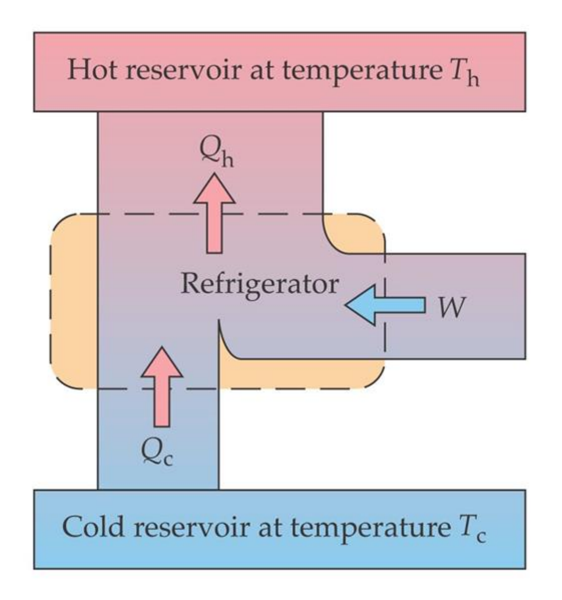
PHAS1000 – THERMAL PHYSICS

Lecture 14

Heat Engines, Refrigerators and Heat Pumps



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Overview

This lecture covers:

- 2^{nd} law
- Refrigerator/ air conditioning
- Heat pump
- Heat engine
- Entropy
- Carnot cycle
- Maximum possible efficiency

2^{nd} Law of Thermodynamics

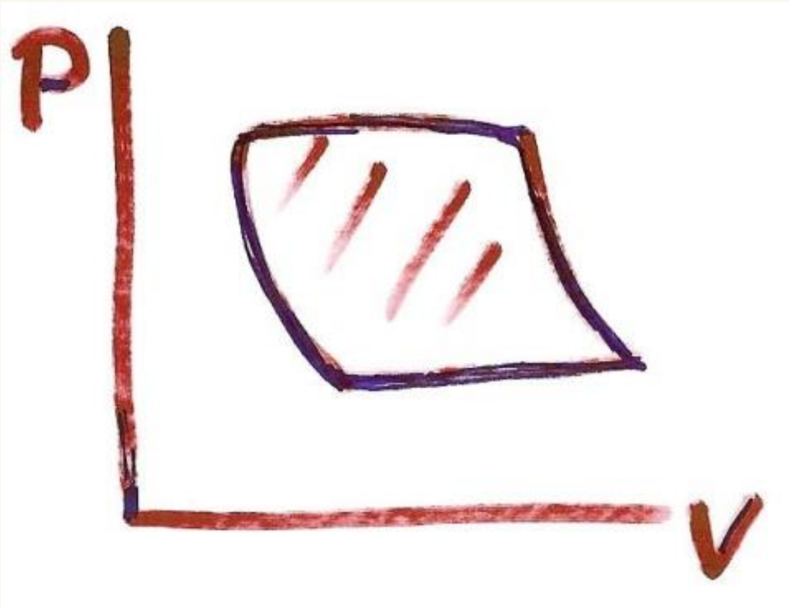
Claussius' statement of 2^{nd} law

No process is possible whose sole result is the transfer of heat from a colder to a hotter body.

Kelvin's statement of 2^{nd} law

No process is possible whose sole result is the complete conversion of heat into work.

