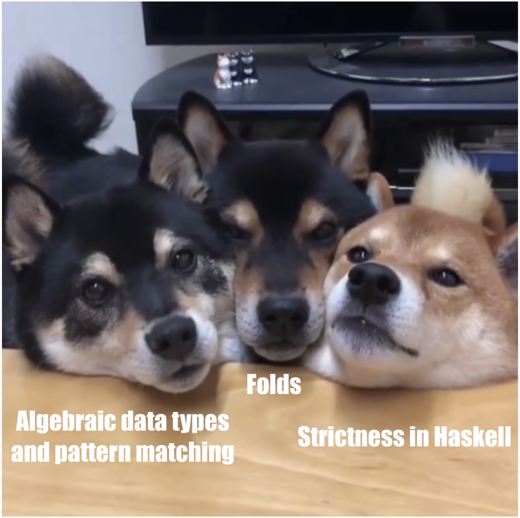
Functional programming, Seminar No. 4

Daniel Rogozin
Institute for Information Transmission Problems, RAS
Serokell OÜ
Higher School of Economics
The Faculty of Computer Science

Today

We will study



Algebraic data types and pattern matching

Pattern matching

Let us take a look at the following functions:

```
swap :: (a, b) -> (b, a)
swap (a, b) = (b, a)

length :: [a] -> Int
length [] = 0
lenght (x : xs) = 1 + length xs
```

Pattern matching

Let us take a look at the following functions:

```
swap :: (a, b) -> (b, a)
swap (a, b) = (b, a)

length :: [a] -> Int
length [] = 0
lenght (x : xs) = 1 + length xs
```

- Such expressions as `(a,b)`, `[]`, and `(x : xs)` are called *patterns*
- One needs to check whether the constructors `(,)` and `( : )` are relevant.
- Consider `swap (45, True)`. Variables `a` and `b` are bound with the values `45` and `True`.
- Consider `lenght [1,2,3]`. Variables `x` and `xs` are bound with the values `1` and `[2,3]`

Algebraic data types. Sums

The simplest example of an algebraic data type is a data type defined with an enumeration of constructors that stores no values.

```haskell
data Colour = Red | Blue | Green | Purple | Yellow
  deriving (Show, Eq)

isRGB :: Colour -> Bool
isRGB Red   = True
isRGB Blue  = True
isRGB Green = True
isRGB _     = False        -- Wild-card
```

Algebraic data types. Products

- An example of a product data type:

```
data Point = Point Double Double
  deriving Show

> :type Point
Point :: Double -> Double -> Point
```

- An example of a function

```
taxiCab :: Point -> Point -> Double
taxiCab (Point x1 y1) (Point x2 y2) =
  abs (x1 - x2) + abs (y1 - y2)
```

Polymorphic data types

- That point data type might be parametrised with a type parameter:

```
data Point a = Point a a
  deriving Show
```

- The `Point` data constructor has the following type. The `Point` from the left (see the definition above) is a type function that has its type (kind).

```
> :type Point
Point :: a -> a -> Point a
> :kind Point
Point :: * -> *
```

Polymorphic data types and type classes

- Suppose we have a function:

```
midPoint
  :: Fractional a => Point a -> Point a -> Point a
midPoint (Pt x1 y1) (Pt x2 y2) =
  Pt ((x1 + x2) / 2) ((y1 + y2) / 2)
```

- Playing with GHCi:

```
> :t midPoint (Pt 3 5) (Pt 6 4)
midPoint (Pt 3 5) (Pt 6 4) :: Fractional a => Point a
> midPoint (Pt 3 5) (Pt 6 4)
Pt 4.5 4.5
> :t it
it :: Fractional a => Point a
```

Inductive data types

- The list is the first example of an inductive data type

```
data List a = Nil | Cons a (List a)
  deriving Show
```

- The data constructors are `Nil :: List a` and `Cons :: a -> List a -> List a`
- Pattern matching and recursion

```
concat :: List a -> List a -> List a
concat Nil ys = ys
concat (Cons x xs) ys = Cons x (xs `concat` ys)
```

Standard lists

- The list data type is already in the standard library, but its approximate definition is the following one:

```
infixr 5 :
data [] a = [] | a : ([] a)
  deriving Show
```

