PHAS1000 – THERMAL PHYSICS

Lecture 7

Depth and Altitude

Image tinyurl.com/yxf34mdb

Overview

We will look at:

- ➢ Pressure with depth
- ➢ Pressure with altitude

using case studies to derive theory and work through examples

Images tinyurl.com/yxf34mdb

Pressure with depth (in a liquid)

How does pressure vary in a liquid?

A reduces linearly with depth

B increases linearly with depth

C increases exponentially with depth

D constant at all points in a liquid

How does pressure vary in a liquid?

A: reduces linearly with depth — 0%

B: increases linearly with depth — 0%

C: increases exponentially with depth — 0%

D: constant at all points in a liquid — 0%

How does pressure vary in a liquid?

A: reduces linearly with depth

0%

B: increases linearly with depth

0%

C: increases exponentially with depth

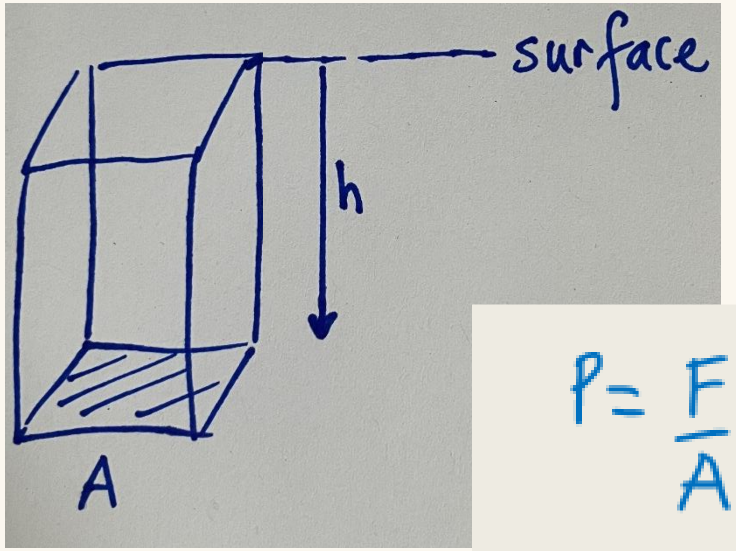
0%

D: constant at all points in a liquid

0%

Pressure with depth (in a liquid)

How deep can you dive in water and still remain safe? Use the fact that oxygen toxicity (and convulsions) sets in when breathing oxygen at a partial pressure greater than about 1.6 bar for a significant time.



$$P = \frac{F}{A} = \frac{Mg}{A} = \frac{\rho V g}{A} = \frac{\rho h A g}{A} = h\rho g$$

Density ρ is constant in a liquid (as liquid incompressible)

So pressure is linear with depth below surface

$$\boxed{P = h\rho g}$$

Pressure with depth (in a liquid)

How deep can you dive in water and still remain safe? Use the fact that oxygen toxicity (and convulsions) set in when breathing oxygen at a partial pressure greater than about 1.6 bar for a significant time.

Scuba diving:

- ❑ Pressure increases with depth under water
- ❑ Air must be supplied at same pressure as external to the body (achieved with special valve)

$$P = h\rho g$$ Pressure due to liquid column above

$$P(h) = P_{atm} + h\rho g$$ Total pressure due to liquid and atmosphere

$$PP_i = \left(\frac{n_i}{n_t}\right) P_t$$ Partial pressure depends on mole fraction

1 bar = 10^5 Pa

1 atm = 1.013 bar

Pressure with depth ANSWER

surface P_{atm}

h

$PP_{O_2} = 0.21 \times P_{atm}$ $\quad P_{ext} = P_{atm} = 1.013 \times 10^5\ Pa$

$PP_{O_2} = 0.21 \times P_{ext}$ $\quad P_{ext} = P_{atm} + h\rho g$

In this question we need h when $PP_{O_2} = 1.6$ bar

$$PP_{O_2} = 0.21 \times (P_{atm} + h\rho g)$$

$$1.6\text{ bar} = 0.21 \times (1.013 \times 10^5 + h \times 1000 \times 9.8)$$

$$\frac{1.6\text{ bar}}{0.21} - 1.013 \times 10^5 = 1000 \times 9.8 \times h$$

$$h = \frac{1}{1000 \times 9.8}\left[\frac{1.6 \times 10^5}{0.21} - 1.013 \times 10^5\right]$$

$$\boxed{h = 67\text{ m}}$$

density of water $1000\ kg\,m^{-3}$

$1\ atm = 1.013 \times 10^5\ Pa$

$1\text{ bar} = 10^5\ Pa$

Question

What would you do/change to allow safe diving even deeper?

