Optimization flow for the complete relative self gravity calculations

Computational Science II - University of Zurich

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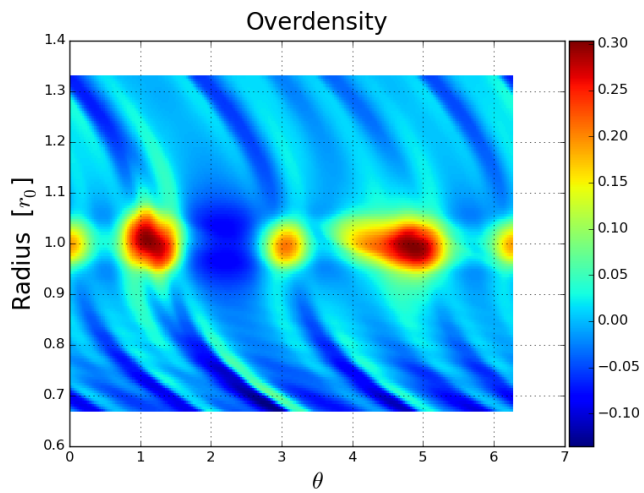
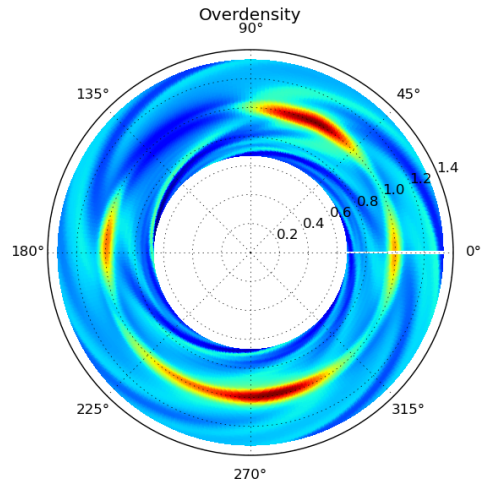




Raw code

$$\vec{a}_r^g(i,j) = -G \cdot \Delta r \cdot \Delta\theta \cdot \sum_{k=1}^{N_r} \sum_{l=1}^{N_t} \frac{\sigma(k,l)\,[r_c(i) - r(k) \cdot (cos(\theta_c(j) - \theta(l))]}{\left[r_c(i)^2 + r(k)^2 - 2 \cdot r_c(i) \cdot r(k) \cdot cos\left(\theta_c(j) - \theta(l)\right)\right]^{3/2}} \tag{1}$$

$$\vec{a}_\theta^g(i,j) = -G \cdot \Delta r \cdot \Delta\theta \cdot \sum_{k=1}^{N_r} \sum_{l=1}^{N_t} \frac{\sigma(k,l)\,[r(k) \cdot (sin(\theta_c(j) - \theta(l))]}{\left[r_c(i)^2 + r(k)^2 - 2 \cdot r_c(i) \cdot r(k) \cdot cos\left(\theta_c(j) - \theta(l)\right)\right]^{3/2}} \tag{2}$$

Raw code

```
G   = 1
dr  = radius(2)-radius(1)
dth = theta(2) - theta(1)
do i = 1, Nr
      do j = 1, Nt


          acc(1) = 0
          acc(2) = 0
          do k = 1,Nr
             do l = 1, Nt
               acc(1) = acc(1) + density(k,l)*dr*dth*radius(k)* &
                        (radius(i) +dr/2 -radius(k)*cos(theta(j)-theta(l)+dth/2))&
                        /(radius(k)**2 + (radius(i)+dr/2)**2 -2*radius(k) &
                        *(radius(i)+dr/2)*cos(theta(l)-theta(j)+dth/2))**(1.5)

               acc(2) = acc(2) + density(k,l)*dr*dth*radius(k)* &
                        sin(theta(j)-theta(l)+dth/2) &
                        /(radius(k)**2 + (radius(i)+dr/2)**2 -2*radius(k) &
                        *(radius(i)+dr/2)*cos(theta(l)-theta(j)+dth/2))**(1.5)
             enddo
           enddo


           acc_r(i,j) = acc(1)
           acc_t(i,j) = acc(2)
           write(5,*) acc_r(i,j)
           write(6,*) acc_t(i,j)
        enddo
enddo
```

Precalculations, reduction and symmetry

Mathematical reformulation

$$\vec{a}_r^g(i,j) = -G \cdot \Delta r \cdot \Delta\theta \cdot \sum_{k=1}^{N_r} \sum_{l=1}^{N_t} \frac{\sigma(k,l) \left[\frac{r_c(i)}{r(k)} - (cos(\theta_c(j) - \theta(l)) \right]}{r(k)^2 \left[1 + \frac{r_c(i)^2}{r(k)^2} - 2 \cdot \frac{r_c(i)}{r(k)} \cdot cos\left(\theta_c(j) - \theta(l)\right) \right]^{3/2}} \tag{3}$$

$$\vec{a}_r^g(i,j) = -G \cdot \Delta r \cdot \Delta\theta \cdot \sum_{k=1}^{N_r} \sum_{l=1}^{N_t} \frac{\sigma(k,l) \left(sin(\theta_c(j) - \theta(l)) \right)}{r(k)^2 \left[1 + \frac{r_c(i)^2}{r(k)^2} - 2 \cdot \frac{r_c(i)}{r(k)} \cdot cos\left(\theta_c(j) - \theta(l)\right) \right]^{3/2}} \tag{4}$$

