

• Tomando $u = x \cdot y$ e $v = z$ temos que $F(x, y, z) = 0$ será equivalente a $F(u, v) = 0$

• Derivando em relação a x :

$$\frac{\partial F}{\partial u} \cdot \frac{\partial u}{\partial x} + \frac{\partial F}{\partial v} \cdot \frac{\partial v}{\partial x} = 0$$

$$\frac{\partial u}{\partial x} = y; \frac{\partial v}{\partial x} = \frac{\partial z}{\partial x}$$

$$y \frac{\partial F}{\partial u} + \frac{\partial F}{\partial v} \cdot \frac{\partial z}{\partial x} = 0 \Rightarrow \frac{\partial z}{\partial x} = -y \frac{\frac{\partial F}{\partial u}}{\frac{\partial F}{\partial v}}$$

• Derivando em y

$$\frac{\partial F}{\partial u} \cdot \frac{\partial u}{\partial y} + \frac{\partial F}{\partial v} \cdot \frac{\partial v}{\partial y} = 0$$

$$x \cdot \frac{\partial F}{\partial u} + \frac{\partial F}{\partial v} \cdot \frac{\partial z}{\partial y} = 0 \Rightarrow \frac{\partial z}{\partial y} = -x \frac{\frac{\partial F}{\partial u}}{\frac{\partial F}{\partial v}}$$

$$I = x \cdot \left(-y \frac{\frac{\partial F}{\partial u}}{\frac{\partial F}{\partial v}} \right) - y \cdot \left(-x \frac{\frac{\partial F}{\partial u}}{\frac{\partial F}{\partial v}} \right)$$

$$I = x \cdot y \cdot \frac{\frac{\partial F}{\partial u}}{\frac{\partial F}{\partial v}} - x \cdot y \cdot \frac{\frac{\partial F}{\partial u}}{\frac{\partial F}{\partial v}} = 0$$

Dessa forma vemos que $x \frac{\partial z}{\partial x} - y \frac{\partial z}{\partial y} = 0$