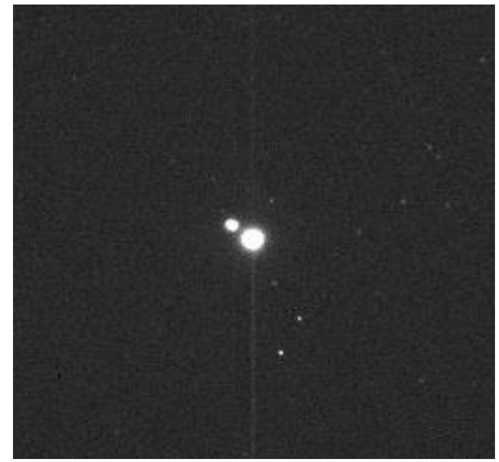
Stellar Masses

- Visual binaries
- Spectroscopic Binaries

Types of Binary System



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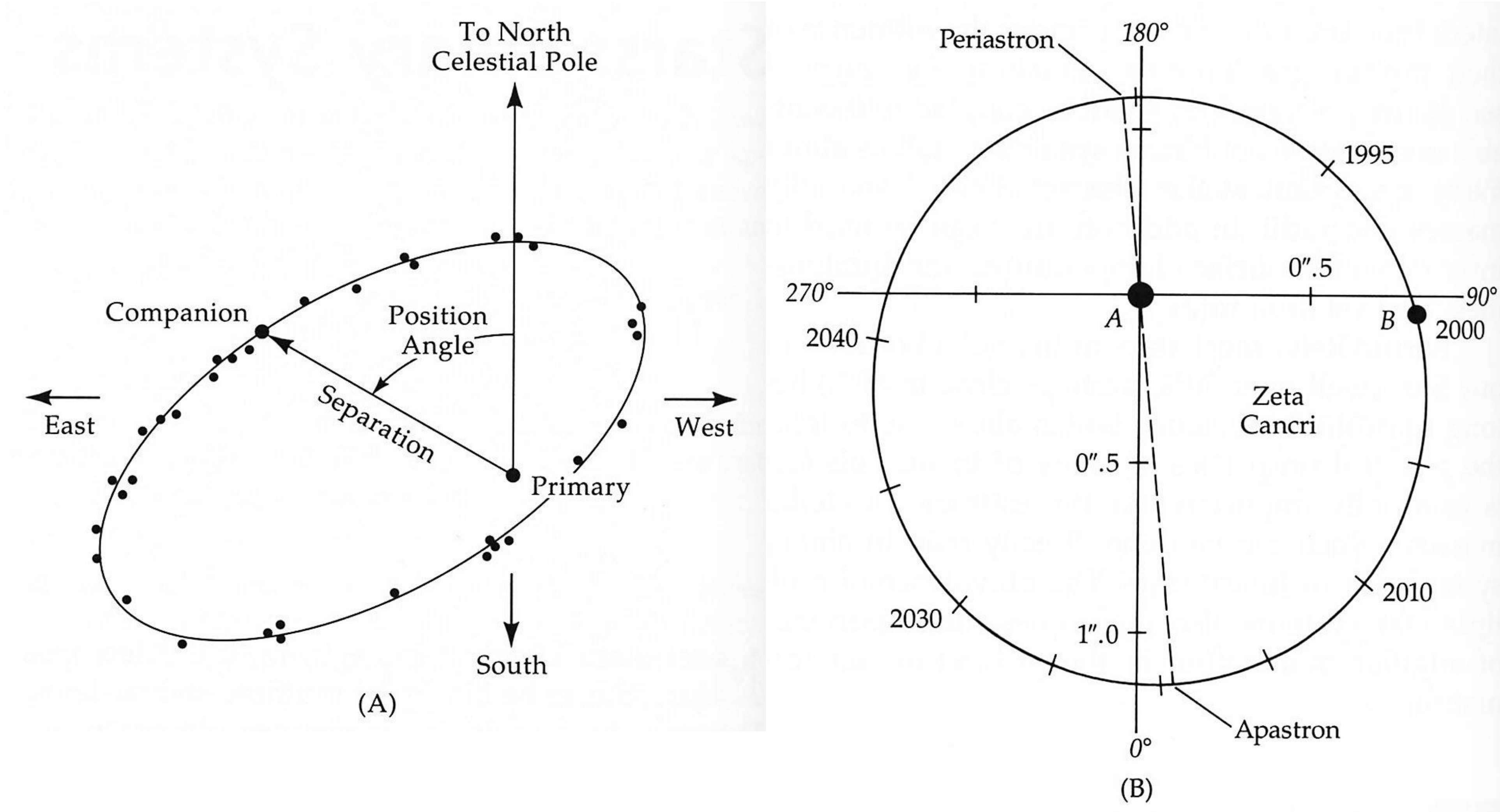
- Visual binaries
  - Two stars spatially resolved on the sky in orbit around each other
- Spectroscopic binaries
  - Two stars not spatially resolved, but orbital motion revealed through periodic Doppler shifts of their spectral lines

Masses from Visual Binaries

- measure period P and separation a (need distance d) $\rightarrow$ M_1+M_2 from Kepler's law
- measure angular semi-major axes of each orbit

