UNIVERSITY OF LEEDS

Mechanics 1

Session 8 – Kinetic & Potential Energies

DR BEN HANSON

Last Lecture

Variable Force, Energy & Work

We discussed:

- That forces might vary over time or through space
- The concept of energy
- The concept of work done

You should be able to:

- Calculate the (energetic) work done by a force on a particular system with constant forces along straight lines
- Calculate the (energetic) work done by a force on a particular system with changing forces and non-linear paths

This Lecture

Kinetic & Potential Energies

We will:
- Understand how work done translates into kinetic energy
- Understand the concept of power
- Understand the concept of potential energy

You will be able to:
- Calculate the power output of a system
- Calculate potential energy of a spring
- Calculate gravitational potential energy both locally and over large distances

A Quick Note on Units

A Quick Note on Units

SI Units

Quantity	SI Unit
Time	Seconds (s)
Distance	Metres (m)
Speed	Metres per second (ms^{-1})
Acceleration	Metres per second-squared (ms^{-2})
Mass	Kilogram (kg)
Force	Newtons (N), ($kg.ms^{-2}$)
Energy / Work	Joules (J), ($N.m$)
Power	Watts (W), (Js^{-1})

Task 1 (Last Lecture)

Work Done By Tension & Friction

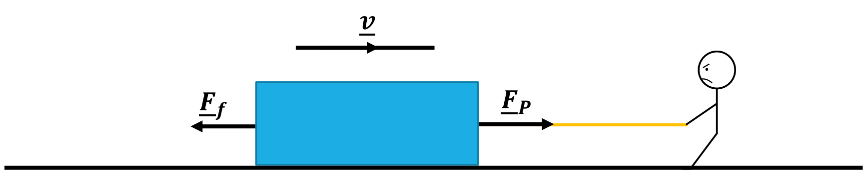
Task 1

Work Done By Tension & Friction

Scenario: Someone is pulling a box from rest along a surface with some friction. They apply a horizontal force $|\underline{F_P}| = 50N$ to the box, which moves from $x = 5m$ to $x = 12m$. The horizontal frictional force on the box $|\underline{F_f}| = 40N$

Tasks:

1. Calculate the work done on the box by the pulling force.
2. Calculate the work done on the box by friction.
3. There is a difference in energy (work done on + work done against). What do you think this energy is? If the box has mass $m = 10kg$, what do you think the speed of the object might be?



MECHANICS 1 - KINETIC & POTENTIAL ENERGIES

Task 1

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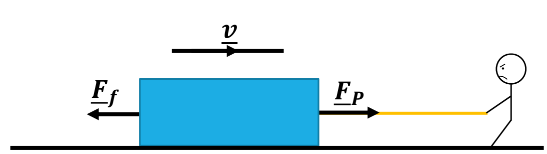
Tasks:

1. Calculate the work done on the box by the pulling force.

$$W_p = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}_p \cdot d\vec{r}$$

$$W_p = \int_{\vec{r}_1}^{\vec{r}_2} (\vec{F}_{p,x}\vec{\imath}) \cdot (dx\vec{\imath} + dy\vec{\jmath})$$

$$W_p = \int_{x_1}^{x_2} \vec{F}_{p,x}\, dx$$



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Tasks:

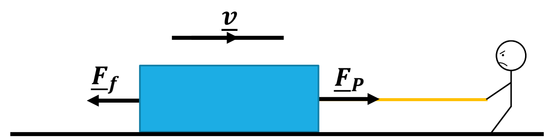
1. Calculate the work done on the box by the pulling force.

$$W_p = \int_{x_1}^{x_2} \vec{F}_{p,x}\, dx$$

$$W_p = \int_{5}^{12} 50\, dx$$

$$W_p = [50x]_5^{12}$$

$$\boxed{W_p = 350J}$$



MECHANICS 1 - KINETIC & POTENTIAL ENERGIES

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Tasks:

2. Calculate the work done on the box by friction.

$$W_f = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}_f \cdot d\vec{r}$$

$$W_f = \int_{\vec{r}_1}^{\vec{r}_2} (\vec{F}_{f,x}\vec{\imath}) \cdot (dx\vec{\imath} + dy\vec{\jmath})$$

