Chapter 24

1. (a) The mass flux is $wd\rho v = (3.22\,\text{m})(1.04\,\text{m})\left(1000\,\text{kg/m}^3\right)(0.207\,\text{m/s}) = 693\,\text{kg/s}$.

   (b) Since water flows only through area wd, the flux through the larger area is still $693\,\text{kg/s}$.

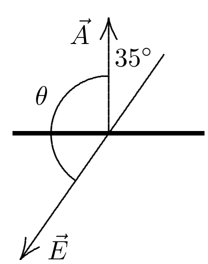
   (c) Now the mass flux is $(wd/2)\rho v = (693\,\text{kg/s})/2 = 347\,\text{kg/s}$.

   (d) Since the water flows through an area $(wd/2)$, the flux is $347\,\text{kg/s}$.

   (e) Now the flux is $(wd\cos\theta)\rho v = (693\,\text{kg/s})(\cos 34^\circ) = 575\,\text{kg/s}$.

2. The vector area $\vec{A}$ and the electric field $\vec{E}$ are shown on the diagram below. The angle θ between them is $180^\circ - 35^\circ = 145^\circ$, so the electric flux through the area is

$$\Phi = \vec{E}\cdot\vec{A} = EA\cos\theta = (1800\,\text{N/C})(3.2\times10^{-3}\,\text{m})^2\cos 145^\circ = -1.5\times10^{-2}\ \text{N}\cdot\text{m}^2/\text{C}\ .$$



3. We use $\Phi = \vec{E}\cdot\vec{A}$, where $\vec{A} = A\,\hat{\jmath} = (1.40\,\text{m})^2\,\hat{\jmath}$.

   (a) $\Phi = (6.00\,\text{N/C})\hat{\imath}\cdot(1.40\,\text{m})^2\hat{\jmath} = 0$.

   (b) $\Phi = (-2.00\,\text{N/C})\hat{\jmath}\cdot(1.40\,\text{m})^2\hat{\jmath} = -3.92\,\text{N}\cdot\text{m}^2/\text{C}$.

   (c) $\Phi = [(-3.00\,\text{N/C})\hat{\imath} + (4.00\,\text{N/C})\hat{k}]\cdot(1.40\,\text{m})^2\hat{\jmath} = 0$.

   (d) The total flux of a uniform field through a closed surface is always zero.

4. We use the fact that electric flux relates to the enclosed charge: $\Phi = q_{\text{enclosed}}/\varepsilon_0$.

   (a) A surface which encloses the charges $2q$ and $-2q$, or all four charges.

   (b) A surface which encloses the charges $2q$ and q.

   (c) The maximum amount of negative charge we can enclose by any surface which encloses the charge $2q$ is $-q$, so it is impossible to get a flux of $-2q/\varepsilon_0$.

5. We use Gauss' law: $\varepsilon_0\Phi = q$, where Φ is the total flux through the cube surface and q is the net charge inside the cube. Thus,

$$\Phi = \frac{q}{\varepsilon_0} = \frac{1.8\times10^{-6}\,\text{C}}{8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2} = 2.0\times10^5\ \text{N}\cdot\text{m}^2/\text{C}\ .$$

6. The flux through the flat surface encircled by the rim is given by $\Phi = \pi a^2 E$. Thus, the flux through the netting is $\Phi' = -\Phi = -\pi a^2 E$.

7. (a) Let $A = (1.40\,\text{m})^2$. Then

$$\begin{aligned}\Phi &= (3.00y\,\hat{\jmath}) \cdot (-A\,\hat{\jmath})|_{y=0} + (3.00y\,\hat{\jmath}) \cdot (A\,\hat{\jmath})|_{y=1.40} \\ &= (3.00)(1.40)(1.40)^2 = 8.23\ \text{N}\cdot\text{m}^2/\text{C}\ .\end{aligned}$$

(b) The electric field can be re-written as $\vec{E} = 3.00y\hat{\jmath} + \vec{E}_0$, where $\vec{E}_0 = -4.00\hat{\imath} + 6.00\hat{\jmath}$ is a constant field which does not contribute to the net flux through the cube. Thus Φ is still $8.23\,\text{N}\cdot\text{m}^2/\text{C}$.

(c) The charge is given by

$$q = \varepsilon_0\Phi = \left(8.85\times10^{-12}\,\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)(8.23\,\text{N}\cdot\text{m}^2/\text{C}) = 7.29\times10^{-11}\ \text{C}$$

in each case.

8. (a) The total surface area bounding the bathroom is

$$A = 2\,(2.5\times3.0) + 2\,(3.0\times2.0) + 2\,(2.0\times2.5) = 37\ \text{m}^2\ .$$

The absolute value of the total electric flux, with the assumptions stated in the problem, is $|\Phi| = |\sum \vec{E}\cdot\vec{A}| = |\vec{E}|\,A = (600)(37) = 22\times10^3\ \text{N}\cdot\text{m}^2/\text{C}$. By Gauss' law, we conclude that the enclosed charge (in absolute value) is $|q_{\text{enc}}| = \varepsilon_0\,|\Phi| = 2.0\times10^{-7}$ C. Therefore, with volume $V = 15\ \text{m}^3$, and recognizing that we are dealing with negative charges (see problem), we find the charge density is $q_{\text{enc}}/V = -1.3\times10^{-8}\ \text{C/m}^3$.

(b) We find $(|q_{\text{enc}}|/e)/V = (2.0\times10^{-7}/1.6\times10^{-19})/15 = 8.2\times10^{10}$ excess electrons per cubic meter.

9. Let A be the area of one face of the cube, E_u be the magnitude of the electric field at the upper face, and E_ℓ be the magnitude of the field at the lower face. Since the field is downward, the flux through the upper face is negative and the flux through the lower face is positive. The flux through the other faces is zero, so the total flux through the cube surface is $\Phi = A(E_\ell - E_u)$. The net charge inside the cube is given by Gauss' law:

$$\begin{aligned}q &= \varepsilon_0\Phi = \varepsilon_0 A(E_\ell - E_u) = (8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)(100\,\text{m})^2(100\,\text{N/C} - 60.0\,\text{N/C}) \\ &= 3.54\times10^{-6}\ \text{C} = 3.54\ \mu\text{C}\ .\end{aligned}$$

10. There is no flux through the sides, so we have two "inward" contributions to the flux, one from the top (of magnitude $(34)(3.0)^2$) and one from the bottom (of magnitude $(20)(3.0)^2$). With "inward" flux conventionally negative, the result is $\Phi = -486\ \text{N}\cdot\text{m}^2/\text{C}$. Gauss' law, then, leads to $q_{\text{enc}} = \varepsilon_0\,\Phi = -4.3\times10^{-9}$ C.

11. The total flux through any surface that completely surrounds the point charge is q/ε_0. If we stack identical cubes side by side and directly on top of each other, we will find that eight cubes meet at any corner. Thus, one-eighth of the field lines emanating from the point charge pass through a cube with a corner at the charge, and the total flux through the surface of such a cube is $q/8\varepsilon_0$. Now the field lines are radial, so at each of the three cube faces that meet at the charge, the lines are parallel to the face and the flux through the face is zero. The fluxes through each of the other three faces are the same, so the flux through each of them is one-third of the total. That is, the flux through each of these faces is $(1/3)(q/8\varepsilon_0) = q/24\varepsilon_0$.

