



KTH Royal Institute of Technology

# Omogen Heap

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- 2 Data structures
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Contest (1)

template.cpp10 lines

```
#include <bits/stdc++.h>
#define all(x) begin(x), end(x)
using namespace std;

using ll = long long;

int main() {
    cin.tie(0)->sync_with_stdio(0);
    cin.exceptions(cin.failbit);
}
```

Data structures (2)

BIT.h  
Description: Query [l, r] sums, and point updates. kth() returns the smallest index i s.t. query(0, i) >= k  
Time:  $\mathcal{O}(\log n)$  for all ops.

33f78c, 22 lines

```
template <typename T>
struct BIT {
    vector<T> s;
    int n;
    BIT(int n): s(n + 1), n(n) {}
    void update(int i, T v) {
        for (i++; i <= n; i += i & -i) s[i] += v;
    }
    T query(int i) {
        T ans = 0;
        for (i++; i; i -= i & -i) ans += s[i];
        return ans;
    }
    T query(int l, int r) { return query(r) - query(l - 1); }
    int kth(T k) { // returns n if k > sum of tree
        if (k <= 0) return -1;
        int i = 0;
        for (int pw = 1 << __lg(n); pw; pw >>= 1)
            if (i + pw <= n && s[i + pw] < k) k -= s[i + pw];
        return i;
    }
};
```

KDBIT.h

Description:  $k$ -dimensional BIT. BIT<int, N, M> gives an  $N \times M$  BIT.  
Query: bit.query(x1, x2, y1, y2) Update: bit.update(x, y, delta)  
Time:  $\mathcal{O}(\log^k n)$  Status: Tested

3b9692, 28 lines

```
template <class T, int... Ns>
struct BIT {
    T val = 0;
    void update(T v) { val += v; }
    T query() { return val; }
};

template <class T, int N, int... Ns>
struct BIT<T, N, Ns...> {
    BIT<T, Ns...> bit[N + 1];
    // map<int, BIT<T, Ns...>> bit;
    // if the memory use is too high
    template <class... Args>
    void update(int i, Args... args) {
        for (i++; i <= N; i += i & -i) bit[i].update(args...);
    }
    template <class... Args>
    T query(int i, Args... args) {
        T ans = 0;
        for (i++; i; i -= i & -i) ans += bit[i].query(args...);
        return ans;
    }
    template <class... Args,
              enable_if_t<(sizeof...(Args) ==
                          2 * sizeof...(Ns))>* = nullptr>
    T query(int l, int r, Args... args) {
        return query(r, args...) - query(l - 1, args...);
    }
};
```

DSU.h  
Description: Maintains union of disjoint sets  
Time:  $\mathcal{O}(\alpha(N))$

c22586, 14 lines

```
struct DSU {
    vector<int> s;
    DSU(int n): s(n, -1) {}
    int find(int i) { return s[i] < 0 ? i : s[i] = find(s[i]); }
    bool join(int a, int b) {
        a = find(a), b = find(b);
        if (a == b) return false;
        if (s[a] > s[b]) swap(a, b);
        s[a] += s[b], s[b] = a;
        return true;
    }
    int size(int i) { return -s[find(i)]; }
    bool same(int a, int b) { return find(a) == find(b); }
};
```

RMQ.h  
Description: Constant time subarray min/max queries for a fixed array  
Time:  $\mathcal{O}(n \log n)$  initialization and  $\mathcal{O}(1)$  queries. Status: Tested

336eac, 15 lines

```
template <typename T, class Compare = less<T>>
struct RMQ {
    vector<vector<T>> t;
    Compare cmp;
    RMQ(vector<T>& a): t(__lg(a.size()) + 1, a) {
        int n = a.size(), lg = __lg(n);
        for (int k = 1, len = 1; k <= lg; k++, len <= 1)
            for (int i = 0; i + 2 * len - 1 < n; i++)
                t[k][i] = min(t[k - 1][i], t[k - 1][i + len], cmp);
    }
    T query(int a, int b) {
        int k = __lg(b - a + 1), len = 1 << k;
        return min(t[k][a], t[k][b - len + 1], cmp);
    }
};
```

