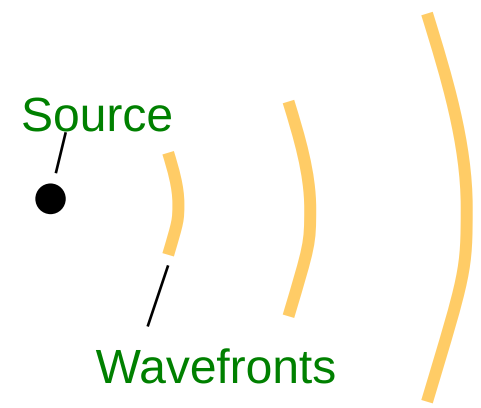
Spherical waves

$$\Delta p = \pm B k s_0 \cos(kx - \omega t) \qquad \mathrm{I} = \frac{\Delta p_0^2}{2\rho v}$$

In a uniform medium the wave moves outwards from the source at a constant speed.



Hence, from a point source (a small object that expands and contracts harmonically), sound waves are produced with spherical wavefronts.

Let $\bar{P}$ = acoustic power emitted by a point source.

At a distance r from the source this power is distributed over a spherical surface of area $4\pi r^2$.

$$\text{Intensity} \quad I = \frac{P}{A} = \frac{P}{4\pi r^2}$$

Intensity diminishes as the inverse-square of distance from a spherical source.

But Intensity $\propto$ Amplitude^2 so Amplitude $\propto$ distance^{-1}.

decibels

The human ear detects sound on an approximately logarithmic scale.

We define the **intensity level** of a sound wave by

$$\boxed{\beta = 10\ \log_{10}\left(\frac{I}{I_o}\right)}$$

Use to convert dB scale to I in units of Wm^{-2}

where I is the intensity of the sound, I_o is the threshold of hearing ($\sim 10^{-12}$ W m^{-2}) and β is measured in decibels (dB).

Examples: breathing	10dB	busy traffic	80dB
whisper	30dB	threshold of pain	120dB
conversation	50dB		

Relationship between displacement and pressure amplitude

$$s_0 = \frac{\Delta p_0}{\rho v \omega} \Rightarrow \Delta p_0 = s_0 \rho v \omega = 2\pi f \rho v s_0$$

$B = 10 \log_{10}\left(\frac{I}{I_0}\right)$

$\therefore I = 10^{\frac{B}{10}} I_0 = 10^{\frac{50}{10}} \cdot 1 \times 10^{-12}$

$= 1 \times 10^{-7}\ Wm^{-2}$

Example, take f = 1 kHz

ρ = 1.3 kgm^{-3}

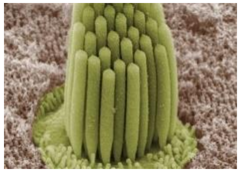
υ = 340 ms^{-1}

50 dB conversation corresponds to sound intensity I = 10^{-7} Wm^{-2}

$$I = \frac{\Delta p_0^2}{2\rho v} \Rightarrow \Delta p_0 = \sqrt{2 I \rho v} = \sqrt{2 \times 10^{-7} \times 1.3 \times 343} \approx 9mPa$$

$$s_0 = \frac{\Delta p_0}{\rho v \omega} \approx 3nm$$

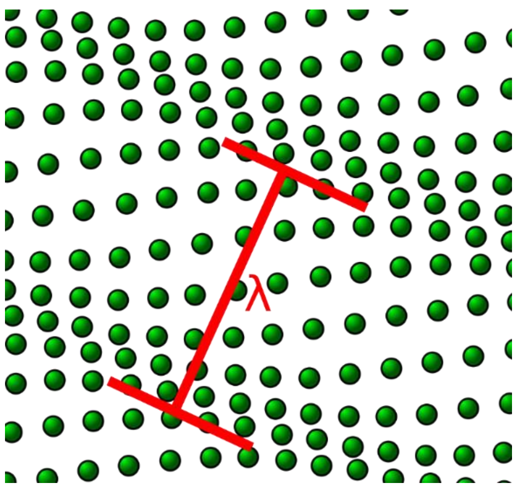
Pain threshold corresponding values are 30 Pa and 11 mm



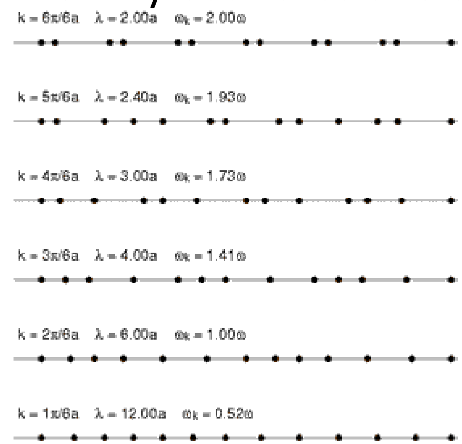
Stereocilia are the mechano-sensing organelles of **hair** cells, which respond to fluid motion for various functions, including hearing and balance. They are about 10–50 micrometers in **length.**

Phonons

A collective excitation in a periodic, elastic arrangement of atoms or molecules (in solids/some liquids). Often referred to as a quasiparticle, it is an excited state in the quantum mechanical quantization of the modes of vibrations for elastic structures of interacting particles. Phonons can be thought of as quantized sound waves, similar to photons as quantized light waves. The phonon is the quantum mechanical description of an elementary vibrational motion in which a lattice of atoms or molecules uniformly oscillates at a single frequency.



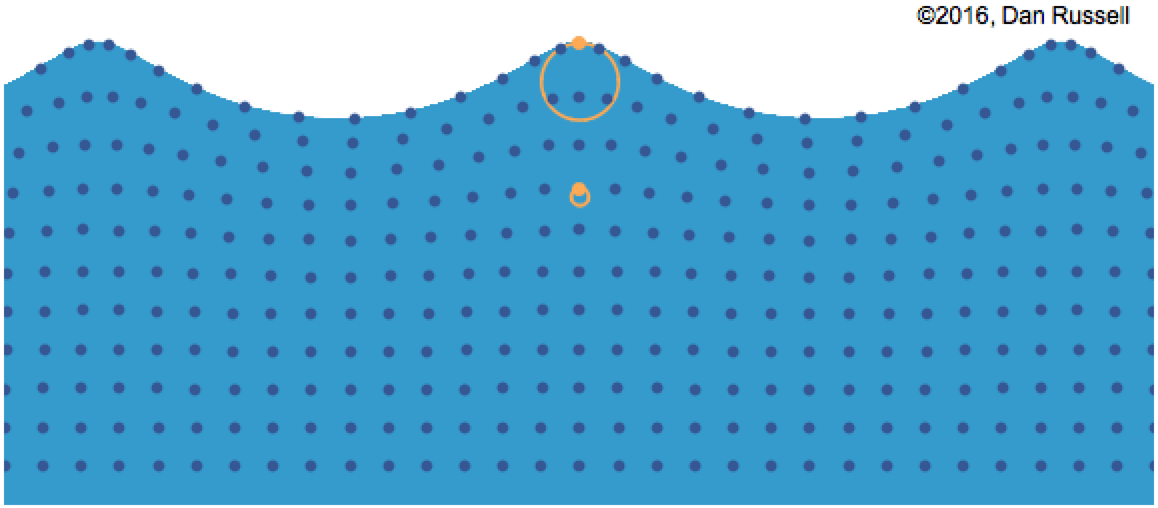
Phonon propagating through a square lattice (atom displacements greatly exaggerated)



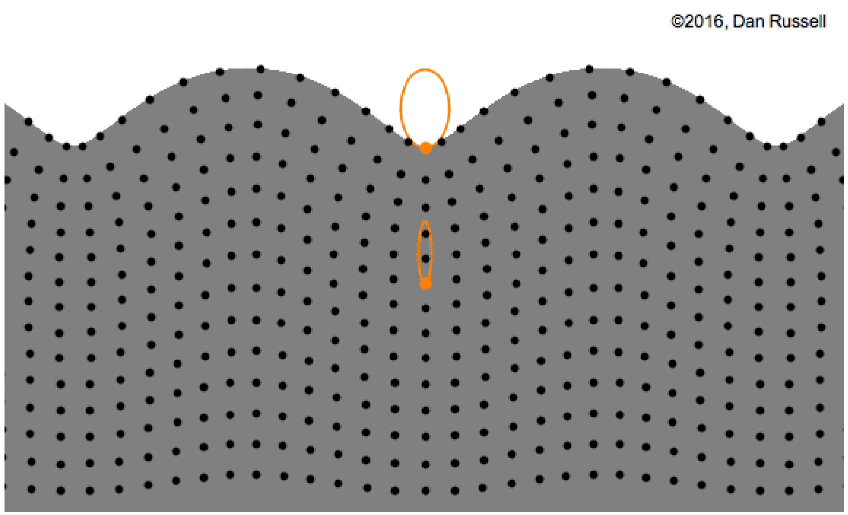
The first 6 normal modes of a one-dimensional lattice: a linear chain of particles. The shortest wavelength is at top, with progressively longer wavelengths below. In the lowest lines the motion of the waves to the right can be seen.

