Nominal Pricing Frictions: Taylor, Rotemberg, and Calvo Models

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February 21, 2018

Money Having Real Effects

1. There needs to be a reason to hold money; it must not be dominated by bonds.
   a) Money in Utility (MIU)
   b) Cash in advance, monetary search, etc.
   c) Just put exogenous money demand function
2. Nominal frictions on prices - either wages or goods
   a) State dependent (e.g., adjustment costs to prices between periods)
   b) Time dependent (e.g., only $\frac{1}{2}$ of firms “allowed" to change)

In general: need both frictions for a model to have a real impact of monetary policy.

We will look at:

1. Taylor 2-period: time dependent
2. Rotemberg adjustment costs: state dependent
3. Calvo pricing: time dependent

1 2-Period Taylor Model

Reference: Walsh 6.2.2.
This explains persistence of monetary shock with some ad-hoc equations (motivated by MIU, Dixit-Stiglitz with nominal frictions, etc.).

1.1 Assumptions and Notations

- Assume a large continuum of firms, each charging their own price. The aggregate price level is the average price.
- Prices are sticky: only $\frac{1}{2}$ firms can change prices in a period, then alternate, etc.

Notation (in % deviations from steady state):

- p_t^* : optimal price that a firm would like to charge at time t
- p_t : aggregate price level at t
- y_t : real output
- m_t : real money supply

1.2 Consumer

Exogenously given money demand. Prior to writing in % deviations:

$$M_t V = P_t Y_t \tag{1}$$

With a constant V, we convert to % deviations:

$$m_t = p_t + y_t \text{ (similar to a MIU demand)} \tag{2}$$

1.3 Firm

Exogenously given optimal price:

$$p_t^* = p_t + \varphi y_t, \ \varphi \in (0, 1) \tag{3}$$

Interpret:

- If output is in a steady state, $y_t = 0 \Rightarrow p_t^* = p_t$.
  (the prices charged are close to the average price.)
- This is similar to monopolistic competition with wage stickiness.
- In a boom: $y_t > 0$, and presumably so are marginal costs, therefore the firm would want to change higher price.
- So φ controls dynamics of marginal cost fluctuations during boom/bust.

1.4 Sticky Prices

- Prices hold for t and $t+1$: "2 period contracts".
- Firms staggered: $\frac{1}{2}$ at t, $\frac{1}{2}$ at $t+1$.
- What is the optimal pricing rule?

Assume:

$$x_t = \frac{1}{2}\left(p_t^* + \mathbb{E}_t\left[p_{t+1}^*\right]\right) \tag{4}$$

x_t represents the price chosen at t; this is a reasonable assumption, but not from optimization. So, the price index is:

$$p_t = \frac{1}{2}\left(x_t + x_{t-1}\right) \tag{5}$$

where x_t are the 'blessed' half where the price is chosen this period, and x_{t-1} are the half chosen last period.

1.5 Money Supply

Exogenous random walk:

$$m_t = m_{t-1} + u_t,\ u_t \sim \mathrm{N}\left(0, \sigma^2\right) \tag{6}$$

All the innovations (shocks) are unanticipated (no autocorrelation).

1.6 Summarizing Assumed Equations

$$\begin{aligned}
p_t^* &= p_t + \varphi y_t && \text{(Friction with marginal costs)} && (7)\\
x_t &= \frac{1}{2}\left(p_t^* + \mathbb{E}_t\left[p_{t+1}^*\right]\right) && \text{(nominal optimal price, staggered)} && (8)\\
p_t &= \frac{1}{2}\left(x_t + x_{t-1}\right) && \text{(pricing aggregate)} && (9)\\
m_t &= m_{t-1} + u_t && \text{(exogenous money supply)} && (10)\\
m_t &= p_t + y_t && \text{(money demand)} && (11)
\end{aligned}$$

We have 5 equations with 5 variables $(p_t, p_t^*, m_t, x_t, y_t)$.

1.7 Solving The Model

1.7.1 Simplifying

Combine (44) and (11):

$$p_t^* = \varphi m_t + (1-\varphi)p_t \tag{12}$$

Equation (12) is a weighted average between real supply and price index. Notice that since $\varphi < 1$, the price level does not fully respond to shock.

