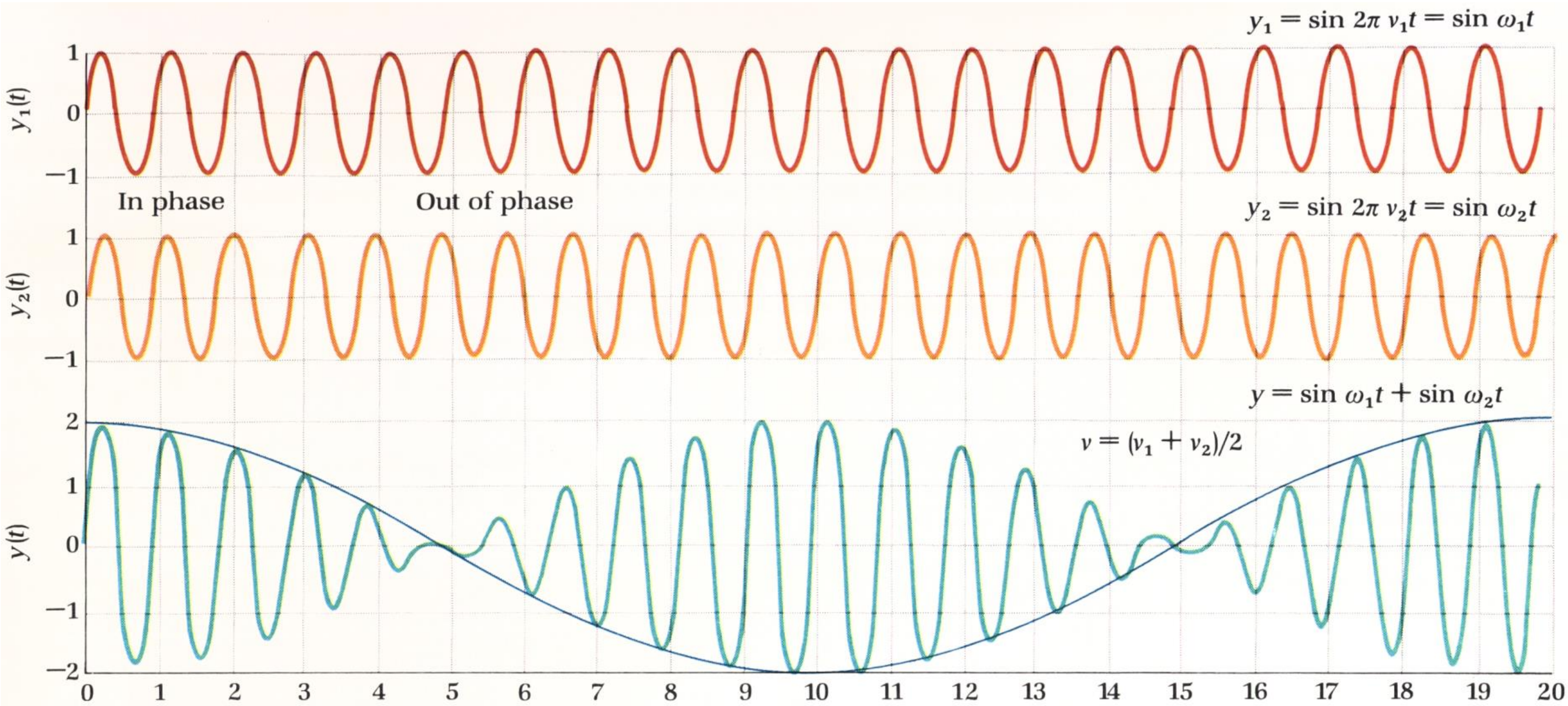
Interference of travelling waves 2: Frequency difference

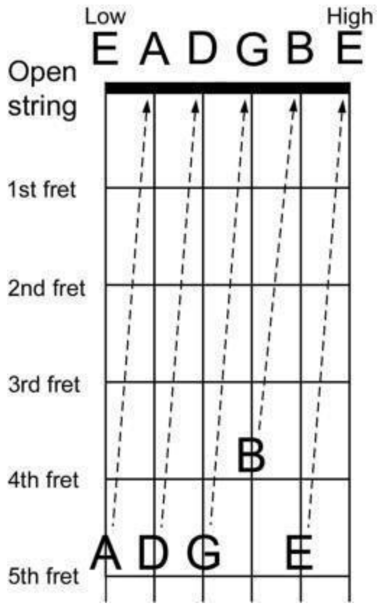
Consider two harmonic waves, passing a fixed point, that have slightly different frequencies. Their superposition is:

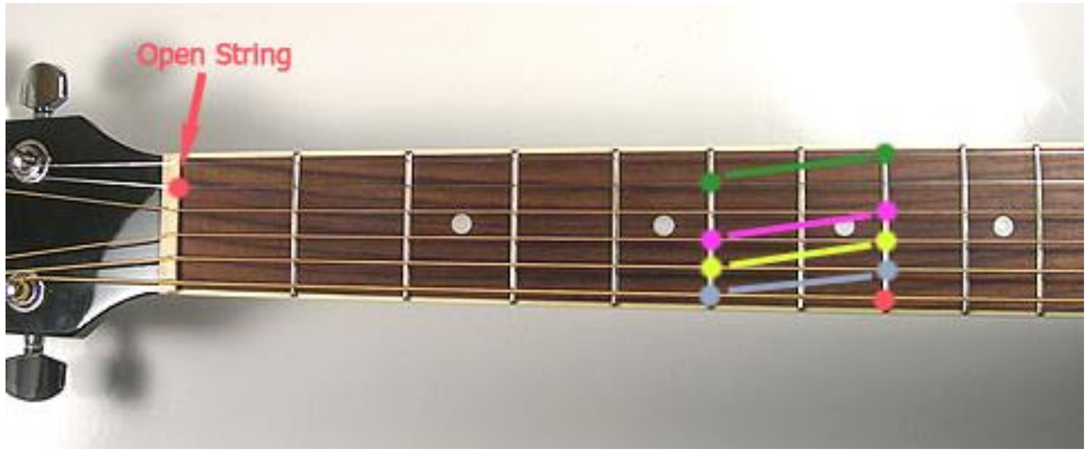


This causes a modulation in amplitude = beats

(n.b. modulation means altering one waveform by adding a second waveform, in this case the result is an amplitude change related at a low frequency)

Interference of travelling waves 2: Frequency difference





or by harmonics

This type of wave interference can be useful for tuning a musical instrument to play the correct note. If the two notes are not perfectly in tune then a wobble in the amplitude is clearly heard. The more out of tune, the faster the wobble. When perfectly in tune the change in amplitude is eliminated.

