Chapter 30

1. (a) The magnitude of the magnetic field due to the current in the wire, at a point a distance r from the wire, is given by

$$B = \frac{\mu_0 i}{2\pi r} \ .$$

With $r = 20\,\text{ft} = 6.10\,\text{m}$, we find

$$B = \frac{(4\pi \times 10^{-7}\,\text{T}\cdot\text{m/A})(100\,\text{A})}{2\pi(6.10\,\text{m})} = 3.3 \times 10^{-6}\,\text{T} = 3.3\,\mu\text{T} \ .$$

(b) This is about one-sixth the magnitude of the Earth's field. It will affect the compass reading.

2. The current i due to the electron flow is $i = ne = (5.6 \times 10^{14}/\text{s})(1.6 \times 10^{-19}\,\text{C}) = 9.0 \times 10^{-5}\,\text{A}$. Thus,

$$B = \frac{\mu_0 i}{2\pi r} = \frac{(4\pi \times 10^{-7})(9.0 \times 10^{-5})}{2\pi(1.5 \times 10^{-3})} = 1.2 \times 10^{-8}\ \text{T} \ .$$

3. (a) The field due to the wire, at a point 8.0 cm from the wire, must be $39\,\mu\text{T}$ and must be directed due south. Since $B = \mu_0 i/2\pi r$,

$$i = \frac{2\pi r B}{\mu_0} = \frac{2\pi(0.080\,\text{m})(39 \times 10^{-6}\,\text{T})}{4\pi \times 10^{-7}\,\text{T}\cdot\text{m/A}} = 16\ \text{A} \ .$$

(b) The current must be from west to east to produce a field which is directed southward at points below it.

4. The points must be along a line parallel to the wire and a distance r from it, where r satisfies

$$B_{\text{wire}} = \frac{\mu_0 i}{2\pi r} = B_{\text{ext}} \ ,$$

or

$$r = \frac{\mu_0 i}{2\pi B_{\text{ext}}} = \frac{(1.26 \times 10^{-6}\,\text{T}\cdot\text{m/A})(100\,\text{A})}{2\pi(5.0 \times 10^{-3}\,\text{T})} = 4.0 \times 10^{-3}\ \text{m} \ .$$

5. We assume the current flows in the $+x$ direction and the particle is at some distance d in the $+y$ direction (away from the wire). Then, the magnetic field at the location of the charge q is

$$\vec{B} = \frac{\mu_0 i}{2\pi d}\ \hat{\text{k}} \ .$$

Thus,

$$\vec{F} = q\vec{v} \times \vec{B} = \frac{\mu_0 i q}{2\pi d}\left(\vec{v} \times \hat{\text{k}}\right) \ .$$

(a) In this situation, $\vec{v} = v(-\hat{\jmath})$ (where v is the speed and is a positive value). Also, the problem specifies $q > 0$. Thus,

$$\vec{F} = \frac{\mu_0 iqv}{2\pi d}\left((-\hat{\jmath}) \times \hat{k}\right) = -\frac{\mu_0 iqv}{2\pi d}(\hat{\imath}) ,$$

which tells us that $\vec{F}_q$ has a magnitude of $\mu_0 iqv/2\pi d$ and is in the direction opposite to that of the current flow.

(b) Now the direction $\vec{v}$ is reversed, and we obtain $\vec{F} = +\mu_0 iqv\hat{\imath}/2\pi d$. The magnitude is identical to that found in part (a), but the direction of the force is now in the same direction as that of the current flow.

6. The straight segment of the wire produces no magnetic field at C (see the *straight sections* discussion in Sample Problem 30-1). Also, the fields from the two semi-circular loops cancel at C (by symmetry). Therefore, $B_C = 0$.

7. Each of the semi-infinite straight wires contributes $\mu_0 i/4\pi R$ (Eq. 30-9) to the field at the center of the circle (both contributions pointing "out of the page"). The current in the arc contributes a term given by Eq. 30-11 pointing into the page, and this is able to produce zero total field at that location if

$$\begin{aligned} B_{\text{arc}} &= 2B_{\text{semi infinite}} \\ \frac{\mu_0 i\phi}{4\pi R} &= 2\left(\frac{\mu_0 i}{4\pi R}\right) \end{aligned}$$

which yields $\phi = 2$ rad.

8. Recalling the *straight sections* discussion in Sample Problem 30-1, we see that the current in segments AH and JD do not contribute to the field at point C. Using Eq. 30-11 (with $\phi = \pi$) and the right-hand rule, we find that the current in the semicircular arc HJ contributes $\mu_0 i/4R_1$ (into the page) to the field at C. Also, arc DA contributes $\mu_0 i/4R_2$ (out of the page) to the field there. Thus, the net field at C is

$$\vec{B} = \frac{\mu_0 i}{4}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{into the page .}$$

9. Recalling the *straight sections* discussion in Sample Problem 30-1, we see that the current in the straight segments colinear with P do not contribute to the field at that point. Using Eq. 30-11 (with $\phi = \theta$) and the right-hand rule, we find that the current in the semicircular arc of radius b contributes $\mu_0 i\theta/4\pi b$ (out of the page) to the field at P. Also, the current in the large radius arc contributes $\mu_0 i\theta/4\pi a$ (into the page) to the field there. Thus, the net field at P is

$$\vec{B} = \frac{\mu_0 i\theta}{4\pi}\left(\frac{1}{b} - \frac{1}{a}\right) \quad \text{out of the page .}$$

10. (a) Recalling the *straight sections* discussion in Sample Problem 30-1, we see that the current in the straight segments colinear with C do not contribute to the field at that point.

(b) Eq. 30-11 (with $\phi = \pi$) indicates that the current in the semicircular arc contributes $\mu_0 i/4R$ to the field at C. The right-hand rule shows that this field is into the page.

(c) The contributions from parts (a) and (b) sum to

$$\vec{B} = \frac{\mu_0 i}{4R} \quad \text{into the page .}$$

11. Our x axis is along the wire with the origin at the midpoint. The current flows in the positive x direction. All segments of the wire produce magnetic fields at P_1 that are out of the page. According to the Biot-Savart law, the magnitude of the field any (infinitesimal) segment produces at P_1 is given by

$$dB = \frac{\mu_0 i}{4\pi}\frac{\sin\theta}{r^2}\,dx$$

where θ (the angle between the segment and a line drawn from the segment to P_1) and r (the length of that line) are functions of x. Replacing r with $\sqrt{x^2+R^2}$ and $\sin\theta$ with $R/r = R/\sqrt{x^2+R^2}$, we integrate from $x=-L/2$ to $x=L/2$. The total field is

$$B = \frac{\mu_0 iR}{4\pi}\int_{-L/2}^{L/2}\frac{dx}{(x^2+R^2)^{3/2}} = \frac{\mu_0 iR}{4\pi}\frac{1}{R^2}\frac{x}{(x^2+R^2)^{1/2}}\bigg|_{-L/2}^{L/2} = \frac{\mu_0 i}{2\pi R}\frac{L}{\sqrt{L^2+4R^2}}\ .$$

If $L \gg R$, then R^2 in the denominator can be ignored and

$$B = \frac{\mu_0 i}{2\pi R}$$

is obtained. This is the field of a long straight wire. For points very close to a finite wire, the field is quite similar to that of an infinitely long wire.

12. The center of a square is a distance $R = a/2$ from the nearest side (each side being of length $L = a$). There are four sides contributing to the field at the center, so the result of problem 11 leads to

$$B_{\text{center}} = 4\left(\frac{\mu_0 i}{2\pi(a/2)}\right)\left(\frac{a}{\sqrt{a^2+4(a/2)^2}}\right) = \frac{2\sqrt{2}\,\mu_0 i}{\pi a}\ .$$

13. Our x axis is along the wire with the origin at the right endpoint, and the current is in the positive x direction. All segments of the wire produce magnetic fields at P_2 that are out of the page. According to the Biot-Savart law, the magnitude of the field any (infinitesimal) segment produces at P_2 is given by

$$dB = \frac{\mu_0 i}{4\pi}\frac{\sin\theta}{r^2}\,dx$$

where θ (the angle between the segment and a line drawn from the segment to P_2) and r (the length of that line) are functions of x. Replacing r with $\sqrt{x^2+R^2}$ and $\sin\theta$ with $R/r = R/\sqrt{x^2+R^2}$, we integrate from $x=-L$ to $x=0$. The total field is

$$B = \frac{\mu_0 iR}{4\pi}\int_{-L}^{0}\frac{dx}{(x^2+R^2)^{3/2}} = \frac{\mu_0 iR}{4\pi}\frac{1}{R^2}\frac{x}{(x^2+R^2)^{1/2}}\bigg|_{-L}^{0} = \frac{\mu_0 i}{4\pi R}\frac{L}{\sqrt{L^2+R^2}}\ .$$

14. We refer to the side of length L as the long side and that of length W as the short side. The center is a distance $W/2$ from the midpoint of each long side, and is a distance $L/2$ from the midpoint of each short side. There are two of each type of side, so the result of problem 11 leads to

$$B = 2\frac{\mu_0 i}{2\pi(W/2)}\frac{L}{\sqrt{L^2+4(W/2)^2}} + 2\frac{\mu_0 i}{2\pi(L/2)}\frac{W}{\sqrt{W^2+4(L/2)^2}}\ .$$

The final form of this expression, shown in the problem statement, derives from finding the common denominator of the above result and adding them, while noting that

$$\frac{L^2+W^2}{\sqrt{W^2+L^2}} = \sqrt{W^2+L^2}\ .$$

15. We imagine the square loop in the yz plane (with its center at the origin) and the evaluation point for the field being along the x axis (as suggested by the notation in the problem). The origin is a distance $a/2$ from each side of the square loop, so the distance from the evaluation point to each side of the square is, by the Pythagorean theorem,

$$R = \sqrt{(a/2)^2+x^2} = \frac{1}{2}\sqrt{a^2+4x^2}\ .$$

Only the x components of the fields (contributed by each side) will contribute to the final result (other components cancel in pairs), so a trigonometric factor of

$$\frac{a/2}{R} = \frac{a}{\sqrt{a^2 + 4x^2}}$$

multiplies the expression of the field given by the result of problem 11 (for each side of length $L = a$). Since there are four sides, we find

$$B(x) = 4\left(\frac{\mu_0 i}{2\pi R}\right)\left(\frac{a}{\sqrt{a^2+4R^2}}\right)\left(\frac{a}{\sqrt{a^2+4x^2}}\right) = \frac{4\,\mu_0\, i\, a^2}{2\pi\left(\frac{1}{2}\right)\left(\sqrt{a^2+4x^2}\right)^2\ \sqrt{a^2+4(a/2)^2+4x^2}}$$

which simplifies to the desired result. It is straightforward to set $x = 0$ and see that this reduces to the expression found in problem 12 (noting that $\frac{4}{\sqrt{2}} = 2\sqrt{2}$).

