PHAS1040 WAVES

Part 1: Vibrations and non-electromagnetic waves (strings, water waves, sound waves)

WEEKS 7-11 Semester 1

Dr S.D. Connell

1.12e Bragg Building

s.d.a.connell@leeds.ac.uk

Part 2: Optics

WEEKS 1-5? Semester 2

Professor S.D. Evans

Tuesday 1pm	WORKSHOP	Roger Stevens LT 02/Chem Eng LT B
Tuesday 4pm	LECTURE	Roger Stevens LT 02
Thursday 3pm	LECTURE	Roger Stevens LT 22

Recommended text:

Physics for scientists and engineers, Tipler & Mosca

Vibrations	-	Chapter 14
Waves	-	Chapter 15
Sound	-	Chapters 15, 16

Further Reading:

Vibrations and Waves in Physics	I G Main
Vibrations and Waves	A P French
Vibrations and Waves	Gough, Richards & Williams

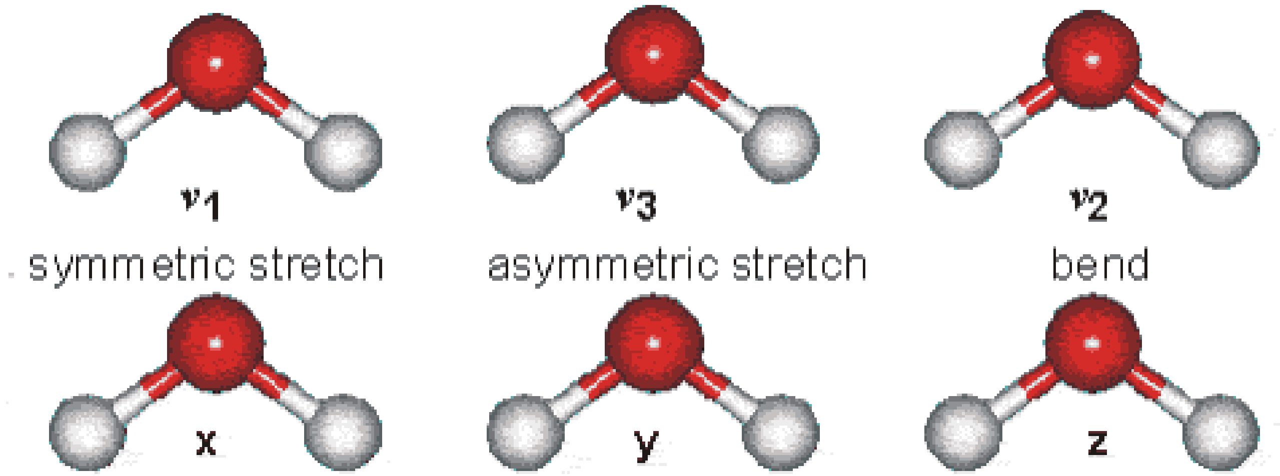
Week		
1	Simple Harmonic Motion	Damped SHM
2	Driven Oscillations	Resonance
3	Travelling Waves, Harmonic waves	Energy and Power of a wave
4	Reflection, Transmission and impedance of waves	Superposition/ Interference
5	Longitudinal waves, sound + seismic waves	The Doppler Effect, shock waves

Driven and damped simple harmonic motion

Harmonic 1D standing wave

Harmonic 2D standing wave

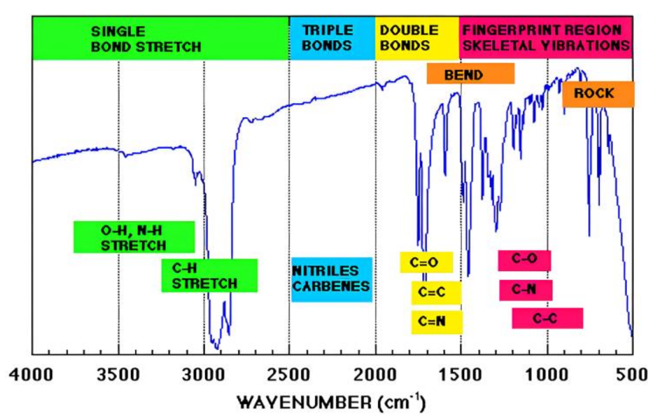
½ k_BT per mode



librations = *small rotational oscillations due to constraints of adjacent molecules*

