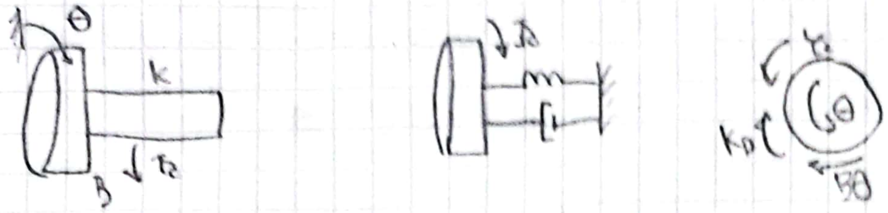
Parcial 2 → Daniel Torres Aguilar



$I_0\ddot\theta + B\dot\theta + K\theta = T_a$ $\qquad q_1 = \theta \qquad q_2 = \dot q_1 = \dot\theta_1 \qquad \dot q_2 = \ddot q_1 = \ddot\theta$

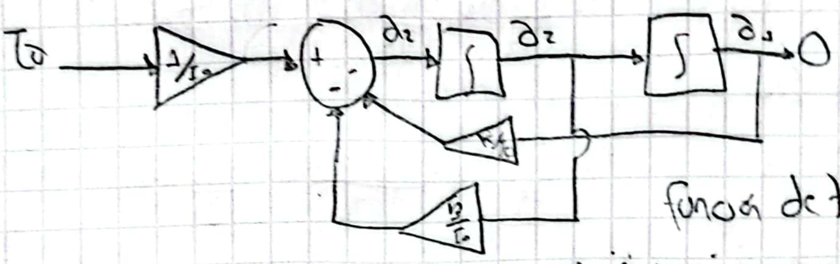
$I_0 \dot q_2 + B q_2 + K q_1 = T_a$

$I_0 \dot q_2 = T_a - B q_2 - K q_1$

$\dot q_2 = \frac{T_a}{I_0} - \frac{B}{I_0} q_2 - \frac{K}{I_0} q_1$

$$\begin{vmatrix} \dot q_1 \\ \dot q_2 \end{vmatrix} = \begin{vmatrix} 0 & 1 \\ -\frac{K}{I_0} & -\frac{B}{I_0} \end{vmatrix} \begin{vmatrix} q_1 \\ q_2 \end{vmatrix} + \begin{vmatrix} 0 \\ 1/I_0 \end{vmatrix} T_a(t)$$

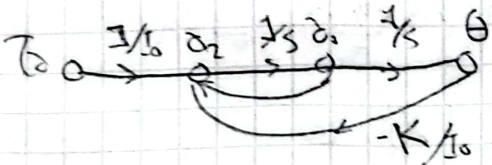
$$\theta = [1\ 0] \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$$



función de transferencia

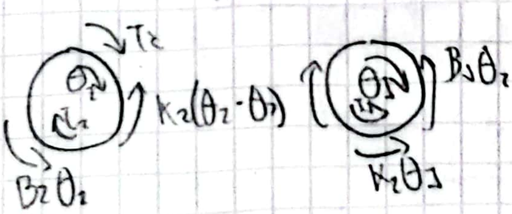
$I_0\ddot\theta + B\dot\theta + K\theta = T_a \qquad I_0 s^2\theta(s) + Bs\theta(s) + K\theta(s) = T_a(s)$

$\theta(s) = (I_0 s^2 + Bs + K) = T_a(s)$



a) $\theta_2 > \theta_1$

a) función de transferencia entre θ_2 y T_a



$T_a - I_2\ddot\theta_2 - K_2(\theta_2 - \theta_1) - B_2\dot\theta_2 = 0$

$T_a = I_2\ddot\theta_2 + K_2(\theta_2 - \theta_1) + B_2\dot\theta_2$

$T_a = J_2\ddot\theta_2 + K_2 - K_2\theta_1 + B_2\theta_2$

$K_2(\theta_2-\theta_1)-J_1\ddot{\theta}_1-K_1\theta_1-B_1\dot{\theta}_1=0 \Rightarrow K_2\theta_2(s)-K_2\theta_1(s)-J_1s^2\theta_1(s)-K_1\theta_1(s)$

