

The Real Business Cycle Model

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1 Elements of the New Keynesian Model

Building up towards a model with non-trivial monetary policy and business cycles:

(1) Neoclassical Growth Model:

- Capital Accumulation
- Labor Supply / Demand

(2) **Real Business Cycles**

- + Stochastic TFP

(3) Monopolistic Competition

- + Differentiated Firms
- + Non-trivial prices (markup)

(4) Stickiness in Price Changes

(5) Reason to hold money (i.e., returns not dominated by bonds)

(6) Monetary Control over Short-term Interest Rates

2 The Real Business Cycle

2.1 Final Goods

Consumer has preferences over final good consumption, C , and labour supplied, L .

$$u(C, L) = \underbrace{\log(C)}_{\text{CRRRA}=1} - \nu \log(L) \tag{1}$$

where $\nu \log(L)$ is the disutility of labour and β the discount factor.

Therefore, the expected welfare is:

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u(C_t, L_t) \right] \quad (2)$$

- Let the consumer's real income in period t be Y_t ; P_t is the price of the final good.
- Nominal income is $P_t Y_t$.
- The consumer can use the final good for consumption, or can use it to add to the capital stock K_t , which depreciates at rate $\delta \in (0, 1)$.
- The consumer rents labour at real price w_t and capital at real price r_t .
- The consumer owns the firms and gains real profits of π_t .

2.2 Consumer's Problem

The consumer's problem and constraints are given by:

$$\max_{\{C_t, L_t, K_{t+1}\}_{t=0}^{\infty}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u(C_t, L_t) \right] \quad (3)$$

$$\text{s.t. } P_t C_t + P_t (K_{t+1} - (1 - \delta)K_t) \leq P_t r_t K_t + P_t w_t L_t + P_t \pi_t \equiv Y_t P_t \quad (4)$$

Divide ?? by P_t to get the budget constraint in real terms:

$$C_t + (K_{t+1} - (1 - \delta)K_t) \leq r_t K_t + w_t L_t + \pi_t \equiv Y_t \quad (5)$$

where:

$$Y_t \equiv r_t K_t + w_t L_t + \pi_t \quad (6)$$

The Lagrange equation is:

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t [u(C_t, L_t) + \lambda_t (r_t K_t + w_t L_t + \pi_t - C_t - K_{t+1} + (1 - \delta)K_t)] \quad (7)$$

The first-order necessary conditions are:

$$[C_t] : \partial_C u(C_t, L_t) = \lambda_t \quad (8)$$

$$[L_t] : \partial_L u(C_t, L_t) = -\lambda_t w_t \quad (9)$$

Dividing ?? by ??

$$\Rightarrow \frac{\partial_L u(C_t, L_t)}{\partial_C u(C_t, L_t)} = w_t \quad (10)$$

Here: $\partial_L u(C_t, L_t) = -\frac{\nu}{L}$, $\partial_C u(C_t, L_t) = \frac{1}{C}$.

The labour supply equation is:

$$\nu \frac{C_t}{L_t} = w_t \quad (11)$$

$$\partial_{K_{t+1}} \mathcal{L} : \mathbb{E}_t [-\lambda_t + \beta \lambda_{t+1} (r_{t+1} + (1 - \delta))] = 0 \quad (12)$$

$$\Rightarrow \frac{\lambda_t}{\mathbb{E}_t [\lambda_{t+1}]} = \mathbb{E}_t [\beta (r_{t+1} + 1 - \delta)] \quad (13)$$

$$1 = \mathbb{E}_t \left[\frac{C_t}{C_{t+1}} \cdot \beta (r_{t+1} + 1 - \delta) \right] \quad (14)$$

Equation ?? represents the asset Euler equation.

Final Goods Resource Constraint:

$$C_t + K_{t+1} = Y_t + (1 - \delta)K_t \quad (15)$$

Later we will show that real income = total production.

2.3 Producer's Problem

Example: RBC with Homogeneous Good

Let there be a competitive representative firm with productivity Z_t , stochastic in nature, and Cobb-Douglas production function with constant returns to scale:

$$F(K_t, L_t, Z_t) = Z_t K_t^\alpha L_t^{1-\alpha}, \quad \alpha \in (0, 1) \quad (16)$$

- The firm is a price taker for w_t, r_t to rent inputs (real prices), and maximizes period profits:

$$P_t \pi_t = \max_{\{L_t, K_t\}} \left[\underbrace{P_t}_{\text{price}} \underbrace{F(K_t, L_t, Z_t)}_{\text{production}} - \underbrace{P_t r_t K_t - P_t w_t L_t}_{\text{rental of inputs}} \right] \quad (17)$$

The first-order conditions are:

$$[K_t] : \partial_K F(K_t, L_t, Z_t) = r_t \quad (18)$$

$$[L_t] : \partial_L F(K_t, L_t, Z_t) = w_t \quad (19)$$

Equation ?? and ?? represent the input demand functions.

Plugging in the form of production function from equation ??:

$$\partial_K F = Z_t \cdot \alpha K_t^{\alpha-1} L_t^{1-\alpha} = \alpha Z_t \left(\frac{K_t}{L_t} \right)^{\alpha-1} \quad (20)$$

$$\partial_L F = (1 - \alpha) Z_t \left(\frac{K_t}{L_t} \right)^{\alpha} \quad (21)$$

Let $k_t \equiv \frac{K_t}{L_t}$: the capital-labor ratio

2.4 Aggregate Productivity Process

Let the stochastic process for aggregate productivity Z_t be:

$$Z_{t+1} = \rho Z_t + \sigma \varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim N(0, 1) \quad (22)$$

The aggregate states are K_t and Z_t .

2.5 Competitive Equilibrium

Definition 1 (Competitive Equilibrium). *It is a set of allocations and conditions as follows:*

Policies: $C_t(K_t, Z_t), K_{t+1}(K_t, Z_t), L_t(K_t, Z_t),$

Prices: $r_t(K_t, Z_t), w_t(K_t, Z_t)$

such that:

1. *Given the prices, the policies solve the consumer's problem*
2. *Given the prices, the policies solve the firm's problem*
3. *Labour and capital markets clear*

The system of equation needed to solve for the competitive equilibrium, summarized from

the previous sections, are:

$$\begin{aligned}
1 &= \mathbb{E}_t \left[\frac{C_t}{C_{t+1}} \cdot \beta (r_{t+1} + 1 - \delta) \right] && \text{(Euler equation)} \\
\nu \frac{C_t}{L_t} &= w_t && \text{(Labour Supply)} \\
r_t &= \alpha Z_t \left(\frac{K_t}{L_t} \right)^{\alpha-1} && \text{(Capital Demand)} \\
w_t &= (1 - \alpha) Z_t \left(\frac{K_t}{L_t} \right)^{\alpha} && \text{(Labor Demand)} \\
C_t + K_{t+1} - (1 - \delta)K_t &= Z_t K_t^{\alpha} L_t^{1-\alpha} && \text{(Resource Constraint)} \\
Z_{t+1} &= \rho Z_t + \sigma \varepsilon_{t+1} && \text{(Technological Process)}
\end{aligned}$$

We can use Dynare to solve for the model with the system of equations above.