Chapter 10

1. If F_{avg} is the magnitude of the average force, then the magnitude of the impulse is $J = F_{\text{avg}}\Delta t$, where Δt is the time interval over which the force is exerted (see Eq. 10-8). This equals the magnitude of the change in the momentum of the ball. Since the ball is initially at rest, J is equal to the magnitude of the final momentum mv. When $F_{\text{avg}}\Delta t = mv$ is solved for the speed, the result is

$$v = \frac{F_{\text{avg}}\Delta t}{m} = \frac{(50\,\text{N})\left(10 \times 10^{-3}\,\text{s}\right)}{0.20\,\text{kg}} = 2.5 \text{ m/s} .$$

2. The magnitude of the average force is

$$\left|\vec{F}_{\text{avg}}\right| = \frac{|\Delta\vec{p}|}{m} = \frac{m|\Delta\vec{v}|}{\Delta t} = \frac{(2300\,\text{kg})(15\,\text{m/s})}{0.56\,\text{s}} = 6.2 \times 10^4 \text{ N} .$$

3. We take the final direction of motion to be the $+\hat{\imath}$ direction (when it is headed back to the pitcher) so that $\vec{v}_f = +60\,\hat{\imath}$ and $\vec{v}_i = -40\,\hat{\imath}$ in SI units. Therefore, $\Delta\vec{v} = 60 - (-40) = 100\,\hat{\imath}$ m/s. The magnitude of the average force is

$$\left|\vec{F}_{\text{avg}}\right| = \frac{|\Delta\vec{p}|}{\Delta t} = \frac{m|\Delta\vec{v}|}{\Delta t} = \frac{(0.150\,\text{kg})(100\,\text{m/s})}{5.0 \times 10^{-3}\,\text{s}} = 3.0 \times 10^3 \text{ N} .$$

4. We estimate his mass in the neighborhood of 70 kg and compute the upward force F of the water from Newton's second law.

$$F - mg = ma$$

where we have chosen $+y$ upward, so that $a > 0$ (the acceleration is upward since it represents a deceleration of his downward motion through the water). His speed when he arrives at the surface of the water is found either from Eq. 2-16 or from energy conservation:

$$v = \sqrt{2gh}$$

where $h = 12$ m, and since the deceleration a reduces the speed to zero over a distance $d = 0.30$ m we also obtain $v = \sqrt{2ad}$. We use these observations in the following.

(a) Equating our two expressions for v leads to $a = gh/d$. Our force equation, then, leads to

$$F = mg + m\left(g\,\frac{h}{d}\right) = mg\left(1 + \frac{h}{d}\right)$$

which yields $F \approx 2.8 \times 10^4$ kg. Since we are not at all certain of his mass, we express this as a guessed-at range (in kN) $25 < F < 30$.

(b) Since $F \gg mg$, the impulse $\vec{J}$ due to the net force (while he is in contact with the water) is overwhelmingly caused by the upward force of the water: $\int F\,dt = \vec{J}$ to a good approximation. Thus, by Eq. 10-2,

$$\int F\,dt = \vec{p}_f - \vec{p}_i = 0 - m\left(-\sqrt{2gh}\right)$$

(the minus sign with the initial velocity is due to the fact that downward is the negative direction) which yields $(70)\sqrt{2(9.8)(12)} = 1.1 \times 10^3$ kg·m/s. Expressing this as a range (in kN·s) we estimate $1.0 < \int F\,dt < 1.2$.

5. We take the initial direction of motion to be positive and use F_{avg} to denote the magnitude of the average force, Δt as the duration of the force, m as the mass of the ball, v_i as the initial velocity of the ball, and v_f as the final velocity of the ball. The force is in the negative direction and the impulse-momentum theorem (Eq. 10-4 with Eq. 10-8) yields $-F_{\text{avg}}\Delta t = mv_f - mv_i$. Thus,

$$v_f = \frac{mv_i - F_{\text{avg}}\Delta t}{m} = \frac{(0.40\,\text{kg})(14\,\text{m/s}) - (1200\,\text{N})(27 \times 10^{-3}\,\text{s})}{0.40\,\text{kg}} = -67\,\text{m/s}\ .$$

The final speed of the ball is 67 m/s. The negative sign indicates that the velocity is opposite to the initial direction of travel.

6. We choose $+y$ upward, which implies $a > 0$ (the acceleration is upward since it represents a deceleration of his downward motion through the snow).

(a) The maximum deceleration a_{max} of the paratrooper (of mass m and initial speed $v = 56$ m/s) is found from Newton's second law

$$F_{\text{snow}} - mg = ma_{\text{max}}$$

where we require $F_{\text{snow}} = 1.2 \times 10^5$ N. Using Eq. 2-15 $v^2 = 2a_{\text{max}}d$, we find the minimum depth of snow for the man to survive:

$$d = \frac{v^2}{2a_{\text{max}}} = \frac{mv^2}{2\,(F_{\text{snow}} - mg)} \approx \frac{(85\,\text{kg})(56\,\text{m/s})^2}{2(1.2 \times 10^5\,\text{N})} = 1.1\,\text{m}\ .$$

(b) His short trip through the snow involves a change in momentum

$$\vec{p}_f - \vec{p}_i = 0 - (85\,\text{kg})(-56\,\text{m/s})$$

(the negative value of the initial velocity is due to the fact that downward is the negative direction) which yields 4.8×10^3 kg·m/s. By the impulse-momentum theorem, this equals the impulse due to the net force $F_{\text{snow}} - mg$, but since $F_{\text{snow}} \gg mg$ we can approximate this as the impulse on him just from the snow.

7. We choose $+y$ upward, which means $\vec{v}_i = -25$ m/s and $\vec{v}_f = +10$ m/s. During the collision, we make the reasonable approximation that the net force on the ball is equal to F_{avg} – the average force exerted by the floor up on the ball.

(a) Using the impulse momentum theorem (Eq. 10-4) we find

$$\vec{J} = m\vec{v}_f - m\vec{v}_i = (1.2)(10) - (1.2)(-25) = 42\ \text{kg}\cdot\text{m/s}\ .$$

(b) From Eq. 10-8, we obtain

$$\vec{F}_{\text{avg}} = \frac{\vec{J}}{\Delta t} = \frac{42}{0.020} = 2.1 \times 10^3\ \text{N}\ .$$

8. We choose the positive direction in the direction of rebound so that $\vec{v}_f > 0$ and $\vec{v}_i < 0$. Since they have the same speed v, we write this as $\vec{v}_f = v$ and $\vec{v}_i = -v$. Therefore, the change in momentum for each bullet of mass m is $\Delta\vec{p} = m\Delta v = 2mv$. Consequently, the total change in momentum for the 100 bullets (each minute) $\Delta\vec{P} = 100\Delta\vec{p} = 200mv$. The average force is then

$$\vec{F}_{\text{avg}} = \frac{\Delta\vec{P}}{\Delta t} = \frac{(200)(3 \times 10^{-3}\,\text{kg})(500\,\text{m/s})}{(1\,\text{min})(60\,\text{s/min})} \approx 5\,\text{N}\ .$$

9. (a) The initial momentum of the car is $\vec{p}_i = m\vec{v}_i = (1400\,\text{kg})(5.3\,\text{m/s})\,\hat{\jmath} = (7400\,\text{kg}\cdot\text{m/s})\,\hat{\jmath}$ and the final momentum is $\vec{p}_f = (7400\,\text{kg}\cdot\text{m/s})\,\hat{\imath}$. The impulse on it equals the change in its momentum:

$$\vec{J} = \vec{p}_f - \vec{p}_i = (7400\,\text{N}\cdot\text{s})(\hat{\imath} - \hat{\jmath}) \ .$$

(b) The initial momentum of the car is $\vec{p}_i = (7400\,\text{kg}\cdot\text{m/s})\,\hat{\imath}$ and the final momentum is $\vec{p}_f = 0$. The impulse acting on it is

$$\vec{J} = \vec{p}_f - \vec{p}_i = -7400\,\hat{\imath}\ \ \text{N}\cdot\text{s} \ .$$

(c) The average force on the car is

$$\vec{F}_{\text{avg}} = \frac{\Delta\vec{p}}{\Delta t} = \frac{\vec{J}}{\Delta t} = \frac{(7400\,\text{kg}\cdot\text{m/s})(\hat{\imath} - \hat{\jmath})}{4.6\,\text{s}} = (1600\,\text{N})(\hat{\imath} - \hat{\jmath})$$

and its magnitude is $F_{\text{avg}} = (1600\,\text{N})\sqrt{2} = 2300\,\text{N}$.

(d) The average force is

$$\vec{F}_{\text{avg}} = \frac{\vec{J}}{\Delta t} = \frac{(-7400\,\text{kg}\cdot\text{m/s})\,\hat{\imath}}{350\times10^{-3}\,\text{s}} = (-2.1\times10^4\,\text{N})\,\hat{\imath}$$

and its magnitude is $F_{\text{avg}} = 2.1\times10^4\,\text{N}$.

(e) The average force is given above in unit vector notation. Its x and y components have equal magnitudes. The x component is positive and the y component is negative, so the force is 45° below the positive x axis.

10. We use coordinates with $+x$ rightward and $+y$ upward, with the usual conventions for measuring the angles (so that the initial angle becomes $180 + 35 = 215°$). Using SI units and magnitude-angle notation (efficient to work with when using a vector capable calculator), the change in momentum is

$$\vec{p}_f - \vec{p}_i = (3.0\ \angle\ 90°) - (3.6\ \angle\ 215°) = (5.9\ \angle\ 60°) \ .$$

This equals the impulse delivered to the ball (by the bat). Then, Eq. 10-8 leads to

$$F_{\text{avg}}\Delta t = 5.9 \quad\Longrightarrow\quad F_{\text{avg}} = \frac{5.9}{2.0\times10^{-3}} \ \approx\ 2.9\times10^3\ \text{N} \ .$$

We note that this force is very much larger than the weight of the ball, which justifies our (implicit) assumption that gravity played no significant role in the collision.

11. We take the magnitude of the force to be $F = At$, where A is a constant of proportionality. The condition that $F = 50\,\text{N}$ when $t = 4.0\,\text{s}$ leads to $A = (50\,\text{N})/(4.0\,\text{s}) = 12.5\,\text{N/s}$. The magnitude of the impulse exerted on the object is

$$J = \int_0^{4.0} F\,dt = \int_0^{4.0} At\,dt = \frac{1}{2}At^2\bigg|_0^{4.0} = \frac{1}{2}(12.5)(4.0)^2 = 100\ \text{N}\cdot\text{s} \ .$$

This equals the magnitude of the change in the momentum of the object (by the impulse-momentum theorem), and since the ball started from rest, we have $J = mv_f$. Therefore, $v_f = J/m = (100\,\text{N}\cdot\text{s})/(10\,\text{kg}) = 10\,\text{m/s}$.

12. (a) The mass of each spherical hailstone of radius $r = 0.5$ cm and density $\rho = 0.92$ g/cm^3 is

$$m = \rho\left(\frac{4\pi r^3}{3}\right) = 0.48\ \text{g} = 4.8\times10^{-4}\ \text{kg} \ .$$

(b) If the final speed is zero, then Eq. 10-4 and Eq. 10-8 (with $+y$ upward) lead to

$$\vec{F}_{\rm avg}\Delta t = -m\vec{v}_i = -\left(4.8\times10^{-4}\right)(-25) = 0.012$$

in SI units (N·s). This gives the impulse imparted to a single hailstone by the roof (and is equal to the magnitude of the force on the roof by the hailstone, by Newton's third law). An imagined "cube" of falling air, $\ell = 1$ m on each side (falling with the hail at $v = 25$ m/s), takes a time of

$$\Delta t = \frac{\ell}{v} = \frac{1\,\rm m}{25\,\rm m/s} = 0.04\ \rm s$$

to fully "collapse" onto a square meter of roof top (delivering its load of 120 hailstones). We can cover an area of $10\,\rm m \times 20\,\rm m$ with 200 of these "collapsing cubes" of air. Therefore, in this time, the total impulse is of magnitude

$$\vec{F}_{\rm avg,total}\Delta t = 200(120)(0.012\,\rm N{\cdot}s) \approx 290\,\rm N{\cdot}s$$

which leads to $\vec{F}_{\rm avg,total} = 290/0.04 = 7.2\times10^3$ N.

13. (a) If m is the mass of a pellet and v is its velocity as it hits the wall, then its momentum is $p = mv = (2.0\times10^{-3}\,\rm kg)(500\,\rm m/s) = 1.0\,\rm kg{\cdot}m/s$, toward the wall.

(b) The kinetic energy of a pellet is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}\left(2.0\times10^{-3}\,\rm kg\right)(500\,\rm m/s)^2 = 2.5\times10^2\ \rm J\ .$$

(c) The force on the wall is given by the rate at which momentum is transferred from the pellets to the wall. Since the pellets do not rebound, each pellet that hits transfers $p = 1.0\,\rm kg{\cdot}m/s$. If ΔN pellets hit in time Δt, then the average rate at which momentum is transferred is

$$F_{\rm avg} = \frac{p\,\Delta N}{\Delta t} = (1.0\,\rm kg{\cdot}m/s)\left(10\,\rm s^{-1}\right) = 10\ \rm N\ .$$

The force on the wall is in the direction of the initial velocity of the pellets.

(d) If Δt is the time interval for a pellet to be brought to rest by the wall, then the average force exerted on the wall by a pellet is

$$F_{\rm avg} = \frac{p}{\Delta t} = \frac{1.0\,\rm kg{\cdot}m/s}{0.6\times10^{-3}\,\rm s} = 1.7\times10^3\,\rm N\ .$$

The force is in the direction of the initial velocity of the pellet.

(e) In part (d) the force is averaged over the time a pellet is in contact with the wall, while in part (c) it is averaged over the time for many pellets to hit the wall. During the majority of this time, no pellet is in contact with the wall, so the average force in part (c) is much less than the average force in part (d).

14. We choose our positive direction in the direction of the rebound (so the ball's initial velocity is negative-valued). We evaluate the integral $J = \int F\,dt$ by adding the appropriate areas (of a triangle, a rectangle, and another triangle) shown in the graph (but with the t converted to seconds). With $m = 0.058\,\rm kg$ and $v = 34\,\rm m/s$, we apply the impulse-momentum theorem:

$$\begin{aligned}
\int F_{\rm wall}\,dt &= m\vec{v}_f - m\vec{v}_i \\
\int_0^{0.002} F\,dt + \int_{0.002}^{0.004} F\,dt + \int_{0.004}^{0.006} F\,dt &= m(+v) - m(-v) \\
\frac{1}{2}F_{\rm max}\,(0.002\,\rm s) + F_{\rm max}\,(0.002\,\rm s) + \frac{1}{2}F_{\rm max}\,(0.002\,\rm s) &= 2mv \\
F_{\rm max}\,(0.004\,\rm s) &= 2\,(0.058\,\rm kg)\,(34\,\rm m/s)
\end{aligned}$$

which yields $F_{\rm max} = 9.9\times10^2$ N.

