PHAS1000 – THERMAL PHYSICS

Lecture 5

Pressure and Kinetic theory

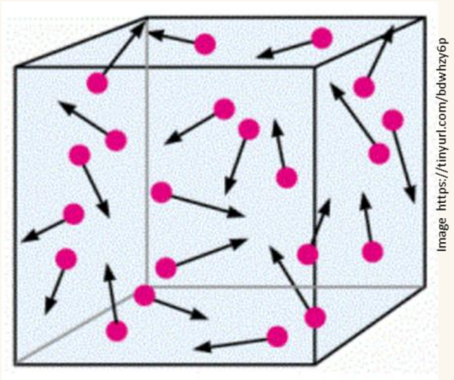


Image https://tinyurl.com/bdwhzy6p

Overview

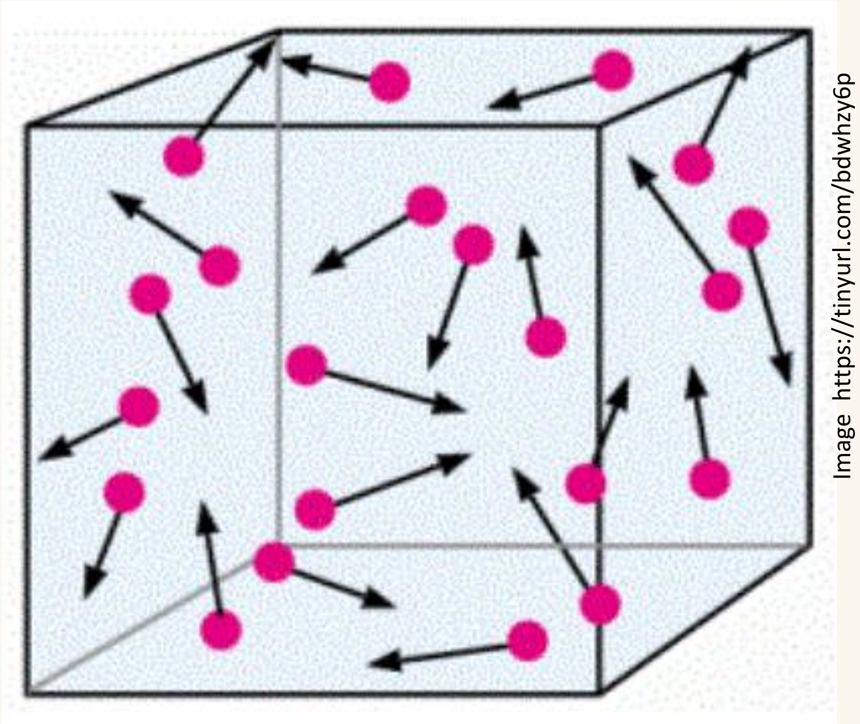


Image https://tinyurl.com/bdwhzy6p

We will look at:

- Pressure
- Modelling at the molecular level
- Ideal gas
- Kinetic theory

Pressure

https://sciencing.com/range-barometric-pressure-5505227.html

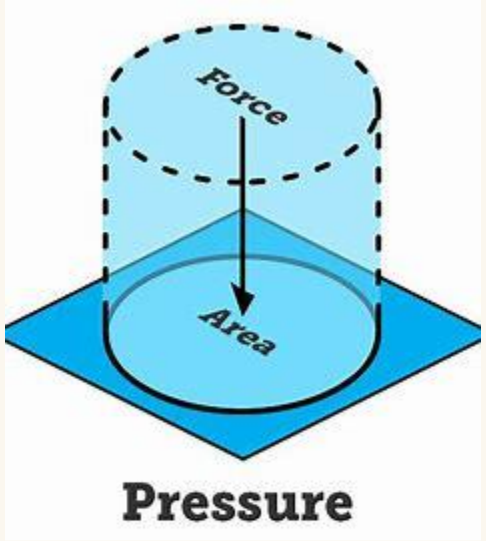
Unsplash.com

Unsplash.com

Unsplash.com

Mohamad Hassan Pixabay.com

Definition

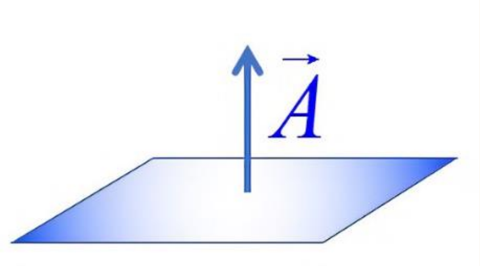


$$Pressure = \frac{Force}{Area}$$

Units

Nm^{-2} = Pa (pascals)

Vector or scalar?



