# Digital Mixers

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I review the theory behind signal mixing, especially that of digital mixing and amplitude detection. The effects of imperfections in analog mixers is discussed, including methods to correct it in the digital hardware. This tutorial is the theoretical basis for the FPGA programming in the GHZADC board.

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## MIXING

The use of mixers is critical for the electronics that drives and measures our qubits, as it allows the control of ∼7 GHz signals with electronics with 1 GHz of total bandwidth. The electronics is split into two parts, a digital section that generates or measures two signals with bandwidths up to 500 MHz, and an analog mixer that converts these signals to higher frequency. In this tutorial, I will consider the downconversion process, taking signals in the several GHz range, and mixing them to a lower frequency that is digitized and then analyzed with digital signal processing techniques.

The main analog component of this system is the IQ mixer, which has a RF input carrier at or around the signal frequency, a RF input signal, and two output signals I and the quadrature Q. When operated properly with the correct carrier power, the mixer can be accurately modeled as a multiplier of the two inputs. Typically, one analyzes the mixer by considering the RF carrier (or local oscillator) given by $A_0 \cos(\omega_0 t)$, and the RF input as $A\cos(\omega t + \phi)$, where $A$ and $\phi$ describe in general the input signal. The direct output I is proportional to the multiplication of these two signals $I = A_0 \cos(\omega_0 t) A \cos(\omega t + \phi)$, whereas the quadrature component has the RF carrier phase-shifted by 90 degrees $Q = A_0 \sin(\omega_0 t) A \cos(\omega t + \phi)$. For calculational simplicity, we combine these two outputs using complex notation

$$\begin{aligned}
Z &= I + iQ && (1)\\
&= A_0 e^{i\omega_0 t} \times A\cos(\omega t + \phi) && (2)\\
&= (A_0 A/2) e^{i\omega_0 t}[e^{i(\omega t+\phi)} + e^{-i(\omega t+\phi)}] && (3)\\
&= (A_0 A/2)[e^{i(\omega_0+\omega)t+i\phi} + e^{i(\omega_0-\omega)t-i\phi}] && (4)\\
&\simeq (A_0/2)\ A e^{-i\phi} e^{i(\omega_0-\omega)t}\ , && (5)
\end{aligned}$$

where in the last equation we have taken only the low frequency part of the signal. This typically is a good approximation since the the difference frequency ($\lesssim 1$ GHz) is well separated from the sum frequency (∼15 GHz), and thus can be extracted easily using a low-pass filter.

Note that the mixer produces a linear transformation of an oscillating input signal with amplitude $A$ and phase $\phi$ to an I and Q output with complex phasor amplitude $Ae^{-i\phi}$ that oscillates at a lower (mixed-down) frequency $\omega_0 - \omega$. This frequency can be positive or negative.

To determine the amplitude A, one mixes the input signal down to zero frequency by setting $\omega_0 = \omega$, and then measures the DC values of I and Q. Typically, this is measured over a finite time $\tau$, which corresponds to measuring the signal with a bandwidth $\sim 1/\tau$ around the oscillation frequency.

Digital mixing is easy to understand since the signal is already in the complex form $Z$. A input signal given by $Ze^{i\omega t}$ can be converted to one at a different (typically lower) frequency with the multiplication

$$\begin{aligned}
Z' &= Ze^{i\omega t} \times e^{-i\omega' t} && (6)\\
&= Ze^{i(\omega-\omega')t}\ . && (7)
\end{aligned}$$

Note this relation is exact. For computation, it requires the multiplication of 4 terms, 2 each for the real and imaginary components.

## ERROR CORRECTION OF ANALOG MIXERS

Digital mixing is exact to within the numerical precision of a number representation. Analog mixers have errors that are typically modeled as a phase error $\theta$ between the I and Q mixers and amplitude error $1-\beta$ in the gain between the two ports. Errors in the mixer output is thus given by the general form

$$Z_e = \cos(\omega t) + i\beta \sin(\omega t + \theta)\ , \quad (8)$$

where for no error we take $\beta = 1$ and $\theta = 0$. It turns out that this error can be expressed in terms of a signal with positive and negative frequency with complex amplitudes $A$ and $B$

$$Z_e = Ae^{i\omega t} + Be^{-i\omega t}\ . \quad (9)$$

By equating the two expressions for $Z_e$, one finds

$$\begin{aligned}
A + B &= 1 + i\beta \sin\theta && (10)\\
A - B &= \beta \cos\theta\ , && (11)
\end{aligned}$$

from which the complex amplitudes can be found

$$A = [1 + \beta e^{+i\theta}]/2 \tag{12}$$
$$B = [1 - \beta e^{-i\theta}]/2\ . \tag{13}$$

Note that for no error, one finds the expected results $A = 1$ and $B = 0$.

