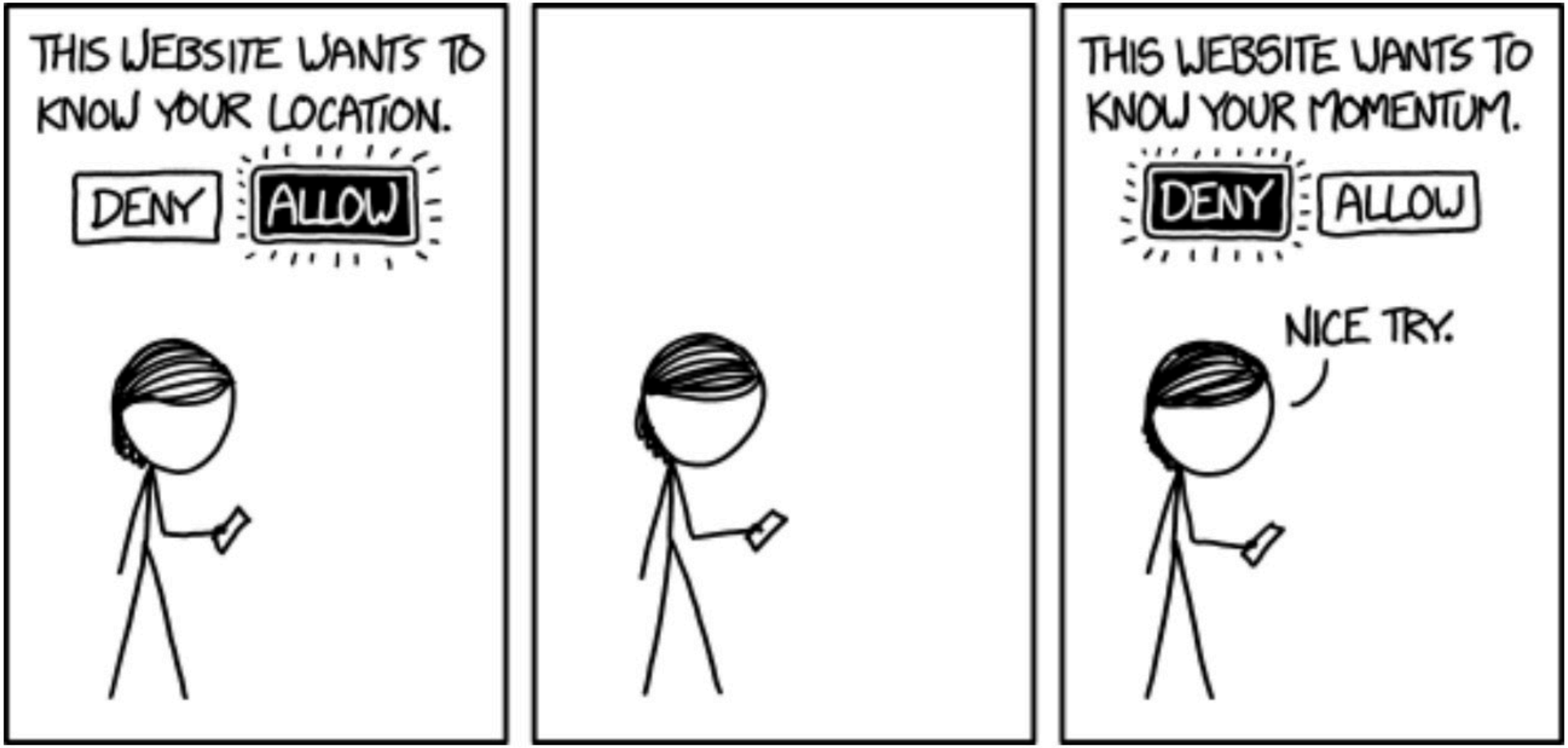
THIS WEBSITE WANTS TO KNOW YOUR LOCATION.
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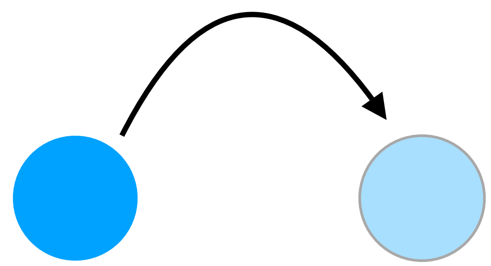
NICE TRY.

Heisenberg Uncertainty Relation

Random Variables

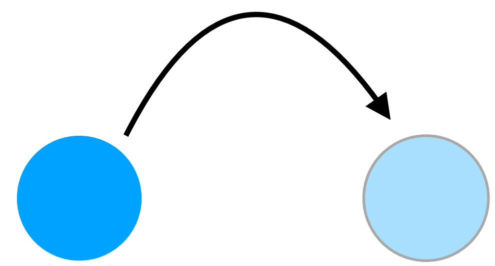
A random walk



Suppose that every second a particle either hops 1 unit forward with probability **p**, or stays where it is with probability **(1-p).**

After N seconds how far has it travelled?

A random walk

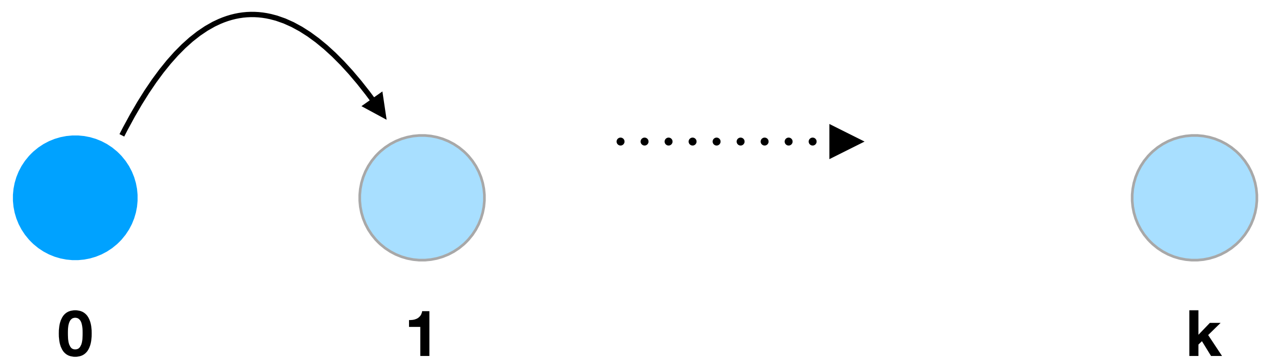


Suppose that every second a particle either hops 1 unit forward with probability **p**, or stays where it is with probability **(1-p).**

After N seconds how far has it travelled?

1. **Repeated “yes/no” event.**
2. **Constant probabilities.** → **Binomial Distribution!**
3. **Independent.**

A random walk

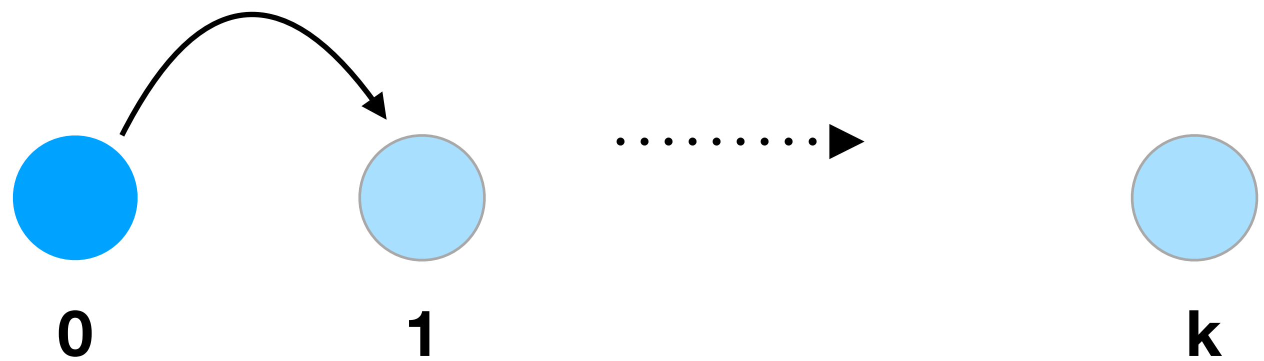


Suppose that every second a particle either hops 1 unit forward with probability **p**, or stays where it is with probability **(1-p).**

After N seconds how far has it travelled?

$$X = k \text{ with probability } \binom{N}{k} p^k (1-p)^{N-k}$$

A random walk



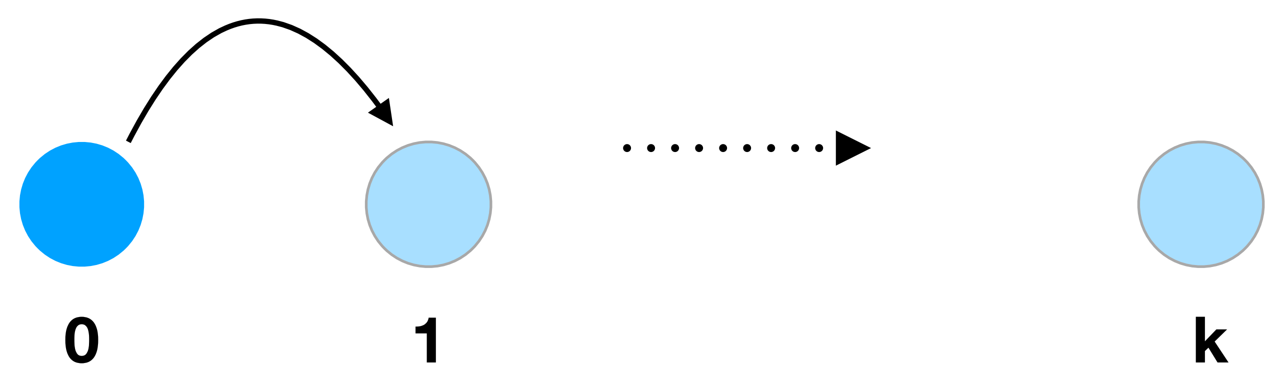
Suppose that every second a particle either hops 1 unit forward with probability **p**, or stays where it is with probability **(1-p).**

After N seconds how far has it travelled?

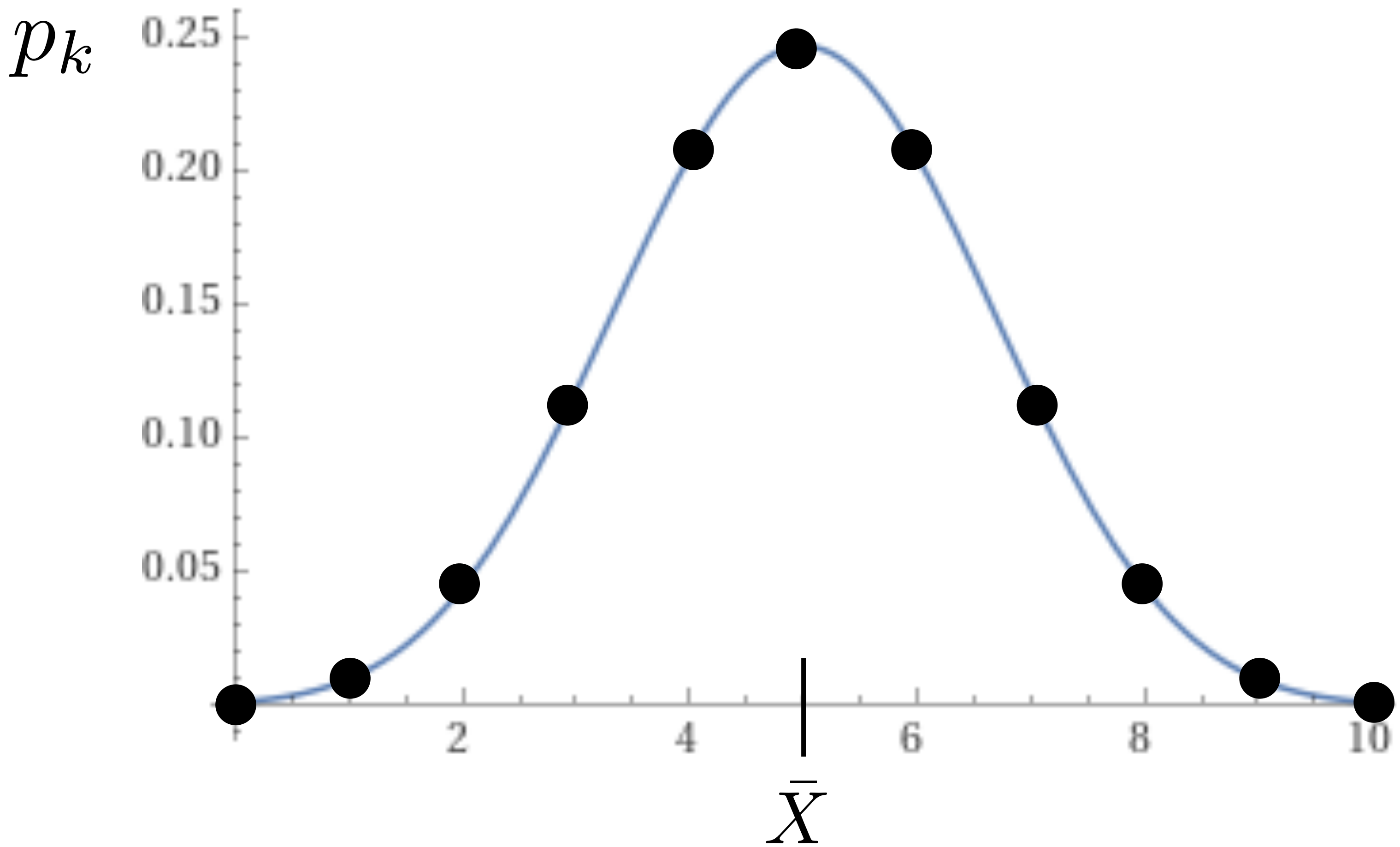
$$X = k \text{ with probability } \binom{N}{k} p^k (1-p)^{N-k}$$

X is a <u>random variable</u>
= a quantity that has an associated probability distribution.