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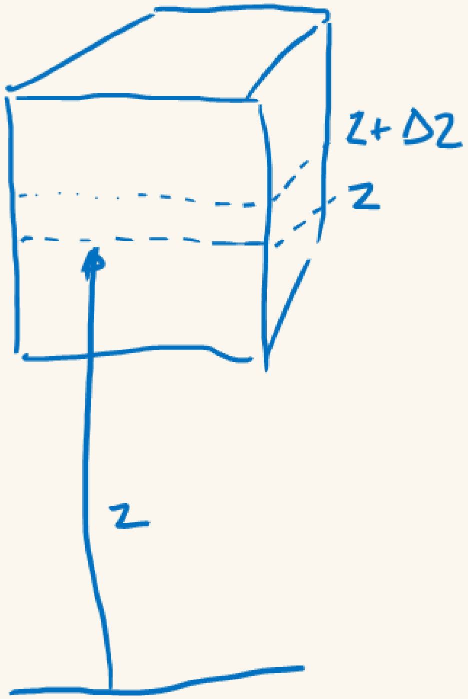
Submarine	Trimix	Control the pressure of the oxygen in the tank	Increase the density of oxygen
Hop in a submarine like the Octonauts	Increase the concentration of oxygen in the tank.	Trimix	Submarine with Decompression chamber
Higher air pressure in tank	increase internal pressure, submersible	higher percentage of oxygen in tank	Get lots of oxygen tanks
Helium mixed with oxygen	Trimix	Use a sealed submersible. Preferably not like how Ocean gate	+4 more messages

Pressure with altitude

What is the highest altitude at which you can survive without a pressurised suit or oxygen mask? Use the fact that hypoxia (and unconsciousness) sets in rapidly when breathing oxygen at a partial pressure less than 0.12 bar.

Felix Baumgartner set the world record for skydiving an estimated 24 miles (39 km), reaching a speed of 834 mph, (Mach 1.24) on 14 October 2012, and became the first person to break the sound barrier (without vehicular power) on his descent.

Pressure with altitude - answer (1)



$P_z > P_{z+\Delta z}$ due to weight of atmosphere in slice between z and z+ Δz

$$P_{z+\Delta z} - P_z = \Delta P = -\rho g \Delta z \qquad (\text{from } P = h\rho g)$$

$$\frac{\Delta P}{\Delta z} = -\rho g$$

Pressure with altitude – answer (2)

$$\rho = \frac{\text{mass}}{\text{volume}} = \frac{nM}{\left(\frac{nRT}{P}\right)} = \frac{M}{RT} \times P$$

notice density depends on pressure

we will assume temp is constant in atmosphere

$$\frac{\Delta P}{\Delta z} = -\frac{Mg}{RT} P \qquad \frac{dP}{dz} = -\frac{Mg}{RT} P \qquad \frac{dP}{P} = -\frac{Mg\,dz}{RT}$$

$$\int \frac{dP}{P} = -\frac{Mg}{RT} \int dz$$

When $z = 0$ $P = P_0 = 1$ atm

$$\ln P = -\frac{Mg}{RT} z + c$$

$$P = e^{-\frac{Mgz}{RT}} \times e^{c}$$

$P_0 = P_{atm} = e^{c}$

$$\boxed{P = P_0\, e^{-\frac{Mgz}{RT}}}$$

Pressure with altitude – answer (3)