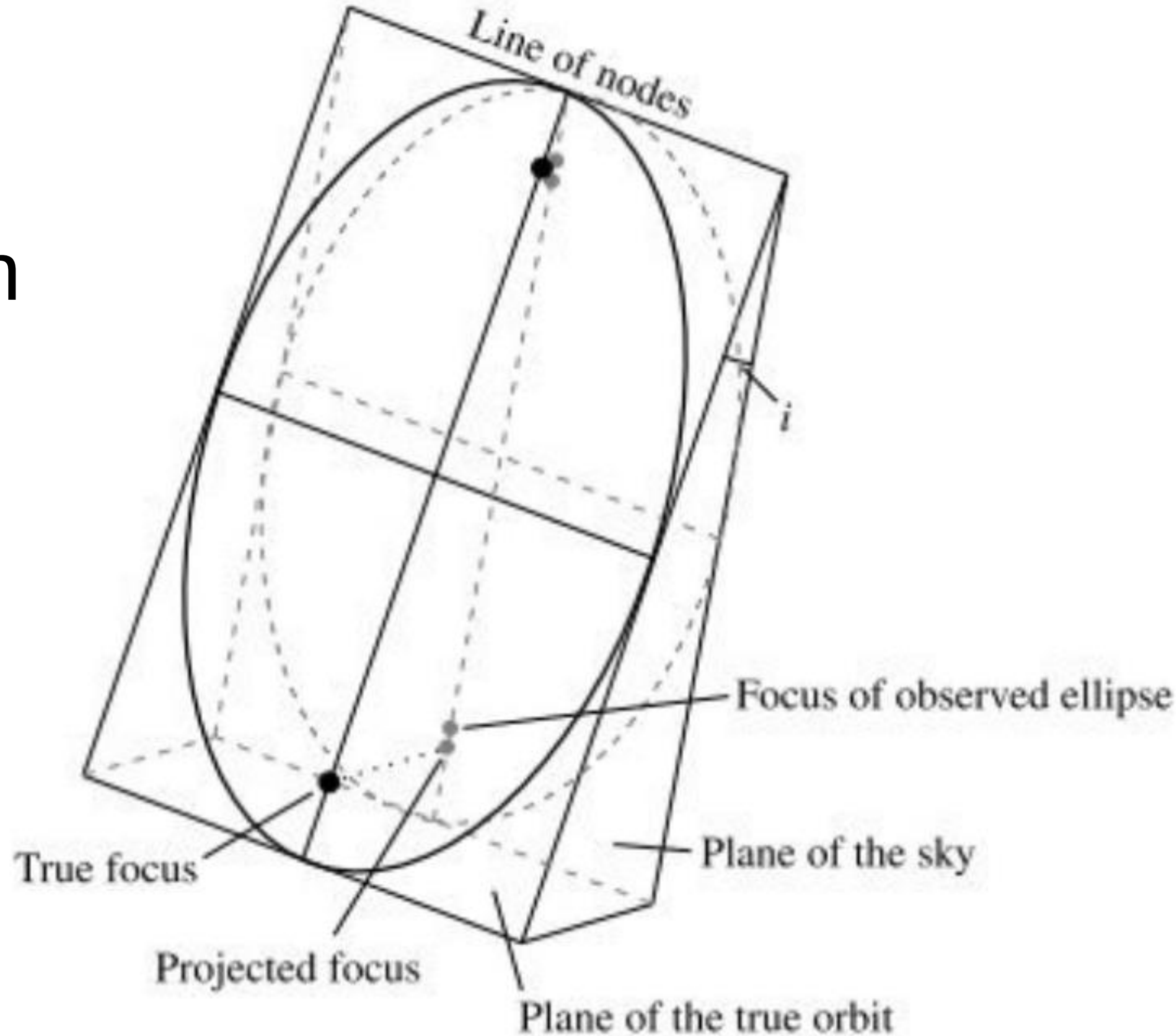
$$\frac{\theta_1}{\theta_2} = \frac{r_1}{r_2} = \frac{M_2}{M_1}$$

