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13.2

37) Resolvendo a integral temos:

$$\text{(I)}\left(\int_0^1 \frac{1}{t+1}\,dt\right)i + \text{(II)}\left(\int_0^1 \frac{1}{t^2+1}\,dt\right)j + \text{(III)}\left(\int_0^1 \frac{2t}{t^2+1}\,dt\right)k$$

$$\text{(I)}: \left(\int_0^1 \frac{1}{t+1}\,dt\right)i = \left[\ln(t+1)\,i\right]_0^1 = \left[\ln(1+1) - \ln(0+1)\right]i = \ln(2)\,i$$

$$\text{(II)}: \left(\int_0^1 \frac{1}{t^2+1}\,dt\right)j = \left[\tan^{-1}\left(\frac{t}{1}\right)j\right]_0^1 = \left[\tan^{-1}\left(\frac{1}{1}\right) - \tan^{-1}\left(\frac{0}{1}\right)\right]j = \left(\frac{\pi}{4}\right)j$$

$$\text{(III)}: \left(\int_0^1 \frac{2t}{(t^2+1)}\right)k = \left(\int_1^2 \frac{1}{u}\,du\right)k \qquad \int \to t^2+1 = u \;;\; du = 2dt \cdot dt \;;\; \begin{matrix} t=0 \to u=1 \\ t=1 \to u=2 \end{matrix}$$

$$\Rightarrow \left(\int_0^1 \frac{2t}{t^2+1}\right)k = \left(\int_1^2 \frac{1}{u}\,du\right)k = [\ln u \cdot k]_1^2 = [\ln 2 - \ln 1]k = \ln 2 \cdot k$$

$\therefore$

$$\int_0^1 \left(\frac{1}{t+1}i + \frac{1}{t^2+1}j + \frac{t}{t^2+1}k\right)dt = \ln 2\, i + \frac{\pi}{4}j + \ln 2\, k$$