Pressure is a scalar.

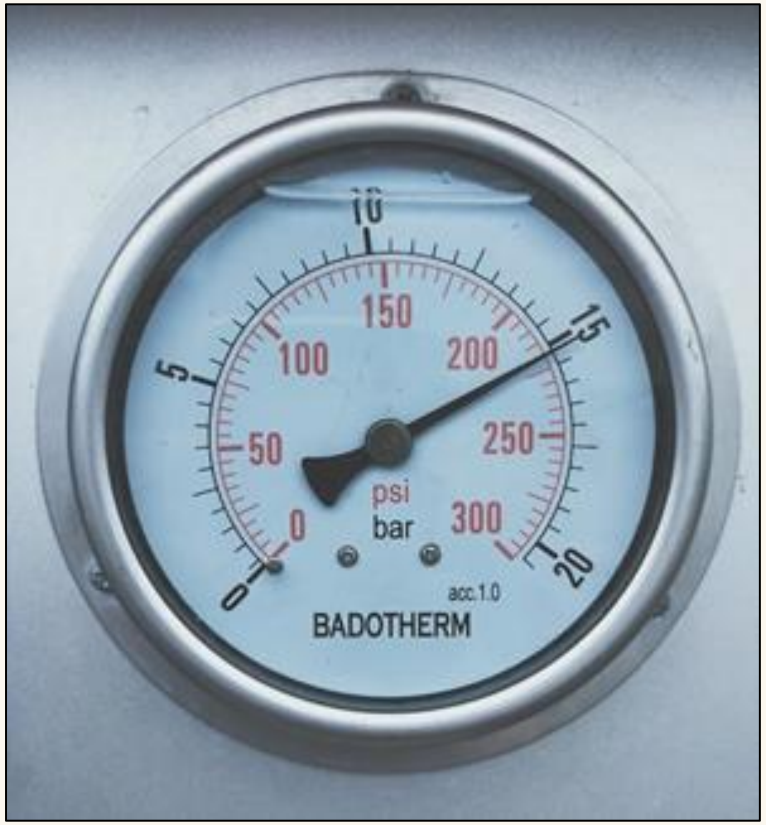
- Force is vector
- Area element is vector (normal to surface)

F = P**A**

Pressure is the scalar constant of proportionality.

Pressure acts in all directions

Other Units



Meow Meow on Pexels.com

1 bar = 10^5 Pa

1 atm = 1.013 bar = 1013 millibar = 1.013 x 10^5 Pa

Atmospheric pressure

What is the force of the atmosphere on your head?

$F = PA$ $\qquad P_{atm} = 1.013 \times 10^5$ Pa

Area head $= \pi r^2$ $\qquad r = 10$ cm

$F = 1.013 \times 10^5 \times \pi \times 0.1^2$

$= 3182$ N $\qquad \sim 3$ kN

Why do we not feel it?