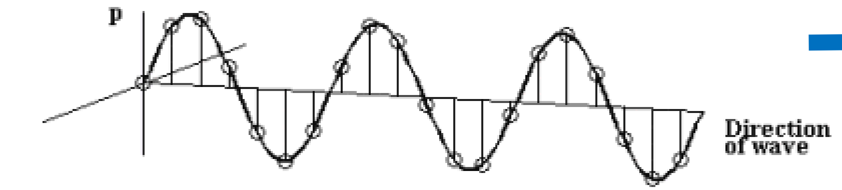
Vibrating bonds in molecules, Simple Harmonic Motion. Example here is water, and the multiple vibrational modes add to the other degrees of freedom (3 x translational and 3 x rotational), resulting in very high heat capacity – see Voice Thermo lectures.

These vibrational modes are high frequency, visible using Infra-Red Spectroscopy

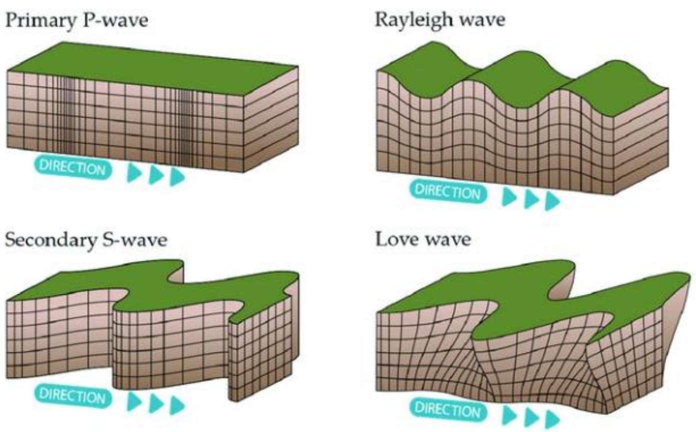


Travelling waves

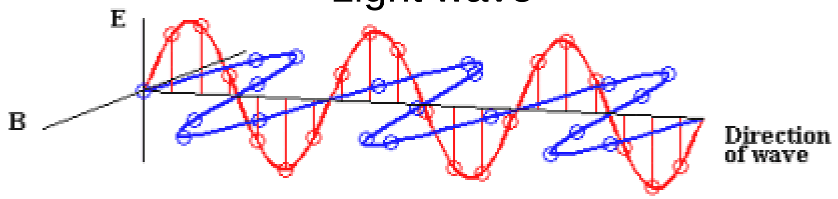
Mechanical wave

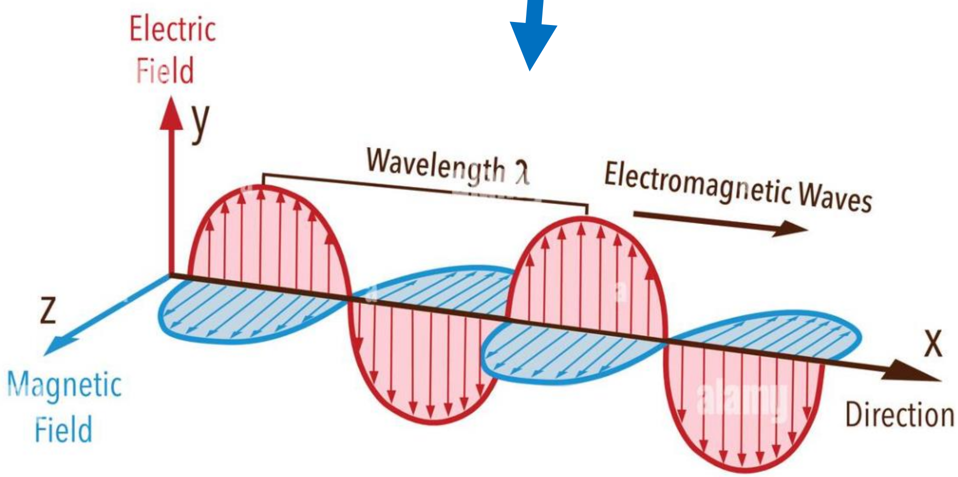


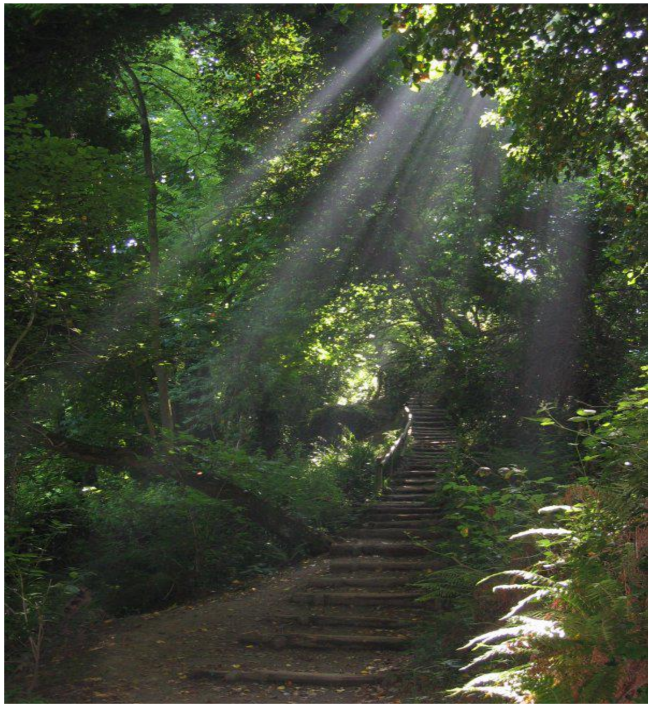
Seismic waves



