Aula 17 - 4.2/2ƒ

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- $x = \rho \cdot \cos(\theta)$ e $y = \rho \cdot \mathrm{sen}(\theta)$
- $(x,y) = \varphi(\rho,\theta)$

$$dx\,dy = \left| \frac{\partial(x,y)}{\partial(\rho,\theta)} \right| d\rho\, d\theta$$

$$\frac{\partial(x,y)}{\partial(\rho,\theta)} = \begin{vmatrix} \cos(\theta) & -\rho\cdot\mathrm{sen}(\theta) \\ \mathrm{sen}(\theta) & \rho\cdot\cos(\theta) \end{vmatrix} = \rho\left(\cos^2(\theta) + \mathrm{sen}^2(\theta)\right) = \rho$$

$$dx\,dy = \rho\, d\rho\, d\theta$$

- Mudando variável

$$\iint_B f(x)\,dx\,dy = \iint_{B_{\rho\theta}} (\rho\cdot\cos\theta)\,\rho\,d\rho\,d\theta = \int_{-\pi/6}^{\pi/6} \cos\theta \left[\int_0^{\cos(3\theta)} (\rho^2)\,d\rho\right] d\theta =$$

$$\int_{-\pi/6}^{\pi/6} \cos\theta \left[\frac{\rho^3}{3}\right]\Bigg|_0^{\cos(3\theta)} d\theta = \frac{1}{3}\cdot\int_{-\pi/6}^{\pi/6} \cos(\theta)\left[\cos^3(3\theta)\right] d\theta$$

- Reescrevendo:

$$\cos\theta\cos^3 3\theta = \underbrace{(\cos 3\theta \cos 3\theta)}_{\mathrm{I}}\cdot\underbrace{(\cos 3\theta\cdot\cos\theta)}_{\mathrm{II}}$$

$$\mathrm{I} = \frac{1}{2}\left[\cos((3+3)\theta) + \cos((3-3)\theta)\right] = \frac{1}{2}\left[\cos(6\theta) + 1\right]$$

$$\mathrm{II} = \frac{1}{2}\left[\cos((3+1)\theta) + \cos((3-1)\theta)\right] = \frac{1}{2}\left[\cos 4\theta + \cos 2\theta\right]$$

$$\cos\theta\cos^3 3\theta = \frac{1}{2}\left[\cos(6\theta)+1\right]\cdot\frac{1}{2}\left[\cos 4\theta + \cos 2\theta\right] \Rightarrow$$

$$\cos\theta\cdot\cos^3(3\theta) = \frac{1}{4}\left[\cos 6\theta\cdot\cos 4\theta + \cos 6\theta\cdot\cos 2\theta + \cos(4\theta) + \cos(2\theta)\right]$$

* $\cos(6\theta)\cdot\cos(4\theta) = \frac{1}{2}\left[\cos((6+4)\theta) + \cos((6-4)\theta)\right] = \frac{1}{2}\left(\cos(10\theta) + \cos(2\theta)\right)$

* $\cos(6\theta)\cdot\cos(2\theta) = \frac{1}{2}\left[\cos((6+2)\theta) + \cos((6-2)\theta)\right] = \frac{1}{2}\left(\cos(8\theta) + \cos(4\theta)\right)$

* $\cos(\theta)\cdot\cos^3(3\theta) = \frac{1}{4}\left[\frac{1}{2}\left[\cos(10\theta) + \cos(2\theta)\right] + \frac{1}{2}\left[\cos(8\theta) + \cos(4\theta)\right] + \cos(4\theta) + \cos(2\theta)\right]$

$$= \frac{1}{8}\left[\cos(10\theta) + \cos(8\theta) + 3\cdot\cos(4\theta) + 3\cdot\cos(2\theta)\right]$$

- Integrando:

$$\int_{\pi/6}^{\pi/6} \frac{1}{8}\left[\cos(10\theta) + \cos(8\theta) + 3\cos(4\theta) + 3\cos(2\theta)\right] d\theta = \frac{1}{8}\left[\frac{\mathrm{sen}(10\theta)}{10} + \frac{\mathrm{sen}(8\theta)}{8} + \frac{3}{4}\cdot\mathrm{sen}(4\theta) + \frac{3}{2}\,\mathrm{sen}(2\theta)\right]_{\pi/6}^{\pi/6}$$

$$= \frac{1}{8}\left[\frac{1}{10}\left(\mathrm{sen}\,\frac{10\pi}{6} - \mathrm{sen}\left(\frac{-10\pi}{6}\right)\right) + \frac{1}{8}\left(\mathrm{sen}\left(\frac{8\pi}{6}\right) - \mathrm{sen}\left(\frac{-8\pi}{6}\right)\right) + \frac{3}{4}\left(\mathrm{sen}\,\frac{4\pi}{6} - \mathrm{sen}\left(\frac{4\pi}{6}\right) + \frac{3}{2}\left(\mathrm{sen}\,\frac{2\pi}{3} - \mathrm{sen}\left(\frac{-2\pi}{3}\right)\right)\right)\right]$$

$$= \frac{1}{8}\left[\frac{1}{10}\,2\cdot\mathrm{sen}\left(\frac{5\pi}{3}\right) + \frac{1}{8}\,2\cdot\mathrm{sen}\left(\frac{4\pi}{3}\right) + \frac{3}{4}\cdot 2\,\mathrm{sen}\left(\frac{2\pi}{3}\right) + \frac{3}{2}\cdot 2\,\mathrm{sen}\left(\frac{2\pi}{3}\right)\right]$$

$$= \frac{1}{40}\,\mathrm{sen}\left(\frac{5\pi}{3}\right) + \frac{1}{32}\,\mathrm{sen}\left(\frac{4\pi}{3}\right) + \frac{9}{32}\,\mathrm{sen}\left(\frac{2\pi}{3}\right)$$

$$= \frac{1}{40}\left(-\frac{\sqrt{3}}{2}\right) + \frac{1}{32}\left(-\frac{\sqrt{3}}{2}\right) + \frac{9}{32}\cdot\frac{\sqrt{3}}{2} = \frac{9}{40}\cdot\frac{\sqrt{3}}{2} = \frac{9\sqrt{3}}{80}$$