UNIVERSITY OF LEEDS

Mechanics 1

Session 4 – Friction & Forces

DR BEN HANSON

1

Last Lecture

Newton's Laws of Motion

We Learned:

- The conceptual importance of Newton's 3 Laws of Motion
- Real-world examples of motion for which each of these laws applies
- How to mathematically analyse multiple forces acting simultaneously

You should be able to:

- Describe each of Newton's 3 Laws of Motion, with examples to illustrate
- Draw force diagrams for various physical systems
- Mathematically "resolve" force vectors into perpendicular directions (we'll have lot's more practice at this)

This Lecture

Friction & Forces

We will:

- Learn how to resolve forces into different directions
- Learn how to determine the net force vector on an object and thus, its acceleration
- Learn about the microscopic source of friction
- Learn about the two different types of friction (static friction and kinetic friction)

You will be able to:

- Calculate the acceleration vector of an object by determining the net force acting upon it
- Calculate the frictional forces experienced by an object
- Calculate the friction constants of surfaces based on their motion

Resolving Forces

Mechanical Equilibrium

Resolving Forces

Mechanical Equilibrium

Mechanical equilibrium is when the net force <u>vector</u> on an object is zero (i.e. zero in every direction)

$$\underline{F}_{Net} = \sum_i \underline{F}_i = \underline{0}$$

As such, the object may either be moving at a constant velocity or not moving at all (i.e. a constant velocity $\underline{v} = \underline{0}$)

Task 1

Resolving Forces

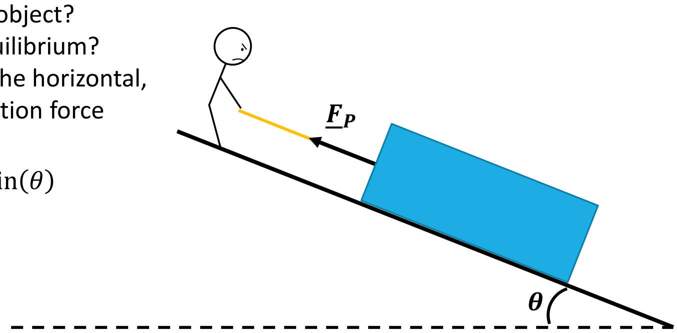
Task 1

Resolving Forces

Scenario: A box is at being pulled uphill by someone. They are exerting a constant force $\underline{F}_p$ on the box via a rope. The box is not moving at all.

Tasks:

1. What forces are acting on this object?
2. Is this object in mechanical equilibrium?
3. If the ramp is at an angle θ to the horizontal, derive an equation for the reaction force magnitude, $|\underline{R}|$.
4. Show that $|\underline{\boldsymbol{F}}_{\boldsymbol{p}}| = |\underline{\boldsymbol{F}}_{\boldsymbol{f}}| + mg\sin(\theta)$



Resolving Forces

In Depth

Resolving Forces

What's the Use?

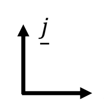
Why do we resolve forces parallel (and perpendicular) to the ramp?

Simply because it's mathematically easier. Let's have a look.

Resolving Forces

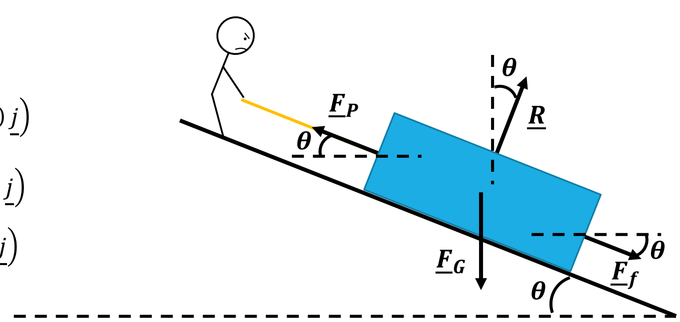
In x and y

Calculate net force, $\underline{F}_{Net} = \sum_i \underline{F}_i$



$$\underline{F}_{Net} = \underline{F}_G + \underline{F}_P + \underline{F}_f + \underline{R}$$

