Measuring the Efficacy of Magnetic Halbach Arrays

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MATH4581, April 22, 2024

I. INTRODUCTION

Halbach arrays are arrangements of magnets that have significant benefits over simpler magnet layouts. They consist of at least three magnets where one magnet's north end is pointed towards a chosen direction, and two magnets sit on opposite sides of the first magnet while pointing their magnetic north faces towards the central magnet. This pattern naturally continues for the south end of the magnets and continues to form strong magnetic peaks of alternating north-south directions. The strong magnetic fields of this Halbach array pattern allow engineers in industry to enhance their electric motors with more torque, power, and passive stability. However, the ideal properties of Halbach arrays require specific materials that many hobbyists do not have access to. Therefore, this paper uses ANOVAs and Bonferroni tests to attempt to answer whether cheap Amazon N52 magnets can create a noticeably stronger magnetic field in a Halbach configuration. The question this paper addresses is: **With cheap N52 magnets, is there a noticeable difference between magnets laid out in a typical array and magnets laid out in a Halbach array? Are the Halbach arrays stronger?**

This paper tests four configurations: Single Magnet, 3-Magnet Halbach, 5-Magnet Halbach, and 3-Magnet No Halbach. Each configuration theoretically amplifies magnetic fields differently. The Single Magnet is the control. The 3-Magnet Halbach amplifies only one north face, the 5-Magnet Halbach amplifies a north face and partially amplifies two south faces. This could affect the magnetic strength of the single north end. The 3-Magnet No Halbach removes amplification from the 5-Magnet Halbach configuration. It should not be significantly different from the Single Magnet. Figures 1-4 below show the placement and orientation of each array.

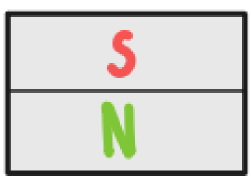

Figure 1 Polar configuration of Single Magnet

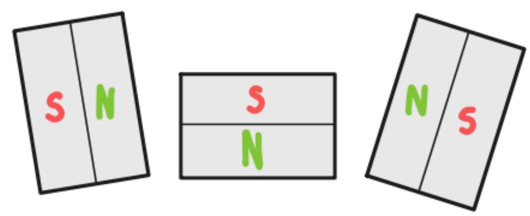

Figure 2 Polar configuration of 3-Magnet Halbach

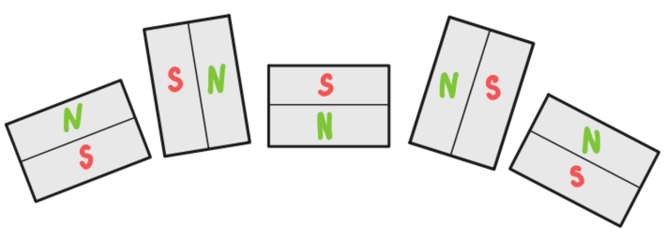

Figure 3 Polar configuration of 5-Magnet Halbach

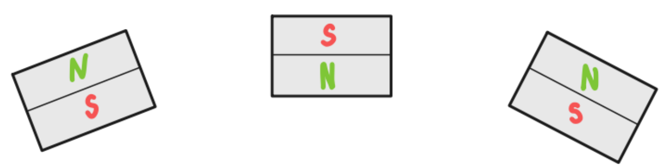

Figure 4 Polar configuration of 3-Magnet No Halbach

The configurations were tested using an ANOVA to check whether there exists a significant difference between the groups. Then, a Bonferroni test was done to assess how each pair of configurations differed from others. The Bonferroni test utilized the following equation for calculating the test statistic, where A and B are two different datasets, x-bar is the dataset's average, s_p is the standard deviation of all available data in the experiment, and n is the number of samples per dataset:

$$t.s. = \frac{\overline{x_A} - \overline{x_B}}{s_p\sqrt{\frac{1}{n_A} + \frac{1}{n_B}}} \tag{1}$$

The test statistic (t.s.) for each pair could be compared to the critical value (t*) from the t-distribution table. If the test statistic is greater than the critical value, then the null hypothesis, which states that there is no significant difference between groups, is rejected.

