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MASTER OF CIVIL AND INDUSTRIAL ENGINEERING IN SEISMIC ZONES

Probabilistic assessment of stress losses in post-tensioned elements for reinforced concrete slabs

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Pisa-Bucharest
2015

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June 25th 2015

University of Pisa

Department of Energy, Systems, Territory and Construction Engineering

DISSERTATION THESIS

Thesis title: Probabilistic assessment of stress losses in post-tensioned elements for reinforced concrete slabs

Date of issue: 22th June

Deadline: 25th June

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Standard statement on the originality of the work

I hereby declare that the dissertation thesis entitled " Probabilistic assessment of stress losses in post-tensioned elements for reinforced concrete slabs" is written by me and never has been presented at another faculty or higher educational institution in the country or abroad.

Bucharest, June 25th 2015

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Signature

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MCS Monte Carlo Simulation

FORM First Order Reliability Method

SORM Second Order Reliability Method

A_c Cross sectional area of concrete

A_p Area of a prestressing tendon or tendons

$E_{c,eff}$ Effective modulus of elasticity of concrete

E_{cm} Secant modulus of elasticity of concrete

E_p Design value of modulus of elasticity of prestressing steel

P Prestressing force

P_0 Initial force at the active end of the tendon immediately after stressing

f_c Compressive strenght of concrete

f_{cd} Design value of concrete compressive strenght

f_{ck} Characteristic compressive cylinder compressive strenght

f_{cm} Mean value of concrete cylinder compressive strenght

f_p Tensile strenght of prestressing steel

f_{pk} Characteristic tensile strenght of prestresing steel

$f_{p0,1}$ 0,1% proof-stress of prestressing steel

$f_{p0,1k}$ Characteristic 0,1% proof-stress of prestresing steel

$f_{p,2k}$ Characteristic 0,2% proof-stress of reinforcement

h Overall depth of cross section

k Coefficient

t_0 The age of concrete at the time of loading

u Perimeter of concrete cross-section, having area A_c

α Angle

μ Coefficient of friction between the thendons and their ducts

$\varphi(t,t_0)$ Creep coefficient, defining creep between times t and t_0, related to elastic deformation at 28 days

$\varphi(\infty,t_0)$ Final value of creep coefficient

Abstract

The main aim of research is to formulate and evaluate a probabilistic model for post-tensioned slab systems as an effective mean to achieve informations about stress losses which occure during the lifetime of concrete slab.

The probabilistic model is used to objectively assess the fragilities for structural components, in particular, for post-tensioned concrete slabs.

The probabilistic model developed is similar to deterministic model (Eurocode 2) or demand methods used in engineering practice, but has additional correction terms which take account of inherent systematic and random errors.

The probabilistic capacity model is combined with the probabilistic demand model to construct limit-state function that is used to construct point and interval estimates of the fragilities of structural components (slabs).

The probabilistic capacity model is used to estimate the fragilities of structural elements in terms of maximum stress losses, and the probabilistic demand model is used in conjugation with the capacity model to assess the fragilities for a given set of stress.

More specificaly, the fragility of a failure criterion is defined as the given criterion limit (capacity) is equal or exceeds the performance criterion(demand).

The corresponding probability will be computed by using the methods of structural system reliability.

Chapter 1. Introduction

1.1.Problem statement

The main aim of research is to formulate and evaluate a probabilistic model for post-tensioned slab systems as an effective mean to achieve informations about stress losses which occure during the lifetime of concrete slab.

The probabilistic model is used to objectively assess the fragilities for structural components, in particular, for post-tensioned concrete slabs.

Rationale for proposed research

Post tensioning is a technique for reinforced concrete. Post-tensioned tendons, which are prestressing steel cables inside plastic ducts or sleeves, are positioned in the forms before the concrete is placed.

Adding post-tensioning reinforcing combines the action of reinforcing tensile zones with the advantages of compressing the concrete structure.

Optimum efficiency is obtained because the post-tensioning reinforcing is located in the tensile zones, the concrete is being compressed and the post-tensioning reinforcing is creating an uplift forces in the middle of the spans, where it is needed the most.

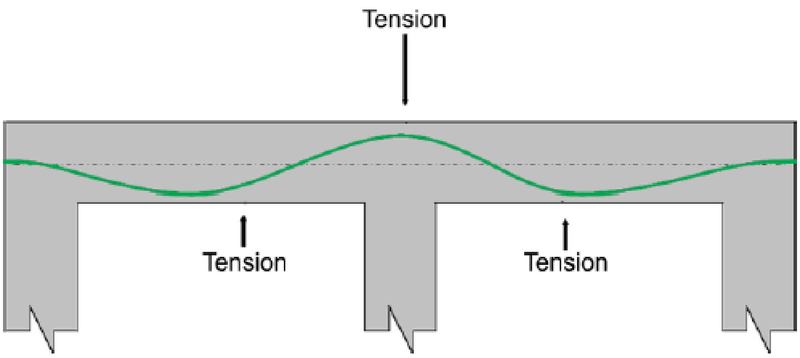


Fig.1.1. The advantages of a post-tensioned slab system

The major disadvantage with the post-stressing system in the concrete slabs is that the tendon forces will decrease with the time due to various long term mechanisms in both the concrete and the tendons.

Recent crushes of bridges and concrete slabs have emphasized the vulnerability of post-tensioned systems and the need to mitigate the risk consequent to the failure of these systems.

Post-tensioned slab systems critical elements for the entire strucuture and their functionality is a primary importance for life safety and economic recovery of a community.

The estimation of consequent losses provide valuble information for risk mitigation and recovery purposes.

This can be achieved by the development of predictive models in civil engineering , in order to determine the capacity of elements. Example of predictive capacity models are Eurocodes.

Such current models are usually deterministic and on the conservative side. They were typically developed using simplified mechanics rules and conservatively fitting to avaible experimental data.

As a result, they do not explicitly take account for the uncertainty inherent in the model and provide biased estimates of the capacity.

While these deterministic models have been successfully used to design safe structures, the needs of modern structural engineering practice, and especially the implementation of the performance-based design concept, require predictive capacity models that are unbiased and explicitly account for all the prevailing uncertainties.

In the context of this paper, a "model" is a mathematical expression relating more quantities of interest, e.g., the capacities of structural component, to a set of measureable variables:

x = $(x_1, x_2 \ldots x_n)$, e.g. material properties constants, member dimension, and imposed boundary conditions.

The main purpouse of the model is to provide a means for predicting the quantitites of interest for given deterministic or random values of variables **x.**

The model is univariate because only one quantity is to be predicted.

Un univariate capacity model has the general form:

$$C(x,\Theta) = \hat{c}(x) + \gamma(x,\Theta) + \sigma\varepsilon \qquad (1.1)$$

where Θ denotes a set of parameters introduced to fit the model to deterministic model and C is the capacity quantity of interest.

The function C(**x,Θ**) have a general form algebraic expressions.

$$C(x,Q) = \hat{c}(x) + \gamma(x,q) + \sigma\varepsilon \qquad (1.2)$$

Where $\Theta(\boldsymbol{\theta},\boldsymbol{\sigma})$, $\theta(\theta_1, \theta_2, \ldots \theta_n)$, denotes the set of unknown model parameters,

$\hat{c}(x) =$ selected deterministic model

$\gamma(x,q)$ correction term for the bias inherent in the deterministic model that is expressed as a function of the variables x and parameters θ, ε= random variables with zero mean and unit variance, and σ represents the standard deviation of the model error.

In formulating the model, we employ a suitable transformation of the quantity of interest to justify the following assumptions:

- the model variance σ^2 is independent of x (homoskedasticy assumption)
- ε has the normal distribution (normality as assumption)

The probabilistic model is used to objectively assess the fragilities for structural components, in particular, for post-tensioned concrete slabs.

In recent years, fragility curves have become popular for characterizing vulnerability of civil structures.Fragility is defined as the conditional probability of attaining or exceeding a specific damage state for a given set of input variables.

The fragility of a post-tensioning system is an importan ingredient in assessing the vulnerability of the hole strucutre from which the element makes part.

More specificaly, the fragility of a failure criterion is defined as the given criterion limit (capacity) is equal or exceeds the performance criterion(demand).

The probabilistic model is used to objectively assess the fragilities for structural components, in particular, for post-tensioned concrete slabs.

In this goal, we formulate the problem in terms of limit state function.

The performance function (the limit state function) according to our performance criterion or failure criterion is defined:

M = performance criterion – given criterion limit = $g(X_1, X_2 \ldots X_n)$

Where :

- the performance criterion is the total stress loss
- the given criterion limit is the stress in the element already known.

1.2.Literature review

In the past, there have been several studies on seismic fragility of structures. Different authors have followed different strategies and approaches . Thus, Karim and Yamazaki (2001) used Monte Carlo simulation for specific structural models.

Hwang and Huo (1994) developed an analytical method for generating fragility curves based on numerical simulations of the dynamic behaviour of specific structures.

In order to save computational time in the Monte Carlo simulations, Fukushima et. al(1996) and Kai and Fukushima (1996) proposed a fragility analysis method where random vibration theory in the frequency domain is used to evaluate the structural response.

Gardoni and Der Kiureghian (2002) developed a methodology for constructing component and system fragility estimates by solving reliability problems that involve the structural capacity at the component level and the corresponding demands, due to an earthquake ground motion.

The assessment methodology explicitly takes account for all the prevailing uncertains, including uncertainties in structural properties and loading characteristics, statistical uncertainty, measurement errors , modeling errors arising from inccurate models' forms or missing variables, and inaccuracies in the methodology itself.

A common charachteristic of these approaches is that the modeling and estimation is carried out at the structural system level. The fragility estimate for a specific structural system cannot be used to asses the fragility of another structure, except when the two structures are of similar type or by an arbitrary combination of fragility curves .

Also, these approaches do not properly take account for all the uncertainties that are involved.

1.3.Limitations

In assessing a model, we deal with two types of uncertainties: aleatory (known also as inherent variability or randomness) and epistemic uncertainties.

The first ones are those which are inherent in nature and they cannot be influenced by the observer or the manner of the observation.

Referring to the model formulation, this type of uncertainy is present in the variables $\mathbf{x}$ and partly in the error terms ε_n.

The epistemic uncertainties are those which arise from our lack of knowledge, our free choice to simplify matters, from errors in measuring observations and from finite size of observation samples.

This type of uncertainty is present in the model parameters $\mathbf{\Theta}$ and partly in the error terms ε_n.

The fundamental difference between the two types of uncertainties is that the aleatory uncertainties are irreductible and the epistemic uncertainties are reductible, e.g. more accurate measurements and collection of additional samples.

The specific type of uncertainties that arise in assesing models are the type of uncertainty which arise when approximations are introduced in the derivation of the deterministic model.

It has two essential components: error in the form of the model, e.g. a linear expression is used when the actual relation is nonlinear, and the missing variables, i.e. $\mathbf{x}$ contains only a subset of the variables that influence the quantity of interest.

In our case, the probabilistic model has been simplified, by using a smaller number of variables **x,** which can significantly influence the accuracy of our results.

We choosed to redeuce the number of variables in order to save computational time.

The simplifications we made are related to terms which are determinated by empirical formulas (and which we can consider being constant) or are statisticaly dependent.

1.4.Our contribution

1.4.1. Importance Sampling for Super Computers

A lot of methods exist to study the influence of uncertainties on the results of severe accident computer codes.

In this paper, the term “influence of uncertainties” means sensitivity analysis or evaluation of the probability that a response exceeds a threshold.

Many of these methods could not be suitable, from the theoretical point of view, when the phenomena that are modelled by the computer code are discontinuous in the variation range of influent parameters or some input variables are statistically dependent.

In order to consider the limitations imposed by literature regarding the uncertainties which arise, one of the probabilistic method which we used is Importance sampling.

Importance sampling is a variance reduction technique which is used in the Monte Carlo method. Variance reduction is a procedure used to increase the precision of the estimates that can be obtained for a given number of iterations. Every output random variable from the simuation is associated with a variance which limits the precision of the simulation results.

The idea behind importance sampling is that certain values of the imput random variables in a simulation have more on the parameter being estimated than others. If these important values are emphasized by sampling more frequently, then the estimator variance can be reduced. Hence, the basic methodology in importance sampling is to choose a distribution which encourages the important values.

