Development Economics, Homework 2 - CEMFI

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Question 1, Part 1

(a) Compute the welfare gains of removing the seasonal component from the stream of consumption separately for each degree of seasonality. **(η = 1)**

We need to compute the welfare gains of moving from the consumption with all the shocks to the consumption after removing the seasonal component. Following the class notes, I want to find the compensation that should be given to an individual (all months and all years) facing seasonality such that his or her life-time utility is the same as without seasonality. Therefore, the compensation must satisfy the following equation:

$$\sum_{t=1}^{40} \beta^{12t} \sum_{m=1}^{12} \beta^{m-1} u((1+comp.) z_i e^{g(m)} e^{-\sigma_\varepsilon^2/2} \varepsilon_{t,i}) = \sum_{t=1}^{40} \beta^{12t} \sum_{m=1}^{12} \beta^{m-1} u(z_i e^{-\sigma_\varepsilon^2/2} \varepsilon_{t,i})$$

Welfare gains of removing the seasonal component from the stream of consumption separately for each degree of seasonality:

Low: 0.0042
Mid: 0.0086
High: 0.0171

Note that the seasonal component is the same across individuals, but it is different in each season. Therefore, in each degree of seasonality the compensation is the same for all individuals. The negative impact is more pronounced when the seasonality is higher. Therefore, the welfare gains of removing the seasonal component are increasing with the degree of seasonality.

(b) Compute the welfare gains of removing the non-seasonal consumption risk.

To compute the welfare gains of removing the non-seasonal component from the stream of consumption, I find the compensation that should be given to individuals facing the non-seasonal component (shocks do not depend on the degree of seasonality) such that his or her life-time utility is the same as without it. Therefore, the compensation must satisfy the following equation:

$$\sum_{t=1}^{40} \beta^{12t} \sum_{m=1}^{12} \beta^{m-1} u((1+comp.) z_i e^{g(m)} e^{-\sigma_\varepsilon^2/2} \varepsilon_{t,i}) = \sum_{t=1}^{40} \beta^{12t} \sum_{m=1}^{12} \beta^{m-1} u(z_i e^{g(m)})$$

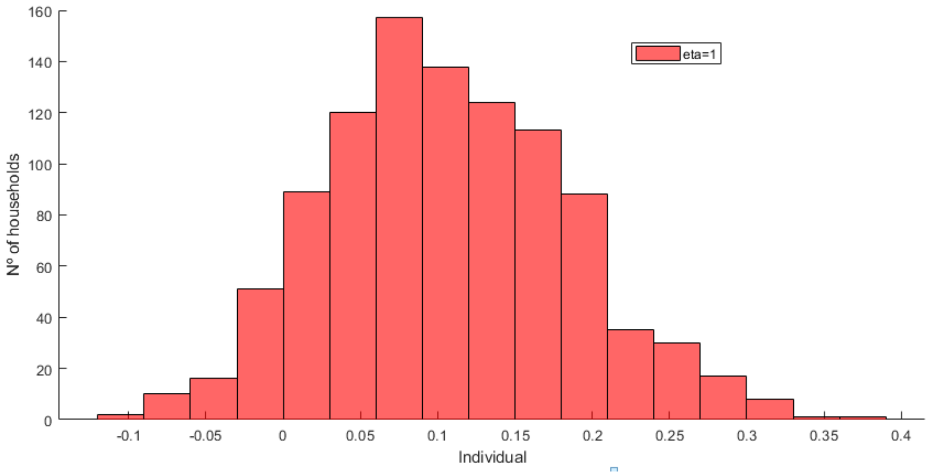
In this case there is going to be a different result for each individual, given that the realization of individual shocks is different across individuals. Therefore, I present the mean value, the standard errors and a histogram to have an intuition of the distribution.

Average and standard deviation of the welfare gains of removing non-seasonal consumption risk:

Mean: 0.1082; Standard errors: 0.0792

The following graph presents the histogram of the welfare gains of removing non-seasonal consumption risk. We can see that depending on the individual the compensation needed. Note that there are a few individuals for which the value is negative, implying they have received a positive shock and are therefore are worst-off by removing the non-seasonal component.

Histogram: Welfare gains of removing non-seasonal consumption risk



(c) Compare and discuss your results in (a) and (b).

