UNIVERSITY OF LEEDS

Mechanics 1

Session 3 – Newton's Laws of Motion: Rethinking Forces

DR BEN HANSON

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Last Lecture

Trajectories

We learned:

- The physics of projectile trajectories
- How to represent a trajectory as a vector equation
- How to represent the components of the vector using kinematic equations

You should be able to:

- Describe the physical path taken by a projectile in terms of (x, y) coordinates
- Describe the physical path taken by a projectile in terms of a position vector $\underline{r}$
- Calculate the position of a projectile at some time t, given its initial position, velocity and acceleration

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This Lecture

Question: What causes acceleration (and therefore motion)?

Question: What is a force?

This Lecture

Newton's Laws of Motion

We will learn:

- The conceptual importance of Newton's 3 Laws of Motion
- Real-world examples of motion for which each of these laws applies
- How to mathematically analyse multiple forces acting simultaneously

You will be able to:

- Describe each of Newton's 3 Laws of Motion, with examples to illustrate
- Draw force diagrams for various physical systems
- Mathematically "resolve" force vectors into perpendicular directions

Newton’s Laws of Motion

A Conceptual Journey

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Newton’s Laws of Motion

A Conceptual Journey

Newton’s Law’s of Motion take us from a kinematic perspective to a mechanical perspective

The development of the concept of “force” as a physical, measurable quantity, consistent between different types of interaction (mechanical, electromagnetic, gravitation etc) is a profound moment in the history of physics. So let’s walk the path again together…

Disclaimer: This is not going to be historically accurate. It’s a conceptual story

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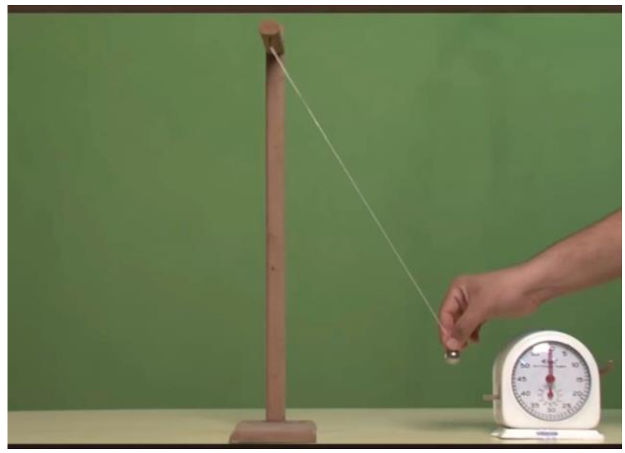
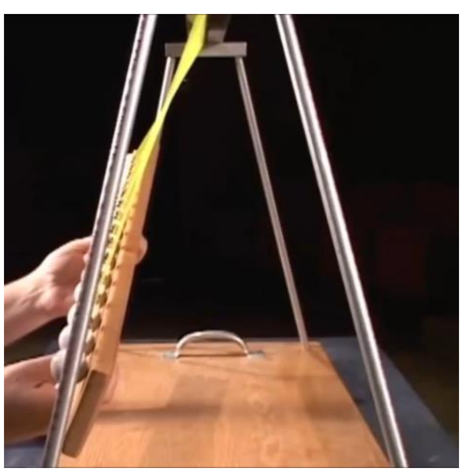
Newton's Laws of Motion

Kinematics



Newton's Laws of Motion

Kinematics



We can see that things move. But why?
What is the "cause" of that motion?

Newton’s Laws of Motion

Kinematics

Imagine an object, sliding across a frozen surface…

1. First observation
   a) Objects move, and slow down
   b) The thicker or rougher the external substance, the faster the object slows down
2. Second Observation
   a) Objects move, and change direction
   b) The sharper the turn or larger the contact angle, the greater the change of direction

Objects appear to keep moving unless there is “something” to stop them. Let’s call that something a “force”.

https://www.youtube.com/watch?v=Sic7CckO-js

Newton’s Laws of Motion

Newton’s First Law

A body remains at rest, <u>or in motion at a constant speed in a straight line</u>, unless acted upon by a force.

Newton's Laws of Motion

Newton's First Law

A body remains at rest, <u>or in motion at a constant speed in a straight line</u>, unless acted upon by a force.

Forces change an object's speed and direction? Force must be a vector!