16. Our y axis is along the wire with the origin at the top endpoint, and the current is in the positive y direction. All segments of the wire produce magnetic fields at P that are into the page. According to the Biot-Savart law, the magnitude of the field any (infinitesimal) segment produces at P is given by

$$dB = \frac{\mu_0 i}{4\pi}\frac{\sin\theta}{r^2}\,dy$$

where θ (the angle between the segment and a line drawn from the segment to P) and r (the length of that line) are functions of y. Replacing r with $\sqrt{y^2+a^2}$ and $\sin\theta$ with $a/r = a/\sqrt{y^2+a^2}$, we integrate from $y = -a$ to $y = 0$. The total field is

$$B = \frac{\mu_0 i a}{4\pi}\int_{-a}^{0}\frac{dy}{(y^2+a^2)^{3/2}} = \frac{\mu_0 i a}{4\pi}\,\frac{1}{a^2}\,\frac{y}{(y^2+a^2)^{1/2}}\bigg|_{-a}^{0} = \frac{\mu_0 i}{4\pi a}\,\frac{a}{\sqrt{a^2+a^2}}$$

which simplifies to the desired result (noting that $\frac{1}{4\sqrt{2}} = \frac{\sqrt{2}}{8}$).

17. Using the result of problem 12 and Eq. 30-12, we wish to show that

$$\frac{2\sqrt{2}\,\mu_0 i}{\pi a} > \frac{\mu_0 i}{2R} \quad , \quad \text{or} \quad \frac{4\sqrt{2}}{\pi a} > \frac{1}{R} \ ,$$

but to do this we must relate the parameters a and R. If both wires have the same length L then the geometrical relationships $4a = L$ and $2\pi R = L$ provide the necessary connection:

$$4a = 2\pi R \quad\Longrightarrow\quad a = \frac{\pi R}{2}\ .$$

Thus, our proof consists of the observation that

$$\frac{4\sqrt{2}}{\pi a} = \frac{8\sqrt{2}}{\pi^2 R} > \frac{1}{R}\ ,$$

as one can check numerically (that $8\sqrt{2}/\pi^2 > 1$).

18. Recalling the *straight sections* discussion in Sample Problem 30-1, we see that the current in the straight segments colinear with P do not contribute to the field at that point. We use the result of problem 16 to evaluate the contributions to the field at P, noting that the nearest wire-segments (each of length a) produce magnetism into the page at P and the further wire-segments (each of length $2a$) produce magnetism pointing out of the page at P. Thus, we find (into the page)

$$B_P = 2\left(\frac{\sqrt{2}\mu_0 i}{8\pi a}\right) - 2\left(\frac{\sqrt{2}\mu_0 i}{8\pi(2a)}\right) = \frac{\sqrt{2}\mu_0 i}{8\pi a}\ .$$

19. Consider a section of the ribbon of thickness dx located a distance x away from point P. The current it carries is $di = i\,dx/w$, and its contribution to B_P is

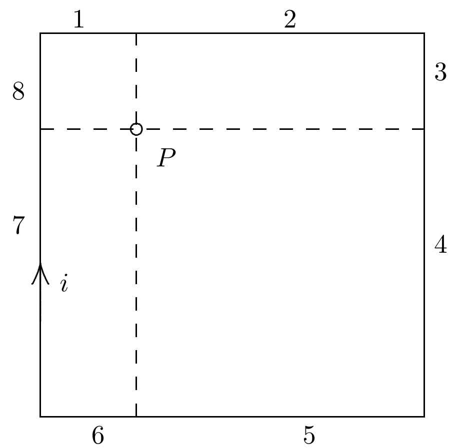
$$dB_P = \frac{\mu_0 di}{2\pi x} = \frac{\mu_0 i dx}{2\pi x w} \ .$$

Thus,

$$B_P = \int d\,B_P = \frac{\mu_0 i}{2\pi w}\int_d^{d+w}\frac{dx}{x} = \frac{\mu_0 i}{2\pi w}\,\ln\Big(1+\frac{w}{d}\Big) \ ,$$

and $\vec{B}_P$ points upward.

20. The two small wire-segments, each of length $a/4$, shown in Fig. 30-39 nearest to point P, are labeled 1 and 8 in the figure below.



Let $\vec{e}$ be a unit vector pointing into the page. We use the results of problems 13 and 16 to calculate B_{P1} through B_{P8}:

$$\begin{aligned}
B_{P1} &= B_{P8} = \frac{\sqrt{2}\mu_0 i}{8\pi(a/4)} = \frac{\sqrt{2}\mu_0 i}{2\pi a} \ , \\
B_{P4} &= B_{P5} = \frac{\sqrt{2}\mu_0 i}{8\pi(3a/4)} = \frac{\sqrt{2}\mu_0 i}{6\pi a} \ , \\
B_{P2} &= B_{P7} = \frac{\mu_0 i}{4\pi(a/4)}\cdot\frac{3a/4}{[(3a/4)^2+(a/4)^2]^{1/2}} = \frac{3\mu_0 i}{\sqrt{10}\pi a} \ ,
\end{aligned}$$

and

$$B_{P3} = B_{P6} = \frac{\mu_0 i}{4\pi(3a/4)}\cdot\frac{a/4}{[(a/4)^2+(3a/4)^2]^{1/2}} = \frac{\mu_0 i}{3\sqrt{10}\pi a} \ .$$

Finally,

$$\begin{aligned}
\vec{B}_P &= \sum_{n=1}^{8} B_{Pn}\vec{e} \\
&= 2\,\frac{\mu_0 i}{\pi a}\left(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{6}+\frac{3}{\sqrt{10}}+\frac{1}{3\sqrt{10}}\right)\vec{e} \\
&= \frac{2(4\pi\times10^{-7}\,\mathrm{T\cdot m/A})(10\,\mathrm{A})}{\pi(8.0\times10^{-2}\,\mathrm{m})}\left(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{6}+\frac{3}{\sqrt{10}}+\frac{1}{3\sqrt{10}}\right)\vec{e} \\
&= (2.0\times10^{-4}\,\mathrm{T})\,\vec{e} \ ,
\end{aligned}$$

where $\vec{e}$ is a unit vector pointing into the page.

21. (a) If the currents are parallel, the two fields are in opposite directions in the region between the wires. Since the currents are the same, the total field is zero along the line that runs halfway between the wires. There is no possible current for which the field does not vanish.

(b) If the currents are antiparallel, the fields are in the same direction in the region between the wires. At a point halfway between they have the same magnitude, $\mu_0 i/2\pi r$. Thus the total field at the midpoint has magnitude $B = \mu_0 i/\pi r$ and

$$i = \frac{\pi r B}{\mu_0} = \frac{\pi(0.040\,\mathrm{m})(300 \times 10^{-6}\,\mathrm{T})}{4\pi \times 10^{-7}\,\mathrm{T{\cdot}m/A}} = 30\ \mathrm{A}\ .$$

22. Since they carry current in the same direction, then (by the right-hand rule) the only region in which their fields might cancel is between them. Thus, if the point at which we are evaluating their field is r away from the wire carrying current i and is $d - r$ away from the wire carrying current $3i$, then the canceling of their fields leads to

$$\frac{\mu_0 i}{2\pi r} = \frac{\mu_0 (3i)}{2\pi(d-r)} \quad \Longrightarrow \quad r = \frac{d}{4}\ .$$

23. Using the right-hand rule, we see that the current i_2 carried by wire 2 must be out of the page. Now, $B_{P1} = \mu_0 i_1/2\pi r_1$ where $i_1 = 6.5\,\mathrm{A}$ and $r_1 = 0.75\,\mathrm{cm} + 1.5\,\mathrm{cm} = 2.25\,\mathrm{cm}$, and $B_{P2} = \mu_0 i_2/2\pi r_2$ where $r_2 = 1.5\,\mathrm{cm}$. From $B_{P1} = B_{P2}$ we get

$$i_2 = i_1\left(\frac{r_2}{r_1}\right) = (6.5\,\mathrm{A})\left(\frac{1.5\,\mathrm{cm}}{2.25\,\mathrm{cm}}\right) = 4.3\ \mathrm{A}\ .$$

24. We label these wires 1 through 5, left to right, and use Eq. 30-15 (divided by length). Then,

$$\begin{aligned}\vec{F}_1 &= \frac{\mu_0 i^2}{2\pi}\left(\frac{1}{d} + \frac{1}{2d} + \frac{1}{3d} + \frac{1}{4d}\right)\hat{\jmath} = \frac{25\mu_0 i^2}{24\pi d}\,\hat{\jmath} \\ &= \frac{(13)(4\pi \times 10^{-7}\,\mathrm{T{\cdot}m/A})(3.00\,\mathrm{A})^2(1.00\,\mathrm{m})\hat{\jmath}}{24\pi(8.00 \times 10^{-2}\,\mathrm{m})} \\ &= 4.69 \times 10^{-5}\,\mathrm{N/m}\ \hat{\jmath}\ ;\end{aligned}$$

$$\vec{F}_2 = \frac{\mu_0 i^2}{2\pi}\left(\frac{1}{2d} + \frac{1}{3d}\right)\hat{\jmath} = \frac{5\mu_0 i^2}{12\pi d}\,\hat{\jmath} = 1.88 \times 10^{-5}\,\mathrm{N/m}\ \hat{\jmath}\ ;$$

$F_3 = 0$ (because of symmetry); $\vec{F}_4 = -\vec{F}_2$; and $\vec{F}_5 = -\vec{F}_1$.