In part (a) we removed the same risk for all individuals for each degree of seasonality. The compensation required is higher in the case that the deterministic seasonal component is higher. All individuals are better by removing the seasonal risk.

In part (b) we removed the stochastic component risk (different for each consumer), calculating the mean and standard deviation. In this case the compensation is different for each individual. Most individuals are better by removing the seasonal risk (some exceptions).

(d) Redo for $\eta = 2$ and $\eta = 4$.

(d.a) Compute the welfare gains of removing the seasonal component from the stream of consumption separately for each degree of seasonality.

Mean welfare gains of removing seasonality ($\eta = 2$)

Low: 0.0066

Mid: 0.0185

High: 0.0601

Mean welfare gains of removing seasonality **(η =4)**

Low: 0.0118

Mid: 0.0426

High: 0.1867

The higher the η the more risk adverse the individual is (we can see this from the utility function of the agent). Therefore, given a particular season, the higher the η the higher the compensation needed to make the individual indifferent.

(d.b) Compute the welfare gains of removing the non-seasonal consumption risk.

As discussed before, the welfare gains will be different for each individual and the same for any level of seasonality. Therefore, I present the mean value, the standard errors and a histogram to have an intuition of the distribution for **η = 2** and **η = 3**.

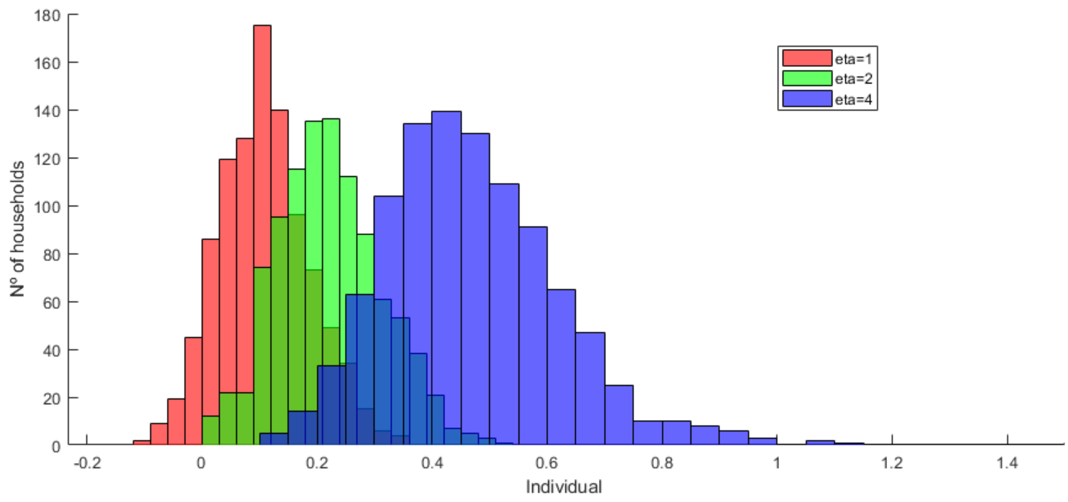
Average and standard deviation of the welfare gains of removing non-seasonal consumption risk:

η = 2: Mean: 0.2198 - Standard Deviation: 0.0913

η =4: Mean: 0.4746 - Standard Deviation: 0.1594

The following histogram presents the differences in welfare gains across individuals for η = 1, η = 2 and η = 4.

Histogram: Welfare gains of removing non-seasonal consumption risk



As expected, from the histogram we can see that for larger values of η (more risk adverse individuals), the welfare gains of removing the stochastic components of consumption is higher. Note that even though the distribution has a different mean, the shape is quite similar for the different values of η.

Question 1, Part 2: Now add a stochastic seasonal component to consumption.

(a) Compute the welfare gains of removing the seasonal component from the stream of consumption separately for each degree of seasonality

In this section, to avoid extra calculation, I establish the deterministic seasonal component to be always "medium degree of seasonality" and allow for the stochastic seasonal component to differ.

