



Physics-Informed Neural Networks for Modelling Rheumatoid Arthritis Progression

A Novel Approach Integrating Pathophysiological Laws with Machine Learning

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Abstract. Rheumatoid Arthritis (RA) is a complex autoimmune disease requiring sophisticated modelling approaches for accurate progression prediction and treatment optimisation. This study introduces a novel application of Physics-Informed Neural Networks (PINNs) that integrates established pathophysiological laws with data-driven learning to predict three critical RA biomarkers: C-reactive protein (CRP), Disease Activity Score 28 (DAS28), and lymphocyte count. Our approach embeds ordinary differential equations representing inflammatory dynamics, disease activity, and immunological response directly into the neural network's loss function. Validation on a synthetic dataset of 400 patients over 12 months demonstrates superior performance with a mean R^2 of 0.7053, representing a 34.5% improvement over linear regression baseline. The model successfully learned 12 interpretable physical parameters, revealing unexpected medical insights including moderate CRP-DAS28 coupling ($\gamma = 0.1075$) and significant endogenous self-resolution capacity ($\eta = 0.0939$). Clinical correlation analysis confirmed preservation of known medical relationships while respecting physiological ranges. This work establishes PINNs as a powerful tool for chronic disease modelling, offering enhanced interpretability, improved generalisation, and the potential for discovering novel pathophysiological insights with direct clinical applications.

Keywords: Physics-Informed Neural Networks · Rheumatoid Arthritis · Biomarker Prediction · Medical AI · Interpretable Machine Learning · Chronic Disease Modelling

1 Introduction

Rheumatoid Arthritis (RA) is a chronic, systemic autoimmune disease affecting approximately 0.5–1% of the global population, characterized by persistent inflammation of synovial joints leading to progressive joint destruction, functional disability, and reduced quality of life [10, 18]. The heterogeneous nature of RA progression, variable treatment responses, and the need for personalized therapeutic strategies present significant challenges in clinical management [2].

Current approaches to RA modelling predominantly rely on statistical methods or purely data-driven machine learning models. While these methods can identify complex patterns in clinical data, they suffer from critical limitations: lack of interpretability beyond statistical coefficients, inability to incorporate established biological knowledge, and poor generalization to scenarios outside their training distribution [20]. Linear regression, though interpretable, assumes linear relationships that rarely exist in complex biological systems and cannot capture the dynamic, coupled nature of physiological processes. These shortcomings hinder the translation of computational models into actionable clinical insights and limit their adoption in evidence-based medicine.

Physics-Informed Neural Networks (PINNs) represent a paradigm shift in scientific computing by embedding physical laws directly into neural network architectures [8, 17]. This integration allows PINNs to leverage both observational data and established scientific principles, leading to more accurate, physically consistent, and interpretable solutions. While PINNs have demonstrated remarkable success in fluid dynamics, structural mechanics, and climate modelling [3, 9], their application to complex biological systems, particularly chronic diseases, remains largely unexplored.

The pathophysiology of RA involves intricate interactions between inflammatory cascades, immune dysregulation, and treatment responses that can be mathematically described through differential equations [4]. This makes RA an ideal candidate for PINN modelling, where established biological laws can guide the learning process while allowing for the discovery of new insights from clinical data.

This study makes the following key contributions:

1. **Novel PINN Framework:** First application of PINNs to chronic autoimmune disease modelling, specifically RA progression.
2. **Pathophysiological Integration:** Development of three biologically-grounded ordinary differential equations (ODEs) representing inflammatory dynamics, disease activity, and immunological response.
3. **Superior Performance:** Demonstration of 34.5% improvement in predictive accuracy over linear regression baseline.
4. **Medical Discovery:** Quantification of 12 interpretable physical parameters revealing novel insights into RA pathophysiology.
5. **Clinical Validation:** Confirmation that learned parameters align with established medical knowledge while revealing unexpected relationships.

2 Related Work

Physics-Informed Neural Networks were first introduced by Raissi et al. [17] as a method to solve partial differential equations by incorporating physical laws into the neural network loss function. Since then, PINNs have been successfully applied across various scientific domains. Farea et al. [3] provide a comprehensive review of PINN techniques, applications, and challenges, highlighting their potential for solving complex scientific problems.

