Monetary Policy, Commitment, and Time Consistency

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1 Objective for Monetary Policy

Up until now, we have assumed an exogenously given monetary policy, which the agents in the economy take as given. Instead, consider a monetary authority attempting to maximize the utility of consumer, but who can only operate through manipulations of the nominal interest rates.[1]

Consider that the planner can only choose the deviation from the stationary nominal interest rate, i_t (i.e., leaving off the 'hat' for convenience). Then by taking a second-order approximation around steady state in the "New Keynesian" setup, we can show that the objective function is

$$\approx -\Omega \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \left[\pi_{t+1}^2 + \lambda(x_{t+i} - x^*)^2\right] + (\text{terms with no policy})$$

where x_t is the output gap, and x^* is the steady state output gap, and π_{t+1} is the inflation rate. All in % deviations.[2]

[1] References: Woodford (2011) and Woodford (1998)

[2] In order to setup the objective of the planner, it turns out to be convenient to nest the problem with a variation on the consumer utility, with the disutility of work being a composite of the disutility of working for every variety. - Assume that the household consumes composite good C_t and has disutility, $D(\cdot)$ from working to produce variety c_{jt} with some exogenous shock Z_t:

$$V_t \equiv U(Y_t; Z_t) - \underbrace{\int_0^1 D(c_{jt}; Z_t)\, \mathrm{d}j}_{\substack{\text{disutility from} \\ \text{all varieties}}} \tag{1}$$

We can show that the second order approximation of the welfare function, local to the steady state is:

$$\mathbb{E}_t \sum_{i=0}^{\infty} \beta^i V_{t+i} \approx -\Omega \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \left[\pi_{t+1}^2 + \lambda(x_{t+i} - x^*)^2\right] + (\text{terms with no policy}) \tag{2}$$

The monetary authority chooses the nominal interest rate i_t to maximize this objective, given that the economy will evolve according to the "new keynesian" IS curve and the "new keynesian" Phillips curve. If we drop the terms invariant to policy, this can be written as a quadratic loss function with Gaussian, mean zero, shocks, and linear evolution equations. That is, a Linear Quadratic Gaussian Control problem,

$$\min_{\{i_t\}} \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \left(\frac{1}{2} \pi_{t+i}^2 + \lambda x_{t+i} \right) \tag{3}$$

$$\text{s.t. } x_t = \mathbb{E}_t \left[x_{t+1} \right] - \frac{1}{\sigma} \left(i_t - \mathbb{E}_t \left[\pi_{t+1} \right] \right) + u_t \tag{4}$$

$$\pi_t = \beta \mathbb{E}_t \left[\pi_{t+1} \right] + \kappa x_t + e_t \tag{5}$$

Naively, given what we know about LQ control, this appears to have a very simple solution following a linear policy rule, $i_t = \delta_\pi \pi_t + \delta_x x_t$, i.e. a Taylor rule. However, Keep in mind that the linearized equations from the new Keynesian model have $\mathbb{E}_t \left[x_{t+1} \right]$ and $\mathbb{E}_t \left[\pi_{t+1} \right]$ from the firms and households. But those depend on expectations of future monetary authority policy. So, if we could find the δ_π, δ_x rule above which the government sticks to. Would they have any reason to deviate from policy at some state given the choice? If so, then the policy is **time-inconsistent**, and a game is being played between the monetary authority and the agents in the economy. We will come back to this after isolating policy versus discretion.

2 Monetary Policy: Discretion versus Commitment

Following variations of Walsh (2010). Assume some Phillips-style equation:

$$y = \bar{y} + \alpha(\pi - \pi^e) + \varepsilon, \ \varepsilon \sim \mathrm{N}\left(0, \sigma_\varepsilon^2\right) \tag{6}$$

where:

- $\bar{y}$: natural rate of output
- π^e: expectations (last sub-period) of inflation by agents
- π: actual inflation
- y: output

Note that the objective here is leaving out any "money demand". The idea is that this follows form our MIU notes that if the utility is separable in real money demand, then the monetary authority has some way to change nominal interest rates, but we don't worry about the method. A cashless setup.