12. Using Eq. 24-11, the surface charge density is

$$\sigma = E\varepsilon_0 = \left(2.3\times10^5\,\text{N/C}\right)\left(8.85\times10^{-12}\,\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right) = 2.0\times10^{-6}\ \text{C/m}^2\ .$$

13. (a) The charge on the surface of the sphere is the product of the surface charge density σ and the surface area of the sphere (which is $4\pi r^2$, where r is the radius). Thus,

$$q = 4\pi r^2 \sigma = 4\pi \left(\frac{1.2\,\mathrm{m}}{2}\right)^2 \left(8.1\times 10^{-6}\,\mathrm{C/m^2}\right) = 3.66\times 10^{-5}\ \mathrm{C}\ .$$

(b) We choose a Gaussian surface in the form of a sphere, concentric with the conducting sphere and with a slightly larger radius. The flux is given by Gauss' law:

$$\Phi = \frac{q}{\varepsilon_0} = \frac{3.66\times 10^{-5}\,\mathrm{C}}{8.85\times 10^{-12}\,\mathrm{C^2/N\cdot m^2}} = 4.1\times 10^{6}\ \mathrm{N\cdot m^2/C}\ .$$

14. (a) The area of a sphere may be written $4\pi R^2 = \pi D^2$. Thus,

$$\sigma = \frac{q}{\pi D^2} = \frac{2.4\times 10^{-6}\,\mathrm{C}}{\pi (1.3\,\mathrm{m})^2} = 4.5\times 10^{-7}\ \mathrm{C/m^2}\ .$$

(b) Eq. 24-11 gives

$$E = \frac{\sigma}{\varepsilon_o} = \frac{4.5\times 10^{-7}\,\mathrm{C/m^2}}{8.85\times 10^{-12}\,\mathrm{C^2/N\cdot m^2}} = 5.1\times 10^{4}\ \mathrm{N/C}\ .$$

15. (a) Consider a Gaussian surface that is completely within the conductor and surrounds the cavity. Since the electric field is zero everywhere on the surface, the net charge it encloses is zero. The net charge is the sum of the charge q in the cavity and the charge q_w on the cavity wall, so $q + q_w = 0$ and $q_w = -q = -3.0\times 10^{-6}\,\mathrm{C}$.

(b) The net charge Q of the conductor is the sum of the charge on the cavity wall and the charge q_s on the outer surface of the conductor, so $Q = q_w + q_s$ and

$$q_s = Q - q_w = (10\times 10^{-6}\,\mathrm{C}) - (-3.0\times 10^{-6}\,\mathrm{C}) = +1.3\times 10^{-5}\ \mathrm{C}\ .$$

16. (a) The side surface area A for the drum of diameter D and length h is given by $A = \pi D h$. Thus

$$\begin{aligned} q &= \sigma A = \sigma \pi D h = \pi \varepsilon_0 E D h \\ &= \pi \left(8.85\times 10^{-12}\,\frac{\mathrm{C^2}}{\mathrm{N\cdot m^2}}\right)\left(2.3\times 10^{5}\,\mathrm{N/C}\right)(0.12\,\mathrm{m})(0.42\,\mathrm{m}) \\ &= 3.2\times 10^{-7}\ \mathrm{C}\ . \end{aligned}$$

(b) The new charge is

$$\begin{aligned} q' &= q\left(\frac{A'}{A}\right) = q\left(\frac{\pi D' h'}{\pi D h}\right) \\ &= (3.2\times 10^{-7}\mathrm{C})\left[\frac{(8.0\,\mathrm{cm})(28\,\mathrm{cm})}{(12\,\mathrm{cm})(42\,\mathrm{cm})}\right] = 1.4\times 10^{-7}\ \mathrm{C}\ . \end{aligned}$$

17. The magnitude of the electric field produced by a uniformly charged infinite line is $E = \lambda/2\pi\varepsilon_0 r$, where λ is the linear charge density and r is the distance from the line to the point where the field is measured. See Eq. 24-12. Thus,

$$\lambda = 2\pi\varepsilon_0 E r = 2\pi(8.85\times 10^{-12}\,\mathrm{C^2/N\cdot m^2})(4.5\times 10^{4}\,\mathrm{N/C})(2.0\,\mathrm{m}) = 5.0\times 10^{-6}\ \mathrm{C/m}\ .$$

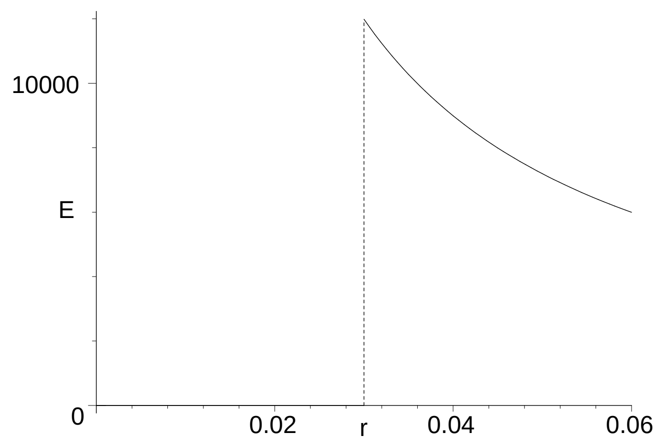
18. We imagine a cylindrical Gaussian surface A of radius r and unit length concentric with the metal tube. Then by symmetry

$$\oint_A \vec{E}\cdot d\vec{A} = 2\pi r E = \frac{q_{\mathrm{enclosed}}}{\varepsilon_0}\ .$$

(a) For $r > R$, $q_{\text{enclosed}} = \lambda$, so $E(r) = \lambda/2\pi r\varepsilon_0$.

(b) For $r < R$, $q_{\text{enclosed}} = 0$, so $E = 0$. The plot of E vs r is shown below. Here, the maximum value is

$$E_{\text{max}} = \frac{\lambda}{2\pi r\varepsilon_0} = \frac{(2.0 \times 10^{-8}\,\text{C/m})}{2\pi(0.030\,\text{m})\,(8.85 \times 10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)} = 1.2 \times 10^4 \text{ N/C} .$$



19. We assume the charge density of both the conducting cylinder and the shell are uniform, and we neglect fringing. Symmetry can be used to show that the electric field is radial, both between the cylinder and the shell and outside the shell. It is zero, of course, inside the cylinder and inside the shell.

(a) We take the Gaussian surface to be a cylinder of length L, coaxial with the given cylinders and of larger radius r than either of them. The flux through this surface is $\Phi = 2\pi r L E$, where E is the magnitude of the field at the Gaussian surface. We may ignore any flux through the ends. Now, the charge enclosed by the Gaussian surface is $q - 2q = -q$. Consequently, Gauss' law yields $2\pi r\varepsilon_0 LE = -q$, so

$$E = -\frac{q}{2\pi\varepsilon_0 Lr} .$$

The negative sign indicates that the field points inward.

(b) Next, we consider a cylindrical Gaussian surface whose radius places it within the shell itself. The electric field is zero at all points on the surface since any field within a conducting material would lead to current flow (and thus to a situation other than the electrostatic ones being considered here), so the total electric flux through the Gaussian surface is zero and the net charge within it is zero (by Gauss' law). Since the central rod is known to have charge q, then the inner surface of the shell must have charge $-q$. And since the shell is known to have total charge $-2q$, it must therefore have charge $-q$ on its outer surface.

(c) Finally, we consider a cylindrical Gaussian surface whose radius places it between the outside of conducting rod and inside of the shell. Similarly to part (a), the flux through the Gaussian surface is $\Phi = 2\pi r L E$, where E is the field at this Gaussian surface, in the region between the rod and the shell. The charge enclosed by the Gaussian surface is only the charge q on the rod. Therefore, Gauss' law yields

$$2\pi\varepsilon_0 r L E = q \implies E = \frac{q}{2\pi\varepsilon_0 L r} .$$

The positive sign indicates that the field points outward.

20. We denote the radius of the thin cylinder as $R = 0.015$ m. Using Eq. 24-12, the net electric field for $r > R$ is given by

$$E_{\text{net}} = E_{\text{wire}} + E_{\text{cylinder}} = \frac{-\lambda}{2\pi\varepsilon_0 r} + \frac{\lambda'}{2\pi\varepsilon_0 r}$$

where $-\lambda = -3.6\,\mathrm{nC/m}$ is the linear charge density of the wire and λ' is the linear charge density of the thin cylinder. We note that the surface and linear charge densities of the thin cylinder are related by

$$q_{\text{cylinder}} = \lambda' L = \sigma(2\pi RL) \implies \lambda' = \sigma(2\pi R) \ .$$

Now, E_{net} outside the cylinder will equal zero, provided that $2\pi R\sigma = \lambda$, or

$$\sigma = \frac{\lambda}{2\pi R} = \frac{3.6 \times 10^{-9}\,\mathrm{C/m}}{(2\pi)(0.015\,\mathrm{m})} = 3.8 \times 10^{-8}\ \mathrm{C/m^2} \ .$$

21. We denote the inner and outer cylinders with subscripts i and o, respectively.

    (a) Since $r_i < r = 4.0\,\mathrm{cm} < r_o$,

    $$E(r) = \frac{\lambda_i}{2\pi\varepsilon_0 r} = \frac{5.0 \times 10^{-6}\,\mathrm{C/m}}{2\pi\,(8.85 \times 10^{-12}\,\mathrm{C^2/N{\cdot}m^2})\,(4.0 \times 10^{-2}\,\mathrm{m})} = 2.3 \times 10^6\ \mathrm{N/C} \ .$$

    $\vec{E}(r)$ points radially outward.

