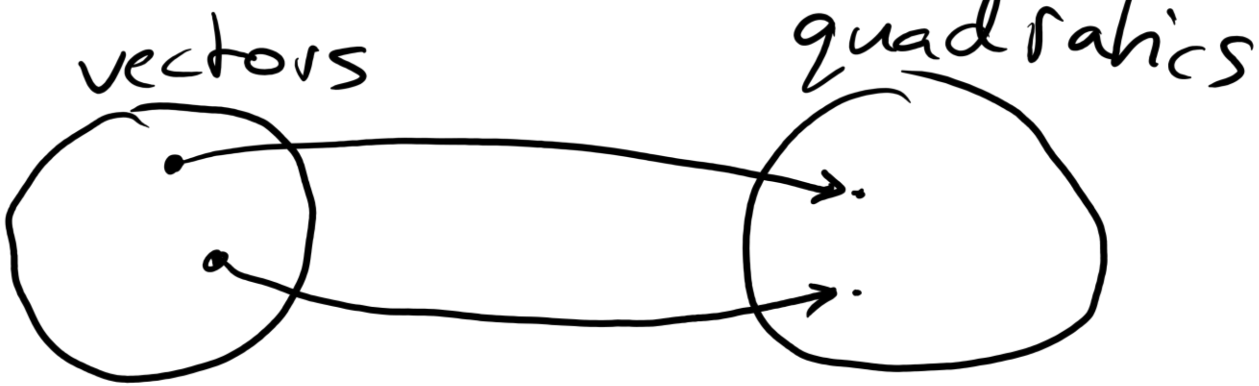
Functions

What is a Function?

- Map from set A to set B



$$\mathbb{R} \longrightarrow \mathbb{R}$$

Definitions

$f : A \longrightarrow B$

$f : x \longmapsto f(x)$

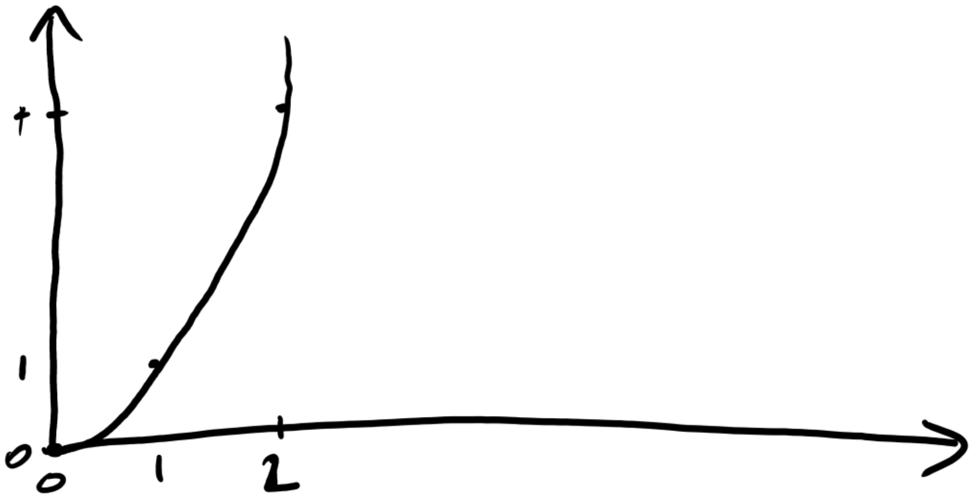
- Domain starting set A
- Codomain set mapped to (B)
- Range set of object in B actually mapped to
- Argument Particular point in A considered

Value = output

Graphs of Functions

- For functions $\mathbb{R} \to \mathbb{R}$, can graph functions as points $(x, f(x))$ in $\mathbb{R}^2$

Eg.
$f(x) = x^2$



- Sketching graphs is **still** a useful skill!