Precalculations, reduction and symmetry

Precompute

- $\frac{r_c(i)}{r(k)}$
- $\frac{r_c(i)^2}{r(k)^2}$
- $\frac{1}{r(k)^2}$
- $cos\left(\theta_c(j) - \theta(l)\right)$
- $sin\left(\theta_c(j) - \theta(l)\right)$
- other $N_\theta \cdot N_r$, N_r^2, N_θ^2

Symmetry

- $cos(\alpha + \pi) = -cos(\alpha)$
- $sin(\alpha + \pi) = -sin(\alpha)$

Reduce the number of calculations

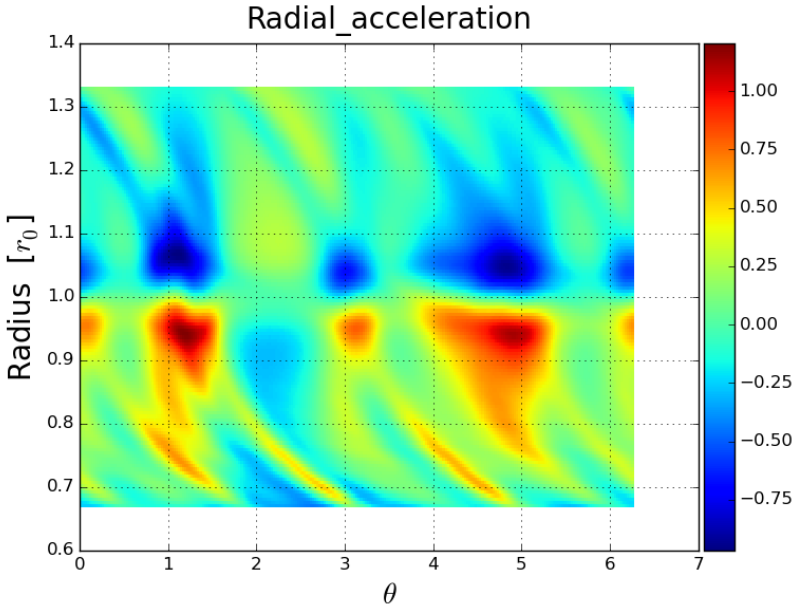
Final code

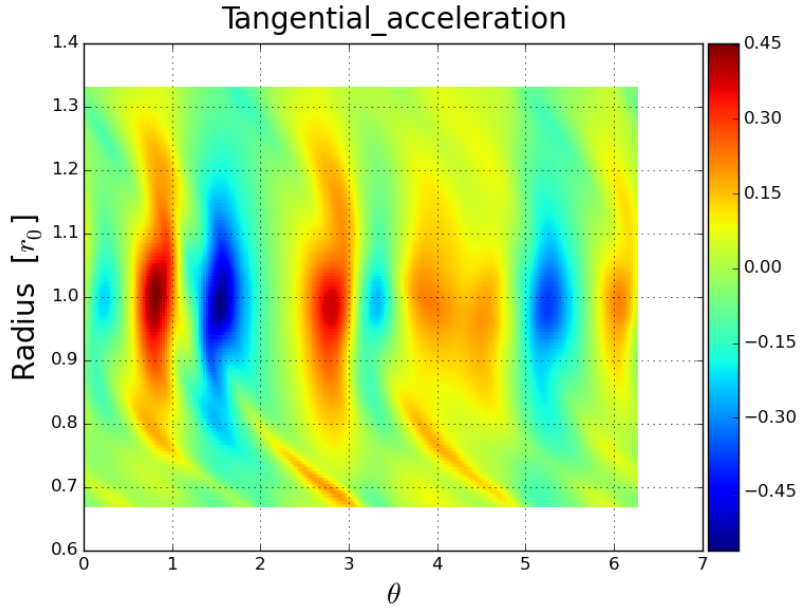
```
do i = 1, nr
        do j = 1, nt/2
        !acceleration calculations for a single cell
                acc(1) = 0
                acc(2) = 0
                acc(3) = 0
                acc(4) = 0
                do k = 1,nr
                        do l = 1, nt
                                !reducing double operations
                                prod    = ratio(k,i)*2*cos_diff(l,j)

                                rad     = ratio_f(k,i) - prod

                                den     = rad * sqrt(rad)
                                comm    = mass(k,l)*radius_corn_2_inv(i)/den
                                !acc calculations

                                acc(1) = acc(1) + comm* &
                                        (ratio(k,i) - cos_diff(l,j))

                                acc(2) = acc(2) + comm* &
                                        sin_diff(l,j)
```

