Lecture 4: AC Circuits and Impedance

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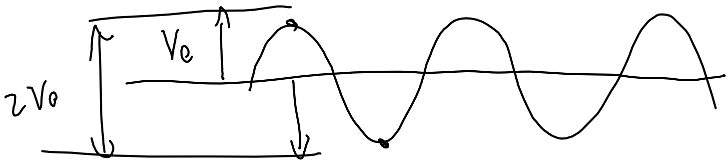
Introduction to AC Circuits

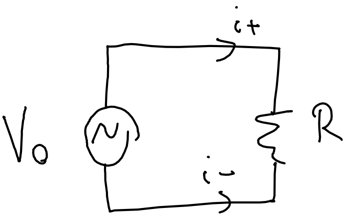
In Alternating Current (AC) circuits, the current and voltage vary sinusoidally over time. A typical AC voltage source can be expressed as:

$$V(t) = V_0 \sin(\omega t)$$

where:

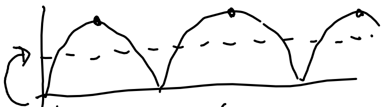
- V_0 is the peak voltage (maximum amplitude),
- ω is the angular frequency in radians per second ($\omega = 2\pi f$, where f is the frequency in Hz),
- t is the time in seconds.





how do we calculate the power lost in the resistor?

$$P = I^2 R = VI = \frac{V^2}{R}$$



$$V_{RMS} = V_0/\sqrt{2}$$

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The root-mean-square (RMS) value of the voltage is given by:

$$V_{\mathrm{RMS}} = \frac{V_0}{\sqrt{2}}$$

Do it Yourself: AC Voltage and Frequency

Exercise:

Given an AC source with a peak-to-peak voltage of 680V and a frequency of 50Hz:

1. Write the equation for $V(t)$. $= V_0 \sin(\omega t)$
2. Calculate the RMS voltage.

$$= \frac{680}{2} \sin(100\pi t)$$

$$V_{RMS} = 240V.$$

Inductors and Capacitors: Opposition to Change

Key Concepts:

- Inductors oppose changes in **current**.
- Capacitors oppose changes in **voltage**.

Equations:

Capacitor	Inductor
$i = C\frac{dV}{dt}$	$v = L\frac{di}{dt}$

$$\frac{V}{R} = C\frac{dV}{dt}$$

$$IR = L\frac{dI}{dt}$$

$$\frac{1}{RC}V = \frac{dV}{dt}$$

guess $V = e^{-t/\tau}$

$\tau = RC$

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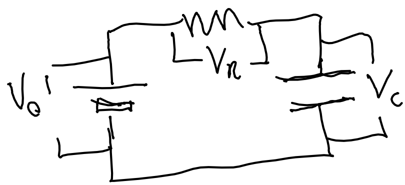
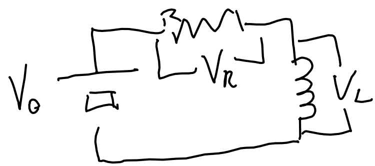




$V_0 = V_C$

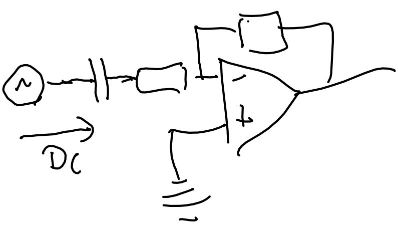
$V_R = 0$

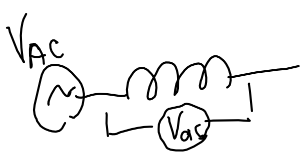
$I = 0$

$V_L = 0$

$V_R = V_0$

$I = \frac{V_0}{R}$





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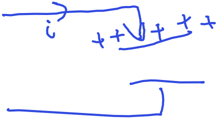
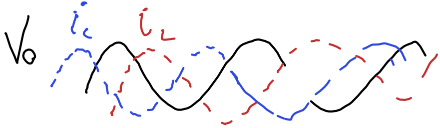
Passes High Frequency	Blocks High Frequency
reactance: $X_c = \frac{1}{j\omega C} = \frac{-j}{\omega C}$	reactance: $X_L = j\omega L$

