Electroweak Corrections to DIS

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Abstract

We discuss the complete Electroweak Corrections to DIS, both Neutral Current (NC) and Charged Current (CC) interactions. To match the experimental data we exclude purely QED corrections. This is straightforward in the NC case and requires a dipole subtraction scheme for the CC.

1 Introduction - Definitions

For DIS the unpolarized cross section for neutral current interaction (i = NC) and charged current interaction (i = CC) can be written as

$$\frac{d^2\sigma^i}{dxdy} = \frac{2\pi\alpha^2}{xyQ^2}\omega^i\left[2x(2-2y+y^2)F_1 \mp xy(2-y)F_3 + 2(1-y)F_L\right] \tag{1}$$

where the upper sign is for a positron scattering and the lower, plus sign is for an electron.
The prefactor is given by

$$\omega^{NC} = 1 \qquad \omega^{CC} = \frac{1}{4}(1\pm\lambda_e)\left(\frac{1}{2\sqrt{2}s_W^2}\right)^2\left(\frac{Q^2}{Q^2+M_W^2}\right)^2 \tag{2}$$

furthermore it is helpful to introduce

$$\eta_{\gamma Z} = \left(\frac{1}{2s_W c_W}\right)^2\frac{Q^2}{Q^2+M_Z^2} \quad \text{and} \quad \eta_Z = \eta_{\gamma Z}^2, \tag{3}$$

In the case of the neutral current interaction the structure functions can be written, at least to leading order in the parton model, as

$$\begin{aligned} F_{1,L}^{NC} =& F_{1,L}^{\gamma} - (g_{V,e}\pm\lambda_e g_{A,e})\eta_{\gamma Z}F_{1,L}^{\gamma Z} + (g_{V,e}^2+g_{A,e}^2\pm 2\lambda_e g_{V,e}g_{A,e})\eta_Z F_{1,L}^Z \\ F_3^{NC} =& \lambda_e F_3^{\gamma} - (g_{A,e}\pm\lambda_e g_{V,e})\eta_{\gamma Z}F_3^{\gamma Z} + (2g_{V,e}g_{A,e}\pm\lambda_e(g_{V,e}^2+g_{A,e}^2))\eta_Z F_3^Z \end{aligned} \tag{4}$$

where

$$\begin{aligned} \left[F_1^{\gamma}, F_1^{\gamma Z}, F_1^Z\right] &= \frac{1}{2}\sum_q\left(e_q^2, 2e_q g_{V,q}, g_{V,q}^2+g_{A,q}^2\right)\left(f_q(x)+\overline{f}_q(x)\right) \\ \left[F_3^{\gamma}, F_3^{\gamma Z}, F_3^Z\right] &= \sum_q\left(0, 2e_q g_{A,q}, 2g_{V,q}g_{A,q}\right)\left(f_q(x)-\overline{f}_q(x)\right). \end{aligned} \tag{5}$$

Analogously one finds for the charged current reaction

$$\begin{aligned} F_1^{CC} =&(f_u + \overline{f}_d + f_c + \overline{f}_s) \\ F_3^{CC} =&2(f_u - \overline{f}_d + f_c - \overline{f}_s). \end{aligned} \tag{6}$$

One can use the Callan-Gross relation $F_L = F_2 - 2xF_1$ to switch the basis of structure functions.

When scattering off polarized hadrons we write the cross section as

$$\frac{d^2\Delta\sigma^i}{dxdy} = \frac{8\pi\alpha^2}{xyQ^2}\omega^i \left[(2-2y+y^2)xg_5 \mp xy(2-y)g_1 + (1-y)g_L\right] \tag{7}$$

where the upper, minus sign is for positrons scattering and the lower, plus sign is for electrons.
The polarized structure functions are decomposed in the neutral current case as

$$\begin{aligned} g_1^{NC} =& \pm \lambda_e g_1^\gamma - (g_{A,e} \pm \lambda_e g_{V,e})\eta_{\gamma Z} g_1^{\gamma Z} + (2g_{A,e}g_{V,e} \pm \lambda_e(g_{V,e}^2 + g_{A,e}^2))\eta_Z g_1^Z \\ g_{5,L}^{NC} =& g_{5,L}^\gamma - (g_{V,e} \pm \lambda_e g_{A,e})\eta_{\gamma Z} g_{5,L}^{\gamma Z} + (g_{V,e}^2 + g_{A,e}^2 \pm 2\lambda_e g_{V,e}g_{A,e})\eta_Z g_{5,L}^Z \end{aligned} \tag{8}$$

with the upper sign being the positron again.

$$\begin{aligned} \left[g_1^\gamma, g_1^{\gamma Z}, g_1^Z\right] &= \frac{1}{2}\sum_q \left(e_q^2, 2e_q g_{V,q}, g_{V,q}^2 + g_{A,q}^2\right)\left(\Delta f_q(x) + \Delta\overline{f}_q(x)\right) \\ \left[g_5^\gamma, g_5^{\gamma Z}, g_5^Z\right] &= \sum_q \left(0, e_q g_{A,q}, g_{V,q}g_{A,q}\right)\left(\Delta f_q(x) - \Delta\overline{f}_q(x)\right). \end{aligned} \tag{9}$$

In the same way we can write for the charged current, polarized scattering

$$\begin{aligned} g_1^{CC} =& \Delta f_u + \Delta\overline{f}_d + \Delta f_c + \Delta\overline{f}_s \\ g_5^{CC} =& -\Delta f_u + \Delta\overline{f}_d - \Delta f_c + \Delta\overline{f}_s \end{aligned} \tag{10}$$

In all these and the following definitions we employ the $\{\alpha, M_Z, M_W\}$ scheme. The renormalization happens in the On-Shell scheme. The weak mixing angle is therefor defined as

$$s_W^2 = 1 - \frac{M_W^2}{M_Z^2} \qquad \text{and} \qquad c_W = \frac{M_W}{M_Z} \tag{11}$$

2 Electroweak Radiative Corrections

In this section we discuss the NLO electroweak radiative corrections. They fall into three categories: self-energy type corrections, vertex corrections and box type corrections. While in the neutral current case photonic QED corrections form a gauge-invariant subset that is trivially subtracted, the charge current case requires a subtraction prescription.

2.1 Self-Energy contributions

The self-energy corrections to the t-channel propagator do generally speaking factorize. We give the renormalization conditions here and quote the complete self-energy expressions in the Appendix.

2.1.1 Charged Current

In the case of the charged current the renormalized W self energy factorizes according to

$$d\sigma_{SE} = 2d\sigma_B \frac{\hat{\Sigma}_{WW}(-Q^2)}{Q^2 + M_W^2} \tag{12}$$

where

$$\hat{\Sigma}_{WW}(p^2) = \Sigma_{WW}(p^2) + \delta Z_W(p^2 - M_W^2) - \delta M_W^2. \tag{13}$$

The charged current structure functions can therefor just be amended by

$$F_i^{CC} \to F_i^{CC}\left(1 + 2\frac{\hat{\Sigma}_{WW}(-Q^2)}{Q^2 + M_W^2}\right). \tag{14}$$

2.1.2 Neutral Current

The neutral current distinguishes three cases. Firstly corrections to the Z propagator depend on the renormalized Z self energy

$$\hat{\Sigma}_{ZZ}(p^2) = \Sigma_{ZZ}(p^2) + \delta Z_{ZZ}(p^2 - M_Z^2) - \delta M_Z^2 \tag{15}$$

while the photon propagator similarly depends on the renormalized photon self energy

$$\hat{\Sigma}_{\gamma\gamma}(p^2) = \Sigma_{\gamma\gamma}(p^2) + \delta Z_{\gamma\gamma} p^2. \tag{16}$$

And finally the photon-Z mixing becomes

$$\hat{\Sigma}_{\gamma Z}(p^2) = \Sigma_{\gamma Z}(p^2) + \frac{\delta Z_{Z\gamma}}{2}(p^2 - M_Z^2) + \frac{\delta Z_{\gamma Z}}{2} p^2 \tag{17}$$

The self-energy contributions to the structure functions can be written as

$$\begin{aligned}
F_1^{\gamma} =& e_q^2 \frac{\hat{\Sigma}_{\gamma\gamma}(-Q^2)}{Q^2} - e_q g_V^q (2 s_W c_W) \eta_{\gamma Z} \frac{\hat{\Sigma}_{\gamma Z}(-Q^2)}{Q^2} \\
F_1^{\gamma Z} =& e_q g_V^q \left[\frac{\hat{\Sigma}_{\gamma\gamma}(-Q^2)}{Q^2} + \frac{\hat{\Sigma}_{ZZ}(-Q^2)}{Q^2 + M_Z^2}\right] - (e_q^2 + \eta_{\gamma Z}({g_V^q}^2 + {g_A^q}^2))(2 s_W c_W) \frac{\hat{\Sigma}_{\gamma Z}(-Q^2)}{Q^2} \\
F_1^{Z} =& ({g_V^q}^2 + {g_A^q}^2) \frac{\hat{\Sigma}_{ZZ}(-Q^2)}{Q^2 + M_Z^2} - e_q g_V^q \eta_{\gamma Z} (2 s_W c_W) \frac{\hat{\Sigma}_{\gamma Z}(-Q^2)}{Q^2}
\end{aligned} \tag{18}$$

for F_1 and for F_3 as

$$\begin{aligned}
F_3^{\gamma} =& 2e_q g_A^q (2s_W c_W)\eta_{\gamma Z}\frac{\hat{\Sigma}_{\gamma Z}(-Q^2)}{Q^2}\\
F_3^{\gamma Z} =& 2e_q g_A^q \left[\frac{\hat{\Sigma}_{\gamma\gamma}(-Q^2)}{Q^2}+\frac{\hat{\Sigma}_{ZZ}(-Q^2)}{Q^2+M_Z^2}\right]-g_A^q g_V^q \eta_{\gamma Z}(2s_W c_W)\frac{\hat{\Sigma}_{\gamma Z}(-Q^2)}{Q^2}\\
F_3^{Z} =& 4g_A^q g_V^q \frac{\hat{\Sigma}_{ZZ}(-Q^2)}{Q^2+M_Z^2}-2e_q g_A^q (2s_W c_W)\frac{\hat{\Sigma}_{\gamma Z}(-Q^2)}{Q^2}.
\end{aligned}\tag{19}$$