- solve for individual masses

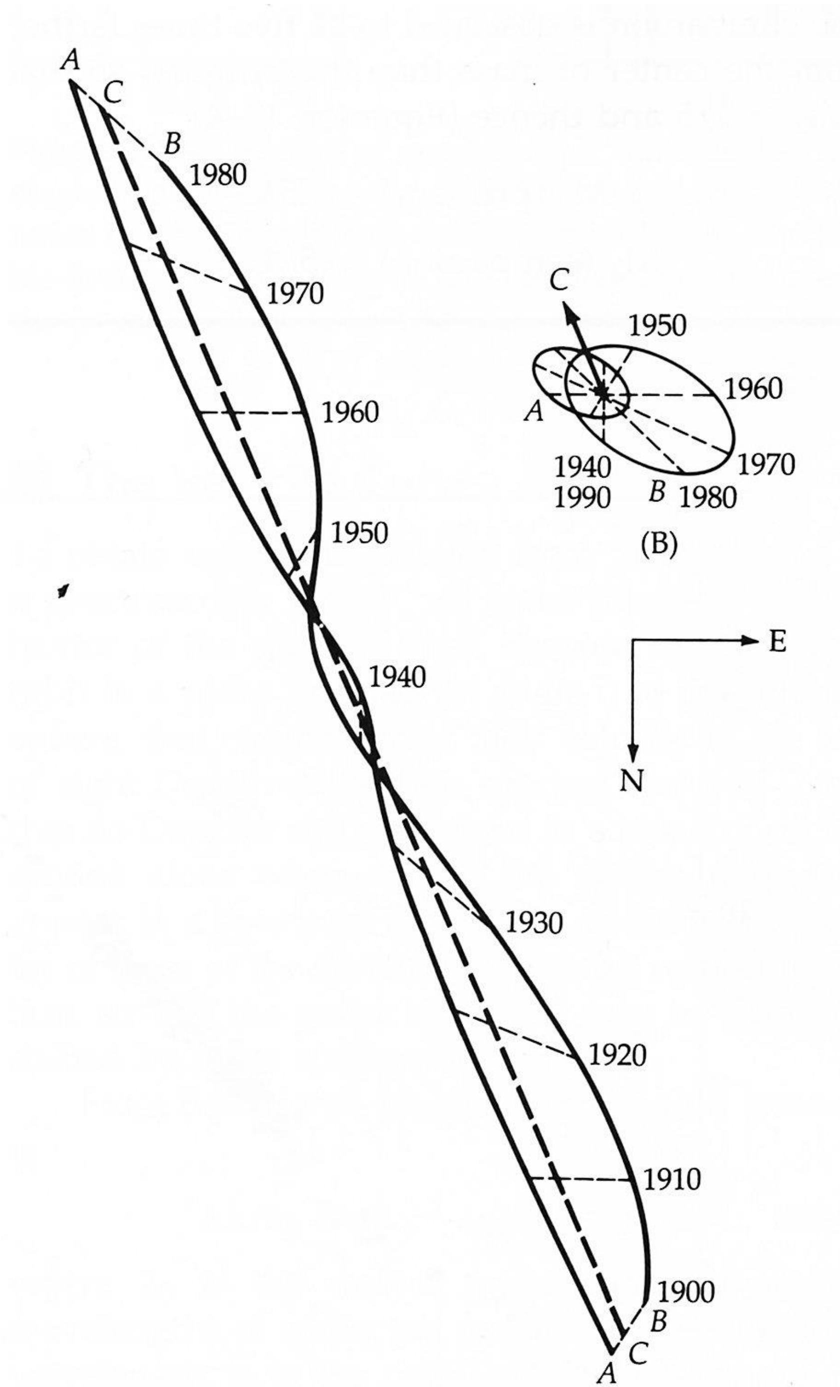


Zeilik Fig 12-1

- Unique fit of ellipse to data gives inclination



From Carroll & Ostlie



Sirius A and B

Zeilik Fig 12-2

Class Example

- The semi-major axes of the orbits of Sirius A and B are 2.5” and 5.0” respectively. With a period of 50 years and a distance of 2.6 pc what are the masses of A and B in solar masses?

Angular separation

$$q = q_1 + q_2 = 2.5 + 5.0 = 7.5¢$$

Physical separation

$$a = qd = \frac{7.5}{206265} ´\ 2.6 ´\ 3.1 ´\ 10^{16} = 2.9 ´\ 10^{12}\ \mathrm{m}$$

$$M_1 + M_2 = \frac{4p^2 a^3}{GP^2}$$

$$= \frac{4p^2 (2.9 ´\ 10^{12})^3}{6.7 ´\ 10^{-11} (50 ´\ 3.1 ´\ 10^7)^2} = 6.0 ´\ 10^{30}\ \mathrm{kg} = 3.0\ \mathrm{M}_{Sun}$$