Splay.h  
Description: An implicit balanced BST. You only need to change update() and prop().  
If used for link-cut tree, code everything up to splay(). Time: amortized  $\mathcal{O}(\log n)$  for all operations

0d0cee, 92 lines

```
struct node {
    node *ch[2] = {0}, *p = 0;
    int cnt = 1, val;

    node(int val, node* l = 0, node* r = 0):
        ch{l, r}, val(val) {}
};

int cnt(node* x) { return x ? x->cnt : 0; }
int dir(node* p, node* x) { return p && p->ch[0] != x; }
void setLink(node* p, node* x, int d) {
    if (p) p->ch[d] = x;
    if (x) x->p = p;
}

node* update(node* x) {
    if (!x) return 0;
    x->cnt = 1 + cnt(x->ch[0]) + cnt(x->ch[1]);
    setLink(x, x->ch[0], 0);
    setLink(x, x->ch[1], 1);
    return x;
}

void prop(node* x) {
    if (!x) return;
    // update(x); // needed if prop() can change subtree sizes
}

void rotate(node* x, int d) {
    if (!x || !x->ch[d]) return;
    node *y = x->ch[d], *z = x->p;
    setLink(x, y->ch[d ^ 1], d);
    setLink(y, x, d ^ 1);
    setLink(z, y, dir(z, x));
    update(x);
    update(y);
}

node* splay(node* x) {
    while (x && x->p) {
        node *y = x->p, *z = y->p;
        // prop(z), prop(y), prop(x); // needed for LCT
        int dy = dir(y, x), dz = dir(z, y);
        if (!z)
            rotate(y, dy);
        else if (dy == dz)
            rotate(z, dz), rotate(y, dy);
        else
            rotate(y, dy), rotate(z, dz);
    }
    return x;
}

// the returned node becomes the new root, update the root pointer!
node* nodeAt(node* x, int pos) {
    if (!x) return 0;
    while (prop(x), cnt(x->ch[0]) != pos)
        if (pos < cnt(x->ch[0]))
            x = x->ch[0];
    else
```

```
        pos -= cnt(x->ch[0]) + 1, x = x->ch[1];
        return splay(x);
    }

node* merge(node* l, node* r) {
    if (!l || !r) return l ?: r;
    l = nodeAt(l, cnt(l) - 1);
    setLink(l, r, 1);
    return update(l);
}

// first is everything < pos, second is >= pos
pair<node*, node*> split(node* t, int pos) {
    if (pos <= 0 || !t) return {0, t};
    if (pos > cnt(t)) return {t, 0};
    node *l = nodeAt(t, pos - 1), *r = l->ch[1];
    if (r) l->ch[1] = r->p = 0;
    return {update(l), update(r)};
}

// insert a new node between pos-1 and pos
node* insert(node* t, int pos, int val) {
    auto [l, r] = split(t, pos);
    return update(new node(val, l, r));
}

// apply lambda to all nodes in an inorder traversal
template <class F>
void each(node* x, F f) {
    if (x) prop(x), each(x->ch[0], f), f(x), each(x->ch[1], f);
}
```

358d18, 37 lines

## Geometry (3)

## Graphs (4)

### SCCTarjan.h

**Description:** Finds strongly connected components of a directed graph. Visits/indexes SCCs in reverse topological order.  
**Usage:** scc(graph) returns an array that has the ID of each node’s SCC. scc(graph, [&](vector<int>& v) { ... }) calls the lambda on each SCC, and returns the same array.  
**Time:**  $\mathcal{O}(|V| + |E|)$

```
namespace SCCTarjan {
    vector<int> val, comp, z, cont;
    int Time, ncomps;
    template <class G, class F>
    int dfs(int j, G& g, F& f) {
        int low = val[j] = ++Time, x;
        z.push_back(j);
        for (auto e : g[j])
            if (comp[e] < 0) low = min(low, val[e] ?: dfs(e, g, f));
        if (low == val[j]) {
            do {
                x = z.back();
                z.pop_back();
                comp[x] = ncomps;
                cont.push_back(x);
            } while (x != j);
            f(cont);
            cont.clear();
            ncomps++;
        }
        return val[j] = low;
    }
    template <class G, class F>
```

```
vector<int> scc(G& g, F f) {
    int n = g.size();
    val.assign(n, 0);
    comp.assign(n, -1);
    Time = ncomps = 0;
    for (int i = 0; i < n; i++)
        if (comp[i] < 0) dfs(i, g, f);
    return comp;
}

template <class G> // convenience function w/o lambda
vector<int> scc(G& g) {
    return scc(g, [](auto& v) {});
}
} // namespace SCCTarjan
```

