Stat 30200: Mathematical Statistics 2

Recommended Reading

[1] J.O. Berger, *Statistical decision theory and Bayesian analysis*, Springer, 1985.

[2] J.O. Berger and T. Sellke, *Testing a point null hypothesis: the irreconcilability of p values and evidence*, Journal of the American Statistical Association (1987), 112–122.

[3] J.O. Berger and R.L. Wolpert, *The likelihood principle*, 1988.

[4] J M Bernardo and A F M Smith, *Bayesian theory*, Wiley, Chichester, 1994.

[5] W. Edwards, H. Lindman, and L.J. Savage, *Bayesian statistical inference for psychological research.*, Psychological Review **70** (1963), no. 3, 193.

[6] D.V. Lindley, *Understanding uncertainty*, John Wiley and Sons, 2006.

[7] L.J. Savage, *Reading suggestions for the foundations of statistics*, The American Statistician **24** (1970), no. 4, 23–27.

[8] ______, *The foundations of statistics*, Dover Pubns, 1972.

[9] T. Sellke, MJ Bayarri, and J.O. Berger, *Calibration of p values for testing precise null hypotheses*, The American Statistician **55** (2001), no. 1, 62–71.

1 Outline

This course continues the development of Mathematical Statistics, with an emphasis on Bayesian inference. Topics include Bayesian Inference and Computation, Frequentist Inference, Decision theory, admissibility and Stein's paradox, the Likelihood principle, Exchangeability and De Finetti's theorem, multiple comparisons and False Discovery Rates. The mathematical level will generally be at that of an easy advanced calculus course. We will assume familiarity with standard statistical distributions (e.g. Normal, Poisson, Binomial, Exponential), with the laws of probability, expectation, conditional expectation etc, as well as standard statistical concepts such as likelihood, p values, and confidence intervals.

Concepts will be illustrated by instructive "toy" examples, where calculations can be done by hand, as well as some more complex, practical applications of Bayesian statistics. Familiarity with the R statistical language will be assumed.

2 Grading

Final Grades will be based on average grade from weekly homeworks (60%), an in-class midterm (15%), a final exam (15%) and a take-home final/literature report (10%).

The take-home final will involve the preparation of a brief report on a paper on Bayesian statistics chosen from the literature. A good starting list of potential "classic" papers is given by Savage (1970) in "Reading suggestions for the Foundations of Statistics", American Statistician 24 (4). See also Chapter 2 of [4], which includes some more recent references. I will also give suggestions on more applied papers. If time permits the take-home final will also include a brief presentation to the class of the main points of the report.

3 Tentative Topic Schedule

The following is a tentative outline of the order I intend to cover topics (subject to change)

1. (Lec 1) Review of syllabus, and what type of course this is; what students find difficult; encourage questions.
2. Simple example of Bayesian inference (assignment problem; handout).
3. Problems with frequentist approaches to inference. Examples of crazy confidence procedures and hypothesis tests (Berger p 24-25); difficulty of calibrating p values (Sellke paper).
4. (Lec 2) Subjective probability: card tricks, measurement of probability, and axiomatic approach (Lindley chapter 1 examples; chapter 3 handout; notes handout).
5. (Lec 3) More simple examples of Bayesian inference (handout: beta prior for frequencies; normal prior for normals, continuation of classification example). Conjugacy. Summarizing posterior distributions: means, medians, modes, credible intervals (symmetric and HPD). Savage's potato.
6. (Lec 4) Prior Distributions (handout). Jeffrey's priors. Improper priors and "non-informative" priors. Empirical Bayes and Hierarchical models. Examples. Rules of thumb.
7. Exchangeability and de Finnetti's theorem (handout); examples, including Kreps's thumb tack.
8. Posterior computation (handout). Monte Carlo and Importance Sampling. Metropolis–Hastings algorithm, including proof of correct stationary distribution; coordinate-wise MH and Gibbs.
9. Computer Practical, Gibbs sampling examples (inbreeding coefficient; introducing latent variables).
10. Midterm
11. Decision theory: action space, and loss (0-1 loss, squared loss, absolute loss); Examples, including label-switching in mixtures.
12. Bayesian Conditional Decision Principle, of choosing action to minimize posterior expected loss. Examples Loss functions for prediction. Proper scoring rules. Examples. Utility and axiomatic approaches (Berger Ch 2)
13. Qualitative properties of utility; St Petersburg Paradox (Berger pp56-57). Frequentist decision principles: Risk Function, Integrated Risk, Minimax decision rules. Bayes Decision Rule. Admissibility. James-Stein Estimators.
14. Connection between Bayes Decision Rules and conditional Bayes decision principle. Admissibility of Bayes Rules. Complete Class Theorem. Neyman-Pearson Lemma. (handout photocopies of relevant pages from Berger).
15. Likelihood Principle (handouts).
16. Hypothesis testing, Bayesian and Frequentist. Bayes Factors. False Discovery Rates and multiple testing.
17. Bayesian model averaging.