$P = P_0 e^{-\frac{Mgz}{RT}}$

$P_0 = P_{atm} = 1\,atm = 1.013 \times 10^5 Pa$

$T = 273\,K$

$M_{N_2} = 28g$

$M_{O_2} = 32g$

$M_{air} = 29g$

$PP_{O_2} = \frac{n_i}{n_t} \times P$

$PP_{O_2} = 0.21 \times P_0 e^{-\frac{Mgz}{RT}}$

solve for z when $PP_{O_2} = 0.12\,bar$

$\ln\left(\frac{PP_{O_2}}{0.21 P_0}\right) = -\frac{Mgz}{RT}$

$z = -\frac{RT}{Mg} \ln\left[\frac{PP_{O_2\,limit}}{0.21\ P_0}\right]$

$= -\frac{8.31 \times 273}{29 \times 10^{-3} \times 9.8} \ln\left(\frac{0.12 \times 10^5}{0.21 \times 1.013 \times 10^5}\right) = 4570\,m$

$\sim 4.5\,km$

~ 19 km

Patm = 0.063 bar

Armstrong Limit: All bodily fluids boil, as water has SVP of 0.063 bar at 37°C. No survival without pressurised suit !!!

Need pressurised suit to deliver oxygen at greater than external pressure

~ 15 km

Patm = 0.12 bar

PP_{O_2} = 0.03 bar

Need **100% oxygen** at external pressure.

At PP_{O_2} < 0.05 bar Instant blackout and death !

Need a mask to deliver more than 21% oxygen at external pressure

~ 5 km

Patm = 0.57 bar

PP_{O_2} = 0.12 bar

Breathe air

Commercial airliners are pressurised to equivalent of 2.4 km altitude.

ground

Patm = 1 bar

PP_{O_2} = 0.21 bar

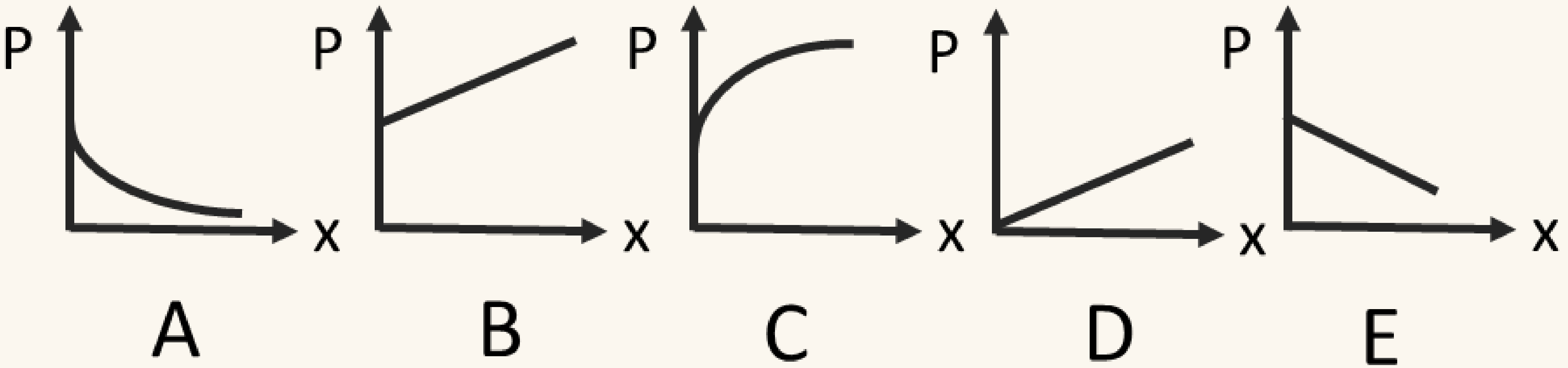
Note: Altitudes only approximate. If temperature drop with altitude is included, all distances are reduced.

