UNIVERSITY OF LEEDS

Mechanics 1

Session 11 – Momentum and Its Conservation

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Last Lecture

Energy Conservation

We learned:

- How relative motion works in classical physics
- That acceleration, and therefore force, is sometimes measured differently in different reference frames
- To derive the location of the centre of mass of a collection of particles

You should be able to:

- Calculate the relative speed between two moving objects
- Calculate the centre of mass of a collection of particles
- Calculate the velocity (and potentially the acceleration) of a collection of particles using its centre of mass dynamics

This Lecture

Momentum

We will:

- Understand what momentum is conceptually
- See that momentum is a vector
- Understand why momentum is always conserved following collisions
- See that everything becomes mathematically easier in the centre of mass reference frame

You will be able to:

- Use the concept of momentum conservation to calculate the subsequent kinetic properties (velocities) following a collision

Momentum

What is it?

Momentum

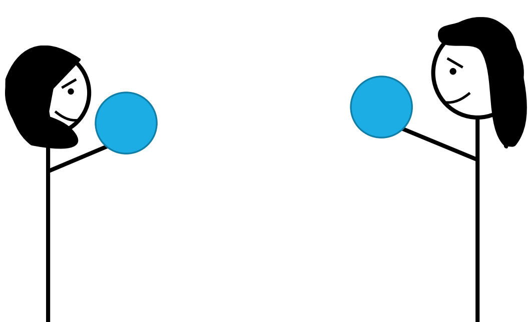
What is it?

The momentum of an object is, simply, the mass multiplied by the velocity.

But because it is a conserved quantity, it has a wealth of uses in physics calculations.

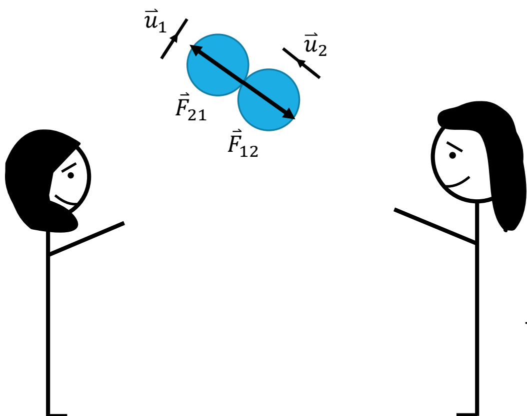
Momentum

Why is it Conserved?



Momentum

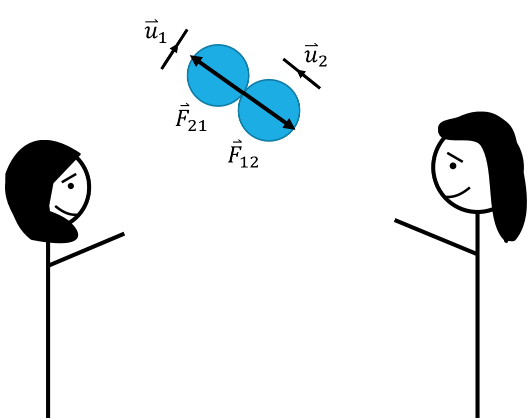
Why is it Conserved?



Impulse over small (collision) time,	$I = \vec{F}\Delta t$
Impulse is also momentum change,	$I = \Delta\vec{p}$
Combine,	$\Delta\vec{p} = \vec{F}\Delta t$
Object 1,	$\Delta\vec{p}_1 = \vec{F}_{21}\Delta t$
Object 2,	$\Delta\vec{p}_2 = \vec{F}_{12}\Delta t$
Total momentum change,	$\Delta\vec{p}_T = \Delta\vec{p}_1 + \Delta\vec{p}_2$
	$\Delta\vec{p}_T = \vec{F}_{21}\Delta t + \vec{F}_{12}\Delta t$

Momentum

Why is it Conserved?

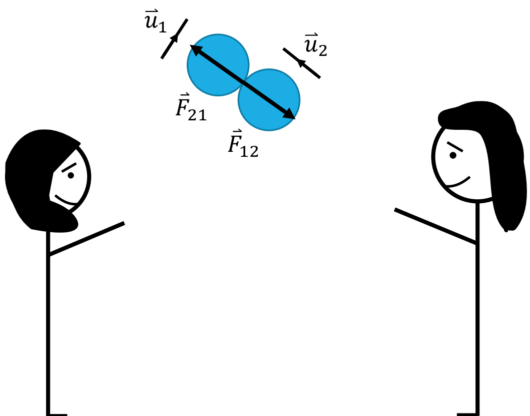


Total momentum change,	$\Delta\vec{p}_T = \vec{F}_{21}\Delta t + \vec{F}_{12}\Delta t$
Factorise,	$\Delta\vec{p}_T = \left(\vec{F}_{21} + \vec{F}_{12}\right)\Delta t$
Newton's 3rd Law,	$\vec{F}_{21} = -\vec{F}_{12}$
Hence,	$\boxed{\Delta\vec{p}_T = 0}$

The change in the total momentum of all objects after a collision, $\Delta\vec{p}_T$, is zero. Hence, momentum is conserved throughout collisions!