$$\frac{M_2}{M_1} = \frac{a_1}{a_2} = \frac{q_1}{q_2} = \frac{2.5}{5.0} = 0.5$$

so

$$M_2 = 0.5 M_1$$

$$M_1 + 0.5 M_1 = 1.5 M_1 = 3.0\ \mathrm{M}_{Sun}$$

and

$$M_1 = \frac{3.0}{1.5} = 2.0\ \mathrm{M}_{Sun}$$

$$M_2 = 0.5\text{´}\ 2.0 = 1.0\ \mathrm{M}_{Sun}$$

Masses from Spectroscopic Binaries

- Doppler shifts of their spectral lines

$$\frac{v_r}{c} = \frac{\Delta\lambda}{\lambda_0}$$

where v_r is the observed relative radial velocity

- for circular orbits the orbital velocities are

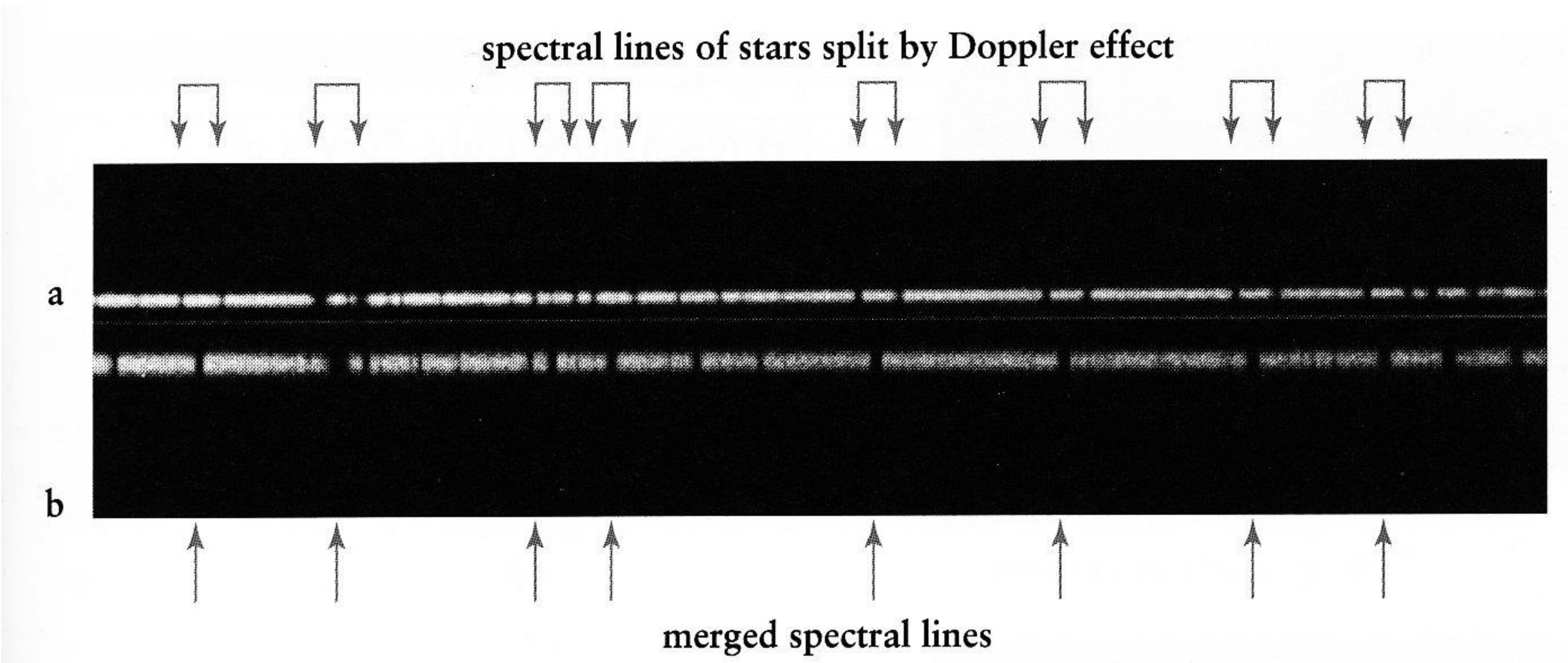
$$v_1 = \frac{2pr_1}{P} \quad \text{and} \quad v_2 = \frac{2pr_2}{P}$$