15. We first consider the 1200 kg part. The impulse has magnitude J and is (by our choice of coordinates) in the positive direction. Let m_1 be the mass of the part and v_1 be its velocity after the bolts are exploded. We assume both parts are at rest before the explosion. Then $J = m_1 v_1$, so

$$v_1 = \frac{J}{m_1} = \frac{300\,\mathrm{N{\cdot}s}}{1200\,\mathrm{kg}} = 0.25 \text{ m/s} .$$

The impulse on the 1800 kg part has the same magnitude but is in the opposite direction, so $-J = m_2 v_2$, where m_2 is the mass and v_2 is the velocity of the part. Therefore,

$$v_2 = -\frac{J}{m_2} = -\frac{300\,\mathrm{N{\cdot}s}}{1800\,\mathrm{kg}} = -0.167 \text{ m/s} .$$

Consequently, the relative speed of the parts after the explosion is $0.25\,\mathrm{m/s} - (-0.167\,\mathrm{m/s}) = 0.417\,\mathrm{m/s}$.

16. We choose our positive direction in the direction of the rebound (so the ball's initial velocity is negative-valued: $\vec{v}_i = -5.2$ m/s).

(a) The speed of the ball right after the collision is

$$\begin{aligned} v_f &= \sqrt{\frac{2K_f}{m}} \\ &= \sqrt{\frac{2\left(\frac{1}{2}K_i\right)}{m}} \\ &= \sqrt{\frac{\frac{1}{2}mv_i^2}{m}} \\ &= \frac{v_i}{\sqrt{2}} \approx 3.7 \text{ m/s} . \end{aligned}$$

(b) With $m = 0.15$ kg, the impulse-momentum theorem (Eq. 10-4) yields

$$\vec{J} = m\vec{v}_f - m\vec{v}_i = (0.15)(3.7) - (0.15)(-5.2) = 1.3$$

in SI units (N·s).

(c) Eq. 10-8 leads to $F_{\text{avg}} = J/\Delta t = 1.3/0.0076 = 1.8 \times 10^2$ N.

17. We choose $+y$ in the direction of the rebound (directly away from the wall) and $+x$ towards the right in the figure (parallel to the wall; see Fig. 10-30). Using unit-vector notation, the the ball's initial and final velocities are

$$\begin{aligned} \vec{v}_i &= v\cos\theta\,\hat{\imath} - v\sin\theta\,\hat{\jmath} = 5.2\,\hat{\imath} - 3.0\,\hat{\jmath} \\ \vec{v}_f &= v\cos\theta\,\hat{\imath} + v\sin\theta\,\hat{\jmath} = 5.2\,\hat{\imath} + 3.0\,\hat{\jmath} \end{aligned}$$

respectively (with SI units understood).

(a) With $m = 0.30$ kg, the impulse-momentum theorem (Eq. 10.4) yields

$$\vec{J} = m\vec{v}_f - m\vec{v}_i = 2(0.30)(3.0\,\hat{\jmath})$$

so that the magnitude of the impulse delivered on the ball by the wall is 1.8 N·s and its direction is directly away from the wall (which, in terms of Fig. 10-30, is "up").

(b) Using Eq. 10-8, the force on the ball by the wall is $\vec{J}/\Delta t = 1.8\hat{\jmath}/0.010 = 180\,\hat{\jmath}$ N. By Newton's third law, the force on the wall by the ball is $-180\,\hat{\jmath}$ N (that is, its magnitude is 180 N and its direction is directly into the wall, or "down" in the view provided by Fig. 10-30).

18. (a) Regardless of the direction of the thrust, the change in linear momentum of the space probe is given by the impulse-momentum theorem (also using Eq. 10-8):

$$\Delta p = (3000\,\mathrm{N})(65.0\,\mathrm{s}) = 1.95 \times 10^5\,\mathrm{kg{\cdot}m/s}\ .$$

(b) The change in speed for the probe of mass m is

$$\Delta v = \frac{\Delta p}{m} = \frac{1.95 \times 10^5\,\mathrm{kg{\cdot}m/s}}{2500\,\mathrm{kg}} = 78.0\,\mathrm{m/s}\ .$$

Let the initial and final speeds of the probe be v_i and v_f, respectively. Then, the change in its kinetic energy is $\Delta K = \frac{1}{2}m(v_f^2 - v_i^2)$. If the thrust is backward then $v_f = v_i - \Delta v$, and

$$\begin{aligned}\Delta K &= \frac{1}{2}m\left((v_i - \Delta v)^2 - v_i^2\right)\\ &= \frac{1}{2}(2500\,\mathrm{kg})\left((300\,\mathrm{m/s} - 78.0\,\mathrm{m/s})^2 - (300\,\mathrm{m/s})^2\right)\\ &= -5.09 \times 10^7\ \mathrm{J}\end{aligned}$$

If the thrust is forward then $v_f = v_i + \Delta v$, and

$$\begin{aligned}\Delta K &= \frac{1}{2}m\left((v_i + \Delta v)^2 - v_i^2\right)\\ &= \frac{1}{2}(2500\,\mathrm{kg})\left((300\,\mathrm{m/s} + 78.0\,\mathrm{m/s})^2 - (300\,\mathrm{m/s})^2\right)\\ &= 6.61 \times 10^7\ \mathrm{J}\ .\end{aligned}$$

If the thrust is sideways then $v_f = \sqrt{(\Delta v)^2 + v_i^2}$, and

$$\Delta K = \frac{1}{2}m\left((\Delta v)^2 + v_i^2 - v_i^2\right) = \frac{1}{2}(2500\,\mathrm{kg})(78.0\,\mathrm{m/s})^2 = 7.61 \times 10^6\,\mathrm{J}\ .$$

19. (a) We take the force to be in the positive direction, at least for earlier times. Then the impulse is

$$\begin{aligned}J &= \int_0^{3.0\times10^{-3}} F\,dt\\ &= \int_0^{3.0\times10^{-3}} \left(6.0 \times 10^6\right)t - \left(2.0 \times 10^9\right)t^2\ dt\\ &= \left[\frac{1}{2}(6.0 \times 10^6)t^2 - \frac{1}{3}(2.0 \times 10^9)t^3\right]_0^{3.0\times10^{-3}}\\ &= 9.0\ \mathrm{N{\cdot}s}\ .\end{aligned}$$

(b) Since $J = F_{\text{avg}}\,\Delta t$, we find

$$F_{\text{avg}} = \frac{J}{\Delta t} = \frac{9.0\,\mathrm{N{\cdot}s}}{3.0 \times 10^{-3}\,\mathrm{s}} = 3.0 \times 10^3\ \mathrm{N}\ .$$

(c) To find the time at which the maximum force occurs, we set the derivative of F with respect to time equal to zero – and solve for t. The result is $t = 1.5 \times 10^{-3}$ s. At that time the force is

$$F_{\max} = (6.0 \times 10^6)(1.5 \times 10^{-3}) - (2.0 \times 10^9)(1.5 \times 10^{-3})^2 = 4.5 \times 10^3\,\mathrm{N}\ .$$

(d) Since it starts from rest, the ball acquires momentum equal to the impulse from the kick. Let m be the mass of the ball and v be its speed as it leaves the foot. Then,

$$v = \frac{p}{m} = \frac{J}{m} = \frac{9.0\,\mathrm{N}\cdot\mathrm{s}}{0.45\,\mathrm{kg}} = 20\ \mathrm{m/s}\ .$$

20. (a) We choose $+x$ along the initial direction of motion and apply momentum conservation:

$$\begin{aligned} m_{\text{bullet}}\vec{v}_i &= m_{\text{bullet}}\vec{v}_1 + m_{\text{block}}\vec{v}_2 \\ (5.2\,\text{g})(672\,\text{m/s}) &= (5.2\,\text{g})(428\,\text{m/s}) + (700\,\text{g})\vec{v}_2 \end{aligned}$$

which yields $v_2 = 1.81$ m/s.

(b) It is a consequence of momentum conservation that the velocity of the center of mass is unchanged by the collision. We choose to evaluate it before the collision:

$$\vec{v}_{\text{com}} = \frac{m_{\text{bullet}}\vec{v}_i}{m_{\text{bullet}} + m_{\text{block}}} = \frac{(5.2\,\text{g})(672\,\text{m/s})}{5.2\,\text{g} + 700\,\text{g}}$$

which gives the result $\vec{v}_{\text{com}} = 4.96$ m/s.

21. We examine the horizontal components of the momenta of the package and sled. Let m_s be the mass of the sled and v_s be its initial velocity. Let m_p be the mass of the package and let v be the final velocity of the sled and package together. The horizontal component of the total momentum is conserved, so $m_s v_s = (m_s + m_p)v$ and

$$v = \frac{v_s m_s}{m_s + m_p} = \frac{(9.0\,\text{m/s})(6.0\,\text{kg})}{6.0\,\text{kg} + 12\,\text{kg}} = 3.0 \text{ m/s} .$$

22. We refer to the discussion in the textbook (see Sample Problem 10-2, which uses the same notation that we use here) for many of the important details in the reasoning. Here we only present the primary computational step (using SI units):

$$v = \frac{m+M}{m}\sqrt{2gh} = \frac{2.010}{0.010}\sqrt{2(9.8)(0.12)} = 3.1 \times 10^2 \text{ m/s} .$$

23. Let m_m be the mass of the meteor and m_e be the mass of Earth. Let v_m be the velocity of the meteor just before the collision and let v be the velocity of Earth (with the meteor) just after the collision. The momentum of the Earth-meteor system is conserved during the collision. Thus, in the reference frame of Earth before the collision, $m_m v_m = (m_m + m_e)v$, so

$$v = \frac{v_m m_m}{m_m + m_e} = \frac{(7200\,\text{m/s})(5 \times 10^{10}\,\text{kg})}{5.98 \times 10^{24}\,\text{kg} + 5 \times 10^{10}\,\text{kg}} = 6 \times 10^{-11}\,\text{m/s} .$$

We convert this as follows:

$$\left(6 \times 10^{-11}\,\frac{\text{m}}{\text{s}}\right)\left(\frac{1000\,\text{mm}}{\text{m}}\right)\left(\frac{3.2 \times 10^7\,\text{s}}{\text{y}}\right) = 2 \text{ mm/y} .$$

24. (a) To relate the sliding distance to the speed V of the bullet-plus-block at the instant it has finished embedding itself in the block, we can either use Eq. 2-16 and $\vec{F}_{\text{net}} = m\vec{a}$, or energy conservation as expressed by Eq. 8-31 (with $W = 0$ and $f_k = \mu_k(m+M)g$ using Eq. 6-2). We choose the latter approach:

$$\begin{aligned} K_{\text{bullet plus block}} &= \Delta E_{\text{th}} \\ \frac{1}{2}(m+M)V^2 &= \mu_k(m+M)gd \end{aligned}$$

which yields $V = \sqrt{2\mu_k\,gd} = 2.7$ m/s.

(b) For the collision itself, we use momentum conservation (with the direction of motion being positive).

$$\begin{aligned} m_{\text{bullet}}v_i &= (m+M)V \\ (0.0045\,\text{kg})v_i &= (2.4045\,\text{kg})(2.7\text{m/s}) \end{aligned}$$

which gives the result $v_i = 1.4 \times 10^3$ m/s.

25. (a) The magnitude of the deceleration of each of the cars is $a = f/m = \mu_k mg/m = \mu_k g$. If a car stops in distance d, then its speed v just after impact is obtained from Eq. 2-16:

$$v^2 = v_0^2 + 2ad \quad \Longrightarrow \quad v = \sqrt{2ad} = \sqrt{2\mu_k gd}$$

since $v_0 = 0$ (this could alternatively have been derived using Eq. 8-31). Thus,

$$v_A = \sqrt{2(0.13)(9.8)(8.2)} = 4.6 \text{ m/s} \quad , \quad \text{and}$$

(b) $v_B = \sqrt{2(0.13)(9.8)(6.1)} = 3.9$ m/s.

(c) Let the speed of car B be v just before the impact. Conservation of linear momentum gives $m_B v = m_A v_A + m_B v_B$, or

$$v = \frac{(m_A v_A + m_B v_B)}{m_B} = \frac{(1100)(4.6) + (1400)(3.9)}{1400} = 7.5 \text{ m/s} .$$

The conservation of linear momentum during the impact depends on the fact that the only significant force (during impact of duration Δt) is the force of contact between the bodies. In this case, that implies that the force of friction exerted by the road on the cars is neglected during the brief Δt. This neglect would introduce some error in the analysis. Related to this is the assumption we are making that the transfer of momentum occurs at one location – that the cars do not slide appreciably during Δt – which is certainly an approximation (though probably a good one). Another source of error is the application of the friction relation Eq. 6-2 for the sliding portion of the problem (after the impact); friction is a complex force that Eq. 6-2 only partially describes.

26. We note that the "(a)" and "(b)" in Fig. 10-32 do not correspond to parts (a) and (b) (in fact, it's somewhat the reverse). Our $+x$ direction is to the right (so all velocities are positive-valued).

(a) We apply momentum conservation to relate the situation just before the bullet strikes the second block to the situation where the bullet is embedded within the block.

$$(0.0035\,\text{kg})v = (1.8035\,\text{kg})(1.4\,\text{m/s}) \quad \Longrightarrow \quad v = 721 \text{ m/s} .$$

(b) We apply momentum conservation to relate the situation just before the bullet strikes the first block to the instant it has passed through it (having speed v found in part (a)).

$$(0.0035\,\text{kg})v_0 = (1.2\,\text{kg})(0.63\,\text{m/s}) + (0.0035\,\text{kg})(721\,\text{m/s})$$

which yields $v_0 = 937$ m/s.

27. (a) We want to calculate the force that the scale exerts on the marbles. This is the sum of the force that holds the marbles already on the scale against the downward force of gravity and the force that brings the falling marbles to rest. At the end of time t, the number of marbles on the scale is Rt. At this moment, the gravitational force on them is $Rtmg$ and the upward force of the scale that holds them is $F_1 = Rtmg$. Just before striking the scale, a marble that fell from height h has speed $v = \sqrt{2gh}$ and momentum $p = m\sqrt{2gh}$. To stop the falling marbles, the scale must exert an upward force $F_2 = Rp = Rm\sqrt{2gh}$. The total force of the scale on the marbles is

$$F = F_1 + F_2 = Rtmg + Rm\sqrt{2gh} = Rm\left(gt + \sqrt{2gh}\right) .$$

(b) For the given data (using SI units, so $m = 0.0045$ kg), we find

$$F = (100)(0.0045)\left((9.8)(10.0) + \sqrt{2(9.8)(7.60)}\right)$$

which yields $F = 49.6$ N. Assuming the scale is calibrated to read in terms of an equivalent mass, its reading is $F/g = 49.6/9.8 = 5.06$ kg.

28. We choose $+x$ in the direction of (initial) motion of the blocks, which have masses $m_1 = 5$ kg and $m_2 = 10$ kg. Where units are not shown in the following, SI units are to be understood.

(a) Momentum conservation leads to

$$\begin{aligned} m_1\vec{v}_{1i} + m_2\vec{v}_{2i} &= m_1\vec{v}_{1f} + m_2\vec{v}_{2f} \\ (5)(3) + (10)(2) &= 5\vec{v}_{1f} + (10)(2.5) \end{aligned}$$

which yields $\vec{v}_{1f} = 2$. Thus, the speed of the 5 kg block immediately after the collision is 2.0 m/s.

