Stellar Masses

- Binary systems
- Kepler's 3rd Law
- Orbits

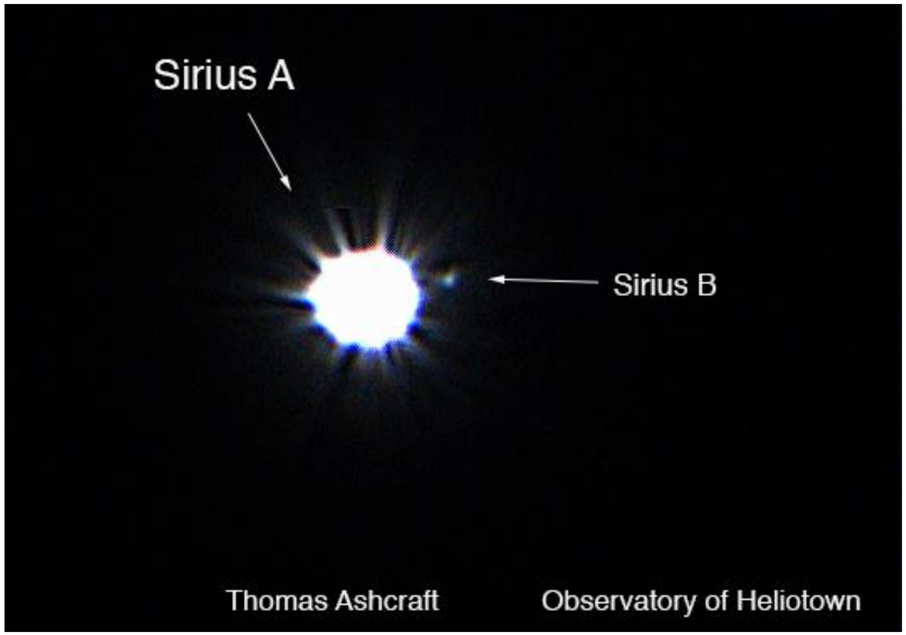
Mass Determination

- Mass of a star is difficult to infer from stellar spectra
- Instead use gravitational influence in a binary star system
- Most stars are in binary or multiple systems

Class Example

- How far is Sirius B from Sirius A when their separation is 11” and the distance to the system is 2.6 pc? Express answer in au.

A. 2.6 au
B. 11 au
C. 29 au



- How far is Sirius B from Sirius A?

$$l = qd$$

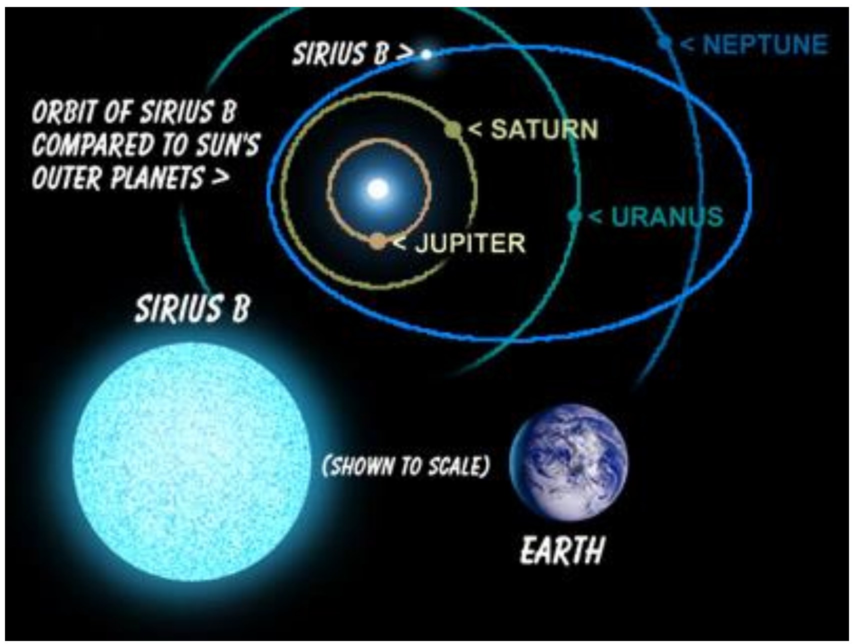
$$= \frac{11}{206265} 2.6 \times 3.1 \times 10^{16}$$