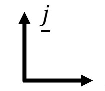
$$\underline{F}_{Net} = -mg\underline{j} + \left(-|\underline{F}_P|\cos(\theta)\underline{i} + |\underline{F}_P|\sin(\theta)\,\underline{j}\right)$$
$$+ \left(|\underline{F}_f|\cos(\theta)\,\underline{i} - |\underline{F}_f|\sin(\theta)\,\underline{j}\right)$$
$$+ \left(|\underline{R}|\sin(\theta)\,\underline{i} + |\underline{R}|\cos(\theta)\,\underline{j}\right)$$



Resolving Forces

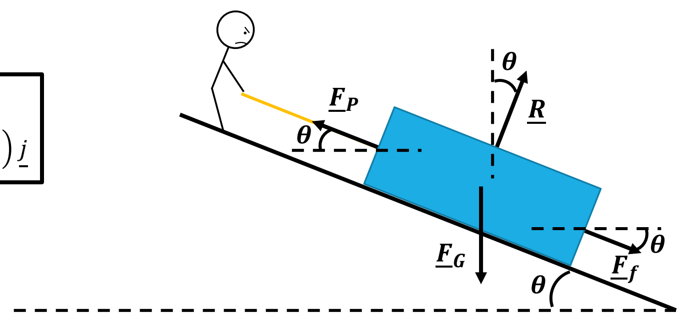
In x and y

Calculate net force, $\underline{F}_{Net} = \sum_i \underline{F}_i$



$$\underline{F}_{Net} = \underline{F}_G + \underline{F}_P + \underline{F}_f + \underline{R}$$

$$\boxed{\begin{aligned}\underline{F}_{Net} = &\left(\left(|\underline{F}_f| - |\underline{F}_p|\right)\cos(\theta) + \underline{R}\sin(\theta)\right)\underline{i} \\ &+ \left(\left(|\underline{F}_p| - |\underline{F}_f|\right)\sin(\theta) + \underline{R}\cos(\theta) - mg\right)\underline{j}\end{aligned}}$$

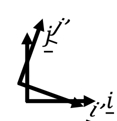


Complicated, load of angles, difficult to manipulate and solve ☹

Resolving Forces

In x' and y'

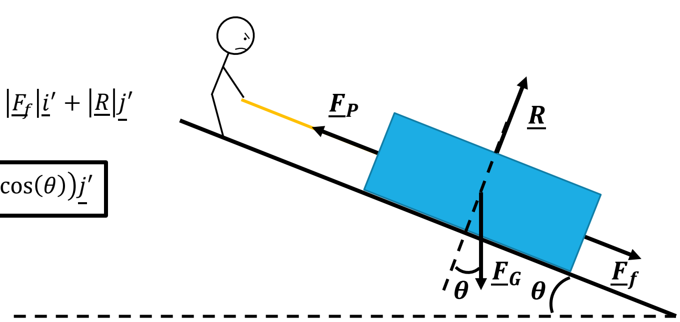
Calculate net force, $\underline{F}_{Net} = \sum_i \underline{F}_i$



$$\underline{F}_{Net} = \underline{F}_G + \underline{F}_P + \underline{F}_f + \underline{R}$$

$$\underline{F}_{Net} = \left(mg\sin(\theta)\,\underline{i}' - mg\cos(\theta)\underline{j}'\right) \; -|\underline{F}_P|\underline{i}' + |\underline{F}_f|\underline{i}' + |\underline{R}|\underline{j}'$$