The fundamental issue in implementing importance sampling simulation is the choice of the baised distribution which encourages the important regions of the imput variables. Coosing or designing a good data baised distribution is the art of importance sampling. The reward for a good distribution can be huge run-time savings and the penalty for a bad distribution can be longer run times than for a general Monte Carlo simulation without importance sampling.

The use of a response surface method (FORM/SORM) introduces an additional error on the result of the uncertainty and sensitivity analysis.

The estimation of that error is not easy to compute in most cases.

1.4.2. Probabilistic formulation

For a structural component, fragility is defined as the conditional probability of attaining or exceeding a presctibed limit state function for a given set of boundary variables.

Considering $g(x,\Theta)$ to be a mathemnatical model which describes the limit state of interest for the structural component, where **x** denotes a vector of measurable variables and **Θ** denotes a vector of model parameters.

Usually **x** can be divided like form , $x=(r,s)$, where **r** is a vector of material and geometrical variables, and **s** is a vector of demand variables such as boundary forces or deformations.

Using the capacity model, the limit state function can be formulated as:

$$g(x,\Theta)=C(r,s,\Theta)-D(r,s,\Theta) \tag{1.3}$$

where $D(r,s,\Theta)$ denotes the demand .

The fragility of the structural component is formulated as:

$$F(s,\Theta)=P[\bigcup\{g(r,s,\Theta)\leq 0\}s,\Theta] \tag{1.4}$$

Uncertainty inherent in the fragility estimate due to the epistemic uncertainties is reflected in the probability distribution of relative to the parameters .

An exact evaluation of this probability distribution requires nested reliability calculations.

Approximate confidence bounds can be optained using first-order analysis (the reliability index corresponding to the conditional fragility will be optain propagating FORM).

Unsing a first-order Taylor series expansion around the mean point . the variance of reliability index is given by

$$\sigma_\beta^2 \cong \nabla^T{}_{\underline{\theta}}\beta\cdot\Sigma_{\underline{\theta\theta}}\cdot\nabla_{\underline{\theta}}\beta \tag{1.5}$$

Where $\nabla_{\underline{\theta}}\beta$ is the gradient of the geometric reliability index, which is a vector containing the mean value β for each of the 13 variables used (or gradient row vector of β at the mean point)

$$\Sigma_{\underline{\theta\theta}}=\underset{\sim}{D}^T\underset{\sim}{R}\underset{\sim}{D} \text{ posterior covariance matrix} \tag{1.6}$$

$$P^{up}=\Phi(-\beta+\sigma_\beta) \text{ the upper bound} \tag{1.7}$$

$$P^{down}=\Phi(-\beta-\sigma_\beta) \text{ the lower bound} \tag{1.8}$$

1.4.3. Structure of the paper

Following the general introduction given in this chapter, Chapter 2 disscuses a probabilistic frame work in order to obtain fragility estimates regarding the stress losses in post-tensioned elements for reinforced concrete slabs.

In chapter 3, the deterministic model from Eurocode 2 is presented in the manner of stress losses.

Each mechanisms causing the post-stress losses is described along with methods for estimating and measuring losses in concrete structures.

In Chapter 4, the methods of structural system reliability used to compute the corresponding probability of having the occurance of a of the total stress losses less or equal to a certain threshold are presented.

The mathematical methods used for the calculation of failure probabilities and generalized reliability indices are FORM (First Order Reliability Method-without correction for curvature of the limit state surface at the most central point), SORM (Second Order Reliability Method- that include the curvature correction) and Monte Carlo Simulation **(**Importance sampling - variance reduction technique).

For first- and second-order reliability methods (FORM/SORM) are presented the steps trough wich are going when the methods are propagated.

The transformation of the space of the basic random variables $X_1, X_2, \ldots, X_3$ into a space of standard normal variables is made using the Nataf model (the probabilistic model is only made up of the marginal densities and of the matrix of covariance).

The Nataf model is implemented in Ferum, open source Matlab.

In this transformed space, the point of the minimum distance from the origin on the limit state surface (this point is called the design point) is researched, followed by an approximation of the failure surface near the design point.

Like result, we optain the computation of the failure probability corresponding to the approximation failure surface.

In Chapter 5, the fragility curves obtained propagating the three methods which are presented in Chapter 4.

The fragility curves have been obtained by variating the given stress σ (demand) and time t set equal to different periods.

Chapter 6 contains the overall summary and conclusion of the framework.

The Appendix contains the stress losses Matlab code .

Chapter 2. Probabilistic formulation of the problem

Probabalistic model for stress losses are developed based on similar deterministic procedures used in practice but they will further include novel elements relating to correction terms that explicitly describe the inherent systematic and random errors.

In order to achieve this, a set of explanatory functions have been used in order to provide means to gain insight into underlying behavioral phenomena and to select the parameters that are most relevant to the demand.

The approach takes into account information gained from scientific/engineering laws, observational data from laboratory experiments or field investigations, and engineering experience and engineering judgement.

The probabilistic capacity model is be combined with the probabilistic demand model to construct limit-state function that is used to construct point and interval estimates of the fragilities of structural components and systems. The probabilistic capacity model is used to estimate the fragilities of structural elements in terms of maximum stress losses, and the probabilistic demand model is used in conjugation with the capacity model to assess the fragilities for a given set of tension.

We want determine the probability of having the occurance of a of the total stress losses less or equal to a certain threshold.

To this goal we can formulate the problem in terms of a limit state function, thus we define the function $g(x, \sigma)$.

The corresponding probability will be computed by using the methods of structural system reliability.

The probability of failure P_f to exceed a threshold according to a specified performance criterion of failure criterion is given by :

M = performance criterion – given criterion limit = g (X_1, $X_2 \ldots X_n$)

$$g(x, \sigma) = \Delta\sigma_{tot} - \sigma \qquad (2.1)$$

, where:

$\Delta\sigma_{tot} = \sum \Delta\sigma_i$ is the total losses stress ;

$\Delta\sigma_{tot} = \Delta\sigma_{tot}(x)$ is the total losses stress formulated like a function of ***x*** variables.

The performance function, also named the limit state function, is given by M= 0.

The failure event is defined as the space where M<0, and the success event is defined as the space where M>0.

$g(x,\sigma) = \Delta\sigma_{tot}(x) - \sigma \leq 0$,

Where :

$\Delta\sigma_{tot}(x)$ is the performance criterion;

σ is the given criterion limit.

Thus a probability of failure can be evaluated by the following integral:

$$P_f = \int\int ... \int fx(x_1, x_2 ... x_n) dx_1 dx_2 ... dx_n \quad (2.2)$$

where f_x is the joint density function of $X_1, X_2, \ldots, X_n$ and the integration is performed over the region where M < 0.

$$P(C \leq D \mid D) = P(g(x,\sigma) \leq 0 \mid \sigma) \quad (2.3)$$

The corresponding probability will be computed by using the methods of structural system reliability.

The simple variables which we used are listed in table Table 1.

Table 1.1. The list of simple variables

Simple variable	Type of distribution	Mean, μ	Standard deviation, σ	Coefficient of variation, V
fpk = x1	Lognormal	1420	284	0.2
θi = x2	Lognormal	0.85	0.087	0.2
Ep = x3	Lognormal	195000	5850	0.03
Ecm = x4	Lognormal	34000	6800	0.2
Ap = x5	Lognormal	600	120	0.2
Ac = x6	Lognormal	240000	48000	0.2
zcp=x7	Normal	70	14	0.2
RH=x8	Beta	0.5	0.224	0.447
fcd= x9	Lognormal	22.67	5	0.22
u= x10	Lognormal	2480	496	0.2
Δσpr= x11	Normal	100	20	0.2
Δσc = x12	Normal	6.8	1.36	0.2
Ic= x13	Lognormal	1.15E+09	230400000	0.2

Thus, each type of stress loss will become a function of simple variables:

$$\Delta\sigma_1 = \Delta\sigma_\mu = 0.8 \cdot x_1 \cdot 0.2 \cdot (x_2 + 0.01 \cdot 7) \quad (2.4)$$

$$\Delta\sigma_1 = \Delta\hat{\sigma}_1(x_1, x_2) \quad (2.5)$$

$$\Delta\sigma_2 = \Delta\sigma_{sl} = 0.8 \cdot x_1 \cdot (1 - \exp(-0.2 \cdot (x_2 + 0.01 \cdot 1))) \quad (2.6)$$

$$\Delta\sigma_2 = \Delta\hat{\sigma}_2(x_1, x_2) \tag{2.7}$$

$$\Delta\sigma_3 = \Delta\sigma_{el} = x_3 \cdot \frac{(0.5 \cdot x_{12})}{x_4} \tag{2.8}$$

$$\Delta\sigma_3 = \Delta\hat{\sigma}_3(x_3, x_4, x_{12}) \tag{2.9}$$

$$\Delta x_4 = \Delta\sigma_{p,s} = x_3 \cdot \frac{\left(\frac{7-2}{(7-2)+0.04\cdot(\frac{2\cdot x_6}{x_{10}})^{0.667}} \cdot 0.85 \cdot 0.0039 + (1 - e^{(-0.2\cdot 7^{0.5})} \cdot (2.5 \cdot (x_9 - 10) \cdot 10^{-6}))\right)}{1 + \frac{x_3}{x_4} \cdot \frac{x_5}{x_6} \cdot (1 + \frac{x_5}{x_{13}} \cdot x_7^2)} \tag{2.10}$$

$$\Delta\sigma_4 = \Delta\hat{\sigma}_4(x_3, x_4, x_5, x_6, x_7, x_9, x_{10}, x_{13}) \tag{2.11}$$

$$\Delta\sigma_5 = \Delta\sigma_{p,c} = x_{12} \cdot \frac{\dfrac{\frac{x_3}{x_4} \cdot x_8}{(0.1 \cdot (2 \cdot \frac{x_6}{x_{10}})^{0.333} \cdot \frac{16.8}{x_9^{0.5}} \cdot \frac{1}{(1+2^{0.2})} \cdot \frac{7-2}{(1.5 \cdot (1 + (0.012 \cdot x_8 \cdot 100)^{18}} \cdot \frac{2 \cdot x_6}{x_{10}} + 250 + 7 - 2)^{0.3}}}{1 + \frac{x_3}{x_4} \cdot \frac{x_5}{x_6} \cdot (1 + \frac{x_5}{x_{13}} \cdot x_7^2)} \tag{2.12}$$

$$\Delta\sigma_5 = \Delta\hat{\sigma}_5(x_3, x_4, x_5, x_6, x_7, x_8, x_{10}, x_{13}) \tag{2.13}$$

$$\Delta\sigma_6 = \Delta\sigma_{p,r} = \frac{x_{11}}{1 + \frac{x_3}{x_4} \cdot \frac{x_5}{x_6} \cdot (1 + \frac{x_5}{x_{13}} \cdot x_7^2)} \tag{2.14}$$

$$\Delta\sigma_6 = \Delta\hat{\sigma}_6(x_3, x_4, x_5, x_6, x_7, x_{11}, x_{13}) \tag{2.15}$$

$$\begin{aligned} g(x,\sigma) = {} & \Delta\hat{\sigma}_1(x_1, x_2) + \Delta\hat{\sigma}_2(x_1, x_2) + \Delta\hat{\sigma}_3(x_3, x_4, x_{12}) + \Delta\hat{\sigma}_4(x_3, x_4, x_5, x_6, x_7, x_9, x_{10}, x_{13}) + \\ & + \Delta\hat{\sigma}_5(x_3, x_4, x_5, x_6, x_7, x_8, x_{10}, x_{13}) + \Delta\hat{\sigma}_6(x_3, x_4, x_5, x_6, x_7, x_{11}, x_{13}) - \sigma \end{aligned} \tag{2.16}$$

The lognormal distribution is used for material charchteristics, because lognormally distributed variables cannot assume values below zero. The non-negative property is consistent with phisical reality.

Beta distribution was used for humidity because has a finit range (0% - 100%) and can be scalled as interval [0 1].

Because each of the basic random variables has a unique distribution and they interact, the integral cannot be easy evaluated.

Two types of methods can be used for assessing the probability to exceed a threshold: Monte Carlo Simulation with or without variance reduction techniques and the approximate methods (FORM/SORM).

Chapter 3. Description of the deterministic model from Eurocode 2

3.1. Introduction

The major disadvantage with the post-stressing system in the concrete slabs is that the tendon forces will decrease with the time due to various long term mechanisms in both the concrete and the tendons.

The long term post stress losses are due to contraction of the concrete, creep and shrinkage and relaxation of the steel tendons. In addition, initial losses occur during the post-tensioning process, these are the elastic shortening of the concrete, friction between the steel tendons and the ducts and losses at the achorages.

