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ISIS-1104-07 MAtemática Estructural y Lógica
Midterm Exam 1
Date: September 25, 2017

- Esta prueba es INDIVIDUAL.
- Está permintido el uso de las “cheat sheets” que se encuentran en sicua+.
- Está prohibido el uso de cualquier otro material como cuadernos, libros o fotocopias.
- Está prohibido el uso de cualquier dispositivo electrónico.
- El intercambio de información relevante a esta prueba con otro estudiante está terminantemente prohibido.
- Responda el examen en los espacios proporcionados. No se aceptarán hojas adicionales.
- No olvide marcar el examen antes de entregarlo.

Nombre	Carné

NO ESCRIBIR NADA BAJO ESTA LÍNEA

1	35 %	
2	35 %	
3	40 %	
Total	110 %	

1. Propositional Calculus

Suppose we add the following function to the propositional calculus:

$$says(K, P) \equiv (K \Rightarrow P) \wedge (\neg K \Rightarrow \neg P) \tag{1}$$

1.1. Find a simpler expression that is equivalent to $says(K, P)$. Prove that the simpler expression is equivalent to $says(K, P)$ using axioms and theorems or with a truth table:

$says(K, P) \equiv (K \equiv P)$

$$\begin{array}{ll}
 & says(K, P) \\
= & \qquad \langle \text{ (1) } \rangle \\
 & (K \Rightarrow P) \wedge (\neg K \Rightarrow \neg P) \\
= & \qquad \langle \text{ Contrapositive and double } \neg \rangle \\
 & (K \Rightarrow P) \wedge (P \Rightarrow K) \\
= & \qquad \langle \text{ Def. } \equiv \rangle \\
 & K \equiv P
\end{array}$$

1.2. Prove or refute:

$$says(K, K \equiv S) \equiv S$$

You may use the expression from problem 1.1.

$$
\begin{array}{ll}
 & says(K, K \equiv S) \\
= & \quad \langle \text{ Exercise 1.1 } \rangle \\
 & K \equiv (K \equiv S) \\
= & \quad \langle \text{ Assoc} \equiv \rangle \\
 & (K \equiv K) \equiv S \\
= & \quad \langle \text{ Identity } \rangle \\
 & true \equiv S \\
= & \quad \langle \text{ Identity } \rangle \\
 & S
\end{array}
$$

1.3. Prove or refute the validity of the following inference rule:

$$\frac{\begin{array}{c} says(P, \neg Q) \\ says(Q, \neg Q \wedge \neg P) \end{array}}{\neg Q}$$

You may use the expression from problem 1.1.

Hypothesis 1: $says(P, \neg Q)$. By problem 1.1 this is $P \equiv \neg Q$

Hypothesis 2: $says(Q, \neg Q \wedge \neg P)$

$$\begin{array}{ll}
 & \quad \langle \text{ Hypotesis 2 } \rangle \\
 & says(Q, \neg Q \wedge \neg P) \\
\Rightarrow & \quad \langle \text{ Hypothesis 1 } \rangle \\
 & says(Q, P \wedge \neg P) \\
= & \quad \langle \text{ Contradiction } \rangle \\
 & says(Q, false) \\
= & \quad \langle \text{ Problem 1.1 } \rangle \\
 & Q \equiv false \\
= & \quad \langle\ Def_2\ false\ \rangle \\
 & \neg Q
\end{array}$$

2. Deduction in the propositional calculus

The bank was robbed during the night and we know the following facts:

1. The suspects are Adam, Bret, Charlie, and David. We know that at least one is guilty and no one else could have been involved.
2. Either Bret or Adam are guilty, but not both
3. If Charlie is guilty then Adam is innocent.
4. If David is innocent then Charlie is guilty.
5. Bret always works alone.

2.1. Modeling

Given the variables listed below to represent the facts of the problem, model the hypotheses and your conclusion regarding the guilt or innocence of the four suspects.

A: Adam is guilty

B: Bret is guilty

C: Charlie is guilty

D: David is guilty

If Bret is guilty then, by (5), Charlie, Adam and David would be innocent. However, if Charlie is innocent then by (4), David would be guilty, generating a contradiction. Therefore, Bret must be innocent.

If Bret is innocent, then by (2) Adam is guilty. Then by (3) Charlie is innocent. Finally, since charlie is innocent, by (4), David is guilty.

Hypotesis :

1. $A \lor B \lor C \lor D$
2. $A \not\equiv B$
3. $C \Rightarrow \neg A$
4. $\neg D \Rightarrow C$
5. $B \Rightarrow \neg A \land \neg C \land \neg D$

Conclusion: $A \land \neg B \land \neg C \land D$

2.2. Proofs in Propositional Calculus

First we prove $\neg B$ by contradiction:

$$
\begin{array}{ll}
 & \quad \langle \text{ Assumption } \rangle \\
 & B \\
\Rightarrow & \quad \langle \text{ Modus Ponens with Hypothesis 5 } \rangle \\
 & \neg A \ \wedge \ \neg C \ \wedge \ \neg D \\
\Rightarrow & \quad \langle \text{ Modus Ponens with Hypothesis 4 } \rangle \\
 & \neg A \ \wedge \ \neg C \ \wedge \ C \\
= & \quad \langle \text{ Contradiction } \rangle \\
 & \neg A \ \wedge \ false \\
= & \quad \langle \text{ Domination } \rangle \\
 & false
\end{array}
$$

We proved $\neg B \Rightarrow false$, which means B is false.

