Vectors

What is a vector?

- Objects that obey same rules of arithmetic as ordinary numbers
- Multi-dimensional
- Numbers with direction

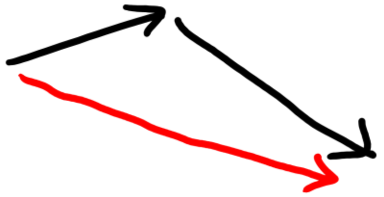
- A vector is NOT a set of components

Vector Addition

- Can add vectors “nose to tail”

$\underline{a} =$ 

$\underline{b} =$ 

Define: $\underline{a} + \underline{b}$ 

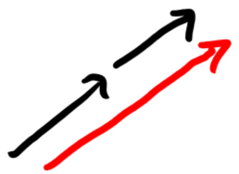
- Do not need components for this!

Scalar Multiplication

- Can define multiplication by ordinary numbers – scalars – intuitively

$$2 \times x = x + x$$

$$2\underline{a} = \underline{a} + \underline{a} =$$



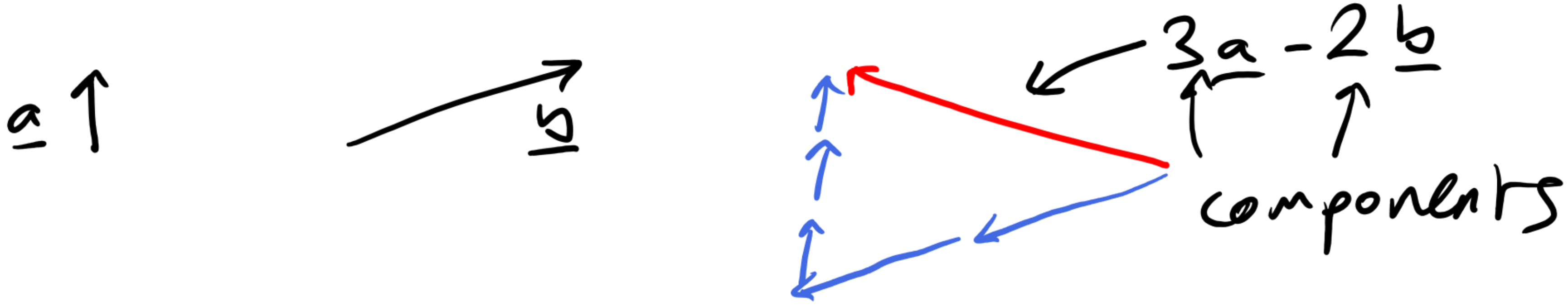
- Vectors are parallel if multiples of each other

$-3\underline{a}$ parallel to $\underline{a}$

- Do not need components to multiply by scalars!

Basis Vectors and Coordinate Systems

- Can choose arbitrary set of vectors and write all vectors in terms of these
- This defines a coordinate system or basis
- Can choose a basis however we like as long as not parallel
- The number of basis vectors we need is the **dimension** of the space
- The scalars multiplying the basis vectors are the **components** of the vector



Cartesian Coordinates

- Simplest basis to use
- Mutually perpendicular basis vectors – “orthogonal”
- All length 1 – “normal”

$$\underline{\hat{i}} = \xrightarrow[1]{} \qquad \underline{\hat{j}} = \uparrow 1$$

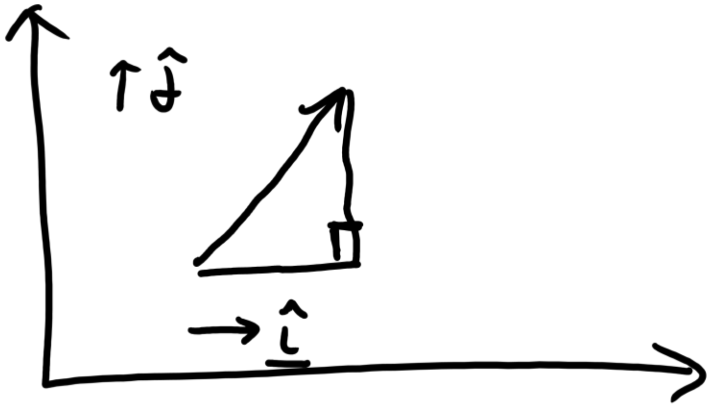
- In this case we say the basis is orthonormal

Length and direction of a vector

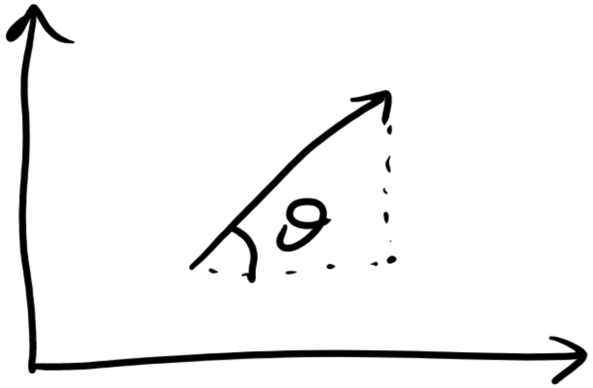
Pythagoras

Eg. $\underline{a} = 2\hat{\underline{i}} + 3\hat{\underline{j}}$

Length $\sqrt{2^2+3^2} = \sqrt{13}$



Only in Cartesian



$2\hat{i} + 3\hat{j}$

$\tan\theta = \frac{3}{2}$

Angle with x axis $\quad \tan^{-1}\left(\frac{y}{x}\right)$

$$\underline{a} = -5\,\underline{\hat{i}} + 7\,\hat{j}$$

Length: $\sqrt{5^2+7^2} = \sqrt{74}$

Angle: $54°$

Column vectors

- Can simplify notation by placing components in a column

$$\begin{pmatrix} x \\ y \end{pmatrix} \equiv x\underline{\hat{i}} + y\underline{\hat{j}}$$

Changing coordinate systems

- Always work in a system that will simplify your problem!

- Can change coordinate systems by substitution

$$\hat{\underline{l}} = \sin\theta\,\hat{\underline{i}} + \cos\theta\,\hat{\underline{j}}$$

Kinematics in >1 Dimension

- Position vector

$$\underline{r} = \underline{r}(t) = r_1(t)\,\hat{\underline{i}} + r_2(t)\,\hat{\underline{j}}$$

- Velocity vector

$$\underline{v} = \frac{d}{dt}\underline{r}(t)$$

$$= \frac{d}{dt}\left(r_1(t)\,\hat{\underline{i}} + r_2(t)\,\hat{\underline{j}}\right)$$

- Only this simple because basis is fixed!

$$= \left(\frac{d}{dt}r_1(t)\right)\hat{\underline{i}} + \left(\frac{d}{dt}r_2(t)\right)\hat{\underline{j}}$$

$$= v_1\,\hat{\underline{i}} + v_2\,\hat{\underline{j}}$$

Other Vectors

$$ax^2 + bx + c + fx^3 + gx^4$$