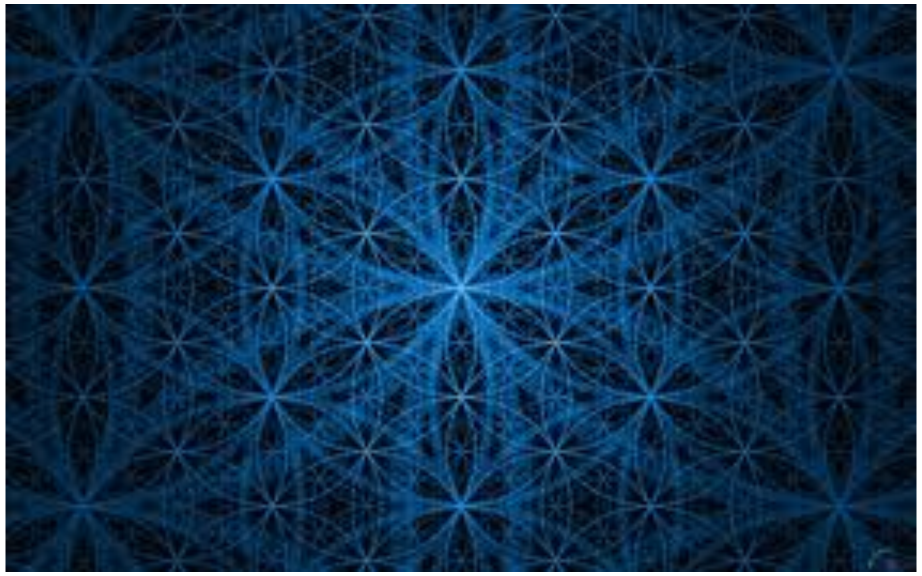
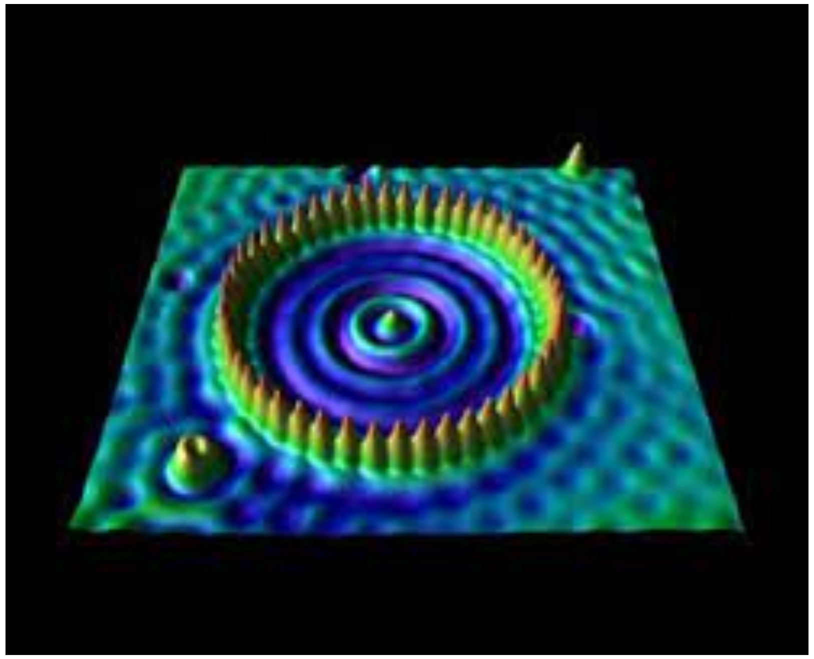
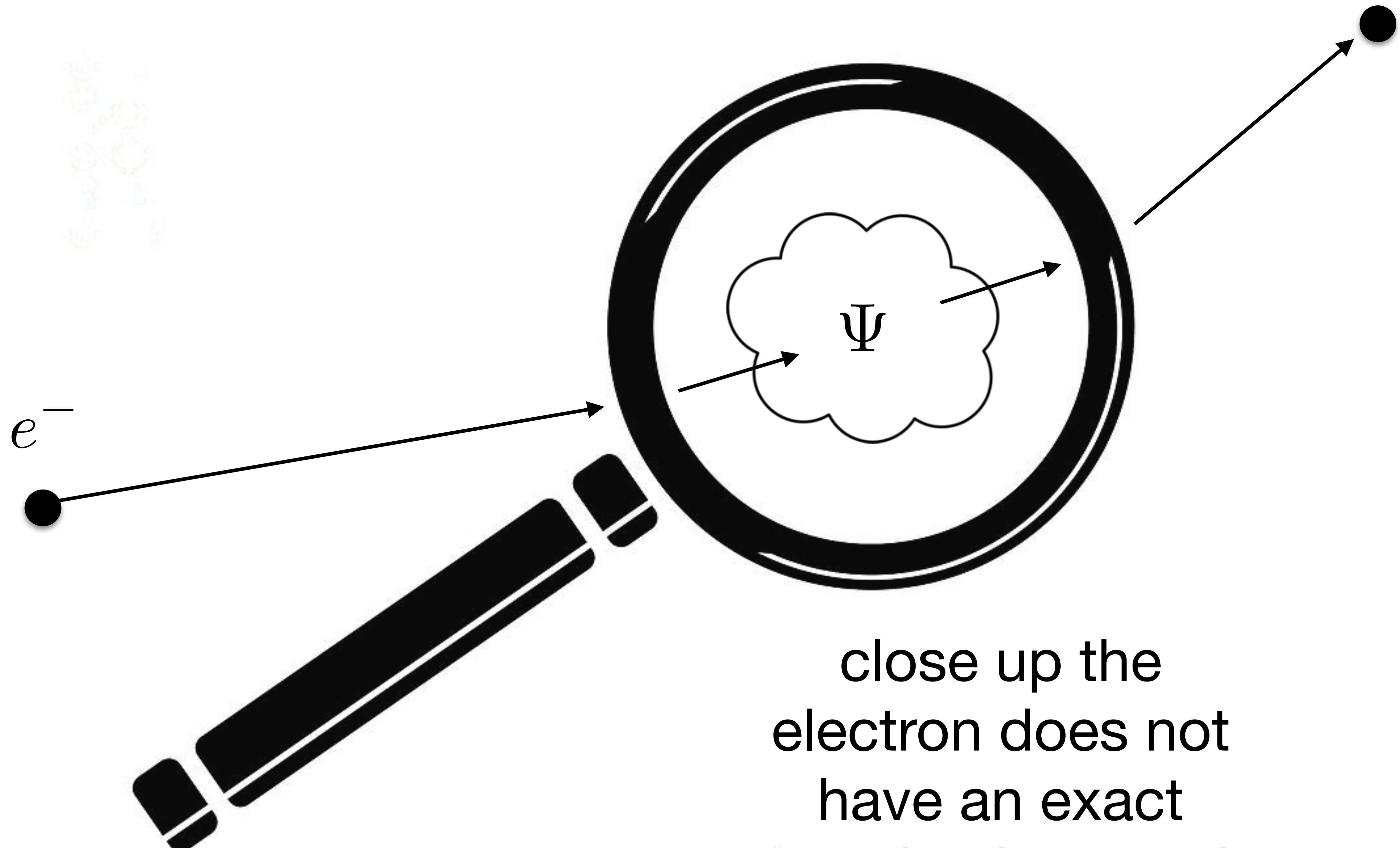
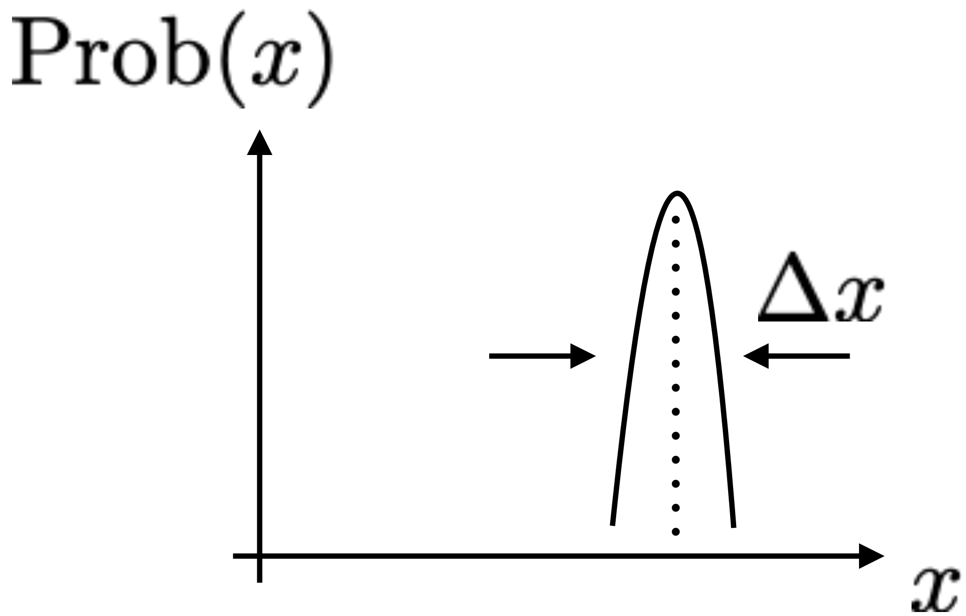