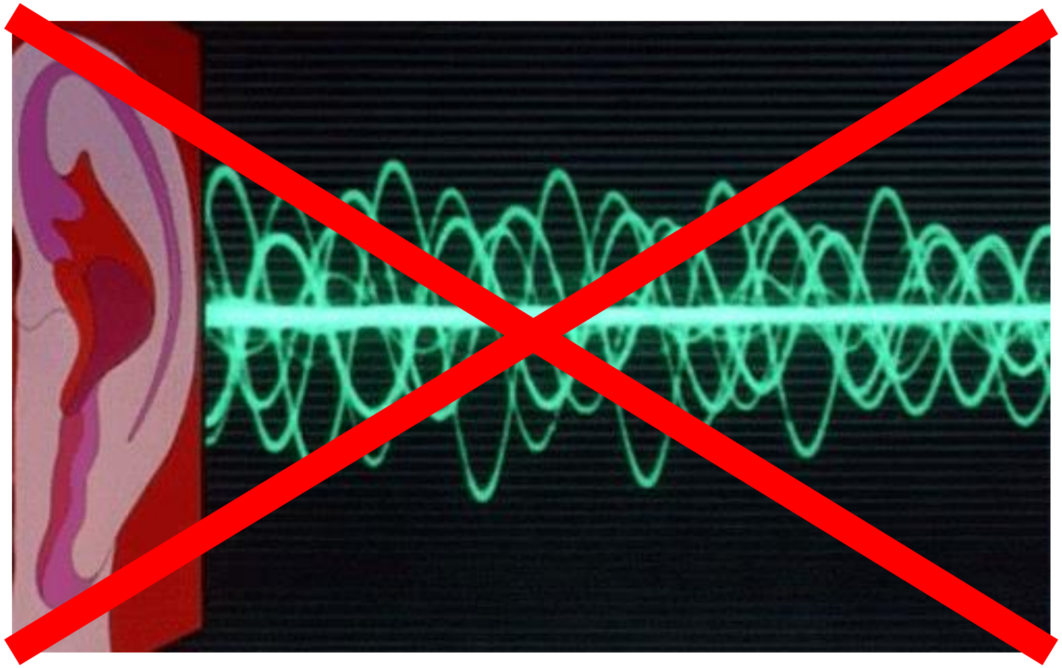
Light wave

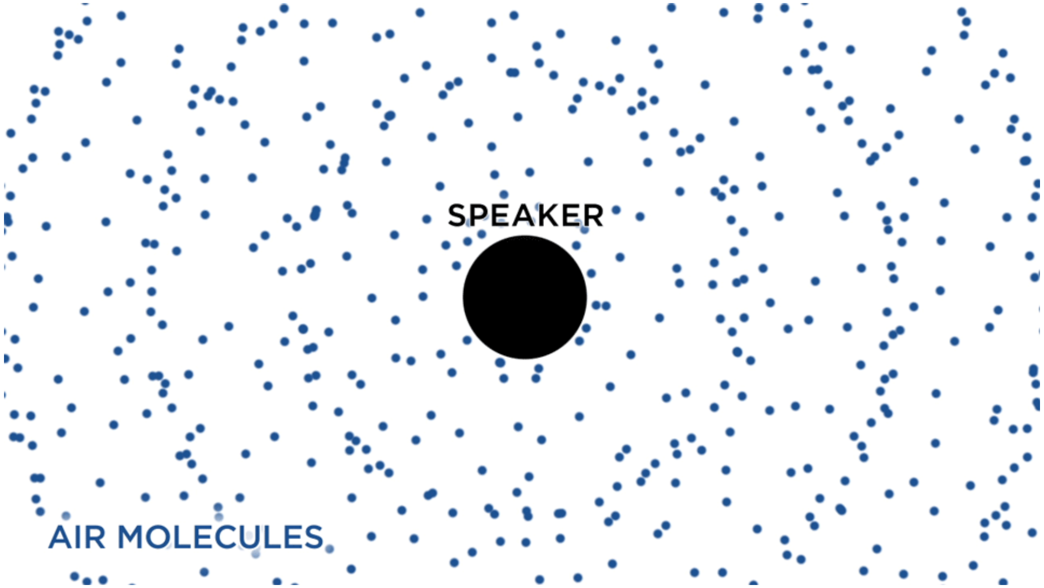




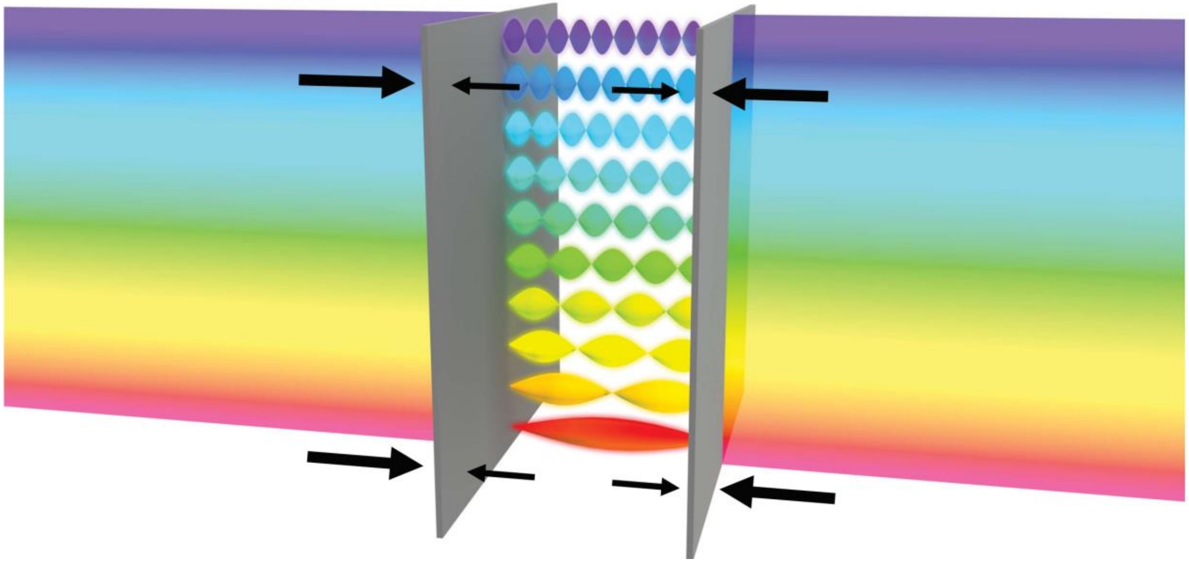


Sound Waves





Sounds waves are longitudinal compression travelling waves



Casimir effect (1948)
Standing wave of
vacuum fluctuations

Shock wave