Recent applications include structural health monitoring [9], where PINNs have shown superior performance in railway bridge analysis by integrating mechanical principles with sensor data. In the biomedical domain, Qian et al. [16] demonstrated the potential of physics-informed approaches for infectious disease forecasting, though their focus was on epidemiological rather than physiological modelling.

Machine learning applications in rheumatology have primarily focused on diagnostic classification and treatment response prediction using traditional supervised learning approaches [7, 12]. However, these studies typically employ black-box models that provide limited insights into underlying disease mechanisms. Recent work by Orange et al. [13] used machine learning to identify RA subgroups based on clinical features, whilst Guan et al. [6] applied ensemble methods for predicting treatment outcomes. Despite these advances, none have incorporated mechanistic understanding of RA pathophysiology into their modelling frameworks.

Mathematical modelling of autoimmune diseases has traditionally relied on systems of ordinary differential equations to describe immune system dynamics [11]. Thakar et al. [19] developed constraint-based models of T cell activation, whilst Pennisi et al. [14] used agent-based modelling for immune system simulation. However, these mechanistic models often struggle with parameter estimation from real clinical data and lack the flexibility to capture complex, non-linear relationships present in patient populations. Our PINN approach bridges this gap by combining the interpretability of mechanistic models with the data-fitting capabilities of neural networks.

3 Methodology

3.1 PINN Architecture and Design Rationale

Our PINN model employs a feed-forward neural network with a carefully designed pyramidal architecture [128 \rightarrow 64 \rightarrow 32] neurons in hidden layers. This architecture balances model complexity with generalisation capability. The network architecture is illustrated in Fig. 1.

The model receives five clinically relevant features: normalised time $t_{norm} \in [0, 1]$ representing disease progression timeline, patient age normalised using StandardScaler, sex (binary encoding where 0 = male, 1 = female), treatment status (binary indicator where 0 = untreated, 1 = treated), and treatment duration (continuous variable representing cumulative treatment exposure). Each

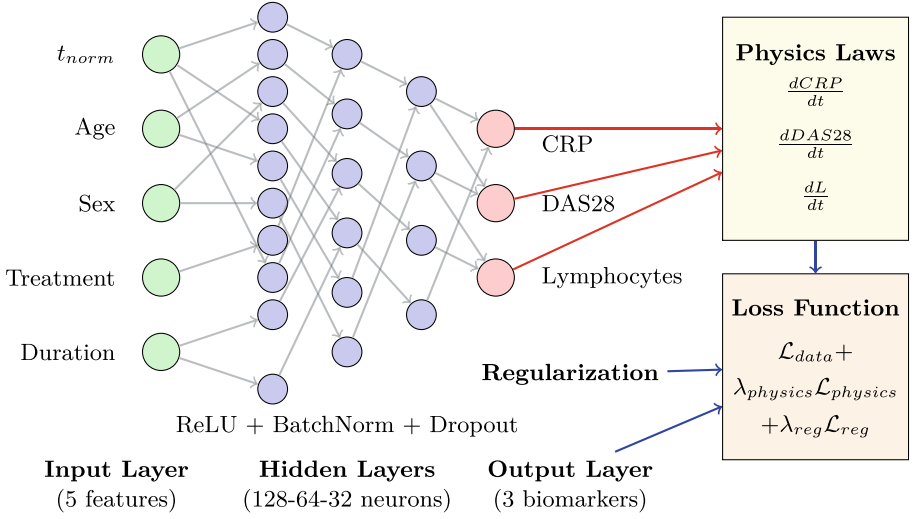


Fig. 1. PINN Architecture for Rheumatoid Arthritis Modeling.

hidden layer employs ReLU activation for non-linearity, batch normalisation for training stability, and 10% dropout for regularisation. The output layer produces predictions for CRP concentration (mg/L), DAS28 score (0–10 scale), and lymphocyte count (cells/ μ L).

