ATIVIDADE 3 - CÁLCULO I

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1)

① $f(x) = \text{arctg}\left(\frac{x+7}{1-7x}\right)$

$$\frac{1}{1+\left(\frac{x+7}{1-7x}\right)^2} \cdot \frac{(x+7)' \cdot (1-7x) - (x+7) \cdot (1-7x)'}{(1-7x)^2}$$

$$1+\left(\frac{x+7}{1-7x}\right)^2 \Rightarrow 1+\frac{x^2+14x+49}{1-14x+49x^2} \Rightarrow \frac{1-\cancel{14x}+49x^2+x^2+\cancel{14x}+49}{1-14x+49x^2}$$

$$\boxed{\frac{50x^2+50}{1-14x+49x^2}}$$

$$\frac{(x+7)' \cdot (1-7x) - (x+7) \cdot (1-7x)'}{(1-7x)^2} \Rightarrow \frac{(1-7x) \cdot (1) - (x+7) \cdot (-7)}{49x^2-14x+1}$$

$$\frac{1-\cancel{7x}+\cancel{7x}+49}{49x^2-14x+1} \Rightarrow \boxed{\frac{50}{49x^2-14x+1}}$$

$$\frac{\frac{50}{49x^2-14x+1}}{\frac{50x^2+50}{49x^2-14x+1}} \Rightarrow \frac{50}{\cancel{49x^2-14x+1}} \cdot \frac{\cancel{49x^2-14x+1}}{50x^2+50} \Rightarrow \frac{\cancel{50}}{\cancel{50}(x^2+1)}$$

$$\boxed{\frac{1}{x^2+1}}$$

2)

② $f(x)=\sqrt{5-x}$ $\quad (5-x)^{1/2}\cdot(5-x)' \Rightarrow \dfrac{1}{2(5-x)^{1/2}}\cdot(-1)$

$f'(x)=-\dfrac{1}{2\sqrt{5-x}} \Rightarrow f'(x)=\dfrac{\Delta y}{\Delta x} \Rightarrow -\dfrac{1}{2\sqrt{5-x}}=\dfrac{y-y_0}{x-x_0}$

$-\dfrac{1}{2\sqrt{5-x}}=\dfrac{\sqrt{5-x}-0}{x-9} \Rightarrow 2\sqrt{5-x}^{\,2}=-x+9 \Rightarrow 2(5-x)=x+9$

$10-2x=x+9$

$\boxed{x=1}$ $\quad f(1)=\sqrt{5-1}$ $\quad \boxed{f(1)=2}$

$f'(x)=-\dfrac{1}{2\sqrt{5-x}}$

$f'(1)=-\dfrac{1}{2\sqrt{4}} \Rightarrow \boxed{f'(1)=-\dfrac{1}{4}}$

reta tangente

$y-y_0=f'(x_0)(x-x_0)$

$y-2=-\dfrac{1}{4}(x-1)$

$y=\dfrac{-x+1}{4}+2$

$\boxed{y=\dfrac{-x+9}{4}}$

reta normal

$y-y_0=\dfrac{-1}{f'(x_0)}\cdot(x-x_0)$

$y-2=\dfrac{-1}{-\frac{1}{4}}\cdot(x-1)$

$y-2=\dfrac{-x+1}{-\frac{1}{4}} \Rightarrow \dfrac{-x+1}{1}\cdot\dfrac{4}{-1}$

$y=\dfrac{-4x+1}{-1}+2$

$\boxed{y=4x+1}$

3)

③ $f(x)=\sqrt{2x^2+5}$

$\lim_{h\to0} f(x)=\dfrac{\sqrt{2(x+h)^2+5}-\sqrt{2x^2+5}}{h}\cdot\dfrac{(\sqrt{2(x+h)^2+5}+\sqrt{2x^2+5})}{(\sqrt{2(x+h)^2+5}+\sqrt{2x^2+5})}$

$\lim_{h\to0}\dfrac{2(x+h)^2+5-(2x^2+5)}{h(\sqrt{2(x+h)^2+5}+\sqrt{2x^2+5})} \Rightarrow \lim_{h\to0}\dfrac{2(x^2+2xh+h^2)+5-(2x^2+5)}{h(\sqrt{2(x+h)^2+5}+\sqrt{2x^2+5})}$

$\lim_{h\to0}\dfrac{\cancel{2x^2}+4xh+2h^2\cancel{+5}-\cancel{2x^2}\cancel{-5}}{h(\sqrt{2(x+h)^2+5}+\sqrt{2x^2+5})} \Rightarrow \lim_{h\to0}\dfrac{\cancel{h}(4x+2h)}{\cancel{h}(\sqrt{2(x+h)^2+5}+\sqrt{2x^2+5})}$

$\lim_{h\to0}\dfrac{4x+2h^{0}}{\sqrt{2(x+h^{0})^2+5}+\sqrt{2x^2+5}} \Rightarrow \lim_{h\to0}\dfrac{4x}{\sqrt{2(x)^2+5}+\sqrt{2x^2+5}}$

$\lim_{h\to0}\dfrac{4x}{\cancel{2}\sqrt{2x^2+5}} \Rightarrow \boxed{\lim_{h\to0}=\dfrac{2x}{\sqrt{2x^2+5}}}$

4)

④ $f(x) = \ln\left(\frac{3}{x+2} - x\right)$

$$\frac{3}{x+2} - x > 0 \Rightarrow \frac{3 - x(x+2)}{x+2} > 0 \Rightarrow \frac{3 - x^2 - 2x}{x+2} > 0 \Rightarrow \frac{x^2 + 2x - 3}{x+2} < 0$$

$$\frac{(x+3)(x-1)}{x+2} < 0$$

		-3	1
$x+3$	$-$	$+$	$+$
$x-1$	$-$	$-$	$+$
	$+$	$-$	$+$

$$\lim_{x \to -3^+} \ln\left(\frac{3}{x+2} - x\right) \Rightarrow \ln\left(\frac{3}{-3+2} - (-3)\right) \Rightarrow \ln(-3+3) \Rightarrow \ln(0) = -\infty$$

Como -3 e 1 são raizes da função, esses valores irão dar ln(0), para f(-2) não tem solução.

5)

⑤ $f(x) = 4x - 10$

$f(3) = 2$

Para valores na qual a derivada e menor que 4, dara resultados menores que 2 para $f(3)$.