More complex examples

Longitudinal and transverse at the same time



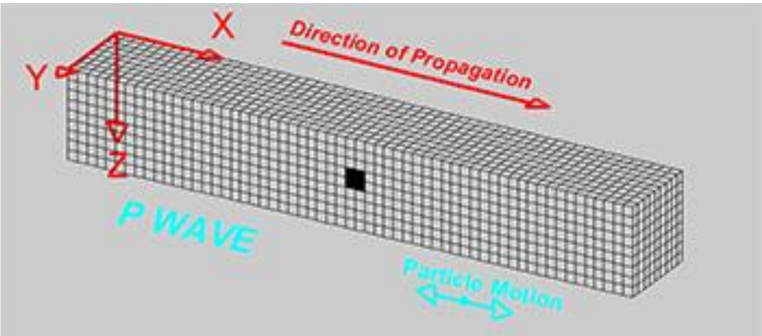
Wave on water
Circular motion

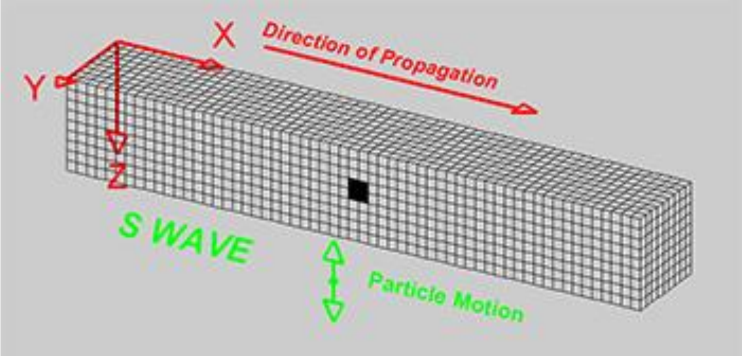


Seismic Rayleigh wave (P wave vertical transverse S wave combined) = Elliptical motion

Example of waves: Seismic waves

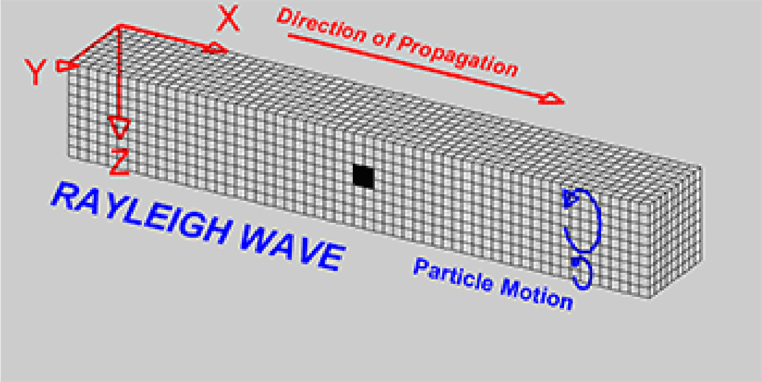
P-wave = Primary wave, longitudinal pressure wave (like sound) very fast (3-13 km s^{-1} through solid material.

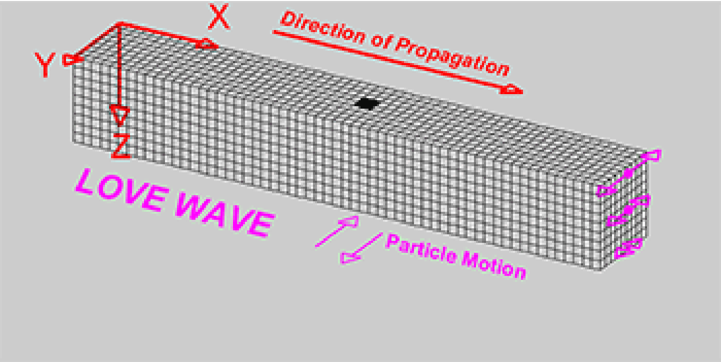




S-wave = Secondary wave, vertical transverse waves. Slower 3-5 km s^{-1}.

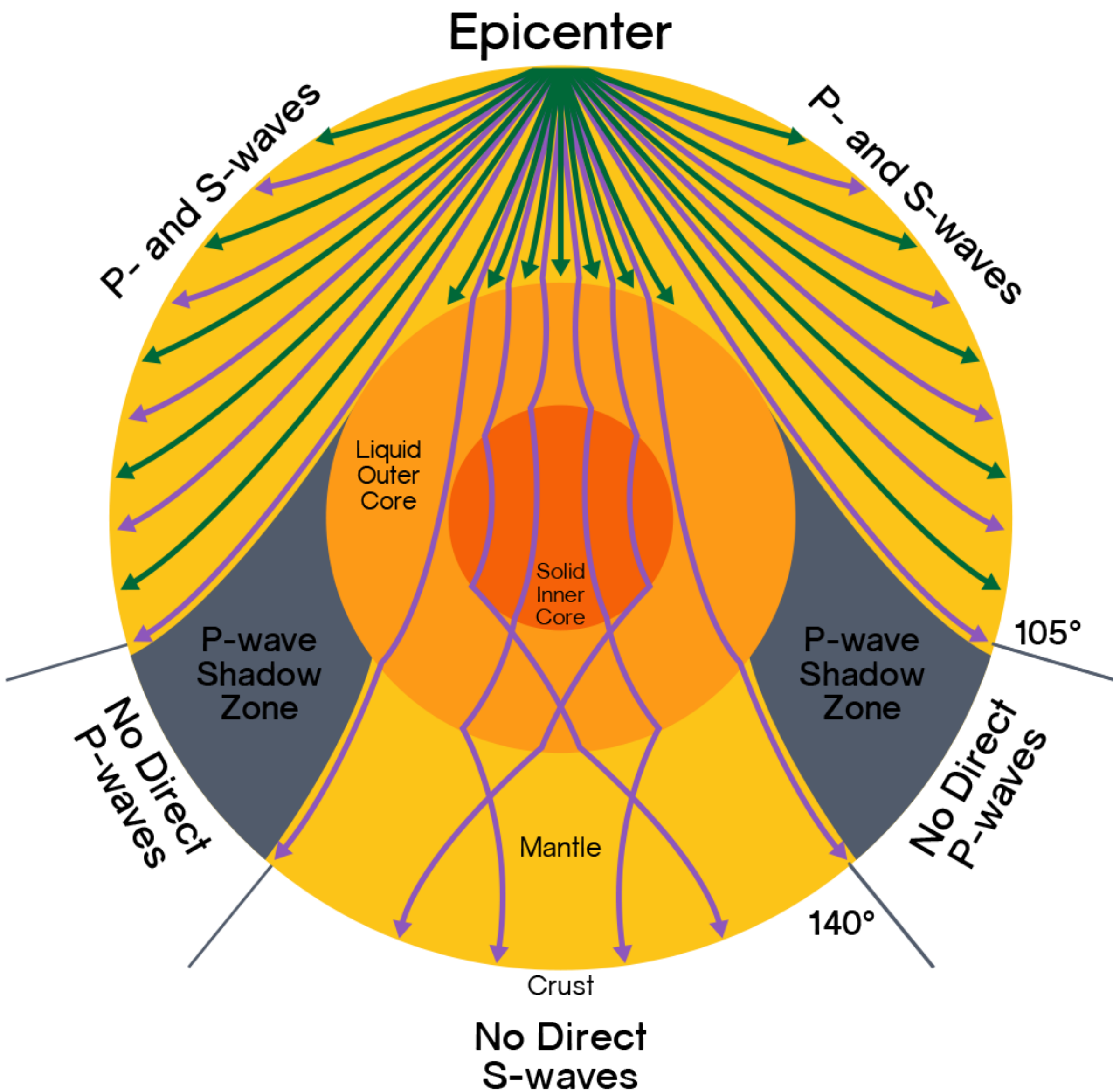
Rayleigh wave = P and S waves combined, like an ocean wave, surface moves a vertical circular path (vertically polarised).





