PHAS1000 – THERMAL PHYSICS

Lecture 13

Adiabatic Processes

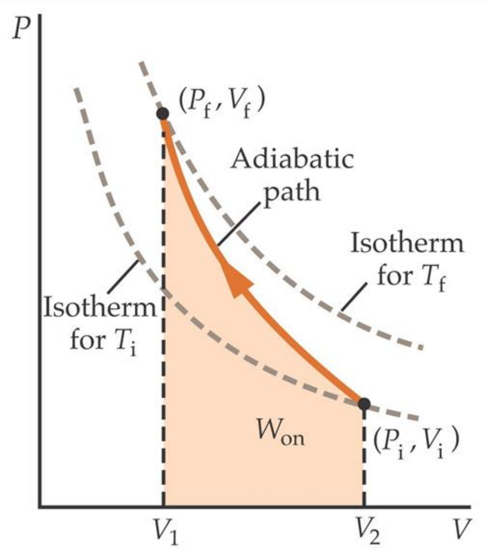
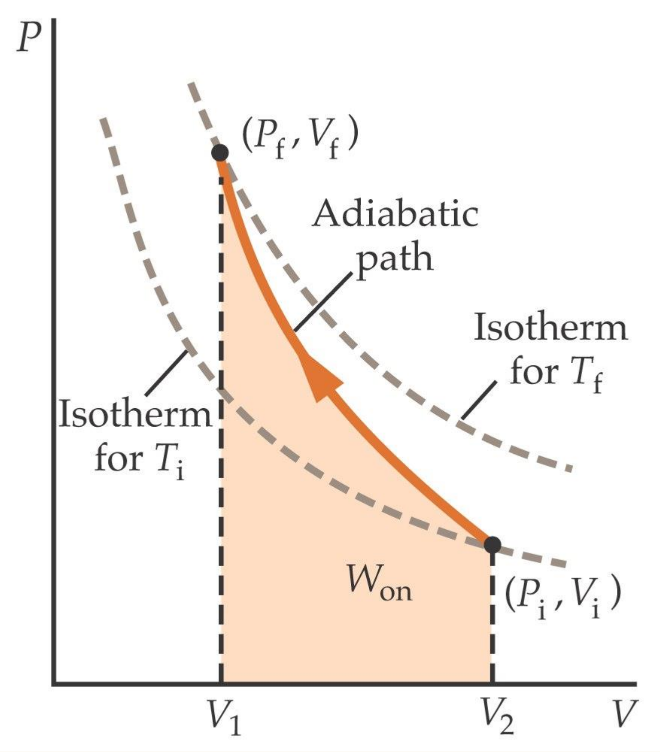


Image Tipler

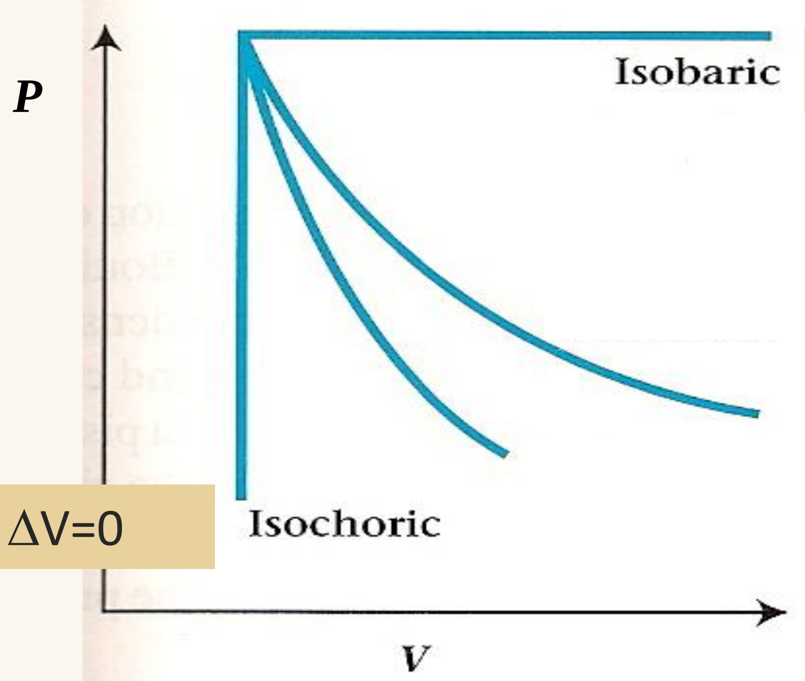
Overview

This lecture covers:

- ➢ Adiabatic process
- ➢ Combinations of processes



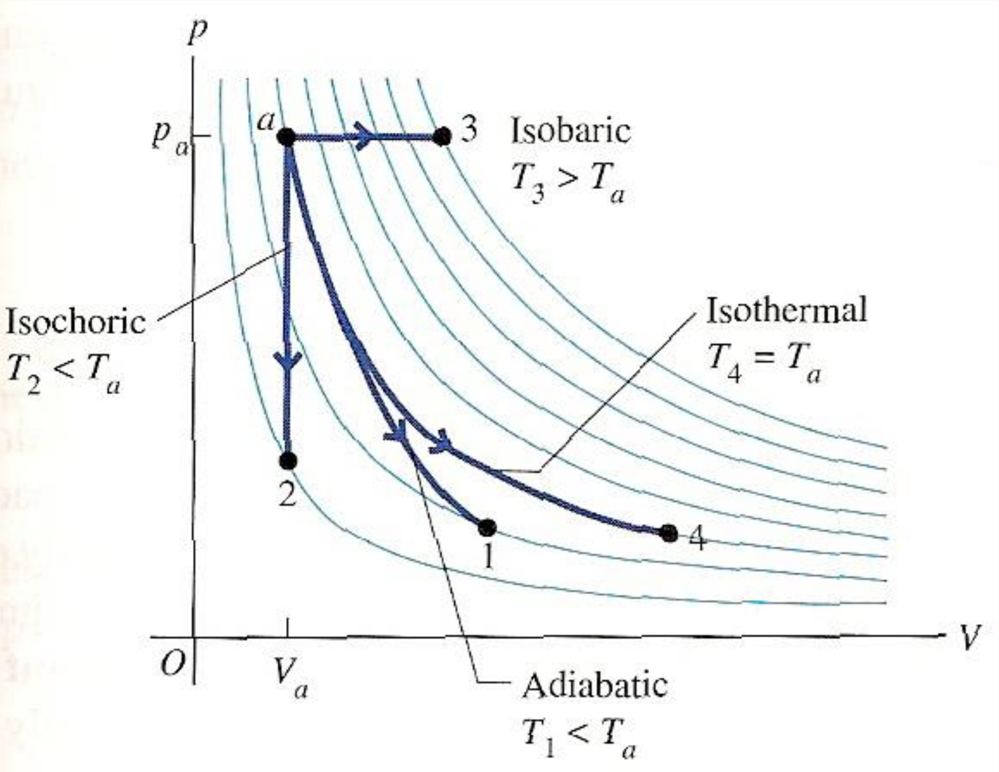
Different Processes



$\Delta P=0$

$\Delta V=0$

Isothermal	$\Delta T=0$
Adiabatic	$Q_{in}=0$



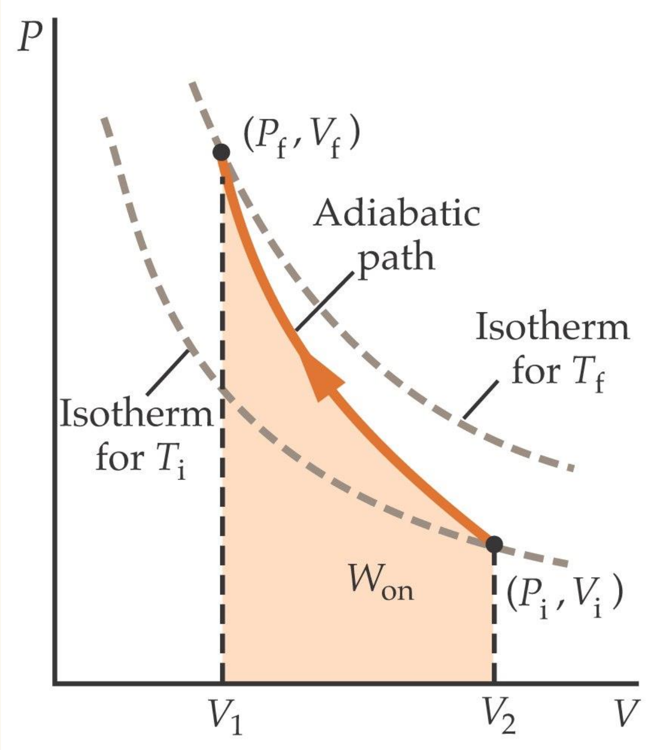
Adiabats are steeper than isothermals

Adiabatic compression

$Q_{in} = 0$ e.g. Insulated containers, rapid processes, large masses

Adiabatic Compression

http://www.youtube.com/watch?v=c4eZ3K1jHiA&NR=1



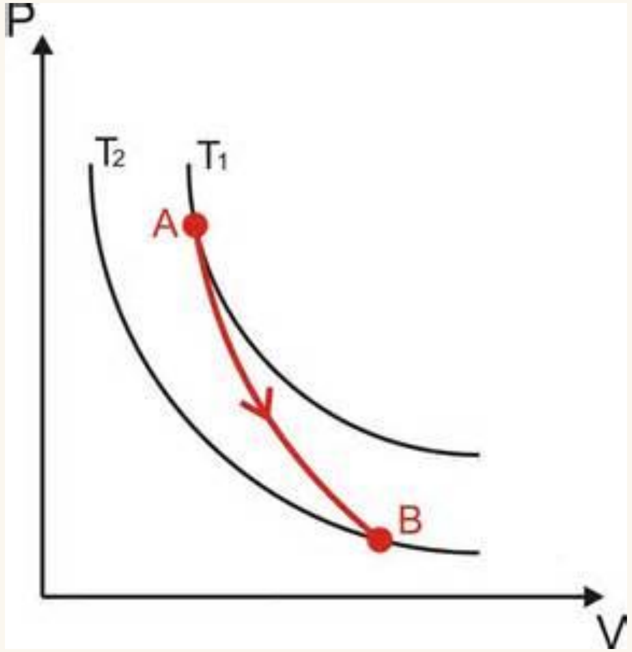
Adiabatic compression

V reduced
P increased
T increased

Adiabatic expansion

Adiabatic Expansion

[Popping Water Bottle Caps - YouTube]