A way to measure the strength of each configuration is to see where a hall-effect sensor measures a magnetic field that is sufficiently strong. This method is utilized in section II. EXPERIMENT, where a robot, which was built to answer this paper's question, moves the sensor and records where the sensor detects a threshold magnetic field along a single axis in space.

II. EXPERIMENT

This paper runs an experiment on different Halbach combinations by measuring where a hall-effect sensor detects the field along an axis. For this paper, the author built a single degree-of-freedom robot with one stepper motor moving a hall-effect sensor back and forth. The hall-effect sensor (HES) sends a discrete "on" signal when it detects a sufficiently strong magnetic field. So, the machine that moves the HES tracks at what position the sensor turns "on" and collects data to compare for each magnetic configuration. Figure 5 below shows the high level design of the robot.

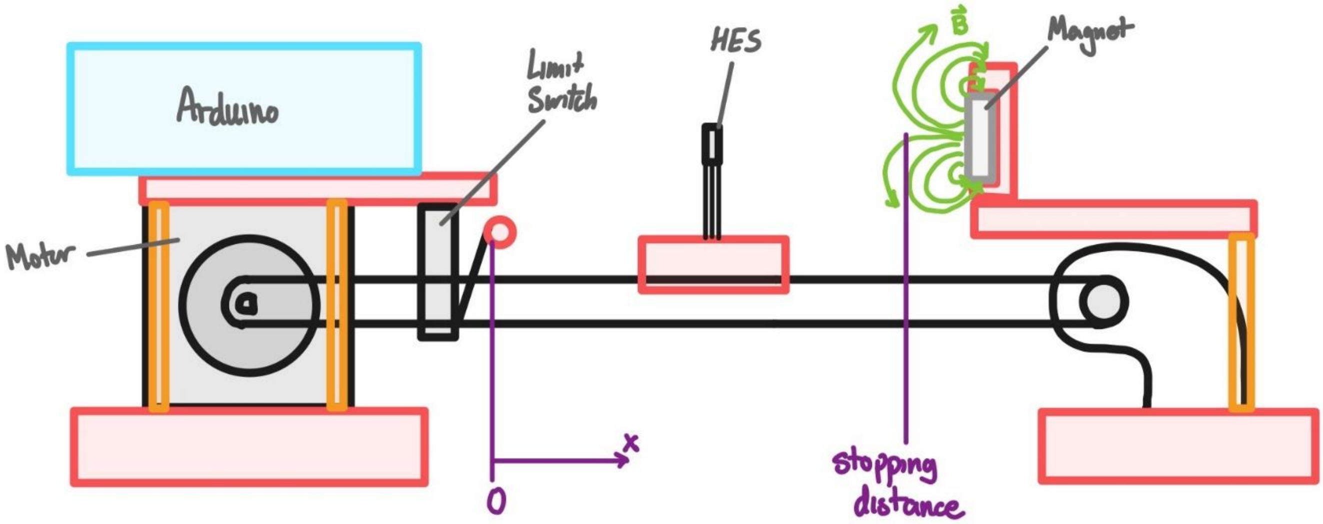

Figure 5 Sketch of the machine, its components, and locations of the origin and stopping position

The author built this machine by using hobbyist electronics. A NEMA17 42SHDC3025-4B stepper motor was mounted to a 3D printed bracket and held a GT-2 timing belt using a toothed pulley. This bracket also held a limit switch which would act as an origin point for positional measurements. This belt is wrapped down to another 3D printed mount which was connected to the motor mount with a 12mm diameter carbon fiber rod. This other mount held rectangular N52 magnets in open slots, such that each of the four configurations could be easily tested. The sensor that measured the presence of the magnet's fields clamped to the belt with a 3D printed part. With this combination of plastic and carbon fiber pieces, the stepper motor was successfully able to translate the HES towards and away from the magnets.