- for inclination angle i the observed radial velocities are

$$v_{r1} = v_1 \sin i \quad \text{and} \quad v_{r2} = v_2 \sin i$$

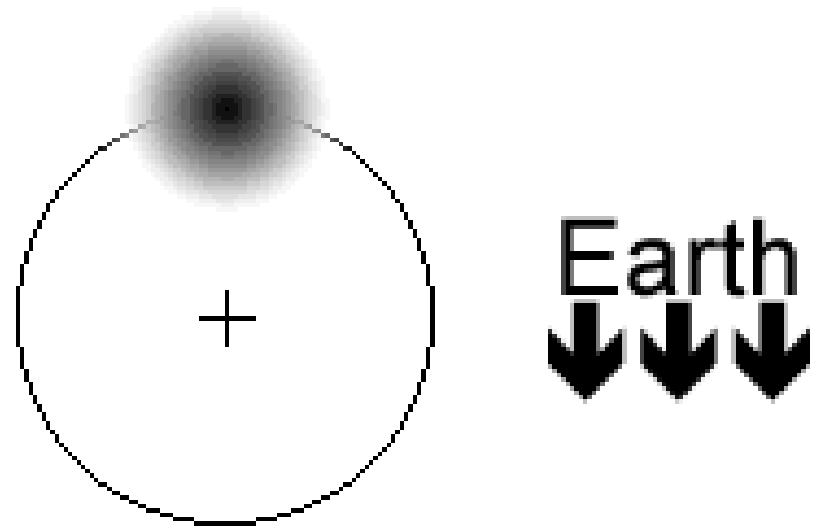
Double-lined Spectroscopic Binaries

- Spectral lines of both stars observed

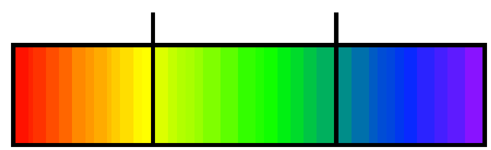


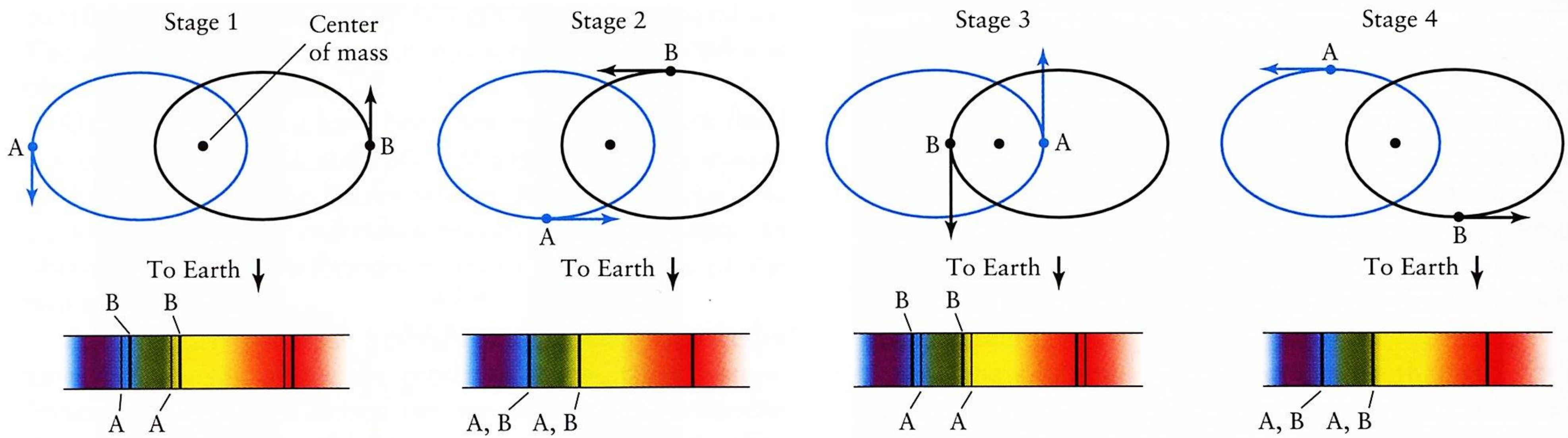
From Universe textbook

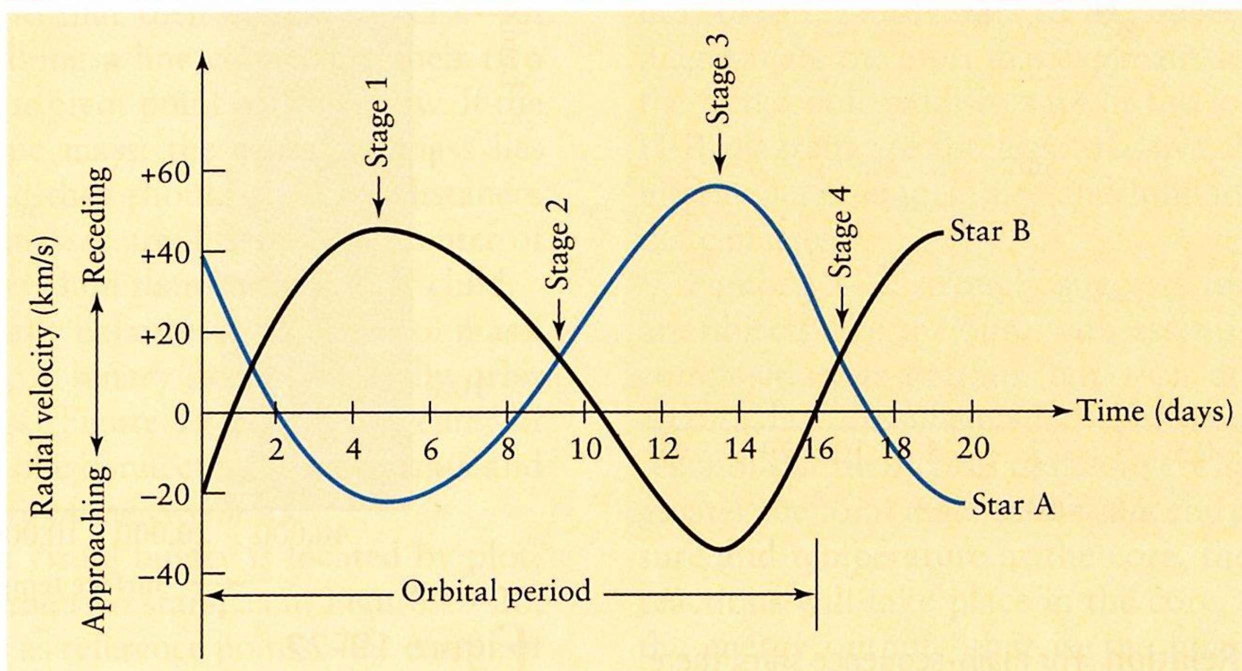
Orbit of Star Around System's Center of Mass (Viewed from above)



Doppler Shift (Detects movement *along* line of sight)







From Universe textbook

- can determine the mass ratio from

$$\frac{v_{r1}}{v_{r2}} = \frac{v_1}{v_2} = \frac{r_1}{r_2} = \frac{M_2}{M_1}$$

Also

$$a = r_1 + r_2 = \frac{P}{2p}(v_1 + v_2) = \frac{P}{2p}\left(\frac{v_{r1} + v_{r2}}{\sin i}\right)$$

so from Kepler's law

$$M_1 + M_2 = \frac{4p^2 a^3}{GP^2} = \frac{P}{2pG}\left(\frac{v_{r1} + v_{r2}}{\sin i}\right)^3$$

i.e. only a lower limit to the sum of the masses

Class Example

- The spectral lines in a double-lined spectroscopic binary exhibit sinusoidal motion with amplitudes of 150 and 350 kms^{-1} in a period of 8 hours. Assuming we view the system close to edge-on, calculate the individual masses in solar masses.