(b) We find the reduction in total kinetic energy:

$$K_i - K_f = \frac{1}{2}(5)(3)^2 + \frac{1}{2}(10)(2)^2 - \frac{1}{2}(5)(2)^2 - \frac{1}{2}(10)(2.5)^2$$

which gives the result 1.25 J. Rounding to two figures and recalling that $\Delta K = K_f - K_i$ then our answer is $\Delta K = -1.3$ J.

(c) In this new scenario where $\vec{v}_{2f} = 4.0$ m/s, momentum conservation leads to $\vec{v}_{1f} = -1.0$ m/s and we obtain $\Delta K = +40$ J.

(d) The creation of additional kinetic energy is possible if, say, some gunpowder were on the surface where the impact occurred (initially stored chemical energy would then be contributing to the result).

29. Let m_F be the mass of the freight car and v_F be its initial velocity. Let m_C be the mass of the caboose and v be the common final velocity of the two when they are coupled. Conservation of the total momentum of the two-car system leads to $m_F v_F = (m_F + m_C)v$, so $v = v_F m_F/(m_F + m_C)$. The initial kinetic energy of the system is

$$K_i = \frac{1}{2}m_F v_F^2$$

and the final kinetic energy is

$$K_f = \frac{1}{2}(m_F + m_C)v^2 = \frac{1}{2}(m_F + m_C)\frac{m_F^2 v_F^2}{(m_F + m_C)^2} = \frac{1}{2}\frac{m_F^2 v_F^2}{(m_F + m_C)} .$$

Since 27% of the original kinetic energy is lost, we have $K_f = 0.73K_i$. Thus,

$$\frac{1}{2}\frac{m_F^2 v_F^2}{(m_F + m_C)} = (0.73)\left(\frac{1}{2}m_F v_F^2\right) .$$

Simplifying, we obtain $m_F/(m_F + m_C) = 0.73$, which we use in solving for the mass of the caboose:

$$m_C = \frac{0.27}{0.73}\,m_F = 0.37\,m_F = (0.37)\left(3.18 \times 10^4\ \text{kg}\right) = 1.18 \times 10^4\ \text{kg} .$$

30. We think of this as having two parts: the first is the collision itself – where the bullet passes through the block so quickly that the block has not had time to move through any distance yet – and then the subsequent "leap" of the block into the air (up to height h measured from its initial position). The first part involves momentum conservation (with $+y$ upward):

$$(0.01\,\text{kg})(1000\,\text{m/s}) = (5.0\,\text{kg})\vec{v} + (0.01\,\text{kg})(400\,\text{m/s})$$

which yields $\vec{v} = 1.2$ m/s. The second part involves either the free-fall equations from Ch. 2 (since we are ignoring air friction) or simple energy conservation from Ch. 8. Choosing the latter approach, we have

$$\frac{1}{2}(5.0\,\text{kg})(1.2\,\text{m/s})^2 = (5.0\,\text{kg})\left(9.8\,\text{m/s}^2\right)h$$

which gives the result $h = 0.073$ m.

31. (a) Let v be the final velocity of the ball-gun system. Since the total momentum of the system is conserved $mv_i = (m+M)v$. Therefore, $v = mv_i/(m+M)$.

(b) The initial kinetic energy is $K_i = \frac{1}{2}mv_i^2$ and the final kinetic energy is $K_f = \frac{1}{2}(m+M)v^2 = \frac{1}{2}m^2v_i^2/(m+M)$. The problem indicates $\Delta E_{\text{th}} = 0$, so the difference $K_i - K_f$ must equal the energy U_s stored in the spring:

$$U_s = \frac{1}{2}mv_i^2 - \frac{1}{2}\frac{m^2v_i^2}{(m+M)} = \frac{1}{2}mv_i^2\left(1 - \frac{m}{m+M}\right) = \frac{1}{2}mv_i^2\frac{M}{m+M} \ .$$

Consequently, the fraction of the initial kinetic energy that becomes stored in the spring is $U_s/K_i = M/(m+M)$.

32. For a picture of this one-dimensional example of an "explosion" involving two objects ($m_1 = 4.0$ kg and $m_2 = 6.0$ kg), see Fig. 9-40 (but reverse the velocity arrows). Since the system was initially at rest, momentum conservation leads to

$$0 = m_2\vec{v}_2 + m_1\vec{v}_1 \quad \Longrightarrow \quad |\vec{v}_1| = \frac{m_2}{m_1}\,|\vec{v}_2|$$

which yields 6.0 m/s for the speed of the physics book. Mechanical energy conservation tells us that the initial potential energy is

$$U_i = K_{f\,\text{total}} = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

which gives the result $U_i = 120$ J.

33. As hinted in the problem statement, the velocity v of the system as a whole – when the spring reaches the maximum compression x_{m} – satisfies $m_1v_{1i} + m_2v_{2i} = (m_1+m_2)v$. The change in kinetic energy of the system is therefore

$$\begin{aligned}\Delta K &= \frac{1}{2}(m_1+m_2)v^2 - \frac{1}{2}m_1v_{1i}^2 - \frac{1}{2}m_2v_{2i}^2 \\ &= \frac{(m_1v_{1i}+m_2v_{2i})^2}{2\,(m_1+m_2)} - \frac{1}{2}m_1v_{1i}^2 - \frac{1}{2}m_2v_{2i}^2\end{aligned}$$

which yields $\Delta K = -35$ J. (Although it is not necessary to do so, still it is worth noting that algebraic manipulation of the above expression leads to $|\Delta K| = \frac{1}{2}\left(\frac{m_1m_2}{m_1+m_2}\right)v_{\text{rel}}^2$ where $v_{\text{rel}} = v_1 - v_2$). Conservation of energy then requires

$$\frac{1}{2}kx_{\text{m}}^2 = -\Delta K \quad \Longrightarrow \quad x_{\text{m}} = \sqrt{\frac{-2\Delta K}{k}} = \sqrt{\frac{-2(-35)}{1120}}$$

which gives the result $x_{\text{m}} = 0.25$ m.

34. We think of this as having two parts: the first is the collision itself – where the blocks "join" so quickly that the 1.0-kg block has not had time to move through any distance yet – and then the subsequent motion of the 3.0 kg system as it compresses the spring to the maximum amount x_{m}. The first part involves momentum conservation (with $+x$ rightward):

$$(2.0\,\text{kg})(4.0\,\text{m/s}) = (3.0\,\text{kg})\vec{v}$$

which yields $\vec{v} = 2.7$ m/s. The second part involves mechanical energy conservation:

$$\frac{1}{2}(3.0\,\text{kg})(2.7\,\text{m/s})^2 = \frac{1}{2}(200\,\text{N/m})x_{\text{m}}^2$$

which gives the result $x_{\text{m}} = 0.33$ m.

35. (a) Let m_1 be the mass of the block on the left, v_{1i} be its initial velocity, and v_{1f} be its final velocity. Let m_2 be the mass of the block on the right, v_{2i} be its initial velocity, and v_{2f} be its final velocity. The momentum of the two-block system is conserved, so $m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$ and

$$v_{1f} = \frac{m_1v_{1i} + m_2v_{2i} - m_2v_{2f}}{m_1} = \frac{(1.6)(5.5) + (2.4)(2.5) - (2.4)(4.9)}{1.6}$$

which yields $v_{1f} = 1.9$ m/s. The block continues going to the right after the collision.

(b) To see if the collision is elastic, we compare the total kinetic energy before the collision with the total kinetic energy after the collision. The total kinetic energy before is

$$K_i = \frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_2v_{2i}^2 = \frac{1}{2}(1.6)(5.5)^2 + \frac{1}{2}(2.4)(2.5)^2 = 31.7 \text{ J} .$$

The total kinetic energy after is

$$K_f = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2 = \frac{1}{2}(1.6)(1.9)^2 + \frac{1}{2}(2.4)(4.9)^2 = 31.7 \text{ J} .$$

Since $K_i = K_f$ the collision is found to be elastic.

(c) Now $v_{2i} = -2.5\,\text{m/s}$ and

$$v_{1f} = \frac{m_1v_{1i} + m_2v_{2i} - m_2v_{2f}}{m_1} = \frac{(1.6)(5.5) + (2.4)(-2.5) - (2.4)(4.9)}{1.6}$$

which yields $v_{1f} = -5.6$ m/s. Thus, the velocity is opposite to the direction shown in Fig. 10-37.

36. We use m_1 for the mass of the electron and $m_2 = 1840m_1$ for the mass of the hydrogen atom. Using Eq. 10-31,

$$v_{2f} = \frac{2m_1}{m_1 + 1840m_1}\, v_{1i} = \frac{2}{1841}\, v_{1i}$$

we compute the final kinetic energy of the hydrogen atom:

$$K_{2f} = \frac{1}{2}(1840m_1)\left(\frac{2\,v_{1i}}{1841}\right)^2 = \frac{(1840)(4)}{1841^2}\left(\frac{1}{2}(1840m_1)v_{1i}^2\right)$$

so we find the fraction to be $(1840)(4)/1841^2 \approx 2.2 \times 10^{-3}$, or 0.22%.

37. (a) Let m_1 be the mass of the cart that is originally moving, v_{1i} be its velocity before the collision, and v_{1f} be its velocity after the collision. Let m_2 be the mass of the cart that is originally at rest and v_{2f} be its velocity after the collision. Then, according to Eq. 10-30,

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2}v_{1i} .$$

Using SI units (so $m_1 = 0.34$ kg), we obtain

$$m_2 = \frac{v_{1i} - v_{1f}}{v_{1i} + v_{1f}}m_1 = \left(\frac{1.2 - 0.66}{1.2 + 0.66}\right)(0.34) = 0.099 \text{ kg} .$$

(b) The velocity of the second cart is given by Eq. 10-31:

$$v_{2f} = \frac{2m_1}{m_1 + m_2}v_{1i} = \left(\frac{2(0.34)}{0.34 + 0.099}\right)(1.2) = 1.9 \text{ m/s} .$$

(c) The speed of the center of mass is

$$v_{\text{com}} = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2} = \frac{(0.34)(1.2) + 0}{0.34 + 0.099} = 0.93 \text{ m/s} .$$

Values for the initial velocities were used but the same result is obtained if values for the final velocities are used.

38. No mechanical energy is "lost" in this encounter, so we analyze it with the elastic collision equations, particularly Eq. 10-38. Thus,

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i} \approx -v_{1i} + 2v_{2i}$$

where we have made the (certainly reasonable) approximation that $m_2 \gg m_1$ and simplified accordingly. Thus, $v_{1f} = -12 + 2(-13) = -38$, resulting in a final speed (relative to the Sun) of 38 km/s.

39. We use $m_1 = 4$ u for the mass of the alpha particle and $m_2 = 197$ u for the mass of the gold nucleus in Eq. 10-31:

$$v_{2f} = \frac{2(4)}{4 + 197} v_{1i} = \frac{8}{201} v_{1i}$$

we compute the final kinetic energy of the gold nucleus (which must be the same as the kinetic energy lost by the alpha particle – since this is an elastic collision)

$$K_{2f} = \frac{1}{2} m_2 v_{2f}^2 = \frac{1}{2}(197\,\text{u}) \left(\frac{8\,v_{1i}}{201}\right)^2 .$$

We divide this by the initial alpha particle energy $K_i = \frac{1}{2}(4\,\text{u})v_{1i}^2$ to obtain

$$\frac{K_{2f}}{K_i} = \frac{(197)(8)^2}{(4)(201)^2} \approx 0.078$$

so we find the percentage is 7.8%.

40. First, we find the speed v of the ball of mass m_1 right before the collision (just as it reaches its lowest point of swing). Mechanical energy conservation (with $h = 0.700$ m) leads to

$$m_1 g h = \frac{1}{2} m_1 v^2 \implies v = \sqrt{2gh} = 3.7 \text{ m/s} .$$

(a) We now treat the elastic collision (with SI units) using Eq. 10-30:

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v = \frac{0.5 - 2.5}{0.5 + 2.5}(3.7) = -2.47$$

which means the final speed of the ball is 2.47 m/s.

(b) Finally, we use Eq. 10-31 to find the final speed of the block:

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v = \frac{2(0.5)}{0.5 + 2.5}(3.7) = 1.23 \text{ m/s} .$$

41. (a) Let m_1 be the mass of the body that is originally moving, v_{1i} be its velocity before the collision, and v_{1f} be its velocity after the collision. Let m_2 be the mass of the body that is originally at rest and v_{2f} be its velocity after the collision. Then, according to Eq. 10–30,

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} .$$

We solve for m_2 to obtain

$$m_2 = \frac{v_{1i} - v_{1f}}{v_{1f} + v_{1i}} m_1 .$$

We combine this with $v_{1f} = v_{1i}/4$ to obtain $m_2 = 3m_1/5 = 3(2.0)/5 = 1.2\,\text{kg}$.

(b) The speed of the center of mass is

$$v_{\text{com}} = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2} = \frac{(2.0)(4.0)}{2.0 + 1.2} = 2.5 \text{ m/s} .$$

42. We refer to the discussion in the textbook (Sample Problem 10-4, which uses the same notation that we use here) for some important details in the reasoning. We choose rightward in Fig. 10-15 as our $+x$ direction. We use the notation $\vec{v}$ when we refer to velocities and v when we refer to speeds (which are necessarily positive). Since the algebra is fairly involved, we find it convenient to introduce the notation $\Delta m = m_2 - m_1$ (which, we note for later reference, is a positive-valued quantity).

(a) Since $\vec{v}_{1i} = +\sqrt{2gh_1}$ where $h_1 = 9.0$ cm, we have

$$\vec{v}_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} = -\frac{\Delta m}{m_1 + m_2} \sqrt{2gh_1}$$

which is to say that the *speed* of sphere 1 immediately after the collision is $v_{1f} = (\Delta m/(m_1 + m_2))\sqrt{2gh_1}$ and that $\vec{v}_{1f}$ points in the $-x$ direction. This leads (by energy conservation $m_1 g h_{1f} = \frac{1}{2} m_1 v_{1f}^2$) to

$$h_{1f} = \frac{v_{1f}^2}{2g} = \left(\frac{\Delta m}{m_1 + m_2}\right)^2 h_1 .$$

With $m_1 = 50$ g and $m_2 = 85$ g, this becomes $h_{1f} \approx 0.6$ cm.

(b) Eq. 10-31 gives

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} = \frac{2m_1}{m_1 + m_2} \sqrt{2gh_1}$$

which leads (by energy conservation $m_2 g h_{2f} = \frac{1}{2} m_2 v_{2f}^2$) to

$$h_{2f} = \frac{v_{2f}^2}{2g} = \left(\frac{2m_1}{m_1 + m_2}\right)^2 h_1 .$$

With $m_1 = 50$ g and $m_2 = 85$ g, this becomes $h_{2f} \approx 4.9$ cm.