Adiabatic expansion

V increased
P reduced
T reduced

Adiabatic – evaluating the work

1st law $Q_{in} = \Delta U + W_{by}$ but $Q_{in} = 0$ for adiabatic

$\therefore W_{by} = -\Delta U$

$$\boxed{W_{by} = -nc_v' \Delta T}$$

other ways to express this:

$W_{by} = -nc_v'(T_f - T_i) = +nc_v'(T_i - T_f)$

For ideal gas $PV = nRT$ so $T = \frac{PV}{nR}$

$\therefore W_{by} = \frac{\cancel{n}c_v'}{\cancel{n}R}\left(P_iV_i - P_fV_f\right)$

$$\boxed{W_{by} = \frac{c_v'}{R}\left(P_iV_i - P_fV_f\right)}$$

but Mayer's eqn gives $c_p' - c_v' = R$ and $\frac{c_p'}{c_v'} = \gamma$

$\therefore c_p' = \gamma c_v'$

$\therefore \gamma c_v' - c_v' = R$

$c_v'(\gamma - 1) = R$

or $\frac{c_v'}{R} = \frac{1}{(\gamma - 1)}$

$$\therefore \boxed{W_{by} = \frac{1}{(\gamma - 1)}\left(P_iV_i - P_fV_f\right)}$$

Summary of adiabatic

	ISOBARIC $\Delta P=0$	ISOCHORIC $\Delta V=0$	ISOTHERMAL $\Delta T=0$	ADIABATIC $Q_{in}=0$
W_{by}	$P\Delta V$	0	$nRTln\left(\frac{V_f}{V_i}\right)$	$\frac{1}{\gamma-1}\left(P_iV_i-P_fV_f\right)$
Q_{in}	$\Delta U+W_{by}$ $=nc_v'\Delta T+P\Delta V$	ΔU	W_{by}	0
ΔU	$nc_v'\Delta T$	$nc_v'\Delta T$	0	$nc_v'\Delta T$

Question

When an ideal gas is allowed to expand *isothermally* from volume V_1 to a larger volume V_2, the gas does an amount of work W_{12}.

If instead it was allowed to expand *adiabatically* from volume V_1 to V_2, the work done by the gas is....

A equal to W_{12}

B less than W_{12}

C greater than W_{12}

Answer

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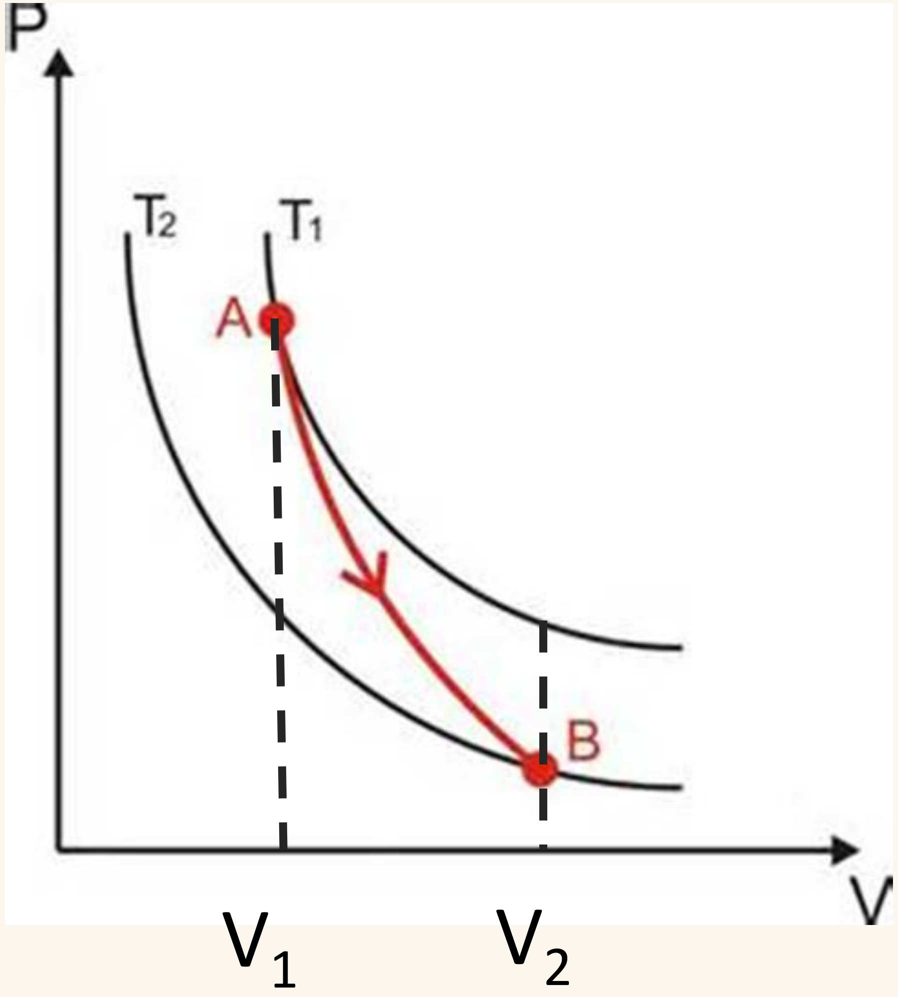

Expansion:
Between same volumes, area under adiabatic curve is less than under isothermal curve.

ANS B

Same Question – now about compression

When an ideal gas is compressed *isothermally* from volume V_1 to a smaller volume V_2, the gas does an amount of work W_{12}.

