Monopolistic Competition

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1 Monopolistic Competition: Overview

Two equivalent approaches:

(1) Consumer has preferences for variety of *differentiated* products (with constant elasticity of substitutions)

(2) Consumer has preferences for a final good. The homogeneous consumption good produced by competitive aggregator of *differentiated* intermediate goods (with constant elasticity of substitution).

We use a Dixit-Stiglitz CES aggregator for consumption and production.

Why?

1) Monetary and trade models require multiple products and/or firms

2) We can model market power and enable sticky prices

3) Setup for decreasing demand functions of firms, which is "required" for price-setting behaviour.

- Heterogeneity means Dynare, etc. may be insufficient, unless aggregate stochastic difference equations exist.

2 Consumer's Problem

2.1 Defining the problem

Consumer with differentiated goods:
Let $\omega \in [0, N] \equiv \Omega$ be "labels" for differentiated goods.

The consumer has preferences (or aggregator has production):

$$U = \left[\int_\Omega y(\omega)^\rho \, \mathrm{d}\omega\right]^{\frac{1}{\rho}}, \; \rho \in (0,1) \text{ (substitutability)} \tag{1}$$

where we are integrating the quantity demanded of variety ω over the set of all varieties Ω

$$\text{s.t. } \int_\Omega \underbrace{p(\omega)}_{\substack{\text{price}\\\text{faced}\\\text{for } \omega}} y(\omega) \, \mathrm{d}\omega \leq \underbrace{P.Y}_{\substack{\text{nominal}\\\text{income}}} \tag{2}$$

Lagrangian:

$$\mathcal{L} = \left[\int_\Omega y(\omega)^\rho \, \mathrm{d}\omega\right]^{\frac{1}{\rho}} + \lambda\left(PY - \int_\Omega y(\omega)p(\omega) \, \mathrm{d}\omega\right) \tag{3}$$

2.2 Optimality Conditions

Taking the first-order conditions from the Lagrangian equation (3):

$$[y(\omega)] : \frac{1}{\rho}\left[\int_\Omega y(\tilde{\omega})^\rho \, \mathrm{d}\tilde{\omega}\right]^{\frac{1}{\rho}-1} \rho y(\omega)^{\rho-1} = \lambda p(\omega) \tag{4}$$

Take the ratio of (4) any two products ω, ω'

$$\left(\frac{y(\omega)}{y(\omega')}\right)^{\rho-1} = \frac{p(\omega)}{p(\omega')} \tag{5}$$

Let $\kappa \equiv \frac{1}{1-\rho}$

$$\Rightarrow y(\omega') = y(\omega)p(\omega)^\kappa p(\omega')^{-\kappa} \tag{6}$$

Multiply by $p(\omega')$

$$y(\omega')p(\omega') = y(\omega)p(\omega)^\kappa p(\omega')^{1-\kappa} \tag{7}$$

Integrate over $\omega' \in \Omega$

$$\int_\Omega y(\omega')p(\omega') \, \mathrm{d}\omega' = y(\omega)p(\omega)^\kappa \int_\Omega p(\omega')^{1-\kappa} \, \mathrm{d}\omega' \tag{8}$$

From the budget constraint in equation (2), the LHS is total spending: PY

$$\int_\Omega PY = y(\omega)p(\omega)^\kappa \int_\Omega p(\omega')^{1-\kappa} \, \mathrm{d}\omega' \tag{9}$$

Define $P \equiv [\int_\Omega p(\omega')^{1-\kappa} \, \mathrm{d}\omega']^{\frac{1}{1-\kappa}}$ as the price index, then

$$\int_\Omega PY = y(\omega)p(\omega)^\kappa \cdot P^{1-\kappa} \tag{10}$$

$$\boxed{y(\omega) = \left(\frac{p(\omega)}{P}\right)^{-\kappa} Y} \tag{11}$$

where

- $y(\omega)$: demand for variety ω
- $\frac{p(\omega)}{P}$: price relative to the price index
- Y: real income