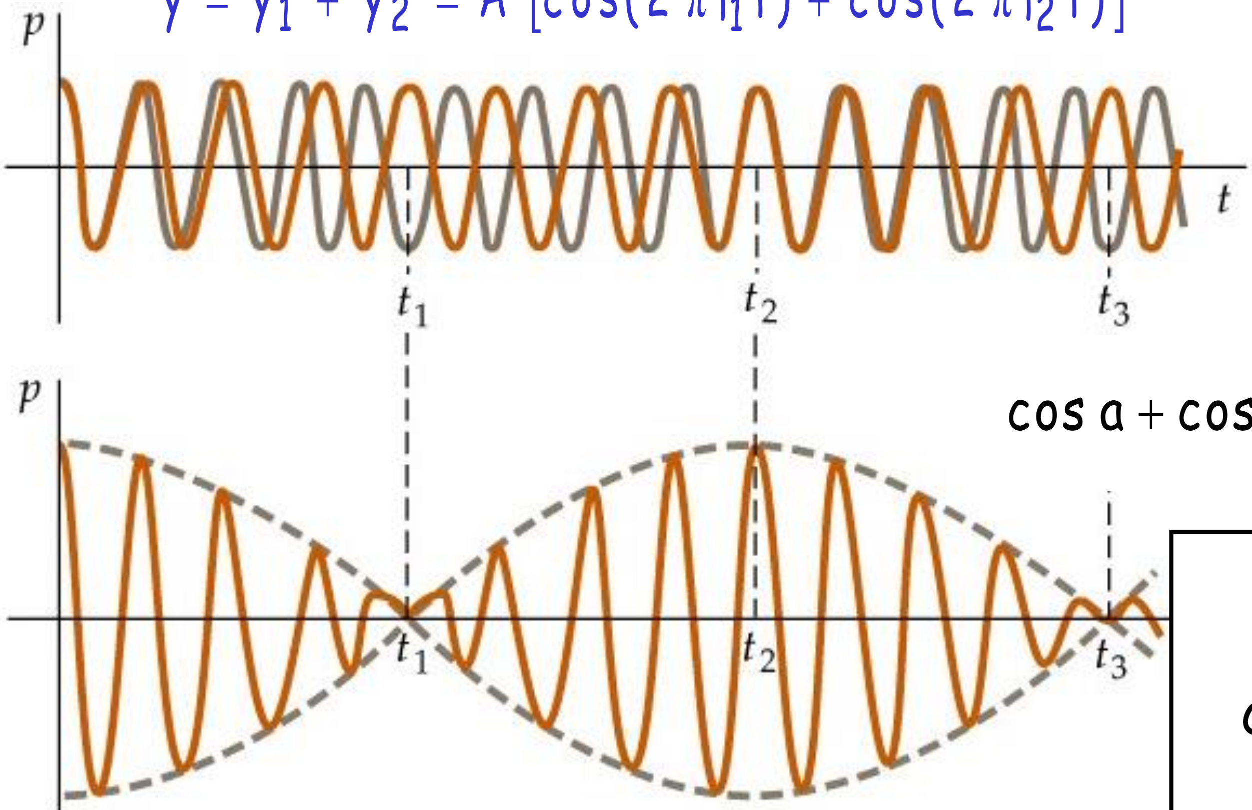
https://www.youtube.com/watch?v=hCFMbh2IsPQ

= BEATS

Beats

Consider two harmonic waves, passing a fixed point, that have slightly different frequencies. Their superposition is:

$$y = y_1 + y_2 = A\left[\cos(2\pi f_1 t) + \cos(2\pi f_2 t)\right]$$



Simplify using

$$\cos a + \cos b \equiv 2\cos\left(\frac{a-b}{2}\right)\cos\left(\frac{a+b}{2}\right)$$

A beat is heard when

$$\cos\left[2\pi t\left(\frac{f_1 - f_2}{2}\right)\right] = \pm 1$$

Two beats per cycle, so beat frequency = f_1-f_2

$$y = 2A\cos\left[2\pi t\left(\frac{f_1 - f_2}{2}\right)\right]\cos\left[2\pi t\left(\frac{f_1 + f_2}{2}\right)\right]$$

slow amplitude modulation

rapidly oscillating carrier wave

Summary

Superposition of two harmonic waves with equal amplitude & frequency but offset with a phase shift

$$y = 2A\cos\left(\frac{\phi}{2}\right)\sin\left(kx - \omega t + \frac{\phi}{2}\right)$$

Superposition of two harmonic waves of slightly different frequencies

$$y = 2A\cos\left[2\pi t\left(\frac{f_1 - f_2}{2}\right)\right]\cos\left[2\pi t\left(\frac{f_1 + f_2}{2}\right)\right]$$

The Doppler effect

The Doppler effect is most familiar in the sound of sirens from passing police cars or ambulances. The note of the siren seems to drop as the vehicle passes. This change in frequency, known as the Doppler effect, occurs when either the source of a sound, or the observer, is moving relative to the wave medium (the air).

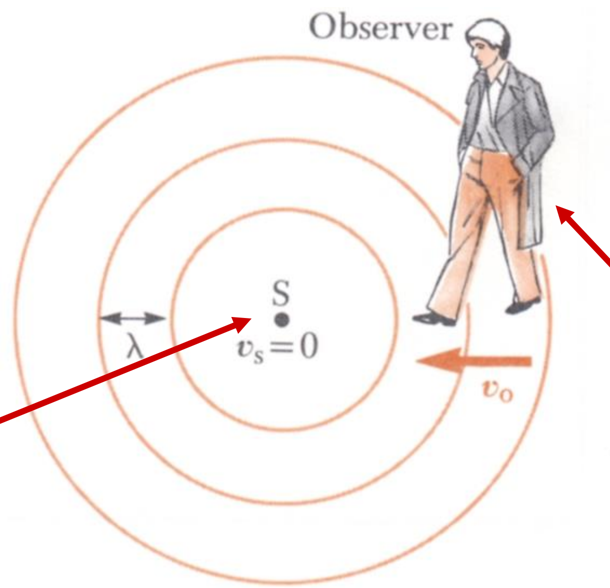
https://www.youtube.com/watch?v=p-hBCcmCUPg

We shall address only the non-relativistic Doppler effect, although the effect is also observable in light waves, where there is no wave medium, and time dilation and Lorentz contraction must be taken into account.