Combine (12) and (8):

$$x_t = \frac{1}{2}\left[\varphi m_t + (1-\varphi)p_t + \mathbb{E}_t\left[\varphi m_{t+1} + (1-\varphi)p_{t+1}\right]\right] \tag{13}$$

Given that m_t follows a random walk process:

$$= \frac{1}{2}\left[\varphi m_t + (1-\varphi)p_t + \varphi m_t + (1-\varphi)\mathbb{E}_t\left[p_{t+1}\right]\right] \tag{14}$$

Use equation (9) for $\mathbb{E}_t\left[p_{t+1}\right]$ and p_t

$$x_t = \left[2\varphi m_t + \frac{1-\varphi}{2}\left(x_t + x_{t-1}\right) + \frac{1-\varphi}{2}\left(x_t + \mathbb{E}_t\left[x_{t+1}\right]\right)\right] \tag{15}$$

Rearrange to get:

$$x_t = (1-2A)m_t + A(x_{t-1} + \mathbb{E}_t\left[x_{t+1}\right]) \tag{16}$$

where $A = \frac{1}{2}\frac{1-\varphi}{1+\varphi}$.

Equation (16) is a 2nd-order difference equation, where m_t is exogenous.

1.7.2 Guess and Verify

A Guess:

$$x_t = \Lambda x_{t-1} + (1-\Lambda)m_t \text{ (for some } \Lambda) \tag{17}$$

Substitute guess to $t+1$ period to find expectation:

$$\mathbb{E}_t\left[x_{t+1}\right] = \Lambda x_t + (1-\Lambda)\underbrace{\mathbb{E}_t\left[m_{t+1}\right]}_{=m_t} \tag{18}$$
$$= \Lambda x_{t-1} + (1-\Lambda)m_t \tag{19}$$
$$= \Lambda^2 x_{t-1} + (1-\Lambda^2)m_t \tag{20}$$

Substitute into equation (16):

$$x_t = (1-2A)m_t + Ax_{t-1} + A\left[\Lambda^2 x_{t-1} + (1-\Lambda^2)m_t\right] \tag{21}$$
$$= \left[A + A\Lambda^2\right] x_{t-1} + \left(1-2A+A(1-\Lambda^2)\right) m_t \tag{22}$$

Now we can verify that if $\Lambda = A + A\Lambda^2$ and use with the guess in equation (17):

$$\Lambda x_{t-1} + (1-\Lambda)m_t = \left[A + A\Lambda^2\right] x_{t-1} + \left(1-2A+A(1-\Lambda^2)\right) m_t \tag{23}$$

Using the method of undetermined coefficients:

$$\Rightarrow A + A\Lambda^2 = \Lambda \text{ (if quadratic form)} \tag{24}$$

Solving for the quadratic $\Lambda(A)$ and picking the stable root:

$$\Rightarrow \Lambda = \frac{1-\sqrt{1-4A^2}}{2A} \tag{25}$$

Substitute for $A = \frac{1}{2}\frac{1-\varphi}{1+\varphi}$:

$$\boxed{\Lambda = \frac{1-\sqrt{\varphi}}{1+\sqrt{\varphi}}} \tag{26}$$

- We can show that if $0 < \varphi < 1$, then $\Lambda \in (0,1)$, i.e., partial response to a shock.
- So, with $x_t = \Lambda x_{t-1} + (1-\Lambda)m_t$, x_t responses only partially to a shock to m_t.

Now, going back to the price index in equation (9), we see that it takes > 2 periods to adjust to a m_t shock, even though prices all adjust in 2 periods.
Finally, substitute x_t formula into (9), then into (11) with (10) to get:

$$y_t = \Lambda y_{t-1} + \frac{1+\Lambda}{2}u_t \tag{27}$$

So, if $\Lambda > 0$: monetary shock u_t has a persistent effect on real output.

2 Rotemberg (1982, 1996): Adjustment Costs

Rotemberg's quadratic cost of nominal price adjustment is derived within a simple state based model (Walsh 6.2).

2.1 Model and Assumptions

- The price for firm j depends on the aggregate level, P_t, and real economic variables, x_t (in log terms).
- If prices could adjust costlessly, then assume the prices would be:

$$P_t^*(j) = P_t + \alpha x_t \text{ (optimal)} \tag{28}$$

  This requires some degree of market power and decreasing demand.

- Also, assume that the log profits are decreasing quadratic in deviation from $P_t^*(j)$:

$$\pi_t(j) = -\delta \left[P_t(j) - P_t^*(j)\right]^2 = -\delta \left[P_t(j) - P_t - \alpha x_t\right]^2 \tag{29}$$

- This could be derived with a 2nd order approximation to monopolistic competition, etc.
- Very common style in older papers.
  (We can add $\mathrm{N}(0, \sigma^2)$ shocks to get Linear Quadratic Gaussian states; still very tractable.)
- But, prices are also costly to adjust; assume quadratic cost:

$$c_t(j) = \varphi \left(P_t(j) - P_{t-1}(j)\right)^2 \tag{30}$$

2.2 Firm's Problem for Firm j

Firm's j's problem is:

$$\max_{P_t(j)} \sum_{t=0}^{\infty} \beta^t \mathbb{E}_0 \left[\pi_t(j) - c_t(j)\right] \tag{31}$$

$$= \sum_{t=0}^{\infty} \beta^t \mathbb{E}_0 \left[-\delta \left(P_t(j) - P_t - \alpha x_t\right)^2 - \varphi \left(P_t(j) - P_{t-1}(j)\right)^2\right] \tag{32}$$