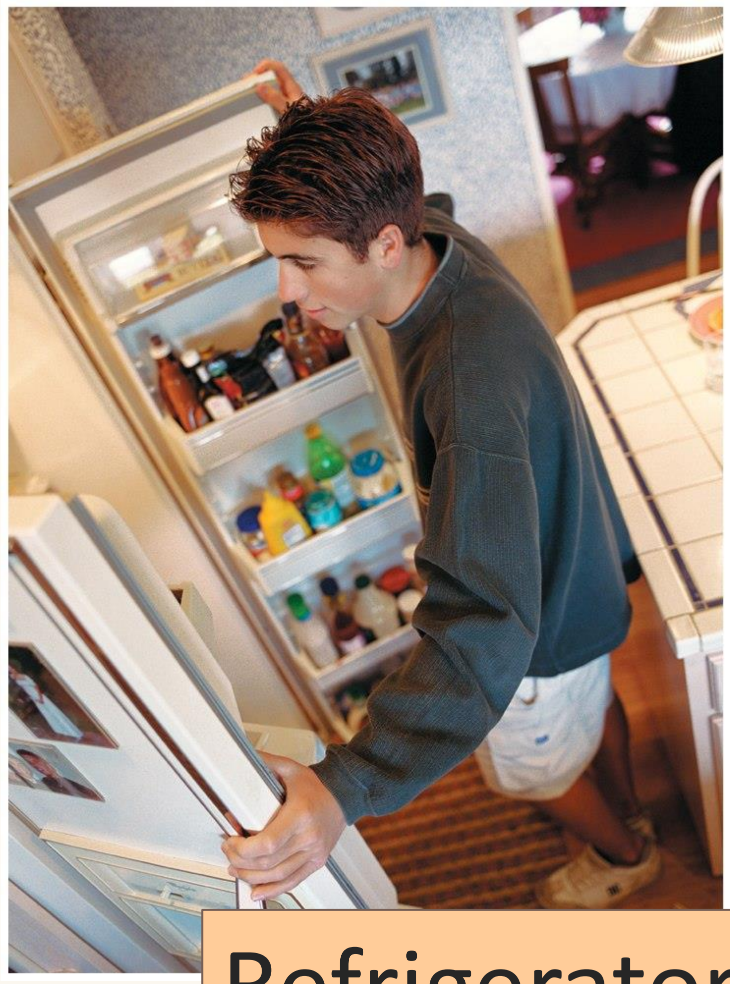
Complete cycles - reminder



In a complete cycle

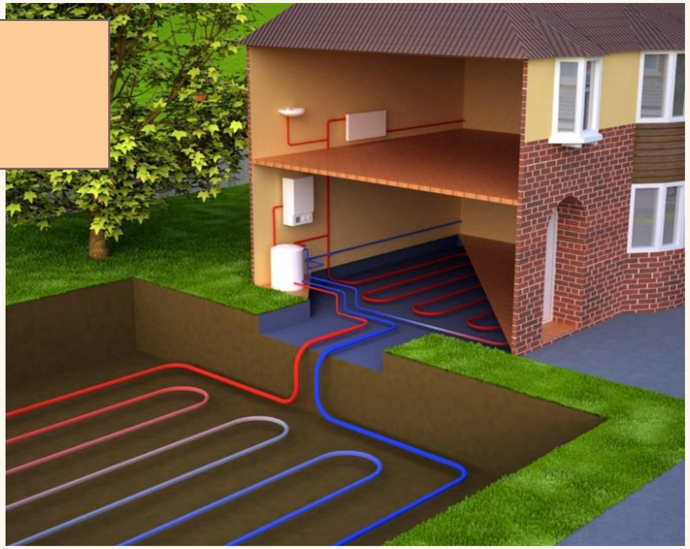
- $\Delta U = 0$
- net work done = area enclosed
- clockwise W_{by} is +ve Q_{in} is +ve
- anticlockwise W_{by} is -ve Q_{in} is -ve

Application of thermodynamics cycles



Refrigerator

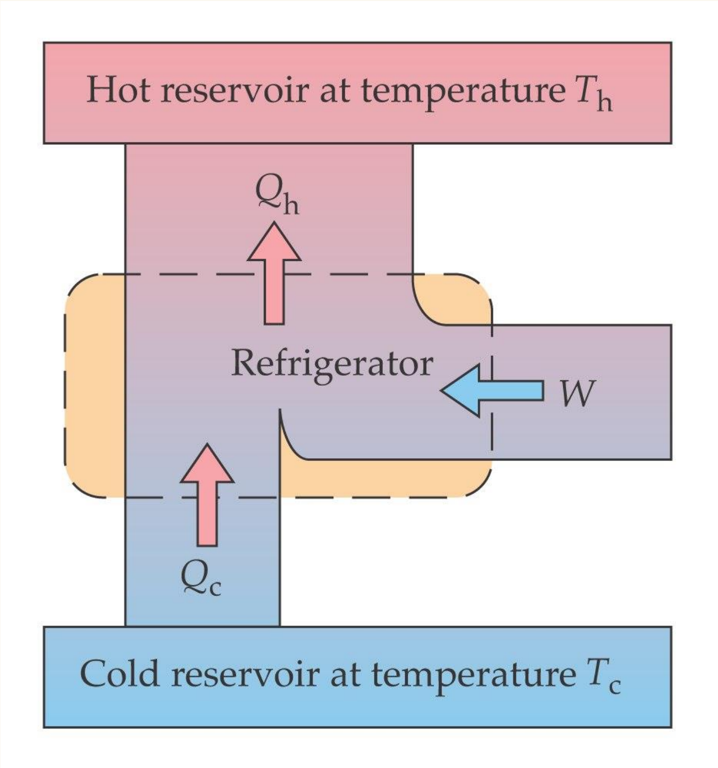
Heat Pump



Heat Engine

Image https://tinyurl.com/353vy7hx

Refrigerator / Air conditioning



Conservation of energy yields

$$Q_h = W + Q_c$$

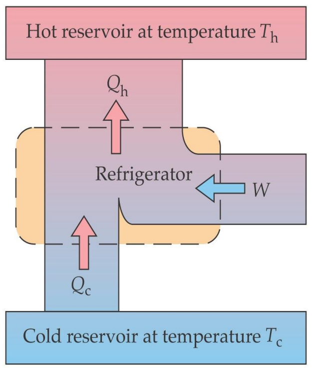
Coefficient of Performance

$$\mathrm{COP} = \frac{Q_c}{W}$$

$$\mathrm{COP} = \frac{\text{desired effect}}{\text{necessary input}}$$

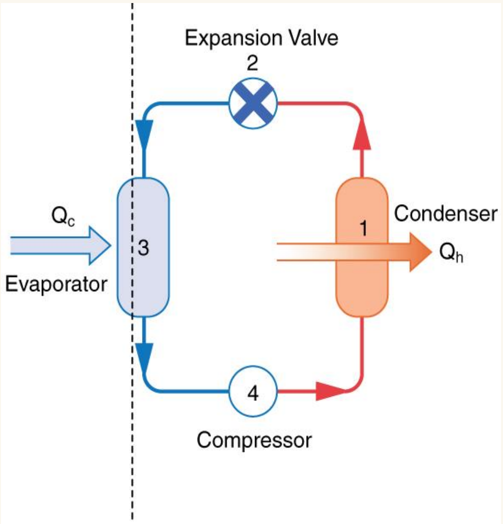
How a fridge works

Food at T_c
Room at T_h



Adiabatic expansion makes vapour cool to below the food temperature ($T < T_c$)