Observer moving **towards** stationary source

N.B. All velocities will be **relative to the medium**.

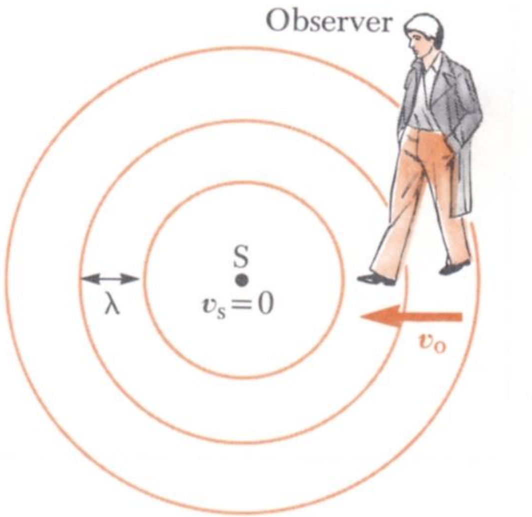
Point source S at rest ie $v_s = 0$



Observer moving **TOWARDS** S at speed v_o

frequency of source = f
velocity of sound = v
wavelength of sound = λ

frequency observed = f'



If observer O was stationary, he would detect f wave-fronts per unit time

ie: if $v_o = 0$ and $v_s = 0$ f' = f

Moving observer O travels a distance $v_o t$ in time t.

During this time O detects an additional $\frac{v_o t}{\lambda}$ wave-fronts

ie: an additional $\frac{v_o}{\lambda}$ wave-fronts per unit time.

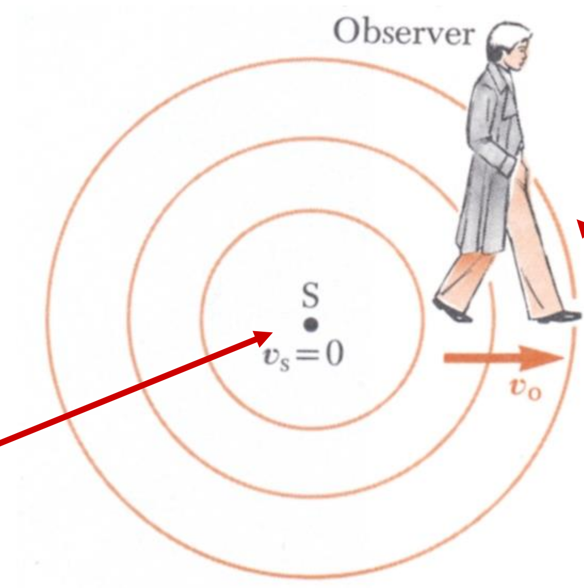
Since more wave-fronts are heard in a given time interval, a higher frequency is observed. $f' = f + \frac{v_o}{\lambda}$

Substituting $\frac{1}{\lambda} = \frac{f}{v}$ gives $f' = f + \frac{v_o}{v} f$

$$\boxed{f' = f \left[\frac{v + v_o}{v} \right]}$$

Note, $(v + v_o)$ = speed of waves relative to O (the observer).

Observer moving **away** from stationary source



Point source S at rest ie $v_s = 0$

Observer moving **AWAY FROM** S at speed v_o

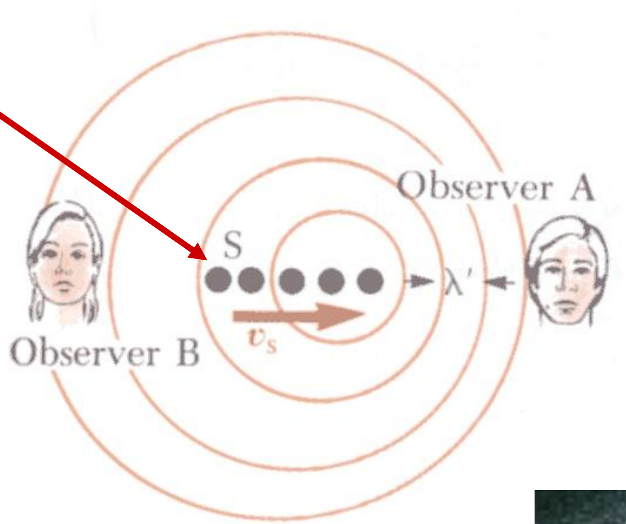
O now detects fewer wave-fronts per unit time, so the frequency is lowered.

The speed of the wave relative to O is $(v - v_o)$, and we find

$$\boxed{f' = f\left[\frac{v - v_o}{v}\right]}$$

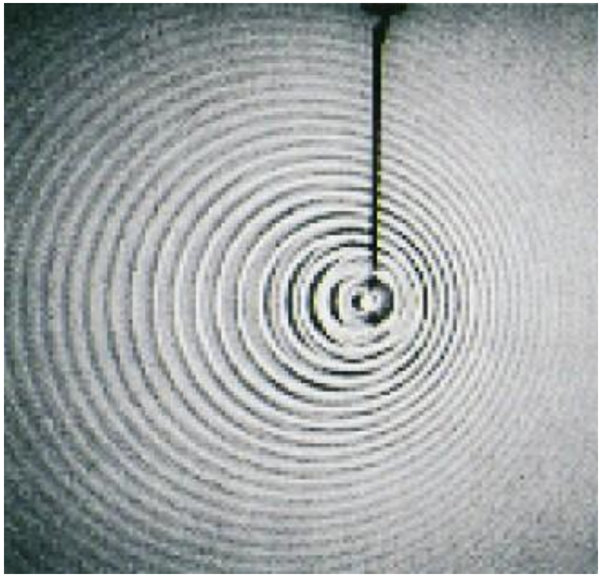
Source moving **towards** stationary observer

Point source S moving with speed v_s



Source is moving towards observer A at speed v_s.

Wave-fronts are closer together as a result of the motion of the source.



The wavelength λ' is shorter than the wavelength λ that a stationary source would emit at this frequency.

