UNIVERSITY OF LEEDS

Mechanics 1

Session 6 – Electric and Gravitational Forces

DR BEN HANSON

Last Lecture

Tension & Newton's Third Law

We learned:

- What tension is
- That all objects are at least partially elastic
- How the level of elasticity in an object determines how we model its motion

You should be able to:

- Identify where in a system tension is acting
- Calculate the acceleration of coupled objects under tension

This Lecture

Electric and Gravitational Forces

We will:

- Learn what the gravitational force is and how it is mathematically modelled
- Briefly discuss Taylor series expansions
- Learn what the electric force is and how it is mathematically modelled
- Compare and contrast electrical and gravitational forces

You will be able to:

- Calculate the (linear) motion of an object experiencing an electric or gravitational force
- Understand how vectors are used to represent forces in arbitrary directions

Gravity

Classical Gravity at Least (no relativity yet!)

Gravity

Classical Gravity

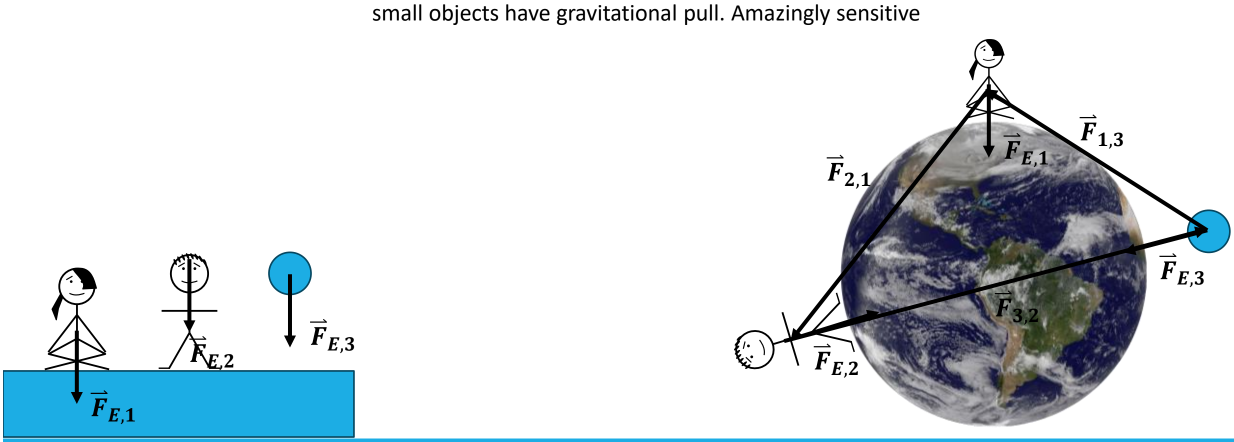
If two objects have mass, they will experience an (equal and opposite) gravitational attraction towards one another

This is not at all obvious. Gravity is a relatively modern idea. After all, you may have seen an apple fall from a tree, but have you ever seen an apple fall towards another apple?

Gravity

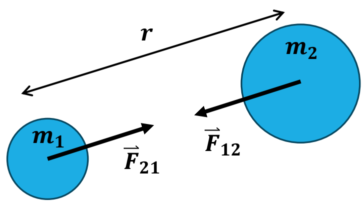
Classical Gravity

There's a really good experiment in labs where you show that small objects have gravitational pull. Amazingly sensitive



Gravity

Classical Gravity



$$|\vec{F}_{12}| = G\frac{m_1 m_2}{r^2}$$

Universal Gravitational Constant,
$G \approx 6.67 \times 10^{-11} Nm^2 kg^{-2}$

Gravitational force proportional to each mass

Gravitational force inversely proportional to distance between masses squared

Task 1

Little $g = 9.81$...mostly

Task 1

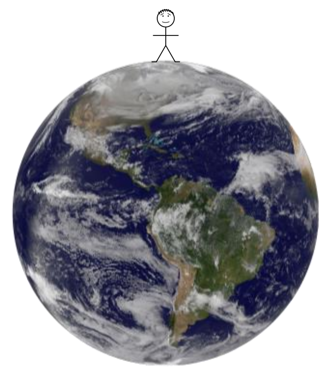
Little $g = 9.81$...mostly

Scenario: A person of mass m is stood on the surface of the Earth. Earth has a radius R and a mass m_e.