    (b) Since $r > r_o$,

    $$E(r) = \frac{\lambda_i + \lambda_o}{2\pi\varepsilon_0 r} = \frac{5.0 \times 10^{-6}\,\mathrm{C/m} - 7.0 \times 10^{-6}\,\mathrm{C/m}}{2\pi\,(8.85 \times 10^{-12}\,\mathrm{C^2/N{\cdot}m^2})\,(8.0 \times 10^{-2}\,\mathrm{m})} = -4.5 \times 10^5\ \mathrm{N/C} \ ,$$

    where the minus sign indicates that $\vec{E}(r)$ points radially inward.

22. To evaluate the field using Gauss' law, we employ a cylindrical surface of area $2\pi\, r\, L$ where L is very large (large enough that contributions from the ends of the cylinder become irrelevant to the calculation). The volume within this surface is $V = \pi\, r^2 L$, or expressed more appropriate to our needs: $dV = 2\pi\, r\, L\, dr$. The charge enclosed is, with $A = 2.5 \times 10^{-6}\ \mathrm{C/m^5}$,

    $$q_{\text{enc}} = \int_0^r A\, r^2\, 2\pi\, r\, L\, dr = \frac{\pi}{2}\, A\, L\, r^4 \quad .$$

    By Gauss' law, we find $\Phi = |\vec{E}|(2\pi r L) = q_{\text{enc}}/\varepsilon_0$; we thus obtain

    $$\left|\vec{E}\right| = \frac{A\, r^3}{4\,\varepsilon_0} \quad .$$

    (a) With $r = 0.030$ m, we find $|\vec{E}| = 1.9$ N/C.

    (b) Once outside the cylinder, Eq. 24-12 is obeyed. To find $\lambda = q/L$ we must find the total charge q. Therefore,

    $$\frac{q}{L} = \frac{1}{L}\int_0^{0.04} A\, r^2\, 2\pi\, r\, L\, dr = 1.0 \times 10^{-11}\ \mathrm{C/m} \ .$$

    And the result, for $r = 0.050$ m, is $|\vec{E}| = \lambda/2\pi\varepsilon_0\, r = 3.6$ N/C.

23. The electric field is radially outward from the central wire. We want to find its magnitude in the region between the wire and the cylinder as a function of the distance r from the wire. Since the magnitude of the field at the cylinder wall is known, we take the Gaussian surface to coincide with the wall. Thus, the Gaussian surface is a cylinder with radius R and length L, coaxial with the wire. Only the charge on the wire is actually enclosed by the Gaussian surface; we denote it by q. The area of the Gaussian surface is $2\pi RL$, and the flux through it is $\Phi = 2\pi RLE$. We assume there is no flux through the ends of the cylinder, so this Φ is the total flux. Gauss' law yields $q = 2\pi\varepsilon_0 RLE$. Thus,

    $$q = 2\pi\left(8.85 \times 10^{-12}\,\frac{\mathrm{C^2}}{\mathrm{N{\cdot}m^2}}\right)(0.014\,\mathrm{m})(0.16\,\mathrm{m})\left(2.9 \times 10^4\,\mathrm{N/C}\right) = 3.6 \times 10^{-9}\ \mathrm{C} \ .$$

24. (a) In Eq. 24-12, $\lambda = q/L$ where q is the net charge enclosed by a cylindrical Gaussian surface of radius r. The field is being measured outside the system (the charged rod coaxial with the neutral cylinder) so that the net enclosed charge is only that which is on the rod. Consequently,

$$|\vec{E}| = \frac{\lambda}{2\pi\varepsilon_0\, r} = \frac{2.0 \times 10^{-9}}{2\pi\varepsilon_0\,(0.15)} = 240 \text{ N/C} .$$

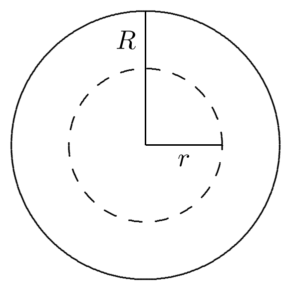
(b) and (c) Since the field is zero inside the conductor (in an electrostatic configuration), then there resides on the inner surface charge $-q$, and on the outer surface, charge $+q$ (where q is the charge on the rod at the center). Therefore, with $r_i = 0.05$ m, the surface density of charge is

$$\sigma_{\text{inner}} = \frac{-q}{2\pi r_i L} = -\frac{\lambda}{2\pi r_i} = -6.4 \times 10^{-9} \text{ C/m}^2$$

for the inner surface. And, with $r_o = 0.10$ m, the surface charge density of the outer surface is

$$\sigma_{\text{outer}} = \frac{+q}{2\pi r_o L} = \frac{\lambda}{2\pi r_o} = +3.2 \times 10^{-9} \text{ C/m}^2 .$$

25. (a) The diagram below shows a cross section (or, perhaps more appropriately, "end view") of the charged cylinder (solid circle). Consider a Gaussian surface in the form of a cylinder with radius r and length ℓ, coaxial with the charged cylinder. An "end view" of the Gaussian surface is shown as a dotted circle. The charge enclosed by it is $q = \rho V = \pi r^2 \ell \rho$, where $V = \pi r^2 \ell$ is the volume of the cylinder.



If ρ is positive, the electric field lines are radially outward, normal to the Gaussian surface and distributed uniformly along it. Thus, the total flux through the Gaussian cylinder is $\Phi = EA_{\text{cylinder}} = E(2\pi r\ell)$. Now, Gauss' law leads to

$$2\pi\varepsilon_0 r\ell E = \pi r^2 \ell\rho \implies E = \frac{\rho r}{2\varepsilon_0} .$$

(b) Next, we consider a cylindrical Gaussian surface of radius $r > R$. If the external field E_{ext} then the flux is $\Phi = 2\pi r\ell E_{\text{ext}}$. The charge enclosed is the total charge in a section of the charged cylinder with length ℓ. That is, $q = \pi R^2 \ell\rho$. In this case, Gauss' law yields

$$2\pi\varepsilon_0 r\ell E_{\text{ext}} = \pi R^2 \ell\rho \implies E_{\text{ext}} = \frac{R^2\rho}{2\varepsilon_0 r} .$$

26. According to Eq. 24-13 the electric field due to either sheet of charge with surface charge density σ is perpendicular to the plane of the sheet (pointing *away* from the sheet if the charge is positive) and has magnitude $E = \sigma/2\varepsilon_0$. Using the superposition principle, we conclude:

(a) $E = \sigma/\varepsilon_0$, pointing up;

(b) $E = 0$;

(c) and, $E = \sigma/\varepsilon_0$, pointing down.

27. (a) To calculate the electric field at a point very close to the center of a large, uniformly charged conducting plate, we may replace the finite plate with an infinite plate with the same area charge density and take the magnitude of the field to be $E = \sigma/\varepsilon_0$, where σ is the area charge density for the surface just under the point. The charge is distributed uniformly over both sides of the original plate, with half being on the side near the field point. Thus,

$$\sigma = \frac{q}{2A} = \frac{6.0 \times 10^{-6}\,\mathrm{C}}{2(0.080\,\mathrm{m})^2} = 4.69 \times 10^{-4}\ \mathrm{C/m^2}\ .$$

The magnitude of the field is

$$E = \frac{4.69 \times 10^{-4}\,\mathrm{C/m^2}}{8.85 \times 10^{-12}\,\mathrm{C^2/N{\cdot}m^2}} = 5.3 \times 10^7\ \mathrm{N/C}\ .$$

The field is normal to the plate and since the charge on the plate is positive, it points away from the plate.