Question

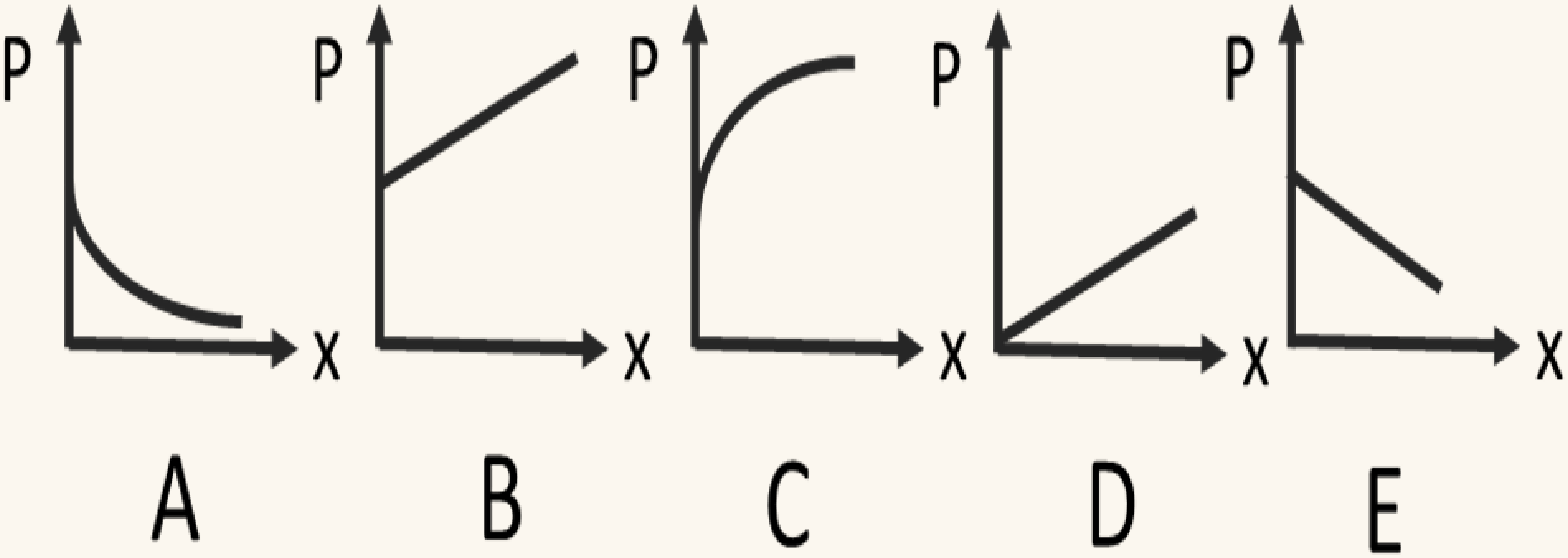
Chose the correct graph from below for each of the following:

a) Variation of pressure with depth (x) in a lake

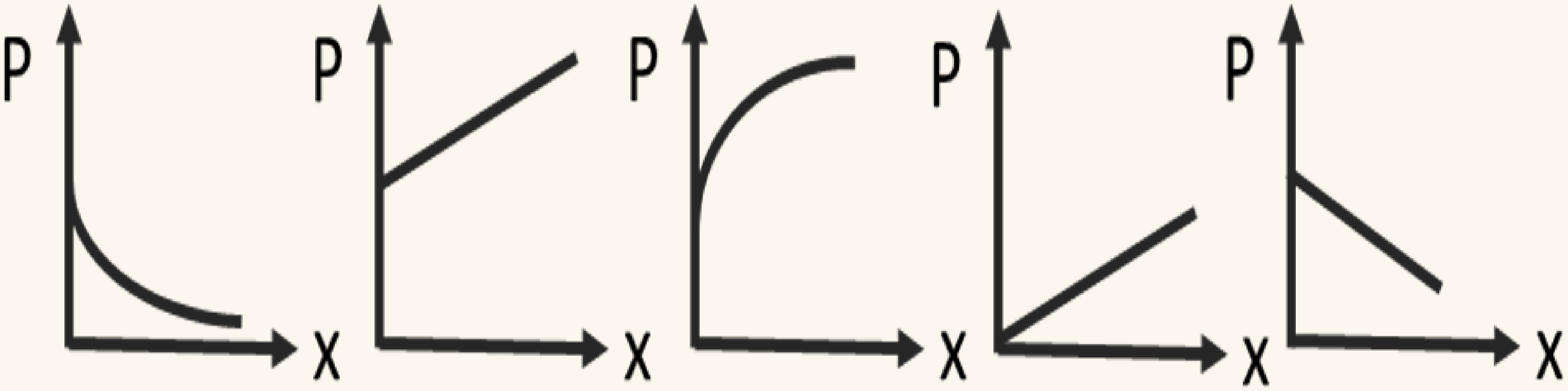
b) Variation of pressure with altitude (x) in air



Which graph shows variation of pressure with depth (x) in a lake? Touch the correct graph.