25. Each wire produces a field with magnitude given by $B = \mu_0 i/2\pi r$, where r is the distance from the corner of the square to the center. According to the Pythagorean theorem, the diagonal of the square has length $\sqrt{2}a$, so $r = a/\sqrt{2}$ and $B = \mu_0 i/\sqrt{2}\pi a$. The fields due to the wires at the upper left and lower right corners both point toward the upper right corner of the square. The fields due to the wires at the upper right and lower left corners both point toward the upper left corner. The horizontal components cancel and the vertical components sum to

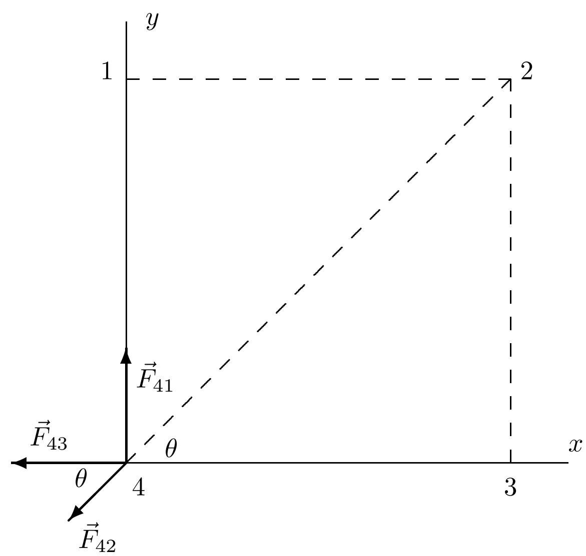
$$\begin{aligned}B_{\text{total}} &= 4\frac{\mu_0 i}{\sqrt{2}\pi a}\cos 45^\circ = \frac{2\mu_0 i}{\pi a} \\ &= \frac{2(4\pi \times 10^{-7}\,\mathrm{T{\cdot}m/A})(20\,\mathrm{A})}{\pi(0.20\,\mathrm{m})} = 8.0 \times 10^{-5}\ \mathrm{T}\ .\end{aligned}$$

In the calculation $\cos 45^\circ$ was replaced with $1/\sqrt{2}$. The total field points upward.

26. Using Eq. 30-15, the force on, say, wire 1 (the wire at the upper left of the figure) is along the diagonal (pointing towards wire 3 which is at the lower right). Only the forces (or their components) along the diagonal direction contribute. With $\theta = 45°$, we find

$$\begin{aligned} F_1 &= \left|\vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14}\right| \\ &= 2F_{12}\cos\theta + F_{13} \\ &= 2\left(\frac{\mu_0 i^2}{2\pi a}\right)\cos 45° + \frac{\mu_0 i^2}{2\sqrt{2}\pi a} \\ &= 0.338\left(\frac{\mu_0 i^2}{a}\right) . \end{aligned}$$

27. We use Eq. 30-15 and the superposition of forces: $\vec{F}_4 = \vec{F}_{14} + \vec{F}_{24} + \vec{F}_{34}$. With $\theta = 45°$, the situation is as shown below:



The components of $\vec{F}_4$ are given by

$$\begin{aligned} F_{4x} &= -F_{43} - F_{42}\cos\theta \\ &= -\frac{\mu_0 i^2}{2\pi a} - \frac{\mu_0 i^2 \cos 45°}{2\sqrt{2}\pi a} \\ &= -\frac{3\mu_0 i^2}{4\pi a} \end{aligned}$$

and

$$\begin{aligned} F_{4y} &= F_{41} - F_{42}\sin\theta \\ &= \frac{\mu_0 i^2}{2\pi a} - \frac{\mu_0 i^2 \sin 45°}{2\sqrt{2}\pi a} \\ &= \frac{\mu_0 i^2}{4\pi a} . \end{aligned}$$

Thus,

$$F_4 = (F_{4x}^2 + F_{4y}^2)^{1/2} = \left[\left(-\frac{3\mu_0 i^2}{4\pi a}\right)^2 + \left(\frac{\mu_0 i^2}{4\pi a}\right)^2\right]^{1/2} = \frac{\sqrt{10}\mu_0 i^2}{4\pi a} ,$$

and $\vec{F}_4$ makes an angle ϕ with the positive x axis, where

$$\phi = \tan^{-1}\left(\frac{F_{4y}}{F_{4x}}\right) = \tan^{-1}\left(-\frac{1}{3}\right) = 162^\circ \ .$$

28. (a) Consider a segment of the projectile between y and $y + dy$. We use Eq. 30-14 to find the magnetic force on the segment, and Eq. 30-9 for the magnetic field of each semi-infinite wire (the top rail referred to as wire 1 and the bottom as wire 2). The current in rail 1 is in the $+\hat{\imath}$ direction, and the current in the rail 2 is in the $-\hat{\imath}$ direction. The field (in the region between the wires) set up by wire 1 is into the paper (the $-\hat{k}$ direction) and that set up by wire 2 is also into the paper. The force element (a function of y) acting on the segment of the projectile (in which the current flows in the $-\hat{\jmath}$ direction) is given below. The coordinate origin is at the bottom of the projectile.

$$\begin{aligned} d\vec{F} &= d\vec{F}_1 + d\vec{F}_2 \\ &= i\,dy(-\hat{\jmath}) \times \vec{B}_1 + \,dy(-\hat{\jmath}) \times \vec{B}_2 \\ &= i[B_1 + B_2]\,\hat{\imath}\,dy \\ &= i\left[\frac{\mu_0 i}{4\pi(2R + w - y)} + \frac{\mu_0 i}{4\pi y}\right]\hat{\imath}\,dy \ . \end{aligned}$$

Thus, the force on the projectile is

$$\vec{F} = \int d\vec{F} = \frac{i^2\mu_0}{4\pi}\int_R^{R+w}\left(\frac{1}{2R + w - y} + \frac{1}{y}\right)dy\,\hat{\imath} \ = \frac{\mu_0 i^2}{2\pi}\ln\left(1 + \frac{w}{R}\right)\hat{\imath} \ .$$

(b) Using the work-energy theorem, we have $\Delta K = \frac{1}{2}mv_f^2 = W_{\text{ext}} = \int \vec{F}\cdot d\vec{s} = FL$. Thus, the final speed of the projectile is

$$\begin{aligned} v_f &= \left(\frac{2W_{\text{ext}}}{m}\right)^{1/2} = \left[\frac{2}{m}\,\frac{\mu_0 i^2}{2\pi}\ln\left(1 + \frac{w}{R}\right)L\right]^{1/2} \\ &= \left[\frac{2(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(450\times10^3\,\text{A})^2\,\ln(1 + 1.2\,\text{cm}/6.7\,\text{cm})(4.0\,\text{m})}{2\pi(10\times10^{-3}\,\text{kg})}\right]^{1/2} \\ &= 2.3\times10^3\ \text{m/s} \ . \end{aligned}$$

29. The magnitudes of the forces on the sides of the rectangle which are parallel to the long straight wire (with $i_1 = 30$ A) are computed using Eq. 30-15, but the force on each of the sides lying perpendicular to it (along our y axis, with the origin at the top wire and $+y$ downward) would be figured by integrating as follows:

$$F_{\perp\,\text{sides}} = \int_a^{a+b}\frac{i_2\mu_0 i_1}{2\pi y}\,dy \ .$$

Fortunately, these forces on the two perpendicular sides of length b cancel out. For the remaining two (parallel) sides of length L, we obtain

$$\begin{aligned} F &= \frac{\mu_0 i_1 i_2 L}{2\pi}\left(\frac{1}{a} - \frac{1}{a+d}\right) = \frac{\mu_0 i_1 i_2 b}{2\pi a(a+b)} \\ &= \frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(30\,\text{A})(20\,\text{A})(8.0\,\text{cm})(30\times10^{-2}\,\text{m})}{2\pi(1.0\,\text{cm} + 8.0\,\text{cm})} \\ &= 3.2\times10^{-3}\ \text{N} \ , \end{aligned}$$

and $\vec{F}$ points toward the wire.

30. A close look at the path reveals that only currents 1, 3, 6 and 7 are enclosed. Thus, noting the different current directions described in the problem, we obtain

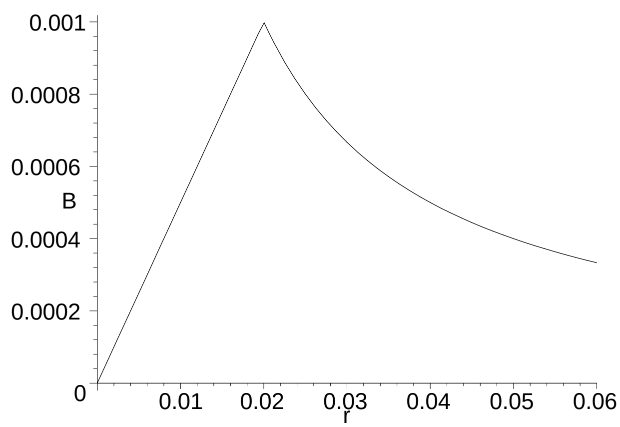
$$\oint \vec{B} \cdot d\vec{s} = \mu_0(7i - 6i + 3i + i) \quad = \quad 5\mu_0 i \ .$$

31. (a) Two of the currents are out of the page and one is into the page, so the net current enclosed by the path is 2.0 A, out of the page. Since the path is traversed in the clockwise sense, a current into the page is positive and a current out of the page is negative, as indicated by the right-hand rule associated with Ampere's law. Thus,

$$\oint \vec{B} \cdot d\vec{s} = -\mu_0 i = -(2.0\,\text{A})(4\pi \times 10^{-7}\,\text{T·m/A}) = -2.5 \times 10^{-6}\,\text{T·m} \ .$$

(b) The net current enclosed by the path is zero (two currents are out of the page and two are into the page), so $\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{\text{enc}} = 0$.

32. We use Eq. 30-22 for the B-field inside the wire and Eq. 30-19 for that outside the wire. The plot is shown below (with SI units understood).



33. We use Ampere's law. For the dotted loop shown on the diagram $i = 0$. The integral $\int \vec{B} \cdot d\vec{s}$ is zero along the bottom, right, and top sides of the loop. Along the right side the field is zero, along the top and bottom sides the field is perpendicular to $d\vec{s}$. If ℓ is the length of the left edge, then direct integration yields $\oint \vec{B} \cdot d\vec{s} = B\ell$, where B is the magnitude of the field at the left side of the loop. Since neither B nor ℓ is zero, Ampere's law is contradicted. We conclude that the geometry shown for the magnetic field lines is in error. The lines actually bulge outward and their density decreases gradually, not discontinuously as suggested by the figure.

