ISIS-1104-01 Matemática Estructural y Lógica
Mid Term 3
Fecha: December 5, 2016

- Esta prueba es INDIVIDUAL.
- Está permintido el uso de las "cheat sheets" que se encuentran en sicua+.
- Está prohibido el uso de cualquier otro material como cuadernos, libros o fotocopias.
- Está prohibido el uso de cualquier dispositivo electrónico.
- El intercambio de información relevante a esta prueba con otro estudiante está terminantemente prohibido.
- Cualquier irregularidad con respecto a estas reglas podría ser considerada fraude.
- Responda el examen en los espacios proporcionados. No se aceptarán hojas adicionales.
- No olvide marcar el examen antes de entregarlo.

IMPORTANTE: Soy consciente de que cualquier tipo de fraude en los exámenes es considerado como una falta grave en la Universidad. Al firmar y entregar este examen doy expreso testimonio de que este trabajo fue desarrollado de acuerdo con las normas establecidas. Del mismo modo, aseguro que no participé en ningún tipo de fraude.

Nombre	Carné
Firma	Fecha

NO ESCRIBIR NADA BAJO ESTA LÍNEA

1.1	10 %	
1.2	20 %	
2.1	15 %	
2.2	20 %	
3.1	15 %	
3.2	20 %	
4.	10 %	***

1. Integers

Do not use induction for any of the problems in this section.

1.1. Prove the following statement:

$$((m \mid a) \wedge (m \nmid (a+b))) \Rightarrow (m \nmid b)$$

1.2. Prove the following statement:

For any integers: a, b, c, d, n, with $n > 0$, if $(a \equiv_n b)$ and $((a+c) \equiv_n d)$ then $((b+c) \equiv_n d)$.

2. Induction on Natural Numbers

2.1. F_N: We define: Fibonacci: F_n

Basis case 0: $F_0 = 0$

Basis case 1: $F_1 = 1$

Inductive case: $F_{n+1} = F_n + F_{n-1}$ for $n > 0$

Prove that for all $n \geq 1$: $(F_{n+1} \cdot F_{n-1}) - F_n^2 = (-1)^n$

2.2. H_N

Given the following recursive definition:

Basis case 1: $H_1 = 1$

Inductive case: $H_{n+1} = H_n + \frac{1}{n+1}$, for $n \ > \ 1$

Use induction to prove that for $n \geq 1$: $(+i|1 \leq i \leq n : H_i) = (n+1) \cdot H_n - n$
Hint: Note that $H_n = H_{n+1} - \frac{1}{n+1}$ for $n \geq 1$

3. Structural Induction

We can define a well formed formula of sums of x's (wff_{sx}) inductively as follows.

- x is a wff_{sx}
- if α and β are wff_{sx}, $(\alpha + \beta)$ is also a wff_{sx}.

These are examples of wff_{sx}:

- `x`
- `(x + x)`
- `(x + (x + x))`

To prove that a property , P is true for all wff_{sx}, you have to use structural induction, and you must use the following pattern.

Basis Case: The property is *true* for `x`

Inductive Case: If the property is *true* for α and β then it is true for $(\alpha + \beta)$

> **I.H. 1:** $P(\alpha)$
> **I.H 2:** $P(\beta)$
> **Prove:** $P((\alpha + \beta))$

We define the following functions and predicates recursively for f, a wff_{sx}.

$Xs(f)$**:** Number of x's in f.

$$Xs(x) \ = 1$$
$$Xs(\alpha + \beta) \ = Xs(\alpha) + Xs(\beta)$$

$sums(f)$**:** Number of +'s in a f.

$$Sums(x) \ = 0$$
$$Sums(\alpha + \beta) \ = Sums(\alpha) + Sums(\beta) + 1$$

$D(x)$**:** Depth of a f.

$$D(x) \ = 0$$
$$D(\alpha + \beta) \ = max(D(\alpha), D(\beta)) + 1$$

$Balanced(f)$**:** Is f balanced?.

$$Balanced(x) \ = \text{true}$$
$$Balanced(\alpha + \beta) \ = Balanced(\alpha) \wedge Balanced(\beta) \wedge (D(\alpha) = D(\beta))$$

3.1. Using structural induction prove: $Xs(f) = Sums(f) + 1$

3.2. Using structural induction prove: $Blanced(f) \Rightarrow Xs(f) = 2^{D(f)}$

4. Counting

Suppose you have an organization with 6 Americans, 6 French and 6 Colombians.

1. How many different committees of 5 people can you form.
2. How many different committees of 5 people can you form in which there is at least one person from each nationality.
3. How many different ways can you seat 4 people in a row so that no two people of the same nationality are seated side by side.
4. How many committees of 5 people can you form in which they are all of different nationalities.