Aula 17 - 4.2/1d

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* Fubini: $\iint_B f(x,y)\,dx\,dy$

· Mudando Variáveis

$x = \rho \cdot \cos\theta$ e $y = \rho \cdot \operatorname{sen}\theta$

· $(x,y) = \varphi(\rho,\theta)$

$$dx\,dy = \left|\frac{\partial(x,y)}{\partial(\rho,\theta)}\right| d\cdot\rho\cdot d\theta$$

· $$\frac{\partial(x,y)}{\partial(\rho,\theta)} = \begin{vmatrix} \frac{\partial x}{\partial \rho} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial \rho} & \frac{\partial y}{\partial \theta} \end{vmatrix} = \begin{vmatrix} \frac{1}{2}\cos(\theta) & -\frac{\rho}{2}\operatorname{sen}(\theta) \\ \operatorname{sen}(\theta) & \rho\cdot\cos(\theta) \end{vmatrix} = \frac{\rho}{2}\left(\cos^2\theta + \operatorname{sen}^2(\theta)\right) = \frac{\rho}{2}$$

$\therefore$ $dx\,dy = \frac{\rho}{2}\cdot d\cdot\rho\cdot d\theta$

· Encontrando o conjunto $B_{\rho\theta}$.

$0 \le \rho \le 1$ e $0 \le \theta \le \pi$

· Calculando integral

$$\iint_B \operatorname{sen}(4x^2+y^2)\,dx\,dy = \iint_{B_{\rho\theta}} \operatorname{sen}(\rho^2\cdot\cos^2(\theta) + \rho^2\operatorname{sen}^2\theta)\,\frac{\rho}{2}\,d\rho d\theta \Rightarrow$$

$$\int_0^{\pi}\left[\int_0^1 \operatorname{sen}(\rho^2)\,\frac{\rho}{2}\,d\rho\right]d\theta$$

$$\int_0^{\pi}\left[\int_0^1 \operatorname{sen}(u)\,\frac{1}{2}\left(\frac{du}{2}\right)\right]d\theta = \int_0^{\pi}\left[-\frac{1}{4}\cos(u)\right]\Big|_0^1 d\theta$$

$\rho^2 = u$

$du = 2\rho d\rho$

$$\int_0^{\pi} -\frac{1}{4}\left[\cos 1 - \cos 0\right]d\theta = \int_0^{\pi}\frac{1}{4}\left[1-\cos 1\right]d\theta$$

$$\int_0^{\pi}\frac{1}{4}\left[1-\cos 1\right]\,d\theta = \frac{1}{4}(1-\cos 1)[\theta]\Big|_0^{\pi} = \frac{\pi}{4}(1-\cos 1)$$