During one cycle (which lasts for period T) the source moves a distance $v_s T$ ($= v_s/f$)

In one cycle the wavelength is shortened by v_s/f

$$\lambda' = \lambda - \frac{v_s}{f} \qquad \text{but } \lambda = \frac{v}{f} \quad \& \quad \lambda' = \frac{v}{f'}$$

$$\text{So } \frac{v}{f'} = \frac{v}{f} - \frac{v_s}{f}.$$

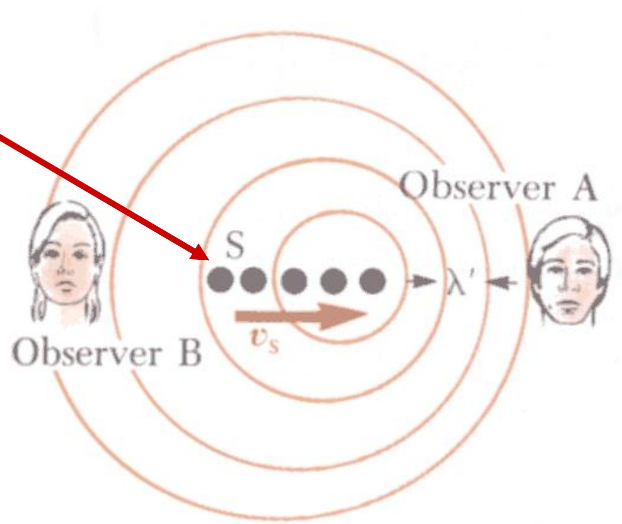
$$\boxed{\Rightarrow \quad f' = f\left[\frac{v}{v - v_s}\right]}$$

ie: Observed frequency is increased when source moves towards observer.

Note: Singular behaviour when $v_s \rightarrow v$. We'll discuss this situation later.

Source moving **away** from stationary observer

Point source S
moving with speed v_s



Source is moving away from observer B at speed v_s.

Wave-fronts are further apart, λ' is greater and B hears a decreased frequency given by

$$\boxed{f' = f\left[\frac{v}{v + v_s}\right]}$$

Summary

Frequency heard when observer is in motion

$$f' = f\left[\frac{v \pm v_o}{v}\right]$$

+ : towards source
- : away from source

Frequency heard when source is in motion

$$f' = f\left[\frac{v}{v \mp v_s}\right]$$

- : towards observer
+ : away from observer

Frequency heard when both source and observer are in motion

$$f' = f\left[\frac{v \pm v_o}{v \mp v_s}\right]$$

upper signs = towards
lower signs = away from

observer - over

observer and source moving towards each other increases frequency

ASIDE

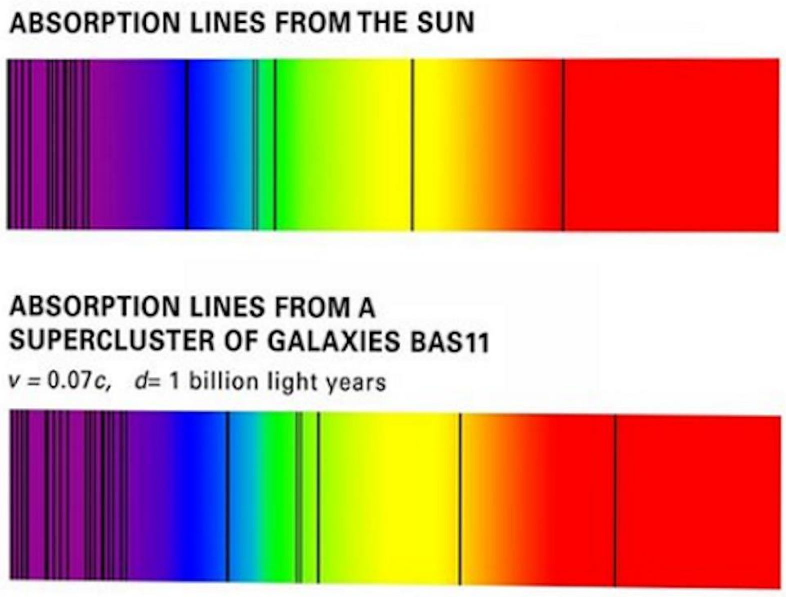
Not examined in Waves section

Doppler Effect - Red shift

The Doppler Effect also applies to light.

The light from a source moving towards the observer appear to have a higher frequency and shorter wavelength. The colour shifts towards the blue end of the spectrum.

A light source moving away from the observer has a lower frequency and longer wavelength – colour shifts red.



The full relativistic doppler equation is similar but takes into account time dilation via the Lorentz factor (not here), but as long as v is much less than c, then the equation is approximately correct.

$$z = \frac{\Delta\lambda}{\lambda_o} \approx \frac{v}{c} \quad \text{for } v \ll c$$

e.g. He absorption line 468.6 nm shifts to 499.3 nm, giving red shift of 6.55×10^{-2}, so recessional speed = $6.55\times10^{-2} \times 3\times10^{8} \approx 1.97\times10^{7}$ ms^{-1}

However, other factors also play a part, cosmic scale factor R, Hubble constant, gravitational red-shift, must be introduced as well as the Lorentz factor

Shock Waves

Consider a source moving with velocity v_s which exceeds the sound velocity v .

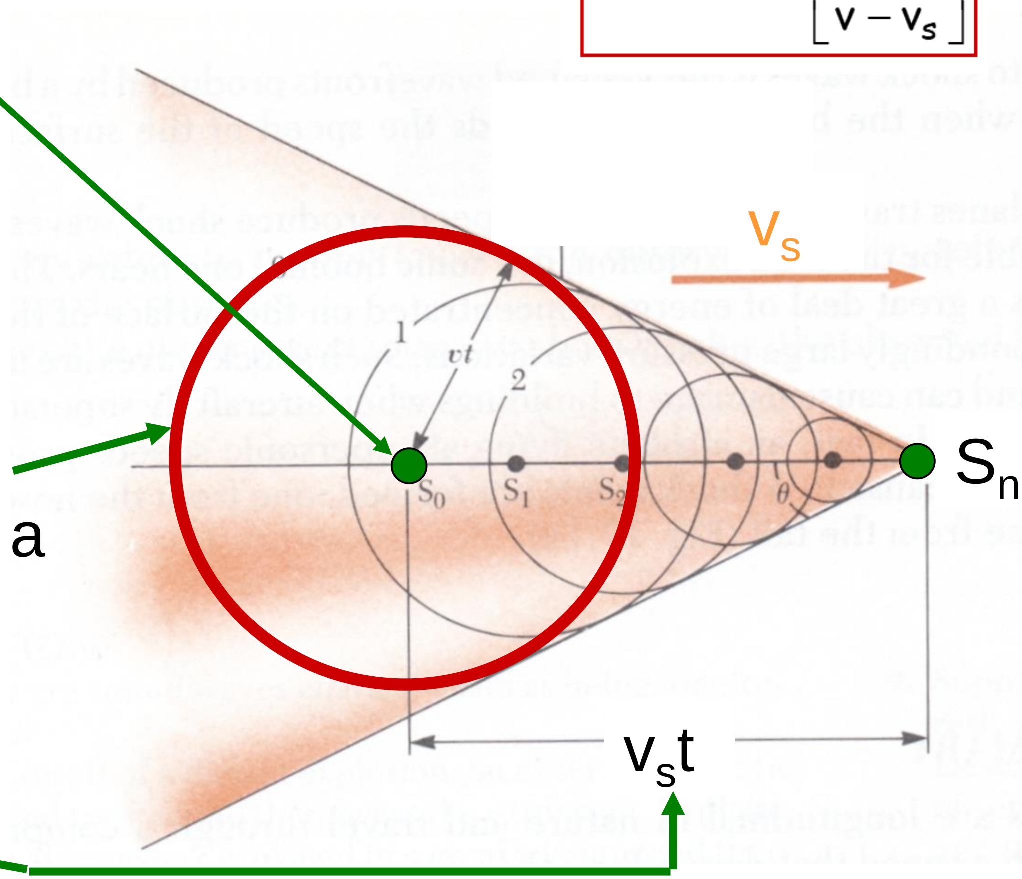
$$\Rightarrow \quad f' = f\left[\frac{v}{v - v_s}\right]$$