In this chapter the different mechanisms causing the post-stress losses are described along with methods for estimating and measuring losses in concrete structures.

3.2. Initial losses

Elastic shortening

The elastic shortening is the elastic response of the concrete during the post-tensioning process. Since the deformation is elastic, it depends on the modulus of elasticity of the concrete and the contraction of the concrete will recover if the load is removed. To prevent the experience for high losses, normal reinforcing will be added, that the initially tensioned tendon will experience the highest losses and the tendon which was tensioned last will be not affected. The contribution from the elastic shortening of the total poststressed losses for a single tendon can be estimate accordin to equation ().

$$\Delta\sigma_{el} = A_p E_p \times \sum [\frac{j \times \Delta\sigma_c(t)}{E_{cm}(t)}] \qquad (3.1)$$

Losses due to friction

Friction between the tendon and the duct can lead to poststressed losses and uneven force distribution along the length of the tendon. Usually the losses due to friction are considered to originate from two different parts; the curvature of the tendon ad wobbling, Collin and Mitchel (1991). The curvature losses are due to the change in bending angle of curved tendons introducing a normal acting on the duct and thus multiplying this normal force with the friction coefficient between tendon and duct yields the loss of tendon force. Since the bending angles usually are quite small, the friction losses are obtained as a function of the change in angle. Losses due to wobbling are caused by unintentional changes of the angle of the tendon, see fig…. and depend mainly on the properties of the duct, e.g. type and diameter of the duct and the spacing od the duct supports but also on the type of tendon and the form of structure. The influence of wobbling is described by an emphirically determined wobble coefficient **k** describing the unintentional changes in angle per unit length. The wobble losses over the length of the tendon are then obtained as the wobble coefficient multiplied with the length of the tendon. In Eurocode 2, the total loss due to friction is estimated as:

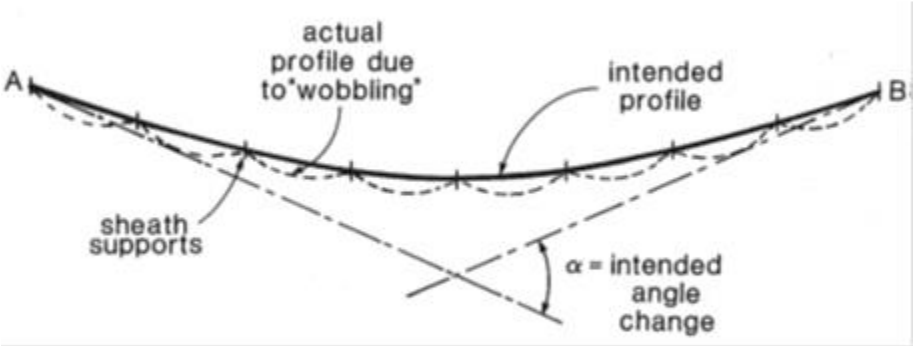


Fig.3.1. Influence of wobbling on tendons according to Eurocode 2, Collins and Mitchell (1991)

(Note: sheath=metal duct)

$$\Delta\sigma_{\mu} = \Delta\sigma_{p,\max} \times \mu\left(\sum \frac{L_i}{r_i} + k \times x\right) \tag{3.2}$$

Losses at anchorage

$$\Delta\sigma_{sl} = \sigma_{p,\max}\left[1 - e^{-2\left(kx + \mu \sum_i^j \frac{L_i}{r_i}\right)}\right] \tag{3.3}$$

3.3. Long-term losses

The phenomena which contribute to the long-term loss of postsressed force are creep and shrinkage of concrete and relaxation of the steel tendons. In the figure .., the principal contribution from different mechanisms on the total long-term deformation of concrete that is allowed to dry out is shown, Brown and Hope (1972).

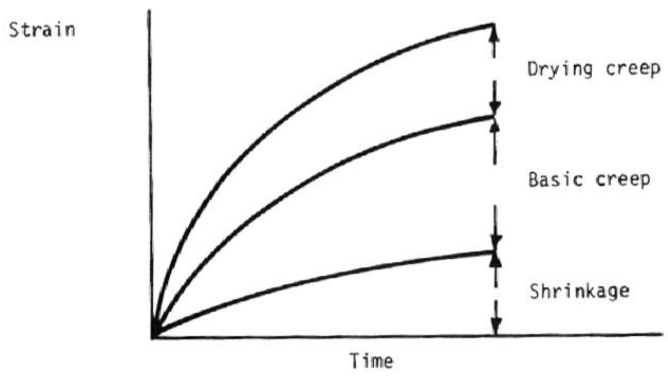


Fig.3.2. Different mechanisms which cause long-term deformation of a loaded concrete specimen subjected to drying, Brown and Hope (1972).

Shrinkage of concrete

Shrinkage can be defined as “the time-dependent strain measured in an unloaded and unstrained specimen at constant temperature”, Kover and Zhutovsky (2006), and includes both drying shrinkage and chemical shrinkage. Drying shrinkage is the contraction due to moisture losses in the concrete and includes both the shrinkage due to loss of moisture to the ambient medium, i.e. drying, and autogenus shrinkage which is the shrinkage due to the self-desiccation of the concrete, and is approximately proportional to the moisture loss in concrete.

The drying shrinkage is partly irreversible, i.e. upon rewetting the swelling strains will be less than the preceding shrinkage strains. Chemical shrinkage is the resulting contraction du to various chemical reactions in the cement paste, such as carbonation shrinkage and hydration shrinkage.

The chemical shrinkage is very difficult or even impossible to predict since the cement reactions are impossible to estimate, even though the cement’s mineral composition could be well known, Kovler and Zhutovsky (2006).

With time, it is assumed that the drying shrinkage approaches a final value, i.e. reaching the state of moisture equilibrium with the ambient medium, which means that the lower the relative humidity of the surrounding air, the greater the shrinkage. The shrinkage strain at an arbitrary point in time, can be expressed as:

$$\Delta\sigma_{p,s} = \frac{\varepsilon_{cs} E_p}{1 + \frac{E_p}{E_{c,ef}\, f} \times \frac{A_p}{A_c}\left(1 + \frac{Ap}{Ac} \times z_{cp}^2\right)} \tag{3.4}$$

Where ε_{cs} is the total shrink strain and can be evaluated as:

$$\varepsilon_{cs}(t) = \varepsilon_{cd} + \varepsilon_{ca} \tag{3.5}$$

Where :

$$\varepsilon_{cd}(t) = \beta_{ds}(t, t_s) \cdot k_h \cdot \varepsilon_{cd,0} \tag{3.6}$$

where:

$$\beta_{ds}(t, t_s) = \frac{t - ts}{(t - ts) + 0.04\sqrt{h_0^3}}$$

$$\varepsilon_{cd,0} = 0.85[(220 + 110 \cdot \alpha_{ds1}) \cdot e^{(-\alpha_{ds2}\frac{f_{cm}}{f_{cm0}})}] \cdot 10^{-6} \cdot \beta_{RH}$$

$$\beta_{RH} = 1.55[1 - (\frac{RH}{RH_0})^3]$$

$$\varepsilon_{ca}(t) = \beta_{as}(t) \cdot \varepsilon_{ca}(\infty) \tag{3.7}$$

where:

$$\beta_{as}(t) = 1 - e^{(-0.2t^{0.5})}$$

$$\varepsilon_{ca}(\infty) = 2.5(f_{ck} - 10)10^{-6}$$

Creep of concrete

The definition of creep is the time dependent increase of strain under constant load. Creep is usually expressed in the form of a creep coefficient, which is defined as the ratio of creep deformation to elastic deformation, but can also be epresses as specific creep or creep compliance, defined as creep per unit stress, for instance MPa^{-1}.

Concrete's creep can be devided into two different parts: basic creep and drying creep.

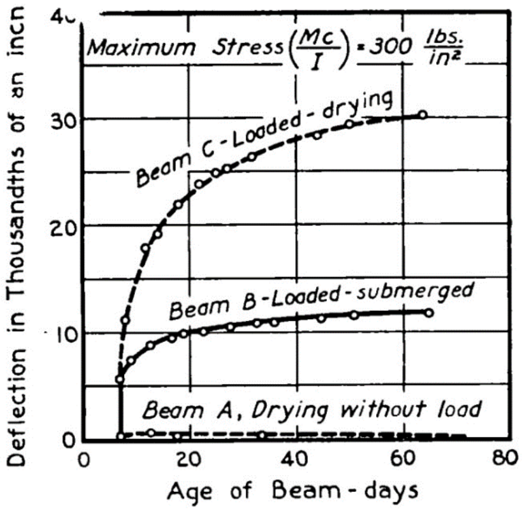


Fig.3.3. The two components of concrete's creep, Brown and Hope (1972).

$$\sigma_5 = \frac{\frac{E_p}{E_c} \times \varphi(t,t_0) \times \sigma_c}{1 + \frac{E_p}{E_{c,ef}\, f} \times \frac{A_p}{A_c}(1 + \frac{Ap}{Ac} \times z_{cp}^2)} \quad (3.8)$$

where:

$$\varphi(t,t_0) = \varphi_0 \times \beta_c(t,t_0)$$

$$\varphi_0 = \varphi_{RH} \times \beta(f_{cm}) \times \beta(t_0)$$

$$\varphi_{RH} = 1 + \frac{1 - RH/100}{0.1\sqrt[3]{h_0}}$$

$$h_0 = \frac{2A_c}{u}$$

$$\beta(f_{cm}) = \frac{16.8}{\sqrt{f_{cm}}}$$

Assumptions:

- Four layers of prestressing steel type T15S;
- One concrete type for the entire cross section, 35-40 strenght class;
- Any cross-sectional shape;
- Prestressing and dead load are applied at the same timp t_0 to the concrete section;
- Assume the prestressing is applied at the same time of end of curing of concrete;

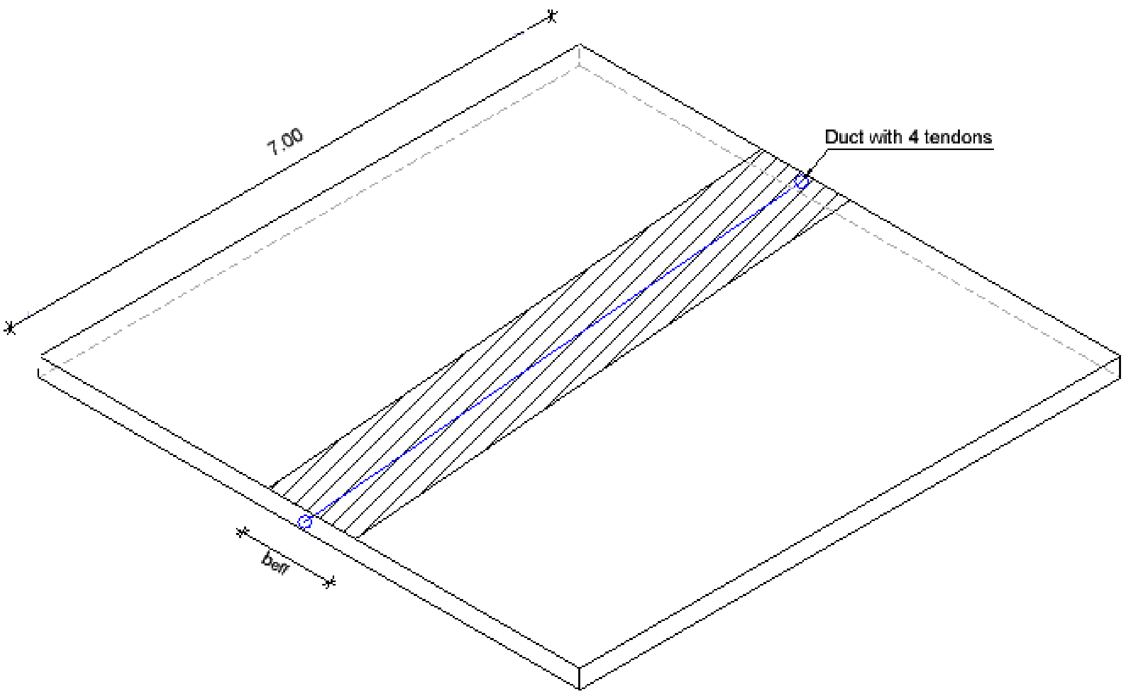


Fig.3.4. The slab model

As a result, we combine Eq…. and obtain the total losses stress, $\Delta\sigma_{tot}$.

$$\Delta\sigma_{tot} = \Delta\sigma_{\mu} + \Delta\sigma_{sl} + \Delta\sigma_{el} + \Delta\sigma_{p,s} + \Delta\sigma + \Delta\sigma_{p,r} \qquad (3.9)$$

For comparation with probabilistic model , we computed the stress losses manually.

