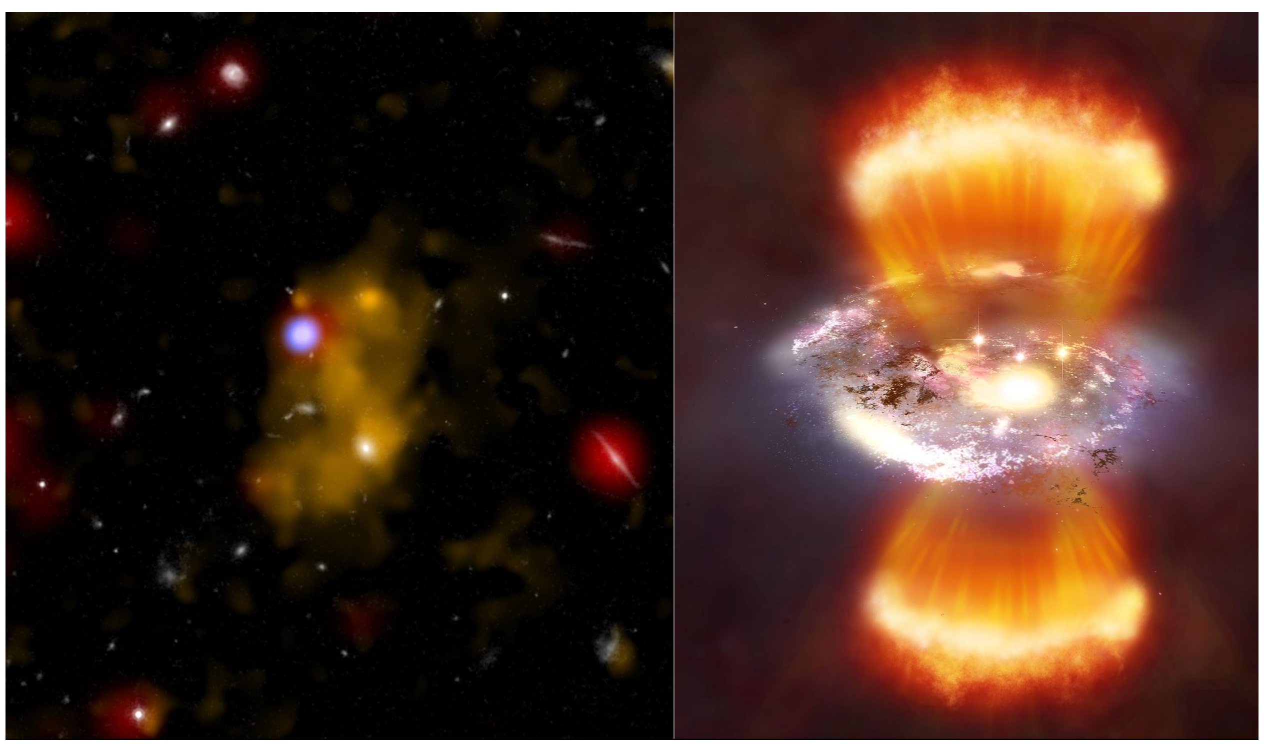
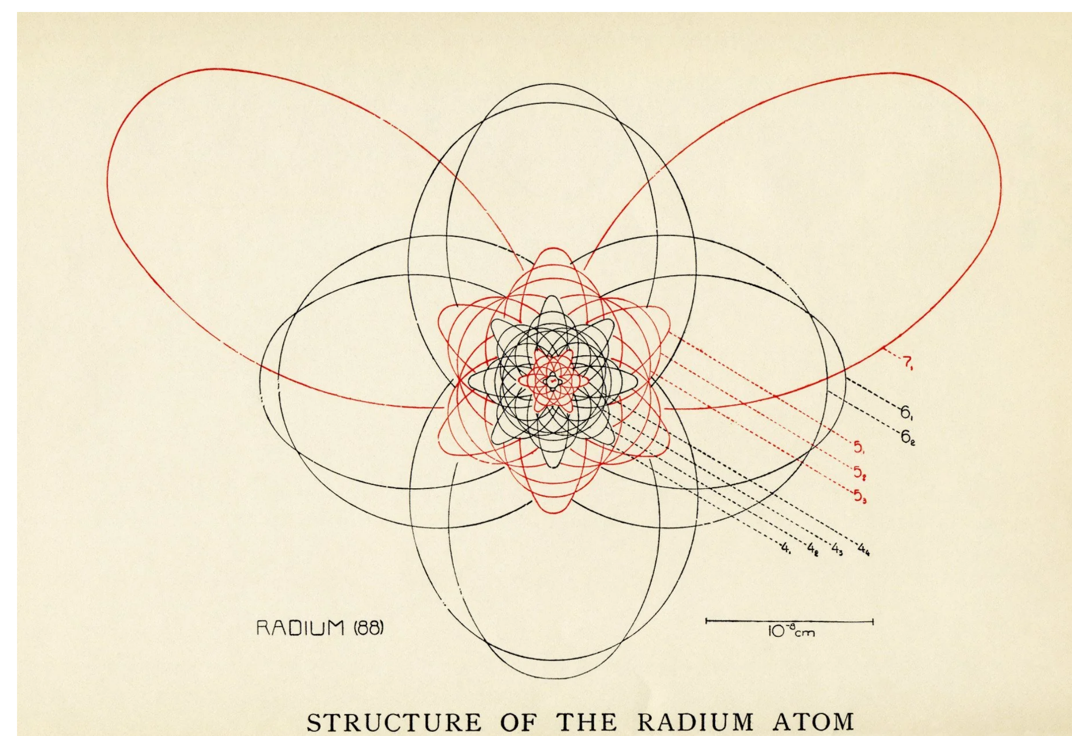
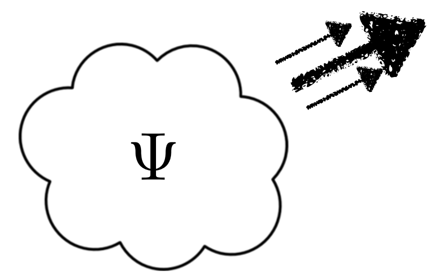
The Bohr model of the atom

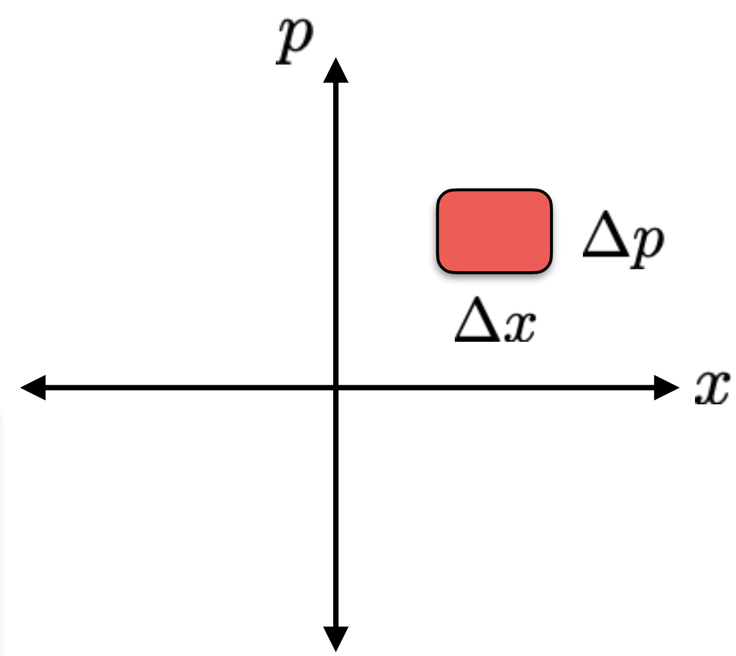




Particles in space

Spatially localised:

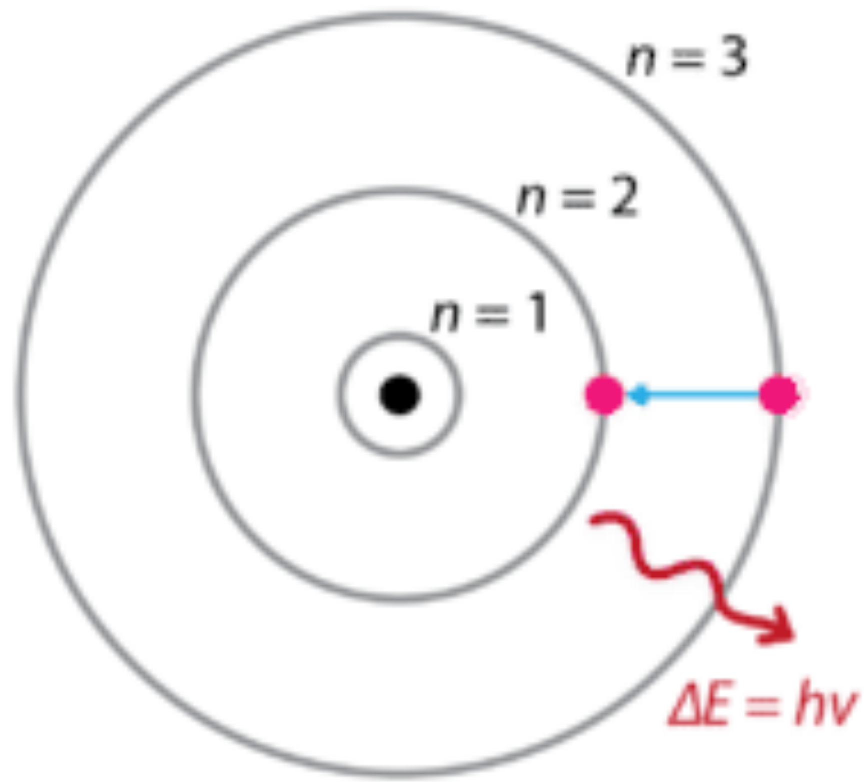




Heisenberg
Uncertainty Relation

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

Simple model of an atom

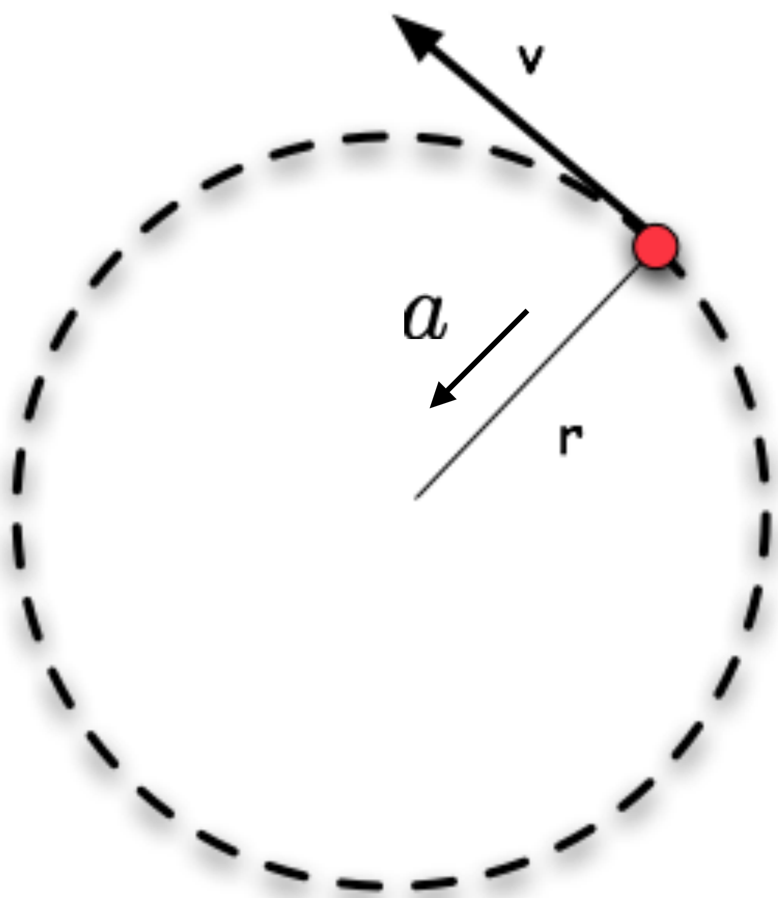


Discrete energy levels

Classical instability

Model an atom as electrons circling a nucleus

$$a = -\frac{v^2}{r}$$



BUT: classical electromagnetism tells us that an accelerating charge radiates energy

Therefore electrons should lose energy and spiral into centre.

$$t \sim 10^{-11} s$$

Central Question: So how can matter be stable??