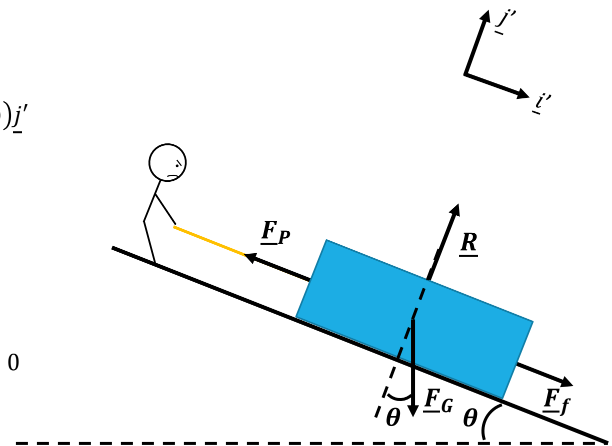
$$\boxed{\underline{F}_{Net} = \left(|\underline{F}_f| + mg\sin(\theta) - |\underline{F}_p|\right)\underline{i}' + \left(|\underline{R}| - mg\cos(\theta)\right)\underline{j}'}$$



Simpler, minimal angles, easier to manipulate and solve ☺

Resolving Forces

Mechanical Equilibrium



Calculate net force, $\underline{F}_{Net} = \sum_i \underline{F}_i$

$$\underline{F}_{Net} = (|\underline{F}_f| + mg\sin(\theta) - |\underline{F}_p|)\underline{i}' + (|\underline{R}| - mg\cos(\theta))\underline{j}'$$

$\underline{F}_{Net} = \underline{0}$,

$\underline{j}'$direction, $|\underline{R}| - mg\cos(\theta) = 0$

$$\boxed{|\underline{R}| = mg\cos(\theta)}$$

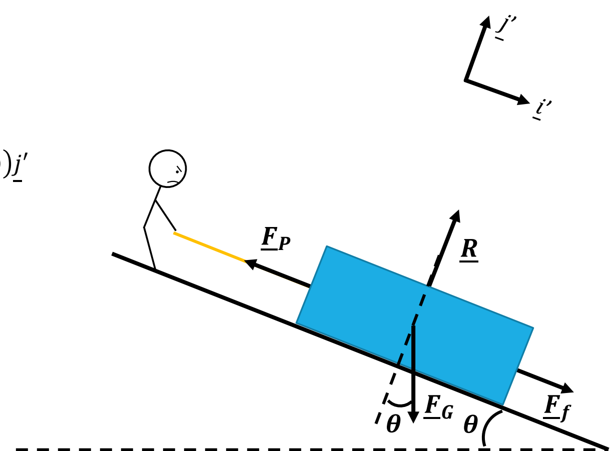
$\underline{i}'$direction, $|\underline{F}_f| + mg\sin(\theta) - |\underline{F}_p| = 0$

$$\boxed{|\underline{F}_p| = |\underline{F}_f| + mg\sin(\theta)}$$

MECHANICS 1 – FRICTION & FORCES

Resolving Forces

Mechanical Equilibrium



Calculate net force, $\underline{F}_{Net} = \sum_i \underline{F}_i$

$$\underline{F}_{Net} = (|\underline{F}_f| + mg\sin(\theta) - |\underline{F}_p|)\underline{i}' + (|\underline{R}| - mg\cos(\theta))\underline{j}'$$

$$\underline{F}_{Net} = \Big((|\underline{F}_f| - |\underline{F}_p|)\cos(\theta) + \underline{R}\sin(\theta)\Big)\underline{i} + \Big((|\underline{F}_p| - |\underline{F}_f|)\sin(\theta) + \underline{R}\cos(\theta) - mg\Big)\underline{j}$$

Key point: Both representations are exactly the same object: $\underline{F}_{Net}$. In other words, **they are equivalent to one another!**

Resolving Forces

Not Mechanical Equilibrium

Resolving Forces

Not Mechanical Equilibrium

Calculate net force, $\underline{F}_{Net} = \sum_i \underline{F}_i$

$\underline{F}_{Net} = \left(|\underline{F}_f| + mg\sin(\theta) - |\underline{F}_p|\right)\underline{i}' + \left(|\underline{R}| - mg\cos(\theta)\right)\underline{j}'$