### SCCKosaraju.h

**Description:** Finds strongly connected components of a directed graph. Visits/indexes SCCs in topological order.  
**Usage:** scc(graph) returns an array that has the ID of each node’s SCC.  
**Time:**  $\mathcal{O}(|V| + |E|)$

```
namespace SCCKosaraju {
    vector<vector<int>> adj, radj;
    vector<int> todo, comp;
    vector<bool> vis;
    void dfs1(int x) {
        vis[x] = 1;
        for (int y : adj[x])
            if (!vis[y]) dfs1(y);
        todo.push_back(x);
    }
    void dfs2(int x, int i) {
        comp[x] = i;
        for (int y : radj[x])
            if (comp[y] == -1) dfs2(y, i);
    }
    vector<int> scc(vector<vector<int>>& _adj) {
        adj = _adj;
        int time = 0, n = adj.size();
        comp.resize(n, -1), radj.resize(n), vis.resize(n);
        for (int x = 0; x < n; x++)
            for (int y : adj[x]) radj[y].push_back(x);
        for (int x = 0; x < n; x++)
            if (!vis[x]) dfs1(x);
        reverse(todo.begin(), todo.end());
        for (int x : todo)
            if (comp[x] == -1) dfs2(x, time++);
        return comp;
    }
}; // namespace SCCKosaraju
```

## Mathematics (5)

### Fraction.h

**Description:** Struct for representing fractions/rationals. All ops are  $\mathcal{O}(\log N)$  due to GCD in constructor. Uses cross multiplication.

```
template <typename T>
struct Q {
    T a, b;
    Q(T p, T q = 1) {
        T g = gcd(p, q);
        a = p / g;
        b = q / g;
        if (b < 0) a = -a, b = -b;
    }
    T gcd(T x, T y) const { return __gcd(x, y); }
```

```
Q operator+(const Q& o) const {
    return {a * o.b + o.a * b, b * o.b};
}
Q operator-(const Q& o) const {
    return *this + Q(-o.a, o.b);
}
Q operator*(const Q& o) const { return {a * o.a, b * o.b}; }
Q operator/(const Q& o) const { return *this * Q(o.b, o.a); }
Q recip() const { return {b, a}; }
int signum() const { return (a > 0) - (a < 0); }
bool operator<(const Q& o) const {
    return a * o.b < o.a * b;
}
friend ostream& operator<<(ostream& cout, const Q& o) {
    return cout << o.a << "/" << o.b;
}
};
```

### FractionOverflow.h

**Description:** Safer struct for representing fractions/rationals. Comparison is 100% overflow safe; other ops are safer but can still overflow. All ops are  $\mathcal{O}(\log N)$ .

```
template <typename T>
struct QO {
    T a, b;
    QO(T p, T q = 1) {
        T g = gcd(p, q);
        a = p / g;
        b = q / g;
        if (b < 0) a = -a, b = -b;
    }
    T gcd(T x, T y) const { return __gcd(x, y); }
    QO operator+(const QO& o) const {
        T g = gcd(b, o.b), bb = b / g, obb = o.b / g;
        return {a * obb + o.a * bb, o.b * obb};
    }
    QO operator-(const QO& o) const {
        return *this + QO(-o.a, o.b);
    }
    QO operator*(const QO& o) const {
        T g1 = gcd(a, o.b), g2 = gcd(o.a, b);
        return {(a / g1) * (o.a / g2), (b / g2) * (o.b / g1)};
    }
    QO operator/(const QO& o) const {
        return *this * QO(o.b, o.a);
    }
    QO recip() const { return {b, a}; }
    int signum() const { return (a > 0) - (a < 0); }
    static bool lessThan(T a, T b, T x, T y) {
        if (a / b != x / y) return a / b < x / y;
        if (x % y == 0) return false;
        if (a % b == 0) return true;
        return lessThan(y, x % y, b, a % b);
    }
    bool operator<(const QO& o) const {
        if (this->signum() != o.signum() || a == 0) return a < o.a;
        if (a < 0)
            return lessThan(abs(o.a), o.b, abs(a), b);
        else
            return lessThan(a, b, o.a, o.b);
    }
    friend ostream& operator<<(ostream& cout, const QO& o) {
        return cout << o.a << "/" << o.b;
    }
};
```

