Rational and Adaptive Expectations

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Types of expectations:

- **None**: No consideration of agent forecasts of the future in the economic model.
- **Adaptive Expectations**: Ad-hoc decisions based on previous data (i.e., looking backwards)
- **Rational Expectations**: Dynamic decisions need to be based on some expectation of the future (e.g., workers, financial markets, etc.)

1 First Generation

- They never suggested that adaptive expectation was optimal
- Possibility of systematic forecast errors, e.g. update new expectation based on a proportion of the previous forecast error.

- Let p_t be realized log price level.
- Let $p^e_{t,t+1}$ be the expected price level at time $t+1$ given time t information.

1.1 Adaptive Expectation

Take a fraction $\lambda \in (0,1)$:

$$p^e_{t,t+1} = p^e_{t-1,t} + \lambda(p_t - p^e_{t-1,t}) \tag{1}$$

where $(p_t - p^e_{t-1,t})$ is the forecast error last year, and λ is externally given.

1.2 Rational Expectation

$$p^e_{t,t+1} = \mathbb{E}\left[p_{t+1} \mid \Omega_t\right] \equiv \mathbb{E}\left[p_{t+1}\right] \tag{2}$$

where Ω_t is the information set available to agents at time t.
- Model dependent: Ω_t has present and past information
- $\mathbb{E}$ is the mathematical expectation
- Rational expectations only makes sense in the context of a specific model and information set.

1.3 Forecast Error

Note that the forecast error is given by:

$$\mathbb{E}_t\left[p_{t+1} - \mathbb{E}_t\left[p_{t+1}\right]\right] = 0 \tag{3}$$

No systematic errors: Not the same as no errors.
- Constraining the information sets or adding uncertainty on the underlying processes can be called “bounded rationality", but it is philosophically the same:

$$p_{t+1} - \mathbb{E}_t\left[p_{t+1}\right] = \text{ (the “surprise")} \tag{4}$$

- Market imperfections focus on information imperfections.

1.4 Example: Cagan-Sargent-Wallace

(Adapted from Bagliano 2010 notes) Take some ad-hoc money supply-demand equation (Keynesian style Money Demand LM) where p_t is the log price level, m_t is the log money supply, α is some elasticity of money demand, and $p^e_{t,t+1}$ is the expected price level tomorrow.

[1]

$$\underbrace{m_t - p_t}_{\text{money supply}} = -\alpha \underbrace{\left(p^e_{t,t+1} - p_t\right)}_{\text{money demand}} \text{ (in real terms)} \tag{8}$$

$$\boxed{p_t = \left(\frac{1}{1+\alpha}\right) m_t + \left(\frac{\alpha}{1+\alpha}\right) p^e_{t,t+1}} \tag{9}$$

Where expected inflation is defined as (in terms of non-logged P),

$$\pi^e_{t,t+1} = \frac{P^e_{t,t+1}}{P_t} = p^e_{t,t+1} - p_t \tag{10}$$

This is the equilibrium price level today in terms of money supply and expected price tomorrow. The dynamics then depend on how agents form expectations.

1.4.1 Adaptive Expectations

Assume (from equation (1)):

$$p^e_{t,t+1} = \underbrace{\lambda p_t + (1-\lambda) p^e_{t-1,t}}_{\substack{\text{weighting between realized} \\ \text{and previous forecast}}} \tag{11}$$

Iterating backwards:

$$p^e_{t,t+1} = \lambda p_t + (1-\lambda)\left[\lambda p_{t-1} + (1-\lambda) p_{t-2,t-1}\right] \tag{12}$$

$$\Rightarrow \boxed{p^e_{t,t+1} = \lambda \sum_{i=0}^{\infty} (1-\lambda)^i p_{t-i}} \text{ (looks backwards for all data)} \tag{13}$$

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[1] This often starts in absolute, rather than real terms, such as

$$\frac{M_t}{P_t} = \overline{Y} \exp(-\alpha(\bar{r} + \pi^e_{t,t+1})) \tag{5}$$

where M_t and P_t are not logged, $\overline{Y}$ is the constant real output, and $i = \bar{r} + \pi^e_{t,t+1}$. Take logs of equation (5):

$$\log M_t - \log P_t = \left(\log \overline{Y} - \alpha \bar{r}\right) - \alpha \pi^e_{t,t+1} \tag{6}$$

Let lower case be the log terms; approxmiate expected inflation as P^e:

$$\pi^e_{t,t+1} = \frac{P^e_{t,t+1}}{P_t} = p^e_{t,t+1} - p_t \tag{7}$$

Omit the constant through demeaning to get (9)

Plug into equation (9)

$$p_t = \frac{1}{1+\alpha} m_t + \frac{\alpha}{1+\alpha} \cdot \lambda \sum_{i=0}^{\infty} (1-\lambda)^i p_{t-i} \tag{14}$$

$$\boxed{p_t = \frac{1}{1+\alpha(1-\lambda)} m_t + \frac{\alpha\lambda}{1-\alpha(1-\lambda)} \cdot \lambda \sum_{i=1}^{\infty} (1-\lambda)^i p_{t-i}} \tag{15}$$

This is an example of adaptive expectations, which depends on current money supply and previous history (information set).
- Note that a temporary change in m_t has the same effect as a permanent one at time t.