Tasks:

1. Show that the acceleration due to gravity at the surface of the Earth, $g \approx 9.81$
2. Show that if the person is instead stood on top of Mt Everest, the acceleration due to gravity $g_{ev} < g$.
3. Calculate the acceleration due to gravity on the surface of the moon

   Note: Use the internet to find the values you need



Gravity

It's not constant...so why use g?

Gravity

Why can we use g?

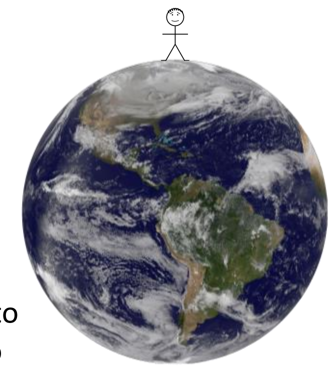
$$|\vec{F}_{12}| = G\frac{m_1 m_2}{r^2}$$

$$|\vec{F}_{12}| = m_2\left(G\frac{m_1}{r^2}\right)$$

$$|\vec{F}_{12}| = m_2 a_2$$

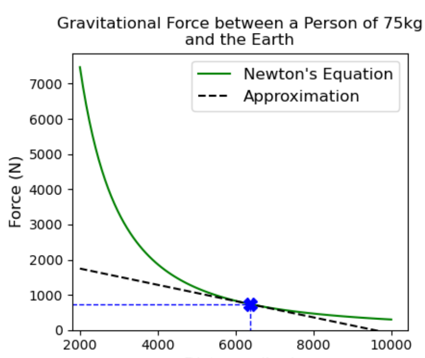
Acceleration is clearly not constant, so why can we treat it as though it is?

Something called a “Taylor Series” will never stop being useful to you. I use it literally every day, including in my real life! And so do you. I’m going to do a video on it, please watch ☺



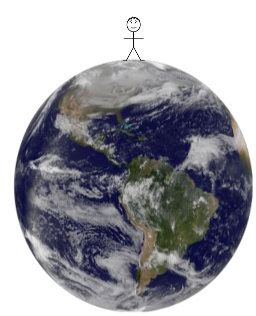
Gravity

Why can we use g?



$$|\vec{F}_{12}|(r) = G\frac{m_1 m_2}{r^2}$$

$$|\vec{F}_{12}|(R_e + \delta h) \approx G\frac{m_1 m_2}{R_e^2}\left(1 - 2\frac{\delta h}{R_e}\right)$$



$\delta h \ll R_e$, therefore g is effectively constant at human length scales ☺

Task 2

Calculating Motion Due to Gravity

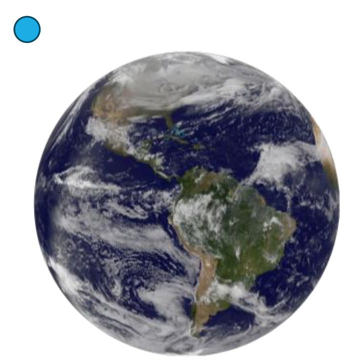
Task 2

Calculating Motion Due to Gravity

Scenario: A ball is dropped from a height h above the surface of the Earth. Assuming no air resistance:

Tasks:

1. Calculate the time taken for the ball fall $20m$ Earth if $h = 50m$
2. Calculate the time taken for the ball fall $20m$ Earth if $h = R_e$, the radius of the Earth



Electricity

Coulomb's Law

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Electricity

Coulomb's Law

If two objects have charge, they will experience an (equal and opposite) electrical attraction or repulsion towards one another. Whether it's attraction or repulsion depends on the charge

The electric force is extremely similar to classical gravity. This is a big deal in modern theoretical physics (i.e. why are they similar?)