The author also chose and programmed electronics to enable successful control of the stepper motor. An Arduino UNO was used to store and run programs. On top of the UNO was an Adafruit Motor Shield V2, which enabled easy interfacing with the stepper motor, hall sensor, and limit switch. The entire machine was powered over a USB cable which connected to a laptop. This laptop also ran the Arduino IDE, which was used to develop the code for the project (see Appendix A).

The machine needed its distance measurements to be calibrated before collecting data. This was especially important for each of the two bult-in turning methods: INTERLEAVE and MICROSTEP. INTERLEAVE would utilize both coils in the motor to turn it smoothly and quickly while MICROSTEP would slowly partition steps amongst coils, resulting in extremely fine resolution movement. So, the starting position of the hall-effect sensor was marked with masking tape on the carbon fiber tube, then again after moving 700 steps for each method. The distance between these tapes gave a distance traveled per 700 steps for each method and resulted in calibrated conversion factors. In the program, these conversion factors were multiplied by the number of steps traveled to track the total distance traveled by the HES.

Once the machine was built, wired, and programmed the procedure was relatively automated. While the machine was off, the magnets were gently placed in the desired configuration. Then, the USB cable would plug into the machine, and it would automatically home its position using the limit switch. Once it homed and established an origin, it would move the hall sensor forward at 50 rotations per minute (from the perspective of the stepper motor) using the INTERLEAVE turning method. After 3000 steps, where each step on the stepper motor is 1.8 degrees, the stepper would switch modes. It would move the hall sensor forward at 10 rotations per minute using the MICROSTEPS turning method until the HES detected the magnetic field. Once the magnetic field was detected, it would display the distance traveled in millimeters on a Serial Monitor running on the laptop and repeat the procedure. After 30 samples, the machine was turned off, data was recorded, and a new configuration of magnets was set up. This procedure would repeat until 30 samples were recorded for each of the four configurations.

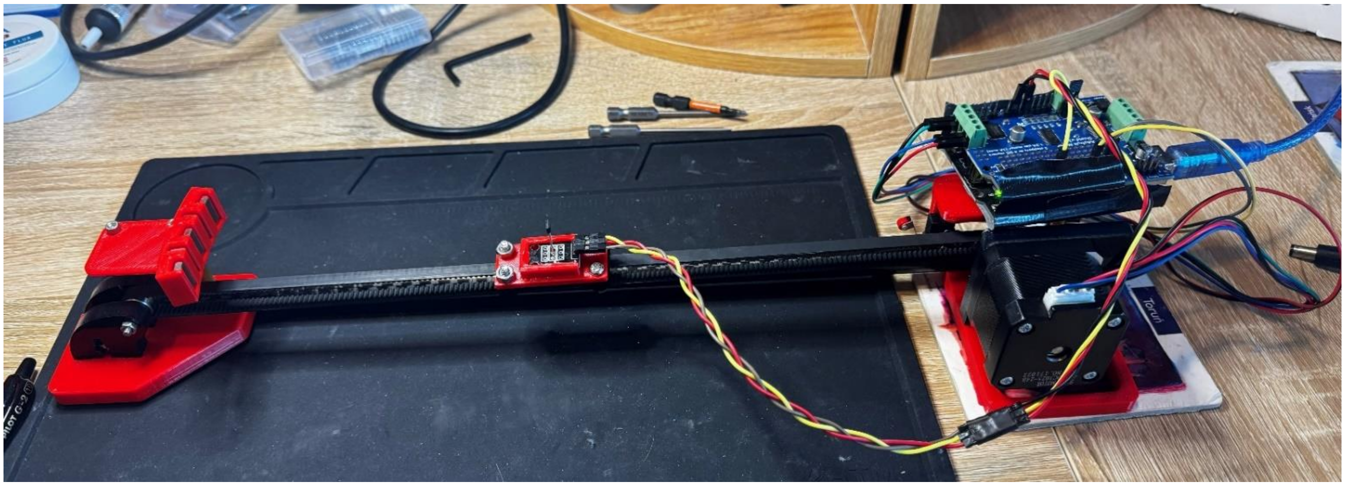

Figure 6 Photo of the robot

III. RESULTS & ANALYSIS

The machine performed 30 samples for each configuration (see Appendix B for the table of raw data). Since this experiment tested whether one variable, which was the category of magnetic fields, since the datasets had equal spreads and since the datasets had no outliers, a Single-Factor ANOVA was performed. The results of the ANOVA, shown in Table 1, showed that there was a significant difference between the four groups. With a chosen significance level of 0.05, the null hypothesis, which says there is no difference between the four groups, can be rejected. This is because the p-value was calculated to be $3.2 * 10^{-162}$. The test also showed the following means and standard deviations amongst each configuration and across the entire sample set, shown in Table 2.