$$= 4.3 \times 10^{12} \text{ m} = 29 \text{ au}$$

$$l(au) = q('')d(pc)$$

$$= 11 \times 2.6$$

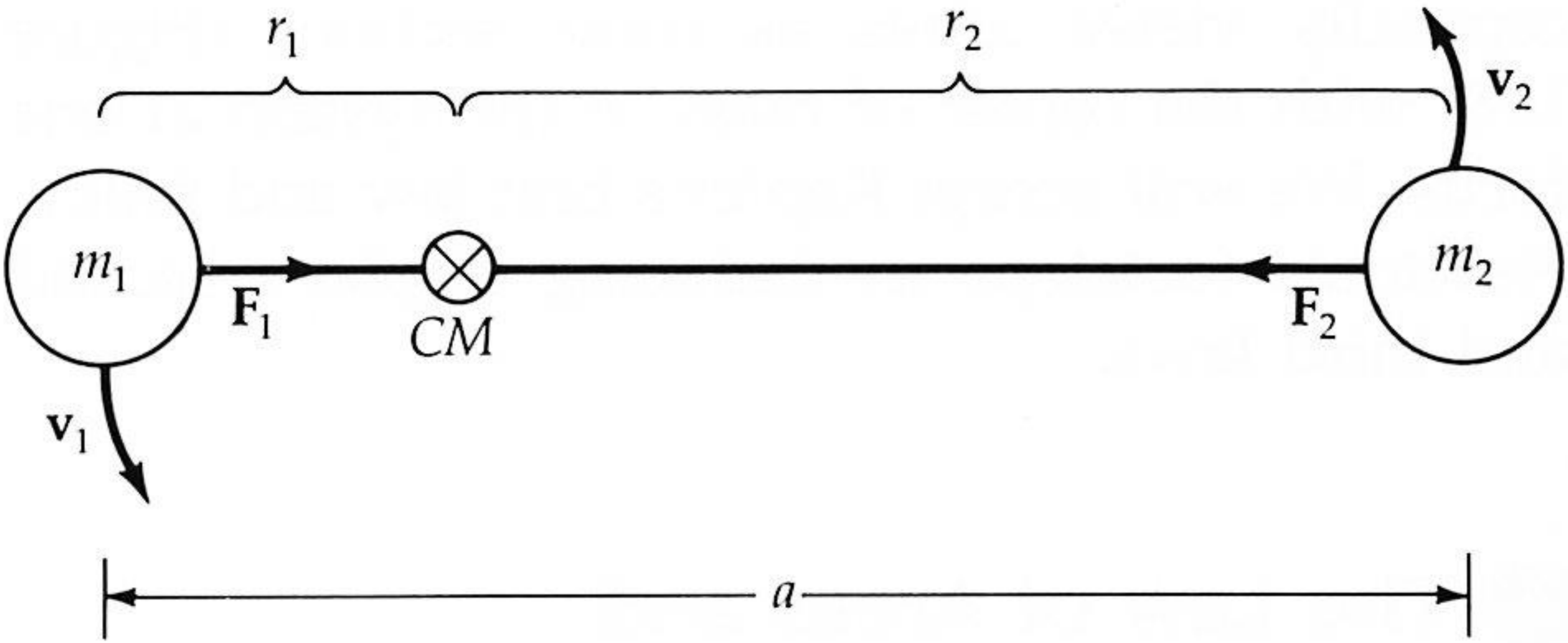
$$= 29 \text{ au}$$



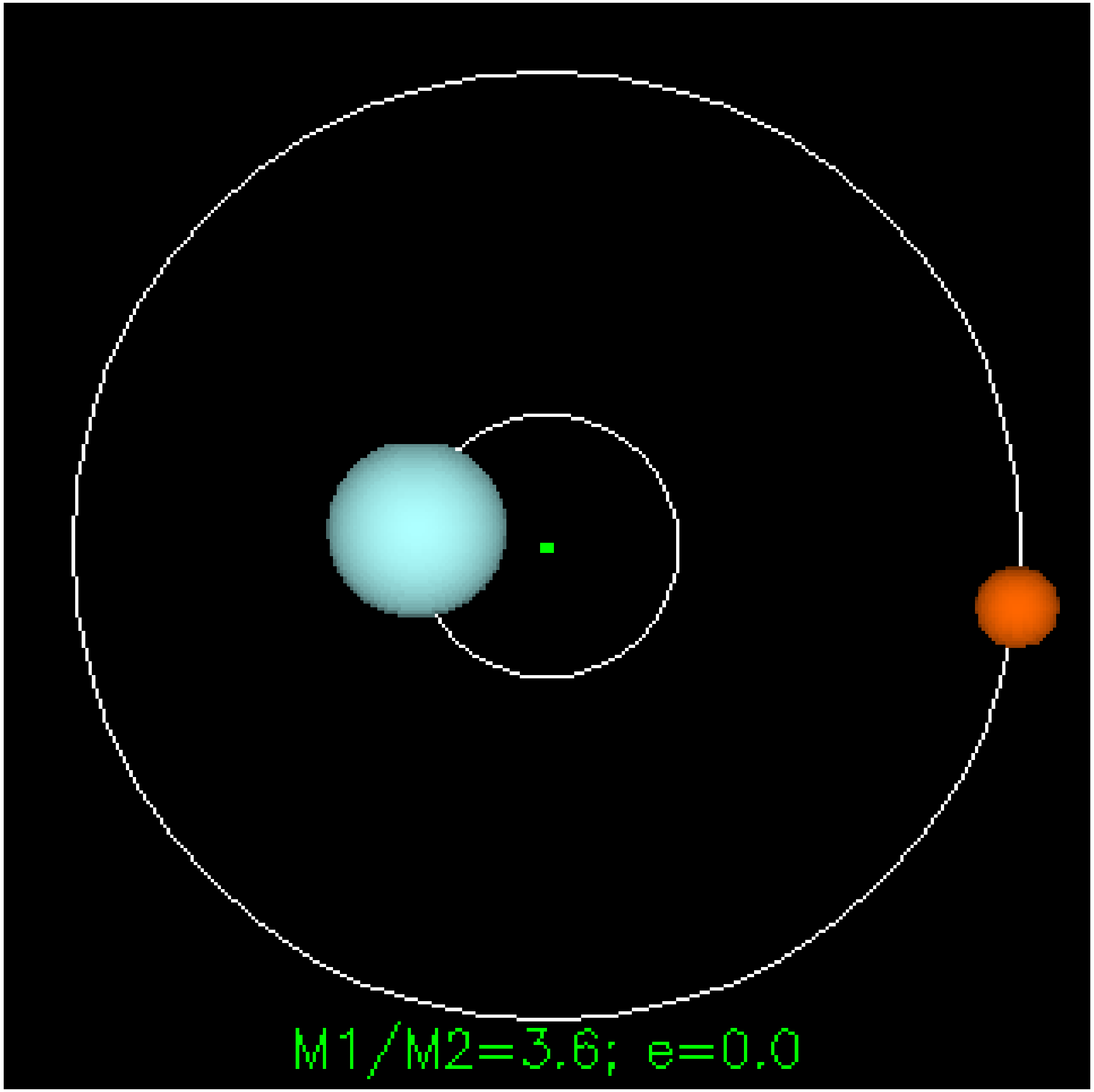
https://www.sciencecenter.net/whatsup/03/cm-stars.htm

Binary Systems

- consider two stars with masses M_1 and M_2 in circular orbits around their centre of mass (CM)
- radius of each orbit is r_1 and r_2 respectively and the total separation is a
- can use Newton's Laws and circular motion to determine masses



Zeilik Fig 1-14



http://www.astronomy.ohio-state.edu/~pogge/Ast162/Movies/visbin.html

Circular Motion

$$F_1 = \frac{M_1 v_1^2}{r_1} = \frac{4p^2 M_1 r_1}{P^2} \qquad v_1 = \frac{2pr_1}{P}$$