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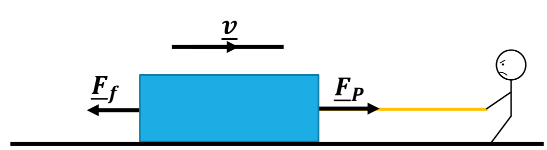

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Tasks:

2. Calculate the work done on the box by friction.

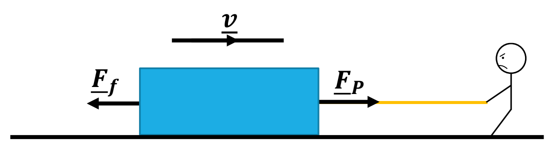
$$W_f = \int_{x_1}^{x_2} \vec{F}_{f,x}\, dx$$

$$W_f = \int_{5}^{12} -40\, dx$$

$$W_f = -[40x]_5^{12}$$

$$\boxed{W_f = -280J}$$

Negative result, so work is done *against* the motion i.e. energy is used (work is done) in opposition to the motion



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Work Done By Tension & Friction

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Tasks:

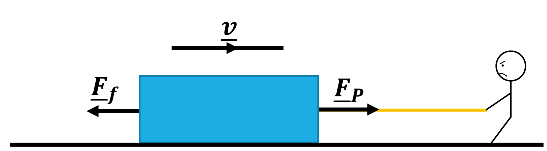
3. There is a difference in energy (work done on + work done against). What do you think this energy is? If the box has mass $m = 10kg$, what do you think the speed of the object might be?

$$\Delta W = Work\ on + Work\ against$$

$$\Delta W = 350J + (-280J)$$

$$\boxed{\Delta W = 70J}$$

I've given, in total, ΔW energy to this box. So, shouldn't it now have ΔW's worth of kinetic energy?



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Tasks:

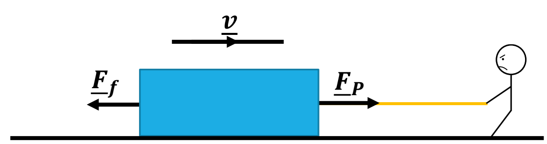
3. There is a difference in energy (work done on + work done against). What do you think this energy is? If the box has mass $m = 10kg$, what do you think the speed of the object might be?

$$\Delta W = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2\Delta W}{m}}$$

$$\boxed{v \approx 3.74ms^{-1}}$$

I've given, in total, ΔW energy to this box. So, shouldn't it now have $\Delta W'$s worth of kinetic energy?



Try the SUVAT equations. See if you get the same velocity ☺

Work – Kinetic Energy Theorem

Work – Kinetic Energy Theorem

What is it?

The net work done on an object is equal to the change in the object's kinetic energy

If we do work to an object, we give it kinetic energy. If we do work against an object (negative work on), we take kinetic energy away. If we do no work to the object, its kinetic energy does not change.

$$W = \Delta E_k$$

$$\int_{\vec{r}_1}^{\vec{r}_2} \vec{F}_{Net} \cdot d\vec{r} = \Delta\left(\frac{1}{2}mv^2\right)$$

The velocity changes!

Task 2 (Last Lecture)

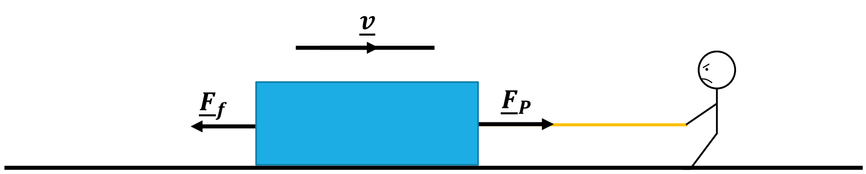
Work Done By Not-Constant Tension & Friction

Task 2

Work Done By Not-Constant Tension & Friction

Scenario: Someone is pulling a box of mass $m = 30kg$ along a surface with some friction. It begins with a speed $v_i = 3ms^{-1}$. They apply a horizontal force $|\underline{F_P}| = (50x)N$ to the box, which moves from $x = 5m$ to $x = 12m$. The horizontal frictional force on the box $|\underline{F_f}| = 40N$.