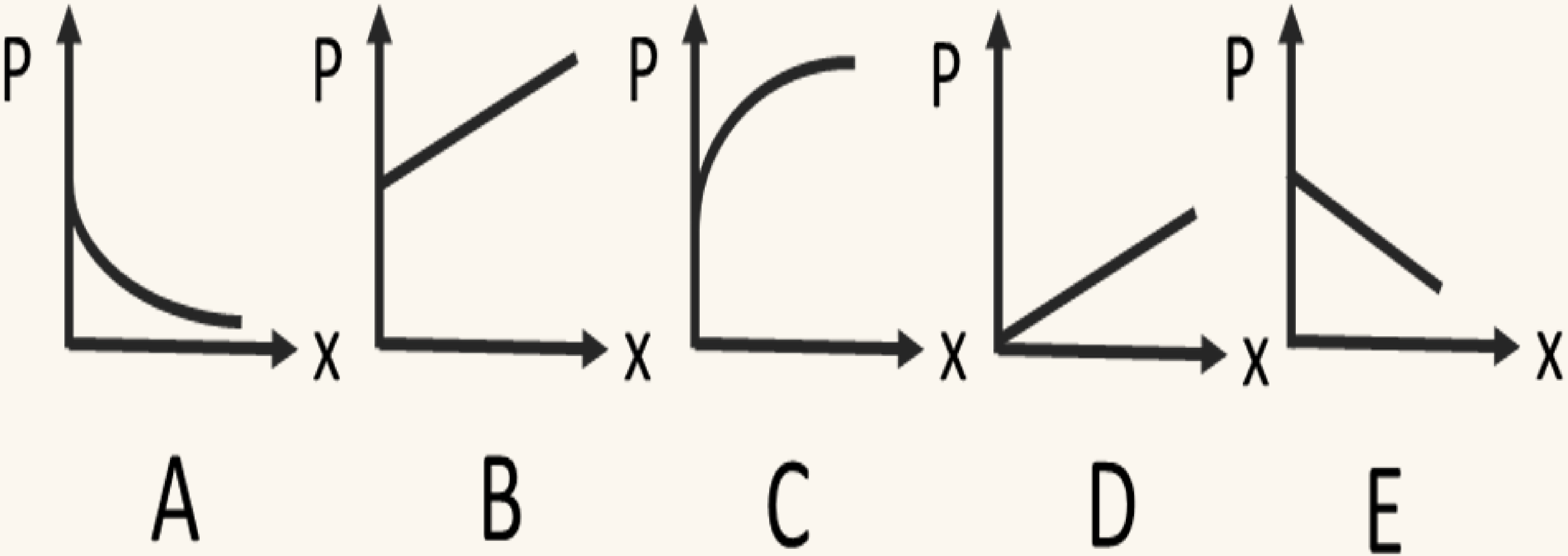


Which graph shows variation of pressure with depth (x) in a lake? Touch the correct graph.