(b) At a point far away from the plate, the electric field is nearly that of a point particle with charge equal to the total charge on the plate. The magnitude of the field is $E = q/4\pi\varepsilon_0 r^2 = kq/r^2$, where r is the distance from the plate. Thus,

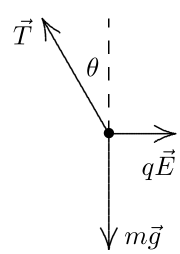
$$E = \frac{\left(8.99 \times 10^9\,\mathrm{N{\cdot}m^2/C^2}\right)\left(6.0 \times 10^{-6}\,\mathrm{C}\right)}{(30\,\mathrm{m})^2} = 60\ \mathrm{N/C}\ .$$

28. The charge distribution in this problem is equivalent to that of an infinite sheet of charge with surface charge density σ plus a small circular pad of radius R located at the middle of the sheet with charge density $-\sigma$. We denote the electric fields produced by the sheet and the pad with subscripts 1 and 2, respectively. The net electric field $\vec{E}$ is then

$$\begin{aligned}\vec{E} &= \vec{E}_1 + \vec{E}_2 = \left(\frac{\sigma}{2\varepsilon_0}\right)\hat{\mathrm{k}} + \frac{(-\sigma)}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right)\hat{\mathrm{k}} \\ &= \frac{\sigma z}{2\varepsilon_0\sqrt{z^2+R^2}}\,\hat{\mathrm{k}}\end{aligned}$$

where Eq. 23-26 is used for $\vec{E}_2$.

29. The forces acting on the ball are shown in the diagram below. The gravitational force has magnitude mg, where m is the mass of the ball; the electrical force has magnitude qE, where q is the charge on the ball and E is the magnitude of the electric field at the position of the ball; and, the tension in the thread is denoted by T. The electric field produced by the plate is normal to the plate and points to the right. Since the ball is positively charged, the electric force on it also points to the right. The tension in the thread makes the angle θ $(= 30^\circ)$ with the vertical.



Since the ball is in equilibrium the net force on it vanishes. The sum of the horizontal components yields $qE - T\sin\theta = 0$ and the sum of the vertical components yields $T\cos\theta - mg = 0$. The expression $T = qE/\sin\theta$, from the first equation, is substituted into the second to obtain $qE = mg\tan\theta$. The electric field produced by a large uniform plane of charge is given by $E = \sigma/2\varepsilon_0$, where σ is the surface charge density. Thus,

$$\frac{q\sigma}{2\varepsilon_0} = mg\tan\theta$$

and

$$\begin{aligned}\sigma &= \frac{2\varepsilon_0 mg\tan\theta}{q} \\ &= \frac{2(8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2})(1.0\times10^{-6}\,\mathrm{kg})(9.8\,\mathrm{m/s^2})\tan 30^\circ}{2.0\times10^{-8}\,\mathrm{C}} \\ &= 5.0\times10^{-9}\ \mathrm{C/m^2}\ .\end{aligned}$$

30. Let î be a unit vector pointing to the left. We use Eq. 24-13.

(a) To the left of the plates: $\vec{E} = (\sigma/2\varepsilon_0)\hat{\imath}$ (from the right plate)$+(-\sigma/2\varepsilon_0)\hat{\imath}$ (from the left one)$= 0$.

(b) To the right of the plates: $\vec{E} = (\sigma/2\varepsilon_0)(-\hat{\imath})$ (from the right plate)$+(-\sigma/2\varepsilon_0)(-\hat{\imath})$ (from the left one)$= 0$.

(c) Between the plates:

$$\begin{aligned}\vec{E} &= \left(\frac{\sigma}{2\varepsilon_0}\right)\hat{\imath} + \left(\frac{-\sigma}{2\varepsilon_0}\right)(-\hat{\imath}) = \left(\frac{\sigma}{\varepsilon_0}\right)\hat{\imath} \\ &= \left(\frac{7.0\times10^{-22}\,\mathrm{C/m^2}}{8.85\times10^{-12}\,\frac{\mathrm{N\cdot m^2}}{\mathrm{C^2}}}\right)\hat{\imath} = \left(7.9\times10^{-11}\ \mathrm{N/C}\right)\hat{\imath}\ .\end{aligned}$$

31. The charge on the metal plate, which is negative, exerts a force of repulsion on the electron and stops it. First find an expression for the acceleration of the electron, then use kinematics to find the stopping distance. We take the initial direction of motion of the electron to be positive. Then, the electric field is given by $E = \sigma/\varepsilon_0$, where σ is the surface charge density on the plate. The force on the electron is $F = -eE = -e\sigma/\varepsilon_0$ and the acceleration is

$$a = \frac{F}{m} = -\frac{e\sigma}{\varepsilon_0 m}$$

where m is the mass of the electron. The force is constant, so we use constant acceleration kinematics. If v_0 is the initial velocity of the electron, v is the final velocity, and x is the distance traveled between the initial and final positions, then $v^2 - v_0^2 = 2ax$. Set $v = 0$ and replace a with $-e\sigma/\varepsilon_0 m$, then solve for x. We find

$$x = -\frac{v_0^2}{2a} = \frac{\varepsilon_0 m v_0^2}{2e\sigma}\ .$$

Now $\frac{1}{2}mv_0^2$ is the initial kinetic energy K_0, so

$$x = \frac{\varepsilon_0 K_0}{e\sigma}\ .$$

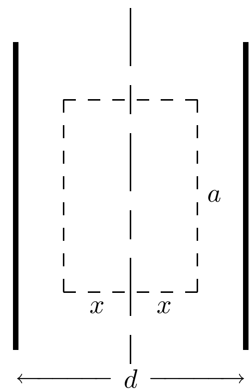
We convert the given value of K_0 to Joules. Since $1.00\,\mathrm{eV} = 1.60\times10^{-19}\,\mathrm{J}$, $100\,\mathrm{eV} = 1.60\times10^{-17}\,\mathrm{J}$. Thus,

$$x = \frac{(8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2})(1.60\times10^{-17}\,\mathrm{J})}{(1.60\times10^{-19}\,\mathrm{C})(2.0\times10^{-6}\,\mathrm{C/m^2})} = 4.4\times10^{-4}\ \mathrm{m}\ .$$

32. We use the result of part (c) of problem 30 to obtain the surface charge density.

$$E = \sigma/\varepsilon_0 \implies \sigma = \varepsilon_0 E = \left(8.85\times10^{-12}\,\frac{\mathrm{C^2}}{\mathrm{N\cdot m^2}}\right)(55\,\mathrm{N/C}) = 4.9\times10^{-10}\ \mathrm{C/m^2}\ .$$

33. (a) We use a Gaussian surface in the form of a box with rectangular sides. The cross section is shown with dashed lines in the diagram below. It is centered at the central plane of the slab, so the left and right faces are each a distance x from the central plane. We take the thickness of the rectangular solid to be a, the same as its length, so the left and right faces are squares. The electric field is normal to the left and right faces and is uniform over them. If ρ is positive, it points outward at both faces: toward the left at the left face and toward the right at the right face. Furthermore, the magnitude is the same at both faces. The electric flux through each of these faces is Ea^2. The field is parallel to the other faces of the Gaussian surface and the flux through them is zero. The total flux through the Gaussian surface is $\Phi = 2Ea^2$.



The volume enclosed by the Gaussian surface is $2a^2x$ and the charge contained within it is $q = 2a^2x\rho$. Gauss' law yields $2\varepsilon_0 Ea^2 = 2a^2x\rho$. We solve for the magnitude of the electric field:

$$E = \frac{\rho x}{\varepsilon_0} .$$

(b) We take a Gaussian surface of the same shape and orientation, but with $x > d/2$, so the left and right faces are outside the slab. The total flux through the surface is again $\Phi = 2Ea^2$ but the charge enclosed is now $q = a^2 d\rho$. Gauss' law yields $2\varepsilon_0 Ea^2 = a^2 d\rho$, so

$$E = \frac{\rho d}{2\varepsilon_0} .$$

34. (a) The flux is still $-750\,\mathrm{N\cdot m^2/C}$, since it depends only on the amount of charge enclosed.

(b) We use $\Phi = q/\varepsilon_0$ to obtain the charge q:

$$q = \varepsilon_0 \Phi = \left(8.85 \times 10^{-12}\,\frac{\mathrm{C^2}}{\mathrm{N\cdot m^2}}\right)\left(-750\,\mathrm{N\cdot m^2/C}\right) = -6.64 \times 10^{-10}\ \mathrm{C} .$$

35. Charge is distributed uniformly over the surface of the sphere and the electric field it produces at points outside the sphere is like the field of a point particle with charge equal to the net charge on the sphere. That is, the magnitude of the field is given by $E = q/4\pi\varepsilon_0 r^2$, where q is the magnitude of the charge on the sphere and r is the distance from the center of the sphere to the point where the field is measured. Thus,

$$q = 4\pi\varepsilon_0 r^2 E = \frac{(0.15\,\mathrm{m})^2(3.0 \times 10^3\,\mathrm{N/C})}{8.99 \times 10^9\,\mathrm{N\cdot m^2/C^2}} = 7.5 \times 10^{-9}\,\mathrm{C} .$$

The field points inward, toward the sphere center, so the charge is negative: $-7.5 \times 10^{-9}\,\mathrm{C}$.

