PROVA 3 - CÁLCULO II

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① $f(x,y) = 16x^2y+5$ $\quad$ $g(x,y) = x^2+2y^2=6 \rightarrow g(x,y) \Rightarrow x^2+2y^2-6=0$

$\nabla f=(32xy,\ 16x^2)$ $\quad$ $\nabla f = \lambda \nabla g$

$\nabla g=(2x,\ 4y)$ $\quad$ $(32xy,\ 16x^2) = \lambda(2x, 4y)$

$$\begin{cases} 32xy = \lambda 2x \\ 16x^2 = \lambda 4y \\ x^2+2y^2=6 \end{cases}$$

$32xy = \lambda 2x$ $\quad$ $\lambda = \dfrac{\overset{16}{\cancel{32x}}y}{\cancel{2x}}$ $\quad$ $\boxed{\lambda = 16y}$ $\quad$ $\boxed{y = \dfrac{\lambda}{16}}$

$16x^2 = \lambda 4y$ $\quad$ $x^2 = \dfrac{4\lambda.\lambda}{16.16}$ $\quad$ $\boxed{x^2 = \dfrac{4\lambda^2}{16^2}}$

$\dfrac{4\lambda^2}{16^2} + 2\left(\dfrac{\lambda}{16}\right)^2 = 6$

$\dfrac{4\lambda^2}{16^2} + \dfrac{2\lambda^2}{16^2} = 6$ $\quad$ $\lambda^2 = 16^2$ $\quad$ $\boxed{\lambda = \pm 16}$

$\dfrac{6\lambda^2}{16^2} = 6$

$y = \dfrac{\lambda}{16} \Rightarrow \boxed{y = \pm 1}$

$x^2 = \dfrac{4\lambda^2}{16^2} \Rightarrow x^2 = \dfrac{4.\cancel{16^2}}{\cancel{16^2}} \Rightarrow \boxed{x = \pm 2}$

$32xy - \lambda 2x = 0$

$2x(16y - \lambda) = 0$

$\boxed{x=0}$

$x^2+2y^2=6$

$0+2y^2=6$

$\boxed{y = \pm\sqrt{3}}$

Pontos:

$(-2,-1)$ $\quad$ $(0, \sqrt{3})$

$(-2,1)$ $\quad$ $(0, -\sqrt{3})$

$(2,-1)$

$(2,1)$

$f(x,y) = 16x^2y+5$

$f(-2,-1) = 16.4.(-1)+5 \Rightarrow \boxed{-59}$

$f(-2,1) = 16.4.1+5 \Rightarrow \boxed{69}$

$f(2,-1) = 16.4.(-1)+5 \Rightarrow \boxed{-59}$

$f(2,1) = 16.4.1+5 \Rightarrow \boxed{69}$

$f(0,\sqrt{3}) = 16.0.\sqrt{3}+5 \Rightarrow \boxed{5}$

$f(0,-\sqrt{3}) = 16.0(-\sqrt{3})+5 \Rightarrow \boxed{5}$

O máximo da função é nos pontos $(-2,1)$ e $(2,1)$.
O mínimo da função é nos pontos $(-2,-1)$ e $(2,-1)$.

② $f(x,y) = -x^4y - y^5$ $\quad P_0 = (1,1)$

$\nabla f = (-4x^3y, -x^4 - 5y^4) \Rightarrow \nabla f_{(1,1)} = (-4, -1-5) \Rightarrow \nabla f_{(1,1)} = (-4, -6)$

ⓐ $u = (-1,-1)$

$\nabla f(x,y) \cdot \frac{u}{\|u\|} \Rightarrow (-4x^3y, -x^4 - 5y^4)_{(1,1)} \cdot \frac{(-1,-1)}{\sqrt{(-1)^2 + (-1)^2}} \Rightarrow (-4,-6) \cdot \frac{(-1,-1)}{\sqrt{2}}$

$\frac{4+6}{\sqrt{2}} \Rightarrow \frac{\cancel{10}\sqrt{2}}{\cancel{2}} \Rightarrow \boxed{5\sqrt{2}}$

Como a derivada direcional é positiva para o vetor $(-1,-1)$, estaremos subindo.

ⓒ $f(x,y,z) = -x^4y - y^5 - z.0$

$\nabla f = (-4x^3y, -x^4 - 5y^4, 0)$

$\nabla f(1,1,2) = (-4,-6,0)$

Plano Tangente é

$\boxed{-4x - 6y = -10}$

$-4x - 6y - z.0 + D = 0$

$-4 - 6 + D = 0$

$\boxed{D = 10}$