1. Heisenberg fine-print.
2. Quantizations.

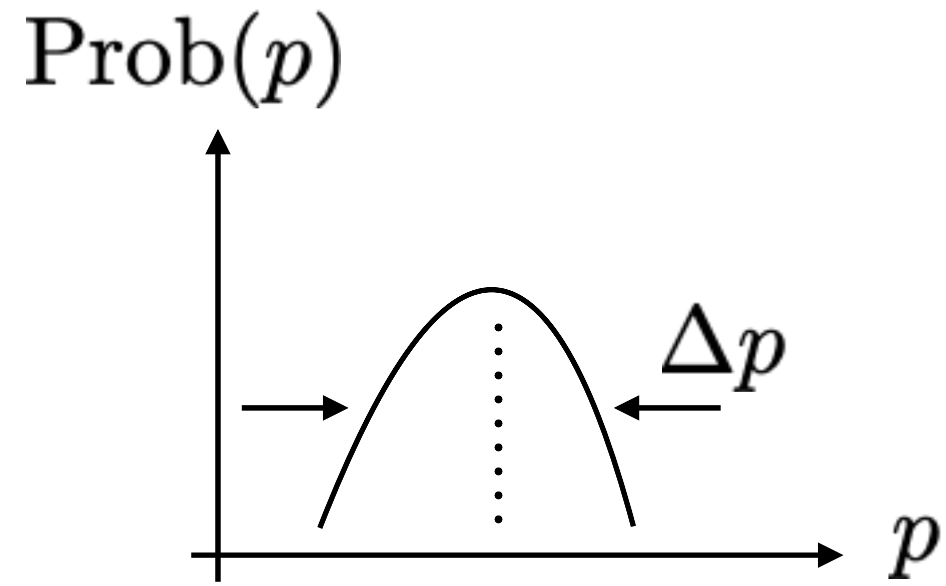




close up the
electron does not
have an exact
location in space!



Δx : width of x distribution



Δp : width of p distribution

$$\boxed{\Delta x \Delta p \geq \frac{\hbar}{2}}$$

$$\hbar = 1.055 \times 10^{-34} (kgm^2s^{-1})$$

A common (incorrect) account of the Heisenberg Uncertainty Relation

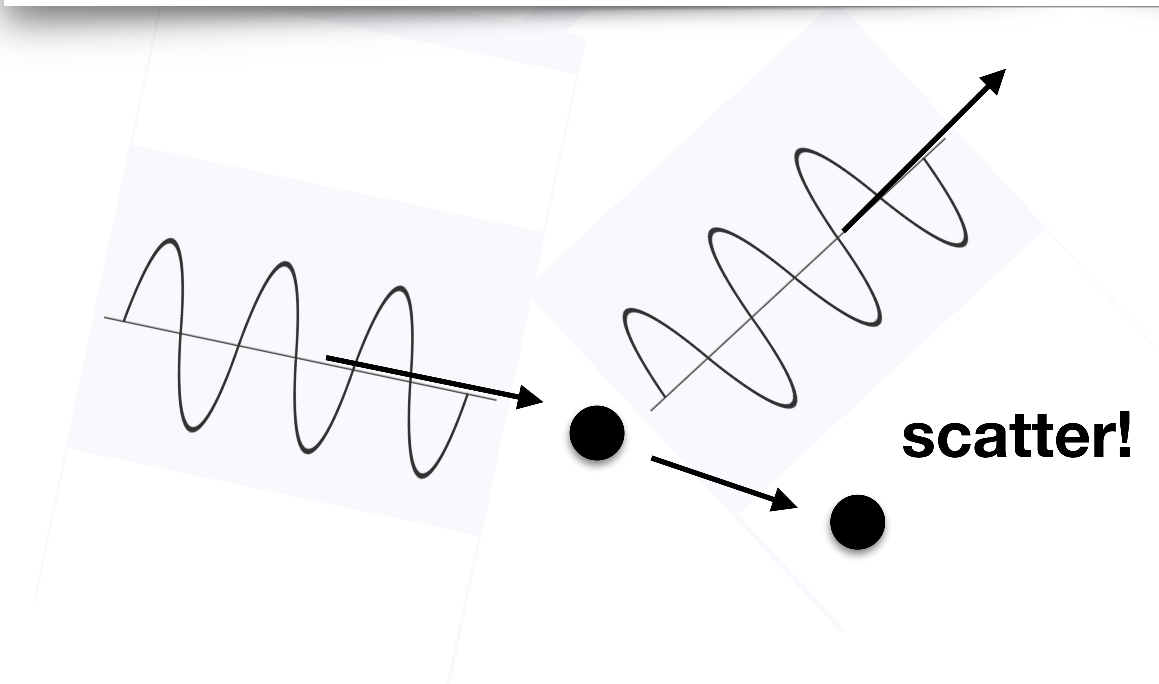
Tipler Chapter 34

The Uncertainty Principle

An important principle consistent with the wave–particle duality of nature is the uncertainty principle. It states that, in principle, it is impossible to simultaneously measure both the position and the momentum of a particle with unlimited precision. A common way to measure the position of an object is to look at the object with light. If we do this, we scatter light from the object and determine the position by the direction of the scattered light. If we use light of wavelength λ, we can measure the position x only to an uncertainty Δx of the order of λ because of diffraction effects.

$$\Delta x \sim \lambda$$

To reduce the uncertainty in position, we therefore use light of very short wavelength, perhaps even X rays. In principle, there is no limit to the accuracy of such a position measurement, because there is no limit on how small the wavelength λ can be.