An observation

Heisenberg
Uncertainty Relation

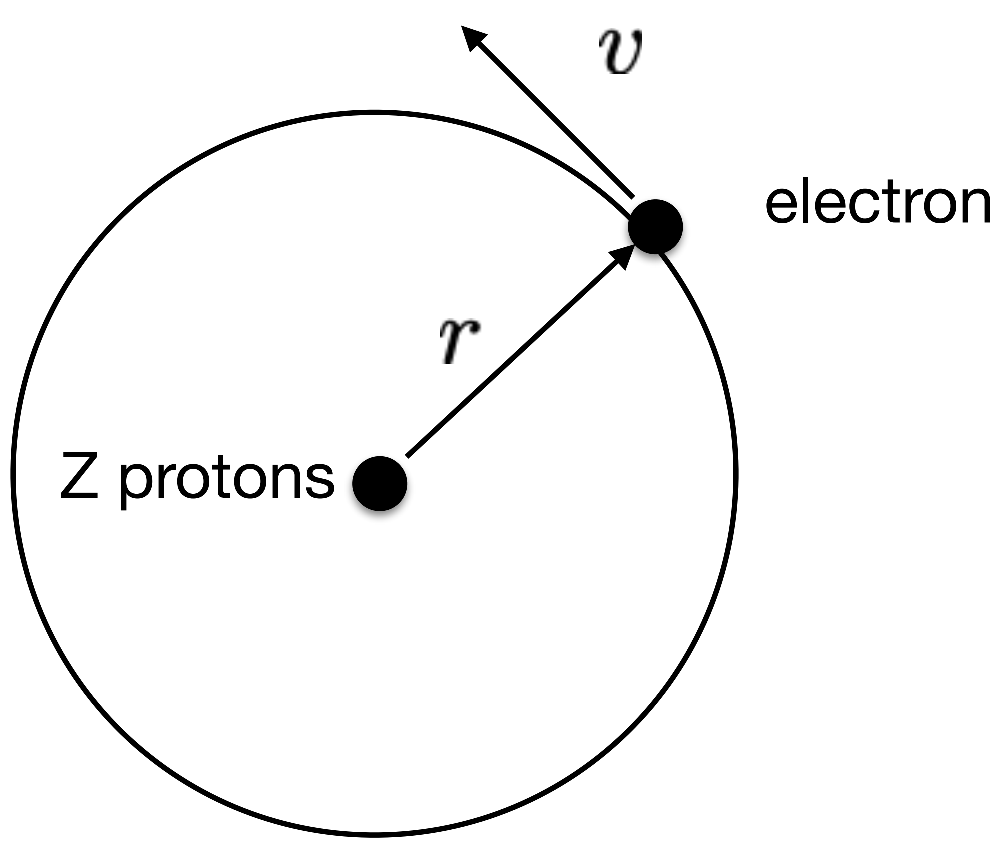
$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

LHS has units of
angular momentum

So $\hbar$ also has **units of angular momentum**

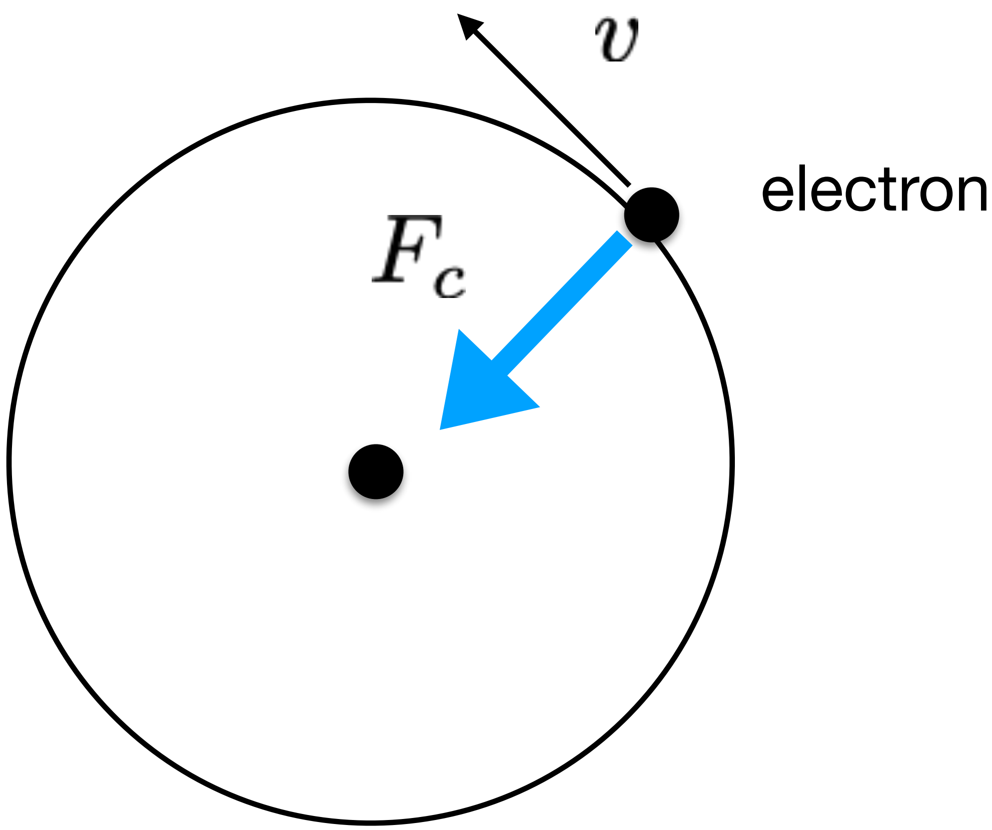
Bohr's Postulate

The angular momentum of an electron orbiting the nucleus is quantised in multiples of $\hbar$.



$$mvr = n\hbar$$

$$n = 1, 2, 3, 4, \ldots$$



$$F_c = \text{ centripetal force on electron}$$

$$F_c = \frac{mv^2}{r}$$

The only force acting on electron is the Coulomb force.

$$F_{\text{Coul}} = -\frac{1}{4\pi\epsilon_0}\frac{Qq}{r^2}$$

$$\Rightarrow \frac{mv^2}{r} = -\frac{1}{4\pi\epsilon_0}\frac{Qq}{r^2}$$

The derivation for Hydrogen

$$\frac{mv^2}{r} = -\frac{1}{4\pi\epsilon_0}\frac{Qq}{r^2}$$

$$Qq = -e^2$$

$$\Rightarrow \boxed{v^2 = \frac{1}{4\pi\epsilon_0}\frac{e^2}{mr}}$$

Now include Bohr's fundamental assumption: $L = r(mv) = n\hbar$

$$n = 1, 2, 3, 4, \ldots$$

$$\Rightarrow \boxed{mvr = n\hbar}$$

We have the following two equations:

$$mvr = n\hbar \qquad n = 1, 2, 3, 4, \ldots$$

$$v^2 = \frac{1}{4\pi\epsilon_0}\frac{e^2}{mr}$$

Eliminate r and solve for v:

$$r = \frac{n\hbar}{mv} \Rightarrow v^2 = \frac{1}{4\pi\epsilon_0}\frac{e^2}{\left(\frac{n\hbar}{v}\right)}$$

$$\Rightarrow \boxed{v = \frac{1}{4\pi\epsilon_0}\frac{e^2}{n\hbar}}$$

Velocity quantised!

$$n = 1, 2, 3, 4, \ldots$$

Now solve for r: $$v = \frac{1}{4\pi\epsilon_0}\frac{e^2}{n\hbar} \qquad r = \frac{n\hbar}{mv}$$

$$\Rightarrow r = \frac{n\hbar}{m}\left(\frac{4\pi\epsilon_0 n\hbar}{e^2}\right)$$

$$\Rightarrow \boxed{r = \frac{4\pi\epsilon_0 (n\hbar)^2}{me^2}}$$

quantised orbits!

$$n = 1, 2, 3, 4, \ldots$$

$$r_n = \frac{4\pi\epsilon_0 n^2\hbar^2}{me^2} = a_1 n^2$$

$$a_1 = \frac{4\pi\epsilon_0\hbar^2}{me^2} = 0.0529nm$$

radius of innermost orbit

Energy spectrum of the Bohr atom

Total energy of the electron:

$$E = \frac{1}{2}mv^2 - \frac{1}{4\pi\epsilon_0}\frac{e^2}{r}$$

The quantised velocity and radius:

$$r_n = \frac{4\pi\epsilon_0 n^2\hbar^2}{me^2} \qquad v_n = \frac{1}{4\pi\epsilon_0}\frac{e^2}{n\hbar}$$

$$\Rightarrow E_n = \frac{1}{2}\frac{me^4}{(4\pi\epsilon_0)^2 n^2\hbar^2} - \frac{e^2}{4\pi\epsilon_0}\left(\frac{me^2}{4\pi\epsilon_0 n^2\hbar^2}\right)$$

Energy Spectrum

$$E_n = \frac{1}{2}\frac{me^4}{(4\pi\epsilon_0)^2 n^2 \hbar^2} - \frac{e^2}{4\pi\epsilon_0}\left(\frac{me^2}{4\pi\epsilon_0 n^2 \hbar^2}\right)$$

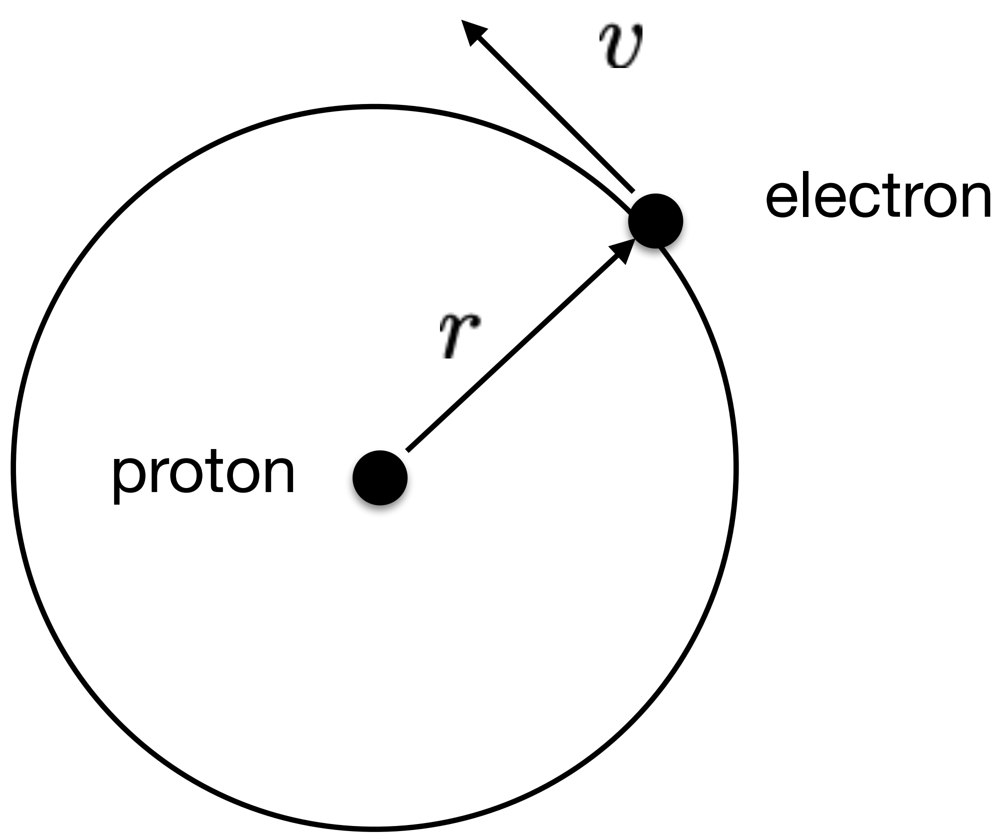
$$\Rightarrow E_n = -\frac{me^4}{32\pi^2\epsilon_0^2}\frac{1}{n^2} \qquad n = 1, 2, 3, 4, \ldots$$

$$\Rightarrow E_n = -\frac{E_1}{n^2} \qquad E_1 = 13.6eV$$

Recap: the key assumption

Bohr's assumption:

The angular momentum of an electron orbiting the nucleus is quantised in multiples of $\hbar$.



$$mvr = n\hbar$$
$$n = 1, 2, 3, 4, \ldots$$

Key Bohr Atom properties

Circular motion + Bohr's assumption:

Quantised energies

$$E_n = -\frac{me^4}{32\pi^2\epsilon_0^2}\frac{1}{n^2}$$

$$\Rightarrow E_n = -\frac{E_1}{n^2}$$

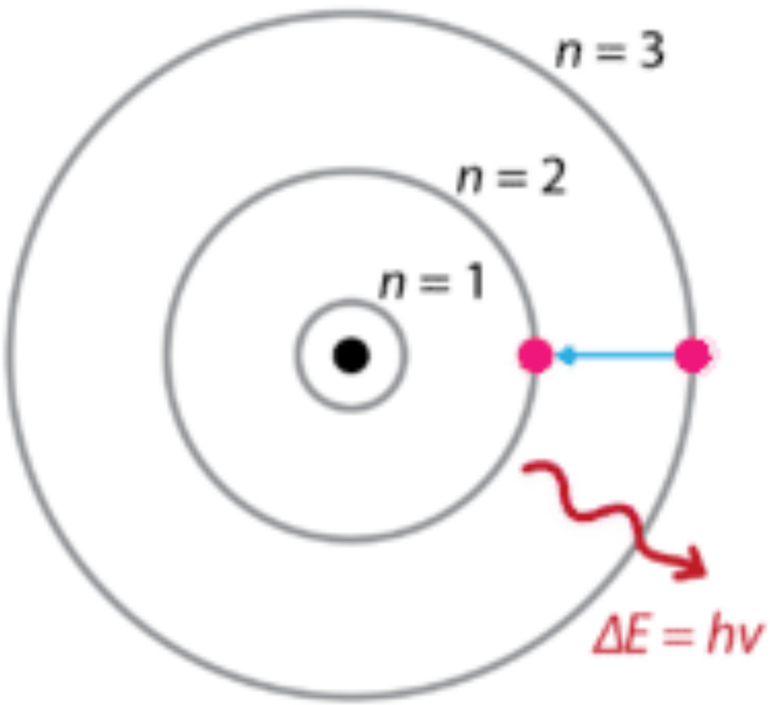
$$E_1 = 13.6eV$$

Quantised orbits

$$r_n = \frac{4\pi\epsilon_0 n^2\hbar^2}{me^2} = a_1 n^2$$

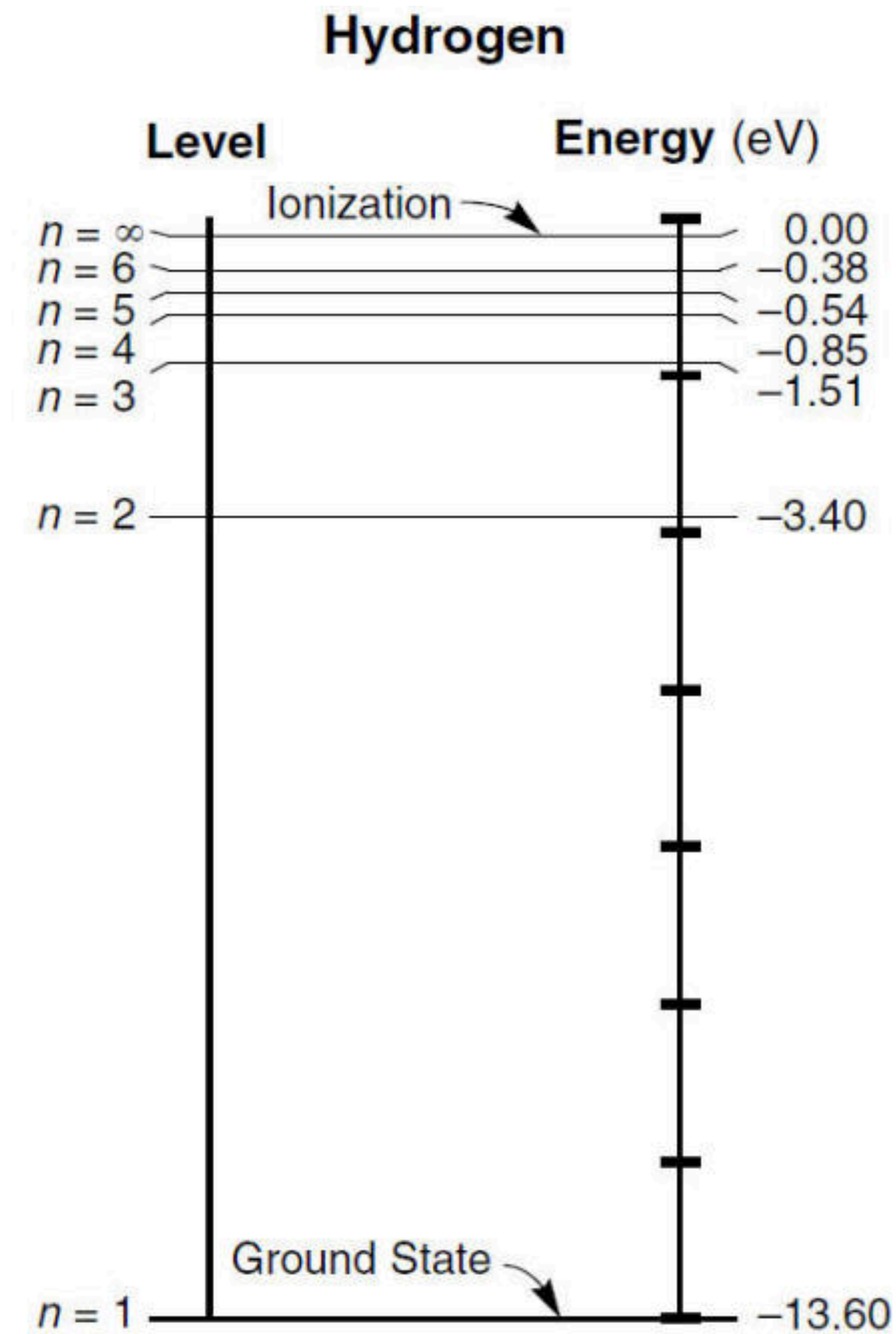
$$a_1 = \frac{4\pi\epsilon_0\hbar^2}{me^2} = 0.0529nm$$

$$n = 1, 2, 3, 4, \ldots$$



$$E_n = -\frac{13.6eV}{n^2}$$

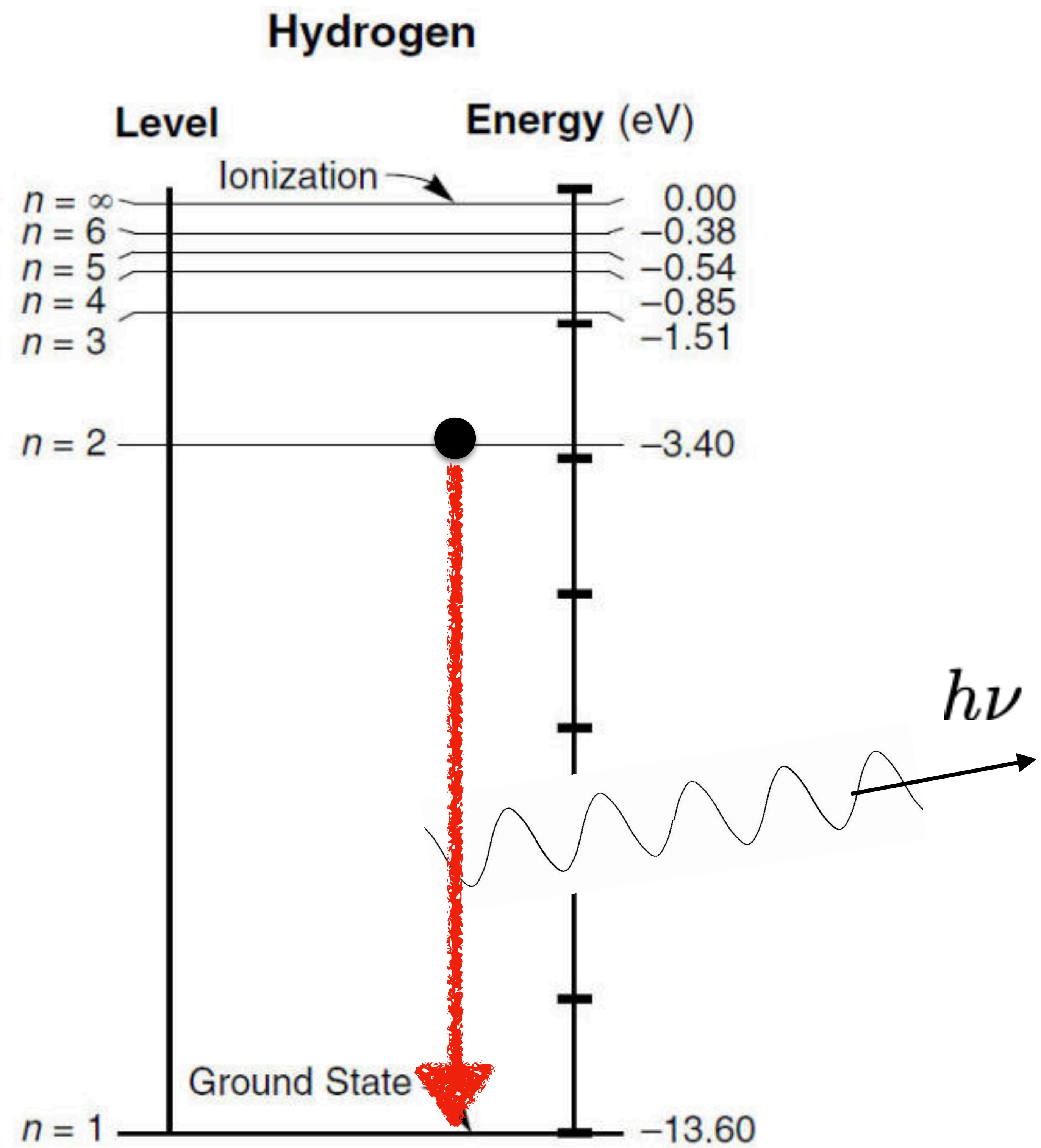
$$r_n = a_1 n^2$$



Energy Levels for the Hydrogen Atom

Emission spectra

Electron in first excited state drops down to ground state and a photon is emitted.



Energy Levels for the Hydrogen Atom

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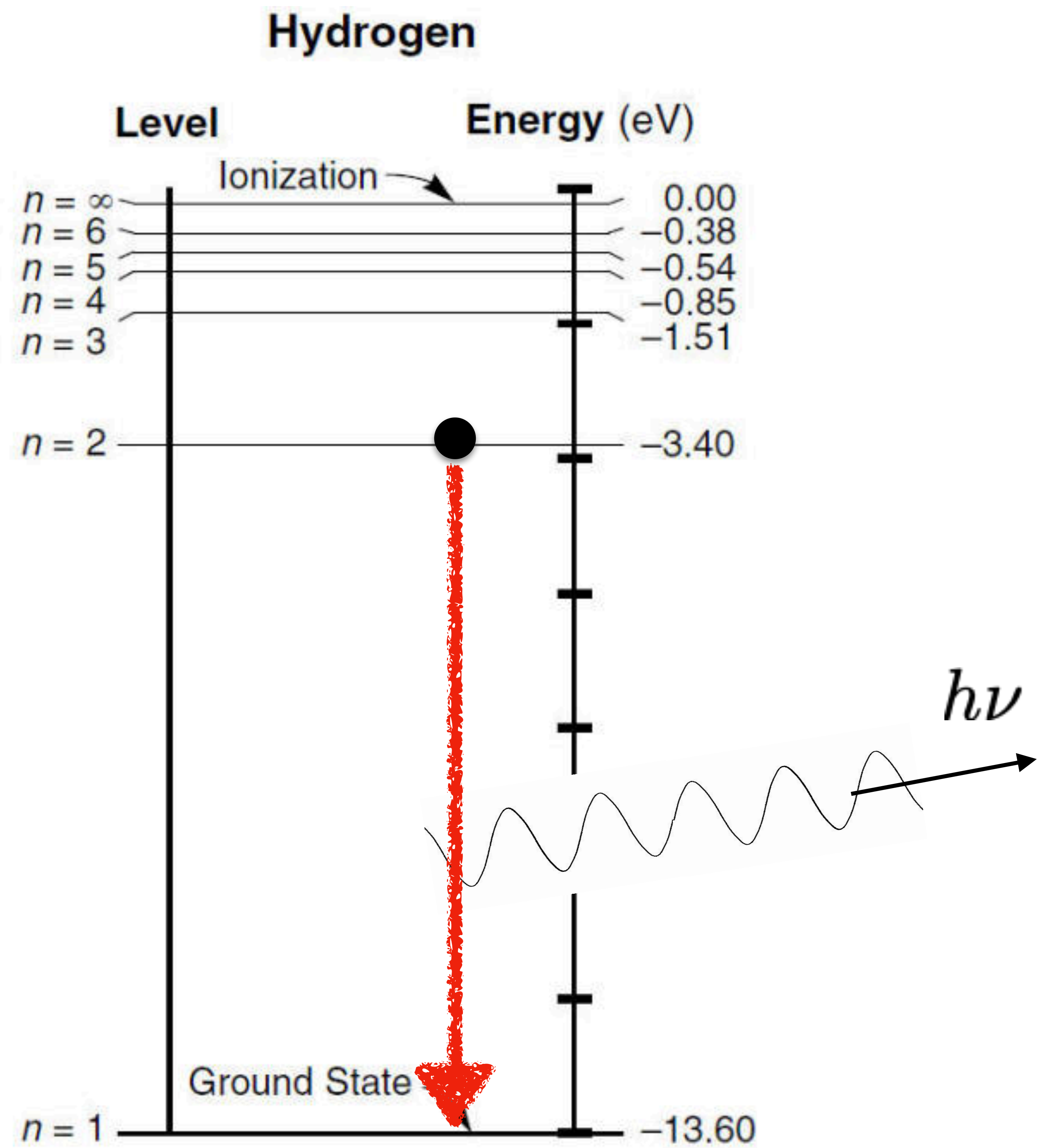
Electron in first excited state drops down to ground state and a photon is emitted.