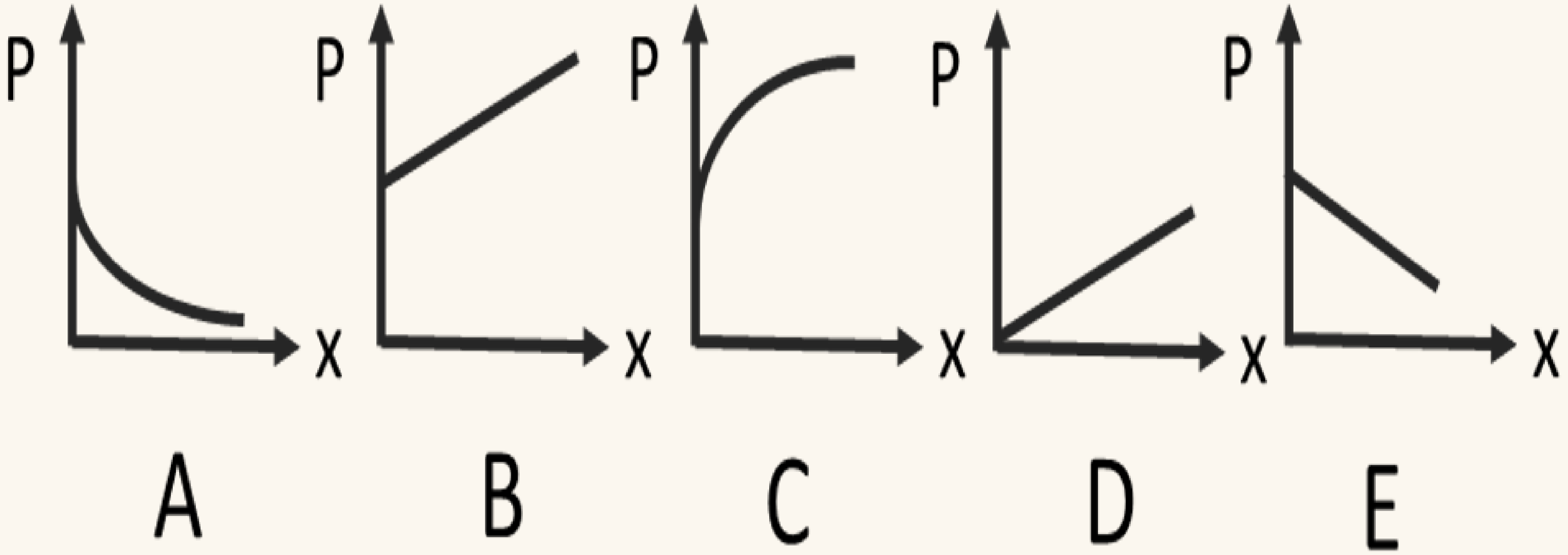


A B C D E

Touch the graph that shows variation of pressure with altitude (x) in air.



Touch the graph that shows variation of pressure with altitude (x) in air.



Question

What is the density of air at altitude of 10 km?

Take sea level temperature to be 15 °C. Temp reduces at linear rate of 6.5° per km.

Consider variations in both pressure, and temperature.

Summary

Pressure with depth
in liquid

Constant density

$$P = h\rho g$$

$$P(h) = P_{atm} + h\rho g$$

Pressure with altitude
in atmosphere

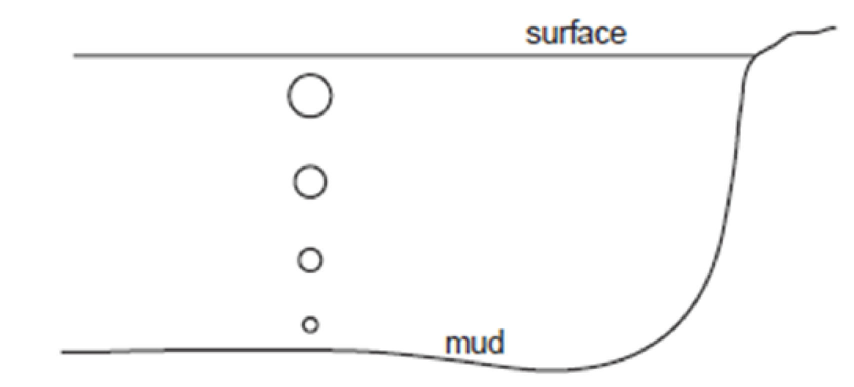
Density reduces with altitude

$$P = P_0 e^{-\frac{Mg}{RT}z}$$

Practice Questions

Question 1

A spherical bubble rises from the bottom of a lake. If the temperature is uniform and the bubble doubles its volume by the time it reaches the surface, how deep is the lake?



Question 2

Calculate the height at which the atmospheric pressure is reduced to half its value at sea level value, for an atmosphere consisting solely of nitrogen, at uniform temp of 300K.

Question 3

In reality the temperature drops off with altitude, at about 6.5°C per km.

(a) Would this increase or reduce the height calculated for the 0.12 bar oxygen limit in our derivation above?