(c) Fortunately, they hit again at the lowest point (as long as their amplitude of swing was "small" – this is further discussed in Chapter 16). At the risk of using cumbersome notation, we refer to the *next* set of heights as h_{1ff} and h_{2ff}. At the lowest point (before this second collision) sphere 1 has velocity $+\sqrt{2gh_{1f}}$ (rightward in Fig. 10-15) and sphere 2 has velocity $-\sqrt{2gh_{1f}}$ (that is, it points in the $-x$ direction). Thus, the velocity of sphere 1 immediately after the second collision is, using Eq. 10-38,

$$\begin{aligned}
\vec{v}_{1ff} &= \frac{m_1 - m_2}{m_1 + m_2} \sqrt{2gh_{1f}} + \frac{2m_2}{m_1 + m_2} \left(-\sqrt{2gh_{2f}}\right) \\
&= \frac{-\Delta m}{m_1 + m_2} \left(\frac{\Delta m}{m_1 + m_2} \sqrt{2gh_1}\right) - \frac{2m_2}{m_1 + m_2} \left(\frac{2m_1}{m_1 + m_2} \sqrt{2gh_1}\right) \\
&= -\frac{(\Delta m)^2 + 4m_1 m_2}{(m_1 + m_2)^2} \sqrt{2gh_1} .
\end{aligned}$$

This can be greatly simplified (by expanding $(\Delta m)^2$ and $(m_1 + m_2)^2$) to arrive at the conclusion that the speed of sphere 1 immediately after the second collision is simply $v_{1ff} = \sqrt{2gh_1}$ and that $\vec{v}_{1ff}$ points in the $-x$ direction. Energy conservation ($m_1 g h_{1ff} = \frac{1}{2} m_1 v_{1ff}^2$) leads to

$$h_{1ff} = \frac{v_{1ff}^2}{2g} = h_1 = 9.0 \text{ cm} .$$

(d) One can reason (energy-wise) that $h_{1\,ff} = 0$ simply based on what we found in part (c). Still, it might be useful to see how this shakes out of the algebra. Eq. 10-39 gives the velocity of sphere 2 immediately after the second collision:

$$\begin{aligned} v_{2\,ff} &= \frac{2m_1}{m_1+m_2}\sqrt{2gh_{1\,f}} + \frac{m_2-m_1}{m_1+m_2}\left(-\sqrt{2gh_{2\,f}}\right) \\ &= \frac{2m_1}{m_1+m_2}\left(\frac{\Delta m}{m_1+m_2}\sqrt{2gh_1}\right) + \frac{\Delta m}{m_1+m_2}\left(\frac{-2m_1}{m_1+m_2}\sqrt{2gh_1}\right) \end{aligned}$$

which vanishes since $(2m_1)(\Delta m) - (\Delta m)(2m_1) = 0$. Thus, the second sphere (after the second collision) stays at the lowest point, which basically recreates the conditions at the start of the problem (so all subsequent swings-and-impacts, neglecting friction, can be easily predicted – as they are just replays of the first two collisions).

43. (a) Let m_1 be the mass of one sphere, v_{1i} be its velocity before the collision, and v_{1f} be its velocity after the collision. Let m_2 be the mass of the other sphere, v_{2i} be its velocity before the collision, and v_{2f} be its velocity after the collision. Then, according to Eq. 10–38,

$$v_{1f} = \frac{m_1-m_2}{m_1+m_2}v_{1i} + \frac{2m_2}{m_1+m_2}v_{2i}\,.$$

Suppose sphere 1 is originally traveling in the positive direction and is at rest after the collision. Sphere 2 is originally traveling in the negative direction. Replace v_{1i} with v, v_{2i} with $-v$, and v_{1f} with zero to obtain $0 = m_1 - 3m_2$. Thus $m_2 = m_1/3 = (300\,\text{g})/3 = 100\,\text{g}$.

(b) We use the velocities before the collision to compute the velocity of the center of mass:

$$v_{\text{com}} = \frac{m_1v_{1i} + m_2v_{2i}}{m_1+m_2} = \frac{(300\,\text{g})(2.0\,\text{m/s}) + (100\,\text{g})(-2.0\,\text{m/s})}{300\,\text{g} + 100\,\text{g}}$$

which yields $v_{\text{com}} = 1.0$ m/s.

44. The velocities of m_1 and m_2 just after the collision with each other are given by Eq. 10-38 and Eq. 10-39 (setting $v_{1i} = 0$).

$$\begin{aligned} v_{1f} &= \frac{2m_2}{m_1+m_2}\,v_{2i} \\ v_{2f} &= \frac{m_2-m_1}{m_1+m_2}\,v_{2i} \end{aligned}$$

After bouncing off the wall, the velocity of m_2 becomes $-v_{2f}$ (see *a massive target* in §10-5). In these terms, the problem requires

$$\begin{aligned} v_{1f} &= -v_{2f} \\ \frac{2m_2}{m_1+m_2}\,v_{2i} &= -\frac{m_2-m_1}{m_1+m_2}\,v_{2i} \end{aligned}$$

which simplifies to

$$2m_2 = -\left(m_2 - m_1\right) \implies m_2 = \frac{m_1}{3}\,.$$

45. (a) We use conservation of mechanical energy to find the speed of either ball after it has fallen a distance h. The initial kinetic energy is zero, the initial gravitational potential energy is Mgh, the final kinetic energy is $\frac{1}{2}Mv^2$, and the final potential energy is zero. Thus $Mgh = \frac{1}{2}Mv^2$ and $v = \sqrt{2gh}$. The collision of the ball of M with the floor is an elastic collision of a light object with a stationary massive object. The velocity of the light object reverses direction without change in magnitude. After the collision, the ball is traveling upward with a speed of $\sqrt{2gh}$. The ball of

mass m is traveling downward with the same speed. We use Eq. 10-38 to find an expression for the velocity of the ball of mass M after the collision:

$$\begin{aligned} v_{Mf} &= \frac{M-m}{M+m}\,v_{Mi} + \frac{2m}{M+m}\,v_{mi} \\ &= \frac{M-m}{M+m}\sqrt{2gh} - \frac{2m}{M+m}\sqrt{2gh} \\ &= \frac{M-3m}{M+m}\sqrt{2gh}\ . \end{aligned}$$

For this to be zero, $M = 3m$.

(b) We use the same equation to find the velocity of the ball of mass m after the collision:

$$v_{mf} = -\frac{m-M}{M+m}\sqrt{2gh} + \frac{2M}{M+m}\sqrt{2gh} = \frac{3M-m}{M+m}\sqrt{2gh}$$

which becomes (upon substituting $M = 3m$) $v_{mf} = 2\sqrt{2gh}$. We next use conservation of mechanical energy to find the height h' to which the ball rises. The initial kinetic energy is $\frac{1}{2}mv_{mf}^2$, the initial potential energy is zero, the final kinetic energy is zero, and the final potential energy is mgh'. Thus

$$\frac{1}{2}mv_{mf}^2 = mgh' \quad\Longrightarrow\quad h' = \frac{v_{mf}^2}{2g} = 4h$$

where $2\sqrt{2gh}$ is substituted for v_{mf}.

46. (a) Conservation of linear momentum implies $m_A\vec{v}_A + m_B\vec{v}_B = m_A\vec{v}\,'_A + m_B\vec{v}\,'_B$. Since $m_A = m_B = m = 2.0\,\mathrm{kg}$, the masses divide out and we obtain (in m/s)

$$\begin{aligned} \vec{v}\,'_B &= \vec{v}_A + \vec{v}_B - \vec{v}\,'_A \\ &= (15\hat{\imath} + 30\hat{\jmath}) + (-10\hat{\imath} + 5\hat{\jmath}) - (-5\hat{\imath} + 20\hat{\jmath}) \\ &= 10\hat{\imath} + 15\hat{\jmath}\ . \end{aligned}$$

(b) The final and initial kinetic energies are

$$\begin{aligned} K_f &= \frac{1}{2}mv_A'^2 + \frac{1}{2}mv_B'^2 = \frac{1}{2}(2.0)\left((-5)^2 + 20^2 + 10^2 + 15^2\right) = 8.0\times10^2\ \mathrm{J} \\ K_i &= \frac{1}{2}mv_A^2 + \frac{1}{2}mv_B^2 = \frac{1}{2}(2.0)\left(15^2 + 30^2 + (-10)^2 + 5^2\right) = 1.3\times10^3\ \mathrm{J}\ . \end{aligned}$$

The change kinetic energy is then $\Delta K = -5.0\times10^2$ J (that is, 500 J of the initial kinetic energy is lost).

47. We orient our $+x$ axis along the initial direction of motion, and specify angles in the "standard" way – so $\theta = +64°$ for the alpha (α) particle (after collision) and $\phi = -51°$ for the oxygen nucleus (o) (which is going into the fourth quadrant, in our scenario). We apply the conservation of linear momentum to the x and y axes respectively.

$$\begin{aligned} m_\alpha v_\alpha &= m_\alpha v'_\alpha\cos\theta + m_o v'_o\cos\phi \\ 0 &= m_\alpha v'_\alpha\sin\theta + m_o v'_o\sin\phi \end{aligned}$$

We are given $v'_o = 1.2\times10^5$ m/s, which leaves us two unknowns and two equations, which is sufficient for solving.

(a) We solve for the final alpha particle speed using the y-momentum equation:

$$v'_\alpha = -\frac{m_\alpha v'_\alpha\sin\theta}{m_o\sin\phi} = -\frac{(16)\left(1.2\times10^5\right)\sin(-51°)}{(4)\sin(64°)}$$

which yields $v'_\alpha = 4.15\times10^5$ m/s.

(b) Plugging our result from part (a) into the x-momentum equation produces the initial alpha particle speed:

$$\begin{aligned} m_\alpha v_\alpha &= \frac{m_\alpha v'_\alpha \cos\theta + m_o v'_o \cos\phi}{m_\alpha} \\ &= \frac{(4)\left(4.15 \times 10^5\right)\cos(64^\circ) + (16)\left(1.2 \times 10^5\right)\cos(-51^\circ)}{4} \\ &= 4.84 \times 10^5 \text{ m/s} . \end{aligned}$$

48. We orient our $+x$ axis along the initial direction of motion, and specify angles in the "standard" way – so $\theta = +60^\circ$ for the proton (1) which is assumed to scatter into the first quadrant and $\phi = -30^\circ$ for the target proton (2) which scatters into the fourth quadrant (recall that the problem has told us that this is perpendicular to θ). We apply the conservation of linear momentum to the x and y axes respectively.

$$\begin{aligned} m_1 v_1 &= m_1 v'_1 \cos\theta + m_2 v'_2 \cos\phi \\ 0 &= m_1 v'_1 \sin\theta + m_2 v'_2 \sin\phi \end{aligned}$$

We are given $v_1 = 500$ m/s, which provides us with two unknowns and two equations, which is sufficient for solving. Since $m_1 = m_2$ we can cancel the mass out of the equations entirely.

(a) Combining the above equations and solving for v'_2 we obtain

$$v'_2 = \frac{v_1 \sin(\theta)}{\sin(\theta - \phi)} = \frac{500\sin(60^\circ)}{\sin(90^\circ)} = 433$$

in SI units (m/s). We used the identity $\sin(\theta)\cos(\phi) - \cos(\theta)\sin(\phi) = \sin(\theta - \phi)$ in simplifying our final expression.

(b) In a similar manner, we find

$$v'_1 = \frac{v_1 \sin(\phi)}{\sin(\phi - \theta)} = \frac{500\sin(-30^\circ)}{\sin(-90^\circ)} = 250 \text{ m/s} .$$

49. (a) We use Fig. 10-16 of the text (which treats both angles are positive-valued, even though one of them is in the fourth quadrant; this is why there is an explicit minus sign in Eq. 10-43 as opposed to it being implicitly in the angle). We take the cue ball to be body 1 and the other ball to be body 2. Conservation of the x component of the total momentum of the two-ball system leads to $mv_{1i} = mv_{1f}\cos\theta_1 + mv_{2f}\cos\theta_2$ and conservation of the y component leads to $0 = -mv_{1f}\sin\theta_1 + mv_{2f}\sin\theta_2$. The masses are the same and cancel from the equations. We solve the second equation for $\sin\theta_2$:

$$\sin\theta_2 = \frac{v_{1f}}{v_{2f}}\sin\theta_1 = \left(\frac{3.50 \text{ m/s}}{2.00 \text{ m/s}}\right)\sin 22.0^\circ = 0.656 .$$

Consequently, the angle between the second ball and the initial direction of the first is $\theta_2 = 41.0^\circ$.

(b) We solve the first momentum conservation equation for the initial speed of the cue ball.

$$\begin{aligned} v_{1i} &= v_{1f}\cos\theta_1 + v_{2f}\cos\theta_2 \\ &= (3.50 \text{ m/s})\cos 22.0^\circ + (2.00 \text{ m/s})\cos 41.0^\circ \\ &= 4.75 \text{ m/s} . \end{aligned}$$

(c) With SI units understood, the initial kinetic energy is

$$K_i = \frac{1}{2}mv_i^2 = \frac{1}{2}m(4.75)^2 = 11.3m$$

and the final kinetic energy is

$$K_f = \frac{1}{2}mv_{1f}^2 + \frac{1}{2}mv_{2f}^2 = \frac{1}{2}m\left((3.50)^2 + (2.00)^2\right) = 8.1m \ .$$

Kinetic energy is not conserved.

50. We orient our $+x$ axis along the initial direction of motion, and specify angles in the "standard" way – so $\theta = -90°$ for the particle B which is assumed to scatter "downward" and $\phi > 0$ for particle A which presumably goes into the first quadrant. We apply the conservation of linear momentum to the x and y axes respectively.

$$\begin{aligned} m_B v_B &= m_B v'_B \cos\theta + m_A v'_A \cos\phi \\ 0 &= m_B v'_B \sin\theta + m_A v'_A \sin\phi \end{aligned}$$

(a) Setting $v_B = v$ and $v'_B = v/2$, the y-momentum equation yields

$$m_A v'_A \sin\phi = m_B \, \frac{v}{2}$$

and the x-momentum equation yields

$$m_A v'_A \cos\phi = m_B \, v \ .$$

Dividing these two equations, we find $\tan\phi = \frac{1}{2}$ which yields $\phi = 27°$. If we choose to measure this from the final direction of motion for B, then this becomes $90° + 27° = 117°$.

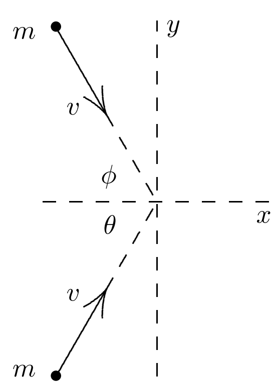
(b) We can *formally* solve for v'_A (using the y-momentum equation and the fact that $\sin\phi = 1/\sqrt{5}$)

$$v'_A = \frac{\sqrt{5}}{2} \frac{m_B}{m_A} v$$

but lacking numerical values for v and the mass ratio, we cannot fully determine the final speed of A. Note: substituting $\cos\phi = 2/\sqrt{5}$, into the x-momentum equation leads to exactly this same relation (that is, no new information is obtained which might help us determine an answer).

51. Suppose the objects enter the collision along lines that make the angles $\theta > 0$ and $\phi > 0$ with the x axis, as shown in the diagram below. Both have the same mass m and the same initial speed v.

We suppose that after the collision the combined object moves in the positive x direction with speed V. Since the y component of the total momentum of the two-object system is conserved, $mv\sin\theta - mv\sin\phi = 0$. This means $\phi = \theta$. Since the x component is conserved, $2mv\cos\theta = 2mV$. We now use $V = v/2$ to find that $\cos\theta = 1/2$. This means $\theta = 60°$. The angle between the initial velocities is $120°$.