At $t=0$, source is at S_o

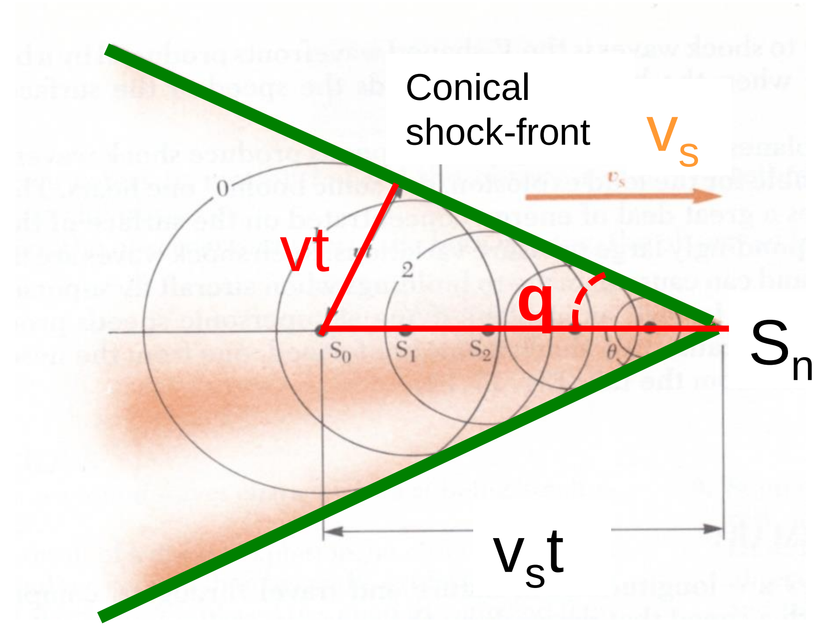
At time t, source is at S_n , & about to produce a wave-front (pressure maximum).

Meanwhile the wave-front centred on S_o has reached a radius vt.

Source has travelled distance v_st from S_o to S_n



The line (conical surface in 3D) drawn from S_n to the wave-front centred on S_o is tangential to all wave-fronts generated at intermediate times.



This surface is the **shock-front**.

The medium outside the shock-front is undisturbed by the source.

$$\sin\theta = \frac{vt}{v_s t} = \frac{v}{v_s} = \frac{1}{M}$$

where M is the Mach number

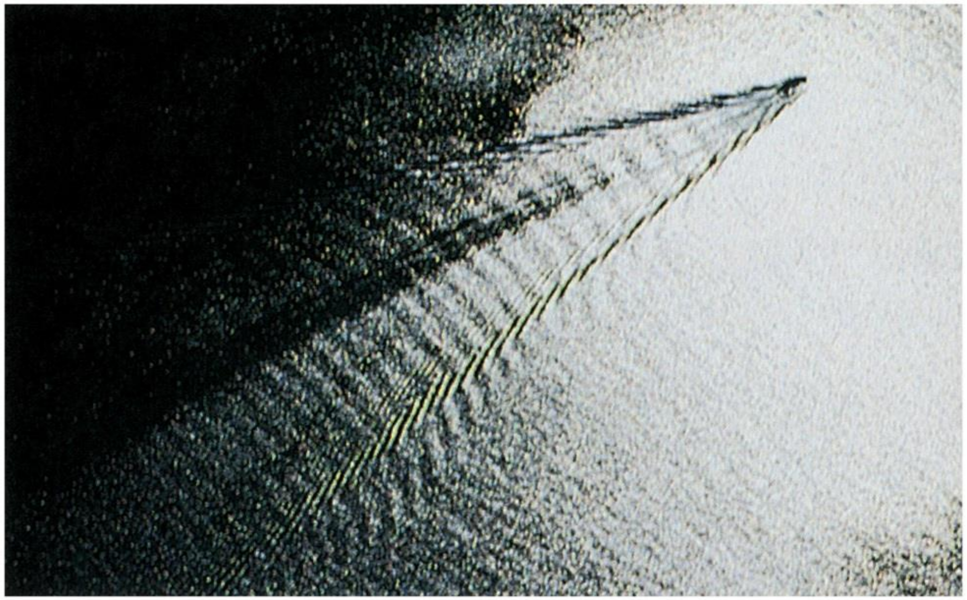
It is a **large amplitude** discontinuity in the pressure*

When the shock meets your ear, you hear a **sonic boom**.

* n.b. in examples online you often clearly see the shockwave. This is usually due to the local drop in air pressure causing water vapour to condense. On a dry day the change in air pressure and hence density causes the optical distortion seen in the picture above.

https://www.youtube.com/watch?v=6B4IVcCuIZE
https://www.youtube.com/watch?v=26Pb6FZ4uUk

Example of shockwaves: wave-fronts in water



A V-shaped bow-wave is produced when a boat's speed exceeds the speed of the surface water waves.