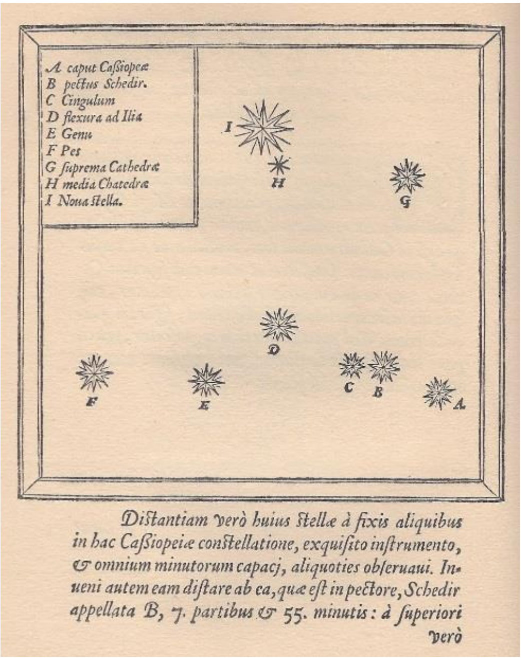
Transverse travelling wave

Shock wave of the Tycho Type 1a supernova

Nov 2nd 1572.

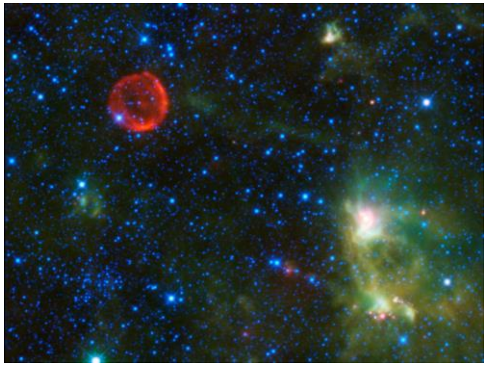
Nov 16th brighter than Venus

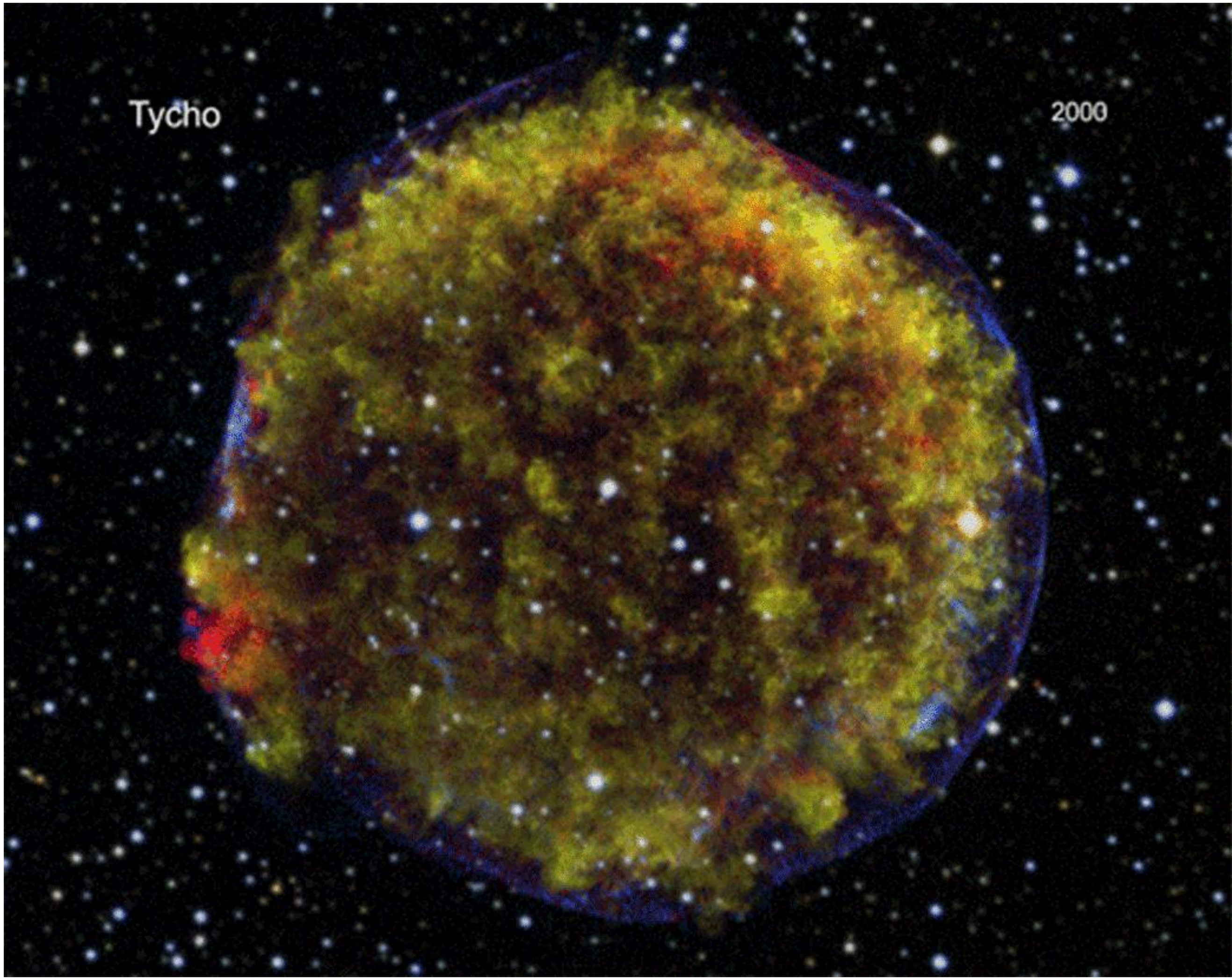
Remnant rediscovered in x-ray at Jodrell Bank 1952



