Note: This assignment will be evaluated after the deadline passes. You will get your score 48 hrs after the deadline. Until then the score will be shown as Zero.

Multiple Select Questions (MSQ):

1) A function $T : V \to W$ between two vector spaces V and W is said to be a linear transformation if the following conditions hold: *1 point*

Condition 1: $T(v_1 + v_2) = T(v_1) + T(v_2)$ for all $v_1, v_2 \in V$.
Condition 2: $T(cv) = cT(v)$ for all $v \in V$ and $c \in \mathbb{R}$.

Consider the following function:

$$T : \mathbb{R}^2 \to \mathbb{R}$$
$$T(x, y) = \begin{cases} 3x & \text{if } y = 0 \\ 4y & \text{if } y \neq 0 \end{cases}$$

Which of the following statements is true?

- [ ] Condition 1 holds.
- [x] Condition 1 does not hold.
- [x] Condition 2 holds.
- [ ] Condition 2 does not hold.

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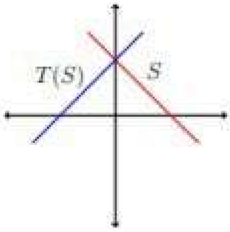
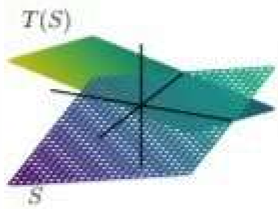
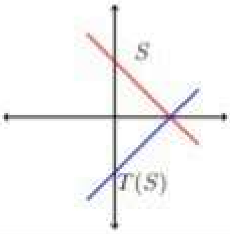
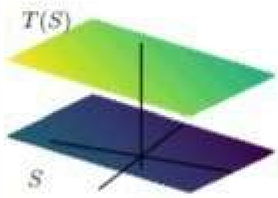
2) Suppose the matrix representation of a linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ with respect to ordered bases $\beta = \{(1,0,1), (0,1,0), (0,0,1)\}$ for the domain and $\gamma = \{(1,0,0), (0,1,0), (1,0,1)\}$ for the range, is $I_{3\times3}$, i.e., the identity matrix of order 3. Let A denote the matrix representation of the linear transformation T with respect to the standard ordered basis of $\mathbb{R}^3$ for both domain and range. Which of the following are true? *1 point*

- [ ] $A = I_{3\times3}$ i.e., identity matrix of order 3.
- [ ] A is a singular matrix
- [ ] $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix}$
- [x] $det(A) = 1.$
- [ ] $det(A) = -1.$
- [x] $A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$

3) Match the linear transformations and sets of vectors in column A with the images of those sets under the linear transformation in column B and the geometric representations of both sets in column C. ***1 point***

	Matrix form of linear transformation (Column A)		Image of the given set (Column B)		Geometric representations (Column C)
i)	$T : \mathbb{R}^3 \to \mathbb{R}^3$ $T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 2 & 3 \end{bmatrix}$ Set: $S = \{(x, y, z) \mid x + y + z = 1\}$	a)	$T(S) = \{(x, y) \mid x - y = 1\}$	1)	
ii)	$T : \mathbb{R}^3 \to \mathbb{R}^3$ $T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ Set: $S = \{(x, y, z) \mid x + y + z = 1\}$	b)	$T(S) = \{(x, y, z) \mid x + y + z = 3\}$	2)	
iii)	$T : \mathbb{R}^2 \to \mathbb{R}^2$ $T = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ Set: $S = \{(x, y) \mid x + y = 1\}$	c)	$T(S) = \{(x, y) \mid x - y = -1\}$	3)	
iv)	$T : \mathbb{R}^2 \to \mathbb{R}^2$ $T = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$ Set: $S = \{(x, y) \mid x + y = 1\}$	d)	$T(S) = \{(x, y, z) \mid x + y - z = 1\}$	4)	

Choose the correct option from the following.

- [ ] i $\longrightarrow$ d $\longrightarrow$ 2, ii $\longrightarrow$ b $\longrightarrow$ 4.
- [x] i $\longrightarrow$ b $\longrightarrow$ 4, ii $\longrightarrow$ d $\longrightarrow$ 2.
- [ ] iii $\longrightarrow$ a $\longrightarrow$ 1, iv $\longrightarrow$ c $\longrightarrow$ 3

- [x] iii $\to$ a $\to$ 3, iv $\to$ c $\to$ 1

4) Consider a linear transformation $S : M_2(\mathbb{R}) \to M_2(\mathbb{R})$ such that $S(A) = A^T$. Let B be the matrix representation of S with respect to the ordered bases: **1 point**

$$\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$$

of $M_2(\mathbb{R})$. Choose the set of correct options:

- [ ] The order of the matrix B is 2×2.
- [x] The order of the matrix B is 4×4.
- [x] The dimension of the row space of the the matrix B is 4.
- [ ] The dimension of the column space of the matrix B is 3.
- [ ] The nullity of the the matrix B is 1.
- [x] The rank of the the matrix B is 4.
- [x] S is surjective.