Tasks:
1. Calculate the work done on the box by the pulling force.
2. Calculate the work done by the net force.
3. Calculate the change in velocity



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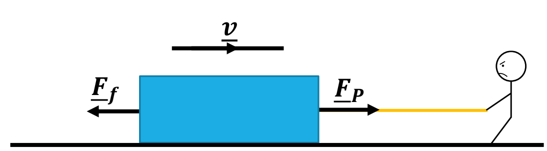
Tasks:
1. Calculate the work done on the box by the pulling force.

$$W = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}_p \cdot d\vec{r}$$

$$W = \int_{\vec{r}_1}^{\vec{r}_2} (\vec{F}_{p,x}\vec{\imath}) \cdot (dx\vec{\imath} + dy\vec{\jmath})$$

$$W = \int_{x_1}^{x_2} \vec{F}_{p,x} dx$$



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Tasks:

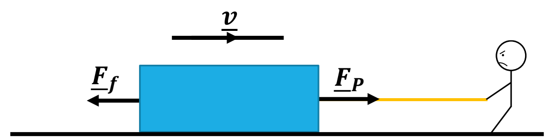
1. Calculate the work done on the box by the pulling force.

$$W_p = \int_{x_1}^{x_2} \vec{F}_{p,x} dx$$

$$W_p = \int_{5}^{12} 50x \, dx$$

$$W_p = [25x^2]_5^{12}$$

$$\boxed{W_p = 2975J}$$



MECHANICS 1 - VARIABLE FORCE, ENERGY & WORK

Task 2

Work Done By Not-Constant Tension & Friction

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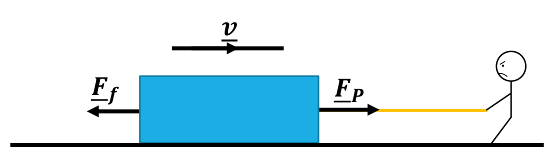
Tasks:

2. Calculate the work done by the net force.

$$W_{Net} = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}_{Net} \cdot d\vec{r}$$

$$W_{Net} = \int_{\vec{r}_1}^{\vec{r}_2} \left((\vec{F}_{p,x} + \vec{F}_{f,x})\vec{\imath} \right) \cdot (dx\vec{\imath} + dy\vec{\jmath})$$

$$W_{Net} = \int_{x_1}^{x_2} (\vec{F}_{p,x} + \vec{F}_{f,x})dx$$



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Tasks:

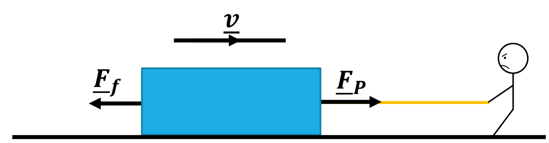
2. Calculate the work done by the net force.

$$W_{Net} = \int_{x_1}^{x_2} (\vec{F}_{p,x} + \vec{F}_{f,x})dx$$

$$W_{Net} = \int_{5}^{12} 50x - 40\, dx$$

$$W_{Net} = [25x^2 - 40x]_5^{12}$$

$$\boxed{W_{Net} = 2695J}$$



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Tasks:

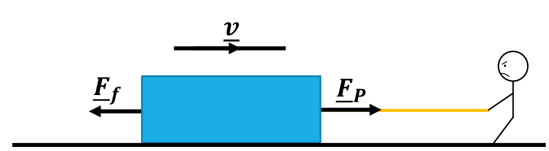
3. Calculate the change in velocity of the object

$$W_{Net} = \Delta\left(\frac{1}{2}mv^2\right)$$

$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = W_{Net}$$

$$v_f^2 - v_i^2 = \frac{2W_{Net}}{m}$$

$$v_f = \sqrt{v_i^2 + \frac{2W_{Net}}{m}}$$



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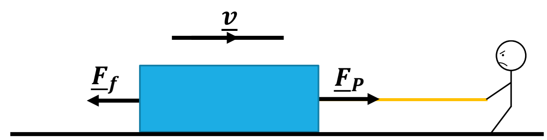
Tasks:

3. Calculate the change in velocity of the object

$$v_f = \sqrt{v_i^2 + \frac{2W_{Net}}{m}}$$

$$\boxed{v_f \approx 13.73ms^{-1}}$$

Note: This answer is not physically realistic.
It's just practice for calculating work done ☺



Power

The Rate of Work

Power

The Rate of Work

Often, the total amount of work done is not a useful quantity. It is often more useful to consider the rate of work done over time

Just like how sometimes we're more interested in speed than we are in the total distance travelled.

