PHAS1000 – THERMAL PHYSICS

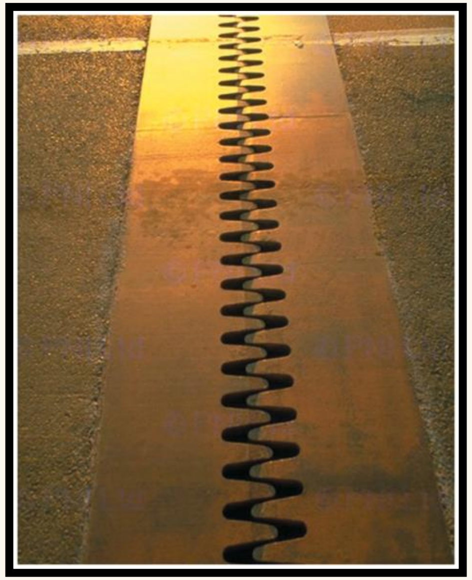
Lecture 3

Expansion

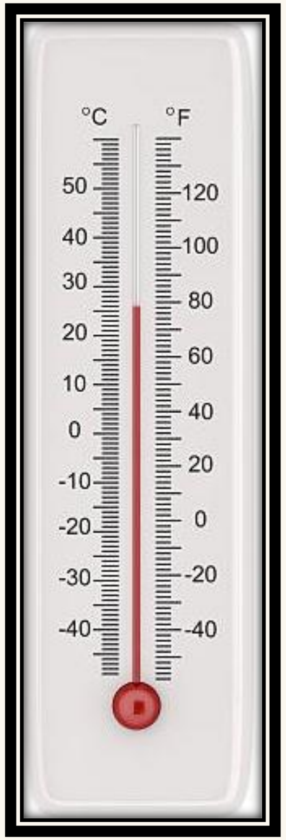
Thermal Expansion

Problem

Solution



Desirable

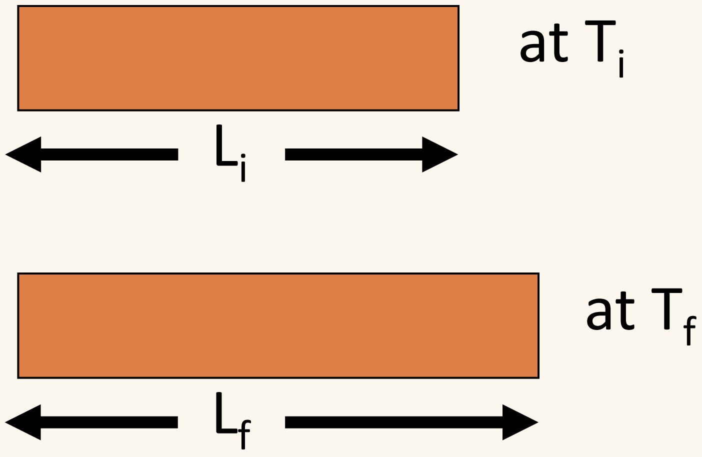


Topics covered:

- Coefficient of linear expansion
- Coefficient of volume expansion
- Differential expansion

Images tinyurl.com/y56447zo tinyurl.com/y6zxfr6s Tipler

Definition



Fractional change in length is proportional to change in temperature

$$\frac{\Delta L}{L} = \frac{(L_f - L_i)}{L_i} = \alpha \Delta T$$

α = coefficient of linear expansion

$$\alpha = \frac{\Delta L / L}{\Delta T}$$

What are the units of α?

Units K^{-1}

$$\alpha = \lim_{\Delta T \to 0} \frac{\Delta L / L}{\Delta T} = \frac{1}{L} \frac{dL}{dT}$$

Coefficient of Linear Expansion

Usual ways to express the equation

$$\Delta L = L_0 \alpha \Delta T$$

$$L = L_0(1 + \alpha \Delta T)$$

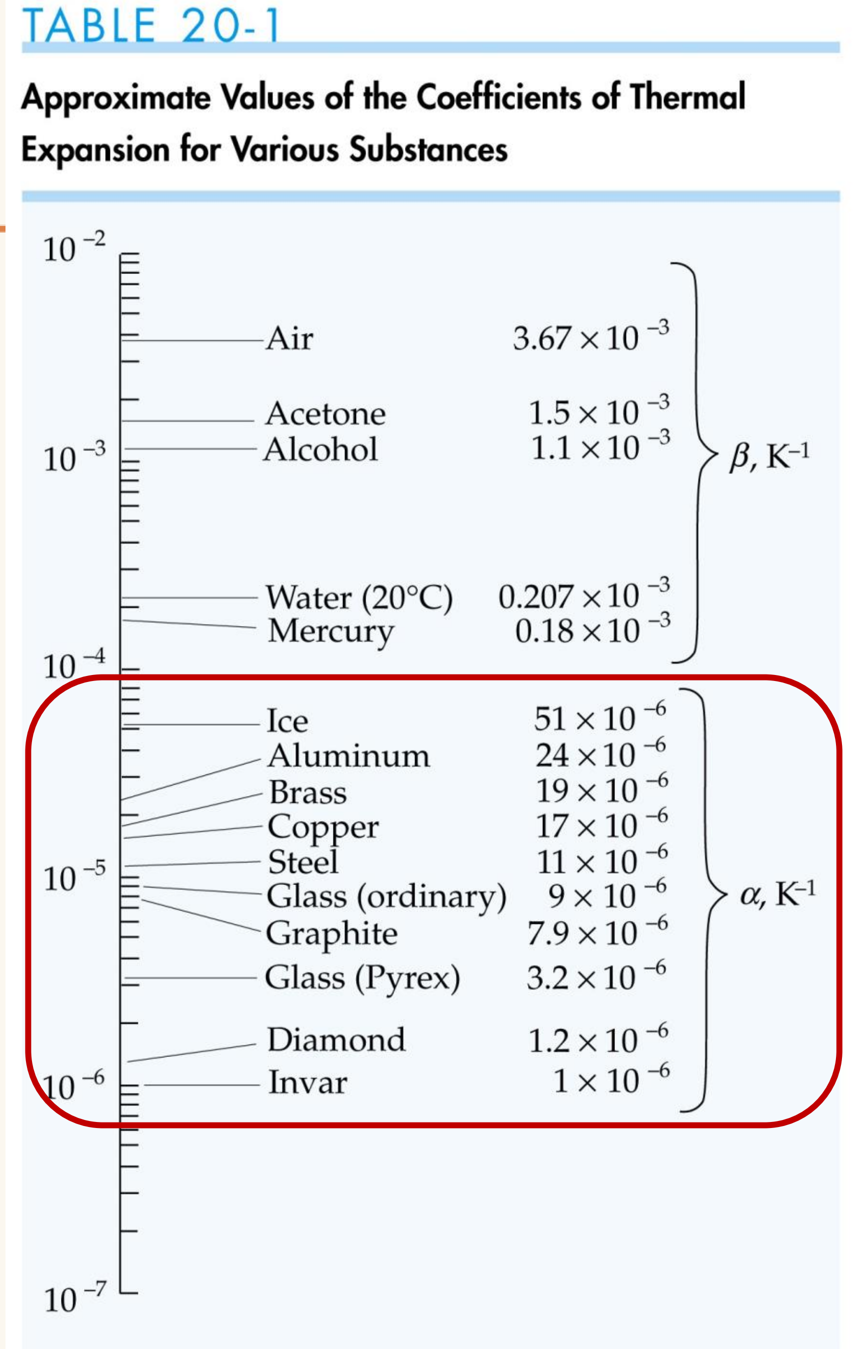


TABLE 20-1

Approximate Values of the Coefficients of Thermal Expansion for Various Substances