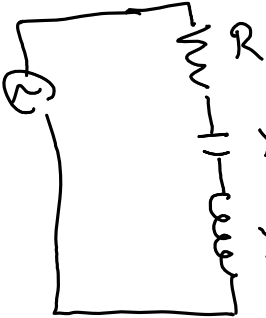
$X_c = \frac{1}{j\omega C}$

Impedance in RLC Circuits





reactance: $X_c = \frac{1}{j\omega C} = \frac{-j}{\omega C}$, $X_L = \underline{j\omega L}$



$X_C = \frac{-j}{\omega C}$

$X_L = j\omega L$

$$X_{LC} = X_C + X_L = j\omega L - \frac{j}{\omega C} = j\left(\omega L - \frac{1}{\omega C}\right)$$

$$X = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

Impedance $\Rightarrow Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$

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Impedance, denoted by Z, is the total opposition to current flow in an AC circuit. The impedance of a series RLC circuit is given by:

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

where:

- R is the resistance in ohms (Ω),
- L is the inductance in henrys (H),
- C is the capacitance in farads (F),
- ω is the angular frequency of the AC source.

Do it Yourself: Impedance Calculation

Exercise:

For a series RLC circuit with:

$$R = 50\,\Omega, \quad L = 0.2\,\text{H}, \quad C = 20\,\mu\text{F}, \quad f = 60\,\text{Hz}$$

1. Calculate inductive reactance X_L and capacitive reactance X_C.
2. Determine the total impedance Z.

Resonance and Bandwidth

Resonance and Bandwidth

Resonance occurs when the inductive reactance X_L and capacitive reactance X_C cancel each other out, i.e.,

$$\omega L = \frac{1}{\omega C}$$

At resonance, the impedance of the circuit is purely resistive ($Z = R$) and the resonant frequency is given by:

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

Do it Yourself: Resonant Frequency

Exercise:

For a series RLC circuit with:

$$L = 1\,\mathrm{H}, \quad C = 100\,\mu\mathrm{F}$$

Calculate the resonant frequency f_0.

Example 1: Series RLC Circuit with Sinusoidal Input

Consider a series RLC circuit with:

$$R = 10\,\Omega, \quad L = 0.1\,\mathrm{H}, \quad C = 100\,\mu\mathrm{F}$$

The circuit is driven by a sinusoidal voltage source

$$V(t) = 50\sin(1000t).$$

Task:

- Calculate the total impedance.
- Find the current in the circuit.

Solution

Reactance of each component:

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$$Z_R = 10\,\Omega, \quad X_L = j100\,\Omega, \quad X_C = -j10\,\Omega$$

Total reactance:

Solution

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$$Z_R = 10\,\Omega, \quad X_L = j100\,\Omega, \quad X_C = -j10\,\Omega$$

Total reactance:

$$Z_{\text{total}} = Z_R + X_L + X_C = 10 + j90\,\Omega$$

Magnitude of impedance:

Solution

Reactance of each component:

$$Z_R = 10\,\Omega, \quad X_L = j100\,\Omega, \quad X_C = -j10\,\Omega$$

Total reactance:

$$Z_{\text{total}} = Z_R + X_L + X_C = 10 + j90\,\Omega$$

Magnitude of impedance:

$$|Z_{\text{total}}| = \sqrt{10^2 + 90^2} = 90.55\,\Omega$$

Current in the circuit:

Solution

Reactance of each component:

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Magnitude of impedance:

$$|Z_{\text{total}}| = \sqrt{10^2 + 90^2} = 90.55\,\Omega$$

Current in the circuit:

$$I(t) = \frac{V(t)}{|Z_{\text{total}}|} = 0.552\sin(1000t)\,\text{A}$$

Applications of AC Circuits

- **Power Distribution:** AC is used in power systems for ease of voltage transformation.
- **Tuned Circuits:** RLC circuits are used in communication systems for tuning to specific frequencies.
- **Filters:** AC circuits with capacitors and inductors filter unwanted frequencies in signal processing.