Momentum

The Full Equations



Hence, $\Delta\vec{p}_T = 0$

Initially, $\vec{p}_T = m_1\vec{u}_1 + m_2\vec{u}_2$

After collision, $\vec{p}_T = m_1\vec{v}_1 + m_2\vec{v}_2$

Hence, $(m_1\vec{v}_1 + m_2\vec{v}_2) - (m_1\vec{u}_1 + m_2u_2) = 0$

$$\boxed{m_1\vec{u}_1 + m_2\vec{u}_2 = m_1\vec{v}_1 + m_2\vec{v}_2}$$

In general, for N colliding objects,

$$\boxed{\sum_{i=1}^{N} m_i\vec{u}_i = \sum_{i=1}^{N} m_i\vec{v}_i}$$

Task 1

Momentum Calculations

Task 1

Momentum Calculations

Scenario: Two children are playing marbles. A circle with radius $R = 5cm$ is drawn on the ground in chalk, and the first child's marble, mass $m_1 = 100g$ and radius $R_1 = 0.75cm$, is placed at the centre of the circle. The second child rolls their larger marble, mass $m_2 = 250g$ and radius $R_2 = 1.25cm$ toward the first and hits it. Just before it hits, the second marble has velocity $\vec{u}_2 = 30cms^{-1}\vec{j}$. Following the collision, the second marble has a new velocity $\vec{v}_2 = 2.8cms^{-1}\vec{\imath} + 16.6cms^{-1}\vec{j}$.

Tasks:

1. Calculate the velocity of the first marble after the collision.
2. If the ground has co-efficient of kinetic friction $\mu_k = 0.15$, will the first marble still be in the circle when it comes to a stop? *Hint: You can treat a rolling marble as a simple sliding marble for this question (i.e. friction acts as normal).*

Task 2

An Interesting Derivation with Energy Conservation

Task 2

An Interesting Derivation with Energy Conservation

When (kinetic) energy is also conserved during a collision, it is called an "elastic collision".

Tasks:

1. By considering both conserved kinetic energy and momentum during a 1D collision between two objects with initial speeds u_1 and u_2, and final speeds v_1 and v_2, show that:

$$\frac{v_2 - v_1}{u_1 - u_2} = 1$$

Task 3

Newton's Cradle

Task 3

Newton's Cradle

Tasks:
1. Explain how this device works from fundamental principles. Use concepts such as impulse and momentum conservation.
2. Prove to yourself mathematically how this device works
3. Would this device continue to move forever? If not, why not?

Momentum & Centre of Mass

Frames of Reference

Momentum & Centre of Mass

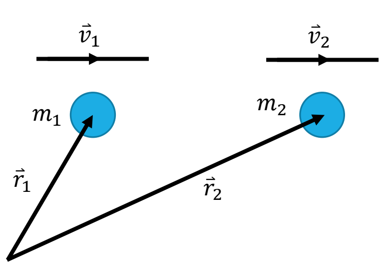
Frames of Reference

The laws of physics are fundamentally the same in all inertial reference frames. But some are easier to deal with mathematically than others

Let's switch to the centre of mass reference frame and see what happens

Momentum & Centre of Mass

Frames of Reference



$$M_T \vec{r}_{CM} = \sum_{i=1}^{N} m_i \vec{r}_i \qquad M_T \vec{v}_{CM} = \sum_{i=1}^{N} m_i \vec{v}_i$$

$$\vec{r}_{CM} = \frac{1}{M_T} \sum_{i=1}^{N} m_i \vec{r}_i \qquad \vec{v}_{CM} = \frac{1}{M_T} \sum_{i=1}^{N} m_i \vec{v}_i$$

Recall that the centre of mass is the location at which all internal forces cancel out, and the only forces that appear to be acting on the CoM are external forces (i.e. forces from outside the system)