Table 1: ANOVA Results

ANOVA						
Source of Variation	*SS*	*Df*	*MS*	*F*	*P-value*	*F crit*
Between Groups	279.2643	3	93.08810986	24385.69	3.2241E-162	2.682809407
Within Groups	0.44281	116	0.003817325			
Total	279.7071	119				

Table 2: Group Result Averages and Standard Deviations

Groups	*Count*	*Average*	*Standard Deviation*
Single Magnet	30	244.10	0.05439
3-Magnet	30	240.96	0.06750
5-Magnet	30	241.35	0.07524
3-Magnet No Halbach	30	244.29	0.03981
Total Set	120	242.68	1.52673

Simply by looking at the averages in Table 2 above, there is a noticeable difference between each of the groups. The 3-Magnet configuration has the strongest magnetic field with the hall sensor stopping at the shortest distance from its starting point. Second in strength is the 5-magnet, followed by a significant drop in strength with the Single Magnet, and the 3-Magnet No Halbach being the weakest. It's also clear that the averages are significantly distinct from each other given the smallness of the standard deviations. However, to truly assess whether each configuration is significantly different from each other, a Bonferroni test was performed for each pair. Table 3 below shows the results of the Bonferroni tests:

Table 3: Bonferroni Test Results Between Each Pair of Configurations

	Single Magnet + 3-Magnet Halbach	Single Magnet + 5-Magnet Halbach	Single Magnet + 3-Magnet No Halbach	3-Magnet Halbach + 5-Magnet Halbach	3-Magnet Halbach + 3-Magnet No Halbach	5-Magnet Halbach + 3-Magnet No Halbach
t.s.	7.9526	6.9675	0.4817	0.9852	8.4343	7.4491
*t**	2.2709	2.2709	2.2709	2.2709	2.2709	2.2709
	Reject H0	*Reject H0*	*Fail to Reject H0*	*Fail to Reject H0*	*Reject H0*	*Reject H0*

Table 3 above shows that some pairs differ significantly while others do not. The pairs, Single Magnet + 3-Magnet No Halbach and 3-Magnet Halbach + 5-Magnet Halbach, do not pass the Bonferroni test, meaning these groups do not significantly differ. All other groups, however, do pass and do significantly differ. The 3-Magnet Halbach configuration has the greatest difference amongst all the pairs when compared to the 3-Magnet No Halbach configuration, followed by the Single Magnet + 3-Magnet Halbach pair, 5-Magnet Halbach + 3-Magnet No Halbach pair, and finally the Single Magnet + 5-Magnet Halbach pair.

IV. DISCUSSION

The ANOVA and Bonferroni Tests show that there is a significant difference between Halbach and non-Halbach arrays, and that Halbach arrays are significantly stronger. When observing averages from each group from the ANOVA test, the average hall-sensor distance increases for Halbach arrays. Figure 5 below shows that the two Halbach arrays result in smaller distances, which signifies the configuration had a strong enough magnetic field to trip the HES at a shorter position down the length of the machine.