Love wave = P and S waves combined, but this time horizontally polarised, surface moves side to side and back and forth. The most damaging.

Example of waves: Seismic waves

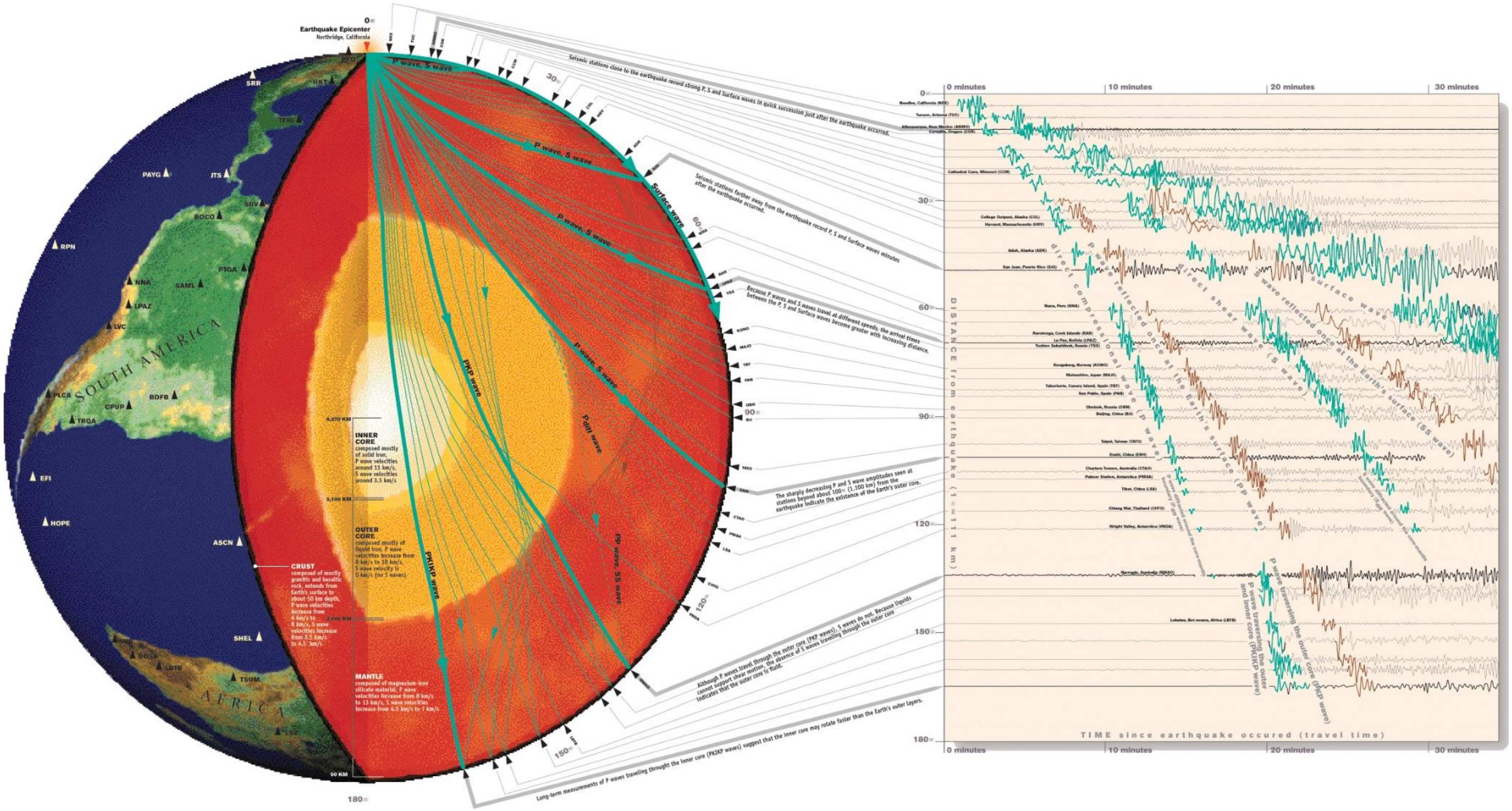


P-waves are diffracted at the interface between zones, a change in density/refractive index (i.e. wave impedance!) causing a shadow of P-waves when low angle incident waves hit liquid core.

The transverse waves are shear waves which cannot propagate through liquid (liquid has zero elastic shear modulus), but longitudinal pressure waves can transmit though liquid.

Measurement of Earth’s interior structure using Seismic Waves

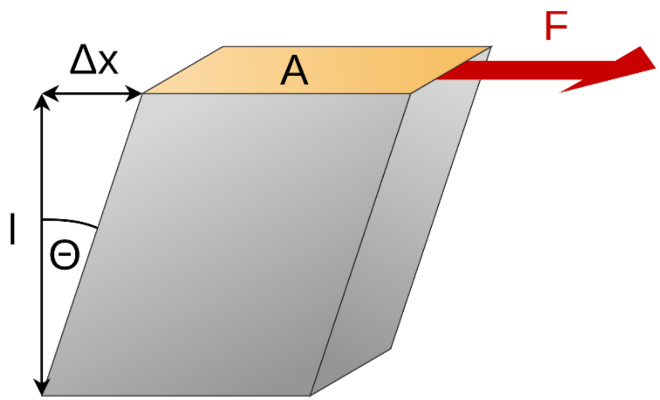
Example of waves: Seismic waves



Also used in **oil exploration**, an oil reservoir will also only transmit longitudinal pressure waves, not transverse waves.

Speed of transverse Seismic wave moving through solid

Requires shear modulus **G** rather than compressibility modulus B



$$G = \frac{F/A}{\Delta x/l} = \frac{Fl}{A\Delta x}$$

Where

F is the force
A is the area across which force acts
F/A = γ_{xy} = shear strain
$\Delta x/l$ = tan Θ (= Θ usually, small angle)
Δx = transverse displacement
l = initial length of the volume

In the same way the velocity of a longitudinal pressure wave (seismic primary wave) is given by

$$v = \sqrt{\frac{B}{\rho}}$$

The velocity of the shear (transverse) wave is given by

$$v_s = \sqrt{\frac{G}{\rho}}$$

Standing waves, a reflected travelling wave

$$y = 2A_0 \sin(kx)\cos(\omega t)$$

Amplitude = $2A_0 \sin(kx)$

Amplitude depends on x.

Every part of the string vibrates in SHM with the same frequency and phase.