Like results, the values computed with the deterministic model are closed to values computed with the probabilistic model.

Chapter 4. Probabilistic methods

4.1. Monte Carlo Simulation

Direct Monte Carlo Simulation techniques can be use to estimate the probability P_f defined in equation (-). Monte Carlo simulation consist of drawings samples of the basic variables according to their probabilistic characteristics and then feeding them into the performance function.

An estimate of the probability of P_f can be found by:

$$P_f = \frac{N_f}{N} \quad (4.1)$$

Where N_f is the number of simulation cycles in which $g(x) \le 0$, and N the total number of simulations cycles. As N approaches infinity, P_f approaches the true probability of

failure. The accuracy of the estimation can be evaluated in terms of its variance computed approximately as

$$Var(P_f) \cong \frac{(1-P_f)P_f}{N} \quad (4.2)$$

It is recommended to measure the statistical accuracy of the estimated probability of failure by computing its coefficient of variation as

$$COV(P_f) \cong \frac{\sqrt{\frac{(1-P_f)P_f}{N}}}{P_f} \quad (4.3)$$

The smaller the coefficient of variation, the better the accuracy of the estimated probability of failure.

It is evident from () that as N approaches infinity, $Var(P_f)$ and $COV(P_f)$ approach zero.

For a small probability of failure and a small number of simulation cycles, the variance of P_f can be quite large. Consequently, it may take a large number of simuation cycles to achive a specific accuracy.

The amount of computer time needed for the direct Monte Carlo method is large, specially in the case where each simuation cycle involves a long calculation performed by a severe accident computer code.

More efficient Monte Carlo methods have been developed and define the family of "Variance redduction techniques".

In comparison with Monte Carlo method, for the same number of runs, the accuracy of the failure probability with a variance reduction techniques is better.

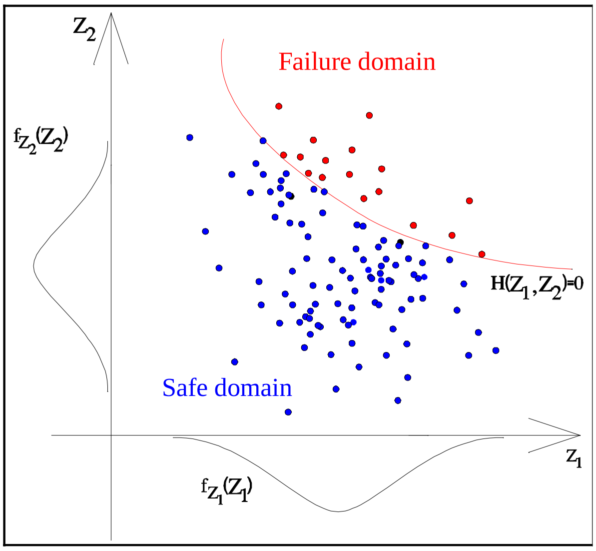
These methods are: Importance sampling, Stratified Sampling, Latin hypercurbe sampling, Adaptive sampling, Conditional expectation, Directional simulation, etc.

For this work, we choosed to use the Importance sampling method.

Importance sampling is a technique that gets around the problem of variance reduction by changing the probability of the model so as to make the rare event happen often instead of rarely.

To understand the basic idea, suppose we wish to compute $E(h(X)) = \int h(x) f(x) dx$ for a continuous random variable X distributed with density f(x).

Fig.4.1. Reliability assessment by Monte-Carlo simulation



For example, if $h(x) = I\{x > b\}$ for a given large b, then $h(X) = I\{X > b\}$ and $E(h(X)) = P(X > b)$. Now let $g(x)$ be any other density such that $f(x) = 0$ whenever $g(x) = 0$, and observe that we can rewrite

$$E(h(X)) = \int h(x) f(x) dx = \int [h(x) \frac{f(x)}{g(x)}] g(x) = \tilde{E}[h(X) \frac{f(X)}{g(X)}] \qquad (4.4)$$

Where $\tilde{E}$ denotes expected value when g is used as the distribution of X (as opposite to the original distribution f).

In other words: if X has distribution g, then the expected value of $h(X)\frac{f(X)}{g(X)}$ is the same as what we originally wanted. The ratio $L(X)=\frac{f(X)}{g(X)}$ is called the likehood ratio. We can write

$$E(h(X))=\tilde{E}(h(X)L(X)) \ ;$$

The left-hand side uses distribution f for X, while the right-hand side uses distribution g for X.

To make this work in our favour, we would want to choose g so that the variance of $h(X)L(x)$ (under g) is small relative to its mean.

We can easily generalise this idea to multi-dimensions: suppose $h=h(X_1,...,X_k)$ is real-valued where $(X_1,...,X_k)$ has joint density $f(x_1,...,x_k)$. Then for an alternative joint density $g(x_1,...,x_k)$, we once can write

$$E(h(X_1,...,X_k))=\tilde{E}(h(X_1,...,X_k)L(X_1,...,X_k)) \ ,$$

where $L(X_1,...,X_k)=\frac{f(X_1,...,X_k)}{g(X_1,...,X_k)}$, and $\tilde{E}$ denotes expected value when g is used as the joint of distribution of $(X_1,...,X_k)$.

4.2. Approximated methods (FORM & SORM)

The first- and second-order reliability methods (FORM/SORM) consist of 4 steps:

- the transformation of the space of the basic random variables $X_1, X_2, ..., X_3$ into a space of standard normal variables,
- the research, in this transformed space, of the point of the minimum distance from the origin on the limit state surface (this point is called the design point),
- an approximation of the failure surface near the design point,
- a computation of the failure probability corresponding to the approximation failure surface.

FORM and SORM apply to problems where the set of basic variables are continuous.

4.3. Transformation of space

The possibilities to construct a sufficiently good convex polyhedral approximation to the safe set in a Gaussian formulation space depends not just on the shape of the safe set in the free ohysical formulation space, but also on the properties of the transformation T.

The choice of the transformation to be used depends on the characteristics of the joint density of random imput vector X.

The most current transformations are the transformations of Rosenblatt when the joint density is known, and the transformation of Nataf, when the probabilistic model is only made up of the marginal desities of the matrix of covariance.

In the present work, we used the Nataf model of transformation, which is implemented in FERUM, open source Matlab.

The Nataf model

Multivariate distribution models are frequentely needed in engineering to describe dependent random quantities.

Because of the nature of engineering problems and the inadequancy of statistical data, the avaible information on the dependence is often limited to the knowledge of covariances between random variables.

In many cases, marginal distributions of variables can be prescribed based on relatively limited data or on physical or mathematical grounds. In such situations, there is a need for multivariate distributions models which are consistent with the set of known marginal an covariances.

One of these is the Nataf model.

Define stabdard narmal variates $Z = (Z_1, ..., Z_n)$ obtained by marginal transformation of $X = (X_1, ..., X_n)$.

$$Z_i = \Phi^{-1}[F_{X_i}(X_i)],\ i = 1, ..., n \qquad (4.5)$$

where $\Phi(.)$ is the standard cumulative normal probability.

Nataf’s distribution for **X** is obtained bu assuming that **Z** is jointly normal. Using the rules of probability transformation, the joint **PDF** of **X** is

$$f_x(X) = f_{x_1}(x_1) f_{x_2}(x_2) ... f_{x_n}(x_n) \frac{\varphi_n(z, R')}{\varphi(z_1)\varphi(z_2)...\varphi(z_n)} \qquad (4.6)$$

Where $z_i = \Phi^{-1}[F_{x_i}(x_i)]$, $\varphi(.)$ is the standard normal PDF, and $\varphi_n(z, R')$ in the n-dimensional normal PDF of zero means, unit standard deviation, and correlation matrix **R**’.

The elements φ'_{ij} or **R'** are defined in terms of the correlation coeffcients φ_{ij} through the integral relation

$$\varphi_{ij} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(\frac{x_i-\mu_i}{\sigma_i})(\frac{x_j-\mu_j}{\sigma_j})f_{x_i}(x_i)f_{x_j}(x_j)\times\frac{\varphi_2(z_i,z_j,\varphi'_{ij})}{\varphi(z_i)\varphi(z_j)}dx_i dx_j$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(\frac{x_i-\mu_i}{\sigma_i})(\frac{x_j-\mu_j}{\sigma_j})\varphi_2(z_i,z_j,\varphi'_{ij})dz_i dz_j \quad (4.7)$$

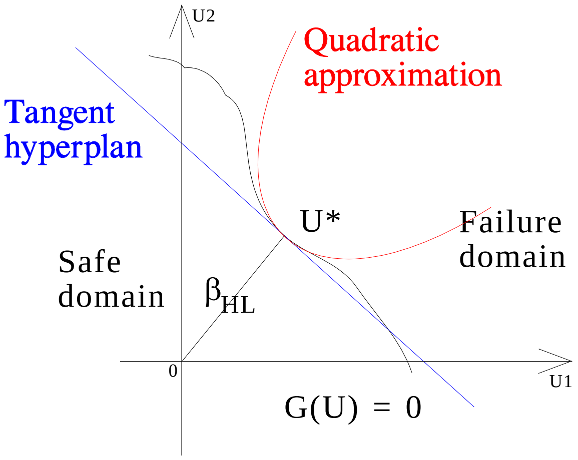
The preceding distribution model is valid provided the mappings in equation () are one to one and the correlation matrix **R**'is positive definite. The first condition is satisfied when $F_{x_i}(x_i)$ are continuous and strictly increasing. The second condition is satisfied in nearly all cases of practical interest, since the correlation matric of X by definition is positive defined and, as subsequently shown, the differences $|\varphi_{ij}-\varphi'_{ij}|$ are small.

For each pair of marginal distributions and with known φ_{ij} , equation () can be iteratively solved for φ'_{ij} . To avoid such tedious calculations, a set of empirical formulate relating φ'_{ij} to φ_{ij} are developed. These are based on the following fundamentals properties of equation ():

Lemma 1: φ_{ij} *is a strictly increasing function of* φ'_{ij}.

Lemma 2: $\varphi'_{ij}=0$ *for* $\varphi_{ij}=0$.

Lemma 3: $|\varphi_{ij}|\le|\varphi'_{ij}|$ *where the equlity stands when* $\varphi_{ij}=0$ *or when both marginals*



are normal.

Lemma 4: *F is independent of* φ_{ij} *if one of the variables is normal.*

Lemma 5: *F is invariant to increasing linear transformations of X_i and X_j .*

Fig.6.

Lemma 6: *F is independent of the parameters of Group 1 distributions*.

Lemma 7: *F is a function of the coefficient of variation, $\delta = \frac{\sigma}{\mu}$, of Group 2 distributions*.

Based on Lemmas 4-7, five categories of formulas for two-parameter distributions are developed: (1) *F=constant* for X_j belonging to Group 1 and X_i normal; (2) $F = F(\delta_j)$ for X_j belonging to Group 1and X_i normal; (3) $F = F(\varphi_{ij})$ for both X_i and X_j belonging to Group 1;(4) $F = F(\varphi_{ij}, \delta_j)$ for X_i belonging to Group1and X_j belonging to Group 2; and (5) $F = F(\varphi_{ij}, \delta_i, \delta_j)$ for both X_i and X_j belonging to Group 2.

Fig.4.2. Reliability assessment with FORM/SORM methods

For the selected distributions in the Table (), the formulae are developed by least-square fitting of polynomial expressions to exact values computed by numerical integration of equation (). The results are listed in Tables together with maximum errors resulting from the least square fitting.