$T(s)=\theta_2(s)-K_2\theta_1(s)-J_1s^2\theta_1(s)-K_1\theta_1(s)-B_2\theta_1(s)=0$

$\theta_1(s)(-B_s-K_1-K_2-Js^2)+K_2\theta_2(s)=0$

$$\begin{bmatrix}-K_2 & J_2s^2+K_2+B_2s \\ -B_s-K_1-K_2-J_1s^2 & K_2\end{bmatrix}\begin{bmatrix}\theta_1(s)\\ \theta_2(s)\end{bmatrix}=\begin{bmatrix}T_2\\ 0\end{bmatrix}$$

$$\frac{\theta_2(s)}{T(s)}=\frac{J_2s^2+K_2+B_2s}{-K_2^2\,(J_2s^2+K_2+B_2s)(B_s+K_1+K_2+J_1s^2)}$$

b) Espacio de estados $\quad K_2\theta_2-K_2\theta_1-J_1\ddot{\theta}_1-K_1\theta_1-B_1\dot{\theta}_1=0$

$\theta_1=x_1 \quad x_2=\dot{\theta}_1 \quad \dot{x}_2=\ddot{\theta}_1 \quad \cdot \quad \ddot{\theta}_1=\frac{K_2}{J_1}\theta_2-\frac{K_2}{J_1}\theta_1-\frac{K_1}{J_1}\theta_1-\frac{B_1}{J_1}\dot{\theta}_1$

$\theta_2=x_3 \quad x_4=\dot{\theta}_2 \quad \dot{x}_4=\ddot{\theta}_2$

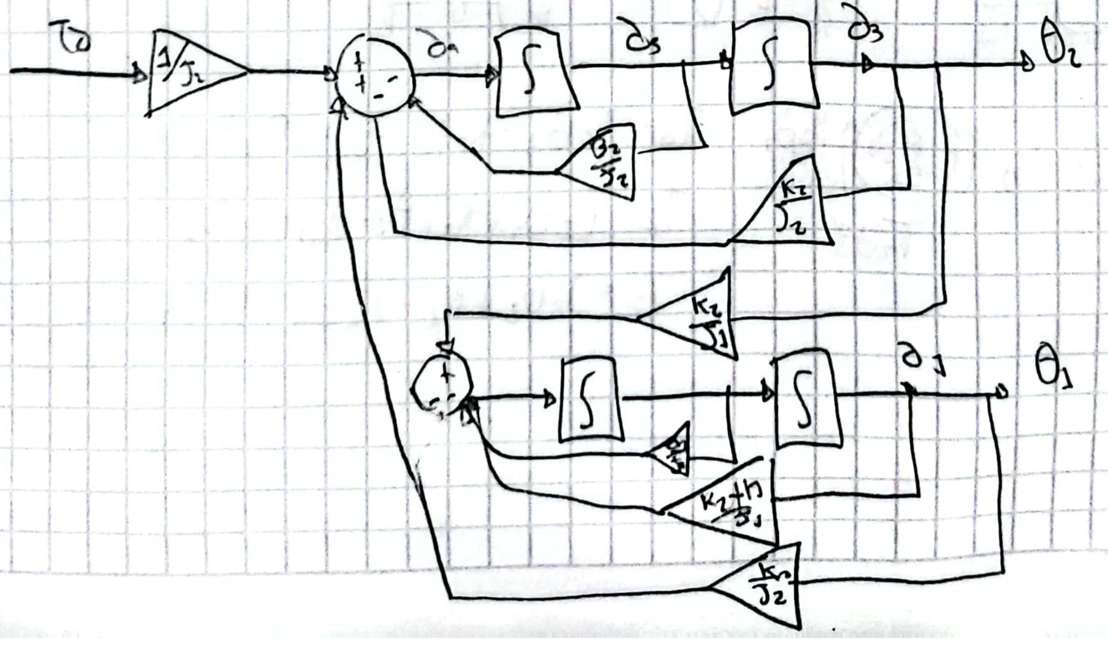
$T_a=J_2\ddot{\theta}_2+K_2(\theta_2-\theta_1)+B_2\dot{\theta}_2 \qquad \dot{x}_2=\frac{K_2}{J_1}x_3-\frac{(K_2+K_1)}{J_1}x_1-\frac{B_1}{J_1}x_2$