	Expresión	Justificación
1	$A \not\equiv B$	Hypothesis
2	$\neg B$	Lemma
3	A	Deduction (1,2)
4	$C \Rightarrow \neg A$	Hypothesis
5	$\neg C$	Modus Tollens (3,4): ($A \equiv \neg\neg A$)
6	$\neg D \Rightarrow C$	Hypothesis
7	$\neg\neg D$	Modus Tollens (5,6):
8	D	Double $\neg$ (7)
9	$A \wedge \neg B \wedge \neg C \wedge D$	Composition (3,2,5,8)

3. Predicate calculus

3.1. Prove or refute the validity of the following inference rule:

$$\frac{\begin{array}{c}(\forall\, x \mid P \wedge Q \;:\; R\,) \\ P(X) \\ \neg R(X)\end{array}}{\neg Q(X)}$$

Hypothesis:

1. $(\forall\, x \mid P \wedge Q \;:\; R\,)$
2. $P(X)$
3. $\neg R(X)$

$$\begin{array}{ll}
 & \quad\langle \text{ Hypothesis 1 } \rangle \\
 & (\forall\, x \mid P \wedge Q \;:\; R\,) \\
= & \quad\langle \text{ Trading-}\forall_1 \rangle \\
 & (\forall\, x \mid P \;:\; Q \Rightarrow R\,) \\
\Rightarrow & \quad\langle \text{ Universal Modus Ponens with Hypothesis (2) } \rangle \\
 & Q(X) \Rightarrow R(X) \\
\Rightarrow & \quad\langle \text{ Modus Tollens with Hypothesis (3) } \rangle \\
 & \neg Q(X)
\end{array}$$

3.2. Modeling

You land on the island of alphas, betas and deltas. You know that:

1. There are three kinds of inhabitants: alphas, betas, and deltas. However, inhabitants are not necessarily of a single kind.

2. They can be red or blue, but not red and blue. $(\forall\, n \mid \; : \; red(n) \not\equiv blue(n)\,)$ This is the same as saying (using definition of $\not\equiv$ and negation of $\equiv$): $(\forall\, n \mid \; : \; \neg red(n) \equiv blue(n)\,)$

3. Blue inhabitants are Alphas$(\forall\, n \mid blue(n) \; : \; alpha(n)\,)$

4. Deltas are red$(\forall\, n \mid delta(n) \; : \; red(n)\,)$

5. Alpha's parents are always Deltas $(\forall\, n, m \mid alpha(n) \wedge parent(m, n) \; : \; delta(m)\,)$

You meet two blue natives: A and B, and you conclude that B cannot be A's parent.

Premise 1 $blue(A)$

Premise 2 $blue(B)$

Conclusion $\neg parent(B, A)$

Model the hypotheses, the premises and the conclusion of the problem using the following predicates.

- alpha(d) : d is alpha
- beta(d) : d is beta
- delta(d) : d is delta
- blue(d) : d is blue
- red(d): d is red
- parent(p,c): p is c's parent

This is the translation of the first item: $(\forall\, h \mid \; : \; alpha(h) \vee beta(h) \vee delta(h)\,)$

3.3. Deduction in the predicate calculus

Formally prove that you can reach the conclusion from the hypotheses and the premises.

	Expresión	Justificación
1	$blue(A)$	Premise
2	$(\forall\ n \mid blue(n)\ :\ alpha(n)\ )$	Hypothesis
3	$alpha(A)$	Generalized Modus Ponens (1,2)
4	$blue(B)$	Premise
5	$(\forall\ n \mid\ \ :\ red(n) \not\equiv blue(n)\ )$	Hypothesis
6	$\neg red(B)$	Leibniz (4,5)
7	$(\forall\ n \mid delta(n)\ :\ red(n)\ )$	Hypothesis
8	$\neg delta(B)$	Generalized Modus Tollens (6,7)
9	$(\forall\ n, m \mid alpha(n) \wedge parent(m,n)\ :\ delta(m)\ )$	Hypothesis
10	$\neg parent(B, A)$	Problem (3.1) - (9,3,8)

Another approach:

$$
\begin{array}{ll}
 & \qquad \langle \text{ Premises } \rangle \\
 & blue(A)\ \wedge\ blue(B) \\
\Rightarrow & \qquad \langle \text{ Generalized Modus Ponens with hypothesis 3 } \rangle \\
 & alpha(A)\ \wedge\ blue(B) \\
= & \qquad \langle \text{ Leibniz with hypothesis 2 } \rangle \\
 & alpha(A)\ \wedge\ \neg red(B) \\
\Rightarrow & \qquad \langle \text{ Generalized Modus Tollens with Hypothesis 4 } \rangle \\
 & alpha(A) \wedge \neg delta(B) \\
\Rightarrow & \qquad \langle \text{ Problem 3.1, with hypothesis 5 } \rangle \\
 & \neg parent(B, A)
\end{array}
$$