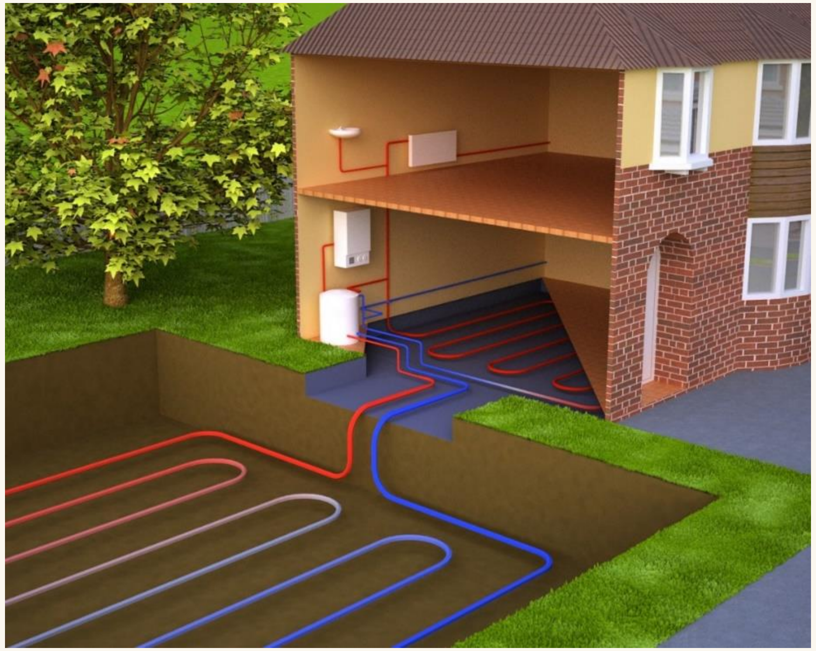
Heat (Q_c) absorbed from the food evaporates the fluid

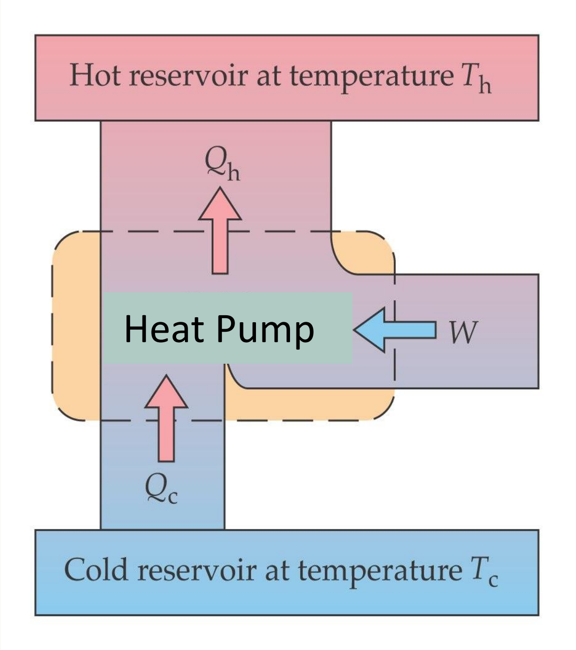


Heat (Q_h) radiates to room and vapour cools

Vapour compressed adiabatically by motor (work W) and temperature rises above room temperature (i.e. $T > T_h$)

Heat Pump





Conservation of energy yields

$$Q_h = W + Q_c$$

Coefficient of Performance

$$\text{COP} = \frac{Q_h}{W}$$

Like a refrigerator, a heat pump takes heat from cold place and delivers heat to hot place, by addition of work.

Question 1

(a) An air conditioning system has a COP of 2.5 and uses a motor rated at 1600 W.

i. What is the maximum rate that heat can be removed from the room?

ii. When working at maximum, at what rate is heat exhausted to the outside?

(b) If this unit is ‘turned round’ and used as a heat pump in the winter, delivering heat at the same rate as part ii above

i. What is the COP of the heat pump ?

Answer to Q1

(a) An air conditioning system has a COP of 2.5 and uses a motor rated at 1600 W.

i. What is the maximum rate that heat can be removed from the room?

ii. When working at maximum, at what rate is heat exhausted to the outside?

(b) If this unit is 'turned round' and used as a heat pump in the winter, delivering heat at the same rate as part ii above

i. What is the COP of the heat pump ?