36. (a) Since $r_1 = 10.0\,\mathrm{cm} < r = 12.0\,\mathrm{cm} < r_2 = 15.0\,\mathrm{cm}$,

$$E(r) = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r^2} = \frac{(8.99 \times 10^9\,\mathrm{N\cdot m^2/C^2})(4.00 \times 10^{-8}\,\mathrm{C})}{(0.120\,\mathrm{m})^2} = 2.50 \times 10^4\ \mathrm{N/C} .$$

(b) Since $r_1 < r_2 < r = 20.0\,\text{cm}$,

$$\begin{aligned} E(r) &= \frac{1}{4\pi\varepsilon_0}\frac{q_1+q_2}{r^2} = \frac{(8.99\times 10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(4.00+2.00)(1\times 10^{-8}\,\text{C})}{(0.200\,\text{m})^2} \\ &= 1.35\times 10^4\ \text{N/C}\ . \end{aligned}$$

37. The field is radially outward and takes on equal magnitude-values over the surface of any sphere centered at the atom's center. We take the Gaussian surface to be such a sphere (of radius r). If E is the magnitude of the field, then the total flux through the Gaussian sphere is $\Phi = 4\pi r^2 E$. The charge enclosed by the Gaussian surface is the positive charge at the center of the atom plus that portion of the negative charge within the surface. Since the negative charge is uniformly distributed throughout the large sphere of radius R, we can compute the charge inside the Gaussian sphere using a ratio of volumes. That is, the negative charge inside is $-Zer^3/R^3$. Thus, the total charge enclosed is $Ze - Zer^3/R^3$ for $r \le R$. Gauss' law now leads to

$$4\pi\varepsilon_0 r^2 E = Ze\left(1 - \frac{r^3}{R^3}\right) \implies E = \frac{Ze}{4\pi\varepsilon_0}\left(\frac{1}{r^2} - \frac{r}{R^3}\right)\ .$$

38. We interpret the question as referring to the field *just* outside the sphere (that is, at locations roughly equal to the radius r of the sphere). Since the area of a sphere is $A = 4\pi r^2$ and the surface charge density is $\sigma = q/A$ (where we assume q is positive for brevity), then

$$E = \frac{\sigma}{\varepsilon_0} = \frac{1}{\varepsilon_0}\left(\frac{q}{4\pi r^2}\right) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

which we recognize as the field of a point charge (see Eq. 23-3).

39. The proton is in uniform circular motion, with the electrical force of the sphere on the proton providing the centripetal force. According to Newton's second law, $F = mv^2/r$, where F is the magnitude of the force, v is the speed of the proton, and r is the radius of its orbit, essentially the same as the radius of the sphere. The magnitude of the force on the proton is $F = eq/4\pi\varepsilon_0 r^2$, where q is the magnitude of the charge on the sphere. Thus,

$$\frac{1}{4\pi\varepsilon_0}\frac{eq}{r^2} = \frac{mv^2}{r}$$

so

$$\begin{aligned} q &= \frac{4\pi\varepsilon_0 m v^2 r}{e} = \frac{(1.67\times 10^{-27}\,\text{kg})(3.00\times 10^5\,\text{m/s})^2(0.0100\,\text{m})}{(8.99\times 10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times 10^{-19}\,\text{C})} \\ &= 1.04\times 10^{-9}\ \text{C}\ . \end{aligned}$$

The force must be inward, toward the center of the sphere, and since the proton is positively charged, the electric field must also be inward. The charge on the sphere is negative: $q = -1.04\times 10^{-9}\,\text{C}$.

40. We imagine a spherical Gaussian surface of radius r centered at the point charge $+q$. From symmetry consideration E is the same throughout the surface, so

$$\oint \vec{E}\cdot d\vec{A} = 4\pi r^2 E = \frac{q_{\text{encl}}}{\varepsilon_0}\ ,$$

which gives

$$E(r) = \frac{q_{\text{encl}}}{4\pi\varepsilon_0 r^2}\ ,$$

where q_{encl} is the net charge enclosed by the Gaussian surface.

(a) Now $a < r < b$, where $E = 0$. Thus $q_{\text{encl}} = 0$, so the charge on the inner surface of the shell is $q_i = -q$.

(b) The shell as a whole is electrically neutral, so the outer shell must carry a charge of $q_o = +q$.

(c) For $r < a \quad q_{\text{encl}} = +q$, so

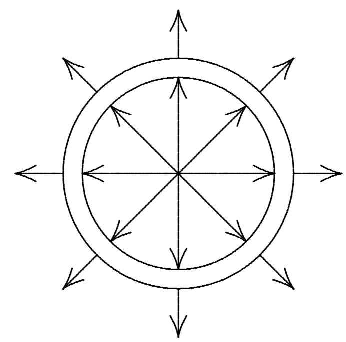
$$E\bigg|_{r<a} = \frac{q}{4\pi\varepsilon_0 r^2}\ .$$

(d) For $b > r > a \quad E = 0$, since this region is inside the metallic part of the shell.

(e) For $r > b \quad q_{\text{encl}} = +q$, so

$$E\bigg|_{r<a} = \frac{q}{4\pi\varepsilon_0 r^2}\ .$$

The field lines are sketched to the right.

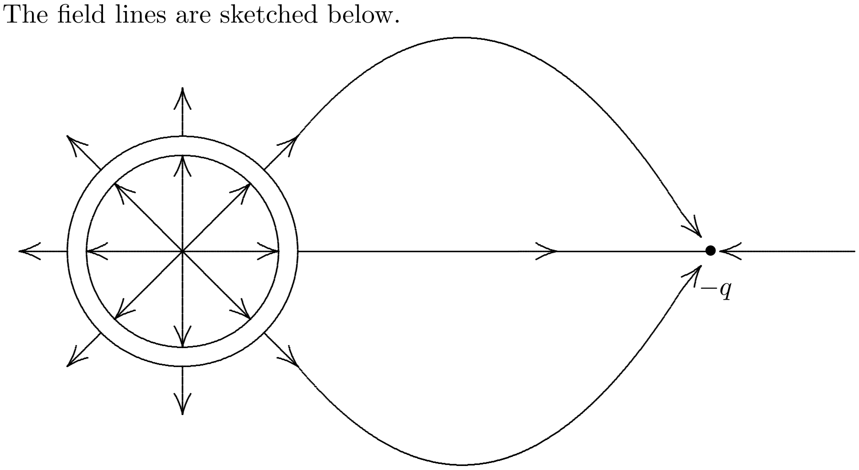


(f) The net charge of the central point charge-inner surface combination is zero. Thus the electric field it produces is also zero.

(g) The outer shell has a spherically symmetric charge distribution with a net charge $+q$. Thus the field it produces for $r > b$ is $E = q/(4\pi\varepsilon_0 r^2)$.

(h) Yes. In fact there will be a distribution of induced charges on the outer shell, as a result of a flow of positive charges toward the side of the surface that is closer to the negative point charge outside the shell.

(i) No. The change in the charge distribution on the outer shell cancels the effect of the negative point charge. The field lines are sketched below.



(j) Yes, there is a force on the $-q$ point charge, as expected from Eq. 22-4.

(k) The field lines around the first charge at the center of the spherical shell is unchanged. The implication, then, is that there is still no net force on that charge.

(l) We assume there is some non-electrical force holding the spherical shell in place, which compensates for the force of the $-q$ point charge exerted on the outside surface charges on the shell. Newton's third law applies to this situation, as far as the $-q$ point charge and the surface charges on the sphere are concerned. There is no direct force between the central $+q$ charge and the external $-q$ point charge, so we would not apply Newton's third law to their interaction.