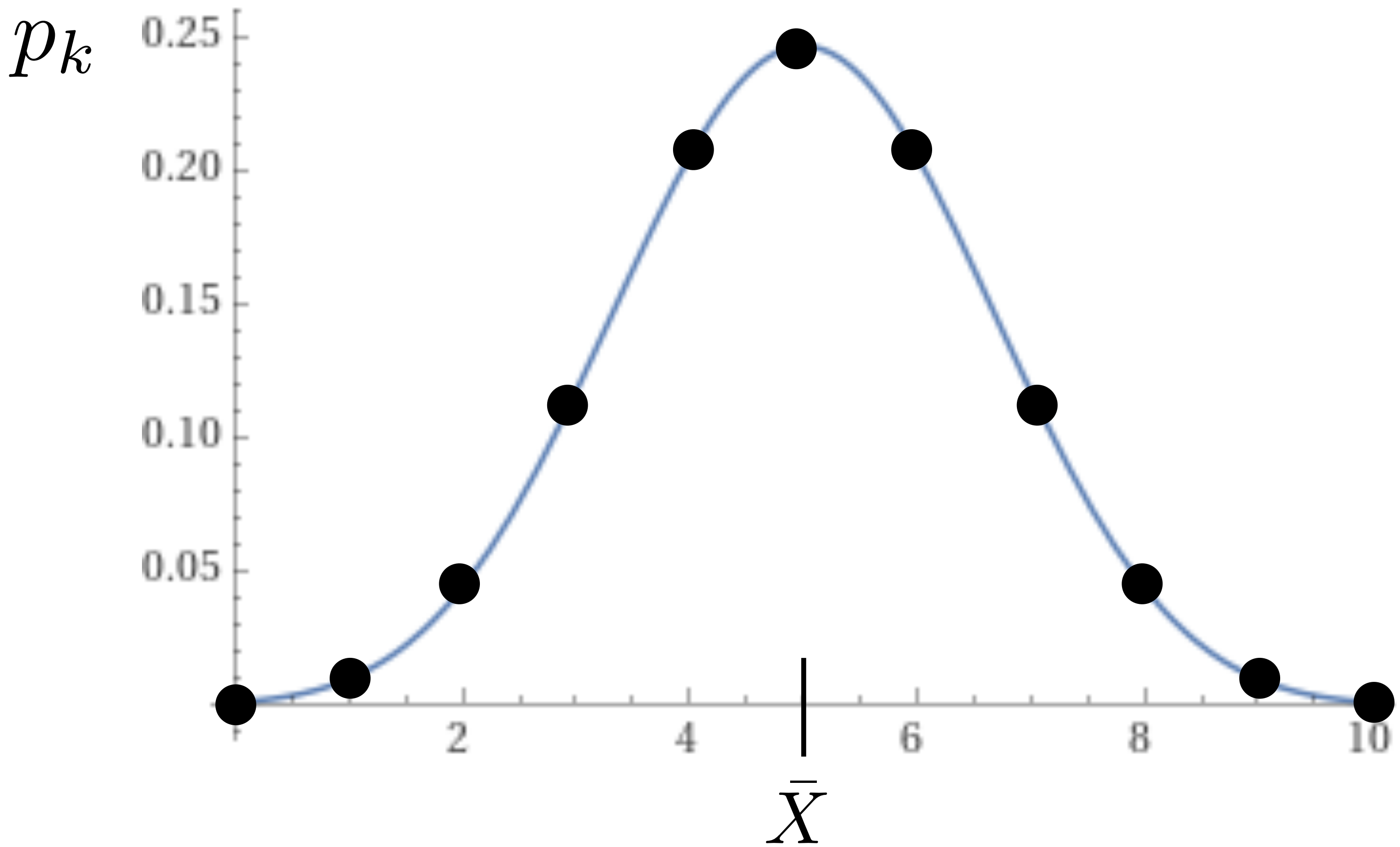
A random walk



Suppose p=1/2. What is its **average position** after N=10 seconds?
What is the **standard deviation** in position after N=10 seconds?



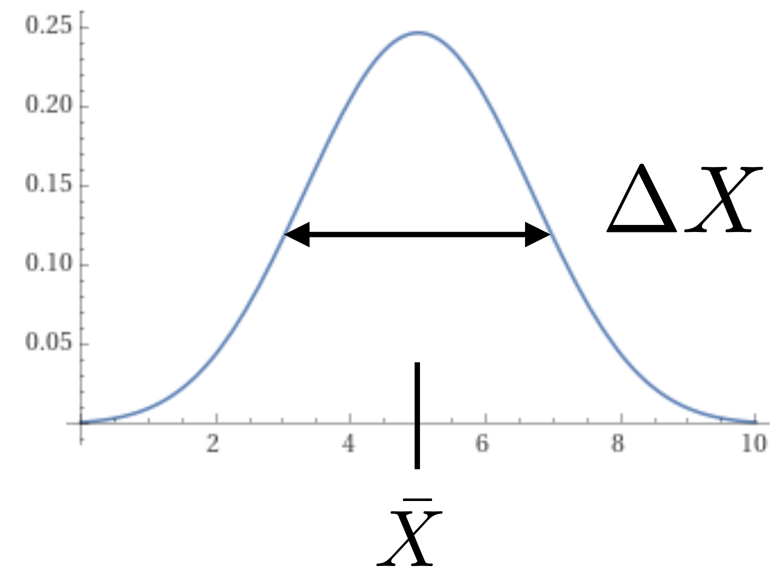
$$X = k \text{ with probability } \binom{N}{k} p^k (1-p)^{N-k}$$



$$X = k \text{ with probability } \binom{N}{k} p^k (1-p)^{N-k}$$

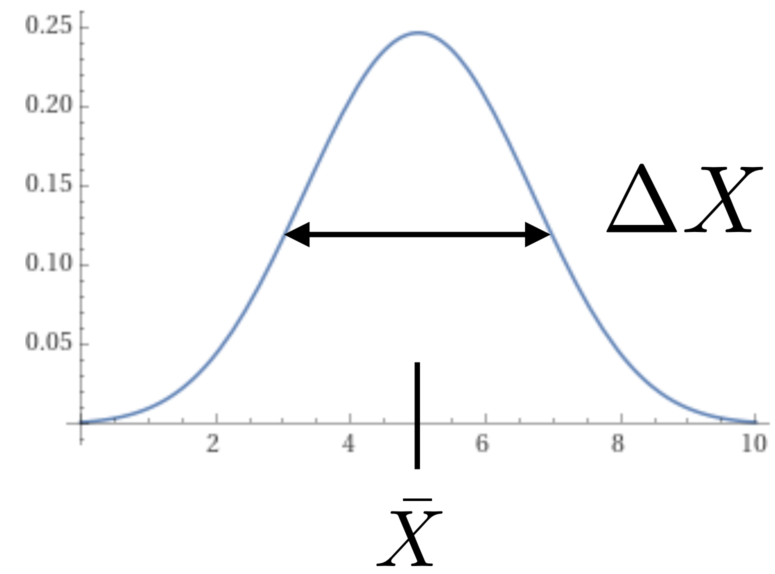
$$\text{Average: } \bar{X} = \sum_k k p_k = \sum_{k=0}^{10} k \binom{10}{k} \frac{1}{2^{10}} = 5$$

$$\Delta X^2 = \text{Average}(X^2) - \bar{X}^2$$



$$\Delta X^2 = \text{Average}(X^2) - \bar{X}^2$$

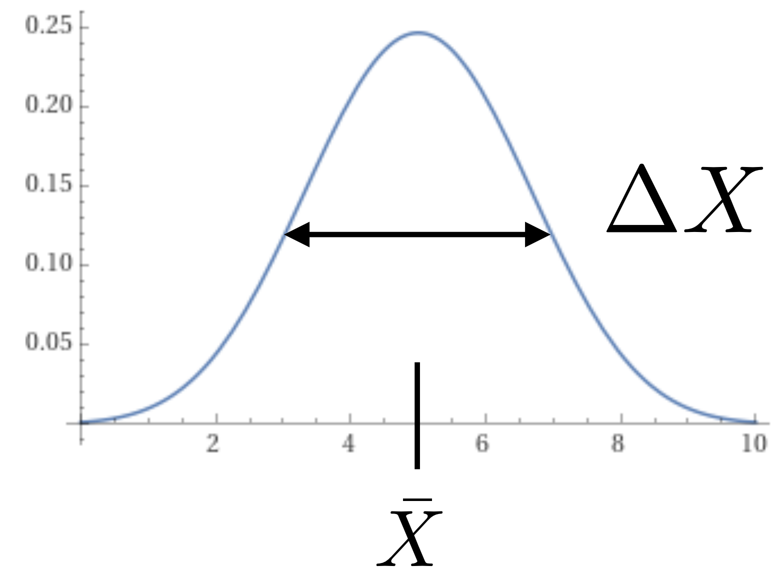
$$= \sum_k k^2 p_k - \bar{X}^2$$



$$\Delta X^2 = \text{Average}(X^2) - \bar{X}^2$$

$$= \sum_k k^2 p_k - \bar{X}^2$$

$$= \sum_{k=0}^{10} k^2 \binom{10}{k} \frac{1}{2^{10}} - 25$$



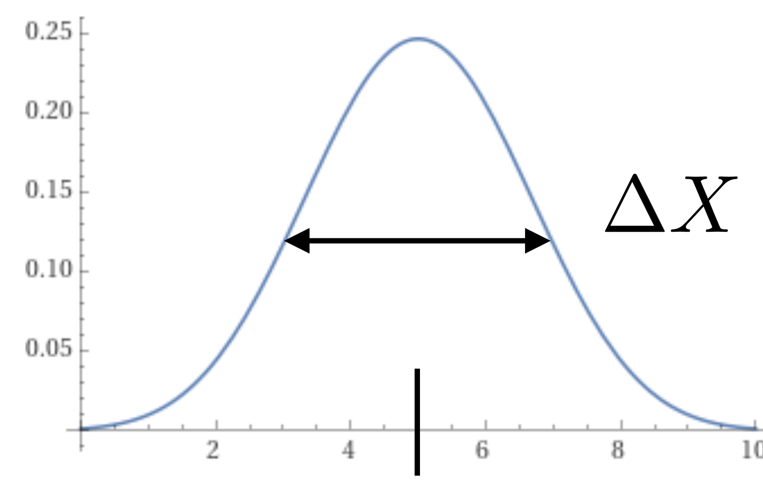
$$\Delta X^2 = \text{Average}(X^2) - \bar{X}^2$$