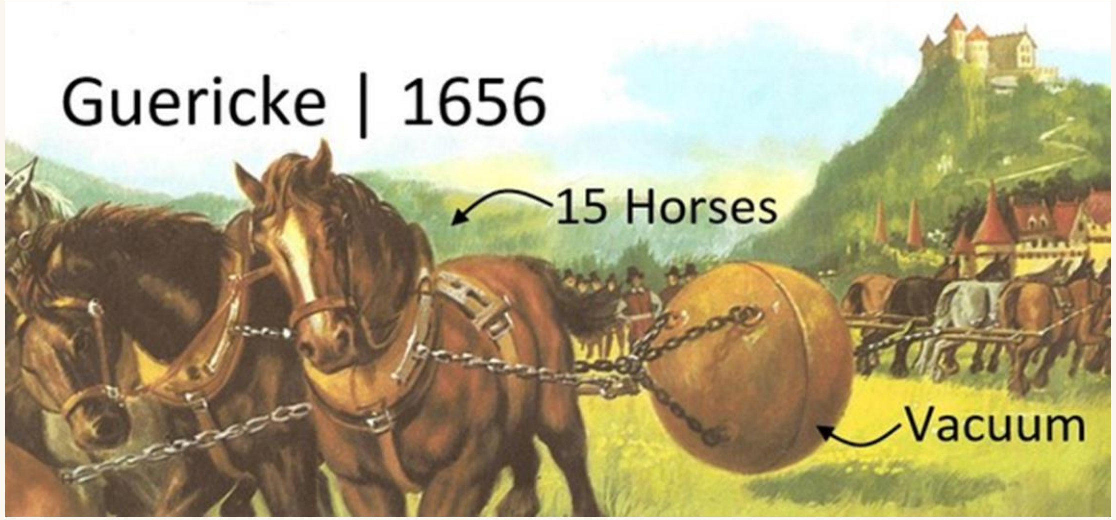
Balanced by internal pressure in body.

Atmospheric pressure

How can you demonstrate the pressure of the atmosphere?

[Can Crush Experiment]

Magdeburg hemispheres



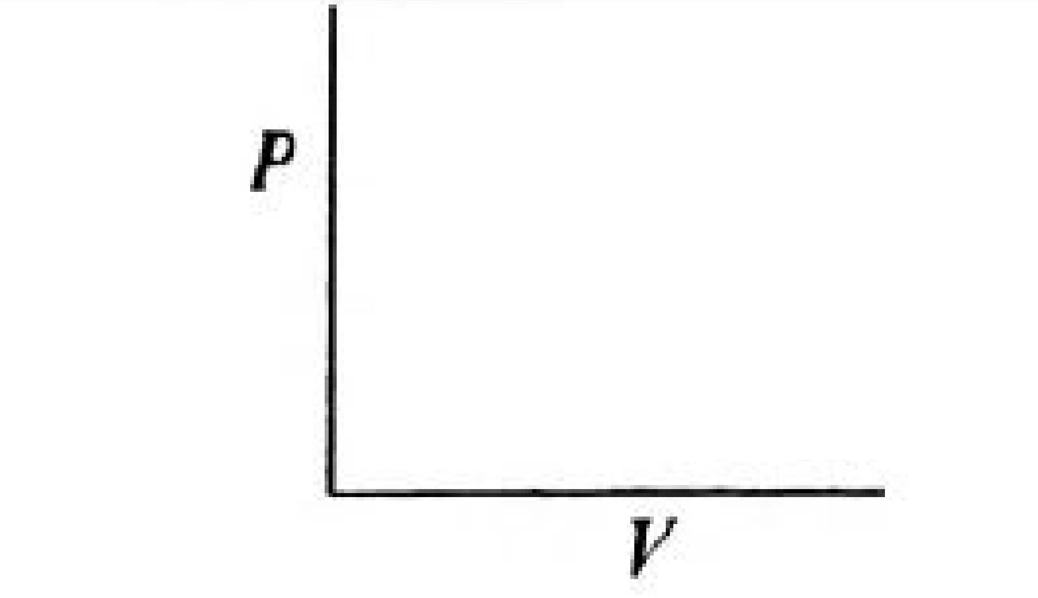
Otto van Guericke

Boyle's law

Robert Boyle discovered experimentally that for a fixed mass of gas at constant temperature:

PV = constant

What does this graph of P versus V look like?



