UNIVERSITY OF LEEDS

Mechanics 1

Session 14: Circular Motion – Force & Torque

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Last Lecture

Circular Motion – Acceleration in Radial Co-ordinates

We:

- Described the different components of velocity in circular coordinates
- Derived the vector form of centripetal acceleration
- Derived the vector form of velocity and acceleration with variable angular speed
- Considered what it means to have rotation in these directions

You should be able to:

- Reproduce the derivation of the *full* kinematic equations for velocity and acceleration in circular co-ordinates with constant radius, R, and variable angular speed, ω
- Calculate velocities and accelerations in circular coordinates
- Transform from cartesian coordinates to circular coordinates

This Lecture

Circular Motion – Force & Torque

We will:

- Describe different causes of centripetal force for different physical systems
- Understand the concept of torque, and how torques cause changes in angular speed

You will be able to:

- Calculate accelerations and forces in circular coordinates
- Calculate the torques about any axis

Centripetal and Angular Forces

What do Forces Do in Circular Motion?

Centripetal and Angular Forces

All objects that exhibit circular motion have a centripetal force $\left(\underline{\hat{r}}\ \text{component}\right)$.

Some may also have an angular force $\left(\underline{\hat{\theta}}\ \text{component}\right)$.

However, the cause / source of that force is different for different systems.

Task 1

Forces in Circular Motion

Task 1

Forces in Circular Motion

Scenario: Ben is swinging a mass tied to an (approximately) massless rope in a horizontal circle over his head. The mass, $m \approx 3kg$, and the radius of the circle $R \approx 1.2m$. The object is moving with an angular speed $\omega \approx 4rads.s^{-1}$. The object is exhibiting no vertical motion.

Tasks:

1. Calculate the centripetal force acting on the object.
2. What type of force is the centripetal force?
3. Now, assuming the force of gravity is not-negligible, but the angular speed is still the same ($\omega \approx 4rads.s^{-1}$) calculate the tension in the rope. *Hint: Gravity will pull the object down, giving us two components of tension. There are simultaneous equations here for tension and an angle…*

Angles always measured in radians when considering circular motion!

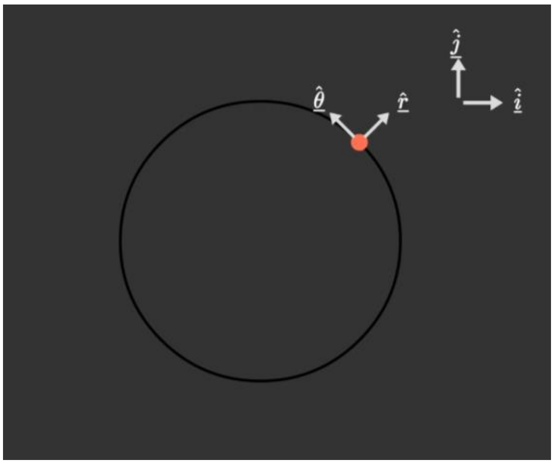
MECHANICS 1: CIRCULAR MOTION – FORCE & TORQUE

Task 2

More forces in Circular Motion

Task 2

More forces in Circular Motion



Scenario: A car of mass $m = 1000kg$ is driving around a roundabout with $R = 12m$. Unfortunately, the driver does not know when to leave the roundabout so just keeps driving around at a constant speed of $v_\theta = 9ms^{-1}$.

Tasks:
1. Calculate the centripetal force acting on the car
2. What type of force is the centripetal force?
3. The car accelerates in the $\underline{\hat{\theta}}$ direction such that to $v_\theta = 10ms^{-1}$, but remains on the same circular path. Calculate the new centripetal force acting on the car.
4. If the co-efficient of static friction of the car on the road, $\mu_s = 0.9$, what is the maximum speed the car can reach before it begins to skid? *Hint: Consider what happens when the centripetal force is greater than the maximum frictional force.*

Angles always measured in radians when considering circular motion!

Centripetal and Angular Forces

If an object is travelling on a circular path, the centripetal force is the cause of the change of direction (i.e. keeps the object on the circular path).