(a) (i) $COP_{ref} = \frac{Q_c}{W}$ $Q_c = W \times COP$ $\frac{dQ_c}{dt} = \frac{dW}{dt} \times COP$

$= 1600 \times 2.5$

$= \boxed{4000\ J/s}$

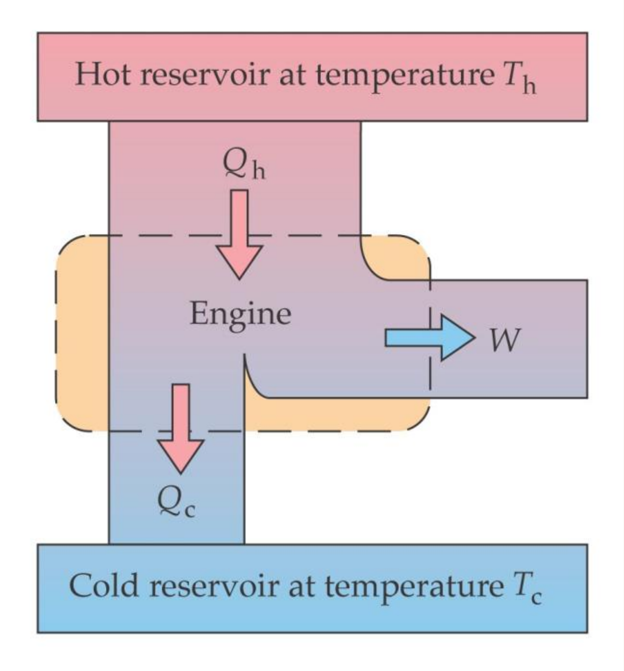
(ii) $Q_h = W + Q_c$ $\frac{dQ_h}{dt} = \frac{dW}{dt} + \frac{dQ_c}{dt}$

$= 1600 + 4000 = \boxed{5600\ J/s}$

(b) $COP_{hp} = \frac{Q_h}{W} = \frac{\frac{dQ_h}{dt}}{\frac{dW}{dt}} = \frac{5600}{1600} = \boxed{3.5}$