52. We orient our $+x$ axis along the initial direction of motion, and specify angles in the "standard" way – so $\theta = +60°$ for one ball (1) which is assumed to go into the first quadrant with speed $v'_1 = 1.1$ m/s, and $\phi < 0$ for the other ball (2) which presumably goes into the fourth quadrant. The mass of each ball is m, and the initial speed of one of the balls is $v_0 = 2.2$ m/s. We apply the conservation of linear momentum to the x and y axes respectively.

$$\begin{aligned} mv_0 &= mv'_1 \cos\theta + mv'_2 \cos\phi \\ 0 &= mv'_1 \sin\theta + mv'_2 \sin\phi \end{aligned}$$

The mass m cancels out of these equations, and we are left with two unknowns and two equations, which is sufficient to solve.

(a) With SI units understood, the y-momentum equation can be rewritten as

$$v_2' \sin\phi = -v_1' \sin 60° = -0.95$$

and the x-momentum equation yields

$$v_2' \cos\phi = v_0 - v_1' \cos 60° = 1.65$$

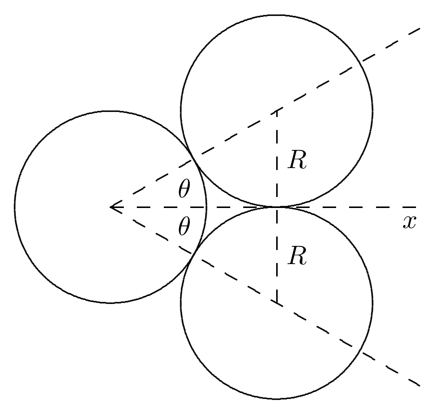
Dividing these two equations, we find $\tan\phi = -0.577$ which yields $\phi = -30°$. If we choose to measure this as a positive-valued angle (as the textbook does in §10-6), then this becomes 30°. We plug $\phi = -30°$ into either equation and find $v_2' \approx 1.9$ m/s.

(b) One can check to see if this an elastic collision by computing

$$\frac{2K_i}{m} = v_0^2 \quad \text{and} \quad \frac{2K_f}{m} = {v_1'}^2 + {v_2'}^2$$

and seeing if they are equal (they are), but one must be careful not to use rounded-off values. Thus, it is useful to note that the answer in part (a) can be expressed "exactly" as $v_2' = \frac{1}{2}v_0\sqrt{3}$ (and of course $v_1' = \frac{1}{2}v_0$ "exactly" – which makes it clear that these two kinetic energy expressions are indeed equal).

53. The diagram below shows the situation as the incident ball (the left-most ball) makes contact with the other two. It exerts an impulse of the same magnitude on each ball, along the line that joins the centers of the incident ball and the target ball. The target balls leave the collision along those lines, while the incident ball leaves the collision along the x axis. The three dotted lines that join the centers of the balls in contact form an equilateral triangle, so both of the angles marked θ are 30°. Let v_0 be the velocity of the incident ball before the collision and V be its velocity afterward. The two target balls leave the collision with the same speed. Let v represent that speed. Each ball has mass m.



Since the x component of the total momentum of the three-ball system is conserved,

$$mv_0 = mV + 2mv\cos\theta$$

and since the total kinetic energy is conserved,

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mV^2 + 2\left(\frac{1}{2}mv^2\right) .$$

We know the directions in which the target balls leave the collision so we first eliminate V and solve for v. The momentum equation gives $V = v_0 - 2v\cos\theta$, so $V^2 = v_0^2 - 4v_0v\cos\theta + 4v^2\cos^2\theta$ and the energy equation becomes $v_0^2 = v_0^2 - 4v_0v\cos\theta + 4v^2\cos^2\theta + 2v^2$. Therefore,

$$v = \frac{2v_0\cos\theta}{1 + 2\cos^2\theta} = \frac{2(10\,\text{m/s})\cos 30°}{1 + 2\cos^2 30°} = 6.93 \text{ m/s} .$$

(a) The discussion and computation above determines the final velocity of ball 2 (as labeled in Fig. 10-41) to be 6.9 m/s at 30° counterclockwise from the $+x$ axis.

(b) Similarly, the final velocity of ball 3 is 6.9 m/s at 30° clockwise from the $+x$ axis.

(c) Now we use the momentum equation to find the final velocity of ball 1:

$$V = v_0 - 2v\cos\theta = 10\,\text{m/s} - 2(6.93\,\text{m/s})\cos 30^\circ = -2.0\,\text{m/s} \ .$$

The minus sign indicates that it bounces back in the $-x$ direction.

54. The problem involves the completely inelastic collision of the two children followed by their completely inelastic collision with the (already moving) man. Speeds are given but no angles, so we are free to orient our $-x$ axis along the direction of motion of the man before his collision with the children (so his angle is 180°). The magnitude of the man's momentum before that collision is $(75\,\text{kg})(2.0\,\text{m/s}) = 150$ kg·m/s. Thus, with SI units understood, the second collision is described by momentum conservation:

$$\vec{p} + (150 \angle 180^\circ) = 0$$

which yields the momentum of the stuck-together children $\vec{p} = (150 \angle 0^\circ)$ in magnitude-angle notation. We now describe the first collision (of the two children) using momentum conservation:

$$\vec{p}_1 + \vec{p}_2 = (150 \angle 0^\circ) \quad \text{or} \quad 150\,\hat{\imath}$$

where the unit-vector notation has also been used, in case the magnitude-angle notation is less familiar. Now, since $m_1 = m_2 = 30$ kg and $|\vec{p}_1| = |\vec{p}_2| = 120$ kg·m/s, we see that the y components of the children's initial velocities must be equal and opposite. Therefore, if child 1 has an initial velocity angle θ then child 2 has an initial velocity angle $-\theta$. The previous equation becomes

$$120\cos(\theta) + 120\cos(-\theta) = 150$$

which has the solution $\theta = 51^\circ$. The angle between the children (initially) is therefore $2\theta \approx 103^\circ$.

55. Let $m_n = 1.0$ u be the mass of the neutron and $m_d = 2.0$ u be the mass of the deuteron. In our manipulations we treat these masses as "exact", so, for instance, we write $m_n/m_d = \frac{1}{2}$ in our simplifying steps. We assume the neutron enters with a velocity $\vec{v}_\text{o}$ pointing in the $+x$ direction and leaves along the positive y axis with speed v_n. The deuteron goes into the fourth quadrant with velocity components $v_{dx} > 0$ and $v_{dy} < 0$. Conservation of the x component of momentum leads to

$$m_n v_\text{o} = m_d v_{dx} \quad \Longrightarrow \quad v_{dx} = \frac{1}{2} v_\text{o}$$

and conservation of the y component leads to

$$0 = m_n v_n + m_d v_{fy} \quad \Longrightarrow \quad v_{dy} = -\frac{1}{2} v_\text{n} \ .$$

Also, the collision is elastic, so kinetic energy "conservation" leads to

$$\frac{1}{2} m_n v_\text{o}^2 = \frac{1}{2} m_n v_n^2 + \frac{1}{2} m_d v_d^2$$

which we simplify by multiplying through with $2/m_n$ and using $v_d^2 = v_{dx}^2 + v_{dy}^2$

$$v_\text{o}^2 = v_n^2 + \frac{m_d}{m_n}\left(v_{dx}^2 + v_{dy}^2\right) \ .$$

Now we substitute in the relations found from the momentum conditions:

$$v_\text{o}^2 = v_n^2 + 2\left(\frac{v_\text{o}^2}{4} + \frac{v_n^2}{4}\right) \quad \Longrightarrow \quad v_n = v_\text{o}\sqrt{\frac{1}{3}} \ .$$

Finally, we set up a ratio expressing the (relative) loss of kinetic energy (by the neutron).

$$\frac{K_\text{o} - K_n}{K_\text{o}} = 1 - \frac{v_n^2}{v_\text{o}^2} = 1 - \frac{1}{3} = \frac{2}{3} \ .$$

56. (a) Choosing upward as the positive direction, the momentum change of the foot is

$$\Delta\vec{p} = 0 - m_{\text{foot}}\,\vec{v}_i = -(0.003\,\text{kg})(-1.5\text{m/s})$$

which yields an impulse of 4.50×10^{-3} N·s.

(b) Using Eq. 10-8 and now treating *downward* as the positive direction, we have

$$\vec{J} = \vec{F}_{\text{avg}}\Delta t = m_{\text{lizard}}\,g\,\Delta t = (0.090)(9.8)(0.6)$$

which yields $\vec{J} = 0.529$ N·s.

(c) Considering the large difference between the answers for part (a) and part (b), we see that the slap cannot account for the support; we infer, then, that the push does the job.

57. From mechanical energy conservation (or simply using Eq. 2-16 with $\vec{a} = g$ downward) we obtain

$$v = \sqrt{2gh} = \sqrt{2(9.8)(1.5)} = 5.4 \text{ m/s}$$

for the speed just as the body makes contact with the ground.

(a) During the compression of the body, the center of mass must decelerate over a distance $d = 0.30$ m. Choosing $+y$ downward, the deceleration a is found using Eq. 2-16

$$0 = v^2 + 2ad \quad\Longrightarrow\quad a = -\frac{v^2}{2d} = -\frac{5.4^2}{2(0.30)}$$

which yields $a = -49$ m/s^2. Thus, the magnitude of the net (vertical) force is $m|a| = 49m$ in SI units, which (since $49 = 5(9.8)$) can be expressed as $5mg$.

(b) During the deceleration process, the forces on the dinosaur are (in the vertical direction) $\vec{N}$ and $m\vec{g}$. If we choose $+y$ upward, and use the final result from part (a), we therefore have $N - mg = 5mg$, or $N = 6mg$. In the horizontal direction, there is also a deceleration (from $v_o = 19$ m/s to zero), in this case due to kinetic friction $f_k = \mu_k N = \mu_k(6mg)$. Thus, the net force exerted by the ground on the dinosaur is

$$F_{\text{ground}} = \sqrt{f_k^2 + N^2} \approx 7mg\ .$$

(c) We can applying Newton's second law in the horizontal direction (with the sliding distance denoted as Δx) and then use Eq. 2-16, or we can apply the general notions of energy conservation. The latter approach is shown:

$$\frac{1}{2}mv_o^2 = \mu_k(6mg)\Delta x \quad\Longrightarrow\quad \Delta x = \frac{19^2}{2(6)(0.6)(9.8)} \approx 5 \text{ m}\ .$$

58. (a) As explained in the problem, the height of the n^{th} domino is $h_n = 1.5^{n-1}$ in centimeters. Therefore, $h_{32} = 1.5^{31} = 2.9 \times 10^5$ cm $= 2.9$ km (!).

(b) When the center of the domino is directly over the corner, the height of the center-point is

$$h_c = \sqrt{\left(\frac{h}{2}\right)^2 + \left(\frac{d}{2}\right)^2} = \frac{d}{2}\sqrt{101}$$

where $h = 10d$ has been used in that last step. While the domino is in its usual resting position, the height of that point is only $h_o = h/2$ which can be written as $5d$. Since the answer is requested to be in terms of U_1 then

$$U_1 = mg(5d) \quad\Longrightarrow\quad d = \frac{U_1}{5mg}\ .$$

Therefore, the energy needed to push over the domino is

$$\Delta U = mgh_c - U_1 = mg\left(\frac{d}{2}\sqrt{101}\right) - U_1 = \frac{U_1}{10}\sqrt{101} - U_1$$

which yields approximately $0.005U_1$; the problem refers to this as $\Delta E_{1,\text{in}}$.

(c) The "loss" of potential energy equal to

$$mgh_c - mg\left(\frac{h}{2}\sin\theta\right)$$

becomes the kinetic energy (denoted $\Delta E_{1,\text{out}}$ in the problem). Therefore, we obtain

$$\Delta E_{1,\text{out}} = mg\left(\frac{d}{2}\sqrt{101}\right) - mg\left(\frac{10d}{2}\sin\theta\right)$$

which (using $\theta = 45°$) simplifies to $1.49mgd$. Since $d = U_1/5mg$ this becomes roughly $\Delta E_{1,\text{out}} = 0.30U_1$.

(d) We see from part (b) that $\Delta E_{n,\text{in}}$ is directly proportional to $m_n d_n$ and consequently (since the density is assumed the same for all of them and the volume of a domino is hdw where w is the width) is proportional to $w_n h_n d_n^2$. The width also scales like the other quantities, so $\Delta E_{n,\text{in}}$ is proportional to $1.5^{4(n-1)}$. Therefore, $\Delta E_{2,\text{in}} = 1.5^4 \Delta E_{1,\text{in}}$ which implies $\Delta E_{2,\text{in}} = 0.025U_1$.

(e) Therefore,

$$\frac{\Delta E_{1,\text{out}}}{\Delta E_{2,\text{in}}} = \frac{0.30U_1}{0.025U_1} = 12\ .$$

59\. (a) We choose $+x$ to be away from the armor (pointing back towards the gun). The velocity is there negative-valued and the acceleration is positive-valued. Using Eq. 2-11,

$$0 = \vec{v}_0 + \vec{a}t \implies \vec{a} = -\frac{\vec{v}_0}{t} = -\frac{-300}{40\times10^{-6}} = 7.5\times10^6\ \text{m/s}^2\ .$$

(b) Since the final momentum is zero, the momentum change is

$$\Delta\vec{p} = 0 - m\vec{v}_0 = -(0.0102\,\text{kg})(-300\,\text{m/s}) = 3.1\ \text{kg·m/s}\ .$$

(c) We compute $K_f - K_i = 0 - \frac{1}{2}mv_0^2$ and obtain $-\frac{1}{2}(0.0102)(300)^2 \approx -460$ J.

(d) If we assume uniform deceleration, Eq. 2-17 gives

$$\Delta x = \frac{1}{2}(\vec{v}_0 + 0)\,t = \frac{1}{2}(-300)\left(40\times10^{-6}\right)$$

so that the distance is $|\Delta x| = 0.0060$ m.

(e) By the impulse-momentum theorem, the impulse of the armor on the bullet is $\vec{J} = \Delta\vec{p} = 3.1$ N·s. By Newton's third law, the impulse of the bullet on the armor must have that same magnitude.

(f) Using Eq. 10-8, we find the magnitude of the (average) force exerted by the bullet on the armor:

$$F_{\text{avg}} = \frac{J}{t} = \frac{3.1}{40\times10^{-6}} = 7.7\times10^4\,\text{N}\ .$$

(g) From Newton's second law, we find $a_p = F_{\text{avg}}/M$ (where $M = 65$ kg) to be 1.2×10^3 m/s^2.

(h) Momentum conservation leads to $V = mv_0/M = 0.047$ m/s. (This result can be gotten a number of ways, given the information available at this point in the problem.)

(i) Shortening the distance means decreasing the stopping time (Eq. 2-17 shows this clearly) which (recalling our calculation in part (a)) means the magnitude of the bullet's deceleration increases. It does not change the answer to part (b) (for the change in momentum), nor does it affect part (c) (the change in kinetic energy). Since $\vec{J}$ is determined by $\Delta\vec{p}$, part (e) is unchanged. But with t smaller, $J/t = F_{\text{avg}}$ is larger, as is a_p. Finally, v_p is the same as before since momentum conservation describes the input/output of the collision and not the inner dynamics of it.

