Aula 08 - 14.1/4

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$$\frac{\partial z}{\partial y} = \frac{\partial}{\partial y}\left[(x+y)\cdot e^{\frac{x}{y}}\right] = (x+y)\frac{\partial}{\partial y}\left[e^{\frac{x}{y}}\right] + e^{\frac{x}{y}}\cdot\frac{\partial}{\partial y}\left[(x+y)\right] = (x+y)\cdot e^{\frac{x}{y}}\left(\frac{-x}{y}\right) + e^{\frac{x}{y}}$$

$$\frac{\partial z}{\partial y} = e^{\frac{x}{y}}\cdot\left(1 - \frac{x^2}{y^2} - \frac{x}{y}\right)$$

$$\frac{\partial^2 z}{\partial x\partial y} = \frac{\partial}{\partial x}\left[\frac{\partial z}{\partial x}\right] = \frac{\partial}{\partial x} = \left[e^{\frac{x}{y}}\cdot\left(1 - \frac{x^2}{y^2} - \frac{x}{y}\right)\right] =$$

$$e^{\frac{x}{y}}\frac{\partial}{\partial x}\left[1 - \frac{x^2}{y^2} - \frac{x}{y}\right] + \left(1 - \frac{x^2}{y^2} - \frac{x}{y}\right)\cdot\frac{\partial}{\partial x}\left[e^{\frac{x}{y}}\right] =$$

$$e^{\frac{x}{y}}\left(-2\frac{x}{y^2} - \frac{1}{y}\right) + \left(1 - \frac{x^2}{y^2} - \frac{x}{y}\right)\cdot e^{\frac{x}{y}}\cdot\frac{1}{y}$$

$$\frac{\partial^2 z}{\partial y^2} = \frac{\partial}{\partial y}\left[\frac{\partial z}{\partial y}\right] = \frac{\partial}{\partial y}\left[e^{\frac{x}{y}}\cdot\left(1 - \frac{x^2}{y^2} - \frac{x}{y}\right)\right] =$$

$$e^{\frac{x}{y}}\frac{\partial}{\partial y}\left[\left(1 - \frac{x^2}{y^2} - \frac{x}{y}\right)\right] + \left(1 - \frac{x^2}{y^2} - \frac{x}{y}\right)\frac{\partial}{\partial y}\left[e^{\frac{x}{y}}\right] =$$

$$e^{\frac{x}{y}}\left(\frac{2x^2}{y^3} + \frac{x}{y^2}\right) + \left(1 - \frac{x^2}{y^2} - \frac{x}{y}\right)\cdot e^{\frac{x}{y}}\cdot\left(\frac{-x}{y^2}\right)$$

Simplificando:

$$\frac{\partial^2 z}{\partial x\partial y} = e^{\frac{x}{y}}\left[\frac{-x^2}{y^3} - \frac{3x}{y}\right] \quad ;$$

$$\frac{\partial^2 z}{\partial y^2} = e^{\frac{x}{y}}\left[3\cdot\frac{x^2}{y^3} + \frac{x^3}{y^4}\right]$$

$$x\frac{\partial^2 z}{\partial x\partial y} + y\frac{\partial^2 z}{\partial y^2} = x\cdot e^{\frac{x}{y}}\left[\frac{-x^2}{y^3} - \frac{3x^2}{y^2}\right] + y\cdot e^{\frac{x}{y}}\left[3\cdot\frac{x^2}{y^3} + \frac{x^3}{y^4}\right]$$

$$= e^{\frac{x}{y}}\left(\left[\frac{-x^3}{y^3} - \frac{3x^2}{y^2}\right] + \left[3\cdot\frac{x^2}{y^2} + \frac{x^3}{y^3}\right]\right)$$

$$\therefore\ x\frac{\partial^2 z}{\partial x\partial y} + y\frac{\partial^2 z}{\partial y^2} = 0$$