Note: This assignment will be evaluated after the deadline passes. You will get your score 48 hrs after the deadline. Until then the score will be shown as Zero.

Multiple Select Questions (MSQ):

1) An inner product on $\mathbb{R}^3$ is defined as: ***1 point***

$$\langle .,. \rangle : \mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{R}$$

$$\langle (x_1, x_2, x_3), (y_1, y_2, y_3) \rangle = x_1y_1 + x_2y_2 + x_3y_3.$$

Match the sets of vectors in column A with their properties of orthogonality or orthonormality in column B with respect to the above inner product.

	Set of vectors (Column A)		Properties (Column B)
a)	$\{(2,3,4),(-1,2,-1)\}$	i)	Forms a basis but not orthogonal
b)	$\{\frac{1}{\sqrt{2}}(1,0,-1), \frac{1}{\sqrt{2}}(-1,0,-1)\}$	ii)	Forms an orthogonal basis
c)	$\{(2,3,4),(-1,2,-1),(0,4,-3)\}$	iii)	Orthogonal but not orthonormal, and does not form a basis of $\mathbb{R}^3$
d)	$\{(2,3,4),(-1,2,-1),(11,2,-7)\}$	iv)	Orthonormal, but does not form a basis of $\mathbb{R}^3$

Table : M2W8G1

Choose the set of correct options.

- [ ] a $\to$ iv)
- [x] a $\to$ iii)
- [x] b $\to$ iv)
- [ ] b $\to$ iii)
- [ ] c $\to$ ii)
- [x] c $\to$ i)
- [ ] d $\to$ i)

- [x] d $\rightarrow$ ii)

2) Choose the set of correct options. **1 point**

- [x] Suppose $\beta = \{v_1, v_2, \ldots, v_n\}$ is an orthogonal basis of an inner product space V. If there exists some $v \in V$, such that $\langle v, v_i \rangle = 0$ for all $i = 1, 2, \ldots, n$, then $v = 0$.
- [x] There exists an orthonormal basis for $\mathbb{R}^n$ with the standard inner product.
- [x] If P_W denotes the linear transformation which projects the vectors of an inner product space V to a subspace W of V, then $range(P_W) \cap null\ space(P_W) = \{0\}$, where 0 denotes the zero vector of V.
- [x] $\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$ cannot represent a matrix corresponding to some projection.

3) Let V be an inner product space of dimension n. Given a subspace $W \subseteq V$ of dimension k, let $P_W : V \to V$ denote the projection transformation onto W. Choose the correct statements from the following. **1 point**

- [ ] There is a unique subspace $W \subseteq V$ such that P_W is a linear isomorphism.
- [ ] For any subspace W, there exists $v \in V$ such that $P_W(v) = -v$.
- [ ] If $\mathcal{B} = \{v_1, v_2, \ldots, v_n\}$ is a basis of V such that $v_i \in W$ for $i \in \{1, 2, \ldots, k\}$, and $v_i \in W^\perp$ for $i \in \{k+1, \ldots, n\}$, then the matrix of P_W with respect to $\mathcal{B}$ for both the domain and codomain is a diagonal matrix.
- [ ] Given any $x \in \mathbb{R}$, there exists a subspace W such that the matrix of the linear transformation P_W with respect to the same basis for both the domain and the codomain has determinant equal to x.
- [ ] Given any $m \in \{0, 1, \ldots, n\}$, there exists a subspace $W \subseteq V$ such that the nullity of P_W is m.

Numerical Answer Type (NAT):

4) If A is an orthogonal matrix of order 5, then find nullity of the matrix A.

0

1 point

5)

Let $v \in \mathbb{R}^3$ be a vector such that $||v|| = 5$. If u is the vector obtained from v after the anti- clock wise rotation of XY-plane with angle 70° about the Z- axis, then find the length of the vector u.

5

1 point

6) Let $v = (1, 2, 2)$ be a vector in $\mathbb{R}^3$. If (a, b, c) is the vector obtained from v after the anti- clock wise rotation of YZ-plane with angle 60° about the X- axis, then find the value of $a + b + c$.

3

1 point

7) Consider a vector space $M_{2\times2}(\mathbb{R})$ and a norm on the vector space defined as

$||A|| = \max\{|a_{11}| + |a_{21}|, |a_{12}| + |a_{22}|\}$, where $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \in M_{2\times2}(\mathbb{R})$.

Let $B = \begin{bmatrix} x & \sqrt{16}\, x \\ -\sqrt{16}\, y & y \end{bmatrix}$ be an orthogonal matrix i.e. $BB^T = B^TB = I$ and assume $x, y > 0$.

Then find the norm of the matrix $C = \begin{bmatrix} \sqrt{17}\, x & \sqrt{17}\, x \\ \sqrt{17}\, y & \sqrt{17}\, y \end{bmatrix}$ (i.e., $||C||$)

2

1 point

8) Let $V = \mathbb{R}^2$ be the inner product space with usual inner product and a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined as $T(x, y) = (\frac{a}{\sqrt{a^2+8^2}}x + \frac{16}{\sqrt{b^2+16^2}}y, \frac{8}{\sqrt{a^2+8^2}}x + \frac{b}{\sqrt{b^2+16^2}}y)$. If T is an orthogonal linear transformation, then find the value of $16a + 8b$.

0

1 point

9) Find the number of 2×2 orthogonal matrices whose $(1, 1)$-th entry is $\frac{1}{\sqrt{2}}$.

4

1 point

Comprehension Type Question:

With a particular frame of reference (in $\mathbb{R}^3$), position of a target is given as the vector $(3, 4, 5)$. Three shooters S_1, S_2, and S_3 are moving along the lines $x = y$, $x = -y$, and $x = 2y$, on the XY-plane (i.e., $z = 0$) to shoot the target. Suppose that, there is another shooter S_4, who is moving on the plane $x + y + z = 0$. Suppose all of them shoot the target so that the target is at the closest distance from the respective path or plane on which they are travelling. Answer questions 10,11 and 12 using the given information.

10) Choose the set of correct options. **1 point**

- [ ] S_1 will shoot the target from the point $(\frac{7}{2}, -\frac{7}{2}, 0)$.
- [x] S_1 will shoot the target from the point $(\frac{7}{2}, \frac{7}{2}, 0)$.
- [ ] S_1 will shoot the target from the point $(1, 1, 0)$.
- [ ] S_2 will shoot the target from the point $(-\frac{1}{2}, -\frac{1}{2}, 0)$.
- [ ] S_2 will shoot the target from the point $(1, -1, 0)$.
- [x] S_2 will shoot the target from the point $(-\frac{1}{2}, \frac{1}{2}, 0)$.
- [x] S_3 will shoot the target from the point $(4, 2, 0)$.
- [ ] S_3 will shoot the target from the point $(2, 1, 0)$.
- [ ] S_3 will shoot the target from the point $(0, 0, 0)$.

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11) If (a, b, c) is the point from which the shooter S_4 will shoot the target, then find the value of $a + 2b + 3c$.

2

1 point

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12) Let d_i be the distance of the target from the point where the shooter S_i shoots the target, for i = 1,2,3,4 and let d be the minimum amongst the d_i. Find the value of d^2.

25.5

1 point