Functional programming, Seminar No. 2

Danya Rogozin
Institute for Information Transmission Problems, RAS
Serokell OÜ
Higher School of Economics
The Faculty of Computer Science

Intro

On the previous seminar, we:

- discussed the general aspects of Haskell
- took a look at the Haskell ecosystem

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Today, we:

- study the basic Haskell syntax
- study the list data type as one of the Haskell data structures
- realise why Haskell is a lazy language

Bindings

The equality sign in Haskell denotes binding:

Example

```haskell
fortyTwo = 42
coolString = "coolString"
```

Local binding with the `let`-keyword:

Example

```haskell
fortyTwo = let number = 43 in number - 1
```

Function definitions

The following functions are also defined as bindings:

Example

```
add x y = x + y
userName name = "Username: " ++ name
id x = x
```

The same functions defined with lambda:

Example

```
add = \x y -> x + y
userName = \name -> "Username: " ++ name
id = \x -> x
```

Function application

As in the lambda calculus, function application is left associative by default

Example

```
{-
foo x y z = f x y z = ((f x) y) z
-}
```

One may use the dollar infix operator. That would allow us to avoid too many brackets. For example, the following functions are equivalent:

Example

```
function f x y z = f ((x y) z)
function1 f x y z = f $ x y $ z
```

Prefix and infix notation

Any operator or function might be called in prefix and infix:

Example

```
> map (\x -> x * pi * 100) [1..3]
[314.159265358979,628.3185307179587,942.4777960769379]
> (\x -> x * pi * 100) `map` [1..3]
[314.159265358979,628.3185307179587,942.4777960769379]
```

One may declare an operator defining its priority and associativity explicitly. Here's an example:

Example

```
(^) :: (Num a, Integral b) => a -> b -> a
infixr 8 ^
```

Currying and partial application

Recall the function `add` once more. Here is an example of partial application:

Example

```
add x y = x + y
addFive = add 5
twentyEight = addFive 23
  -- 28
```

Partial application is well-defined since all many-argument functions in Haskell are curried by default.

Immutability and laziness

In Haskell, values are immutable. A small example:

Example

```
> list = [1,2,3,4]
> reverse list
[4,3,2,1]
> list
[1,2,3,4]
> 10 : list
[10,1,2,3,4]
> list
[1,2,3,4]
```

The straighforward factorial and the tail-recursive one:

Example

```
factorial n
  = if n == 0 then 1 else n * factorial (n - 1)

tailFactorial n = helper 1 n
  where
  helper acc x =
    if x > 1
    then helper (acc * x) (x - 1)
    else acc
```

Guards

Let us take a look at the factorial implementation via guards:

Example

```
tailFactorial n = helper 1 n
  where
  helper acc x | x > 1 = helper (acc * x) (x - 1)
               | otherwise acc
```

Basic datatypes

The basic datatypes are:

- `Bool`: Boolean values
- `Int`: Bounded integer datatype
- `Integer`: Unbounded integer datatype
- `Char`: Unicode characters
- `()`: Unit value datatype
- If `a` and `b` are types, then `a -> b` is a type
- If `a` and `b` are types, then `(a,b)` is a type
- If `a` is a type, then `[a]` is a type

A type declaration has the following form:

```
term :: type
```

Datatypes and constructors

We take the list of basic data types and associate constructors with these types. A constructor is a term that allows one to obtain a value of the desired type.

`Bool`	`True` and `False`
`Int`	Integers from -2^{29} to $2^{29}-1$
`Integer`	The set of integers
`Char`	Characters `'0'`, ..., `'9'`, `'a'`, ..., `'z'`, etc
`()`	`()` only
`a -> b`	$\lambda x \to m$
`(a,b)`	if `x :: a` and `y :: b`, then `(x, y) :: (a,b)`
`[a]`	the empty list `[]`
`[a]`	if `x :: a` and `xs :: [a]`, then `x : xs :: [a]`

Types in GHCi

Use the GHCi command `:t` to know a type of an expression:

Example

```
> :t 5
5 :: Num p => p
> :t not
not :: Bool -> Bool
> :t [0.5, 0.6, 0.7]
[0.5, 0.6, 0.7] :: Fractional a => [a]
> :t (\x -> "dratuti, " ++ x)
(\x -> "dratuti, " ++ x) :: [Char] -> [Char]
> :t 'x'
'x' :: Char
```

Function declaration with datatypes

Let us recall the examples of function declarations:

Example

```
add x y = x + y
userName name = "Username: " ++ name
```

One may annotate these functions with types as follows:

Example

```
add :: Int -> Int -> Int
add x y = x + y

userName :: String -> String
userName name = "Username: " ++ name
```

Lists

Let's talk about lists. In Haskell, a list is a homogeneous collection of elements.

Example

```haskell
empty :: [Int]
empty = []

ten :: [Int]
ten = [10]

tenEleven :: [Int]
tenEleven = 11 : ten

tenElevenTwelve :: [Int]
tenElevenTwelve = 12 : tenEleven
-- 12 : (11 : [])
```

Lists. Ranges

Example

```
oneToFive :: [Int]
oneToFive = [1..5]

oneToSevenOdd :: [Int]
oneToSevenOdd = [1,3..7]

nat :: [Int]
nat [0,1..]

evens :: [Int]
evens = [0,2,4..]
```

Lists. Heads and Tails

Example

```
> tail [1..3]
[2,3]
> head [1..3]
1
> head []
*** Exception: Prelude.head: empty list
> tail []
*** Exception: Prelude.tail: empty list
```