- Syntax sugar:

```
[1,2,3,4] == 1 : 2 : 3 : 4 : []
```

- The example of a definition with built-in lists:

```
infixr 5  ++
(++) :: [a] -> [a] -> [a]
(++) []     ys = ys
(++) (x:xs) ys = x : xs ++ ys
```

- `case ... of ...` expressions allows one to patternmatch everywhere

```
filter :: (a -> Bool) -> [a] -> [a]
filter p [] = []
filter p (x : xs) =
  case p x of
    True  -> x : filter p xs
    False -> filter p xs
```

- The pattern matching from the previous slide is a syntax sugar for the corresponding `case ... of ...` expression

Semantic aspects of pattern matching

- Pattern matching is performed from up to down and from left to right after that.
- A pattern match is either
  - succeed
  - or failed
  - or diverged
- Here is an example:

```
foo (1,4) = 7
foo (0,_) = 8
```

- `(0, undefined)` fails in the first case and it succeeds in the second one
- `(undefined, 0)` diverges during a match
- What about `(1,7-3)`?

As-patterns

- Suppose we have the following function (a quite bad one)

```haskell
dupHead :: [a] -> [a]
dupHead (x : xs) = x : x : xs
```

- One may rewrite this function as follows:

```haskell
dupHead :: [a] -> [a]
dupHead s@(x : xs) = x : s
```

- Here, the name `s` is assigned to the whole pattern `x : xs`

Irrefutable patterns

- Irrefutable patterns are wild-cards, variables, and lazy patterns
- An example of a lazy pattern:

```
> f *** g (a,b) = (f a, g b)
> (const 2) *** (const 1) $ undefined
*** Exception: Prelude.undefined
> f *** g ~(a,b) = (f a, g b)
> (const 2) *** (const 1) $ undefined
(2,1)
```

`newtype` and `type` declarations

- The keyword `type` introduces type synonyms.

```haskell
type String = [Char]
```

- In Haskell, the string data type type is merely a type synonym for the list of characters
- The keyword `newtype` defines a new type with the single constructor that packs a value of a given type

```haskell
newtype Age = Age Int
```

- The same type `Age` defined with the accessor `runAge`

```haskell
newtype Age = Age { runAge :: Int }
-- where runAge :: Age -> Int
```

Field labels

- Sometimes product data types are rather cumbersome:

```haskell
data Person = Person String String Int Float String
```

- As an alternative, one may define a data type with field labels

```haskell
data Person =
  Person { firstName :: String
         , lastName :: String
         , age :: Int
         , height :: Float
         , phoneNumber :: String
         }
```

- Such a data type is a record with accessors such as `firstName :: Person -> String`

Field labels and type classes

- Let us recall the `Eq` type class once more

```
class Eq a where
  (==) :: a -> a -> Bool
  (/=) :: a -> a -> Bool

instance Eq Int where
  x == y = x `eqInt` y

eqFunction :: Eq a => a -> a -> Int
eqFunction x y =
  case x == y of
    True -> 42
    False -> 0
```

- In fact, type classes are sugar for data types with field labels
- The constraint `Eq a` is an additional argument

Field labels and type classes

- The previous listing, an unsugared version (but very roughly):

```haskell
data Eq a =
  Eq { eq :: a -> a -> Bool
     , neq :: a -> a -> Bool
     }

intInstance :: Eq Int
intInstance = Eq eqInt (\x y -> not $ x `eqInt` y)

eqFunction :: Eq a -> a -> a -> Int
eqFunction eqInst x y =
  case ((eq eqInst) x y) of
    True -> 42
    False -> 0
```

Some standard algebraic data types

- The `Maybe a` data type is a type of optional values:

```haskell
data Maybe a = Nothing | Just a

maybe :: b -> (a -> b) -> Maybe a -> b
maybe b _ Nothing = b
maybe b f (Just x) = f x
```

- A simple example

```haskell
safeHead :: [a] -> Maybe a
safeHead [] = Nothing
safeHead (x : _) = Just x
```

Some standard algebraic data types

- The `Either` data type describes one or the other value

```
data Either e a = Left e | Right a

either :: (a -> c) -> (b -> c) -> Either a b -> c
either f _ (Left x) = f x
either _ g (Right x) = g x
```

- An example:

```
safeTail :: [a] -> Either String [a]
safeTail [] = Left "I have no tail, mate"
safeTail (_ : xs) = Right xs
```