Key Features

- Intercept
- Roots
- Stationary Points
- Points of inflection
- Range
- Asymptotes

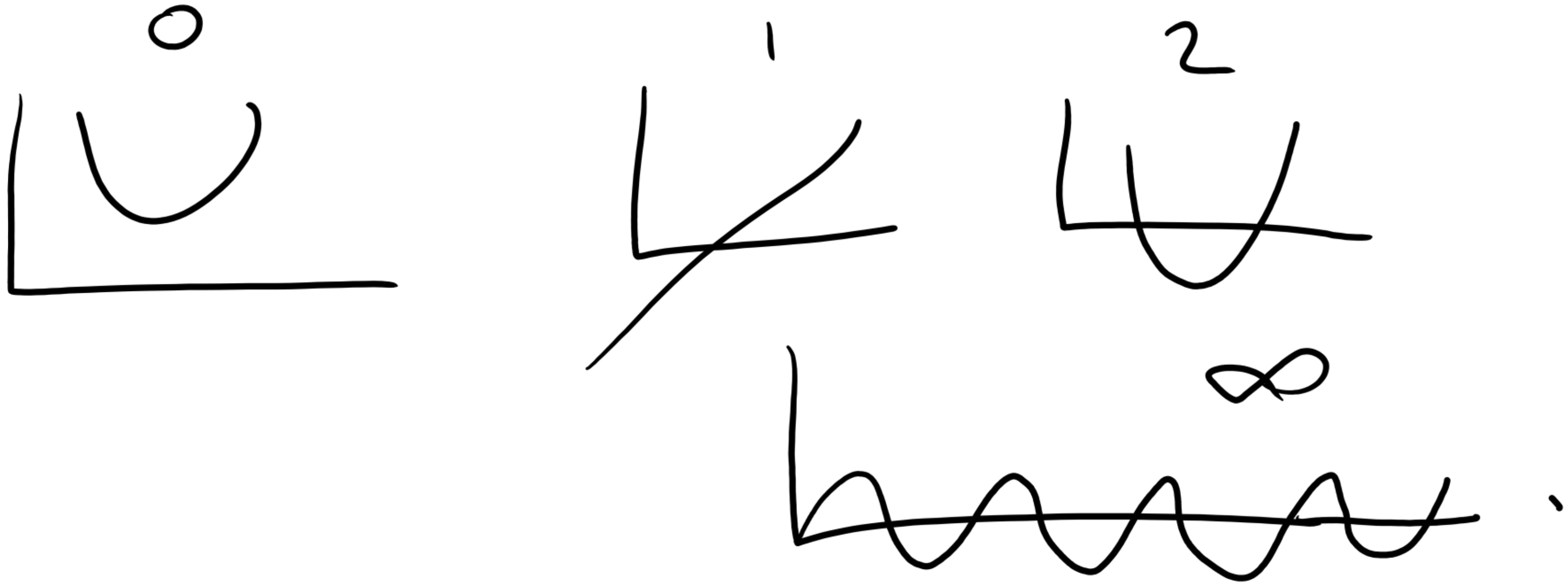
Intercept

- $f(0)$

Point where graph meets y axis

Roots

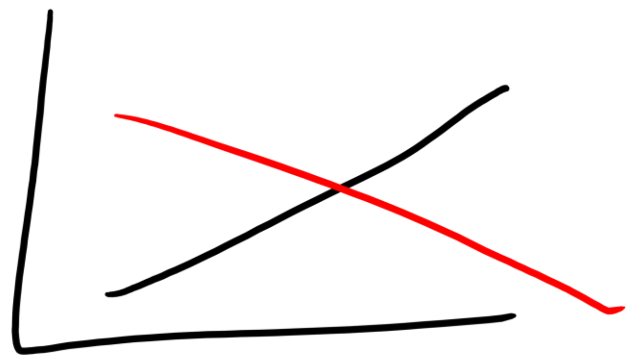
- Solutions of $f(x) = 0$
- Functions can have any number of roots from 0 to ∞



Stationary Points

- Solutions of $f'(x) = 0$

- Positive derivative $\Rightarrow$ increasing function
- Negative derivative $\Rightarrow$ decreasing function
- 0 derivative $\Rightarrow$ stationary function