If instead it was compressed *adiabatically* from volume V_1 to V_2, the work done by the gas is....

A equal to W_{12}

B less than W_{12}

C greater than W_{12}

Answer

When an ideal gas is compressed *isothermally* from volume V_1 to a smaller volume V_2, the gas does an amount of work W_{12}.

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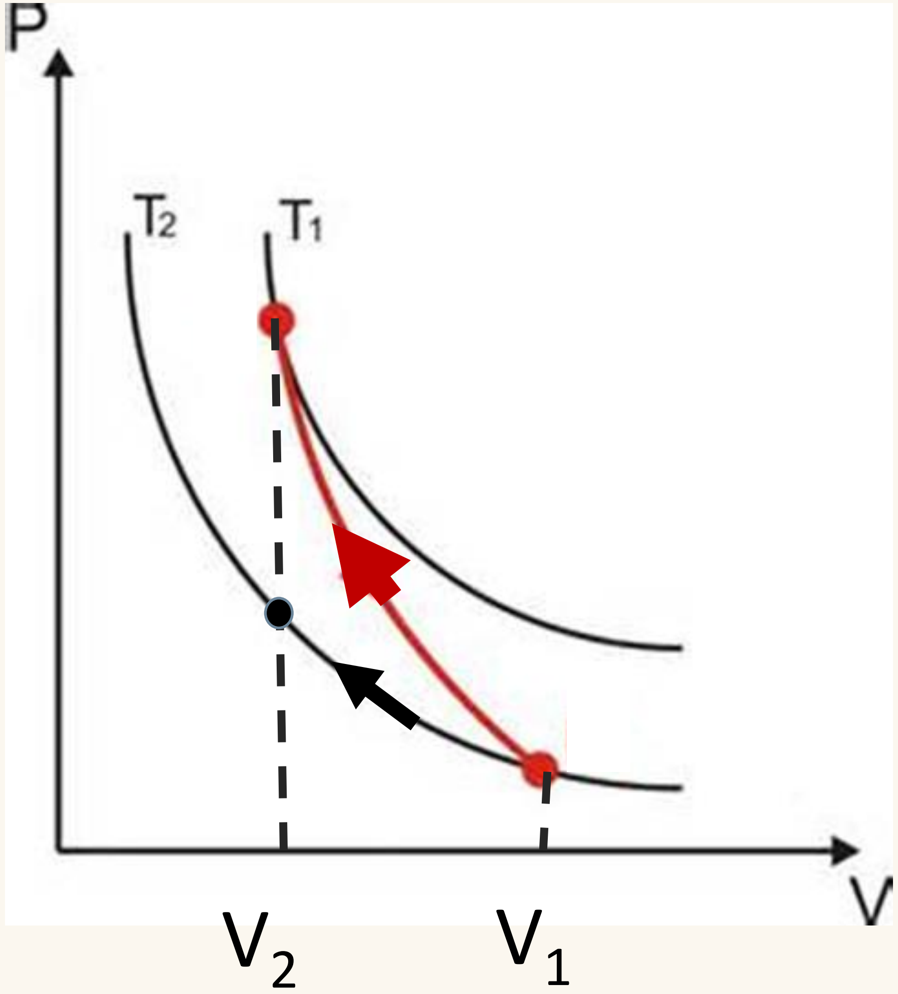
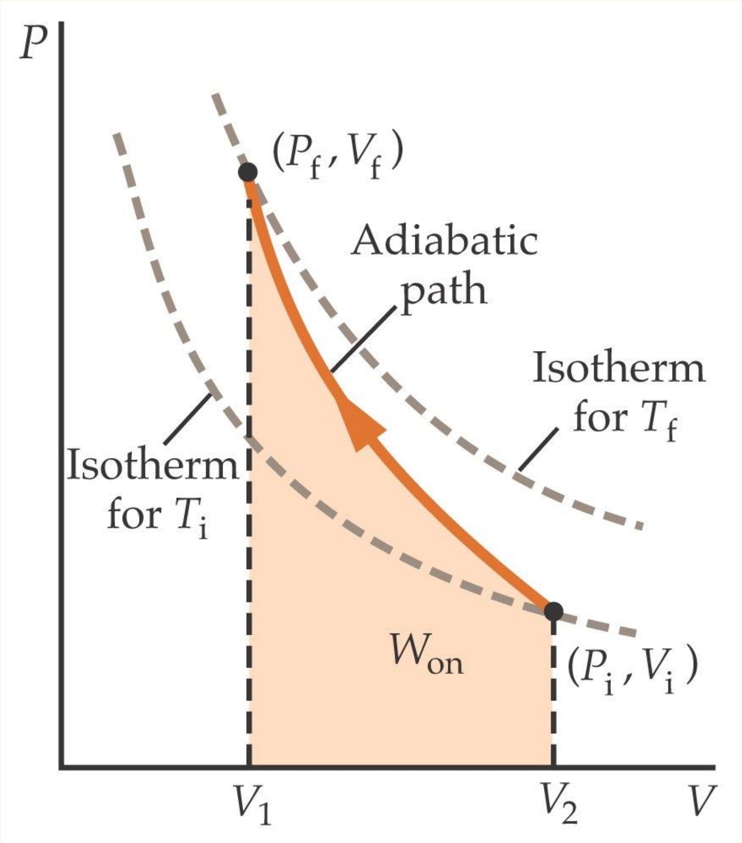


Compression:
Between same volumes, area under adiabatic curve is more than under isothermal curve.