(i) To find the welfare gains of **removing only the deterministic seasonal component**, we get the compensation by solving the following equation:

$$\sum_{t=1}^{40}\beta^{12t}\sum_{m=1}^{12}\beta^{m-1}u((1+comp.)z_i e^{g(m)}e^{-\sigma_m^2/2}\varepsilon_{m,i}e^{-\sigma_\varepsilon^2/2}\varepsilon_{t,i}) = \sum_{t=1}^{40}\beta^{12t}\sum_{m=1}^{12}\beta^{m-1}u(z_i e^{-\sigma_m^2/2}\varepsilon_{m,i}e^{-\sigma_\varepsilon^2/2}\varepsilon_{t,i})$$

Welfare gains of removing deterministic seasonal component **(η = 1)**:

	Low	**Mid**	**High**
Mean	0.1066	0.1066	0.1066

As before, the result is the same in each season as the deterministic seasonal component have been removed.

(ii) To find the welfare gains of **removing only the stochastic seasonal component**, we get the compensation by solving the following equation:

$$\sum_{t=1}^{40}\beta^{12t}\sum_{m=1}^{12}\beta^{m-1}u((1+comp.)z_i e^{g(m)}e^{-\sigma_m^2/2}\varepsilon_{m,i}e^{-\sigma_\varepsilon^2/2}\varepsilon_{t,i}) = \sum_{t=1}^{40}\beta^{12t}\sum_{m=1}^{12}\beta^{m-1}u(z_i e^{g(m)}e^{-\sigma_\varepsilon^2/2}\varepsilon_{t,i})$$

There is going to be a different effect for each individual. Therefore, I present the mean value, the standard errors and a histogram to have an intuition of the distribution.

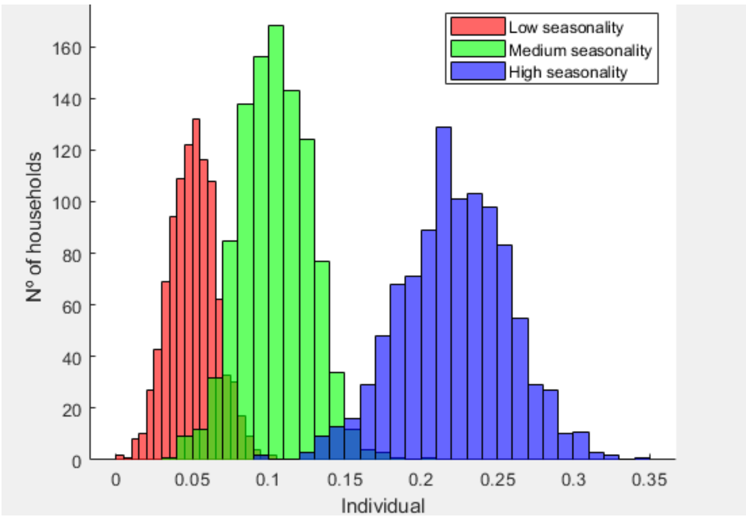
Average and standard deviation of the welfare gains of removing stochastic seasonal components for different level of variance of the stochastic component (**η = 1)**:

	Low	**Mid**	**High**
Mean	0.0505	0.1044	0.2223
Standard errors	0.0156	0.0227	0.0361

From the histogram and the table, we can see that the higher is the level of variance of the stochastic component, the higher are the welfare gains of removing seasonality. This makes sense as a risk adverse individual gains from reducing risk and be able to smooth consumption.

Histogram: Welfare gains of removing stochastic seasonal risk.

(For high, medium and low level of variance of the stochastic component)



(iii) To find the welfare gains of **removing both stochastic and deterministic components**, we get the compensation by solving the following equation:

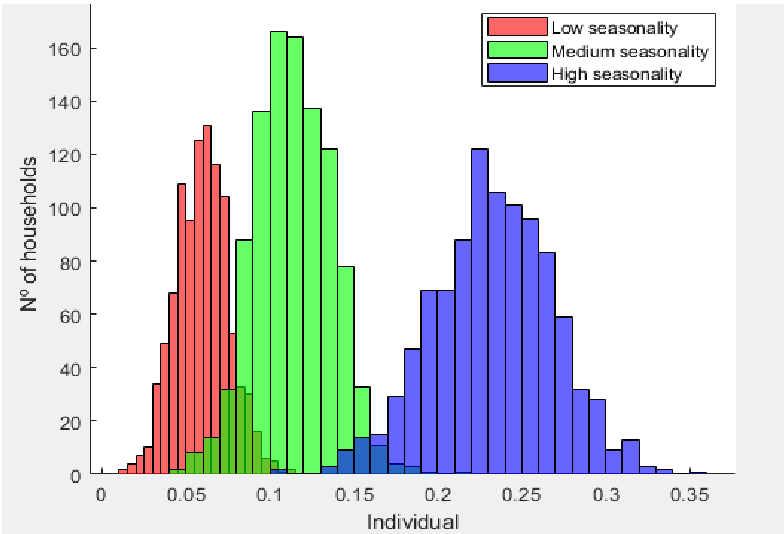
$$\sum_{t=1}^{40}\beta^{12t}\sum_{m=1}^{12}\beta^{m-1}u((1+comp.)z_i e^{g(m)}e^{-\sigma_m^2/2}\varepsilon_{m,i}e^{-\sigma_\varepsilon^2/2}\varepsilon_{t,i}) = \sum_{t=1}^{40}\beta^{12t}\sum_{m=1}^{12}\beta^{m-1}u(z_i e^{-\sigma_\varepsilon^2/2}\varepsilon_{t,i})$$

Average and standard deviation of the welfare gains of removing both components **(η = 1)**:

	Low	**Mid**	**High**
Mean	0.0596	0.1139	0.2328
Standard errors	0.0157	0.0229	0.0364

Histogram: Welfare gains of removing stochastic seasonal risk.

(For high, medium and low level of variance of the stochastic component)



(b) Compute

To find the welfare gains of removing non-seasonal consumption risk, we get the compensation by solving the following equation:

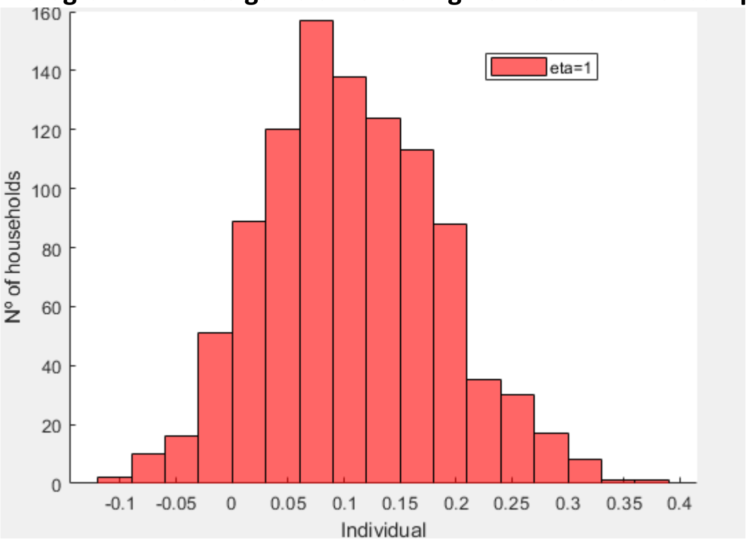
$$\sum_{t=1}^{40} \beta^{12t} \sum_{m=1}^{12} \beta^{m-1} u((1+comp.)z_i e^{g(m)} e^{-\sigma_m^2/2} \varepsilon_{m,i} e^{-\sigma_\varepsilon^2/2} \varepsilon_{t,i}) = \sum_{t=1}^{40} \beta^{12t} \sum_{m=1}^{12} \beta^{m-1} u(z_i e^{g(m)} e^{-\sigma_m^2/2} \varepsilon_{m,i})$$

Average and standard deviation of the welfare gains of removing the nonseasonal consumption risk components **(η = 1)**:

	Low	**Mid**	**High**
Mean	0.1066	0.1066	0.1066
Standard errors	0.0785	0.0785	0.0785

As before, the level of seasonality does not matter. From the histogram we can see the results for the different consumers.

Histogram: Welfare gains of removing non- seasonal consumption risk.



(c) Compare and discuss your results in (a) and (b)

Given that in part (a) we are removing the stochastic seasonal component, the higher is the seasonality variance the higher the welfare gains. However, in part (b) the seasonal component does not change and therefore the seasonality variance does not have any effect in the compensation.