60. From mechanical energy conservation (or simply using Eq. 2-16 with $\vec{a} = g$ downward) we obtain

$$v = \sqrt{2gh} = \sqrt{2(9.8)(6.0)} = 10.8 \text{ m/s}$$

for the speed just as the $m = 3000$-kg block makes contact with the pile. At the moment of "joining", they are a system of mass $M = 3500$ kg and speed V. With downward positive, momentum conservation leads to

$$mv = MV \implies V = \frac{(3000)(10.8)}{3500} = 9.3 \text{ m/s} .$$

Now this block-pile "object" must be rapidly decelerated over the small distance $d = 0.030$ m. Using Eq. 2-16 and choosing $+y$ downward, we have

$$0 = V^2 + 2ad \implies a = -\frac{9.3^2}{2(0.030)} = -1440$$

in SI units (m/s^2). Thus, the net force during the decelerating process has magnitude $M\,|a| = 5.0\times10^6$ N.

61. Using Eq. 10-31 with $m_1 = 3.0$ kg, $v_{1i} = 8.0$ m/s and $v_{2f} = 6.0$ m/s, then

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} \implies m_2 = m_1 \left(\frac{2v_{1i}}{v_{2f}} - 1 \right)$$

leads to $m_2 = M = 5.0$ kg.

62. In the momentum relationships, we could as easily work with weights as with masses, but because part (b) of this problem asks for kinetic energy – we will find the masses at the outset: $m_1 = 280 \times 10^3/9.8 = 2.86 \times 10^4$ kg and $m_2 = 210 \times 10^3/9.8 = 2.14 \times 10^4$ kg. Both cars are moving in the $+x$ direction: $v_{1i} = 1.52$ m/s and $v_{2i} = 0.914$ m/s.

(a) If the collision is completely elastic, momentum conservation leads to a final speed of

$$V = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2} = 1.26 \text{ m/s} .$$

(b) We compute the total initial kinetic energy and subtract from it the final kinetic energy.

$$K_i - K_f = \frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 - \frac{1}{2}(m_1 + m_2) V^2 = 2.25 \times 10^3 \text{ J} .$$

(c) and (d) Using Eq. 10-38 and Eq. 10-39, we find

$$\begin{aligned} v_{2\,f} &= \frac{2m_1}{m_1 + m_2} v_{1\,i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2\,i} = 1.61 \text{ m/s} \quad \text{and} \\ v_{1\,f} &= \frac{m_1 - m_2}{m_1 + m_2} v_{1\,i} + \frac{2m_2}{m_1 + m_2} v_{2\,i} = 1.00 \text{ m/s} . \end{aligned}$$

63. We choose coordinates with $+x$ East and $+y$ North, with the standard conventions for measuring the angles. With SI units understood, we write the initial magnitude of the man's momentum as $(60)(6.0) = 360$ and the final momentum of the two of them together as $(98)(3.0) = 294$. Using magnitude-angle notation (quickly implemented using a vector capable calculator in polar mode), momentum conservation becomes

$$\begin{aligned} \vec{p}_{\text{man}} + \vec{p}_{\text{child}} &= \vec{p}_{\text{together}} \\ (360 \angle 90^\circ) + \vec{p} &= (294 \angle 35^\circ) \end{aligned}$$

Therefore, the momentum of the 38 kg child before the collision is $\vec{p} = (308 \angle -38^\circ)$. Thus, the child's velocity has magnitude equal to $308/38 = 8.1$ m/s and direction of 38° south of east.

64. (a) We choose a coordinate system with $+x$ downriver and $+y$ in the initial direction of motion of the second barge. The velocities in component forms are $\vec{v}_{1i} = (6.2\,\text{m/s})\hat{\imath}$ and $\vec{v}_{2i} = (4.3\,\text{m/s})\hat{\jmath}$ before collision. After the collision, barge 2 has velocity

$$\vec{v}_{2f} = (5.1\,\text{m/s})\left((\sin 18^\circ)\hat{\imath} + (\cos 18^\circ)\hat{\jmath}\right) \; .$$

Writing $\vec{v}_{1f} = v_{1f}\left((\cos\theta)\hat{\imath} + (\sin\theta)\hat{\jmath}\right)$, with θ we express the component form of the conservation of momentum:

$$\begin{aligned} m_1 v_{1i} &= m_1 v_{1f}\cos\theta + m_2 v_{2f}\sin 18^\circ \\ m_2 v_{2i} &= m_1 v_{1f}\sin\theta + m_2 v_{2f}\cos 18^\circ \; . \end{aligned}$$

Substituting $v_{1i} = 6.2\,\text{m/s}$, $v_{2i} = 4.3\,\text{m/s}$, and $v_{2f} = 5.1\,\text{m/s}$, we find: $v_{1f} = 3.4\,\text{m/s}$, $\theta = 17^\circ$ (from the point of view of someone on that barge, this deflection is toward the left).

(b) The loss of kinetic energy is

$$K_i - K_f = \left(\frac{1}{2}m_1 v_{1i}^2 + \frac{1}{2}m_2 v_{2i}^2\right) - \left(\frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2\right)$$

which yields 9.5×10^5 J.

65. Let the mass of each ball be m. Conservation of (kinetic) energy in elastic collisions requires that $K_i = K_f$ which leads to

$$\frac{1}{2}mV^2 = \frac{1}{2}(16m)v^2$$

which yields $v = V/4$.

66. The speed of each particle of mass m upon impact with the scale is found from mechanical energy conservation (or simply using Eq. 2-16 with $\vec{a} = g$ downward): $v = \sqrt{2gh}$, where $h = 3.5\,\text{m}$. With $+y$ upward, the change in momentum for the particle is therefore

$$\Delta\vec{p} = m\Delta\vec{v} = 2mv = 2m\sqrt{2gh} \; .$$

During a time interval Δt, the number of collisions is $N = R\Delta t$ where $R = 42\,\text{s}^{-1}$. Thus, using the impulse-momentum theorem and Eq. 10-8, the average force is

$$\begin{aligned} \vec{F}_{\text{avg}} &= \frac{N\Delta\vec{p}}{\Delta t} \\ &= 2mR\sqrt{2gh} \\ &= 2(0.110)(42)\sqrt{2(9.8)(3.5)} \\ &= 77 \text{ N} \end{aligned}$$

which corresponds to a mass reading of $77/9.8 = 7.8\,\text{kg}$.

67. The momentum before the collision (with $+x$ rightward) is

$$(6.0 \text{ kg})(8.0 \text{ m/s}) + (4.0 \text{ kg})(2.0 \text{ m/s}) = 56 \text{ kg}\cdot\text{m/s} \; .$$

(a) The total momentum at this instant is $(6.0\,\text{kg})(6.4\,\text{m/s}) + (4.0\,\text{kg})\vec{v}$. Since this must equal the initial total momentum (56, using SI units), then we find $\vec{v} = 4.4$ m/s.

(b) The initial kinetic energy was

$$\frac{1}{2}(6.0 \text{ kg})(8.0 \text{ m/s})^2 + \frac{1}{2}(4.0 \text{ kg})(2.0 \text{ m/s})^2 = 200 \text{ J} \; .$$

The kinetic energy at the instant described in part (a) is

$$\frac{1}{2}(6.0\text{ kg})(6.4\text{ m/s})^2 + \frac{1}{2}(4.0\text{ kg})(4.4\text{ m/s})^2 = 162\text{ J} .$$

The "missing" 38 J is not dissipated since there is no friction; it is the energy stored in the spring at this instant when it is compressed. Thus, $U_e = 38$ J.

68. This is a completely inelastic collision, followed by projectile motion. In the collision, we use momentum conservation.

$$\begin{aligned} \vec{p}_{\text{shoes}} &= \vec{p}_{\text{together}} \\ (3.2\,\text{kg})(3.0\,\text{m/s}) &= (5.2\,\text{kg})\vec{v} \end{aligned}$$

Therefore, $\vec{v} = 1.8$ m/s toward the right as the combined system is projected from the edge of the table. Next, we can use the projectile motion material from Ch. 4 or the energy techniques of Ch. 8; we choose the latter.

$$\begin{aligned} K_{\text{edge}} + U_{\text{edge}} &= K_{\text{floor}} + U_{\text{floor}} \\ \frac{1}{2}(5.2\,\text{kg})(1.8\,\text{m/s})^2 + (5.2\,\text{kg})(9.8\text{ m/s}^2)(0.40\,\text{m}) &= K_{\text{floor}} + 0 \end{aligned}$$

Therefore, the kinetic energy of the system right before hitting the floor is $K_{\text{floor}} = 29$ J.

69. We use the impulse-momentum theorem $\vec{J} = \Delta\vec{p}$ where $\vec{J} = \int \vec{F}\,dt$. Integrating the given expression for force from the moment it starts from rest up to a variable upper limit t, we have $\vec{J} = \left(16t - \frac{1}{3}t^3\right)\hat{\imath}$ with SI units understood.

(a) Since $\left(16t - \frac{1}{3}t^3\right)\hat{\imath} = m\vec{v}$ with $m = 1.6$, we obtain $\vec{v} = 24\,\hat{\imath}$ in meters-per-second, for $t = 3.0$ s.

(b) Setting $\left(16t - \frac{1}{3}t^3\right)\hat{\imath} = m\vec{v}$ equal to zero leads to $t = 6.9$ s as the positive root.

(c) We can work through the $\frac{d\vec{v}}{dt} = 0$ condition using our $\left(16t - \frac{1}{3}t^3\right)\hat{\imath} = m\vec{v}$ relation, or more simply observe, from the outset, that this is equivalent to finding when the acceleration, hence the force, is zero. We obtain $t = 4.0$ s as the positive root, which we plug into the $\left(16t - \frac{1}{3}t^3\right)\hat{\imath} = m\vec{v}$ relation and find $\vec{v}_{\text{max}} = 27\,\hat{\imath}$ m/s.

70. (a) We use coordinates with $+x$ rightward and $+y$ upward, with the usual conventions for measuring the angles (so that the final angle is written $90° - 40° = 50°$). With SI units understood, the magnitude of the diver's momentum before contact is $(60.0)(3.00) = 180$ and after contact is $(60.0)(5.00) = 300$. Using magnitude-angle notation (quickly implemented using a vector capable calculator in polar mode), the change in momentum is

$$(300\ \angle\ 50°) - (180\ \angle -90°) = (453\ \angle\ 65°) .$$

This equals the *total* impulse delivered to the diver (by the board and by gravity). If F_{net} denotes the magnitude of the average *net* force exerted on the diver, then we have

$$F_{\text{net}}\Delta t = 453 \implies F_{\text{net}} = \frac{453}{1.2} = 377\text{ N} .$$

(b) Since $\vec{F}_{\text{net}} = (377\ \angle\ 65°)$ and the weight of the diver is $(588\ \angle -90)$, we obtain

$$(377\ \angle\ 65°) - (588\ \angle -90°) = (943\ \angle\ 80°) .$$

Therefore, the magnitude of the average force exerted by the board on the diver is 943 N.

71. The magnitude of the impulse exerted by the gunner on the gun per minute is $J = F_{\text{avg}}\Delta t$, where $F_{\text{avg}} = 180\,\text{N}$ and $\Delta t = 60\,\text{s}$. The impulse exerted on the gun by each bullet of mass m and speed v is $J' = mv$. The maximum number of bullets N that he could fire per minute satisfies $J = NJ'$. Thus

$$N = \frac{J}{J'} = \frac{F_{\text{avg}}\Delta t}{mv} = \frac{(180)(60)}{(50\times 10^{-3})(1000)} = 216 \ .$$

72. (a) The magnitude of the force is

$$F = \frac{\Delta p}{\Delta t} = \frac{9.0\times 10^3\,\text{kg}\cdot\text{m/s}}{12\,\text{s}} = 750\,\text{N} \ .$$

(b) Assuming this is one-dimensional motion (so that any acceleration implies a change in the magnitude of the velocity), we find the speed increase to be

$$\Delta v = \frac{\Delta p}{m} = \frac{9.0\times 10^3\,\text{kg}\cdot\text{m/s}}{1500\,\text{kg}} = 6.0\,\text{m/s} \ .$$

73. (a) The momentum conservation equation (for this completely inelastic collision) $m_A\vec{v}_A + m_B\vec{v}_B = (m_A + m_B)\vec{V}$ can be written in terms of weights by multiplying through by g:

$$w_A\vec{v}_A + w_B\vec{v}_B = (w_A + w_B)\vec{V} \ .$$

Our $\hat{\imath}$ direction is West and $\hat{\jmath}$ is South, so we have (with weights in kN and speeds in km/h)

$$\begin{aligned}\vec{V} &= \frac{(12.0)\,(64.4\,\hat{\imath}) + (16.0)\left(96.6\,\hat{\jmath}\right)}{12.0 + 16.0} \\ &= 27.6\,\hat{\imath} + 55.2\,\hat{\jmath}\end{aligned}$$

which implies that the final speed is 61.7 km/h.

(b) And the angle for the final velocity is $\tan^{-1}(55.2/27.6) = 63.4^\circ$ South of West.

74. We choose $\hat{\imath}$ East and $\hat{\jmath}$ North, and use SI units (kg for mass and m/s for speed). The initially moving tin cookie has mass $m_1 = 2.0$ and velocity $\vec{v}_o = 8.0\,\hat{\imath}$, and the initially stationary cookie tin has mass $m_2 = 4.0$.

(a) Momentum conservation leads to

$$\begin{aligned}m_1\vec{v}_o &= m_1\vec{v}_1 + m_2\vec{v}_2 \\ 16\,\hat{\imath} &= 8\cos(37^\circ)\,\hat{\imath} + 8\sin(37^\circ)\,\hat{\jmath} + (4.0)\vec{v}_2\end{aligned}$$

which leads to

$$\vec{v}_2 = 2.4\,\hat{\imath} - 1.2\,\hat{\jmath} \implies \vec{v}_2 = (2.7 \angle\ 27^\circ)$$

where magnitude-angle notation is used. Thus, the speed of the cookie tin is 2.7 m/s.

(b) And its angle is $\tan^{-1}(-1.2/2.4) = -27^\circ$ which can be expressed as 27° south of east.

75. We choose $\hat{\imath}$ East and $\hat{\jmath}$ North, and use SI units. The ball initially moving eastward has mass $m_1 = 5.0$ kg and initial velocity $\vec{v}_{1i} = 4.0\,\hat{\imath}$ m/s, and the ball initially moving westward has mass $m_2 = 4.0$ kg and velocity $\vec{v}_{2i} = -3.0\,\hat{\imath}$ m/s. The final velocity of m_1 is $\vec{v}_{1f} = -1.2\,\hat{\jmath}$.

(a) Momentum conservation leads to

$$\begin{aligned}m_1\vec{v}_{1i} + m_2\vec{v}_{2i} &= m_1\vec{v}_1 + m_2\vec{v}_2 \\ 20\,\hat{\imath} - 12\,\hat{\imath} &= -6\,\hat{\jmath} + 4\vec{v}_2\end{aligned}$$

which leads to

$$\vec{v}_2 = 2.0\,\hat{\imath} + 1.5\,\hat{\jmath} \implies \vec{v}_2 = (2.5 \angle\ 37^\circ)$$

where magnitude-angle notation is used. Thus, the speed of the 4.0 kg ball just after the collision is 2.5 m/s.