$$= \sum_k k^2 p_k - \bar{X}^2$$

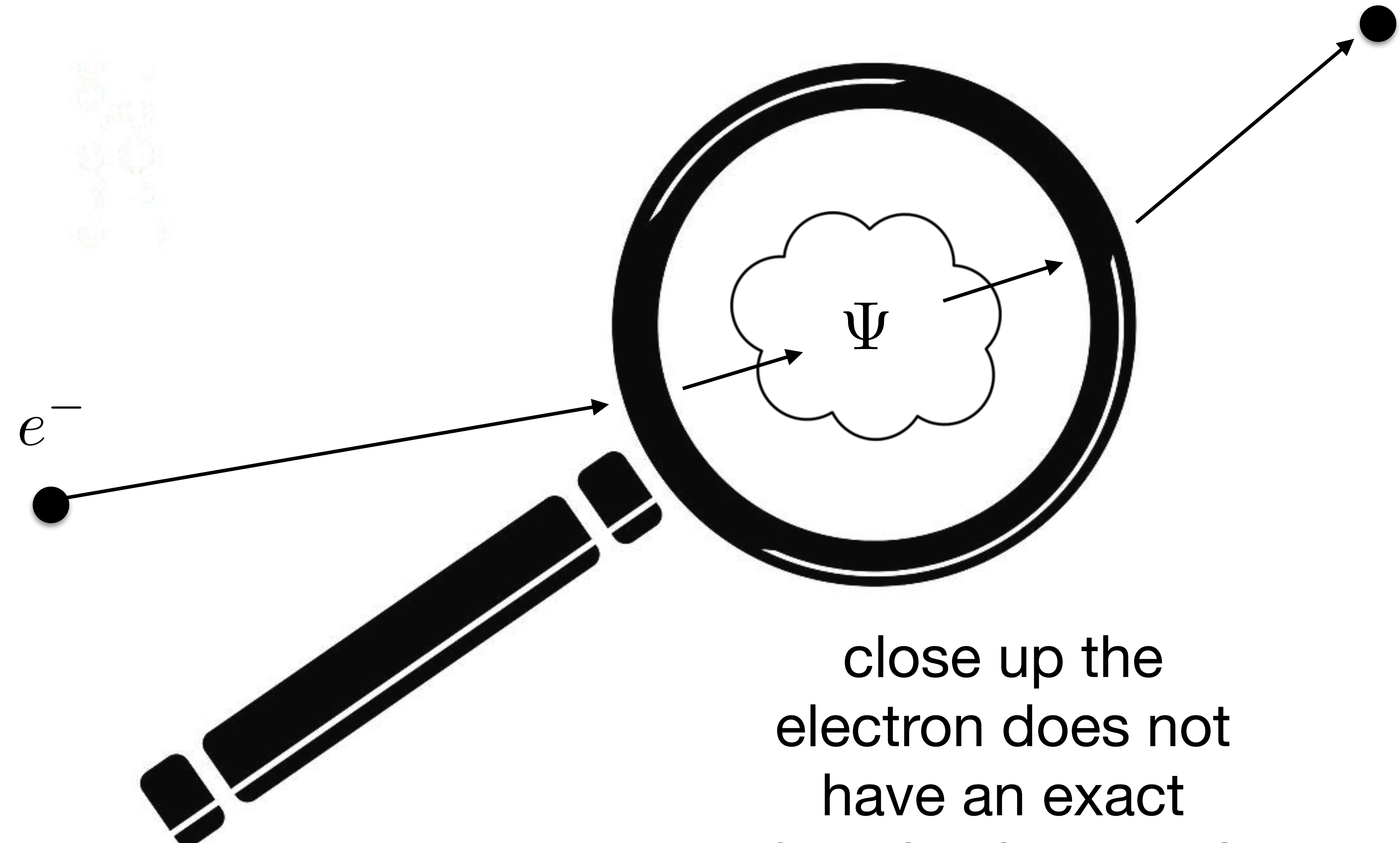
$$= \sum_{k=0}^{10} k^2 \binom{10}{k} \frac{1}{2^{10}} - 25$$

$$= 27.5 - 25$$

$$\Delta X^2 = 2.5 \ \Rightarrow \Delta X = 1.5811$$

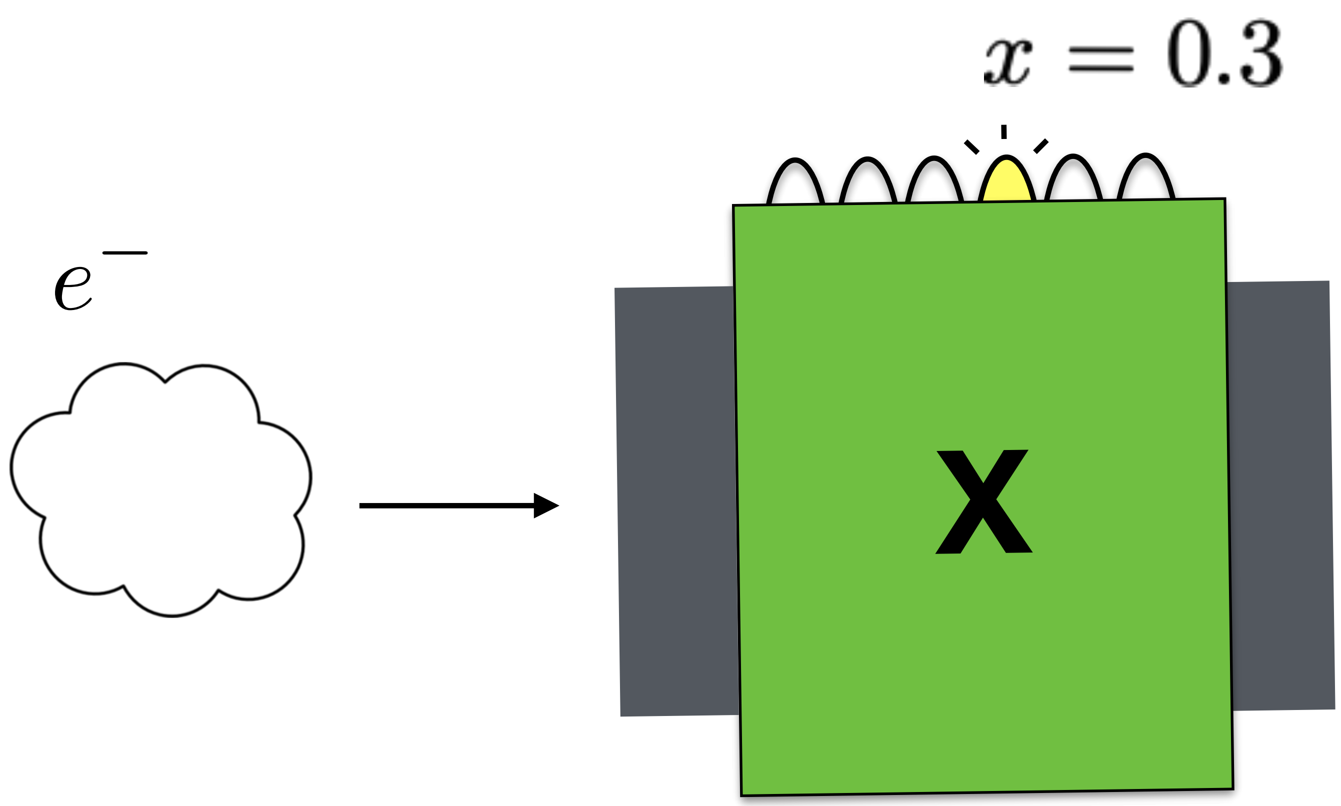


Heisenberg Uncertainty Principle

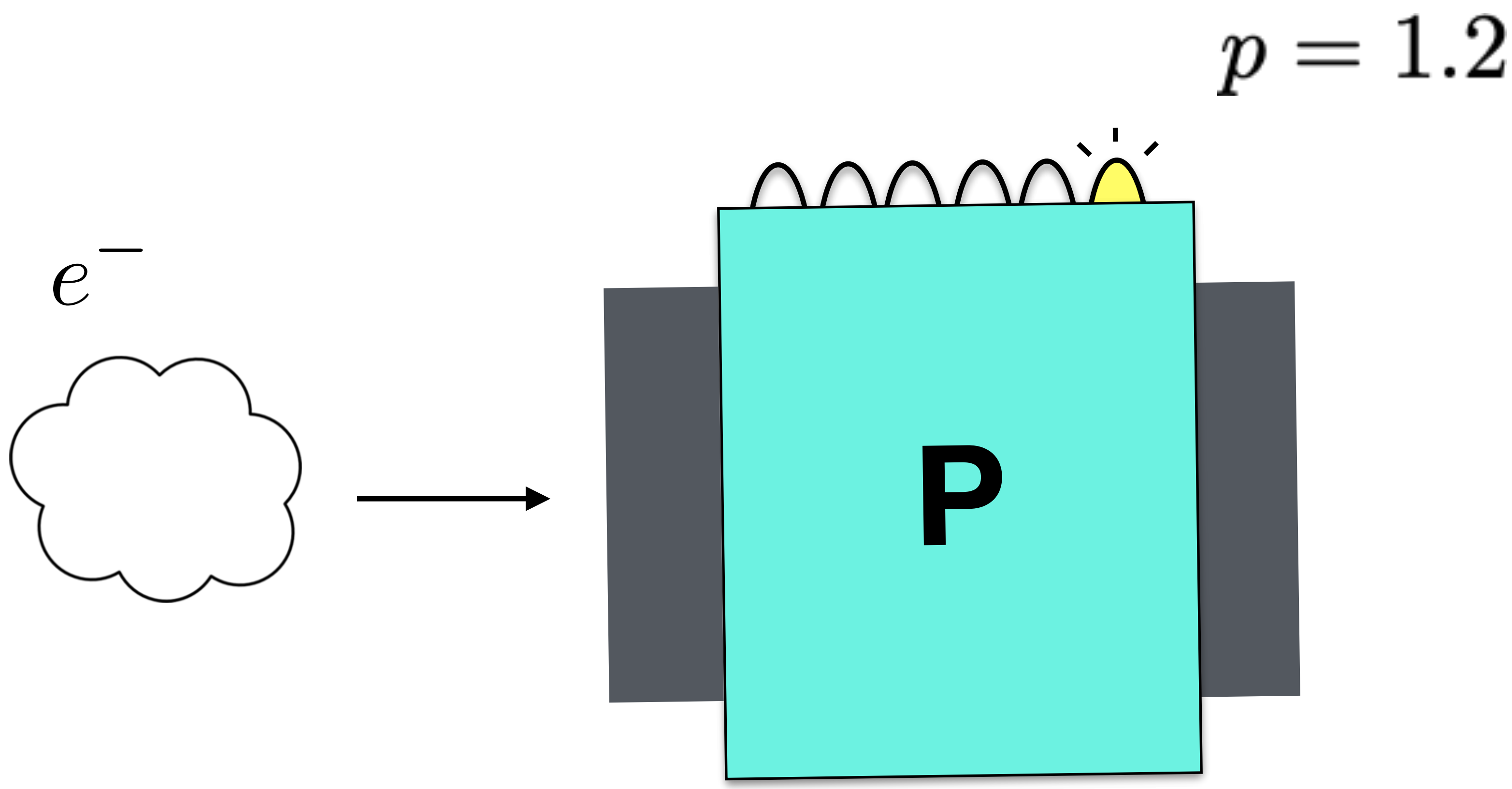


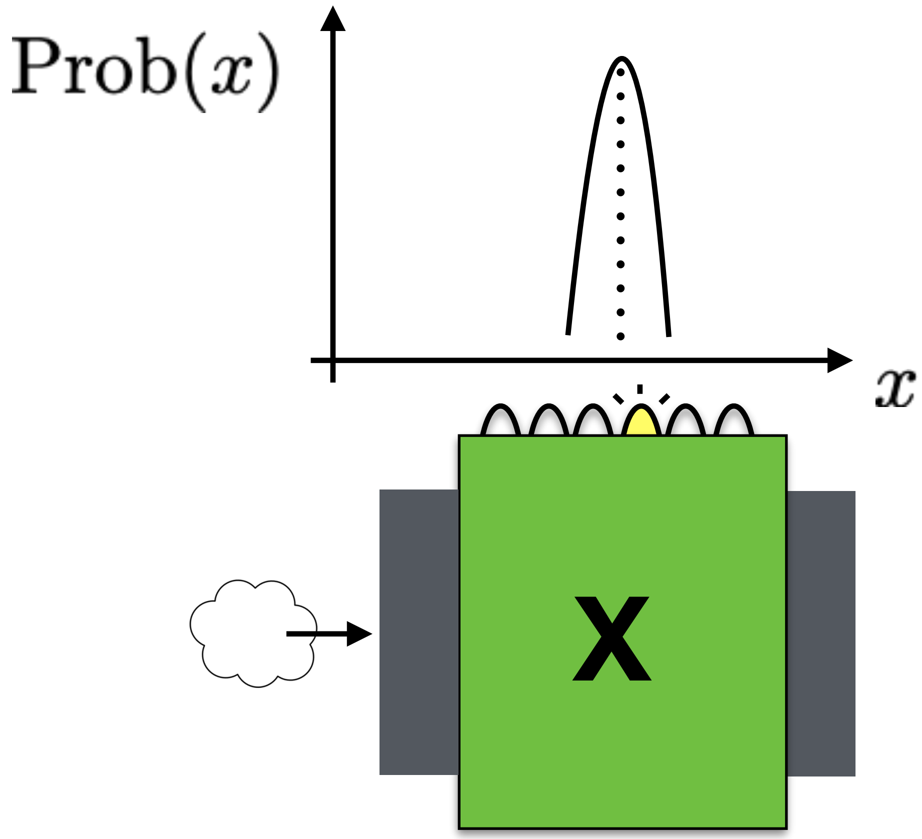
close up the
electron does not
have an exact
location in space!

