ISIS-1104-01 Matemática Estructural y Lógica
Parcial 3
Fecha: May 5, 2016

- Esta prueba es INDIVIDUAL.
- Está permitido el uso de las hojas de fórmulas.
- Está prohibido el uso de cualquier otro material como cuadernos, libros o fotocopias.
- Está prohibido el uso de cualquier dispositivo electrónico.
- El intercambio de información con otro estudiante está terminantemente prohibido.
- Cualquier irregularidad con respecto a estas reglas podría ser considerada fraude.
- Responda el examen en los espacios proporcionados. No se aceptarán hojas adicionales.
- No olvide marcar el examen antes de entregarlo.

IMPORTANTE: Soy consciente de que cualquier tipo de fraude en los exámenes es considerado como una falta grave en la Universidad. Al firmar y entregar este examen doy expreso testimonio de que este trabajo fue desarrollado de acuerdo con las normas establecidas. Del mismo modo, aseguro que no participé en ningún tipo de fraude.

Nombre	Carné
Firma	Fecha

NO ESCRIBIR NADA BAJO ESTA LÍNEA

1	20%	
2	25%	
3	25%	
4	20%	
5	10%	
Total	100%	

Exercise 1. Using induction on n prove the following statement: $(\Sigma\ i \mid 1 \leq i \leq n\ :\ \frac{1}{2^i}\ ) = \frac{2^n - 1}{2^n}$

Exercise 2. Using the recursive definition of the Fibonacci sequence:

Base Case$_0$: $F_0 = 0$

Base Case$_1$: $F_1 = 1$

Inductive Case: $F_{k+1} = F_k + F_{k-1}$

Using induction on n prove that following statement is true for $n \geq 1$:

$$gcd(F_n, F_{n+1}) = 1$$

Exercise 3. We modify our definition of sequences of elements of type T so that the basic constructor operation is append ($\triangleright$).

Basis Clause: The empty sequence represented by the symbol ϵ is a sequence.

Inductive Clause: If $n \in T$, and S is a sequence of T, $S \triangleright n$ (pronounced S append n) is a sequence and represents the sequence obtained by adding n at the end of sequence S.

In this case, this would be the proof pattern:

Basis Case: $P(\epsilon)$

Inductive Case: $P(S) \Rightarrow P(S \triangleright y)$

I.H. $P(S)$

Prove: $P(S \triangleright y)$

Given the following recursively defined functions:

$val : \ Seq_{\{0,\ldots,9\}} \rightarrow \mathbb{Z}$

Base Case: $val(\epsilon) = 0$

Inductive Case: $val(S \triangleright d) = 10 \cdot val(S) + d$

$foo : \ Seq_{\{0,\ldots,9\}} \rightarrow Seq_{\{0,\ldots,9\}}$

Base Case: $foo(\epsilon) = \epsilon \triangleright 1$

Inductive Case:

$$foo(S \triangleright d) = \begin{cases} S \triangleright (d+1) & \text{if } \ d < 9 \\ foo(S) \triangleright 0 & \text{otherwise} \end{cases}$$

Prove the following theorem, using structural induction. Remember you must use the new proof pattern.

$$val(foo(S)) = val(S) + 1$$

Hint: For the inductive case, use proof by cases (last element is 9 or last element is less than 9)

Exercise 4. For this exercise, we also use the modified definition of sequences.

Given the following recursively defined functions:

$sum : Seq_{\mathbb{Z}} \to \mathbb{Z}$

Base Case: $sum(\epsilon) = 0$

Inductive Case: $sum(S \triangleright d) = sum(S) + d$

$odds : Seq_{\mathbb{Z}} \to \mathbb{N}$

Base Case: $odds(\epsilon) = 0$

Inductive Case:

$$odds(S \triangleright d) = \begin{cases} odds(S) & \text{if } \ 2 \mid d \\ odds(S) + 1 & \text{otherwise} \end{cases}$$

Prove the following theorem, using structural induction. Remember: you must use the new proof pattern.

$$odds(S) \ \equiv_2 \ sum(S)$$

Hint: For the inductive case, use proof by cases (last element is odd or last element is even)

Exercise 5. Counting Problems:

1. Suppose you have an organization with 5 women and 4 men.
   (a) How many committees of size 4 that have at least one woman can you form?
   (b) How many ways can you seat 5 people in a row so that no two women nor two men are seated side by side? Notice that in a row, the order does matter. The same people can form more than one row.
2. Give argument to prove that if you have a sequence of integers S such that

$$long(S) > max_s(S) - min_s(S) + 1$$

then there must be an element x of S such that $count(x, S) > 1$