$$E_2 - E_1 = h\nu$$

$$\frac{-13.6eV}{2^2} - \frac{13.6eV}{1^2} = h\nu$$



Energy Levels for the Hydrogen Atom

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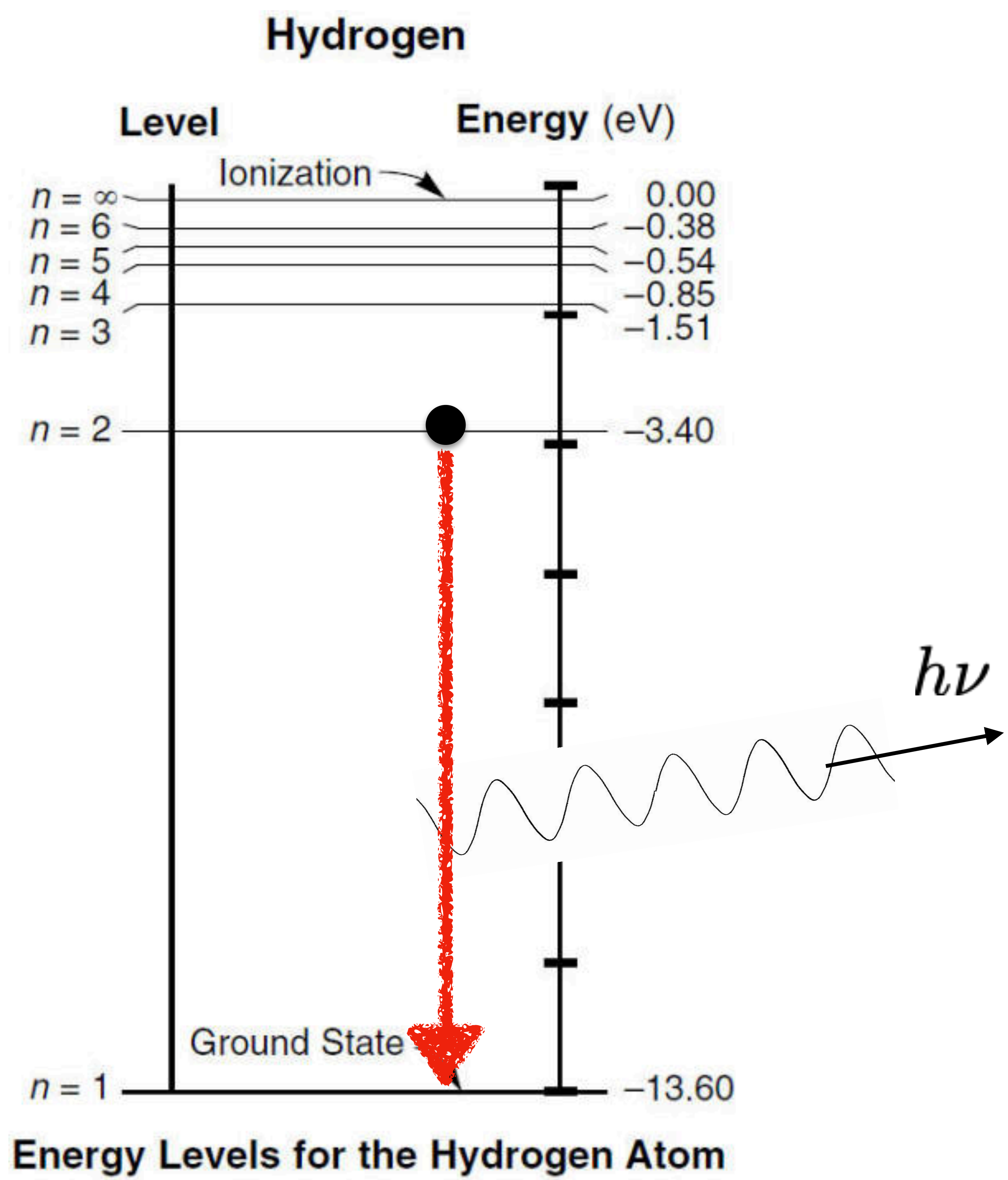
$$E_2 - E_1 = h\nu$$

$$\frac{-13.6eV}{2^2} - \frac{13.6eV}{1^2} = h\nu$$

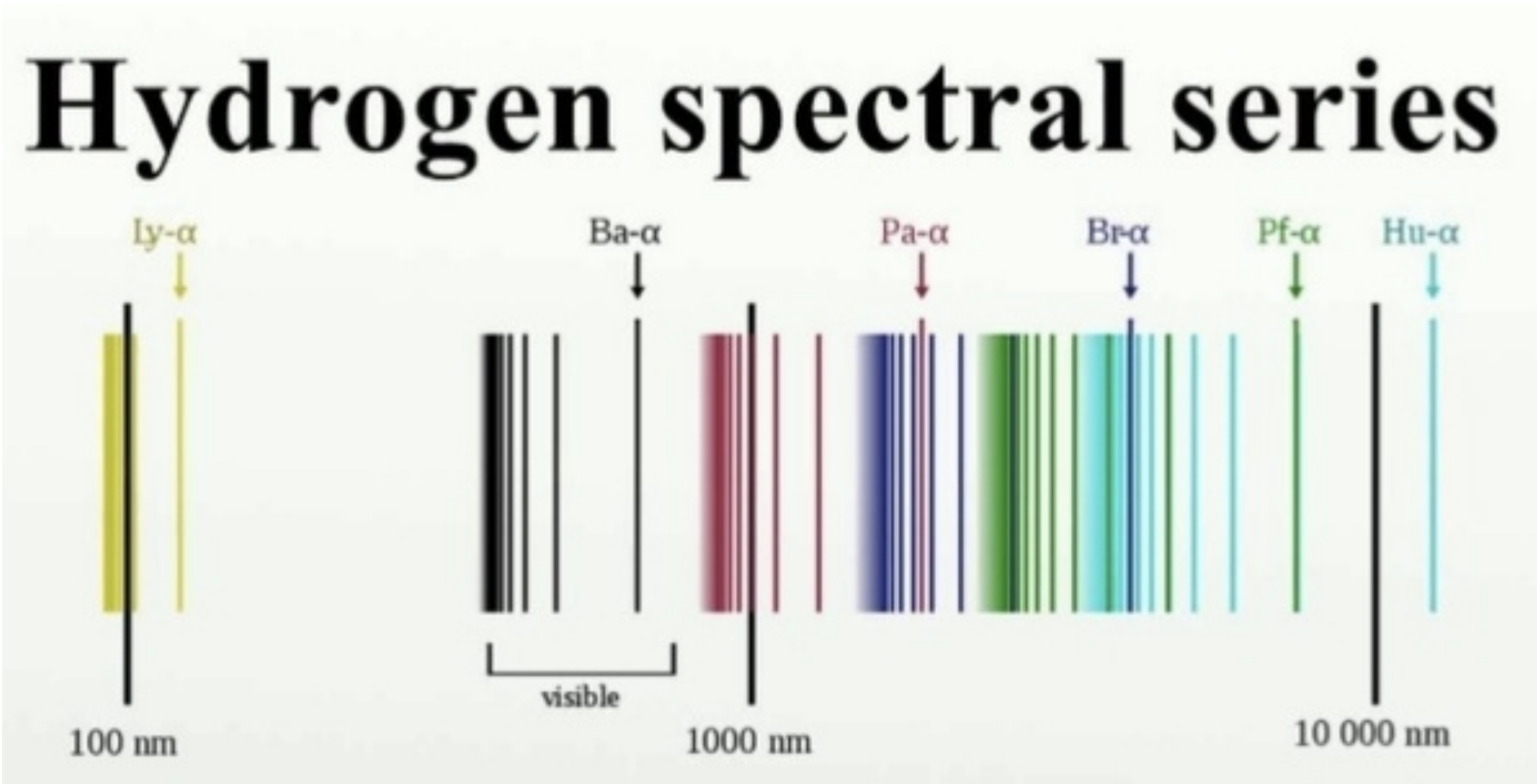
$$\Rightarrow h\nu = 10.2eV$$

$$\Rightarrow \nu = 2.47 \times 10^{15} s^{-1}$$

$$\Rightarrow \lambda = 120nm$$



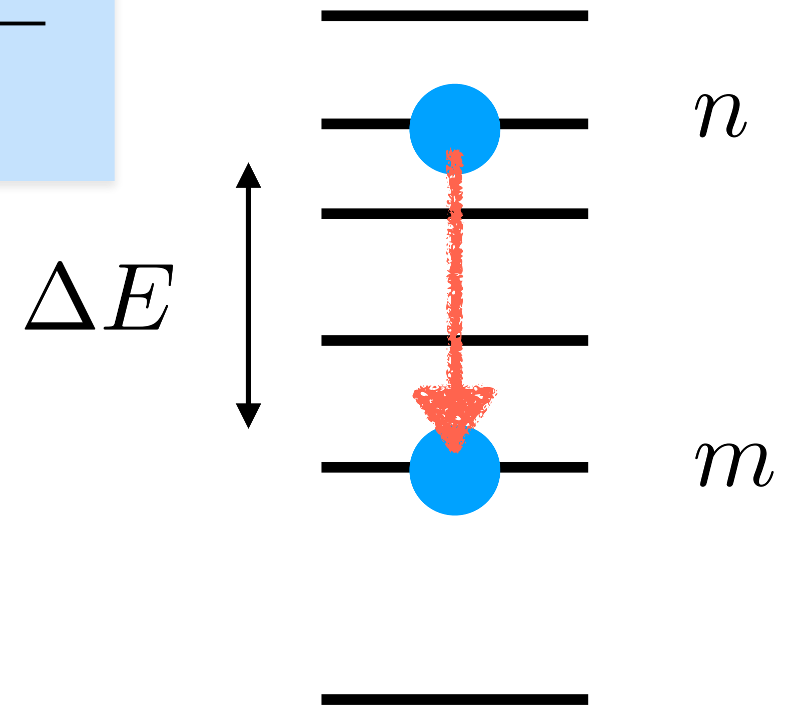
What do we observe?



- Hydrogen atom emits radiation at discrete wavelengths only.
- Electron dropping from upper level to a lower level - emits photon!

Emission wavelengths

$$E_n = -\frac{13.6eV}{n^2}$$



$$\Delta E = h\nu$$

$$\Rightarrow h\nu = E_n - E_m$$

$$\Rightarrow hc\frac{1}{\lambda} = E_n - E_m$$

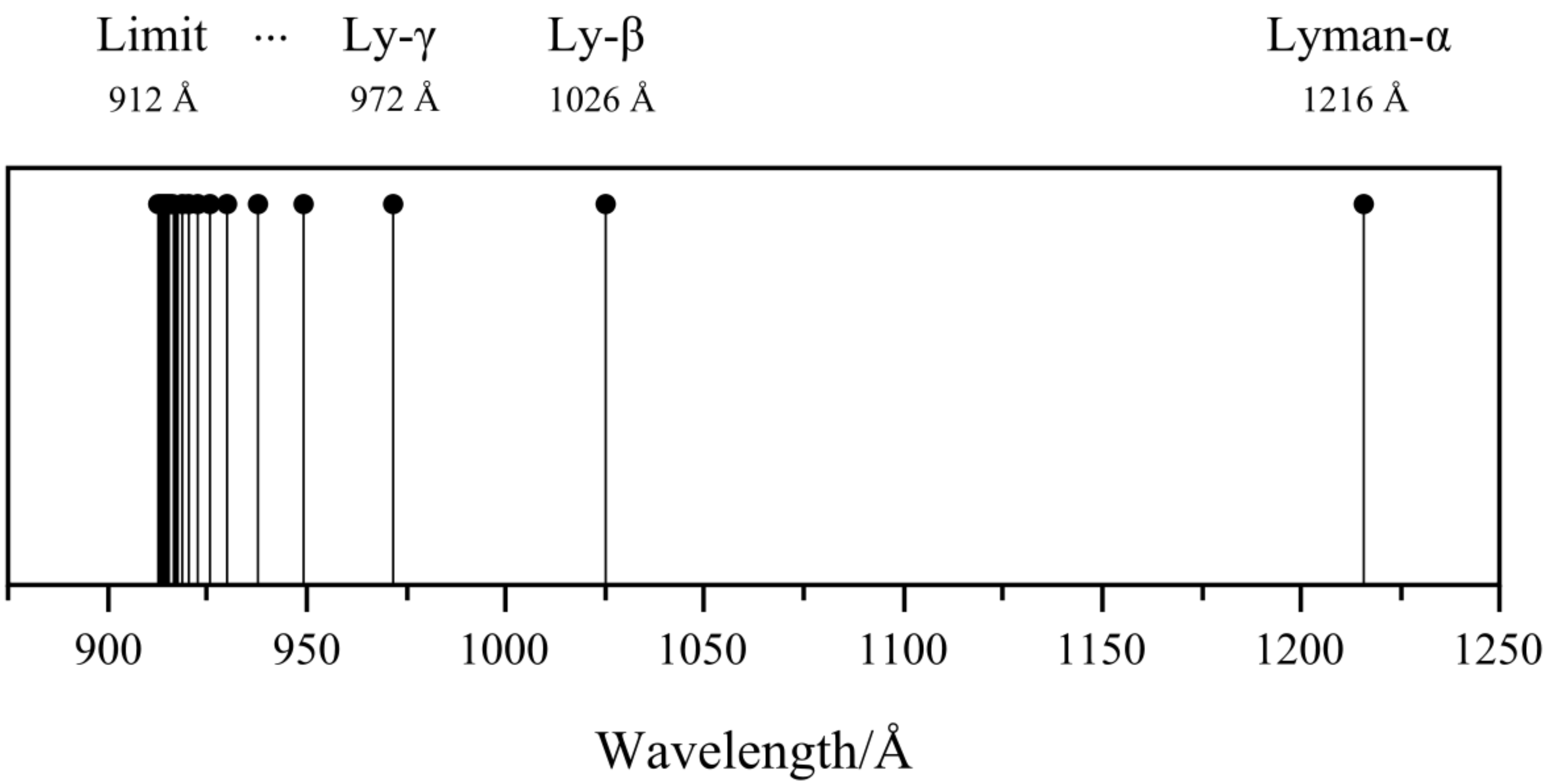
$$\Rightarrow \frac{1}{\lambda} = \frac{E_n - E_m}{hc}$$

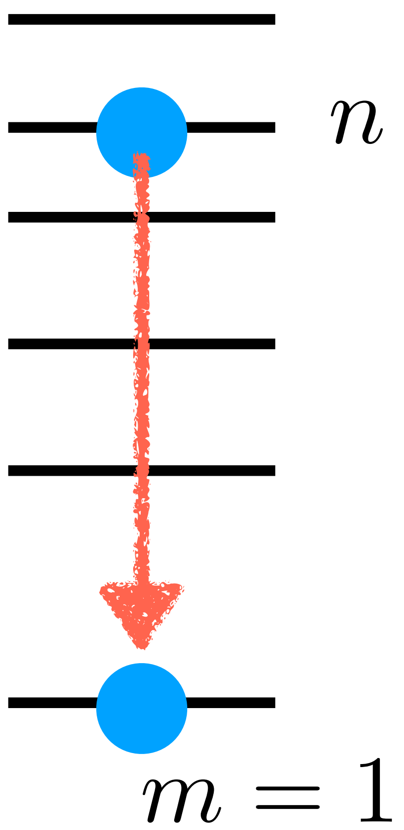
Formula for wavelengths

$$\Rightarrow \boxed{\frac{1}{\lambda} = -\frac{13.6eV}{hc}\left(\frac{1}{n^2} - \frac{1}{m^2}\right)}$$

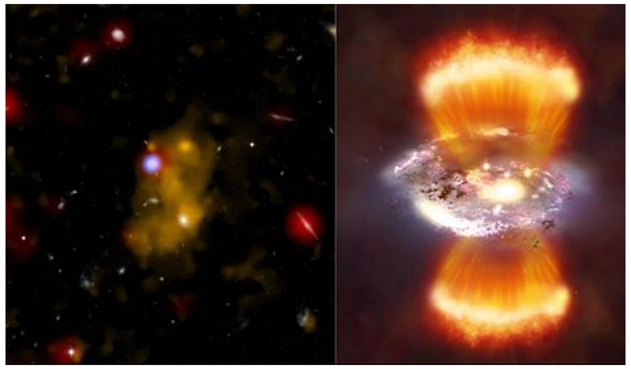
Lyman series

- Electron **drops to the ground state**.