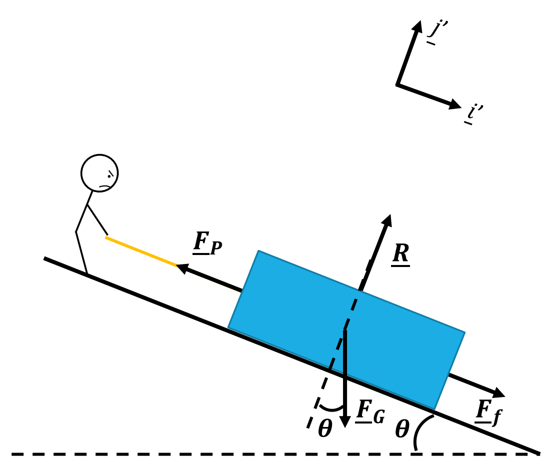
$\underline{F}_{Net} = m\underline{a} = (-ma)\underline{i}',$

$\underline{j}'$direction, $|\underline{R}| - mg\cos(\theta) = 0$

$$\boxed{|\underline{R}| = mg\cos(\theta)}$$

$\underline{i}'$direction, $|\underline{F}_f| + mg\sin(\theta) - |\underline{F}_p| = -ma$

$$\boxed{ma = |\underline{F}_p| - \left(|\underline{F}_f| + mg\sin(\theta)\right)}$$



Resolving Forces

What's the Use?

Resolve forces into whatever directions are easiest!

If there is a direction in which there is no acceleration…use that direction for simplicity!

Task 2

Moving Under Force

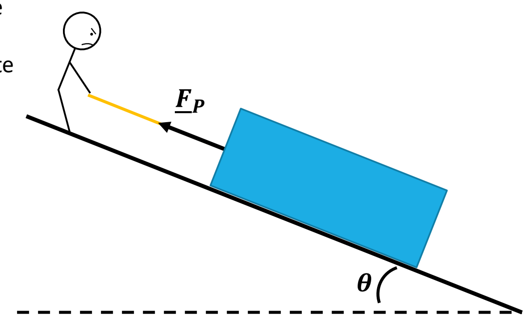
Task 2

Moving Under Force

Scenario: A $50kg$ box is at being pulled uphill by someone. They are exerting a constant force $\underline{F}_p$ on the box via a rope. The box is moving at a constant speed v up the hill.

Tasks:

1. If the reaction force is $400N$, calculate θ, the angle of the slope to the horizontal.
2. If the frictional force against the surface is $280N$, calculate the magnitude of the pulling force $|\underline{F}_P|$
   Hint: Think carefully about this one!
3. Calculate the components of $\underline{F}_P$ in the vertical and horizontal directions.



Task 3

Still Moving Under Force

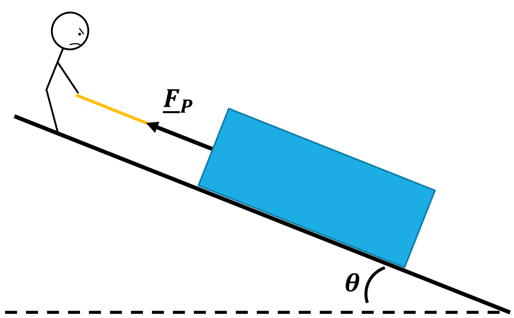
Task 3

Still Moving Under Force

Scenario: A $50kg$ box is still being pulled uphill by someone. The hill is 35.36^o to the horizontal. They increase the force $|\underline{F_P}|$ they are exerting on the box via a rope, such that $F_p = 600N$. The frictional force $\underline{F_f}$ remains the same, such that $|\underline{F_f}| = 280N$. The box now begins to move up the hill.

Tasks:

1. Calculate the time taken to move a distance of $20m$ up the hill



Friction

And Two Different Types

Friction

What is it?

Friction is the force that resists motion between two sliding surfaces

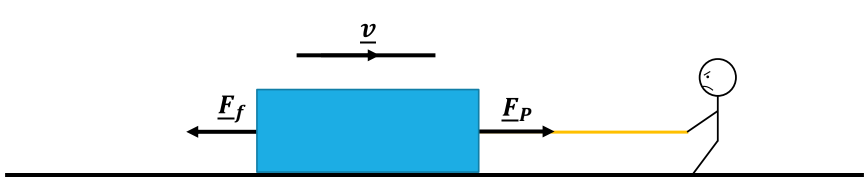
It always acts in the direction opposite of motion (i.e. the opposite direction of the *velocity* vector)

Friction

What direction?