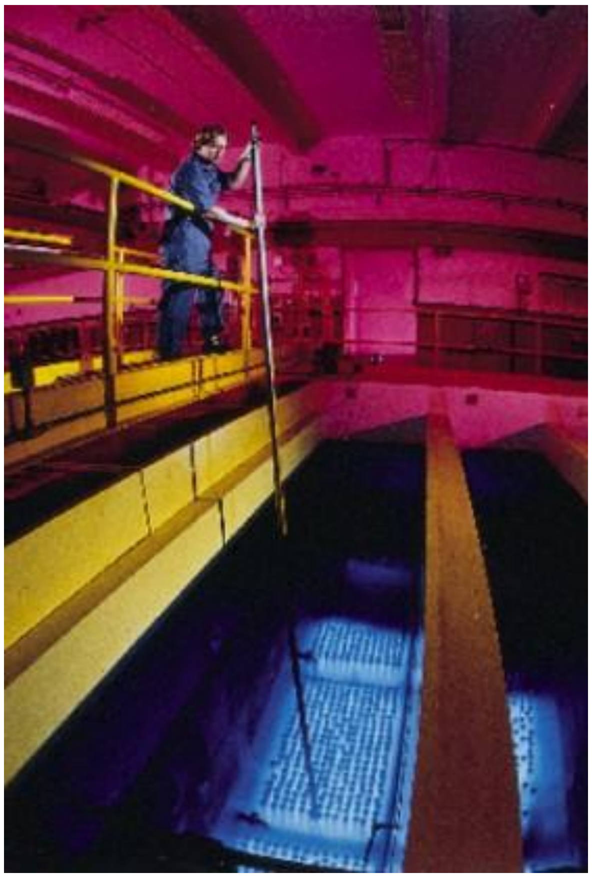
$$\sin\theta = \frac{v}{v_s}$$

v ← Wave speed in the medium (transverse water waves)

v_s ← Velocity of source (the boat)

Example of shockwaves: Cherenkov radiation

$$\sin\theta = \frac{v}{v_s}$$



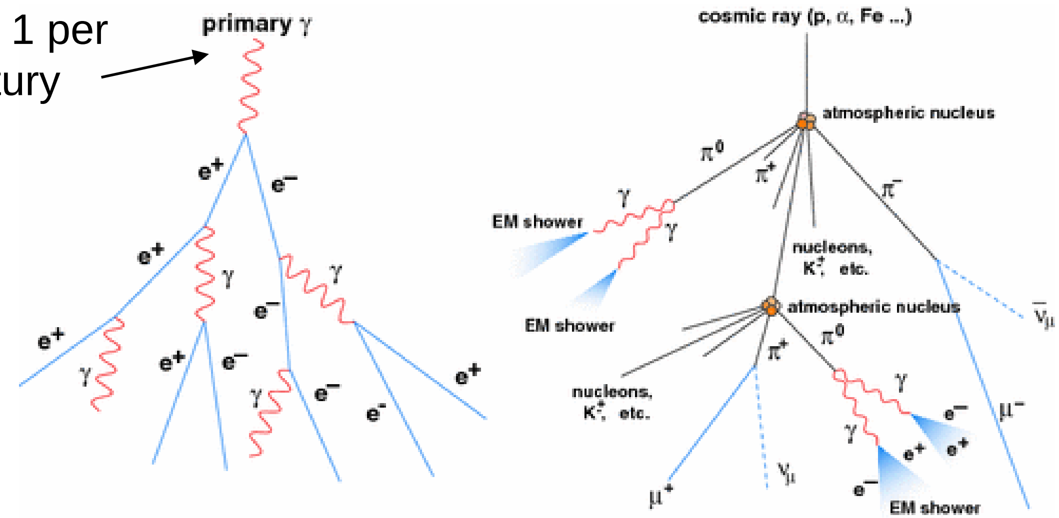
This equation also applies to a form of electromagnetic radiation called Cherenkov radiation.

When a charged particle moves in a medium, with a speed v that is greater than the speed of light **in that medium**, it emits a conical electromagnetic shock-front.

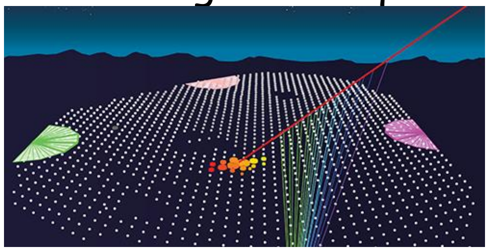
The blue glow surrounding fuel elements in a nuclear reactor is Cherenkov radiation.

Example of shockwaves: Cherenkov radiation

Frequency ≈ 1 per km^2 per century

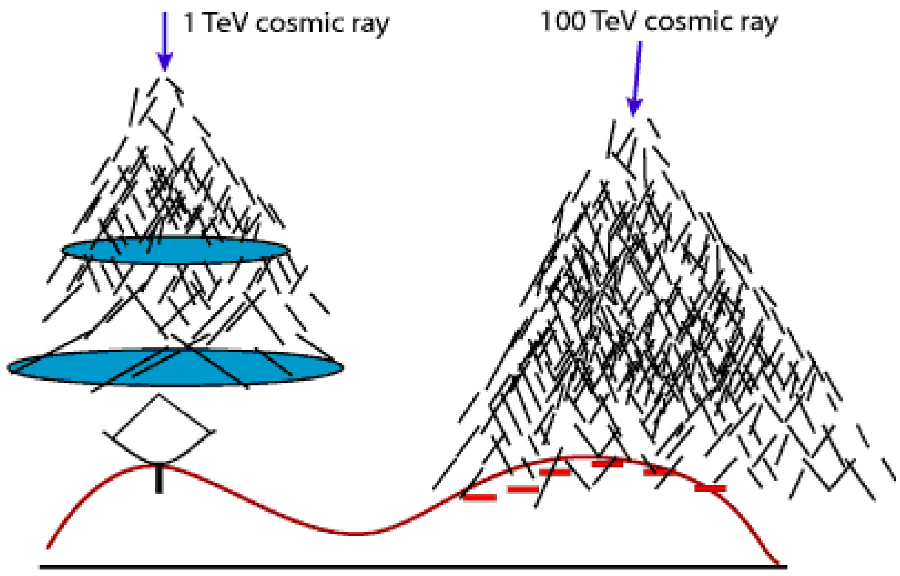


1600 detectors covering 3000 sq km



Very rare ultra-high energy (TeV) gamma photon or cosmic ray (sub-atomic particle) at near speed of light, interacting with atmosphere produces a cascade of Cherenkov radiation. Studied at the Pierre-Auger observatory (Argentina), operational since 2008. Proposed by Prof Alan Watson FRS(Uni of Leeds, School of Physics and Astronomy) in 1992.

Air shower



More complex standing waves Example 2D plate

(we will not be covering Physics of these 2D examples)

Stiff fixed plate, Boundary conditions, zero impedance, antinodes at plate edge.