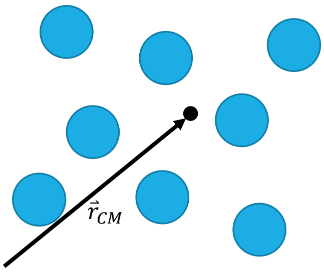
I wonder if that has anything to do with open and closed systems with regards to energy and work done[1]

We now see that the momentum of the centre of mass is equal to the sum over all momenta in the system

[1]It absolutely does. Consider it in your own time ☺

Momentum & Centre of Mass

Frames of Reference



$$M_T\vec{r}_{CM} = \sum_{i=1}^{N} m_i\vec{r}_i \qquad M_T\vec{v}_{CM} = \sum_{i=1}^{N} m_i\vec{v}_i$$

$$\vec{r}_{CM} = \frac{1}{M_T}\sum_{i=1}^{N} m_i\vec{r}_i \qquad \vec{v}_{CM} = \frac{1}{M_T}\sum_{i=1}^{N} m_i\vec{v}_i$$

Total momentum (static reference frame), $\vec{p}_T = \sum_{i=1}^{N} m_i\vec{v}_i$

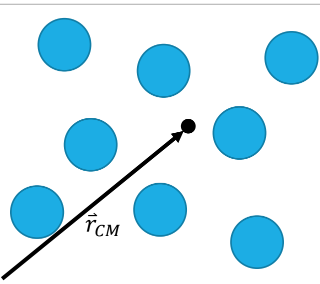
Total momentum (CoM reference frame), $\vec{p}_T = \sum_{i=1}^{N} m_i(\vec{v}_i - \vec{v}_{CM})$

Expand, $\vec{p}_T' = \left(\sum_{i=1}^{N} m_i\vec{v}_i\right) - \left(\sum_{i=1}^{N} m_i\vec{v}_{CM}\right)$

Factorise, $\vec{p}_T' = \left(\sum_{i=1}^{N} m_i\vec{v}_i\right) - \vec{v}_{CM}\left(\sum_{i=1}^{N} m_i\right)$

Momentum & Centre of Mass

Frames of Reference



$$M_T\vec{r}_{CM} = \sum_{i=1}^{N} m_i\vec{r}_i \qquad M_T\vec{v}_{CM} = \sum_{i=1}^{N} m_i\vec{v}_i$$

$$\vec{r}_{CM} = \frac{1}{M_T}\sum_{i=1}^{N} m_i\vec{r}_i \qquad \vec{v}_{CM} = \frac{1}{M_T}\sum_{i=1}^{N} m_i\vec{v}_i$$

Factorise, $\vec{p}_T' = \left(\sum_{i=1}^{N} m_i\vec{v}_i\right) - \vec{v}_{CM}\left(\sum_{i=1}^{N} m_i\right)$

Sum, $\vec{p}_T' = \left(\sum_{i=1}^{N} m_i\vec{v}_i\right) - \vec{v}_{CM}M_T$

Solve, $\boxed{\vec{p}_T' = 0}$

The total momentum of a system as measured from the centre of mass reference frame is zero! Imagine how easy this makes calculations of, say, galaxies colliding…

A Little History

Conserved Quantities in General

A Little History

Conserved Quantities in General

These conserved quantities in physics are useful. Mathematically, they let us set derivatives over time equal to zero, and thus simplify equations. Someone noted this use…

Emmy Noether (1882-1935)

Euler-Lagrange Equations, $$\frac{dL}{dq} - \frac{d}{dt}\left(\frac{dL}{d\dot{q}}\right) = 0$$

More fundamental than Newton's laws…

$$\frac{dL}{dq} = 0, \qquad \frac{d}{dt}\left(\frac{dL}{d\dot{q}}\right) = 0$$

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$$\frac{dL}{dq} = 0, \qquad \frac{d}{dt}\left(\frac{dL}{d\dot{q}}\right) = 0$$

Integrate, $$\boxed{\frac{dL}{d\dot{q}} = c}$$

This thing, $\frac{dL}{d\dot{q}}$, is constant over time. Noether showed that this is due to symmetry in the underlying laws of physics! More on this in 3rd year. For now…

MECHANICS 1 – MOMENTUM

A Little History

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Integrate, $$\frac{dL}{d\dot{q}} = c$$

$$L = K(\dot{q}) - U(q), \qquad \frac{d}{d\dot{q}}\left(\frac{1}{2}m\dot{q}^2 - U(q)\right) = c$$

Differentiate, $$\boxed{m\dot{q} = c}$$

q is just a coordinate, like x. $\dot{q}$, then, is a velocity.
Therefore, momentum is constant over time. It is conserved!