A box to measure position

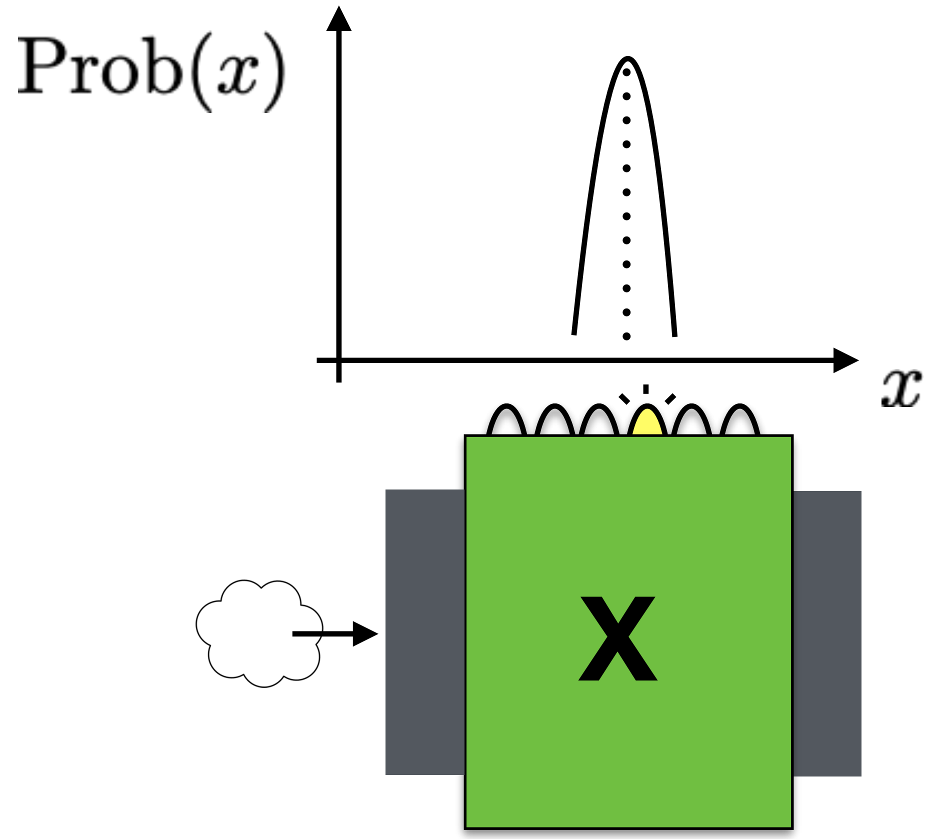


A box to measure momentum

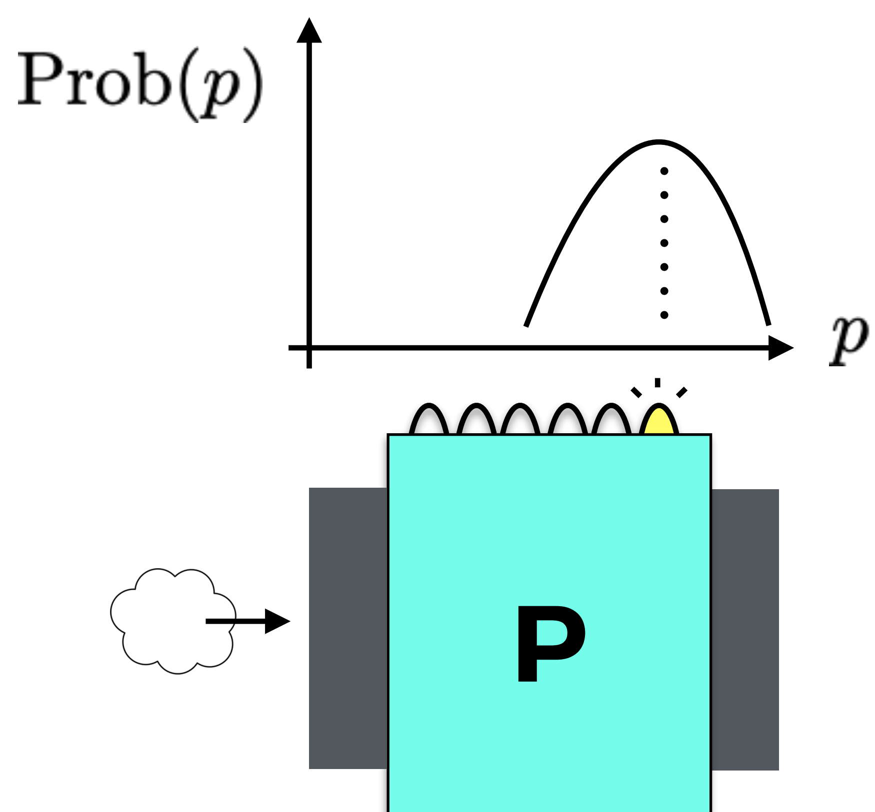




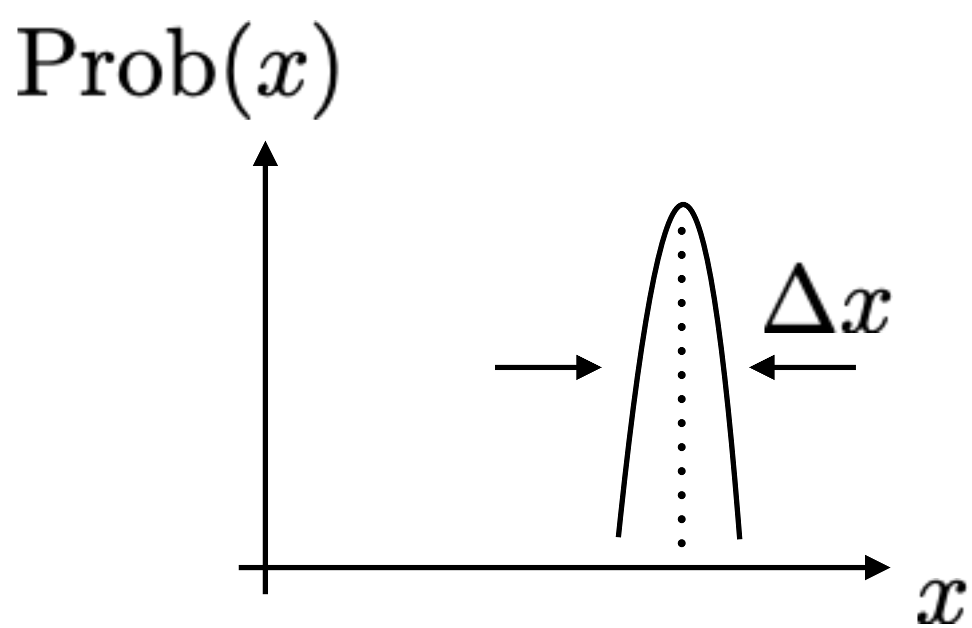
Random outcomes for position of particle



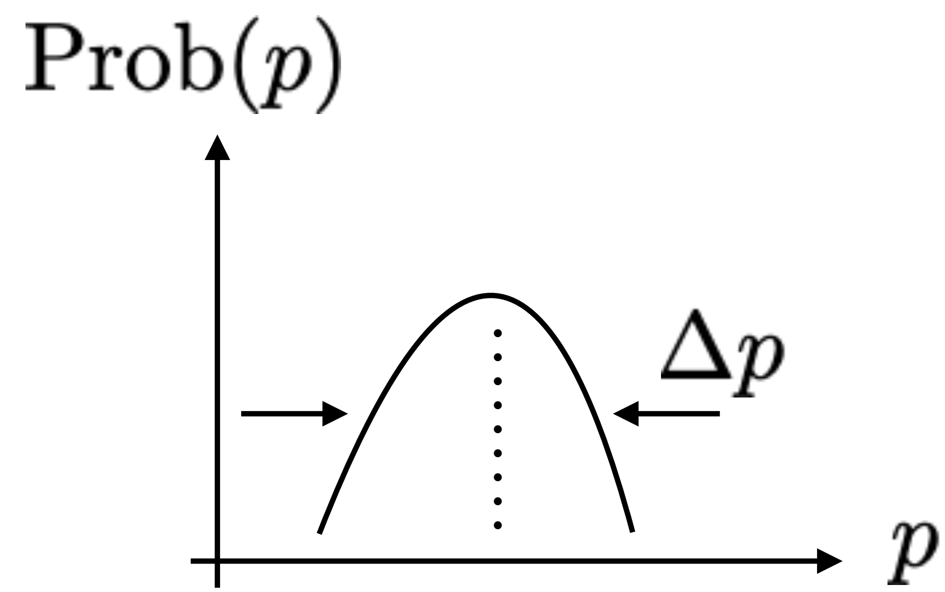
Random outcomes for position of particle



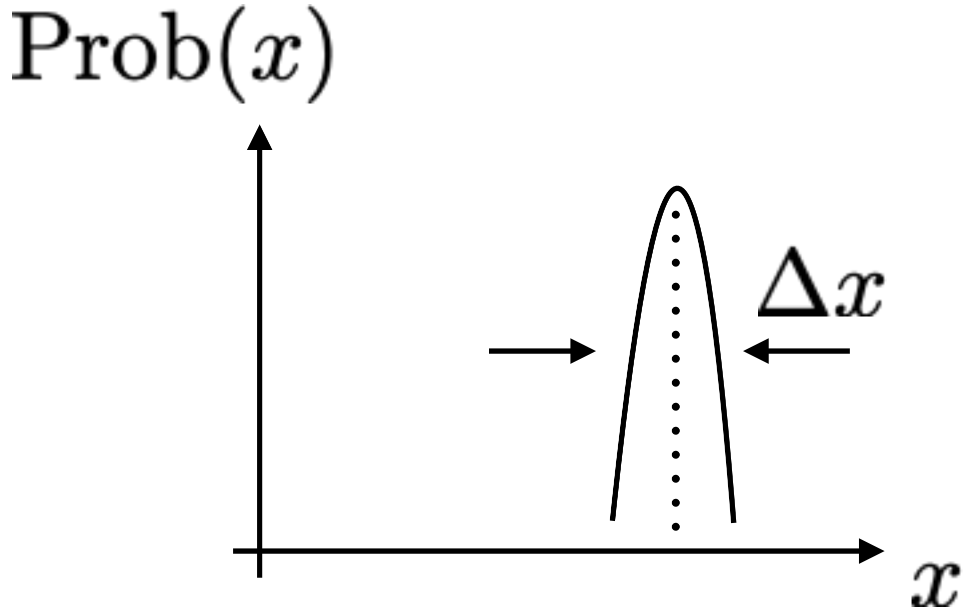
Random outcomes for momentum of particle



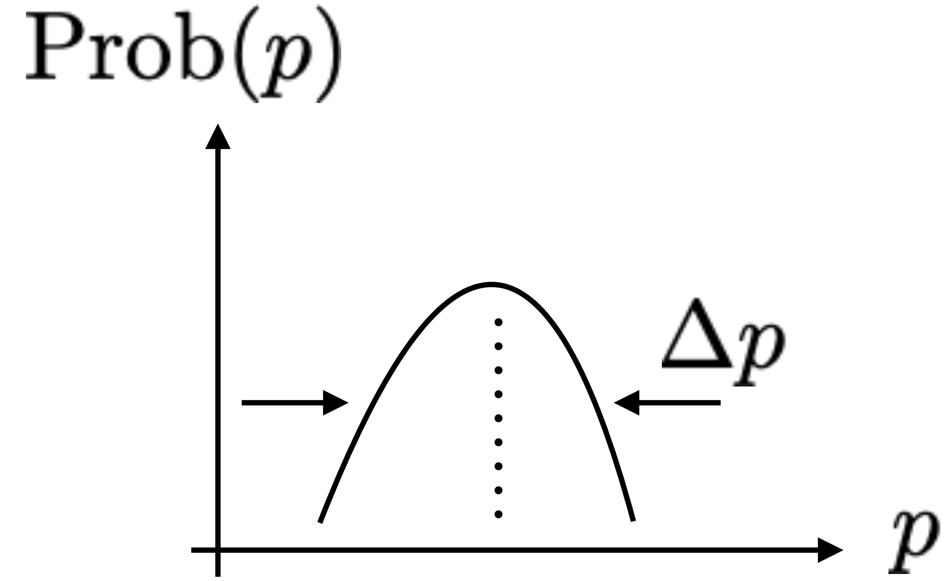
$\Delta x =$ the standard deviation of x



$\Delta p =$ the standard deviation of p



Δx : width of x distribution



Δp : width of p distribution

$$\boxed{\Delta x \Delta p \geq \frac{\hbar}{2}}$$

$$\hbar = 1.055 \times 10^{-34} (kg m^2 s^{-1})$$

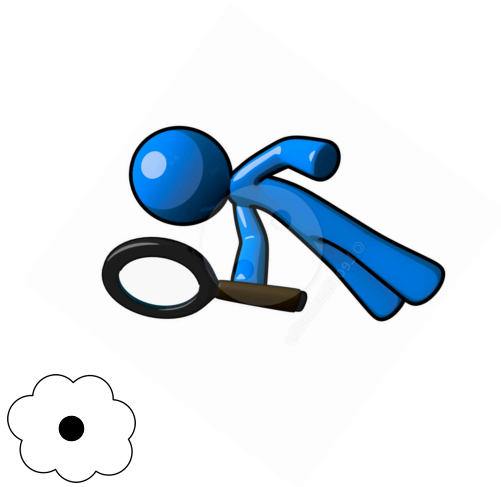
Heisenberg Uncertainty Relation

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

$\hbar = 1.055 \times 10^{-34} (kg m^2 s^{-1})$ (pronounced “h-bar”)