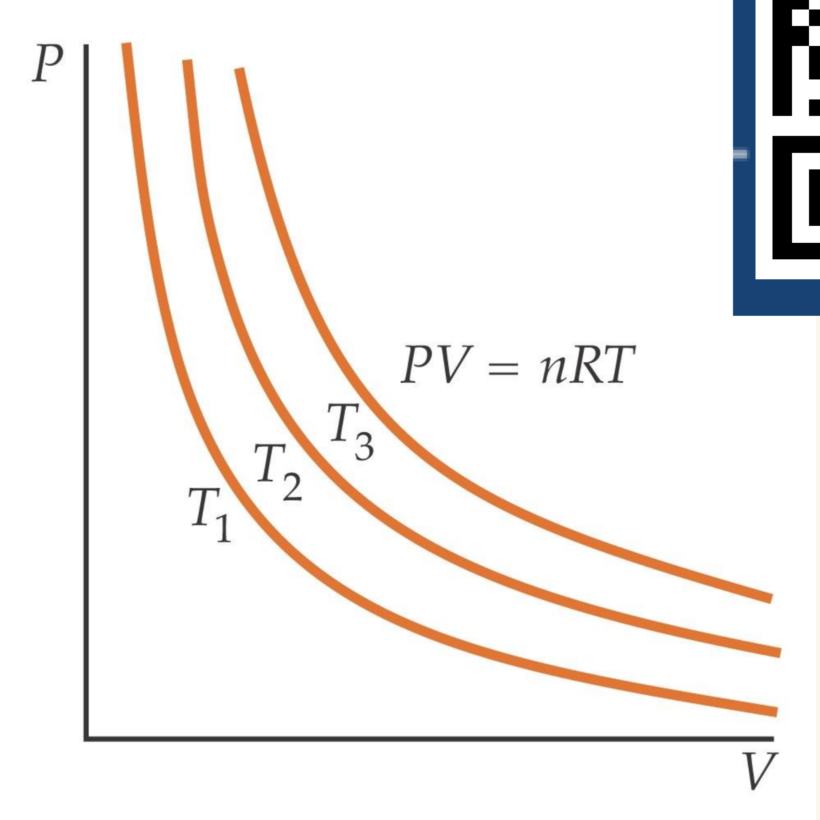
Ideal Gas

Ideal gas equation

$$PV = nRT \qquad PV = NkT$$

P, V, T = pressure, volume, temperature
R = molar gas constant
k = Boltzmann's constant
n = number of moles
N = number of molecules

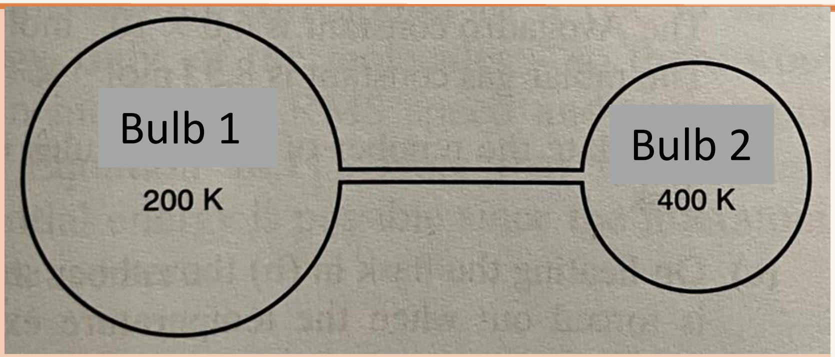
Temperature must be in Kelvin



Starting with T_2
which temp is higher T_1 or T_3

Question

The diagram shows two glass bulbs joined by a very thin capillary tube. This system is filled with an ideal gas, and steady state established with bulb 1 held at 200K, and bulb 2 held at 400K. The volume of bulb 1 is twice that of bulb 2.



If bulb 1 contains x moles of gas, how many moles if gas are in bulb 2?

