Quantum Non-Demolition Measurement of Photon Number in a High-Q Cavity

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MIT 8.421 Term Paper, Spring 2008

Experimental measurements of quantum systems usually result in back action from the observer into the system, who thus alters its natural evolution. This often defeats the purpose of measurement, which is the objective observation of nature. However, quantum theory allows for observation processes with no back action, called quantum non-demolition (QND) measurements. Such measurements have been realized in various systems [1, 2]. In this review paper, we summarize one such realization performed with circular Rydberg Rubidium atoms coupled strongly to a high-Q, high-finesse optical cavity [3]. The occupation number of the cavity Fock state was observed non-destructively via the measurement of an electromagnetically-induced phaseshift in the wavefunction of the atoms.

I. INTRODUCTION

A. Quantum Non-Demolition Measurements

One of the most counter-intuitive aspects of quantum mechanics is the fact that it allows for a physical system to be in a simultaneous superposition of two or more states. Various attempts have been made to provide a physical interpretation which reconciles the theory with classical human intuition. The two main trains of thought are the statistical interpretation and the Copenhagen interpretation. According to the statistical interpretation, superposition is only meaningful when describing an ensemble average of systems, in which case it assigns probabilities to each possible state. It is compatible with the intuitive notion that a system in the ensemble will occupy only one of these states both before and after it is observed in its state.

According to the Copenhagen interpretation, superposition implies something more fundamental than a statistical average; it can be applied to one single system to describe a situation where the system is not occupying either of the involved states completely, but is rather parted among them. Upon measurement, the system is forced to assume one definite state; however, this does not mean that the system occupied that state before the measurement, neither that it will stay in that state after the measurement. Rather, the act of measurement introduced coupling between the quantum system and the rest of the (classical) environment, thus changing temporarily the Hamiltonian and effecting the evolution of the system.

From an experimentalist's point of view, it seems that the two interpretations are irresolvable, as by the mere definition of "‘measurement" we are doomed to disturb the system under observation and thus to never find out its true state before or after our intervention. However, according to the mathematical formalism of quantum mechanics, it is possible to perform a benign measurement so long as the observable measured can be related to an operator which commutes with the system's Hamiltonian. In that case, when operated upon, the system may remain in an unperturbed eigenstate of its own Hamiltonian. Such measurements, called quantum non-demolition (QND) measurements, are of great interest, since they provide a way to truly observe a quantum system without disturbing it.

B. Conditions for QND

Let H_S be the Hamiltonian of the system to be observed, H_P be the Hamiltonian of the probe with which it will be observed and H_I be the Hamiltonian of the interaction between the system and the probe. Moreover, let A_S be the quantity which we want to measure and A_P be the quantity that we detect. In order to perform a measurement, we need to satisfy the following conditions:

a) H_I must depend on A_S, $\frac{\partial A_S}{\partial H_I} \neq 0$;

b) H_I must affect the evolution of A_P, $[H_I, A_P] \neq 0$

so that information about A_S can be obtained via the measurement of A_P. Furthermore, in order to perform a non-demolition measurement, it need be the case that

c) H_I should not affect the evolution of A_S, $[H_I, A_S] = 0$;

d) H_S should not depend on the conjugate variable of A_S, $\frac{\partial A_S}{\partial H_S} = 0$.

The last criterion protects the system from the consequences of the uncertainty principle. Measurement of A_S may affect its conjugate variable; therefore, in order to ensure the unperturbed evolution of our system after measurement, it is necessary that this effect of "back action" does not alter the system's Hamiltonian [4].