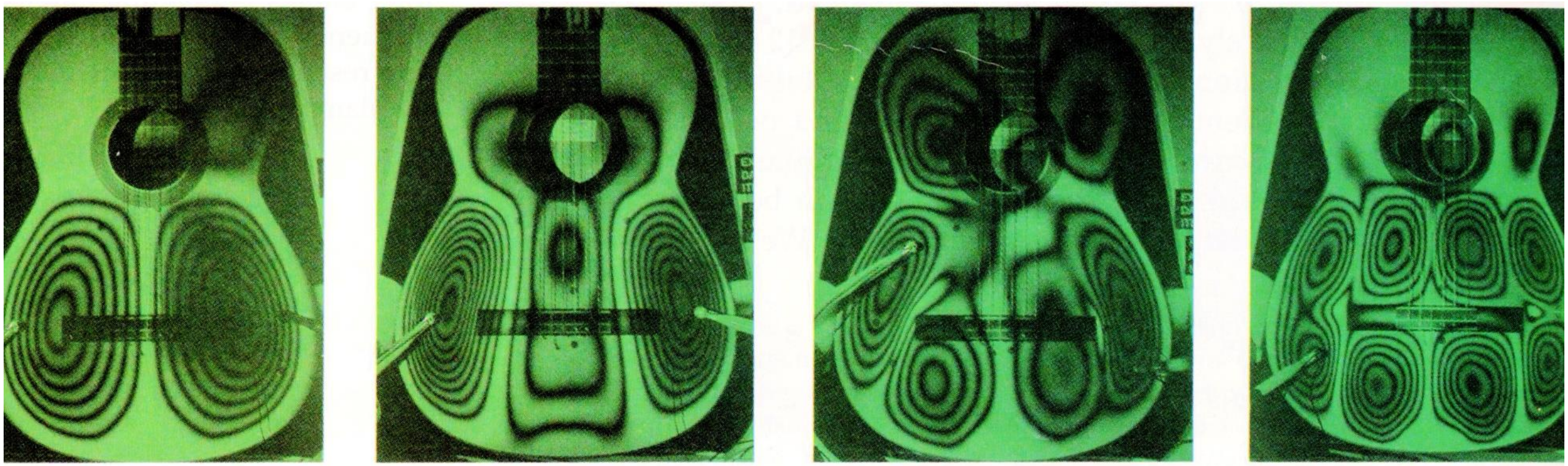
2D standing waves using salt to visualise nodes, the same idea as sound tube experiment in second year lab

https://www.youtube.com/watch?v=jhaTULO2Zkc

Shorter wavelength standing waves at higher frequencies. Position and pattern in 2D is determined by superposition of standing waves according to path length (phase difference) of waves reflecting from different points along the edges.

More complex standing waves Acoustic Guitar

Boundary conditions imposed by stiff support struts and shape of the guitar with outside edge supported by the sides