A $\frac{x}{4}$ B $\frac{x}{2}$ C x D $2x$

The capillary tube allows movement of gas molecules. This will happen until PRESSURES are equal.

$$PV = nRT$$

$$n = \frac{PV}{RT}$$

$$n_1 = \frac{P_1V_1}{RT_1} \qquad n_2 = \frac{P_2V_2}{RT_2}$$

$$\frac{n_2}{n_1} = \frac{P_2V_2}{RT_2} \times \frac{RT_1}{P_1V_1}$$

$$\frac{n_2}{n_1} = \frac{V_2}{V_1} \times \frac{T_1}{T_2} = \frac{1}{2} \times \frac{200}{400} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

$$n_2 = \frac{n_1}{4} = \frac{x}{4} \qquad \boxed{A}$$

Molecules and Moles

n = number of moles
N = number of molecules
M = mass of mole
m = mass of molecule
ρ = density
N_A = Avogadro's number 6.02×10^{23}
k = Boltzmann's constant = 1.38×10^{-23} JK^{-1}
R = molar gas constant = 8.31 $Jmol^{-1}K^{-1}$

$$N = nN_A$$

$$M = mN_A$$

$$R = kN_A$$

Which expression would allow us to calculate density ?

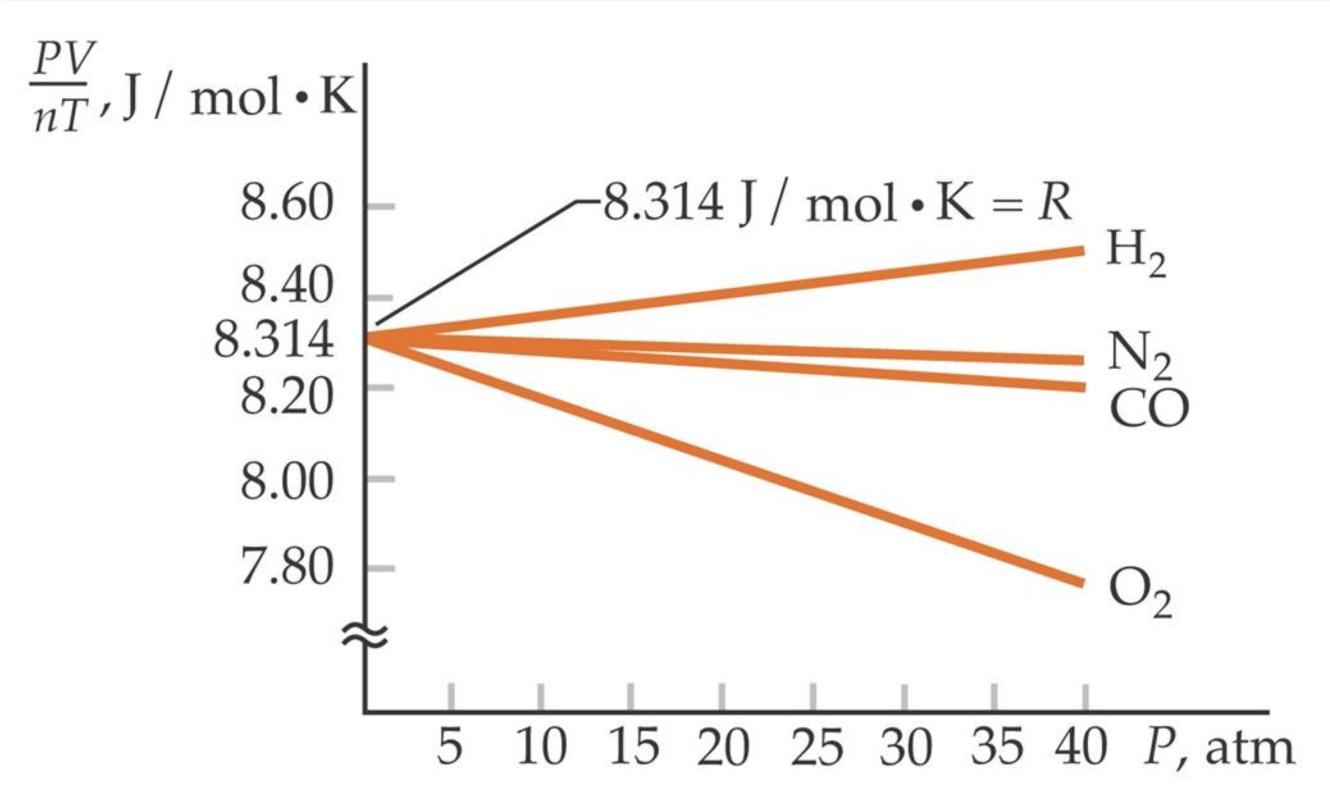
A $\frac{NM}{V}$

B $\frac{nm}{V}$

C $\frac{nM}{V}$

D $\frac{nN_AM}{V}$

Ideal gas constant, R

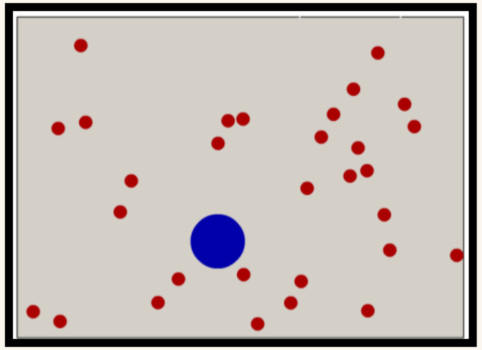


Real gases behave as ideal when molecules well separated (i.e. at low pressure).

Kinetic Theory of Gases

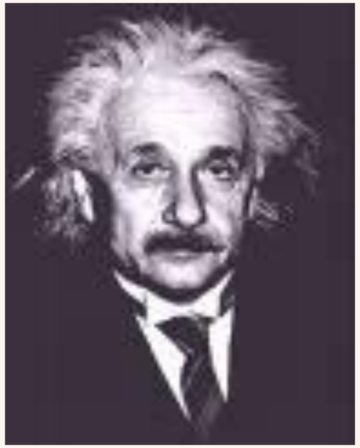
Modelling an ideal gas

Robert Brown
1773-1858



<u>Assumptions of kinetic theory</u>

1. Gas contains *many* molecules
2. Molecules well separated
3. Direction of motion of molecules is random
4. Molecules exert no force on each other, except when collide
5. Elastic collisions between molecules and with walls



Albert Einstein
1879-1955

Pressure

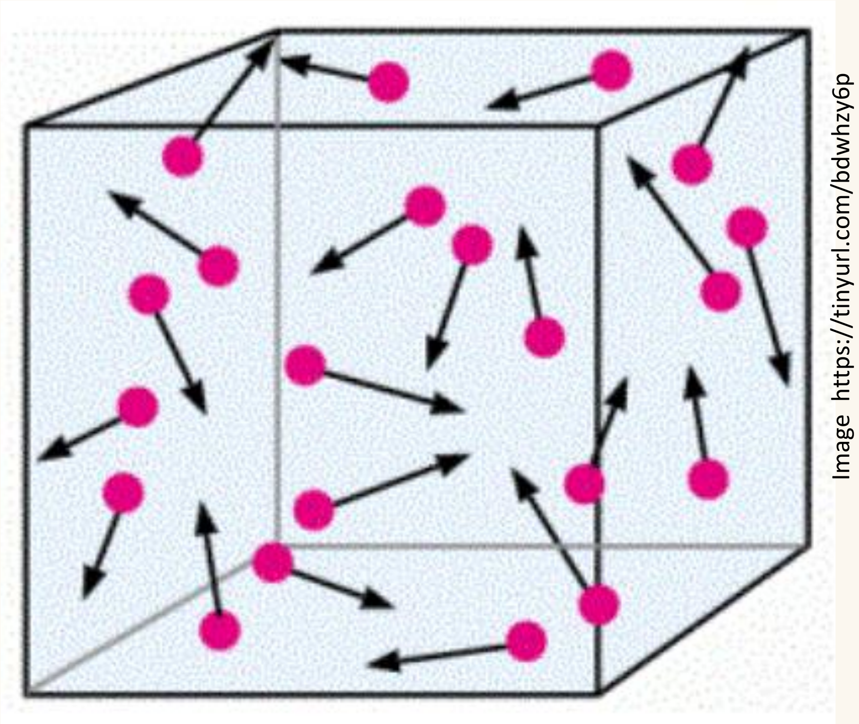
How does kinetic theory explain pressure?