Numerical Answer Type (NAT):

5) Consider the following statements:

Statement 1: Consider a linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^4$ such that T is not injective. Then $rank(T) < 3$.
Statement 2: If $T : V \to W$ is a linear transformation, whose matrix representation with respect to some ordered bases is given by the matrix $\begin{bmatrix} 0 & \alpha & \gamma \\ 1 & 0 & \gamma \\ 0 & \beta & \frac{\gamma\beta}{\alpha} \end{bmatrix}$, where $\alpha, \beta, \gamma \in \mathbb{R} \setminus \{0\}$, then the rank of the linear transformation T is 3.
Statement 3: If $T : \mathbb{R}^3 \to \mathbb{R}^4$ is a linear transformation such that $T(x, y, z) = (2x - z, 3y - 2z, z, 0)$, then $\{(-3, 1, 1, 0), (1, -5, 1, 0), (3, 5, -1, 0)\}$ is a basis of the image space.
Statement 4: If $T : M_2(\mathbb{R}) \to M_2(\mathbb{R})$ is a linear transformation such that $T(A) = PA$, where $A \in M_2(\mathbb{R})$ and $P = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$, then $\left\{ \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \right\}$ is a basis of the kernel.

Write down the statement numbers corresponding to the correct statements in increasing order.

[Note: Suppose Statement 1, Statement 2, and Statement 4 are correct then your answer should be 124. Similarly, if Statement 2 and Statement 3 are correct then your answer should be 23. In this list one or more than one statement can be correct. Do not add any space between the digits.]

13

1 point

6) Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be a linear transformation. The matrix of the linear transformation T with respect to the standard bases for the domain and codomain is given by $\begin{bmatrix} 12 & k & -9 \\ 8 & -2 & 2k \end{bmatrix}$ where $k \in \mathbb{R}$. Find the value of k for which T is a linear transformation which is **not** surjective.

-3

1 point

Comprehension Type Question:

Suppose a bread-making machine B makes 6 breads from 2 eggs, 3 (in hundreds) grams of wheat, and 1 (in hundred) grams of sugar. B also makes 8 breads from 3 eggs, 4 (in hundreds) grams of wheat, and 2 (in hundreds) grams of sugar, and 10 breads from 5 eggs, 5 (in hundreds) grams of wheat, and 3 (in hundreds) grams of sugar. Suppose the production of breads is a linear function of the amount of eggs, wheat (in hundreds), and sugar (in hundreds) used as raw ingredients. Based on the above data answer the following questions. Suppose x eggs, y (in hundreds) grams of wheat, and z (in hundreds) grams of sugar are used as the raw materials to produce $ax + by + cz$ number of breads. We can express this as follows:

$$T : \mathbb{R}^3 \to \mathbb{R}$$

$$T(x, y, z) = ax + by + cz$$

where the co-ordinates in $\mathbb{R}^3$ denote the number of eggs, amount (in grams) of wheat (in hundreds), and amount (in grams) of sugar (in hundreds). Observe that T is a linear transformation.

Use the above information for Question 6, 7 and 8.

7) Choose the correct set of options from the the following. (MSQ) ***1 point***

- [ ] $Nullity(T) = 1$
- [x] $Rank(T) = 1$
- [x] $Nullity(T) = 2$
- [ ] $Rank(T) = 2$
- [ ] $Nullity(T) = 3$
- [ ] $Rank(T) = 3$
- [ ] T is neither one to one nor onto.
- [ ] T is one to one but not onto.
- [x] T is onto but not one to one.
- [ ] T is an isomorphism.

8) Choose the set of correct statements. (MSQ) ***1 point***

- [ ] If 4 eggs and 2 (in hundreds) grams of sugar is used, and no wheat is used, then 9 breads are produced.
- [x] If 4 eggs and 2 (in hundreds) grams of sugar is used, and no wheat is used, then no bread is produced.
- [x] If only 3 (in hundreds) grams of wheat is used, then 6 breads are produced.
- [ ] If 3 (in hundreds) grams of wheat and 1 (in hundred) grams of sugar is used, and no egg is used, then no bread is produced.
- [x] If 3 (in hundreds) grams of wheat and 1 (in hundred) grams of sugar is used, and no egg is used, then 6 breads are produced.

9) How many breads are produced by the machine from 6 eggs, 10 (in hundreds) grams of wheat, and 5 (in hundreds) grams of sugar? (NAT)

20

1 point

10) Let T be a linear transformation from $\mathbb{R}^8$ to $\mathbb{R}^7$. Suppose a basis for the null space of T has 6 vectors. How many linearly independent vectors are needed to form a basis for the range of T?

3

1 point

11) Let $\mathcal{M}$ be the set of all skew-symmetric matrices of order 10. Then $\mathcal{M}$ forms a vector space under matrix addition and scalar multiplication. Let T be a linear transformation from $\mathcal{M}$ to $\mathbb{R}$ defined by $T(A) = 8\, tr(A)$, where $tr(A) =$ trace of A. What is the nullity of T?

45

1 point

12) Let $T : \mathbb{R}^3 \longrightarrow \mathbb{R}^2$ be defined by $T(x, y, z) = (2x + 3y, 6z)$. Choose the correct options about T. ***1 point***

- [x] The matrix of T is $\begin{bmatrix} 2 & 3 & 0 \\ 0 & 0 & 6 \end{bmatrix}$.
- [ ] A basis for the range of T is $\{(2, 2)\}$.

- [ ] The matrix of T is $\begin{bmatrix} 2 & 0 \\ 3 & 0 \\ 0 & 6 \end{bmatrix}$.
- [x] A basis for the null space of T is $\{(1, -\frac{2}{3}, 0)\}$.
- [ ] A basis for the range of T is $\{(2, 3), (0, 6)\}$.
- [ ] A basis for the null space of T is $\{(1, -\frac{2}{3}, 0), (0, 0, 6)\}$.