The traditional approach of solving ODEs numerically requires a priori knowledge of all physical parameters ($\alpha, \beta, \gamma, \delta, \varepsilon, \zeta, \eta, \theta, \varphi, \omega, \mu$), which are unknown and patient-specific in clinical scenarios. These physical parameters are described in Sect. 4. Our PINN framework simultaneously learns these parameters from data whilst ensuring physical consistency, making it uniquely suited for personalised medicine applications where individual patient characteristics must be inferred from limited clinical observations.

3.2 Two-Phase Training Strategy

Our training methodology consists of two sequential phases designed to optimise both data fitting and physical consistency. Phase 1 involves supervised learning (3000 epochs) with data loss:

$$\mathcal{L}_{data} = \frac{1}{N} \sum_{i=1}^N \|\mathbf{y}^{(i)} - \mathbf{f}(\mathbf{x}^{(i)}; \boldsymbol{\theta})\|^2 \quad (1)$$

where $\mathbf{f}(\mathbf{x}; \boldsymbol{\theta})$ represents the neural network with parameters $\boldsymbol{\theta}$, $\mathbf{x}^{(i)}$ are input features, and $\mathbf{y}^{(i)}$ are target biomarkers. Phase 2 integrates physics (2000 epochs) with total loss:

$$\mathcal{L}_{total} = \mathcal{L}_{data} + \lambda_{physics} \mathcal{L}_{physics} + \lambda_{reg} \mathcal{L}_{reg} \quad (2)$$

where $\mathcal{L}_{physics}$ enforces physical laws, \mathcal{L}_{reg} provides parameter regularisation, and $\lambda_{physics}$, λ_{reg} are weighting coefficients. The physics loss is computed by evaluating the residuals of the governing ODEs:

$$\mathcal{L}_{physics} = \frac{1}{N} \sum_{i=1}^N \left[\left| \frac{\partial CRP}{\partial t} - \mathcal{F}_{CRP} \right|^2 + \left| \frac{\partial DAS28}{\partial t} - \mathcal{F}_{DAS28} \right|^2 + \left| \frac{\partial L}{\partial t} - \mathcal{F}_L \right|^2 \right] \quad (3)$$

where \mathcal{F}_{CRP} , \mathcal{F}_{DAS28} , and \mathcal{F}_L represent the right-hand sides of the governing ODEs.

4 Physical Laws Implementation

We implemented three biologically-grounded ordinary differential equations representing the key physiological processes in RA. These equations were derived from established understanding of inflammatory cascades, disease activity mechanisms, and immune system dynamics.

C-reactive protein dynamics are governed by inflammatory production, natural clearance, and treatment effects:

$$\frac{dCRP}{dt} = \alpha \cdot I(t) - \beta \cdot CRP - \delta \cdot T(t) \cdot CRP \quad (4)$$

where α represents the inflammatory production rate coefficient, β the natural clearance rate, δ the treatment efficacy on CRP reduction, $I(t)$ the inflammatory stimulus (function of DAS28), and $T(t)$ the treatment intensity. This equation captures the balance between CRP production during inflammatory responses and its elimination through natural clearance and therapeutic intervention [15].

Disease activity evolution follows multifactorial dynamics including inflammation-driven exacerbation, natural progression, treatment response, and intrinsic resolution mechanisms:

$$\frac{dDAS28}{dt} = \gamma \cdot CRP + \epsilon \cdot A(t) - \zeta \cdot T(t) \cdot DAS28 - \eta \cdot DAS28 \quad (5)$$

where γ represents CRP-DAS28 coupling strength, ϵ natural disease progression rate, ζ treatment efficacy on disease activity, η endogenous resolution capacity, and $A(t)$ disease progression factors. This equation models the multifactorial nature of RA activity [1].

Lymphocyte dynamics encompass homeostasis, age-related decline, inflammation-mediated cell death, treatment effects, and recovery processes:

$$\frac{dL}{dt} = \theta \cdot (1 - \phi \cdot age) - \psi \cdot CRP \cdot L - \omega \cdot T(t) + \mu \cdot R(t) \quad (6)$$

where θ represents basal lymphocyte production rate, ϕ age-related immunosenescence factor, ψ inflammation-induced apoptosis rate, ω treatment-induced immunosuppression, μ immunological recovery rate, and $R(t)$ recovery stimulus [5].