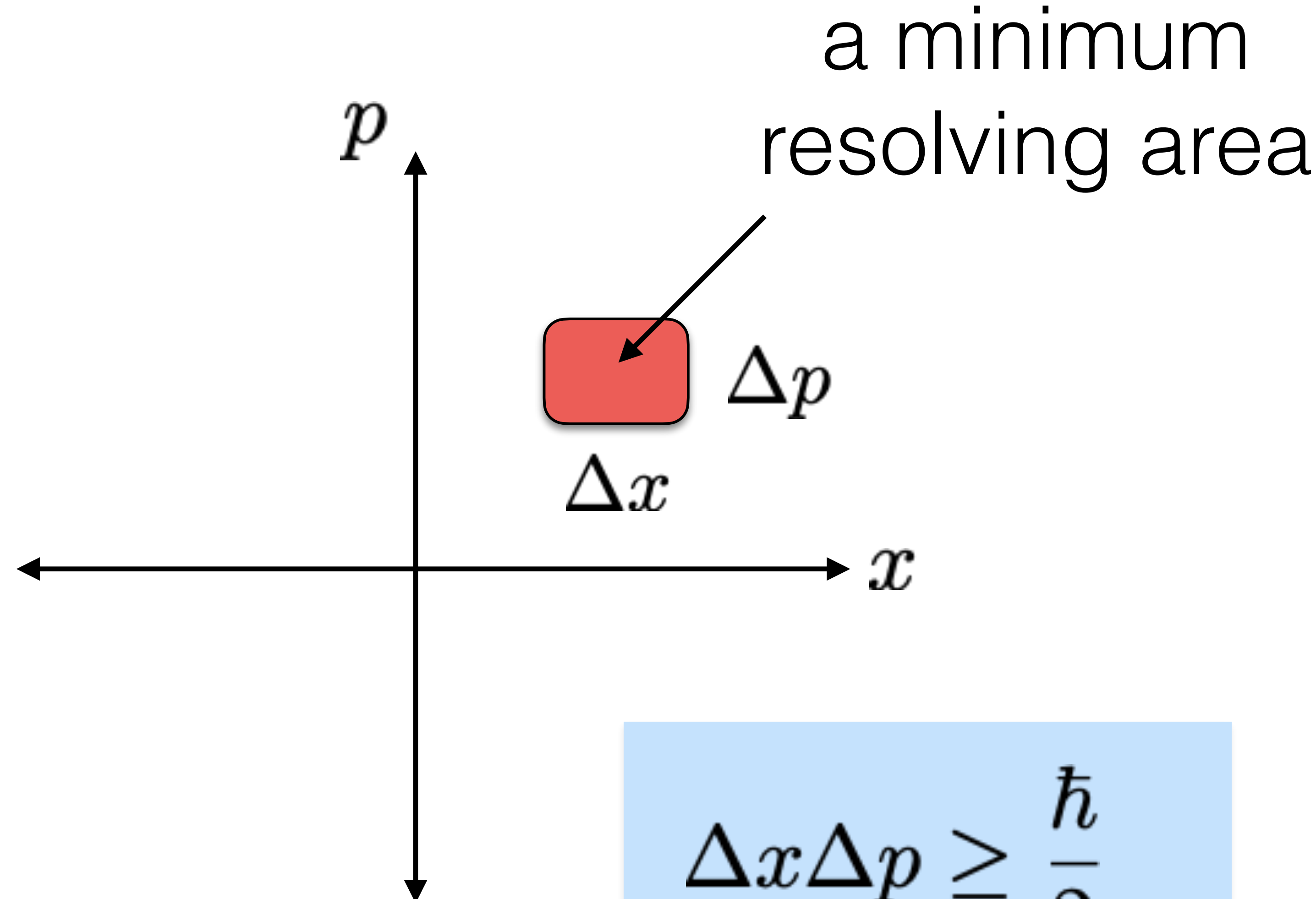
$$\hbar = \frac{h}{2\pi}$$

h is called “Planck’s constant”. This is a **fundamental constant of Nature**, like the speed of light!



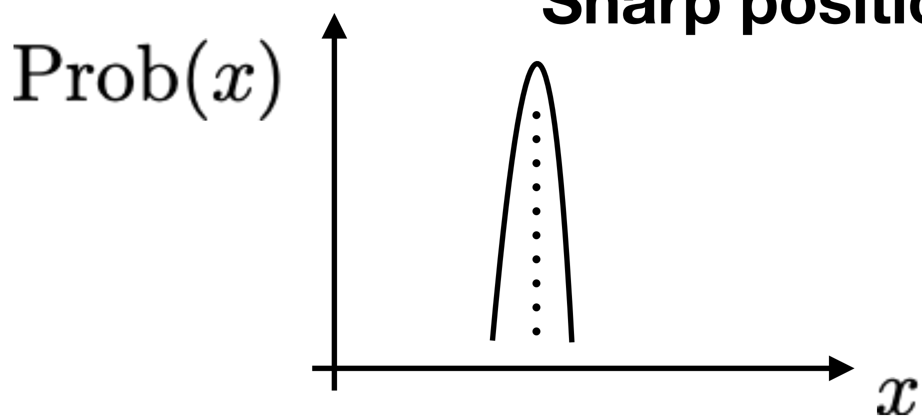
$$(x(t), p(t))$$

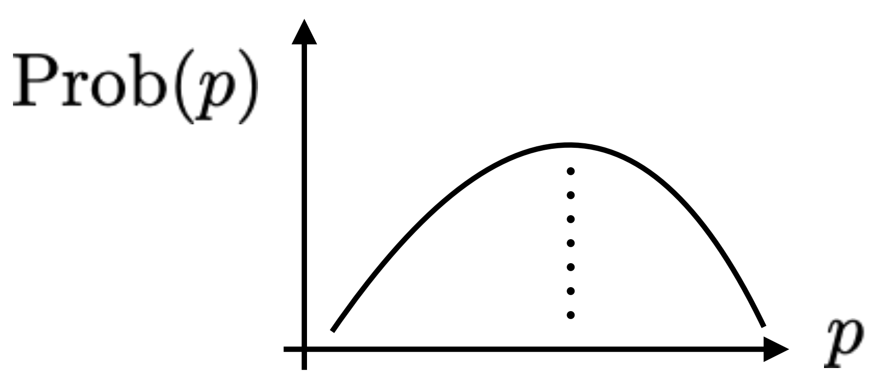
x and p cannot **both** have sharp values in quantum physics!

