Question 1: Adjustment Costs

Take the example from class for a firm. A competitive firm sells output y_t at price p_t and chooses a production plan to maximize

$$\mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t R_t\right] \tag{1}$$

where

$$R_t = p_t y_t - \kappa y_t^2 - \frac{\delta}{2}(y_{t+1} - y_t)^2 \tag{2}$$

subject to initial condition y_0.[1] The price p_t lies on the demand curve

$$p_t = A_0 - A_1 Y_t + \nu_t \tag{3}$$

and ν_t is a demand shock evolving according to:

$$\nu_{t+1} = \rho\nu_t + \sigma_\nu w_{t+1} \tag{4}$$

for $w_{t+1} \sim \mathrm{N}(0,1)$.

The firm believes that market-wide output follows the law of motion:

$$Y_{t+1} = H_0 + H_1 Y_t + H_2 \nu_t \tag{5}$$

subject to the initial condition Y_0. The firm observes p_t, Y_t and y_t at time t when choosing y_{t+1}

(a) Form the Bellman equation for our firm in our canonical form as a linear quadratic programming problem (i.e. `https://lectures.quantecon.org/jl/lqcontrol.html`.

(b) Assume that: $\beta = .95, \delta = 2, A_0 = 1010, A_1 = 1, H_0 = 200, H_1 = .8, H_2 = 2, \rho = .9, \sigma_\nu = .05, \kappa = .3$. Find the optimal policy for y_{t+1} as a function of the state (i.e. solve the LQ problem)

(c) Find the stationary distribution for Y_t and ν_t.[2]

(d) Set Y_0 and ν_0 to the mean of the stationary distribution from the previous part. Simulate a sequence of y_t using the solution to the Linear Quadratic problem:

1. Set $y_0 = 50$ and draw a sequence of shocks w_{t+1} for $T = 20$
2. Simulate the sequence y_t for these shocks
3. Plot y_t, p_t, ν_t, Y_t, for this sequence

(e) (**OPTIONAL**) Simulate the stationary distribution of y_t. Take $N = 100$ and $T = 50$. Choose an y_0 as the y_T in the previous question (though it shouldn't matter much)

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[1] **Note:** There is a cost of operation here of κy_t^2 compared to our class example.

[2] Hint: You can simulate numerically using the stochastic process, use equations from theory, or just use the QuantEcon library with the appropriate linear state space.

1. Draw a new sequence of $\{w_{t+1}\}_{t=0^T}$ shocks for $n = 1, \ldots N$.
2. From this use the solution to the problem to find y_T for each N
3. Plot the histogram.
4. Find the mean, 5th, and 95th quantiles of y_T from these simulated paths to get a sense of the stationary distribution of y_t
5. Is there a way you could calculate this stationary distribution from theory instead?