p = \int_0^h \rho(g+a)dh = \rho h(g+a) .$$

(b) We reverse the direction of the acceleration, from that in part (a). This amounts to changing a to $-a$. Thus,

$$p = \rho(g-a) .$$

(c) In a free fall, we use the above equation with $a = g$, which gives $p = 0$. The internal pressure p in the water totally disappears, because there is no force of interaction among different portions of the water in the bucket to make their acceleration different from g.

83. The absolute pressure is

$$\begin{aligned} p &= p_0 + \rho g h \\ &= 1.01\times 10^5\,\text{N/m}^2 + (1.03\times 10^3\,\text{kg/m}^3)(9.8\,\text{m/s}^2)(150\,\text{m}) = 1.62\times 10^6 \text{ Pa} . \end{aligned}$$

84. The area facing down (and up) is $A = (0.050\,\text{m})(0.040\,\text{m}) = 0.0020\,\text{m}^2$. The submerged volume is $V = Ad$ where $d = 0.015\,\text{m}$. In order to float, the downward pull of gravity mg must equal the upward buoyant force exerted by the seawater of density ρ:

$$mg = \rho V g \implies m = \rho V = (1025)(0.0020)(0.015) = 0.031\,\text{kg} .$$

85. Using Eq. 15-8, the maximum depth is

$$h_{\max} = \frac{\Delta p}{\rho g} = \frac{(0.050)\left(1.01\times 10^5\,\text{Pa}\right)}{(1000\,\text{kg/m}^3)\,(9.8\,\text{m/s}^2)} = 0.52 \text{ m} .$$

86. Note that "surface area" refers to the *total* surface area of all six faces, so that the area of each (square) face is $24/6 = 4$ m^2. From Archimedes' principle and the requirement that the cube (of total volume V and density ρ) floats, we find

$$\rho V g = \rho_w V_{\text{sub}} g \quad \Longrightarrow \quad \frac{\rho}{\rho_w} = \frac{V_{\text{sub}}}{V}$$

for the fraction of volume submerged. The assumption that the cube floats upright, as described in this problem, simplifies this relation to

$$\frac{\rho}{\rho_w} = \frac{h_{\text{sub}}}{h}$$

where h is the length of one side, and $\rho_w = 4\rho$ is given. With $h = \sqrt{4} = 2$ m, we find $h_{\text{sub}} = h/4 = 0.50$ m.

87. We equate the buoyant force F_b to the combined weight of the cork and sinker:

$$\rho_w V_w g = \rho_c V_c g + \rho_s V_s g$$

With $V_w = \frac{1}{2} V_c$ and $\rho_w = 1.00$ g/cm^3, we obtain

$$V_c = \frac{2\rho_s V_s}{\rho_w - 2\rho_c} = \frac{2(11.4)(0.4)}{1 - 2(0.2)} = 15.2 \text{ cm}^3 \ .$$

Using the formula for the volume of a sphere (Appendix E), we have

$$r = \left(\frac{3V_c}{4\pi}\right)^{1/3} = 1.54 \text{ cm} \ .$$

88. The equation of continuity is

$$A_i v_i = A_f v_f$$

where $A = \pi r^2$. Therefore,

$$v_f = v_i \left(\frac{r_i}{r_f}\right)^2 = (0.09)\left(\frac{0.2}{0.6}\right)^2 \ .$$

Consequently, $v_f = 1.00 \times 10^{-2}$ m/s.

89. (a) This is similar to the situation treated in Sample Problem 15-8, and we refer to some of its steps (and notation). Combining Eq. 15-35 and Eq. 15-36 in a manner very similar to that shown in the textbook, we find

$$R = A_1 A_2 \sqrt{\frac{2\Delta p}{\rho\,(A_1^2 - A_2^2)}} \ .$$

for the flow rate expressed in terms of the pressure difference and the cross-sectional areas. Note that this reduces to Eq. 15-38 for the case $A_2 = A_1/2$ treated in the Sample Problem. Note that $\Delta p = p_1 - p_2 = -7.2 \times 10^3$ Pa and $A_1^2 - A_2^2 = -8.66 \times 10^{-3}$ m^4, so that the square root is well defined. Therefore, we obtain $R = 0.0776$ m^3/s.

(b) The mass rate of flow is $\rho R = 68.9$ kg/s.

90. (a) The equation of continuity is $A_1 v_1 = A_2 v_2$ where $A_1 = \pi r_1^2$ and $A_2 = \pi r_2^2 = \pi\,(r_1/2)^2$. Consequently, we find $v_2 = 4v_1$.

(b) $\Delta\left(\frac{1}{2}\rho v^2\right)$ is equal to

$$\frac{1}{2}\rho\left(v_2^2 - v_1^2\right) = \frac{1}{2}\rho\left(16v_1^2 - v_1^2\right) = \frac{15}{2}\,\rho v_1^2 \ .$$