II. QND MEASUREMENT OF A SINGLE PHOTON IN A HIGH-Q CAVITY

There have been various proposals for systems and detection methods satisfying the requirements of a QND measurement. Such proposals are based on non-linear effects, since in order for the interaction Hamiltonian to preserve the quantum number of interest, it must be at

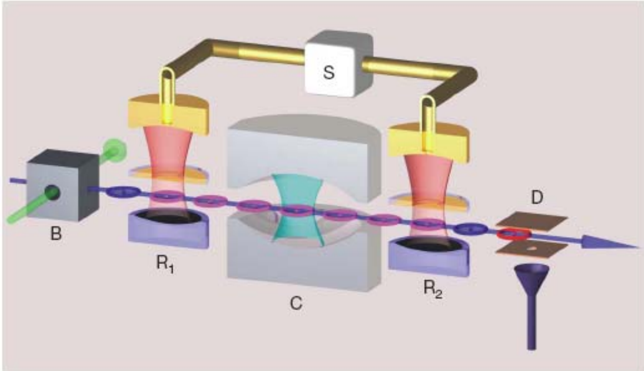

FIG. 1: The experimental setup. Velocity-selected circular Rydberg atoms are prepared in B. They are subjected to a $\pi/2$ pulse in Ramsey cavities $R1$, $R2$, each of which consists of two cavities coupled by a partly reflected mirror. This setup achieves a well-localized mode field which does not couple to the middle cavity C simultaneously with low Q-factor, thus inhibiting spontaneous emission. Cavity C, containing mostly zero or one photons, is connected to microwave source S. The atoms are detected in D via state selective field ionization. Figure from [3]

least quadratic in the creation and annihilation operators associated with the observed variables. The experiment which we will focus on in this review paper is based on the non-linear coupling between a high-Q Fabry-Perot cavity and Rubidium atoms in a circular Rydberg state for non-destructive counting of the number of photons in the cavity (zero or one) [3]. It was performed at the Laboratoire Kastler Brossel of the Ecole Normale Superieure by Haroche et al. Quantum non-demolition was demonstrated by repeated measurements of the cavity Fock state, which monitored the lifetime of a single photon in agreement with theoretical expectations based on the cavity ringdown time. A schematic of the experimental setup is shown in Figure 1.

The setup is immersed in a cryogenic environment at $0.8K$. A stream of 900 s^{-1} circular Rydberg atoms ($n = 50$) with velocity $250m/s$ is first sent through a Ramsey cavity $R1$, where they receive a $\pi/2$ pulse in resonance with the $n = 50 \rightarrow n = 51$ ($g \rightarrow e$) transition at $51.1GHz$. They then proceed to cavity C, which is slightly detuned from resonance and randomly occupied by zero or one photons from the ambient thermal field (the probability for higher occupation numbers is, according to the authors, on the order of 3×10^{-3} [**?**]). The atomic stream is dilute enough so that there is only one atom crossing C at a time. The atoms couple to C and thus acquire a Stark shift which depends on the field amplitude in the cavity, i.e. on the number of photons. Upon their exit from C the atoms are subjected to a second $\pi/2$ resonant pulse in Ramsey cavity $R2$. Their final state is detected in field-ionization detector D.

A. Mathematical Description

Using the nomenclature introduced above for the system, the probe, and the interaction Hamiltonian, let H_S be the Hamiltonian for the field in the cavity, H_P be the Hamiltonian for the atoms, and H_I be the Hamiltonian describing the atom-cavity interaction. The observable we want to measure, A_S, is the photon number in the cavity, and the observable we are detecting in D, A_P, is the atomic phase.

Initially the atoms are prepared in state g. After they experience a resonant $\pi/2$ pulse in R1, they exit in a superposition of states $\frac{1}{\sqrt{2}}(|e\rangle + |g\rangle)$. They then enter C, where the combined atom-cavity system is described by the Jaynes-Cummings model [5]. The interaction Hamiltonian is of the form

$$H_I = \begin{pmatrix} \Delta & \omega_1\sqrt{n+1} \\ \omega_1\sqrt{n+1} & \Delta \end{pmatrix}$$