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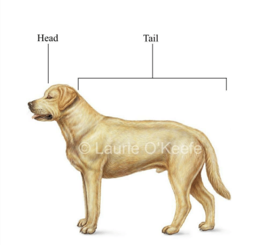

Other helpful list functions

Example

```
Prelude> drop 3 [1..7]
[4,5,6,7]
Prelude> take 4 ['a'..'h']
"abcd"
Prelude> replicate 3 "d"
["d","d","d"]
Prelude> replicate 3 'd'
"ddd"
Prelude> zip [1,2,3] "this is a word"
[(1,'t'),(2,'h'),(3,'i')]
Prelude> unzip [(1,'t'),(2,'h'),(3,'i')]
([1,2,3],"thi")
Prelude> ['a'..'h'] !! 3
'd'
```

List compeherension

Example

```
> take 4 [(i, j) | i <- [1..10], j <- [1..10], i == j*j]
[(1,1),(4,2),(9,3),(16,4)]
> [ i | i <- "a cool sentence", i < 'h']
"a c eece"
> [ i | i <- "a cool sentence", fromEnum i < 100 ]
"a c c"
```

Higher order functions

Function is a first-class object and one may pass any function as an argument:

Example

```haskell
inc :: Int -> Int
inc x = x + 1

changeTwiceBy :: (Int -> Int) -> Int -> Int
changeTwiceBy operation value
  = operation (operation value)

seven :: Int
seven = changeTwiceBy inc 5
```

Case-expressions

Case-expressions allows one to perform case analysis within a function.

Example

```
getFont :: Int -> String
getFont n =
  case n of
    0 -> "PLAIN"
    1 -> "BOLD"
    2 -> "ITALIC"
    _ -> "UNKNOWN"
```

We recall a couple of definitions related to the lambda calculus:

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2. A term M is called *strongly normalisable* (SN), if any reduction path that starts from M terminates

It is clear, that SN implies WN, not vice versa. In other words, there exists a term that has an infinite reduction path, but it has a finite reduction path at the same time.

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Let us consider the example of the following ridiculous term: $(\lambda xy.x)(\lambda z.z)((\lambda x.xx)(\lambda x.xx))$. One may reduce this term in two ways:

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\begin{aligned}
&(\lambda xy.x)(\lambda z.z)((\lambda x.xx)(\lambda x.xx)) \rightarrow_\beta \\
&(\lambda y.[x := (\lambda z.z)])((\lambda x.xx)(\lambda x.xx)) \rightarrow_\beta \\
&(\lambda y.\lambda z.z)((\lambda x.xx)(\lambda x.xx)) \rightarrow_\beta \\
&(\lambda z.z)[y := (\lambda x.xx)(\lambda x.xx)] \rightarrow_\beta \\
&\lambda z.z
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$$\lambda z.z$$

From the other hand:

$$(\lambda xy.x)(\lambda z.z)((\lambda x.xx)(\lambda x.xx)) \to_\beta$$
$$(\lambda xy.x)(\lambda z.z)(xx)(x := [\lambda x.xx]) \to_\beta$$
$$(\lambda xy.x)(\lambda z.z)((\lambda x.xx)(\lambda x.xx)) \to_\beta \dots$$

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- Can we distinguish all possible ways of term reduction?

In fact, we need to distinguish possible ways of application reduction, so far as we have no other options in the remaining cases:

1. If x is a variable, then x is already in normal form
2. If a term has the form $\lambda x.M$, then we reduce M

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The first way is called *applicative order reduction*, the second one is normal order reduction. In the following sense, the second is better:

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Theorem
Let M be a term such that M has a normal form M', then M might be reduced to M' with normal order reduction.

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- The most mainstream programming languages you know (Java, Python, Kotlin, etc) have call-by-value semantics
- The Haskell reduction has a call-by-name strategy. Informally, such a stragety is called *lazy*. Laziness denotes that Haskell doesn't compute a value if it's not needed at the moment
- Call-by-name reduction reduces reducible terms to the bitter end, but it's not always optimal, unfortunately

Haskell reduction

Suppose we have the following trivial function:

Example

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square :: Int -> Int
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If we call this function on $(1+2)$, then we would have the following story:

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The notion of a weak head normal form

In Haskell, reduction evalutates a term to its weak head normal form, where the outermost must be either constructor or lambda. Here are examples: WHNFs from the left and non-WHNFs from the right

`78`	
`2 : [1,2]`	`1 + 665`
`'p' : ("ri" ++ "vet")`	`(\x -> x ++ "ab") "cd"`
`[1, 1 + 2, 1 + 3]`	`length [1..145]`
`("hel" ++ "lo", "world")`	`(\f g x -> f (g x)) id`
`\x -> (x + 2) + 2`	

Pure functions and side-effects

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- It means that, such a function has the same behaviour at every point. This principle is also called *referential transparency*
- A side-effect function is a function that may yield different values passing the same arguments. Mathematically, such a function is not function at all.
- Haskell functions are (mostly) pure ones, but Haskell isn't confluent as a version of the lambda calculus

The failure of the Church-Rosser property

Let us consider the following quite simple example. In Haskell one has a function called `seq`. According to Hackage, “The value of `seq a b` is bottom if a is bottom, and otherwise equal to b.” This function is a sort of instrument to introduce the restricted strictness to Haskell. The listing below demostrates the failure of the CRP:

```
seq :: a -> b -> b
seq _|_ _ = _|_
seq _ b   = b

dno = undefined

seq dno 14         == dno
seq (dno . id) 14 == 14
```

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On the next seminar, we will

- start to learn polymorphism and its advantages
- introduce typeclasses
- study the first examples of crucially important examples of typeclasses

Thank you!