ANS C

More about adiabatic processes

P, *V*, *T*, all change



$$PV = nRT$$

$$PV^{\gamma} = constant$$

$$TV^{\gamma-1} = constant$$

Deriving the adiabatic equations

For adiabatic processes we had $\Delta U = -W_{by}$ so for small changes we can write $dU = -dW_{by}$

But $\Delta U = nc_v'\Delta T$ so $dU = nc_v'dT$ and $W_{by} = P\Delta V$ so $dW_{by} = PdV$

So we have $nc_v'dT = -PdV$ or $nc_v'dT = -\frac{nRT}{V}dV$ (from the ideal gas equation)

Rearranging gives $\frac{dT}{T} = -\frac{R}{c_v'}\frac{dV}{V}$

From a previous slide $c_v'(\gamma - 1) = R$ and so we can substitute for $\frac{R}{c_v'} = (\gamma - 1)$

Thus $\frac{dT}{T} = -(\gamma - 1)\frac{dV}{V}$ or $\frac{dT}{T} = (1 - \gamma)\frac{dV}{V}$

Integrating $\int \frac{dT}{T} = (1 - \gamma)\int \frac{dV}{V}$ gives $ln\left(\frac{T_2}{T_1}\right) = (1 - \gamma)ln\left(\frac{V_2}{V_1}\right)$ and hence $\left(\frac{T_2}{T_1}\right) = \left(\frac{V_1}{V_2}\right)^{\gamma-1}$

Which can be expressed as $T_2V_2^{\gamma-1} = T_1V_1^{\gamma-1}$

Deriving the adiabatic equations - cont

$T_2V_2^{\gamma-1} = T_1V_1^{\gamma-1}$ = constant

Using the ideal gas equation we can derive the other adiabatic equation…….

$PV = nRT$ yields $T = \frac{PV}{nR}$ so substituting in equation at top ($TV^{\gamma-1}$ = constant) gives $\frac{PV}{nR}V^{\gamma-1} = constant$

Which simplifies to $PV^{\gamma} = constant$ i.e. $P_2V_2^{\gamma} = P_1V_1^{\gamma}$

Question 1

The engine of a Ferrari F355 F1 sports car takes in air at 20°C and 1 atm and compresses it adiabatically to 0.090 times the original volume. The air may be treated as an ideal gas with $\gamma = 1.40$.

a) Find the final temperature and pressure.

b) How much work is done on the gas per mole?

c) What is the change in internal energy of the gas per mole?

Answer

(a) Find the final temperature and pressure.

(a) $T_i = 20^\circ C \quad P_i = 1 atm$

$TV^{\gamma-1} = const \quad T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$

$$T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{\gamma-1} = (273 + 20)\left(\frac{1}{0.09}\right)^{(1.4-1)}$$

$$= \frac{293}{0.09^{0.4}} = 768K$$

$= 495^\circ C$

$PV^\gamma = const \quad P_1 V_1^\gamma = P_2 V_2^\gamma$

$$P_2 = P_1 \left(\frac{V_1}{V_2}\right)^\gamma = (1 atm)\left(\frac{1}{0.09}\right)^{1.4}$$

$$= P_2 = 29 atm$$

Answer cont

(b) How much work is done on the gas per mole?

(b) $W_{on} = (-W_{by}) = \frac{1}{(\gamma-1)}(P_2V_2 - P_1V_1)$

$PV = nRT$

$W_{on} = \frac{1}{(\gamma-1)}(nRT_2 - nRT_1)$

$W_{on} \text{ per mole} = \frac{R}{(\gamma-1)}(T_2 - T_1)$

$= \frac{8.31}{0.4}(495 - 20) = 9865\,J$

$9.9\,kJ/\text{mole}$

Answer cont

(c) What is the change in internal energy of the gas per mole?