2.1 Barro-Gordon (1983) Style Model

Versions of this model include a monetary policy shock after the central planner has chosen their π. This has been removed since it didn't seem necessary for the core results.

2.1.1 Model Setup

Policy maker has preferences to minimize a loss function:

$$L = \frac{1}{2}\delta\left(y - (\bar{y} + \kappa)\right)^2 + \frac{1}{2}\pi^2, \ \delta, \kappa > 0 \tag{7}$$

Key: $\kappa > 0$ means that the goal is to attain higher than natural level of output, e.g., monopoly price friction in natural output. The central bank would also desire to achieve $\pi = 0$ inflation. Therefore, the bliss point is given by: $(\bar{y} + \kappa, 0)$.

2.1.2 Timing

1. Agents form expectations of inflation π^e, e.g., set 1-period wage contracts.
2. ε is realized (agents can act on it).
3. Policy maker uses some instruments (e.g., i, M) to set π, which can be conditioned on ε:
4. y determined for all the agents in the economy

2.1.3 Policy Intuition

- Timing allows for “surprise inflation".
- If π^e set nominal wages, then:

a) $\pi > \pi^e$: real wages decline and $y \uparrow$

b) $\pi < \pi^e$: real wages increase and $y \downarrow$

- Because $\kappa > 0$, policy makers have incentives to increase inflation to get $y > \bar{y}$, but there is also a cost of inflation.
- ε shock after agent's action leaves room for stabilization policy.

2.2 Discretion

Assume: no commitment to a rule, hence this is a ‘sequential moves’ game: use backward induction.

2.2.1 Policy Maker

Given π^e and ε, the policy maker minimizes the loss function:

$$\min_{\pi,y} \mathbb{E}\left[\frac{1}{2}\delta\left(y-(\bar{y}+\kappa)\right)^2+\frac{1}{2}\pi^2\right] \tag{8}$$

$$\text{s.t. } y=\bar{y}+\alpha(\pi-\pi^e)+\varepsilon \tag{9}$$

The Lagrangian is:

$$\mathcal{L}=\frac{1}{2}\delta\left(y-(\bar{y}+\kappa)\right)^2+\frac{1}{2}\pi^2+\lambda\left(y-\bar{y}-\alpha(\pi-\pi^e)-\varepsilon\right) \tag{10}$$

The set of FOCs are:

$$\boldsymbol{\partial}_\pi : \pi=\alpha\lambda \tag{11}$$

$$\boldsymbol{\partial}_y : \delta(y-\bar{y}+\kappa)+\lambda=0 \tag{12}$$

$$\boldsymbol{\partial}_\lambda : y=\bar{y}+\alpha(\pi-\pi^e)+\varepsilon \tag{13}$$

Solve for π:

$$\pi=\frac{\delta\alpha^2\pi^e+\delta\alpha(\kappa-\varepsilon)}{1+\delta\alpha^2} \tag{14}$$

2.2.2 Agents

Set π^e knowing the response, but <u>before</u> ε is realized. From equation (14):

$$\pi^e=\mathbb{E}(\pi)=\frac{\delta\alpha^2\pi^e+\delta\kappa\alpha}{1+\delta\alpha^2} \tag{15}$$

Solving for π^e:

$$\pi^e=\alpha\delta\kappa>0 \tag{16}$$

Given equation (16), use equation (14) to find the policy:

$$\pi=\underbrace{\alpha\delta\kappa}_{\substack{\text{inflation}\\\text{bias}}}-\underbrace{\frac{\delta\alpha\varepsilon}{1+\delta\alpha^2}}_{\text{stabilization}} \tag{17}$$

Plug into the output in equation (9):

$$y = \bar{y} + \frac{\varepsilon}{1+\delta\alpha^2} \tag{18}$$

2.2.3 Analysis of Discretion

1. Inflation bias, because central bank has an incentive to increase inflation to boost output.

2. But agents expect it, so has no effect of output, and hurts welfare.

3. The ability to increase output through inflation is hurting welfare.

2.3 Commitment

What if the central bank could commit to a rule? (i.e., state rule before agents act, and be constrained by it?). Since this is a LQ problem, the rule will be linear:

$$\pi = \phi_0 + \phi_1\varepsilon \tag{19}$$

The rule will include stabilization objective.