(b) We compute the decrease in total kinetic energy:

$$K_i - K_f = \frac{1}{2}(5)(4)^2 + \frac{1}{2}(4)(3)^2 - \frac{1}{2}(5)(1.2)^2 - \frac{1}{2}(4)(2.5)^2$$

which gives the result 42 J.

76. Using mechanical energy conservation, we find the speed v of a pendulum at the bottom of its swing is related to the height h it was released from (or that it swings up to) by $v^2 = 2gh$. Thus, the conservation of momentum at the instant they collide can be expressed as

$$m_1\sqrt{2gd} = (m_1 + m_2)\sqrt{2gh_f}\ .$$

Therefore, the "final" height of the system (which it swings to shortly after the collision) is

$$h_f = \left(\frac{m_1}{m_1 + m_2}\right)^2 d\ .$$

77. If we neglect the time required for the spring to decelerate the leftward moving glider m_2 and re-accelerate it (rightward), then we are effectively assuming that glider bounces elastically off the wall (with the spring playing no dynamic role). Thus, we assume the time t required for m_2 to travel distance $d + x$ (to the wall and then rightward to position x, assuming the origin is at the wall) is simply $t = (d + x)/v$ where $v = |v_{2f}|$ is its speed resulting from the first elastic collision. This velocity is found from Eq. 10-31:

$$v_{2f} = \frac{2m_1}{m_1 + m_2}\, v_{1i} = \frac{2(590)}{940}(-75)$$

which yields -94 cm/s. Thus, with $d = 53$ cm, we have the relation $t = (53 + x)/94$ with x in cm and t in s. During that time, glider m_1 has a displacement $\Delta x = x - d$ due its velocity v_{1f} where

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2}\, v_{1i} = \frac{240}{940}(-75)$$

which yields $v_{1f} = -19$ cm/s. This provides another relation between t and x: $t = (x - d)/v_{1f} = (53 - x)/19$. Equating these to relations, we obtain

$$\frac{53 + x}{94} = \frac{53 - x}{19} \quad\Longrightarrow\quad x = 35 \text{ cm}\ .$$

78. Eq. 10-31, for situations where $m_1 \gg m_2$, reduces simply to $v_{2f} \approx 2v_{1i}$. Thus, the speed of the fly after the collision is $2(2.1) = 4.2$ m/s.

79. (a) We find the velocity $\vec{v}_{1f}$ of the 1200 kg car after the collision (taking the direction of motion as positive) using momentum conservation (with mass in kg and speed in km/h).

$$\begin{aligned} m_1\vec{v}_{1i} + m_2\vec{v}_{2i} &= m_1\vec{v}_{1f} + m_2\vec{v}_{2f} \\ (1200)(70) + (900)(60) &= (1200)\vec{v}_{1f} + (900)(70) \end{aligned}$$

This gives the result $\vec{v}_{1f} = 62.5$ km/h.

(b) We compute the reduction of total kinetic energy in the collision:

$$Q = K_i - K_f = \frac{1}{2}(1200)(70)^2 + \frac{1}{2}(900)(60)^2 - \frac{1}{2}(1200)(62.5)^2 - \frac{1}{2}(900)(70)^2$$

which gives the result 11250 in mixed units (kg·km^2/h^2). We set up the requested ratio (where $v_o = 5$ km/h):

$$\frac{Q}{\frac{1}{2}m_1 v_o^2} = \frac{11250}{15000} = \frac{3}{4}\ .$$

80. We refer to the discussion in the textbook (see Sample Problem 10-2, which uses the same notation that we use here) for many of the important details in the reasoning. Here we only present the primary computational step (using SI units).

(a) The bullet's initial kinetic energy is

$$\frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{m+M}{m}\sqrt{2gh}\right)^2 = \frac{m+M}{m}U_f$$

where $U_f = (m+M)gh$ is the system's final potential energy (equal to its total mechanical energy since its speed is zero at height h). Thus,

$$\frac{U_f}{\frac{1}{2}mv^2} = \frac{m}{m+M} = \frac{0.008}{7.008} = 0.00114 \ .$$

(b) The fraction $m/(m+M)$ shown in part (a) has no v-dependence. The answer remains the same.

(c) As we found in part (a), the fraction is $m/(m+M)$. The numerical value of h given in the problem statement has not been used in this solution.

81. (a) Since $\vec{F}_{\text{net}} = \frac{d\vec{p}}{dt}$ (Eq. 9-23), we read from value of F_x (see graph) that the rate of change of momentum is 4.0 kg·m/s^2 at $t = 3.0$ s.

(b) The impulse, which causes the change in momentum, is equivalent to the area under the curve in this graph (see Eq. 10-3). We break the area into that of a triangle $\frac{1}{2}(2.0\,\text{s})(4.0\,\text{N})$ plus that of a rectangle $(1.0\,\text{s})(4.0\,\text{N})$, which yields a total of 8.0 N·s. Since the car started from rest, its momentum at $t = 3.0$ s must therefore be 8.0 kg·m/s.

82. We use $J = \int F dt = m\Delta v = mv_f$. The integral $\int F\,dt$ is estimated from the area under the curve in Fig. 10-61 as approximately $4\,\text{N·s}$. (If one doesn't want to "count squares" one can assume the curve to be a parabola, in which case $F = \xi(t - 3.25)(t - 0.35)$ (with t in milliseconds) will fit it once the parameter ξ is adjusted so that $F = 2200$ N when t is midway between 0.35 ms and 3.25 ms. Then the integral can be done explicitly.) Thus, the final speed of the ball is

$$v_f = \frac{J}{m} = \frac{4\,\text{N·s}}{0.5\,\text{kg}} = 8 \text{ m/s} \ .$$

83. (a) The impulse on the ball is

$$\vec{J} = \Delta\vec{p} = m\vec{v} - 0 = (46 \times 10^{-3}\,\text{kg})(50\,\text{m/s})\hat{\imath} = (2.3\,\text{N·s})\hat{\imath}$$

where we choose $\hat{\imath}$ to be in the direction of the velocity $\vec{v}$ of the ball as it leaves the club (at 30° above horizontal – so it is like the x axis of an inclined plane problem).

(b) The impulse on the club is, by Newton's third law, $\vec{J}' = -\vec{J} = -(2.3\,\text{N·s})\hat{\imath}$. We note that it is directed opposite to the direction of motion.

(c) Using Eq. 10-8, the average force on the ball is

$$\vec{F}_{\text{avg}} = \frac{\vec{J}}{\Delta t} = \frac{(2.3)\hat{\imath}}{1.7 \times 10^{-3}} = 1400\,\hat{\imath} \text{ N} \ .$$

(d) The work done on the ball is

$$W = \Delta K = \frac{1}{2}mv^2 = \frac{1}{2}(46 \times 10^{-3})(50)^2 = 58 \text{ J} \ .$$

84. We first note that when the the velocity of a projectile is simply reversed as a result of collision, its change in momentum (in magnitude) is $2mv$ (where v is its speed). If this collision takes time Δt, then the average force involved is (using Eq. 10-8) $F_{\text{avg}} = 2mv/\Delta t$. To relate this observation to the present situation, we replace m with Δm (representing just that amount of the water stream which is in contact with the blade during Δt, and since the impinging flow rate dm/dt is constant (and no water is lost or "splattered away" in the process) then we conclude $dm/dt = \Delta m/\Delta t$. Therefore,

$$F_{\text{avg}} = 2v\,\frac{dm}{dt}\ .$$

85. One could reason as in §9-7 (with the thrust concept) or proceed with Eq. 10-8. Choosing the latter approach, we note that (with the final momentum being zero) the average force is (in magnitude)

$$F_{\text{avg}} = v\,\frac{\Delta m}{\Delta t}$$

where Δm is the portion of the water that is decelerated (by the wall) from speed $v = 500$ cm/s to zero during time Δt. If the impinging mass flow rate dm/dt is constant, then we conclude $dm/dt = \Delta m/\Delta t$. Thus, $F_{\text{avg}} = v\,dm/dt$. We are given the volume flow rate $dV/dt = 300\ \text{cm}^3/\text{s}$, and we use the concept of density to relate mass and volume: $m = \rho V$ where $\rho = 1.0\ \text{g/cm}^3$ for water (most students have seen density in previous courses). Thus,

$$F_{\text{avg}} = v\,\frac{dm}{dt} = \rho\,v\,\frac{dV}{dt} = (1.0)(500)(300)$$

which yields $F_{\text{avg}} = 1.5 \times 10^5\ \text{g}\cdot\text{cm/s}^2$ which we convert to SI, giving the result $F_{\text{avg}} = 1.5$ N.

86. Although we do not present problems and solutions here, we share a few thoughts on the matter.

    (a) This might be more like part (b) of problem 80, in which energy is "liberated" in the collision, but this depends on what particular sort of pinball collision one has in mind.

    (b) This is a good example of an inelastic (but not completely so) collision and might be similar to part (a) of problem 80.

    (c) This might be similar to problem 85, finding the average force on the car in the hailstorm. Instead of having the hail be halted completely by the collision (as is done with the water in problem 85) there should be some small rebound speed.

    (d) An interesting comparison can be made here between the impact of fist with face with glove, and without the glove. The increase in contact time with the glove certainly decreases the force of impact.

    (e) If baseball is chosen as one's example, it might be of interest to refer to the article by Howard Brody in the August 1990 issue of the *American Journal of Physics*, where he considers that the bat may be viewed as a relatively free body in the batting process.

87. (First problem in **Cluster 1**)
Instead of using V for final speed in completely inelastic collisions (as is used in Eq. 10-18), we use $v_{1f} = v_{2f}$, since that facilitates comparison of the results of parts (a) and (b). When we make comparisons, we assume $v_{1i} > 0$.

    (a) When they stick together, we have

$$v_{1f} = v_{2f} = \frac{m_1}{m_1 + m_2}\,v_{1i}\ .$$

(b) Eq. 10-30 and Eq. 10-31 provide the elastic collision results:

$$\begin{aligned} v_{1f} &= \frac{m_1 - m_2}{m_1 + m_2} v_{1i} \\ v_{1f} &= \frac{2m_1}{m_1 + m_2} v_{1i} \end{aligned}$$

from which it is evident that $v_{1f\,\text{elastic}} < v_{1f\,\text{inelastic}}$ and $v_{2f\,\text{elastic}} > v_{2f\,\text{inelastic}}$.

88. (Second problem in **Cluster 1**)
We note that the problem has implicitly chosen the initial direction of motion (of m_1) as the positive direction. The questions to find "greatest" and "least" values are understood in terms of that axis choice (*greatest* = largest positive value, and *least* = the negative value of greatest magnitude or the smallest non-negative value). In addition to the assumptions mentioned in the problem, we also assume that m_1 cannot pass through m_2 (like a bullet might be able to). We are only able to use momentum conservation, since no assumptions about the total kinetic energy can be made.

$$m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f}$$

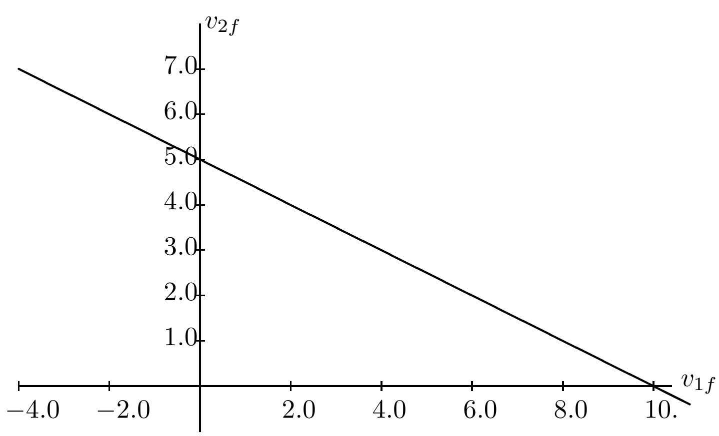
This (since $m_2 = 2.00 m_1$) simplifies to

$$v_{1i} = v_{1f} + 2.00 v_{2f} \ .$$

(a) Using $v_{1i} = 10.0$ m/s, we have

$$v_{2f} = (5.00\,\text{m/s}) - 0.500 v_{1f} \ .$$

(b) Ignoring physics considerations, our function is a line of infinite extent with negative slope.



(c) The greatest possible value of v_{1f} occurs in the completely inelastic case (reasons mentioned in the next several parts) where (see solution to part (a) of previous problem) its value would be $(10.0)(1/3) \approx 3.33$ m/s.

(d) Clearly, this is also the value of v_{2f} in this case.

(e) They stick together (completely inelastic collision).

(f) As mentioned above, we assume m_1 does not pass through m_2 and the problem states that there's no energy production so that $K_{1f} \le K_{1i}$ which implies $v_{1f} \le v_{1i}$.

(g) The plot is shown below, in part (ℓ).

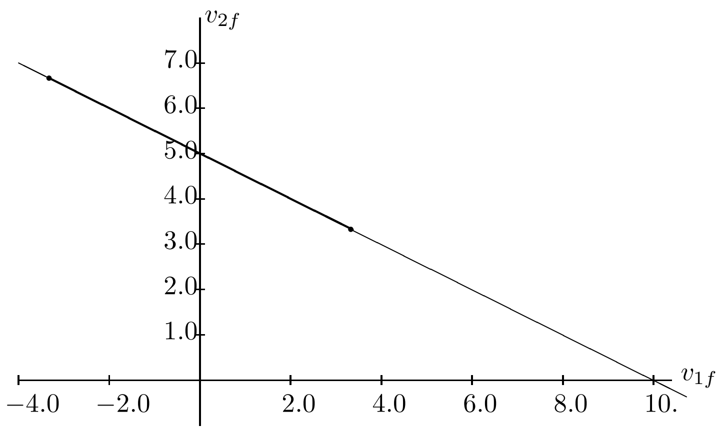
(h) With energy production not a possibility, then the "hardest rebound" m_1 can suffer is in an elastic collision, in which its final velocity (see part (b) of the previous problem) is $(10.0)(1-2)/3 \approx -3.33$ m/s.

(i) Eq. 10-31 gives the velocity of m_2 as $(10.0)(2/3) \approx 6.67$ m/s (see also part (b) of previous problem).

(j) As mentioned, this is an elastic collision (no "loss" of kinetic energy).

(k) The problem states that there's no energy production so that $K_{1i} - K_{1f} = K_{2f}$ and any greater value of $|v_{2f}|$ would violate this condition.

(l) The above graph is redrawn here, with the dark part representing the physically allowed region; the small circles bounding the dark segment correspond to the values calculated in the previous parts of this problem.