Utility and Aggregate Output? From equation (1)

$$U = \left[\int_\Omega y(\omega)^\rho \, \mathrm{d}\omega\right]^{\frac{1}{\rho}}$$

Substituting in the demand function from equation (11):

$$= \left[\int_\Omega p(\omega)^{-\rho\kappa} \, \mathrm{d}\omega\right]^{\frac{1}{\rho}} \cdot P^\kappa Y \tag{12}$$

Note: $-\rho\kappa = -\left(1 - \frac{1}{\kappa}\right)\kappa = 1 - \kappa$

$$= \left[\int_\Omega p(\omega)^{1-\kappa} \, \mathrm{d}\omega\right]^{\frac{1-\kappa}{1-\kappa}\cdot\frac{1}{\rho}} \cdot P^\kappa Y \tag{13}$$

Using the definition of the price index:

$$= P^{\frac{1-\kappa}{\rho}+\kappa} \cdot Y = Y \tag{14}$$

$$U = Y \tag{15}$$

Utility is the real income. Hence, P is a cost of living index.

2.3 Adding Risk Aversion and Labor Supply

Change the utility,

$$U = \underbrace{\frac{\left([\int_\Omega y(\omega)^\rho \ \mathrm{d}\omega]^{\frac{1}{\rho}}\right)^{1-\gamma}}{1-\gamma}}_{\text{CRRA } \gamma} - \underbrace{\frac{AL^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}}}_{\text{Frisch elasticity } \nu} \tag{16}$$

Stochastic Discount Factor: Let $Y_t \equiv [\int_\Omega y_t(\omega)^\rho \ \mathrm{d}\omega]^{\frac{1}{\rho}}$.
Then, from the RBC Euler equation in lecture notes 9:

$$\Lambda_t \equiv \beta \mathbb{E}_t \left[\left(\frac{Y_{t+1}}{Y_t} \right)^{-\gamma} \right] \tag{17}$$

- A real business cycle with homogeneous good.
- This is how the household discounts future income, e.g. if $\frac{Y_{t+1}}{Y_t} = (1+g_t)$ deterministically:

$$\Lambda_t = \beta \underbrace{(1+g_t)^{-\gamma}}_{\text{risk aversion}} \tag{18}$$

3 Firm's Problem

3.1 Defining the problem

- Firm chooses production and <u>prices</u>, subject to demand function. Assume that own price deviation has no impact on other firms' prices.
- Maximizes PDV of <u>real</u> profits, discounting according to the consumer's stochastic discount factor (alternatively, at the interest rate of a risk-free bond by a no-arbitrage argument: equivalent).
- Variety ω has productivity $Z_t(\omega)$. Assume the only input is labor l at real wage cost.

The firm's problem is to maximize profits discounted by Λ:

$$\max_{\{y_t, \ell_t, p_t\}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \left(\prod_{\tau=0}^{t} \Lambda_t \right) \left[\frac{p_t(\omega)}{P_t} y_t(\omega) - w_t \ell_t(\omega) \right] \right] \qquad \text{(profits)} \tag{19}$$

$$\text{s.t. } y_t(\omega) = Z_t(\omega) \ell_t(\omega) \qquad \text{(unit productivity, CRS)} \tag{20}$$

$$y_t(\omega) = \left(\frac{p_t(\omega)}{P} \right)^{-\kappa} Y \qquad \text{(demand from consumer)} \tag{21}$$

Note: If the discount factor was not stochastic or with linear utility, i.e.:

$$\text{If } \lambda_t = \beta \;\Rightarrow \prod_{\tau=0}^{t} \Lambda_t = \beta^t$$

Without pricing frictions: (e.g., adjustment costs), this is solved statically period by period.