As in part 1, when we remove some components it has different effect across individuals. Specifically, the stochastic seasonal component or the non-seasonal stochastic annual component of consumption are different for each individual, therefore the gains of removing them will be different.

(d) Redo for η = 2 and η = 4.

Mean welfare gains of removing both seasonal components.

	Low	**Mid**	**High**
η = 2	0.1211	0.2355	0.5060
η = 4	0.2429	0.5037	1.2742

Welfare gains of removing stochastic seasonal components.

(η =2)	**Low**	**Mid**	**High**
Mean	0.1008	0.2131	0.4787
Standard errors	0.0166	0.0283	0.0541

(η =4)	**Low**	**Mid**	**High**
Mean	0.1921	0.4423	1.1813
Standard errors	0.0393	0.0918	0.3353

Welfare gains of removing stochastic consumption risk

	Low	**Mid**	**High**
(η =2)	0.2209	0.2207	0.2221
(η =4)	0.4724	0.4659	0.4561

In all cases, the more risk adverse the individuals (higher η), in average the higher the gains of removing the seasonality risk.

Question 2. Adding Seasonal Labor Supply.

Calibrate κ to get the monthly hours that match the average weekly hours worked per adult in poor countries, as reported by Bick et al (2018).

(a) Assume a **deterministic seasonal component and a stochastic seasonal component** for labor supply both of which are **highly positively correlated** with their consumption counterparts. Compute the welfare gains of removing seasons isolating the effects of consumption and leisure.

In order to generate **highly positively correlated stochastic** components for consumption and labor it is assumed that $ln\ \varepsilon_{m,i}^{c}$ and $ln\varepsilon_{m,i}^{h}$ are distributed as follows:

$$\begin{pmatrix} ln\ \varepsilon_{m,i}^{c} \\ ln\ \varepsilon_{m,i}^{h} \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_m^2 & 0.04 \\ 0.04 & \sigma_m^2 \end{pmatrix}\right)$$

Moreover, it is assumed that $g(m)$ has the same values for consumption and labor. This assumption is in order to generate highly positively correlated deterministic seasonal components.

Once this is defined, to measure each effect, I find the compensation that should be given to each individual by solving the following equations that would leave them indifferent between each scenario (similar procedure than in Question 1):

Consumption effect:

$$\sum_{t=1}^{40} \beta^{12t} \sum_{m=1}^{12} \beta^{m-1} u\left((1+comp.) z^c{}_i e^{g(m)} e^{-\frac{\sigma_m^2}{2}} \varepsilon^c{}_{m,i} e^{-\frac{\sigma_\varepsilon^2}{2}} \varepsilon^c{}_{t,i}\, , z^h{}_i e^{g(m)} e^{-\frac{\sigma_m^2}{2}} \varepsilon^h{}_{m,i} e^{-\frac{\sigma_\varepsilon^2}{2}} \varepsilon^h{}_{t,i}\right)$$

$$= \sum_{t=1}^{40} \beta^{12t} \sum_{m=1}^{12} \beta^{m-1} u(z^c{}_i e^{-\frac{\sigma_\varepsilon^2}{2}} \varepsilon^c{}_{t,i}\, , z^h{}_i e^{g(m)} e^{-\frac{\sigma_m^2}{2}} \varepsilon^h{}_{m,i} e^{-\frac{\sigma_\varepsilon^2}{2}} \varepsilon^h{}_{t,i})$$

Labor effects:

$$\sum_{t=1}^{40} \beta^{12t} \sum_{m=1}^{12} \beta^{m-1} u\left((1+comp.) z^c{}_i e^{-\frac{\sigma_\varepsilon^2}{2}} \varepsilon^c{}_{t,i}\, , z^h{}_i e^{g(m)} e^{-\frac{\sigma_m^2}{2}} \varepsilon^h{}_{m,i} e^{-\frac{\sigma_\varepsilon^2}{2}} \varepsilon^h{}_{t,i}\right)$$

$$= \sum_{t=1}^{40} \beta^{12t} \sum_{m=1}^{12} \beta^{m-1} u(z^c{}_i e^{-\frac{\sigma_\varepsilon^2}{2}} \varepsilon^c{}_{t,i}\, , z^h{}_i e^{-\frac{\sigma_\varepsilon^2}{2}} \varepsilon^h{}_{t,i})$$