Air	3.67×10^{-3}	β, K^{-1}
Acetone	1.5×10^{-3}	
Alcohol	1.1×10^{-3}	
Water (20°C)	0.207×10^{-3}	
Mercury	0.18×10^{-3}	
Ice	51×10^{-6}	α, K^{-1}
Aluminum	24×10^{-6}	
Brass	19×10^{-6}	
Copper	17×10^{-6}	
Steel	11×10^{-6}	
Glass (ordinary)	9×10^{-6}	
Graphite	7.9×10^{-6}	
Glass (Pyrex)	3.2×10^{-6}	
Diamond	1.2×10^{-6}	
Invar	1×10^{-6}	

solids

Question

(a) A steel bridge is 1000m long. By how much does it expand when the temperature rises from -20°C (winter) to 35°C (summer)?

$$\Delta L = L_0 \alpha \Delta T$$

$$= 1000 \times 11 \times 10^{-6} \times (35 - -20)$$

$$= 0.605 \text{ m}$$

$$= 61 \text{ cm}$$

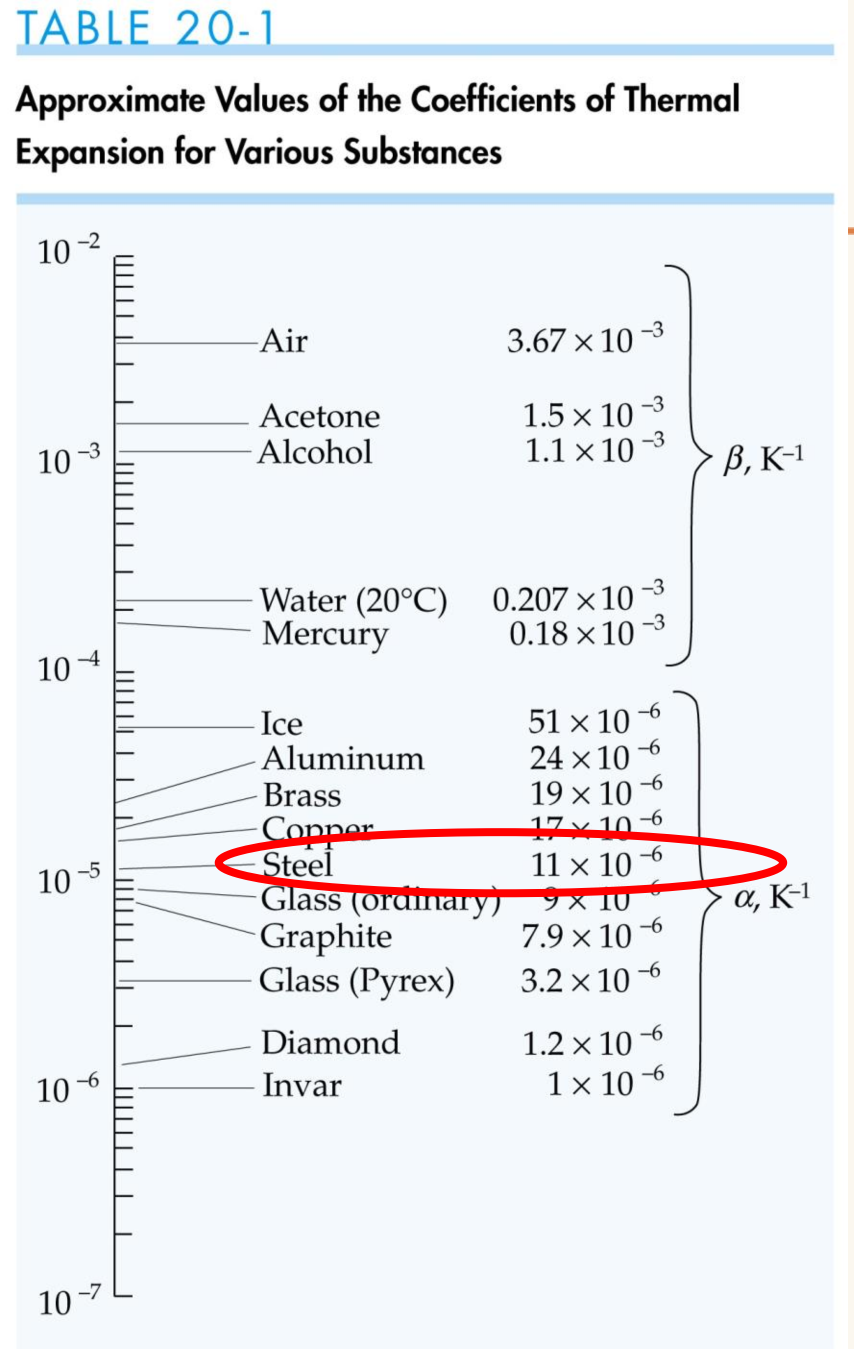


TABLE 20-1

Approximate Values of the Coefficients of Thermal Expansion for Various Substances

Air	3.67×10^{-3}	β, K^{-1}
Acetone	1.5×10^{-3}	
Alcohol	1.1×10^{-3}	
Water (20°C)	0.207×10^{-3}	
Mercury	0.18×10^{-3}	
Ice	51×10^{-6}	α, K^{-1}
Aluminum	24×10^{-6}	
Brass	19×10^{-6}	
Copper	17×10^{-6}	
Steel	11×10^{-6}	
Glass (ordinary)	9×10^{-6}	
Graphite	7.9×10^{-6}	
Glass (Pyrex)	3.2×10^{-6}	
Diamond	1.2×10^{-6}	
Invar	1×10^{-6}	

Question

(b) If each expansion joint allows for up to 2 cm movement, what is the best spacing between joints on the bridge?

