

Note: This assignment will be evaluated after the deadline passes. You will get your score 48 hrs after the deadline. Until then the score will be shown as Zero.

Multiple Select Questions (MSQ)

Match the functions of two variables in Column A with their graphs given in Column B (Use the table to answer questions 1 to 4).

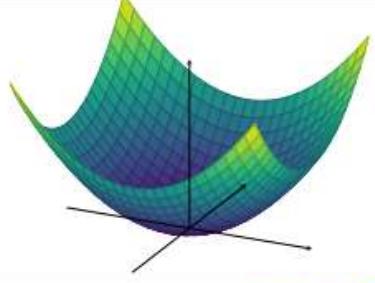
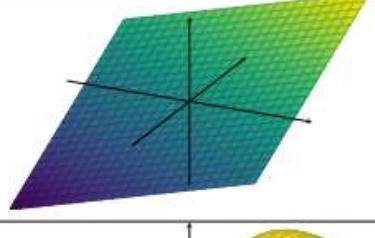
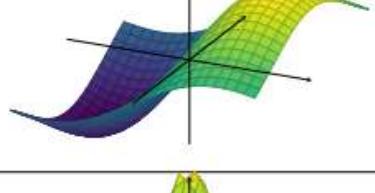
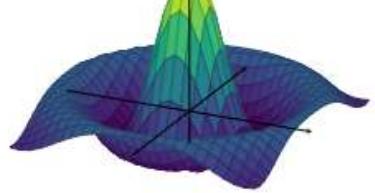
	Function of two variables (Column A)		Graph of the function (Column B)
i)	$f(x, y) = 2x + 3y$	1)	
ii)	$f(x, y) = x^2 + y^2$	2)	
iii)	$f(x, y) = \frac{\sin(\sqrt{x^2+y^2})}{\sqrt{x^2+y^2}}$	3)	
iv)	$f(x, y) = xe^{(-x^2-y^2)}$	4)	

Table: M2W9G1

1) The graph of the function with serial number i) in column A has the serial number _____ in Column B.

2

1 point

2) The graph of the function with serial number ii) in column A has the serial number _____ in Column B.

1

1 point

3) The graph of the function with serial number iii) in column A has the serial number _____ in Column B.

3

1 point

4) The graph of the function with serial number iv) in column A has the serial number _____ in Column B.

4

1 point

5) Consider the function $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ defined as follows:

1 point

$$f(x, y) = \begin{cases} \frac{x+y}{\sqrt{x^2+y^2}} & \text{if } x, y \neq 0, \\ 0 & \text{if } x = y = 0 \end{cases}$$

Choose the correct options from the following.

- $f(x, y) \rightarrow 1$ if $x \rightarrow 0$ and $y \rightarrow b$, for some positive real number b .
- $f(x, y) \rightarrow 0$ if $(x, y) \rightarrow (0, 0)$.
- $f(x, y) \rightarrow 1$ if $(x, y) \rightarrow (0, 0)$ along the positive X -axis.
- $f(x, y) \rightarrow 0$ if $(x, y) \rightarrow (0, 0)$ along the positive Y -axis.
- $f(x, y) \rightarrow 0$ if $(x, y) \rightarrow (0, 0)$ along the line $y = 2x$.
- $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$ does not exist.

6) Consider a function $f : \mathbb{R}^2 \rightarrow \mathbb{R}$, such that $\frac{\partial f}{\partial x} = y \cos(xy)$. Which of the following are possible candidates for the function f ? 1 point

- $f(x, y) = \sin(xy)$.
- $f(x, y) = xy \cos(xy)$.
- $f(x, y) = \sin(xy) + xy$.
- $f(x, y) = \sin(xy) + \cos(y)$.
- $f(x, y) = \sin(xy) + \phi(x)$, for some non-constant function $\phi(x)$.
- $f(x, y) = \sin(xy) + \phi(y)$, for some non-constant function $\phi(y)$.

7) Let $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ be a real valued two-variable function. Let $G(f)$ denote the graph of the function f , i.e., $G(f) = \{(x, y, f(x, y)) \mid x, y \in \mathbb{R}\}$ 1 point

Choose all the correct options from the following.

(Note that the graph of the function is a subset of \mathbb{R}^3 with (x, y, z) coordinates.)