While in the polarized case we find for g_1

$$\begin{aligned}
g_1^{\gamma} =& e_q^2\frac{\hat{\Sigma}_{\gamma\gamma}(-Q^2)}{Q^2}-e_q g_V^q (2s_W c_W)\eta_{\gamma Z}\frac{\hat{\Sigma}_{\gamma Z}(-Q^2)}{Q^2}\\
g_1^{\gamma Z} =& e_q g_V^q \left[\frac{\hat{\Sigma}_{\gamma\gamma}(-Q^2)}{Q^2}+\frac{\hat{\Sigma}_{ZZ}(-Q^2)}{Q^2+M_Z^2}\right]-(e_q^2+\eta_{\gamma Z}(g_A^{q\,2}+g_V^{q\,2}))(2s_W c_W)\frac{\hat{\Sigma}_{\gamma Z}(-Q^2)}{Q^2}\\
g_1^{Z} =& (g_A^{q\,2}+g_V^{q\,2})\frac{\hat{\Sigma}_{ZZ}(-Q^2)}{Q^2+M_Z^2}-e_q g_V^q (2s_W c_W)\frac{\hat{\Sigma}_{\gamma Z}(-Q^2)}{Q^2}
\end{aligned}\tag{20}$$

and for g_5

$$\begin{aligned}
g_5^{\gamma} =& -e_q g_A^q (2s_W c_W)\eta_{\gamma Z}\frac{\hat{\Sigma}_{\gamma Z}(-Q^2)}{Q^2}\\
g_5^{\gamma Z} =& e_q g_A^q \left[\frac{\hat{\Sigma}_{\gamma\gamma}(-Q^2)}{Q^2}+\frac{\hat{\Sigma}_{ZZ}(-Q^2)}{Q^2+M_Z^2}\right]-2g_A^q g_V^q (2s_W c_W)\frac{\hat{\Sigma}_{\gamma Z}(-Q^2)}{Q^2}\\
g_5^{Z} =& 2g_A^q g_V^q \frac{\hat{\Sigma}_{ZZ}(-Q^2)}{Q^2+M_Z^2}-e_q g_A^q (2s_W c_W)\frac{\hat{\Sigma}_{\gamma Z}(-Q^2)}{Q^2}.
\end{aligned}\tag{21}$$

We have suppressed the dependence on parton density functions, which follows the Born-level expressions.

2.2 Vertex contributions

We decompose the $Z - f_i - \overline{f}_i$ vertices according to

$$\frac{e}{2c_W s_W}\left[\left(g_{V,i}^{(0)}+g_{V,i}^{(1)}\right)\gamma^\mu+\left(g_{A,i}^{(0)}+g_{A,i}^{(1)}\right)\gamma^5\gamma^\mu\right]\tag{22}$$

and the $\gamma - f_i - \overline{f}_i$ vertices according to

$$e\left[\left(h_{V,i}^{(0)}+h_{V,i}^{(1)}\right)\gamma^\mu+\left(h_{A,i}^{(0)}+h_{A,i}^{(1)}\right)\gamma^5\gamma^\mu\right]\tag{23}$$

The form factors are most easily calculated considering the decay of a vector boson V according to the reaction $V(p)\to f(k_1)+\overline{f}(k_2)$ and applying the projectors

$$\begin{aligned}
g_{V,f} =& \frac{1}{2(2-d)p^2}\mathrm{Tr}\left[\gamma^\mu \not{k}_1 V_\mu(p^2)\not{k}_2\right]\\
g_{A,f} =& \frac{1}{2(2-d)p^2}\mathrm{Tr}\left[\gamma_5\gamma^\mu \not{k}_1 V_\mu(p^2)\not{k}_2\right]
\end{aligned}\tag{24}$$

for the vector and axial-vector components respectively. The same process can be utilized for off-shell Z-bosons as well as photons. The overall normalization needs to be adjusted to fit our conventions.
Our conventions are such, that the Z-boson Born vertices are

$$
\begin{aligned}
&g^{(0)}_{V,\nu} = \frac{1}{2} && g^{(0)}_{A,\nu} = \frac{1}{2} && g^{(0)}_{V,e} = -\frac{1}{2} + 2s_W^2 && g^{(0)}_{A,e} = -\frac{1}{2} \\
&g^{(0)}_{V,u} = \frac{1}{2} - \frac{4}{3}s_W^2 && g^{(0)}_{A,u} = \frac{1}{2} && g^{(0)}_{V,d} = -\frac{1}{2} + \frac{2}{3}s_W^2 && g^{(0)}_{A,d} = -\frac{1}{2}
\end{aligned} \tag{25}
$$

While for the photon we find at Born level

$$
\begin{aligned}
&h^{(0)}_{V,\nu} = 0 && h^{(0)}_{A,\nu} = 0 && h^{(0)}_{V,e} = -1 && h^{(0)}_{A,e} = 0 \\
&h^{(0)}_{V,u} = \frac{2}{3} && h^{(0)}_{A,u} = 0 && h^{(0)}_{V,d} = -\frac{1}{3} && h^{(0)}_{A,d} = 0
\end{aligned} \tag{26}
$$

Now we have to generalize the structure functions in order to include more general vertex functions beyond leading order

$$
\begin{aligned}
F_1 =& ({h^e_A}^2 + {h^e_V}^2 - 2h^e_A h^e_V \lambda_e) F_1^\gamma + (g^e_V(h^e_V - h^e_A\lambda_e) + g^e_A(h^e_A - h^e_V\lambda_e))\eta_{\gamma Z} F_1^{\gamma Z} \\
&+ ({g^e_A}^2 + {g^e_V}^2 - 2g^e_A g^e_V \lambda_e)\eta_Z F_1^Z \\
F_3 =& (2h^e_A h^e_V - \lambda_e({h^e_A}^2 + {h^e_V}^2)) F_3^\gamma + (g^e_A(h^e_V - h^e_A\lambda_e) + g^e_V(h^e_A - h^e_V\lambda_e))\eta_{\gamma Z} F_3^{\gamma Z} \\
&+ (2g^e_A g^e_V - ({g^e_A}^2 + {g^e_V}^2)\lambda_e)\eta_Z F_3^Z
\end{aligned} \tag{27}
$$

with

$$
\begin{aligned}
\left[F_1^\gamma, F_1^{\gamma Z}, F_1^Z\right] &= \frac{1}{2}\sum_q \left[{h^q_A}^2 + {h^q_V}^2, 2(g^q_A h^q_A + g^q_V h^q_V), {g^q_A}^2 + {g^q_V}^2\right](f_q(x) + \overline{f}_q(x)) \\
\left[F_3^\gamma, F_3^{\gamma Z}, F_3^Z\right] &= \sum_q \left[2h^q_A h^q_V, 2(g^q_V h^q_A + g^q_A h^q_V), 2g^q_A g^q_V\right](f_q(x) - \overline{f}_q(x))
\end{aligned} \tag{28}
$$

while in the polarized case:

$$
\begin{aligned}
g_1 =& (2h^e_A h^e_V - ({h^e_A}^2 + {h^e_V}^2)\lambda_e) g_1^\gamma + (g^e_A(h^e_V - h^e_A\lambda_e) + g^e_V(h^e_A - h^e_V\lambda_e))\eta_{\gamma Z} g_1^{\gamma Z} \\
&+ (2g^e_A g^e_V - ({g^e_A}^2 + {g^e_V}^2)\lambda_e)\eta_Z g_1^Z \\
g_5 =& ({h^e_A}^2 + {h^e_V}^2 - 2h^e_A h^e_V \lambda_e) g_5^\gamma + (g^e_V(h^e_V - h^e_A\lambda_e) + g^e_A(h^e_A - h^e_V\lambda_e))\eta_{\gamma Z} g_5^{\gamma Z} \\
&+ ({g^e_A}^2 + {g^e_V}^2 - 2g^e_A g^e_V \lambda_e)\eta_Z g_5^Z
\end{aligned} \tag{29}
$$

with

$$
\begin{aligned}
\left[g_1^\gamma, g_1^{\gamma Z}, g_1^Z\right] &= \frac{1}{2}\sum_q \left[{h^q_A}^2 + {h^q_V}^2, 2(g^q_A h^q_A + g^q_V h^q_V), {g^q_A}^2 + {g^q_V}^2\right](\Delta f_q(x) + \Delta\overline{f}_q(x)) \\
\left[g_5^\gamma, g_5^{\gamma Z}, g_5^Z\right] &= \sum_q \left[h^q_A h^q_V, (g^q_V h^q_A + g^q_A h^q_V), g^q_A g^q_V\right](\Delta f_q(x) - \Delta\overline{f}_q(x))
\end{aligned} \tag{30}
$$

