# LISTA 4 - CÁLCULO II

Aluno: Daniel Sant' Anna Andrade
Matrícula: 20200036904

1)

① $f(x,y) = 4 + 3x^2 + 7y^2$

$\nabla f = (6x, 14y)$

$\nabla f_{(2,2)} = (12, 28)$

ⓐ $f_{(1,0)} = (6,0)$ desce a montanha em $x$ e $y$

ⓑ $f_{(1,3)} = (6,42)$ desce a montanha em $x$ mas sobe em relação a $y$

2)

② $xy^4 + x^3y^5z = yz^4 + 1 \qquad P=(1,1,1)$

$xy^4 + x^3y^5z - yz^4 = 1$

$\frac{\partial f}{\partial x}_{(1,1,1)} = y^4 + 3x^2y^5z \Rightarrow 1 + 3 \Rightarrow \boxed{4}$

$\nabla f = (4, 8, -3)$

$\frac{\partial f}{\partial y}_{(1,1,1)} = 4xy^3 + 5x^3y^4z - z^4 \Rightarrow 4 + 5 - 1 \Rightarrow \boxed{8}$

$\frac{\partial f}{\partial z}_{(1,1,1)} = x^3y^5 - 4yz^3 \Rightarrow 1 - 4 = \boxed{-3}$

$\boxed{4x + 8y - 3z = 9}$

3)

③ $z = x^3y + 12x^2 - 8y$

$\frac{\partial z}{\partial x} = 3x^2y + 24x$ $\boxed{x=0}$

$\frac{\partial z}{\partial y} = x^3 - 8$ $\boxed{x=2}$

$\boxed{(2,-4)}$

$\begin{cases} 3x^2y + 24x = 0 \\ x^3 - 8 = 0 \end{cases}$

$3x(xy+8) = 0$

$y=2$ ou $y=4$ ou $y=-2$ ou $y=-4$

$x=0$ ou $x=2$ ou $x=4$ ou $x=-2$ ou $x=-4$

$A = \frac{\partial^2 z}{\partial x^2} = 6xy + 24$

$C = \frac{\partial^2 z}{\partial y^2} = 0$

$B = \frac{\partial^2 z}{\partial x \partial y} = 3x^2$

$B = \frac{\partial^2 z}{\partial y \partial x} = 3x^2$

$A = -24$

$B = 12$

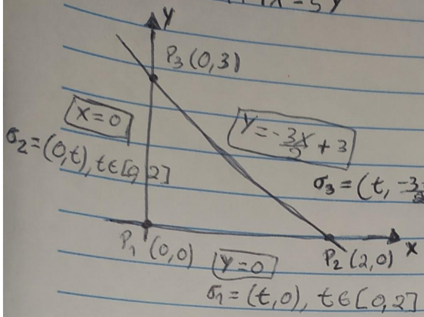
$C = 0$

$\boxed{B^2 - AC = 144 > 0}$

ponto de sela

4)

④ $f(x,y) = 1 + 4x - 5y$




$\sigma_3 = (t, -\frac{3t}{2} + 3),\ t \in [0,2]$

$3 = a0 + b$

$\boxed{b=3}$

$0 = 2a + b$

$2a + 3 = 0$

$\boxed{a = \frac{-3}{2}}$

$\nabla f = (4,-5) \rightarrow \boxed{P.C. \Rightarrow f = (0,0)}$

Não existe

$f(\sigma_1(t)) \Rightarrow f(\sigma_1(t,0)) = 1 + 4t - 5.0 \Rightarrow \boxed{f(\sigma_1(t)) = 1 + 4t}$

$\boxed{f'(\sigma_1(t)) = 4 \neq 0}$

$f(\sigma_2(t)) \Rightarrow f(\sigma_2(0,t)) = 1 + 4.0 - 5.t \Rightarrow \boxed{f(\sigma_2(t)) = 1 - 5t}$

$\boxed{f'(\sigma_2(t)) = -5 \neq 0}$

$f(\sigma_3(t)) = f(\sigma_3(t, \frac{-3t}{2} + 3)) = 1 + 4t - 5(\frac{-3t}{2} + 3) \Rightarrow \frac{1}{1} + \frac{4t}{1} + \frac{15t}{2} - \frac{15}{1} \Rightarrow$

$\boxed{f(\sigma_3(t)) = \frac{23t}{2} - 28} \rightarrow \boxed{f'(\sigma_3(t)) = \frac{23}{2} \neq 0}$

Não existe máximo nem mínimo pois ela é infinita em z tanto positivamente como negativamente

5)

⑤ $f(x,y) = x^2y$ $\qquad x^2+2y^2=6$

$g(x,y) = x^2+2y^2-6=0$

$\nabla f = (2xy, x^2)$ $\qquad \nabla f(x,y) = \lambda \nabla g(x,y)$

$\nabla g = (2x, 4y)$ $\qquad (2xy, x^2) = \lambda(2x, 4y)$

$$\begin{cases} 2xy = \lambda 2x \rightarrow y = \frac{\lambda 2x}{\cancel{2x}} \Rightarrow \boxed{y=\lambda} \\ x^2 = \lambda 4y \rightarrow x^2 = \lambda 4\lambda \Rightarrow \boxed{x^2 = 4\lambda^2} \\ x^2+2y^2=6 \end{cases}$$

$4\lambda^2 + 2\lambda^2 = 6$ $\qquad \boxed{y = \pm 1}$

$6\lambda^2 = 6$

$\boxed{\lambda = \pm 1}$

$x^2 = 4\lambda^2$

$x^2 = 4$

$\boxed{x = \pm 2}$

$2xy - \lambda 2x = 0$

$2x(y-\lambda) = 0$

$\boxed{x=0}$

$y - \lambda = 0$

$y - 1 = 0$

$\boxed{y=1}$

$y + 1 = 0$

$\boxed{y=-1}$

$f(x,y) = x^2y$

$f(1,2) = 1 \cdot 2$

$\boxed{f(1,2) = 2}$

$f(-1,2) = 1 \cdot 2$

$\boxed{f(-1,2) = 2}$

$f(-1,-2) = 1 \cdot (-2)$

$\boxed{f(-1,-2) = -2}$

$f(0,1) = 0 \cdot 1$

$\boxed{f(0,1) = 0}$

$f(0,-1) = 0 \cdot (-1)$

$\boxed{f(0,-1) = 0}$

máximo nos pontos $(1,2)$ e $(-1,2)$
mínimo no ponto $(-1,-2)$