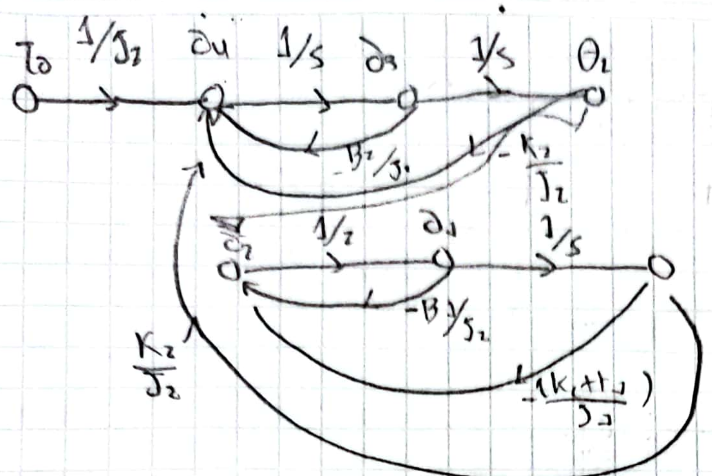
$\ddot{\theta}_2=\frac{T_a}{J_2}-\frac{K_2}{J_2}\theta_2+\frac{K_2}{J_2}\theta_1-\frac{B_2}{J_2}\dot{\theta}_2 \rightarrow \dot{x}_4=\frac{T_a}{J_2}-\frac{K_2}{J_2}x_3+\frac{K_2}{J_2}x_1-\frac{B_2}{J_2}x_4$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix}=\begin{bmatrix}0&1&0&0\\ -\frac{K_2+K_1}{J_1}&-\frac{B_1}{J_1}&\frac{K_2}{J_1}&0\\ 0&0&0&1\\ \frac{K_2}{J_2}&0&-\frac{K_2}{J_2}&-\frac{B_2}{J_2}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_3\end{bmatrix}+\begin{bmatrix}0\\0\\0\\ 1/J_2\end{bmatrix}T_a$$

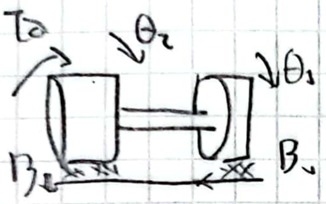
$\theta_2=x_3$

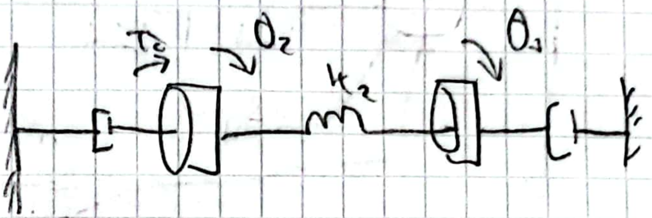
$$y=\begin{bmatrix}0&0&1&0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix}$$





3)





$$T_a = J_2\ddot{\theta}_2 + K_2\theta_2 - K_2\theta_1 + B_2\dot{\theta}_2 \Rightarrow K_2\theta_2 - K_2\theta_1 - J_1\ddot{\theta}_1 - B_1\dot{\theta}_1 = 0$$

