The "New Keynesian" Model

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1 Components of the Basic "New Keynesian" Model

Reference: Walsh Chapter 8 (see 8.3.5 to add a taste shock)

1) Neoclassical growth (the capital dynamics removed for simplicity)
2) RBC shocks to aggregate productivity
3) Monopolistic Competition
4) Money in utility
5) Calvo price stickiness
6) Exogenous nominal interest rate rule (*Taylor rule*) as function of "output gap" and inflation. Money supply is hidden, but assumed implicit to attain interest rate rule.

- Sometimes, "New Keynesian" refers to the resulting linearized difference equations in i_t, π_t, y_t, r_t with shocks.

Why remove capital? Mainly for simplification, but some studies show little connection between capital stock and output at business cycle frequencies.
- Christiano, Eichenbaum, and Evans (2005) shows that variable capital utilization may matter.

2 Household

The preferences of the representative household are defined over a composite consumption good C_t (defined in the next section), real money balances M_t/P_t, and the time devoted to market employment N_t. The household's problem is to maximize the expected present discounted value of lifetime utility:

$$\mathbb{E}_t \left[\sum_{i=0}^{\infty} \beta^i \left(\frac{C_{t+i}^{1-\sigma}}{1-\sigma} + \frac{\gamma}{1-b} \left(\frac{M_{t+i}}{P_{t+i}} \right)^{1-b} - \chi \frac{N_{t+i}^{1+\eta}}{1+\eta} \right) \right] \tag{1}$$

subject to the budget constraint (in real terms):

$$C_t + \frac{M_t}{P_t} + \frac{B_t}{P_t} = \left(\frac{W_t}{P_t} \right) N_t + \frac{M_{t-1}}{P_t} + (1 + i_{t-1}) \left(\frac{B_{t-1}}{P_t} \right) + \Pi_t \tag{2}$$

where P_t is the (aggregate) price index (price of composite good).

2.1 Summary

(1) MIU with separable labour and money demand.

(2) Consumes composite good C_t.

(3) Saves through nominal bonds B_t and holds nominal money M_t.

(4) Owns portfolio of firms and pay wages

(5) Exogenous government consumption of final goods purchased from the market; G_t could be added, and this would change the resource constraint and potentially the firm's demand (e.g., Y_t instead of C_t).

2.2 Composite Good

The composite consumption good consists of *differentiated* products given by the following:

$$C_t \equiv \left[\int_0^1 (c_{jt})^{\frac{\theta-1}{\theta}} \, \mathrm{d}j \right]^{\frac{\theta}{\theta-1}}, \ \theta > 1 \tag{3}$$

which are produced by monopolistically competitive producers (firms): there is a continuum of such firms of measure 1, and firm j produces good c_{jt} in time t.

2.3 Household Problem in Two Stages

2.3.1 CES Preferences Gives Demand and Price Index

$$c_{jt} = \left(\frac{p_{jt}}{P_t}\right)^{-\theta} C_t \qquad \text{(demand curve)} \tag{4}$$

$$P_t \equiv \left[\int_0^1 (p_{jt})^{1-\theta} \, \mathrm{d}j\right]^{\frac{1}{1-\theta}} \qquad \text{(price index)} \tag{5}$$

This is a static setup, as the consumers can costlessly adjust (see Walsh 8.2.1 for the derivation of the indices).

2.3.2 Dynamic Decisions of Consumption and Labour

Setting up the Lagrangian from equations (1) and (2):

$$\begin{aligned}\boldsymbol{\mathcal{L}} = \sum_{i=0}^{\infty} \beta^i \Bigg[& \frac{C_{t+i}^{1-\sigma}}{1-\sigma} + \frac{\gamma}{1-b}\left(\frac{M_{t+i}}{P_{t+i}}\right)^{1-b} - \chi\left(\frac{N_{t+i}^{1+\eta}}{1+\eta}\right) \\ & + \lambda_t\left(C_t + \frac{M_t}{P_t} + \frac{B_t}{P_t} - \left(\frac{W_t}{P_t}\right)N_t - \frac{M_{t-1}}{P_t} - (1+i_{t-1})\left(\frac{B_{t-1}}{P_t}\right) - \Pi_t\right)\Bigg]\end{aligned} \tag{6}$$

Taking the FONCs of C_t and N_t:

$$\boldsymbol{\partial}_{C_t}\mathcal{L}: \; C_t^{-\sigma} = \lambda_t \text{ marginal utility of consumption)} \tag{7}$$

$$\boldsymbol{\partial}_{N_t}\mathcal{L}: \; \chi N_t^{\eta} = \frac{W_t}{P_t}\lambda_t \tag{8}$$