Selected two-parameter distributions

Group	Name	Symbol	CDF	Standard CDF
	Normal	N	$\Phi\left(\frac{x-\mu}{\sigma}\right)$	$\Phi(y)$
	Uniform	U	$\frac{x-a}{b-a}, \quad a \le x \le b$	$y, 0 \le y \le 1$
1	Shifted exponential	SE	$1-\exp[-\lambda(x-x_0)], \quad x_0 \le x$	$1-\exp(-y), \quad 0 \le y$
	Shifted Rayleigh	SR	$1-\exp\left[-\frac{1}{2}\left(\frac{x-x_0}{\alpha}\right)^2\right], \quad x_0 \le x$	$1-\exp\left[-\frac{y^2}{2}\right], \quad 0 \le y$
	Type-I largest value	T1L	$\exp\{-\exp[-\alpha(x-u)]\}$	$\exp[-\exp(-y)]$
	Type-I smallest value	T1S	$1-\exp\{-\exp[\alpha(x-u)]\}$	$1-\exp[-\exp(y)]$
	Lognormal	LN	$\Phi\left[\frac{\ln x-\lambda}{\zeta}\right], \quad 0 < x$	
	Gamma	GM	$\frac{\Gamma(k,\lambda x)}{\Gamma(k)}, \quad 0 \le x$	
2	Type-II largest value	T2L	$\exp\left[-\left(\frac{u}{x}\right)^k\right], \quad 0 < x$	
	Type-III smallest value	T3S	$1-\exp\left[-\left(\frac{x-e}{u-e}\right)^k\right], \quad e \le x$	

Note: $\Phi(.)$ is the standard normal probability; $\Gamma(k,u)=\int_0^u e^{-u}u^{k-1}\,du$ and $\Gamma(k)=\Gamma(k,\infty)$ are the incomplete and complete gamma functions, respectively

Fig.4.3. Selected two-parameter distributions, Multivariate distribution models, P-L. Liu and A. Der Kiureghian

Category 1 formulae, F = constant, for X_j belonging to group 1 and X_i normal

X_j	$F = constant$	Max. error
Uniform	1.023	0.0%
Shifted exponential	1.107	0.0%
Shifted Rayleigh	1.014	0.0%
Type-I largest value	1.031	0.0%
Type-I smallest value	1.031	0.0%

Category 2 formulae, $F = F(\delta_i)$, for X_j belonging to group 2 and X_i normal

X_j	$F = F(\delta_j)$	Max. error
Lognormal	$\dfrac{\delta_j}{\sqrt{\ln(1+\delta_j^2)}}$	(exact)
Gamma	$1.001 - 0.007\delta_j + 0.118\delta_j^2$	0.0%
Type-II largest value	$1.030 + 0.238\delta_j + 0.364\delta_j^2$	0.1%
Type-III smallest value	$1.031 - 0.195\delta_j + 0.328\delta_j^2$	0.1%

Note: range of coefficient of variation is $\delta_j = 0.1 - 0.5$

Fig.4.4. Category 1 and 2 formulae, Multivariate distribution models, P-L. Liu and A. Der Kiureghian

Category 3 formulae. $F = F(\rho_{ij})$, for X_i and X_j both belonging to group 1

X_j \ X_i	U	SE	SR	T1L	T1S
U	$1.047 - 0.047\rho^2$				
(Max. errpr)	(0.0%)				
SE	$1.133 + 0.029\rho^2$	$1.229 - 0.367\rho + 0.153\rho^2$			
(Max. error)	(0.0%)	(1.5%)			
SR	$1.038 - 0.008\rho^2$	$1.123 - 0.100\rho + 0.021\rho^2$	$1.028 - 0.029\rho$		
(Max. error)	(0.0%)	(0.1%)	(0.0%)		
T1L	$1.055 + 0.015\rho^2$	$1.142 - 0.154\rho + 0.031\rho^2$	$1.046 - 0.045\rho + 0.006\rho^2$	$1.064 - 0.069\rho + 0.005\rho^2$	
(Max. error)	(0.0%)	(0.2%)	(0.0%)	(0.0%)	
T1S	$1.055 + 0.015\rho^2$	$1.142 + 0.154\rho + 0.031\rho^2$	$1.046 + 0.045\rho + 0.006\rho^2$	$1.064 + 0.069\rho + 0.005\rho^2$	$1.064 - 0.069\rho + 0.005\rho^2$
(Max. error)	(0.0%)	(0.2%)	(0.0%)	(0.0%)	(0.0%)

Note: $\rho = \rho_{ij}$

Category 4 formulae, $F = F(\rho_{ij}, \delta_j)$, for X_i belonging to group 1 and X_j belonging to group 2

X_j \ X_i	U	SE	SE	T1L	T1S
LN	$1.019 + 0.014\delta_j + 0.010\rho^2 + 0.249\delta_j^2$	$1.098 + 0.003\rho + 0.019\delta_j + 0.025\rho^2 + 0.303\delta_j^2 - 0.437\rho\delta_j$	$1.011 + 0.001\rho + 0.014\delta_j + 0.004\rho^2 + 0.231\delta_j^2 \; 0.130\rho\delta_j$	$1.029 + 0.001\rho + 0.014\delta_j + 0.004\rho^2 + 0.233\delta_j^2 \; 0.197\rho\delta_j$	$1.029 - 0.001\rho + 0.014\delta_j + 0.004\rho^2 + 0.233\delta_j^2 + 0.197\rho\delta_j$
(Max. error)	(0.7%)	(1.6%)	(0.4%)	(0.3%)	(0.3%)
GM	$1.023 - 0.007\delta_j + 0.002\rho^2 + 0.127\delta_j^2$	$1.104 + 0.003\rho - 0.008\delta_j + 0.014\rho^2 + 0.173\delta_j^2 - 0.296\rho\delta_j$	$1.014 + 0.001\rho - 0.007\delta_j + 0.002\rho^2 + 0.126\delta_j^2 - 0.090\rho\delta_j$	$1.031 + 0.001\rho - 0.007\delta_j + 0.003\rho^2 + 0.131\delta_j^2 - 0.132\rho\delta_j$	$1.031 - 0.001\rho - 0.007\delta_j + 0.003\rho^2 + 0.131\delta_j^2 + 0.132\rho$
(Max. error)	(0.1%)	(0.9%)	(0.9%)	(0.3%)	(0.3%)
T2L	$1.033 + 0.305\delta_j + 0.074\rho^2 + 0.405\delta_j^2$	$1.109 - 0.152\rho + 0.361\delta_j + 0.130\rho^2 + 0.455\delta_j^2 - 0.728\rho\delta_j$	$1.036 - 0.038\rho + 0.266\delta_j + 0.028\rho^2 + 0.383\delta_j^2 - 0.229\rho\delta_j$	$1.056 - 0.060\rho + 0.263\delta_j + 0.020\rho^2 + 0.383\delta_j^2 - 0.332\rho\delta_j$	$1.056 + 0.060\rho + 0.263\delta_j + 0.020\rho^2 + 0.383\delta_j^2 + 0.332\rho$
(Max. error)	(2.1%)	(4.5%)	(1.2%)	(1.0%)	(1.0%)
T3S	$1.061 - 0.237\delta_j - 0.005\rho^2 + 0.379\delta_j^2$	$1.147 + 0.145\rho - 0.271\delta_j + 0.010\rho^2 + 0.459\delta_j^2 - 0.467\rho\delta_j$	$1.047 + 0.042\rho \; 0.212\delta_j + 0.353\delta_j^2 - 0.136\rho\delta_j$	$1.064 + 0.065\rho - 0.210\delta_j + 0.003\rho^2 + 0.356\delta_j^2 - 0.211\rho\delta_j$	$1.064 - 0.065\rho - 0.210\delta_j + 0.003\rho^2 + 0.356\delta_j^2 + 0.211\rho$
(Max. error)	(0.5%)	(0.4%)	(0.2%)	(0.2%)	(0.2%)

Note: $\rho = \rho_{ij}$; range of coefficient of variation is $\delta_j = 0.1 - 0.5$

Fig.4.5. Category 3 and 4 formulae, Multivariate distribution models, P-L. Liu and A. Der Kiureghian

Category 5 formulae, $F = F(\rho_{ij}, \delta_i, \delta_j)$, for X_i and X_j both belonging to group 2

X_j \ X_i	LN	GM	T2L	T3S
LN	$\dfrac{\ln(1+\rho\delta_i\delta_j)}{\rho\sqrt{\ln(1+\delta_i^2)\ln(1+\delta_j^2)}}$			
(Max. error)	(exact)			
GM	$1.001+0.033\rho+0.004\delta_i-0.016\delta_j+0.002\rho^2+0.223\delta_i^2+0.130\delta_j^2-0.104\rho\delta_i+0.029\delta_i\delta_j-0.119\rho\delta_j$	$1.002+0.022\rho-0.012(\delta_i+\delta_j)+0.001\rho^2+0.125(\delta_i^2+\delta_j^2)-0.077\rho(\delta_i+\delta_j)+0.014\delta_i\delta_j$		
(Max. error)	(4.0%)	(4.0%)		
T2L	$1.026+0.082\rho-0.019\delta_i+0.222\delta_j+0.018\rho^2+0.288\delta_i^2+0.379\delta_j^2-0.441\rho\delta_i+0.126\delta_i\delta_j-0.277\rho\delta_j$	$1.029+0.056\rho-0.030\delta_i+0.225\delta_j+0.012\rho^2+0.174\delta_i^2+0.379\delta_j^2-0.313\rho\delta_i+0.075\delta_i\delta_j-0.182\rho\delta_j$	$1.086+0.054\rho+0.104(\delta_i+\delta_j)-0.055\rho^2+0.662(\delta_i^2+\delta_j^2)-0.570\rho(\delta_i+\delta_j)+0.203\delta_i\delta_j-0.020\rho^3-0.218(\delta_i^3+\delta_j^3)-0.371\rho(\delta_i^2+\delta_j^2)+0.257\rho^2(\delta_i+\delta_j)+0.141\delta_i\delta_j(\delta_i+\delta_j)$	
(Max. error)	(4.3%)	(4.2%)	(4.3%)	
T3S	$1.031+0.052\rho+0.011\delta_i-0.210\delta_j+0.002\rho^2+0.220\delta_i^2+0.350\delta_j^2+0.005\rho\delta_i+0.009\delta_i\delta_j-0.174\rho\delta_j$	$1.032+0.034\rho-0.007\delta_i-0.202\delta_j+0.121\delta_i^2+0.339\delta_j^2-0.006\rho\delta_i+0.003\delta_i\delta_j-0.111\rho\delta_j$	$1.065+0.146\rho+0.241\delta_i-0.259\delta_j+0.013\rho^2+0.372\delta_i^2+0.435\delta_j^2+0.005\rho\delta_i+0.034\delta_i\delta_j-0.481\rho\delta_j$	$1.063-0.004\rho-0.200(\delta_i+\delta_j)-0.001\rho^2+0.337(\delta_i^2+\delta_j^2)+0.007\rho(\delta_i+\delta_j)-0.007\delta_i\delta_j$
(Max. error)	(2.4%)	(4.0%)	(3.8%)	(2.6%)

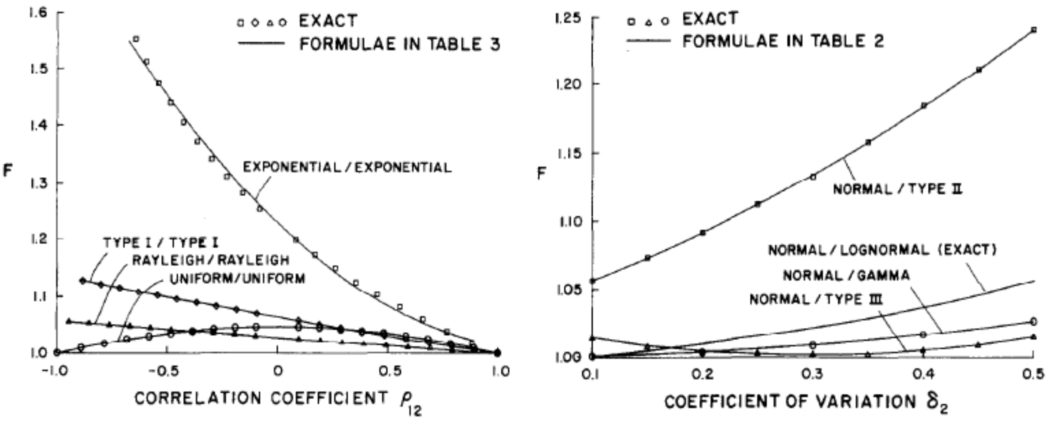
Note: $\rho = \rho_{ij}$; ranges of coefficients of variation are $\delta_i, \delta_j = 0.1-0.5$

Fig.4.6. Category 5 formulae, Multivariate distribution models, P-L. Liu and A. Der Kiureghian

The range of coefficients of variation used in generating the formulae in Tables .. is 0.1-0.15. For values outside this range, the errors in the formulae can be larger than those listed in these tables.