(a) Heisenberg Uncertainty Relation

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

Describes the separate x & p statistics for any **quantum state**

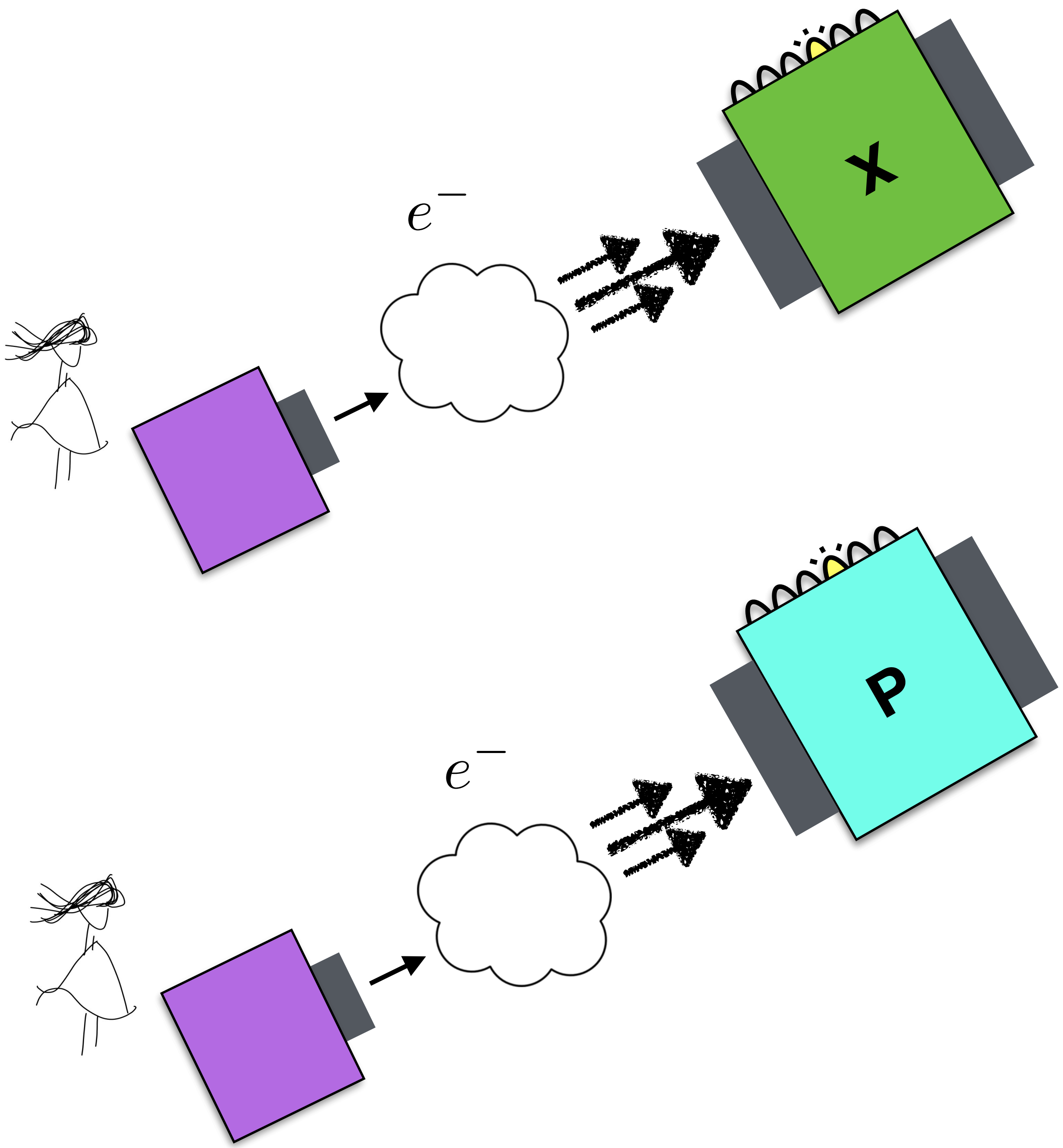
(b) Measurement Disturbance

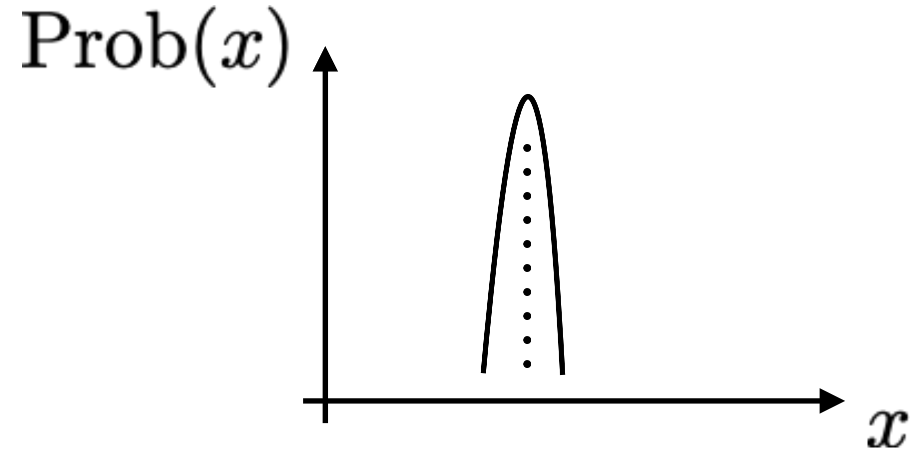
Measuring a quantum system affects the state of the system.

Tipler and others confuse (a) and (b), which is wrong.

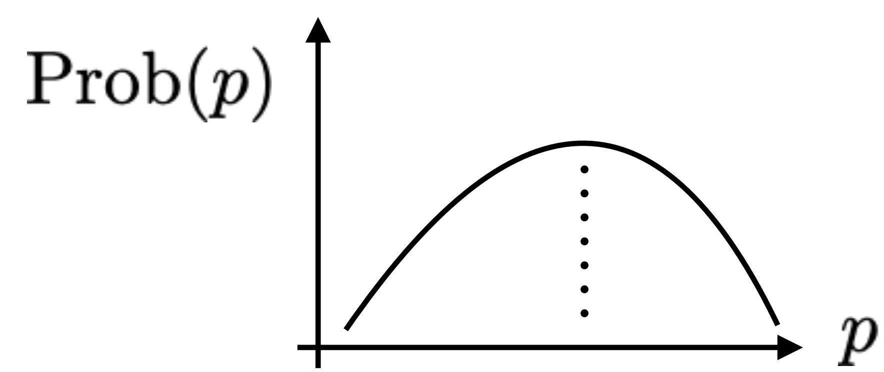
(a) Heisenberg Uncertainty Relation

Run **two separate experiments**, measure x in first and p in second



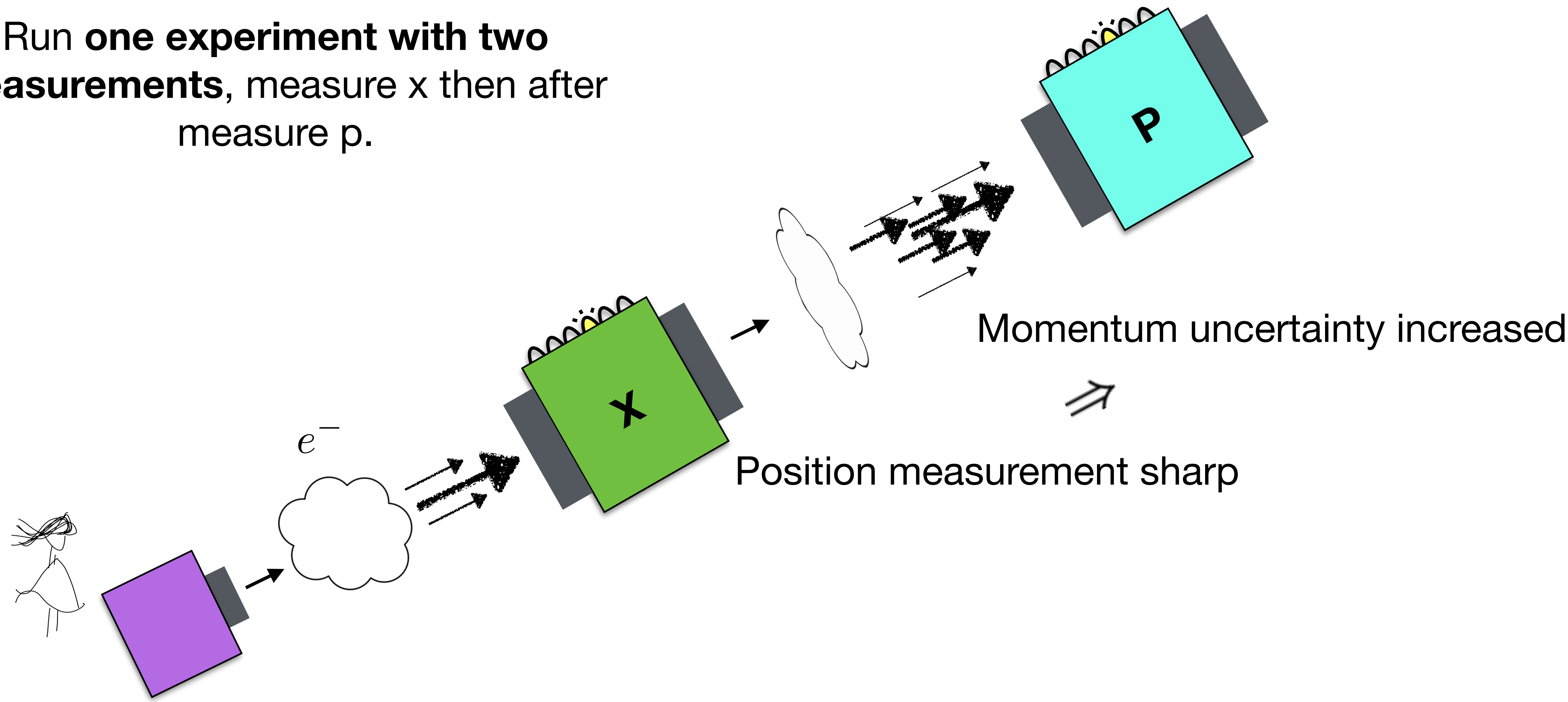


$$\Delta x \Delta p \geq \frac{\hbar}{2}$$



(b) Measurement Disturbance

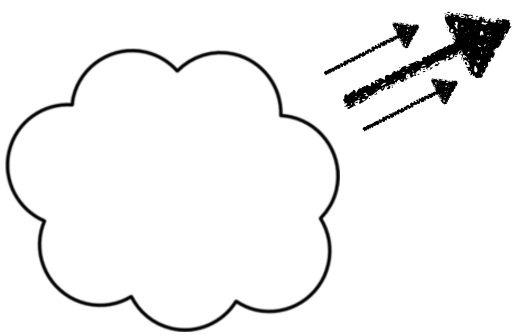
Run **one experiment with two measurements**, measure x then after measure p.