and

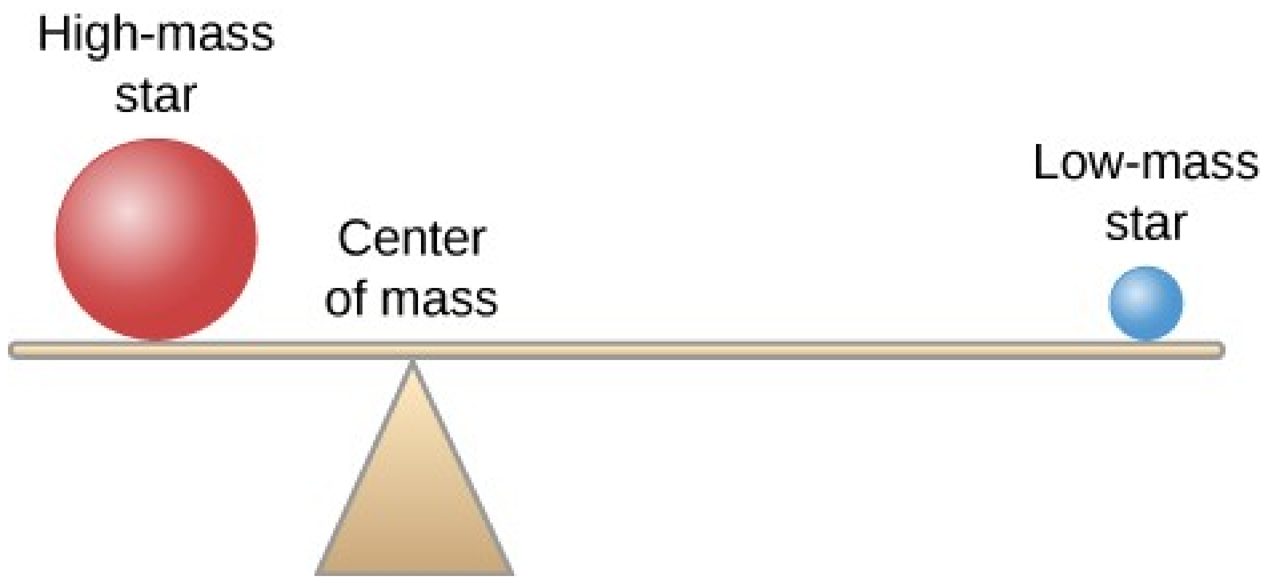
$$F_2 = \frac{M_2 v_2^2}{r_2} = \frac{4p^2 M_2 r_2}{P^2} \qquad v_2 = \frac{2pr_2}{P}$$

where P is the period which is the same for both stars

Centre of Mass

- definition of centre of mass means

$$M_1 r_1 = M_2 r_2$$



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Newton's Law of Gravity

$$F_1 = F_2 = \frac{GM_1M_2}{a^2}$$

where

$$a = r_1 + r_2$$

Newton's form of Kepler's Third Law

- combining these three equations gives

$$\frac{4p^2 M_1 r_1}{P^2} = \frac{G M_1 M_2}{a^2}$$

$$P^2 = \frac{4p^2 a^2 r_1}{G M_2}$$

Eliminate r_1 using

$$a = r_1 + r_2 = r_1 + \frac{M_1}{M_2} r_1 = \left(\frac{M_1 + M_2}{M_2} \right) r_1$$

so

$$P^2 = \frac{4p^2a^3}{G\left(M_1 + M_2\right)}$$

and

$$M_1 + M_2 = \frac{4p^2a^3}{GP^2}$$

Class Example

- What is the period of a binary system consisting of two solar mass stars separated by 30 au in years

$$P = \left[\frac{4\pi^2 a^3}{G(M_1 + M_2)}\right]^{\frac{1}{2}}$$

$$P = \left[\frac{4\pi^2 (30 \times 1.5 \times 10^{11})^3}{6.7 \times 10^{-11}(1+1) \times 2 \times 10^{30}}\right]^{\frac{1}{2}}$$