where Δ is the detuning of the cavity from the atomic natural frequency, $\Delta = \omega - \omega_o$, n is the photon number, and ω_1 is the coupling strength between the atom and the cavity, given by $\omega_1 = d{\cdot}\hat{e}\sqrt{\frac{\hbar\omega_o}{2\epsilon V}}$, d being the electric dipole matrix element of the atoms, $\hat{e}$ the polarization vector of the electromagnetic field in the cavity, ϵ the permittivity of free space, and V the volume of the cavity. This Hamiltonian is based on a quantized description of the electromagnetic field in the rotating wave approximation [**?**]. The eigenstates are

$$\psi_\pm = \pm\sqrt{\frac{\sqrt{\Delta^2 + \omega_1^2(n+1)} \pm \Delta}{2\sqrt{\Delta^2 + \omega_1^2(n+1)}}}\,|en\rangle + \sqrt{\frac{\sqrt{\Delta^2 + \omega_1^2(n+1)} \mp \Delta}{2\sqrt{\Delta^2 + \omega_1^2(n+1)}}}\,|gn+1\rangle$$

and the corresponding eigenvalues are

$$E_\pm = (n + \frac{1}{2})\hbar\omega \pm \hbar\sqrt{\Delta^2 + \omega_1^2(n+1)}$$

It is important to notice that even in the absence of photons ($n = 0$) or in the absence of detuning ($\Delta = 0$) there is always coupling between the atoms and the cavity due to the fact that the state of the atoms has an admixture of the excited state which can potentially de-excite into the cavity (single-photon Rabi flopping).

In this setup the coupling ω_1 has the spatial profile of the Gaussian 900 mode supported by the cavity at 51.1 GHz with width $w = 6\ mm$ [6]: $\omega_1 = \Omega \cdot e^{-\frac{z^2}{w^2}}$, where $\Omega/2\pi = 51\ kHz$ is the maximum coupling at the cavity center and z is the coordinate along the trajectory of the atomic beam [3]. As the atoms pass through 250 m/s, they perceive a time-varying Hamiltonian. In order for the system to evolve smoothly into and out of the dressed Hamiltonian, the condition for adiabatic passage must be satisfied. According to the Adiabatic

Theorem [7], this is ensured if the rate of change of the Hamiltonian is small compared to the minimum energy splitting between its eigenstates. If we express the time varying Jaynes-Cummings Hamiltonian as a function of the atomic velocity v and the cavity length L,

$$H_I(t) = \begin{pmatrix} \Delta & \Omega \cdot e^{-\frac{(vt-\frac{L}{2})^2}{w^2}} \sqrt{n+1} \\ \Omega \cdot e^{-\frac{(vt-\frac{L}{2})^2}{w^2}} \sqrt{n+1} & \Delta \end{pmatrix}$$

it can be readily verified that the relevant dimensionless parameter $\tau = \frac{\omega_1(t)}{\Delta E(t)}$ is small ($\tau \leq \frac{1}{2}$, reaching its maximum value at the center of the cavity where the coupling is strongest).

Having established adiabaticity, we can now proceed to express the full wavefunction of the atom-cavity system as a function of time and photon number:

$$\begin{aligned}\psi_\pm(t,n) &= \pm\sqrt{\frac{\sqrt{\Delta^2+\omega_1(t)^2(n+1)}\pm\Delta}{2\sqrt{\Delta^2+\omega_1(t)^2(n+1)}}}\,|en\rangle\, e^{-iE_+(t)t/\hbar} \\ &+\sqrt{\frac{\sqrt{\Delta^2+\omega_1(t)^2(n+1)}\mp\Delta}{2\sqrt{\Delta^2+\omega_1(t)^2(n+1)}}}\,|gn+1\rangle\, e^{-iE_-(t)t/\hbar}\end{aligned}$$

The combined system evolves according to an adiabatically time-varying Hamiltonian, so the relative magnitudes and phases of the eigenstates are time-dependent as well. As the intensity of the cavity mode dies out, the coupling strength drops to zero, the two systems cease to interact, and the relative phase between the different superimposed states freezes in time. Finally, the atoms are subjected to a second resonant $\pi/2$ pulse which rotates the atomic wavefunction by another 90 degrees on the Bloch sphere, maintaining the phase imprinted upon it by the atom-cavity interaction. This phase depends on the photon number and thus contains the desired information.