3.2 Monopolistically Competitive Firms (Static Problem)

- For simplicity, just assume that the firm rents labor at nominal wage $\tilde{w}$
- One firm + one product, monopoly power
- Let the firm have productivity z, CRS production function.
- Each ω maps to a z The objective function is:

$$P\pi(z) = \max_{\{l,p\}} \left[p(z)z\ell(z) - \tilde{w}\ell(z)\right] \tag{22}$$

$$\text{s.t. } y(z) = z\ell(z) \;\Rightarrow \ell(z) = \frac{y(z)}{z} \tag{23}$$

$$y(z) = \left(\frac{p(z)}{P}\right)^{-\kappa} Y \qquad \text{(consumer's demand)} \tag{24}$$

Plugging the constraints in the objective function (22):

$$P\pi(z) = \max_{\{p\}} \left[p(z)z\left(\frac{y(z)}{z}\right) - \tilde{w}\left(\frac{y(z)}{z}\right)\right] \tag{25}$$

$$= \max_{p} \left[p(z)\cdot\frac{z}{z}\left(\frac{p(z)}{P}\right)^{-\kappa} Y - \frac{\tilde{w}}{z}\left(\frac{p(z)}{P}\right)^{-\kappa} Y\right] \tag{26}$$

$$P\pi(z) = P^{\kappa} Y \max_{p} \left[p(z)^{1-\kappa} - \frac{\tilde{w}}{z} p(z)^{-\kappa}\right] \tag{27}$$

Taking the first-order necessary condition:

$$[p] : (1-\kappa)p(z)^{-\kappa} = \kappa \cdot \frac{\tilde{w}}{z} \cdot p(z)^{-\kappa-1} \tag{28}$$

$$\Rightarrow p(z) = \frac{\kappa}{\kappa - 1} \cdot \frac{\tilde{w}}{z} \tag{29}$$

$$= \frac{1}{\rho} \cdot \frac{\tilde{w}}{z} \tag{30}$$

Let $w = \frac{\tilde{w}}{P}$: the real wage

$$\frac{p(z)}{P} = \frac{1}{\rho} \cdot \frac{w}{z} \tag{31}$$

where:

- $\dfrac{p(z)}{P}$: real price
- $\dfrac{1}{\rho}$: constant markup
- $\dfrac{w}{z}$: marginal cost

Note: Labor market is competitive; production side determines the wage.

3.3 Calculating the Price Index and Real Wages

The price index is:

$$P = \left[\int_{\Omega} p(\omega)^{1-\kappa} \, \mathrm{d}\omega\right]^{\frac{1}{1-\kappa}} \tag{32}$$

Plug in equation (31):

$$P = \left[\int_{\Omega} \left(\frac{wP}{\rho z(\omega)}\right)^{1-\kappa} \, \mathrm{d}\omega\right]^{\frac{1}{1-\kappa}} \tag{33}$$

$$\Rightarrow 1 = \frac{w}{\rho} \left[\int_{\Omega} z(\omega)^{\kappa-1} \, \mathrm{d}\omega\right]^{\frac{1}{1-\kappa}} \tag{34}$$

$$w = \rho \left[\int_{\Omega} z(\omega)^{\kappa-1} \, \mathrm{d}\omega\right]^{\frac{1}{\kappa-1}} \tag{35}$$

Define the aggregate productivity Z as:

$$Z = \left[\int_{\Omega} z(\omega)^{\kappa-1} \, \mathrm{d}\omega\right]^{\frac{1}{\kappa-1}} \tag{36}$$

The real wage is:

$$w = \rho Z \tag{37}$$

(Substitutability = market power).

As $\rho \to 0$: low substitutes, extract high profits.

As $\rho \to 1$: closer to perfect competition.
The model ends up similar to the representative firm model, with a distorting tax. That allows us to add theory of firm heterogeneity with a distribution of ω, but this doesn't help understand pricing or monetary policy.

To summarize, the price indexes for each type of markets are:
In monopolistic competition: $P = \frac{1}{\rho} \cdot \frac{\tilde{w}}{z}$.
In perfect competition: $P = \frac{\tilde{w}}{z}$