A caput Caßiopeæ
B pectus Schedir.
C Cingulum
D flexura ad Ilia
E Genu
F Pes
G ſuprema Cathedræ
H media Chatedræ
I Noua ſtella.

Diſtantiam verò huius ſtellæ à fixis aliquibus in hac Caßiopeiæ conſtellatione, exquiſito inſtrumento, & omnium minutorum capaci, aliquoties obſeruaui. Inueni autem eam diſtare ab ea, quæ eſt in pectore, Schedir appellata B, 7. partibus & 55. minutis: à ſuperiori verò





Time lapse taken by Chandra x-ray observatory in orbit

Oscillatory Motion

If the net force on an object acts always **towards** the equilibrium position (a **restoring** force), a back and forth motion results: **periodic** or **oscillatory** motion, or **vibration.**

Familiar examples of periodic motion

Pendulum	Guitar string
Molecules in a solid	Air molecules in sound wave
Chemical bonds in molecules	Twanging ruler

A special type of oscillation occurs if

restoring force $\propto$ displacement of object :

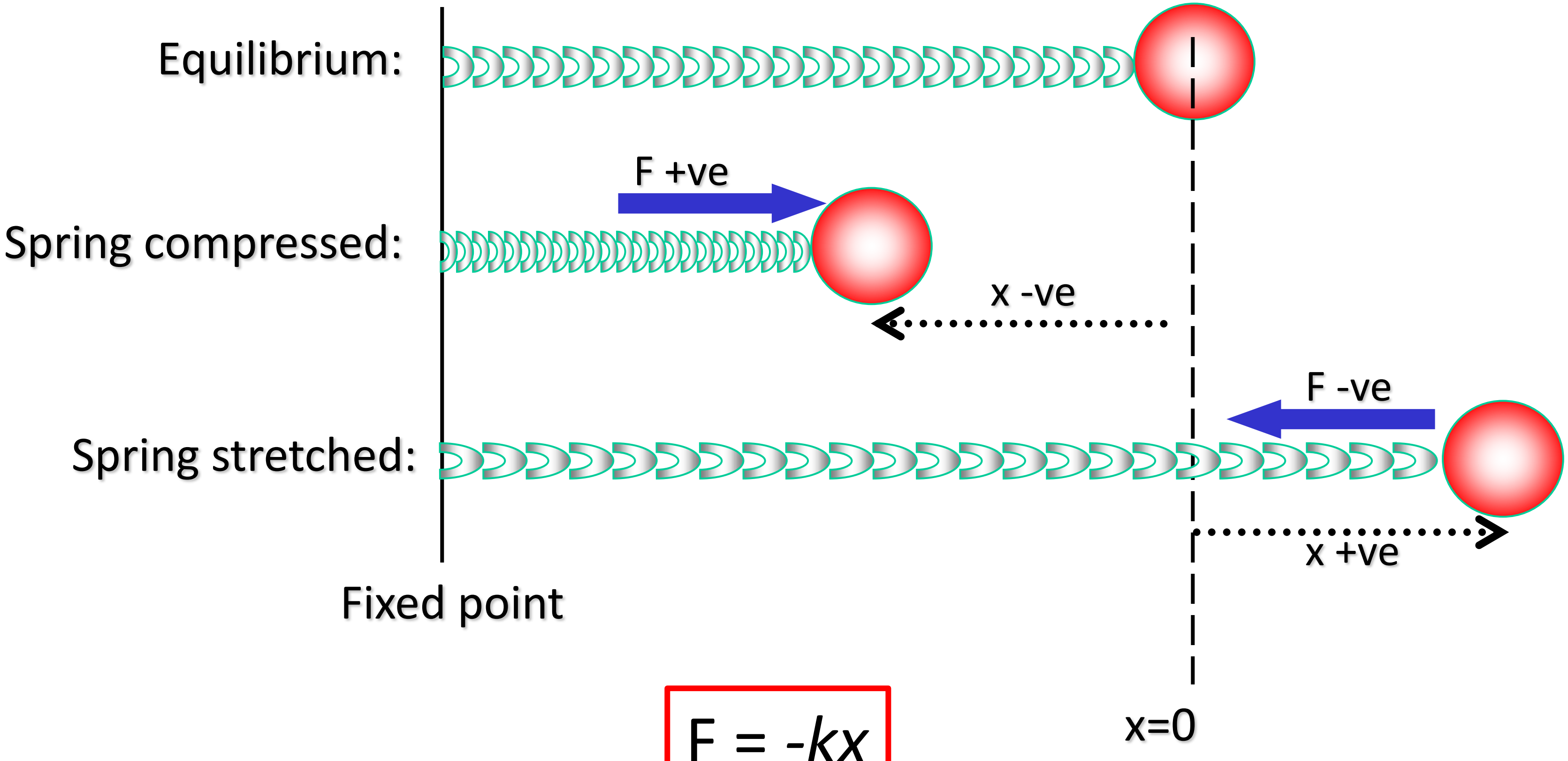
simple harmonic motion (SHM).

> An object moves with simple harmonic motion (SHM) when the acceleration of the object is proportional to its displacement and in the opposite direction.

Simple Harmonic Motion

Example: Mass on a spring
(without gravity)

Displacement = x
Force on mass = F

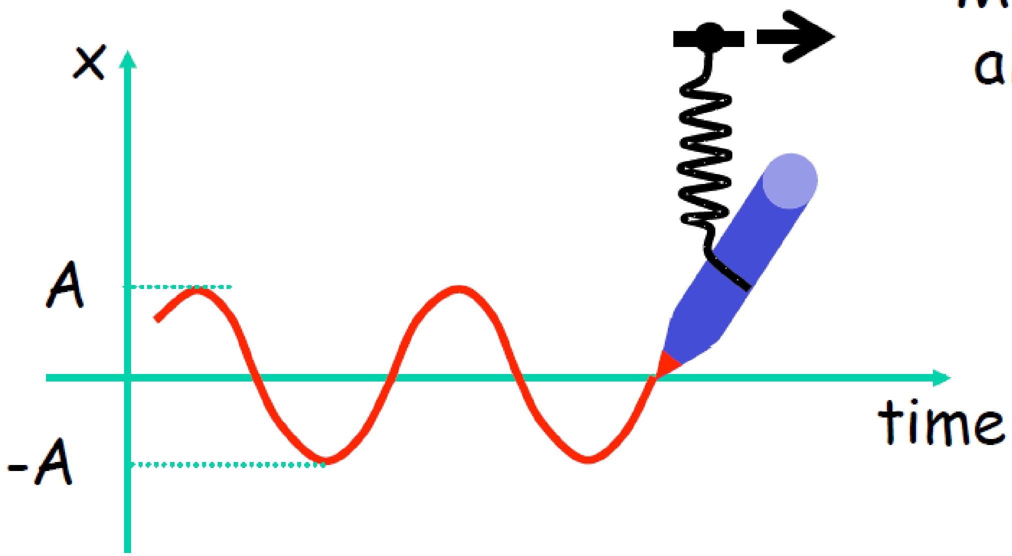


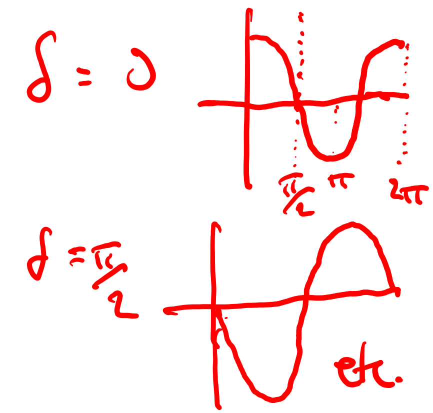
$$\boxed{F = -kx}$$

Derive an expression for the motion of the mass! x = ?