$$M_1 + M_2 = \frac{P}{2pG}\left(\frac{v_{r1} + v_{r2}}{\sin i}\right)^3$$

$$= \frac{8 \times 3600}{2p6.7 \times 10^{-11}}\left(\frac{(150 + 350) \times 10^3}{\sin 90^o}\right)^3$$

$$= 8.6 \times 10^{30}\ \text{kg}$$

$$= 4.3\ \mathrm{M}_{Sun}$$

$$\frac{M_2}{M_1} = \frac{v_{r1}}{v_{r2}} = \frac{150}{350}$$

$$M_1 = \frac{7}{3} M_2$$

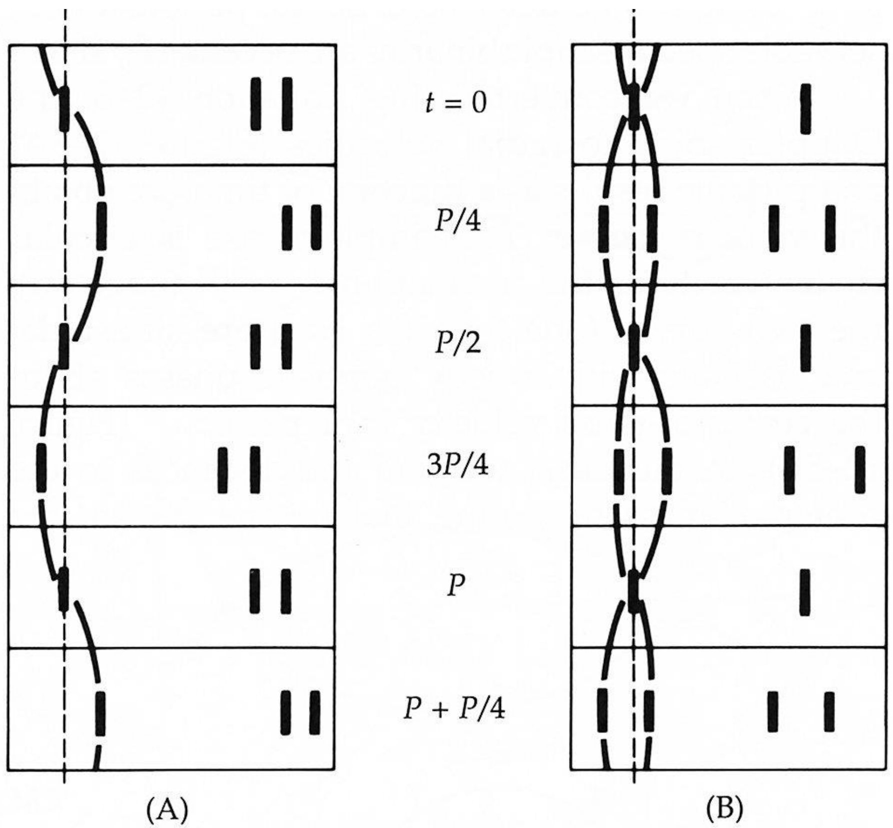
$$\frac{10}{3} M_2 = 4.3\ \mathrm{M}_{Sun}$$

$$M_2 = 1.3\ \mathrm{M}_{Sun}$$

$$M_1 = 3.0\ \mathrm{M}_{Sun}$$

Single-lined Spectroscopic Binaries

- only one spectrum observed



Zeilik Fig 12-4

- Assume only v_{r1} is known so eliminate v_{r2}

$$M_1 + M_2 = \frac{P}{2pG}\left(\frac{v_{r1} + \frac{M_1}{M_2} v_{r1}}{\sin i}\right)^3$$

$$M_1 + M_2 = \frac{P v_{r1}^3}{2 p G} \left(\frac{\frac{M_1 + M_2}{M_2}}{\sin i} \right)^3$$