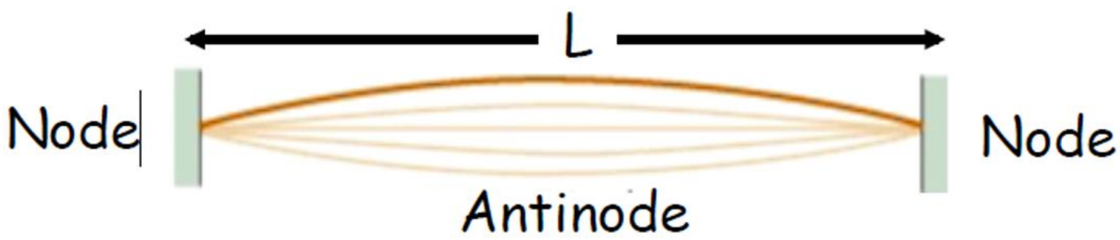
Zero amplitude occurs at $\sin(kx) = 0$, where $kx = \pi, 2\pi, 3\pi...$

& $k = 2\pi / \lambda$ so zeros are at

$$x = \frac{\lambda}{2}, \frac{2\lambda}{2}, \frac{3\lambda}{2} \ldots\ldots = \frac{n\lambda}{2} \quad \text{with } n = 1, 2, 3, \ldots$$

NODES

Fixed ends must be nodes.



$$\frac{n\lambda}{2} = L$$

$$\Rightarrow \boxed{\lambda = \frac{2L}{n}}$$

Standing waves in air columns

Consider a standing wave in closed column of air. Since it is closed, displacement at each end must be zero, so standing wave has a node at each end. This is the same as standing transverse wave on a string. Therefore the frequencies are the same, corresponding to fitting a whole number of *HALF*-wavelengths in the Length.

What happens with an open end?

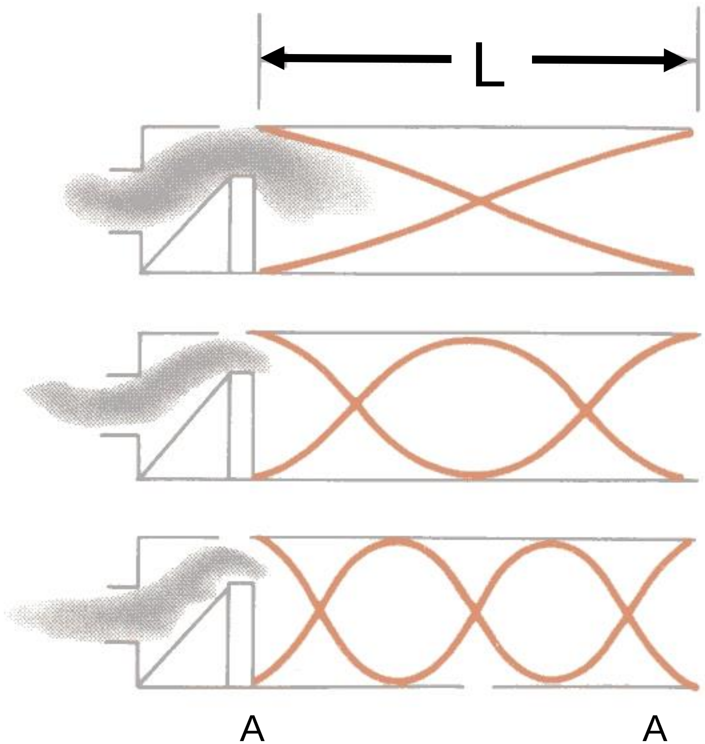
Compare to string, a standing wave is a reflection from a fixed wall with infinite impedance, a node of zero amplitude, and inverted reflected wave.

With a loose string of zero impedance, the reflection is also total, but the end has maximum amplitude and the reflected wave is not inverted. This is an anti-node.

Standing waves in air columns

Standing longitudinal waves can be set up in a tube of air (eg organ pipe).

Consider a pipe open at both ends. Ends are pressure nodes, displacement antinodes.



1[st] normal mode $L = \lambda/2$

2[nd] n. m. $L = \lambda$

3[rd] n. m. $L = 3\lambda/2$

Generally, $L = n\frac{\lambda}{2}$ & $f = \frac{v}{\lambda}$

Hence $f_n = n\frac{v}{2L}$ with $n = 1,2,3,\ldots.$

$$\boxed{f_n = n\, f_1 \qquad \text{with } f_1 = \frac{v}{2L}}$$

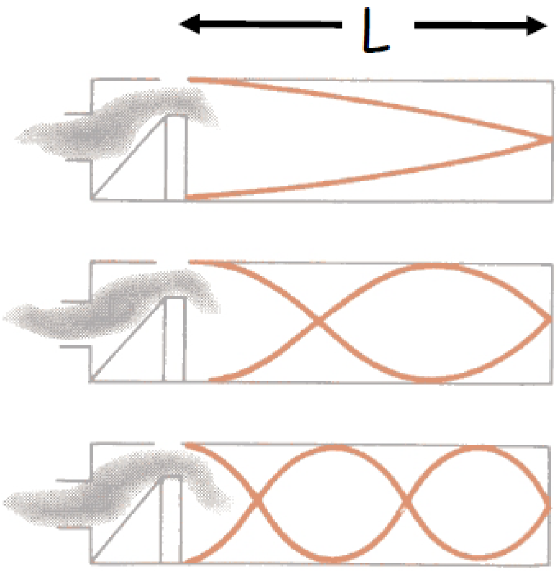
In a pipe open at both ends, the overtones are integer multiples of the fundamental frequency (harmonics).

N.B. These are the **natural frequencies** of the modes of oscillation of the air in the pipe.

$\Rightarrow$ They are **resonant** frequencies of the air column.

Pipe open at one end only

Displacement amplitude:



1st normal mode $\quad L = \lambda/4$

2nd n. m. $\quad L = 3\lambda/4$

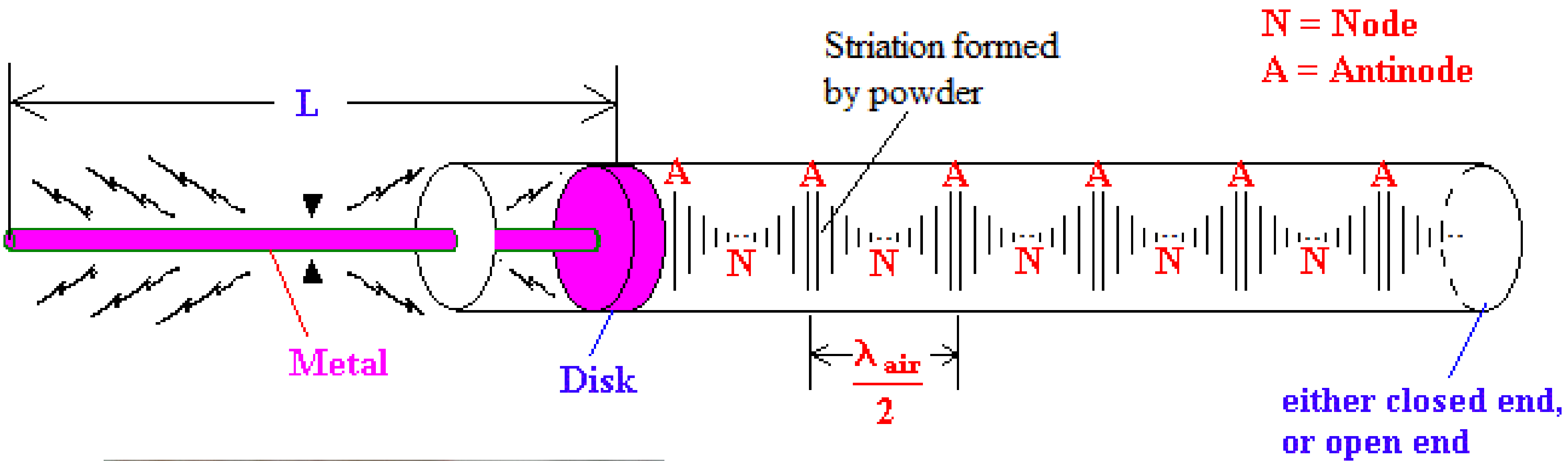
3rd n. m. $\quad L = 5\lambda/4$

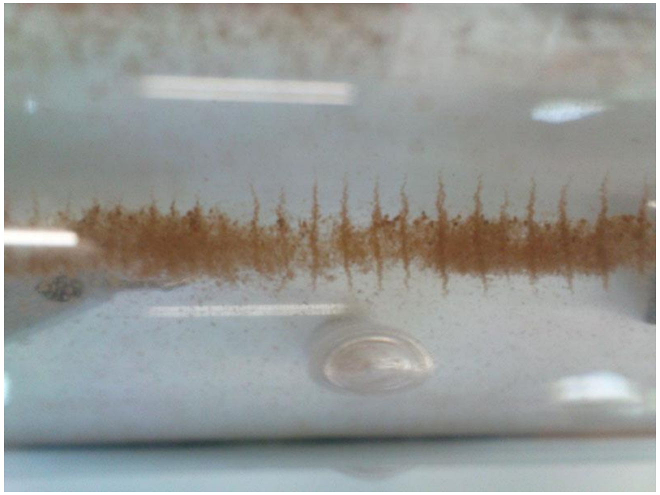
$$L = n\frac{\lambda}{4} \qquad \Rightarrow f_n = n\frac{v}{4L} \qquad with \;\; n = 1,3,5\ldots.$$