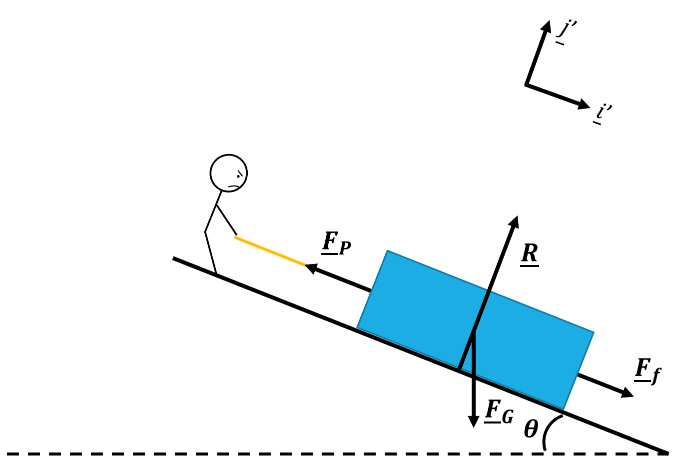
If the centripetal force *cannot get high enough to support a circular path (friction too small, tension so large it snaps string etc)*, then the object will no longer be undergoing circular motion.

Torque

Where do Forces Act?

Torque

Where do Forces Act?



Force	Where does it act?
Rope tension, $\underline{F}_P$	Along the rope
Gravity, $\underline{F}_G$	Vertically down, <u>at the centre of mass</u>
Reaction Force, $\underline{R}$	Perpendicular, <u>at the surface-box interface</u>
Friction, $\underline{F}_f$	Parallel, <u>at the surface-box interface</u>

Torque

Where do Forces Act?

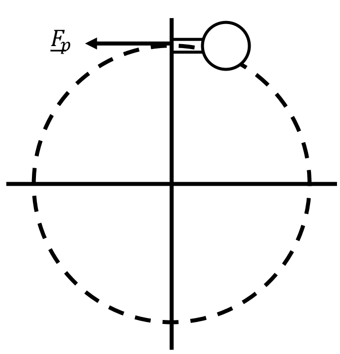
Where forces act is as important as the size and direction of those forces.

Let's consider a circular motion and find out why this is

Torque

Where do Forces Act?

Scenario: Someone is trying to get into the Bragg building via the front revolving door. However, they keep forgetting to leave the revolving door and instead, just keep running around! They can apply a force of magnitude $|\underline{F_p}|$ in any direction they like.



What's going to happen?

1. Initially, $\omega = 0$ and so there is no rotation; the centripetal force is equal to zero.
2. $\underline{F_p}$ applied in the $\underline{\hat{\theta}}$ direction. Angular acceleration α is now non-zero. ω begins to increase.
3. Centripetal force increases with ω. Circular motion is obtained!

Torque

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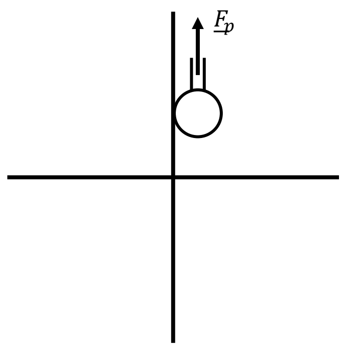

What's going to happen?

1. Initially, $\omega = 0$ and so there is no rotation; the centripetal force is equal to zero.
2. $\underline{F_p}$ applied in the $\underline{\hat{r}}$ direction. Angular acceleration α unaffected. ω does not change.
3. Centripetal force does not exist. Circular motion does not occur.

MECHANICS 1: CIRCULAR MOTION – FORCE & TORQUE

Torque

Where do Forces Act?

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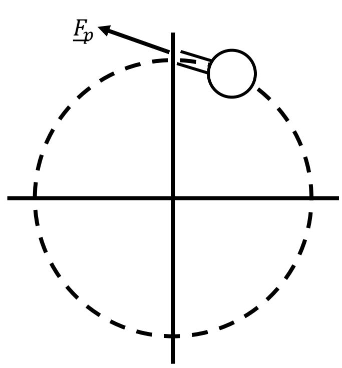


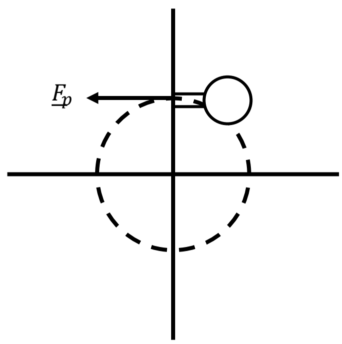
What's going to happen?