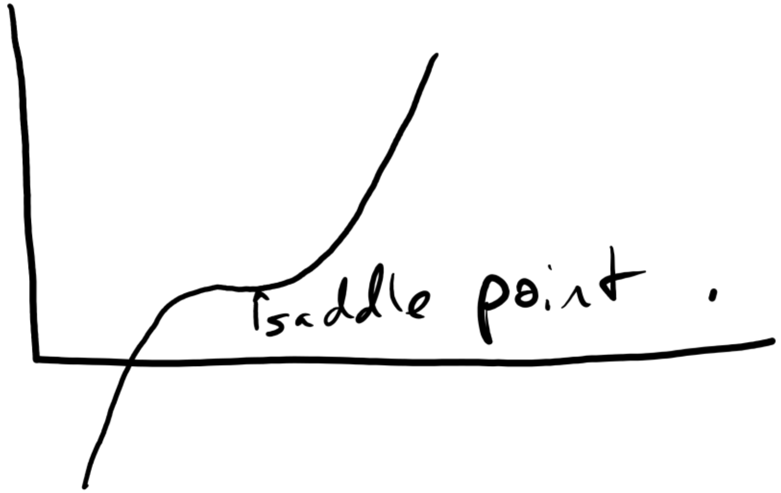


- Points where function is “instantaneously constant”

Stationary Points

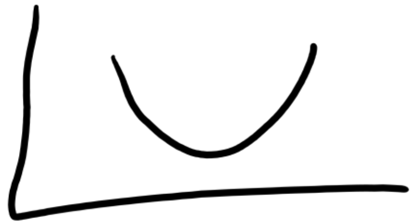
- Extremum: minimum or maximum

- Saddle point



Second Derivatives

- Positive 2^{nd} derivative ⇒ increasing slope ⇒ function curves up

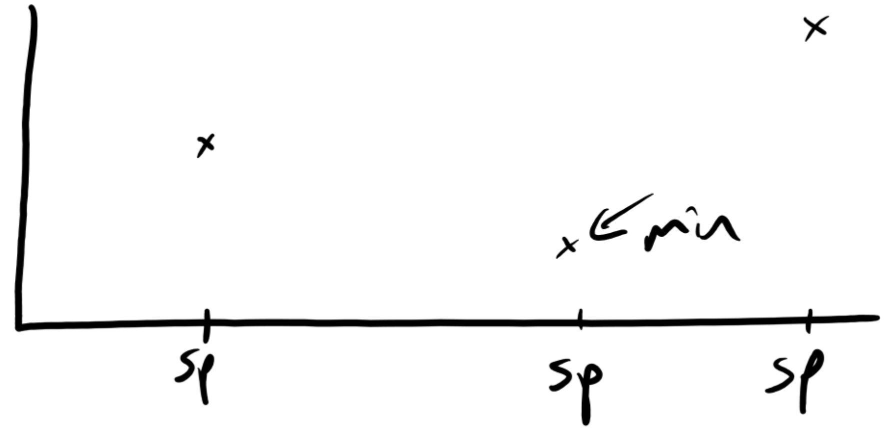


- Negative 2^{nd} derivative ⇒ decreasing slope ⇒ function curves down

- 0 2^{nd} derivative ⇒ stationary slope ⇒ change in curvature

Testing Stationary Points with 2nd Derivative

- For stationary point:
- $f''(x) > 0 \Rightarrow$ minimum
- $f''(x) < 0 \Rightarrow$ maximum
- $f''(x) = 0 \Rightarrow$
- Can determine by considering behaviour **between** stationary points