34. We use Ampere's law: $\oint \vec{B} \cdot d\vec{s} = \mu_0 i$, where the integral is around a closed loop and i is the net current through the loop. For path 1, the result is

$$\begin{aligned} \oint_1 \vec{B} \cdot d\vec{s} &= \mu_0(-5.0\,\text{A} + 3.0\,\text{A}) = (-2.0\,\text{A})(4\pi \times 10^{-7}\,\text{T·m/A}) \\ &= -2.5 \times 10^{-6}\ \text{T·m} \ . \end{aligned}$$

For path 2, we find

$$\begin{aligned} \oint_2 \vec{B} \cdot d\vec{s} &= \mu_0(-5.0\,\text{A} - 5.0\,\text{A} - 3.0\,\text{A}) = (-13.0\,\text{A})(4\pi \times 10^{-7}\,\text{T·m/A}) \\ &= -1.6 \times 10^{-5}\ \text{T·m} \ . \end{aligned}$$

35. For $r < a$,

$$B(r) = \frac{\mu_0 i_{\text{enc}}}{2\pi r} = \frac{\mu_0}{2\pi r} \int_0^r J(r) 2\pi r \, dr = \frac{\mu_0}{2\pi} \int_0^r J_0 \left(\frac{r}{a}\right) 2\pi r \, dr = \frac{\mu_0 J_0 r^2}{3a} \ .$$

36. (a) Replacing $i/\pi R^2$ with $J = 100 \text{ A/m}^2$, in Eq. 30-22, we have

$$\left|\vec{B}\right| = \left(\frac{\mu_0 J}{2}\right) r = 1.3 \times 10^{-7} \text{ T}$$

where $r = 0.0020$ m.

(b) Similarly, writing $i = J\pi R^2$ in Eq. 30-19 yields

$$\left|\vec{B}\right| = \frac{\mu_0 J R^2}{2r} = 1.4 \times 10^{-7} \text{ T}$$

where $r = 0.0040$ m.

37. (a) The the magnetic field at a point within the hole is the sum of the fields due to two current distributions. The first is that of the solid cylinder obtained by filling the hole and has a current density that is the same as that in the original cylinder (with the hole). The second is the solid cylinder that fills the hole. It has a current density with the same magnitude as that of the original cylinder but is in the opposite direction. If these two situations are superposed the total current in the region of the hole is zero. Now, a solid cylinder carrying current i, uniformly distributed over a cross section, produces a magnetic field with magnitude

$$B = \frac{\mu_0 i r}{2\pi R^2}$$

a distance r from its axis, inside the cylinder. Here R is the radius of the cylinder. For the cylinder of this problem the current density is

$$J = \frac{i}{A} = \frac{i}{\pi(a^2 - b^2)} \ ,$$

where $A = \pi(a^2 - b^2)$ is the cross-sectional area of the cylinder with the hole. The current in the cylinder without the hole is

$$I_1 = JA = \pi J a^2 = \frac{ia^2}{a^2 - b^2}$$

and the magnetic field it produces at a point inside, a distance r_1 from its axis, has magnitude

$$B_1 = \frac{\mu_0 I_1 r_1}{2\pi a^2} = \frac{\mu_0 i r_1 a^2}{2\pi a^2 (a^2 - b^2)} = \frac{\mu_0 i r_1}{2\pi(a^2 - b^2)} \ .$$

The current in the cylinder that fills the hole is

$$I_2 = \pi J b^2 = \frac{ib^2}{a^2 - b^2}$$

and the field it produces at a point inside, a distance r_2 from the its axis, has magnitude

$$B_2 = \frac{\mu_0 I_2 r_2}{2\pi b^2} = \frac{\mu_0 i r_2 b^2}{2\pi b^2 (a^2 - b^2)} = \frac{\mu_0 i r_2}{2\pi(a^2 - b^2)} \ .$$

At the center of the hole, this field is zero and the field there is exactly the same as it would be if the hole were filled. Place $r_1 = d$ in the expression for B_1 and obtain

$$B = \frac{\mu_0 i d}{2\pi(a^2 - b^2)}$$

for the field at the center of the hole. The field points upward in the diagram if the current is out of the page.

(b) If $b = 0$ the formula for the field becomes

$$B = \frac{\mu_0 i d}{2\pi a^2} \ .$$

This correctly gives the field of a solid cylinder carrying a uniform current i, at a point inside the cylinder a distance d from the axis. If $d = 0$ the formula gives $B = 0$. This is correct for the field on the axis of a cylindrical shell carrying a uniform current.

(c) Consider a rectangular path with two long sides (side 1 and 2, each with length L) and two short sides (each of length less than b). If side 1 is directly along the axis of the hole, then side 2 would be also parallel to it and also in the hole. To ensure that the short sides do not contribute significantly to the integral in Ampere's law, we might wish to make L *very* long (perhaps longer than the length of the cylinder), or we might appeal to an argument regarding the angle between $\vec{B}$ and the short sides (which is 90° at the axis of the hole). In any case, the integral in Ampere's law reduces to

$$\begin{aligned} \oint_{\text{rectangle}} \vec{B} \cdot d\vec{s} &= \mu_0 i_{\text{enclosed}} \\ \int_{\text{side 1}} \vec{B} \cdot d\vec{s} + \int_{\text{side 2}} \vec{B} \cdot d\vec{s} &= \mu_0 i_{\text{in hole}} \\ (B_{\text{side 1}} - B_{\text{side 2}})\, L &= 0 \end{aligned}$$

where $B_{\text{side 1}}$ is the field along the axis found in part (a). This shows that the field at off-axis points (where $B_{\text{side 2}}$ is evaluated) is the same as the field at the center of the hole; therefore, the field in the hole is uniform.

38. The field at the center of the pipe (point C) is due to the wire alone, with a magnitude of

$$B_C = \frac{\mu_0 i_{\text{wire}}}{2\pi(3R)} = \frac{\mu_0 i_{\text{wire}}}{6\pi R} \ .$$

For the wire we have $B_{P,\,\text{wire}} > B_{C,\,\text{wire}}$. Thus, for $B_P = B_C = B_{C,\,\text{wire}}$, i_{wire} must be into the page:

$$B_P = B_{P,\,\text{wire}} - B_{P,\,\text{pipe}} = \frac{\mu_0 i_{\text{wire}}}{2\pi R} - \frac{\mu_0 i}{2\pi(2R)} \ .$$

Setting $B_C = -B_P$ we obtain $i_{\text{wire}} = 3i/8$.

39. The "current per unit x-length" may be viewed as current density multiplied by the thickness Δy of the sheet; thus, $\lambda = J\Delta y$. Ampere's law may be (and often is) expressed in terms of the current density vector as follows:

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \int \vec{J} \cdot d\vec{A}$$

where the area integral is over the region enclosed by the path relevant to the line integral (and $\vec{J}$ is in the $+z$ direction, out of the paper). With J uniform throughout the sheet, then it clear that the right-hand side of this version of Ampere's law should reduce, in this problem, to $\mu_0 JA = \mu_0 J\Delta y\Delta x = \mu_0 \lambda \Delta x$.

(a) Figure 30-52 certainly has the horizontal components of $\vec{B}$ drawn correctly at points P and P' (as reference to Fig. 30-4 will confirm [consider the current elements nearest each of those points]), so the question becomes: is it possible for $\vec{B}$ to have vertical components in the figure? Our focus is on point P. Fig. 30-4 suggests that the current element just to the right of the nearest one (the one directly under point P) will contribute a downward component, but by the same reasoning the current element just to the left of the nearest one should contribute an upward component to the field at P. The current elements are all equivalent, as is reflected in the horizontal-translational symmetry built into this problem; therefore, all vertical components should cancel in pairs. The field at P must be purely horizontal, as drawn.

(b) The path used in evaluating $\oint \vec{B}\cdot d\vec{s}$ is rectangular, of horizontal length Δx (the horizontal sides passing through points P and P' respectively) and vertical size $\delta y > \Delta y$. The vertical sides have no contribution to the integral since $\vec{B}$ is purely horizontal (so the scalar dot product produces zero for those sides), and the horizontal sides contribute two equal terms, as shown below. Ampere's law yields

$$2B\Delta x = \mu_0\lambda\Delta x \implies B = \frac{1}{2}\mu_0\lambda \ .$$

40. It is possible (though tedious) to use Eq. 30-28 and evaluate the contributions (with the intent to sum them) of all 1200 loops to the field at, say, the center of the solenoid. This would make use of all the information given in the problem statement, but this is not the method that the student is expected to use here. Instead, Eq. 30-25 for the ideal solenoid (which does not make use of the coil radius) is the preferred method:

$$B = \mu_0 in = \mu_0 i\left(\frac{N}{\ell}\right)$$

where $i = 3.60\,\mathrm{A}$, $\ell = 0.950\,\mathrm{m}$ and $N = 1200$. This yields $B = 0.00571$ T.

41. It is possible (though tedious) to use Eq. 30-28 and evaluate the contributions (with the intent to sum them) of all 200 loops to the field at, say, the center of the solenoid. This would make use of all the information given in the problem statement, but this is not the method that the student is expected to use here. Instead, Eq. 30-25 for the ideal solenoid (which does not make use of the coil diameter) is the preferred method:

$$B = \mu_0 in = \mu_0 i\left(\frac{N}{\ell}\right)$$

where $i = 0.30\,\mathrm{A}$, $\ell = 0.25\,\mathrm{m}$ and $N = 200$. This yields $B = 0.0030$ T.

42. We find N, the number of turns of the solenoid, from $B = \mu_0 in = \mu_0 iN/\ell$: $N = B\ell/\mu_0 i$. Thus, the total length of wire used in making the solenoid is

$$2\pi rN = \frac{2\pi rB\ell}{\mu_0 i} = \frac{2\pi(2.60\times10^{-2}\,\mathrm{m})(23.0\times10^{-3}\,\mathrm{T})(1.30\,\mathrm{m})}{2(4\pi\times10^{-7}\,\mathrm{T{\cdot}m/A})(18.0\,\mathrm{A})} = 108\ \mathrm{m}\ .$$

43. (a) We use Eq. 30-26. The inner radius is $r = 15.0\,\mathrm{cm}$, so the field there is

$$B = \frac{\mu_0 iN}{2\pi r} = \frac{(4\pi\times10^{-7}\,\mathrm{T\cdot m/A})(0.800\,\mathrm{A})(500)}{2\pi(0.150\,\mathrm{m})} = 5.33\times10^{-4}\ \mathrm{T}\ .$$

(b) The outer radius is $r = 20.0\,\mathrm{cm}$. The field there is

$$B = \frac{\mu_0 iN}{2\pi r} = \frac{(4\pi\times10^{-7}\,\mathrm{T\cdot m/A})(0.800\,\mathrm{A})(500)}{2\pi(0.200\mathrm{m})} = 4.00\times10^{-4}\ \mathrm{T}\ .$$

44. (a) The ideal solenoid is long enough (and we are evaluating the field at a point far enough inside) such that the open ends of the solenoid are "out of sight" and the situation displays a horizontal-translational symmetry (assuming the axis of the cylindrical shape of the solenoid is horizontal). A view of a "slice" of, say, the bottom of the solenoid would therefore appear similar to that shown in Fig. 30-52, where point P is in the interior of the solenoid and point P' is outside the coil. Now, Fig. 30-52 differs in at least one respect from our "slice" view of the solenoid in that the field at P' would be zero instead of what is shown in that figure. The field vanishes there because the top of the solenoid (similar to that shown in Fig. 30-52, in "slice" view, but with the currents and field directions reversed) would contribute an equal and opposite field to any exterior point, thus canceling it. For interior points, the top and bottom "slices" each contribute $\frac{1}{2}\mu_0\lambda$ (in the same direction) [this is shown in the solution to problem 39] and thus produce an interior field equal to $B = \mu_0\lambda$.