$$P = \left[\frac{3.6 \times 10^{39}}{2.7 \times 10^{20}}\right]^{\frac{1}{2}}$$

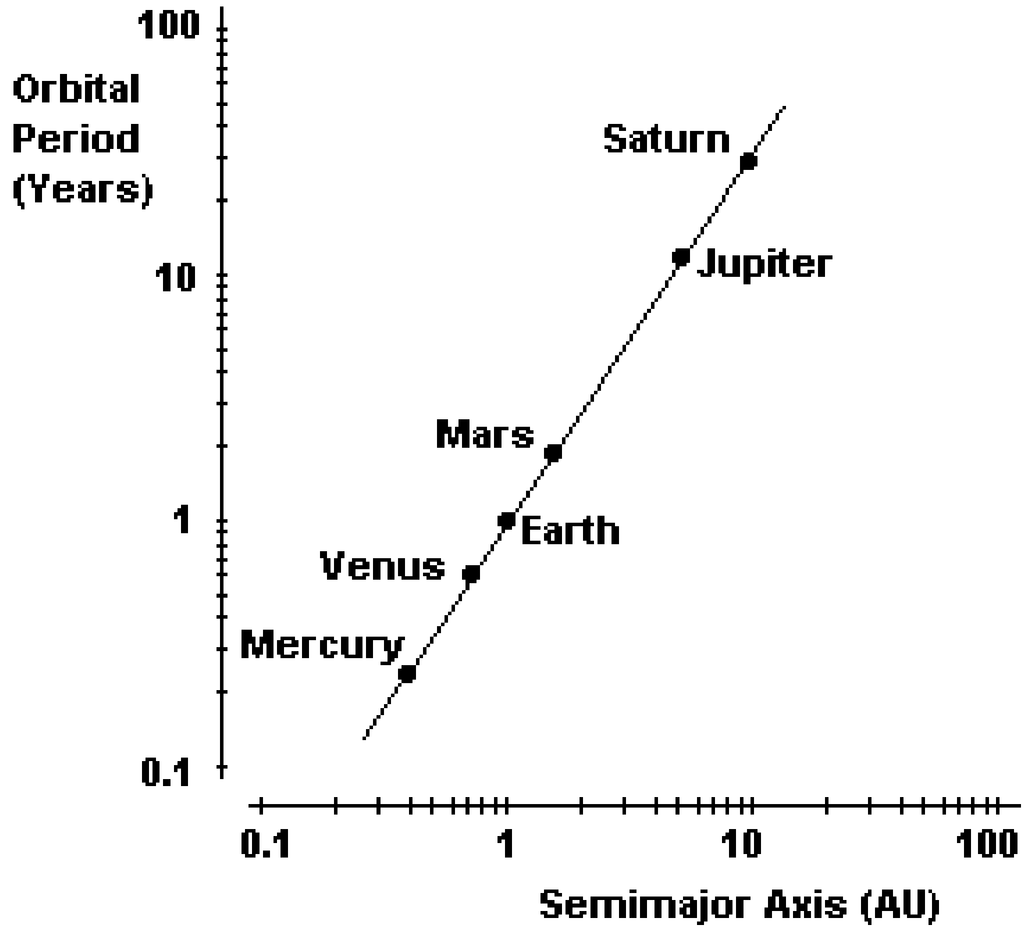
$$P = 3.7 \times 10^{9}\ \mathrm{s}$$

$$P = 120\ \mathrm{years}$$

Kepler’s Third Law

- the planets orbiting the Sun follow the relation

P^2 µ a^3