Task 1

Forces Acting on Objects

Task 1

Forces Acting on Objects

Tasks:

For the following examples, describe with your neighbours in which direction the forces are acting:

1. A trailer is being pulled by a car along the M62 motorway.
   1. The car accelerates from 55mph to 60mph, the speed limit. In what direction is force acting on the trailer?
   2. The car and trailer and moving at a steady 60mph. In what direction is force acting on the trailer?
2. Imagine that you are in a car and a dog runs out into the road. You have to perform an emergency stop, bringing your car to a full stop as quickly as possible.
   1. In what direction would the force be acting on the car? What is the source of that force?
   2. In what direction would the force be acting on you, the driver? What is the source of that force?
3. A child is sliding down a slide. The slide starts off at an angle of 45^o to the ground, but curves to be 15^o at the bottom.
   1. What are the forces acting on the child at the top of the slide?
   2. What are the forces acting on the child at the bottom of the slide?
   3. What force was acting on the child to cause them to change from a direction of 45^o to 15^o?

Newton’s Laws of Motion

Mass & Impulse

Newton's Laws of Motion

Mass

Imagine pushing a pram…

Newton's Laws of Motion

Mass

But what about…a plane!?

Heavier objects are harder to accelerate / decelerate!

https://www.youtube.com/watch?v=B-4NuNeIk3Q

Newton's Laws of Motion

Impulse

Imagine pushing for a short amount of time…

- From this video:
  Large force => Large momentum change
- From the previous videos:
  More contact time => More momentum change

Define Impulse, $\underline{I} = \underline{F}_N \Delta t$

https://www.youtube.com/watch?v=1-a4YJ_JZqo

Newton's Laws of Motion

Newton's Second Law

The impulse acting on an object is equal to its change in momentum

$$\underline{I} = \Delta \underline{p}$$

$$\rightarrow \Delta \underline{p} = \underline{F}_N \Delta t$$

Newton’s Laws of Motion

Newton’s Second Law

The impulse acting on an object is equal to its change in momentum

Impulse equals change in momentum, $\underline{F_N}\Delta t = \Delta \underline{p}$

Rearrange, $\underline{F_N} = \dfrac{\Delta \underline{p}}{\Delta t}$

Take limit, $\underline{F_N} = \dfrac{\mathrm{d}\underline{p}}{\mathrm{d}t}$

Newton’s Laws of Motion

Newton’s Second Law

The net force acting on an object is equal to the rate of change of its momentum

The impulse acting on an object is equal to its change in momentum

Take limit, $\underline{F_N} = \dfrac{\mathrm{d}\underline{p}}{\mathrm{d}t}$

Substitute, $\underline{F_N} = \dfrac{\mathrm{d}}{\mathrm{d}t}(m\underline{v})$

Constant mass, $\underline{F_N} = m\dfrac{\mathrm{d}}{\mathrm{d}t}(\underline{v})$

Hence, $\boxed{\underline{F_N} = m\underline{a}}$

Newton's Laws of Motion

Newton's Second Law

<u>If the mass is constant</u>, the net force on an object is equal to its mass multiplied by its acceleration

$$\underline{F}_N = m\underline{a}$$

A force can be defined simply as the cause of acceleration! Similarly, if an object is accelerating, there must be a non-zero force acting on it!

Newton's Laws of Motion

Net Force & Reaction Forces

Newton’s Laws of Motion

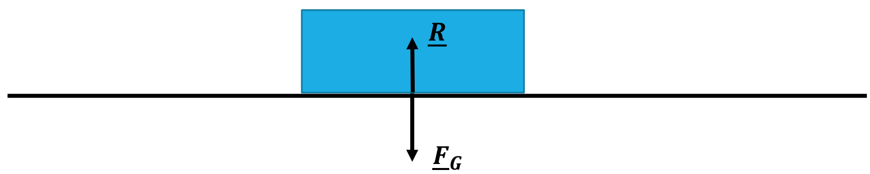
Net Force & Reaction Forces

Scenario: A box is at rest on the floor. It is not accelerating.

Question: What forces are acting upon it, and why?

1. The box is exerting a force on the ground
2. In response, the ground is exerting a force (of the same magnitude) in the opposite direction on the box.