2.2.1 Z Vertices - NLO EW corrections

For the NLO EW contributions to the Z form factors we find (we quote the form factors for the neutrino vertex for completeness sake)

$$
\begin{aligned}
g_{V,\nu}^{(1)}(p^2) = \frac{\alpha}{32\pi s_W^2 M_W^2 p^2} \Big\{ & 4(M_W^2 + 2p^2)A_0[M_W^2] + 2(M_Z^2 + 2p^2)A_0[M_Z^2] \\
& - M_Z^2(2M_Z^2 + 2c_W^2(-1 + 2s_W^2)(2M_W^2 - p^2(d-7)) - p^2(d-7))B_0[p^2, 0, 0] \\
& - 4c_W^2 M_W^2(2M_W^2 + p^2)B_0[p^2, M_W^2, M_W^2] \\
& - 2M_W^2(2s_W^2 - 1)(2M_W^4 + 2p^4 - M_W^2 p^2(d-8))C_0[0, 0, p^2, 0, M_W^2, 0] \\
& - (2M_Z^6 + 2M_Z^2 p^4 - M_Z^4 p^2(d-8))C_0[0, 0, p^2, 0, M_Z^2, 0] \\
& + 8c_W^2 M_W^4(M_W^2 + 2p^2)C_0[0, 0, p^2, M_W^2, 0, M_W^2] \Big\} \\
& + \frac{1}{4}\left(2\delta Z_e + \delta Z_{ZZ} - \frac{2(1 - 2s_W^2)}{c_W^2 s_W}\delta s_W + 2\delta Z_L^\nu\right)
\end{aligned}
\tag{31}
$$

$$
\begin{aligned}
g_{V,e}^{(1)}(p^2) = \frac{\alpha}{32\pi s_W^2 M_W^2 p^2} \Big\{ & 4(1 - 2c_W^2)(M_W^2 + 2p^2)A_0[M_W^2] \\
& + 2(M_Z^2 + 2p^2)(6s_W^2 - 12s_W^4 + 16s_W^6 - 1)A_0[M_Z^2] \\
& - M_Z^2(2c_W^2(2M_W^2 - p^2(d-7)) \\
& + (6s_W^2 - 12s_W^4 + 16s_W^6 - 1)(2M_Z^2 - p^2(d-7)))B_0[p^2, 0, 0] \\
& + 4c_W^4 M_Z^2(2M_W^2 + p^2)B_0[p^2, M_W^2, M_W^2] \\
& - 2c_W^2 M_Z^2(2M_W^4 + 2p^4 - M_W^2 p^2(d-8))C_0[0, 0, p^2, 0, M_W^2, 0] \\
& - M_Z^2(6s_W^2 - 12s_W^4 + 16s_W^6 - 1)(2M_Z^4 + 2p^4 - M_Z^2 p^2(d-8))C_0[0, 0, p^2, 0, M_Z^2, 0] \\
& - 8c_W^4 M_W^2 M_Z^2(M_W^2 + 2p^2)C_0[0, 0, p^2, M_W^2, 0, M_W^2] \Big\} \\
& - \frac{1}{4}\Big((1 - 4s_W^2)(\delta Z_{ZZ} + 2\delta Z_e) - \frac{2(1 + 2s_W^2)}{c_W^2 s_W}\delta s_W - 4c_W s_W \delta Z_{\gamma Z} \\
& + 2(1 - 2s_W^2)\delta Z_L^e - 4s_W^2 \delta Z_R^e\Big)
\end{aligned}
\tag{32}
$$

$$
\begin{aligned}
g_{A,e}^{(1)}(p^2) = \frac{\alpha}{32\pi s_W^2 M_W^2 p^2} \Big\{ & 4(1 - 2c_W^2)(M_W^2 + 2p^2)A_0[M_W^2] - 2(M_Z^2 + 2p^2)(1 - 6s_W^2 + 12s_W^4)A_0[M_Z^2] \\
& - M_Z^2(2c_W^2(2M_W^2 - p^2(d-7)) - (1 - 6s_W^2 + 12s_W^4)(2M_Z^2 - p^2(d-7)))B_0[p^2, 0, 0] \\
& + 4c_W^4 M_Z^2(2M_W^2 + p^2)B_0[p^2, M_W^2, M_W^2] \\
& - 2(2M_W^6 + 2M_W^2 p^4 - M_W^4 p^2(d-8))C_0[0, 0, p^2, 0, M_W^2, 0] \\
& + M_Z^2(1 - 6s_W^2 + 12s_W^4)(2M_Z^4 + 2p^4 - M_Z^2 p^2(d-8))C_0[0, 0, p^2, 0, M_Z^2, 0] \\
& - 8c_W^2 M_W^4(M_W^2 + 2p^2)C_0[0, 0, p^2, M_W^2, 0, M_W^2] \Big\} \\
& - \frac{1}{4}\Big(2\delta Z_e + \delta Z_{ZZ} - 2\frac{1 - 2s_W^2}{c_W^2 s_W}\delta s_W + 2(1 - 2s_W^2)\delta Z_L^e + 2s_W^2 \delta Z_R^e\Big)
\end{aligned}
\tag{33}
$$

$$
\begin{aligned}
g_{V,u}^{(1)}(p^2) = -\frac{\alpha}{864\pi s_W^2 M_W^2 p^2}\Big\{ & 36(M_W^2+2p^2)(3-6c_W^2-2s_W^2)A_0[M_W^2] \\
& + 2(M_Z^2+2p^2)(-27+108s_W^2-144s_W^4+128s_W^6)A_0[M_Z^2] \\
& + M_Z^2(18c_W^2(-3+2s_W^2)(2M_W^2-p^2(d-7)) \\
& + (27-108s_W^2+144s_W^4-128s_W^6)(2M_Z^2+p^2(7-d)))B_0[p^2,0,0] \\
& + 108c_W^2M_W^2(2M_W^2+p^2)B_0[p^2,M_W^2,M_W^2] \\
& + 18M_W^2(-3+2s_W^2)(2M_W^4+2p^4-M_W^2p^2(d-8))C_0[0,0,p^2,0,M_W^2,0] \\
& + M_Z^2(27-108s_W^2+144s_W^4-128s_W^6)(2M_Z^4+2p^4 \\
& - M_Z^2p^2(d-8))C_0[0,0,p^2,0,M_Z^2,0] \\
& - 216c_W^2M_W^4(M_W^2+2p^2)C_0[0,0,p^2,M_W^2,0,M_W^2]\Big\} \\
& - \frac{1}{12}\Big((8s_W^2-3)(2\delta Z_e+\delta Z_{ZZ})+8c_Ws_W\delta Z_{\gamma Z}+2\delta s_W\frac{3+2s_W^2}{c_W^2s_W} \\
& + 2(4s_W^2-3)\delta Z_L^u+8s_W^2\delta Z_R^u\Big)
\end{aligned} \tag{34}
$$

$$
\begin{aligned}
g_{A,u}^{(1)}(p^2) = -\frac{\alpha}{96\pi s_W^2 M_W^2 p^2}\Big\{ & 4(M_W^2+2p^2)(4s_W^2-3)A_0[M_W^2] \\
& - 2(M_Z^2+2p^2)(3-12s_W^2+16s_W^4)A_0[M_Z^2] \\
& + M_Z^2(2c_W^2(-3+2s_W^2)(2M_W^2-p^2(d-7)) \\
& + (3-12s_W^2+16s_W^4)(2M_Z^2-p^2(d-7)))B_0[p^2,0,0] \\
& + 12c_W^2M_W^2(2M_W^2+p^2)B_0[p^2,M_W^2,M_W^2] \\
& + 2M_W^2(-3+2s_W^2)(2M_W^4+2p^4-M_W^2p^2(d-8))C_0[0,0,p^2,0,M_W^2,0] \\
& + M_Z^2(3-12s_W^2+16s_W^4)(2M_Z^4+2p^4-M_Z^2p^2(d-8))C_0[0,0,p^2,0,M_Z^2,0] \\
& - 24c_W^2M_W^4(M_W^2+2p^2)C_0[0,0,p^2,M_W^2,0,M_W^2]\Big\} \\
& + \frac{1}{12}\Big(6\delta Z_e+3\delta Z_{ZZ}-\frac{6(1-2s_W^2)}{c_W^2s_W}\delta s_W+2(3-4s_W^2)\delta Z_L^u+8s_W^2\delta Z_R^u\Big)
\end{aligned} \tag{35}
$$

$$
\begin{aligned}
g_{V,d}^{(1)}(p^2) = -\frac{\alpha}{864\pi s_W^2 M_W^2 p^2}\Big\{ & 36(M_W^2+2p^2)(-3+6c_W^2+4s_W^2)A_0[M_W^2] \\
& - 2(M_Z^2+2p^2)(-27+54s_W^2-36s_W^4+16s_W^6)A_0[M_Z^2] \\
& - M_Z^2(-3+4s_W^2)(18c_W^2(2M_W^2-p^2(d-7)) \\
& - (9-6s_W^2+4s_W^4)(2M_Z^2-p^2(d-7)))B_0[p^2,0,0] \\
& - 108c_W^2M_W^2(2M_W^2+p^2)B_0[p^2,M_W^2,M_W^2] \\
& - 18M_W^2(-3+4s_W^2)(2M_W^4+2p^4-M_W^2p^2(d-8))C_0[0,0,p^2,0,M_W^2,0] \\
& + M_Z^2(-27+54s_W^2-36s_W^4+16s_W^6)(2M_Z^4+2p^4 \\
& - M_Z^2p^2(d-8))C_0[0,0,p^2,0,M_Z^2,0] \\
& + 216c_W^2M_W^4(M_W^2+2p^2)C_0[0,0,p^2,M_W^2,0,M_W^2]\Big\} \\
& - \frac{1}{12}\Big((3-4s_W^2)(2\delta Z_e+\delta Z_{ZZ})-4c_Ws_W\delta Z_{\gamma Z}-2\frac{3-2s_W^2}{c_W^2 s_W}\delta s_W \\
& + 2(3-2s_W^2)\delta Z_L^d-4s_W^2\delta Z_R^d\Big)
\end{aligned}
\tag{36}
$$

$$
\begin{aligned}
g_{A,d}^{(1)}(p^2) = -\frac{\alpha}{96\pi s_W^2 M_W^2 p^2}\Big\{ & 4(M_W^2+2p^2)(-3+6c_W^2+4s_W^2)A_0[M_W^2] \\
& + 2(M_Z^2+2p^2)(3-6s_W^2+4s_W^4)A_0[M_Z^2] \\
& - M_Z^2(2c_W^2(-3+4s_W^2)(2M_W^2-p^2(d-7))+(3-6s_W^2+4s_W^4)(2M_Z^2 \\
& - p^2(d-7)))B_0[p^2,0,0]-12c_W^2M_W^2(2M_W^2+p^2)B_0[p^2,M_W^2,M_W^2] \\
& - 2M_W^2(-3+4s_W^2)(2M_W^4+2p^4-M_W^2p^2(d-8))C_0[0,0,p^2,0,M_W^2,0] \\
& - M_Z^2(3-6s_W^2+4s_W^4)(2M_Z^4+2p^4-M_Z^2p^2(d-8))C_0[0,0,p^2,0,M_Z^2,0] \\
& + 24c_W^2M_W^4(M_W^2+2p^2)C_0[0,0,p^2,M_W^2,0,M_W^2]\Big\} \\
& - \frac{1}{12}\Big(6\delta Z_e+3\delta Z_{ZZ}-6\frac{1-2s_W^2}{c_W^2 s_W}\delta s_W+2(3-2s_W^2)\delta Z_L^d+4s_W^2\delta Z_R^d\Big)
\end{aligned}
\tag{37}
$$

