Capacitor Charging and Discharging

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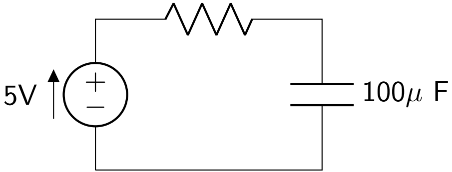
Capacitor Charging

Demonstration: Charging a 100 μF Capacitor

- We charges a 100 μF capacitor using a 5V source.
- Then, we will discharge it through an LED.

Charging Circuit



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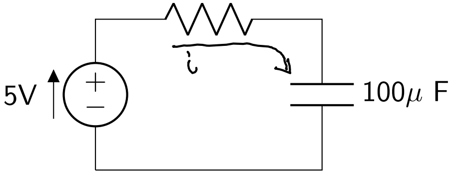
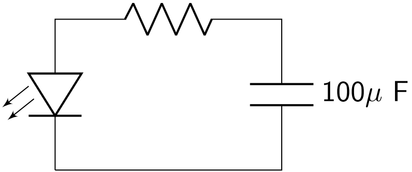
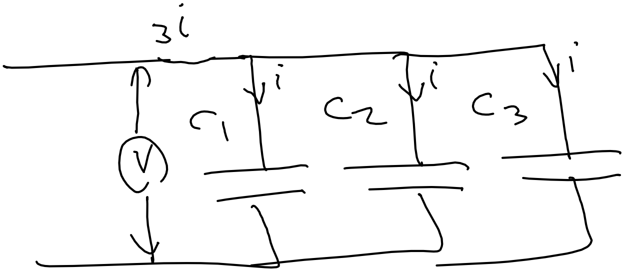


Discharging Circuit

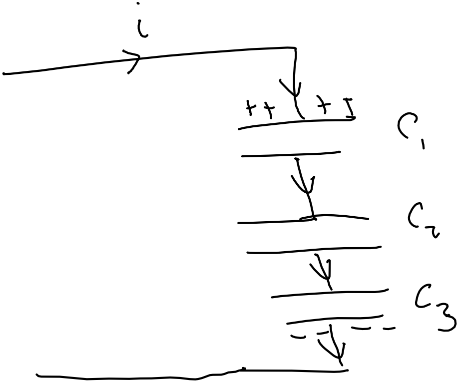


Capacitors in Parallel



$$C_T = C_1 + C_2 + C_3$$

Capacitors in Series



$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}.$$

Formula for Capacitor Charging

The voltage across a charging capacitor in an RC circuit is given by:

$$V_C(t) = V_0 \left(1 - e^{-\frac{t}{RC}}\right)$$

The current during the charging process is:

$$I_C(t) = \frac{V_0}{R} e^{-\frac{t}{RC}}$$

Parameters:

- V_0: Supply voltage
- R: Resistance
- C: Capacitance
- t: Time

$V_R = V_B$

$V_C = V_B\left(1 - e^{-t/RC}\right)$

RC - time constant

Formula for Capacitor Discharging

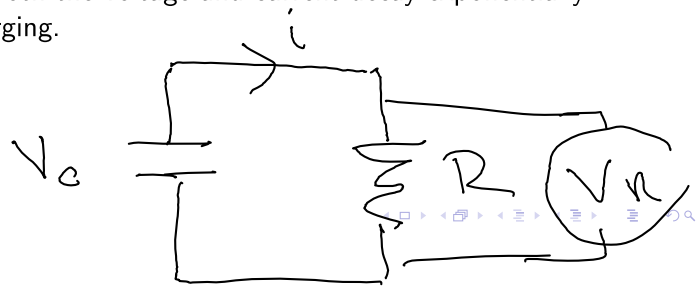
The voltage across a discharging capacitor is given by:

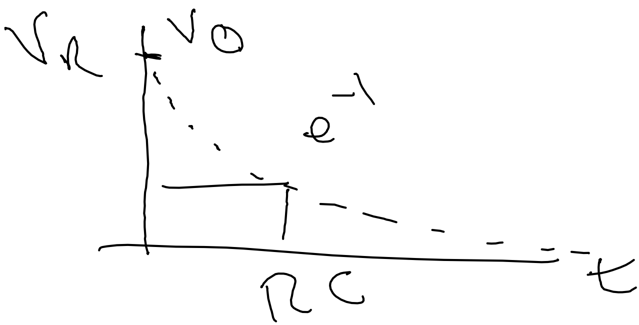
$$V_C(t) = V_0 e^{-\frac{t}{RC}}$$

The current during discharging is:

$$I_C(t) = -\frac{V_0}{R} e^{-\frac{t}{RC}}$$

Key Point: Both the voltage and current decay exponentially during discharging.