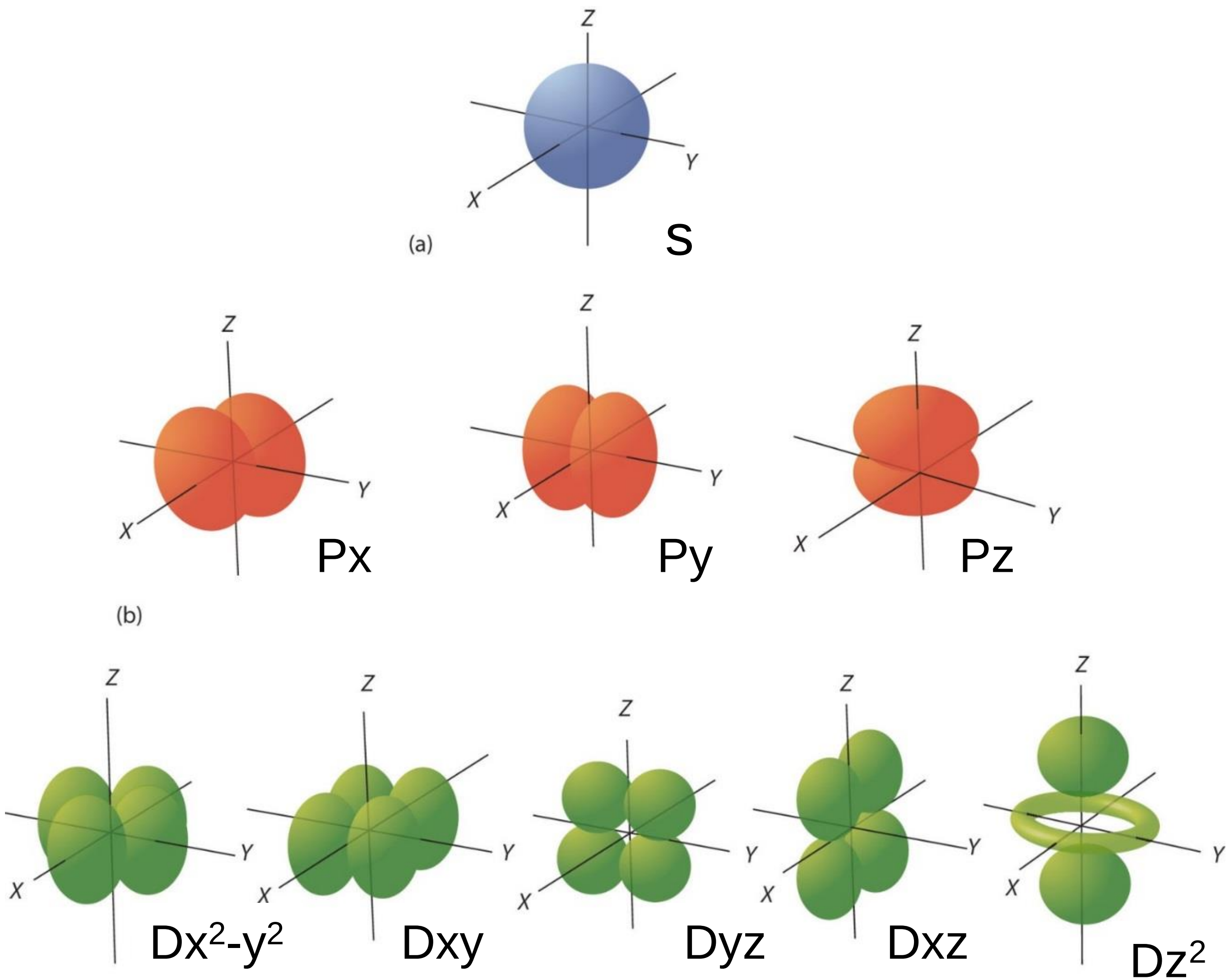


268 Hz ($Q = 52$) 553 Hz ($Q = 66$) 672 Hz ($Q = 61$) 1010 Hz ($Q = 80$)

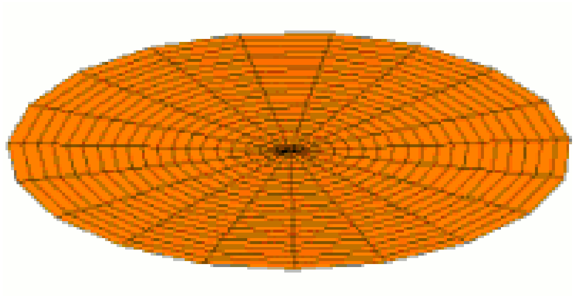
Images obtained using a laser holographic interferometer, which itself uses the interference of coherent lasers of known wavelength (here green laser), the fringes are caused by constructive and destructive interference at different path lengths. Devices like this can measure path length difference of 1nm with spatial resolution of 1um at high speed.

More complex standing waves Example 3D

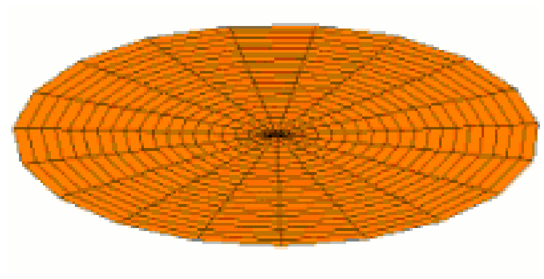
Quantization of electron energy and orbital shapes.
Boundary conditions = confinement to the atom.



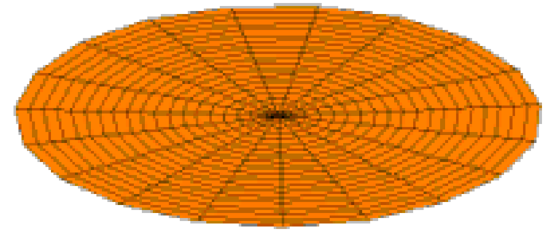
Compare with the standing waves of a drum



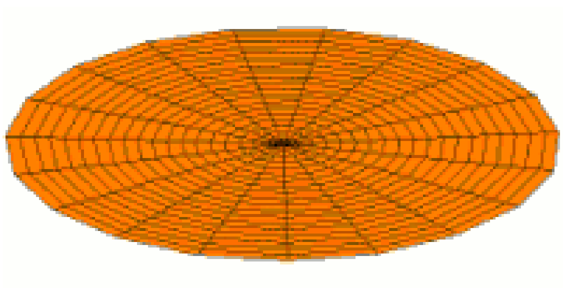
1s



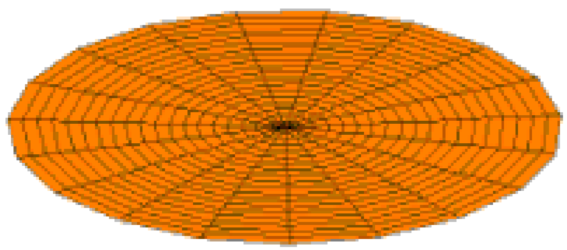
2s



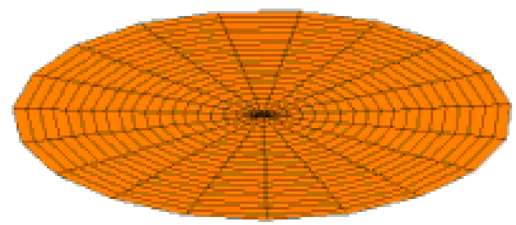
3s



2p



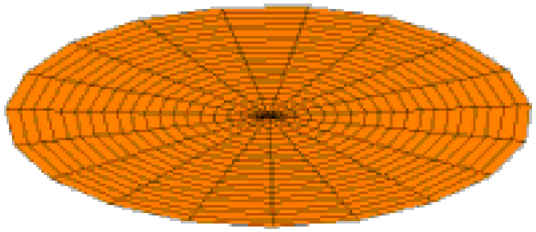
3p



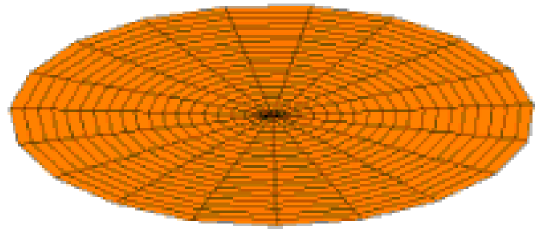
4p

Boundary condition - stretched membrane edge is fixed = node
Modes are analogous to electron orbital (labels)

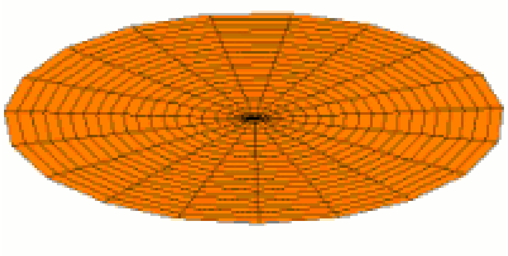
More complex standing waves: Example 2D drum



3s

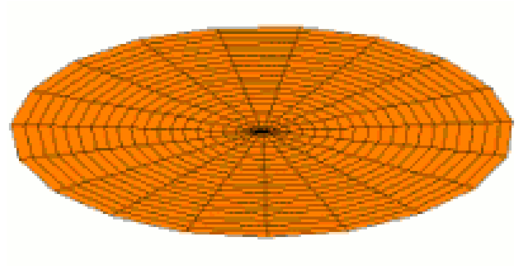


3p

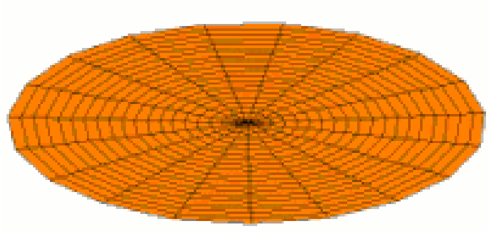


3d

Node at nucleus for all but s-orbitals.



4d



5d

Boundary condition - stretched membrane edge is fixed = node

THE END OF THIS SECTION

NEXT in Semester 2 is **OPTICS**
with
Prof Steve Evans *(current Head - School of Physics and Astronomy*