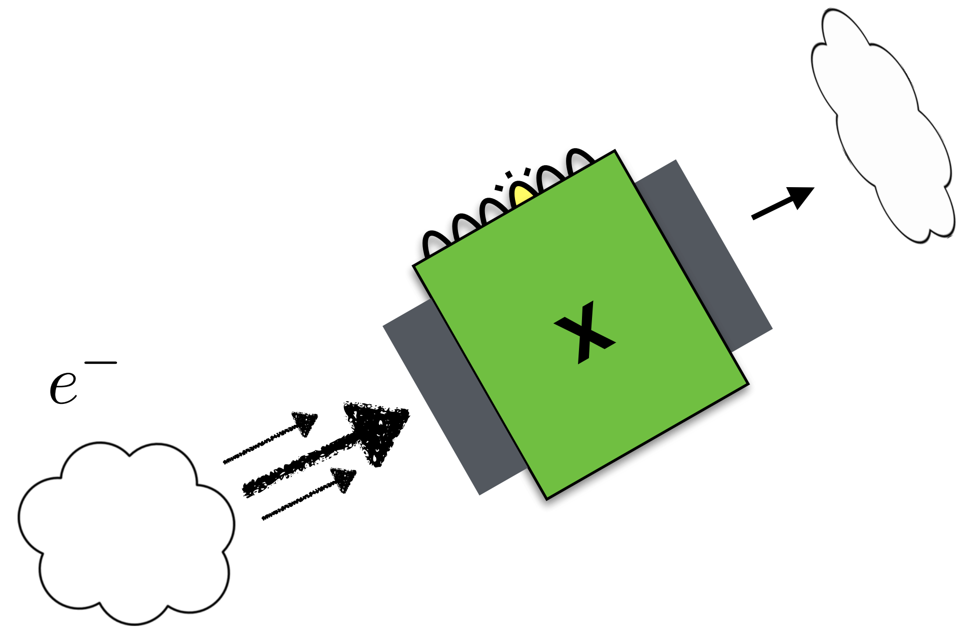


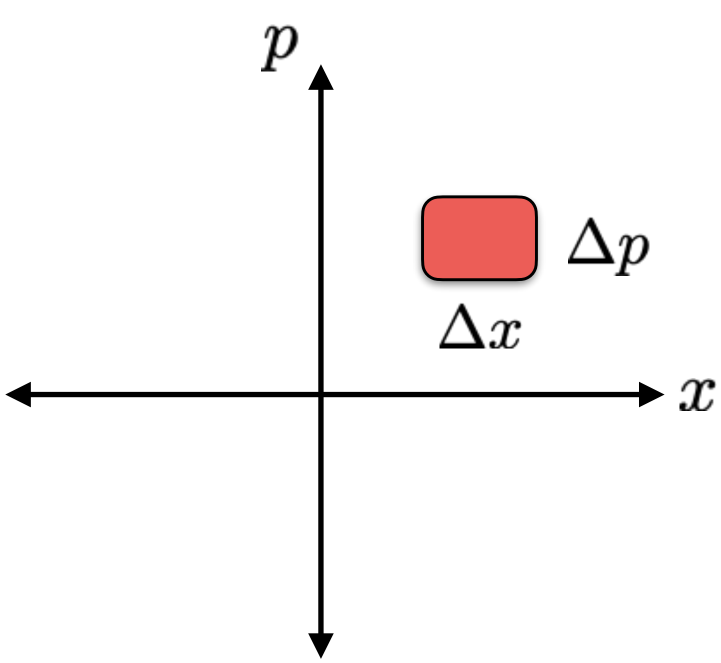
Scenarios (a) and (b) are not the same!