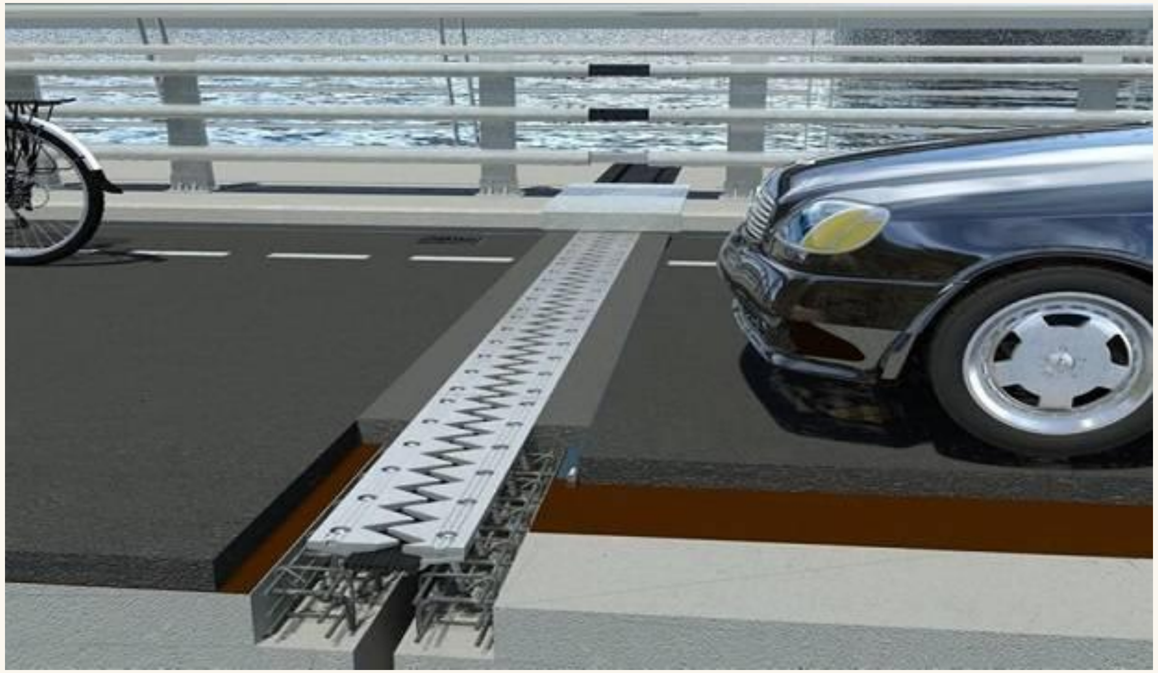
$$\text{number of joints} = \frac{\text{total } \Delta L}{\text{size of joint}}$$

$$= \frac{61}{2} = 30{\cdot}5 \sim 31$$

$$\text{spacing between joints} = \frac{\text{bridge length}}{\text{no. of joints}}$$