Take FONC w.r.t $P_t(j)$:

$$-\delta\left(P_t(j) - P_t - \alpha x_t\right) - \varphi\left(P_t(j) - P_{t-1}(j)\right) + \beta\varphi\left(\mathbb{E}_t\left[P_{t+1}(j)\right] - P_t(j)\right) = 0 \quad (33)$$

For simplicity, assume identical firms: $P_t(j) = P_t$ for all $j, t, ...$

$$\delta\alpha x_t - \varphi\left(P_t - P_{t-1}\right) + \beta\varphi\left(\mathbb{E}_t\left[P_{t+1}\right] - P_t\right) = 0 \quad (34)$$

Using the definition of inflation in logs:

$$\begin{aligned} \pi_t &\equiv P_t - P_{t-1} \\ \mathbb{E}_t\left[\pi_{t+1}\right] &= \mathbb{E}_t\left[P_{t+1}\right] - P_t \end{aligned}$$

Rearrange:

$$\boxed{\pi_t = \beta\mathbb{E}_t\left[\pi_{t+1}\right] + \left(\frac{\alpha\delta}{\varphi}\right)x_t} \quad (35)$$

- Actual inflation depends of real variable x_t and expected inflation.
- Set prices at time t higher than the frictionless price if they expect inflation to increase in the future.
- This is a prototypical “state-based" friction, even if we are setting all firms identical for clarity.
- Tough to connect to data since firms don’t change prices constantly.
- But, adding fixed costs to change gives more empirically consistent behaviour

3 Calvo: Staggered Price Adjustments

- This is a time-dependant variation of Taylor 2-period model with fewer ad-hoc equations
- A good, tractable, baseline model - even if you dislike the assumptions.
- See Walsh 6.2.

3.1 Model and Assumptions

(1) Monopolistic competition (can assume same productivity).

(2) We would want to set the monopolistic price, *but:*
- Arrival rate probability of $1-\omega$ for each period that they are allowed to set the price
- Probability of ω they have to keep old price (sometimes modellers let the price drift indexed to inflation)

(3) They can set the optimal dynamic price, not constrained by an ad-hoc equation.

Why is the model tractable?

(a) If we assume all firms have the same productivity, get same price for those with ω shock.

(b) Since the shock is random and independent of any firm state, the price for the no-shock ω firms is the unconditional average price last period. We just need the single price as the state, instead of the whole distribution of prices and firm types.

3.2 Firm's Choice

3.2.1 Setup

- The state of a firm is its price. Alternatively, if all firms are identical you can just index their state by how many periods ago they set their price.
  But this isn't necessary, as discussed. Just need to look at a firm, i, hit by the ω shock.

- The future prices of the firm are $P_{t+j}(i)$ for $j = 0, \ldots, \infty$. These are dependent on future ω shocks. Profits depend on aggregate φ_t cost shocks.

- The aggregate price index is P_t.

- Period nominal profits from monopolistic competition:

$$\pi_{t+j}(i) = (P_{t+j}(i) - \underbrace{P_{t+j}\varphi_{t+j}}_{\substack{\text{nominal}\\ \text{marginal}\\ \text{cost}}}) \underbrace{\left(\frac{P_{t+j}}{P_{t+j}(i)}\right)^{\theta}}_{\substack{\text{decreasing}\\ \text{demand}\\ \text{from CES}}} \cdot \underbrace{Y_{t+j}}_{\substack{\text{aggregate}\\ \text{final}\\ \text{goods}\\ \text{output}}} \tag{36}$$

$\theta \equiv \kappa$ from the notation in our Monopolistic Competition notes.[1]

[1] This is trying to be consistent with Walsh except $V_{t+j} \to \varphi_{t+j}$ and $\frac{1}{1-q} \to \theta$, both to be consistent with Walsh chapter 8.