Interesting…



Newton’s Laws of Motion

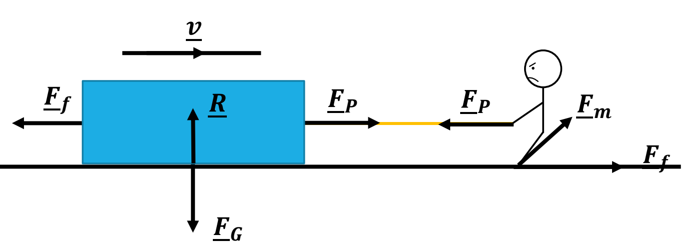
Net Force & Reaction Forces

Scenario: A box is at being pulled by someone. They are exerting a constant force F_p on the box via a rope. The box is moving at a constant speed in the horizontal direction.

Question 1: What forces are acting upon it, and why?

Question 2: Is $\underline{\boldsymbol{F}}_{\boldsymbol{P}} > \underline{\boldsymbol{F}}_{\boldsymbol{f}}$, $\underline{\boldsymbol{F}}_{\boldsymbol{P}} < \underline{\boldsymbol{F}}_{\boldsymbol{f}}$ or $\underline{\boldsymbol{F}}_{\boldsymbol{P}} = \underline{\boldsymbol{F}}_{\boldsymbol{f}}$

Question 3: What forces are acting upon the person, and why?



Newton's Laws of Motion

Newton's Third Law

If body A applies a force to body B, then body B applies and equal and opposite force to body A

MECHANICS 1 – NEWTON'S LAWS OF MOTION

Newton's Laws of Motion

Newton's Laws

1. A body remains at rest, or in motion at a constant speed in a straight line, unless acted upon by a force.

2. The impulse acting on an object is equal to its change in momentum
   a) => The net force on an object is equal to the rate of change of its momentum: $\underline{F}_N = \frac{dp}{dt}$
   b) => If the mass is constant, the net force on an object is equal to its mass multiplied by its acceleration: $\underline{F}_N = m\underline{a}$

3. If body A applies a force to body B, then body B applies and equal and opposite force to body A

Newton’s Laws of Motion

Gravitation

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Newton’s Laws of Motion

Newton’s Gravitation

Wait a minute…

1. Newton’s Second Law implies that, for the same force, lighter objects would accelerate faster, and heavier objects slower.
2. We can observe this experimentally when objects move parallel to the ground

So why does everything fall at the same rate (as originally shown by Galileo)?

Let’s do an experiment!

Newton's Laws of Motion

Newton's Gravitation

Now, imagine being sat under an apple tree…

https://www.youtube.com/watch?v=h48BWDeBLno

Newton's Laws of Motion

Newton's Gravitation

Scenario: An apple falls on your head. You have recently discovered that $\underline{F}_N = m\underline{a}$. This equation says that the acceleration of an object always implies the existence of a force that is causing the acceleration. Further, this acceleration, for a given force, is inversely proportional to the mass $\left(\underline{a} = \frac{\underline{F}_N}{m}\right)$. However, for this ubiquitous downwards force of gravity $\left(\underline{a} = \frac{\underline{F}_G}{m}\right)$, acceleration is somehow independent of mass.

Question: How can this be true? *Hint: Think about the mathematics. How can an equation be independent of a variable?*

Newton's Laws of Motion

Newton's Gravitation

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Question: How can this be true? *Hint: Think about the mathematics. How can an equation be independent of a variable?*

Acceleration is inversely proportional to mass, but everything falls at the same rate. Gravitational force must therefore also be proportional to mass!

We'll do Newtonian gravity properly next week ☺

Task 2

Conceptually Understanding Newton's Laws

Task 2

Conceptually Understanding Newton's Laws

Task: With your neighbours, discuss how each of Newton's Laws apply to various every day situations.

Examples:

1. When a car is accelerating / decelerating, what are the forces acting on the car? In what direction must they be acting.
2. If a bus and a car are moving at the same speed, what can we say about the net force on each of them?
3. When the floor outside is icy, we sometimes slip (and maybe fall ☹). Using Newton's First Law, explain why this is.
4. When you throw a ball (say, a football), it accelerates from a speed of zero to a speed $|\underline{u}|$ before it leaves your hand. Newton's Third Law says the the ball exerts the same force on you as you exerted on it. Explain why you do not go flying backwards at the same speed as the ball.