1. Initially, $\omega = 0$ and so there is no rotation; the centripetal force is equal to zero.
2. $\underline{F_p}$ applied in some combination of the $\underline{\hat{\theta}}$ and $\underline{\hat{r}}$ directions. Angular acceleration α is now non-zero due to the $\underline{\hat{\theta}}$ component only! ω begins to increase.
3. Centripetal force increases with ω. Circular motion is obtained!
4. Centripetal force also affected by the component of $\underline{F_p}$ in the $\underline{\hat{r}}$ direction!

Torque

Where do Forces Act?

Scenario: Someone is trying to get into the Bragg building via the front revolving door. However, they keep forgetting to leave the revolving door and instead, just keep running around! They can apply a force of magnitude $|\underline{F}_p|$ in any direction they like.



What's going to happen?

1. Initially, $\omega = 0$ and so there is no rotation; the centripetal force is equal to zero.
2. $\underline{F}_p$ applied in the $\hat{\underline{\theta}}$ direction. Angular acceleration α is now non-zero. ω begins to increase.
3. Centripetal force increases with ω. Circular motion is obtained!
4. It's harder to get something to angularly accelerate the closer to the pivot you apply the force…

While the centripetal force causes the change in direction, without an angular speed, it remains zero. In some sense, then, the angular force (in the $\hat{\underline{\theta}}$ direction) "causes" the circular motion by making $\omega \neq 0$. We need a new concept to describe how it affects everything…

Torque

Where do Forces Act?

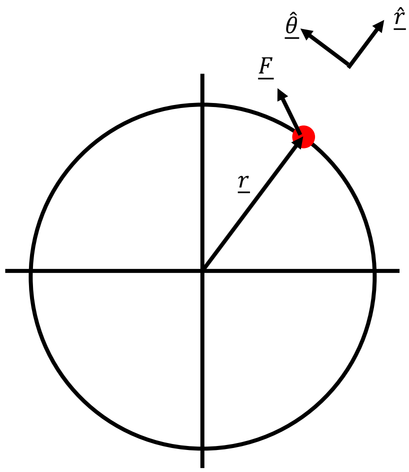
Forces as we have defined them are the "cause" of linear acceleration. If there is an acceleration, then there is a force causing it.

What, then, "causes" angular acceleration? How do we represent forces in angular coordinates?

Let's consider a circular acceleration and find out

Torque

Where do Forces Act?



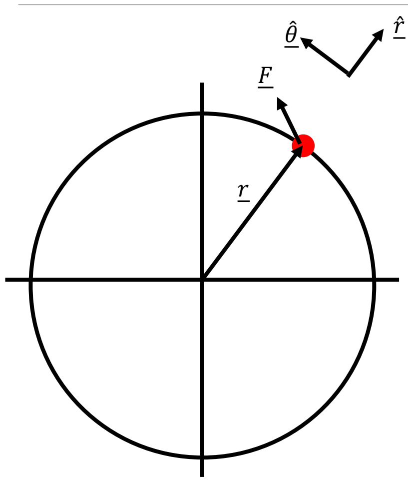
To describe a "rotational force", we need a vector operation between $\underline{r}$ and $\underline{F}$ that is:

1. Zero when the two vectors are in the same direction (as there is no circular motion)
2. Maximum when the two vectors are perpendicular (as this is when all of $\underline{F}$ will be in the $\hat{\underline{\theta}}$ direction, acting to increase α)
3. Some intermediate value when $\underline{r}$ and $\underline{F}$ are neither parallel nor perpendicular
4. Increases with the magnitude of both $\underline{r}$ and $\underline{F}$

The vector cross product!

Torque

Where do Forces Act?



Define the torque, τ, about any point:

$$\boxed{\underline{\tau} = \underline{r} \times \underline{F}}$$

$\underline{r}$: The vector from the point to the location the force is applied

$\underline{F}$: The applied force

Cross product identity:

$$\boxed{\underline{\tau} = \underline{r} \times \underline{F} = |\underline{r}||\underline{F}| \sin(\phi)\, \hat{\underline{n}}}$$

Question: What is $\hat{\underline{n}}$?

Briefly Back in Time

Vectors from Supplementary Material

Vectors

What can we do with vectors?

