# The Real Business Cycle Model

Jesse Perla
University of British Columbia

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## 1 Elements of the New Keynesian Model

Building up towards a model with non-trivial monetary policy and business cycles:

(1) Neoclassical Growth Model:

- Capital Accumulation
- Labor Supply / Demand

(2) **Real Business Cycles**

+ Stochastic TFP

(3) Monopolistic Competition

+ Differentiated Firms
+ Non-trivial prices (markup)

(4) Stickiness in Price Changes

(5) Reason to hold money (i.e., returns not dominated by bonds)

(6) Monetary Control over Short-term Interest Rates

## 2 The Real Business Cycle

### 2.1 Final Goods

Consumer has preferences over <u>final good</u> consumption, $C$, and <u>labour supplied,</u> $L$.

$$u(C, L) = \underbrace{\log(C)}_{\text{CRRA=1}} - \nu \log(L) \tag{1}$$

where $\nu \log(L)$ is the disutility of labour and $\beta$ the discount factor.
Therefore, the expected welfare is:

$$\mathbb{E}_0 \left[ \sum_{t=0}^{\infty} \beta^t u(C_t, L_t) \right] \tag{2}$$

- Let the consumer's real income in period $t$ be $Y_t$; $P_t$ is the price of the final good.
- Nominal income is $P_t Y_t$.
- The consumer can use the final good for consumption, or can use it to add to the capital stock $K_t$, which depreciates at rate $\delta \in (0, 1)$.
- The consumer rents labour at real price $w_t$ and capital at real price $r_t$.
- The consumer owns the firms and gains real profits of $\pi_t$.

## 2.2 Consumer's Problem

The consumer's problem and constraints are given by:

$$\max_{\{C_t, L_t, K_{t+1}\}_{t=0}^{\infty}} \mathbb{E}_0 \left[ \sum_{t=0}^{\infty} \beta^t u(C_t, L_t) \right] \tag{3}$$

$$\text{s.t. } P_t C_t + P_t \left( K_{t+1} - (1 - \delta) K_t \right) \leq P_t r_t K_t + P_t w_t L_t + P_t \pi_t \equiv Y_t P_t \tag{4}$$

Divide (4) by $P_t$ to get the budget constraint in real terms:

$$C_t + (K_{t+1} - (1 - \delta) K_t) \leq r_t K_t + w_t L_t + \pi_t \equiv Y_t \tag{5}$$

where:

$$Y_t \equiv r_t K_t + w_t L_t + \pi_t \tag{6}$$

The Lagrange equation is:

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left[ u(C_t, L_t) + \lambda_t \left( r_t K_t + w_t L_t + \pi_t - C_t - K_{t+1} + (1 - \delta) K_t \right) \right] \tag{7}$$

The first-order necessary conditions are:

$$[C_t] : \boldsymbol{\partial}_C u(C_t, L_t) = \lambda_t \tag{8}$$

$$[L_t] : \boldsymbol{\partial}_L u(C_t, L_t) = -\lambda_t w_t \tag{9}$$

Dividing (9) by (8)

$$\Rightarrow \frac{\boldsymbol{\partial}_L u(C_t, L_t)}{\boldsymbol{\partial}_C u(C_t, L_t)} = w_t \tag{10}$$

Here: $\boldsymbol{\partial}_L u(C_t, L_t) = -\frac{\nu}{L}$, $\boldsymbol{\partial}_C u(C_t, L_t) = \frac{1}{C}$.

The labour supply equation is:

$$\nu \frac{C_t}{L_t} = w_t \tag{11}$$

$$\boldsymbol{\partial}_{K_{t+1}} \mathcal{L} : \mathbb{E}_t \left[ -\lambda_t + \beta \lambda_{t+1} \left( r_{t+1} + (1 - \delta) \right) \right] = 0 \tag{12}$$

$$\Rightarrow \frac{\lambda_t}{\mathbb{E}_t \left[ \lambda_{t+1} \right]} = \mathbb{E}_t \left[ \beta (r_{t+1} + 1 - \delta) \right] \tag{13}$$

$$\boxed{1 = \mathbb{E}_t \left[ \frac{C_t}{C_{t+1}} \cdot \beta \left( r_{t+1} + 1 - \delta \right) \right]} \tag{14}$$

Equation (14) represents the asset Euler equation.