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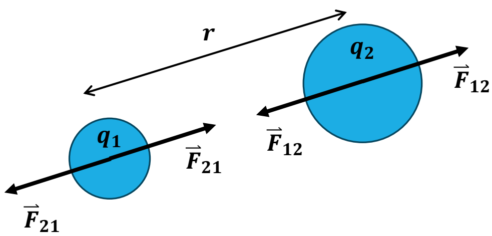
Electricity

Coulomb's Law



$$|\vec{F}_{12}| = k\frac{|q_1||q_2|}{r^2}$$

Coulomb constant,

$$k = \frac{1}{4\pi\epsilon_0} \approx 8.99 \times 10^9 Nm^2C^{-2}$$

Electric force proportional to each mass

Electric force inversely proportional to distance between masses squared

Task 3

Comparing Electricity to Gravity

Task 3

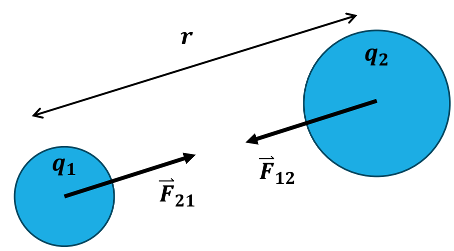
Comparing Electricity to Gravity

Scenario: Two electrons are a distance $r = 0.5 \times 10^{-10} m$ away from one another.

Tasks:

1. Calculate the magnitude of the force on each particle due to the electric force
2. Calculate the magnitude of the force on each particle due to the gravitational force
3. Calculate the ratio of the electric and gravitational forces. What do you notice?
4. Calculate the magnitude of the acceleration of each particle

Note: Use the internet to find the mass and charge of an electron ☺



Electricity and Gravity

A Vector Formalisation

Electricity and Gravity

A Vector Formalisation

We've looked at the force magnitude, but…aren't forces vectors? What direction do these forces act in?

I'm sure you've done questions before where you work out all the charges and directions in your head, and then add them together. Unfortunately, this doesn't work very well with more general forces…

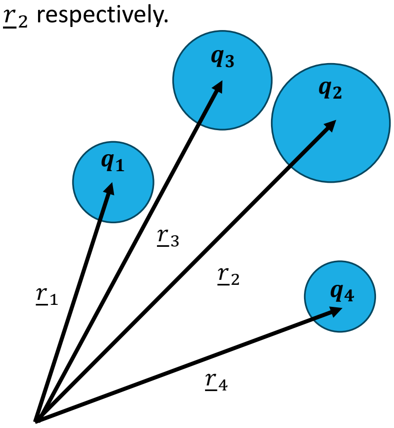
Electricity and Gravity

A Vector Formalisation

Scenario: Two charges a q_1 and q_2 are at positions $\underline{r}_1$ and $\underline{r}_2$ respectively.

Tasks:

1. Calculate the electric force vector, $\vec{F}_{12}$, the electric force on charge 2 due to charge 1.
2. Calculate the electric force vector, $\vec{F}_{21}$, the electric force on charge 1 due to charge 2.
3. Two more charges are added, q_3 and q_4 are at positions $\underline{r}_3$ and $\underline{r}_4$ respectively. Calculate the electric force vector, $\vec{F}_1$, the electric force on charge 1 due to all other charges.
4. Calculate the gravitational force on particle 1 due to all the other masses



Electricity and Gravity

A Vector Formalisation

Scenario: Two charges a q_1 and q_2 are at positions $\underline{r}_1$ and $\underline{r}_2$ respectively.

Tasks:

1. Calculate the electric force vector, $\vec{F}_{12}$, the electric force on charge 2 due to charge 1.

Vector formalisation,

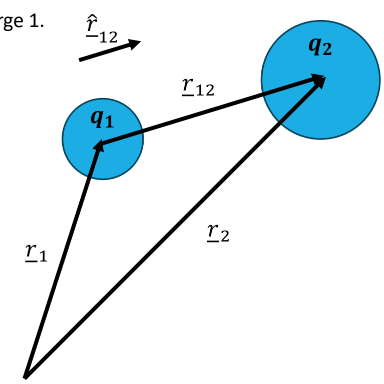
$$\boxed{\vec{F}_{12} = k\frac{q_1 q_2}{|\underline{r}_{12}|^2}\hat{\underline{r}}_{12}}$$

Vector between particles,

$$\underline{r}_{12} = \underline{r}_2 - \underline{r}_1$$

Unit vector gives direction,

$$\hat{\underline{r}}_{12} = \frac{\underline{r}_{12}}{|\underline{r}_{12}|}$$



Notice that if the charges are the same sign, the force $\vec{F}_{12}$ is in the same direction as the unit vector $\hat{\underline{r}}_{12}$ (repulsive). If the signs are different, the force is in the opposite direction to the unit vector (attractive). This formalisation sorts out the directions for you!