(b) Applying Ampere's law to a rectangular path which passes through points P (interior) and P' (exterior) similar to that described in the solution to part (b) of problem 39, we are not surprised to find

$$\oint \vec{B} \cdot d\vec{s} = \left(\vec{B}_P - \vec{B}_{P'}\right) \cdot \hat{\imath} \Delta x = \mu_0 \lambda \Delta x$$

just as we found in part (b) of problem 39 (except that we are now taking the $+x$ direction in the same direction as the field at P, to avoid confusion with signs). The difference with the previous solution is that in 39, $\left(\vec{B}_P - \vec{B}_{P'}\right) \cdot \hat{\imath}$ was equal to $B - (-B) = 2B$, whereas in this case we have $B - 0 = B$. Although the value of B is different in the two problems, we see that the *change* $\left(\vec{B}_P - \vec{B}_{P'}\right) \cdot \hat{\imath}$ is the same: $\mu_0 \lambda$.

45. Consider a circle of radius r, inside the toroid and concentric with it (like either of the loops drawn in Fig. 30-20). The current that passes through the region between this circle and another larger radius circle (well outside the toroid) is Ni, where N is the number of turns and i is the current (note that this region includes a "slice" of the outer rim of the toroid). The current per unit length (of the circle) is $\lambda = Ni/2\pi r$, and $\mu_0 \lambda$ is therefore $\mu_0 Ni/2\pi r$, the magnitude of the magnetic field at the circle (call it B_1). Since the field outside a toroid (call it B_2) is zero, the above result is also the *change* in the magnitude of the field encountered as you move from the circle to the outside (say, to the larger radius circle mentioned above). The equality is not really surprising in light of Ampere's law, particularly if the path used in $\oint \vec{B} \cdot d\vec{s}$ is made to connect the circle in the toroid and the larger radius circle (or portions of each of them, of lengths Δs_1 and Δs_2). The connecting paths (each of size Δr) between the circles can be made perpendicular to the magnetic field lines (so that $\vec{B} \cdot \vec{s} = 0$). In fact, we can keep the connecting paths roughly perpendicular to $\vec{B}$ and manage to have $\Delta s_1 \approx \Delta s_2$ if our Amperian loop is very small (especially if Δr is much smaller than the outer radius of the toroid). Simplifying our notation, the current through the loop is therefore $\Delta s \lambda$, so Ampere's law yields $(B_1 - B_2)\Delta s = \mu_0 \Delta s \lambda$ and $B_2 - B_1 = \mu_0 \lambda$. What this demonstrates is that the change of the magnetic field is $\mu_0 \lambda$ when moving from one point to another (in a direction perpendicular to the field) across a current sheet (as the term is used in problem 39); this principle is useful in any discussion of boundary conditions in electrodynamics applications.

46. The orbital radius for the electron is

$$r = \frac{mv}{eB} = \frac{mv}{e\mu_0 ni}$$

which we solve for i:

$$\begin{aligned} i &= \frac{mv}{e\mu_0 nr} \\ &= \frac{(9.11 \times 10^{-31}\,\text{kg})(0.0460)(3.00 \times 10^8\,\text{m/s})}{(1.60 \times 10^{-19}\,\text{C})(4\pi \times 10^{-7}\,\text{T}\cdot\text{m/A})(100/0.0100\,\text{m})(2.30 \times 10^{-2}\,\text{m})} \\ &= 0.272\ \text{A}\ . \end{aligned}$$

47. (a) We denote the $\vec{B}$-fields at point P on the axis due to the solenoid and the wire as $\vec{B}_s$ and $\vec{B}_w$, respectively. Since $\vec{B}_s$ is along the axis of the solenoid and $\vec{B}_w$ is perpendicular to it, $\vec{B}_s \perp \vec{B}_w$, respectively. For the net field $\vec{B}$ to be at 45° with the axis we then must have $B_s = B_w$. Thus,

$$B_s = \mu_0 i_s n = B_w = \frac{\mu_0 i_w}{2\pi d}\ ,$$

which gives the separation d to point P on the axis:

$$d = \frac{i_w}{2\pi i_s n} = \frac{6.00\,\text{A}}{2\pi(20.0 \times 10^{-3}\,\text{A})(10\,\text{turns/cm})} = 4.77\ \text{cm}\ .$$

(b) The magnetic field strength is

$$\begin{aligned} B &= \sqrt{2}B_s \\ &= \sqrt{2}(4\pi \times 10^{-7}\,\mathrm{T\cdot m/A})(20.0 \times 10^{-3}\,\mathrm{A})(10\,\mathrm{turns}/0.0100\,\mathrm{m}) \\ &= 3.55 \times 10^{-5}\ \mathrm{T}\ . \end{aligned}$$

48. (a) We set $z = 0$ in Eq. 30-28 (which is equivalent using to Eq. 30-12 multiplied by the number of loops). Thus, $B(0) \propto i/R$. Since case b has two loops,

$$\frac{B_b}{B_a} = \frac{2i/R_b}{i/R_a} = \frac{2R_a}{R_b} = 4\ .$$

(b) The ratio of their magnetic dipole moments is

$$\frac{\mu_b}{\mu_a} = \frac{2iA_b}{iA_a} = \frac{2R_b^2}{R_a^2} = 2\left(\frac{1}{2}\right)^2 = \frac{1}{2}\ .$$

49. The magnitude of the magnetic dipole moment is given by $\mu = NiA$, where N is the number of turns, i is the current, and A is the area. We use $A = \pi R^2$, where R is the radius. Thus,

$$\mu = (200)(0.30\,\mathrm{A})\pi(0.050\mathrm{m})^2 = 0.47\ \mathrm{A\cdot m^2}\ .$$

50. We use Eq. 30-28 and note that the contributions to $\vec{B}_P$ from the two coils are the same. Thus,

$$B_P = \frac{2\mu_0 iR^2N}{2[R^2 + (R/2)^2]^{3/2}} = \frac{8\mu_0 Ni}{5\sqrt{5}R}\ .$$

$\vec{B}_P$ is in the positive x direction.

51. (a) The magnitude of the magnetic dipole moment is given by $\mu = NiA$, where N is the number of turns, i is the current, and A is the area. We use $A = \pi R^2$, where R is the radius. Thus,

$$\mu = Ni\pi R^2 = (300)(4.0\,\mathrm{A})\pi(0.025\,\mathrm{m})^2 = 2.4\ \mathrm{A\cdot m^2}\ .$$

(b) The magnetic field on the axis of a magnetic dipole, a distance z away, is given by Eq. 30-29:

$$B = \frac{\mu_0}{2\pi}\frac{\mu}{z^3}\ .$$

We solve for z:

$$z = \left(\frac{\mu_0}{2\pi}\frac{\mu}{B}\right)^{1/3} = \left(\frac{(4\pi \times 10^{-7}\,\mathrm{T\cdot m/A})(2.36\,\mathrm{A\cdot m^2})}{2\pi(5.0 \times 10^{-6}\,\mathrm{T})}\right)^{1/3} = 46\ \mathrm{cm}\ .$$

52. (a) For $x \gg a$, the result of problem 15 reduces to

$$B(x) \approx \frac{4\mu_0 ia^2}{\pi(4x^2)(4x^2)^{1/2}} = \frac{\mu_0(ia^2)}{4\pi x^3}\ ,$$

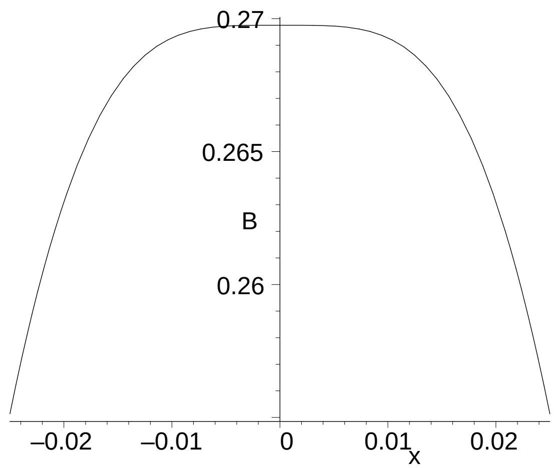
which is indeed the field of a magnetic dipole (see Eq. 30-29).

(b) The magnitude of the magnetic dipole moment is $\mu = ia^2$, by comparison between Eq. 30-29 and the result above.

53. Since the origin is midway between the coils, and the axis is chosen to be x (as opposed to the z used in Eq. 30-28), then the net field of the two coils is

$$B = \frac{\mu_0 N i R^2}{2} \left(\frac{1}{\sqrt{R^2 + (R/2 - x)^2}} + \frac{1}{\sqrt{R^2 + (R/2 + x)^2}} \right)$$

where $i = 50$ A, $N = 300$ and $R = 0.050$ m. The graph of this function (using SI units) is shown below.



54. (a) By imagining that each of the segments bg and cf (which are shown in the figure as having no current) actually has a pair of currents, where both currents are of the same magnitude (i) but opposite direction (so that the pair effectively cancels in the final sum), one can justify the superposition.