Labor effects:

$$\sum_{t=1}^{40} \beta^{12t} \sum_{m=1}^{12} \beta^{m-1} u\left((1+comp.) z^c{}_i e^{g(m)} e^{-\frac{\sigma_m^2}{2}} \varepsilon^c{}_{m,i} e^{-\frac{\sigma_\varepsilon^2}{2}} \varepsilon^c{}_{t,i}\, , z^h{}_i e^{g(m)} e^{-\frac{\sigma_m^2}{2}} \varepsilon^h{}_{m,i} e^{-\frac{\sigma_\varepsilon^2}{2}} \varepsilon^h{}_{t,i}\right)$$

$$= \sum_{t=1}^{40} \beta^{12t} \sum_{m=1}^{12} \beta^{m-1} u(z^c{}_i e^{-\frac{\sigma_\varepsilon^2}{2}} \varepsilon^c{}_{t,i}\, , z^h{}_i e^{-\frac{\sigma_\varepsilon^2}{2}} \varepsilon^h{}_{t,i})$$

From the following procedure we get:

Total effects	Low	Mid	High
Mean	0.1737	0.2579	0.3862
Standard errors	0.0301	0.0382	0.0477

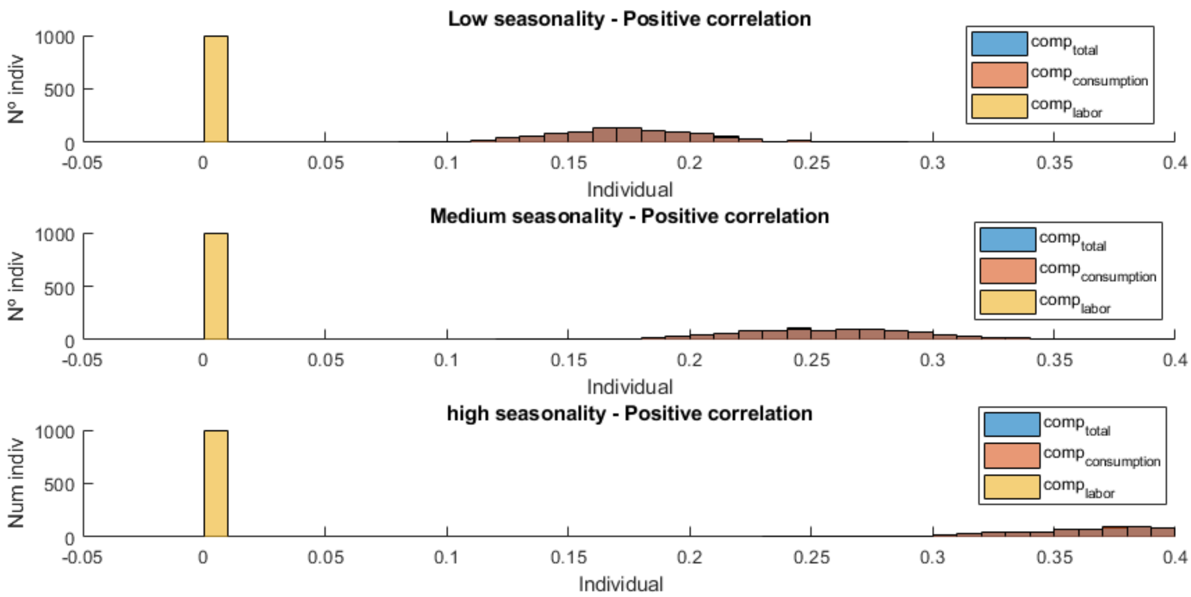
Consumption effects	Low	Mid	High
Mean	0.1737	0.2579	0.3861
Standard errors	0.0301	0.0382	0.0477

Labor effects	Low	Mid	High
Mean	0.0299	0.0425	0.0765
Standard errors	0.0419	0.0641	0.1375

As in the previous question, the higher is the season the higher are the gains of removing the risk. Note that the total effect and the consumption effect are almost the same (I find this a little strange). Moreover, the labor effects are much smaller compared to the consumption and total effects.

The following graph presents the histogram of the welfare gains in each case (depending on the individual the compensation needed).