Scenario: A box is at being pulled by someone. They are exerting a constant force $\underline{F}_p$ on the box via a rope. The box is moving at a constant speed in the horizontal direction.

Question: In which direction is friction acting?

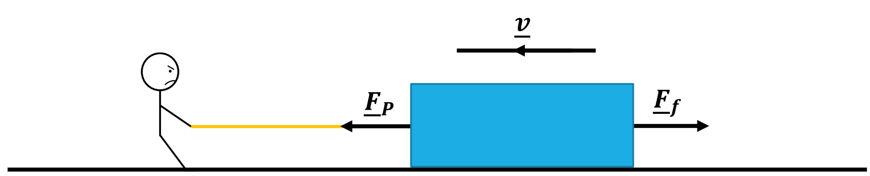


Friction

What direction?

Scenario: A box is at being pulled by someone. They are exerting a constant force $\underline{F}_p$ on the box via a rope. The box is moving at a constant speed in the horizontal direction.

Question: In which direction is friction acting?

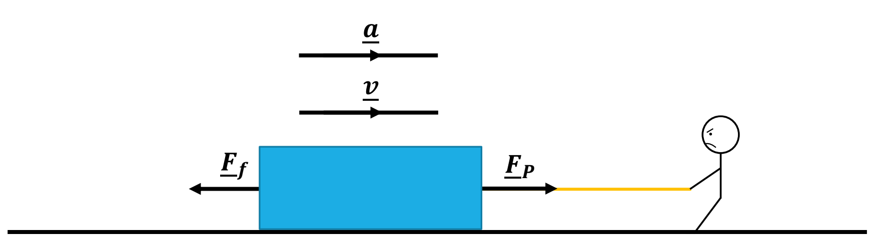


Friction

What direction?

Scenario: A box is at being pulled by someone. They are exerting a constant force $\underline{F}_p$ on the box via a rope. The box is accelerating in the horizontal direction, and the box is currently moving in that direction too.

Question: In which direction is friction acting?

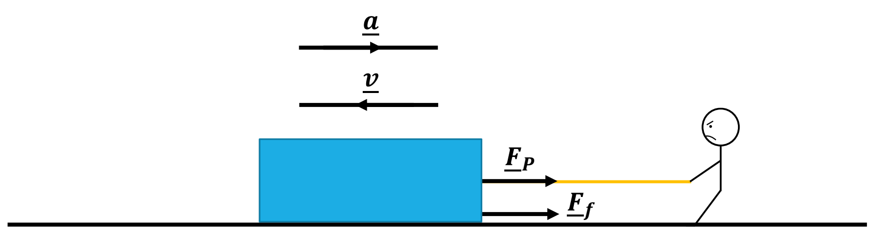


Friction

What direction?

Scenario: A box is at being pulled by someone. They are exerting a constant force $\underline{F}_p$ on the box via a rope. The box is accelerating in the horizontal direction, but currently, the box is moving in the other direction (i.e. they are slowing it down)!

Question: In which direction is friction acting?
Hint: Think carefully about this one



Friction

What direction?

Scenario: A box is on a hill. No one is pulling anything. The box is not moving at all.

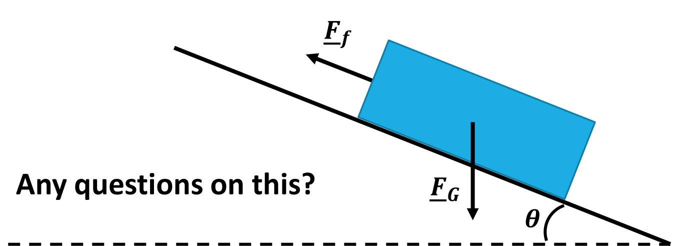
Question: In which direction is friction acting?

Answer: Uphill! Friction opposes motion <u>and the onset of motion!</u>

If an object is not moving (zero velocity), the type of friction acting is called ***static friction***. It acts to prevent motion starting, and keep the velocity at zero.