Molecules collide with walls (change of momentum)

Force = rate of change of momentum

Many collisions per second

Pressure = force/area



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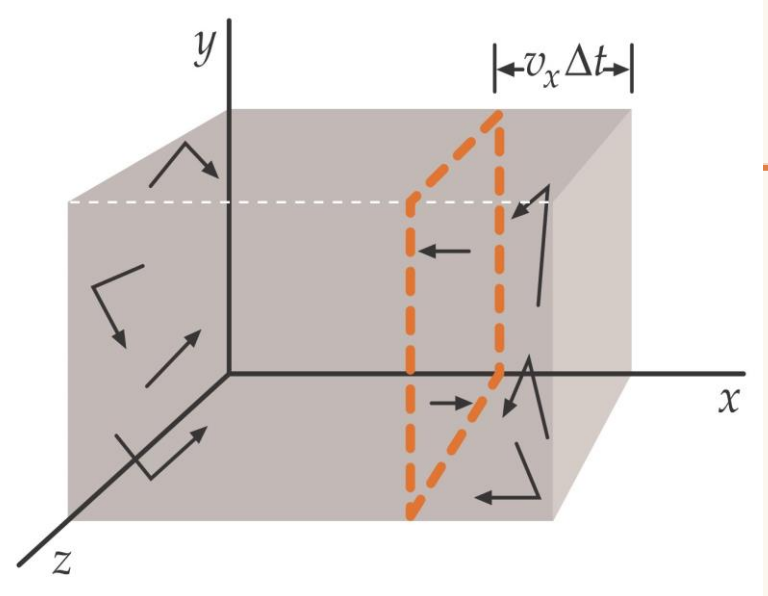
Pressure from kinetic theory



Derivation of pressure equation

Consider rectangular box, volume V, containing N gas molecules, each of mass m, moving with speed v.

Molecules that will hit right-hand wall, of area A, in time interval Δt are those within $v_x \Delta t$ of the wall and moving to the right along x-axis.

Number of these molecules is $\frac{1}{2}\frac{N}{V}v_x \Delta t A$

½ since only half molecules in that volume are moving to the right.

Initial momentum of molecule = $+mv_x$

Final momentum of molecule = $-mv_x$

Change of momentum per molecule = $2mv_x$

Pressure from kinetic theory

$$P = \frac{F}{A} \qquad F = \frac{\left|\Delta \overline{p}\right|}{\Delta t}$$

So change of momentum of all such molecules $\left|\Delta \overline{p}\right| = (2mv_x) \times \left(\frac{1}{2}\frac{N}{V} v_x \Delta t A\right) = \frac{N}{V} m v_x^2 A \Delta t$

Then pressure exerted by gas molecules on right-hand wall = $\frac{N}{V} m v_x^2$ Dividing by $A\Delta t$

To allow for the fact that not all molecules will have exactly the same speed we replace v_x^2 with $\left(v_x^2\right)_{av}$

Thus pressure on right-hand wall = $P = \frac{N}{V} m \left(v_x^2\right)_{av}$

Extending this to y and z directions: $\left(v_x^2\right)_{av} = \left(v_y^2\right)_{av} = \left(v_z^2\right)_{av}$ $\qquad \left(v^2\right)_{av} = \left(v_x^2\right)_{av} + \left(v_y^2\right)_{av} + \left(v_z^2\right)_{av} = 3\left(v_x^2\right)_{av}$

i.e. $\left(v_x^2\right)_{av} = \frac{1}{3}\left(v^2\right)_{av}$ and hence $\boxed{P = \frac{1}{3}\frac{N}{V} m \left(v^2\right)_{av}}$ or $\boxed{P = \frac{1}{3}\rho\left(v^2\right)_{av}}$ since $\rho = \frac{Nm}{V}$

