UNIVERSITY OF LEEDS

Mechanics 1

Session 9 – Energy Conservation

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Last Lecture

Kinetic & Potential Energies

We discussed:

- How work done translates into kinetic energy
- The concept of power
- The concept of potential energy

You should be able to:

- Calculate the power output of a system
- Calculate potential energy of a spring
- Calculate gravitational potential energy both locally and over large distances

This Lecture

Energy Conservation

We will:

- Understand that the total energy of a closed system is always conserved
- Understand that moving objects retain energy in the form of kinetic energy
- Understand that conservative forces retain energy in the form of potential energy
- Understand that non-conservative forces change energy into heat, sound, light and other forms of radiation

You will be able to:

- Use the concepts of energy conservation to calculate the positions and velocities of moving objects

Work Done & Energy Conservation

Energy Conservation with Conservative Forces

Work Done & Energy Conservation

Conservative Forces

Maths aside, all the work done stuff we've done so far simply shows that forces transfer energy

- The net work done (work done by all forces) changes the kinetic energy
- The work done by *conservative* forces stores energy as potential energy, for later use
- But all the energies you've learned before are still true!

$$\Delta U_{Grav} = mg\Delta h, \qquad \Delta U_{Spring} = \frac{1}{2}k\Delta x^2$$

Work Done & Energy Conservation

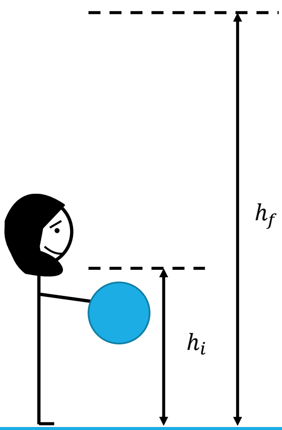
Conservative Forces

Energy conservation arises naturally from the concept of work done

Keep that metaphor of money in mind if it helps!

Work Done & Energy Conservation

Conservative Forces



Kinetic Energy:

$$K_f = \frac{1}{2}m|\vec{v}|^2$$

Potential Energy:

$$U_f = mgh_f$$

Total Energy:

$$E_f = K_f + U_f$$

Total energy can be defined at any position and speed

It is a **state variable**

Kinetic Energy:

$$K_i = \frac{1}{2}m|\vec{u}|^2$$

Potential Energy:

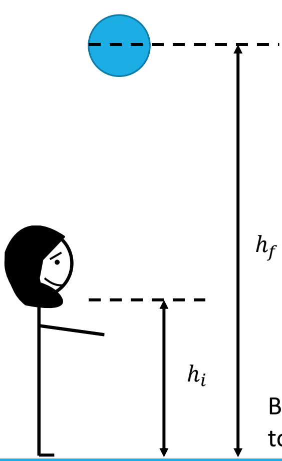
$$U_i = mgh_i$$

Total Energy:

$$E_i = K_i + U_i$$

Work Done & Energy Conservation

Conservative Forces



Work Done by gravity:

$$W = \int_{h_i}^{h_f} -mg\, dy = -mg(h_f - h_i)$$

$$\boxed{W = -mg\Delta h}$$

Work / Kinetic-Energy Theorem:

$$W = \Delta K$$

$$\boxed{W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2}$$

Work against conservative force:

$$\Delta U = -W$$

$$\boxed{\Delta U = mg\Delta h}$$

Hence:

$$\Delta K = -\Delta U$$

$$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = -mg\Delta h$$

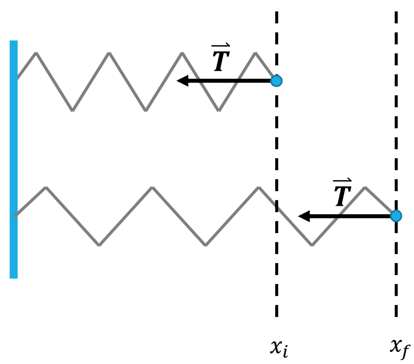
$$\boxed{\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mg\Delta h}$$

$$\boxed{\boldsymbol{E_i = E_f}}$$

Because work done against conservative force equals change in kinetic energy, total energy is conserved!