(b) The dipole moment of path $abcdefgha$ is

$$\begin{aligned} \vec{\mu} &= \vec{\mu}_{bcfgb} + \vec{\mu}_{abgha} + \vec{\mu}_{cdefc} = (ia^2)(\hat{\jmath} - \hat{\imath} + \hat{\imath}) = ia^2\hat{\jmath} \\ &= (6.0\,\text{A})(0.10\,\text{m})^2\hat{\jmath} = 6.0 \times 10^{-2}\ \text{A}\cdot\text{m}^2\ \hat{\jmath}\ . \end{aligned}$$

(c) Since both points are far from the cube we can use the dipole approximation. For $(x,\, y,\, z) = (0,\, 5.0\,\text{m},\, 0)$

$$\begin{aligned} \vec{B}(0,\, 5.0\,\text{m},\, 0) &\approx \frac{\mu_0}{2\pi}\frac{\vec{\mu}}{y^3} \\ &= \frac{(1.26 \times 10^{-6}\,\text{T}\cdot\text{m/A})(6.0 \times 10^{-2}\,\text{m}^2 \cdot \text{A})\hat{\jmath}}{2\pi(5.0\,\text{m})^3} \\ &= 9.6 \times 10^{-11}\ \text{T}\ \ \hat{\jmath}\ \ . \end{aligned}$$

For $(x,\, y,\, z) = (5.0\,\text{m},\, 0,\, 0)$, note that the line joining the end point of interest and the location of the dipole is perpendicular to the axis of the dipole. You can check easily that if an electric dipole is used, the field would be $E \approx (1/4\pi\varepsilon_0)(p/x^3)$, which is half of the magnitude of E for a point on the y axis the same distance from the dipole. By analogy, in our case B is also half the value or $B(0,\, 5.0\,\text{m},\, 0)$, i.e.,

$$B(5.0\,\text{m},\, 0,\, 0) = \frac{1}{2}B(0,\, 5.0\,\text{m},\, 0) = \frac{1}{2}(9.6 \times 10^{-11}\,\text{T}) = 4.8 \times 10^{-11}\,\text{T}\ .$$

Just like the electric dipole case, $\vec{B}(5.0\,\text{m},\, 0,\, 0)$ points in the negative y direction.

55. (a) The magnitude of the magnetic field on the axis of a circular loop, a distance z from the loop center, is given by Eq. 30-28:

$$B = \frac{N\mu_0 i R^2}{2(R^2+z^2)^{3/2}} ,$$

where R is the radius of the loop, N is the number of turns, and i is the current. Both of the loops in the problem have the same radius, the same number of turns, and carry the same current. The currents are in the same sense, and the fields they produce are in the same direction in the region between them. We place the origin at the center of the left-hand loop and let x be the coordinate of a point on the axis between the loops. To calculate the field of the left-hand loop, we set $z = x$ in the equation above. The chosen point on the axis is a distance $s - x$ from the center of the right-hand loop. To calculate the field it produces, we put $z = s - x$ in the equation above. The total field at the point is therefore

$$B = \frac{N\mu_0 i R^2}{2}\left[\frac{1}{(R^2+x^2)^{3/2}} + \frac{1}{(R^2+x^2-2sx+s^2)^{3/2}}\right] .$$

Its derivative with respect to x is

$$\frac{dB}{dx} = -\frac{N\mu_0 i R^2}{2}\left[\frac{3x}{(R^2+x^2)^{5/2}} + \frac{3(x-s)}{(R^2+x^2-2sx+s^2)^{5/2}}\right] .$$

When this is evaluated for $x = s/2$ (the midpoint between the loops) the result is

$$\left.\frac{dB}{dx}\right|_{s/2} = -\frac{N\mu_0 i R^2}{2}\left[\frac{3s/2}{(R^2+s^2/4)^{5/2}} - \frac{3s/2}{(R^2+s^2/4-s^2+s^2)^{5/2}}\right] = 0$$

independently of the value of s.

(b) The second derivative is

$$\begin{aligned}\frac{d^2B}{dx^2} &= \frac{N\mu_0 i R^2}{2}\left[-\frac{3}{(R^2+x^2)^{5/2}} + \frac{15x^2}{(R^2+x^2)^{7/2}}\right.\\ &\qquad \left. -\frac{3}{(R^2+x^2-2sx+s^2)^{5/2}} + \frac{15(x-s)^2}{(R^2+x^2-2sx+s^2)^{7/2}}\right] .\end{aligned}$$

At $x = s/2$,

$$\begin{aligned}\left.\frac{d^2B}{dx^2}\right|_{s/2} &= \frac{N\mu_0 i R^2}{2}\left[-\frac{6}{(R^2+s^2/4)^{5/2}} + \frac{30s^2/4}{(R^2+s^2/4)^{7/2}}\right]\\ &= \frac{N\mu_0 R^2}{2}\left[\frac{-6(R^2+s^2/4)+30s^2/4}{(R^2+s^2/4)^{7/2}}\right] = 3N\mu_0 i R^2\,\frac{s^2-R^2}{(R^2+s^2/4)^{7/2}} .\end{aligned}$$

Clearly, this is zero if $s = R$.

56. (a) By the right-hand rule, $\vec{B}$ points into the paper at P (see Fig. 30-6(c)). To find the magnitude of the field, we use Eq. 30-11 for each semicircle ($\phi = \pi$ rad), and use superposition to obtain the result:

$$B = \frac{\mu_0 i\pi}{4\pi a} + \frac{\mu_0 i\pi}{4\pi b} = \frac{\mu_0 i}{4}\left(\frac{1}{a}+\frac{1}{b}\right) .$$

(b) The direction of $\vec{\mu}$ is the same as the $\vec{B}$ found in part (a): into the paper. The enclosed area is $A = (\pi a^2 + \pi b^2)/2$ which means the magnetic dipole moment has magnitude

$$|\vec{\mu}| = \frac{\pi\, i}{2}\left(a^2+b^2\right) .$$

57. (a) We denote the large loop and small coil with subscripts 1 and 2, respectively.

$$B_1 = \frac{\mu_0 i_1}{2R_1} = \frac{(4\pi \times 10^{-7}\,\mathrm{T\cdot m/A})(15\,\mathrm{A})}{2(0.12\,\mathrm{m})} = 7.9 \times 10^{-5}\ \mathrm{T}\ .$$

(b) The torque has magnitude equal to

$$\begin{aligned}\tau &= \left|\vec{\mu}_2 \times \vec{B}_1\right| &= \mu_2 B_1 \sin 90^\circ \\ &= N_2 i_2 A_2 B_1 &= \pi N_2 i_2 r_2^2 B_1 \\ &= \pi(50)(1.3\,\mathrm{A})(0.82 \times 10^{-2}\,\mathrm{m})^2(7.9 \times 10^{-5}\,\mathrm{T}) &= 1.1 \times 10^{-6}\ \mathrm{N{\cdot}m}\ .\end{aligned}$$

58. (a) The contribution to B_C from the (infinite) straight segment of the wire is

$$B_{C1} = \frac{\mu_0 i}{2\pi R}\ .$$

The contribution from the circular loop is

$$B_{C2} = \frac{\mu_0 i}{2R}\ .$$

Thus,

$$B_C = B_{C1} + B_{C2} = \frac{\mu_0 i}{2R}\left(1 + \frac{1}{\pi}\right)\ .$$

$\vec{B}_C$ points out of the page.

(b) Now $\vec{B}_{C1} \perp \vec{B}_{C2}$ so

$$B_C = \sqrt{B_{C1}^2 + B_{C2}^2} = \frac{\mu_0 i}{2R}\sqrt{1 + \frac{1}{\pi^2}}\ ,$$

and $\vec{B}_C$ points at an angle (relative to the plane of the paper) equal to

$$\tan^{-1}\left(\frac{B_{C1}}{B_{C2}}\right) = \tan^{-1}\left(\frac{1}{\pi}\right) = 18^\circ\ .$$

59. (a) For the circular path L of radius r concentric with the conductor

$$\oint_L \vec{B}\cdot d\vec{s} = 2\pi r B = \mu_0 i_{\,\mathrm{enc}} = \mu_0\, i\, \frac{\pi(r^2 - b^2)}{\pi(a^2 - b^2)}\ .$$

Thus,

$$B = \frac{\mu_0 i}{2\pi(a^2 - b^2)}\left(\frac{r^2 - b^2}{r}\right)\ .$$

(b) At $r = a$, the magnetic field strength is

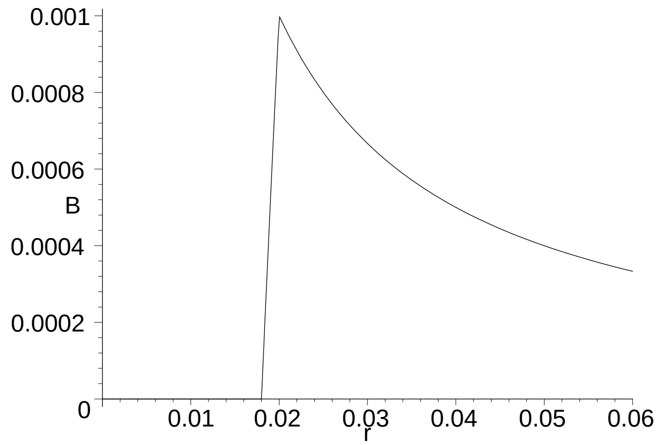
$$\frac{\mu_0 i}{2\pi(a^2 - b^2)}\left(\frac{a^2 - b^2}{a}\right) = \frac{\mu_0 i}{2\pi a}\ .$$

At $r = b$, $B \propto r^2 - b^2 = 0$. Finally, for $b = 0$

$$B = \frac{\mu_0 i}{2\pi a^2}\frac{r^2}{r} = \frac{\mu_0 i r}{2\pi a^2}$$

which agrees with Eq. 30-22.

(c) The field is zero for $r < b$ and is equal to Eq. 30-19 for $r > a$, so this along with the result of part (a) provides a determination of B over the full range of values. The graph (with SI units understood) is shown below.