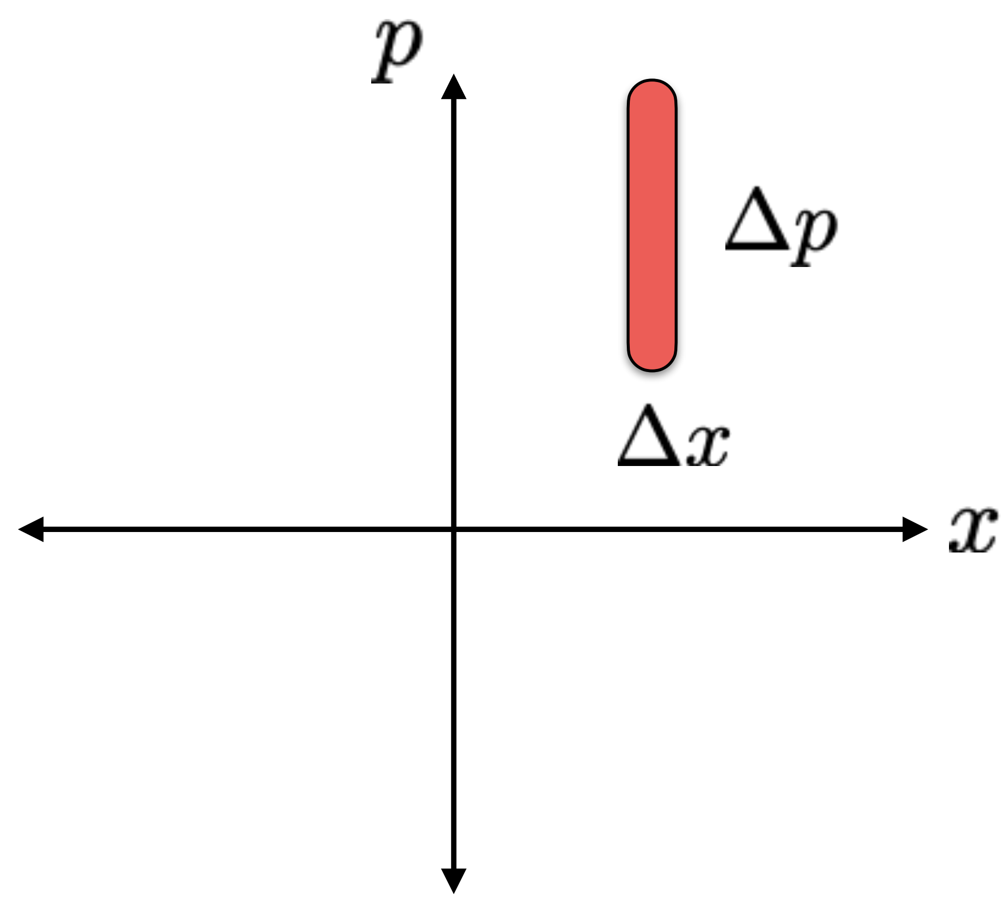


$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

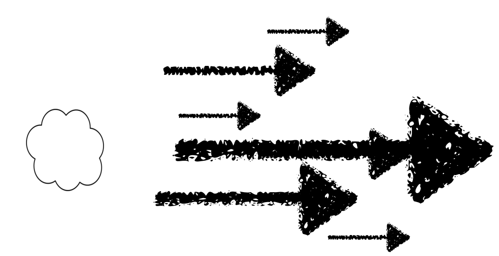
Sharp position

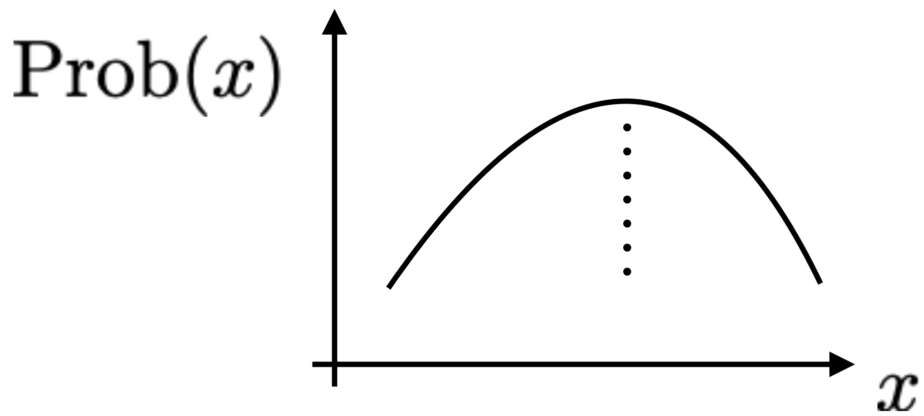


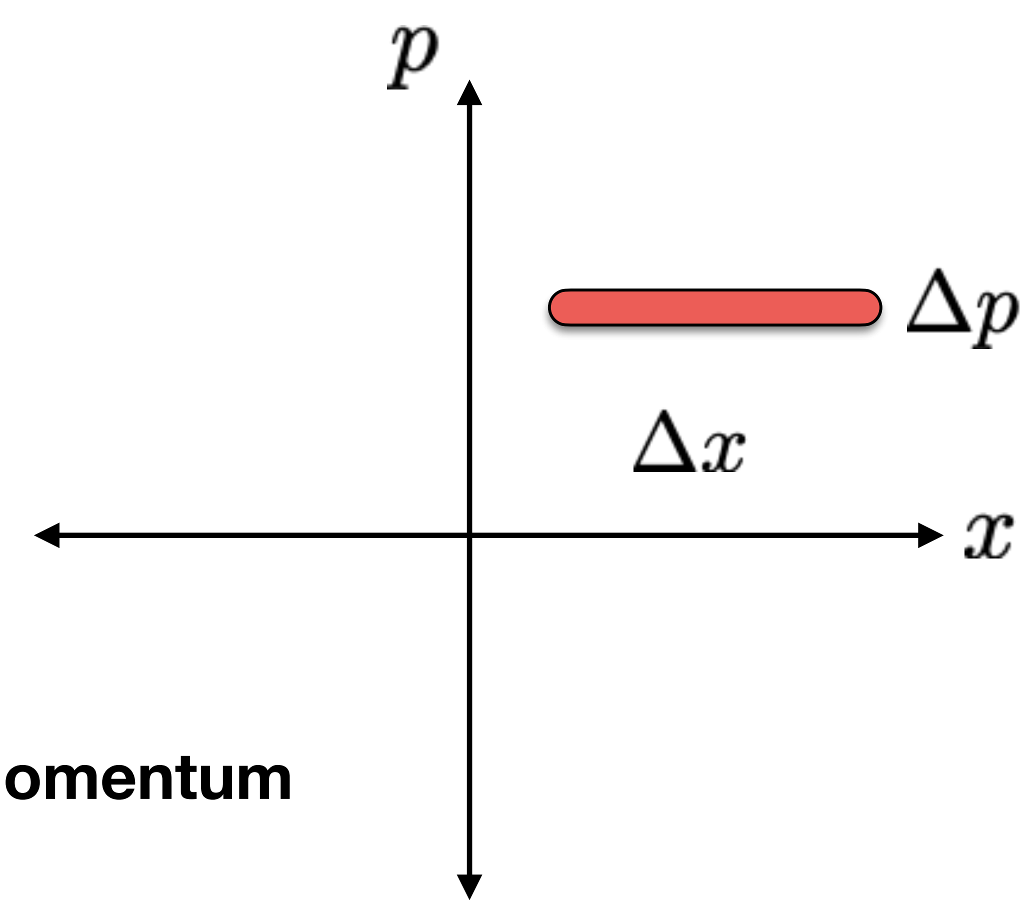


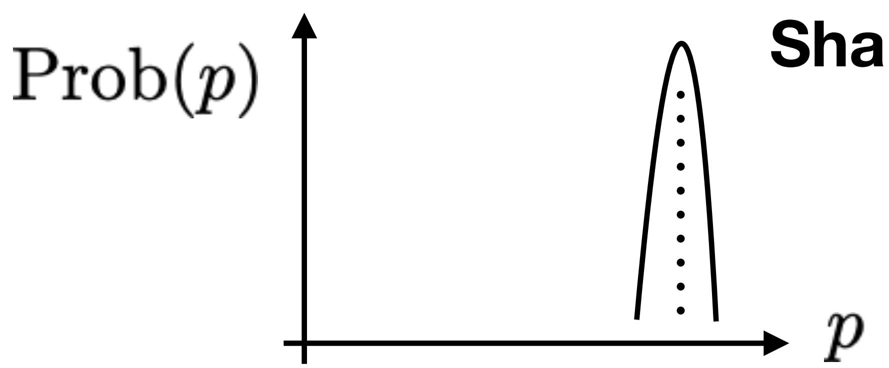


momentum very uncertain



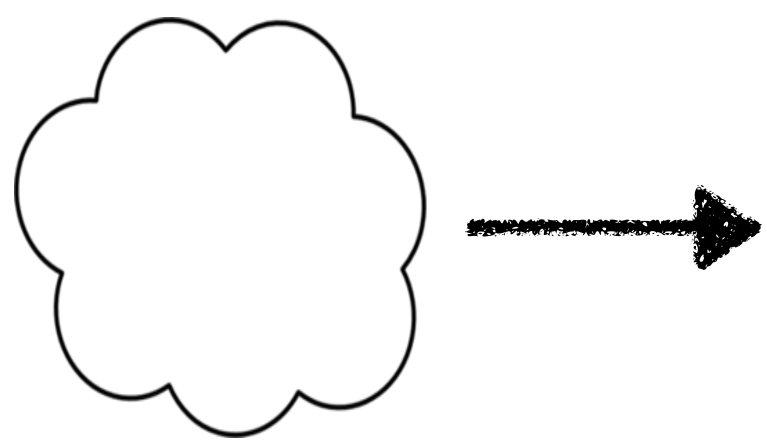


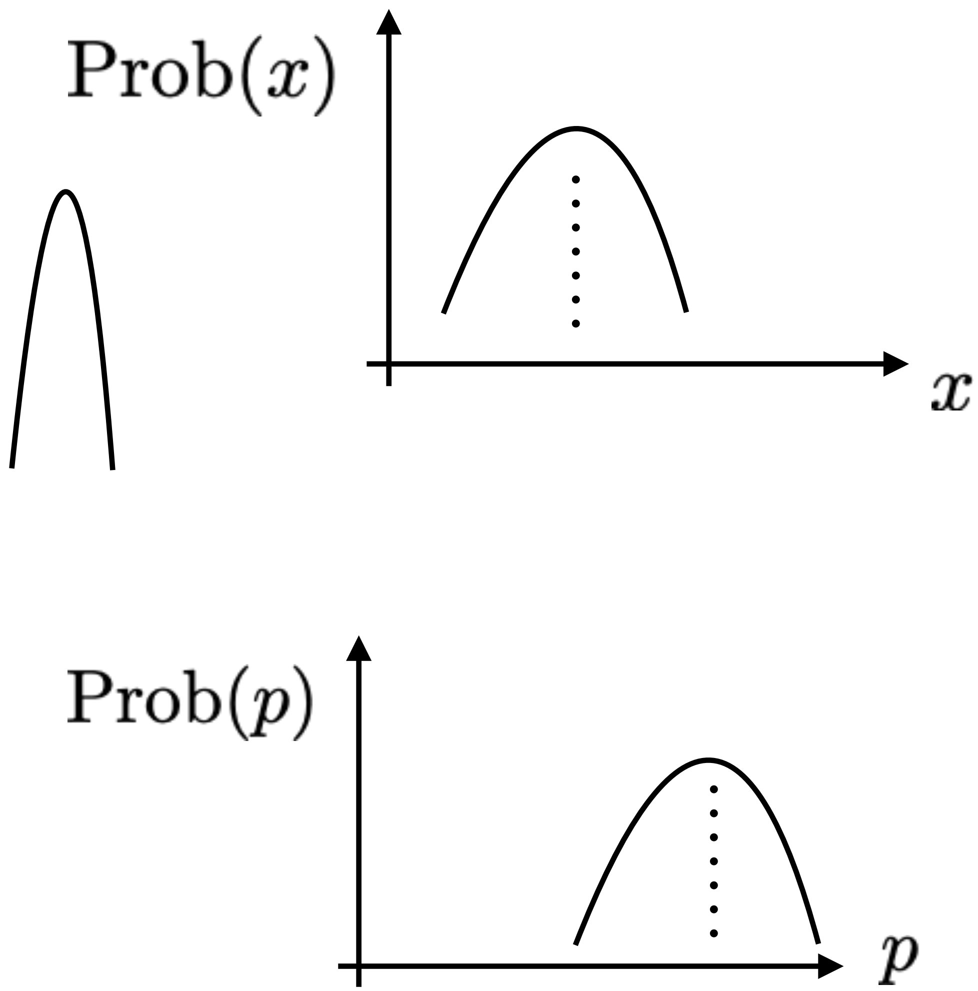


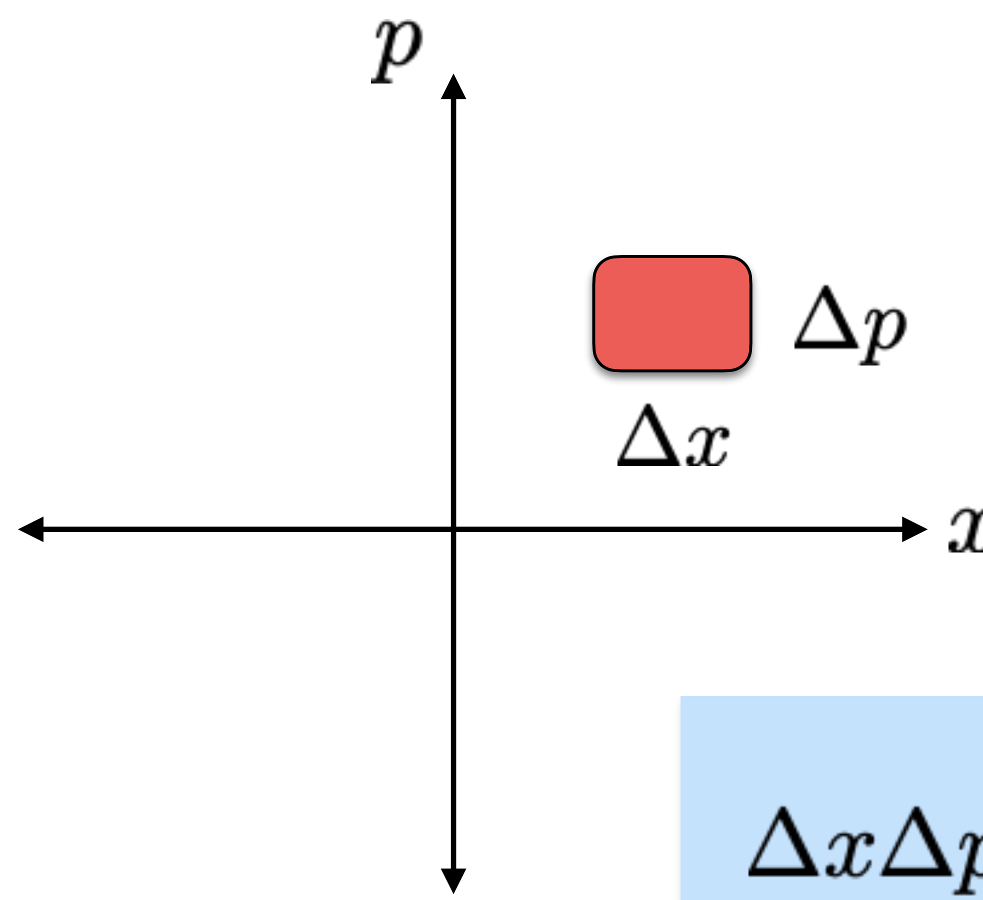


Sharp momentum

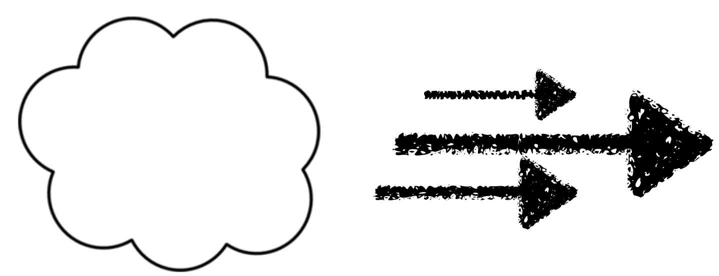
position very uncertain







$$\Delta x \Delta p \geq \frac{\hbar}{2}$$



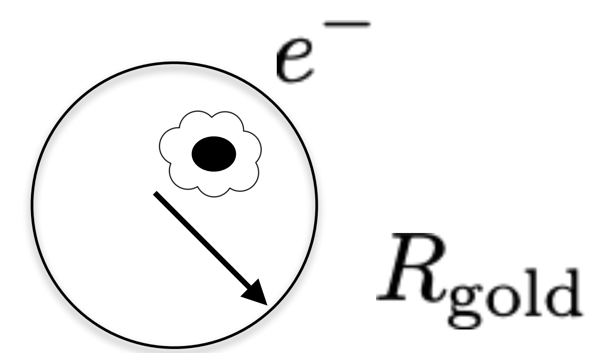
Example

Q: Suppose an electron is localised to within an atom of gold, estimate the Heisenberg uncertainty in its velocity?

$$m_e = 9.1 \times 10^{-31} kg$$

$$R_{\text{gold}} = 135 \times 10^{-10} m$$

$$\hbar = 1.055 \times 10^{-34} (kgm^2 s^{-1})$$



Example

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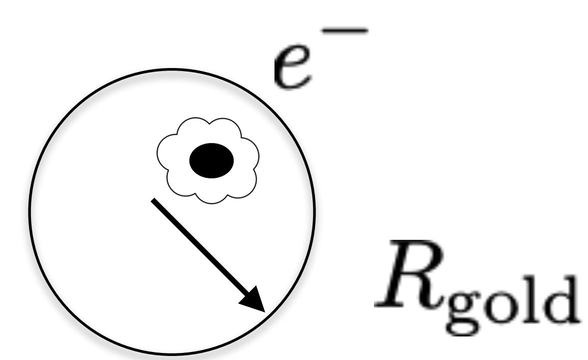
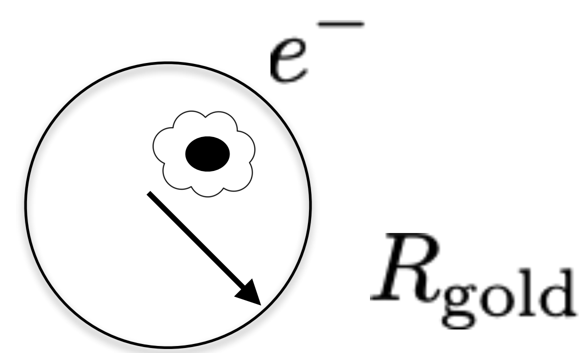
$$\Delta x = 2R_{\text{gold}} = 270 \times 10^{-10} m$$



$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

Example

Q: Suppose an electron is localised to within an atom of gold, estimate the Heisenberg uncertainty in its velocity?



$$m_e = 9.1 \times 10^{-31} kg$$

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$$\Delta x = 2R_{\text{gold}} = 270 \times 10^{-10} m$$

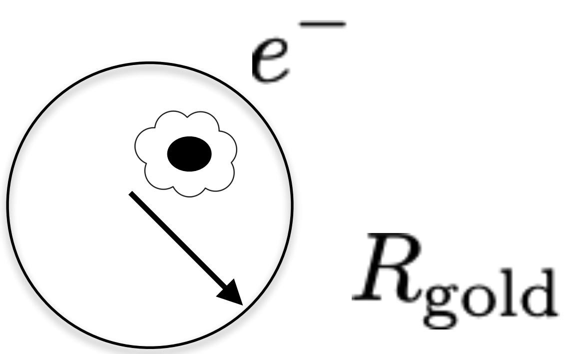
$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

$$\Rightarrow \Delta p \geq \frac{\hbar}{2\Delta x}$$

Example

Q: Suppose an electron is localised to within an atom of gold, estimate the Heisenberg uncertainty in its velocity?

$$\Rightarrow \Delta p \geq \frac{\hbar}{2\Delta x}$$



$$\Rightarrow \Delta p \geq \frac{\hbar}{4R_{\text{gold}}} = 1.95 \times 10^{-27} kgms^{-1}$$

$$\Delta p = m_e \Delta v \quad \Rightarrow \Delta v \geq \frac{1.95 \times 10^{-27}}{(9.1 \times 10^{-31})} = 2,143 ms^{-1}$$

Large uncertainty!