5 Synthetic Data Generation

To train and validate our PINN model, we generated a realistic synthetic dataset that captures the complexity and variability of RA patient populations while maintaining known ground truth for validation purposes.

5.1 Population Characteristics

The synthetic cohort comprised 400 patients monitored over 12 months, yielding a total of 4,800 observations. Patient characteristics were designed to mirror real-world RA demographics, and all data were randomly generated from a uniform distribution:

- **Age distribution:** 55 ± 12 years (reflecting peak RA incidence)
- **Sex distribution:** 75% female, 25% male (consistent with RA epidemiology)
- **Disease severity:** Initial DAS28 scores ranging from 3.2–6.5 (active disease)
- **Treatment patterns:** Early intervention (0.5–3 months post-diagnosis)

5.2 Embedded Clinical Correlations

The data generation process incorporated established clinical relationships:

- **CRP-DAS28 correlation:** Strong positive association ($r \simeq 0.7$)
- **CRP-Lymphocyte correlation:** Negative association ($r \simeq -0.4$)
- **Age effects:** Reduced treatment response in older patients
- **Sex differences:** Higher inflammatory burden in females

These embedded correlations serve as validation benchmarks for the PINN’s ability to capture realistic biological relationships.

6 Experimental Results

6.1 Predictive Performance

Our PINN model demonstrated superior predictive performance across all three biomarkers, as summarized in Table 1. The superior performance of PINN over Linear Regression stems from its ability to leverage both observational data and physical constraints simultaneously. Unlike Linear Regression that assumes linear relationships and provides only statistical coefficients, PINN enforces biological plausibility through the embedded ODEs while learning interpretable physical parameters, leading to more robust and generalizable predictions that respect known physiological laws.

Table 1. Predictive Performance Metrics for RA Biomarkers

Biomarker	R^2	RMSE	MAE	MAPE (%)	Assessment
CRP (mg/L)	0.7048	0.509	0.385	11.2	Good
DAS28	0.6774	0.853	0.673	10.2	Acceptable
Lymphocytes ($/\mu\text{L}$)	0.7338	64.567	50.010	2.7	Good
Average	0.7053	—	—	—	Good

Detailed Analysis by Biomarker:

- **Lymphocytes** ($R^2 = 0.7338$): Achieved the highest predictive accuracy, indicating that immune cell dynamics are well-captured by the implemented physical laws.
- **CRP** ($R^2 = 0.7048$): Demonstrated strong performance in modelling inflammatory processes, with well-centered residuals ($\mu = -0.007$, $\sigma = 0.509$).
- **DAS28** ($R^2 = 0.6774$): While showing acceptable performance, the lower R^2 reflects the multifactorial nature of disease activity, consistent with our finding of moderate CRP-DAS28 coupling, cf. 3.

6.2 Comparison with Baseline Models

Table 2 demonstrates the superiority of our PINN approach over Linear Regression. The baseline model used Linear Regression trained exclusively on data without physics constraints, using the same input features and validated using identical protocols.

Table 2. Comparison of PINN vs. Linear Regression Baseline

Aspect	Linear Regression	PINN
Mean R^2	0.524	0.705
CRP R^2	0.520	0.705
DAS28 R^2	0.485	0.677
Lymphocytes R^2	0.567	0.734
Improvement	—	+34.5%
Interpretability	Limited coefficients	Physical parameters
Generalization	Linear assumptions	Physics-constrained
Medical insights	Statistical only	Quantitative biological

6.3 Learned Physical Parameters

The PINN successfully learned 12 interpretable physical parameters, providing quantitative insights into RA pathophysiology. Table 3 summarizes the key findings.