41. (a) We integrate the volume charge density over the volume and require the result be equal to the total charge:

$$\int dx \int dy \int dz\, \rho = 4\pi \int_0^R dr\, r^2\, \rho = Q\ .$$

Substituting the expression $\rho = \rho_s r/R$ and performing the integration leads to

$$4\pi\left(\frac{\rho_s}{R}\right)\left(\frac{R^4}{4}\right) = Q \quad\Longrightarrow\quad Q = \pi\rho_s R^3\ .$$

(b) At a certain point within the sphere, at some distance r_o from the center, the field (see Eq. 24-8 through Eq. 24-10) is given by Gauss' law:

$$E = \frac{1}{4\pi\varepsilon_0}\,\frac{q_\mathrm{enc}}{r_\mathrm{o}^2}$$

where q_enc is given by an integral similar to that worked in part (a):

$$q_\mathrm{enc} = 4\pi\int_0^{r_\mathrm{o}} dr\, r^2\, \rho = 4\pi\left(\frac{\rho_s}{R}\right)\left(\frac{r_\mathrm{o}^4}{4}\right)\ .$$

Therefore,

$$E = \frac{1}{4\pi\varepsilon_0}\,\frac{\pi\rho_s r_\mathrm{o}^4}{R\, r_\mathrm{o}^2}$$

which (using the relation between ρ_s and Q derived in part (a)) becomes

$$E = \frac{1}{4\pi\varepsilon_0}\,\frac{\pi\left(\frac{Q}{\pi R^3}\right) r_\mathrm{o}^2}{R}$$

and simplifies to the desired result (shown in the problem statement) if we change notation $r_\mathrm{o} \to r$.

42. (a) We note that the symbol "e" stands for the elementary charge in the manipulations below. From

$$-e = \int_0^\infty \rho(r) 4\pi r^2 dr = \int_0^\infty A\exp\left(-2r/a_0\right) 4\pi r^2\, dr = \pi a_0^3 A$$

we get $A = -e/\pi a_0^3$.

(b) The magnitude of the field is

$$\begin{aligned} E &= \frac{q_\mathrm{encl}}{4\pi\varepsilon_0 a_0^2} = \frac{1}{4\pi\varepsilon_0 a_0^2}\left(e + \int_0^{a_0}\rho(r) 4\pi r^2 dr\right) \\ &= \frac{e}{4\pi\varepsilon_0 a_0^2}\left(1 - \frac{4}{a_0^3}\int_0^{a_0}\exp(-2r/a_0)\ r^2\, dr\right) \\ &= \frac{5\, e\, \exp(-2)}{4\pi\varepsilon_0\, a_0^2}\ . \end{aligned}$$

We note that $\vec{E}$ points radially outward.

43. At all points where there is an electric field, it is radially outward. For each part of the problem, use a Gaussian surface in the form of a sphere that is concentric with the sphere of charge and passes through the point where the electric field is to be found. The field is uniform on the surface, so

$$\oint \vec{E}\cdot d\vec{A} = 4\pi r^2 E$$

where r is the radius of the Gaussian surface.

(a) Here r is less than a and the charge enclosed by the Gaussian surface is $q(r/a)^3$. Gauss' law yields

$$4\pi r^2 E = \left(\frac{q}{\varepsilon_0}\right)\left(\frac{r}{a}\right)^3 \implies E = \frac{qr}{4\pi\varepsilon_0 a^3} .$$

(b) In this case, r is greater than a but less than b. The charge enclosed by the Gaussian surface is q, so Gauss' law leads to

$$4\pi r^2 E = \frac{q}{\varepsilon_0} \implies E = \frac{q}{4\pi\varepsilon_0 r^2} .$$

(c) The shell is conducting, so the electric field inside it is zero.

(d) For $r > c$, the charge enclosed by the Gaussian surface is zero (charge q is inside the shell cavity and charge $-q$ is on the shell). Gauss' law yields

$$4\pi r^2 E = 0 \implies E = 0 .$$

(e) Consider a Gaussian surface that lies completely within the conducting shell. Since the electric field is everywhere zero on the surface, $\oint \vec{E} \cdot d\vec{A} = 0$ and, according to Gauss' law, the net charge enclosed by the surface is zero. If Q_i is the charge on the inner surface of the shell, then $q + Q_i = 0$ and $Q_i = -q$. Let Q_o be the charge on the outer surface of the shell. Since the net charge on the shell is $-q$, $Q_i + Q_o = -q$. This means $Q_o = -q - Q_i = -q - (-q) = 0$.

44. The field is zero for $0 \le r \le a$ as a result of Eq. 24-16. Since q_{enc} (for $a \le r \le b$) is related to the volume by

$$q_{\text{enc}} = \rho\left(\frac{4\pi r^3}{3} - \frac{4\pi a^3}{3}\right)$$

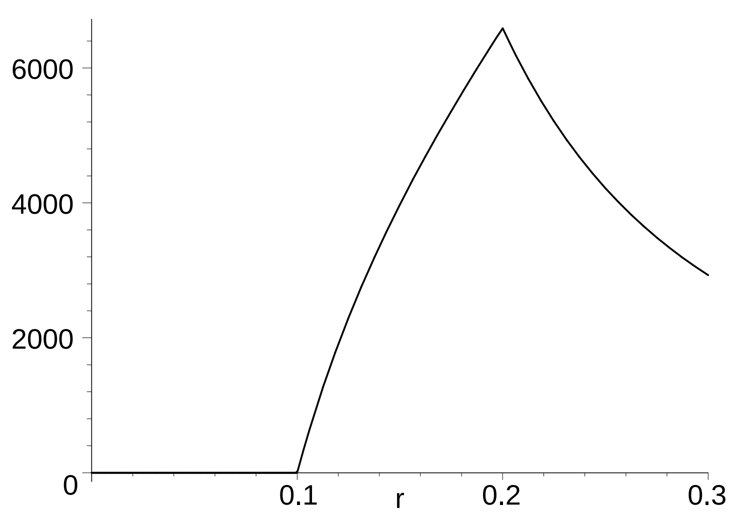
then

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q_{\text{enc}}}{r^2} = \frac{\rho}{4\pi\varepsilon_0 r^2}\left(\frac{4\pi r^3}{3} - \frac{4\pi a^3}{3}\right) = \frac{\rho}{3\varepsilon_0}\frac{r^3 - a^3}{r^2}$$

for $a \le r \le b$. And for $r \ge b$ we have $E = q_{\text{total}}/4\pi\varepsilon_0 r^2$ or

$$E = \frac{\rho}{3\varepsilon_0}\frac{b^3 - a^3}{r^2} \qquad\qquad r \ge b .$$

This is plotted below for r in meters from 0 to 0.30 m. The peak value of the electric field, reached at $r = b = 0.20$ m, is 6.6×10^3 N/C.



45. To find an expression for the electric field inside the shell in terms of A and the distance from the center of the shell, select A so the field does not depend on the distance. We use a Gaussian surface in the form of a sphere with radius r_g, concentric with the spherical shell and within it ($a < r_g < b$). Gauss' law will be used to find the magnitude of the electric field a distance r_g from the shell center. The charge that is both in the shell and within the Gaussian sphere is given by the integral $q_s = \int \rho \, dV$ over the portion of the shell within the Gaussian surface. Since the charge distribution has spherical symmetry, we may take dV to be the volume of a spherical shell with radius r and infinitesimal thickness dr: $dV = 4\pi r^2 \, dr$. Thus,

$$q_s = 4\pi \int_a^{r_g} \rho r^2 \, dr = 4\pi \int_a^{r_g} \frac{A}{r} r^2 \, dr = 4\pi A \int_a^{r_g} r \, dr = 2\pi A(r_g^2 - a^2) \ .$$

The total charge inside the Gaussian surface is $q + q_s = q + 2\pi A(r_g^2 - a^2)$. The electric field is radial, so the flux through the Gaussian surface is $\Phi = 4\pi r_g^2 E$, where E is the magnitude of the field. Gauss' law yields

$$4\pi\varepsilon_0 E r_g^2 = q + 2\pi A(r_g^2 - a^2) \ .$$

We solve for E:

$$E = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{r_g^2} + 2\pi A - \frac{2\pi A a^2}{r_g^2}\right] \ .$$

For the field to be uniform, the first and last terms in the brackets must cancel. They do if $q - 2\pi A a^2 = 0$ or $A = q/2\pi a^2$.