Points of Inflection

$$f''(x) = 0$$

- Saddle Points

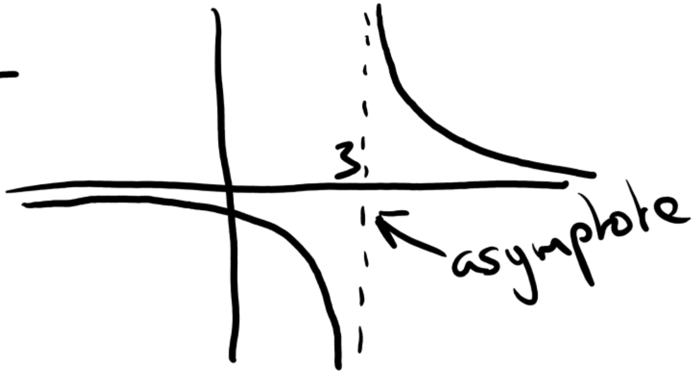
- Non-stationary points of inflection

Asymptotes

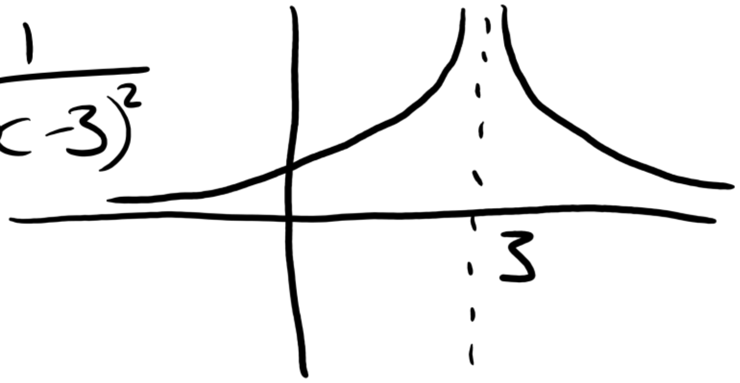
$f : \mathbb{R} \setminus \{3\} \to \mathbb{R}$

- Vertical asymptote / singular point / pole
  - ~~Value~~ Point not in domain
  - Value of function nearby tends to $\pm\infty$

Eg $\frac{1}{x-3}$



Eg $\frac{1}{(x-3)^2}$

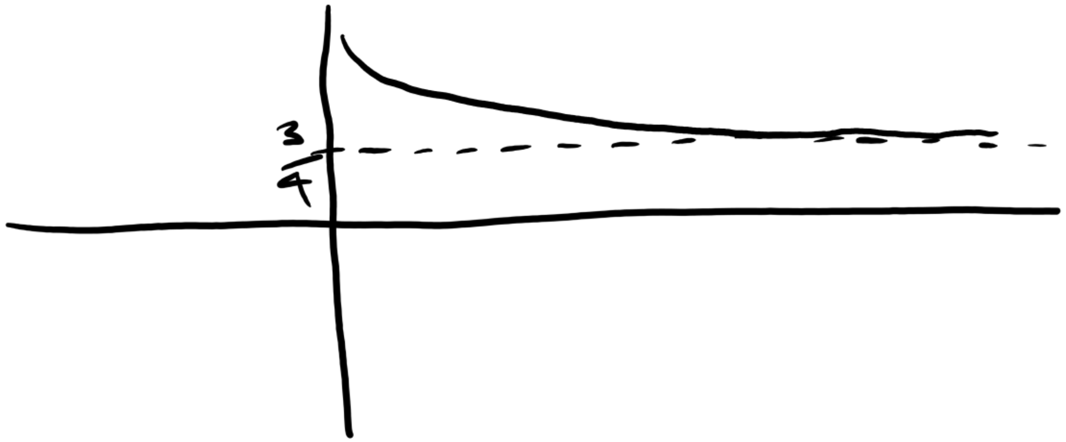


- Can tend to same or different values either side

Asymptotes

- Horizontal asymptote
  - Behaviour of function at large x tends to constant value

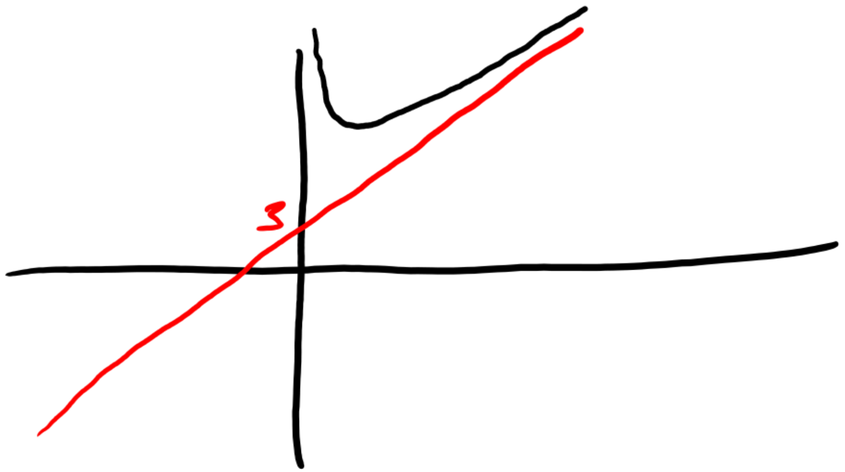
Eg $\quad f(x) = \dfrac{3x-1}{4x+7} = \dfrac{3-\frac{1}{x}}{4+\frac{7}{x}} \rightarrow \dfrac{3-0}{4+0} = \dfrac{3}{4}$