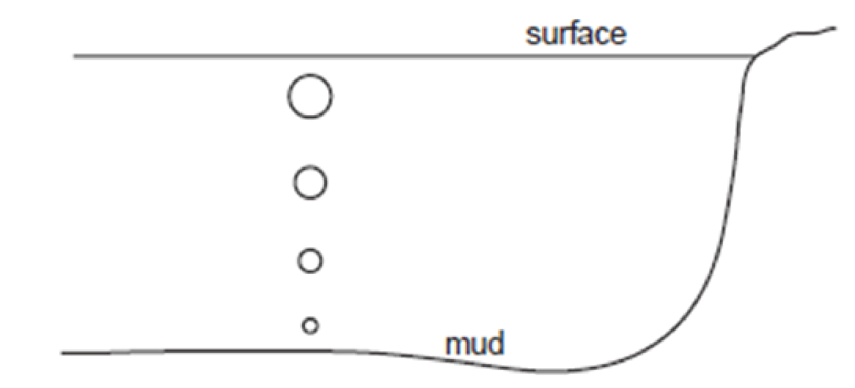
(b) Amend our equation for z (written below) to take into account this temperature drop.

$$z = -\frac{RT}{Mg} ln\left(\frac{PP_{limit}}{PP_{ground}}\right)$$

(c) How different is the calculated altitude when taking temperature into account?

ANSWERS

Answer 1



At the surface of the lake, pressure = P_{atm}

At the bottom of the lake, pressure = $P_{atm} + h\rho g$

Assume bubble is an ideal gas ∴ $PV = nRT$

Assume temp is constant (given in question) ∴ $P_b V_b = P_s V_s$

where b = bottom of lake

s = top of lake

We know $V_s = 2V_b$

∴ from $P_b V_b = P_s V_s$

$$(P_{atm} + h\rho g)V_b = P_{atm} \times 2V_b$$

$$P_{atm} + h\rho g = 2\,P_{atm}$$

$$h\rho g = 1\,P_{atm}$$

$$h = \frac{P_{atm}}{\rho g} = \frac{1.013\times10^5}{1000\times9.8} = \boxed{10.3\,m}$$

(density of water = $1000\ kg/m^3$)

Answer 2

Pressure with altitude in atmosphere $P = P_0 e^{-\frac{Mgz}{RT}}$

Question asks us to solve for z when $P = \frac{P_0}{2}$

ie $\frac{\cancel{P_0}}{2} = \cancel{P_0} e^{-\frac{Mgz}{RT}}$

$$\frac{1}{2} = e^{-\frac{Mgz}{RT}}$$

$$\ln\left(\frac{1}{2}\right) = -\frac{Mgz}{RT}$$

$-0.693 = -\frac{Mgz}{RT}$ or $z = \frac{0.693\,RT}{Mg}$

$T = 300K$

$M = 28g/mol$ (all nitrogen)

so $z = \frac{0.693 \times 8.31 \times 300}{28 \times 10^{-3} \times 9.8} = 6296\,m$

$\sim \boxed{6.3\,km}$

Answer 3

(a) if T reduced, then z reduced

(b) $$z = -\frac{RT}{Mg} \ln\left(\frac{PP_{limit}}{PP_{ground}}\right)$$

remove −ve sign by inverting log ratio

$$z = \frac{RT}{Mg} \ln\left(\frac{PP_{ground}}{PP_{limit}}\right)$$

insert $T = T_{ground} - \frac{6.5}{1000} z$ $\qquad T = \left(T_{ground} - 6.5\times10^{-3} z\right)$

thus full eqn becomes

$$z = \frac{R}{Mg}\left(T_{ground} - 6.5\times10^{-3} z\right) \ln\left(\frac{PP_{ground}}{PP_{limit}}\right)$$

collecting together the terms in z

$$z + 6.5\times10^{-3} \frac{R}{Mg} \ln\left(\frac{PP_{ground}}{PP_{limit}}\right) z = \frac{RT_{ground}}{Mg} \ln\left(\frac{PP_{ground}}{PP_{limit}}\right)$$

$$z\left(1 + 6.5\times10^{-3} \frac{R}{Mg} \ln\left(\frac{PP_{ground}}{PP_{limit}}\right)\right) = \frac{RT_{ground}}{Mg} \ln\left(\frac{PP_{ground}}{PP_{limit}}\right)$$

$$z = \frac{\frac{RT_{ground}}{Mg} \ln\left(\frac{PP_{ground}}{PP_{limit}}\right)}{\left[1 + 6.5\times10^{-3} \frac{R}{Mg} \ln\left(\frac{PP_{ground}}{PP_{limit}}\right)\right]}$$

solving for z at $PP_{limit} = 0.12$ bar

$z = \frac{4570}{1.11}$ → as found before

→ new denominator

$z = 4117$ m

reduction by almost 500m