1) Which of the following statements are correct? ***1 point***

- [ ] The domain of $f(x) = \sqrt{-(x-2)(x-3)(x-4)(x-5)}$ is $[2,3] \cup [4,5]$.
- [ ] If $f \circ g(x) = g \circ f(x) = x$ then f and g are inverses of each other.
- [ ] The function $f(x) = x^3 + 5$ is differentiable everywhere.
- [ ] If the polynomials $x^3 + ax^2 + 5x + 7$ and $x^3 + 2x^2 + 3x + 2a$ leave the same remainder when divided by $(x-2)$, then the value of a is $\frac{-3}{2}$.

No, the answer is incorrect.

Score: 0

Accepted Answers:

The domain of $f(x) = \sqrt{-(x-2)(x-3)(x-4)(x-5)}$ is $[2,3] \cup [4,5]$.
If $f \circ g(x) = g \circ f(x) = x$ then f and g are inverses of each other.
The function $f(x) = x^3 + 5$ is differentiable everywhere.
If the polynomials $x^3 + ax^2 + 5x + 7$ and $x^3 + 2x^2 + 3x + 2a$ leave the same remainder when divided by $(x-2)$, then the value of a is $\frac{-3}{2}$.

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2) Consider the function defined as follows with $m, n \in \mathbb{R}$: $f(x) = \begin{cases} 5e^x + mx & \text{if x} < 0 \\ 4x^2 - 3x + n & \text{if} x \geq 0 \end{cases}$ Choose the set of correct options. ***1 point***

- [ ] f is continuous but not differentiable for any choice of values of m and n.
- [ ] If $n = 5$ and $m = -8$, then f is continuous and differentiable everywhere.
- [ ] If f is differentiable everywhere, then $\lim\limits_{x \to 0^-} f'(x) = -3$.
- [ ] If f is differentiable everywhere, then $\lim\limits_{x \to 0^-} f'(x) = 5$.

No, the answer is incorrect.

Score: 0

Accepted Answers:

If $n = 5$ and $m = -8$, then f is continuous and differentiable everywhere.
If f is differentiable everywhere, then $\lim\limits_{x \to 0^-} f'(x) = -3$.

3) Table M1Q3T3T-1 gives functions in Column A, lines through the point (0, 1) in column B and plots in Column C. ***1 point***

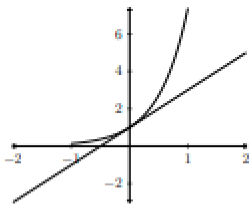
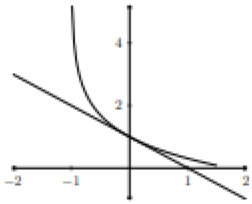
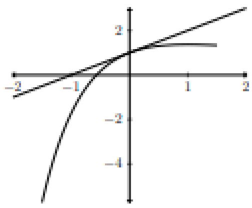
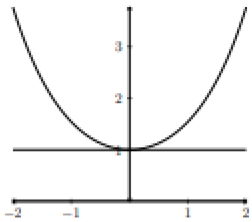
	Functions (Column A)		Lines through (0,1) (Column B)		Plots (Column C)
i)	$f(x) = xe^{-x} + 1$	a)	$y = 1 - x$	1)	
ii)	$f(x) = \frac{e^{x}+e^{-x}}{2}$	b)	$y = 2x + 1$	2)	
iii)	$f(x) = 1 - ln(x + 1)$	c)	$y = 1$	3)	
		d)	$y = x + 1$	4)	

Table: M1Q3T3T-1

Based on the given Table M1Q3T3T-1, choose the options which represent the correct matching of a given function in column A with its tangent line at (0,1) in column B and the plot of the graph and tangent line at (0,1) in column C.

- i) $\rightarrow$ d) $\rightarrow$ 3)
- ii) $\rightarrow$ c) $\rightarrow$ 4)
- iii) $\rightarrow$ b) $\rightarrow$ 1)
- i) $\rightarrow$ a) $\rightarrow$ 2)
- ii) $\rightarrow$ c) $\rightarrow$ 4)
- iii) $\rightarrow$ a) $\rightarrow$ 2)

No, the answer is incorrect.