$$f_n = n\, f_1 \qquad \text{with } f_1 = \frac{v}{4L} \;\; \& \; n \text{ odd}$$

- Lower notes than open pipe!
- Only odd harmonics.

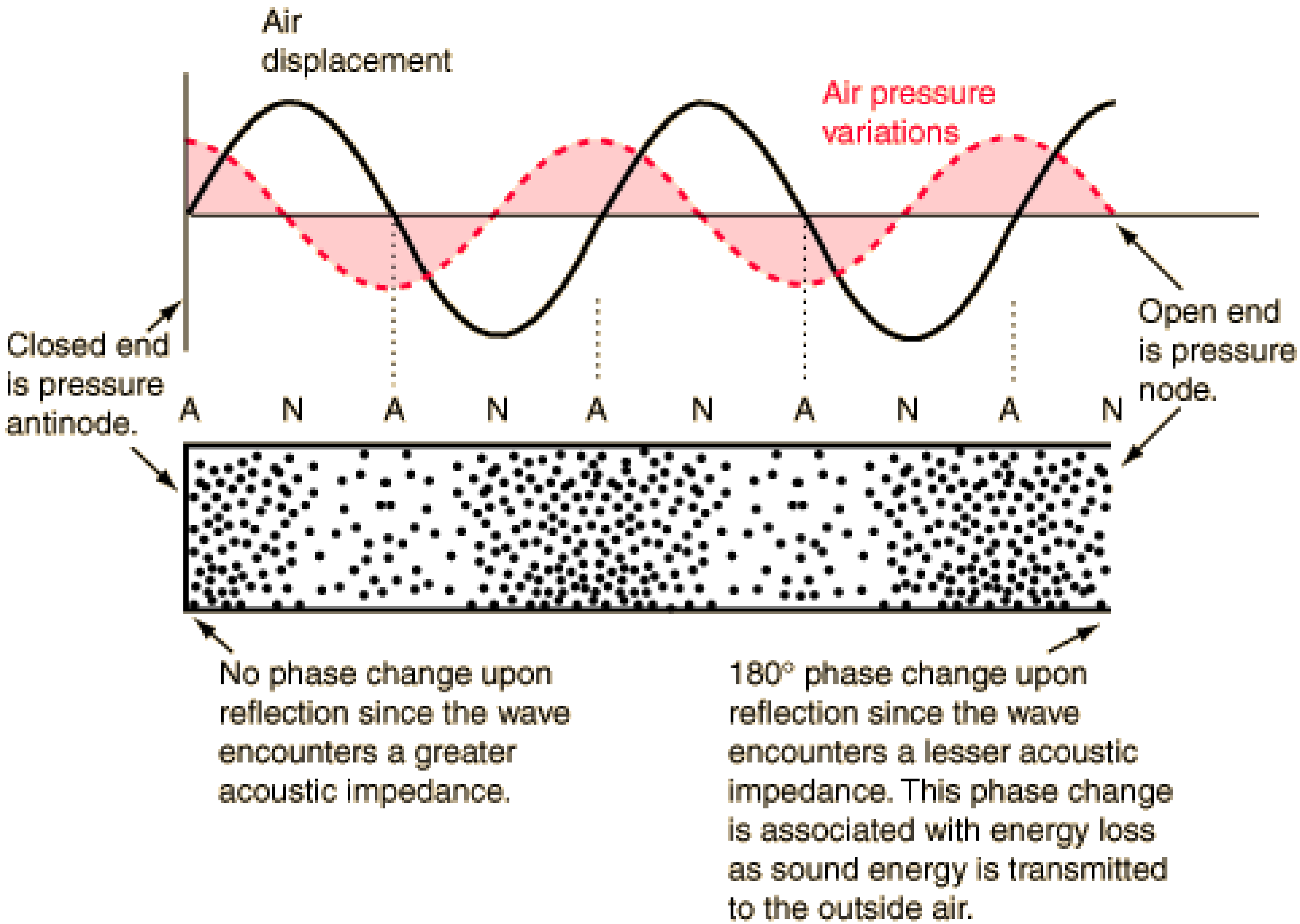
Visualising standing waves
Second-year lab: speed of sound





Origin of striations?

Production of a standing wave in an air column involves reflections from both the closed end and the open end of the column.



SUMMARY: Sound

Sound waves:

$$s = s_0 \sin(kx - \omega t)$$ amplitude wave (displacement from equilibrium in x-direction)

$$\Delta p = (\pm) B k s_0 \cos(kx - \omega t)$$ pressure wave

B is adiabatic bulk modulus $B = \gamma p \Rightarrow v = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{\gamma R T}{M}}$

pressure wave is 90° out of phase with displacement wave

$$\text{Power} = \frac{1}{2} \rho v \omega^2 s_0{}^2 \times \text{Area}$$

cf. string $P_{av} = \frac{1}{2} \mu v \omega^2 A^2$

$$\text{Intensity} \equiv \frac{\text{Power}}{\text{Area}} = \frac{1}{2} \rho v \omega^2 s_0{}^2 = \frac{\Delta p_0^2}{2 \rho v}$$ (watts/m^2)

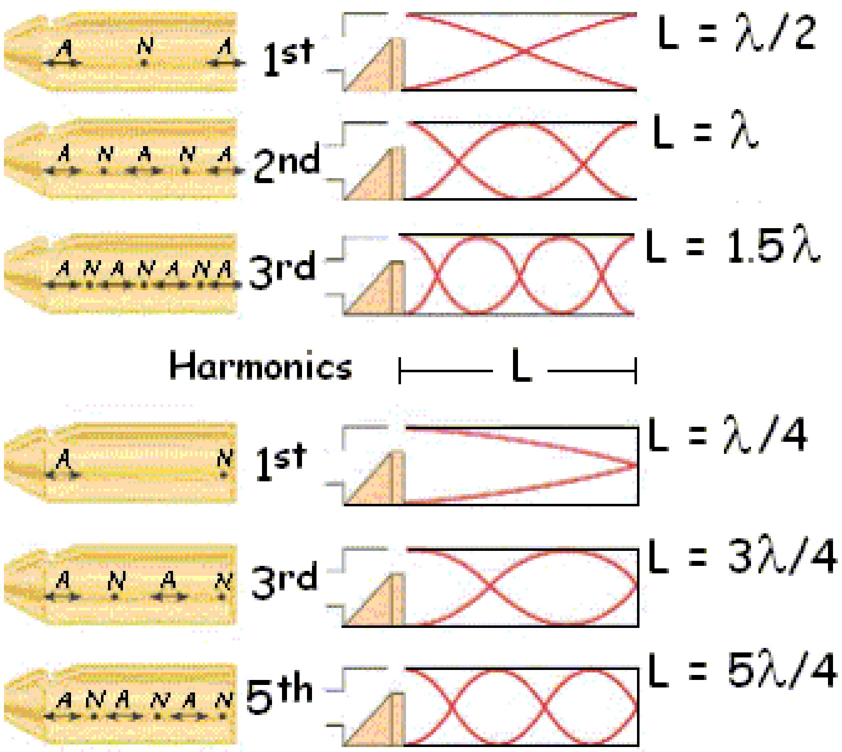
$$\boxed{\beta = 10\,\log_{10}\left(\frac{I}{I_o}\right)}$$

