1) Suppose that the number of buses reaching a particular stop in an one-hour time period follows the Poisson distribution with an unknown parameter λ. Previous records suggest that the prior probabilities of λ are $P(\lambda = 8) = 0.3$ and $P(\lambda = 10) = 0.7$. If in a particular one-hour time period 9 buses reach the bus stop, find the posterior mode of λ.

10

1 point

2) Call duration of daily stand up meetings of employees of a certain company follows the exponential distribution with an unknown parameter λ. Duration (in minutes) of last ten meetings are 24, 35, 30, 20, 26, 16, 18, 30, 27, 20. Find the Bayesian estimate (posterior mean) of λ using the prior distribution of Exp $(\frac{1}{15})$ for λ. Write your answer correct to two decimal places.

0.04

1 point

3) The outcomes on tossing a coin ten times are: T, T, H, T, H, H, H, H, H, T. Let p be the probability of heads. Previous records show that heads appear on an average 50.0% of the time. Find the posterior mean of p using the Beta$(2, \beta)$ prior. Write your answer correct to two decimal places.

0.57

1 point

4) Rainfall in the monsoon season in Delhi follows normal distribution with mean μ and variance 256. Rainfall (in mm) registered in the 2021 monsoon are 228, 414, 486, 534, 428, 659, 655, 733, 333, 900. Prior information about the average rainfall is that it has mean 700 mm and variance 289. Use the normal prior that matches your prior information and find the posterior mean. Write your answer correct to two decimal places.

550.26

1 point

5) Suppose length of a phone call (in minutes) made by Kapil is exponentially distributed with an unknown parameter λ. Consider a sample (in minutes) $20, 23, 50, 2, 7, 10, 15, 70, 30, 29$ from his previous call records.
Find the method of moments estimate of λ for the given sample. Enter the answer correct to three decimal places.

0.039

1 point

6) Suppose length of a phone call (in minutes) made by Kapil is exponentially distributed with an unknown parameter λ. Consider a sample (in minutes) $20, 23, 50, 2, 7, 10, 15, 70, 30, 29$ from his previous call records.
Using a Uniform[0, 1] prior, find the posterior mean of λ. Enter the answer correct to three decimal places.

0.043

1 point

7)

We are given that the number of emergency patients arriving at a hospital each night follows a Poisson distribution with an unknown parameter λ. Historical data gives prior beliefs that $P(\lambda = 7) = 0.25$ and $P(\lambda = 5) = 0.75$. If 9 emergency patients arrived on a particular night, then find the posterior mode of λ.

5

1 point

8) A doctor is testing patients for a rare disease. She continues testing until finding the first positive case. Let p be the probability that a patient tests negative. The prior distribution of p is Beta(α, β), with prior mean equal to 0.8. After testing 12 patients, she finds that the first 11 test negative and the 12th tests positive. What is the posterior mean of p in terms of β? Assume independence of test results for different patients. ***1 point***

- (selected) $\dfrac{4\beta + 11}{5\beta + 12}$
- $\dfrac{4\beta + 11}{5\beta + 1}$
- $\dfrac{4\beta + 10}{5\beta + 11}$
- $\dfrac{4\beta + 11}{\beta + 1}$

9) Let $X_1, X_2, ..., X_n \sim$ i.i.d. Bernoulli (p) distribution. The prior distribution of p has the PMF as follows:

p	0.2	0.6
$f_P(p)$	0.3	0.7

For $n = 5$ consider the samples $\{1, 1, 0, 0, 1\}$.

Find the posterior mode of p. Enter the answer correct to one decimal place.

0.6

1 point

10) Let $X_1, X_2, \ldots, X_n \sim$ i.i.d. Bernoulli(p) distribution. The prior distribution of p has the PMF as follows:

p	0.2	0.6
$f_P(p)$	0.3	0.7

For $n = 5$ consider the samples $\{1, 1, 0, 0, 1\}$.

Find the posterior mean of p. Enter the answer correct to two decimal places.

0.58

1 point