1.5 Rational Expectations

$$p^e_{t,t+1} = \mathbb{E}_t \left[p_{t+1} \right] \tag{16}$$

Plug into equation (9)

$$p_t = \left(\frac{1}{1+\alpha} \right) m_t + \left(\frac{\alpha}{1+\alpha} \right) \mathbb{E}_t \left[p_{t+1} \right] \tag{17}$$

Iterate forward and take expectations of future periods:

$$\mathbb{E}_t \left[p_{t+1} \right] = \frac{1}{1+\alpha} \mathbb{E}_t \left[m_{t+1} \right] + \frac{\alpha}{1+\alpha} \mathbb{E}_t \left[p_{t+2} \right] \tag{18}$$

Plug into equation (17)

$$p_t = \left(\frac{1}{1+\alpha} \right) m_t + \left(\frac{\alpha}{1+\alpha} \right) \mathbb{E}_t \left[p_{t+1} + \frac{1}{1+\alpha} m_{t+1} + \frac{\alpha}{1+\alpha} \mathbb{E}_{t+1} \left[p_{t+2} \right] \right] \tag{19}$$

$$p_t = \left(\frac{1}{1+\alpha} \right) m_t + \left(\frac{\alpha}{1+\alpha} \right) \mathbb{E}_t \left[p_{t+1} \right] + \frac{\alpha}{1+\alpha} \frac{1}{1+\alpha} \mathbb{E}_t \left[m_{t+1} \right] + \left(\frac{\alpha}{1+\alpha} \right)^2 \mathbb{E}_t \left[\mathbb{E}_{t+1} \left[p_{t+2} \right] \right] \tag{20}$$

By law of iterated expectations: $\mathbb{E}_t \left[\mathbb{E}_{t+1} \left[p_{t+2} \right] \right] = \mathbb{E}_t \left[p_{t+2} \right]$

$$p_t = \left(\frac{1}{1+\alpha} \right) m_t + \left(\frac{\alpha}{1+\alpha} \right) \mathbb{E}_t \left[p_{t+1} \right] + \frac{\alpha}{1+\alpha} \frac{1}{1+\alpha} \mathbb{E}_t \left[m_{t+1} \right] + \left(\frac{\alpha}{1+\alpha} \right)^2 \mathbb{E}_t \left[p_{t+2} \right] \tag{21}$$

Repeat to $t \to \infty$:

$$\boxed{p_t = \frac{1}{1+\alpha} \sum_{j=0}^{\infty} \left(\frac{\alpha}{1+\alpha}\right)^j \mathbb{E}_t\left[m_{t+j}\right]} \tag{22}$$

Equation (22) is forward looking with forecasts.
The past only matters since it is in the information set at time t.
- Temporary vs. permanent changes in m_t have very different effects.

Differences between adaptive and rational expectations:

1. p_{t-i} enters through conditional information set
2. p_{t-i} may <u>not</u> change p_t, unless part of expectations (pricing sequence affected only in adaptive expectations)
3. Forward versus Backward ($t-i$ and $t+j$)

1.6 Example: Permanent Increase in Money Supply

Let there be a permanent increase in $\bar{m}$ to $\bar{m}+k$.

1.6.1 Rational Expectations

Before change:

$$p_{t-1} = \frac{1}{1+\alpha} \cdot \frac{1}{1-\frac{\alpha}{1+\alpha}} \bar{m} \tag{23}$$

After announcement, $\bar{m} \to \bar{m}+k$. Today:

$$p_t = \frac{1}{1+\alpha} \cdot \frac{1}{1-\frac{\alpha}{1+\alpha}} (\bar{m}+k) \tag{24}$$

1.6.2 Adaptive Expectations

The price adjustment will be slow depending on λ.

If the general population had and adaptive expectations, investors would have an arbitrage due to the predictable increase in the price level if they used <u>all available information</u>.
- They could pump enormous amounts of money out of the “unsophisticated".
- This enormous arbitrage might be seen by the individuals, but they are powerless to change anything.

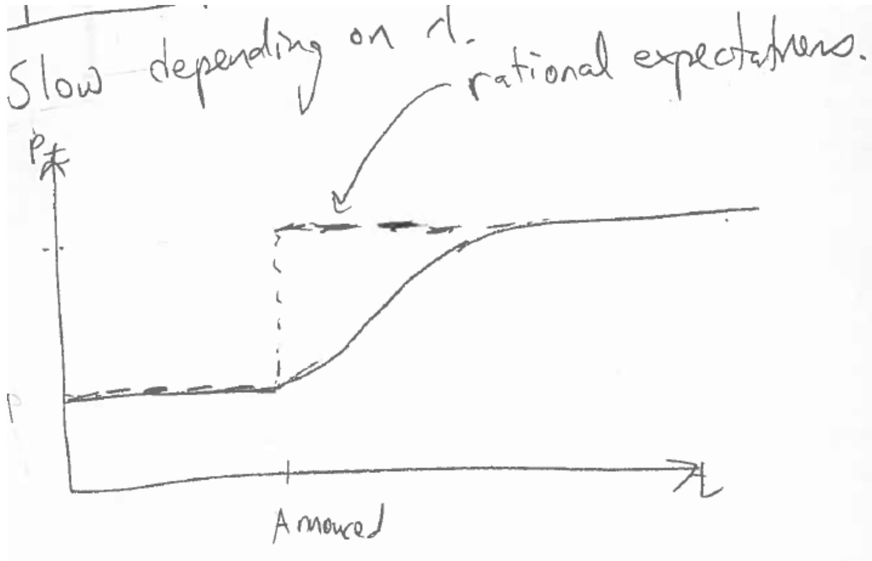

Figure 1: Adaptive Expectations Adjustment

- You don't need many deep pocketed investors for this to be true. Asset prices reveal information.

> *"You can fool some of the people some of the time, but not all of the people all of the time!"*
> *"If you are so smart, why aren't you rich?"*