89. (Third problem in **Cluster 1**)
We note that the problem has implicitly chosen the initial direction of motion (of m_1) as the positive direction. The questions to find "greatest" and "least" values are understood in terms of that axis choice (*greatest* = largest positive value, and *least* = the negative value of greatest magnitude or the smallest non-negative value). In addition to the assumptions mentioned in the problem, we also assume that m_1 cannot pass through m_2 (like a bullet might be able to). We are only able to use momentum conservation, since no assumptions about the total kinetic energy can be made.

$$m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f}$$

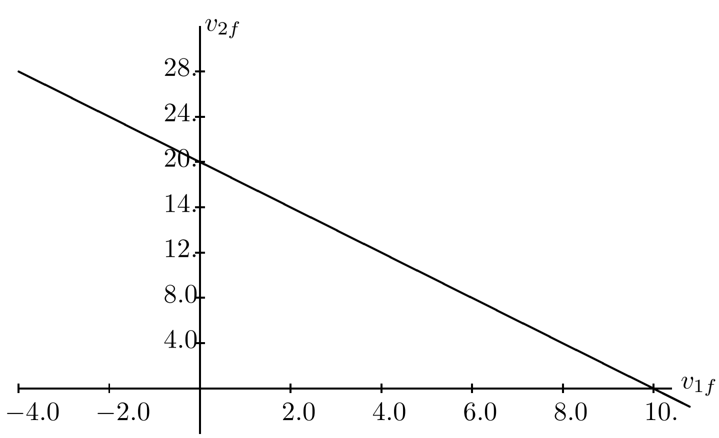
This (since $m_2 = 0.500 m_1$) simplifies to

$$v_{1i} = v_{1f} + 0.500 v_{2f} \ .$$

(a) Using $v_{1i} = 10.0$ m/s, we have

$$v_{2f} = (20.0\,\text{m/s}) - 2.00 v_{1f} \ .$$

(b) Ignoring physics considerations, our function is a line of infinite extent with negative slope.



(c) The greatest possible value of v_{1f} occurs in the completely inelastic case (reasons mentioned in the next several parts) where (see solution to part (a) of previous problem) its value would be $(10.0)(2/3) \approx 6.67$ m/s.

(d) Clearly, this is also the value of v_{2f} in this case.

(e) They stick together (completely inelastic collision).

(f) As mentioned above, we assume m_1 does not pass through m_2 and the problem states that there's no energy production so that $K_{1f} \le K_{1i}$ which implies $v_{1f} \le v_{1i}$.

(g) The plot is shown below, in part (ℓ).

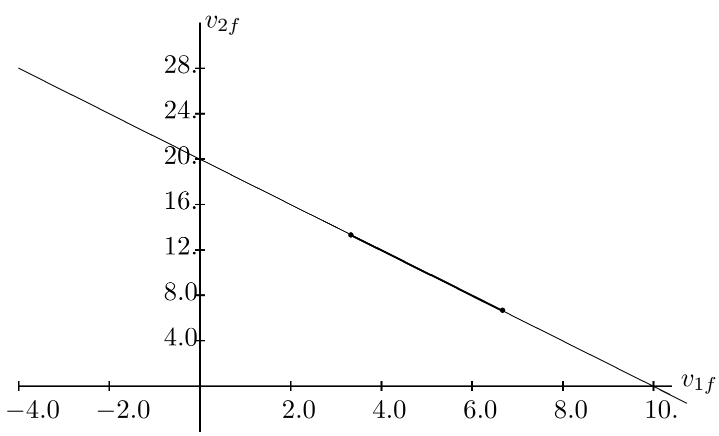
(h) With energy production not a possibility, then the "hardest rebound" m_1 can suffer is in an elastic collision, in which its final velocity (see part (b) of the previous problem) is $(10.0)(2-1)/3 \approx 3.33$ m/s.

(i) Eq. 10-31 gives the velocity of m_2 as $(10.0)(4/3) \approx 13.3$ m/s (see also part (b) of previous problem).

(j) As mentioned, this is an elastic collision (no "loss" of kinetic energy).

(k) The problem states that there's no energy production so that $K_{1i} - K_{1f} = K_{2f}$ and any greater value of $|v_{2f}|$ would violate this condition.

(l) The above graph is redrawn here, with the dark part representing the physically allowed region; the small circles bounding the dark segment correspond to the values calculated in the previous parts of this problem.



90. (First problem in **Cluster 2**)
The setup for this cluster refers to Fig. 10-16 in the chapter that assumes both angles are positive (at least, this is what is assumed in writing down Eq. 10-43) regardless of whether they are measured clockwise or counterclockwise. In this solution, we adopt that same convention.

(a) We first examine conservation of the y components of momentum:

$$\begin{aligned} 0 &= -m_1 v_{1f} \sin\theta_1 + m_2 v_{2f} \sin\theta_2 \\ 0 &= -m_1 (5.00 \text{ m/s}) \sin 30^\circ + (2m_1)\, v_{2f} \sin\theta_2 \end{aligned}$$

Next, we examine conservation of the x components of momentum.

$$\begin{aligned} m_1 v_{1i} &= m_1 v_{1f} \cos\theta_1 + m_2 v_{2f} \cos\theta_2 \\ m_1 (10.0 \text{ m/s}) &= m_1 (5.00 \text{ m/s}) \cos 30^\circ + (2m_1)\, v_{2f} \cos\theta_2 \end{aligned}$$

From the y equation, we obtain $1.25 = v_{2f} \sin\theta_2$ with SI units understood; similarly, the x equation yields $2.83 = v_{2f} \cos\theta_2$. Squaring these two relations and adding them leads to

$$1.25^2 + 2.83^2 = v_{2f}^2 \left(\sin^2\theta_2 + \cos^2\theta_2\right)$$

and consequently to $v_{2f} = \sqrt{1.25^2 + 2.83^2} = 3.10$ m/s. Plugging back in to either the x or y equation yields the angle $\theta_2 = 23.8°$.

(b) We compute decrease in total kinetic energy:

$$K_i - K_f = 27.9\, m_1$$

so that the collision is seen to be inelastic. We find that

$$\frac{27.9\, m_1}{\frac{1}{2}\, m_1\, 10^2} = 0.558\ ,$$

or roughly 56%, of the initial energy has been "lost."

91. (Second problem in **Cluster 2**)
As explained in the previous solution, we take both angles θ_1 and θ_2 to be positive-valued.

(a) We first examine conservation of the y components of momentum.

$$\begin{aligned} 0 &= -m_1\, v_{1f}\, \sin\theta_1 + m_2\, v_{2f}\, \sin\theta_2 \\ 0 &= -m_1 v_{1f}\, \sin 30° + 2m_1\, v_{2f}\, \sin\theta_2 \end{aligned}$$

Next, we examine conservation of the x components of momentum.

$$\begin{aligned} m_1\, v_{1i} &= m_1\, v_{1f}\, \cos\theta_1 + m_2\, v_{2f}\, \cos\theta_2 \\ m_1(10.0 \text{ m/s}) &= m_1 v_{1f}\, \cos 30° + 2m_1\, v_{2f}\, \cos\theta_2 \end{aligned}$$

From the y equation, we obtain $v_{1f} = 4\, v_{2f}\, \sin\theta_2$; similarly, the x equation yields $20 - v_{1f}\sqrt{3} = 4v_{2f}\, \cos\theta_2$ with SI units understood (also, $\cos 30° = \sqrt{3}/2$ has been used). Squaring these two relations and adding them leads to

$$v_{1f}^2\, (1+3) - 40 v_{1f}\sqrt{3} + 400 = 16\, v_{2f}^2\, \left(\sin^2\theta_2 + \cos^2\theta_2\right)$$

and thus to $v_{2f}^2 = v_{1f}^2/4 - 5v_{1f}\sqrt{3}/2 + 25$. We plug this into the condition of total kinetic energy "conservation."

$$\begin{aligned} K_i &= K_f \\ \frac{1}{2}\, m_1\, v_{1i}^2 &= \frac{1}{2}\, m_1\, v_{1f}^2 + \frac{1}{2}\, m_2\, v_{2f}^2 \\ \frac{1}{2} m_1 \left(10\ \frac{\text{m}}{\text{s}}\right)^2 &= \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2}\,(2m_1)\left(\frac{v_{1f}^2}{4} - \frac{5\sqrt{3}}{2} v_{1f} + 25\right) \end{aligned}$$

This leads to an equation of second degree (in the variable v_{1f}):

$$\frac{3}{4}\, v_{1f}^2 - \frac{5\sqrt{3}}{2}\, v_{1f} - 25 = 0$$

which has a positive root $v_{1f} = \frac{5}{3}\sqrt{3}\left(1 + \sqrt{5}\right) \approx 9.34$ m/s.

(b) We plug our result for v_{1f} into the relation $v_{2f} = \sqrt{v_{1f}^2/4 - 5v_{1f}\sqrt{3}/2 + 25}$ derived above and obtain $v_{2f} = \frac{5}{6}\sqrt{6}\left(\sqrt{5} - 1\right) \approx 2.52$ m/s.

(c) Plugging these values of v_{1f} and v_{2f} into, say, the $v_{1f} = 4\, v_{2f}\, \sin\theta_2$ relation, we find $\theta_2 = 67.8°$.

92. (Second problem in **Cluster 2**)
As explained in the first solution in this cluster, we take both angles θ_1 and θ_2 to be positive-valued.

(a) We first examine conservation of the y components of momentum.

$$\begin{aligned} 0 &= -m_1\, v_{1f}\, \sin\theta_1 + m_2\, v_{2f}\, \sin\theta_2 \\ 0 &= -m_1 v_{1f}\, \sin 30^\circ + 2m_1\, v_{2f}\, \sin\theta_2 \end{aligned}$$

Next, we examine conservation of the x components of momentum.

$$\begin{aligned} m_1\, v_{1i} &= m_1\, v_{1f}\, \cos\theta_1 + m_2\, v_{2f}\, \cos\theta_2 \\ m_1(10.0 \text{ m/s}) &= m_1 v_{1f}\, \cos 30^\circ + 2m_1\, v_{2f}\, \cos\theta_2 \end{aligned}$$

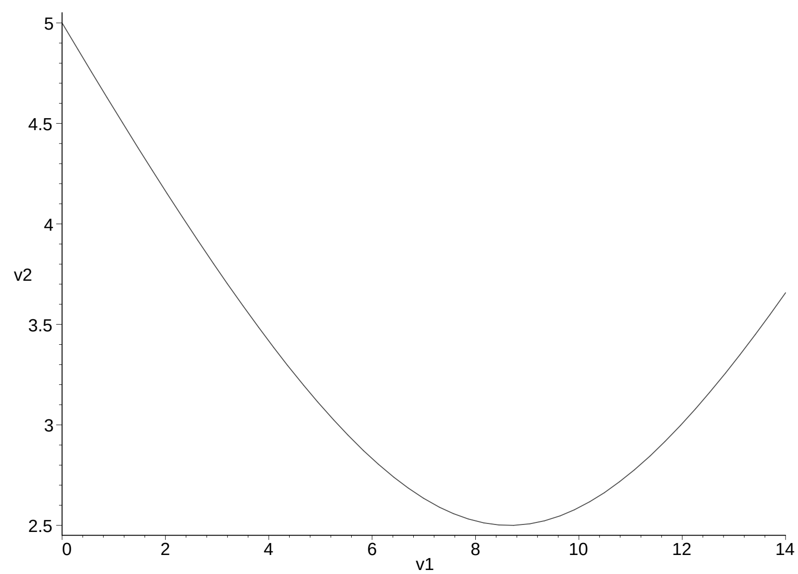
From the y equation, we obtain $v_{1f} = 4\, v_{2f}\, \sin\theta_2$; similarly, the x equation yields $20 - v_{1f}\sqrt{3} = 4v_{2f}\, \cos\theta_2$ with SI units understood (and the fact that $\cos 30^\circ = \sqrt{3}/2$ has been used). Squaring these two relations and adding them leads to

$$v_{1f}^2\,(1+3) - 40 v_{1f}\sqrt{3} + 400 = 16\, v_{2f}^2\left(\sin^2\theta_2 + \cos^2\theta_2\right)$$

and thus to

$$v_{2f}^2 = v_{1f}^2/4 - 5v_{1f}\sqrt{3}/2 + 25\ .$$

(b) The plot (v_{2f} versus v_{1f}) is shown below. The units for both axes are meters/second.



(c) Simply from the total kinetic energy requirement that $K_i \geq K_f$ we see immediately that $v_{1f} \leq v_{1i} = 10.0$ m/s (where the upper bound represents the trivial case where it passes m_2 by completely with $K_i = K_f$), and with the more stringent requirement that it does strike m_2 and scatters at $\theta_1 = 30^\circ$ we again find that it is bounded by the $K_i = K_f$ case. The elastic collision scenario was worked in the previous problem with the result $v_{1f} = 9.34$ m/s.

(d) And we also found the result $v_{2f} = 2.52$ m/s.

(e) As mentioned, this is an elastic collision.

(f) A higher speed for v_{1f} would require energy conversion into kinetic form (say, from an explosion) since $K_i < K_f$ would be the result.

(g) To save space, a separate graph for this part is not shown.

(h) Returning to the x and y equations derived in part (a), we divide them to obtain

$$\frac{v_{1f}}{20 - v_{1f}\sqrt{3}} = \frac{4v_{2f}\sin\theta_2}{4v_{2f}\cos\theta_2} = \tan\theta_2$$

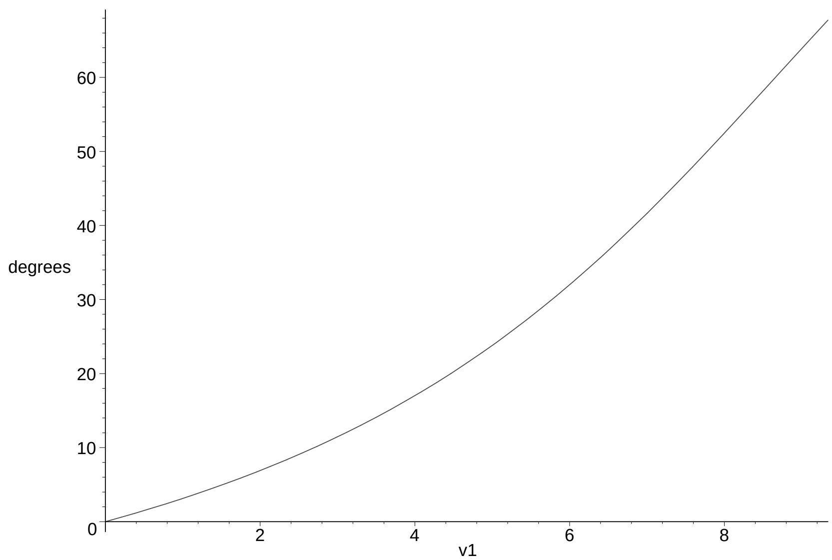
which leads to

$$\theta_2 = \tan^{-1}\left(\frac{v_{1f}}{20 - v_{1f}\sqrt{3}}\right) .$$

(i) See part (k).

(j) The value for the elastic case was computed in the previous problem; we find $\theta_2 = 67.8°$ when $v_{1f} = 9.34$ m/s.

(k) This corresponds to the upper righthand point of the curve shown below.



(l) , (m), (n), and (o)
Now, unlike the notation used in the one-dimensional collisions, this v_{1f} cannot be negative (it is the *magnitude* of the velocity). This suggests that its smallest value is zero, but the requirement that it scatter at $\theta_1 = 30°$ might seem to conflict with this. However, if one considers the (smooth) limit

of $v_{1f} \to 0$, we find there is nothing inconsistent with $\theta_1 = 30°$ in setting $v_{1f} = 0$. It is certainly inelastic (but not completely so! A completely inelastic collision *would* be inconsistent with the $\theta_1 = 30°$ condition!); we find from $v_{2f} = 5.00$ m/s (see the graph for part (b)) that $K_i < K_f$ in this case. Clearly, $\theta_2 = 0°$ in this circumstance (see, e.g., the graph for part(i)).