Power

The Rate of Work

Total work done,

$$W = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r}$$

Infinitesimal,

$$dW = \vec{F} \cdot d\vec{r}$$

Define power,

$$\boxed{P = \frac{dW}{dt}}$$

Substitute,

$$P = \vec{F} \cdot \frac{d\vec{r}}{dt}$$

$$\boxed{P = \vec{F} \cdot \vec{v}}$$

Task 1

Some Quick Questions on Power

Task 1

Some Quick Questions on Power

Tasks:

1. If I'm pushing a box on a frictionless surface for 5 seconds and in total, I do $50J$ of work, what is my power output?

2. A car is moving horizontally with a speed $|\vec{v}| = 20ms^{-1}$. It is towing a trailer, and the force of tension along the connecting tether, $\vec{T} = 5000N$. The tether, however, is at an angle of 10^o to the horizontal. What is the power output of the car on the trailer?

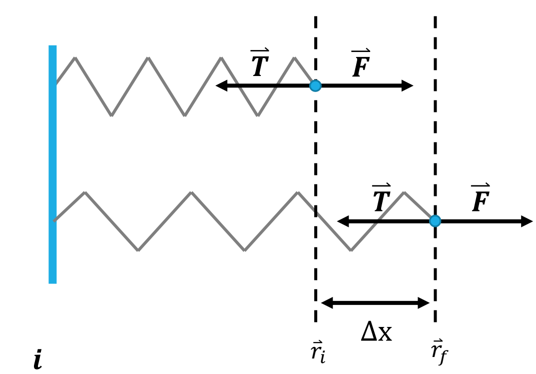
3. A train begins to accelerate. Over the course of 25 seconds, the train increases in speed from rest to $v = 26.8ms^{-1}$. If the train has mass $m = 300 \times 10^3 kg$, what is the power output of the train engine?

4. (For the anime fans and meme-lovers). Goku is using a Kamehameha on Vegeta. The beam of energy hits Vegeta and sends him flying. Over the course of 7 seconds, Vegeta increases in speed from rest to $v = 37.8ms^{-1}$. If Vegeta has mass $m = 90kg$, what is the power of Goku's Kamehameha blast?

Work Done by Springs

Work Done by Springs

Where Does My Energy Go?



Work done by spring, $$W_T = \int_{\vec{r}_i}^{\vec{r}_f} \vec{T} \cdot d\vec{r}$$

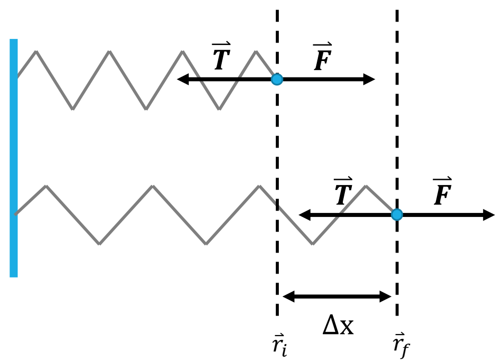
$$W_T = \int_0^{\Delta x} -kx\,\underline{i} \cdot dx\,\underline{i}$$

Hooke's Law, $$W_T = \int_0^{\Delta x} -kx.\,dx$$

Integrate, $$W_T = -\left[\frac{1}{2}kx^2\right]_0^{\Delta x}$$

Work Done by Springs

Where Does My Energy Go?



Integrate, $W_T = -\left[\frac{1}{2}kx^2\right]_0^{\Delta x}$

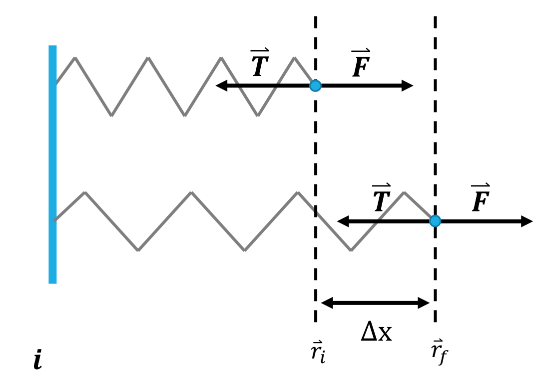
$$\boxed{W_T = -\frac{1}{2}k\Delta x^2}$$

Energy is transferred into the spring itself, stored. Unlike with friction, if $\vec{F}$ is removed, energy can be regained. This is **potential energy.**

MECHANICS 1 – KINETIC & POTENTIAL ENERGIES

Work Done by Springs

Where Does My Energy Go?



