ISIS-1104-01 Matemática Estructural y Lógica
Parcial 1
Fecha: Marzo 3, 2016

- Esta prueba es INDIVIDUAL.
- Sólo está permitido el uso de las hojas de fórmulas publicadas en Sicua+.
- Está prohibido el uso de cualquier dispositivo electrónico.
- El intercambio de información con otro estudiante está terminantemente prohibido.
- Cualquier irregularidad con respecto a estas reglas podría ser considerada fraude.
- Responda el examen en los espacios proporcionados. No se aceptarán hojas adicionales.
- No olvide marcar el examen antes de entregarlo.
- Las preguntas son en inglés, pero si lo desea, puede responder en español.

IMPORTANTE: Soy consciente de que cualquier tipo de fraude en los exámenes es considerado como una falta grave en la Universidad. Al firmar y entregar este examen doy expreso testimonio de que este trabajo fue desarrollado de acuerdo con las normas establecidas. Del mismo modo, aseguro que no participé en ningún tipo de fraude.

Nombre	Carné
Firma	Fecha

NO ESCRIBIR NADA BAJO ESTA LÍNEA

1.1	15 %	
1.2	15 %	
2.1	15 %	
2.2	20 %	
3.1	15 %	
3.2	20 %	
Total	100 %	

1. [30 %] Propositional Calculus

Suppose we add the following functions to the propositional calculus:

$$nn(P, Q) \equiv \neg P \wedge \neg Q \tag{1}$$

$$says(K, P) \equiv (nn(K, P) \vee nn(\neg K, \neg P)) \tag{2}$$

1.1. [15 %] Prove or refute: $p \wedge q \equiv nn(nn(p, p), nn(q, q))$

1.2. [15 %] Prove or refute:

$$says(K, K \equiv S) \equiv S$$

2. Deduction in the propositional calculus [35 %]

There was a robbery in which a lot of goods were stolen. The robber(s) left in a truck. It is known that:

1. The suspects are Adam, Bret and Charlie. We know that at least one is guilty and no one else could have been involved.
2. Charlie never commits a crime without Adam's participation.
3. Bret does not know how to drive. (This means he could not have acted alone).

We are only interested in deducing whether or not Adam is guilty.

2.1. Modeling [15 %]

Given the variables listed below to represent the facts of the problem, model the hypotheses and your conclusion regarding Adam's guilt or innocence. Remember: we don't care about the other two.

A: Adam is guilty

B: Bret is guilty

C: Charlie is guilty

This is the translation of the first item: $A \vee B \vee C$

2.2. Proofs in Propositional Calculus [20 %]

Formally prove your conclusion.

Poof by contradiction:

	Expresión	Justificación
1	$\neg A$	Supposition
2	$C \Rightarrow A$	Hypothesis
3	$\neg C$	Modus Tollens (1,2)
4	$A \vee B \vee C$	Hypothesis
5	$A \vee B$	Disjunctive Syllogism (3,4)
6	B	Disjunctive Syllogism (1,5)
7	$B \Rightarrow A \vee C$	Hypothesis
8	$A \vee C$	Modus Ponens (6,7)
9	A	Disjunctive Syllogism (3,8)

So we proved:

$$\neg A \Rightarrow A$$

which, by the definition of $\Rightarrow$, is:

$$\neg\neg A \vee A$$

This last expression is, by double negation and idempotency of $\vee$, equivalent to A.

This proof technique is also called "consequentia mirabillis".

3. Predicate calculus [35 %]

You land on the island of alphas, betas and deltas. You know that:

1. There are three kinds of inhabitants: alpha, beta or delta. However, inhabitants are not necessarily of a single kind.

2. Three eyed inhabitants are not alphas.

3. Betas are not purple.

4. Deltas' children are not deltas.

You meet two inhabitants of the island: A and B.

1. Both A and B have three eyes

2. A es purple

3. B is A's child.

We can conclude that B is a Beta.

3.1. Modeling [15 %]

Model the problem, using the following predicates:

- alpha(d) : d is alpha
- beta(d) : d is beta
- delta(d) : d is delta
- purple(d) : d is purple
- parent(p,c): c is p's child
- three(d): d has three eyes

This is the translation of the first item: $(\forall\ h \mid \ :\ alpha(h) \vee beta(h) \vee delta(h)\ )$

3.2. Deduction in the predicate calculus [20 %]

Formally prove that you can reach the conclusion from the hypotheses and the premises.