Aula 23 - 9.3/1d

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· Projetando e calculando área:

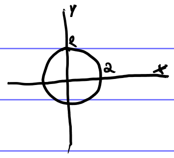
$$\iint_S dS = \iint_{R'} \sqrt{1+\left(\frac{\partial z}{\partial y}\right)^2+\left(\frac{\partial z}{\partial x}\right)^2}\, dy\, dx$$

· $z = 4 - u^2 - v^2$

$z_u = -2u$

$z_v = -2v^2$

· $$\iint_{R'} \sqrt{1+\left(\frac{\partial z}{\partial u}\right)^2+\left(\frac{\partial z}{\partial v}\right)^2}\, du\, dv = \iint \sqrt{1+(-2u)^2+(-2v)^2}\, du\, dv$$



· $u^2 + v^2 = r^2$

$J = r$

· Intervalos de integração

$0 < r < e^{-\theta}$

$0 < \theta < \pi$

· $$\int_0^{\pi}\int_0^{e^{-\theta}} \sqrt{1+4r^2} \cdot r\, dr\, d\theta$$

$\therefore\ u = 1 + 4r^2$

$du = 8r\, dr$

$$\int_0^{\pi}\int_0^{e^{-\theta}} \sqrt{1+4r^2} \cdot r\, dr\, d\theta = \frac{1}{12}\int_0^{\pi} (1+4r^2)^{3/2} \Big|_{r=0}^{e^{-\theta}} d\theta \Rightarrow$$

$$\int_0^{\pi}\int_0^{e^{-\theta}} \sqrt{1+4r^2} \cdot r\, dr\, d\theta = \frac{1}{12}\int_0^{\pi} (1+4e^{-2\theta})^{3/2} - 1\, d\theta \Rightarrow$$

$$\int_0^{\pi}\int_0^{e^{-\theta}} \sqrt{1+4r^2} \cdot r\, dr\, d\theta = \frac{1}{12} \cdot 4 = \frac{1}{3}$$