2.2.2 γ Vertices - NLO EW corrections

For the NLO EW contributions to the γ form factors we find

$$
\begin{aligned}
h_{V,\nu}^{(1)}(p^2) = \frac{\alpha}{16\pi s_W p^2}\Big\{ & (2M_W^2-p^2(d-7))B_0[p^2,0,0]-(2M_W^2+p^2)B_0[p^2,M_W^2,M_W^2] \\
& + (2M_W^4+2p^4-M_W^2p^2(d-8))C_0[0,0,p^2,0,M_W^2,0] \\
& + 2M_W^2(M_W^2+2p^2)C_0[0,0,p^2,M_W^2,0,M_W^2]\Big\} - \frac{1}{8c_W s_W}\delta Z_{Z\gamma}
\end{aligned}
\tag{38}
$$

$$
\begin{aligned}
h^{(1)}_{V,e}(p^2) = \frac{\alpha}{32\pi s_W^2 M_W^2 p^2}\Big\{ & -4(M_W^2+2p^2)A_0[M_W^2] - 2(M_Z^2+2p^2)(1-4s_W^2+8s_W^4)A_0[M_Z^2] \\
& + M_Z^2(1-4s_W^2+8s_W^4)(2M_Z^2 - p^2(d-7))B_0[p^2,0,0] \\
& + 2M_W^2(2M_W^2+p^2)B_0[p^2,M_W^2,M_W^2] \\
& + M_Z^2(1-4s_W^2+8s_W^4)(2M_Z^4+2p^4-M_Z^2p^2(d-8))C_0[0,0,p^2,0,M_Z^2,0] \\
& - 4M_W^4(M_W^2+2p^2)C_0[0,0,p^2,M_W^2,0,M_W^2]\Big\} \\
& + \frac{1}{8c_W s_W}\Big((1-4s_W^2)\delta Z_{Z\gamma} - 4c_W s_W(\delta Z_{\gamma\gamma}+2\delta Z_e+\delta Z_L^e+\delta Z_R^e)\Big)
\end{aligned} \tag{39}
$$

$$
\begin{aligned}
h^{(1)}_{A,e}(p^2) = \frac{\alpha}{32\pi s_W M_W^2 p^2}\Big\{ & -4(M_W^2+2p^2)A_0[M_W^2] + 2(M_Z^2+2p^2)(-1+4s_W^2)A_0[M_Z^2] \\
& - M_Z^2(-1+4s_W^2)(2M_Z^2 - p^2(d-7))B_0[p^2,0,0] \\
& + 2M_W^2(2M_W^2+p^2)B_0[p^2,M_W^2,M_W^2] \\
& - M_Z^2(-1+4s_W^2)(2M_Z^4+2p^4-M_Z^2p^2(d-8))C_0[0,0,p^2,0,M_Z^2,0] \\
& - 4M_W^4(M_W^2+2p^2)C_0[0,0,p^2,M_W^2,0,M_W^2]\Big\} \\
& + \frac{1}{8c_W s_W}(\delta Z_{Z\gamma} + 4c_W s_W(-\delta Z_L^e+\delta Z_R^e))
\end{aligned} \tag{40}
$$

$$
\begin{aligned}
h^{(1)}_{V,u}(p^2) = \frac{\alpha}{432\pi s_W^2 M_W^2 p^2}\Big\{ & 36(M_W^2+2p^2)A_0[M_W^2] + 2(M_Z^2+2p^2)(9-24s_W^2+32s_W^4)A_0[M_Z^2] \\
& + M_Z^2(9c_W^2(2M_W^2-p^2(d-7)) \\
& - (9-24s_W^2+32s_W^4)(2M_Z^2-p^2(d-7)))B_0[p^2,0,0] \\
& - 27M_W^2(2M_W^2+p^2)B_0[p^2,M_W^2,M_W^2] \\
& + 9M_W^2(2M_W^4+2p^4-M_W^2p^2(d-8))C_0[0,0,p^2,0,M_W^2,0] \\
& - M_Z^2(9-24s_W^2+32s_W^4)(2M_Z^4+2p^4-M_Z^2p^2(d-8))C_0[0,0,p^2,0,M_Z^2,0] \\
& + 54M_W^4(M_W^2+2p^2)C_0[0,0,p^2,M_W^2,0,M_W^2]\Big\} \\
& + \frac{1}{24c_W s_W}((-3+8s_W^2)\delta Z_{Z\gamma} + 8c_W s_W(\delta Z_{\gamma\gamma}+2\delta Z_e+\delta Z_L^u+\delta Z_R^u))
\end{aligned} \tag{41}
$$

$$
\begin{aligned}
h^{(1)}_{A,u}(p^2) = \frac{\alpha}{144\pi s_W M_W^2 p^2}\Big\{ & 12(M_W^2+2p^2)A_0[M_W^2] - 2(M_Z^2+2p^2)(-3+8s_W^2)A_0[M_Z^2] \\
& + M_Z^2(3c_W^2(2M_W^2-p^2(d-7)) - (3-8s_W^2)(2M_Z^2-p^2(d-7)))B_0[p^2,0,0] \\
& - 9M_W^2(2M_W^2+p^2)B_0[p^2,M_W^2,M_W^2] \\
& + 3M_W^2(2M_W^4+2p^4-M_W^2p^2(d-8))C_0[0,0,p^2,0,M_W^2,0] \\
& + M_Z^2(-3+8s_W^2)(2M_Z^4+2p^4-M_Z^2p^2(d-8))C_0[0,0,p^2,0,M_Z^2,0] \\
& + 18M_W^4(M_W^2+2p^2)C_0[0,0,p^2,M_W^2,0,M_W^2]\Big\} \\
& + \frac{1}{24c_W s_W}(-3\delta Z_{Z\gamma} + 8c_W s_W(\delta Z_L^u-\delta Z_R^u))
\end{aligned} \tag{42}
$$

$$h^{(1)}_{V,d}(p^2) = \frac{\alpha}{864\pi s_W^2 M_W^2 p^2}\Big\{ -36(M_W^2+2p^2)A_0[M_W^2] - 2(M_Z^2+2p^2)(9-12s_W^2+8s_W^4)A_0[M_Z^2]$$
$$- M_Z^2(36c_W^2(2M_W^2-p^2(d-7))$$
$$-(9-12s_W^2+8s_W^4)(2M_Z^2-p^2(d-7)))B_0[p^2,0,0]$$
$$+54M_W^2(2M_W^2+p^2)B_0[p^2,M_W^2,M_W^2]$$
$$-36M_W^2(2M_W^4+2p^4-M_W^2p^2(d-8))C_0[0,0,p^2,0,M_W^2,0]$$
$$+M_Z^2(9-12s_W^2+8s_W^4)(2M_Z^4+2p^4-M_Z^2p^2(d-8))C_0[0,0,p^2,0,M_Z^2,0]$$
$$-108M_W^4(M_W^2+2p^2)C_0[0,0,p^2,M_W^2,0,M_W^2]\Big\}$$
$$+\frac{1}{24c_Ws_W}((3-4s_W^2)\delta Z_{Z\gamma}-4c_Ws_W(\delta Z_{\gamma\gamma}+2\delta Z_e+\delta Z_L^d+\delta Z_R^d)) \tag{43}$$

$$h^{(1)}_{A,d}(p^2) = \frac{\alpha}{288\pi s_W M_W^2 p^2}\Big\{ -12(M_W^2+2p^2)A_0[M_W^2] + 2(M_Z^2+2p^2)(-3+4s_W^2)A_0[M_Z^2]$$
$$-M_Z^2(12c_W^2(2M_W^2-p^2(d-7))-(3-4s_W^2)(2M_Z^2-p^2(d-7)))B_0[p^2,0,0]$$
$$+18M_W^2(2M_W^2+p^2)B_0[p^2,M_W^2,M_W^2]$$
$$-12M_W^2(2M_W^4+2p^4-M_W^2p^2(d-8))C_0[0,0,p^2,0,M_W^2,0]$$
$$-M_Z^2(-3+4s_W^2)(2M_Z^4+2p^4-M_Z^2p^2(d-8))C_0[0,0,p^2,0,M_Z^2,0]$$
$$-36M_W^4(M_W^2+2p^2)C_0[0,0,p^2,M_W^2,0,M_W^2]\Big\}$$
$$+\frac{1}{24}\Big(\frac{3}{c_Ws_W}\delta Z_{Z\gamma}-4\delta Z_L^d+4\delta Z_R^d\Big) \tag{44}$$