Lyman-alpha as a probe



Cosmological Applications:

- Lyman-alpha is crucial for understanding cosmic structures.
- An effective probe for studying high-redshift galaxies and intergalactic medium conditions.
- The absorption features in the spectra of distant quasars create what is known as the "Lyman-alpha forest," which helps astronomers map neutral hydrogen distribution in the universe.

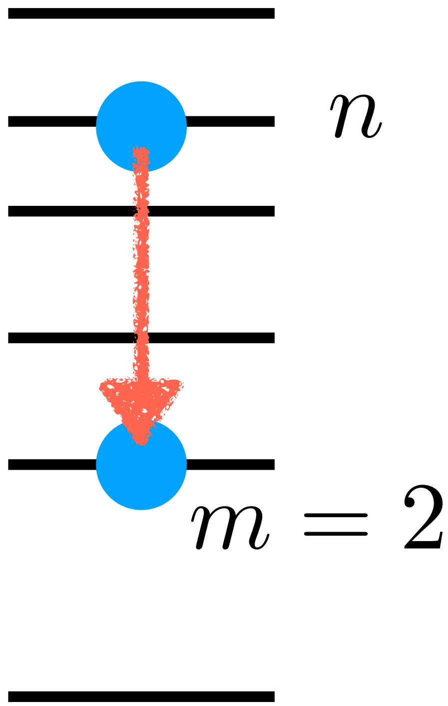
Solar Observations:

- In solar physics, the Lyman-alpha line is the most intense UV spectral line observed on the solar disk.
- Plays a vital role in illuminating solar chromospheric and coronal structures, influencing models of solar activity and space weather phenomena.

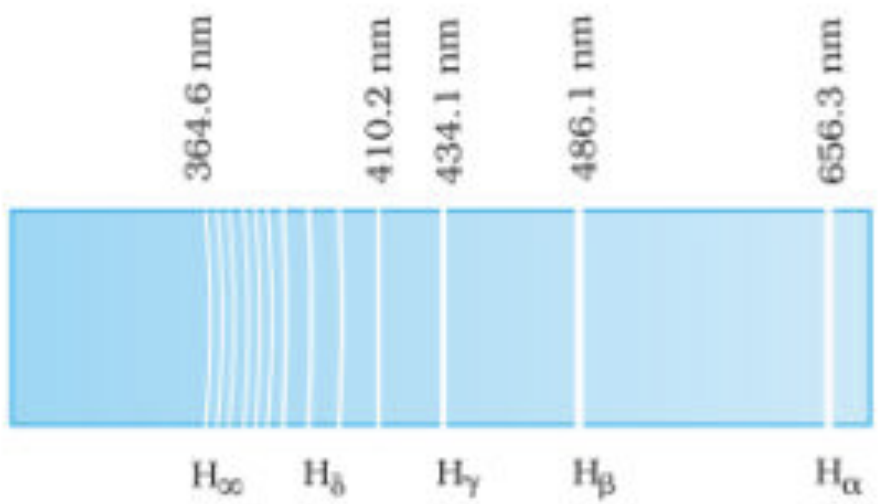
Radiative Transfer Models (E&M fields moving through media):

- The complex interaction of Lyman-alpha photons with neutral hydrogen leads to significant radiative transfer effects.
- These interactions are critical for interpreting observations of star-forming regions and understanding galaxy formation dynamics.

Balmer series



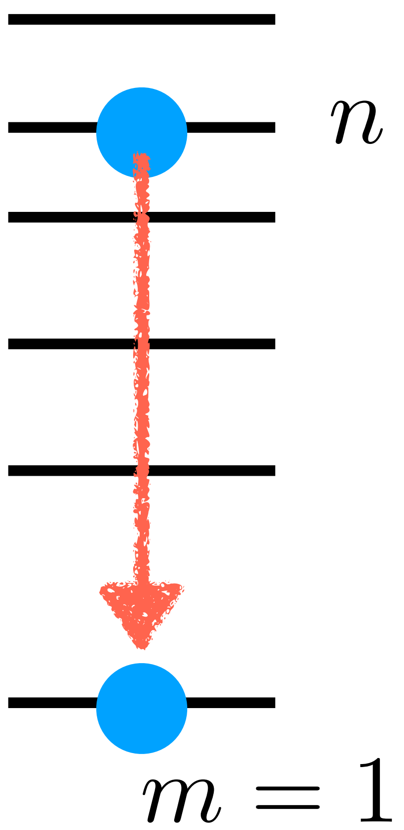
- Electron **drops to the first excited state.**



Balmer series in the emission spectrum of hydrogen.

The Balmer series is very useful in astronomy (determining stellar motion, exoplanets, etc…)

Computing lines for Lyman Series



- Electron **drops to the ground state.**

$$\frac{1}{\lambda} = -\frac{13.6eV}{hc}\left(\frac{1}{n^2} - \frac{1}{1^2}\right)$$

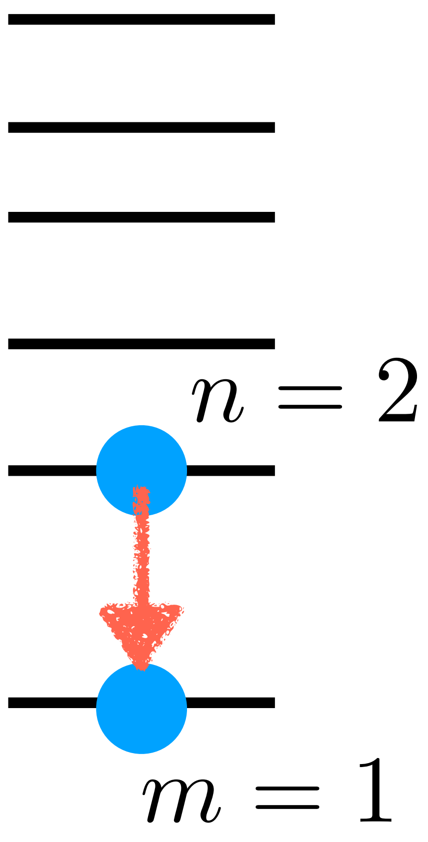
$$\Rightarrow \frac{1}{\lambda} = \frac{13.6eV}{hc}\left(1 - \frac{1}{n^2}\right)$$

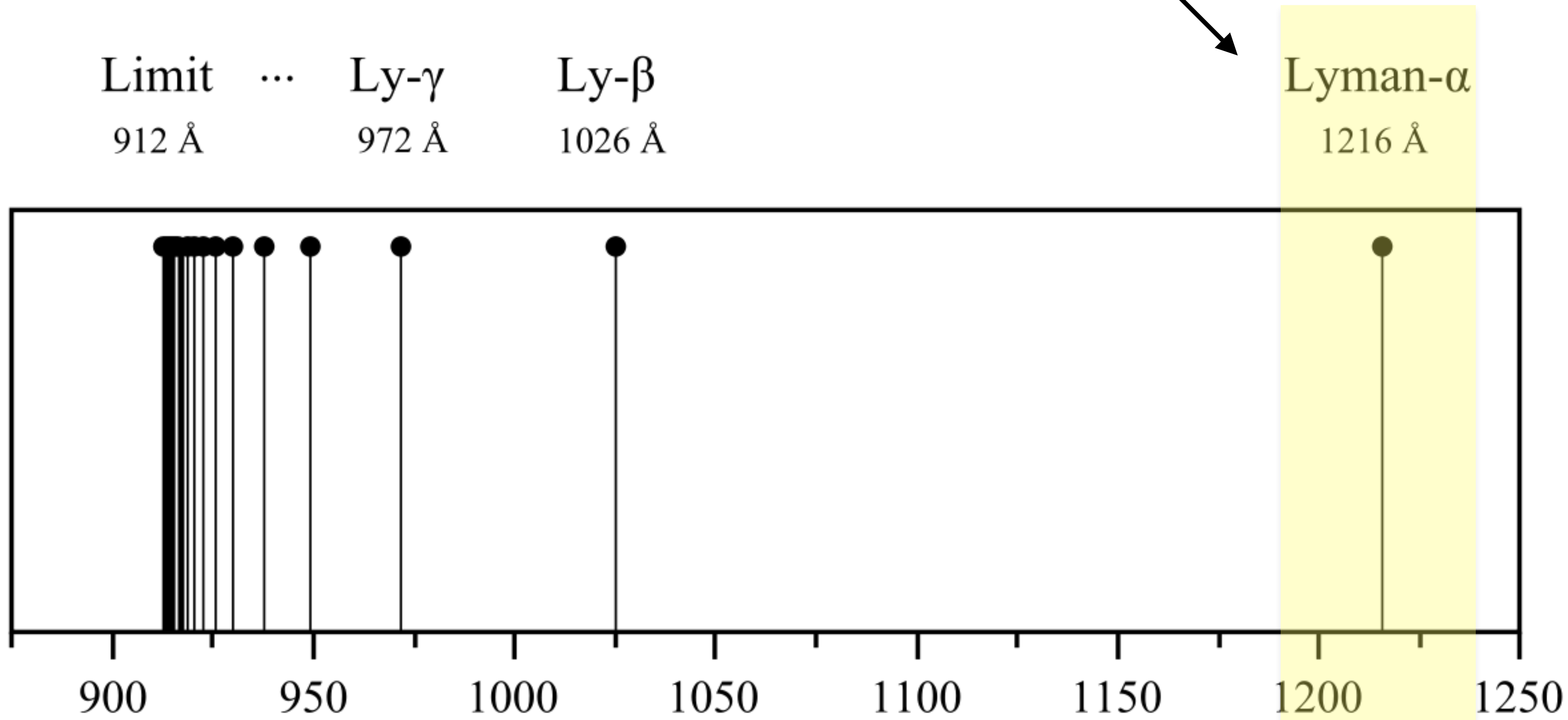
$$\Rightarrow \lambda = \frac{hc}{13.6eV}\left(\frac{1}{1-\frac{1}{n^2}}\right) \Rightarrow \lambda = 9.1 \times 10^{-8} m \left(\frac{1}{1-\frac{1}{n^2}}\right)$$

E.g. the Lyman alpha-line

$$n = 2 \quad \Rightarrow \lambda = 9.1 \times 10^{-8} (\frac{4}{3}) m$$

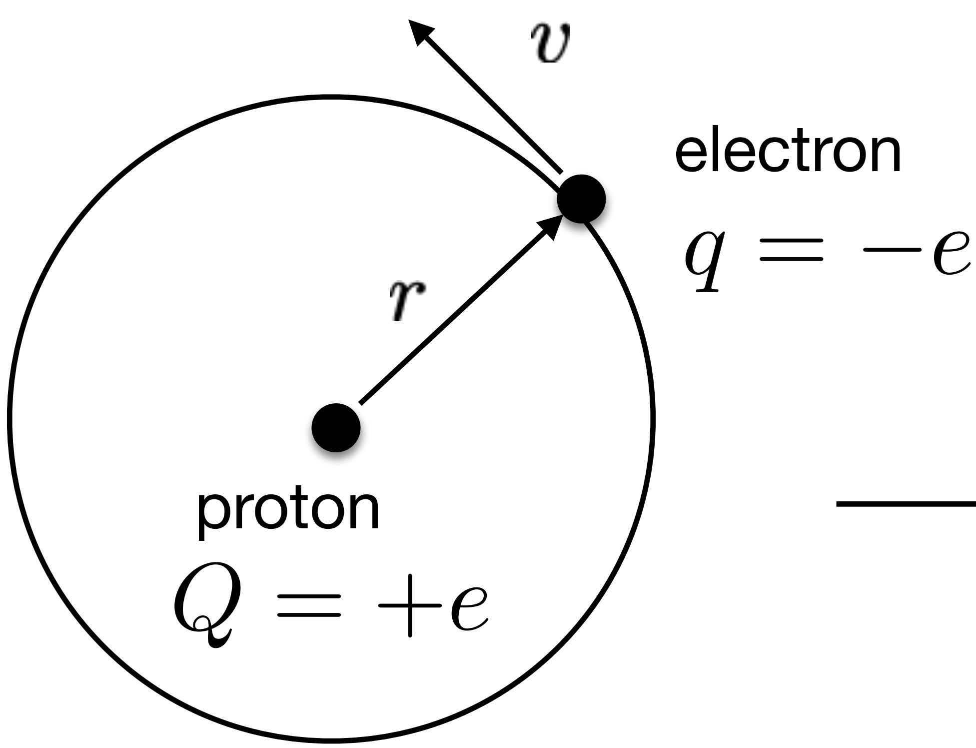
$$\Rightarrow \lambda = 12.1 \times 10^{-8} m$$