Folds

Take a look at these functions

```haskell
sum :: Num a => [a] -> a
sum [] = 0
sum (x : xs) = x + sum xs

product :: Num a => [a] -> a
product [] = 1
product (x : xs) = x * product xs

concat :: [[a]] -> [a]
concat [] = []
concat (x : xs) = x ++ concat xs
```

The definition of a right fold

- The definition of a right fold is the:

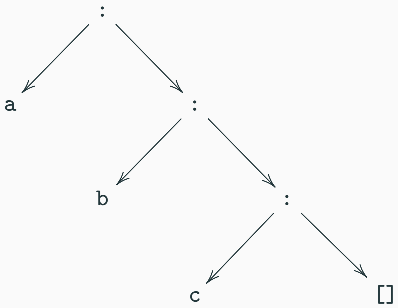
```
foldr :: (a -> b -> b) -> b -> [a] -> b
foldr _ ini [] = ini
foldr f ini (x : xs) = f x (foldr f ini xs)
```

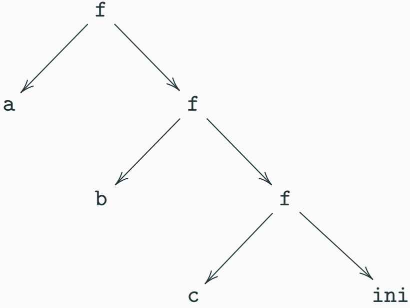
- An informal explanation:

```
foldr f z [x1, x2, ..., xn] ==
  x1 `f` (x2 `f` ... (xn `f` z)...)
```

The definition of a right fold

One may visualise that for some list `[a,b,c]`. The list from the left and its right fold from the right





Functions `sum`, `product`, and `concat` with `foldr`

```
sum :: Num a => [a] -> a
sum = foldr (+) 0

product :: Num a => [a] -> a
product = foldr (*) 1

concat :: [[a]] -> [a]
concat = foldr (++) []
```

The universal property of a right fold

The universal property
Let `g` be a function defined by the following equations:

```haskell
g []       = v
g (x : xs) = f x (g xs)
```

then one has $\forall$ `xs :: [a]` $($`g xs` $\equiv$ `foldr f v xs`$)$

- The universal property is proved inductively
- This property implies `foldr f v` and `g` are equivalent in this case

The definition of a left fold

- In addition to the right fold, one also has the left one

```
foldl :: (b -> a -> b) -> b -> [a] -> b
foldl _ ini [] = ini
foldl f ini (x : xs) = foldl f (f ini x) xs
```

- Informally:

```
foldl f ini [x1, x2, ..., xn]
  == (...((ini `f` x1) `f` x2) `f`...) `f` xn
```

The definition of a left fold

- In addition to the right fold, one also has the left one

```
foldl :: (b -> a -> b) -> b -> [a] -> b
foldl _ ini [] = ini
foldl f ini (x : xs) = foldl f (f ini x) xs
```

- Informally:

```
foldl f ini [x1, x2, ..., xn]
  == (...((ini `f` x1) `f` x2) `f`...) `f` xn
```

- We can optimise the implementation of `foldl`.
- `foldl` is the most optimal function, but we are not capable of processing infinite lists using the left fold function.

Are `foldr` and `foldl` equivalent?

- Note that `foldr` and `foldl` are not equivalent generally

```
> foldl (/) 64 [4,2,4]
2.0
> foldr (/) 64 [4,2,4]
0.125
> foldl (\x y -> 2*x + y) 4 [1,2,3]
43
> foldr (\x y -> 2*x + y) 4 [1,2,3]
16
```

- `foldr` and `foldl` are equivalent if the folding operation is associative and commutative

The right scan

- The right scan is the foldr that yields a list that contains all intermediate values

```haskell
scanr :: (a -> b -> b) -> b -> [a] -> [b]
scanr _ ini [] = [ini]
scanr f ini (x:xs) = f x q : qs
  where qs@(q:_) = scanr f ini xs
```

- `foldr` and `scanr` are connected with each other as follows

$$\texttt{head (scanr f z xs)} \equiv \texttt{foldr f z xs}$$

- The examples are

```haskell
> scanr (:) [] [1,2,3]
[[1,2,3],[2,3],[3],[]]
> scanr (+) 0 [1..10]
[55,54,52,49,45,40,34,27,19,10,0]
> scanr (*) 1 [1..5]
[120,120,60,20,5,1]
```