Work Done & Energy Conservation

Springs As Well



Kinetic Energy:

$$K_i = \frac{1}{2}mu^2$$

Potential Energy:

$$U_i = \frac{1}{2}kx_i^2$$

Kinetic Energy:

$$K_f = \frac{1}{2}mv^2$$

Potential Energy:

$$U_f = \frac{1}{2}kx_f^2$$

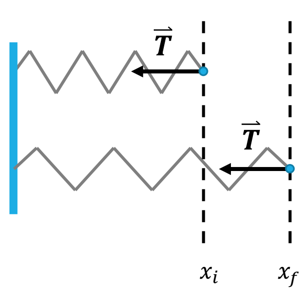
Energy Conserved:

$$\boxed{\frac{1}{2}mu^2 + \frac{1}{2}kx_i^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx_f^2}$$

MECHANICS 1 – ENERGY CONSERVATION

Work Done & Energy Conservation

Springs As Well



Work Done by spring:

$$W = \int_{x_i}^{x_f} -kx\,dx = -\frac{1}{2}k\left(x_f^2 - x_i^2\right)$$

$$\boxed{W = -\frac{1}{2}k\left(x_f^2 - x_i^2\right)}$$

Work / Kinetic-Energy Theorem:

$$W = \Delta K$$

$$\boxed{W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2}$$

Work against conservative force:

$$\Delta U = -W$$

$$\boxed{\Delta U = \frac{1}{2}k\left(x_f^2 - x_i^2\right)}$$

Hence:

$$\Delta K = -\Delta U$$

$$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = -\frac{1}{2}k\left(x_f^2 - x_i^2\right)$$

$$\boxed{\frac{1}{2}mu^2 + \frac{1}{2}kx_i^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx_f^2}$$

$$\boxed{\boldsymbol{E_i = E_f}}$$

Because work done against conservative force equals change in kinetic energy, total energy is conserved!

Task 1

Energy Conservation with Conservative Forces

Task 1

Energy Conservation with Conservative Forces

Scenario:
A ball is thrown straight upwards from a height $h = 1.5m$ and with an initial speed $|\vec{u}| = 5ms^{-1}$. Assuming air resistance is negligible:

Tasks:

1. Calculate the maximum height of the ball using the principles of conservation of energy.
   Hint: You do not need work done for this!

2. Check that the maximum height of the ball calculated using the SUVAT equations gives the same result.

Task 2

More Energy Conservation with Conservative Forces

Task 2

More Energy Conservation with Conservative Forces

Scenario:
A ball is thrown from a height $h = 1.5m$ at an angle 50^o to the horizontal and with an initial speed $|\vec{u}| = 5ms^{-1}$.
Assuming air resistance is negligible:

Tasks:

1. Calculate the maximum height of the ball using the principles of conservation of energy.
   Hint: You do not need work done for this!

2. Check that the maximum height of the ball calculated using the SUVAT equations gives the same result.

Task 3

Even more Energy Conservation with Conservative Forces

Task 3

Even more Energy Conservation with Conservative Forces

Scenario:
A mass $m = 2kg$ is attached to the end of a spring with spring constant $k = 20N/m$. The spring is hung vertically and the mass is released from rest at the spring's equilibrium position.

Tasks:
1. Calculate the maximum extension of the spring using whatever method you see fit.

Work Done & Energy Conservation

Energy Conservation with Non-Conservative Forces

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Work Done & Energy Conservation Non-Conservative Forces

When friction / air resistance does work, and energy is conserved, where does the energy go?

Friction, and non-conservative forces generally, extract energy from the object and transfer it to the environment, often as heat. This concept is part of one of the most fundamental ideas in all of physics:

The 2nd Law of Thermodynamics

You'll learn about this idea over the next couple of years.

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Task 4

Energy Conservation with Non-Conservative Forces

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Task 4

Energy Conservation with Non-Conservative Forces

Scenario:
A block with mass $m = 50kg$ is released at the top of a hill, height $h = 30m$ and angled $\theta = 40^o$ to the horizontal. It begins to slide down the hill from rest. It is accelerating, so it picks up speed, but the kinetic co-efficient of the hill, $\mu_k = 0.2$.

Tasks:
1. Calculate the speed of the block when it reaches the bottom of the hill ($h = 0m$)
   Hint: Consider, Initial energy = final energy + energy lost to non-conservative forces (i.e. friction)

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A Small Glimpse of Thermodynamics

Total Energy Conservation

A Glimpse of Thermodynamics

Total Energy Conservation

Up to now, we have been thinking of our systems in isolation, with energy E_{Sys}. But what if we considered the entire energy of the Universe, E_T: the system plus it's environment, E_{Env}…

Total Energy,	$\Delta E_T = \Delta E_{Sys} + \Delta E_{Env}$
We know about the system,	$\Delta E_T = \Delta K + \Delta U + \Delta E_{Env}$
Write in terms of work done,	$\Delta E_T = W_{Net} - W_{Conservative} + \Delta E_{Env}$
Non-conservative work,	$\Delta E_T = W_{Non-Conservative} + \Delta E_{Env}$
Total energy conserved, $\Delta E_T = 0$,	$\boxed{\Delta E_{Env} = -W_{Non-Conservative}}$

All non-conservative work done (in any process) is added to the environment as heat. Friction is literally the cause of climate change. But on a much grander scale, it is the cause of universal disorder. Very philosophically interesting…