$$= \frac{1000}{31}$$

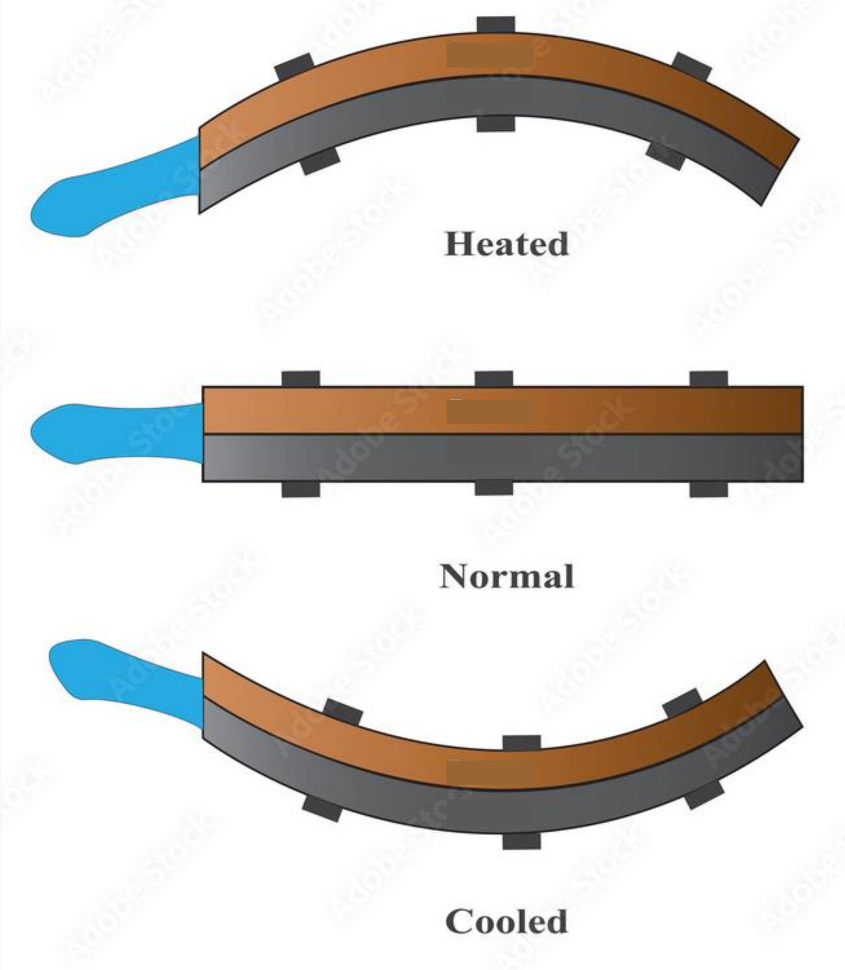
$$= 32{\cdot}2\,\text{m}$$



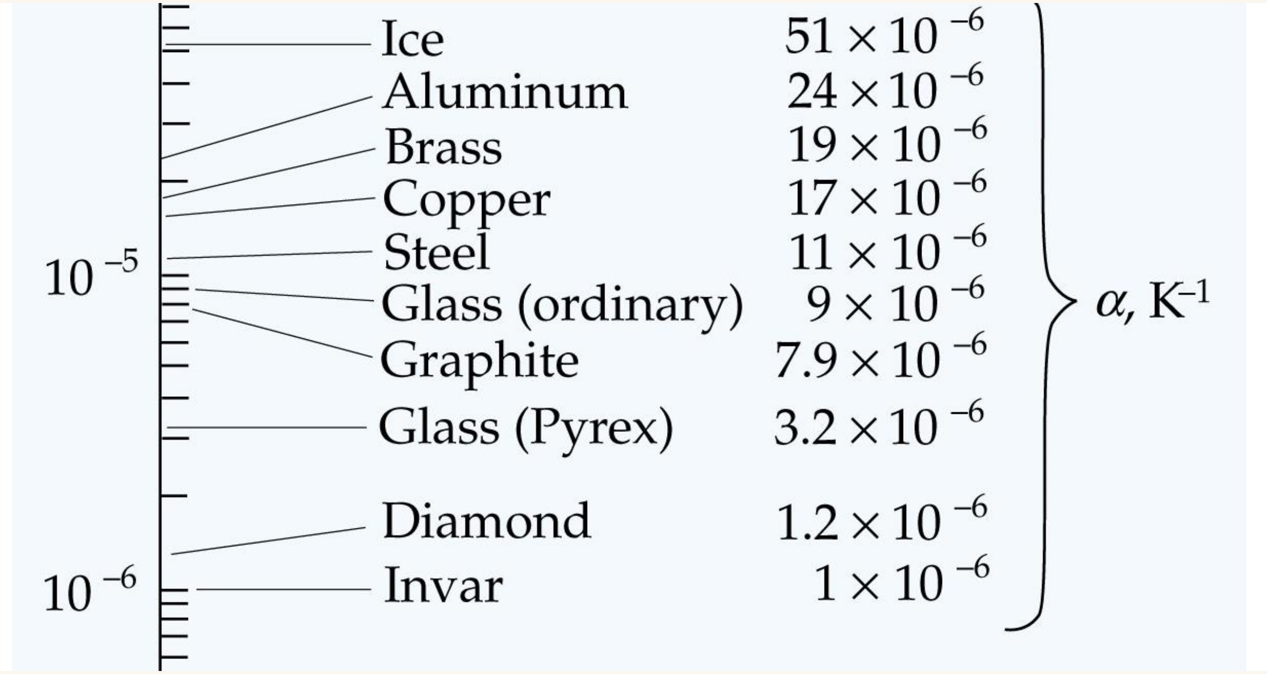
(c) What would happen if there were no expansion joints?

Differential expansion

199-145-020



Which strip is brass and which steel ?

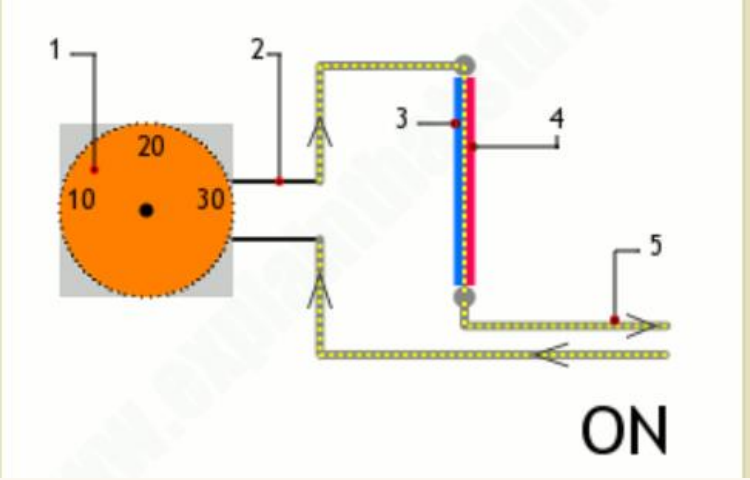


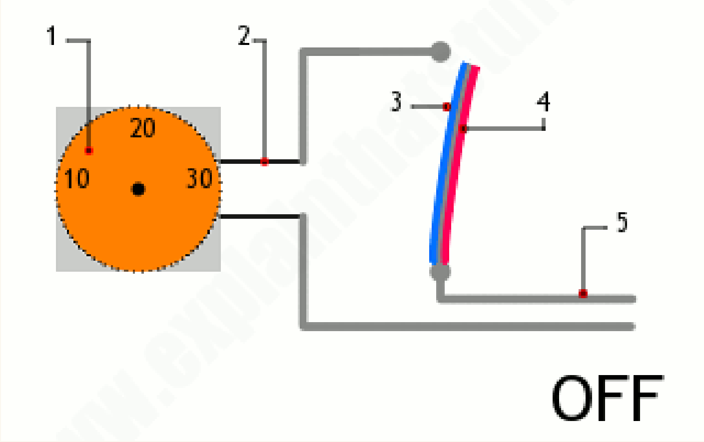
Ice	51×10^{-6}
Aluminum	24×10^{-6}
Brass	19×10^{-6}
Copper	17×10^{-6}
Steel	11×10^{-6}
Glass (ordinary)	9×10^{-6}
Graphite	7.9×10^{-6}
Glass (Pyrex)	3.2×10^{-6}
Diamond	1.2×10^{-6}
Invar	1×10^{-6}