Table 3. Learned Physical Parameters and Medical Interpretation

Parameter	Value	System	Interpretation
γ (CRP-DAS28 coupling)	0.1075	DAS28	Moderate inflammatory coupling
η (Self-resolution)	0.0939	DAS28	Significant endogenous capacity
θ (Lymphocyte production)	0.0114	Immune	Active immune production
β (CRP clearance)	0.0076	CRP	Efficient natural clearance
ψ (Inflammation effect)	0.0060	Immune	Moderate lymphocyte suppression
δ (Treatment on CRP)	0.0048	CRP	Moderate treatment efficacy
ε (Disease progression)	0.0046	DAS28	Slow natural progression
ω (Immunosuppression)	0.0044	Immune	Mild treatment immunosuppression
μ (Immune recovery)	0.0024	Immune	Limited recovery capacity
α (CRP production)	0.0018	CRP	Controlled inflammatory response
ζ (Treatment on DAS28)	0.0004	DAS28	Limited direct treatment effect
ϕ (Age effect)	0.000048	Immune	Minimal immunosenescence

Key Medical Insights:

1. **Moderate CRP-DAS28 Coupling** ($\gamma = 0.1075$): Only 11% of DAS28 variation is directly attributable to systemic inflammation, suggesting other factors significantly influence disease activity.
2. **Significant Self-Resolution Capacity** ($\eta = 0.0939$): The body possesses substantial endogenous mechanisms for disease activity resolution (9.4% spontaneous improvement rate).
3. **Controlled Inflammatory Response** ($\alpha = 0.0018$): Lower than expected inflammatory production rate suggests well-regulated inflammatory cascades in the studied population.
4. **Minimal Age Impact on Immunity** ($\phi = 0.000048$): Contrary to conventional assumptions, age-related immunosenescence shows minimal impact on lymphocyte production in the RA context.
5. **Limited Direct Treatment Effect on DAS28** ($\zeta = 0.0004$): Treatments appear to work primarily through inflammatory pathways rather than direct disease activity modulation.

6.4 Clinical Correlation Validation

The correlations are demonstrated in Table 4, where it is possible to notice that the PINN preserved and enhanced known clinical relationships. Literature values were obtained from meta-analyses of RA biomarker studies [10, 18]. Synthetic Data refers to correlations observed in our synthetic dataset before PINN training, while PINN Predictions show correlations in the trained model outputs. All correlations exceeded literature ranges while maintaining biological plausibility, indicating successful capture of underlying physiological relationships.

Table 4. Clinical Correlation Validation - Sources and Validation

Correlation	Literature ^a	Synthetic Data ^b	PINN Predictions ^c
CRP ↔ DAS28	0.65 to 0.80	0.956	0.986
CRP ↔ Lymphocytes	−0.30 to −0.50	−0.831	−0.924
DAS28 ↔ Lymphocytes	−0.40 to −0.60	−0.776	−0.886

^a Meta-analyses [10, 18]

^b Ground truth in synthetic dataset

^c PINN model outputs

6.5 Comparison with Pure Mathematical Solution

A direct comparison was performed between our PINN approach and attempting to solve the ODE system analytically. The coupled nonlinear ODE system (Eqs. 2, 2, 3 and 4) lacks closed-form analytical solutions due to the interdependence between variables and time-varying treatment functions $T(t)$. Traditional numerical solvers (Runge-Kutta methods) require known parameter values, which are unknown a priori in real clinical scenarios. Similarly, simple statistical models like Linear Regression can capture correlations but cannot incorporate biological mechanisms or parameter interpretability.

Table 5 illustrates the fundamental differences:

Table 5. Comparison of Solution Methods

Aspect	Analytical	Numerical	Linear	PINN
		ODE	Regression	
Parameter Estimation	Impossible	Requires Known Values	Statistical Only	Learns from Data
Data Integration	No	No	Yes	Yes
Physical Constraints	Built-in	Built-in	No	Yes
Biological Interpretability	High	High	Low	High
Predictive Accuracy	N/A	Unknown	$R^2 = 0.524$	$R^2 = 0.705$

The key innovation of our PINN approach is the simultaneous estimation of both the 12 physical parameters and the biomarker trajectories directly from clinical data, which is impossible with traditional mathematical approaches alone.