Score: 0

Accepted Answers:

i) $\rightarrow$ d) $\rightarrow$ 3)

ii) $\to$ c) $\to$ 4)
iii) $\to$ a) $\to$ 2)

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4) Suppose $L(x)$ is the best linear approximation to the function $f(x) = sin(3x)cos(4x) + \sqrt{1+2x}$ at $x = 0$. Choose the set of correct options. ***1 point***

- [ ] $L(x) = 4x + 1$
- [ ] $L(x) = 5x + 1$
- [ ] There is a unique linear approximation to $f(x)$ at $x = 0$
- [ ] There are two possible linear approximations to $f(x)$ at $x = 0$

No, the answer is incorrect.
Score: 0

Accepted Answers:

$L(x) = 4x + 1$
There is a unique linear approximation to $f(x)$ at $x = 0$

Nisha researches on various cosmic signals. She found that a harmful cosmic signal of the form $y = 3^{ax}$ has the potential to harm the entire life form on the planet Earth. For her interest she found two cosmic signals which are of form $f(x) = \log_{2x+3}(6x^2 + 23x + 21)$ (where $2x + 3 > 0$ and $2x + 3 \neq 1$) and $g(x) = -\log_{3x+7}(4x^2 + 12x + 9) + 4$ (where $3x + 7 > 0$ and $3x + 7 \neq 1$) which can destroy this harmful signal. She found that when a harmful cosmic signal passes through the intersection point of $f(x)$ and $g(x)$, its effect nullifies before reaching the Earth. She has to find the value of a so that she can prevent this harmful cosmic signal from reaching the Earth. Based on this information answer the following questions

5) Which of the following statements is (are) correct? ***1 point***

- ○ The equation $f(x) = g(x)$ has only one real root.
- ○ The equation $f(x) = g(x)$ has two real roots.
- ○ $y = 3^{ax}$ must pass through the intersection point of $f(x)$ and $g(x)$ to protect the life form on Earth.
- ○ The equation $f(x) = g(x)$ has three real roots.

No, the answer is incorrect.
Score: 0

Accepted Answers:

The equation $f(x) = g(x)$ has only one real root.
$y = 3^{ax}$ must pass through the intersection point of $f(x)$ and $g(x)$ to protect the life form on Earth.

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6) What will be the value of a.

No, the answer is incorrect.

Score: 0

Accepted Answers:

(Type: Numeric) -4

1 point

A group of Biotechnology students were creating a Genetically Modified Plant (GMP). They found that the expression $f(x) = \frac{a}{1+e^{-0.5x}}$ gives the increase in the number of leaves on the plant as a function of days. On 0^{th} day there were 10 leaves. By the end of 36^{th} day, the number of leaves started decreasing as function of $g(x) = -10 \times 2^{\frac{x}{b}} + 100$ and eventually there were no leaf on that plant after some days (Refer Figure 2). Consider $f(x)$ and $g(x)$ represents the number of leaves on that plant by the end of x^{th} day.

Note:

(1) Take $19.9...$ as 20.

(2) For simplicity consider a leaf is fully grown when $f(x)$ is an integer value.

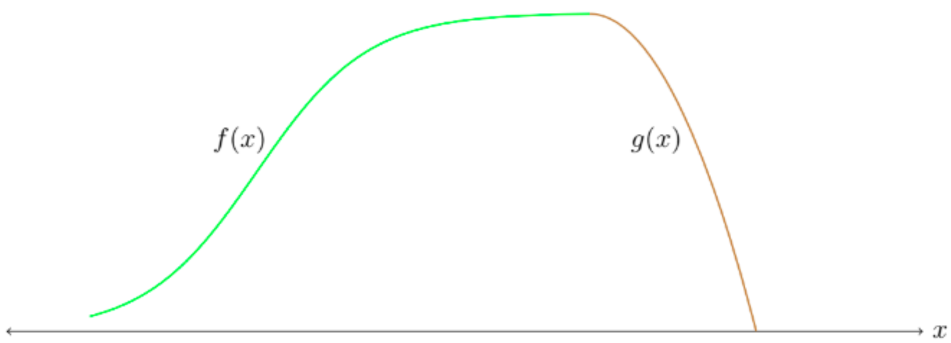

Figure 2

7) What will be the value of a.

No, the answer is incorrect.

Score: 0

Accepted Answers:

(Type: Numeric) 20

0.5 points

8) Which of the following statements is (are) correct?

0.5 points

- b can be found using $f(36) = g(36)$
- b cannot be determined.

- [ ] 36^{th} day, there are roughly 20 leaves.
- [ ] 36^{th} day, there are roughly 30 leaves.