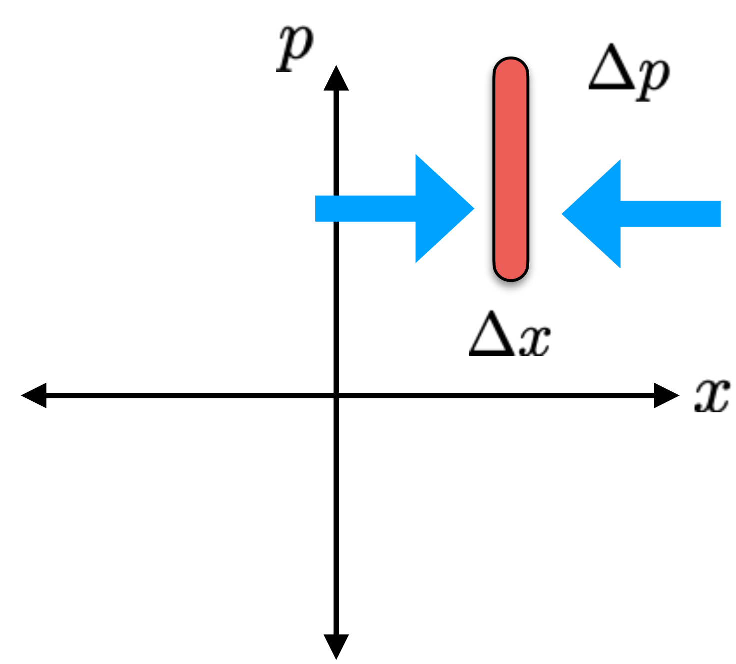
Heisenberg Uncertainty

Measurement Disturbance







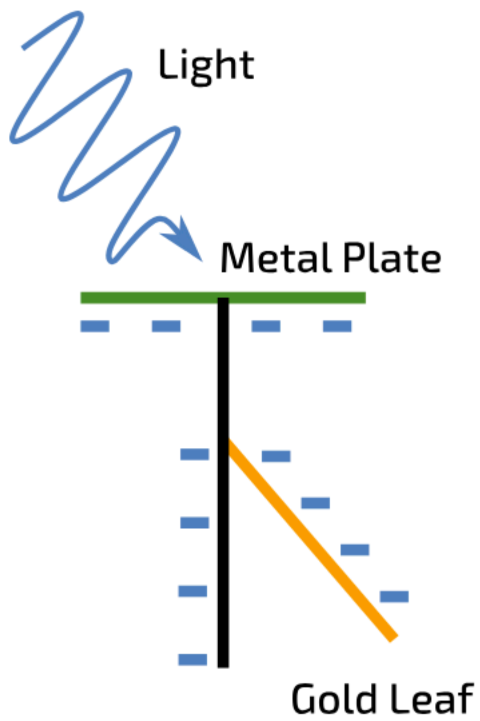


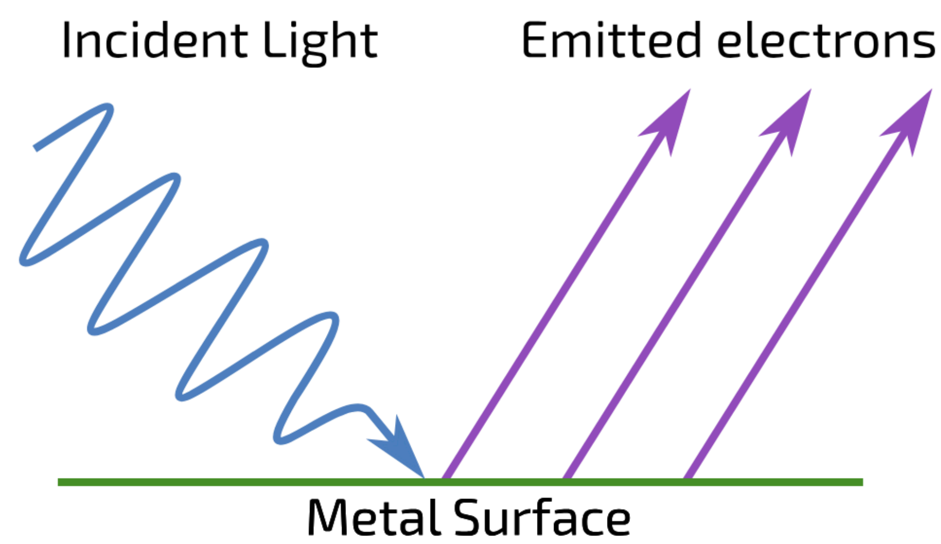
Heisenberg

Not Heisenberg

The Photoelectric effect

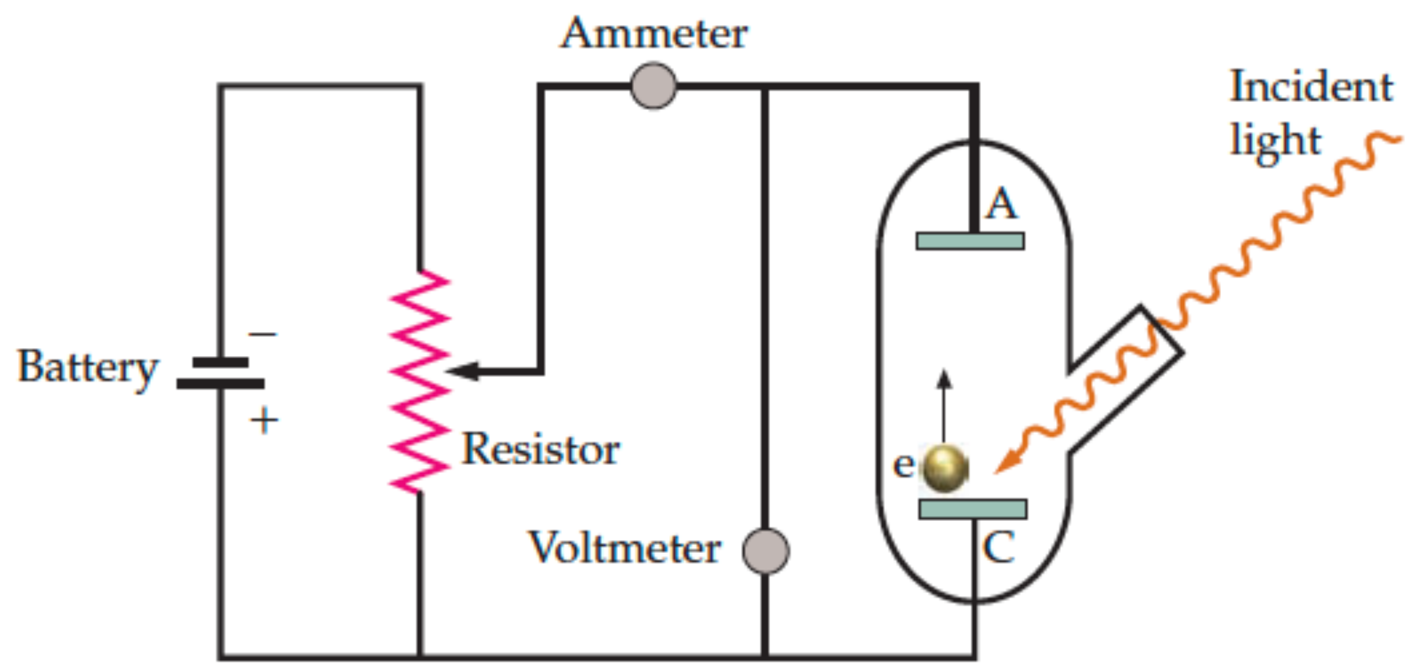
The Dawn of Quantum Physics





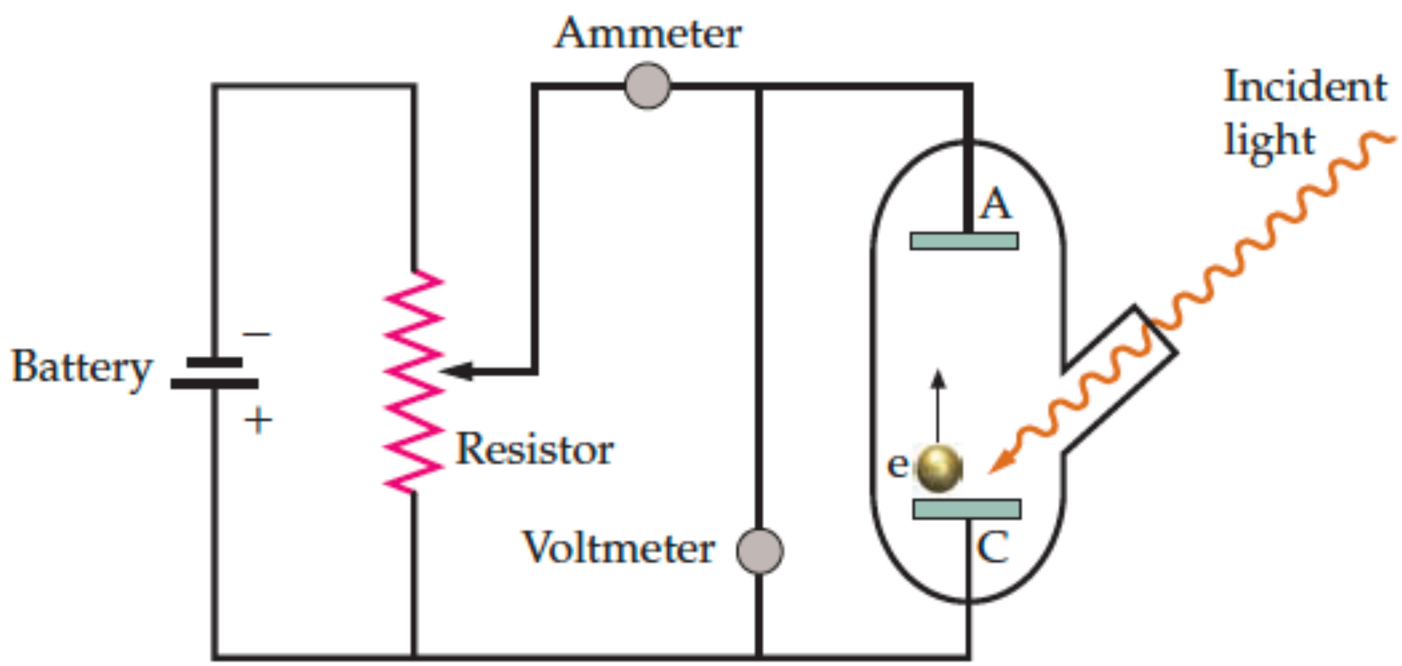
- Classical wave theory of light predicts **energy of light** proportional to its **intensity**.
  Therefore, if we use **low frequency light**, but with **sufficiently high intensity** we can eject electrons.
- Does this happen in reality?

Testing the wave theory of light



- We use monochromatic light (single frequency light **f**) of varying intensity.
- Light strikes the cathode.
- Ammeter measures the current = number of electrons flowing.
- We vary the potential difference **V** between plates.