the **intensity level** of a sound wave relative to I_0, the threshold of hearing ($\sim 10^{-12}$ W m^{-2})

$$\text{Intensity} \quad I = \frac{P}{A} = \frac{P}{4\pi r^2}$$

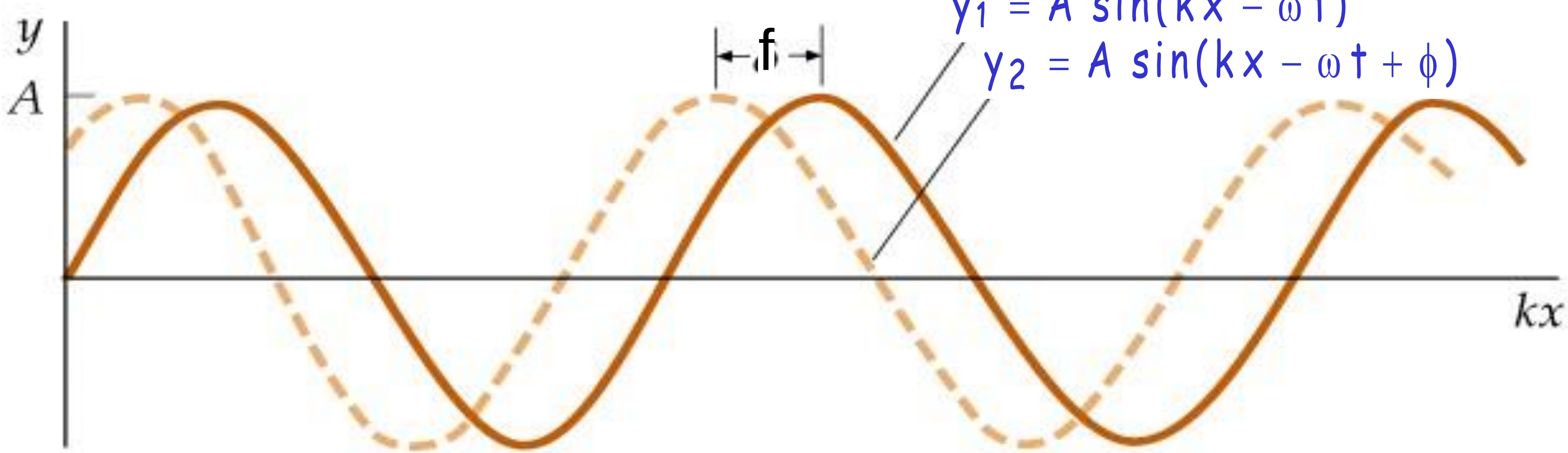
Intensity falls of as distance squared

standing waves in air column



Interference of travelling waves 1 : path difference

Consider two harmonic waves with **equal amplitude & frequency but out of phase**



$y_1 = A\sin(kx - \omega t)$

$y_2 = A\sin(kx - \omega t + \phi)$

Resulting wave function is their superposition:

$$y = y_1 + y_2 = A\left[\sin(kx - \omega t) + \sin(kx - \omega t + \phi)\right]$$

$$\sin a + \sin b \equiv 2\cos\left(\frac{a-b}{2}\right)\sin\left(\frac{a+b}{2}\right)$$

gives

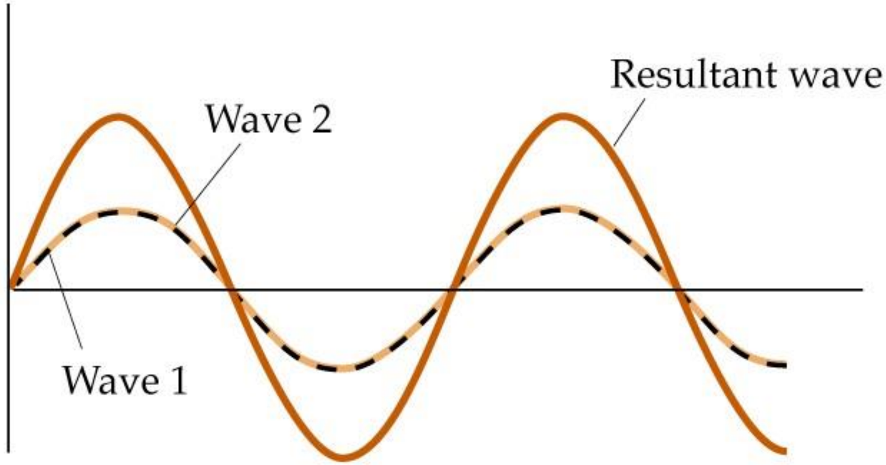
$$\boxed{y = 2A\cos\left(\frac{\phi}{2}\right)\sin\left(kx - \omega t + \frac{\phi}{2}\right)}$$

A Harmonic travelling wave with same frequency and wavelength as originals

When $\underline{\phi = 0}$, or ***even*** multiple of π, cos (ϕ/2) = ±1, so |amplitude| = 2A.

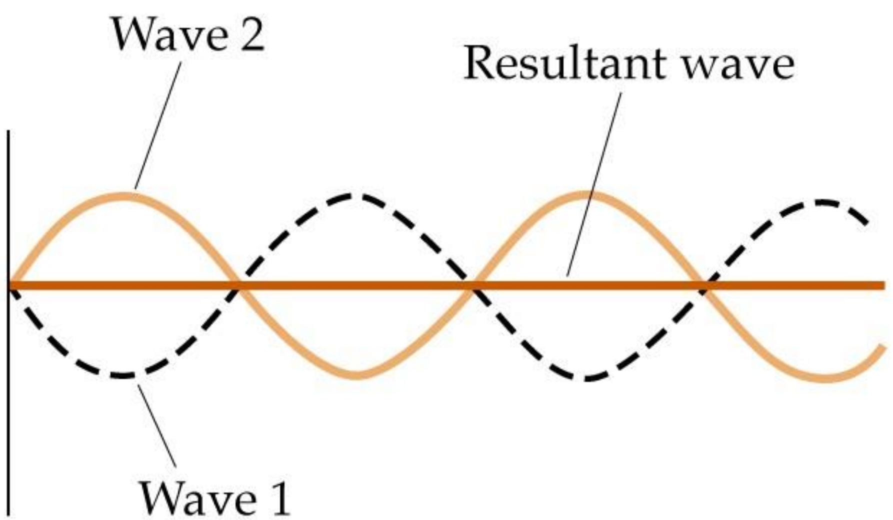
$$y = 2A\cos\left(\frac{\phi}{2}\right)\sin\left(kx - \omega t + \frac{\phi}{2}\right)$$

The waves are **in phase** and interfere **constructively.**



When $\underline{\phi = \pi}$ or any ***odd*** multiple of π cos (ϕ/2) = 0, so amplitude of the resultant wave = 0