Analyse all other forces first, determine which way the object "should" be moving. Static friction will exactly counter this, keeping the net force zero.

Any questions on this?



Friction

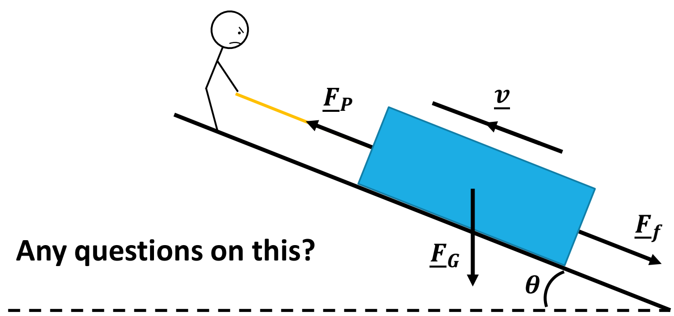
What direction?

Scenario: A box is at being pulled uphill by someone. They are exerting a constant force $\underline{F}_p$ on the box via a rope. The box is moving up the hill.

Question: In which direction is friction acting?

Answer: Downhill! Friction opposes motion and the onset of motion!

If an object is moving (non-zero velocity), the type of friction acting is called ***kinetic friction***. It always acts in the opposite direction to the velocity vector, regardless of what other forces are present



Any questions on this?

Friction

What direction?

Scenario: Someone is attempting to pull a box up a hill. They are exerting a constant force $\underline{F}_p$ on the box via a rope. The box is not moving at all.

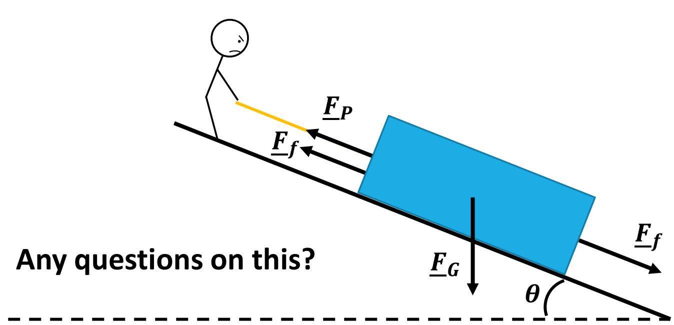
Question: In which direction is friction acting?
Hint: Think very carefully about this one

Answer: It depends!

If $|\underline{F}_P| > |\underline{F}_G|\sin(\theta)$, static friction must be downhill to resist motion.

If $|\underline{F}_P| < |\underline{F}_G|\sin(\theta)$, static friction must be uphill to resist motion.

If $|\underline{F}_P| = |\underline{F}_G|\sin(\theta)$, static friction must be therefore be zero!



Any questions on this?

Friction

The Source of Friction

Friction emerges from microscopic interactions

Consider a block being placed on a horizontal table.

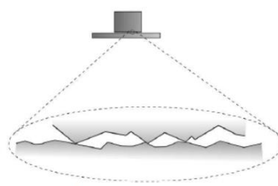


Figure: Microscopic bumps in the surfaces reduce the area over which the objects make contact.

Rather than trying to calculate all forces pushing the block into the surface, we can simplify the statement of this fact by invoking Newton's Third Law. The force pushing the block into the surface is equal to the normal reaction force of the surface on the block.

Friction

The Types of Friction

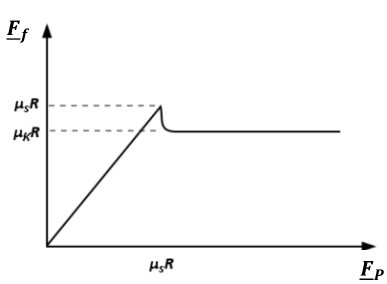
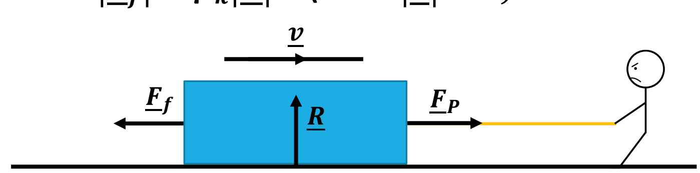


Figure: Graph of friction force against applied force

$\mu_k < \mu_s$
Always true!