2.2.3 W Vertices - NLO EW corrections

We can parameterize the leptonic vertex contributions as

$$F^{lep}_{1,e^-u} = \frac{\alpha}{16\pi M_W^2 Q^2 s_W^2}(d-2)\Big\{4(2Q^2-M_W^2)A_0[M_W^2] - 2(M_Z^2-2Q^2)(1-2s_W^2+2s_W^4)A_0[M_Z^2]$$
$$+M_Z^2(-1+2s_W^2)(2M_Z^2+Q^2(d-7))B_0[-Q^2,0,0]$$
$$+4M_W^2(M_W^2-Q^2)s_W^2B_0[-Q^2,0,M_W^2]$$
$$+4c_W^2M_W^2(M_W^2+M_Z^2-Q^2)B_0[-Q^2,M_W^2,M_Z^2]$$
$$+8M_W^4Q^2s_W^2C_0[0,0,-Q^2,0,0,M_W^2]$$
$$-M_Z^2(1-2s_W^2)(2M_Z^4+2Q^4+M_Z^2Q^2(d-8))C_0[0,0,-Q^2,0,M_Z^2,0]$$
$$+8c_W^2M_W^2(M_Z^2Q^2+M_W^2(Q^2-M_Z^2))C_0[0,0,-Q^2,M_W^2,0,M_Z^2]\Big\} \tag{45}$$

$$
\begin{aligned}
F^{lep}_{L,e^-u} = \frac{\alpha x}{8\pi M_W^2 Q^2 s_W^2}(d-4)\Big\{ & 4(M_W^2-2Q^2)A_0[M_W^2]+2(M_Z^2-2Q^2)(1-2s_W^2+2s_W^4)A_0[M_Z^2] \\
& + M_Z^2(1-2s_W^2)(2M_Z^2+Q^2(d-7))B_0[-Q^2,0,0] \\
& - 4M_W^2(M_W^2-Q^2)s_W^2 B_0[-Q^2,0,M_W^2] \\
& - 4c_W^2 M_W^2(M_W^2+M_Z^2-Q^2)B_0[-Q^2,M_W^2,M_Z^2] \\
& - 8M_W^4 Q^2 s_W^2 C_0[0,0,-Q^2,0,0,M_W^2]\Big\}
\end{aligned} \tag{46}
$$

$$
\begin{aligned}
F^{lep}_{3,e^-u} = \frac{\alpha}{4\pi M_W^2 Q^2 s_W^2}\Big\{ & 4(2Q^2-M_W^2)A_0[M_W^2]-2(M_Z^2-2Q^2)(1-2s_W^2+2s_W^4)A_0[M_Z^2] \\
& + M_Z^2(2M_Z^2-3Q^2)(2s_W^2-1)B_0[-Q^2,0,0] \\
& + 4M_W^2(M_W^2-Q^2)s_W^2 B_0[-Q^2,0,M_W^2] \\
& + 4c_W^2 M_W^2(M_W^2+M_Z^2-Q^2)B_0[-Q^2,M_W^2,M_Z^2] + 8M_W^4 Q^2 s_W^2 C_0[0,0,-Q^2,0,0,M_W^2] \\
& + 2M_Z^2(M_Z^2-Q^2)^2(-1+2s_W^2)C_0[0,0,-Q^2,0,M_Z^2,0] \\
& + 8c_W^2 M_W^2(M_Z^2Q^2+M_W^2(Q^2-M_Z^2))C_0[0,0,-Q^2,M_W^2,0,M_Z^2]\Big\}
\end{aligned} \tag{47}
$$

$$
\begin{aligned}
F^{q}_{1,e^-u} = \frac{\alpha}{144\pi M_W^2 Q^2 s_W^2}(d-2)\Big\{ & 36(2Q^2-M_W^2)A_0[M_W^2]-2(M_Z^2-2Q^2)(9-18s_W^2+10s_W^4)A_0[M_Z^2] \\
& - M_Z^2(2M_Z^2(9-18s_W^2+8s_W^4)+Q^2(9-10s_W^2)(d-7))B_0[-Q^2,0,0] \\
& + 36M_W^2(M_W^2-Q^2)s_W^2 B_0[-Q^2,0,M_W^2] \\
& + 36c_W^2 M_W^2(M_W^2+M_Z^2-Q^2)B_0[-Q^2,M_W^2,M_Z^2] \\
& - 16M_W^2 Q^4 s_W^2 C_0[0,0,-Q^2,0,0,0] \\
& + 72M_W^4 Q^2 s_W^2 C_0[0,0,-Q^2,0,0,M_W^2] \\
& - M_Z^2(9-18s_W^2+8s_W^4)(2M_Z^4+2Q^4+M_Z^2Q^2(d-8))C_0[0,0,-Q^2,0,M_Z^2,0] \\
& + 72c_W^2 M_W^2(M_Z^2Q^2-M_W^2(M_Z^2-Q^2))C_0[0,0,-Q^2,M_W^2,0,M_Z^2]\Big\}
\end{aligned} \tag{48}
$$

$$
\begin{aligned}
F^{q}_{L,e^-u} = \frac{\alpha x}{72\pi M_W^2 Q^2 s_W^2}(d-4)\Big\{ & 36(M_W^2-2Q^2)A_0[M_W^2]+2(M_Z^2-2Q^2)(9-18s_W^2+10s_W^4)A_0[M_Z^2] \\
& + M_Z^2(2M_Z^2(9-18s_W^2+8s_W^4)+Q^2(9-10s_W^2)(d-7))B_0[-Q^2,0,0] \\
& - 4M_W^2(9(M_W^2-Q^2)s_W^2 B_0[-Q^2,0,M_W^2] \\
& + 9c_W^2(M_W^2+M_Z^2-Q^2)B_0[-Q^2,M_W^2,M_Z^2] \\
& - 2Q^2s_W^2(2Q^2C_0[0,0,-Q^2,0,0,0]-9M_W^2C_0[0,0,-Q^2,0,0,M_W^2]))\Big\}
\end{aligned} \tag{49}
$$

$$
\begin{aligned}
F^q_{3,e^-u} = \frac{\alpha}{36\pi M_W^2 Q^2 s_W^2}\Big\{&36(2Q^2-M_W^2)A_0[M_W^2] - 2(M_Z^2-2Q^2)(9-18s_W^2+10s_W^4)A_0[M_Z^2] \\
&+ M_Z^2(3Q^2(9-10s_W^2) - 2M_Z^2(9-18s_W^2+8s_W^4))B_0[-Q^2,0,0] \\
&+ 36M_W^2(M_W^2-Q^2)s_W^2 B_0[-Q^2,0,M_W^2] \\
&+ 36c_W^2 M_W^2(M_W^2+M_Z^2-Q^2)B_0[-Q^2,M_W^2,M_Z^2] \\
&- 16M_W^2 Q^4 s_W^2 C_0[0,0,-Q^2,0,0,0] + 72M_W^4 Q^2 s_W^2 C_0[0,0,-Q^2,0,0,M_W^2] \\
&- 2(M_Z^3 - M_Z Q^2)^2(9-18s_W^2+8s_W^4)C_0[0,0,-Q^2,0,M_Z^2,0] \\
&+ 72c_W^2 M_W^2(M_Z^2Q^2 + M_W^2(-M_Z^2+Q^2))C_0[0,0,-Q^2,M_W^2,0,M_Z^2]\Big\}
\end{aligned} \tag{50}
$$

F_1 and F_L are the same for all quark/electron combinations.
For the leptonic and quark Vertex respectively we switch according to

$$
F^{lep}_{3,e^-\bar{d}} = -F^{lep}_{3,e^-u} \quad \text{and} \quad F^{lep}_{3,e^+\bar{u}} = -F^{lep}_{3,e^-u} + \frac{\alpha}{2\pi}\left[\frac{1+2s_W^2}{s_W^2}\right] \tag{51}
$$

$$
F^q_{3,e^-\bar{d}} = -F^q_{3,e^-u} + \frac{7\alpha}{9\pi} \quad \text{and} \quad F^q_{3,e^+\bar{u}} = -F^q_{3,e^-u} + \frac{7\alpha}{9\pi} \tag{52}
$$

The polarized contributions can easily be found via

$$
g_5 = F_1, \quad g_L = F_L \quad \text{and} \quad g_1 = \frac{1}{2}F_3 \tag{53}
$$

2.3 Neutral Box-type contributions

The Box corrections do not easily factorize in any of the scenarios. We therefor quote them here wholesale. In the case of the neutral current they are however separately finite and gauge-invariant.

$$
\begin{aligned}
F_2^{WW\gamma} =& -\frac{\alpha}{32\pi s_W^4}(1-\lambda_e)Q_q Q^2(2C_0[0,0,-Q^2,M_W^2,0,M_W^2] \\
&- SxD_0[0,0,0,0,-Q^2,Sx,M_W^2,0,M_W^2,0])
\end{aligned} \tag{54}
$$

$$
\begin{aligned}
F_1^{WW\gamma} =& -\frac{\alpha}{32\pi s_W^4}(1-\lambda_e)Q_q Q^2(2C_0[0,0,-Q^2,M_W^2,0,M_W^2] \\
&- SxD_0[0,0,0,0,-Q^2,Sx,M_W^2,0,M_W^2,0])F_2^{WW\gamma} = 0F_3^{WW\gamma} = -F_1^{WW\gamma}
\end{aligned} \tag{55}
$$