Waves are **in antiphase** & interfere **destructively.**



Interpretation of superposition of two out-of-phase travelling waves

Harmonic travelling wave with the same frequency and wavelength as original

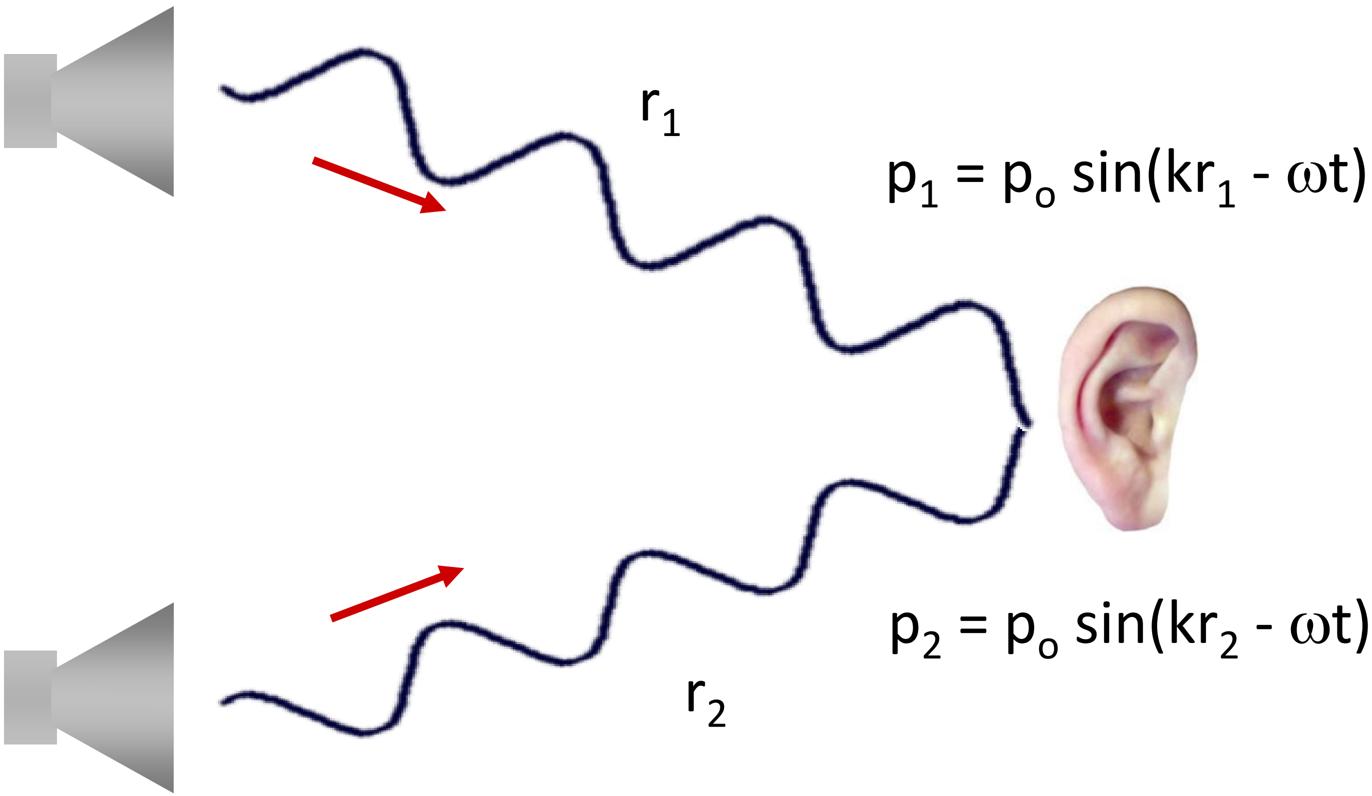
$$\boxed{y = 2A\cos\left(\frac{\phi}{2}\right)\sin\left(kx - \omega t + \frac{\phi}{2}\right)}$$

Amplitude $2A\cos(\phi/2)$
so amplitude is controlled by the phase difference between the two waves, and hence the path difference

Phase offset is an average of the phase offset of each original wave

Example: Waves from two sources

Consider two loudspeakers generating the same frequency:



$p_1 = p_o \sin(kr_1 - \omega t)$

$p_2 = p_o \sin(kr_2 - \omega t)$

Pressure at ear, $p \;=\; p_1+p_2 \;=\; p_o\,[\sin(kr_1 - \omega t) + \sin(kr_2 - \omega t)]$

The **phase difference** ϕ between the waves is $\quad kr_1 - kr_2$

$$\boxed{y = 2A\cos\left(\frac{\phi}{2}\right)\sin\left(kx - \omega t + \frac{\phi}{2}\right)}$$

Define **path difference** $\Delta = r_1 - r_2$. Then $\boxed{\phi = 2\pi\Delta/\lambda}$

<u>Constructive interference:</u> A maximum in the sound will be heard when the path difference is zero or an integral multiple of the wavelength.

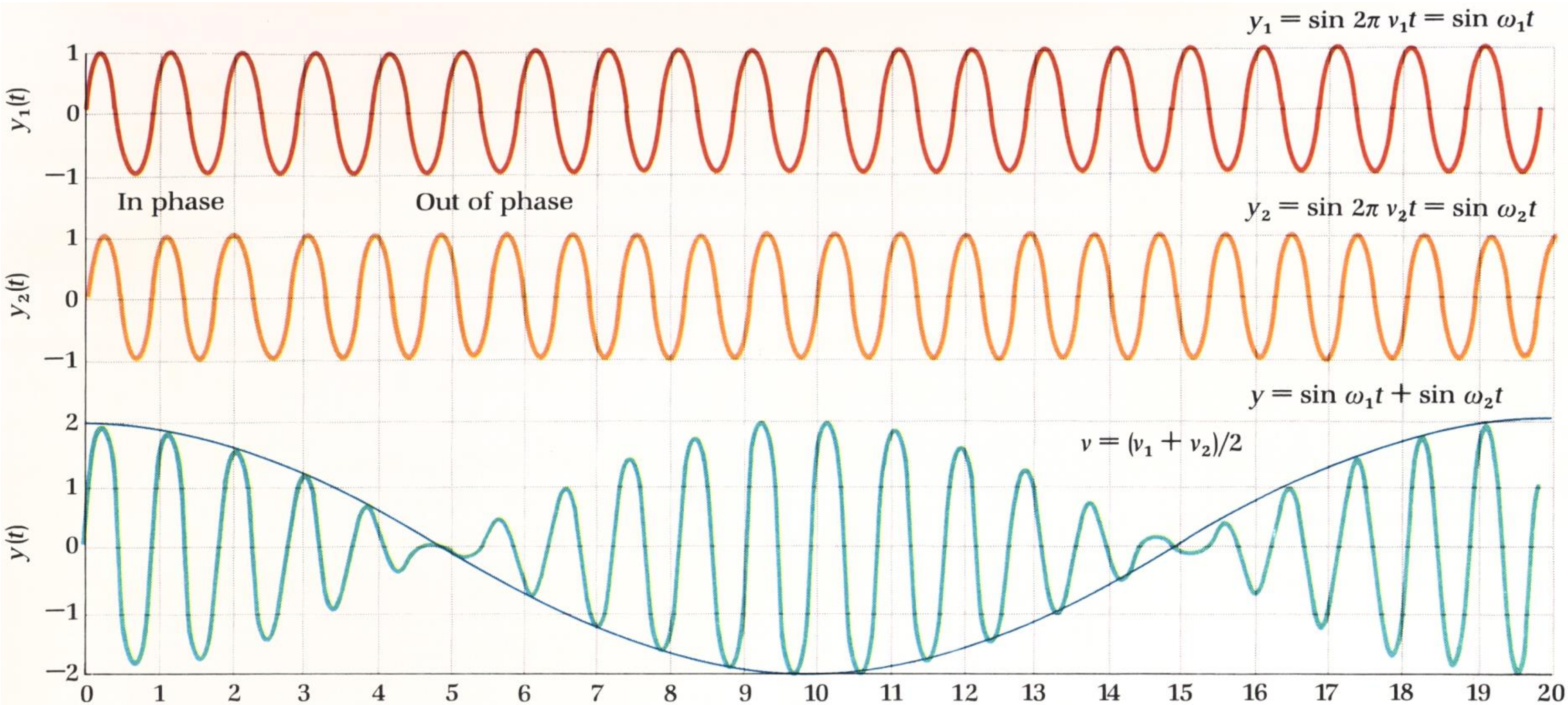
<u>Destructive interference:</u> If the path difference is a half-integral multiple of the wavelength the waves will be in antiphase so no sound will be heard.

Hence, passing across in front of the speakers, we hear patches of loud and quiet **if the sources are coherent** (remain synchronised, so phase differences do not drift randomly).