2.3.1 Approach

If the Taylor equation is the agent's response, then with commitment, the policy maker solves the following (using certainty equivalence):

$$\min_{\phi_0,\phi_1} \mathbb{E}\left[\frac{1}{2}\left(\delta\left[y-(\bar{y}+\kappa)\right]^2+\pi^2\right)\right] \tag{20}$$

$$\text{s.t. } y = \bar{y} + \alpha(\pi - \pi^e) + \varepsilon \tag{21}$$

$$\pi = \phi_0 + \phi_1\varepsilon \tag{22}$$

Therefore,

$$\pi = \frac{-\delta\alpha}{1+\delta\alpha^2}\varepsilon,\ \pi^e = 0 \text{ (no inflation bias)} \tag{23}$$

We can calculate the expected value of these as:

$$\mathbb{E}(\text{value under discretion}) - \mathbb{E}(\text{value under commitment}) = \frac{\alpha\delta\kappa}{2} > 0 \quad \text{(from loss function)}$$

This gives a sense of welfare cost due to inflation bias.

3 Monetary Policy in the New Keynesian Model

As derived before to a 2nd order, the monetary authority has the objective of minimizing the loss function:

$$\min_{\{i_{t+i},\pi_{t+i},x_{t+i}\}} \frac{1}{2}\,\mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \left(\pi_{t+1}^2 + \lambda x_{t+i}^2\right) \text{ (for some } \lambda) \tag{24}$$

$$\text{s.t. } \pi_{t+i} = \beta \mathbb{E}_{t+i}\left[\pi_{t+1+i}\right] + \kappa x_{t+i} + e_{t+i} \tag{25}$$

$$x_{t+i} = \mathbb{E}_{t+i}\left[x_{t+1+i}\right] - \sigma^{-1}\left(i_{t+i} - \mathbb{E}_{t+i}\left[\pi_{t+1+i}\right]\right) + u_{t+i} \tag{26}$$

Set up as a Lagrangian:

$$\begin{aligned}\mathcal{L} = \; & \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \Big[\frac{1}{2}\left(\pi_{t+i}^2 + \lambda x_{t+i}^2\right) \\ & + \theta_{t+i}\left(x_{t+i} - x_{t+i+1} + \sigma^{-1}(i_{t+i} - \pi_{t+i+1}) - u_{t+i}\right) \\ & + \psi_{t+i}\left(\pi_{t+i} - \beta\pi_{t+i+1} - \kappa x_{t+i} - e_{t+i}\right)\Big]\end{aligned} \tag{27}$$

Take the FONC:

$$\boldsymbol{\partial}_{i_{t+i}}\mathcal{L} \; : \sigma^{-1}\mathbb{E}_t\left[\theta_{t+i}\right] = 0 \tag{28}$$

Since $\mathbb{E}_t\left[\theta_{t+i}\right] = 0$, this Lagrange multiplier in equation (28) is not binding and causes no real cost on changing i_t. So, we can assume that $\theta_{t+i} = 0$, which is when the policy is actually the output gap x_t, where i_t works behind the scenes.