2.4 Charged Box-Type Contributions

In order to include charged boxes we have to modify the original cross section by two extra terms:

$$
\frac{d^2\sigma^i}{dxdy} = \frac{2\pi\alpha^2}{xyQ^2}\omega^i\left[2x(2-2y+y^2)F_1 \pm xy(2-y)F_3 + 2(1-y)F_L + \frac{x}{y}F_{\text{new}}\right] \tag{56}
$$

where

$$
\begin{aligned}
F_1 = \frac{\alpha}{12\pi c_W^2 s_W^2 Q^2}(M_W^2 + Q^2)\Big\{ & 4c_W^2 s_W^2 B_0[-Q^2, 0, M_W^2] + 2(3 - 6s_W^2 + 2s_W^4)B_0[-Q^2, M_W^2, M_Z^2] \\
& - 2(3 - 4s_W^2)B_0[Q^2 - Sx, 0, 0] \\
& - 8c_W^2(M_W^2 + 3Q^2)s_W^2 C_0[0, 0, -Q^2, 0, 0, M_W^2] \\
& - 4(6c_W^4 Q^2 + M_W^2(3 - 6s_W^2 + 2s_W^4) + M_Z^2(3 - 6s_W^2 + 2s_W^4))C_0[0, 0, -Q^2, M_W^2, 0 \\
& + 2c_W^2(M_W^2 + Q^2)s_W^2 C_0[0, 0, Q^2 - Sx, 0, 0, 0] \\
& - (M_W^2(-3 + 4s_W^2) + Q^2(-3 + 4s_W^2) + M_Z^2(-3 + 6s_W^2 - 2s_W^4))C_0[0, 0, Q^2 - Sx, \\
& + 2c_W^2(M_W^4 + 2M_W^2 Q^2 + 3Q^4)s_W^2 D_0[0, 0, 0, 0, Q^2 - Sx, -Q^2, 0, 0, 0, M_W^2] \\
& + (M_W^4 + M_Z^4 + 2M_Z^2 Q^2 + 3Q^4 + 2M_W^2(2M_Z^2 + Q^2))(3 - 6s_W^2 + 2s_W^4)D_0[0, 0, 0,
\end{aligned}
\tag{57}
$$

$$F_3 = \tag{58}$$

$$F_L = \tag{59}$$

$$
\begin{aligned}
F_{\text{new}} = \frac{2\alpha Q^2}{3\pi c_W^2 s_W^2}(M_W^2 + Q^2)\Big\{ & 4c_W^2 s_W^2 D_0[0, 0, 0, 0, Sx, -Q^2, 0, 0, 0, M_W^2] \\
& + (3 - 6s_W^2 + 4s_W^4)D_0[0, 0, 0, 0, Sx, -Q^2, 0, M_W^2, 0, M_Z^2] \\
& - 2c_W^2 s_W^2 D_0[0, 0, 0, 0, Q^2 - Sx, -Q^2, 0, 0, 0, M_W^2] \\
& + (-3 + 6s_W^2 - 2s_W^4)D_0[0, 0, 0, 0, Q^2 - Sx, -Q^2, 0, M_W^2, 0, M_Z^2]\Big\}
\end{aligned}
\tag{60}
$$

2.5 QED Dipole Subtraction for Charged Current

In the charged current case we also have to account for the experimentalists excluding additional photon radiation. Unlike in the neutral current case the QED corrections do not represent a gauge-invariant subset so we have to consider them together with the electroweak corrections.

The Catani-Seymour dipole represents a gauge invariant quantity and can therefor be used to subtract the infrared divergences from the loop corrected cross section to obtain a physically meaningful prediction.

We define the quantity

$$d\sigma^{\text{CC}}_{\text{NLO}} = d\sigma^{\text{CC}}_{\text{loop}} + \frac{\alpha}{2\pi} d\sigma^{\text{CC}}_{\text{Born}} I^{\text{R.S.}} \tag{61}$$

where the Born cross section is calculated in $d = 4 - 2\epsilon$ dimensions and the dipole term is given by

$$I^{\text{R.S.}} = \sum_{i \neq j} c_\pm Q_i Q_j \Big(\frac{\mu^2}{-|s_{ij}|}\Big)^\epsilon \frac{1}{\epsilon^2} + \frac{3}{2}\sum_i Q_i^2 \frac{1}{\epsilon} \tag{62}$$

where the sum extends over all external charged fermion legs and $c_\pm$ is -1 if fermion i and j are both incoming/outgoing and $+1$ if one is incoming and one is outgoing.
In our DIS case we can write the Dipole factor as

$$\begin{aligned} I^{\text{R.S.}}_{\text{DIS}} =& \frac{2}{\epsilon^2}\left[-q_e q_q^{\text{in}}\left(-\frac{\mu^2}{s}\right)^\epsilon + q_q^{\text{out}} q_q^{\text{in}}\left(\frac{\mu^2}{t}\right)^\epsilon + q_e q_q^{\text{out}}\left(-\frac{\mu^2}{s+t}\right)^\epsilon\right] \\ &+ \frac{3}{2}\frac{1}{\epsilon}\left[q_e^2 + q_q^{\text{in}2} + q_q^{\text{out}2}\right] \end{aligned} \tag{63}$$

A Renormalization Constants

We collect here the analytic formulae for the 1-loop SM renormalization constants in the on-shell scheme. The conventions follow [5].
The weak mixing angle can be expressed through the mass renormalization of W and Z as:

$$\delta s_W = \frac{c_W^2}{2 s_W}\left[\frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2}\right]. \tag{64}$$

From the $U(1)$ Ward identity we find the charge renormalization to be

$$\delta Z_e = -\frac{1}{2}\delta Z_{\gamma\gamma} - \frac{s_W}{2 c_W}\delta Z_{Z\gamma}. \tag{65}$$

Through the complex pole of the W and Z boson propagators we find

$$\delta M_Z^2 = \Re\{\Sigma_{ZZ}(M_Z^2)\} \qquad \delta M_W^2 = \Re\{\Sigma_{WW}(M_W^2)\}. \tag{66}$$

The wavefunction renormalizations for external fermions are next. Note that these are manifestly real for massless fermions

$$\begin{aligned} \delta Z_L^\nu =& -\frac{\alpha(d-2)^2}{16\pi M_W^2 s_W^2 d}(2A_0[M_W^2] + A_0[M_Z^2]) \\ \delta Z_L^e =& -\frac{\alpha(d-2)^2}{16\pi M_W^2 s_W^2 d}(2A_0[M_W^2] + (1-2s_W^2)^2 A_0[M_Z^2]) \\ \delta Z_R^e =& -\frac{\alpha(d-2)^2}{4\pi M_W^2 d}A_0[M_Z^2] \\ \delta Z_L^u =& -\frac{\alpha(d-2)^2}{144\pi M_W^2 s_W^2 d}(18A_0[M_W^2] + (3-4s_W^2)^2 A_0[M_Z^2]) \\ \delta Z_R^u =& -\frac{\alpha(d-2)^2}{9\pi M_W^2 d}A_0[M_Z^2] \\ \delta Z_L^d =& -\frac{\alpha(d-2)^2}{144\pi M_W^2 s_W^2 d}(18A_0[M_W^2] + (3-2s_W^2)^2 A_0[M_Z^2]) \\ \delta Z_R^d =& -\frac{\alpha(d-2)^2}{36\pi M_W^2 d}A_0[M_Z^2]. \end{aligned} \tag{67}$$

For the gauge boson wavefunction renormalizations we find

$$\delta Z_{\gamma\gamma} = \frac{\alpha}{72\pi m_t^2 M_W^2 M_Z^2(1-d)} \Big\{ 24 m_t^2 M_W^2 (2-d) A_0[m_b^2] + 16 M_W^2 M_Z^2 (2 - 3d + d^2) A_0[m_t^2] \\ - 3m_t^2 (M_Z^2 (26 - 41d + 16d^2 - d^3) A_0[M_W^2] \\ + 4M_W^2 (19 M_Z^2 (2-d) B_0[M_Z^2, 0, 0] \\ - (4m_b^2 - M_Z^2(2-d)) B_0[M_Z^2, m_b^2, m_b^2])) \Big\} - \Delta\alpha \qquad (68)$$

$$\delta Z_{Z\gamma} = \frac{\alpha}{2\pi s_W c_W M_Z^2} (d-2) A_0[M_W^2] \qquad (69)$$

$$\delta Z_{\gamma Z} = \frac{\alpha}{12\pi c_W s_W M_Z^2 (d-1)} \Big\{ 2(3 - 4s_W^2)(2-d) A_0[m_b^2] + 4(3 - 8s_W^2)(2-d) A_0[m_t^2] \\ + 6(3 - 2d - 2s_W^2(1-d))(2-d) A_0[M_W^2] \\ - M_Z^2 (27 - 76 s_W^2)(2-d) B_0[M_Z^2, 0, 0] + (3 - 4s_W^2)(4m_b^2 - M_Z^2(2-d)) B_0[M_Z^2, m_b^2, \\ + 2(3 - 8s_W^2)(4m_t^2 - M_Z^2(2-d)) B_0[M_Z^2, m_t^2, m_t^2] \\ - 3(4M_W^2(3d - 4 + 2s_W^2(1-d)) + M_Z^2(6d - 5 + 6s_W^2(1-d))) B_0[M_Z^2, M_W^2, M_W^2] \Big\} \qquad (70)$$

