

• Calculando derivadas parciais em x :

$$\frac{\partial z}{\partial x} = \frac{\partial(x^2+y^2) \ln(x^2+y^2)}{\partial x} = \ln(x^2+y^2) \cdot \frac{\partial(x^2+y^2)}{\partial x} + (x^2+y^2) \cdot \frac{\partial \ln(x^2+y^2)}{\partial x}$$

(I) (II)

• Resolvendo I:

$$\frac{\partial(x^2+y^2)}{\partial x} = 2x$$

• Resolvendo II:

$$\ln(u)' = \frac{1}{u} u' \quad \therefore \quad \frac{\partial \ln(x^2+y^2)}{\partial x} = \frac{1}{x^2+y^2} \cdot \frac{\partial(x^2+y^2)}{\partial x} = \frac{2x}{x^2+y^2}$$

$$\therefore \frac{\partial z}{\partial x} = 2x \ln(x^2+y^2) + (x^2+y^2) \cdot \frac{2x}{x^2+y^2}$$

$$\left| \frac{\partial z}{\partial x} = 2x \ln(x^2+y^2) + 2x = 2x(\ln(x^2+y^2) + 1) \right.$$

• Calculando em relação a y :

$$\frac{\partial z}{\partial y} = \frac{\partial(x^2+y^2) \ln(x^2+y^2)}{\partial y} = \ln(x^2+y^2) \cdot \frac{\partial(x^2+y^2)}{\partial y} + (x^2+y^2) \cdot \frac{\partial \ln(x^2+y^2)}{\partial y}$$

(III) (IV)

$$\text{III: } \frac{\partial(x^2+y^2)}{\partial y} = 2y$$

IV: Pelas mesmas propriedades vistas acima temos:

$$\frac{\partial \ln(x^2+y^2)}{\partial y} = \frac{2y}{x^2+y^2}$$

• Substituindo, temos:

$$\frac{\partial z}{\partial y} = 2y \ln(x^2+y^2) + (x^2+y^2) \cdot \frac{2y}{x^2+y^2}$$

$$\left| \therefore \frac{\partial z}{\partial y} = 2y(\ln(x^2+y^2) + 1) \right.$$

Então a resposta fica:

$$\frac{\partial z}{\partial x} = 2x(\ln(x^2+y^2) + 1) \quad \text{e} \quad \frac{\partial z}{\partial y} = 2y(\ln(x^2+y^2) + 1)$$