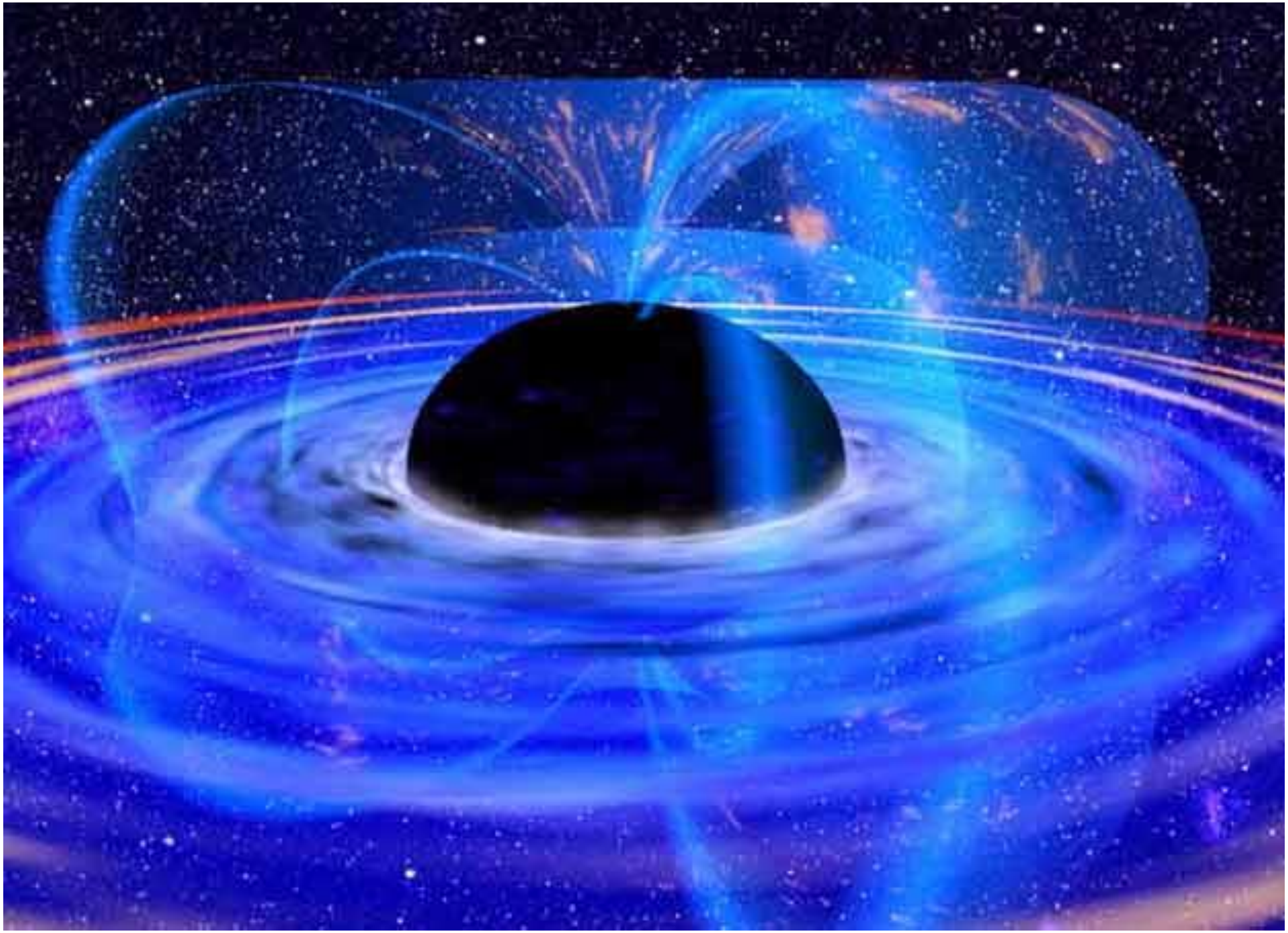
so $$\frac{M_2^3 \sin^3 i}{(M_1 + M_2)^2} = \frac{P v_{r1}^3}{2 p G}$$