$$
\begin{aligned}
\delta Z_{ZZ} =& \frac{\alpha(2-d)}{48M_W^2(M_Z^2-4m_b^2)\pi s_W^2(1-d)}(2m_b^2(27-24s_W^2+16s_W^4-9d)-M_Z^2(9-12s_W^2+8s_W^4)(2-d))A_0[m_b^2] \\
&+ \frac{\alpha(2-d)}{48M_W^2(M_Z^2-4m_t^2)\pi s_W^2(1-d)}(2m_t^2(27-48s_W^2+64s_W^4-9d)-M_Z^2(9-24s_W^2+32s_W^4)(2-d))A_0[m \\
&+ \frac{\alpha}{32M_W^4(m_H^2-M_Z^2)^2\pi s_W^2(-1+d)}(c_W^2(-3m_H^6+12m_H^4M_Z^2-14m_H^2M_Z^4+2M_Z^6) \\
&- 4M_W^2M_Z^2(m_H^2-2M_Z^2)(-1+d))A_0[m_H^2] \\
&+ \frac{\alpha(2-d)}{16M_W^2(4M_W^2-M_Z^2)\pi s_W^2(1-d)}(2c_W^2(-4M_W^2+M_Z^2)s_W^2 \\
&- s_W^4(M_Z^2+4M_W^2(-3+2d))+3c_W^4(4M_W^2(-3+2d)+M_Z^2(-3+4d)))A_0[M_W^2] \\
&+ \frac{\alpha}{64M_W^4(m_H^2-M_Z^2)^2\pi s_W^2(1-d)}(c_W^2(4M_Z^6+m_H^4M_Z^2(34-5d)+4m_H^2M_Z^4(-9+d) \\
&+ m_H^6(-8+d))+4M_W^2M_Z^2(M_Z^2(d-6)+m_H^2(4-d))(1-d))A_0[M_Z^2] \\
&+ \frac{\alpha(27-54s_W^2+76s_W^4)(d-2)^2B_0[M_Z^2,0,0]}{48c_W^2\pi s_W^2(1-d)} \\
&+ \frac{\alpha}{96M_W^2(4m_b^2-M_Z^2)\pi s_W^2(-1+d)}(8m_b^4(27-24s_W^2+16s_W^4-9d) \\
&+ M_Z^4(9-12s_W^2+8s_W^4)(d-2)^2-2m_b^2M_Z^2(-48s_W^2+32s_W^4+9(8-5d+d^2)))B_0[M_Z^2,m_b^2,m_b^2] \\
&+ \frac{\alpha}{32M_W^4\pi s_W^2(-1+d)}(c_W^2(3m_H^4-8m_H^2M_Z^2)-4M_W^2M_Z^2(1-d))B_0[M_Z^2,m_H^2,M_Z^2] \\
&+ \frac{\alpha}{96M_W^2(4m_t^2-M_Z^2)\pi s_W^2(-1+d)}(8m_t^4(27-48s_W^2+64s_W^4-9d) \\
&+ M_Z^4(9-24s_W^2+32s_W^4)(2-d)^2+2m_t^2M_Z^2(96s_W^2-128s_W^4-9(8-5d+d^2)))B_0[M_Z^2,m_t^2,m_t^2] \\
&+ \frac{\alpha}{32M_W^2(4M_W^2-M_Z^2)\pi s_W^2(-1+d)}(2c_W^2(4M_W^2-M_Z^2)s_W^2(4M_W^2-M_Z^2(2-d)) \\
&+ s_W^4(-M_Z^4(2-d)-16M_W^4(3-2d)+4M_W^2M_Z^2(11-11d+2d^2)) \\
&- 3c_W^4(16M_W^4(-3+2d)+4M_W^2M_Z^2(15-15d+2d^2)+M_Z^4(6-11d+4d^2)))B_0[M_Z^2,M_W^2,M_W^2]
\end{aligned}
\tag{71}
$$

$$
\begin{aligned}
\delta Z_{WW} =& \frac{3\alpha}{16(m_b^2-m_t^2)^2 M_W^4 \pi s_W^2(-1+d)}(3m_b^6 - 3m_t^6 + m_t^2 M_W^4 + m_b^2(9m_t^4 + 2m_t^2 M_W^2 + M_W^4(5-2d)) \\
&+ m_b^4(-9m_t^2 + M_W^2(-4+d)) + m_t^4 M_W^2(2-d) - M_W^6(2-d))A_0[m_b^2] \\
&\frac{\alpha}{32M_W^4(m_H^2-M_W^2)^2\pi s_W^2(1-d))}(3m_H^6 - 12m_H^4 M_W^2 + 2M_W^6(3-4d) + 2m_H^2 M_W^4(5+2d))A_0[m_H^2] \\
&+ \frac{3\alpha}{32m_t^2(-m_b^2+m_t^2)^2 M_W^4 \pi s_W^2(-1+d)}(6m_t^8 + m_b^6(-6m_t^2 + M_W^2(-2+d)) \\
&+ m_b^4(18m_t^4 + m_t^2 M_W^2(14-5d) + M_W^4(-3+d)(-2+d)) \\
&- m_b^2(18m_t^6 + m_t^4 M_W^2(10-3d) + 2m_t^2 M_W^4(5-2d) + M_W^6(d-2)^2) \\
&+ m_t^6 M_W^2(-2+d) + m_t^2 M_W^6(-4+d)(-2+d) - m_t^4 M_W^4(8+(-5+d)d))A_0[m_t^2] \\
&+ \frac{\alpha}{64M_W^4(m_H^2-M_W^2)^2\pi s_W^6(-3+d)(-1+d))}((-3+d)(-m_H^6 s_W^4(-8+d) \\
&+ 8c_W^2(m_H^2-M_W^2)^2 M_Z^2 s_W^4(-1+d) - 8c_W^2(m_H^2-M_W^2)^2 M_Z^2(6-c_W^4(-7+d) \\
&+ 2c_W^6(-2+d) - 3d + c_W^2(-21+11d))) + 2c_W^4 M_Z^6(3(-3+d) + c_W^4(42-98d \\
&+ 40d^2 - 4d^3 - 16s_W^2(-2-d+d^2)) + 2c_W^6(-12+25d-10d^2+d^3+4s_W^2(-2-d+d^2)) \\
&+ 2c_W^2(d(21-10d+d^2) + 4s_W^2(-2-d+d^2))) \\
&- 4c_W^2 m_H^2 M_Z^4(3(-3+d) + c_W^2(45-3d-7d^2+d^3+8s_W^2(-2-d+d^2)) \\
&- 2c_W^4(24+4d-7d^2+d^3+8s_W^2(-2-d+d^2)) + c_W^6(21+5d-7d^2+d^3+8s_W^2(-2-d+d^2))) \\
&- m_H^4 M_Z^2(-6(-3+d) + c_W^6(-114+53d-5d^2-16s_W^2(-2-d+d^2)) \\
&+ c_W^2(-162+69d-5d^2-16s_W^2(-2-d+d^2)) + 2c_W^4(5(24-11d+d^2) \\
&+ 16s_W^2(-2-d+d^2)))))A_0[M_W^2] \\
&+ \frac{\alpha}{64M_W^4 M_Z^2(M_W^2-M_Z^2)^2\pi s_W^2(-1+d)}(-6M_Z^8 + 4M_W^6 M_Z^2(-1+d) + M_W^2 M_Z^6(18+d) \\
&- M_W^4 M_Z^4(2+5d) + 4c_W^2(2M_Z^8(6-s_W^4(-1+d)-3d) + 5M_W^8(2-3d+d^2) \\
&+ M_W^2 M_Z^6(-38+18d+d^2+s_W^4(2-3d+d^2)) \\
&- M_W^4 M_Z^4(8-21d+6d^2+s_W^4(2-3d+d^2))))A_0[M_Z^2] \\
&+ \frac{\alpha(2-d)}{4M_W^2 d(-3+d))}A_0^{IR}[M_W^2] - \frac{9\alpha(d-2)^2}{16\pi s_W^2(-1+d))}B_0[M_W^2,0,0] \\
&+ \frac{3\alpha}{16M_W^4\pi s_W^2(1-d)}(3m_b^4 + 3m_t^4 + m_b^2(-6m_t^2 + M_W^2(-3+d)) + m_t^2 M_W^2(-3+d) \\
&+ M_W^4(-2+d))B_0[M_W^2, m_b^2, m_t^2] \\
&+ \frac{\alpha}{32M_W^4\pi s_W^2(-1+d)}(3m_H^4 - 8m_H^2 M_W^2 + 4M_W^4(-1+d))B_0[M_W^2, m_H^2, M_W^2] \\
&- \frac{\alpha}{32M_W^4\pi s_W^2(-1+d)}(-3M_Z^4 - 4M_W^2 M_Z^2(-8+s_W^4(-1+d)+3d) \\
&+ 4M_W^4(-14+3s_W^2(-1+d)+6d))B_0[M_W^2, M_W^2, M_Z^2].
\end{aligned}
\tag{72}
$$

We quote those here for completeness and convenience, though they should drop out in the total, renormalized cross sections.

B Self-Energy functions

The self-energies once needs to calculate the W and Z mass renormalizations, as well as the self-energy corrections to the cross sections we give in this appendix. We only give the transversal parts, since those are the only relevant contributions.

$$
\begin{aligned}
\Sigma^T_{ZZ}(p^2) = \frac{\alpha}{48\pi c_W^2 s_W^2 p^2 (d-1)} \Big\{ & 3(M_Z^2 - M_H^2 - p^2(2-d))A_0[M_H^2] \\
& + 2p^2(2-d)((9 - 12s_W^2 + 8s_W^4)A_0[m_b^2] + (9 - 24s_W^2 + 32s_W^4)A_0[m_t^2]) \\
& + 6p^2(2c_W^2 s_W^2(2-d) - s_W^4(2-d) + c_W^4(14 - 15d + 4d^2))A_0[M_W^2] \\
& + 3(M_H^2 - M_Z^2 + p^2(-2+d))A_0[M_Z^2] \\
& - 2p^4(27 - 54s_W^2 + 76s_W^4)(2-d)B_0[p^2, 0, 0] \\
& + p^2(2m_b^2(27 - 24s_W^2 + 16s_W^4 - 9d) - p^2(9 - 12s_W^2 + 8s_W^4)(2-d))B_0[p^2, m_b^2, m_b^2 \\
& + 3(M_H^4 + M_Z^4 + p^4 - 2M_H^2(M_Z^2 + p^2) - 2M_Z^2 p^2(3-2d))B_0[p^2, M_H^2, M_Z^2] \\
& + p^2(2m_t^2(27 - 48s_W^2 + 64s_W^4 - 9d) - p^2(9 - 24s_W^2 + 32s_W^4)(2-d))B_0[p^2, m_t^2, m \\
& + 3p^2(2c_W^2(4M_W^2 - p^2)s_W^2 + s_W^4(p^2 - 4M_W^2(3-2d)) \\
& + 3c_W^4(4M_W^2(3-2d) + p^2(3-4d)))B_0[p^2, M_W^2, M_W^2] \Big\}
\end{aligned}
\tag{73}
$$