(c) $Q_{in} = 0$ adiabatic

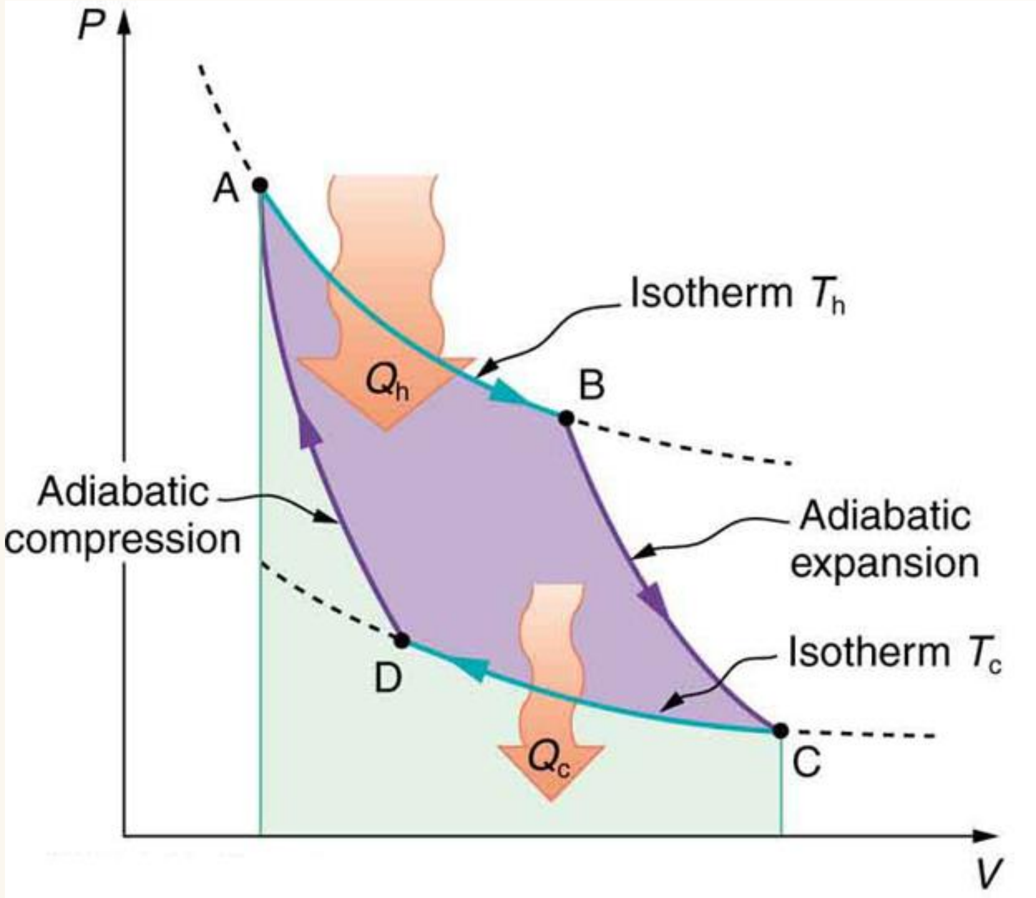
$Q_{in} = \Delta U + W_{by}$

$\Delta U = - W_{by}$

$= + W_{on}$ $\underline{9.9 \text{ kJ/mole}}$

Carnot Cycle

A Carnot cycle has 2 isothermal steps and 2 adiabatic steps.



$$Q_{in} = \Delta U + W_{by}$$

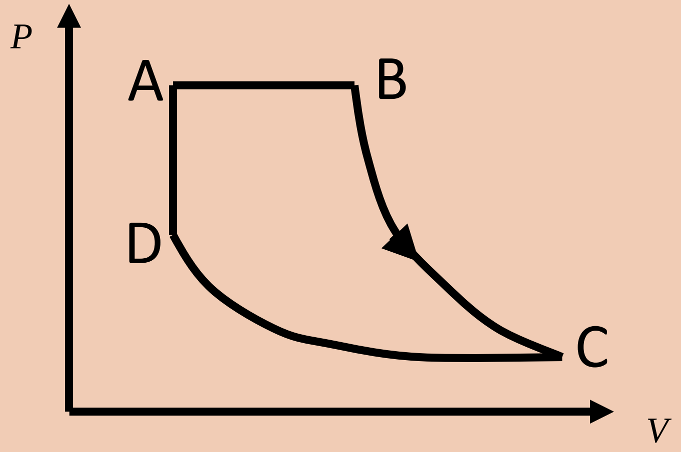
Heat enters or leaves the gas in the isothermal steps

Temperature falls or rises in adiabatic steps.

Work is done in all steps.

Everything you could possibly want to calculate from a P-V diagram

P-V cycle for an ideal gas with $\gamma = 1.40$



At A: V = 1L, P = 5 atm, T = 20°C

At B: V = 1.2L

At C: V = 3L

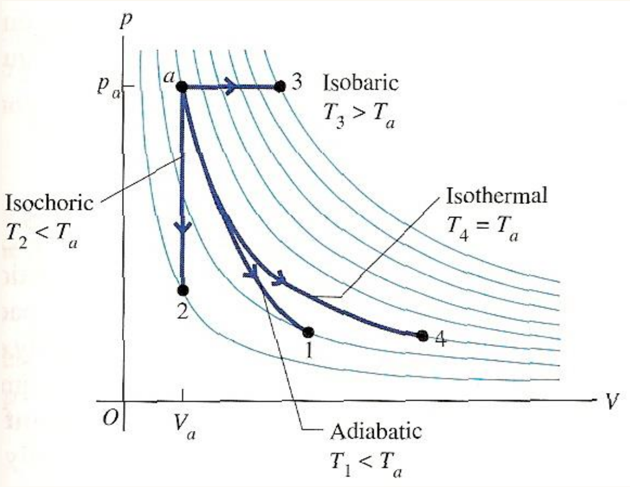
(a) By looking at the shape of the P-V diagram, label each step as ISOBARIC, ISOCHORIC, ISOTHERMAL or ADIABATIC (one of each).

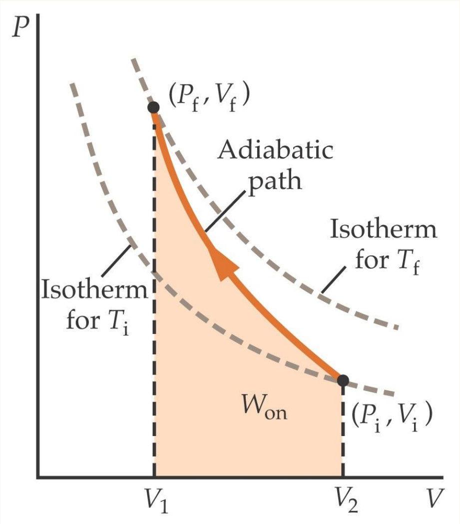
(b) Find all the missing P, V and T for each point (A, B, C, D)

(c) Find all the Q_{in}, W_{by} and ΔU for all steps (A-B, B-C, C-D, D-A)

(d) Find the net Q_{in}, W_{by} and ΔU for the complete cycle (A-B-C-D-A)

Summary





Isobaric $\Delta P = 0$
Isochoric $\Delta V = 0$
Isothermal $\Delta T = 0$
Adiabatic $Q_{in} = 0$

In an adiabatic process
P, V and T all change.

$$PV = nRT \qquad \gamma = \frac{c_p'}{c_v'}$$

$$PV^{\gamma} = const$$

$$TV^{\gamma - 1} = const$$

Questions

Question Q1

In an isothermal expansion, an ideal gas at an initial pressure P_0 expands until its volume has doubled.