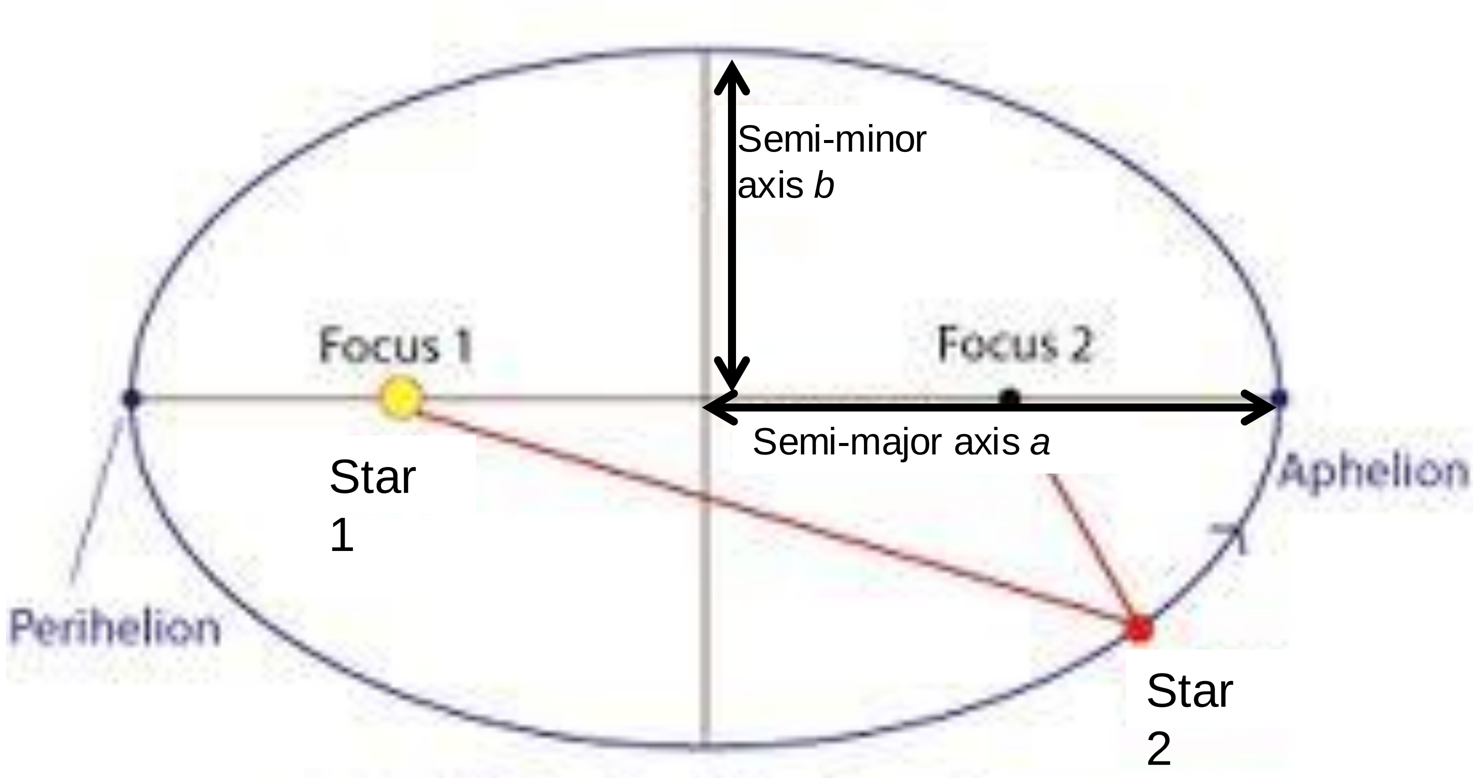
Kepler's Third Law

- Can transform into useful units:

$$\left(P/_{yr}\right)^2 \left(M_1 + M_2 /_{M_{Sun}}\right) = \left(a/_{au}\right)^3$$

Real Orbits

- orbits are generally elliptical described by their semi-major axis a and semi-minor axis b