$$T(s) = \theta_2(J_2 s^2 + K_2 + B_2 s) + \theta_1(s)(-K_2)$$

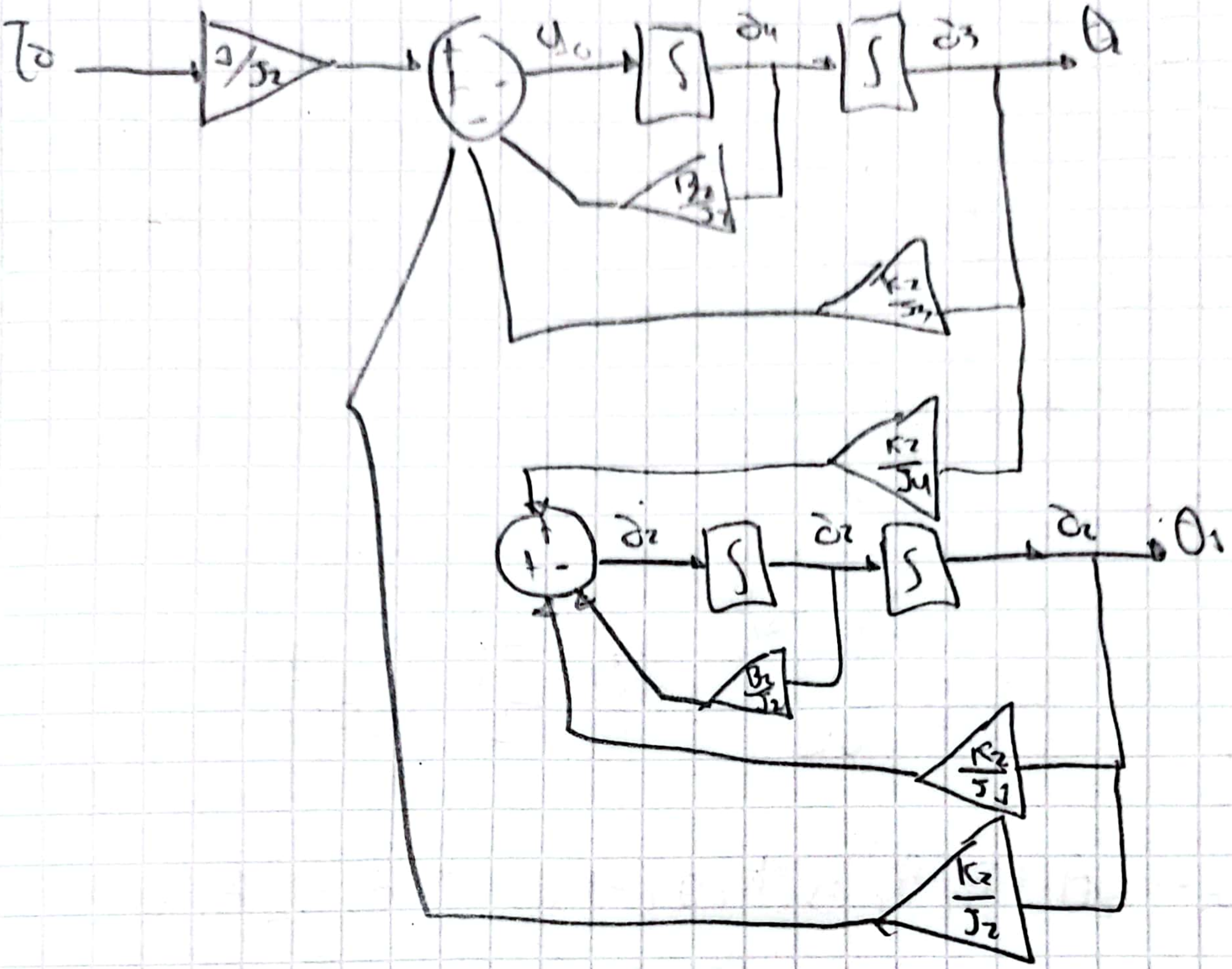
$$\begin{bmatrix} -K_2 & J_2 s^2 + K_2 + B_2 s \\ -B_1(s) - K_2 - J_1 s^2 & K_2 \end{bmatrix}\begin{bmatrix} \theta_1(s) \\ \theta_2(s) \end{bmatrix} = \begin{bmatrix} T(s) \\ 0 \end{bmatrix}$$

$$\frac{\theta_2(s)}{T(s)} = \frac{J_2 s^2 + K_2 + B_2 s}{-K_2^2 (J_2 s^2 + K_2 + B_2 s)(B_1 s + K_2 + J_1 s^2)}$$