(a) Find the pressure after the expansion.

(b) The gas is then compressed adiabatically back to its original volume, at which point the pressure is 1.32 P_0. Determine whether the gas is monatomic, diatomic or polyatomic.

ANSWERS

ANS Q1 In an isothermal expansion, an ideal gas at an initial pressure P_0 expands until its volume has doubled.

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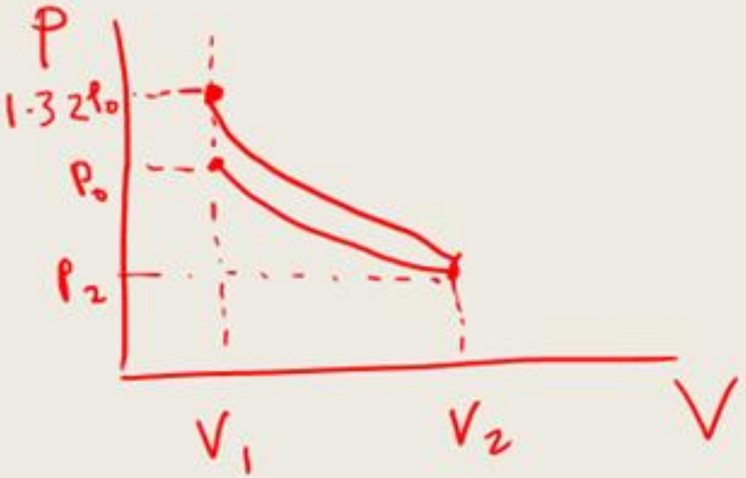


(a) $PV = nRT$ T const

$P_1V_1 = P_2V_2$

$P_2 = \frac{P_0}{2}$

(b) γ

adiabatic $PV^\gamma = \text{const}$

$P_2V_2^\gamma = P_3V_3^\gamma$

$$\frac{P_3}{P_2} = \left(\frac{V_2}{V_3}\right)^\gamma$$

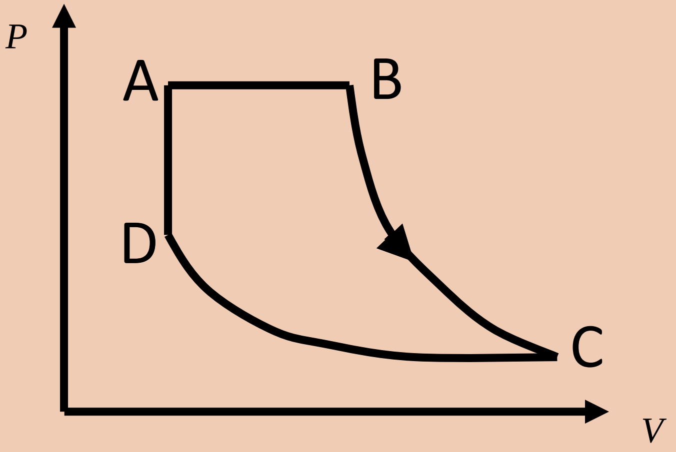
$$\ln\left(\frac{P_3}{P_2}\right) = \gamma \ln\left(\frac{V_2}{V_3}\right)$$

$$\gamma = \frac{\ln(P_3/P_2)}{\ln(V_2/V_3)} = \frac{\ln\left(\frac{1.32P_0}{P_0/2}\right)}{\ln 2} = 1.40$$

γ monatomic	$\frac{5}{3}$	1.67
diatomic	$\frac{7}{5}$	1.40
polyatomic	$\frac{8}{6}$	$= 1.33$

Everything you could possibly want to calculate from a P-V diagram

P-V cycle for an ideal gas with $\gamma = 1.40$



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(b) Find all the missing P, V and T for each point (A, B, C, D)

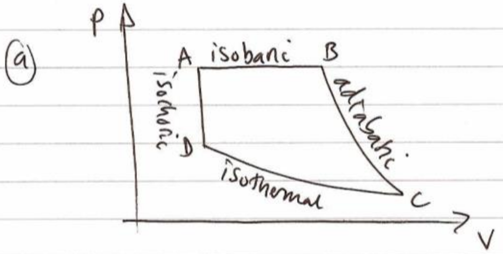
(c) Find all the Q_{in}, W_{by} and ΔU for all steps (A-B, B-C, C-D, D-A)

(d) Find the net Q_{in}, W_{by} and ΔU for the complete cycle (A-B-C-D-A)

Answer

Everything you could possibly want to calculate from a P-V diagram !

