By flux quantization

$$\delta_L - \delta_R + \phi = 0 . \quad (*)$$

If $\delta_L = \delta + \phi/2$, then by $(*)$ we have that

$$\delta_R = \delta + 3\phi/2$$

$$\neq \delta + \tfrac{1}{2}\phi .$$

If instead we define δ by

$$\delta \equiv \delta_L + \phi/2$$

$$\Rightarrow \quad \delta = (\delta_L + \delta_R)/2 \quad \text{by } (*).$$

Using $\quad \delta_L = \delta - \phi/2$ &

$$\delta_R = \delta + \phi/2 ,$$

the total current is given by

$$I = I_L \sin(\delta - \phi/2) + I_R \sin(\delta + \phi/2) \quad (\Delta)$$

Differentiating w.r.t. ϕ, one finds that

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$$\frac{dI}{d\phi} = \frac{1}{2}\left\{ -I_L \cos(\delta - \phi/2) + I_R \cos(\delta + \phi/2) \right\}$$

$$= \frac{1}{2}\Big\{ I_R \cos\delta \cos\phi/2 - I_R \sin\delta \sin\phi/2$$

$$- I_L \cos\delta \cos\phi/2 - I_L \sin\delta \sin\phi/2 \Big\}$$

$$= \frac{1}{2}\Big\{ (I_R - I_L) \cos\delta \cos\phi/2$$

$$- (I_R + I_L) \sin\delta \sin\phi/2 \Big\} \quad (0)$$

$$I = 0 \Rightarrow I_L \sin(\delta - \phi/2) = -I_R \sin(\delta + \phi/2)$$

or that

$$I_L(\sin\delta \cos\phi/2 - \cos\delta \sin\phi/2)$$

$$= -I_R(\sin\delta \cos\phi/2 + \cos\delta \sin\phi/2)$$

$$\Rightarrow (I_L + I_R)\sin\delta \cos\phi/2 = -(I_R - I_L)\cos\delta \sin\phi/2$$

$$\Rightarrow \sin\delta = \frac{-(I_R - I_L)}{(I_R + I_L)} \frac{\cos\delta \sin\Phi/2}{\cos\Phi/2} \qquad (\square)$$

Plugging $(\square)$ into (o), we have that

$$\frac{dI}{d\phi} = \frac{1}{2}\Big\{ (I_R - I_L)\cos\delta \cos\Phi/2$$

$$+ (I_R - I_L) \frac{\cos\delta \sin\Phi/2}{\cos\Phi/2} \Big\}$$

evaluating @ $\sin(\bar{\delta}) = 0$ and $\cos(\bar{\delta}) = 1$

$$\Rightarrow \left.\frac{dI}{d\phi}\right|_{\delta=\bar{\delta}} = \frac{1}{2}(I_R - I_L)\left\{\cos\Phi/2 + \tan\left(\frac{\Phi}{2}\right)\right\}$$