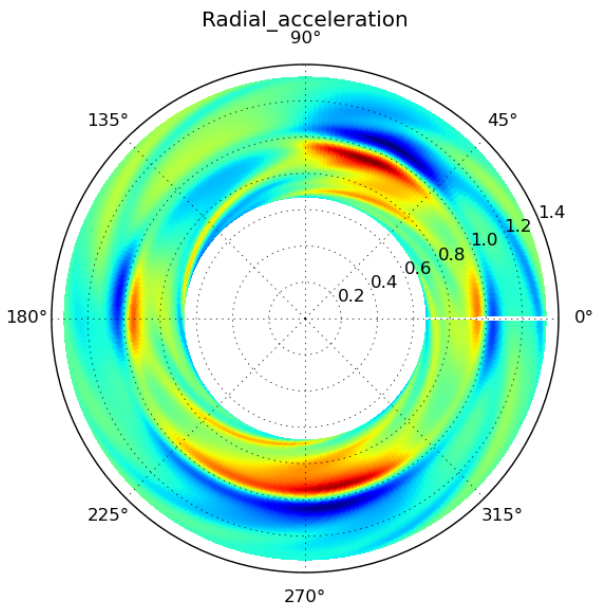
Final code

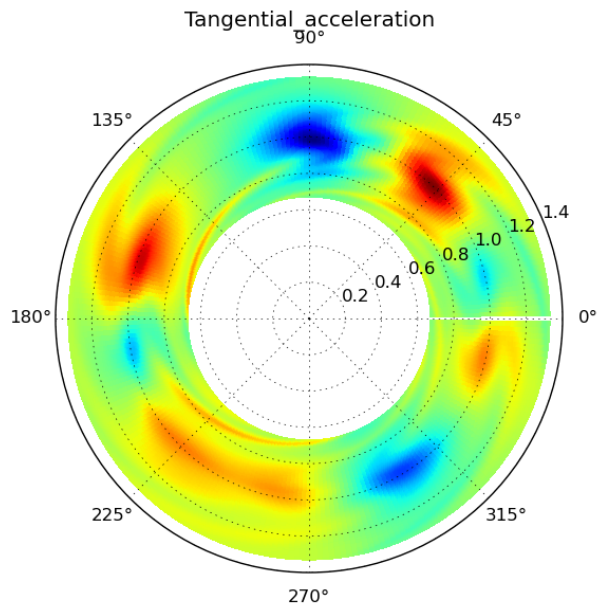
```
                                    !exploiting symmetry in theta along j
                                    rad      = ratio_f(k,i) + prod
                                    den      = rad * sqrt(rad)
                                    comm     = mass(k,l)*radius_corn_2_inv(i)/den

                                    acc(3) = acc(3) + comm* &
                                             (ratio(k,i) + cos_diff(l,j))

                                    acc(4) = acc(4) - comm* &
                                             sin_diff(l,j)

                          enddo
                    enddo

                    acc_r(i,j) = acc(1)
                    acc_t(i,j) = acc(2)

                    acc_r(i,j+nt/2) = acc(3)
                    acc_t(i,j+nt/2) = acc(4)


          enddo
enddo
```

Results


Radial_acceleration
Radius $[r_0]$
θ


Tangential_acceleration
Radius $[r_0]$
θ


Radial_acceleration


Tangential_acceleration

Time measurements and number of calculations

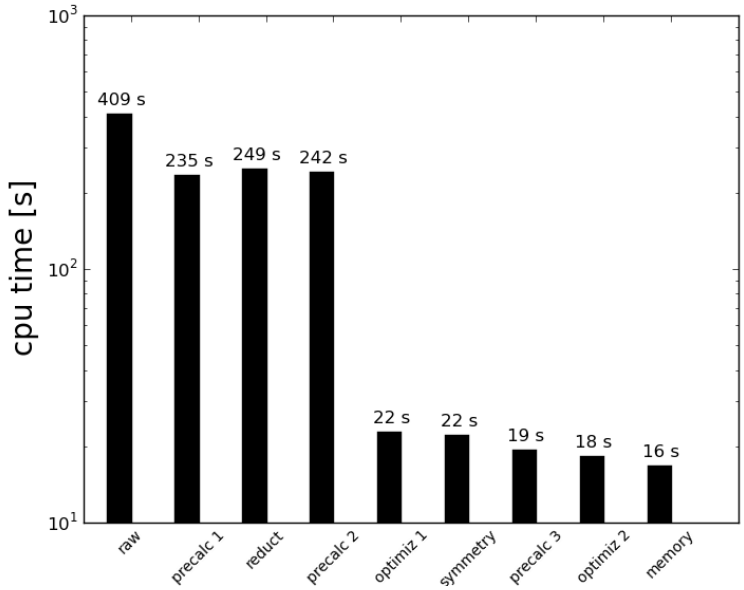


Table: Number of operations $\times \frac{N_r^2 \cdot N_\theta^2}{2}$

Operation type	+	-	*	/	cos	sin	**	sqrt	var
Raw	26	14	30	24	6	2	6	0	2
Final	5	3	10	2	0	0	0	2	11