(a)



adiabatic always steeper than isothermal

(b) For isobaric, isochoric, isothermal $PV = nRT$

& $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$

For adiabatic $PV^{\gamma} = \text{const}$ $TV^{\gamma-1} = \text{const}$

	P	V	T
A	5 atm	1L	293 K
B	5 atm	1.2L	352 K
C	1.39 atm	3L	244 K
D	4.17 atm	1L	244 K

See calculations below

using data at A to find out about B

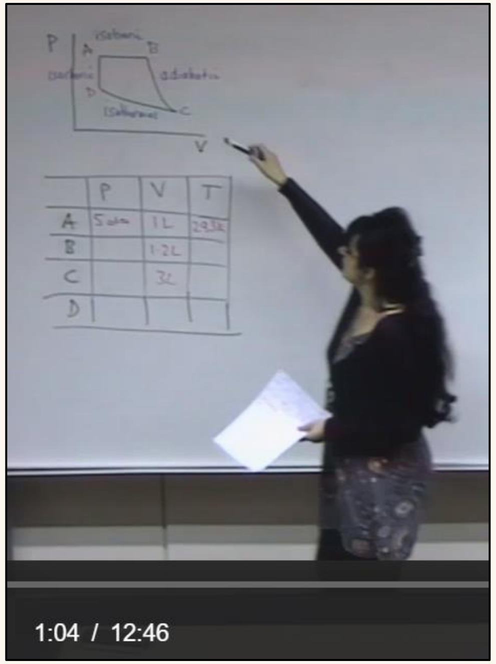
$P_A = P_B$ $\frac{V_A}{T_A} = \frac{V_B}{T_B}$ $T_B = \frac{V_B T_A}{V_A} = \frac{1.2}{1} \times 293 = 352K$

Using data at B to find out about C

$P_BV_B^{\gamma} = P_CV_C^{\gamma}$ $P_C = P_B\left(\frac{V_B}{V_C}\right)^{\gamma} = 5 \times \left(\frac{1.2}{3}\right)^{1.4} = 1.39 \text{ atm}$

$T_BV_B^{\gamma-1} = T_CV_C^{\gamma-1}$ $T_C = T_B\left(\frac{V_B}{V_C}\right)^{\gamma-1} = 352\left(\frac{1.2}{3}\right)^{0.4} = 244K$

I recorded myself solving this a few years ago and you can watch on link below.
https://mymedia.leeds.ac.uk/Mediasite/Play/2f82d93db53140bfa129cadff9a4cf5b1d
Note in the video I used W_{on} rather than W_{by}, so all signs for work will be opposite to what you use.
And I used ΔE_{int} instead of ΔU for change in internal energy.



using data at C and A to find out about D

$V_D = V_A$ $T_D = T_C$ (isothermal)

$P_CV_C = P_DV_D$ $P_D = P_C\frac{V_C}{V_D} = 1.39 \times \frac{3}{1} = 4.17 \text{ atm}$

(c)

	Q_{in}	W_{by}	ΔE_{int}
AB	358 J	101 J	257 J
BC	0	464 J	−471 J
CD	−464 J	−464 J	0
DA	214 J	0	214 J
TOTAL	108 J	101 J	0

$\rightarrow W_{by} = -\Delta E_{int}$ within rounding errors

within rounding errors

WORK DONE

$P_{atm} = 1.013\times10^5\ Pa$ $\quad 1L = 10^{-3}\ m^3$

(AB) $W_{by} = P\Delta V = -5\times1.013\times10^5 \times (1.2-1)\times10^{-3}$
$= \boxed{101\ J}$

(BC) $W_{by} = \frac{1}{\gamma-1}\left(P_BV_B - P_CV_C\right) = \frac{1.013\times10^5}{0.4}\times\left(5\times1.2 - 1.39\times3\right)\times10^{-3}$

$= \boxed{464\ J}$

(CD) $W_{by} = nRT\ln\left(\frac{V_D}{V_C}\right) = P_CV_C\ln\left(\frac{V_D}{V_C}\right) = 1.39\times1.013\times10^5\times3\times10^{-3}\ln\left(\frac{1}{3}\right)$
$= \boxed{-464\ J}$

use $nRT = PV$

or use this to calculate number of moles $(= 0.2)$

ΔE_{int}

$\boxed{\frac{C_p}{C_v} = \gamma}$ $\boxed{C_p - C_v = nR}$ $\therefore \boxed{C_v = \frac{nR}{\gamma-1}}$

(AB) $\Delta E_{int} = C_v\Delta T$

$\Delta E_{int} = \frac{nR}{\gamma-1}\Delta T$

$n = \frac{PV}{RT} = \frac{5\times1.013\times10^5\times10^{-3}}{8.31\times293} = \boxed{0.21\ \text{moles}}$

(evaluated from pt A)

$\therefore \Delta E_{int} = \frac{0.21\times8.31\times(352-293)}{0.4}$

$= 257\ J$

(BC) $\Delta E_{int} = C_v\Delta T = \frac{nR\Delta T}{\gamma-1} = \frac{0.21\times8.31\times(244-352)}{0.4}$

$= -471\ J$ (or should be same as W_{by} within rounding errors

(DA) $\Delta E_{int} = C_v\Delta T = \frac{nR\Delta T}{\gamma-1} = \frac{0.21\times8.31\times(293-244)}{0.4}$

$= 214\ J$

$$Q_{in} = \Delta E_{int} + W_{by}$$

For complete cycle total up for each step:

within rounding errors $\Delta E_{net} = 0$

$$(Q_{in})_{net} = (W_{by})_{net}$$