It is straightforward to verify that the criteria for a QND measurement described in the introduction are satisfied. The atom-cavity interaction Hamiltonian H_I clearly depends on the photon number and affects the phase of the atomic wavefunction, thus connecting the two variables A_S, A_P. Moreover, the Jaynes-Cummings interaction as presented above (i.e. in the rotating wave approximation) conserves total photon number (A_S) and is independent of the atomic probability amplitudes (conjugate of A_P).

In mathematical terms, the time evolution of the atom-cavity wavefunction can be described in the following sequence:
Initially,

$$|initial\rangle = |g\rangle \times |cavity\rangle$$

After the first $\pi/2$ pulse,

$$|R1\rangle = \frac{1}{\sqrt{2}}(|g\rangle + |e\rangle) \times |cavity\rangle$$

. During atom-cavity coupling,

$$\begin{aligned}|C\rangle &= \frac{1}{\sqrt{2}}(A(t,n)+B(t,n))\,|\psi_+\rangle\, e^{\frac{-i\int_0^t E_+(t')dt'}{\hbar}} \\ &\quad +(B(t,n)-A(t,n))\,|\psi_-\rangle\, e^{\frac{-i\int_0^t E_-(t')dt'}{\hbar}}\end{aligned}$$

where

$$A(t,n) = \sqrt{\frac{\sqrt{\Delta^2+\omega_1(t)^2(n+1)}\pm\Delta}{2\sqrt{\Delta^2+\omega_1(t)^2(n+1)}}}$$

$$B(t,n) = \sqrt{\frac{\sqrt{\Delta^2+\omega_1(t)^2(n+1)}\mp\Delta}{2\sqrt{\Delta^2+\omega_1(t)^2(n+1)}}}$$

Once out of the cavity ($\Delta = 0, \omega_1 = 0, \partial t = 0$),

$$|Cout\rangle = -\frac{1}{\sqrt{2}}(|g\rangle + e^{-i\phi(n)}\,|e\rangle) \times |cavity\rangle$$

where $\phi(n) = 2\int_0^{L/v}\sqrt{\Delta^2+(n+1)\Omega^2 e^{-\frac{(vt'-\frac{L}{2})^2}{w^2}}}\,dt'$, L being the cavity length ($L = 27.57\ mm$)[6]. After the second $\pi/2$ pulse,

$$|R2\rangle = \frac{1}{2}((1-e^{-i\phi(n)})\,|g\rangle + (1+e^{-i\phi(n)}\,|e\rangle) \times |cavity\rangle$$

By proper detuning of the cavity, which can be achieved either by moving the mirrors with piezoelectric translators, or by applying a small static electric field across the cavity to induce a Stark Shift on the atoms, it is possible to adjust ϕ so that after the second $\pi/2$ pulse the atom is in $|g\rangle$ if $n = 0$ and in $|e\rangle$ if $n = 1$.

B. Experimental Challenges

1. *High-Finesse Cavity*

The realization of this elegant experiment poses many challenges. First of all, in order to demonstrate non-demolition it is necessary to perform multiple measurements on a system, and thus to be able to maintain the system isolated from the rest of the environment for a time much longer than the duration of the measurement. Photons are evasive and their spatial control requires optical cavities of very high finesse to ensure a long ringdown time. The experiment described above was performed in a Fabry-Perot cavity of diamond-machined copper mirrors coated with superconducting niobium. It is characterized by a finesse of 4.6×10^9 and a Q-factor of 4.2×10^{10} at 51 GHz and 0.8 K, corresponding to a cavity ringdown time of 130 ms [6]. Given the velocity of the atomic beam $v = 250\ m/s$ and the cavity length $L = 27.57\ mm$, it is possible to perform as many as 900 measurements of the same photon before it leaks out of the cavity. Additionally, the presence of a small but finite detuning Δ between the atomic transition frequency and the cavity resonance inhibits spontaneous emission from the excited state, thus enhancing coherence in the dressed states.