$COP_{hp} = COP_{ref} + 1$

Heat Engine



Conservation of energy yields

$$Q_h = W + Q_c$$

Efficiency

$$\varepsilon = \frac{\text{desired effect}}{\text{necessary input}}$$

$$\varepsilon = \frac{W}{Q_h} = \frac{Q_h - Q_c}{Q_h} = 1 - \frac{Q_c}{Q_h}$$

Converts heat to work, but not with 100% efficiency

If the door of a fridge is left open for a few hours, the room will

A be cooled slightly

B warm up gradually

C remain at the same temperature

Answer

If the door of a fridge is left open for a few hours, the room will

A be cooled slightly

B warm up gradually

C remain at the same temperature

conservation of energy

$Q_h = W + Q_c$

Fridge Q_c = heat taken out of food

Q_h = heat exhausted to room

W = work done by motor

In this example

Q_c = heat removed from room

Q_h = heat exhasted to room

$Q_h = W + Q_c$

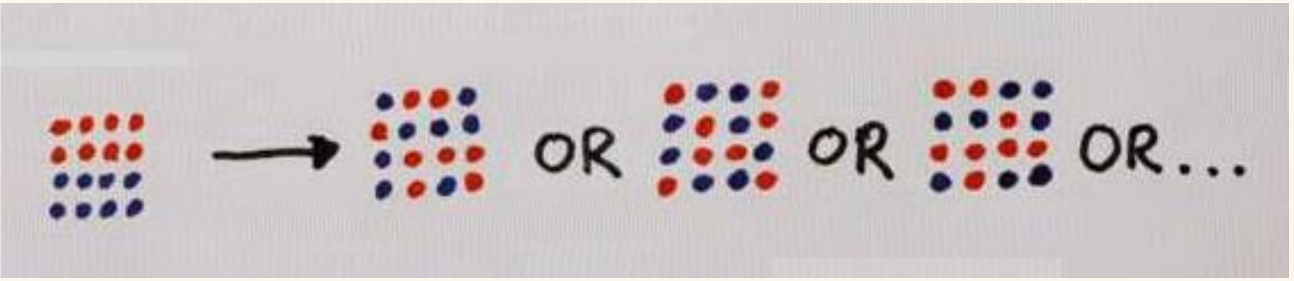
$\therefore Q_h > Q_c$

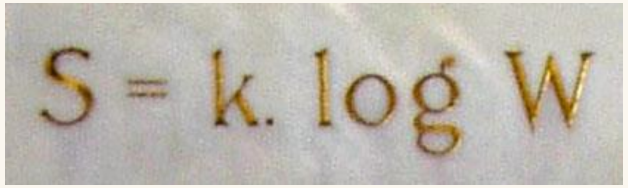
$\therefore$ net heat into room

ANS $\boxed{B}$

Entropy – Statistical Mechanics approach

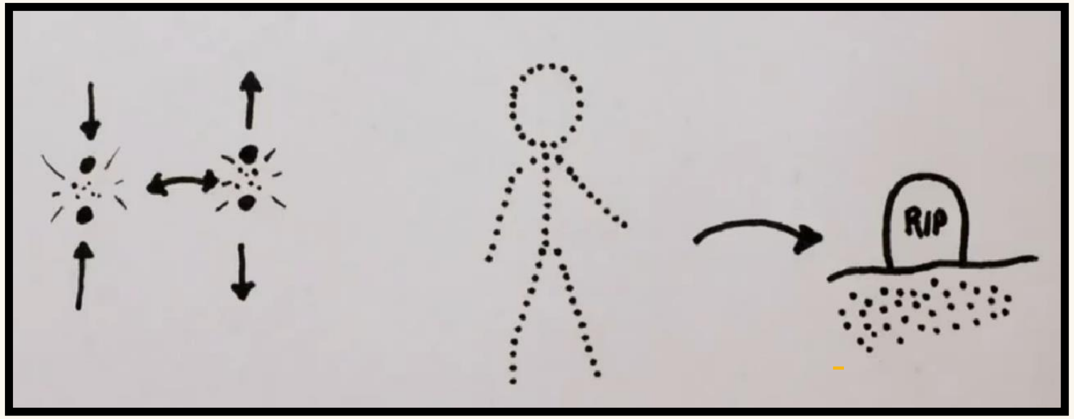
Entropy is a measure of the disorder of a system.





Boltzmann's formula.

W = no of ways to arrange the molecules.



Watch this short video by clicking the picture or loading the URL http://www.youtube.com/watch?v=GdTMuivYF30

Systems always proceed in the direction of increasing disorder.

Entropy – Classical Thermodynamics approach

For a reversible process:-

$$dS = \frac{dQ}{T}$$

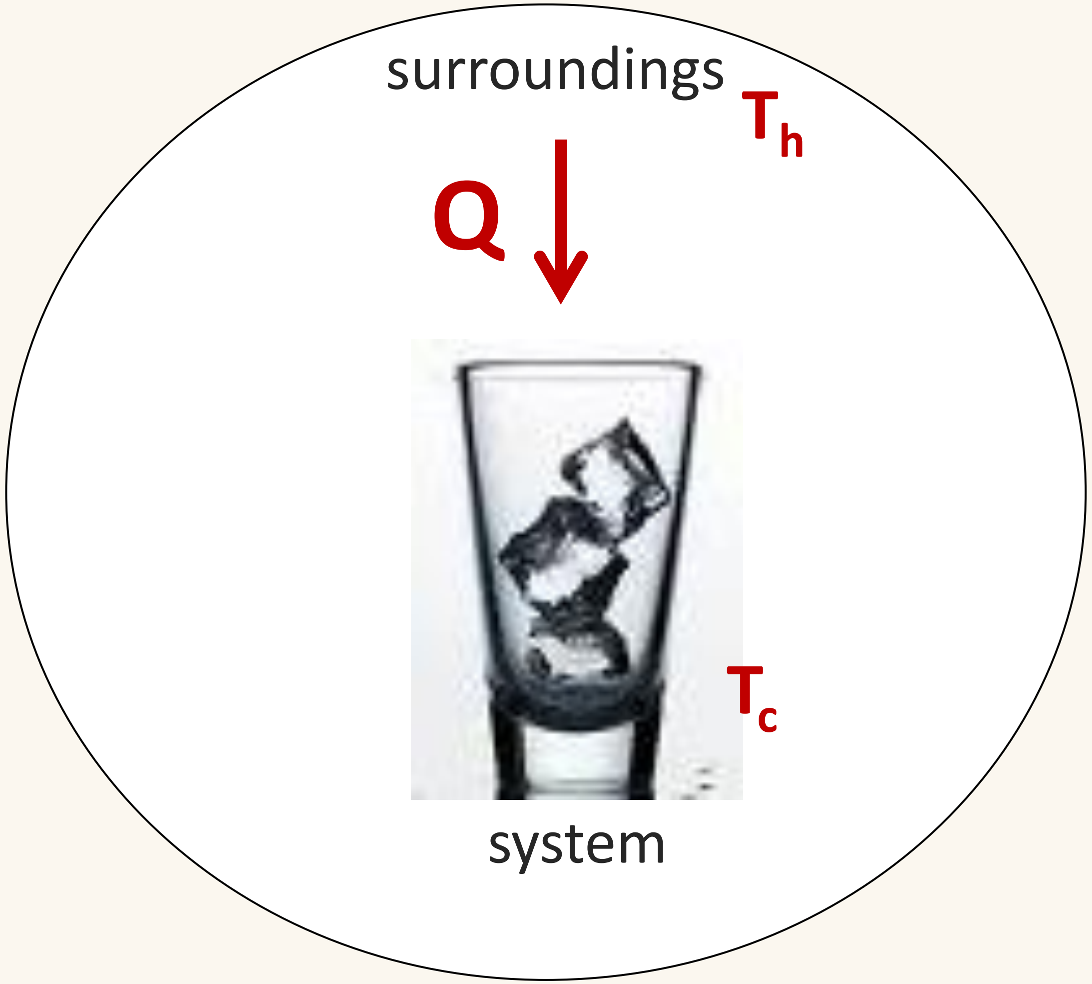
Entropy, S measured in J/K

Entropy is a state variable.

Thus ΔS depends only on the initial and final states and not on the path.

$$\Delta S = \int dS = \int \frac{dQ}{T}$$

Entropy of the Universe



Universe = system + surroundings

$$\Delta S_{sys} = \frac{Q}{T_c}$$ For heat added to glass at T_c

$$\Delta S_{Sur} = \frac{-Q}{T_h}$$ For heat removed from air at T_h

$$\Delta S_{universe} = \frac{Q}{T_c} + \frac{-Q}{T_h}$$ adding all terms

But $T_c < T_h$

So $\Delta S_{universe} \geq 0$

The entropy of the universe always tends to a maximum

What limits efficiency?