Electricity and Gravity

A Vector Formalisation

Scenario: Two charges a q_1 and q_2 are at positions $\underline{r}_1$ and $\underline{r}_2$ respectively.

Tasks:

2. Calculate the electric force vector, $\vec{F}_{21}$, the electric force on charge 1 due to charge 2.

Vector formalisation,

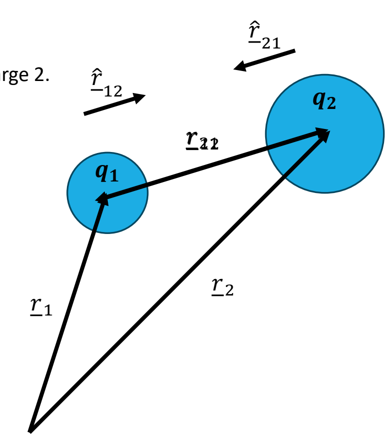
$$\boxed{\vec{F}_{21} = k\frac{q_2 q_1}{|\underline{r}_{21}|^2}\hat{\underline{r}}_{21}}$$

Vector between particles,

$$\underline{r}_{21} = \underline{r}_1 - \underline{r}_2$$

Unit vector gives direction,

$$\hat{\underline{r}}_{21} = \frac{\underline{r}_{21}}{|\underline{r}_{21}|}$$



Notice that it's similar to before, but because $\underline{r}_{21} = -\underline{r}_{12}$, we pick up an extra minus sign. This means that the force $\vec{F}_{21} = -\vec{F}_{12}$…Newton's Third Law!

Electricity and Gravity

A Vector Formalisation

Scenario: Two charges a q_1 and q_2 are at positions $\underline{r}_1$ and $\underline{r}_2$ respectively.

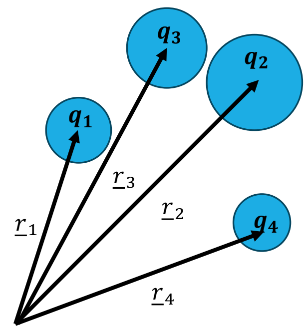
Tasks:

3. Two more charges are added, q_3 and q_4 are at positions $\underline{r}_3$ and $\underline{r}_4$ respectively. Calculate the electric force vector, $\vec{F}_1$, the electric force on charge 1 due to all other charges.

Sum over all forces,
$$\vec{F}_1 = \vec{F}_{21} + \vec{F}_{31} + \vec{F}_{41}$$

Expand,
$$\vec{F}_1 = k\frac{q_2 q_1}{|\underline{r}_{21}|^2}\hat{\underline{r}}_{21} + k\frac{q_3 q_1}{|\underline{r}_{31}|^2}\hat{\underline{r}}_{31} + k\frac{q_4 q_1}{|\underline{r}_{41}|^2}\hat{\underline{r}}_{41}$$

Factorise,
$$\vec{F}_1 = kq_1\left(\frac{q_2}{|\underline{r}_{21}|^2}\hat{\underline{r}}_{21} + \frac{q_3}{|\underline{r}_{31}|^2}\hat{\underline{r}}_{31} + \frac{q_4}{|\underline{r}_{41}|^2}\hat{\underline{r}}_{41}\right)$$



Electricity and Gravity

A Vector Formalisation

Scenario: Two charges a q_1 and q_2 are at positions $\underline{r}_1$ and $\underline{r}_2$ respectively.

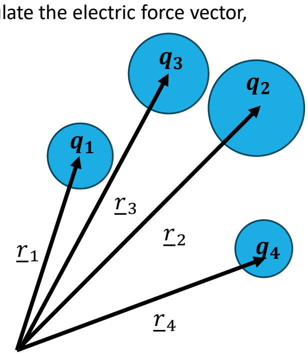
Tasks:

3. Two more charges are added, q_3 and q_4 are at positions $\underline{r}_3$ and $\underline{r}_4$ respectively. Calculate the electric force vector, $\vec{F}_1$, the electric force on charge 1 due to all other charges.