i.e. if we can estimate M_1

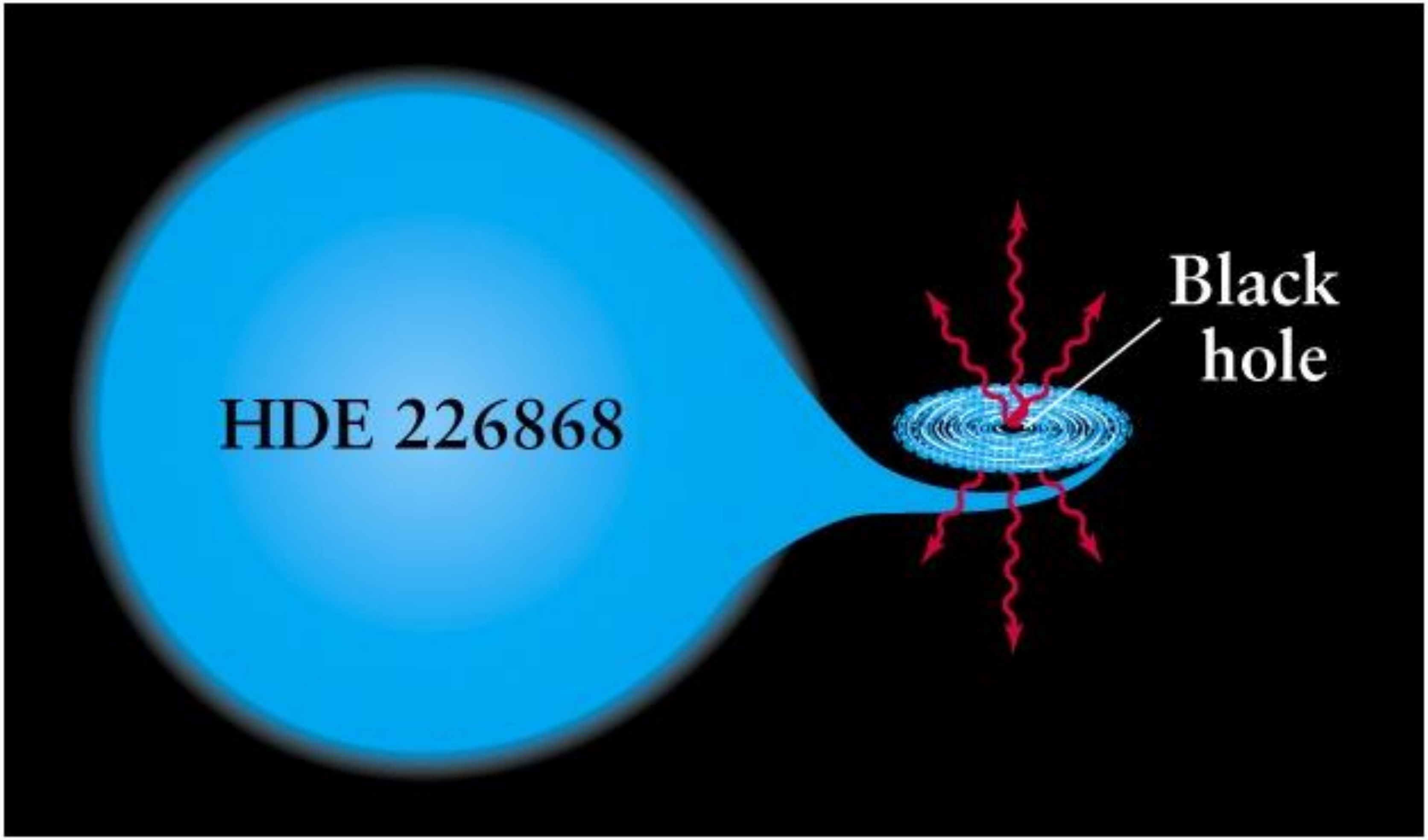
we can constrain M_2

Unseen Companions

- What kind of unseen companions?
  - Neutron stars
  - Black holes
  - Exoplanets



From Universe textbook



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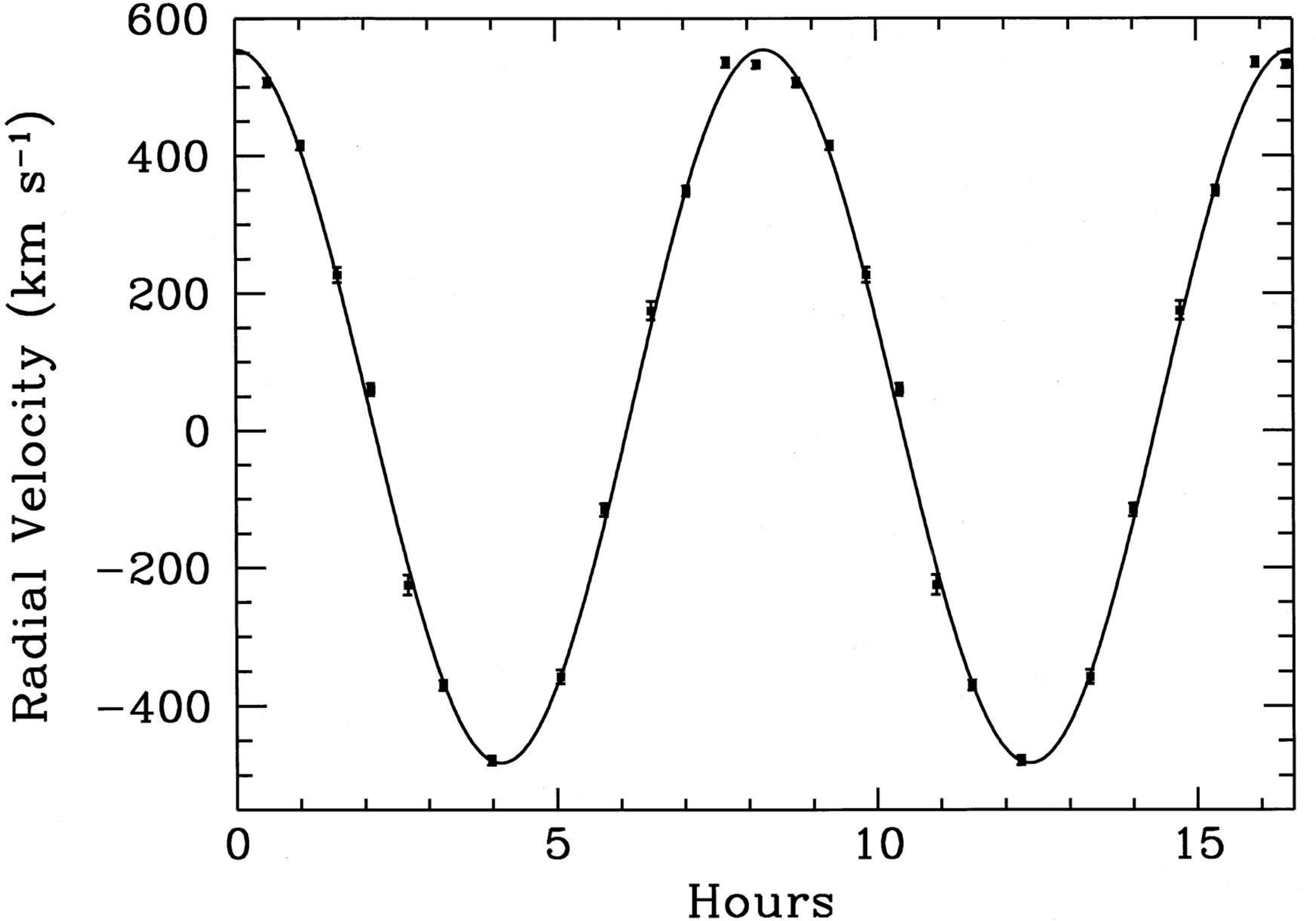
From Universe textbook



Radial-velocity curve of the visible star in the X-ray binary GS 2000 + 25
Fillipenko et al. (1999) www.pnas.org/content/96/18/9993.full
Shows that invisible compact companion star is a 5 solar mass black hole

Summary

- visual binaries provide accurate masses, but not many known
- spectroscopic binaries only usually constrain the masses with inclination the greatest uncertainty unless the system is eclipsing
- spectroscopic binaries used to find black holes and planets orbiting other stars