2. *Probe Atoms*

Circular Rydberg atoms are highly excited atoms in stretched states of maximum orbital angular momentum. Since their valence electron is in an orbit far away from the nucleus and the remaining electrons, which shield the nuclear charge, the effective charge it sees is $+e$. Therefore, the behavior of these atoms is analogous to that of the hydrogen atom, and many of their properties can be described simply by rescaling the hydrogen model by their principal quantum number n. When prepared in circular states, i.e. states with maximum orbital and magnetic quantum numbers, they further exhibit very high electric dipole matrix elements due to the toroidal shape of the wavefunction of the valence electron. This is easy to understand using simple rescaling laws. The average distance of a valence electron in a Rydberg atom $<r>$ is proportional to $<r> \propto n^2$. Perturbation theory tells us that the DC Stark Shift induced in an atom due to en external field E is (for low fields) proportional to $\frac{1}{2}\alpha E^2 \propto \sum_i \frac{d^2E^2}{E_n - E_i}$, where α is the polarizability and d is the atomic dipole matrix element. By the Bohr formula $E_n \propto \frac{1}{n^2}$, $(E_n - E_i) \propto \frac{1}{n^3}$, and therefore $\alpha \propto n^7$.

The Rydberg atoms used in this experiment are Rubidium atoms manipulated on the $n = 50 \rightarrow n = 51$ transition. Their dipole matrix element $d \propto n^2\alpha_o e \approx 1250$ atomic units, where α_o is the Bohr radius and e the elementary charge [8]. They are velocity-selected via a Doppler-selective pumping technique [9, 10], after which the atomic beam is characterized by a velocity width of $\pm 2\ m/s$ which allows knowledge of the atomic position within 1 mm [8]. This level of specification is necessary in order to ensure that there is always at most one atom coupled to the cavity.

3. *Detection*

Detection of the final atomic wavefunction, $|R2\rangle$, is performed via Ramsey interferometry. The observed Ramsey fringes result from interference between the two possible final states for the atoms

$$|R2\rangle = \frac{1}{2}((1 - e^{-i\phi(n)})\,|g\rangle + (1 + e^{-i\phi(n)}\,|e\rangle) \times |cavity\rangle$$

for photon numbers $n = 0, 1$. For cavity detuning $\frac{\Delta}{2\pi} = 67\ kHz$, the phase difference $\phi(1) - \phi(0)$ can be tuned to π [3], thus maximizing contrast in the Ramsey fringes. Measurement of the atomic wavefunction is performed via ionization in detector D, after which the resulting electrons are accelerated and counted. Since

