Mechanics 1

Session 5 – Tension & Newton's Third Law

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Last Lecture

Friction & Forces

We learned:
- How to resolve forces into different directions
- How to determine the net force vector on an object and thus, its acceleration
- About the microscopic source of friction
- About the two different types of friction (static friction and kinetic friction)

You should be able to:
- Calculate the acceleration vector of an object by determining the net force acting upon it
- Calculate the frictional forces experienced by an object
- Calculate the friction constants of surfaces based on their motion

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This Lecture

Tension & Newton's Third Law

We will:
- Learn what tension is
- Begin to understand that all objects are at least partially elastic
- Understand how the level of elasticity in an object determines how we model it's motion

You will be able to:
- Identify where in a system tension is acting
- Calculate the acceleration of coupled objects under tension

Tension

In Elastic Objects

Tension

In Elastic Objects

When a force is applied to an elastic object, tension is the force within that object

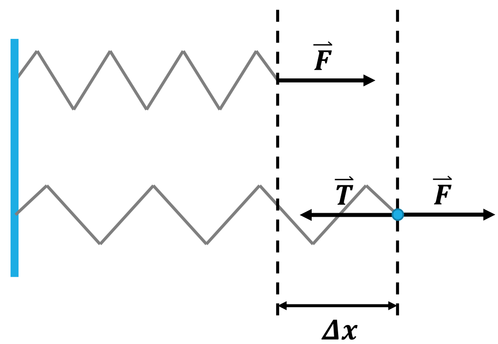
But secretly, every object (and I mean every single object in the known Universe) is at least a little bit elastic. Let's think about a few examples…

Tension

Hooke's Law

Tension

In Strings / Ropes



Hooke's Law:

$$|\vec{F}| = k\Delta x$$

Applied force

Spring constant

Spring extension

$$|\vec{T}| = -k\Delta x$$

Newton's Second Law implies that $\vec{T}$ must exist!

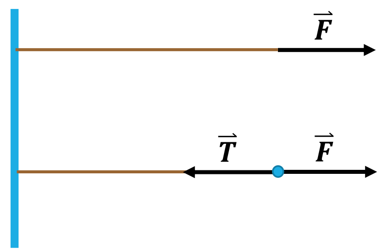
MECHANICS 1 – TENSION & NEWTON'S THIRD LAW

Tension

In Strings / Ropes

Tension

In Strings / Ropes



Hooke's Law:

$$\left|\vec{F}\right| = k\Delta x$$
$$\left|\vec{T}\right| = -k\Delta x$$

Still applies!

- Large k implies small Δx for same applied force.
- Large k also implies fast "relaxation", so force is equilibrated quickly along the rope.
- So, we ignore extension and just go straight into the equilibrium conditions

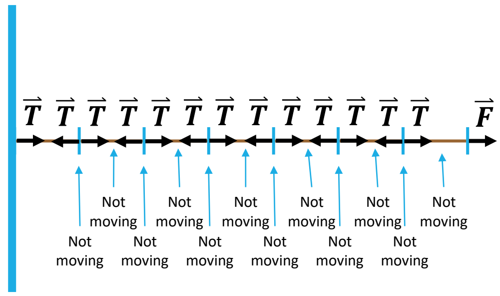
MECHANICS 1 – TENSION & NEWTON'S THIRD LAW

Tension

Springs, Strings & Ropes in Equilibrium

Tension

Strings & Ropes in Equilibrium

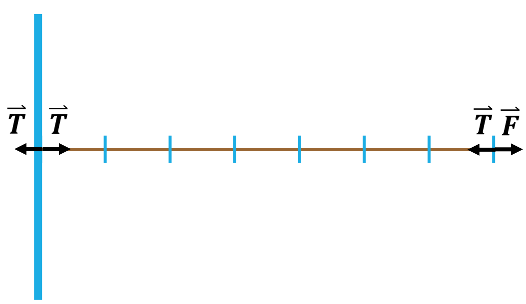


At every point along a rope (or any continuous object with no extension), Newton's Second & Third Laws imply that the forces must be equal.

If the forces were not in equilibrium at any point, extension would occur at that point!

Tension

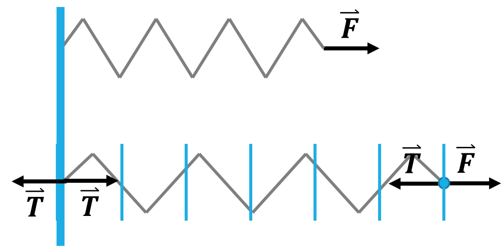
Strings & Ropes in Equilibrium



Tension is transmitted along the rope!