Friction: dissipates energy as waste heat

- external friction of moving parts
- internal friction of gas / fluid

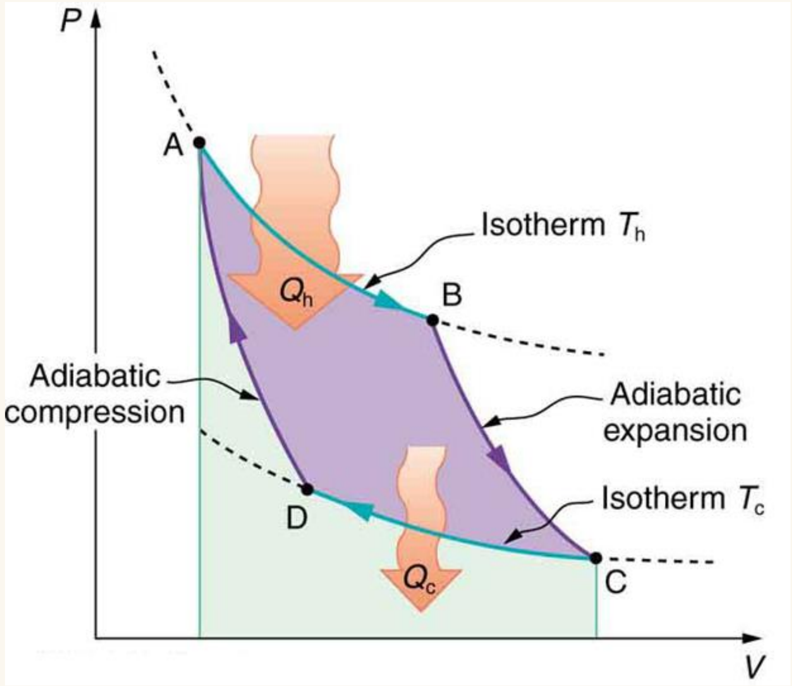
Heat loss: conduction or radiation of heat out of the system

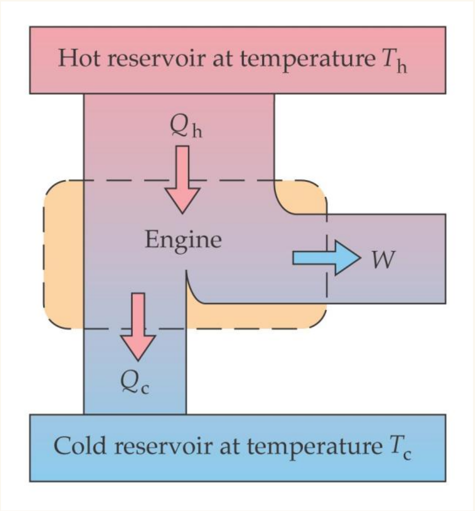
Sadi Carnot
1796 - 1832

If we could eliminate friction and heat loss then the cycle would be reversible. But still efficiency would be limited by thermodynamics

Carnot said the maximum theoretical efficiency is when we use reversible processes.