Example

Q: Estimate the Heisenberg uncertainty in velocity for a **neutron** inside a gold atom.

Same calculation, but now the mass is different.

$$m_n = 1.67 \times 10^{-27} kg$$

$$\Rightarrow \Delta v \geq \frac{1.95 \times 10^{-27}}{(1.67 \times 10^{-27})} = 1.17 ms^{-1}$$

$$\Delta v_e \geq 2,143 ms^{-1}$$

$$\Delta v_n \geq 1.17 ms^{-1}$$

Electrons are "more quantum" than neutrons in gold.

Example

Q: How much can we localise a proton before relativistic effects come into play?

Example

Q: How much can we localise a proton before relativistic effects come into play?

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

$$m_p = 1.67 \times 10^{-27} kg$$

$$\Rightarrow \Delta v_p \geq \frac{\hbar}{2 m_p \Delta x}$$

$$\Rightarrow \Delta v_p \geq \frac{3.17 \times 10^{-8}}{\Delta x}$$

Making Δx **smaller**,
means **bigger** velocity uncertainty

Example

Q: How much can we localise a proton before relativistic effects come into play?

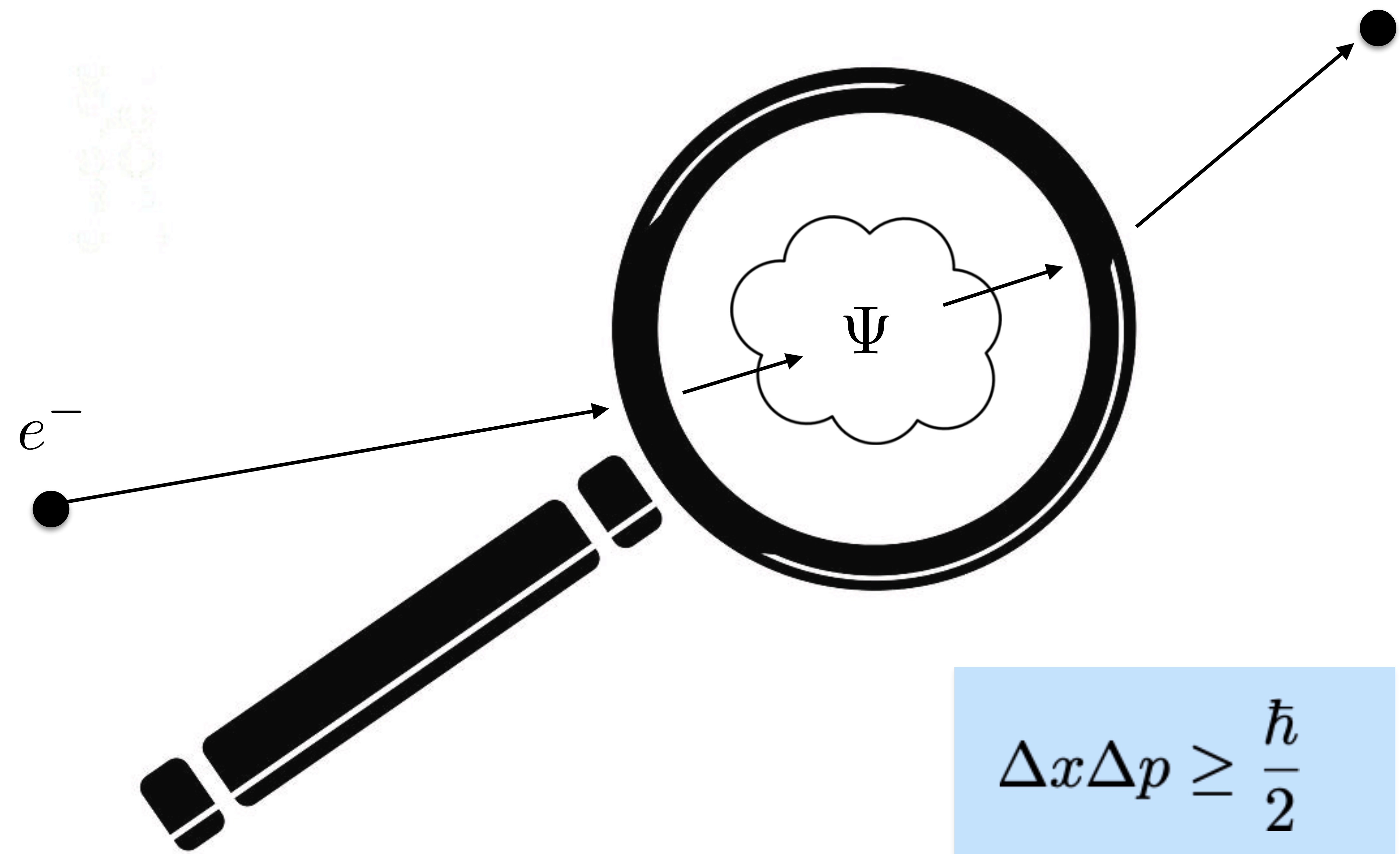
$$\Rightarrow \Delta v_p \geq \frac{3.17 \times 10^{-8}}{\Delta x}$$

$$\Delta v_p \approx c = 3 \times 10^8 ms^{-1}$$

about 1/10 of an atomic nucleus!

$$\Delta x \geq \frac{3.17 \times 10^{-8}}{3 \times 10^8} \Rightarrow \Delta x \geq 1.06 \times 10^{-16} m$$

Heisenberg Uncertainty Relation



$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

A common (incorrect) account of the Heisenberg Uncertainty Relation

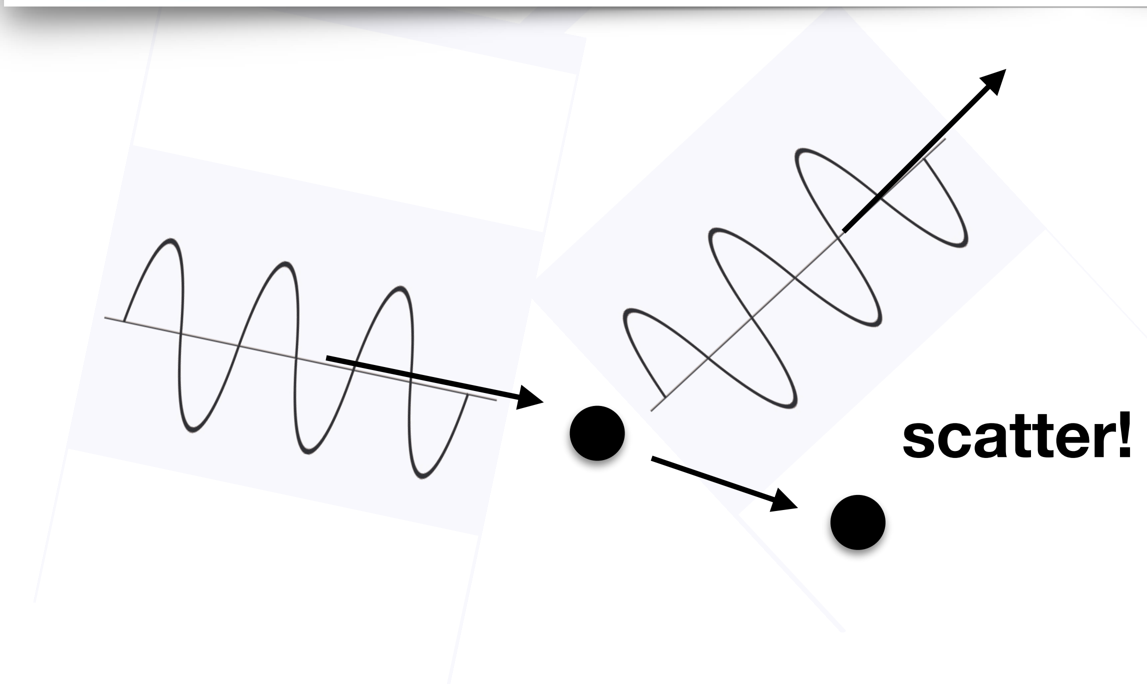
Tipler Chapter 34

The Uncertainty Principle

An important principle consistent with the wave–particle duality of nature is the uncertainty principle. It states that, in principle, it is impossible to simultaneously measure both the position and the momentum of a particle with unlimited precision. A common way to measure the position of an object is to look at the object with light. If we do this, we scatter light from the object and determine the position by the direction of the scattered light. If we use light of wavelength λ, we can measure the position x only to an uncertainty Δx of the order of λ because of diffraction effects.

$$\Delta x \sim \lambda$$

To reduce the uncertainty in position, we therefore use light of very short wavelength, perhaps even X rays. In principle, there is no limit to the accuracy of such a position measurement, because there is no limit on how small the wavelength λ can be.