The left scan

- One also has a scan function for the `foldl` function:

```
scanl :: (b -> a -> b) -> b -> [a] -> [b]
scanl f q ls = q : (case ls of
                        []   -> []
                        x:xs -> scanl f (f q x) xs)
```

- `foldl` and `scanl` are connected with each other as follows:

$$\texttt{last (scanl f z xs)} \equiv \texttt{foldl f z xs}$$

- The examples:

```
> scanl (++) "!" ["a","b","c"]
["!","!a","!ab","!abc"]
> scanl (*) 1 [1..] !! 5
120
```

- In contrast to `foldl`, `scanl` works with infinite lists.

Strictness in Haskell

Bottom

- Any well-formed expression in Haskell has a type
- Prima facie, the `Bool` data type has two values: `False` and `True` according to its definition:

```haskell
data Bool = False | True
```

- One may define an expession `dno :: Bool` which is defined recursively as `dno = not dno`
- `dno` is neither `False` nor `True`, but it's a Boolean value!
- This value is a bottom ($\perp$). In Haskell, $\perp$ is a value that has a type `forall a. a`. Such errors as `undefined` have this type.

Strict functions

- Haskell is lazy. That's why `const 42 undefined == 42`
- Lazy functions are non-strict ones

Strict functions

- Haskell is lazy. That's why `const 42 undefined == 42`
- Lazy functions are non-strict ones
- In constrast to lazy functions, strict functions satisfy this equation

$$\mathtt{f}\ x_1\ x_2\ \ldots\ \perp\ \ldots\ x_n = \perp$$

- For this reason `constStrict 42 undefined = undefined`

Strictness in Haskell. The `seq` function

- We've already had a look at the `seq` function.
- `seq` is a combinator that enforces computation. It evaluates the first argument to its WHNF.
- This combinator has a type $a \to b \to b$.
- It's quite close to something like $\lambda xy.y$, but `seq` satisfies the following equations:

$$\texttt{seq} \perp x = \perp$$
$$\texttt{seq}\, v\, x = x, v \neq \perp$$

- This function "breaks" our laziness! But this enforcing with `seq` is not so far-reaching. Data constructors and lambdas put a barrier for the $\perp$ expansion:

```
> seq (4,undefined) 5
5
> seq (\x -> undefined) 5
5
> seq (id . undefined) 5
5
```

Strictness in Haskell. The strict application

- One may implement the strict appication using `seq`

```
infixr 0 $!
($!) :: (a -> b) -> a -> b
f $! x = x `seq` f x
```

- That is, this application behaves as usual unless the second argument is the bottom.

Strictness in Haskell. The strict application

- Let us recall the tail-recursive factorial. The second version is strict:

```haskell
tailFactorial :: Integer -> Integer
tailFactorial n = helper 1 n
  where
  helper acc x =
    if x > 1
    then helper (acc * x) (x - 1)
    else acc

tailFactorialStrict :: Integer -> Integer
tailFactorialStrict n = helper 1 n
  where
    helper acc x =
      if x > 1
      then (helper $! (acc * x)) (x - 1)
      else acc
```

The strict `foldl`

- The strict version of `foldl`

```
foldl' :: (a -> b -> a) -> a -> [b] -> a
foldl' f ini [] = ini
foldl' f ini (x:xs) = foldl' f arg xs
  where arg = (f ini) $! x
```

Strictness in Haskell. Bang patterns

- A data type might contain strict values with the strictness flag `!`, e.g.

```
data Complex a = !a :+ !a
  deriving Show
infix 6 :+

im :: Complex a -> a
im (x :+ y) = y
```

  > im (undefined :+ 5) *** Exception: Prelude.undefined

- The `BangPatterns` extension allows one to make pattern a strict one

```
> :set -XBangPatterns
> foo !x = True
> foo undefined
*** Exception: Prelude.undefined
```

Summary

Today we

- discussed the data type landscape and together with pattern matching
- studied folds
- realised how one can enforce lazy evaluation

Summary

Today we

- discussed the data type landscape and together with pattern matching
- studied folds
- realised how one can enforce lazy evaluation

On the next seminar, we will

- study such type classes as `Functor`, `Foldable`, and `Monoid`