α, K^{-1}

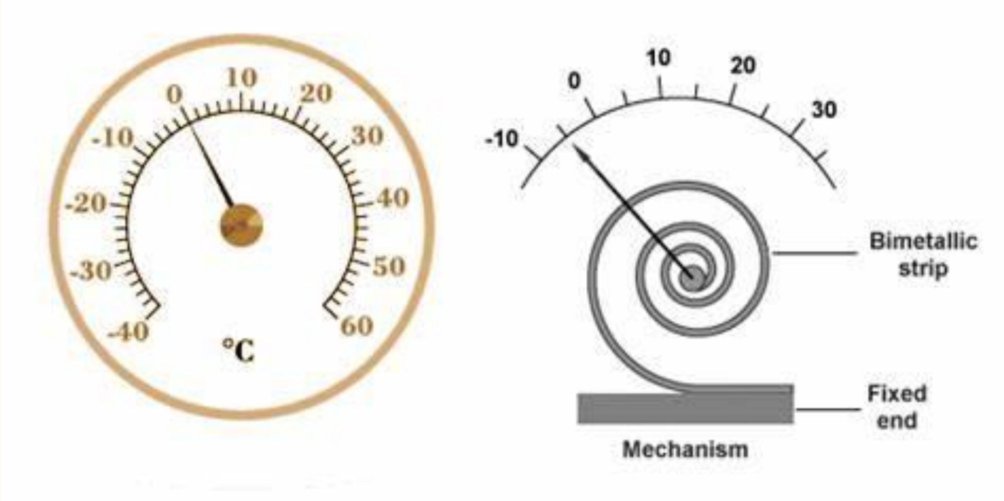
Uses of bimetal strips

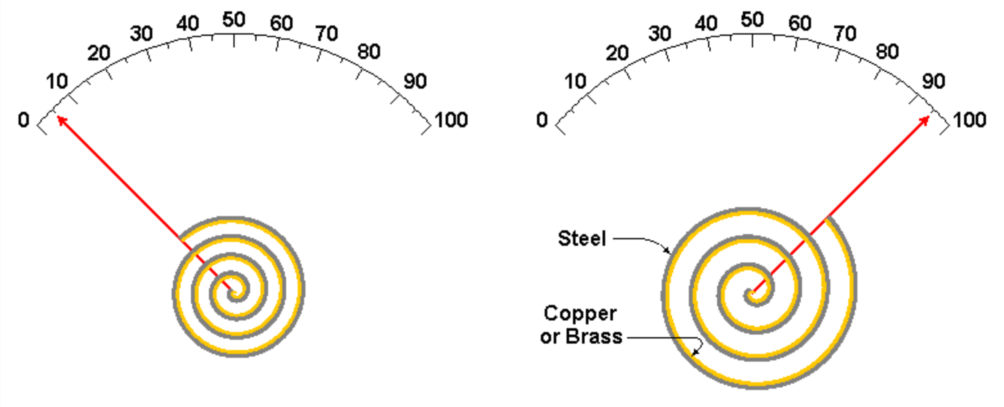
Thermostat circuit





Bimetallic thermometer





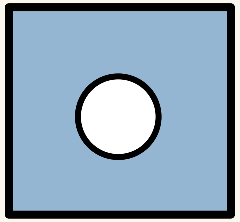
Applications

What is happening here? And why?

Image hyperphysics

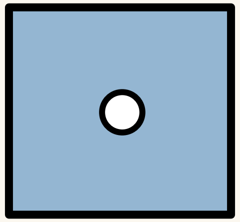
What happens to holes?

COLD



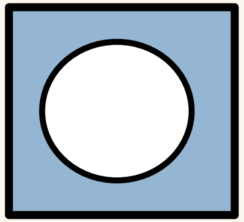
HOT

A



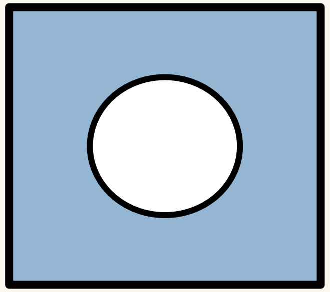
Square = same size
Hole = smaller

B



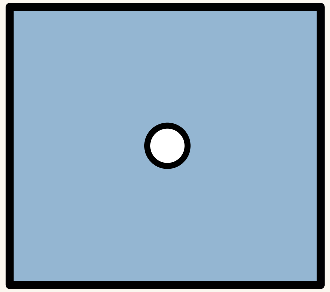
Square = same size
Hole = bigger

C



Square = bigger
Hole = bigger

D

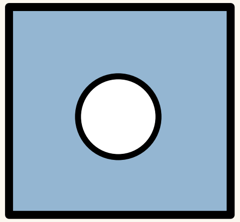


Square = bigger
Hole = smaller

199-145-020

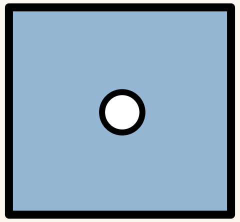
What happens to holes? ANSWER

COLD



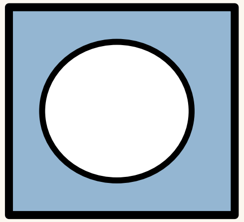
HOT

A



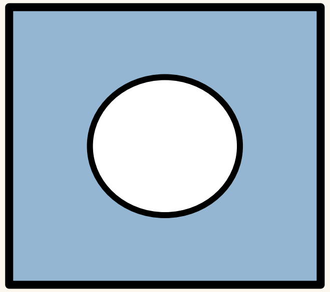
Square = same size
Hole = smaller

B



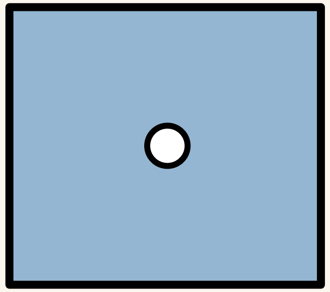
Square = same size
Hole = bigger

[✓ C]



Square = bigger
Hole = bigger

D



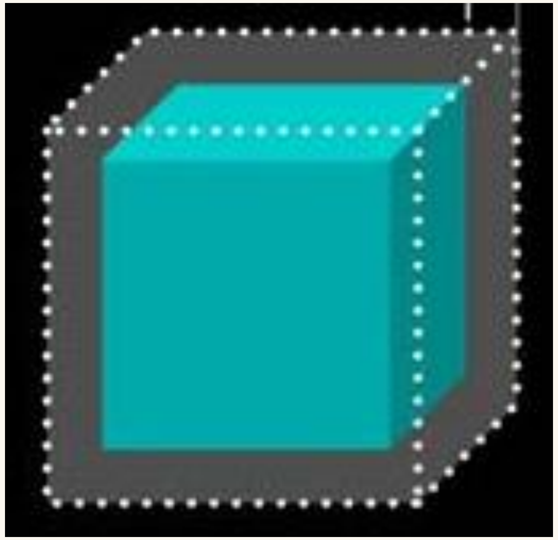
Square = bigger
Hole = smaller

Volume Expansion

$$\frac{\Delta V}{V} = \beta \Delta T$$

$$V = V_0(1 + \beta \Delta T)$$

β = coefficient of volume expansion



For linear expansion we had…….

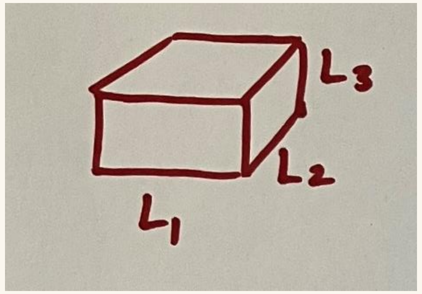
$$\alpha = \lim_{\Delta T \to 0} \frac{\Delta L / L}{\Delta T} = \frac{1}{L} \frac{dL}{dT}$$

So for volume expansion we have…….

$$\beta = \lim_{\Delta T \to 0} \frac{\Delta V / V}{\Delta T} = \frac{1}{V} \frac{dV}{dT}$$

Image tinyurl.com/yxsfwvvr

How is β related to α?



$$\frac{dV}{dT} = \frac{dV}{dL_3}\frac{dL_3}{dT}$$

$$\beta = \frac{1}{V}\frac{dV}{dT}$$

Volume at temp T is $\boxed{V = L_1 \times L_2 \times L_3}$

change of volume with temp:

$$\frac{dV}{dT} = L_1 L_2 \frac{dL_3}{dT} + L_1 L_3 \frac{dL_2}{dT} + L_2 L_3 \frac{dL_1}{dT}$$