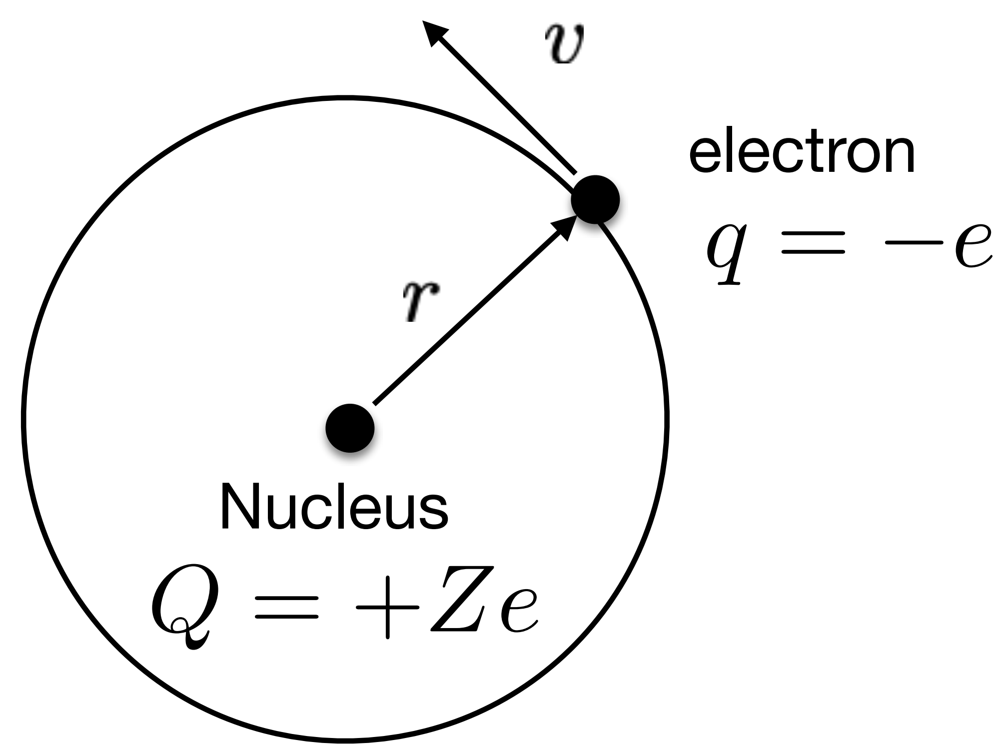


Beyond Hydrogen

Heavier atoms



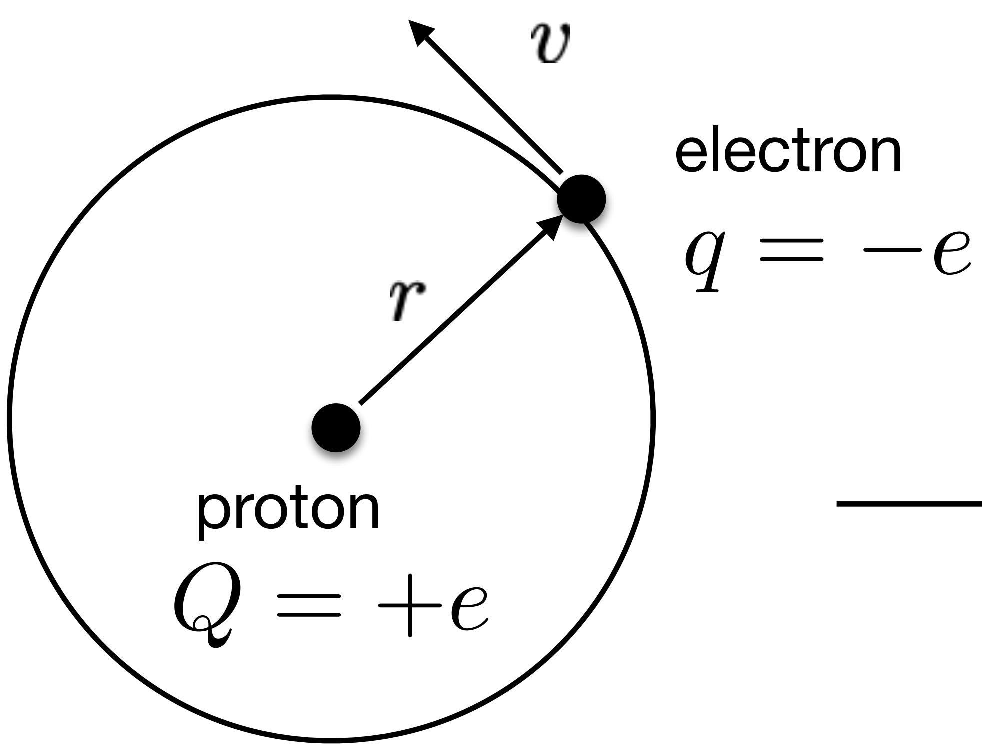
$\longrightarrow$



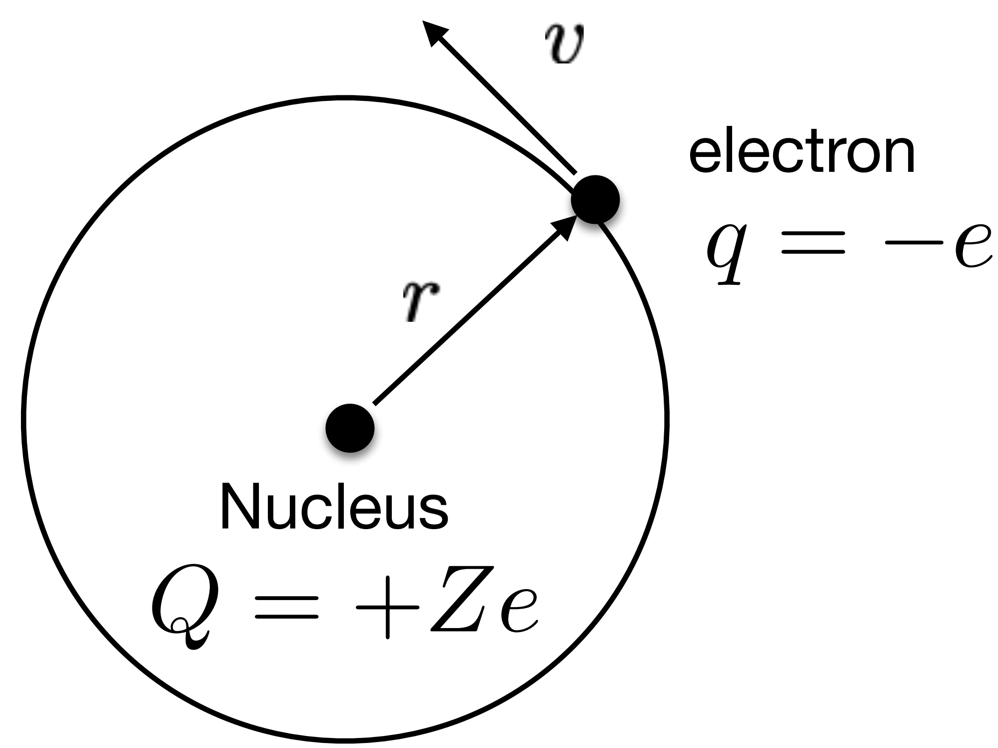
$$e \times e \to (Ze) \times e$$

$$v_n = \frac{1}{4\pi\epsilon_0}\frac{e^2}{n\hbar} \longrightarrow v_n = \frac{1}{4\pi\epsilon_0}\frac{Ze^2}{n\hbar}$$