(a) Heisenberg Uncertainty Relation

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

Describes the separate x & p statistics for any **quantum state**

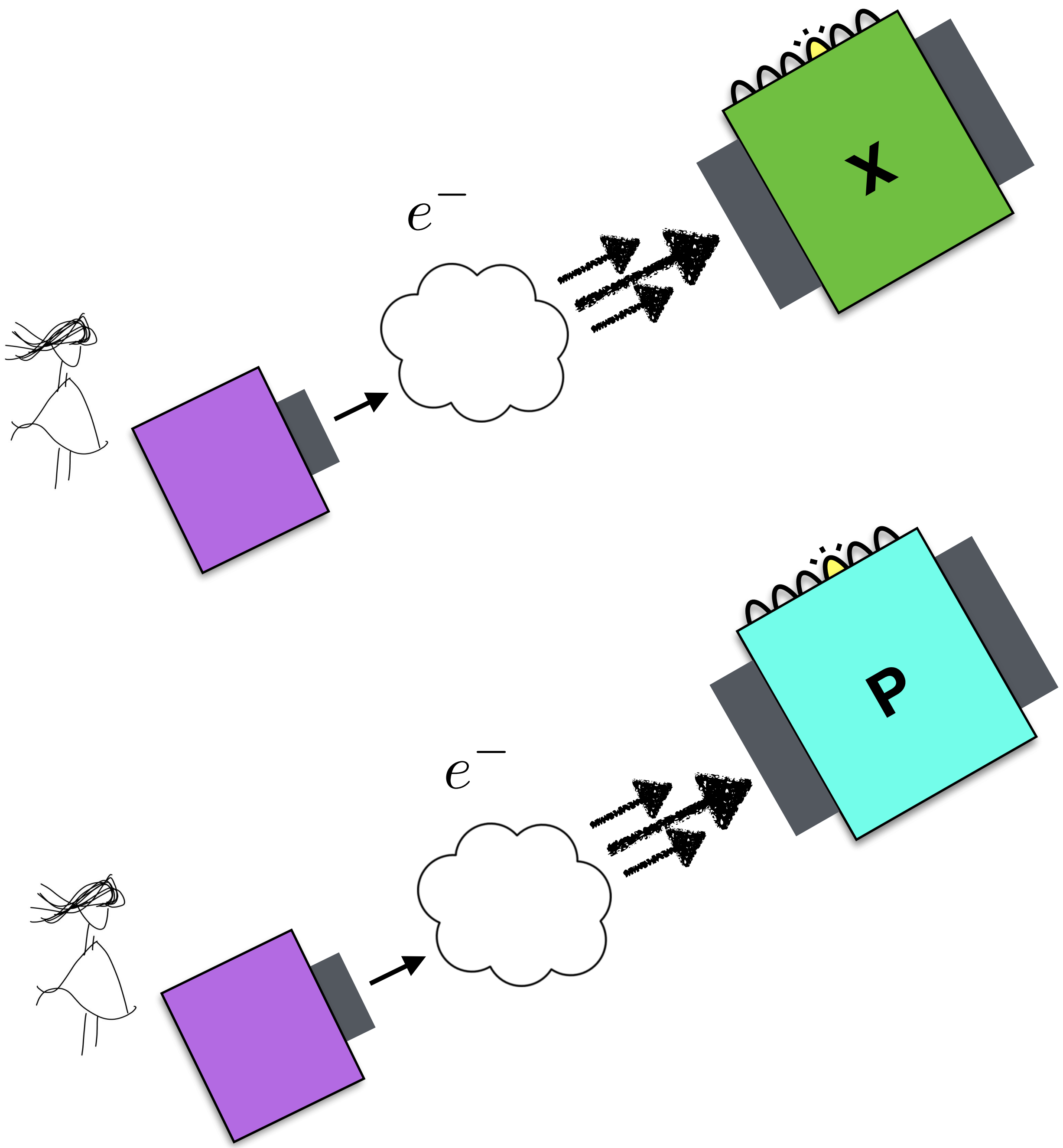
(b) Measurement Disturbance

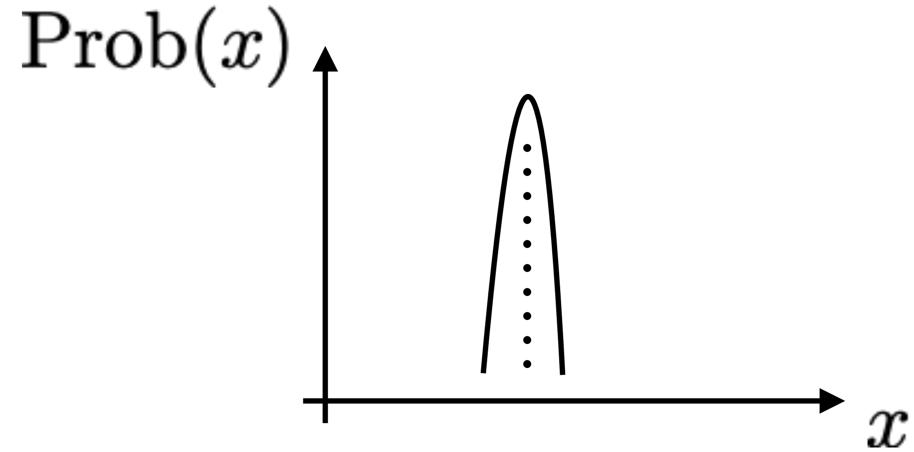
Measuring a quantum system affects the state of the system.

Tipler and others confuse (a) and (b), which is <u>wrong.</u>

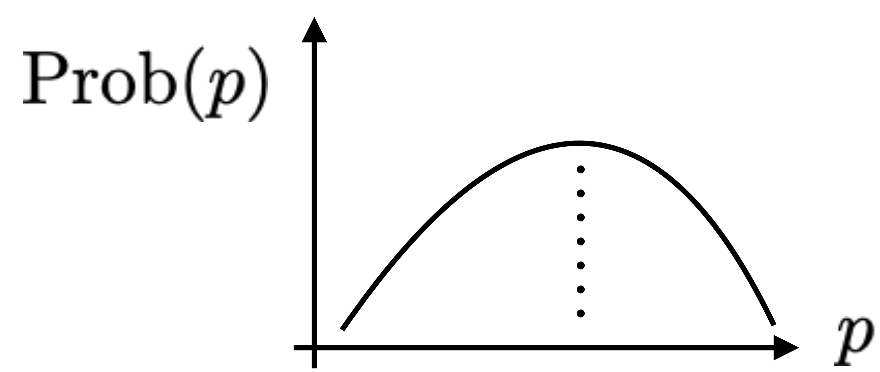
(a) Heisenberg Uncertainty Relation

Run **two separate experiments**, measure x in first and p in second



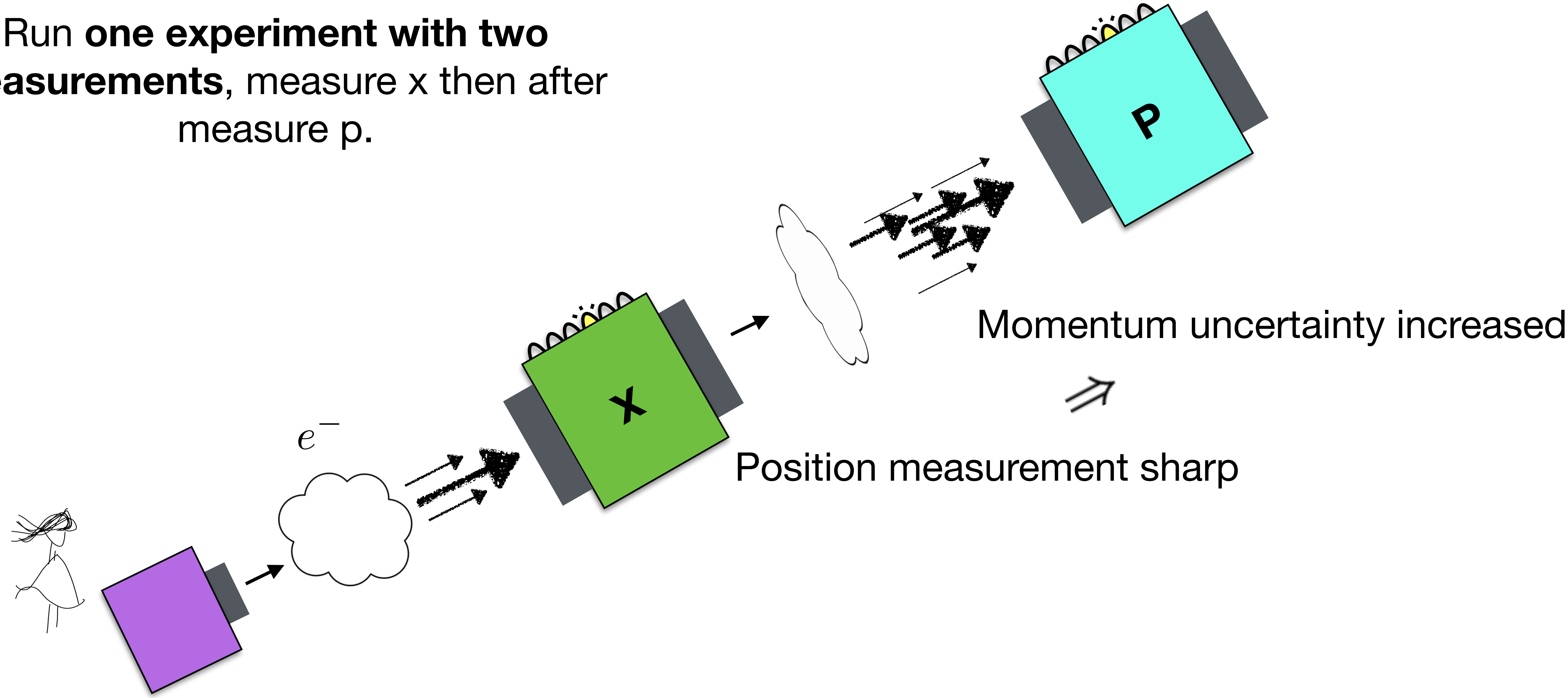


$$\Delta x \Delta p \geq \frac{\hbar}{2}$$



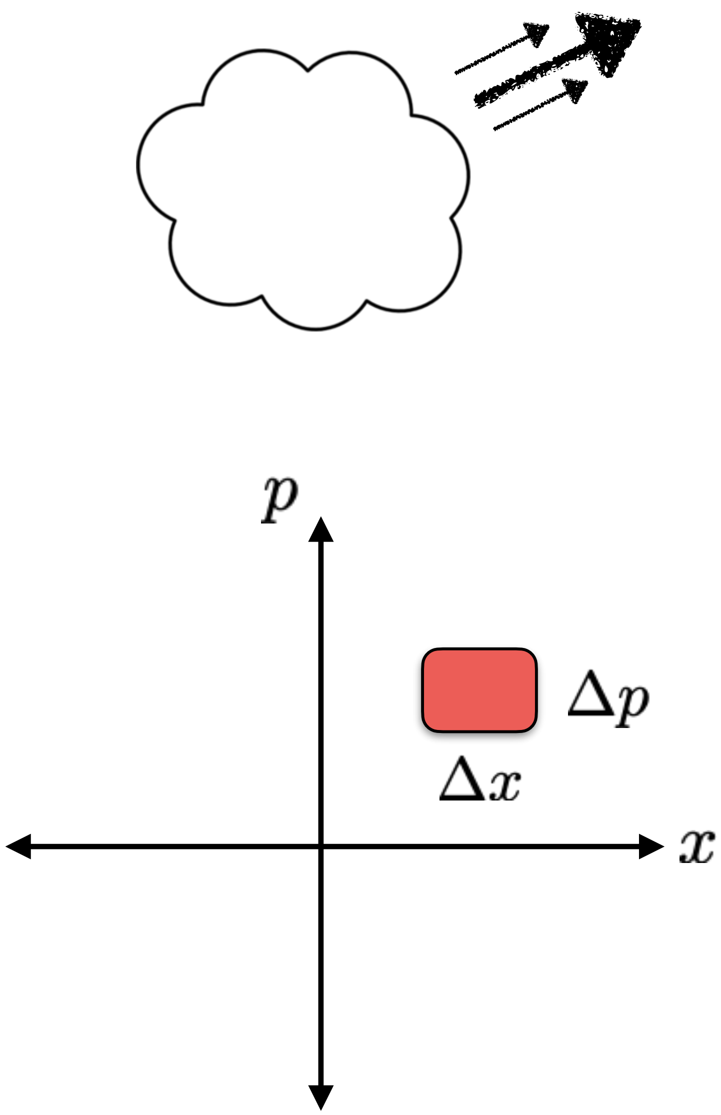
(b) Measurement Disturbance

Run **one experiment with two measurements**, measure x then after measure p.

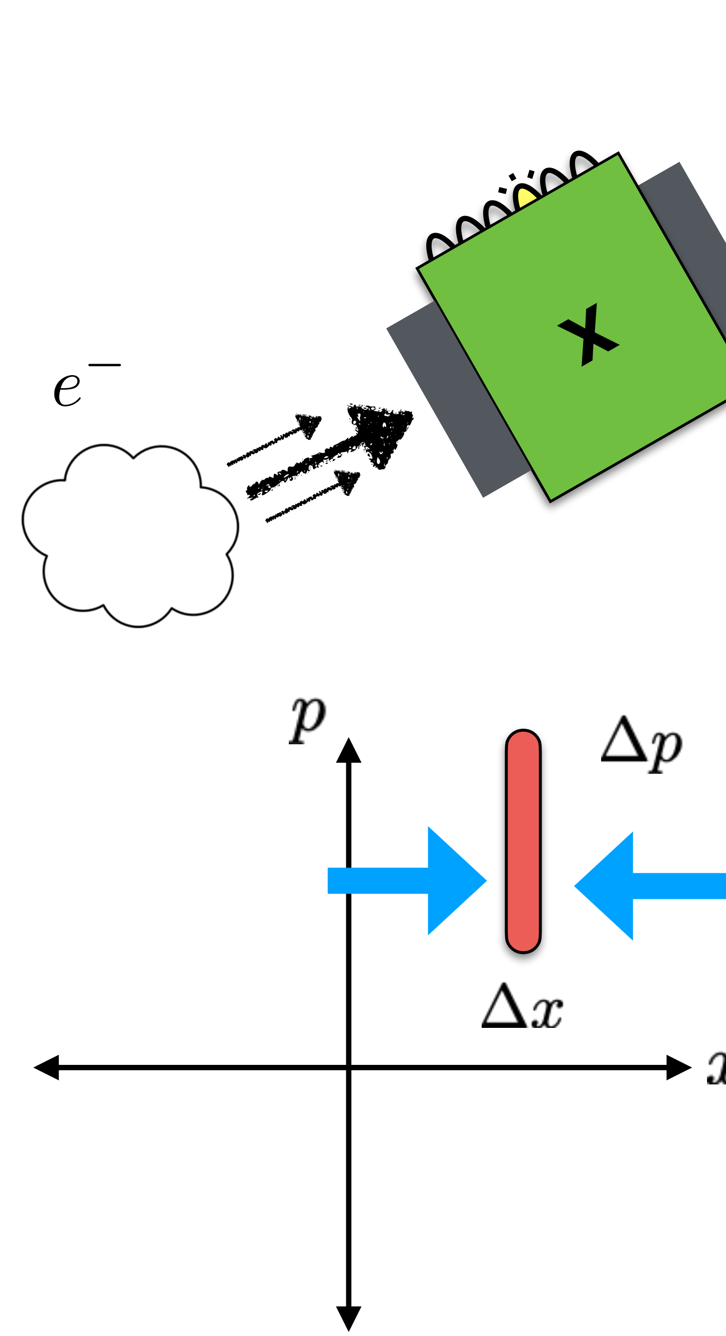


Scenarios (a) and (b) are not the same!

Heisenberg Uncertainty



Measurement Disturbance



Heisenberg

Not Heisenberg