$$
\begin{aligned}
\Sigma^T_{\gamma Z}(p^2) = \frac{\alpha}{24\pi c_W s_W (d-1)} \Big\{ & 2(3 - 4s_W^2)(d-2)A_0[m_b^2] - 4(3 - 8s_W^2)(2-d)A_0[m_t^2] \\
& - 6(2-d)(s_W^2 + c_W^2(3-2d))A_0[M_W^2] + p^2(27 - 76s_W^2)(2-d)B_0[p^2, 0, 0] \\
& - (3 - 4s_W^2)(4m_b^2 + p^2(-2+d))B_0[p^2, m_b^2, m_b^2] \\
& - 2(3 - 8s_W^2)(4m_t^2 - p^2(2-d))B_0[p^2, m_t^2, m_t^2] \\
& + 3(s_W^2(p^2 - 4M_W^2(2-d)) - c_W^2(4M_W^2(4-3d) + p^2(5-6d)))B_0[p^2, M_W^2, M_W^2] \Big\}
\end{aligned}
\tag{74}
$$

$$
\begin{aligned}
\Sigma^T_{\gamma\gamma}(p^2) = \frac{\alpha}{12\pi(d-1)} \Big\{ & 4(2-d)(A_0[m_b^2] + 4A_0[m_t^2]) + 6(2 - 3d + d^2)A_0[M_W^2] \\
& - 38p^2(2-d)B_0[p^2, 0, 0] + (8m_b^2 - 2p^2(2-d))B_0[p^2, m_b^2, m_b^2] \\
& + 8(4m_t^2 - p^2(2-d))B_0[p^2, m_t^2, m_t^2] + 3(4M_W^2 + 3p^2)(1-d)B_0[p^2, M_W^2, M_W^2] \Big\}
\end{aligned}
\tag{75}
$$

$$
\begin{aligned}
\Sigma^T_{WW}(p^2) = \frac{\alpha}{16\pi s_W^2 p^2(1-d)} \Big\{ & 6(m_t^2 - m_b^2 - p^2(2-d))A_0[m_b^2] + (M_H^2 - M_W^2 + p^2(2-d))A_0[M_H^2] \\
& + 6(m_b^2 - m_t^2 + p^2(-2+d))A_0[m_t^2] - (M_H^2 + M_Z^2 + 12c_W^2 p^2 + 12p^2 s_W^2 \\
& - 2M_W^2(5 - 2c_W^2(2-d) - 2s_W^2(2-d) - 2d) - 2d(7-2d)p^2)A_0[M_W^2] \\
& - (1 + 4c_W^2(-2+d))(M_W^2 - M_Z^2 - p^2(2-d))A_0[M_Z^2] + 18p^4(2-d)B_0[p^2, 0, 0] \\
& - 4s_W^2(p^4(2-3d) + M_W^4(-2+d) - 2M_W^2 p^2(-2+d))B_0[p^2, 0, M_W^2] \\
& + 6(m_b^4 + m_b^2(-2m_t^2 + p^2(-3+d)) + (m_t^2 - p^2)(m_t^2 + p^2(-2+d)))B_0[p^2, m_b^2, m_t^2] \\
& - (M_H^4 + M_W^4 + p^4 - 2M_H^2(M_W^2 + p^2) + 2M_W^2 p^2(-3+2d))B_0[p^2, M_H^2, M_W^2] \\
& + (-M_W^4 - (M_Z^2 - p^2)^2 + 2M_W^2(M_Z^2 + p^2) - 4c_W^2(M_W^2(p^2(5-3d) + 2M_Z^2(2-d)) \\
& + p^4(2-3d) + M_Z^2 p^2(5 - 3d + 4(1-d)s_W^4) - (M_W^4 + M_Z^4)(2-d)))B_0[p^2, M_W^2, M
\end{aligned}
\tag{76}
$$

C Necessary Integrals

To perform an expansion in $d = 4 - 2\epsilon$ we give analytic expressions for all 1-loop functions that appear in our calculation. The integrals are calculated as described in [4].

We find for the tadpole

$$
A_0[m^2] = m^2 \left[\frac{1}{\epsilon} + 1 - \log \frac{m^2}{\mu^2}, \right] \tag{77}
$$

while for the Bubble

$$
\begin{aligned}
B_0[p^2, m_1^2, m_2^2] = & \frac{1}{\epsilon} + 2 - \frac{m_1^2 - m_2^2 + s}{2s} \log\left(\frac{m_1^2}{m_2^2}\right) - \log\left(\frac{m_2^2}{\mu^2}\right) \\
& + \frac{\sqrt{2m_1^2(m_2^2 + s) - m_1^4 - (m_2^2 - s)^2}}{s} \tan^{-1}\left(\frac{\sqrt{2m_1^2(m_2^2 + s) - m_1^4 - (m_2^2 - s)^2}}{s - m_1^2 - m_2^2}\right).
\end{aligned}
\tag{78}
$$

Only two special cases of the triangle integral appear, both of which are finite:

$$
\begin{aligned}
C_0[0, 0, p^2, 0, m^2, 0] = \frac{1}{p^2} \Bigg[& \log\left(1 + \frac{p^2}{m^2}\right) \log\left(1 + \frac{m^2}{p^2}\right) + \mathrm{Li}_2\left(\frac{m^2}{p^2 + m^2}\right) \\
& - \mathrm{Li}_2\left(\frac{p^2 + m^2}{m^2}\right) + \mathrm{Li}_2\left(\frac{p^2}{p^2 + m^2}\right)\Bigg]
\end{aligned}
\tag{79}
$$

and

$$C_0[0,0,p^2,m_1^2,0,m_2^2] = \frac{1}{p^2}\Bigg[\log\left(\frac{m_1^2}{p^2}\right)\log\left(1-\frac{p^2}{m_1^2}\right) - \mathrm{Li}_2\left(\frac{p^2}{m_1^2}\right)$$
$$+ \mathrm{Li}_2\left(\frac{2m_1^2}{m_1^2+m_2^2-p^2-\sqrt{m_1^4+(m_2^2-p^2)^2-2m_1^2(m_2^2+p^2)}}\right)$$
$$- \mathrm{Li}_2\left(\frac{2(m_1^2-p^2)}{m_1^2+m_2^2-p^2-\sqrt{m_1^4+(m_2^2-p^2)^2-2m_1^2(m_2^2+p^2)}}\right)$$
$$+ \mathrm{Li}_2\left(\frac{2m_1^2}{m_1^2+m_2^2-p^2+\sqrt{m_1^4+(m_2^2-p^2)^2-2m_1^2(m_2^2+p^2)}}\right)$$
$$-\mathrm{Li}_2\left(\frac{2(m_1^2-p^2)}{m_1^2+m_2^2-p^2+\sqrt{m_1^4+(m_2^2-p^2)^2-2m_1^2(m_2^2+p^2)}}\right)\Bigg]. \tag{80}$$

The only Box integral appearing is

$$D_0[0,0,0,0,s,t,0,m_1^2,0,m_2^2] = \int_0^1 dx \frac{2\log\left(-\frac{m_1 m_2}{Sx}\right) - d_0(x)}{2(m_1^2(m_2^2+sx)+Sx(m_2^2+t(x-1)+sx))} \tag{81}$$

where

$$d_0(x) = \frac{1}{\gamma_s}(m_1^2+m_2^2-(1-x)t+2sx)\log\left(\frac{m_1^2+m_2^2-(1-x)t+\gamma_s}{m_1^2+m_2^2-(1-x)t-\gamma_s}\right)$$
$$\gamma_s = \sqrt{m_1^4+(m_2^2-t(1-x))^2-2m_1^2(m_2^2+t(1-x))} \tag{82}$$

For convenience we evaluate the last integral numerically.

C.1 Infrared divergent integrals

In the case of charged current scattering, where we subtracted a dipole term to render the expression IR-finite, without calculating the real emission contributions, one also encounters a handful of IR-divergent loop integrals which we take from [6]. They are

$$C_0[0,0,s,0,0,0] = \frac{1}{s}\left[\frac{1}{\epsilon^2} - \frac{1}{\epsilon}\log\left(\frac{s}{\mu^2}\right) + \frac{1}{2}\log^2\left(\frac{s}{\mu^2}\right) - \frac{7\pi^2}{12}\right] \tag{83}$$

$$C_0[0,0,t,0,0,m^2] = \frac{1}{t}\left[\left(-\frac{1}{\epsilon}+\log\left(\frac{m^2}{\mu^2}\right)\right)\log\left(1-\frac{t}{m^2}\right) + \mathrm{Li}_2\left(\frac{t}{m^2}\right) + \log^2\left(1-\frac{t}{m^2}\right)\right] \tag{84}$$

$$D_0[0,0,0,s,t,0,0,0,m^2] = \frac{1}{s(m^2-t)}\Bigg[-\frac{1}{\epsilon^2} + \frac{1}{\epsilon}\left(\log\left(\frac{s}{\mu^2}\right) + 2\log\left(1-\frac{t}{m^2}\right)\right)$$
$$+\frac{1}{2}\log\left(\frac{s}{m^2}\right)^2 - \frac{1}{2}\log\left(\frac{s}{\mu^2}\right)^2 - 2\log\left(\frac{s}{\mu^2}\right)\log\left(1-\frac{t}{m^2}\right)$$
$$+\mathrm{Li}_2\left(1+\frac{m^2}{s}\right) + 4\mathrm{Li}_2\left(\frac{t}{t-m^2}\right) + \frac{\pi^2}{4}\Bigg] \tag{85}$$

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