Heavier atoms



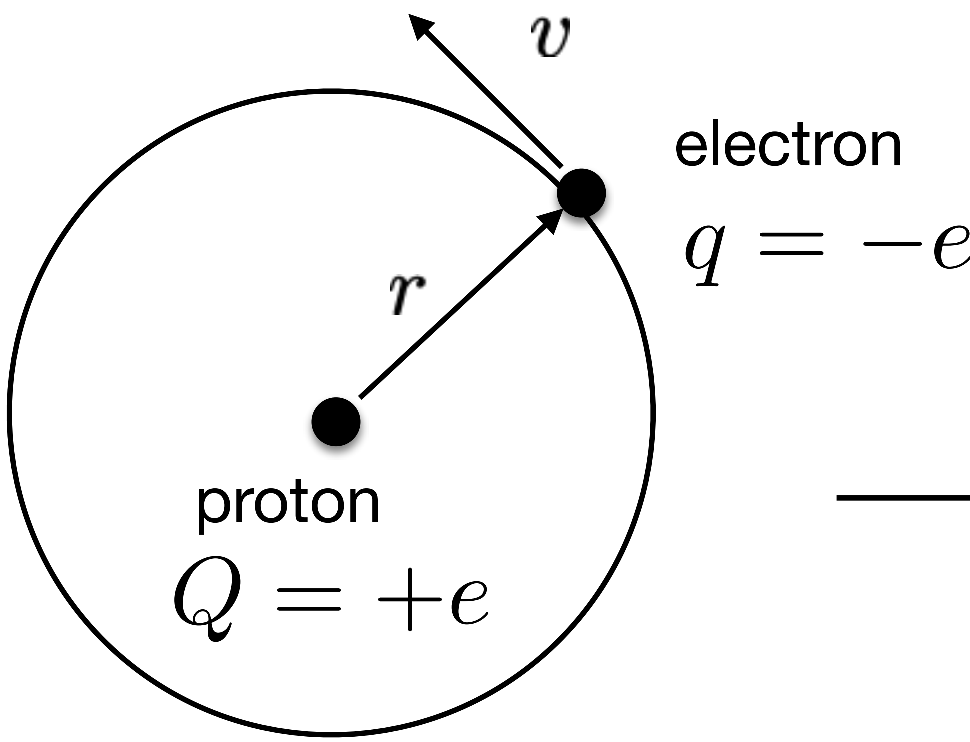
$\longrightarrow$



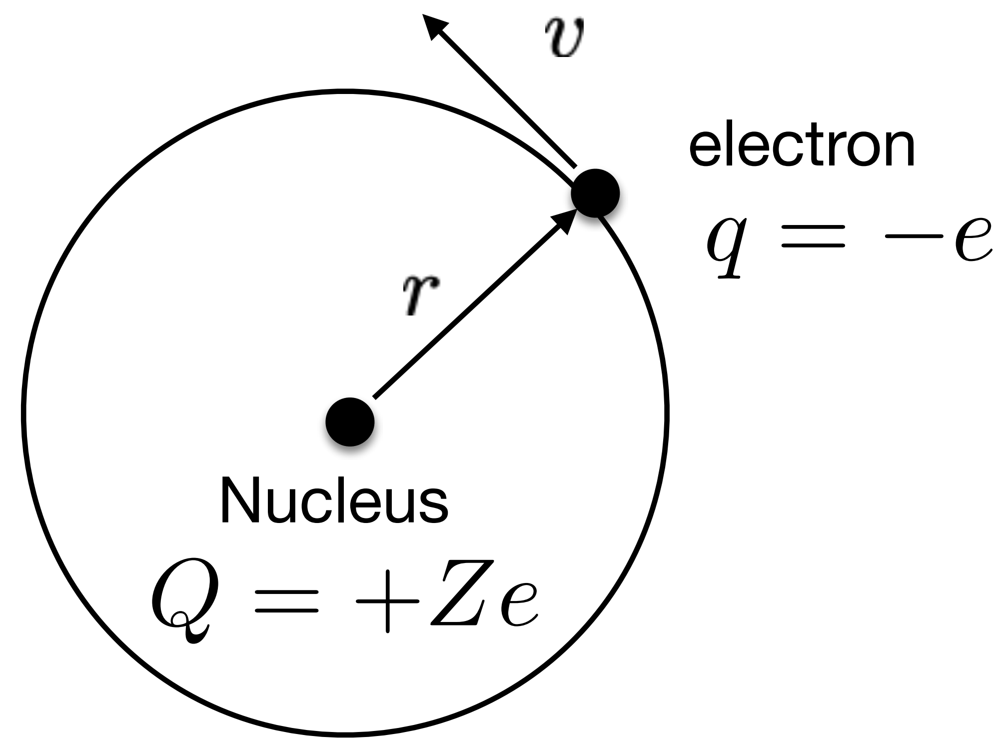
$$e \times e \to (Ze) \times e$$

Replace e^2 with Ze^2

Heavier atoms



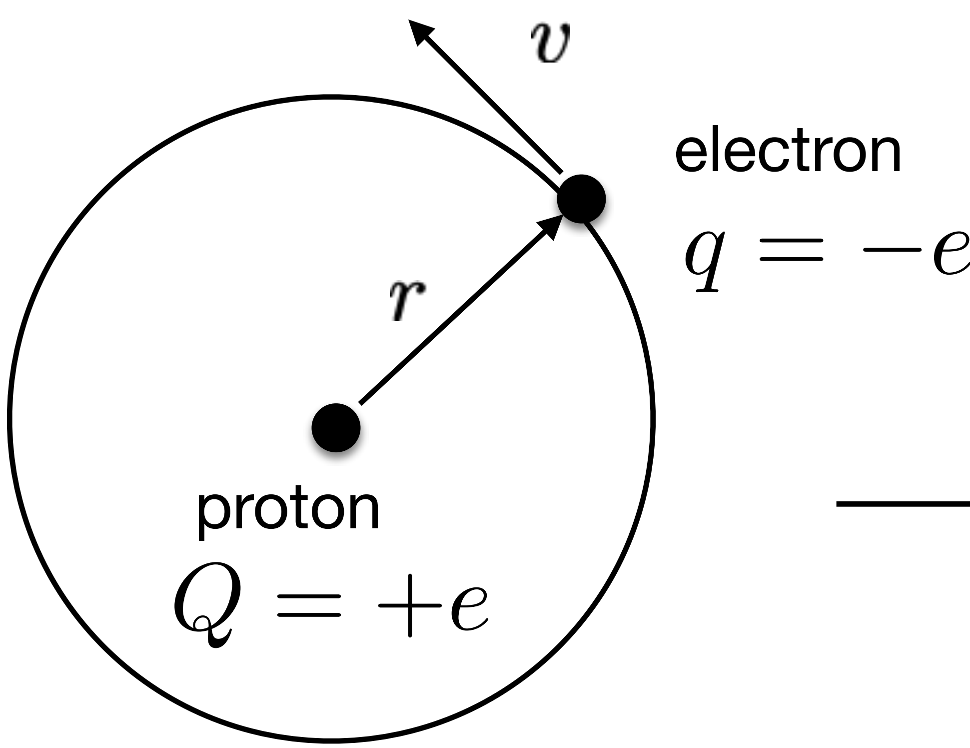
$\longrightarrow$



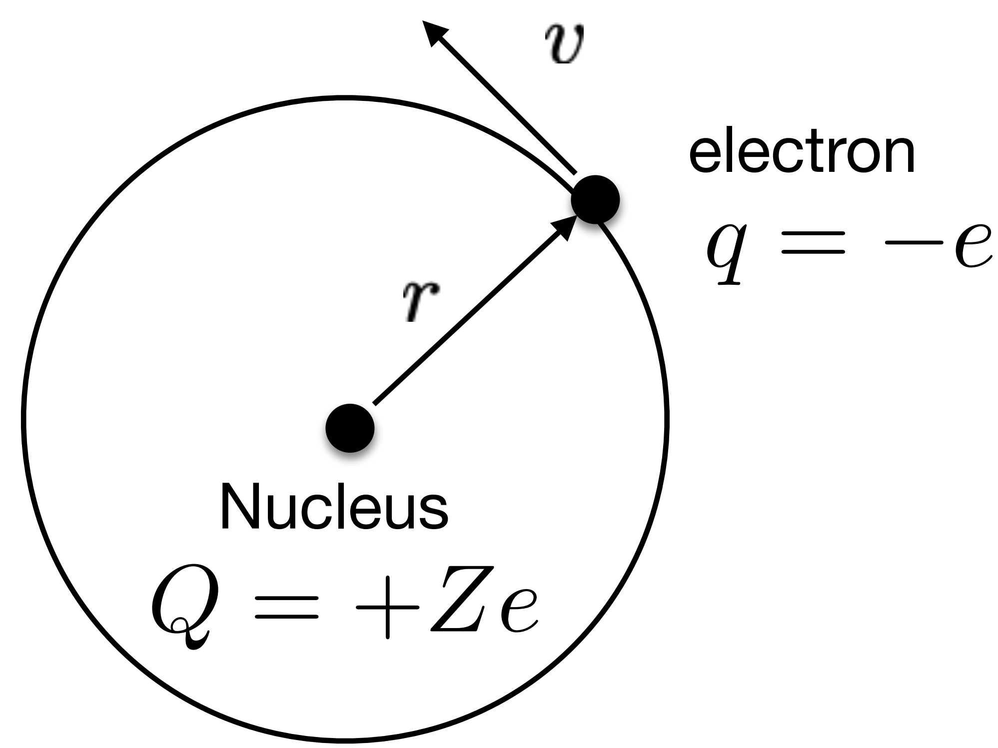
$$e \times e \to (Ze) \times e$$

$$v_n = \frac{1}{4\pi\epsilon_0}\frac{e^2}{n\hbar} \longrightarrow v_n = \frac{1}{4\pi\epsilon_0}\frac{Ze^2}{n\hbar}$$

Heavier atoms



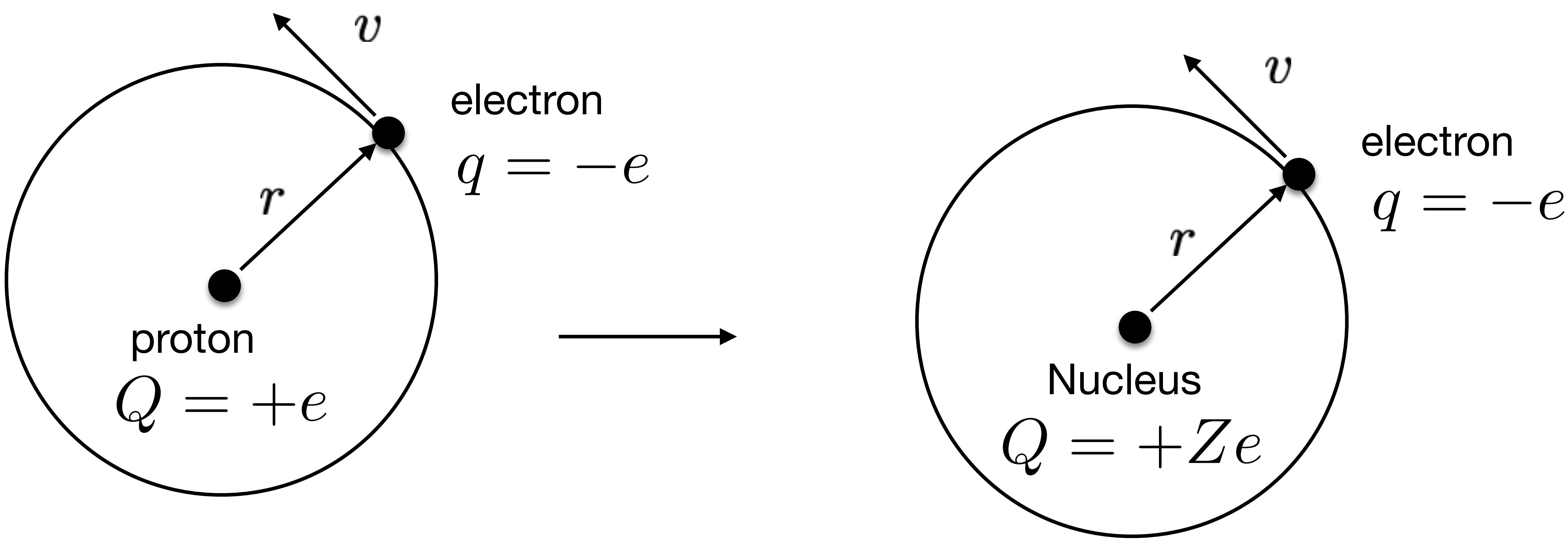
$\longrightarrow$



$$e \times e \to (Ze) \times e$$

$$r_n = \frac{4\pi\epsilon_0 n^2 \hbar^2}{me^2} \longrightarrow r_n = \frac{4\pi\epsilon_0 n^2 \hbar^2}{mZe^2}$$

Heavier atoms

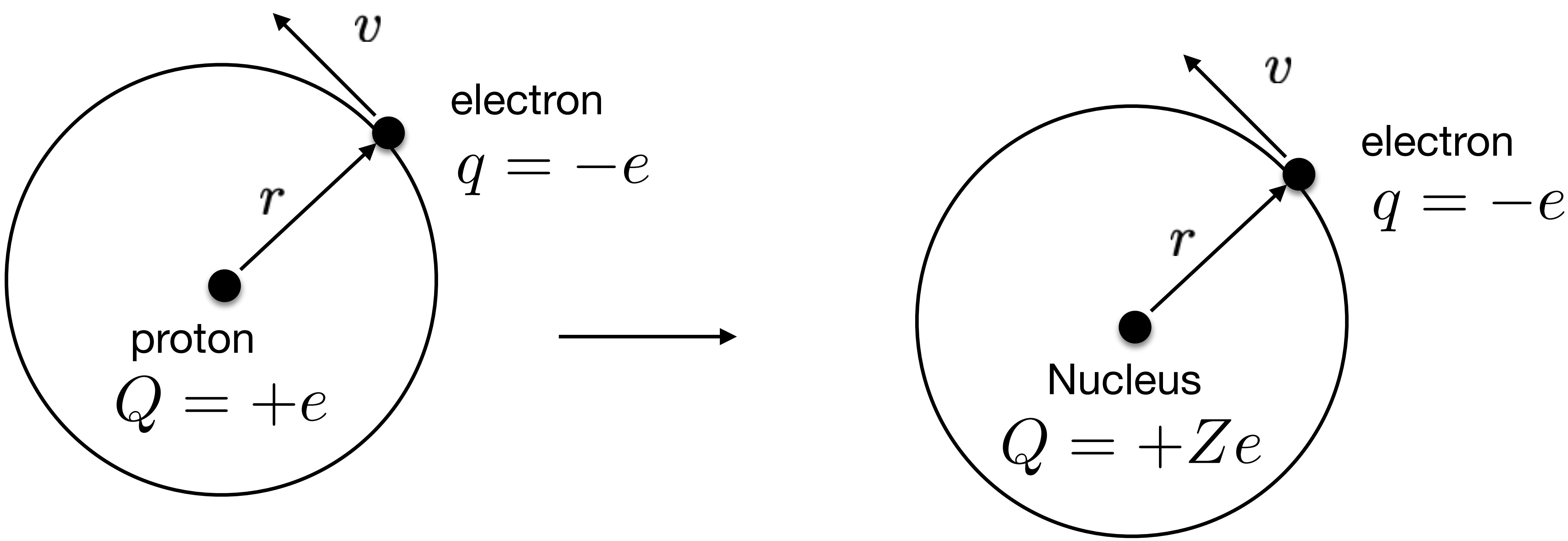


$$e \times e \to (Ze) \times e$$

$$E_n = -\frac{me^4}{32\pi^2\epsilon_0^2}\frac{1}{n^2} \longrightarrow$$

??

Heavier atoms

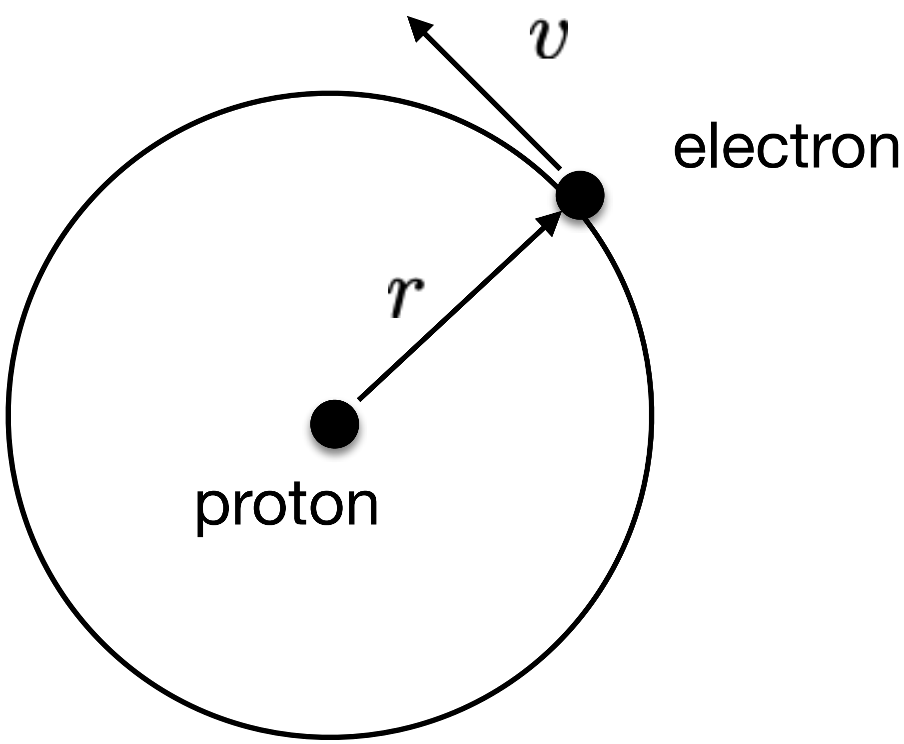


$$e \times e \to (Ze) \times e$$

$$E_n = -\frac{me^4}{32\pi^2\epsilon_0^2}\frac{1}{n^2} \longrightarrow E_n = -\frac{mZ^2e^4}{32\pi^2\epsilon_0^2}\frac{1}{n^2}$$

What *is* the Bohr postulate?