46. (a) From Gauss' law,

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{q_{\text{encl}}}{r^3}\vec{r} = \frac{1}{4\pi\varepsilon_0}\frac{(4\pi\rho r^3/3)\vec{r}}{r^3} = \frac{\rho\vec{r}}{3\varepsilon_0} \ .$$

(b) The charge distribution in this case is equivalent to that of a whole sphere of charge density ρ plus a smaller sphere of charge density $-\rho$ which fills the void. By superposition

$$\vec{E}(\vec{r}) = \frac{\rho\vec{r}}{3\varepsilon_0} + \frac{(-\rho)(\vec{r} - \vec{a})}{3\varepsilon_0} = \frac{\rho\vec{a}}{3\varepsilon_0} \ .$$

47. We use

$$E(r) = \frac{q_{\text{encl}}}{4\pi\varepsilon_0 r^2} = \frac{1}{4\pi\varepsilon_0 r^2}\int_0^r \rho(r) 4\pi r^2 \, dr$$

to solve for $\rho(r)$:

$$\rho(r) = \frac{\varepsilon_0}{r^2}\frac{d}{dr}[r^2 E(r)] = \frac{\varepsilon_0}{r^2}\frac{d}{dr}(Kr^6) = 6K\varepsilon_0 r^3 \ .$$

48. (a) We consider the radial field produced at points within a uniform cylindrical distribution of charge. The volume enclosed by a Gaussian surface in this case is $L\pi r^2$. Thus, Gauss' law leads to

$$E = \frac{|q_{\text{enc}}|}{\varepsilon_0 A_{\text{cylinder}}} = \frac{|\rho|\left(L\pi r^2\right)}{\varepsilon_0\,(2\pi r L)} = \frac{|\rho| r}{2\varepsilon_0} \ .$$

(b) We note from the above expression that the magnitude of the radial field grows with r.

(c) Since the charged powder is negative, the field points radially inward.

(d) The largest value of r which encloses charged material is $r_{\max} = R$. Therefore, with $|\rho| = 0.0011\ \text{C/m}^3$ and $R = 0.050$ m, we obtain

$$E_{\max} = \frac{|\rho| R}{2\varepsilon_0} = 3.1 \times 10^6\ \text{N/C} \ .$$

(e) According to condition 1 mentioned in the problem, the field is high enough to produce an electrical discharge (at $r = R$).

49. (a) At A, the only field contribution is from the $+5.00Q$ particle in the hollow (this follows from Gauss' law – it is the only charge enclosed by a Gaussian spherical surface passing through point A, concentric with the shell). Thus, using k for $1/4\pi\varepsilon_0$, we have $\vec{E} = k(5Q)/(0.5)^2 = 20kQ$ directed radially outward.

(b) Point B is in the conducting material, where the field must be zero in any electrostatic situation.

(c) Point C is outside the sphere where the net charge at smaller values of radius is $-3.00Q + 5.00Q = 2.00Q$. Therefore, we have $\vec{E} = k(2Q)/(2)^2 = \frac{1}{2}kQ$ directed radially outward.

50. Since the fields involved are uniform, the precise location of P are not relevant. Since the sheets are oppositely charged (though not equally so), the field contributions are additive (since P is between them). Using Eq. 24-13, we obtain

$$\vec{E} = \frac{\sigma_1}{2\varepsilon_0} + \frac{3\sigma_1}{2\varepsilon_0} = \frac{2\sigma_1}{\varepsilon_0}$$

directed towards the negatively charged sheet.

51. (a) We imagine a Gaussian surface A which is just outside the inner surface of the spherical shell. Then $\vec{E}$ is zero everywhere on surface A. Thus

$$\oint_A \vec{E} \cdot d\vec{A} = \frac{(Q' + Q)}{\varepsilon_0} = 0 \ ,$$

where Q' is the charge on the inner surface of the shell. This gives $Q' = -Q$.

(b) Since $\vec{E}$ remains zero on surface A the result is unchanged.

(c) Now,

$$\oint_A \vec{E} \cdot d\vec{A} = \frac{(Q' + q + Q)}{\varepsilon_0} = 0 \ ,$$

so $Q' = -(Q + q)$.

(d) Yes, since $\vec{E}$ remains zero on surface A regardless of where you place the sphere inside the shell.

52. We choose a coordinate system whose origin is at the center of the flat base, such that the base is in the xy plane and the rest of the hemisphere is in the $z > 0$ half space.

(a) $\Phi = \pi R^2(-\hat{\mathrm{k}}) \cdot E\hat{\mathrm{k}} = -\pi R^2 E$.

(b) Since the flux through the entire hemisphere is zero, the flux through the curved surface is $\vec{\Phi}_c = -\Phi_{\text{base}} = \pi R^2 E$.

53. Let $\Phi_0 = 10^3\,\mathrm{N{\cdot}m^2/C}$. The net flux through the entire surface of the dice is given by

$$\Phi = \sum_{n=1}^{6} \Phi_n = \sum_{n=1}^{6} (-1)^n n\, \Phi_0 = \Phi_0(-1 + 2 - 3 + 4 - 5 + 6) = 3\Phi_0 \ .$$

Thus, the net charge enclosed is

$$q = \varepsilon_0 \Phi = 3\varepsilon_0\Phi_0 = 3\left(8.85 \times 10^{-12}\,\frac{\mathrm{C^2}}{\mathrm{N{\cdot}m^2}}\right)\left(10^3\,\mathrm{N{\cdot}m^2/C}\right) = 2.66 \times 10^{-8}\ \mathrm{C} \ .$$

54. We use $\Phi = \int \vec{E} \cdot d\vec{A}$. We note that the side length of the cube is $3.0\,\mathrm{m} - 1.0\,\mathrm{m} = 2.0\,\mathrm{m}$.

(a) On the top face of the cube $y = 2.0\,\mathrm{m}$ and $d\vec{A} = (dA)\hat{\mathrm{j}}$. So $\vec{E} = 4\hat{\mathrm{i}} - 3((2.0)^2 + 2)\hat{\mathrm{j}} = 4\hat{\mathrm{i}} - 18\hat{\mathrm{j}}$. Thus the flux is

$$\begin{aligned} \Phi &= \int_{\text{top}} \vec{E} \cdot d\vec{A} = \int_{\text{top}} (4\hat{\mathrm{i}} - 18\hat{\mathrm{j}}) \cdot (dA)\hat{\mathrm{j}} \\ &= -18 \int_{\text{top}} dA = (-18)(2.0)^2\,\mathrm{N{\cdot}m^2/C} = -72\ \mathrm{N{\cdot}m^2/C} \ . \end{aligned}$$

(b) On the bottom face of the cube $y = 0$ and $d\vec{A} = (dA)(-\hat{\jmath})$. So $\vec{E} = 4\hat{\imath} - 3(0^2 + 2)\hat{\jmath} = 4\hat{\imath} - 6\hat{\jmath}$. Thus, the flux is

$$\begin{aligned}\Phi &= \int_{\text{bottom}} \vec{E} \cdot d\vec{A} = \int_{\text{bottom}} (4\hat{\imath} - 6\hat{\jmath}) \cdot (dA)(-\hat{\jmath}) \\ &= 6\int_{\text{bottom}} dA = 6(2.0)^2\,\text{N}\cdot\text{m}^2/\text{C} = +24\ \text{N}\cdot\text{m}^2/\text{C}\ .\end{aligned}$$

(c) On the left face of the cube $d\vec{A} = (dA)(-\hat{\imath})$. So

$$\begin{aligned}\Phi &= \int_{\text{left}} \vec{E} \cdot d\vec{A} = \int_{\text{left}} (4\hat{\imath} + E_y\hat{\jmath}) \cdot (dA)(-\hat{\imath}) \\ &= -4\int_{\text{bottom}} dA = -4(2.0)^2\,\text{N}\cdot\text{m}^2/\text{C} = -16\ \text{N}\cdot\text{m}^2/\text{C}\ .\end{aligned}$$

(d) On the back face of the cube $d\vec{A} = (dA)(-\hat{k})$. But since $\vec{E}$ has no z component $\vec{E} \cdot d\vec{A} = 0$. Thus, $\Phi = 0$.

(e) We now have to add the flux through all six faces. One can easily verify that the flux through the front face is zero, while that through the right face is the opposite of that through the left one, or $+16\,\text{N}\cdot\text{m}^2/\text{C}$. Thus the net flux through the cube is $\Phi = (-72 + 24 - 16 + 0 + 0 + 16)\,\text{N}\cdot\text{m}^2/\text{C} = -48\ \text{N}\cdot\text{m}^2/\text{C}$.