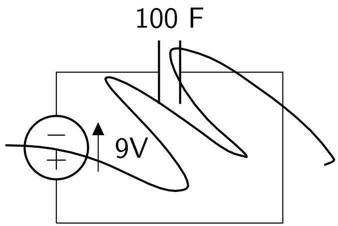




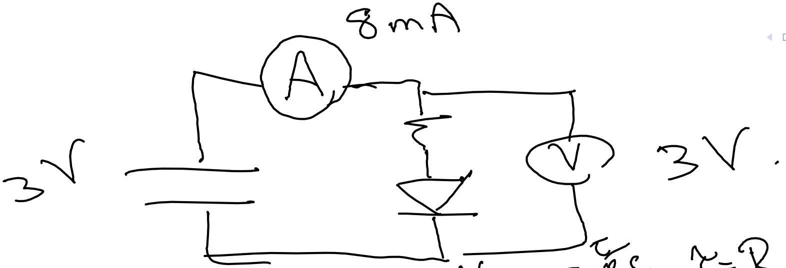
Demonstration: Measuring Charge/Discharge

Demonstration: Using a Large 100F Capacitor

- We will charge a large 100F capacitor.
- The long charge/discharge time will allow real-time measurement.
- We will use a digital multimeter (DMM) to track the voltage.



Observe: The slow discharge can be measured and observed using the DMM in real time.



$$I = \frac{V}{R} e^{-t/RC} \qquad \tau = RC$$

$$= 375 \times 100$$

$$= 37{,}500$$

$$V = IR$$

$$\frac{V}{I} = R \qquad \frac{3}{8\times10^{-3}} = R = 375\,\Omega$$

Key Takeaways

- Capacitors store electrical energy and release it over time.
- Parallel capacitors increase total capacitance, leading to longer discharge times.
- The charging and discharging behavior is governed by exponential functions.
- Demonstrations helped us visualize these concepts with real-time measurements.

Energy Storage in a Capacitor

- A capacitor stores energy in the form of an electric field between its plates.
- The energy stored in a capacitor is related to the charge Q on the plates and the voltage V across the capacitor.
- The energy stored in a capacitor is given by:

$$\boxed{E_C = \frac{1}{2}CV^2}$$

$50 \times 9 \simeq 450 \text{ J}$

where:

- E_C is the energy stored (in joules),
- C is the capacitance (in farads),
- V is the voltage across the capacitor (in volts).

Key Concept

The capacitor stores energy when charged, and this energy is released when the capacitor discharges.

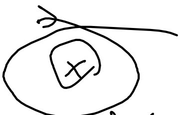
Inductors in Circuits

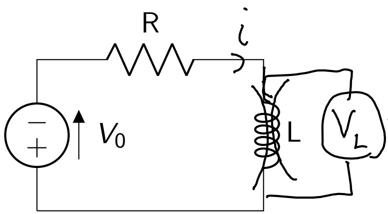
- An inductor opposes changes in current by storing energy in its magnetic field.
- Voltage across an inductor is proportional to the rate of change of current:

IR

$$\boxed{V_L = L\frac{di}{dt}}$$

where L is the inductance and $i(t)$ is the current.
- Inductors store energy when a current flows through them.





$= L\frac{dI}{dt}$

Inductance

$$IR = -L\frac{dI}{dt}$$

$$\frac{R}{L} I = \frac{dI}{dt}$$

$$\text{if } I = e^{-at}$$

$$\frac{R}{L}\cancel{e^{at}} = a\cancel{e^{at}}$$

Time constant

$$a = \frac{R}{L}$$

$$I_L = I_0 e^{-\frac{Rt}{L}}$$

Energy Stored in an Inductor

- When a current $i(t)$ flows through an inductor, energy is stored in its magnetic field.
- The energy stored is given by:

$$E_L = \frac{1}{2} L i(t)^2$$

- This energy is released when the current decreases.

Key Points

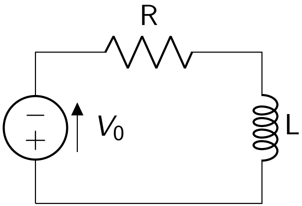
- Energy storage in inductors is analogous to energy storage in capacitors.
- Inductors resist changes in current (unlike capacitors, which resist changes in voltage).

Charging an Inductor

- Consider an RL circuit with a resistor R, an inductor L, and a DC voltage source V_0.
- Kirchhoff's Voltage Law (KVL) applied to the loop gives:

$$V_0 = V_R + V_L = i(t)R + L\frac{di}{dt}$$

- This differential equation governs the charging process of the inductor.



Solving the Inductor Charging Equation

- From KVL:

$$V_0 = i(t)R + L\frac{di}{dt}$$

  Rearrange:

$$\frac{di}{dt} = \frac{V_0 - i(t)R}{L}$$

- Solving this gives the current as a function of time:

$$i(t) = \frac{V_0}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$

Energy Stored During Charging

- As current builds up in the inductor, energy is stored in its magnetic field.
- The energy stored at any time t is:

$$E_L = \frac{1}{2} L i(t)^2$$

- At steady state ($t \to \infty$), the current reaches $i_\infty = \frac{V_0}{R}$, and the maximum energy stored is:

$$E_{L,\max} = \frac{1}{2} L \left(\frac{V_0}{R} \right)^2$$

Example Problem: Inductor Charging

- Given a circuit with $V_0 = 12\,\mathrm{V}$, $R = 10\,\Omega$, and $L = 5\,\mathrm{H}$:
- Find the current $i(t)$ at $t = 2\,\mathrm{s}$.

$$i(t) = \frac{V_0}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$

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- Given a circuit with $V_0 = 12\,\mathrm{V}$, $R = 10\,\Omega$, and $L = 5\,\mathrm{H}$:
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$$i(t) = \frac{V_0}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$

$$i(2) = \frac{12}{10}\left(1 - e^{-\frac{10}{5}\times 2}\right) = 1.2\,\mathrm{A}$$