Re-phrasing of Bohr's postulate



$$mvr = n\hbar = n\frac{h}{2\pi}$$

$$\Rightarrow 2\pi r = n\frac{h}{mv}$$

But left-hand side is the *circumference* of a circle of radius r!
Also mv is the *momentum* of the electron.

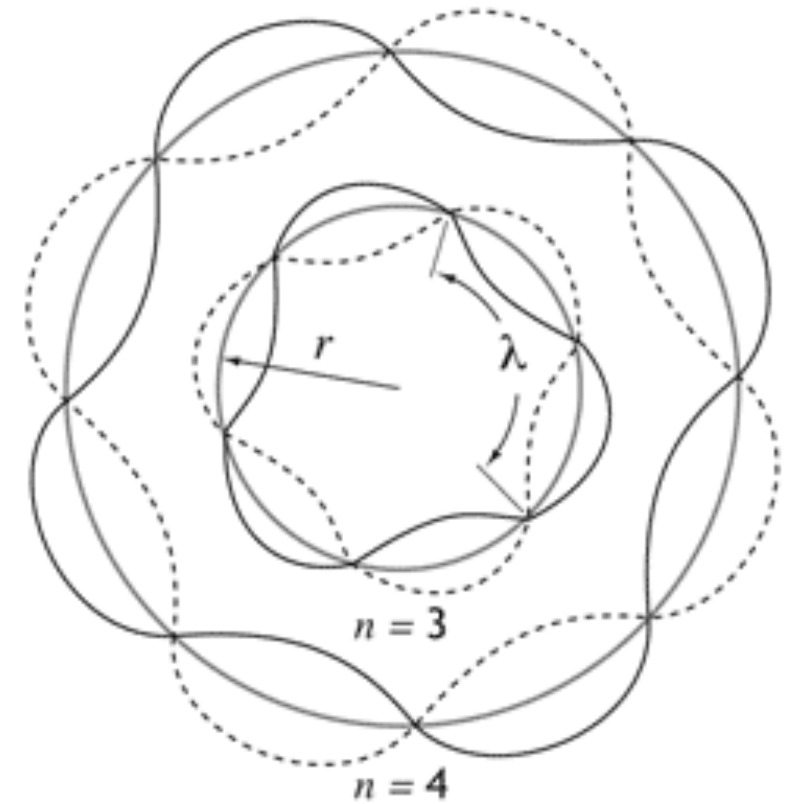
$$\text{Momentum} \;\leftrightarrow\; \text{Length scale}$$

Re-phrasing of Bohr's postulate

"Wavelike nature"

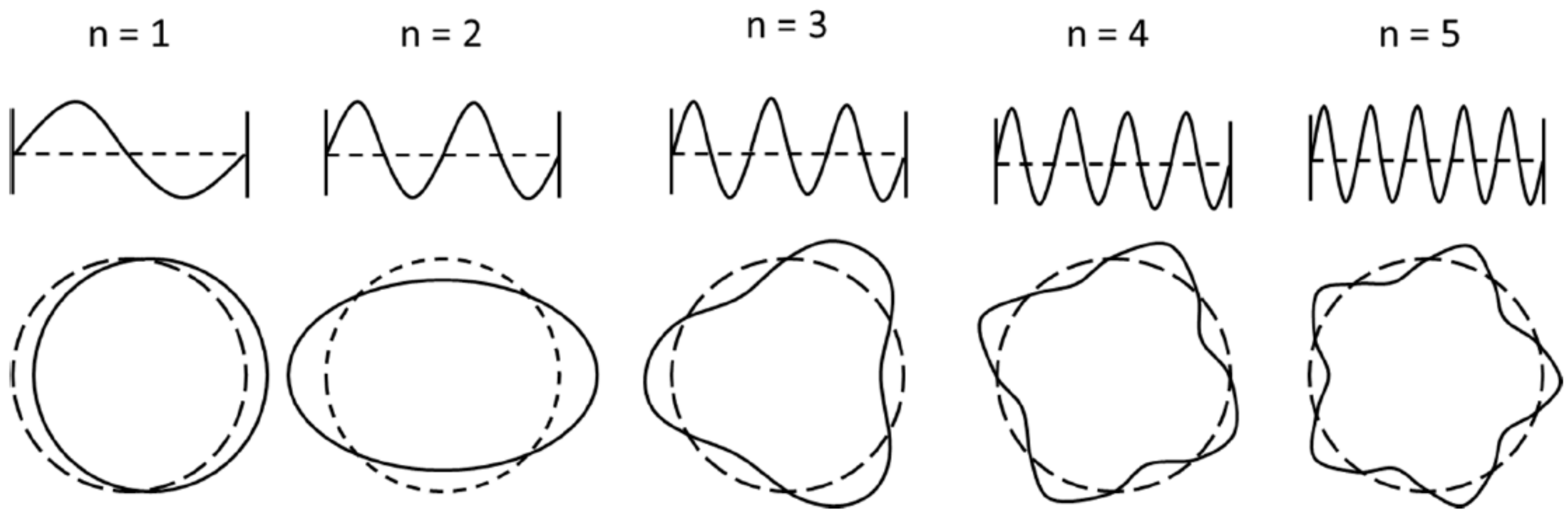
Define the de Broglie Wavelength: $$\lambda_{\mathrm{dB}} = \frac{h}{p}$$

$$\Rightarrow 2\pi r = n\lambda_{\mathrm{dB}}$$



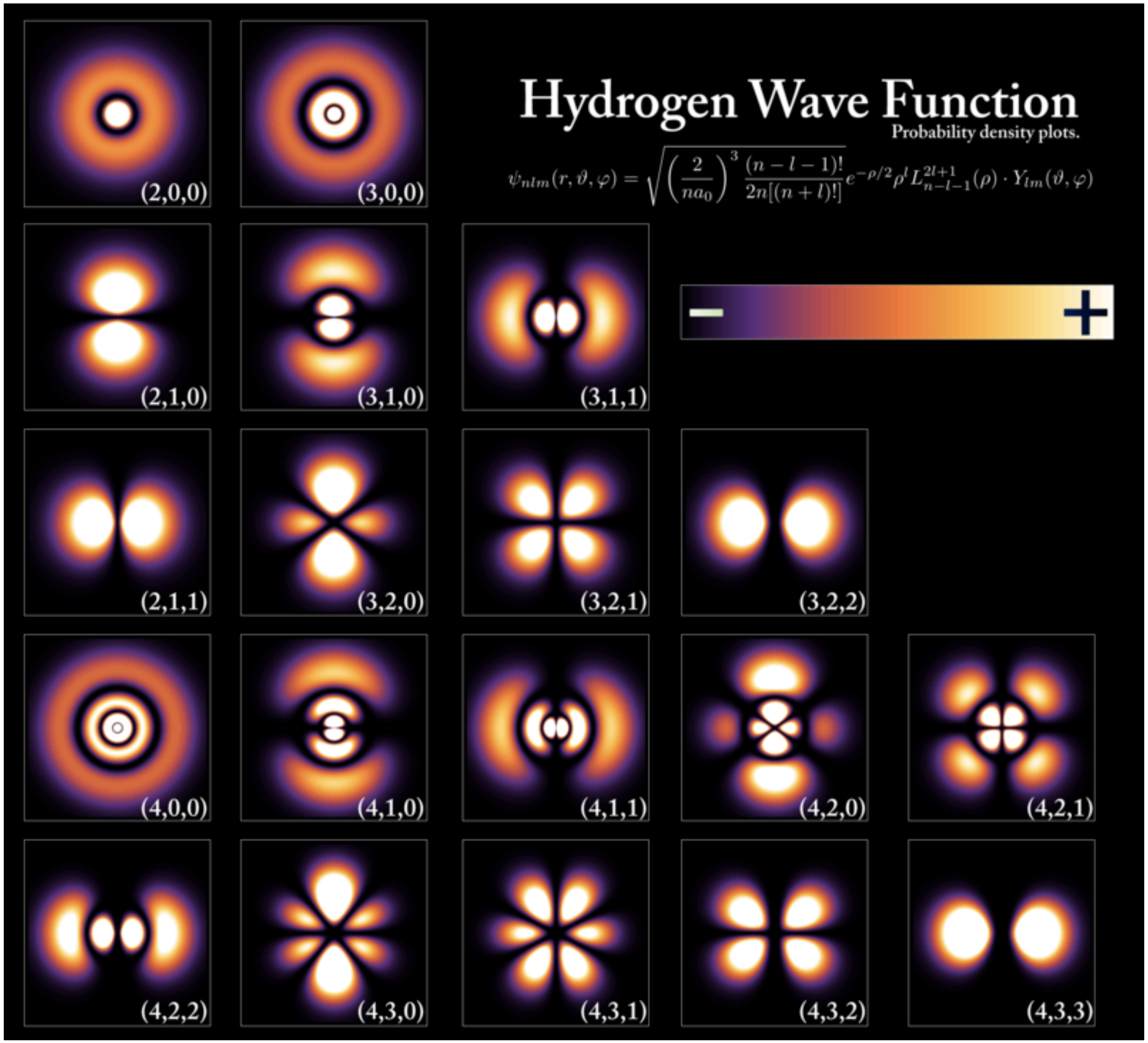
Bohr's postulate can be re-phrased as the assumption that an *integer number of de Broglie wavelengths* loop around the circular orbit!

Note: Bohr atom is only an approximation



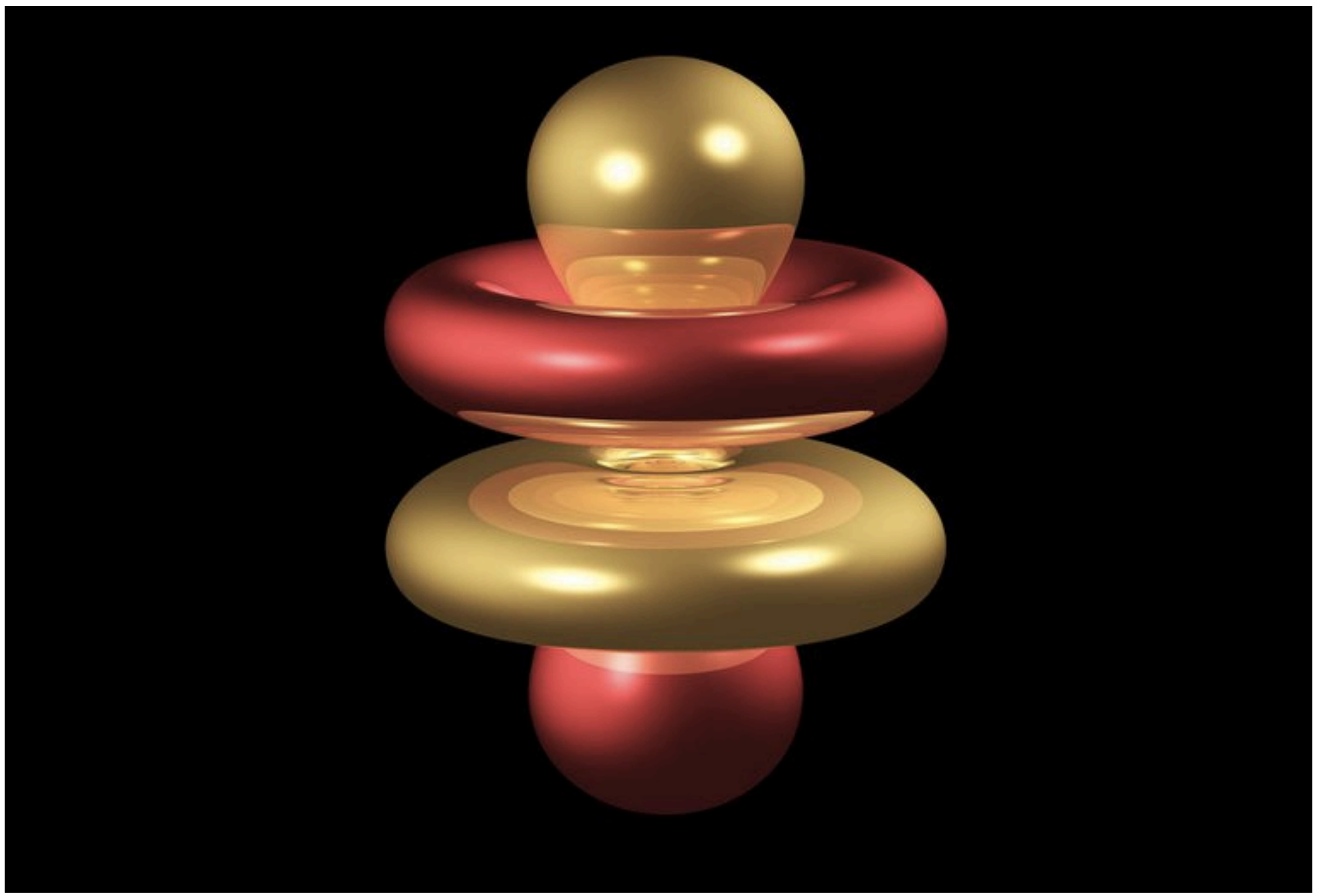
- Bohr atom doesn't respect the Heisenberg Uncertainty Principle in **the radial direction**, since it assumes a sharp radial distance, which should imply complete uncertainty in radial motion.
- Doesn't predict exactly the right energy levels of larger atoms.
- Can't describe multiple electron scenarios.

Actual Hydrogen states



Spatial probabilities for the quantum state

Actual Hydrogen states



Spatial probabilities for the quantum state

Electron ‘smeared’ over space in a quantum superposition.