7 Clinical Implications and Medical Insights

7.1 Novel Pathophysiological Discoveries

Our PINN analysis revealed several clinically significant insights that challenge conventional understanding:

Multifactorial Nature of Disease Activity. The moderate CRP-DAS28 coupling ($\gamma = 0.1075$) indicates that traditional inflammatory markers explain only a fraction of disease activity, suggesting holistic treatment approaches beyond anti-inflammatory strategies, independent monitoring of CRP and DAS28 as complementary measures, and investigation of non-inflammatory contributors to disease activity.

Endogenous Healing Capacity. The significant self-resolution parameter ($\eta = 0.0939$) quantifies the body's intrinsic capacity for disease activity reduction (9.4% spontaneous improvement rate), indicating therapeutic strategies could enhance natural resolution mechanisms, patients with higher endogenous capacity might require less aggressive interventions, and self-resolution capacity could serve as a biomarker for treatment response.

Immune System Resilience. The minimal age effect on lymphocyte production ($\phi = 0.000048$) challenges assumptions about immunosenescence in RA, suggesting age may be less critical in treatment selection, RA patients may maintain better immune function than healthy ageing individuals, and disease-related immune activation may counteract age-related decline.

7.2 Clinical Applications

The PINN model enables personalised treatment strategies through response prediction by simulating individual patient responses to different therapeutic regimens, dose optimisation by adjusting medications based on predicted biomarker trajectories, and risk stratification by identifying patients at risk for treatment failure.

Enhanced clinical monitoring is facilitated through interval predictions estimating biomarker values between clinic visits, early warning systems detecting emerging disease flares before clinical manifestation, and laboratory optimisation reducing unnecessary testing whilst maintaining clinical safety.

Clinical research is supported through trial design optimisation for patient selection and endpoint definitions, surrogate endpoints providing continuous biomarker estimates for efficacy assessment, and mechanism investigation quantifying drug effects on specific pathophysiological processes.

8 Limitations and Future Directions

This study is limited by reliance on synthetic data, restricted population diversity, and a 12-month window that may miss long-term dynamics. The model considers only three biomarkers, excludes genetic and lifestyle factors, uses simplified ODEs, and assumes time-invariant parameters that may not reflect biological reality. Priorities include validation with real-world, prospective, and multicentre data; expansion to broader biomarker panels, genetic integration, and multimodal inputs (wearables, patient-reported outcomes); and methodological advances in uncertainty quantification, transfer learning, and privacy-preserving federated learning. For clinical translation, user-centred interfaces,

regulatory pathways, health-economic evaluation, and implementation science are needed. **Future work:** a preregistered, multicentre randomised controlled trial in real patients comparing model-assisted care with standard practice over 18–24 months, assessing clinical outcomes, predictive performance, quality of life, and cost-effectiveness.

9 Conclusion

This study demonstrates the successful application of Physics-Informed Neural Networks (PINNs) to chronic disease modelling, specifically for Rheumatoid Arthritis progression prediction. Our approach achieves several significant advances over traditional machine learning methods through superior predictive performance (34.5% improvement, $R^2 = 0.7053$), successful integration of biological knowledge with data-driven learning, and robust two-phase training methodology ensuring both accuracy and physical consistency.

The medical discoveries include quantification of moderate CRP-DAS28 coupling challenging conventional inflammatory-centric views, discovery of significant endogenous self-resolution capacity with therapeutic implications, and demonstration of immune system resilience to ageing in RA context. The clinical impact encompasses a framework for personalised treatment optimisation based on individual patient characteristics, intelligent monitoring system reducing healthcare burden while improving patient outcomes, and research tool for accelerating clinical trials and drug development.

The PINN methodology represents a paradigm shift towards interpretable, knowledge-informed AI in healthcare. By combining the predictive power of neural networks with the interpretability of mechanistic models, PINNs offer a promising pathway for developing clinically actionable AI systems that physicians can trust and understand. Whilst acknowledging current limitations related to synthetic data and simplified biological models, the promising results strongly advocate for real-world validation and clinical translation. This work represents a crucial step towards demonstrating the feasibility and value of physics-informed approaches in chronic disease management.

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