Combining (7) and (8):

$$\frac{\chi N_t^{\eta}}{C_t^{-\sigma}} = \frac{W_t}{P_t} \text{ (Labour supply)} \tag{9}$$

and, following the MIU notes, the Euler condition for the optimal intertemporal allcoation of consumption is given by:

$$C_t^{-\sigma} = \beta(1+i_t)\mathbb{E}_t\left[\left(\frac{P_t}{P_{t+1}}\right)C_{t+1}^{-\sigma}\right] \tag{10}$$

and finally, the (less important) money holding versus bonds Euler equation is:

$$\frac{\gamma\left(\frac{M_t}{P_t}\right)^{-b}}{C_t^{-\sigma}} = \frac{i_t}{1+i_t} \tag{11}$$

2.3.3 Stochastic Discount Factor

If the discount factor itself follows a stochastic process, then from the first order conditions, the stochastic discount factor $\triangle_{i,t+i}$ is given by:

$$\triangle_{i,t+i} = \beta^i \left(\frac{C_{t+i}}{C_t} \right)^{-\sigma} \tag{12}$$

3 Firms

Let the aggregate productivity of the continuum of firms be Z_t. Firms maximize profits, subject to the following constraints.

- The production function follows a constant returns to scale in labour input N_{jt} for firm j in time t:

$$c_{jt} = Z_t N_{jt} \tag{13}$$

- Calvo pricing friction: firms have monopoly power, but can only change prices with probability $1 - \omega$ each period, $i.i.d$ over time and across firms.
- A firm's marginal (real) cost is:

$$\varphi_t = \frac{W_t/P_t}{Z_t} \tag{14}$$

- The firm, with arrival $1-\omega$, chooses p_{jt} to maximize the expected present discounted value of real profits, discounting with the consumer's stochastic discount factor.

3.1 Firm's Problem

Since the firm can change the price with the next $1 - \omega$ arrival, their expectation is also over the number of periods until the arrival. Therefore, the firm's pricing decision problem is given by:

$$\max_{p_{jt}} \mathbb{E}_t \left[\sum_{i=0}^{\infty} \overbrace{\omega^i \triangle_{i,t+i}}^{\substack{\text{only terms} \\ \text{at } P_{t+i}}} \left(\frac{p_{jt}}{P_{t+i}} - \varphi_{t+i} \right) c_{jt+i} \right] \tag{15}$$

where $\frac{p_{jt}}{P_{t+i}}$ is the real price, and φ is the real marginal cost and c_{jt+i} is from the demand function, both at $t+i$. Now, substituting in the demand function from equation (4):

$$\max_{p_{jt}} \mathbb{E}_t \left[\sum_{i=0}^{\infty} \omega^i \triangle_{i,t+i} \left[\left(\frac{p_{jt}}{P_{t+i}} \right)^{1-\theta} - \varphi_{t+i} \left(\frac{p_{jt}}{P_{t+i}} \right)^{-\theta} \right] C_{t+i} \right] \tag{16}$$

Let P_t^* be the arg max, which is identical for all firms able to change their price. Take the FOC of equation (16) and substitute for $p_{jt} = P_t^*$ in equation (12):

$$\left(\frac{P_t^*}{P_t} \right) = \mu \frac{\mathbb{E}_t \left[\sum_{i=0}^{\infty} \omega^i \beta^i C_{t+i}^{1-\sigma} \varphi_{t+i} \left(\frac{P_{t+i}}{P_t^*} \right)^{\theta} \right]}{\mathbb{E}_t \left[\sum_{i=0}^{\infty} \omega^i \beta^i C_{t+i}^{1-\sigma} \left(\frac{P_{t+i}}{P_t^*} \right)^{\theta-1} \right]} \tag{17}$$

where $\mu \equiv \frac{\theta}{\theta-1}$ is the flexible price markup (from CES algebra).
Note: if government has same CES aggregator as consumer above, then replace $C_t \to Y_t$.

3.2 Goods Clearing Condition

$$C_t = Y_t \qquad \text{(or } C_t + G_t = Y_t \text{ with government)} \tag{18}$$

3.3 Aggregate Labour Demand

$$N_t = \frac{Y_t}{Z_t} \qquad \text{(see monopolistic competition notes for aggregation)} \tag{19}$$

3.4 Price Index

Take the price index in equation (5):

$$P_t \equiv \left[\int_0^1 (p_{jt})^{1-\theta} \, \mathrm{d}j \right]^{\frac{1}{1-\theta}}$$

In this integral, consider that there are only two types.: (1) firms able to change their price, who all choose P_t^*; and (2) firms unable to change their price, who keep the existing p_{jt}. With this Calvo pricing friction, the price index can then be calculated in period t as:

$$P_t^{1-\theta} = (1-\omega)(P_t^*)^{1-\theta} + \omega P_{t-1}^{1-\theta} \tag{20}$$

where $(P_t^*)^{1-\theta}$ is the price chosen in period t, and $P_{t-1}^{1-\theta}$ is the average price which couldn't be chosen.