As demonstrations of the formulae in Tables .. , plots of F for several selected pairs of marginal distributions are shown in Figs.. together with the exact results obtained for numerical integration of equation ().

Figs … represent typical cases, whereas Fig. .. represents the case with poorest agreement with exact results (maximum error = 4.5%).



Selected plots of F for X_1 and X_2 both belonging to group 1   *F for X_2 belonging to group 2 and X_1 normal*

Fig.4.7. Typical cases, Multivariate distribution models, P-L. Liu and A. Der Kiureghian

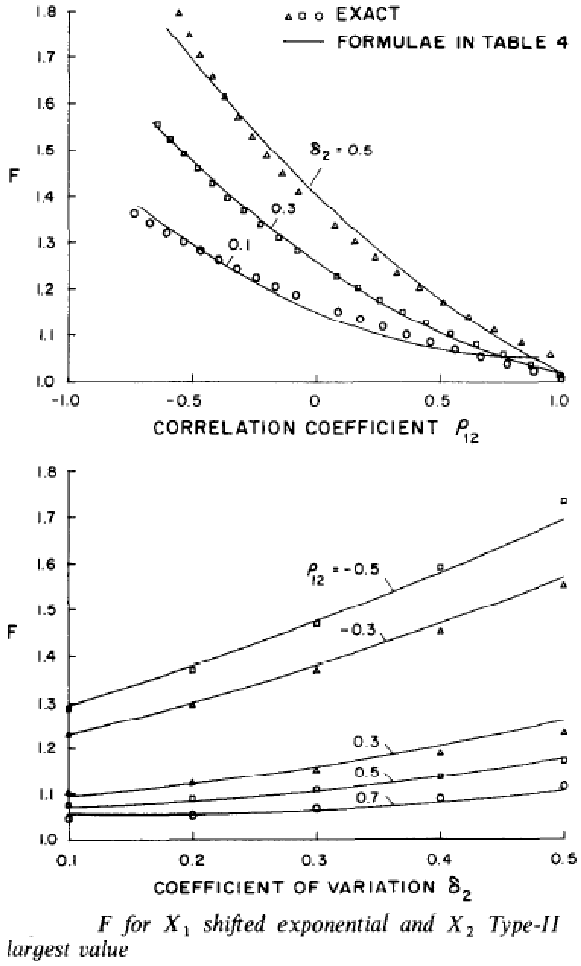


Fig4.8., Case with poorest agreement with exact results, Multivariate distribution models, P-L. Liu and A. Der Kiureghian

For most ditributions F is only slightly higher than unity. However, it can be high as 1.5 or greater when X_i and X_j are negatively correlated and their marginals are skewed in the same direction or they are positively correlated and their marginal are skewed in opposite directions.

4.4. The design point research

The Hasofer-Lind indice β_{BL} is defined in simple words as the minimal distance between the failure surface and the origin of the Gaussian Sapce. The calculation of β_{BL} consists in solving the following problem of optimisation under constraint:

$$\min_{G(y)=0} \sqrt{y^t y}$$

The point associated with this minimal distance is often called the design point.

Let the limit-state function in the space of input variabes $x_1, ..., x_n$ be given by the equation $g(x_1,...,x_n)=0$ and let the input variables be random variables collected in the vector X with the second moment representation E[X] and $Cov[X, X^T]$. To generalize the simple reliability index to normalized random variables $Y_1,...,Y_n$ are introduced by a suitable one to one inhomogeneus linear mapping $X = L(Y), Y = L^{-1}(X)$.

For example, this linear mapping may be composed of a parallel shift and rotation of the coordinate systm followed by an axis parallel affinity.

The corresponding space of points y is then defined by the transformation

$x = L(y)$, $y = L^{-1}(x)$

By this the limit-state equation is mapped into the equation

$h(y_1,...,y_n) = 0$

Where the function *h* is defined by

$$h(y) = g[L(y)] \tag{4.8}$$

Equation 1 defines the limit-state surface in the normalized space. The mean value of **Y** is at the origin and the projection of **Y** on the arbitrary straight line through the origin is a random variable with the standard deviation 1. The geometric reliability index β is then defined as the distance in the normalized space from the origin to the limit surface, that is,

$$\beta = \min\{\sqrt{y^T y} \mid h(y) = 0\}$$

Where the minimum of the distance $\sqrt{y^T y}$ is obtained for **y** varying over the entire limit state surface $h(y) = 0$. (Unsaully the limit-state surface is a closed but not necessarily bounded set in $\mathbb{R}^n$. Therefore the operation “min”is written in stead of the more general “inf”).

A point y on the limit state surface with $\beta = \sqrt{y^T y}$ is called a globally most central limit-state point. There may exist several such globally most central limit-state points.

In particular the limit-state surface may have an infinity of points common with a sphere surface with center at the origin and radius β .

A point z with the property that there is an open neighborhood N(z) or z such that

$$\sqrt{z^T z} = \min\{\sqrt{y^T y} \mid y \in N(z), h(y) = 0\} \tag{4.9}$$

Is called a locally most central limit state point. Obviously the globally the most central limit-state points should be sought among the locally most central limit-state points.

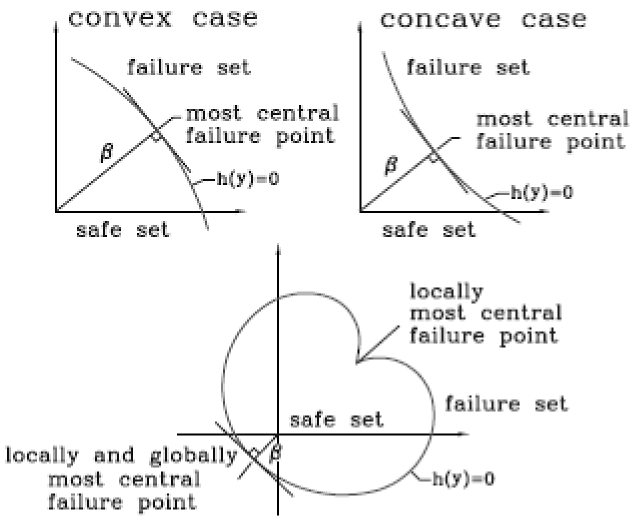


Fig.4.9. Locally and globally most central limit-state point

FORM, SORM

The mapping of the limit state function onto the standard probabilistic transformation $Y = y(x)$ is described by:

$$g(S) = g(S(x)) = g(Sy^{-1}(Y)) = G(Y) \tag{4.10}$$

Hence the probability of failure can be rewritten as:

$$P_f = \int_{G(y)\leq o} \varphi(y)dy \tag{4.11}$$

Where $\varphi(Y)$ denotes the standarad normal PDF of **Y:**

$$\varphi = \frac{1}{(2\pi)^{n/2}} \exp(-\frac{1}{2} \| y \|^2) \tag{4.12}$$

This PDF has two interesting properties, namely it is rotationally and decays exponentially with the square of the norm $\| y \|$. Thus the points making segnificant contributions to the integral $P_f = \int_{G(y)\leq o} \varphi(y)dy$ are those with nearest distance to the origin of the standard normal space.

This leads to the definition of the reliability index β :

$$\beta = \alpha^T y^*$$

$$y^* = \arg\min\{\|y\| \mid G(y) \le 0\}$$

The solution y^* of the constrained optimization problem $\beta = \alpha^T y^*$ is called the design point or the most likely failure point in the standard normal space.

When the limit state function $G(y)$ is linear in y, it is easy to show that:

$$P_f = \Phi(-\beta)$$

where $\Phi(.)$ is the standard normal CDF.

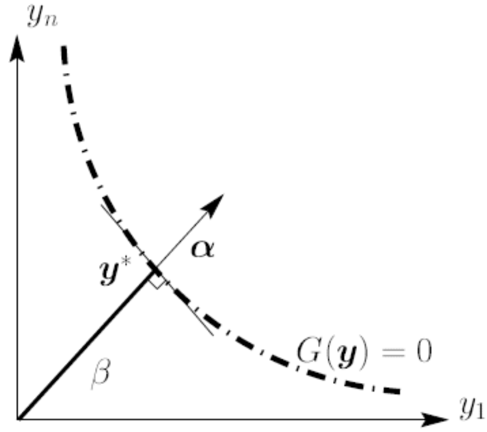


Fig.4.10. Geometrical definition of the design point

When $G(y)$ is non-linear, the *First Order Approximation Method* (FORM) consist in:

- evaluating the reliability index β by solving $\beta = \alpha^T y^*$
- obtaining an approximation of the probability of failure by:

$$P_f \approx P_{f_I} = \Phi(-\beta)$$

Geometrically, this is equivalent to replacing the failure domain by the halfspace outside the hyperplane tangent to the limit state surface at $y = y^*$.

Generally speaking , FORM becomes a better approximation when β is large.

To enhance the precision of $P_f \approx P_{f_I} = \Phi(-\beta)$, *Second Order Approximation Method* (SORM) have been proposed. The idea is to replace the limit state surface by a quadratic surface whose probabilistic content is known analytically.

Two kinds of approximations are usually used, namely the *curvature fitting* which requires the second derivate of $G(y)$ at the design point y^* and the *point fitting* where semi-paraboloids interpolate the limit state surface at given points around the design point.

4.5. Polyhedral approximation at singular points on the limit-state surface (single-point multiple FORM or SORM)

Often certain points on the limit-state surface are singular in the sense that they are intersection points between several differentiable surfaces $\partial S_1,...,\partial S_m$. The sets $S_1,...,S_m$ make up the safe set S for example as the intersection

$$S = \cap_{i=1}^{m} S_i$$

While the failure set is the union

$$F = \cup_{i=1}^{m} F_i$$

where *F* is the complementary set to *S*. This situation is relevant if the limit state is passed if just one of the events $F_1,...,F_m$ occurs. This corresponds to a situation where several different elements function together as series system (chain system) in the sense that failure of the system occurs if just one of the elements fails, that is, if just one link in the chain fails.

It is easy to see that if $\partial S_1,...,\partial S_m$ are differentiable surfaces, then no singular points on ∂S can be locally most central points in case S is defined as the intersesction.

However, this can very well be the case if *S* is defined as the union

$$S = \cup_{i=1}^{m} S_i$$

so that the failure set becomes the intersection

$$F = \cap_{i=1}^{m} F_i$$

This definition of *S* corresponds to a situation where the individual elements work together as a parallel system. Failure on the system requires that all elements fail. At this place we will not go further into system consideration.

Such consideration play an essential role for evaluation of the reliability of statically indeterminate structures.

Let us assume that *q* of the *m* limit-state surface $\partial S_1,...,\partial S_m$ have a nonempty intersection and let us for the sake of simplicity assume that they correspond to the *q* first indices. We will denote the set

$$\kappa = \partial S_1 \cap ... \cap \partial S_2$$

as a rigide of *S* (or of *F*). Choose a point $x \in \kappa$ and replace ∂S_i with the tangent hyperplane ∂H_{ix} to ∂S_i at *x* and let H_{ix} be the half-space with the boundary ∂H_{ix} that approximates F_i for $i = 1,...,q$.

$\cap_{i=1}^{q} F_i$ by $\cap_{i=1}^{q} H_{ix}$

then leads to the approximation

$$P(\cap_{i=1}^{q} F_i) \approx \Phi_q(-\beta; Corr[M, M^T])$$

where Φ_q is the distribution function for the *q*-dimension normal distribution.

In $P(\cap_{i=1}^{q} F_i) \approx \Phi_q(-\beta; Corr[M, M^T])$ the vector $M = (M_{1,x}, ..., M_{q,x})$ is defined as the vector of linear safety margins that correspond to the *q* tangent hyperplanes $\partial H_{1x}, ..., \partial H_{qx}$ at *x* , and β is the corresponding vector of the simple reliability indices.

The most central point of the ridge κ will normally be a good choice of the approximation point x. It is noted that the point generally will not be a locally most central point on ∂S .

Chapter 5. Results

A fragility curve is defined as the conditional probability of attaining or exceeding a specific damage state (failure criterion) for a given set of input variables.

More specificaly, the fragility of a failure criterion is defined as the given criterion limit (demand) is equal or exceeds the performance criterion(capacity).

In our case, the failure criterion represent the damage stage when the total stress loss (the capacity of slab or performance criterion) is equal or higher than the maximum stress in our element (the demand or the given limit). We name this a treshold.