Carnot cycle - clockwise and anticlockwise



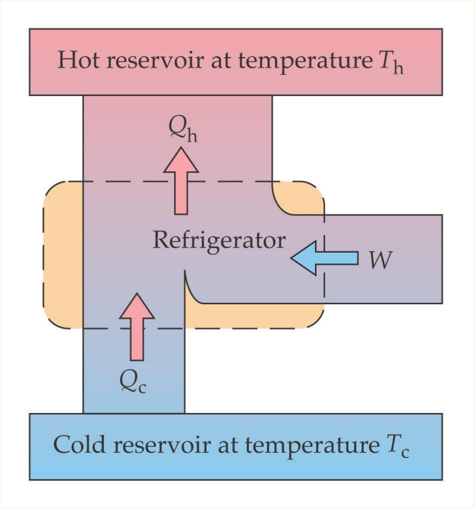


clockwise

Works as a heat engine

$$W = Q_h - Q_c$$

Net work done = net heat gain



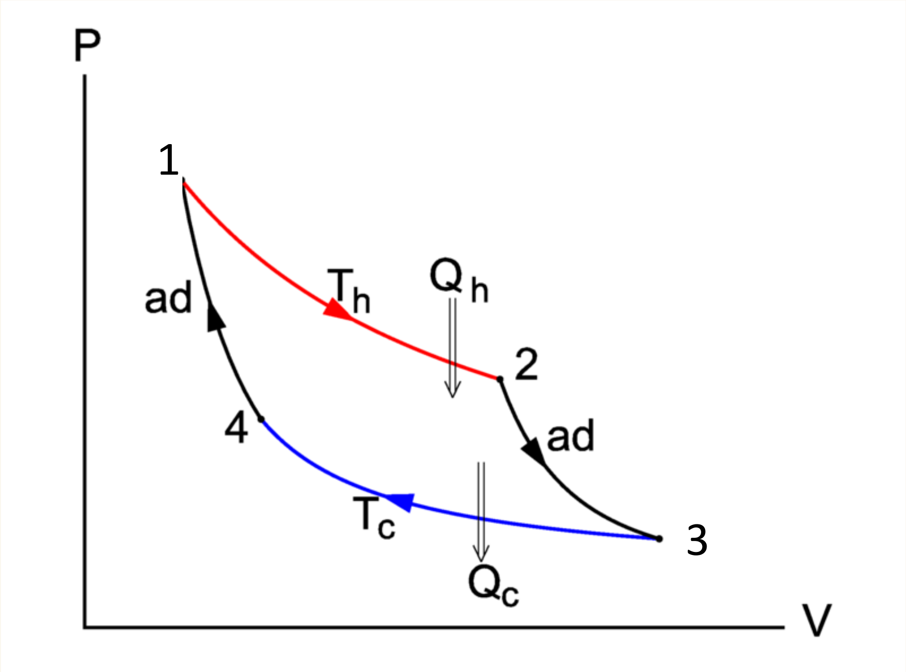
Anti-clockwise

Works as a refrigerator

Q_c absorbed by system

Q_h rejected

Maximum Efficiency



Net entropy change

$$\Delta S = \frac{Q_h}{T_h} - \frac{Q_c}{T_c}$$

1→2 isothermal (at T_h)
Q_h absorbed
Entropy of system increased by $\Delta S = Q_h/T_h$

2→3 adiabatic
$Q_{in} = 0 \quad \Delta S = 0$

3→4 isothermal (at T_c)
Q_c expelled
Entropy of system decreased by $\Delta S = Q_c/T_c$

4→1 adiabatic
$Q_{in} = 0 \quad \Delta S = 0$

Maximum Efficiency

Net entropy change

$$\Delta S = \frac{Q_h}{T_h} - \frac{Q_c}{T_c}$$

To be reversible the net $\Delta S = 0$ in either direction, otherwise $\Delta S \geq 0$ is violated !

So for Carnot cycle

$$\frac{Q_h}{T_h} = \frac{Q_c}{T_c}$$

or rearranging $\dfrac{Q_c}{Q_h} = \dfrac{T_c}{T_h}$

For heat engine we had $\varepsilon = 1 - \dfrac{Q_c}{Q_h}$

For a heat engine running on a Carnot cycle $\dfrac{Q_c}{Q_h} = \dfrac{T_c}{T_h}$ so

$$\boxed{\varepsilon = 1 - \frac{T_c}{T_h}}$$

This is the maximum theoretical efficiency of a heat engine operating between temperatures T_h and T_c

Real machine		Carnot cycle (max efficiency)
Heat Engine	$\varepsilon = \frac{W}{Q_h} = 1 - \frac{Q_c}{Q_h}$	$\varepsilon_{max} = 1 - \frac{T_c}{T_h}$
Heat Pump	$COP_{hp} = \frac{Q_h}{W} = \frac{1}{\varepsilon}$	$(COP_{hp})_{max} = \frac{1}{\varepsilon} = \frac{T_h}{T_h - T_c}$
Refrigerator / air conditioning	$COP_{ref} = \frac{Q_c}{W} = COP_{hp} - 1$	$(COP_{ref})_{max} = \frac{T_c}{T_h - T_c}$
For 'real' machines we express everything in terms heat and work. (allows for friction losses etc)	$Q_h - Q_c = W$	$\frac{Q_c}{Q_h} = \frac{T_c}{T_h}$ For Carnot (ideal reversible machine) we express everything in terms of the two temperatures.

Question 4

a) What is the best possible COP of a heat pump if it works between temperatures of 45°C and -15°C ?

b) For such an ideal heat pump, how much heat transfers into the building if the work done is 3.60×10^7 J?

Question 5

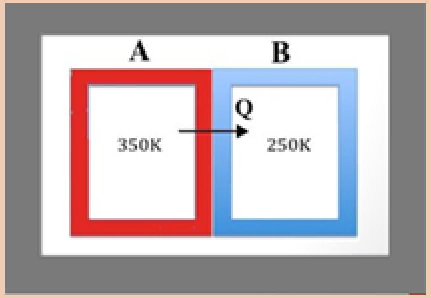
Would you believe an inventor who claimed to have created a device that can do 12 kJ of work by taking in 22 kJ of heat at 600 K, and expelling waste heat to the environment at 300 K?

Question 6

A Carnot engine with efficiency 0.6 has a heat sink at 27°C. To raise the efficiency to 0.7, by how much must the temperature of the heat source be raised?

Question 7

A small amount of heat Q flows out of a hot system A (at 350K) and into a cold system B (at 250 K). Which of the following correctly describes the entropy changes? (The systems are thermally isolated from the rest of the universe).