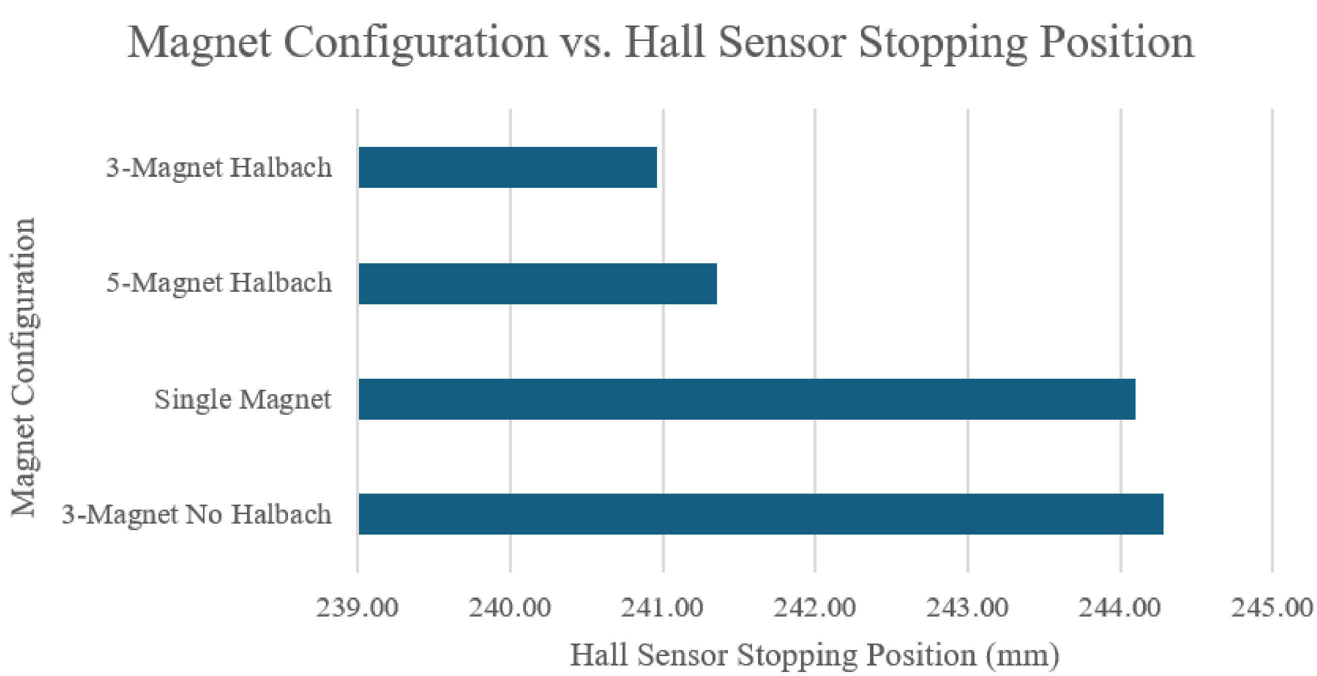

Figure 7 Magnet Configuration vs Stopping Position

This was further supported by the Bonferroni tests, which showed that there were significantly strong differences between any chosen Halbach configuration and any non-Halbach configuration. This means that the Halbach array was consistently and significantly stronger than non-Halbach options. The Bonferroni tests also revealed that 3-Magnet Halbach arrays were not significantly different than 5-Magnet Halbach arrays, which shows that stacking magnets does not provide a significant gain in strength. The same effect goes for a Single Magnet and a 3-Magnet No Halbach groups, where their datasets were not significantly different despite the increase in the number of magnets contributing to the strength of a magnetic field.

V. CONCLUSION

There is indeed a significant difference in magnetic strength between cheap N52 magnets in a typical array and magnets in a Halbach array where Halbach arrays are stronger than typical arrays. This is clear after repeatedly measuring the position of a hall-effect sensor as it approached various magnet combinations. This approach was done using a 1-DOF robot that was custom built by the author specifically for this paper. The robot would establish an origin point for every measurement, and automatically report the distance where the hall-effect sensor first detected an array's magnetic field. These reported values were analyzed using an ANOVA and Bonferroni tests which both revealed the difference in strength between Halbach and non-Halbach arrays. The Halbach arrays were significantly different than typical arrays because each pair of Halbach and non-Halbach in the Bonferroni test showed high test statistics compared to critical values tested at a significance level of 0.05. They were stronger than typical arrays too, because the averages of groups from the ANOVA test showed Halbach arrays would trip the HES 3.0 mm sooner than typical configurations. This can also be seen in Figure 6 from section IV. DISCUSSION. Despite learning that Halbach arrays were significantly stronger than non-Halbach arrays, the Bonferroni tests also revealed that there was no significant gain when adding more magnets to either category of array. The Single Magnet array showed no reasonable difference from the 3-Magnet No Halbach array and the 3-Magnet Halbach array also showed no reasonable difference from the 5-Magnet Halbach array, which contradicted intuition that adding more magnets would improve magnetic strength. Regardless, the initial question of this paper was successfully answered: there is a significant difference between Halbach and non-Halbach arrays when using cheap N52 magnets, and Halbach arrays are significantly stronger than non-Halbach arrays.

