· Multiplicadores de Lagrange :

$\nabla f(x,y) = \lambda \nabla g(x,y)$

$(y^2 z, 2xyz, xy^2) = \lambda(2x, 2y, 2z)$

· $y^2 z = \lambda 2x$

$2xyz = \lambda 2y$

$xy^2 = \lambda 2z$

· $\lambda = \dfrac{y^2 z}{2x} = \dfrac{2xyz}{2y} = \dfrac{xy^2}{2z}$

· $\dfrac{y^2 z}{2x} = \dfrac{2xyz}{2y}$ $\Rightarrow 4x^2 yz = 2y^3 z$ $\Rightarrow 2x^2 = y^2$

· $\dfrac{xy^2}{2z} = \dfrac{2xyz}{2y}$ $\Rightarrow 2xy^3 = 4xyz^2 \Rightarrow y^2 = 2z^2 = 2x^2 \Rightarrow z^2 = x^2$

· Temos: $x^2 + y^2 + z^2 = 4$

$x^2 + 2x^2 + x^2 = 4$

$4x^2 = 4$

$x^2 = 1$

$x = \pm 1$

∴ $y^2 = 2x^2$ $z = \pm 1$

$y = \pm \sqrt{2x^2}$;

$y = \pm\sqrt{2}$

· Pontos considerados:

$(1, \sqrt{2}, 1)$; $(1, -\sqrt{2}, 1)$; $(1, \sqrt{2}, -1)$; $(1, -\sqrt{2}, -1)$; $(-1, -\sqrt{2}, -1)$

$(-1, \sqrt{2}, -1)$; $(-1, -\sqrt{2}, 1)$; $(-1, -\sqrt{2}, 1)$

$f(-1, \sqrt{2}, -1) = 2$ $f(-1, \sqrt{2}, 1) = -2$

$f(-1, \sqrt{2}, -1) = 2$; $f(-1, -\sqrt{2}, 1) = -2$

$f(1, \sqrt{2}, 1) = 2$ $f(1, \sqrt{2}, -1) = -2$

$f(1, -\sqrt{2}, 1) = 2$ $f(1, -\sqrt{2}, -1) = -2$

· O valor mínimo portanto é -2 e o valor máximo 2