ICE– Institutos de Ciências Exatas
DEMAT – Departamento de Matemática

CÁLCULO 1 – SEMANA 10 - INTEGRAIS

Componente Curricular:
IC241 - CÁLCULO I (90h) - Turma: 02 (2020.1)
IC241 - CÁLCULO I (90h) - Turma: 07 (2020.1)
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INTEGRAÇÃO TRIGONOMÉTRICA

Abordagem de integrais que envolvam produtos de funções trigonométricas com argumentos diferentes.

Caso I: $\int \operatorname{sen}(ax) \,.\, \cos(bx)\ dx$

Escreveremos inicialmente as propriedades trigonométricas para:

$$\operatorname{sen}(ax + bx) = \operatorname{sen}(ax).\cos(bx) + \operatorname{sen}(bx).\cos(ax)$$

$$\operatorname{sen}(a + b)x = \operatorname{sen}(ax).\cos(bx)$$
$$+\operatorname{sen}(bx).\cos(ax) \quad \text{eq. }01$$

$$\operatorname{sen}(ax - bx) = \operatorname{sen}(ax).\cos(bx) - \operatorname{sen}(bx).\cos(ax)$$

$$\operatorname{sen}(a - b)x = \operatorname{sen}(ax).\cos(bx)$$
$$-\operatorname{sen}(bx).\cos(ax) \quad \text{eq. }02$$

Somando as eq.01 e a eq.02:

$$\operatorname{sen}(a+b)x + \operatorname{sen}(a-b)x$$

$$= \operatorname{sen}(ax).\cos(bx) + \operatorname{sen}(bx).\cos(ax) +$$
$$\operatorname{sen}(ax).\cos(bx) - \operatorname{sen}(bx).\cos(ax)$$

$$\operatorname{sen}(a+b)x + \operatorname{sen}(a-b)x = 2.(\operatorname{sen}(ax).\cos(bx))$$

$$\operatorname{sen}(ax).\cos(bx) = \frac{1}{2}[\operatorname{sen}(a+b)x + \operatorname{sen}(a-b)x]$$

Calcule a integral:

$$I = \int \operatorname{sen}(5x).\cos(2x)\,dx$$

$$\operatorname{sen}(ax).\cos(bx) = \frac{1}{2}[\operatorname{sen}(a+b)x + \operatorname{sen}(a-b)x]$$

$$a = 5 \ \text{ e } \ b = 2$$

$$I = \int \operatorname{sen}(5x).\cos(2x)\,dx$$
$$= \int \frac{1}{2}[\operatorname{sen}(5+2)x + \operatorname{sen}(5-2)x]\,dx$$
$$= \int \frac{1}{2}[\operatorname{sen}(7)x + \operatorname{sen}(3)x]\,dx$$

$$I = \frac{1}{2}\left[\frac{-\cos(7)x}{7} - \frac{\cos(3)x}{3}\right] + C$$

$$I = \int \operatorname{sen}(5x).\cos(2x)\,dx = \frac{-\cos(7x)}{14} - \frac{\cos(3x)}{6} + C$$

Caso II: $\int \operatorname{sen}(ax).\operatorname{sen}(bx)\ dx$

Escreveremos inicialmente as propriedades trigonométricas para:

$$\cos(ax + bx) = \cos(ax).\cos(bx) - \operatorname{sen}(ax).\operatorname{sen}(bx)$$

$$\cos(a + b)x = \cos(ax).\cos(bx)$$
$$-\operatorname{sen}(ax).\operatorname{sen}(bx) \quad eq.01$$

$$\cos(ax - bx) = \cos(ax).\cos(bx) + \operatorname{sen}(ax).\operatorname{sen}(bx)$$

$$\cos(a - b)x = \cos(ax).\cos(bx)$$
$$+\operatorname{sen}(ax).\operatorname{sen}(bx) \quad eq.02$$

$$\cos(a + b)x - \cos(a - b)x$$

$$= \cos(ax).\cos(bx) - \operatorname{sen}(ax).\operatorname{sen}(bx) -$$
$$\cos(ax).\cos(bx) - \operatorname{sen}(ax).\operatorname{sen}(bx)$$

$$\cos(a + b)x - \cos(a - b)x = -2.(\operatorname{sen}(ax).\cos(bx))$$

$$\operatorname{sen}(ax).\operatorname{sen}(bx) = \frac{1}{2}[\cos(a - b)x - \cos(a + b)x]$$

Calcule a integral:

$$I = \int \operatorname{sen}(3x).\operatorname{sen}(2x)\,dx$$

$$\operatorname{sen}(ax).\operatorname{sen}(bx) = \frac{1}{2}[\cos(a-b)x - \cos(a+b)x]$$

$$a = 3 \;\; e \;\; b = 2$$

$$I = \int \operatorname{sen}(5x).\operatorname{sen}(2x)\,dx$$

$$= \int \frac{1}{2}[\cos(3-2)x - \cos(3+2)x]\,dx$$

$$= \int \frac{1}{2}[\cos(1)x + \cos(5)x]\,dx$$

$$I = \frac{1}{2}\left[\operatorname{sen}x + \frac{\operatorname{sen}(5)x}{5}\right] + C$$

$$I = \int \operatorname{sen}(3x).\operatorname{sen}(2x)\,dx = \frac{\operatorname{sen}x}{2} + \frac{\operatorname{sen}(5x)}{10} + C$$

Caso III: $\int \cos(ax)\,.\cos(bx)\;\,dx$

Escreveremos inicialmente as propriedades trigonométricas para:

$$\cos(ax+bx) = \cos(ax).\cos(bx) - \operatorname{sen}(ax).\operatorname{sen}(bx)$$

$$\cos(a+b)x = \cos(ax).\cos(bx)$$
$$-\operatorname{sen}(ax).\operatorname{sen}(bx) \quad eq.\,01$$

$$\cos(ax - bx) = \cos(ax).\cos(bx) + \operatorname{sen}(ax).\operatorname{sen}(bx)$$

$$\cos(a - b)x = \cos(ax).\cos(bx)$$
$$+ \operatorname{sen}(ax).\operatorname{sen}(bx) \quad eq.02$$

Somando as eq.01 e a eq.02:

$$\cos(a + b)x + \cos(a - b)x$$

$$= \cos(ax).\cos(bx) - \operatorname{sen}(ax).\operatorname{sen}(bx) +$$
$$\cos(ax).\cos(bx) + \operatorname{sen}(ax).\operatorname{sen}(bx)$$

$$\cos(a + b)x + \cos(a - b)x = 2.(\cos(ax).\cos(bx))$$

$$\operatorname{sen}(ax).\cos(bx) = \frac{1}{2}[\cos(a + b)x + \cos(a - b)x]$$

Calcule a integral:

$$I = \int \cos(8x).\cos(2x)\,dx$$

$$\cos(ax).\cos(bx) = \frac{1}{2}[\cos(a + b)x + \cos(a - b)x]$$

$$a = 8 \ \text{ e } \ b = 2$$

$$I = \int \cos(8x).\cos(2x)\,dx$$

$$= \int \frac{1}{2}\left[\cos(8+2)x + \cos(8-2)x\right] dx$$

$$= \int \frac{1}{2}\left[\cos(10)x + \cos(6)x\right] dx$$

$$I = \frac{1}{2}\left[\frac{\operatorname{sen}(10)x}{10} + \frac{\operatorname{sen}(6)x}{6}\right] + C$$

$$I = \int \cos(8x).\cos(2x)\,dx = \frac{\operatorname{sen}(10x)}{20} + \frac{\operatorname{sen}(6x)}{12} + C$$