55. The net enclosed charge q is given by

$$q = \varepsilon_0\Phi = \left(8.85 \times 10^{-12}\,\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)\left(-48\,\text{N}\cdot\text{m}^2/\text{C}\right) = -4.2 \times 10^{-10}\ \text{C}\ .$$

56. Since the fields involved are uniform, the precise location of P is not relevant; what is important is it is above the three sheets, with the positively charged sheets contributing upward fields and the negatively charged sheet contributing a downward field, which conveniently conforms to usual conventions (of upward as positive and downward as negative). The net field is directed upward, and (from Eq. 24-13) is magnitude is

$$\left|\vec{E}\right| = \frac{\sigma_1}{2\varepsilon_0} + \frac{\sigma_2}{2\varepsilon_0} + \frac{\sigma_3}{2\varepsilon_0} = \frac{1.0 \times 10^{-6}}{2 \times 8.85 \times 10^{-12}} = 5.6 \times 10^4\ \text{N/C}\ .$$

57. (a) Outside the sphere, we use Eq. 24-15 and obtain

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\,\frac{q}{r^2} = 1.5 \times 10^4\,\text{N/C outward}\ .$$

(b) With $q = +6.00 \times 10^{-12}$ C, Eq. 24-20 leads to $\vec{E} = 2.5 \times 10^4$ N/C directed outward.

58. (a) and (b) There is no flux through the sides, so we have two contributions to the flux, one from the $x = 2$ end (with $\Phi_2 = +(2+2)(\pi(0.20)^2) = 0.50\ \text{N}\cdot\text{m}^2/\text{C}$) and one from the $x = 0$ end (with $\Phi_0 = -(2)(\pi(0.20)^2)$). By Gauss' law we have $q_{\text{enc}} = \varepsilon_0\,(\Phi_2 + \Phi_0) = 2.2 \times 10^{-12}$ C.

59. (a) The cube is totally within the spherical volume, so the charge enclosed is $\rho V_{\text{cube}} = (500 \times 10^{-9})(0.040)^3 = 3.2 \times 10^{-11}$ C. By Gauss' law, we find $\Phi = q_{\text{enc}}/\varepsilon_0 = 3.6\ \text{N}\cdot\text{m}^2/\text{C}$.

(b) Now the sphere is totally contained within the cube (note that the radius of the sphere is less than half the side-length of the cube). Thus, the total charge is $q_{\text{enc}}\rho V_{\text{sphere}} = 4.5 \times 10^{-10}$ C. By Gauss' law, we find $\Phi = q_{\text{enc}}/\varepsilon_0 = 51\ \text{N}\cdot\text{m}^2/\text{C}$.

60. We use $\Phi = q_{\text{enclosed}}/\varepsilon_0$ and the fact that the amount of positive (negative) charges on the left (right) side of the conductor is $q\,(-q)$. Thus, $\Phi_1 = q/\varepsilon_0$, $\Phi_2 = -q/\varepsilon_0$, $\Phi_3 = q/\varepsilon_0$, $\Phi_4 = (q-q)/\varepsilon_0 = 0$, and $\Phi_5 = (q+q-q)/\varepsilon_0 = q/\varepsilon_0$.

61. (a) For $r < R$, $E = 0$ (see Eq. 24-16).

(b) For r slightly greater than R,

$$\begin{aligned} E_R &= \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \approx \frac{q}{4\pi\varepsilon_0 R^2} = \frac{(8.99\times10^9\,\mathrm{N\cdot m^2/C^2})(2.0\times10^{-7}\mathrm{C})}{(0.25\,\mathrm{m})^2} \\ &= 2.9\times10^4\ \mathrm{N/C}\ . \end{aligned}$$

(c) For $r > R$,

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = E_R\left(\frac{R}{r}\right)^2 = (2.9\times10^4\,\mathrm{N/C})\left(\frac{0.25\,\mathrm{m}}{3.0\,\mathrm{m}}\right)^2 = 200\ \mathrm{N/C}\ .$$

62. The field due to a sheet of charge is given by Eq. 24-13. Both sheets are horizontal (parallel to the xy plane), producing vertical fields (parallel to the z axis). At points above the $z = 0$ sheet (sheet A), its field points upward (towards $+z$); at points above the $z = 2.0$ sheet (sheet B), its field does likewise. However, below the $z = 2.0$ sheet, its field is oriented downward.

(a) The magnitude of the net field in the region between the sheets is

$$\left|\vec{E}\right| = \frac{\sigma_A}{2\varepsilon_0} - \frac{\sigma_B}{2\varepsilon_0} = 2.8\times10^2\ \mathrm{N/C}\ .$$

(b) The magnitude of the net field at points above both sheets is

$$\left|\vec{E}\right| = \frac{\sigma_A}{2\varepsilon_0} + \frac{\sigma_B}{2\varepsilon_0} = 6.2\times10^2\ \mathrm{N/C}\ .$$

63. To exploit the symmetry of the situation, we imagine a closed Gaussian surface in the shape of a cube, of edge length d, with the charge q situated at the inside center of the cube. The cube has six faces, and we expect an equal amount of flux through each face. The total amount of flux is $\Phi_{\text{net}} = q/\varepsilon_0$, and we conclude that the flux through the square is one-sixth of that. Thus, $\Phi = q/6\varepsilon_0$.

64. (a) At $x = 0.040$ m, the net field has a rightward ($+x$) contribution (computed using Eq. 24-13) from the charge lying between $x = -0.050$ m and $x = 0.040$ m, and a leftward ($-x$) contribution (again computed using Eq. 24-13) from the charge in the region from $x = 0.040$ m to $x = 0.050$ m. Thus, since $\sigma = q/A = \rho V/A = \rho\Delta x$ in this situation, we have

$$\left|\vec{E}\right| = \frac{\rho(0.090\ \mathrm{m})}{2\varepsilon_0} - \frac{\rho(0.010\ \mathrm{m})}{2\varepsilon_0} = 5.4\ \mathrm{N/C}\ .$$

(b) In this case, the field contributions from all layers of charge point rightward, and we obtain

$$\left|\vec{E}\right| = \frac{\rho(0.100\ \mathrm{m})}{2\varepsilon_0} = 6.8\ \mathrm{N/C}\ .$$

65. (a) The direction of the electric field at P_1 is away from q_1 and its magnitude is

$$\left|\vec{E}\right| = \frac{q}{4\pi\varepsilon_0 r_1^2} = \frac{(8.99\times10^9\,\mathrm{N\cdot m^2/C^2})(1.0\times10^{-7}\mathrm{C})}{(0.015\,\mathrm{m})^2} = 4.0\times10^6\,\mathrm{N/C}\ .$$

(b) $\vec{E} = 0$, since P_2 is inside the metal.

66. We use Eqs. 24-15, 24-16 and the superposition principle.

(a) $E = 0$ in the region inside the shell.

(b) $E = (1/4\pi\varepsilon_0)(q_a/r^2)$.

(c) $E = (1/4\pi\varepsilon_0)(q_a + q_b)/r^2$.

(d) Since $E = 0$ for $r < a$ the charge on the inner surface of the inner shell is always zero. The charge on the outer surface of the inner shell is therefore q_a. Since $E = 0$ inside the metallic outer shell the net charge enclosed in a Gaussian surface that lies in between the inner and outer surfaces of the outer shell is zero. Thus the inner surface of the outer shell must carry a charge $-q_a$, leaving the charge on the outer surface of the outer shell to be $q_b + q_a$.

67. (a) We use $m_e g = eE = e\sigma/\varepsilon_0$ to obtain the surface charge density.

$$\sigma = \frac{m_e g \varepsilon_0}{e} = \frac{(9.11\times10^{-31}\,\text{kg})\,(9.8\,\text{m/s}^2)\left(8.85\times10^{-12}\,\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)}{1.60\times10^{-19}\,\text{C}} = 4.9\times10^{-22}\ \text{C/m}^2\ .$$

(b) Downward (since the electric force exerted on the electron must be upward).

68. (a) In order to have net charge $-10\ \mu$C when $-14\ \mu$C is known to be on the outer surface, then there must be $+4\ \mu$C on the inner surface (since charges reside on the surfaces of a conductor in electrostatic situations).

(b) In order to cancel the electric field inside the conducting material, the contribution from the $+4\ \mu$C on the inner surface must be canceled by that of the charged particle in the hollow. Thus, the particle's charge is $-4\ \mu$C.