Asymptotes

- Oblique asymptote
  - Function tends to linear function at large x

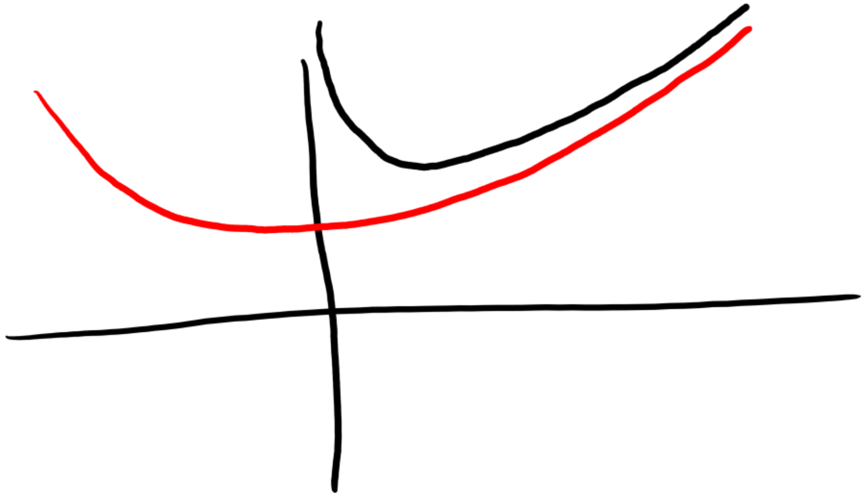
$$f(x) = \frac{x^2+3x+4}{x} = x+3+\frac{4}{x} \rightarrow x+3$$



Asymptotic Curves

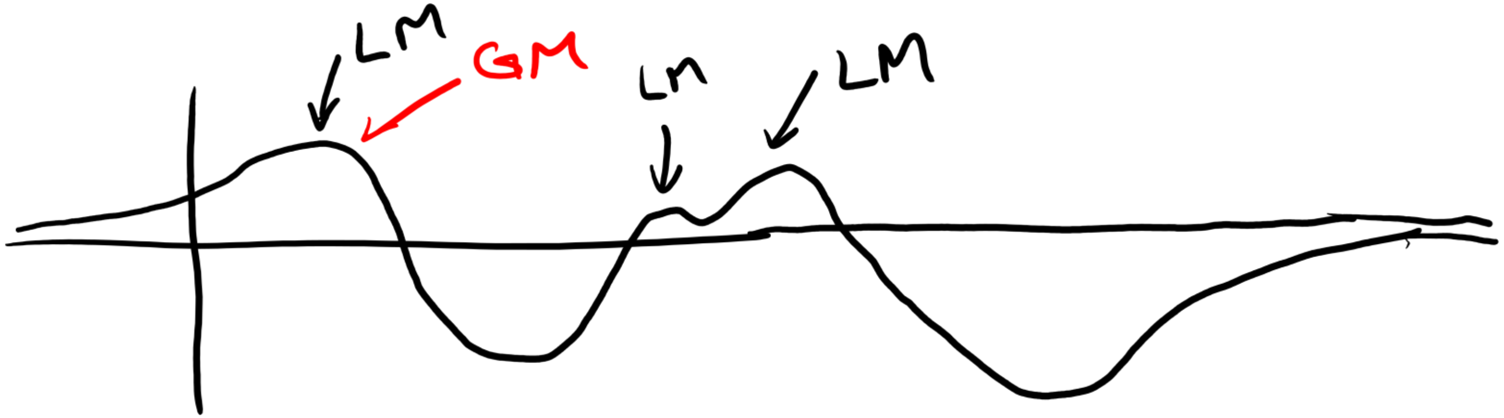
- More general case
- Function can be approximated by other simple function at large x