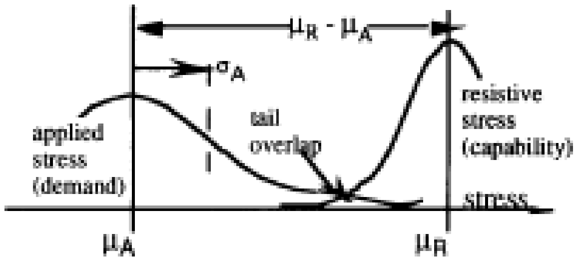


Fig.5.1.Structural failure concept

Thus, we want determine the probability of having the occurance of a of the total stress losses less or equal to a certain threshold:

$$P(C \leq D \mid D) = P(g(x,\sigma) \leq 0 \mid \sigma)$$

The corresponding probability have been computed by using the methods of structural system reliability.

In the context of this paper, we used three methods: Monte Carlo Simulation, FORM and SORM.

The method of Monte-Carlo is used to build *pdf* (probability density function), but also to assess the reliability of components or structures or to evaluate the sensitivity of parameters.

More explicitly, Monte Carlo Simulation consist of drawings samples of the basic variables according to their probabilistic characteristics and then feeding them into the performance function.

First Order Reliability Method (FORM) is a semi-probabilistic reliability analysis method developed to assess the reliability of a system.

The method consist in:

- the transformation of the space of the basic random variables $X_1, X_2, \ldots, X_3$ into a space of standard normal variables,
- the research, in this transformed space, of the point of the minimum distance from the origin on the limit state surface (this point is called the design point),
- an approximation of the failure surface near the design point,
- a computation of the failure probability corresponding to the approximation failure surface.

The difference between First Order Reliability Method and Second Order Reliability Method (SORM) is that SORM includes the correction for curvature of the limit state surface at the most central point ,and FORM doesn't.

The fragility curves have been obtained using the probabilistic capacity model (computed in FERUM, open-spurce Matlab) by variating the given parameter σ and time t set at different values.

Table 5.1. Fragility curves obtained by variating the given stress σ and time set equal to 7 days

Tension, σ	FORM	SORM	MCS	Reliability Index β	(-β + σβ)	(-β - σβ)	P_{up}	P_{down}
120	5.00E-16	2.72E-16	3.71E-16	8.03E+00	-6.79	-9.262	5.61E-12	1E-20
130	8.33E-15	4.65E-15	5.61E-15	7.67E+00	-6.441	-8.908	5.93E-11	2.6E-19
140	1.16E-13	6.63E-14	7.75E-14	7.33E+00	-6.099	-8.559	5.34E-10	5.69E-18
150	1.37E-12	8.04E-13	9.69E-13	6.99E+00	-5.764	-8.216	4.11E-09	1.05E-16
160	1.39E-11	8.33E-12	9.61E-12	6.66E+00	-5.437	-7.879	2.71E-08	1.65E-15
170	1.20E-10	7.41E-11	8.43E-11	6.33E+00	-5.117	-7.549	1.55E-07	2.19E-14
180	8.94E-10	5.68E-10	6.73E-10	6.02E+00	-4.807	-7.225	7.66E-07	2.51E-13
190	5.76E-09	3.78E-09	4.37E-09	5.71E+00	-4.506	-6.908	3.3E-06	2.46E-12
200	3.23E-08	2.19E-08	2.58E-08	5.41E+00	-4.213	-6.598	1.26E-05	2.08E-11
210	1.59E-07	1.12E-07	1.29E-07	5.11E+00	-3.93	-6.295	4.25E-05	1.54E-10
220	6.89E-07	5.03E-07	6.11E-07	4.83E+00	-3.657	-5.999	0.000128	9.93E-10
230	2.65E-06	2.01E-06	2.38E-06	4.55E+00	-3.393	-5.712	0.000346	5.58E-09
240	9.13E-06	7.21E-06	8.65E-06	4.29E+00	-3.138	-5.432	0.000851	2.79E-08
250	2.65E-05	2.13E-05	2.85E-05	4.04E+00	-2.835	-5.25	0.002291	7.6E-08
260	7.95E-05	6.48E-05	8.29E-05	3.78E+00	-2.573	-4.98	0.005041	3.18E-07
270	2.18E-04	1.80E-04	2.21E-04	3.52E+00	-2.321	-4.714	0.010143	1.21E-06
280	5.46E-04	4.57E-04	5.63E-04	3.27E+00	-2.076	-4.455	0.018947	4.19E-06
290	1.26E-03	1.07E-03	1.25E-03	3.02E+00	-1.839	-4.203	0.032958	1.32E-05
300	2.69E-03	2.33E-03	2.70E-03	2.78E+00	-1.61	-3.957	0.053699	3.79E-05
310	5.35E-03	4.75E-03	5.24E-03	2.55E+00	-1.388	-3.717	0.082569	0.000101
320	9.96E-03	9.05E-03	9.82E-03	2.33E+00	-1.173	-3.483	0.120398	0.000248
330	1.74E-02	1.62E-02	1.73E-02	2.11E+00	-0.965	-3.256	0.167272	0.000565
340	2.88E-02	2.72E-02	2.73E-02	1.90E+00	-0.763	-3.034	0.222732	0.001207
350	4.52E-02	4.32E-02	4.34E-02	1.69E+00	-0.569	-2.819	0.284678	0.002409
360	6.75E-02	6.45E-02	6.50E-02	1.49E+00	-0.38	-2.61	0.351973	0.004527
370	9.65E-02	9.15E-02	9.43E-02	1.30E+00	-0.198	-2.406	0.421523	0.008064
380	1.33E-01	1.24E-01	1.28E-01	1.11E+00	-0.021	-2.208	0.491623	0.013622
390	1.75E-01	1.64E-01	1.66E-01	9.33E-01	0.15	-2.015	0.559618	0.021952
400	2.25E-01	2.11E-01	2.15E-01	7.56E-01	0.316	-1.828	0.623999	0.033775
450	5.21E-01	5.01E-01	4.93E-01	-5.37E-02	1.071	-0.963	0.857915	0.167774
500	7.76E-01	7.64E-01	7.85E-01	-7.59E-01	1.72	-0.203	0.957284	0.419568
600	9.73E-01	9.71E-01	9.84E-01	-1.93E+00	2.776	1.077	0.997248	0.85926
700	9.98E-01	9.98E-01	9.98E-01	-2.86E+00	3.595	2.118	0.999838	0.982912
750	9.99E-01	9.99E-01	1.00E+00	-3.26E+00	3.94	2.7	0.999959	0.996533

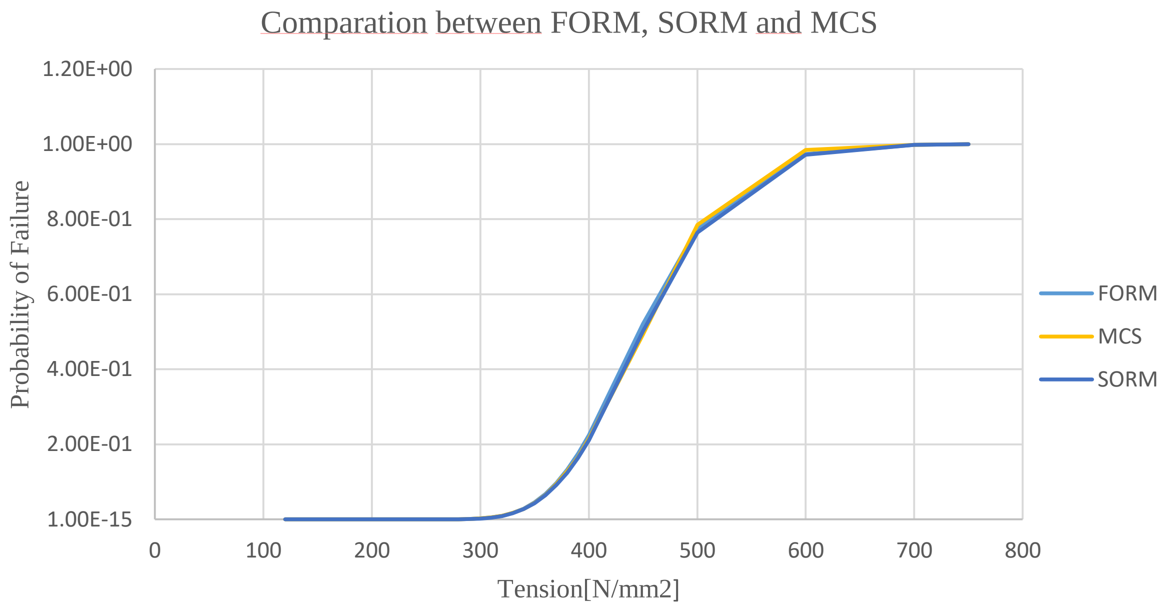


Fig.5.2. Comparation between Fragility Curves, obtained by varying the given tension σ and time t set equal to 7 days

The fragility curves obtained by propagating Monte Carlo Simulation, FORM and SORM are smooth.

In other words, the distribution of the input variables which we designed is a good data baised distribution.

Regarding the inherent uncertainty in the fragility estimate due to the epistemic uncertainties, which is reflected in the probability distribution of relative parameters, we checked the accuracy for FORM method.

For this, we computed the upper and lower bound for the generalized reliability index β.

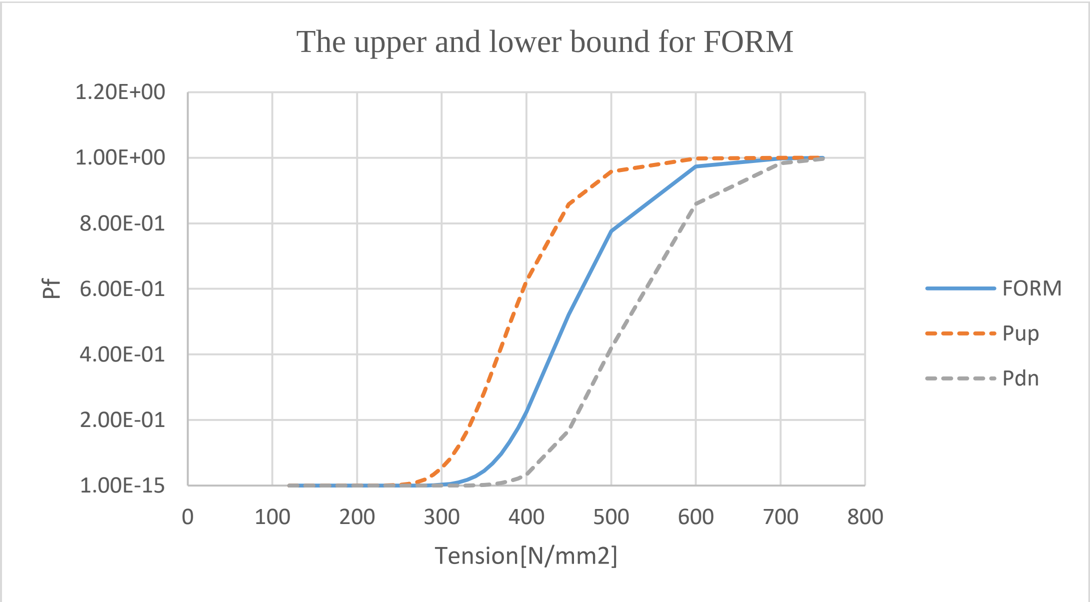


Fig.5.3. The upper and lower bound for FORM, for time set equal to 7 days.

The bounds aproximately correspond to 15% and 85% probability levels.

In order to observe how the element behaves in terms of stress losses during its life time, we computed the fragility curves for different periods of time (7 days, 30 days, 1 year, 5 years, 10 years).