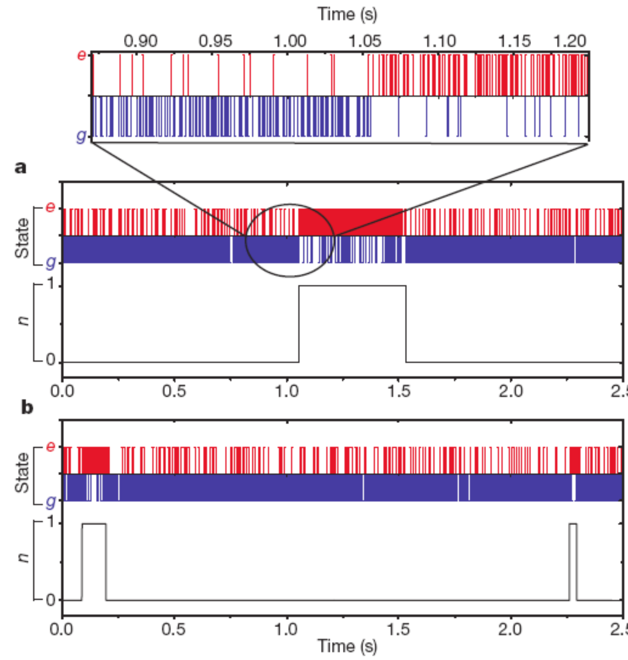

FIG. 2: Sequences of single-photon detection events. The top signal is unprocessed data, e and g denoting an atom in the excited and ground state respectively. The bottom signal is a democratic average over 8-bin data sets tracing the appearance and disappearance of a thermal photon from cavity C. Figure a shows one single-photon event, Figure b shows two single-photon events. Adopted from [3]

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different atomic states require different ionization fields, this process allows the distinction between $n = 1$ and $n = 0$.

4. *Observations*

Two main measurements were made with this QND method. One was the recording of the chronicle of a photon's life in an initially empty cavity which was occasionally filled with a photon from the 0.8K environment. The other was the monitoring of the decay of an artificially inserted photon in the cavity. For the first measurement a stream of Rubidium atoms in the ground state was sent into the cavity before each measurement set in order to clean up the cavity (which was tuned on resonance with the atomic transition for the cleanup process). Then a stream of $\pi/2$-pulsed atoms was sent through, as described above, and the photon number in the cavity was monitored as a function of time from the Ramsey interferometric measurement. Sample traces from these measurements is shown in Figure 2.

For the second measurement, the cavity was cleaned up with resonant ground state atoms and then refilled with one photon by one excited atom. The interaction time of the atoms in the cavity was tuned to match half a

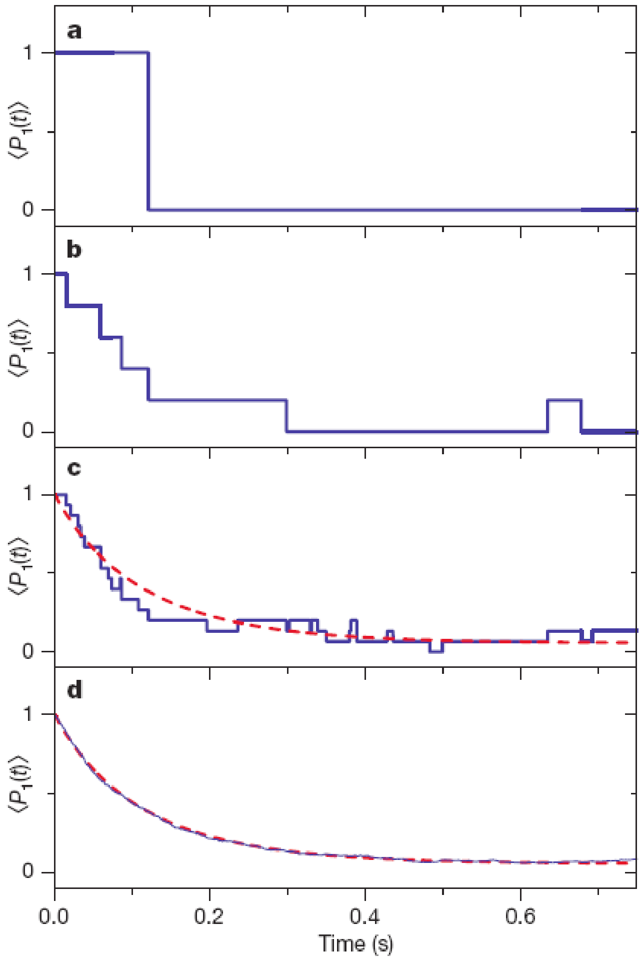

FIG. 3: Photon Decay. Traces a-d show occupation of the $n = 1$ Fock state as a function of time after 1, 5, 15, and 904 measurements. The transition from single quantum events to a smooth continuum which satisfies the equation of motion for the macroscopic cavity fields is remarkable and in excellent agreement with theory. Figure from [3]