$$f(x)\ \frac{x^3+3x+4}{x} = x^2+3+\frac{4}{x} \longrightarrow x^2+3$$

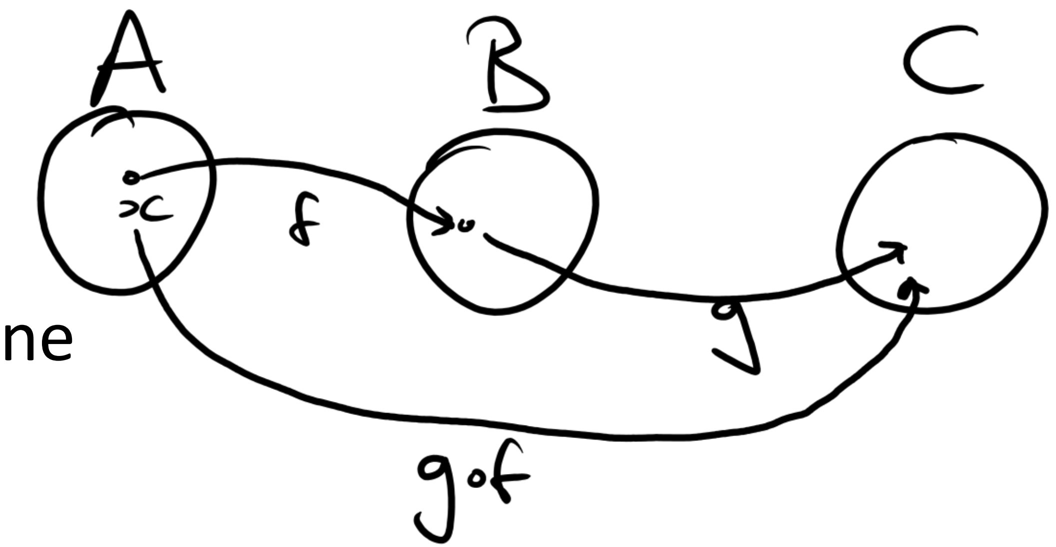


Range

- Consider
  - Behaviour at $\pm\infty$ (if infinite, range is infinite)
  - Any vertical asymptotes (if asymptotes exist, range is infinite)
  - Any stationary points
- If finite range, max of behaviour at $\pm\infty$ and **local maxima** is **global maximum**



Function Composition



- Given $f: A \to B$ and $g: B \to C$, define

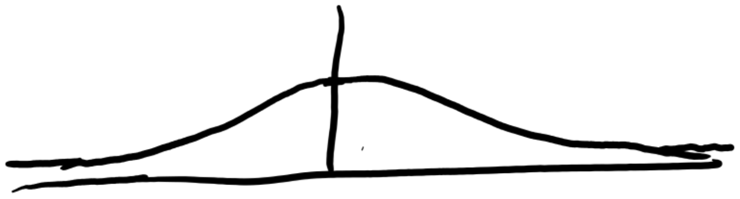
$$g \circ f : A \longrightarrow C$$

$$(g \circ f)(x) = g(f(x))$$

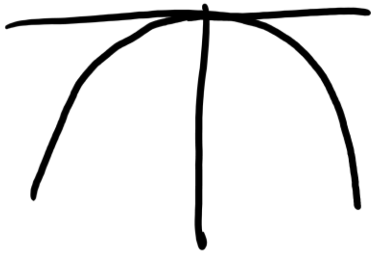
More Complex Sketching

- Can combine functions through
  - Addition $f(x) = g(x) + h(x)$
  - Multiplication $f(x) = g(x) \times h(x)$
  - Composition $f(x) = g(h(x))$

Eg. $f(x) = e^{-x^2}$



$h(x) = -x^2$



$g(x) = e^x$