Histogram: Welfare gains of removing both seasonal risks – Positive correlation.



(b) Assume a **deterministic seasonal component and a stochastic seasonal component** for labor supply both of which are **highly negatively correlated** with their consumption counterparts. Then, compute the welfare gains of removing seasons isolating the effects of consumption and leisure.

To solve this part, I follow the same procedure that in part (a), but in order to generate **highly positively correlated stochastic** components for consumption and labor it is assumed that $ln\ \varepsilon_{m,i}^{c}$ and $ln\varepsilon_{m,i}^{h}$ are distributed as follows:

$$\begin{pmatrix} ln\ \varepsilon_{m,i}^{c} \\ ln\ \varepsilon_{m,i}^{h} \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_m^2 & -0.04 \\ -0.04 & \sigma_m^2 \end{pmatrix}\right)$$

Moreover, it is assumed that $g(m)$ have opposite values for consumption and labor. This assumption is in order to generate highly negatively correlated deterministic seasonal components.

From the following procedure we get**:**

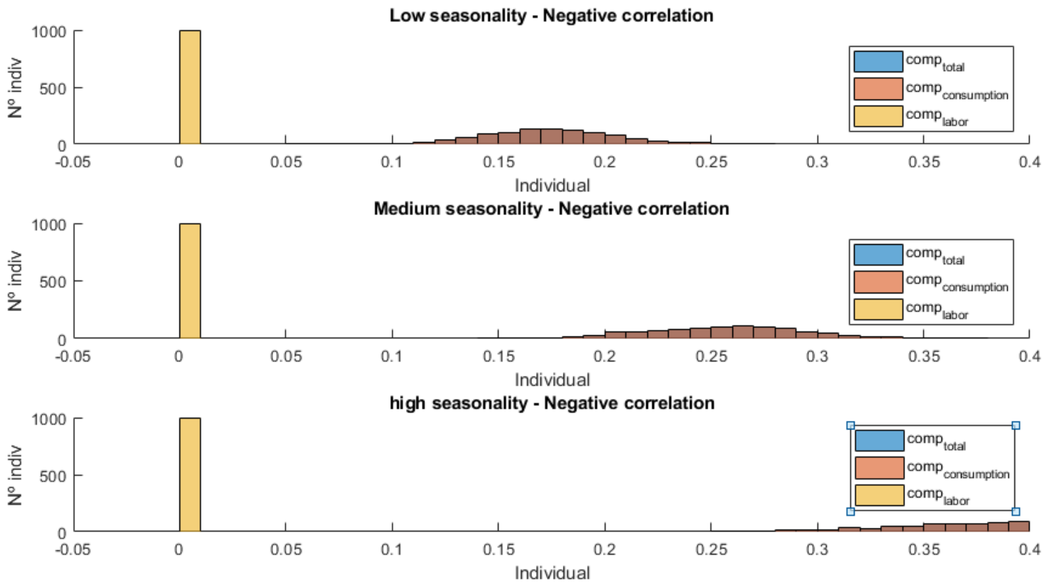
Total effects	**Low**	**Mid**	**High**
Mean	0.1741	0.2570	0.3897
Standard errors	0.0296	0.0383	0.0508

Consumption effects	**Low**	**Mid**	**High**
Mean	0.1741	0.2569	0.3896
Standard errors	0.0296	0.0383	0.0508

Labor effects	**Low**	**Mid**	**High**
Mean	1.0e-03 * 0.0302	1.0e-03 *0.0456	1.0e-03 *0.0780
Standard errors	1.0e-03 * 0.0444	1.0e-03 * 0.0810	1.0e-03 * 0.1423

Results are very similar to part (a). The higher is the season the higher are the gains of removing the risk. Again, note that the total effect and the consumption effect are almost the same and that the labor effects are much smaller compared to the consumption and total effects.

Histogram: Welfare gains of removing both seasonal risks – Positive correlation.



(c) How do your answers to (a) and (b) change if the nonseasonal stochastic component of consumption and leisure are correlated?

Results from part (a) and (b) suggest that the correlation between the seasonal component of consumption and labor does not play an important role as the results are very similar. This result is in line with the fact that the utility function considers each factor separately, not accounting for any complementarity between consumption and labor.