Vector Multiplication – Cross Product

(Vectors In -> Vector Out)

Multiplication	Scalar	Vector
Scalar	✓	✓
Vector	✓	✓

Creating the cross product (also called the vector product)

Imagine we've just invented vectors, and we want an operation similar to scalar multiplication. We've already invented the dot product, but now we want an operation which results in a vector. What do we do? Well…we just make something up!

This is exactly what Joseph-Louis Lagrange did in 1773! Yes, we can invent maths, just like any tool[1] ☺

[1]Some people believe maths is discovered rather than invented. This is a philosophy debate, and we'll talk more about this in tutorials ☺

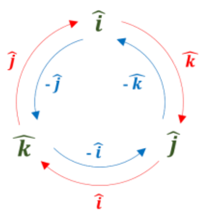
Vectors

What can we do with vectors?

Vector Multiplication – Cross Product **(Vectors In -> Vector Out)**

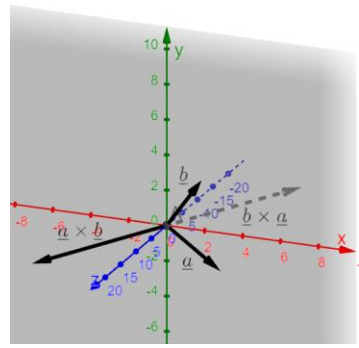
If I have two vectors $\underline{a}$ and $\underline{b}$, and I want a vector output, then the most useful direction for that output is perpendicular to both of the input vectors. Hence, we can *define*:

$$\underline{i} \times \underline{j} = \underline{k}$$
$$\underline{j} \times \underline{k} = \underline{i}$$
$$\underline{k} \times \underline{i} = \underline{j}$$



$$\underline{j} \times \underline{i} = -\underline{k}$$
$$\underline{k} \times \underline{j} = -\underline{i}$$
$$\underline{i} \times \underline{k} = -\underline{j}$$

$$\underline{i} \times \underline{i} = \underline{j} \times \underline{j} = \underline{k} \times \underline{k} = 0$$



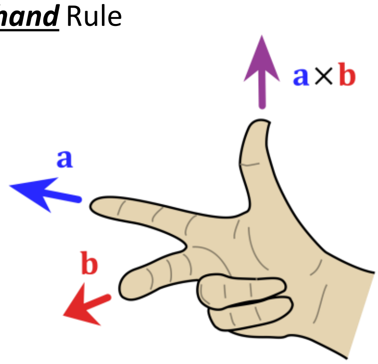
Load the GeoGebra file!

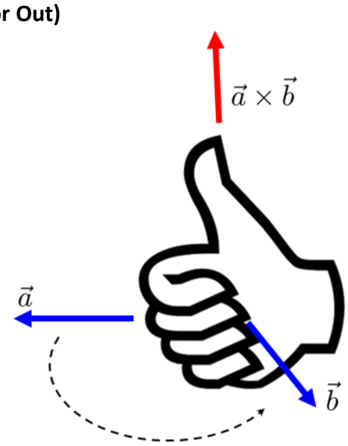
Vectors

What can we do with vectors?

Vector Multiplication – Cross Product **(Vectors In -> Vector Out)**

Right-hand Rule





Vectors

What can we do with vectors?

Vector Multiplication – Cross Product

(Vectors In -> Vector Out)

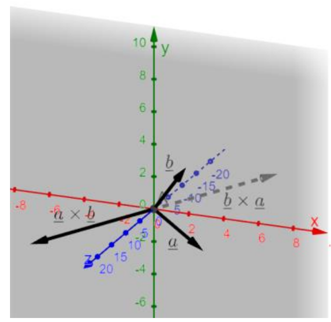
Algebra – Vital Identities

$$\underline{a} \times \underline{b} = |\underline{a}||\underline{b}| \sin\theta \underline{\hat{n}}$$

$$\underline{a} \times \underline{b} = -\underline{b} \times \underline{a}$$

Diagram!

Multiplication	Scalar	Vector
Scalar	✓	✓
Vector	✓	✓



What is the cross product?

The cross product between two vectors $\underline{a}$ and $\underline{b}$ gives a third vector, $\hat{n}$, in a direction parallel to both $\underline{a}$ and $\underline{b}$. This vector is proportional to the magnitude of both $\underline{a}$ and $\underline{b}$, analogous to scalar multiplication.