### PrimeSieve.h

**Description:** Prime sieve for generating all primes up to a certain limit. isprime[i] is true iff i is a prime.  
**Time:** lim=100'000'000  $\approx$  0.8 s. Runs 30% faster if only odd indices are stored.

```
const int MAX_PR = 5'000'000;
bitset<MAX_PR> isprime;
vector<int> primeSieve(int lim) {
    isprime.set();
    isprime[0] = isprime[1] = 0;
    for (int i = 4; i < lim; i += 2) isprime[i] = 0;
    for (int i = 3; i * i < lim; i += 2)
        if (isprime[i])
            for (int j = i * i; j < lim; j += i * 2) isprime[j] = 0;
    vector<int> pr;
    for (int i = 2; i < lim; i++)
        if (isprime[i]) pr.push_back(i);
    return pr;
}
```

### PrimeSieveFast.h

**Description:** Prime sieve for generating all primes smaller than LIM.  
**Time:** LIM=1e9  $\approx$  1.5s

```
const int LIM = 1e8;
bitset<LIM> isPrime;
vector<int> primeSieve() {
    const int S = round(sqrt(LIM)), R = LIM / 2;
    vector<int> pr = {2}, sieve(S + 1);
    pr.reserve((int)(LIM / log(LIM) * 1.1));
    vector<pair<int, int>> cp;
    for (int i = 3; i <= S; i += 2)
        if (!sieve[i]) {
            cp.push_back({i, i * i / 2});
            for (int j = i * i; j <= S; j += 2 * i) sieve[j] = 1;
        }
    for (int L = 1; L <= R; L += S) {
        array<bool, S> block{};
        for (auto& [p, idx] : cp)
            for (int i = idx; i < S + L; idx = (i += p))
                block[i - L] = 1;
        for (int i = 0; i < min(S, R - L); i++)
            if (!block[i]) pr.push_back((L + i) * 2 + 1);
    }
    for (int i : pr) isPrime[i] = 1;
    return pr;
}
```

## Miscellaneous (6)

### NDimensionalVector.h

```
template <int D, typename T>
struct Vec : public vector<Vec<D - 1, T>> {
    static_assert(D >= 1,
        "Vector dimension must be greater than zero!");
    template <typename... Args>
    Vec(int n = 0, Args... args):
        vector<Vec<D - 1, T>>(n, Vec<D - 1, T>(args...)) {}
};
template <typename T>
struct Vec<1, T> : public vector<T> {
    Vec(int n = 0, const T& val = T()): vector<T>(n, val) {}
};
```

### Submasks.h

```
35424b, 3 lines
for (int mask = 0; mask < (1 << n); mask++)
    for (int sub = mask; sub; sub = (sub - 1) & mask)
        // do thing
```

## Strings (7)

### ZValues.h

```
151ee3, 10 lines
vector<int> zValues(string& s) {
    int n = (int)s.length();
    vector<int> z(n);
    for (int i = 1, l = 0, r = 0; i < n; ++i) {
        if (i <= r) z[i] = min(r - i + 1, z[i - l]);
        while (i + z[i] < n && s[z[i]] == s[i + z[i]]) ++z[i];
        if (i + z[i] - 1 > r) l = i, r = i + z[i] - 1;
    }
    return z;
}
```