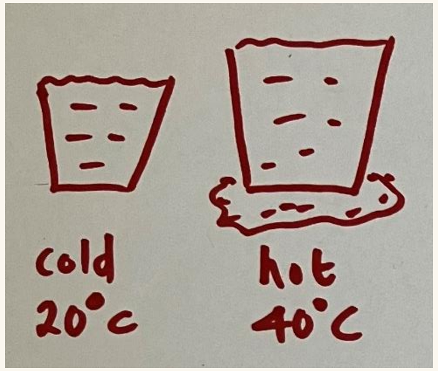
$$\beta = \frac{\cancel{L_1}\cancel{L_2}}{\cancel{L_1}\cancel{L_2}L_3}\frac{dL_3}{dT} + \frac{\cancel{L_1}\cancel{L_3}}{\cancel{L_1}L_2\cancel{L_3}}\frac{dL_2}{dT} + \frac{\cancel{L_2}\cancel{L_3}}{L_1\cancel{L_2}\cancel{L_3}}\frac{dL_1}{dT}$$

$$\beta = \frac{1}{L_3}\frac{dL_3}{dT} + \frac{1}{L_2}\frac{dL_2}{dT} + \frac{1}{L_1}\frac{dL_1}{dT}$$

but $\alpha = \frac{1}{L}\frac{dL}{dT}$ so $\boxed{\beta = 3\alpha}$

Volume and holes

A 1L glass is filled to the brim with water at 20°C. The glass and water are then heated to 40°C. How much water spills out?



$$\text{spilt water} = \Delta V_w - \Delta V_g$$

$$\Delta V_w = \beta_w V \Delta T$$

$$\Delta V_g = \beta_g V \Delta T = 3\alpha_g V \Delta T$$

$$\text{spilt} = (\beta_w - 3\alpha_g) V \Delta T$$

$$= (0.207 \times 10^{-3} - 3 \times 9 \times 10^{-6}) \times 1 \times (40-20)$$

$$= 3.6 \times 10^{-3}\ L$$

$$= 3.6\ mL$$

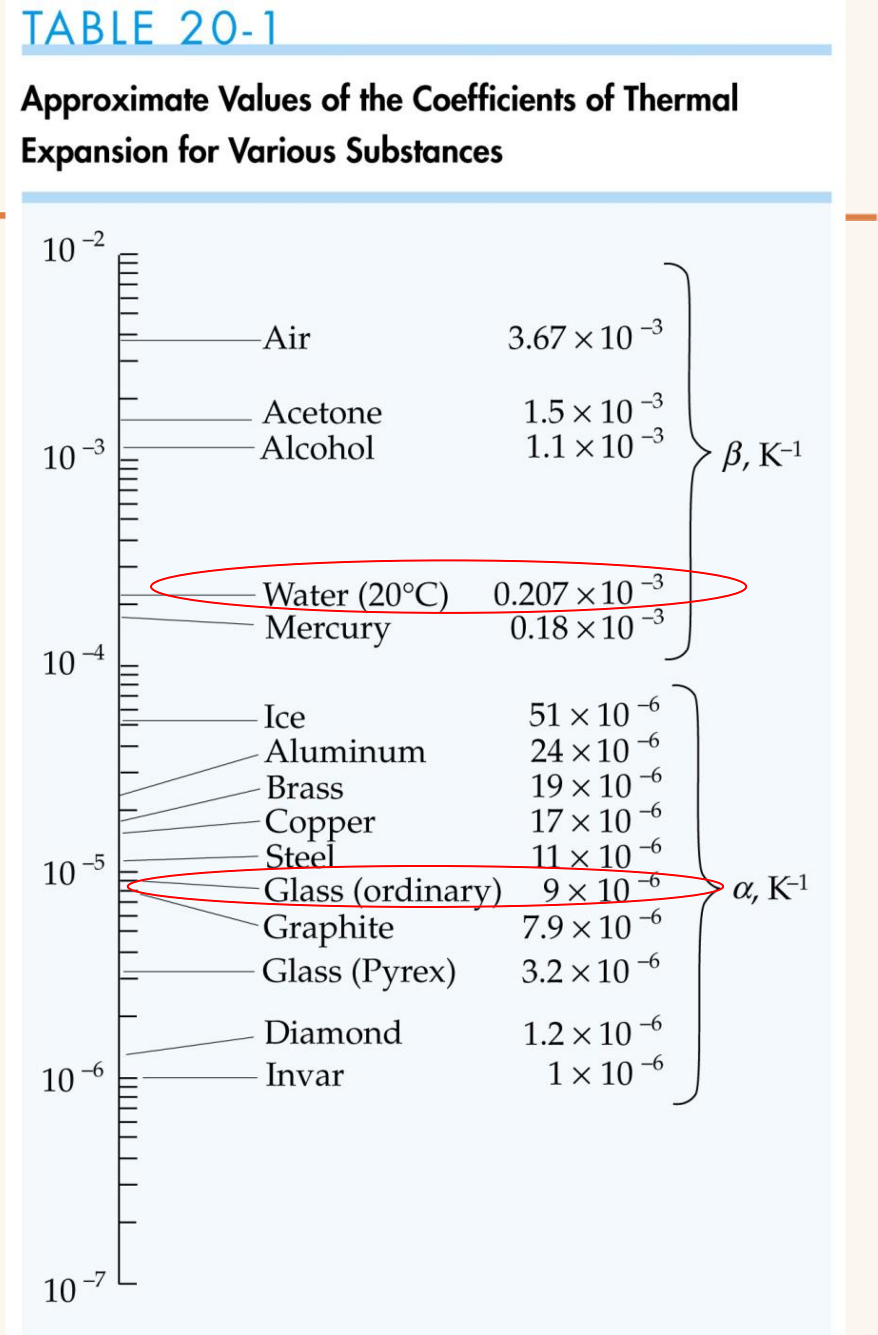


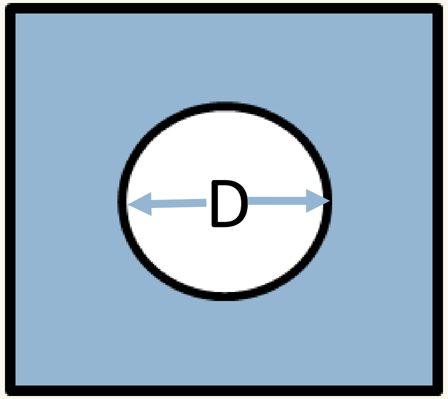
TABLE 20-1

Approximate Values of the Coefficients of Thermal Expansion for Various Substances

Air	3.67×10^{-3}	β, K^{-1}
Acetone	1.5×10^{-3}	
Alcohol	1.1×10^{-3}	
Water (20°C)	0.207×10^{-3}	
Mercury	0.18×10^{-3}	
Ice	51×10^{-6}	α, K^{-1}
Aluminum	24×10^{-6}	
Brass	19×10^{-6}	
Copper	17×10^{-6}	
Steel	11×10^{-6}	
Glass (ordinary)	9×10^{-6}	
Graphite	7.9×10^{-6}	
Glass (Pyrex)	3.2×10^{-6}	
Diamond	1.2×10^{-6}	
Invar	1×10^{-6}	

Linear, area, volume

Linear $L = L_0(1 + \alpha\Delta T)$



Take care:
Diameter is linear
Area really means area.