Kinetic Energy and Temperature

From previous slide : $P = \frac{1}{3}\frac{N}{V}m(v^2)_{av}$

Or maybe from A Level it looked like this: $PV = \frac{1}{3}Nm(c_{rms})^2$

We will rearrange to get expression for KE……

$$3PV = 2N\left(\frac{1}{2}m(v^2)_{av}\right)$$

but $PV = NkT$

$$3\cancel{N}kT = 2\cancel{N}\left(\frac{1}{2}m(v^2)_{av}\right)$$

$$\frac{3}{2}kT = \frac{1}{2}m(v^2)_{av}$$

Thermal energy.
Relates to temp, which
we can measure

Average kinetic energy
of a molecule

Average molecular speeds

kinetic energy per molecule $$\frac{3}{2}kT = \frac{1}{2}m(v^2)_{av}$$

kinetic energy per mole $$\frac{3}{2}RT = \frac{1}{2}M(v^2)_{av}$$ Multiplying both sides by N_A

Rearranging: $$v_{rms} = \sqrt{(v^2)_{av}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3RT}{M}}$$

Trends: $KE_{av} \propto T$ $\quad v_{rms} \propto \sqrt{\frac{T}{M}}$

Question

What is the average (rms) speed of air molecules in still air at room temperature ?

A zero

B 1 mm/s (electron drift speed in metal)

C 2 m/s (walking speed)

D 30 m/s (fast car)

E 500 m/s (supersonic aeroplane)

F 10^8 m/s (close to speed of light)

Estimate

(before we calculate)

Question

What is the average (rms) speed of air molecules in still air at room temperature ?

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$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

$$v_{rms} = \sqrt{\frac{3 \times 8.31 \times 300}{28 \times 10^{-3}}}$$

$$= 517 \text{ m/s}$$

Take room temp = 300 K

Take air as nitrogen, M_{N_2} = 28 g

Question

A gas cylinder has a volume of 20 L. It contains helium at a gauge pressure of 3.0×10^5 Pa, and temperature of 25 °C.

(i) How many moles of gas are in the cylinder?
(ii) What is the mass of gas in the cylinder?

Molar mass of helium = 4 g.

Absolute pressure = gauge pressure + atmospheric pressure

(I)

$$n = \frac{PV}{RT}$$

$$= \frac{(3.0 + 1.013) \times 10^5 \times 20 \times 10^{-3}}{8.31 \times (25 + 273)}$$

$n = 3.24$ moles

(II) mass $= n \times M$

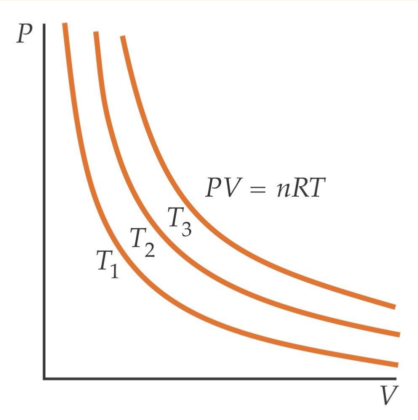
$$= 3.24 \times 4 \times 10^{-3}$$

$$= 0.013 \text{ kg} = 13 \text{g}$$

Summary

1 atm = 1013 millibar = 1.013 x 10^5 Pa

1 bar = 10^5 Pa



$$PV = nRT$$

$$PV = NkT$$

$$\frac{P_1V_1}{n_1T_1} = \frac{P_2V_2}{n_2T_2}$$

$$P = \frac{1}{3}\frac{N}{V}m(v^2)_{av}$$

$$\frac{3}{2}kT = \frac{1}{2}m(v^2)_{av}$$

$$\frac{3}{2}RT = \frac{1}{2}M(v^2)_{av}$$

Assumptions of kinetic theory

Gas contains many molecules
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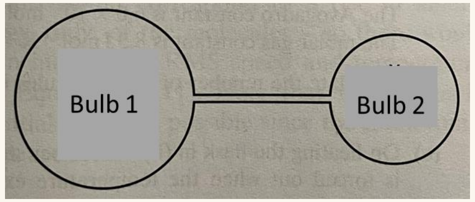

In equilibrium gas pressure will be the same at all points in the system.

$$N = nN_A \qquad M = mN_A \qquad R = kN_A \qquad \rho = \frac{nM}{V}$$

Absolute pressure = gauge pressure + atmospheric pressure