Note: This assignment will be evaluated after the deadline passes. You will get your score 48 hrs after the deadline. Until then the score will be shown as Zero.

1) Which of the following is (are) the critical points of the scalar valued function $f(x, y) = 3x^2y + y^3 - 3x^2 - 3y^2 + 2$? ***1 point***

- [x] $(0, 0)$
- [x] $(0, 2)$
- [x] $(1, 1)$
- [ ] $(1, 2)$
- [ ] $(0, 1)$

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2) Consider a function $f : \mathbb{R}^2 \to \mathbb{R}$ defined as: ***1 point***

$$f(x, y) = \begin{cases} \frac{xy^2}{x^2+y^4}, & \text{if } (x, y) \neq (0, 0) \\ 0, & \text{otherwise} \end{cases}$$

Which of the following is (are) true about $f(x, y)$?

- [ ] The directional derivative at $(0, 0)$ in the direction of a unit vector $u = (u_1, u_2)$ is 1.
- [x] The directional derivative at $(0, 0)$ in the direction of a unit vector $u = (u_1, u_2)$ is $\frac{u_2^2}{u_1}$, where u_1 is non-zero.
- [ ] Amongst all directional derivatives at $(0, 0)$, the maximum occurs in the direction of the vector $(5, 5)$.
- [x] There is no plane which contains all the tangent lines at $(0, 0)$ and hence the tangent plane at $(0, 0)$ does not exist.

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3) Let $f : \mathbb{R}^3 \to \mathbb{R}$ be the function defined by $f(x, y, z) = 2x^3 + y^2 - z^3$, for all $(x, y, z) \in \mathbb{R}^3$. Choose all the directions along which there is no change in the function f at the point $(1, 1, 1)$. ***1 point***

- [x] $\left(\frac{1}{\sqrt{5}}, 0, \frac{2}{\sqrt{5}}\right)$
- [ ] $\left(\frac{6}{7}, \frac{2}{7}, -\frac{3}{7}\right)$
- [ ] $\left(\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}}\right)$
- [x] $\left(0, \frac{3}{\sqrt{13}}, \frac{2}{\sqrt{13}}\right)$

- [x] $\left(\frac{2}{\sqrt{17}}, -\frac{3}{\sqrt{17}}, \frac{2}{\sqrt{17}}\right)$

4) Let $L_f(x,y) = Ax + By + C$ be the linear approximation to the function $f(x,y) = ye^x - \frac{1}{4}(x^2+y^2)$ at $(0,1)$. Then find the value of $A + 2B + 4C$.

3

1 point

5) Consider a function $f(x,y) = 2\sqrt{x^2+4y}$. Let S denote the set of unit vectors u for which the directional derivative of f at $(-2,3)$ in the direction of u is 0. Find the cardinality of the set S.

2

1 point

6) Suppose $f(x,y) = xye^x$ be a scalar valued multivariable function. Then using the linear approximation $L_f(x,y)$ of the function f at $(1,1)$, the estimate value of $f(1.2, 0.9)$ is found to be βe, where β is a real number. The value of β is

1.3

1 point

The temperature T (in degree centigrade, ^0C) in a solid metal sphere is given by the function $e^{-(x^2+y^2+z^2)}$. Answer Questions 6,7 and 8 from the given information.

7) Choose the set of correct options. **_1 point_**

- [x] The rate of change of temperature in the direction of X-axis is continuous at every point.
- [ ] The rate of change of temperature in the direction of Z-axis is not continuous at the origin.
- [x] The rate of change of temperature at the origin from any direction is constant and that is 0.
- [ ] The rate of change of temperature at the origin from any direction is constant and that is e.
- [ ] The rate of change of temperature at the origin from any direction is not constant.

8) The rate of change of the temperature at point $(1,0,0)$ in the direction toward point $(8,6,0)$ is A. Find the value of $10Ae$.

-16

1 point

9) Which of the following statements are true? **_1 point_**

- [x] At a point (a, b, c) on the sphere the maximum rate of change in temperature is given by $2e^{-(a^2+b^2+c^2)}\sqrt{a^2+b^2+c^2}$.

- [ ] At a point (a, b, c) on the sphere the maximum rate of change in temperature is given by $-2e^{-(a^2+b^2+c^2)}\sqrt{a^2+b^2+c^2}$.

- [x] At a point (a, b, c) on the sphere the maximum rate of change in temperature is in the direction of the unit vector $\left(-\frac{a}{\sqrt{a^2+b^2+c^2}}, -\frac{b}{\sqrt{a^2+b^2+c^2}}, -\frac{c}{\sqrt{a^2+b^2+c^2}}\right)$.

- [ ] At a point (a, b, c) on the sphere the maximum rate of change in temperature is in the direction of the unit vector $\left(-\frac{2a}{e^{a^2+b^2+c^2}}, -\frac{2b}{e^{a^2+b^2+c^2}}, -\frac{2c}{e^{a^2+b^2+c^2}}\right)$.

10) Find out the maximum directional derivative at (0,0) of the function $f_1(x, y) = y^4e^{4x}$.

0

1 point

11) Find out the maximum directional derivative at (0,0) of the function $f_2(x, y) = 5 - 4x^2 + 2x - 5y^2$.

2

1 point

12) Find out the maximum directional derivative at (0,0) of the function $f_3(x, y) = 5x\sin(2x) + 2y\cos(5y)$.

2

1 point

13) The equation of the tangent plane to the surface $z = 4 - x^4 - y^2$ at the point $(1, 1, 2)$ is $z = Ax + By + C$. Find the value of $C - A - B$.

14

1 point

14) Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a function such that the partial derivatives of f exist and are continuous at every point. Suppose $f(1, 1) = 3$, and $(0, 1, 5)$ is a point on the tangent line to the graph of f at $(1, 1)$ in the direction $(1, 0)$. Find $f_x(1, 1)$, the partial derivative with respect to x at $(1, 1)$.

-2

1 point