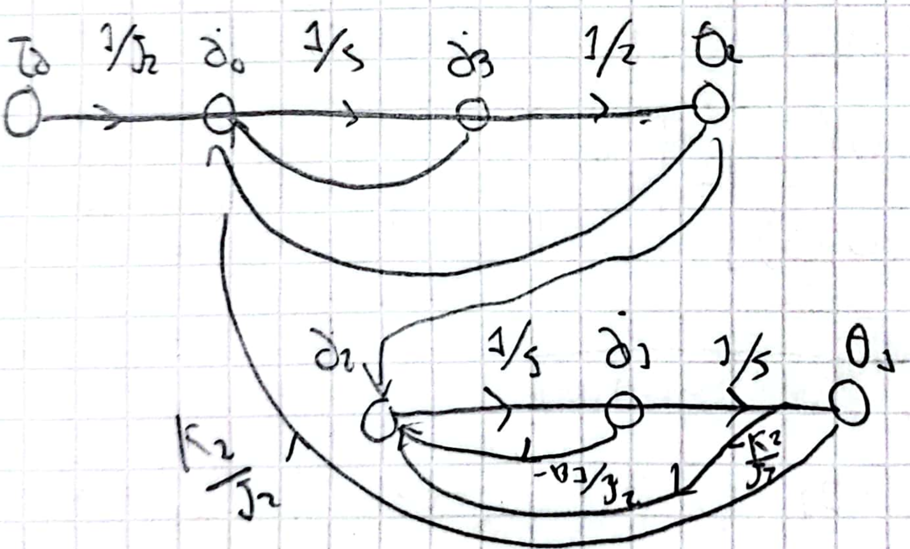
Espacio de estados $\theta_1 = q_1$ $q_2 = \dot{\theta}_1$ $\dot{q}_2 = \ddot{\theta}_1$ $\theta_2 = q_3$ $q_3 = \dot{\theta}_2$ $\dot{q}_3 = \ddot{\theta}_2$

$$\dot{q}_4 = \frac{T_a}{J_2} - \frac{K_2}{J_2}q_3 + \frac{K_2}{J_2} - \frac{B_2}{J_2}q_4 \qquad \dot{q}_2 = \frac{K_2}{J_1}q_3 - \frac{K_2}{J_1}q_1 - \frac{B_1}{J_1}q_2$$

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{K_2}{J_1} & -\frac{B_1}{J_1} & \frac{K_2}{J_1} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{K_2}{J_2} & 0 & -\frac{K_2}{J_2} & -\frac{B_2}{J_2} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1/J_2 \end{bmatrix} T_a \qquad y = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}$$

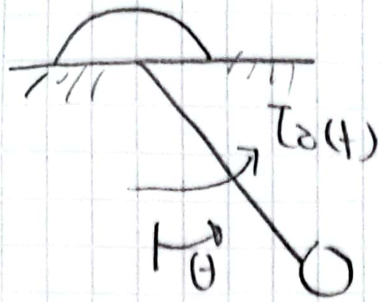


1/J2
K2/J2
θ1

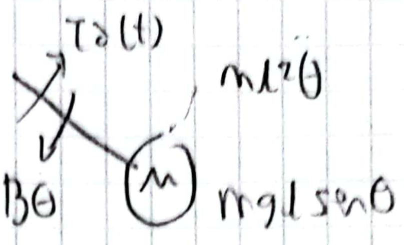


1/J2
1/s
1/s
K2/J2

4)



$I = ml^2$ $\qquad$ $T_a(t) - mgl\,sen\theta$



$$ml^2\ddot\theta + B\dot\theta + mgl\,sen\theta = T_a(t)$$

$\dot\theta = \omega$ $\qquad$ $\ddot\theta = \dot\omega$ $\qquad$ $\dot\omega = \frac{1}{ml^2}[-mgl\,sen\theta - B\omega + T_a(t)]$ $\qquad$ $\theta = 0{,}5\,rad$

$$ml^2\ddot\theta + B\dot\theta + mgl\theta = T_a(t)$$

$\theta = q_1$ $\qquad$ $\dot q_1 = q_2 = \dot\theta = \omega$ $\qquad$ $\dot q_2 = \ddot\theta = \dot\omega$

$\dot q_2 = \frac{1}{ml^2}[-mgl q_1 - Bq_2 + T_a(t)]$ $\qquad$ $\dot q_2 = -\frac{g}{l}q_1 - \frac{B}{ml^2}q_2 + \frac{T_a(t)}{ml^2}$

$$\begin{bmatrix} \dot q_1 \\ \dot q_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -g/l & -B/ml^2 \end{bmatrix}\begin{bmatrix} q_1 \\ q_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/ml^2 \end{bmatrix} T_a(t) \qquad \theta = [1 \quad 0]\begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$$