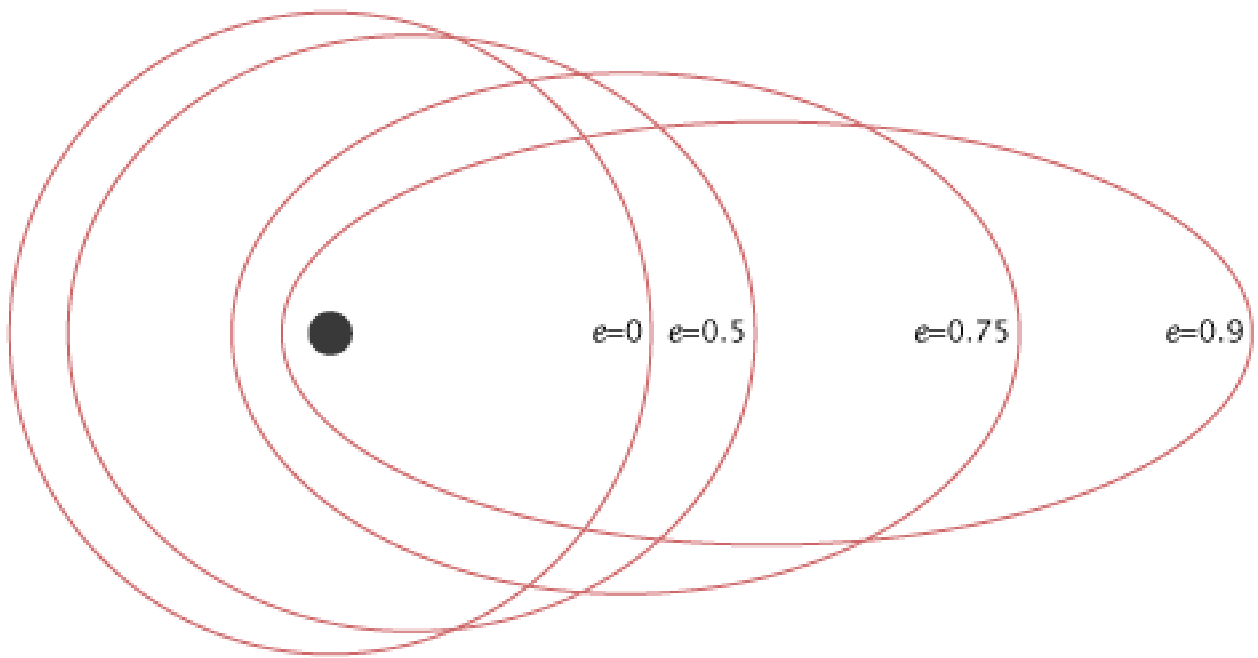


outreach.atnf. csiro.edu

- eccentricity of elliptical orbit is defined by

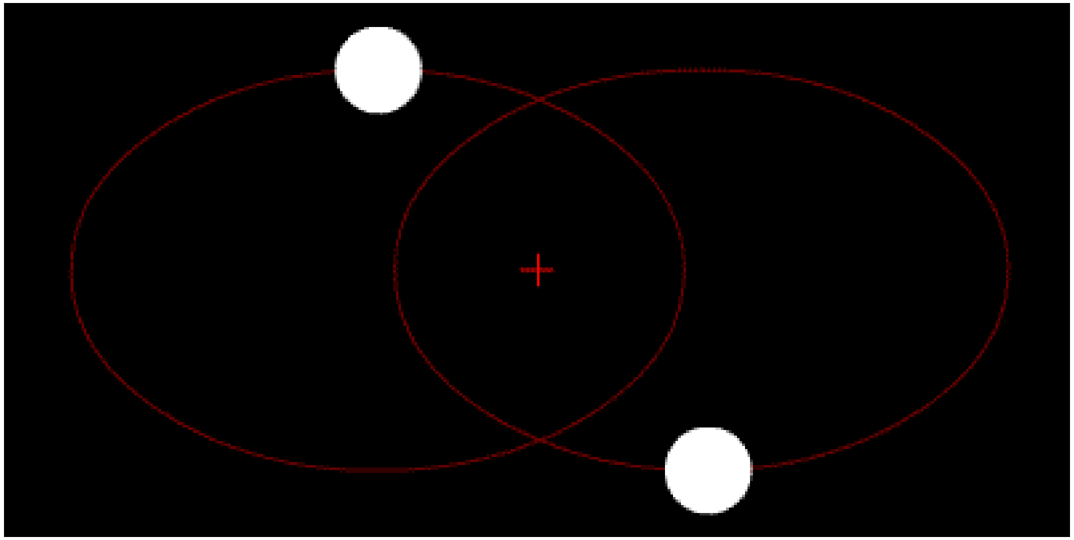
$$e = \frac{\sqrt{a^2 - b^2}}{a}$$

$$e = 0 \Rightarrow \text{ circular orbit}$$



https://www.tutorialspoint.com/satellite_communication/satellite_communication_orbital_mechanics.htm

- Newton’s form of Kepler’s third law also applies to elliptical orbits with *a* the sum of the semi-major axes ($a=a_1+a_2$)



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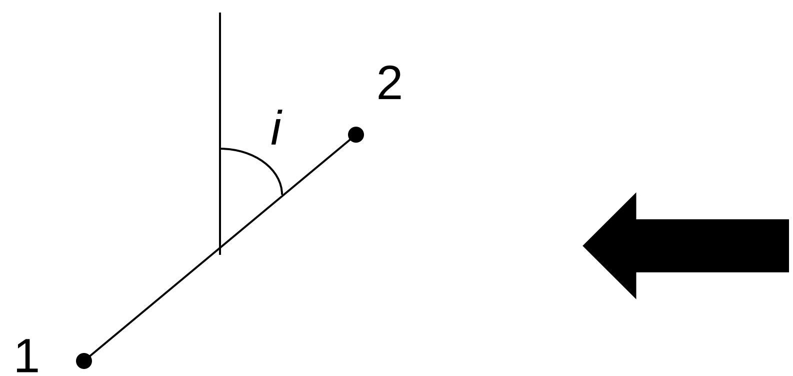
Orbital Inclination

- in general the orbital plane of a binary system will be inclined by some angle *i* to the plane of the sky:

$$i = 0^o \quad \Rightarrow \text{ face on}$$

$$i = 90^o \quad \Rightarrow \text{ edge on}$$



Summary

- Binaries are the only direct way of measuring stellar masses
- Newton's form of Kepler's 3rd law is the starting point for measuring stellar masses

Class Example

- Confirm Kepler's Third law by measuring the slope of this graph