Consider an oscillating pen:

Moving steadily along time axis





The general equation for the curve traced out by the pen is

$$x = A \cos(\omega t + \delta)$$

δ offsets the oscillation along the time axis.

$(\omega t + \delta)$ is the **phase** of the motion
ω is the **angular frequency**
δ is the **phase constant**

n.b. cos is used "by convention", but actually due to the maths of Fourier transform and Euler identity, which can be used to represent oscillations in complex notation, cos is real part, sin imaginary.

$$e^{j\omega t} = cos(\omega t) + j\ sin(\omega t)$$

$$F = -k \cdot x$$

Hooke's law

$$F = m \cdot a = m\frac{d^2x}{dt^2}$$

Equation of motion

$$\boxed{\Rightarrow \frac{d^2x}{dt^2} + \frac{k}{m}x = 0}$$

What function is proportional to its second derivative with opposite sign?

= differential equation, i.e. $-\frac{k}{m}x = \frac{d^2x}{dt^2}$, a function of its own derivative, how to solve?

Try:

$$x = A\cos\omega t \quad \Rightarrow \frac{dx}{dt} = -A\omega \sin\omega t \quad \Rightarrow \frac{d^2x}{dt^2} = -A\omega^2 \cos\omega t$$

$$\boxed{-A\omega^2 \cos\omega t + \frac{k}{m}A\cos\omega t = 0} \quad \text{if} \quad \boxed{\omega^2 = \frac{k}{m} \Rightarrow \omega = \sqrt{\frac{k}{m}}}$$

$x = A\cos\omega t, x = A\sin\omega t, x = A\cos(\omega t + \delta)$ are all solutions

An object moves with simple harmonic motion (SHM) when the acceleration of the object is proportional to its displacement and in the opposite direction.

The displacement x is given by: $$x = A\cos(\omega t + \delta)$$

ω is the **angular frequency** and has units of rad.s^{-1}

$(\omega t + \delta)$ is the **phase** of the motion

δ is the **phase constant**

Time taken for one complete oscillation is the **period** T.

The **frequency** of oscillation, $f = 1/T$ in s^{-1} or Hertz.

The distance from equilibrium to maximum displacement is the **amplitude** of oscillation, A.

The period T is defined by $x(t) = x(t+T)$ — displacement of x' is repeated every cycle ∴ every T

$$x = A\cos(\omega t + \delta) = A\cos(\omega(t+T) + \delta)$$

$= A\cos(\omega t + \omega T + \delta)$

only true if $\omega T = 2\pi$

So $\omega T = 2\pi$.

Relationship between ω, f and the parameters k and m:

$$\boxed{\omega = \frac{2\pi}{T} = 2\pi f}$$ for SHM

$$\omega^2 = \frac{k}{m} \Rightarrow \omega = \sqrt{\frac{k}{m}}$$

& $$\boxed{T = 2\pi\sqrt{\frac{m}{k}}}$$ for mass on spring

IMPORTANT RESULTS: **Period of SHM does not depend on the amplitude!**

Amplitude A and phase constant δ are fixed by the initial position x_o and initial velocity v_o

Aside, where does this come from?

$$\boxed{T = 2\pi\sqrt{\frac{m}{k}}}$$ for mass on spring

Comes from Newton's 2nd law

Same as slide 13, only more clear

$$F = ma$$

$$-kx = m\frac{d^2x}{dt^2}$$

$$-kA\cos(\omega t) = m\frac{d^2A\cos(\omega t)}{dt^2}$$

$$-kA\cos(\omega t) = -m\omega^2 A\cos(\omega t)$$

$$k = m\omega^2$$

and rearrange

$$\omega = \sqrt{\frac{k}{m}}$$