MECHANICS 1: CIRCULAR MOTION – FORCE & TORQUE

Vectors

What can we do with vectors?

Vector Multiplication – Cross Product

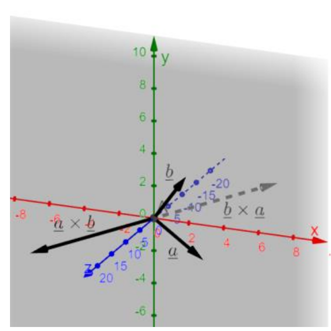
(Vectors In -> Vector Out)

Column Vector

$$\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} \times \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} = \begin{pmatrix} y_1 z_2 - y_2 z_1 \\ x_2 z_1 - x_1 z_2 \\ x_1 y_2 - x_2 y_1 \end{pmatrix}$$

Diagram!

Multiplication	Scalar	Vector
Scalar	✓	✓
Vector	✓	✓



Algebra

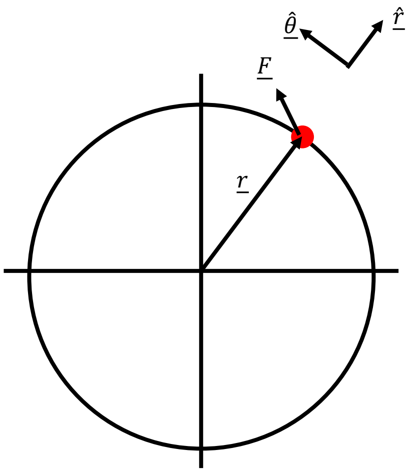
$$\left(x_1 \underline{i} + y_1 \underline{j} + z_1 \underline{k}\right) \times \left(x_2 \underline{i} + y_2 \underline{j} + z_2 \underline{k}\right)$$

$$= (y_1 z_2 - y_2 z_1)\underline{i} - (x_1 z_2 - x_2 z_1)\underline{j} + (x_1 y_2 - x_2 y_1)\underline{k}$$

Just…ok this one is complicated ☹

Torque

Where do Forces Act?



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Cross product identity:

$$\boxed{\underline{\tau} = \underline{r} \times \underline{F} = |\underline{r}||\underline{F}|\sin(\phi)\,\hat{\underline{n}}}$$

Question: What is $\hat{\underline{n}}$? $\hat{\underline{n}}$ is the axis of rotation!

MECHANICS 1: CIRCULAR MOTION – FORCE & TORQUE

Torque

Mechanical Equilibrium

Torque

Mechanical Equilibrium

Now we have this idea of rotational force (i.e. torque), we need a new type of mechanical equilibrium

- If all forces acting on an object (anywhere on the object) sum to zero (i.e. net force is zero), then the object will not linearly accelerate.
- If all torques acting on an object sum to zero (i.e. net torque is zero about an axis), then the object will not rotationally accelerate!

Task 3

Torque on the Bragg Building door

Task 3

Torque on the Bragg Building door

Scenario: Two academics are trying to get into the Bragg building via the revolving door. They are each in a separate quarter applying forces. The first academic applies a force $\underline{F_1} = 200N\underline{\hat{\theta}}$ at a distance of 0.7m from the centre of the door (i.e. at a position $\underline{r_1} = 0.7m\underline{\hat{r}}$). The second academic applies a force $\underline{F_2} = 175N\underline{\hat{\theta}} + 20N\underline{\hat{r}}$ at a distance of 1.1m from the centre of the door.

Tasks:

1. Calculate the net torque on the door about the centre of the door (i.e. the axis of rotation is the vertical axis at the centre of the cross). *Hint:* $\underline{\hat{r}} \times \underline{\hat{\theta}} = \underline{\hat{n}}$*, points out of the page.* $\underline{\hat{\theta}} \times \underline{\hat{r}} = -\underline{\hat{n}}$*, points into the page.*
2. A third academic is striking and doesn't want the first two to be able to enter the building. The apply a force at a position $\underline{r_3} = 0.9m\underline{\hat{r}}$. What force vector could they apply to stop the revolving door from accelerating (i.e. rotational equilibrium conditions).
3. The Bragg building actually has 3 segments, not 4. Explain why, if we use circular co-ordinates, this will not have affected our calculations in any way whatsoever.