Tension

Springs in Equilibrium



Once it has extended, we can make exactly the same argument about springs. The only difference is, because k is relatively small, a small amount of extension occurs first. This is called **equilibration**

Incidentally, this argument only works for *massless* springs and ropes (or approximately massless). If the string has mass, it gets a little more complex. Maybe I'll do a video about this topic (as it is the beginning of something called continuum mechanics!).

MECHANICS 1 – TENSION & NEWTON'S THIRD LAW

Tension

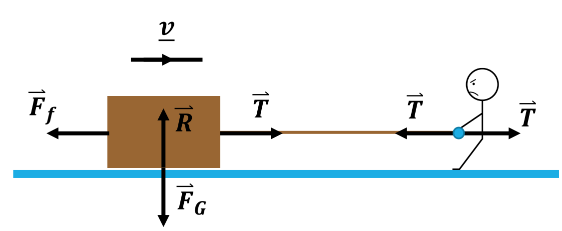
In Strings / Ropes

Scenario: A box is being dragged along the ground with a rope. A person is applying a force to the rope, which is moving the box at a constant speed horizontally.

Tasks:

1. What forces are acting on the box?
2. What force is being applied by the person?
3. What force is acting on the person?

Tension is transmitted along the rope!



Tension

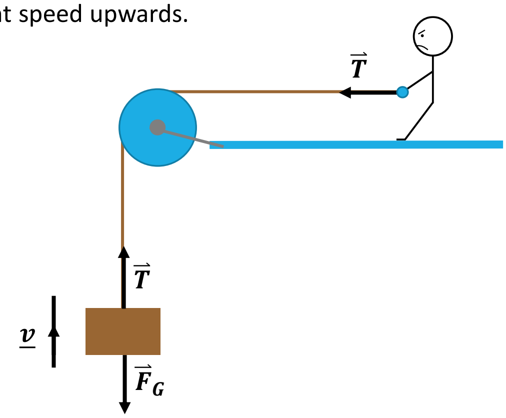
In Strings / Ropes

Scenario: A box is being lifted using a pulley and rope. A person is applying a force to the rope, which is moving the box at a constant speed upwards.

Tasks:

1. What forces are acting on the box?
2. What force is being applied by the person?
3. What force is acting on the person?

Tension is transmitted along the rope!



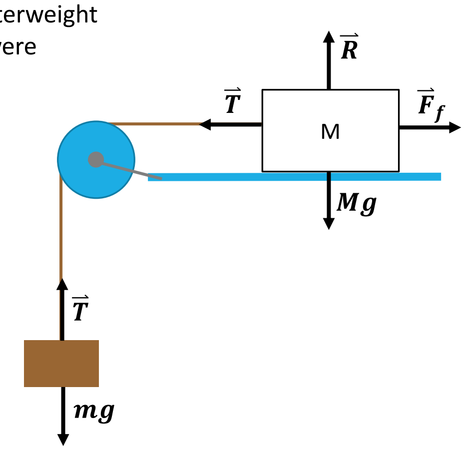
Tension

In Strings / Ropes

Scenario: A box of mass m is attached via a pulley to a counterweight with mass M. The box is not moving, but if any more mass were added to the box, it would begin to move.

Tasks:

1. What forces are acting on the box?
2. What forces are acting on the counterweight?
3. What is the magnitude of the frictional force, $|\vec{F}_f|$?
4. In terms of the masses $\boldsymbol{m}$ and $\boldsymbol{M}$, calculate the value of the static co-efficient of friction



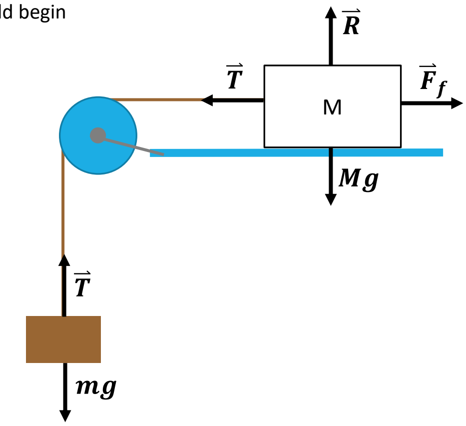
Tension

In Strings / Ropes

Scenario: A box of mass m is attached via a pulley to a counterweight with mass M. The box is not moving, but if any more mass were added to the box, it would begin to move.