Testing the wave theory of light

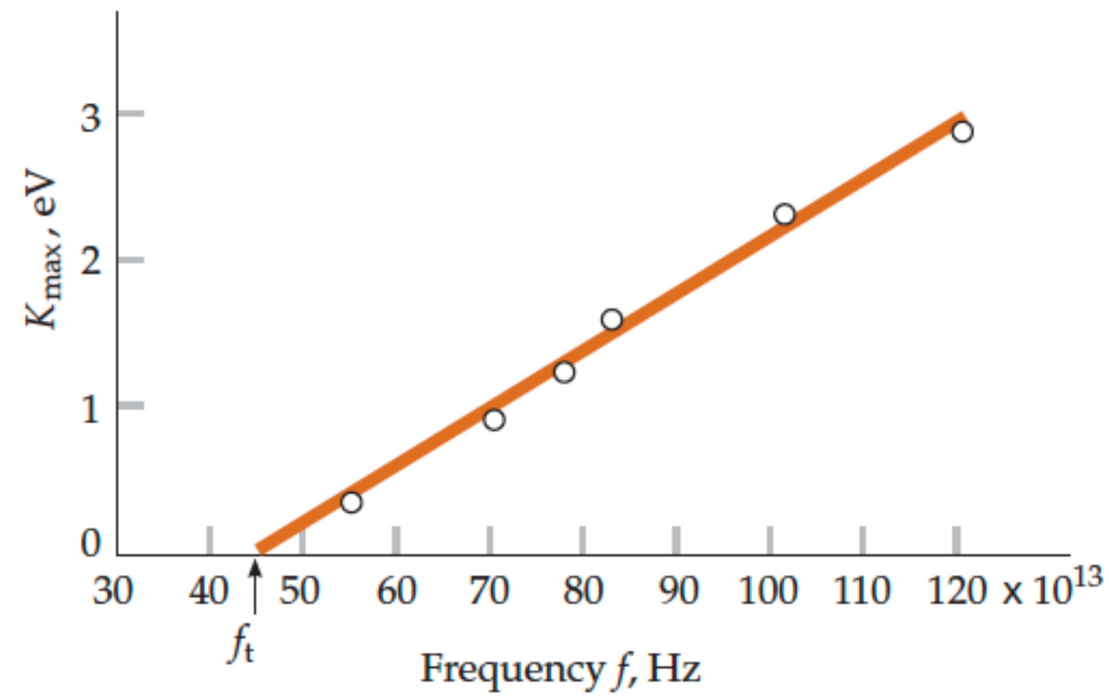


- Discovery 1: For fixed V there is a **threshold frequency** that gives a current.
- Discovery 2: When no current is flowing, increasing the intensity does not generate a current.
- Discovery 3: Energy of electrons grows **linearly** with frequency.

Failure of the wave theory of light

For a *single* electron:

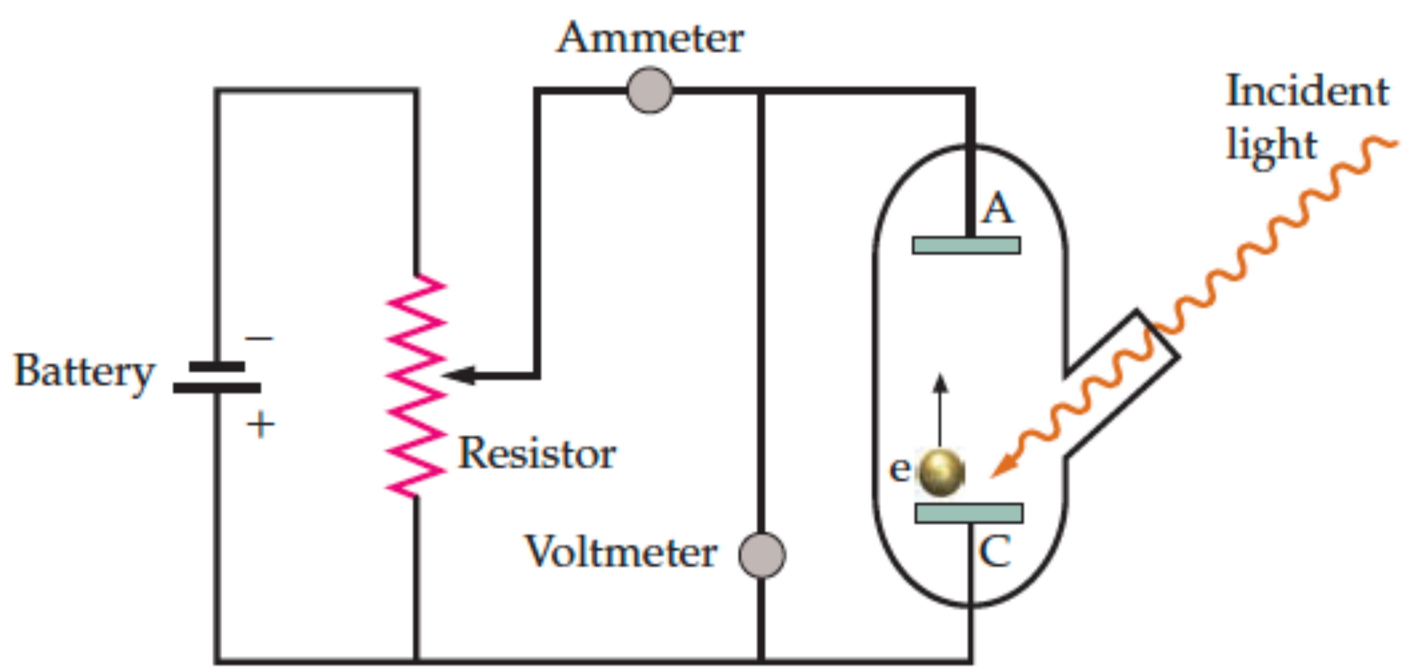
$$E_e = hf - \phi$$



Planck's constant: h
Work function of metal: ϕ

Einstein: the energy of a *single photon* is: $E = hf$

Testing our understanding



- We use light with a **fixed frequency**. A current is flowing.

1. What can we deduce about the **frequency** of the light?
2. If we double the **intensity** of the light what do we expect to happen?
3. What could we do to learn the **work function** of the metal?

Shining light on the matter

How many photons strike a person's face per second?

Given: the intensity of sunlight on Earth is 1,400W/m^2.

$$h = 6.6 \times 10^{-34} Js$$

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Area of a person's face ~ $L^2 = (0.14)^2 = 0.0196m^2$

$$\Rightarrow \text{Energy per second incident: } (1400)(0.0196) = 27.44W$$

$$(\text{Compare with lightbulb: } \sim 60W)$$

Shining light on the matter

How many photons strike a person's face per second?

Given: the intensity of sunlight on Earth is 1,400W/m^2.

$$h = 6.6 \times 10^{-34} Js$$

$$27.44W = 27.44 \text{ Joules per second}$$

$$\text{One photon: } E = hf$$

$$\Rightarrow N \text{ photons: } E_{tot} = NE = Nhf$$

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Given: the intensity of sunlight on Earth is 1,400W/m^2.

$$h = 6.6 \times 10^{-34} Js$$

$$Nhf = 27.44$$

$$\Rightarrow N = \frac{27.44}{hf} = \frac{27.44}{6.6 \times 10^{-34} f}$$

what frequency to use?

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what frequency to use?

Let's take $\lambda \sim 500nm$ $\quad \Rightarrow f = \dfrac{c}{500 \times 10^{-9}} = 6 \times 10^{14} Hz$

Shining light on the matter

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Given: the intensity of sunlight on Earth is 1,400W/m^2.

$$h = 6.6 \times 10^{-34} Js$$

$$Nhf = 27.44$$

$$\Rightarrow N = \frac{27.44}{hf} = \frac{27.44}{6.6 \times 10^{-34}(6 \times 10^{14})}$$

$$= \boxed{7 \times 10^{19} \text{ photons per second}}$$

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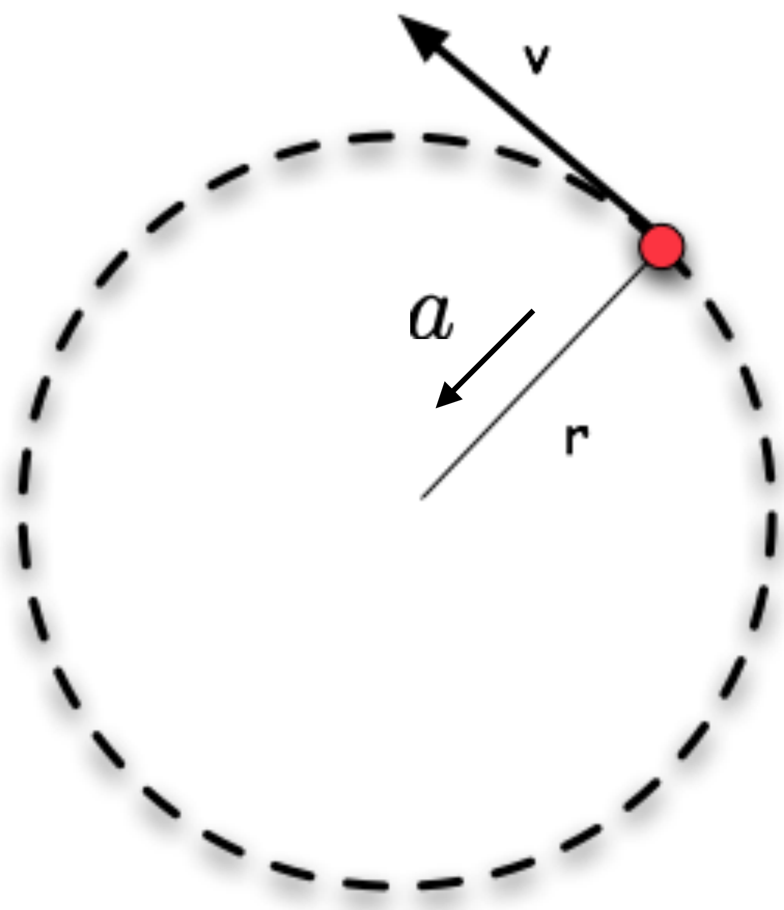
Question: Is this a good estimate? How could we make it better?