60. (a) Eq. 30-22 applies for $r < c$. Our sign choice is such that i is positive in the smaller cylinder and negative in the larger one.

$$B = \frac{\mu_0 i r}{2\pi c^2} \qquad \text{for} \quad r \leq c \quad .$$

(b) Eq. 30-19 applies in the region between the conductors.

$$B = \frac{\mu_0 i}{2\pi r} \qquad \text{for} \quad c \leq r \leq b \quad .$$

(c) Within the larger conductor we have a superposition of the field due to the current in the inner conductor (still obeying Eq. 30-19) plus the field due to the (negative) current in the that part of the outer conductor at radius less than r (see part (a) of problem 59 for more details). The result is

$$B = \frac{\mu_0 i}{2\pi r} - \frac{\mu_0 i}{2\pi r}\left(\frac{r^2 - b^2}{a^2 - b^2}\right) \qquad \text{for} \quad b < r \leq a \quad .$$

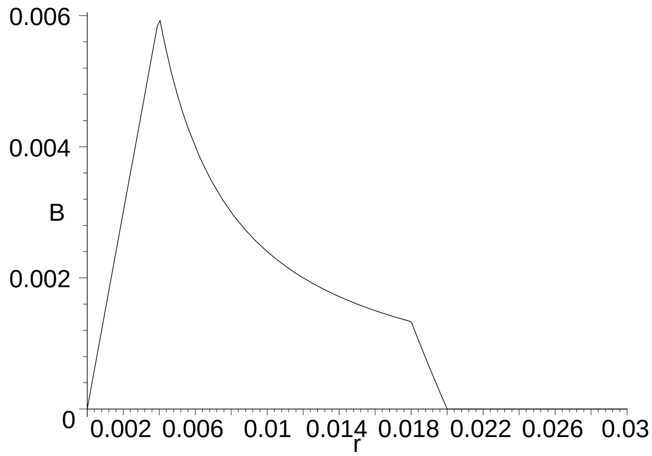
If desired, this expression can be simplified to read

$$B = \frac{\mu_0 i}{2\pi r}\left(\frac{a^2 - r^2}{a^2 - b^2}\right) \quad .$$

(d) Outside the coaxial cable, the net current enclosed is zero. So $B = 0$ for $r \geq a$.

(e) We test these expressions for one case. If $a \to \infty$ and $b \to \infty$ (such that $a > b$) then we have the situation described on page 696 of the textbook.

(f) Using SI units, the graph of the field is shown below:



61. (a) We find the field by superposing the results of two semi-infinite wires (Eq. 30-9) and a semicircular arc (Eq. 30-11 with $\phi = \pi$ rad). The direction of $\vec{B}$ is out of the page, as can be checked by referring to Fig. 30-6(c). The magnitude of $\vec{B}$ at point a is therefore

$$B_a = 2\left(\frac{\mu_0 i}{4\pi R}\right) + \frac{\mu_0 i\pi}{4\pi R} = \frac{\mu_0 i}{2R}\left(\frac{1}{\pi} + \frac{1}{2}\right) .$$

With $i = 10$ A and $R = 0.0050$ m, we obtain $B_a = 1.0 \times 10^{-3}$ T. The direction of this field is out of the page, as Fig. 30-6(c) makes clear.

(b) The last remark in the problem statement implies that treating b as a point midway between two infinite wires is a good approximation. Thus, using Eq. 30-6,

$$B_b = 2\left(\frac{\mu_0 i}{2\pi R}\right) = 8.0 \times 10^{-4}\ \text{T} .$$

This field, too, points out of the page.

62. We use $B(x,\, y,\, z) = (\mu_0/4\pi)i\,\Delta\vec{s} \times \vec{r}/r^3$, where $\Delta\vec{s} = \Delta s\hat{\jmath}$ and $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\vec{k}$. Thus,

$$\vec{B}(x,\, y,\, z) = \left(\frac{\mu_0}{4\pi}\right)\frac{i\,\Delta s\hat{\jmath} \times (x\vec{i} + y\hat{\jmath} + z\vec{k})}{(x^2+y^2+z^2)^{3/2}} = \frac{\mu_0 i\,\Delta s\,(z\hat{\imath} - x\hat{\mathrm{k}})}{4\pi(x^2+y^s+z^2)^{3/2}} .$$

(a) The field on the z axis (at $z = 5.0$ m) is

$$\begin{aligned}\vec{B}(0,\, 0,\, 5.0\,\text{m}) &= \frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(2.0\,\text{A})(3.0\times10^{-2}\,\text{m})(5.0\,\text{m})\hat{\imath}}{4\pi\left(0^2+0^2+(5.0\,\text{m})^2\right)^{3/2}} \\ &= 2.4\times10^{-10}\ \text{T}\ \hat{\imath} .\end{aligned}$$

(b) $\vec{B}(0,\, 6.0\,\text{m},\, 0)$, since $x = z = 0$.

(c) The field in the xy plane, at $(x, y) = (7, 7)$, is

$$\begin{aligned}\vec{B}(7.0\,\text{m},\, 7.0\,\text{m},\, 0) &= \frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(2.0\,\text{A})(3.0\times10^{-2}\,\text{m})(-7.0\,\text{m})\hat{\mathrm{k}}}{4\pi\left((7.0\,\text{m})^2+(7.0\,\text{m})^2+0^2\right)^{3/2}} \\ &= 4.3\times10^{-11}\ \text{T}\ \hat{\mathrm{k}} .\end{aligned}$$

(d) The field in the xy plane, at $(x, y) = (-3, -4)$, is

$$\begin{aligned}\vec{B}(-3.0\,\mathrm{m},\,-4.0\,\mathrm{m},\,0) &= \frac{(4\pi\times10^{-7}\,\mathrm{T\cdot m/A})(2.0\,\mathrm{A})(3.0\times10^{-2}\,\mathrm{m})(3.0\,\mathrm{m})\hat{\mathrm{k}}}{4\pi\left((-3.0\,\mathrm{m})^2+(-4.0\,\mathrm{m})^2+0^2\right)^{3/2}} \\ &= 1.4\times10^{-10}\ \mathrm{T}\ \hat{\mathrm{k}}\ .\end{aligned}$$

63. (a) Eq. 30-19 applies for each wire, with $r = \sqrt{R^2 + (d/2)^2}$ (by the Pythagorean theorem). The vertical components of the fields cancel, and the two (identical) horizontal components add to yield the final result

$$B = 2\left(\frac{\mu_0 i}{2\pi r}\right)\left(\frac{d/2}{r}\right) = \frac{\mu_0 i d}{2\pi\left(R^2 + (d/2)^2\right)}$$

where $(d/2)/r$ is a trigonometric factor to select the horizontal component. It is clear that this is equivalent to the expression in the problem statement.

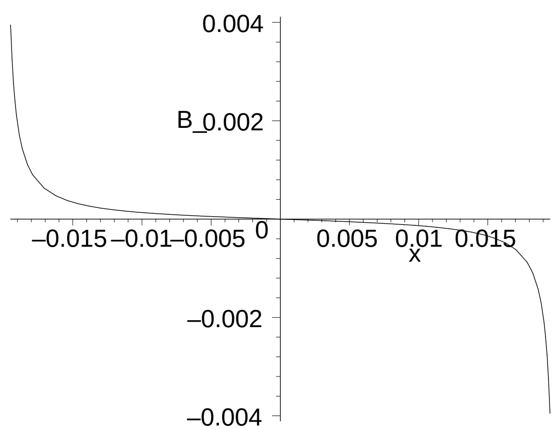
(b) Using the right-hand rule, we find both horizontal components point rightward.

64. (a) The difference between this and Sample Problem 6 is that the current in wire 2 is reversed form what is shown in Fig. 30-59(a). Thus, we replace $i \to -i$ in the expression for $B_2(x)$ and add the fields:

$$B_1(x) + B_2(x) = \frac{\mu_0 i}{2\pi(d+x)} + \frac{\mu_0(-i)}{2\pi(d-x)} = -\frac{\mu_0 i x}{\pi\left(d^2 - x^2\right)}$$

which is equivalent to the desired result.

(b) As remarked in that Sample Problem, this expression does not apply within the wires themselves. If we assume the wires have nearly zero thickness, then the expression applies over nearly all of the range $-0.02 < x < 0.02$ (with SI units understood). To be definite about this issue, we have picked a small wire radius (.005 m) and graphed the field over the range $-.0195 \le x \le 0.0195$.



65. (a) All wires carry parallel currents and attract each other; thus, the "top" wire is pulled downward by the other two:

$$|\vec{F}| = \frac{\mu_0\, L(5.0\ \mathrm{A})(3.2\ \mathrm{A})}{2\pi(0.10\ \mathrm{m})} + \frac{\mu_0\, L(5.0\ \mathrm{A})(5.0\ \mathrm{A})}{2\pi(0.20\ \mathrm{m})}$$

where $L = 3.0$ m. Thus, $|\vec{F}| = 1.7\times10^{-4}$ N.

(b) Now, the "top" wire is pushed upward by the center wire and pulled downward by the bottom wire:

$$|\vec{F}| = \frac{\mu_0\, L(5.0\ \mathrm{A})(3.2\ \mathrm{A})}{2\pi(0.10\ \mathrm{m})} - \frac{\mu_0\, L(5.0\ \mathrm{A})(5.0\ \mathrm{A})}{2\pi(0.20\ \mathrm{m})}$$

so that $|\vec{F}| = 2.1\times10^{-5}$ N.

66. With cylindrical symmetry, we have, external to the conductors,

$$|\vec{B}| = \frac{\mu_0\, i_{\rm enc}}{2\pi\, r}$$

which produces $i_{\rm enc} = 25$ mA from the given information. Therefore, the thin wire must carry 5 mA in a direction opposite to the 30 mA carried by the thin conducting surface.

67. The area enclosed by the loop L is $A = \frac{1}{2}(4d)(3d) = 6d^2$. Thus

$$\begin{aligned}\oint_c \vec{B} \cdot d\vec{s} &= \mu_0 i = \mu_0 jA \\ &= (4\pi \times 10^{-7}\,\mathrm{T \cdot m/A})(15\,\mathrm{A/m^2})(6)(0.20\,\mathrm{m})^2 = 4.5 \times 10^{-6}\ \mathrm{T{\cdot}m}\ .\end{aligned}$$

68. We refer to the center of the circle (where we are evaluating $\vec{B}$) as C. Recalling the *straight sections* discussion in Sample Problem 30-1, we see that the current in the straight segments which are colinear with C do not contribute to the field there. Eq. 30-11 (with $\phi = \pi/2$ rad) and the right-hand rule indicates that the currents in the two arcs contribute

$$\frac{\mu_0 i(\pi/2)}{4\pi R} - \frac{\mu_0 i(\pi/2)}{4\pi R} = 0$$

to the field at C. Thus, the non-zero contributions come from those straight-segments which are not colinear with C. There are two of these "semi-infinite" segments, one a vertical distance R above C and the other a horizontal distance R to the left of C. Both contribute fields pointing out of the page (see Fig. 30-6(c)). Since the magnitudes of the two contributions (governed by Eq. 30-9) add, then the result is

$$B = 2\left(\frac{\mu_0 i}{4\pi R}\right) = \frac{\mu_0 i}{2\pi R}$$

exactly what one would expect from a single infinite straight wire (see Eq. 30-6). For such a wire to produce such a field (out of the page) with a leftward current requires that the point of evaluating the field be below the wire (again, see Fig. 30-6(c)).