Tasks:

3. What is the magnitude of the frictional force, $|\vec{F}_f|$?



Box in equilibrium, $|\vec{T}| = mg$

Weight in equilibrium, $|\vec{F}_f| = |\vec{T}|$

$$\boxed{|\vec{F}_f| = mg}$$

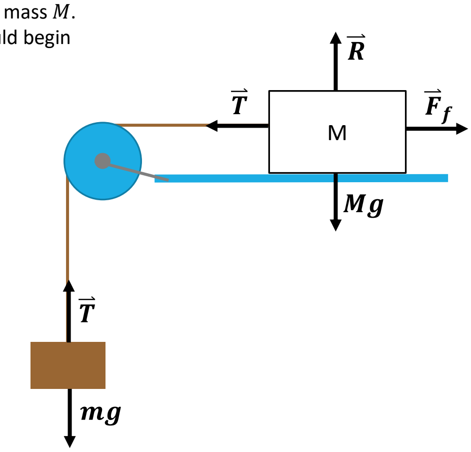
Tension

In Strings / Ropes

Scenario: A box of mass m is attached via a pulley to a counterweight with mass M. The box is not moving, but if any more mass were added to the box, it would begin to move.

Tasks:

4. In terms of the masses $\boldsymbol{m}$ and $\boldsymbol{M}$, calculate the value of the static co-efficient of friction



Previous result, $|\vec{F}_f| = mg$

Expand friction, $\mu_s|\vec{R}| = mg$

$\mu_s Mg = mg$

Simplify, $\boxed{\mu_s = \frac{m}{M}}$

Task 1

Tension in Linear Systems

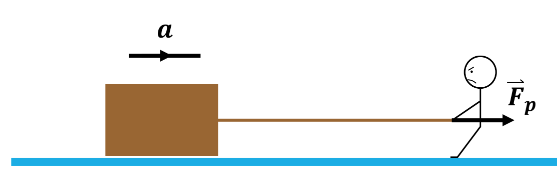
Task 1

Tension in Linear Systems

Scenario: A box of mass $m = 100kg$ is being dragged along the ground by a person using an approximately massless rope. They are applying a force $\vec{F_p} = 300N$ to the rope, and the co-efficient of kinetic friction $\mu_s = 0.25$

Tasks:

1. What forces are acting on the box?
2. Calculate the acceleration of the box



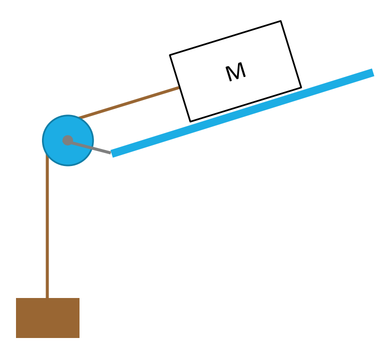
Task 2

Tension in Systems

Task 2

Tension in Systems

Scenario: A box of mass $m = 50kg$ is attached via a pulley and rope to a counterweight with mass $M = 150kg$. The counterweight is on a surface that is 20^o to the horizontal. It is not moving.



Tasks:

1. What forces are acting on this system?
2. Derive an inequality which describes the minimum value of the co-efficient of static friction between the weight and the horizontal surface in terms of the masses of the objects?

   Hint: Treat each object individually

Task 3

More Tension in Systems

Task 3

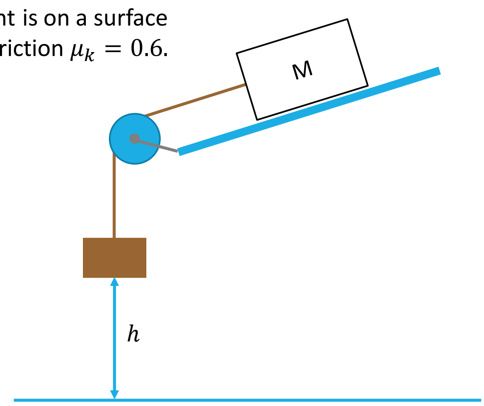
More Tension in Systems

Scenario: A box of mass $m = 50kg$ is a height $h = 1.5m$ above the ground and has just started descending. It is attached via a pulley and rope of negligible mass to a counterweight with mass $M = 100kg$. The counterweight is on a surface that is 40^o to the horizontal and has a co-efficient of kinetic friction $\mu_k = 0.6$.

Tasks:

1. Calculate the time taken to hit the ground
2. Calculate the tension in the rope

Hint: Treat each object individually…but recognise that the tension acting on each object, and the acceleration of each object, must be equal. Linked by simultaneous equations…



Task 4

Springs in Series

Task 4

Springs in Series

Tasks:

1. By considering two identical springs with spring constant k in series under the action of an external force, show that the effective spring constant $k_e = \frac{k}{2}$

Resources

Dr Purdy's Notes

And Examples

I've decided to add all of Dr Purdy's notes each week. They're very, very good and often look at things from a different perspective to me. Also, there are some more practice questions in them ☺