The Bohr atom

Classical instability

Model an atom as electrons circling a nucleus

$$a = -\frac{v^2}{r}$$



BUT: classical electromagnetism tells us that an accelerating charge radiates energy

Classical instability

Model an atom as electrons circling a nucleus

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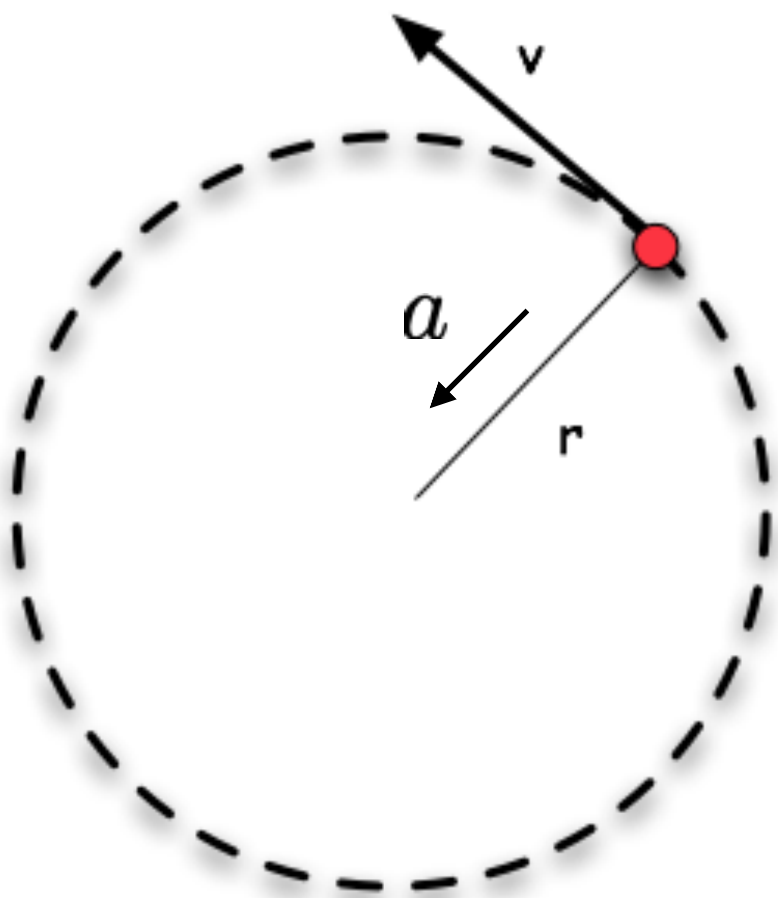


BUT: classical electromagnetism tells us that an accelerating charge radiates energy

Therefore electrons should lose energy and spiral into centre.

$$t \sim 10^{-11} s$$

Central Question: So how can matter be stable??

An observation

Heisenberg
Uncertainty Relation

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

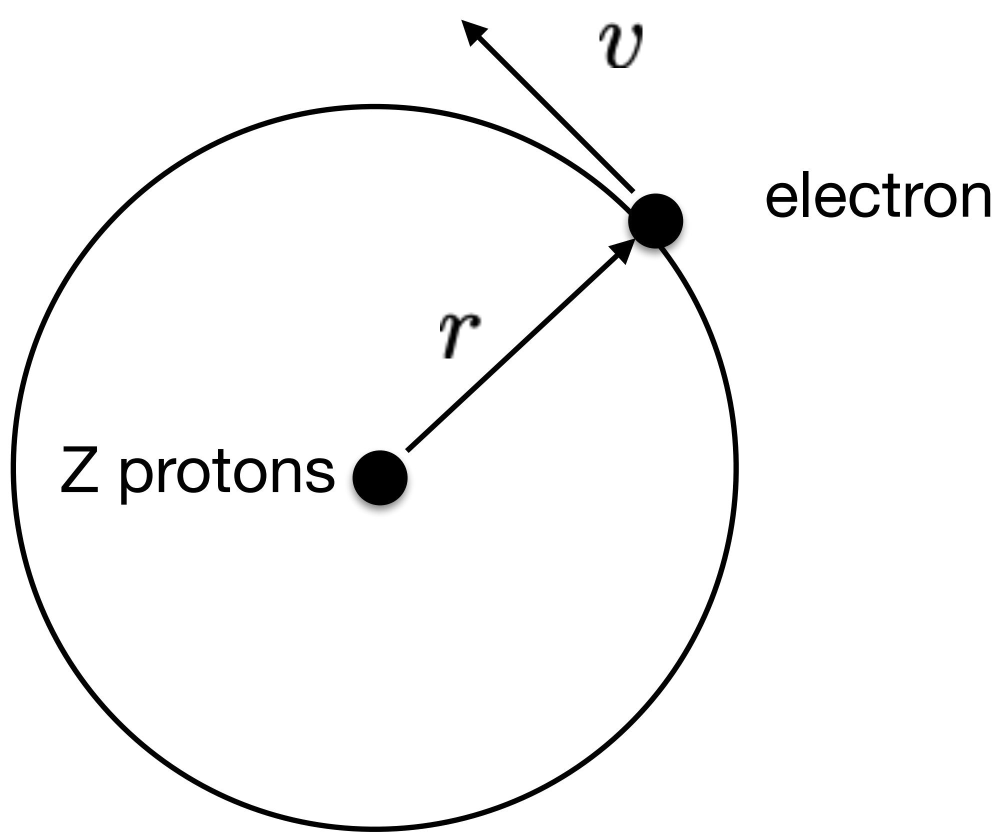
LHS has units of
angular momentum

So $\hbar$ also has **units of angular momentum**

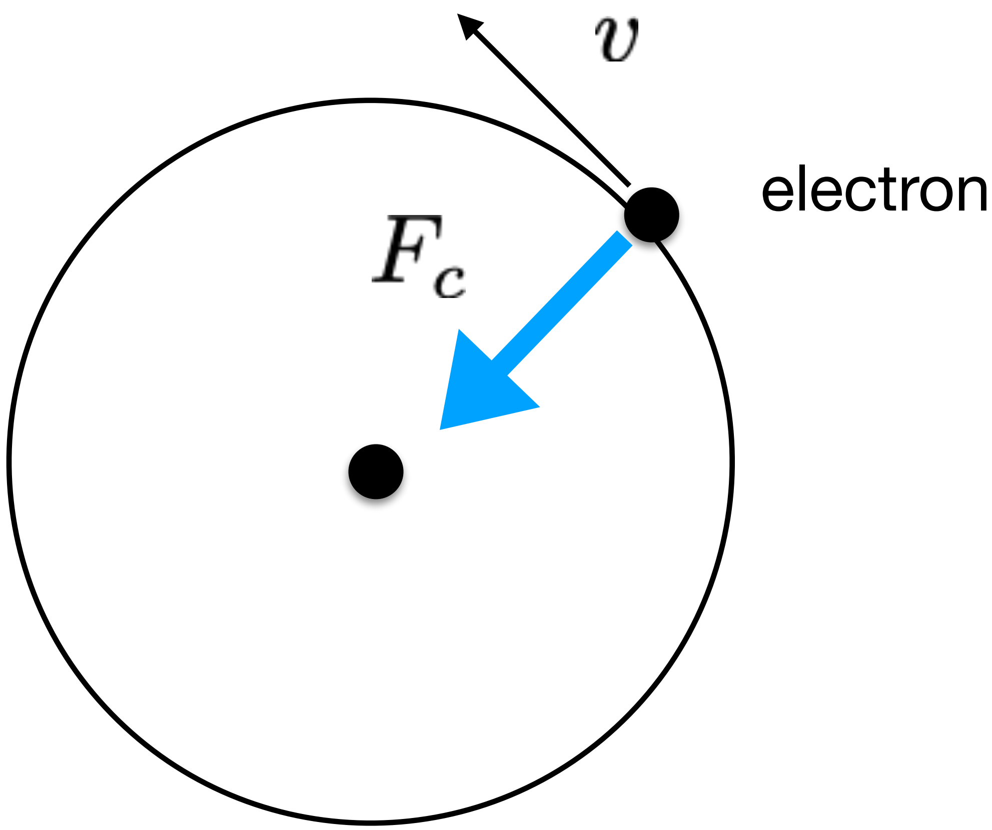
The key assumption

Bohr's assumption:

The angular momentum of an electron orbiting the nucleus is quantised in multiples of $\hbar$.



$$mvr = n\hbar$$
$$n = 1, 2, 3, 4, \ldots$$



$$F_c = \text{ centripetal force on electron}$$

$$F_c = \frac{mv^2}{r}$$

The only force acting on electron is the Coulomb force.

$$F_{\text{Coul}} = -\frac{1}{4\pi\epsilon_0}\frac{Qq}{r^2}$$

$$\Rightarrow \frac{mv^2}{r} = -\frac{1}{4\pi\epsilon_0}\frac{Qq}{r^2}$$

The derivation

$$\frac{mv^2}{r} = -\frac{1}{4\pi\epsilon_0}\frac{Qq}{r^2}$$

$$Qq = -e^2$$

$$\Rightarrow \boxed{v^2 = \frac{1}{4\pi\epsilon_0}\frac{e^2}{mr}}$$

Now include Bohr's fundamental assumption: $L = r(mv) = n\hbar$

$$n = 1, 2, 3, 4, \ldots$$

$$\Rightarrow \boxed{mvr = n\hbar}$$

We have the following two equations:

$$mvr = n\hbar \qquad n = 1, 2, 3, 4, \ldots$$

$$v^2 = \frac{1}{4\pi\epsilon_0}\frac{e^2}{mr}$$

Eliminate r and solve for v:

$$r = \frac{n\hbar}{mv} \Rightarrow v^2 = \frac{1}{4\pi\epsilon_0}\frac{e^2}{\left(\frac{n\hbar}{v}\right)}$$

$$\Rightarrow \boxed{v = \frac{1}{4\pi\epsilon_0}\frac{e^2}{n\hbar}}$$

Velocity quantised!

$$n = 1, 2, 3, 4, \ldots$$

Now solve for r: $$v = \frac{1}{4\pi\epsilon_0}\frac{e^2}{n\hbar} \qquad r = \frac{n\hbar}{mv}$$

$$\Rightarrow r = \frac{n\hbar}{m}\left(\frac{4\pi\epsilon_0 n\hbar}{e^2}\right)$$

$$\Rightarrow \boxed{r = \frac{4\pi\epsilon_0(n\hbar)^2}{me^2}}$$

quantised orbits!

$$n = 1, 2, 3, 4, \ldots$$

$$r_n = \frac{4\pi\epsilon_0 n^2\hbar^2}{me^2} = a_1 n^2$$

$$a_1 = \frac{4\pi\epsilon_0\hbar^2}{me^2} = 0.0529nm$$

radius of innermost orbit

Energy spectrum of the Bohr atom

Total energy of the electron:

$$E = \frac{1}{2}mv^2 - \frac{1}{4\pi\epsilon_0}\frac{e^2}{r}$$

The quantised velocity and radius:

$$r_n = \frac{4\pi\epsilon_0 n^2\hbar^2}{me^2} \qquad v_n = \frac{1}{4\pi\epsilon_0}\frac{e^2}{n\hbar}$$

$$\Rightarrow E_n = \frac{1}{2}\frac{me^4}{(4\pi\epsilon_0)^2 n^2\hbar^2} - \frac{e^2}{4\pi\epsilon_0}\left(\frac{me^2}{4\pi\epsilon_0 n^2\hbar^2}\right)$$

Energy Spectrum

$$E_n = \frac{1}{2}\frac{me^4}{(4\pi\epsilon_0)^2 n^2\hbar^2} - \frac{e^2}{4\pi\epsilon_0}\left(\frac{me^2}{4\pi\epsilon_0 n^2\hbar^2}\right)$$

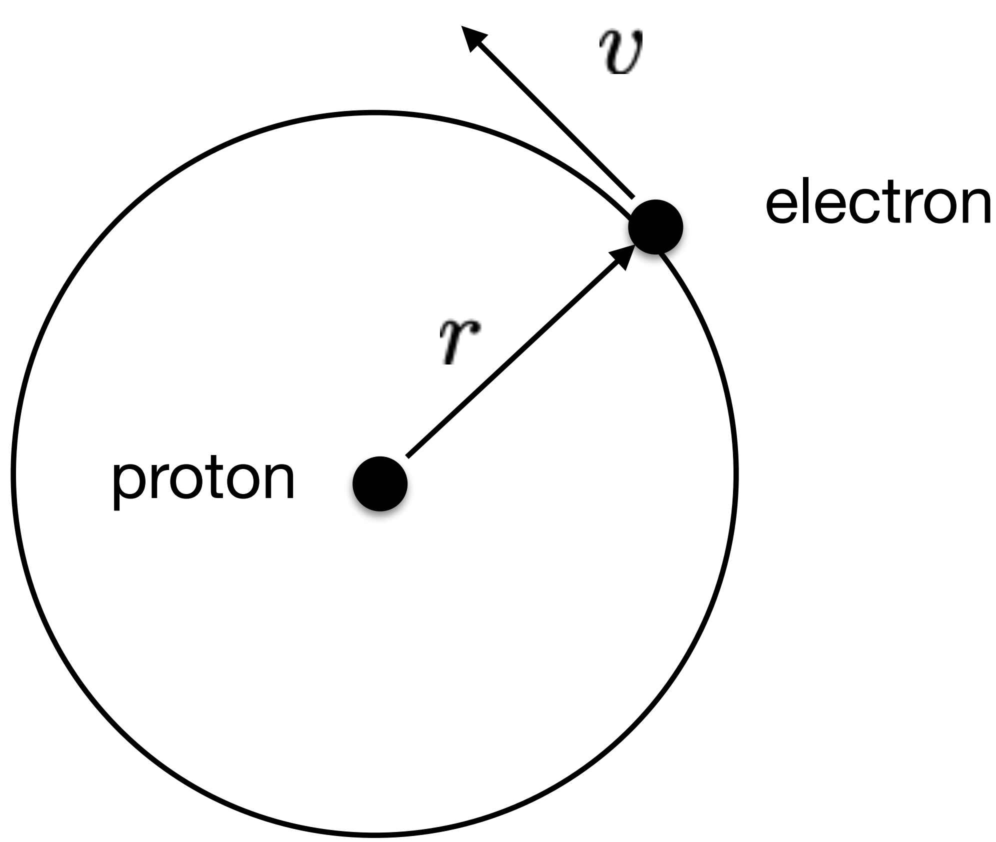
$$\Rightarrow E_n = -\frac{me^4}{32\pi^2\epsilon_0^2}\frac{1}{n^2} \qquad n = 1, 2, 3, 4, \ldots$$

$$\Rightarrow E_n = -\frac{E_1}{n^2} \qquad E_1 = 13.6eV$$

Recap: the key assumption

Bohr's assumption:

The angular momentum of an electron orbiting the nucleus is quantised in multiples of $\hbar$.



$$mvr = n\hbar$$
$$n = 1, 2, 3, 4, \ldots$$

Key Bohr Atom properties

Circular motion + Bohr's assumption:

Quantised energies

$$E_n = -\frac{me^4}{32\pi^2\epsilon_0^2}\frac{1}{n^2}$$

$$\Rightarrow E_n = -\frac{E_1}{n^2}$$

$$E_1 = 13.6eV$$

Quantised orbits

$$r_n = \frac{4\pi\epsilon_0 n^2\hbar^2}{me^2} = a_1 n^2$$

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$$n = 1, 2, 3, 4, \ldots$$