This is an important result: an imperfect analog mixer produces an output signal with frequency opposite in sign of that expected and with a complex amplitude. Apart from the overall magnitude and phase of $A$, the mixed down signal can be simply written in the form

$$Z_e = e^{i\omega t} + \epsilon e^{-i\omega t}\ , \tag{14}$$

where $\epsilon$ is complex and presumably small.

When $Z_e$ is digitally mixed to DC for measurement, the error from the term $\epsilon e^{-i\omega t}$ does not contribute since it mixes to a frequency $-2\omega$ and averages to zero. However, when measuring the spectral amplitude at the frequency $-\omega$, this error signal is seen with amplitude $\epsilon$. For this case ($\omega' = -\omega$), the error can be zeroed by replacing the mixing term $e^{-i\omega' t}$ of Eq. (6) with $e^{i\omega t} - \epsilon e^{-i\omega t}$, giving

$$Z_e'(-\omega) = [e^{i\omega t} + \epsilon e^{-i\omega t}] \times [e^{i\omega t} - \epsilon e^{-i\omega t}] \tag{15}$$
$$= e^{i2\omega t} + (\epsilon - \epsilon) - \epsilon^2 e^{-i2\omega t} \tag{16}$$
$$\simeq 0\ , \tag{17}$$

in the low-pass filter limit. When mixing at a frequency $\omega' = \omega$ with this form of the correction, one finds

$$Z_e'(\omega) = [e^{i\omega t} + \epsilon e^{-i\omega t}] \times [e^{-i\omega t} - \epsilon e^{i\omega t}] \tag{18}$$
$$\simeq 1 - \epsilon^2\ . \tag{19}$$

The correction is small (second order) and calculable.

## HARDWARE ISSUES

Since we need to minimize computational hardware in the FPGA because of its finite resources, I first discuss how the mixing can be efficiently implemented. Starting with the corrected mixing signal of Eq. (14) and writing the error amplitude $\epsilon = |\epsilon| e^{i2\epsilon'}$, one finds

$$Z_e = e^{i\epsilon'} \left[ e^{i(\omega t - \epsilon')} + |\epsilon| e^{-i(\omega t - \epsilon')} \right] \tag{20}$$
$$= e^{i\epsilon'} \Big[\ (1 + |\epsilon|) \cos(\omega t - \epsilon') + i(1 - |\epsilon|) \sin(\omega t - \epsilon')\ \Big]\ . \tag{21}$$

Apart from a trivial overall phase factor, the effect of this correction is to change the amplitudes of the sine and cosine multiplication factors and to add a phase shift to the oscillation terms. These changes can easily be implemented in hardware.

A high-level schematic of the FPGA hardware is shown in Fig. 1. To demodulate the signal, the input from the analog to digital (AD) converter is first multiplied by a time-dependent filter function, followed by the complex exponential $\exp(i\omega t)$ to mix down the signal down to DC. The repetitive summation of the signal then averages to zero all oscillating signals. The waveform from the filter generates the low-pass filter function for the DC signal. A separate computation of the average of the I and Q signals allows a raw snapshot waveform to be analyzed, as well as its repetitive average over many waveforms.

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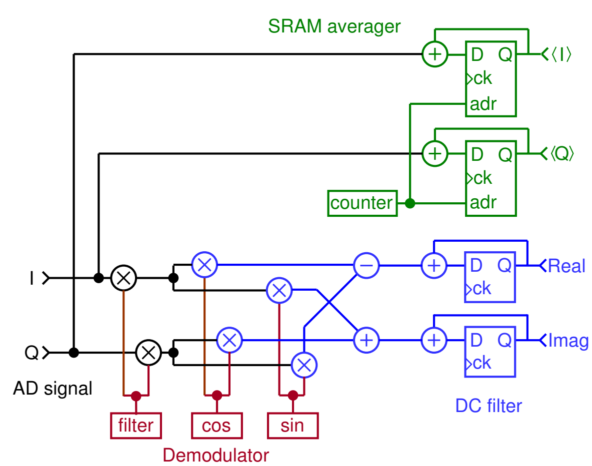


FIG. 1: Schematic diagram of FPGA hardware used to demodulate (blue) and compute the repetitive-trace average (red) of the input IQ signals.