Aula 23 - 9.4/1h

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· $\iint_K f(\sigma(u,v)) \left\| \frac{\partial \sigma}{\partial u} \wedge \frac{\partial \sigma}{\partial v} \right\| du\,dv$

$x = u \; ; \; y = v$

$\sigma(u,v) = (u, v, \sqrt{u^2+v^2})$

· $\frac{\partial \sigma}{\partial u} = \left(1, 0, \frac{u}{\sqrt{u^2+v^2}}\right)$ e $\frac{\partial \sigma}{\partial v} = \left(0, 1, \frac{v}{\sqrt{u^2+v^2}}\right)$

$$\frac{\partial \sigma}{\partial u} \wedge \frac{\partial \sigma}{\partial v} = \begin{bmatrix} i & j & k \\ 1 & 0 & \frac{u}{\sqrt{u^2+v^2}} \\ 0 & 1 & \frac{v}{\sqrt{u^2+v^2}} \end{bmatrix}$$

$$\frac{\partial \sigma}{\partial u} \wedge \frac{\partial \sigma}{\partial v} = \vec{i} \begin{vmatrix} 0 & \frac{u}{\sqrt{u^2+v^2}} \\ 1 & \frac{v}{\sqrt{u^2+v^2}} \end{vmatrix} - \vec{j} \begin{vmatrix} 1 & \frac{u}{\sqrt{u^2+v^2}} \\ 0 & \frac{v}{\sqrt{u^2+v^2}} \end{vmatrix} + \vec{k} \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$$

$$\frac{\partial \sigma}{\partial u} \wedge \frac{\partial \sigma}{\partial v} = - \frac{u}{\sqrt{u^2+v^2}}\vec{i} + \frac{v}{\sqrt{u^2+v^2}}\vec{j} + \vec{k}$$

$$\left\| \frac{\partial \sigma}{\partial u} \wedge \frac{\partial \sigma}{\partial v} \right\| = \sqrt{\left(\frac{u}{\sqrt{u^2+v^2}}\right)^2 + \left(\frac{v}{\sqrt{u^2+v^2}}\right)^2 + 1^2} = \sqrt{\frac{u^2+v^2}{u^2+v^2} + 1} = \sqrt{2}$$

· $\iint_K z\sqrt{2}$

· Utilizando coordenadas polares:

$z^2 - 4z + 3 \geqslant 0$

$1 \leqslant z \leqslant 3$ e $z^2 = \rho^2$ $\therefore$ o intervalo será: $1 \leqslant \rho \leqslant 3$

$$\int_0^{2\pi}\int_1^3 \rho \cdot \sqrt{2}\, \rho d\rho \theta$$

· $\sqrt{2} \int_0^{2\pi} \left[\frac{\rho^3}{3}\right]_1^3 d\theta = \sqrt{2} \int_0^{2\pi} \frac{27}{3} - \frac{1}{3}\, d\theta = \sqrt{2} \left[\frac{26}{3}\right]_0^{2\pi} = \frac{52}{3} \cdot \sqrt{2}\,\pi$