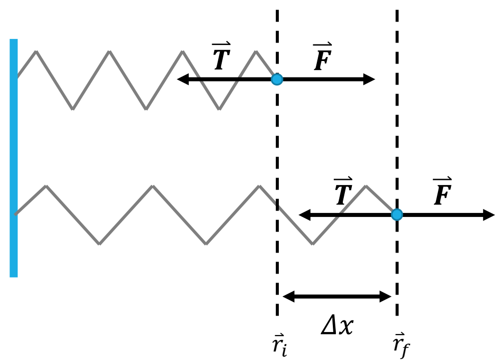
Work done on spring, $W_F = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F} \cdot d\vec{r}$

$$W_F = \int_0^{\Delta x} kx\,\underline{i} \cdot dx\,\underline{i}$$

Hooke's Law, $W_F = \int_0^{\Delta x} kx.dx$

Integrate, $W_F = \left[\frac{1}{2}kx^2\right]_0^{\Delta x}$

Work Done by Springs



Where Does My Energy Go?

Integrate, $W_F = \left[\frac{1}{2}kx^2\right]_0^{\Delta x}$

$$\boxed{W_F = \frac{1}{2}k\Delta x^2}$$

Exactly the same amount of work is done by the applying force as is stored inside the spring. This is because at each point, the net work done was zero (no kinetic energy) and no energy was lost (no frictional forces / forces proportional to velocity!)

In Waves & Vibrations, you will learn exactly what happens if we were to extend the spring, then take $\vec{F}$ away…

MECHANICS 1 – KINETIC & POTENTIAL ENERGIES

Conservative Forces

Conservative Forces

Where Does My Energy Go?

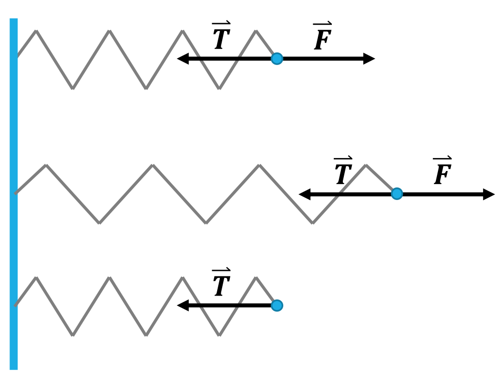
The net work done always equals the change in kinetic energy.

But what about the individual forces?

Some forces add energy to objects (pushing, pulling etc). Some take energy away (friction). But some store energy for later use. The forces that store energy are conservative forces.

Conservative Forces

Elastic Potential Energy



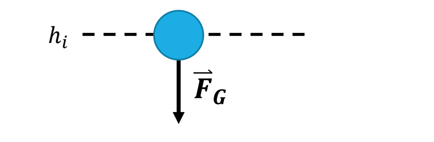
When I stretch a spring, the energy is stored in the spring! It can be reused by letting go of the spring!

$$\Delta U = \frac{1}{2}kx^2$$

MECHANICS 1 – KINETIC & POTENTIAL ENERGIES

Conservative Forces

Gravitational Potential Energy



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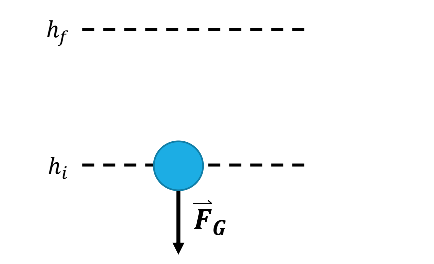
MECHANICS 1 – KINETIC & POTENTIAL ENERGIES

Conservative Forces

Gravitational Potential Energy



Work done by gravity, $$W = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F}_G \cdot d\vec{r}$$

Substitute, $$W = -\int_{h_i}^{h_f} mg \cdot dy$$

Integrate, $$W = -[mgy]_{h_i}^{h_f}$$

Solve, $$W = -mg(h_f - h_i)$$

$$\boxed{W = -mg\Delta h}$$

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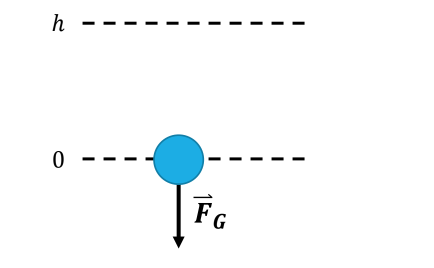
Conservative Forces