<u>Other FONCs:</u>

$$\boldsymbol{\partial}_{\pi_{t+i}}\mathcal{L} \; : \mathbb{E}_t\left[\pi_{t+i} + \psi_{t+i} - \psi_{t+i-1}\right] = 0, \;\; i \geq 1 \tag{29}$$

$$\boldsymbol{\partial}_{x_{t+i}}\mathcal{L} \; : \mathbb{E}_t\left[\lambda x_{t+i} - \kappa\psi_{t+i}\right] = 0, \;\; i \geq 0 \tag{30}$$

But they have complete control of x_t at time t, and so for π_t:

$$\pi_t + \psi_t = 0 \tag{31}$$

The dynamic inconsistency is evident:

1) Sets $\pi_t = -\psi_t$

2) <u>Promises</u> to set $\pi_{t+1} = -(\psi_{t+1} - \psi_t)$

3) <u>But</u> at time $t+1$, would rather set $\pi_{t+1} = -\psi_{t+1}$

3.1 Commitment

Timeless Perspective: The policies were set in the distant past, and the authority is committed to them.
Ignoring the t choice, and solve (29) and (30):

$$\psi_{t+i} = \frac{\lambda}{\kappa} x_{t+i} \tag{32}$$

$$\Rightarrow \pi_t = -\frac{\lambda}{\kappa} \left(x_t - x_{t-1} \right) \tag{33}$$

Note: We used certainty equivalence here.

Substituting into the NKPC in equation (25) and rearrange:

$$\left(1 + \beta + \frac{\kappa^2}{\lambda} \right) x_t = \beta \mathbb{E}_t \left[x_{t+1} \right] + x_{t-1} - \frac{\kappa}{\lambda} e_t \tag{34}$$

Equation (34) is a 2nd order difference equation. We solve it using the **guess and verify** method, with the following guess:

$$x_t = a_x x_{t-1} + b_x e_t \tag{35}$$

where we assume the following process for e_t:

$$e_t = \rho e_{t-1} + \varepsilon_t, \ \varepsilon_t \sim \mathrm{N} \left(0, \sigma_\varepsilon^2 \right) \tag{36}$$

where ρ is the possible persistence from the NKPC. With this process, we can find that:

$$\pi_t = A x_{t-1} + B e_t \tag{37}$$

for some A, B as functions of constant (see Walsh 8.4).
The dependence on the lagged output gap adds persistence to the model. It turns out that even if $\rho = 0$, $A \neq 0$, so it is the commitment itself adding inertia to the model. By committing to inertia, the bank's actions today can affect future variables (in particular, future inflation).
- There may be better perfect commitment rules; this is just one approach.

3.2 Discretion

As the bank cannot influence future variables through its choice without commitment, they would otherwise solve a static problem:

$$\min_{x_t,\pi_t} \frac{1}{2}\left(\pi_t^2 + \lambda x_t^2\right) \tag{38}$$

$$\text{s.t. } \pi_t = \beta\mathbb{E}_t\left[\pi_{t+1}\right] + \kappa x_t + e_t \tag{39}$$

The key here is that expectations of the future, i.e. the $\mathbb{E}_t\left[\pi_{t+1}\right]$ in (39) *cannot be influenced* through any choice today, as promises are not credible. Consequently, the π_{t+1} does not enter the control problem of the monetary authority at time t (unlike a typical dynamic control problem). Plug (39) into (38) to form an unconstrained objective, and take the FOC for x_t:

$$\boldsymbol{\partial}_{x_t} \; : \kappa\pi_t + \lambda x_t = 0 \tag{40}$$

Substitute into (39) and solve the linear difference forwards through normal techniques to find:

$$\pi_t = -\left(\frac{\lambda}{\kappa}\right)x_t = \left[\frac{\lambda}{\lambda(1-\beta\rho)+\kappa^2}\right]e_t \tag{41}$$

where e_t follows the AR(1) process as in equation (36).
- There is no inflation bias here, but a bias on stabilization in the sense that the response to shocks differs from the commitment solution.

Commitment to Rule We finally could assume that the bank can commit, but only to a function of the state, as in Barro 1983. Since the state is e_t, we can find the optimal rule to commit to. The equilibrium inflation (Walsh 8.4.4) is given by:

$$\pi_t = \left[\frac{\lambda(1-\beta\rho)}{\lambda(1-\beta\rho)^2+\kappa^2}\right]e_t \tag{42}$$

This is identical to the discretion rule if the cost shock is serially uncorrelated ($\rho = 0$).

References

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