3.2.2 Firm Maximizes Profits

$$\max_{P_t(i)} \mathbb{E}_t\left[\sum_{j=0}^{\infty}\beta^j \pi_{t+j}(i)\right] \quad \text{(or use consumer SDF } \Lambda_t) \tag{37}$$

$$= \mathbb{E}_t\left[\sum_{j=0}^{\infty}\beta^j (P_{t+j}(i) - P_{t+j}\varphi_{t+j})\left(\frac{P_{t+j}}{P_{t+j}(i)}\right)^{\theta} Y_{t+j}\right] \tag{38}$$

Expand the sum for future states where they do not receive the shock, each with independent ω probability

$$\begin{aligned}
&= (P_t(i) - P_t\varphi_t)\left(\frac{P_t}{P_t(i)}\right)^{\theta} Y_t + \underbrace{\omega}_{\text{no shock}} \beta\mathbb{E}_t\left[(P_t(i) - P_{t+1}\varphi_{t+1})\left(\frac{P_{t+1}}{P_t(i)}\right)^{\theta} Y_{t+1}\right] \\
&\quad + \omega^2\beta^2\mathbb{E}_t\left[(P_t(i) - P_{t+2}\varphi_{t+2})\left(\frac{P_{t+2}}{P_t(i)}\right)^{\theta} Y_{t+2}\right] \\
&\quad + \ldots + (\text{terms with no } P_t(i) \text{ after a } 1-\omega \text{ shock})
\end{aligned} \tag{39}$$

$$= \sum_{j=0}^{\infty}\omega^j\beta^j\mathbb{E}_t\left[(P_t(i) - P_{t+j}\varphi_{t+j})\left(\frac{P_{t+j}}{P_t(i)}\right)^{\theta} Y_{t+j}\right] + (\text{terms with no } P_t(i)) \tag{40}$$

Take the FONC w.r.t $P_t(i)$, and denote $P_t(i) \equiv P_t^*$ since all firms able to choose a price will choose the same one (since they face identical marginal costs):

$$\left(\frac{q}{1-q}\right)\mathbb{E}_t\left[\sum_{j=0}^{\infty}\omega^j\beta^j\left[\left(\frac{P_t^*}{P_{t+j}}\right) - \left(\frac{\theta}{\theta-1}\right)\varphi_{t+j}\right]\left(\frac{1}{P_t^*}\right)\left(\frac{P_{t+j}}{P_t^*}\right)^{\theta} Y_{t+j}\right] = 0 \tag{41}$$

Dropping the (i) since all firms are the same given the ω shock, and rearranging:

$$\frac{P_t^*}{P_t} = \frac{\theta}{\theta-1}\mathbb{E}_t\left[\frac{\sum\limits_{j=0}^{\infty}\omega^j\beta^j\varphi_{t+j}\left(\frac{P_{t+j}}{P_t^*}\right)^{\theta}}{\sum\limits_{j=0}^{\infty}\omega^j\beta^j\left(\frac{P_{t+j}}{P_t^*}\right)^{\theta-1}}\right] \tag{42}$$

Note if $\omega = 0$, then $\frac{P_t^*}{P_t} = \frac{\theta}{\theta-1}\varphi_t$. A constant markup over marginal cost.

Law of Motion: Recall that the price index in monopolistic competition with firms labeled j is,

$$P_t \equiv \left[\int_0^1 (p_{jt})^{1-\theta}\,\mathrm{d}j\right]^{\frac{1}{1-\theta}} \tag{43}$$

With Calvo pricing, there are only two types of firms: (1) firms able to change their price, who all choose P_t^*; and (2) firms unable to change their price, who keep the existing p_{jt}. With this Calvo pricing friction, the price index can then be calculated in period t as:

$$P_t^{1-\theta} = (1-\omega)(P_t^*)^{1-\theta} + \omega P_{t-1}^{1-\theta} \tag{44}$$

where $(P_t^*)^{1-\theta}$ is the price chosen in period t, and $P_{t-1}^{1-\theta}$ is the average price which couldn't be chosen.

Divide by $P_t^{1-\theta}$ and use the definition, $P_t/P_{t-1} \equiv 1 + \pi_t$:

$$1 = (1-\omega)\left(\frac{P_t^*}{P_t}\right)^{1-\theta} + \omega\left(\frac{1}{1+\pi_t}\right)^{1-\theta} \tag{45}$$

But $\dfrac{P_t^*}{P_t}$ is dependent on future $\mathbb{E}_t\left[\dfrac{P_{t+j}}{P_j}\right]$, which includes expected future inflation.

After some painful algebra, we can get a 1st order approximation around steady state (See Walsh Chapter 8 appendix):

$$\pi_t = \beta\mathbb{E}_t\left[\pi_{t+1}\right] + \kappa\hat{\varphi}_t, \;\; \kappa \equiv \frac{(1-\omega)(1-\beta\omega)}{\omega} \tag{46}$$

Equation (46) is typically called a **New Keynesian Phillips Curve** (NKPC), where $\hat{\varphi}_t$ is the deviation from steady state marginal cost.