Area $A = A_0(1 + 2\alpha\Delta T)$

Volume $V = V_0(1 + \beta\Delta T)$ $\beta = 3\alpha$ $V = V_0(1 + 3\alpha\Delta T)$

Summary of thermal expansion

When heated most materials expand

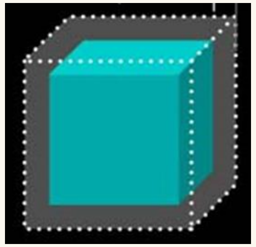
$$\frac{\Delta L}{L} = \alpha \Delta T$$

α = coefficient of linear expansion

$$L = L_0(1 + \alpha \Delta T)$$

$$\frac{\Delta V}{V} = \beta \Delta T$$

β = coefficient of volume expansion

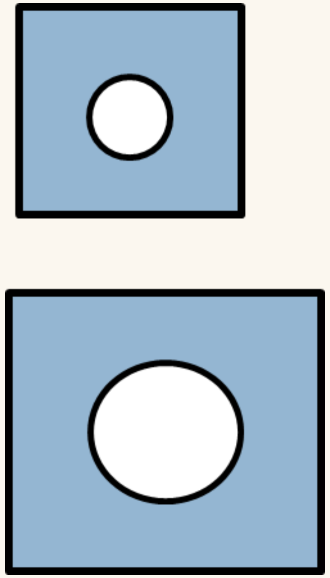


$$V = V_0(1 + \beta \Delta T)$$

$$A = A_0(1 + 2\alpha \Delta T)$$

α and β are material properties. For any given material $\beta = 3\alpha$

All dimensions of a material expand in the same ratio, even holes.



Square = bigger
Hole = bigger

Practice Questions

Practice Question 1

A concrete road has expansion joints at intervals of 20 metres. How wide must each expansion joint be to allow for temperatures as low as -10°C and as high as 40°C?

Take coefficient of thermal expansion of concrete as $10 \times 10^{-6}\ {}^{\circ}C^{-1}$

Practice Question 2

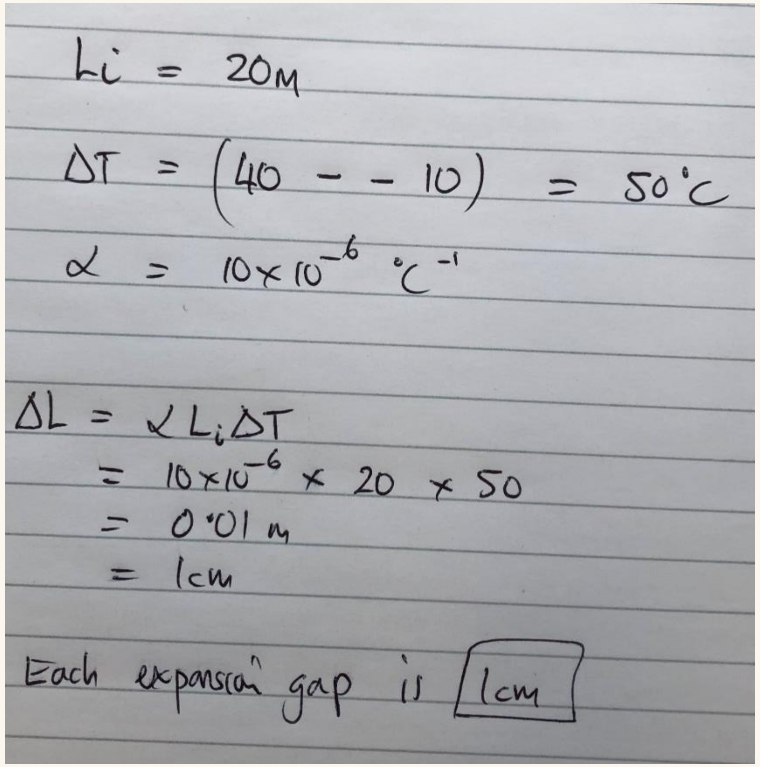
A hole is drilled in an aluminium plate with a steel bit whose diameter at 20°C is 6.245cm. In the process of drilling, the temperature of the drill bit and of the aluminium plate rise to 168°C. (a) What is the diameter of the hole in the aluminium plate when it has cooled to room temperature? (b) Sketch on the same axes graphs of diameter as a function of temperature for the drill bit and for the aluminium plate. $(\alpha_{steel} = 11\times10^{-6}\ K^{-1};\ \alpha_{Al} = 24\times10^{-6}\ K^{-1})$

ANSWERS

Answer Q1

A concrete road has expansion joints at intervals of 20 metres. How wide must each expansion joint be to allow for temperatures as low as -10°C and as high as 4°C?