Mechanical Equilibrium

and also Resolving Forces

Mechanical Equilibrium

Definition

Mechanical equilibrium is when the net force vector on an object is zero (i.e. zero in every direction)

$$\underline{F}_{Net} = \sum_i \underline{F}_i = \underline{0}$$

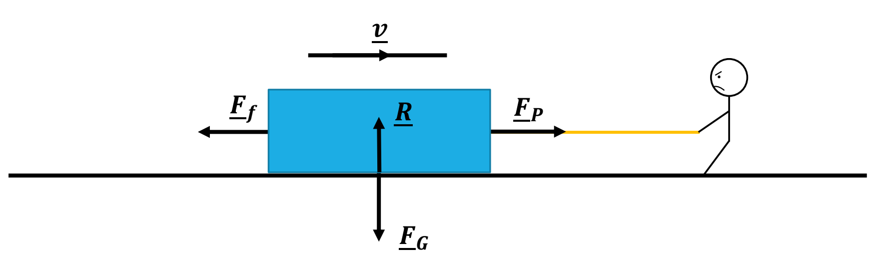
As such, the object may either be moving at a constant velocity or not moving at all (i.e. a constant velocity $\underline{v} = \underline{0}$)

Mechanical Equilibrium

Force Diagrams

Scenario: A box is at being pulled by someone. They are exerting a constant force $\underline{F}_p$ on the box via a rope. The box is moving at a constant speed in the horizontal direction.

Question 1: Is this object in mechanical equilibrium?



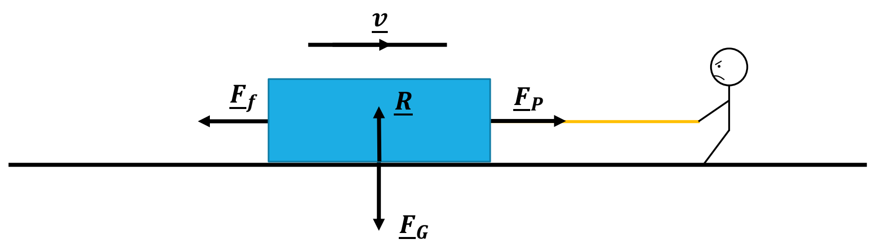
Mechanical Equilibrium

Force Diagrams

Scenario: The person pulling the box stumbles and releases the rope.

Question 1: Is this object in mechanical equilibrium?

Question 2: If the object has a mass of $50kg$ and was initially moving at $0.75m/s$, and $\underline{\boldsymbol{F}}_{\boldsymbol{f}} = \boldsymbol{30N}$, how long will the box take to stop moving?



Mechanical Equilibrium

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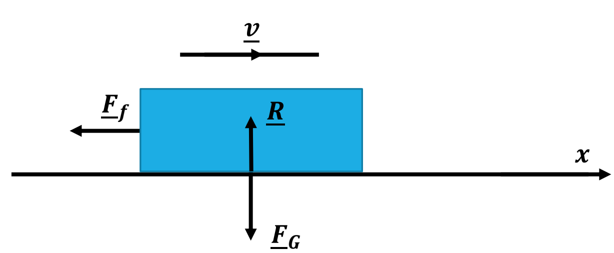
$$\underline{F}_{Net} = m\underline{a}$$

$\rightarrow$ $F_x = ma_x$, $\quad a_x = \frac{F_x}{m} - 0.6ms^{-2}$

1D SUVAT, $\quad v_x(t) = u_x + a_x t$

$v_x(t) = 0$, $\quad 0 = 0.75 - 0.6t$

Solve, $\quad \boxed{t = 1.25s}$



Mechanical Equilibrium

Resolving Forces

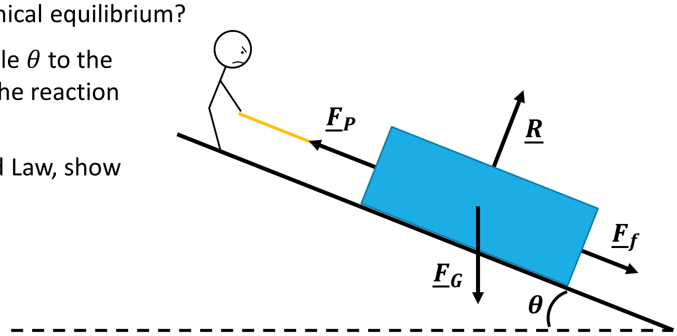
Scenario: A box is at being pulled uphill by someone. They are exerting a constant force $\underline{F}_p$ on the box via a rope. The box is not moving at all.

Question 1: What forces are acting on this object?

Question 2: Is this object in mechanical equilibrium?