**Final Goods Resource Constraint:**

$$C_t + K_{t+1} = Y_t + (1 - \delta) K_t \tag{15}$$

Later we will show that real income = total production.

## 2.3 Producer's Problem

**Example: RBC with Homogeneous Good**

Let there be a competitive representative firm with productivity $Z_t$, stochastic in nature, and Cobb-Douglas production function with constant returns to scale:

$$F(K_t, L_t, Z_t) = Z_t K_t^{\alpha} L_t^{1-\alpha}, \ \alpha \in (0, 1) \tag{16}$$

- The firm is a price taker for $w_t, r_t$ to rent inputs (real prices), and maximizes period profits:

$$P_t \pi_t = \max_{\{L_t, K_t\}} \left[ \underbrace{P_t}_{\text{price}} \underbrace{F(K_t, L_t, Z_t)}_{\text{production}} - \underbrace{P_t r_t K_t - P_t w_t L_t}_{\text{rental of inputs}} \right] \tag{17}$$

The first-order conditions are:

$$[K_t] : \boldsymbol{\partial}_K F(K_t, L_t, Z_t) = r_t \tag{18}$$
$$[L_t] : \boldsymbol{\partial}_L F(K_t, L_t, Z_t) = w_t \tag{19}$$

Equation (18) and (19) represent the input demand functions.

Plugging in the form of production function from equation (16):

$$\boldsymbol{\partial}_K F = Z_t \cdot \alpha K_t^{\alpha-1} L_t^{1-\alpha} = \alpha Z_t \left(\frac{K_t}{L_t}\right)^{\alpha-1} \tag{20}$$
$$\boldsymbol{\partial}_L F = (1-\alpha) Z_t \left(\frac{K_t}{L_t}\right)^{\alpha} \tag{21}$$

Let $k_t \equiv \frac{K_t}{L_t}$: the capital-labor ratio

## 2.4 Aggregate Productivity Process

Let the stochastic process for aggregate productivity $Z_t$ be:

$$Z_{t+1} = \rho Z_t + \sigma \varepsilon_{t+1},\ \varepsilon_{t+1} \sim \mathrm{N}(0, 1) \tag{22}$$

The aggregate states are $K_t$ and $Z_t$.

## 2.5 Competitive Equilibium

**Definition 1** (Competitive Equilibrium)**.** *It is a set of allocations and conditions as follows:*

***Policies:*** $C_t(K_t, Z_t), K_{t+1}(K_t, Z_t), L_t(K_t, Z_t),$

***Prices:*** $r_t(K_t, Z_t), w_t(K_t, Z_t)$

*such that:*

1. *Given the prices, the policies solve the consumer's problem*
2. *Given the prices, the policies solve the firm's problem*
3. *Labour and capital markets clear*

The system of equation needed to solve for the competitive equilibrium, summarized from the previous sections, are:

$$
\begin{aligned}
1 &= \mathbb{E}_t \left[ \frac{C_t}{C_{t+1}} \cdot \beta \left( r_{t+1} + 1 - \delta \right) \right] && \textbf{(Euler equation)} \\
\nu \frac{C_t}{L_t} &= w_t && \textbf{(Labour Supply)} \\
r_t &= \alpha Z_t \left( \frac{K_t}{L_t} \right)^{\alpha-1} && \textbf{(Capital Demand)} \\
w_t &= (1-\alpha) Z_t \left( \frac{K_t}{L_t} \right)^{\alpha} && \textbf{(Labor Demand)} \\
C_t + K_{t+1} - (1-\delta) K_t &= Z_t K_t^{\alpha} L_t^{1-\alpha} && \textbf{(Resource Constraint)} \\
Z_{t+1} &= \rho Z_t + \sigma \varepsilon_{t+1} && \textbf{(Technological Process)}
\end{aligned}
$$

We can use Dynare to solve for the model with the system of equations above.