APPENDIX A: Arduino Code for the Robot

```
#include <Adafruit_MotorShield.h>

Adafruit_MotorShield AFMS = Adafruit_MotorShield();
Adafruit_StepperMotor *myMotor = AFMS.getStepper(200, 2);

int motorDia = 12;
int limitSwitch = 12;
int hallPin = 7;
float distance = 0.0;
float calibratedMICROSTEP = 111.71 / 700.0;
float calibratedINTERLEAVE = 56.03 / 700.0;

void setup() {
  Serial.begin(9600);
  while (!Serial);
  Serial.println("Stepper test!");
  pinMode(limitSwitch, INPUT_PULLUP);
  pinMode(hallPin, INPUT);

  if (!AFMS.begin()) {
    Serial.println("Could not find Motor Shield. Check wiring.");
    while (1);
  }
  Serial.println("Motor Shield found.");
}

void loop() {
  while (digitalRead(limitSwitch) == 1) {
    myMotor->setSpeed(60);
    myMotor->step(1, BACKWARD, INTERLEAVE);
    distance = 0.0;
  }

  myMotor->setSpeed(50);
  myMotor->step(3000, FORWARD, INTERLEAVE);
  distance += calibratedINTERLEAVE * 3000.0;

  while (digitalRead(hallPin) == 1) {
    myMotor->setSpeed(10);
    myMotor->step(1, FORWARD, MICROSTEP);
    distance += calibratedMICROSTEP;
  }

  Serial.println(distance, 4); // mm
  myMotor->release();
  delay(1000);
  distance = 0.0;
}
```

APPENDIX B: RAW DATA

Table A: Stopped Position (in mm) of The Hall-Effect Sensor for Different Magnet Configurations

Trial #	1 Magnet	3 Magnet	5 Magnet	3 Magnet No Halbach
1	244.12	240.9265	241.2457	244.2779
2	244.12	240.9265	241.2457	244.2779
3	244.12	240.9265	241.2457	244.2779
4	244.12	240.9265	241.2457	244.2779
5	244.12	240.9265	241.2457	244.2779
6	243.96	240.9265	241.2457	244.2779
7	244.12	240.9265	241.2457	244.2779
8	243.96	240.9265	241.2457	244.2779
9	243.96	240.9265	241.2457	244.2779
10	244.12	240.9265	241.2457	244.2779
11	243.96	240.9265	241.4053	244.2779
12	244.12	240.9265	241.4053	244.2779
13	244.12	240.9265	241.4053	244.2779
14	244.12	240.9265	241.4053	244.2779
15	244.12	241.0861	241.4053	244.2779
16	244.12	240.9265	241.4053	244.2779
17	244.12	240.9265	241.4053	244.2779
18	244.12	241.0861	241.4053	244.2779
19	244.12	241.0861	241.4053	244.2779
20	244.12	241.0861	241.4053	244.2779
21	244.12	240.9265	241.4053	244.2779
22	244.12	240.9265	241.4053	244.2779
23	244.12	240.9265	241.4053	244.2779
24	244.12	241.0861	241.4053	244.4375
25	244.12	240.9265	241.4053	244.2779
26	244.12	241.0861	241.4053	244.2779
27	244.12	240.9265	241.4053	244.4375
28	244.12	240.9265	241.4053	244.2779
29	244.12	240.9265	241.4053	244.2779
30	244.12	241.0861	241.4053	244.2779