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Rabi oscillation, after which the atom left in the ground state having deposited exactly one photon in the cavity. The photon number was then monitored as a function of time. Results are shown in Figure 3, which depicts the $n = 1$ Fock state occupation number as a function of time for an increasing number of measurements, starting with a single jump from one photon and gradually evolving into a smooth exponentially decaying function after multiple measurements. The agreement between this measurement and the theoretical calculations for the photon lifetime based on the cavity finesse is remarkable.

III. FUTURE PROSPECTS

A. Observing Decoherence

The possibilities that arise with the realization of this experiment are multiple. Performing a QND measurement is equivalent to assuming the role of Maxwell's demon. A task often assigned to this demon is the observation of decoherence. We could imagine, for instance, making a quasi-QND measurement, where by tuning some experimental parameter one could gradually "turn on" the commutator $[H_S, A_P]$ and watch the system evolve out of its Hamiltonian and into a "classical" pure state, as it couples to a mesoscopic system. In an ideal case, the non-commuting part of the interaction Hamiltonian can be treated as a perturbation of known strength and its effects on the time-evolution of the system's wavefunction can be calculated, at least to some order. Then it may be possible to establish whether the phenomenon of the "'collapse" of the wavefunction upon measurement can be accommodated by the theory of quantum mechanics, if this discontinuous evolution is shown to satisfy Schroedinger's equation for the total mesoscopic Hamiltonian acting on the system.

For instance, we could look at the Jaynes-Cummings Hamiltonian presented above in its exact form, before the rotating wave approximation. In terms of the Pauli spin matrices $\hat{\sigma}$, where $\hat{\sigma}_{\pm}$ corresponds to excitation/de-excitation of the atom, and the photon creation and annihilation operators $\hat{a}, \hat{a}^+$, the exact Hamiltonian can be expressed as

$$\hat{H} = \frac{1}{2}\hbar\hat{\sigma}_z - \frac{1}{2}\hbar\omega_1(\hat{\sigma}_+ + \hat{\sigma}_-)(\hat{a} + \hat{a}^+)$$

where we have included the terms $\hat{\sigma}_+\hat{a}^+, \hat{\sigma}_-\hat{a}$ which do not conserve energy to first order (i.e. not via a single-photon process). In the experimental setup we have described, where the probability of there being two or more photons in the system is very small [3], this approximation is valid. It is also because of this approximation that the measurement is non-demolishing, since the remaining terms conserve photon number. However, the interaction processes described by the counter-rotating terms, which involve more than one photon, can become important in a regime where either dipole transitions are forbidden or suppressed, or the atoms can couple strongly to the cavity via more than one photons.

However, it seems challenging to compromise the fact that both the single-photon as well as the multi-photon interactions depend on the same coupling factor $\hbar\omega_1$. This makes it seem impossible to tune the relative strength between first- and second-order processes, which would be desirable if we were to turn our interaction Hamiltonian from non-demolishing to demolishing in a continuous, controlled fashion. Nonetheless, it remains to be investigated whether atom-cavity systems with more than one photons (though with a definite photon number) would allow for such a controlled measurement, if, for instance, by increasing the atom-cavity coupling ω_1 it is possible to reach a very strong coupling regime, where the effects of second-order processes on the atomic wavefunctions would be measurable in addition to the effects of first-order processes.

B. Macroscopic Entanglement

Another fascinating route for investigation is the creation of a Bell state with two macroscopic cavities similar to the one described above. For instance, a simple idea for such a setup could involve sending a photon through a beamsplitter whose outputs are connected to two high-Q cavities. One might expect to observe entanglement between the two cavities by performing a Bell's inequality experiment and measuring the correlation between their occupation numbers. Such experiments are already under way by Haroche et al.

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