Factorise,
$$\vec{F}_1 = kq_1\left(\frac{q_2}{|\underline{r}_{21}|^2}\hat{\underline{r}}_{21} + \frac{q_3}{|\underline{r}_{31}|^2}\hat{\underline{r}}_{31} + \frac{q_4}{|\underline{r}_{41}|^2}\hat{\underline{r}}_{41}\right)$$

Generalise,
$$\vec{F}_1 = kq_1\sum_{j!=1}^{4}\frac{q_j}{|\underline{r}_{j1}|^2}\hat{\underline{r}}_{j1}$$

Generalise more,
$$\boxed{\vec{F}_i = kq_i\sum_{j!=i}^{N}\frac{q_j}{|\underline{r}_{ji}|^2}\hat{\underline{r}}_{ji}, \qquad N - Number\ of\ charges}$$



Electricity and Gravity

A Vector Formalisation

Scenario: Two charges a q_1 and q_2 are at positions $\underline{r}_1$ and $\underline{r}_2$ respectively.

Tasks:

3. Two more charges are added, q_3 and q_4 are at positions $\underline{r}_3$ and $\underline{r}_4$ respectively. Calculate the electric force vector, $\vec{F}_1$, the electric force on charge 1 due to all other charges.

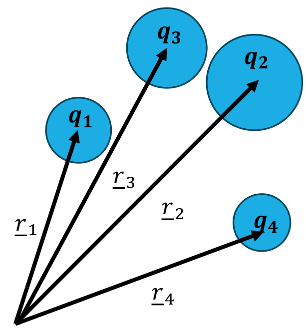
Generalise more,

$$\boxed{\vec{F}_i = kq_i \sum_{j!=i}^{N} \frac{q_j}{|\underline{r}_{ji}|^2} \hat{\underline{r}}_{ji}, \qquad N - Number\ of\ charges}$$

Alternate form,

$$\hat{\underline{r}}_{ij} = -\hat{\underline{r}}_{ji}$$

$$\boxed{\vec{F}_i = -kq_i \sum_{j!=i}^{N} \frac{q_j}{|\underline{r}_{ij}|^2} \hat{\underline{r}}_{ij}, \qquad N - Number\ of\ charges}$$



Electricity and Gravity

A Vector Formalisation

Scenario: Two charges a q_1 and q_2 are at positions $\underline{r}_1$ and $\underline{r}_2$ respectively.

Tasks:

4. Calculate the gravitational force on particle 1 due to all the other masses

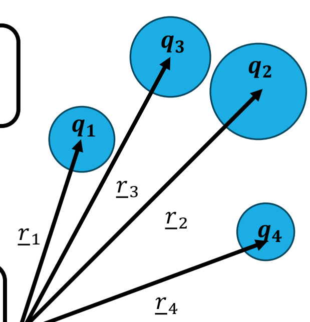
Compare to electric,

$$\boxed{\vec{F}_i = Gm_i \sum_{j!=i}^{N} \frac{m_j}{|\underline{r}_{ij}|^2} \hat{\underline{r}}_{ij}, \qquad N - Number\ of\ masses}$$

The only difference (in form) between the gravitational and electric force is this unit vector. This is because gravity is attractive when the masses are both positive. It's opposite to the electric field, then, so we simply alter the unit vector direction to compensate. But that means…

Alternate form,

$$\boxed{\vec{F}_i = -Gm_i \sum_{j!=i}^{N} \frac{m_j}{|\underline{r}_{ij}|^2} \hat{\underline{r}}_{ji}, \qquad N - Number\ of\ masses}$$



Task 4

Electric forces from multiple sources

Task 4

Electric forces from multiple sources

Scenario: Three objects with charges $q_1 = 5.5\mu C$, $q_2 = 6.5\mu C$ and $q_3 = 4.5\mu C$ are at positions $\underline{r}_1 = \left(0.5\underline{i} + \underline{j}\right) m$, $\underline{r}_2 = 3\left(\underline{i} + \underline{j}\right) m$ and $\underline{r}_3 = \left(5\underline{i} + 2\underline{j}\right) m$ respectively. They each have the same mass, $m = 0.1kg$

Tasks:

1. Calculate the acceleration of each object