For $100 < f < 1000$ Hz and $v = 340\ \mathrm{ms}^{-1} \rightarrow$
$0.34 < \lambda < 3.4$ m
$\rightarrow 0.17 < \Delta < 1.7$ m – distance between adjacent sound maxima (and sound minima)

Interference of travelling waves 2: Frequency difference

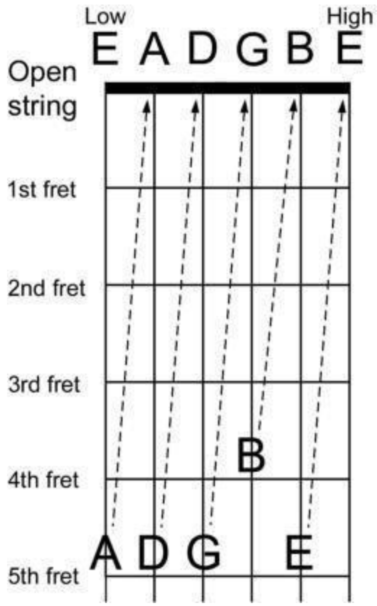
Consider two harmonic waves, passing a fixed point, that have slightly different frequencies. Their superposition is:

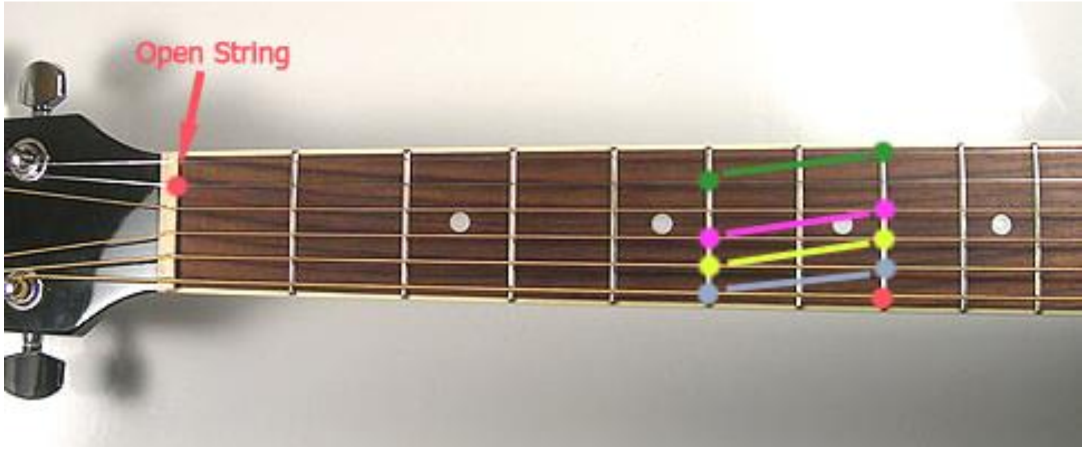


This causes a modulation in amplitude = beats

(n.b. modulation means altering one waveform by adding a second waveform, in this case the result is an amplitude change related at a low frequency)

Interference of travelling waves 2: Frequency difference





or by harmonics

This type of wave interference can be useful for tuning a musical instrument to play the correct note. If the two notes are not perfectly in tune then a wobble in the amplitude is clearly heard. The more out of tune, the faster the wobble. When perfectly in tune the change in amplitude is eliminated.

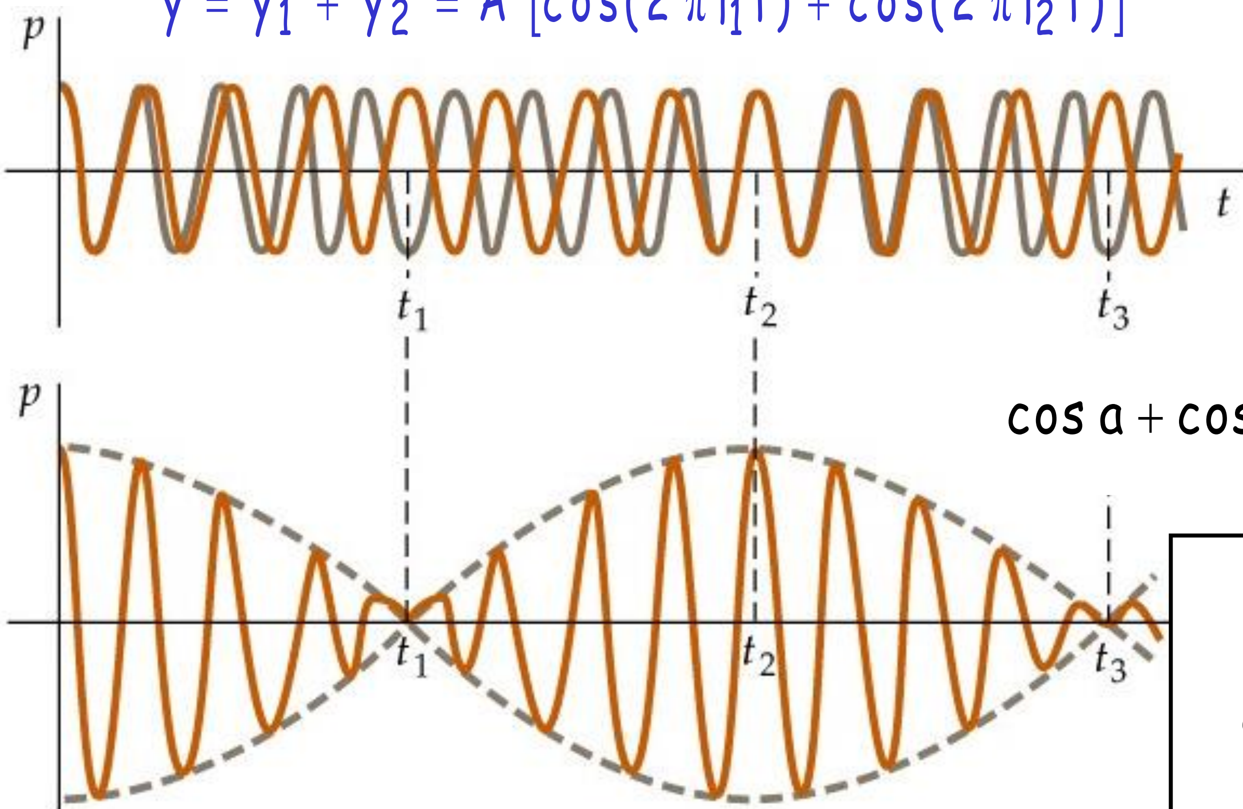
https://www.youtube.com/watch?v=hCFMbh2IsPQ

= BEATS

Beats

Consider two harmonic waves, passing a fixed point, that have slightly different frequencies. Their superposition is:

$$y = y_1 + y_2 = A\left[\cos(2\pi f_1 t) + \cos(2\pi f_2 t)\right]$$



Simplify using

$$\cos a + \cos b \equiv 2\cos\left(\frac{a-b}{2}\right)\cos\left(\frac{a+b}{2}\right)$$

A beat is heard when

$$\cos\left[2\pi t\left(\frac{f_1 - f_2}{2}\right)\right] = \pm 1$$

Two beats per cycle, so
beat frequency = f_1-f_2

$$y = 2A\cos\left[2\pi t\left(\frac{f_1 - f_2}{2}\right)\right]\cos\left[2\pi t\left(\frac{f_1 + f_2}{2}\right)\right]$$

slow amplitude modulation

rapidly oscillating carrier wave

summary

Superposition of two harmonic waves with equal amplitude & frequency

$$y = 2A\cos\left(\frac{\phi}{2}\right)\sin\left(kx - \omega t + \frac{\phi}{2}\right)$$

Superposition of two harmonic waves of slightly different frequencies

$$y = 2A\cos\left[2\pi t\left(\frac{f_1 - f_2}{2}\right)\right]\cos\left[2\pi t\left(\frac{f_1 + f_2}{2}\right)\right]$$