1) Let $X_1, X_2, \ldots, X_{50} \sim$ i.i.d. Poisson(0.04) and let $Y = \sum_{i=1}^{50} X_i$. Use Central Limit theorem to find $P(Y > 3)$.

Enter the answer correct to 2 decimal places.

0.23

1 point

2) Let the moment generating function of a random variable X be given by

1 point

$$M_X(\lambda) = \left(\frac{1}{4}\right) e^{-2\lambda} + \left(\frac{1}{40}\right) + \left(\frac{3}{10}\right) e^{-\lambda} + \left(\frac{3}{40}\right) e^{2\lambda} + \left(\frac{7}{20}\right) e^{\lambda}$$

Find the distribution of X.

- [ ]

X	-2	-1	0	1	2
$P(X=x)$	$\frac{1}{4}$	$\frac{3}{40}$	$\frac{3}{10}$	$\frac{1}{40}$	$\frac{7}{20}$

- [ ]

X	-2	-1	0	1	2
$P(X=x)$	$\frac{1}{4}$	$\frac{1}{40}$	$\frac{3}{10}$	$\frac{3}{40}$	$\frac{7}{20}$

- [x]

X	-2	-1	0	1	2
$P(X=x)$	$\frac{1}{4}$	$\frac{3}{10}$	$\frac{1}{40}$	$\frac{7}{20}$	$\frac{3}{40}$

- [ ]

X	-2	-1	0	1	2
$P(X=x)$	$\frac{1}{4}$	$\frac{3}{40}$	$\frac{1}{40}$	$\frac{7}{20}$	$\frac{3}{10}$

3) A fair coin is tossed 1700 times. Use CLT to compute the probability that head appears at most 560 times. Enter the answer correct to 2 decimal places.

0

1 point

4) Let X be a random variable having the gamma distribution with the parameters $\alpha = 2n$ and $\beta = 1$.

1 point

Hint:

If $X \sim \text{Gamma}(\alpha, \beta)$, $E[X] = \dfrac{\alpha}{\beta}$ and $\text{Var}[X] = \dfrac{\alpha}{\beta^2}$

Sum of n independent $\text{Gamma}(\alpha, \beta)$ is $\text{Gamma}(n\alpha, \beta)$

Use the Weak Law of Large number to find the value of n such that $P\left(\mid \dfrac{X}{2n} - 1 \mid > 0.01\right) < 0.01$

- [x] 505000

- ○ 470000
- ○ 498000
- ○ 482000

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5) Let X be a random variable having the gamma distribution with the parameters $\alpha = 2n$ and $\beta = 1$. **_1 point_**

Hint:
If $X \sim \text{Gamma}(\alpha, \beta), E[X] = \dfrac{\alpha}{\beta}$ and $\text{Var}[X] = \dfrac{\alpha}{\beta^2}$
Sum of n independent $\text{Gamma}(\alpha, \beta)$ is $\text{Gamma}(n\alpha, \beta)$

Use CLT to find the value of n such that $P\left(\mid \dfrac{X}{2n} - 1 \mid > 0.01\right) < 0.01$

- [x] 34570
- [x] 33500
- [ ] 32500
- [ ] 30000

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6) Let the MGF of X be given as $M_X(\lambda) = e^{5\lambda + 7\lambda^2}$. Define a new random variable $Z = 3X + c$. Which of the following are true for the MGF of $Z, M_Z(\lambda)$? **_1 point_**

- [x] $M_Z(\lambda)$ depends on the constant c.
- [ ] $M_Z(\lambda)$ does not depend on the constant c.
- [ ] $M_Z(\lambda) = e^{c\lambda + 15\lambda + 21\lambda^2}$
- [ ] $M_Z(\lambda) = e^{15\lambda + 21\lambda^2}$
- [x] $M_Z(\lambda) = e^{c\lambda + 15\lambda + 63\lambda^2}$

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7) Let X be a random variable with PMF as

x	0	2	3
$f_X(x)$	$1/5$	$3/5$	$1/5$

Suppose $X_1, X_2 \sim$ i.i.d. X. Define a random variable $Y = X_1 + X_2$.
Calculate $E[Y]$. Enter the answer correct to one decimal place.

3.6

1 point

8) The number of online orders received by a bookstore follows a Poisson distribution with a mean rate of 20 orders per hour. The store operates for 10 hours daily. Using the Central Limit Theorem, estimate the probability that the total number of orders received in a 10-hour period is between 180 and 220. Enter the answer correct to two decimal places.

{Hint : Use $F_Z(0.1) = 0.5398, F_Z(1.41) = 0.92$}

0.84

1 point

9) A pharmaceutical company is conducting a clinical trial where historical data shows that 20% of patients experience side effects from a new medication. A research team randomly selects 400 patients for their study. ***1 point***

Let a random variable X model the total number of patients who do not experience side effects in the selected sample. Which of the following is true?

- ○ $X \sim \text{Binomial}(400, 0.20)$
- ◉ $X \sim \text{Binomial}(400, 0.80)$
- ○ $X \sim \text{Binomial}(400, 0.5)$
- ○ $X \sim \text{Binomial}(400, 0.02)$

10) A pharmaceutical company is conducting a clinical trial where historical data shows that 20% of patients experience side effects from a new medication. A research team randomly selects 400 patients for their study.

The research ethics committee requires that if more than 80 patients experience side effects, the trial must be suspended. Using the Central Limit Theorem, find the approximate probability that the trial will need to be suspended. Enter the answer correct to 1 decimal place.

0.5

1 point