- **Static friction** opposes motion before it starts, up to a limit:
  - $|\underline{F}_f| \leq \mu_s |\underline{R}|$ **(when** $|\underline{v}| = \mathbf{0}$)
- **Kinetic friction** opposes motion after it starts, and remains constant
  - $|\underline{F}_f| = \mu_k |\underline{R}|$ **(when** $|\underline{v}| \neq \mathbf{0}$)



Friction

The Types of Friction

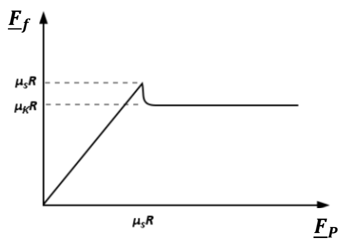
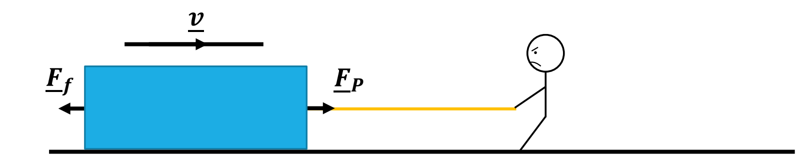


Figure: Graph of friction force against applied force

$\mu_k < \mu_s$
Always true!

- **Static friction** $|\underline{F}_p| = |\underline{F}_f|$
  $|\underline{F}_f| < \mu_s |\underline{R}|$
- **Static friction Limit** $|\underline{F}_p| = |\underline{F}_f|$
  $|\underline{F}_f| = \mu_s |\underline{R}|$
- **Kinetic friction** $|\underline{F}_p| > |\underline{F}_f|$
  $|\underline{F}_f| = \mu_k |\underline{R}|$



Friction

The Types of Friction

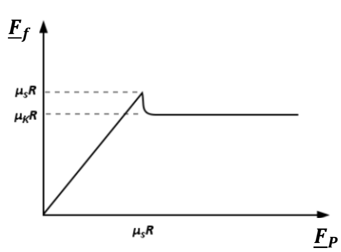
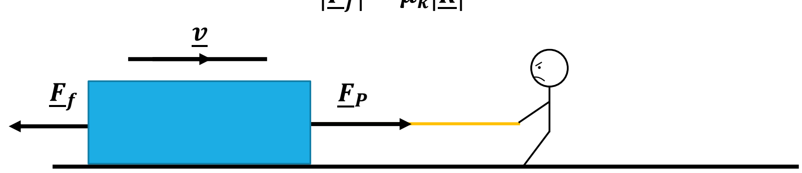


Figure: Graph of friction force against applied force

$\mu_k < \mu_s$
Always true!

- **Kinetic friction** $|\underline{F}_p| > |\underline{F}_f|$
  $|\underline{F}_f| = \mu_k |\underline{R}|$



Task 4

Frictional Forces in Action

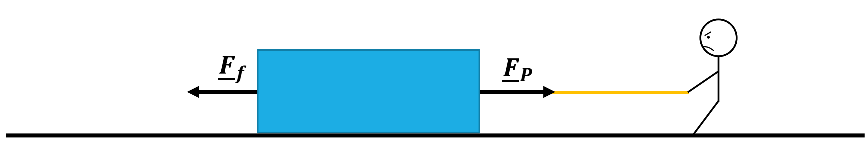
Task 4

Frictional Forces in Action

Scenario: Someone is attempting to pull a $100kg$ box along the ground, but it is not moving. The co-efficient of static friction of the box with the ground, $\mu_s = 0.35$, and the co-efficient of kinetic friction, $\mu_k = 0.25$

Tasks:

1. What is the minimum force $|\underline{F_P}|$ the person must apply to begin moving the box?
2. The person increases their effort, and applies a constant force $|\underline{F_P}| = 360N$
   a. What is the initial acceleration of the box?
   b. How far can they move the box along the ground if they pull at this force for 5s?