Take coefficient of thermal expansion of concrete as 10 x 10^{-6} $°C^{-1}$



$L_i = 20m$

$\Delta T = (40 - - 10) = 50°C$

$\alpha = 10 \times 10^{-6}\ °C^{-1}$

$\Delta L = \alpha L_i \Delta T$

$= 10 \times 10^{-6} \times 20 \times 50$

$= 0·01\ m$

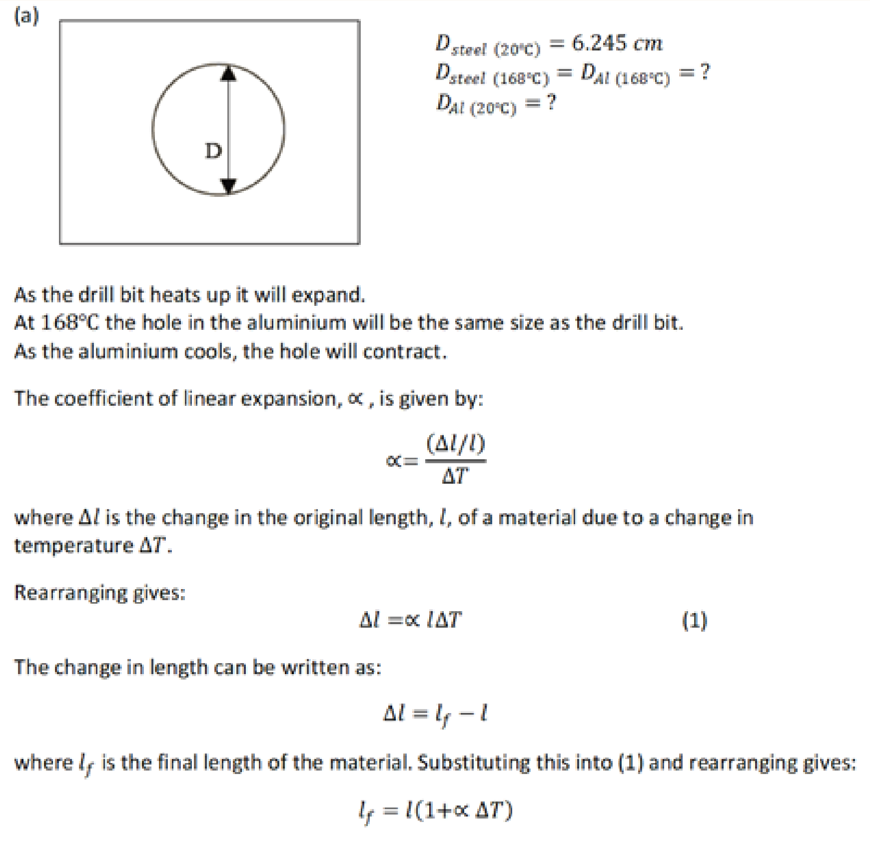
$= 1cm$

Each expansion gap is $\boxed{1cm}$

Answer Q2

A hole is drilled in an aluminium plate with a steel bit whose diameter at 20°C is 6.245cm. In the process of drilling, the temperature of the drill bit and of the aluminium plate rise to 168°C.

(a) What is the diameter of the hole in the aluminium plate when it has cooled to room temperature? (b) Sketch on the same axes graphs of diameter as a function of temperature for the drill bit and for the aluminium plate. $(\alpha_{steel} = 11\times10^{-6}\ K^{-1};\ \alpha_{Al} = 24\times10^{-6}\ K^{-1})$



(a)

$$D_{steel\ (20°C)} = 6.245\ cm$$
$$D_{steel\ (168°C)} = D_{Al\ (168°C)} = ?$$
$$D_{Al\ (20°C)} = ?$$

As the drill bit heats up it will expand.
At 168°C the hole in the aluminium will be the same size as the drill bit.
As the aluminium cools, the hole will contract.

The coefficient of linear expansion, ∝ , is given by:

$$\propto = \frac{(\Delta l/l)}{\Delta T}$$

where Δl is the change in the original length, l, of a material due to a change in temperature ΔT.

Rearranging gives:

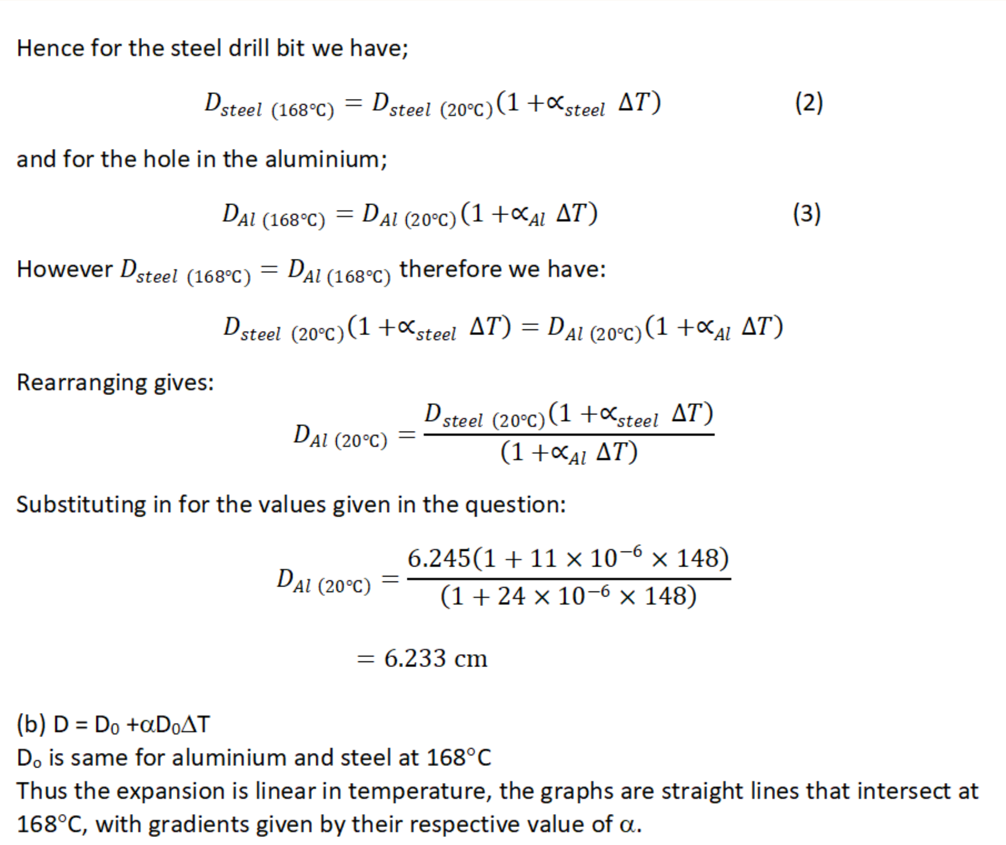
$$\Delta l = \propto l \Delta T \qquad (1)$$

The change in length can be written as:

$$\Delta l = l_f - l$$

where l_f is the final length of the material. Substituting this into (1) and rearranging gives:

$$l_f = l(1 + \propto \Delta T)$$



Hence for the steel drill bit we have;

$$D_{steel\ (168°C)} = D_{steel\ (20°C)}(1 + \propto_{steel}\ \Delta T) \qquad (2)$$

and for the hole in the aluminium;

$$D_{Al\ (168°C)} = D_{Al\ (20°C)}(1 + \propto_{Al}\ \Delta T) \qquad (3)$$

However $D_{steel\ (168°C)} = D_{Al\ (168°C)}$ therefore we have:

$$D_{steel\ (20°C)}(1 + \propto_{steel}\ \Delta T) = D_{Al\ (20°C)}(1 + \propto_{Al}\ \Delta T)$$

Rearranging gives:

$$D_{Al\ (20°C)} = \frac{D_{steel\ (20°C)}(1 + \propto_{steel}\ \Delta T)}{(1 + \propto_{Al}\ \Delta T)}$$

Substituting in for the values given in the question:

$$D_{Al\ (20°C)} = \frac{6.245(1 + 11 \times 10^{-6} \times 148)}{(1 + 24 \times 10^{-6} \times 148)}$$

$$= 6.233\ \text{cm}$$

(b) $D = D_0 + \alpha D_0 \Delta T$

D_o is same for aluminium and steel at 168°C

Thus the expansion is linear in temperature, the graphs are straight lines that intersect at 168°C, with gradients given by their respective value of α.