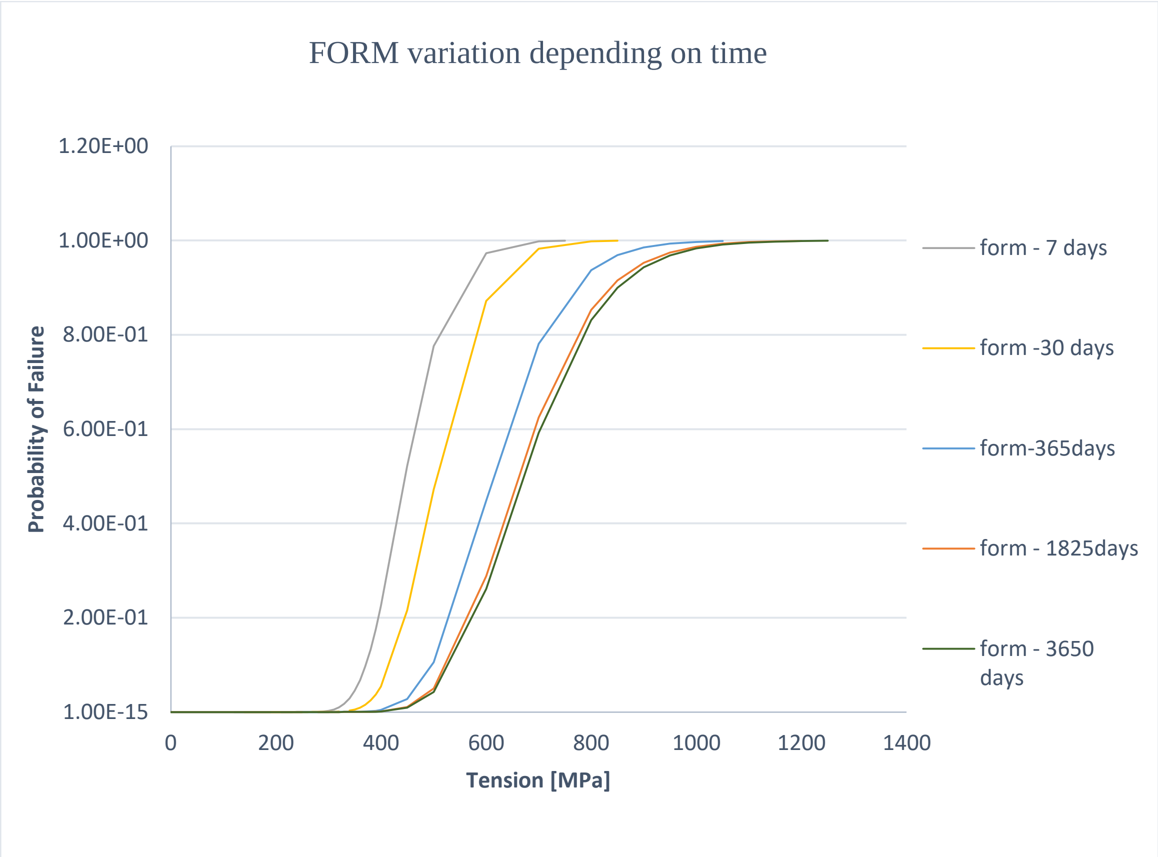


Fig.5.4. FORM variation depending on time

Like we were expected, the losses are increasing due to long-term losses, but after an important period of time, they are starting to stabilize (in our case, after 5 years).

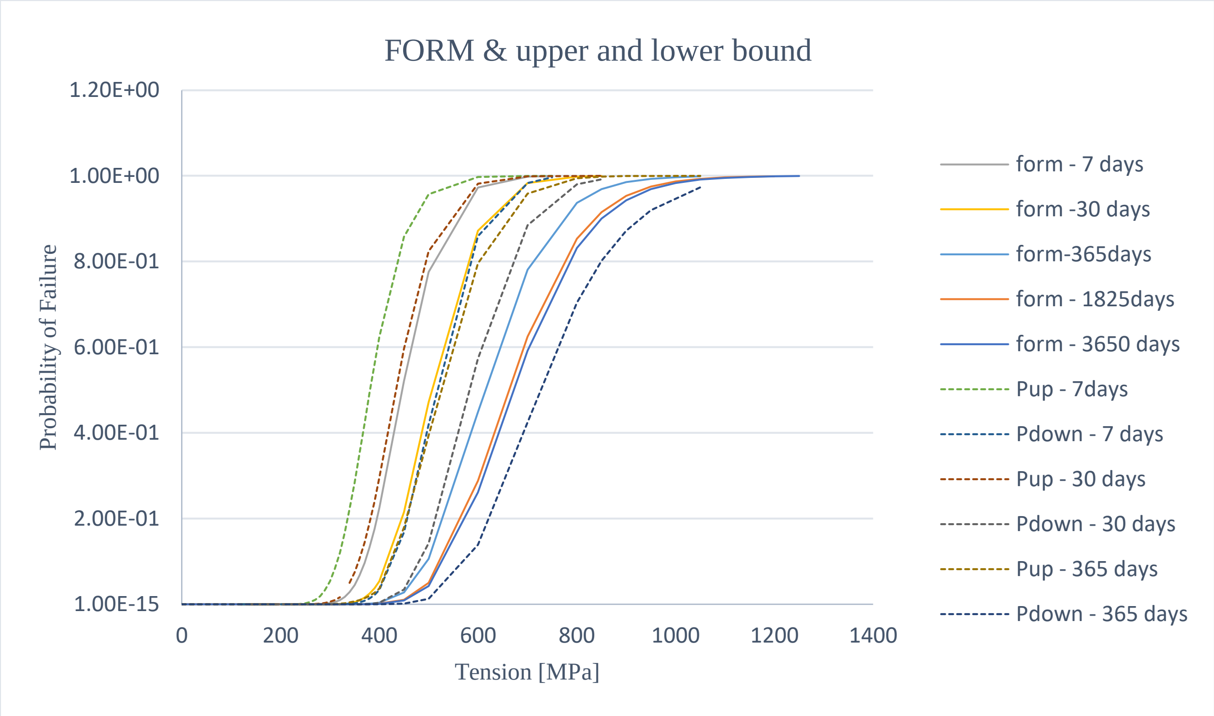


Fig.5.5. FORM & Upper and lower bound variation depending on time

Conclusions

The Monte Carlo simulation method is completely general, and apply to any distribution of the basic random variables, including discrete random variables. Furthermore, there is no requirements on the failure function – only the sign of the failure function is being used.

FORM and SORM are analytical and approximate methods, and their accuracy is generally good for small probabilities (10^{-3}-10^{-8}).

For small order probabilities FORM/SORM are extremely efficient as compared to simulation methods, if the number of random variables is not too high.

For big order probabilities, difficulties in the research of design point are encountered.

In order to solve this problem, we changed the start point, which stands for the starting point of the FORM analysis in the physical space.

The failure functions all three methods are smooth like form.

In the following figure, the advantages and drawbacks are listed.

Simulations	FORM/SORM
RESULTS	***RESULTS***
FAILURE PROBABILITY	**FAILURE PROBABILITY**
ERROR ON THE ESTIMATION	**MOST INFLUENTIAL VARIABLES (PROBABILITY)**
PROBABILITY DISTRIBUTION OF THE RESPONSE	**EFFICIENCY** **(depends on the number of random variables)**
ASSUMPTIONS	***ASSUMPTIONS***
NO ASSUMPTION ON THE RANDOM VARIABLES (DISCRETE, CONTINOUS, DEPENDANCY…)	**CONTINOUS RANDOM VARIABLES**
NO ASSUMPTION ON THE LIMIT STATE FUNCTION	**CONTINOUS LIMIT STATE FUNCTION**
DRAWBACKS	***DRAWBACKS***
COMPUTATION COSTS **(depends on the probability level)**	**NO ERROR ON THE ESTIMATION**
	GLOBAL MINIMUM

Fig.6.1.Comparison of the characteristics of reliability methods

Appendix

Tension losses Matlab Code

```
% Slab tension loss
clear probadata femodel analysisopt gfundata randomfield systems output_filename
output_filename = 'outputfile_tensionloss.txt';
probdata.marg(1,:)= [ 2  1420 284  15 0 0 0 0 0 ]; %fpd
probdata.marg(2,:)= [ 2  0.436 0.087  0.05 0 0 0 0 0 ]; %Teta i
probdata.marg(3,:)= [ 2  195000   5850     8  0 0 0 0 0 ]; %Ep
probdata.marg(4,:)= [ 2  34000   6800      2   0 0 0 0 0 ]; %Ecm
probdata.marg(5,:)= [ 2  600  120      10   0 0 0 0 0 ]; %Ap
probdata.marg(6,:)= [ 2  240000   48000      5   0 0 0 0 0 ]; %Ac
probdata.marg(7,:)= [ 1  70   14      1   0 0 0 0 0 ]; %Zcp
probdata.marg(8,:)= [ 7  0.5   0.224      0.5   0 0 0 1 0 ]; %RH
probdata.marg(9,:)= [ 2  22.67   5      5   0 0 0 0 0 ]; %fck
probdata.marg(10,:)= [ 2  2480   496     2   0 0 0 0 0 ]; %u
probdata.marg(11,:)= [ 1  100   20      0   0 0 0 0 0 ]; %increase of the stress,sigma pr
probdata.marg(12,:)= [ 1  6.8   1.36      0  0 0 0 0 0 ]; %stress in the concrete adjacent to the
tendons,sigma c
probdata.marg(13,:)= [ 2  1152000000   230400000     50  0 0 0 0 0 ]; %Ic

analysisopt.ig_max=100;
analysisopt.il_max=5;
analysisopt.e1=0.001;
analysisopt.e2=0.001;
analysisopt.step_code=0;
analysisopt.grad_flag= 'ffd';
analysisopt.sim_point= 'dspt';
analysisopt.stdv_sim =1;
analysisopt.num_sim = 100000;
analysisopt.target_cov = 0.0125;

gfundata(1).evaluator = 'basic';
gfundata(1).type = 'expression';
gfundata(1).parameter = 'no';
gfundata(1).expression = '0.8*x(1)*0.2*(x(2)+0.01*7)+ (0.8*x(1)*(1-exp(-
0.2*(x(2)+0.01*1))))+ x(3)*((0.5*x(12))/x(4))+ (x(3)*((7-2)/((7-2)+
0.04*((2*x(6)/x(10))^0.667))*0.85*0.00039+(1-exp(-0.2*7^0.5))*(2.5*(x(9)-10)*10^-
6)))/(1+(x(3)/x(4))*(x(5)/x(6))*(1+(x(5)/x(13))*x(7)^2))+(x(12)*((x(3)/x(4))*((1+(1-
x(8)))/(0.1*(2*x(6)/x(10))^0.333))*(16.8/(x(9))^0.5)*(1/(0.1+2^0.2))*((7-
2)/(1.5*(1+(0.012*x(8)*100)^18)*(2*x(6)/x(10))+250+7-
2))^0.3))/(1+(x(3)/x(4))*(x(5)/x(6))*(1+(x(5)/x(13))*x(7)^2))+
x(11)/(1+(x(3)/x(4))*(x(5)/x(6))*(1+(x(5)/x(13))*x(7)^2))-200';
gfundata(1).dgdq = '{1}';
femodel=0;
randomfield.mesh=0;
```

References

[1] O. Ditlevsen and H.O. Madsen, Structural Reliability Methods, July 2005

[2] http://www.ce.berkeley.edu/projects/ferum/

[3] Bruno Sudret and Armen Der Kiureghian, Stohastic Finite Element Methods and Reliability – A State of the Art Report, November 2000

[4] Sami Khan and Martin Williams, Post-tesioned Concrete Floors, 1995

[5] Radu Văcăreanu, Alexandru Aldea, Dan Lungu, Structural Reliability and Risk Analysis, Lecture Notes, 2007

[6] Paolo Gardoni, Amer Der Kiureghian and Khalid M. Mosalam, Probabilistic Models and Fragility Estimates for Bridge Components and Systems, Department of Civil and Environmental Engineering University of California, Berkeley, PEER 2002/ 13 June 2002

[7] Navid H. Jafari, System Reliability of Inner Harbour Navigation Canal Floodwalls during Hurrincane Katrina, Department of Civil and Enviromental Engineering, University of Illinois, Urbana

[8] Eurocode 2, Design of concrete structures, Part 1-1 – General rules and rules for buildings

[9] P. Bamforth, D. Chishlom, J. Gibbs and T. Harrison, How to optimise the engineering properties of concrete in design to Eurocode 2

[10] Nicolas Devictor, Advances in Methodes for Uncertainty and Sensitivity Analysis, CEA/Cadarache, DEN/CAD/DER/SESI/LCFR

[11] Eric C. Anderson, Monte Carlo Methods and Importance Sampling, Lecture Notes – Statistical Genetics, Department of Civil and Environmental Engineering University of California, Berkeley, October 1999

[12] Pei-Ling Liu and Armen Der Kiureghian, Multivariate distribution models with prescribed marginal and covariances – Probabilistic Engineering Mechanics Vol.1-No.2, Department of Civil and Environmental Engineering University of California, Berkeley, 1986