No, the answer is incorrect.

Score: 0

Accepted Answers:

b can be found using $f(36) = g(36)$
36^{th} day, there are roughly 20 leaves.

9) What will be the value of b.

No, the answer is incorrect.

Score: 0

Accepted Answers:

(Type: Numeric) 12

0.5 points

10) find the value of $\log_a(\frac{100b}{3})$

No, the answer is incorrect.

Score: 0

Accepted Answers:

(Type: Numeric) 2

0.5 points

11) Let $f(x)$, $g(x)$, $p(x)$ and $q(x)$ be the functions defined on $\mathbb{R}$. Refer Figure 3 (A and B) and choose the correct option(s) from the following. ***1 point***

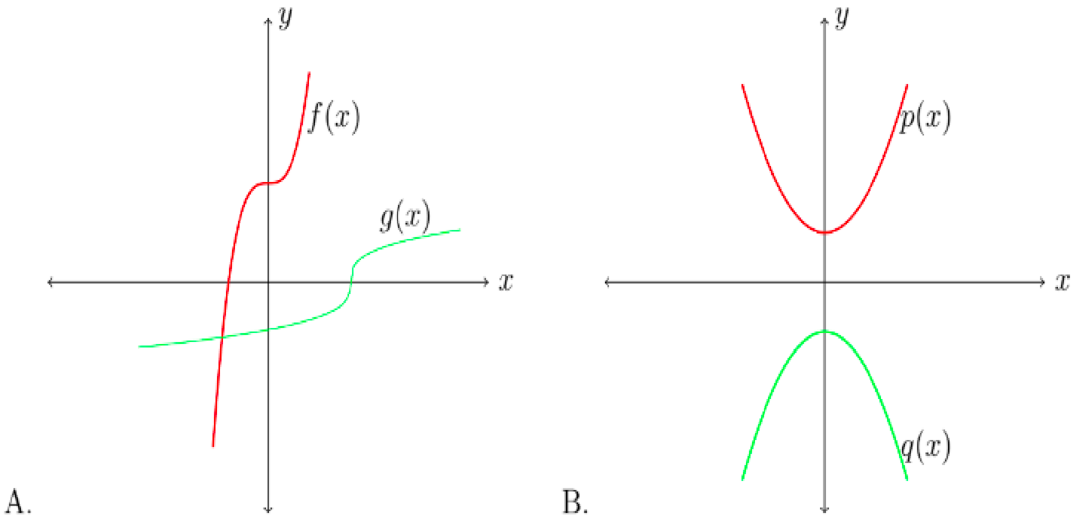

Figure 3

- [ ] $g(x)$ may be the inverse of $f(x)$.
- [ ] $p(x)$ and $q(x)$ are even functions but $f(x)$ and $g(x)$ are neither even functions nor odd functions.
- [ ] $q(x)$ could not be the inverse function of $p(x)$.
- [ ] $p(x)$, $q(x)$ can be an even degree polynomial functions and $f(x)$ can be an odd degree polynomial functions.

No, the answer is incorrect.

Score: 0

Accepted Answers:

$g(x)$ may be the inverse of $f(x)$.
$p(x)$ and $q(x)$ are even functions but $f(x)$ and $g(x)$ are neither even functions nor odd functions.
$q(x)$ could not be the inverse function of $p(x)$.
$p(x)$, $q(x)$ can be an even degree polynomial functions and $f(x)$ can be an odd degree polynomial functions.

12) Evaluate the following limit:

$$\lim_{x \to 0} \frac{9x^4 + 8x^6 + 5e^{\left(\frac{-3}{x^4}\right)}}{x^4}$$

No, the answer is incorrect.

Score: 0

Accepted Answers:

(Type: Numeric) 9

1 point

Consider a sequence $\{a_n\}$ defined as

a_n = $\begin{cases} \frac{\lfloor \frac{n}{2} \rfloor - 4n}{n+3} & \text{when n is odd} \\ \frac{4-7n}{6+2n} & \text{when n is even} \end{cases}$

13) Find the limit of the sequence $\{a_n\}$.

No, the answer is incorrect.

Score: 0

Accepted Answers:

(Type: Numeric) -3.5

1 point

14) Find the limit of the sequence $\{b_n\}$ defined as $b_n = 2a_n^2 + 7a_n$.

No, the answer is incorrect.

Score: 0

Accepted Answers:

(Type: Numeric) 0

1 point