Gravitational Potential Energy



When I raise an object, the energy is stored in the gravitational field! It can be reused by letting go of the object!

$$\Delta U = mg\Delta h$$

Potential Energy

Defining It

Potential Energy

Defining It

For a conservative force, $\vec{F}$, the change in potential energy is equal to the work done against that force i.e.

$$\Delta U = -\int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r}$$

But it's path independent (only initial and final locations matter)

$$\Delta U = U(\vec{r}_2) - U(\vec{r}_1)$$

Energy

Some Conclusions

Energy

Conclusions

Forces do work on systems (i.e. transfer energy to them)

$$W = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r}$$

The net work done on a system is equal to the change in it's kinetic energy

$$\Delta E_k = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}_{Net} \cdot d\vec{r}$$

The power of a force is equal to the rate at which it does work

$$P = \frac{dW}{dt} = \vec{F} \cdot \vec{v}$$

If a force is conservative (i.e. spring restoring force, gravity, electric), then it stores energy as potential energy. This is defined as the work done against that force.

$$\Delta U = -\int_{\vec{r}_1}^{\vec{r}_2} \vec{F}_{con} \cdot d\vec{r}$$

Energy

So…what are they?

A force is conservative if the work done is path-independent. In other words, if the work done integral is only dependent on the initial and final positions.

Hookean springs, gravity, electro-magnetic. Each of these forces are conservative forces. The energy can be regained.

Friction, on the other hand, is not. The amount of work done by friction depends explicitly on the path takes. Hence, that energy is lost.

Very Profound Question: Why is friction path-dependent?

Energy

So…what are they?

Friction is path-dependent because it depends on the velocity vector!

General Statement (extremely important in theoretical physics):

- If a force is dependent only on spatial positions, it is almost certainly a conservative force i.e. energy will be stored as potential energy in an associated *field*
- If a force is dependent only on velocity, it is almost certainly a non-conservative force i.e. energy will be taken away from a system by this force
- **If you know the position of an object over time, and the velocity of an object over time (i.e. you know its energy profile), then in classical physics you know everything there is to know about that object**

Task 2

Work Done and Energy in Springs

Task 2

Work Done and Energy in Springs

Scenario:
Someone is stretching a spring horizontally from its equilibrium position. It has a spring constant $k = 10Nm^{-1}$ and it is attached to a small object with mass $100g$. They apply a constant force $F = 0.5N$ horizontally.

Tasks:
1. By considering the force equilibrium conditions, calculate the new equilibrium extension of the spring when this force is applied.
2. Calculate the work done by the force F as the spring stretches to its new equilibrium.
3. Calculate the work done by the spring restoring force as the spring stretches to its new equilibrium (and thus the potential energy stored in the spring).
4. Using the work-kinetic energy theorem, calculate the speed of the ball when it reaches the equilibrium extension.

Task 3

Work Done by Gravity

Task 3

Work Done by Gravity

Scenario:

The Falcon 9 SpaceX rocket has a total mass $m_r = 550{,}000kg$. The rocket engines can supply a constant force of $F_r = 845kN$ to the rocket itself. The Earth has mass m_e and radius R_e, and G is the universal gravitational constant. When the Falcon9 is being launched into space from the surface of the Earth:

Tasks:

1. Show that the work done by gravity as the rocket travels from the surface of the Earth is given by
$$W_G = -\frac{Gm_r m_e}{R_e}\left(\frac{h}{h+R_e}\right)$$
2. Show that the total work done by the engines and gravity is given by
$$W_T = F_r h - \frac{Gm_r m_e}{R_e}\left(\frac{h}{h+R_e}\right)$$
3. The escape velocity from Earth can be written as $v_e = \sqrt{\frac{2Gm_e}{h+R_e}}$. Show that the height Falcon9 will need to reach to achieve escape velocity is given by
$$h = \frac{Gm_r m_e}{F_r R_e}$$
4. Compare this value to the height of "low earth orbit (LEO)". Note that it is much higher i.e. the force F_r is too small. How, then, do rockets reach orbit?
Hint: Have a read of the SpaceX website: https://www.spacex.com/vehicles/falcon-9/

Resources

Dr Purdy's Notes

And Examples

More notes from Rob Purdy. He has good examples ☺