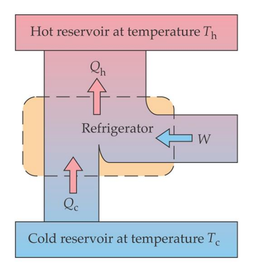


A $|\Delta S_A| > |\Delta S_B|$
B $|\Delta S_A| < |\Delta S_B|$
C $|\Delta S_A| = |\Delta S_B|$
D It cannot be determined from the information given

Summary

Refrigerator / air conditioning $\quad \text{COP} = \dfrac{Q_c}{W}$

Heat Pump $\quad \text{COP} = \dfrac{Q_h}{W}$



Heat Engine $\quad \varepsilon = \dfrac{W}{Q_h} \qquad \varepsilon = 1 - \dfrac{Q_c}{Q_h}$

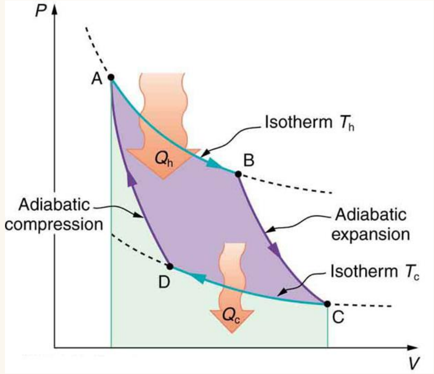
Conservation of energy in a *process* $\quad Q_{in} = \Delta U + W_{by}$

Conservation of energy in a *cycle* $\quad Q_h = W + Q_c$

$$i.e.\ \ Q_h - Q_c = W$$

Entropy (J/K) $\quad dS = \dfrac{dQ}{T} \qquad \Delta S = \displaystyle\int \dfrac{dQ}{T}$

$$\Delta S_{universe} \geq 0$$



Carnot cycle, maximum efficiency

$$\frac{Q_c}{Q_h} = \frac{T_c}{T_h} \qquad \varepsilon = 1 - \frac{T_c}{T_h}$$

Answer Q4

a) What is the best possible COP of a heat pump if it works between temperatures of 45°C and -15°C ?

b) For such an ideal heat pump, how much heat transfers into the building if the work done is 3.60 x 10^7 J?

Equation $COP_{hp} = \frac{Q_h}{W}$ $\quad \varepsilon_{hp} = \frac{W}{Q_h}$ $\quad \therefore COP_{hp} = \frac{1}{\varepsilon_{hp}}$

(a) For Carnot cycle $\varepsilon_{max} = 1 - \frac{T_c}{T_h} = 1 - \frac{(273-15)}{(273+45)}$

$= 0.19$

$\therefore (COP_{hp})_{max} = \frac{1}{0.19} = \underline{\underline{5.26}}$

(b) $COP_{hp} = \frac{Q_h}{W}$

$Q_h = W \times COP$

$= 3.6 \times 10^7 \times 5.26$

$= \underline{\underline{1.89 \times 10^8 \text{ J}}}$

Answer Q5

Would you believe an inventor who claimed to have created a device that can do 12 kJ of work by taking in 22 kJ of heat at 600 K, and expelling waste heat to the environment at 300 K?

Inventor claims $W = 12\,kJ$ $\quad Q_h = 22\,kJ$

$T_h = 600\,K \quad T_c = 300\,K$

max theoretical efficiency between these 2 temps!

$$\varepsilon_{max} = 1 - \frac{T_c}{T_h} = 1 - \frac{300}{600} = 0.5 \text{ ie } 50\%$$

How does the claimed data measure up?

Actual efficiency $\varepsilon = \frac{W}{Q_h} = \frac{12}{22} = 0.55$ ie 55%

So the inventor claims to have produced an engine that is more efficient than allowed by Thermodynamics.

Do NOT believe them!

Answer Q6

A Carnot engine with efficiency 0.6 has a heat sink at 27°C. To raise the efficiency to 0.7, by how much must the temperature of the heat source be raised?

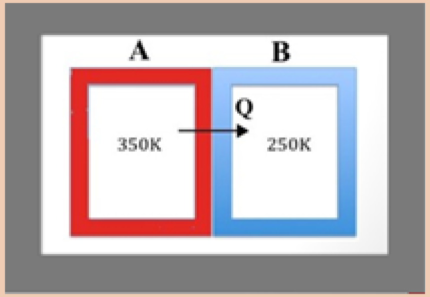
For Carnot $\varepsilon = 1 - \frac{T_c}{T_h}$ so $T_h = \frac{T_c}{(1-\varepsilon)} = \frac{(27+273)}{(1-0.6)} = \underline{750K}$

if $\varepsilon = 0.7$ then $T_h = \frac{T_c}{(1-\varepsilon)} = \frac{(27+273)}{(1-0.7)} = \underline{1000K}$

$\therefore$ rise in temp $= 1000 - 750 = \underline{\underline{250K}}$

Answer Q7

2. A small amount of heat Q flows out of a hot system A (at 350K) and into a cold system B (at 250 K). Which of the following correctly describes the entropy changes? (The systems are thermally isolated from the rest of the universe).



A $|\Delta S_A| > |\Delta S_B|$
B $|\Delta S_A| < |\Delta S_B|$
C $|\Delta S_A| = |\Delta S_B|$
D It cannot be determined from the information given

entropy from A: $\Delta S_A = \frac{-Q}{T_A} = \frac{-Q}{350}$

entropy added to B: $\Delta S_B = \frac{Q}{T_B} = \frac{Q}{250}$

the higher temp (in the denominator) ~~of~~ means that $|\Delta S_A| < |\Delta S_B|$

Ans B