

- Esta prueba es INDIVIDUAL.
- Está permitido el uso de las hojas de teoremas publicadas en sicua+.
- Está prohibido el uso de cualquier dispositivo electrónico.
- El intercambio de información con otro estudiante está terminantemente prohibido.
- Cualquier irregularidad con respecto a estas reglas podría ser considerada fraude.
- Responda el examen en los espacios proporcionados. No se aceptarán hojas adicionales.
- No olvide marcar el examen antes de entregarlo.

IMPORTANTE: Soy consciente de que cualquier tipo de fraude en los exámenes es considerado como una falta grave en la Universidad. Al firmar y entregar este examen doy expreso testimonio de que este trabajo fue desarrollado de acuerdo con las normas establecidas. Del mismo modo, aseguro que no participé en ningún tipo de fraude.

Nombre	Carné
Firma	Fecha

NO ESCRIBIR NADA BAJO ESTA LÍNEA

1.1	10%	
1.2	10%	
1.3	20%	
2.1.1	10%	
2.1.2	10%	
2.2	20%	
3	20%	
Total	100%	

1 [40%] Sets

We define this new function to combine sets:

$$mystery(A, B, C) = (A \setminus B) \cap (C \setminus B)$$

Prove or refute the following statements:

1.1 [10%] $Mystery(A, B, C) = (A \cap C) \setminus B$

1.2 [10%] $Mystery(A, B, C) \subseteq (A \cap C)$

1.3 [20%] *mystery*($A \cup B, A \cap B, A \setminus B$) = $A \setminus B$

2 Relations[40%]

2.1 [20%] Modeling with n-ary relations

Given the following sets and relations:

FightID: Set of all flight IDs.

Dates: Set of Dates (suppose it is in minutes: from 0 (0:00) to 1439 (23:59))

Times: Set of Times

PAssengers: Set of passenger ids

Airlines: Set of airline IDs

Airport: Set of airport codes

PFlights: A relation among *FLightIDs*, *Airlines*, *Times*, *Airports* and *Airports*, where $(f, a, t, from, to) \in PFlight$ if airline a has a daily flight with id f scheduled to leave at t from airport $from$ to airport to .

CFlight: A relation among *FlightIds*, *Dates*, and *Times*, where $(f, d, t) \in CFlight$ if flight f left on date d at time t .

Flown: A relation among passengers, flights and dates, where $(p, f, d) \in Flown$ if passenger p took flight f on date d .

Define the following relations:

2.1.1 [10%] A relation R1 between flights and dates where where $(f, d) \in R1$ if flight f was more that 6 hours late on date d .

2.1.2 [10%] A relation R2 between passengers and airlines where $(p, a) \in R2$ if passenger a was delayed for more than 6 hours on a flight operated by airline a leaving from airport *BOG*

2.2 [25%] Binary relations: properties

Given the following relation between natural numbers:

$$R : \mathbb{N} \times \mathbb{N}$$
$$x R y \equiv (0 \leq (y - x) \leq x)$$

Is this relation:

1. reflexive?
2. symmetric?
3. antisymmetric?
4. transitive?

Prove that your answers are correct.

3 Functions [20%]

Given the following function from integers to natural numbers (integers greater than or equal to zero):

$$z2n : \mathbb{Z} \rightarrow \mathbb{N}$$

$$z2n(x) = \begin{cases} -2x & \text{if } x \leq 0 \\ 2x - 1 & \text{if } x > 0 \end{cases} \quad (1)$$

Prove that this function is both one-to-one and onto.

Hints:

To prove it is onto: Prove by cases: even ($2n$) and odd ($2n + 1$).

To prove it is one-to-one: Prove by cases: positive (> 0), and non positive (≤ 0).