Task 5

More Frictional Forces in Action

37

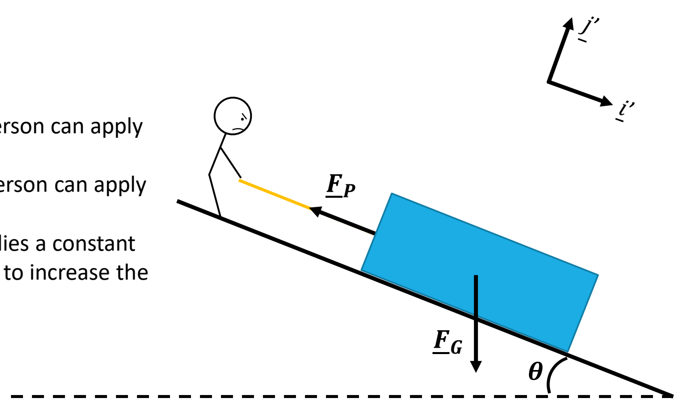
Task 5

More Frictional Forces in Action

Scenario: Someone is attempting to pull a $30kg$ up a hill, angled $\theta = 20^o$. The co-efficient of static friction of the box with the ground, $\mu_s = 0.25$, and the co-efficient of kinetic friction, $\mu_k = 0.20$.

Tasks:

1. Initially the box is not moving.
   a. What is the minimum force $\underline{F}_P$ the person can apply to prevent the box from moving.
   b. What is the maximum force $\underline{F}_P$ the person can apply before the box will begin to move.
2. The person increases their effort, and applies a constant force $\underline{F}_P = -180N\underline{i}'$. How long will it take to increase the vertical height of the box by 15m?



38

Task 6

Even More Frictional Forces in Action

Task 6

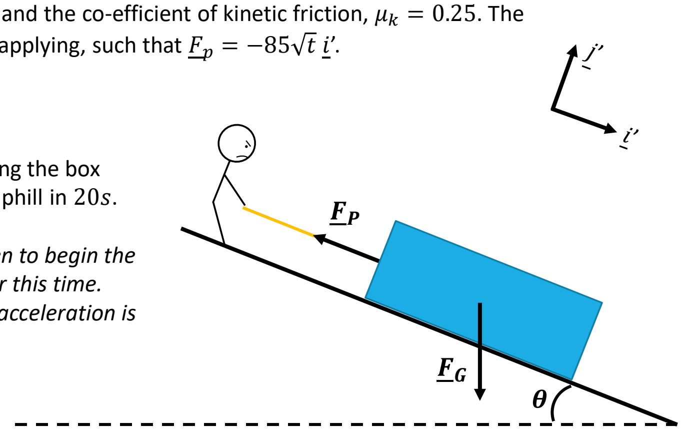
Even More Frictional Forces in Action

Scenario: Someone is attempting to pull a $45kg$ up a hill, angled $\theta = 30^o$. The co-efficient of static friction of the box with the ground, $\mu_s = 0.3$, and the co-efficient of kinetic friction, $\mu_k = 0.25$. The person is slowly increasing the force they are applying, such that $\underline{F_p} = -85\sqrt{t}\ \underline{i}'$.

Tasks:

1. Calculate the time required to begin moving the box
2. Calculate the distance the box will move uphill in $20s$.

Hint: You'll need to calculate the time taken to begin the motion, and then the distance moved after this time. Remember to ask yourselves whether the acceleration is constant…
This is hard, but you can do this!



Task 7
Calculating Coefficients of Friction

Task 7
Coefficients of Friction

We've seen some examples earlier in this lecture. Go through the previous tasks and use the existing information to try and calculate the coefficients of friction (static and kinetic) wherever you can ☺

Resources

43

Dr Purdy’s Notes

And Examples

I’ve added some great notes and examples on Minerva from Rob Purdy. He is a super genius!

44