69. Since the radius is $R = 0.0013$ m, then the $i = 50$ A produces

$$B = \frac{\mu_0 i}{2\pi R} = 0.0077\ \mathrm{T}$$

at the edge of the wire. The three equations, Eq. 30-6, Eq. 30-19 and Eq. 30-22, agree at this point.

70. We note that the distance from each wire to P is $r = d/\sqrt{2} = 0.071$ m. In both parts, the current is $i = 100$ A.

(a) With the currents parallel, application of the right-hand rule (to determine each of their contributions to the field at P) reveals that the vertical components cancel and the horizontal components add – yielding the result:

$$B = 2\left(\frac{\mu_0 i}{2\pi r}\right)\cos 45^\circ = 4.0 \times 10^{-4}\ \mathrm{T}\ .$$

and directed leftward in the figure.

(b) Now, with the currents antiparallel, application of the right-hand rule shows that the horizontal components cancel and the vertical components add. Thus,

$$B = 2\left(\frac{\mu_0 i}{2\pi r}\right)\sin 45^\circ = 4.0 \times 10^{-4}\ \mathrm{T}\ .$$

and directed upward in the figure.

71. (a) As illustrated in Sample Problem 30-1, the radial segments do not contribute to $\vec{B}_P$ and the arc-segments contribute according to Eq. 30-11 (with angle in radians). If $\hat{\mathrm{k}}$ designates the direction "out of the page" then

$$\vec{B} = \frac{\mu_0(0.40\text{ A})(\pi\text{ rad})}{4\pi(0.050\text{ m})}\,\hat{\mathrm{k}} - \frac{\mu_0(0.80\text{ A})\left(\frac{2\pi}{3}\text{ rad}\right)}{4\pi(0.040\text{ m})}\,\hat{\mathrm{k}}$$

which yields $\vec{B} = -1.7\times10^{-6}\,\hat{\mathrm{k}}$ T.

(b) Now we have

$$\vec{B} = -\frac{\mu_0(0.40\text{ A})(\pi\text{ rad})}{4\pi(0.050\text{ m})}\,\hat{\mathrm{k}} - \frac{\mu_0(0.80\text{ A})\left(\frac{2\pi}{3}\text{ rad}\right)}{4\pi(0.040\text{ m})}\,\hat{\mathrm{k}}$$

which yields $\vec{B} = -6.7\times10^{-6}\,\hat{\mathrm{k}}$ T.

72. (a) We designate the wire along $y = r_A = 0.100$ m wire A and the wire along $y = r_B = 0.050$ m wire B. Using Eq. 30-6, we have

$$\begin{aligned}\vec{B}_{\text{net}} &= \vec{B}_A + \vec{B}_B\\ &= -\frac{\mu_0\, i_A}{2\pi\, r_A}\,\hat{\mathrm{k}} - \frac{\mu_0\, i_B}{2\pi\, r_B}\,\hat{\mathrm{k}}\end{aligned}$$

which yields $\vec{B}_{\text{net}} = 52.0\times10^{-6}\hat{\mathrm{k}}$ T.

(b) This will occur for some value $r_B < y < r_A$ such that

$$\frac{\mu_0\, i_A}{2\pi\ (r_A - y)} = \frac{\mu_0\, i_B}{2\pi\ (y - r_B)}\ .$$

Solving, we find $y = 13/160 \approx 0.081$ m.

(c) We eliminate the $y < r_B$ possibility due to wire B carrying the larger current. We expect a solution in the region $y > r_A$ where

$$\frac{\mu_0\, i_A}{2\pi\ (y - r_A)} = \frac{\mu_0\, i_B}{2\pi\ (y - r_B)}\ .$$

Solving, we find $y = 7/40 \approx 0.018$ m.

73. (a) The field in this region is entirely due to the long wire (with, presumably, negligible thickness). Using Eq. 30-19,

$$|\vec{B}| = \frac{\mu_0\, i_w}{2\pi r} = 4.8\times10^{-3}\text{ T}$$

where $i_w = 24$ A and $r = 0.0010$ m.

(b) Now the field consists of two contributions (which are antiparallel) – from the wire (Eq. 30-19) and from a portion of the conductor (Eq. 30-22 modified for annular area):

$$\begin{aligned}|\vec{B}| &= \frac{\mu_0\, i_w}{2\pi r} - \frac{\mu_0\, i_{\text{enc}}}{2\pi r}\\ &= \frac{\mu_0\, i_w}{2\pi r} - \frac{\mu_0\, i_c}{2\pi r}\left(\frac{\pi r^2 - \pi R_i^2}{\pi R_o^2 - \pi R_i^2}\right)\end{aligned}$$

where $r = 0.0030$ m, $R_i = 0.0020$ m, $R_o = 0.0040$ m and $i_c = 24$ A. Thus, we find $|\vec{B}| = 9.3\times10^{-4}$ T.

(c) Now, in the external region, the individual fields from the two conductors cancel completely (since $i_c = i_w$): $\vec{B} = 0$.

74. In this case $L = 2\pi r$ is roughly the length of the toroid so

$$B = \mu_0 i_0 \left(\frac{N}{2\pi r}\right) = \mu_0 n i_0 \ .$$

This result is expected, since from the perspective of a point inside the toroid the portion of the toroid in the vicinity of the point resembles part of a long solenoid.

75. We take the current ($i = 50$ A) to flow in the $+x$ direction, and the electron to be at a point P which is $r = 0.050$ m above the wire (where "up" is the $+y$ direction). Thus, the field produced by the current points in the $+z$ direction at P. Then, combining Eq. 30-6 with Eq. 29-2, we obtain $\vec{F}_e = (-e\mu_0 i/2\pi r)(\vec{v} \times \hat{\mathrm{k}})$.

(a) The electron is moving down: $\vec{v} = -v\,\hat{\mathrm{j}}$ (where $v = 1.0 \times 10^7$ m/s is the speed) so

$$\vec{F}_e = \frac{-e\mu_0 i v}{2\pi r}\,(-\hat{\imath}) = 3.2 \times 10^{-16}\ \mathrm{N}\ \hat{\imath}\ .$$

(b) In this case, the electron in the same direction as the current: $\vec{v} = v\,\hat{\imath}$ so

$$\vec{F}_e = \frac{-e\mu_0 i v}{2\pi r}\,\left(-\hat{\jmath}\right) = 3.2 \times 10^{-16}\ \mathrm{N}\ \hat{\jmath}\ .$$

(c) Now, $\vec{v} = \pm v\hat{\mathrm{k}}$ so $\vec{F}_e \propto \hat{\mathrm{k}} \times \hat{\mathrm{k}} = 0$.

76. Eq. 30-6 gives

$$i = \frac{2\pi R B}{\mu_0} = \frac{2\pi(0.880\,\mathrm{m})(7.30 \times 10^{-6}\,\mathrm{T})}{4\pi \times 10^{-7}\,\mathrm{T\cdot m/A}} = 32.1\ \mathrm{A}\ .$$

77. For $x > 20$ mm, the field due i_2 is downward and thus subtracts from B_1 and is entirely consistent with the given expression for B_2 (note that it becomes negative when $x > d$). Similarly, for $x < -20$ mm, the field due to i_1 is downward and subtracts from B_2 (which is positive and points upward for all $x < d$). This again is consistent with the expression for B_1 which is seen to become negative for x less than $-d$ (that is, x negative and $|x| > |d|$). We conclude that the given expressions are valid over the whole of the x axis, and their answer (Eq. 30-33) holds for all x (other than at the locations of the wires themselves, where it becomes problematic, as discussed in the Sample Problem).

78. By the right-hand rule, the magnetic field $\vec{B}_1$(evaluated at a) produced by wire 1 (the wire at bottom left) is at $\phi = 150°$ (measured counterclockwise from the $+x$ axis, in the xy plane), and the field produced by wire 2 (the wire at bottom right) is at $\phi = 210°$. By symmetry ($\vec{B}_1 = \vec{B}_2$) we observe that only the x-components survive, yielding

$$\vec{B}_1 + \vec{B}_2 = 2\,\frac{\mu_0\,i}{2\pi\,\ell}\,\cos 150°\,\hat{\imath} = -3.46 \times 10^{-5}\,\hat{\imath}\ \mathrm{T}$$

where $i = 10$ A, $\ell = 0.10$ m, and Eq. 30-6 has been used. To cancel this, wire b must carry current into the page (that is, the $-\hat{\mathrm{k}}$ direction) of value

$$i_b = \left(3.46 \times 10^{-5}\right)\frac{2\pi\,r}{\mu_0} = 15\ \mathrm{A}$$

where $r = \sqrt{3}\,\ell/2 = 0.087$ m and and Eq. 30-6 has again been used.

79. Using Eq. 30-22 and Eq. 30-19, we have

$$\begin{aligned} |\vec{B}_1| &= \left(\frac{\mu_0\,i}{2\pi\,R^2}\right) r_1 \\ |\vec{B}_2| &= \frac{\mu_0\,i}{2\pi\,r_2} \end{aligned}$$

where $r_1 = 0.0040$ m, $|\vec{B}_1| = 2.8 \times 10^{-4}$ T, $r_2 = 0.010$ m and $|\vec{B}_2| = 2.0 \times 10^{-4}$ T. Point 2 is known to be external to the wire since $|\vec{B}_2| < |\vec{B}_1|$. From the second equation, we find $i = 10$ A. Plugging this into the first equation yields $R = 5.3 \times 10^{-3}$ m.

80. Using a magnifying glass, we see that all but i_2 are directed into the page. Wire 3 is therefore attracted to all but wire 2. Letting $d = 0.50$ m, we find the net force (per meter length) using Eq. 30-15, with positive indicated a rightward force:

$$\frac{|\vec{F}|}{\ell} = \frac{\mu_0 \, i_3}{2\pi}\left(-\frac{i_1}{2d} + \frac{i_2}{d} + \frac{i_4}{d} + \frac{i_5}{2d}\right)$$

which yields $|\vec{F}|/\ell = 8.0 \times 10^{-7}$ N/m.