4 Monetary Policy Exogenous

Assume that policy adjusts i_t based on inflation and other real variables, while M_t is adjusted in the background. But due to separability, the money demand function is only in the background (see MIU notes).

Summary

Exogenous: Z_t, i_t (and G_t if in function).

Variables: $C_t, Y_t, \frac{W_t}{P_t}, P_t, P_t^*, N_t, \varphi_t$ (i.e. no B_t or M_t needed)

Equations: (9), (10), (14), (17), (18), (19), (20).
- Full set of stochastic difference equations of the NK model.
- Log-linearize using our tools
- Calibrate of estimate parameters (none for money demand, since i_t is determined directly).

5 Output Gap and Flexible Price Equilibrium

It is tough to know the real marginal costs φ_t in the economy. Typically, the model is written in terms of the output gap, i.e. $\frac{Y_t^f}{Y_t}$ where Y_t^f is the economy output if prices were flexible (but still monopolistically competitive).
With flexible price, i.e. $\omega = 0$,

$$\frac{P_t^*}{P_t} = \frac{\theta}{\theta - 1}\, \varphi_t \quad \text{(from equation (17), or monopolistic competition)} \tag{21}$$

$$= \mu\varphi_t = 1 \text{ (since } P_t^* = P_t) \tag{22}$$

From equation (14):

$$\frac{W_t}{P_t} = Z_t\varphi_t = \frac{1}{\mu} Z_t \text{ (also, standard for M.C.)} \tag{23}$$

From labour supply (equation (9)):

$$\frac{\chi N_t^{\eta}}{C_t^{-\sigma}} = \frac{1}{\mu} Z_t \tag{24}$$

From the resource constraint (equation (18)):

$$\frac{\chi N_t^\eta}{Y_t^f - G_t} = \frac{1}{\mu} Z_t \tag{25}$$

Solving for Y_t^f, then substituting with demand in equation (19):

$$\boxed{Y_t^f = G_t + \left(\frac{1}{\chi\mu}\right)^{\frac{1}{\sigma+\eta}} Z_t^{\frac{1+\eta}{\sigma+\eta}}} \tag{26}$$

Equation (26) denotes the function of equilibrium output under flexible prices, defined entirely in parameters and contemporaneous shocks.
Define the *output gap* as the deviation of actual output from the steady state:

$$x_t \equiv \hat{y}_t - \hat{y}_t^f \tag{27}$$

We will derive relationship to φ_t later. The following shows a list of the estimated parameters and how to calibrate them: + exogenous processes

Table 1: Estimated Parameters and How To Calibrate

β	Discount factor; use with σ and real interest rate
σ	CRRA; use risk premium?
η	Frisch elasticity; touch since micro $\neq$ macro
χ	Scale of economy; “free parameter" disappears in % deviations
θ	Relate to markups of firms
ω	$\mathbb{E}$ [periods to change price] $= \frac{1}{1-\omega}$ since Poisson

6 Linearizing Around Steady States

Assume $G_t = 0$ for simplicity. Following standard (but painful) log-linearization will generate the following variables,

- π_t: the % deviation from steady state inflation
- x_t: the % deviation from steady state output gap
- i_t: the % deviation from steady state inflation
- Note: since we are only working with the log-lineared setup, we are changing notation from $\hat{i}_t \to i_t$ and $\hat{\pi}_t \to \pi_t$.

Goal: 2 equations in π_t, x_t, i_t + monetary policy.
- All linear stochastic difference equations.
- The "Keynesian" connection is from the variables in the equations in reduced form
- The "new" part is the dependence on structural parameters and forward-looking, and optimal policies,...

6.1 Linearized Phillips Curve

Reference: Walsh 8.3.1.
Use (17) and (20), and log-linearization, gives the following in terms of steady state % deviation from marginal cost, φ_t

$$\pi_t = \beta \mathbb{E}_t \left[\pi_{t+1}\right] + \hat{\kappa}\hat{\varphi}_t \tag{28}$$

where $\hat{\kappa} \equiv \frac{(1-\omega)(1-\beta\omega)}{\omega}$. Solving (28) forward:

$$\pi_t = \hat{\kappa} \sum_{i=0}^{\infty} \beta^i \mathbb{E}_t \left[\hat{\varphi}_{t+i}\right] \tag{29}$$

Interpretation:
It is the real marginal cost, $\hat{\varphi}_t$, which matters:

- $\uparrow \omega \Rightarrow \downarrow \hat{\kappa}$: more price frictions, less weight on today's marginal cost
- $\uparrow \beta \Rightarrow \downarrow \hat{\kappa}$: More weight to future profits

Combine (13) and (14), use $Y_t = C_t$, and log linearize (from Walsh 8.3.1),

$$\hat{\varphi}_t = (\hat{w}_t - \hat{p}_t) - (\hat{y}_t - \hat{n}_t) \tag{30}$$

Substitute for the labor supply in (9)

$$= (\sigma + \eta)\left[\hat{y}_t - \left(\frac{1+\eta}{\sigma+\eta}\right)\hat{z}_t\right] \tag{31}$$

Recognizing the $\hat{z}_t$ term and log linearize (26):

$$\hat{y}_t^f = \left(\frac{1+\eta}{\sigma+\eta}\right)\hat{z}_t \tag{32}$$

$$\Rightarrow \hat{\varphi}_t = (\sigma+\eta)(\hat{y}_t - \hat{y}_t^f) = (\sigma+\eta)x_t \tag{33}$$

$$\boxed{\pi_t = \beta \mathbb{E}_t \left[\pi_{t+1}\right] + \kappa x_t} \tag{34}$$

where $\kappa \equiv (\sigma + \eta)(1 - \omega)(1 - \beta\omega)/\omega$.
Equation (34) represents the *new Keynesian Phillips curve*, or the equivalent in φ_t.

6.2 Linearized IS Curve

Take the Euler equation in (10) and log-linearized around zero inflation steady state[1]:

$$\hat{y}_t = \mathbb{E}_t\left[\hat{y}_{t+1}\right] - \sigma^{-1}\left(i_t - \mathbb{E}_t\left[\pi_{t+1}\right]\right) \tag{35}$$

or, in terms of the output gap:

$$\boxed{x_t = \mathbb{E}_t\left[x_{t+1}\right] - \sigma^{-1}\left(i_t - \mathbb{E}_t\left[\pi_{t+1}\right]\right) + u_t} \tag{36}$$

where $u_t \equiv \mathbb{E}_t\left[\hat{y}^f_{t+1}\right] - \hat{y}^f_t$ depends only on exogenous shocks, and we could derive the direct process for it. Remember that the central bank is changing i_t through monetary policy. For example, a Taylor rule:

$$i_t = \delta_\pi \pi_t + \delta_x x_t + \nu_t \tag{37}$$

where π_t responds to inflation, x_t responds to output gap and ν_t represents to random monetary shocks.

Zero inflation Equilibrium? It will turn out that the monetary authority could get zero inflation (see Walsh page 349) for a description. To see the intuition in this setup, note that if the central bank could eliminate the output gap, i.e. $x_t = 0$, with an i_t policy for any shock, then the solution to the stochastic difference equation in (34) is $\pi_t = 0$.

Can the central bank achieve an elimination of the output gap? From (36), substitute for the $\pi_{t+1} = 0, x_t = 0$, and $x_{t+1} = 0$ to get $i_t = \sigma u_t$. Hence, with that simple policy the central bank eliminates the output gap and inflation. For this reason, we will add in a shock to the NKPC in (34).

[1]Remember we are using i_t instead of $\hat{i}_t$ to denote percent deviation from steady state since we will always work in the log linearized setup from here.

7 Summarizing The Linearized Equations

The complete set of equations for the linearized NK model (with a Taylor rule for monetary policy) is,

$$x_t = \mathbb{E}_t\left[x_{t+1}\right] - \sigma^{-1}\left(i_t - \mathbb{E}_t\left[\pi_{t+1}\right]\right) + u_t \tag{38}$$

$$\pi_t = \beta \mathbb{E}_t\left[\pi_{t+1}\right] + \kappa x_t + e_t \tag{39}$$

$$i_t = \delta_\pi \pi_t + \delta_x x_t + \nu_t \tag{40}$$

+ shock processes for u_t, e_t, ν_t.
We added a “price shock" with e_t in equation (39), e.g., a (change in) wage markup if labor is not competitive (avoids the trivial zero inflation solution described above).

Parameters: $\beta, \sigma, \eta, \omega, \kappa$, where $\kappa \equiv (\sigma+\eta)(1-\omega)(1-\beta\omega)/\omega$

Policy Parameters: δ_π, δ_x
- We can now estimate, simulate, etc. with fixed or exogenous policies. But what should the monetary authority choose?
- Does the central bank have incentives to deviate from the announced policy?

References

Christiano, L. J., M. Eichenbaum, and C. L. Evans (2005): “Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy,” *Journal of Political Economy*, 113(1), 1–45.