- The intersection of the x -axis with $G(f)$ contains a single element.

- The intersection of the y -axis with $G(f)$ contains a single element.
- The intersection of the z -axis with $G(f)$ contains a single element.
- If every plane parallel to the xy -plane has non-empty intersection with $G(f)$, then f is injective.
- If every plane parallel to the xy -plane has non-empty intersection with $G(f)$, then f is surjective.

Numerical Answer Type (NAT):

8) Which of the following functions f and g from \mathbb{R}^2 to \mathbb{R} are continuous at the origin?

$$f(x, y) = \begin{cases} \frac{x^3}{3x^2y} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } x = 0 \text{ or } y = 0 \end{cases}$$

$$g(x, y) = \begin{cases} x^4 + x^3y + xy^3 + y^4 & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } x = y = 0 \end{cases}$$

Statement 1: $f(x, y)$ is continuous at the origin

Statement 2: $g(x, y)$ is continuous at the origin.

Find out the number of correct statements.

1

1 point

9) Consider the two-variable function defined by

$$f(x, y) = \frac{xy}{x^2+y^2},$$

for all $(x, y) \in \mathbb{R}^2 \setminus \{(0, 0)\}$. Let l_1 denote the limit of $f(x, y)$ as (x, y) approaches $(0, 0)$ along the straight line $y = x$, and let l_2 denote the limit of $f(x, y)$ as (x, y) approaches $(0, 0)$ along the parabola $y = x^2$. Find $l_2 - l_1$.

-0.5

1 point

10) Find the value (up to two decimal places) to which the function $f(x, y) = \frac{xy}{2x^2+y^2}$ approaches, if (x, y) approaches to $(0, 0)$ along the straight line $y = -1x$.

-0.33

1 point

11) Consider the function $f : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ defined as follows:

$$f(x, y) = (f_1(x, y), f_2(x, y))$$

where $f_1(x, y) = xy + y^2$ and $f_2(x, y) = x + xy + 1$. Define a matrix A as follows:

$$\begin{bmatrix} \frac{\partial f_1}{\partial x}(-4, 3) & \frac{\partial f_1}{\partial y}(-4, 3) \\ \frac{\partial f_2}{\partial x}(-4, 3) & \frac{\partial f_2}{\partial y}(-4, 3) \end{bmatrix}$$

What will be the determinant of A ?

-20

1 point

12) Find out the directional derivative of the function $f(x, y) = x^2y^3$ at $(4, 4)$, in the direction of the vector $(0, 5)$.

768

1 point

13) Consider the function $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ defined as follows:

$$f(x, y) = (5x + 5) \sin y$$

Find the value of $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$.

0

1 point

Comprehension Type Question:

The price of a product ($f(x, y)$) depends on the price (x) of the raw materials and the price (y) of transportation of the product to the market according to

$$f : \mathbb{R}^2 \rightarrow \mathbb{R}$$

$$f(x, y) = x^2 + xy + y^2$$

Answer Questions 14,15 and 16 using the information above.

14) Which of the following statements is true if the rate of change of the price of the product with respect to the price of the raw materials is the same as the rate of change of price of the product with respect to the price of transportation of the product to the market?

Statement 1: The price of the raw materials is twice the price of the transportation of the product to the market.

Statement 2: The price of the raw materials is half the price of the transportation of the product to the market.

Statement 3: The price of the raw materials is the same as the price of the transportation of the product to the market.

Statement 4: The price of the raw materials is 3 times the price of the transportation of the product to the market.

(If Statement 2 is correct, then enter 2 as your answer.)

3

1 point

15) Which of the following statements are true?

1 point

- $f(x, y)$ is a linear function.
- $f(cx, cy) = c^2 f(x, y)$ for any real number c .
- $f(x, y)$ is continuous at $(0, 0)$.
- If the price of transportation and raw material of the product approaches 0 and 5 respectively, then the price of the product approaches 30.
- If the price of transportation and raw material of the product approaches 0 and 5 respectively, then the price of the product approaches 25.

16) If the rate of change of the price of the product along the direction of the vector $(1, m)$ is $\frac{1}{\sqrt{1+m^2}}[ka + lb]$, when the price of raw material is a and the price of transportation of the product to the market is b , then find the value of $2k - l$.

3

1 point