Question 3: If the ramp is at an angle θ to the horizontal, derive an equation for the reaction force magnitude, $|\underline{R}|$.

Question 4: Using Newton's Second Law, show that $|\underline{\boldsymbol{F}}_p| = |\underline{\boldsymbol{F}}_f| + |\underline{\boldsymbol{F}}_G| \sin(\theta)$



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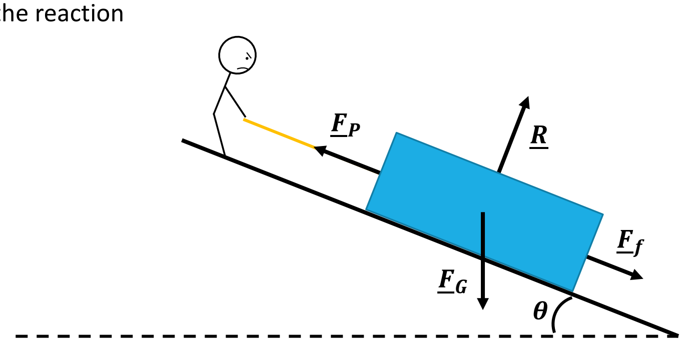

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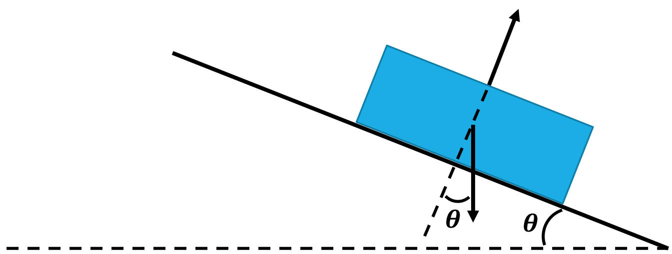

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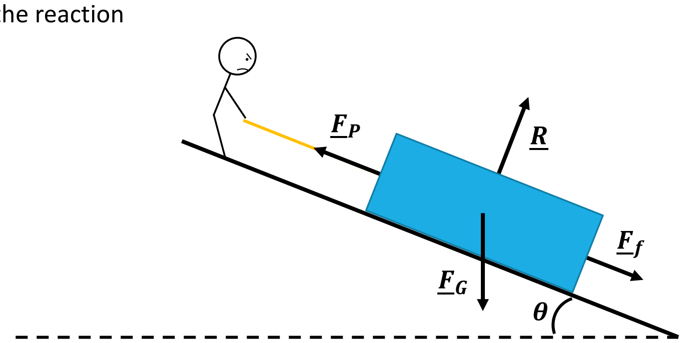
$$\boxed{|\underline{R}| = |\underline{F_G}|\cos(\theta)}$$



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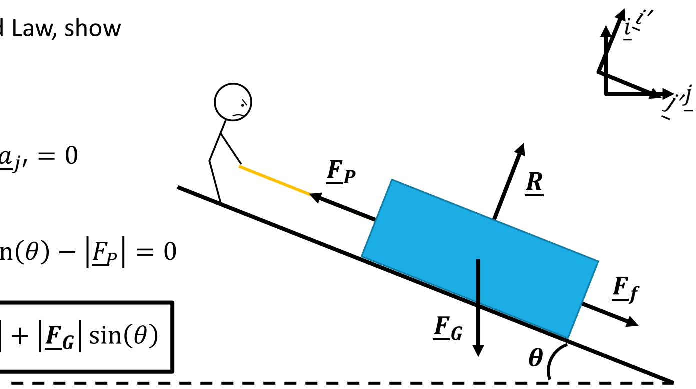

No motion parallel to ramp, $\underline{F}_{j'} = m\underline{a}_{j'} = 0$

Resolve all forces and sum, $|\underline{F}_f| + |\underline{F}_G|\sin(\theta) - |\underline{F}_P| = 0$

Rearrange, $\boxed{|\underline{\boldsymbol{F}}_{\boldsymbol{p}}| = |\underline{\boldsymbol{F}}_{\boldsymbol{f}}| + |\underline{\boldsymbol{F}}_{\boldsymbol{G}}|\sin(\theta)}$

Forces

Just a Small Philosophical Thing

Forces

And Philosophy (it's important)

1. A body remains at rest, or in motion at a constant speed in a straight line, unless acted upon by a force.
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Question: Do forces "exist"?