Computer Vision – TP3
Digital Images

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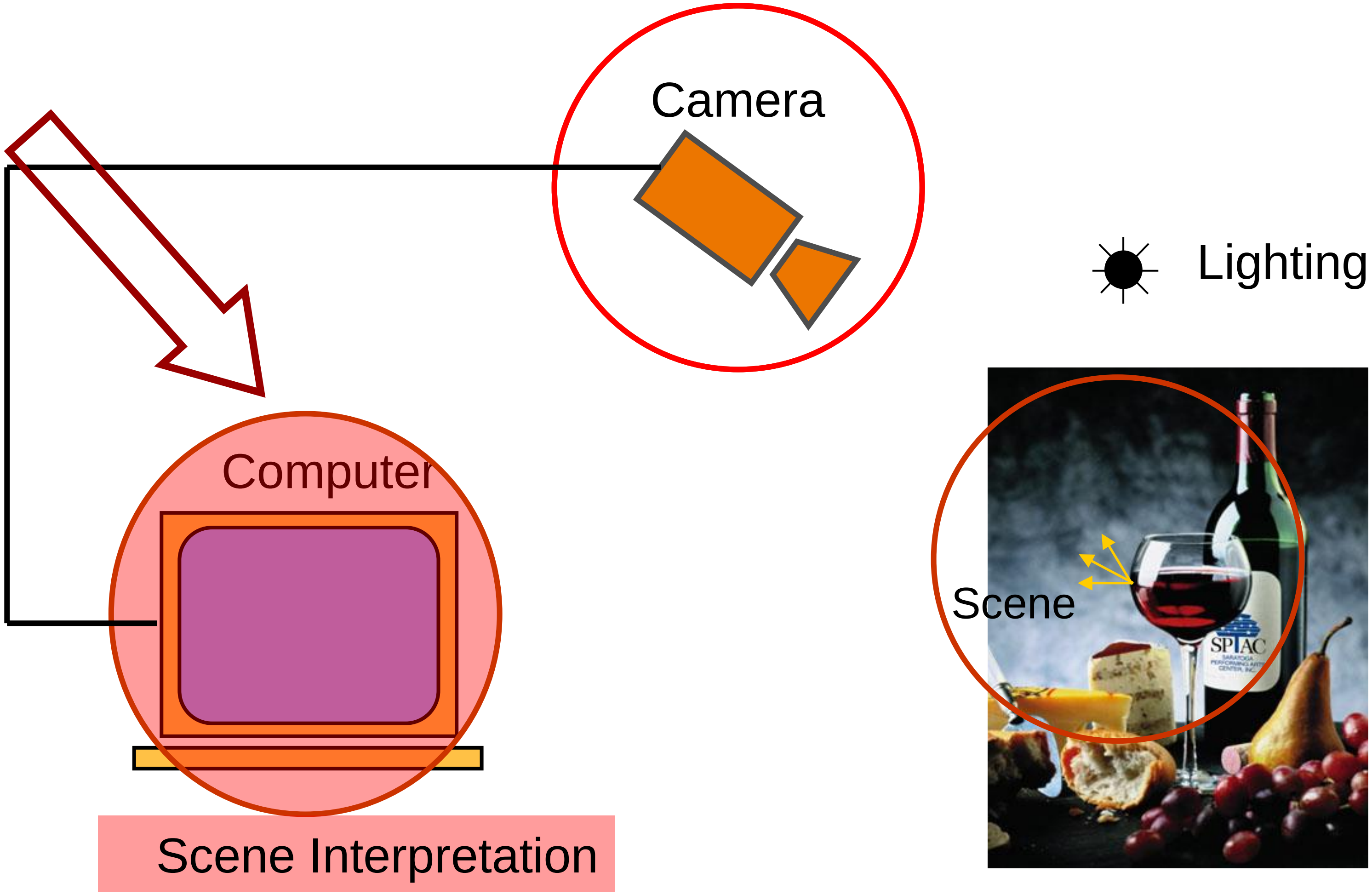
Outline

- Sampling and quantization
- Data structures for digital images

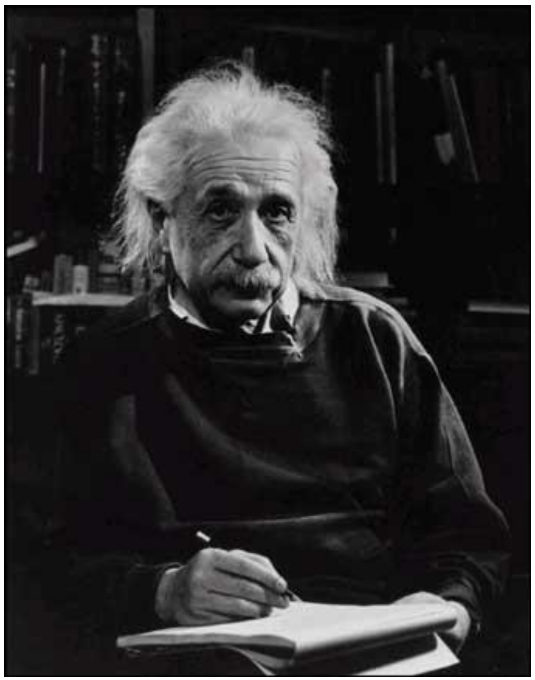
Topic: Sampling and quantization

- Sampling and quantization
- Data structures for digital images

Components of a Computer Vision System



Digital Images

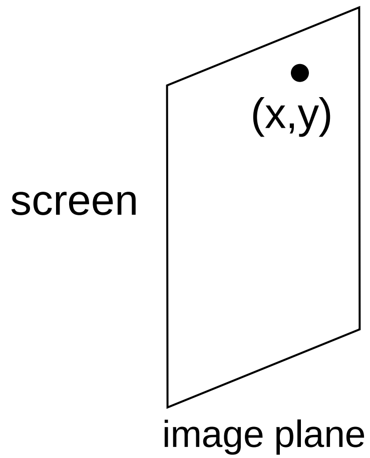


What we see

0	3	2	5	4	7	6	9	8
3	0	1	2	3	4	5	6	7
2	1	0	3	2	5	4	7	6
5	2	3	0	1	2	3	4	5
4	3	2	1	0	3	2	5	4
7	4	5	2	3	0	1	2	3
6	5	4	3	2	1	0	3	2
9	6	7	4	5	2	3	0	1
8	7	6	5	4	3	2	1	0

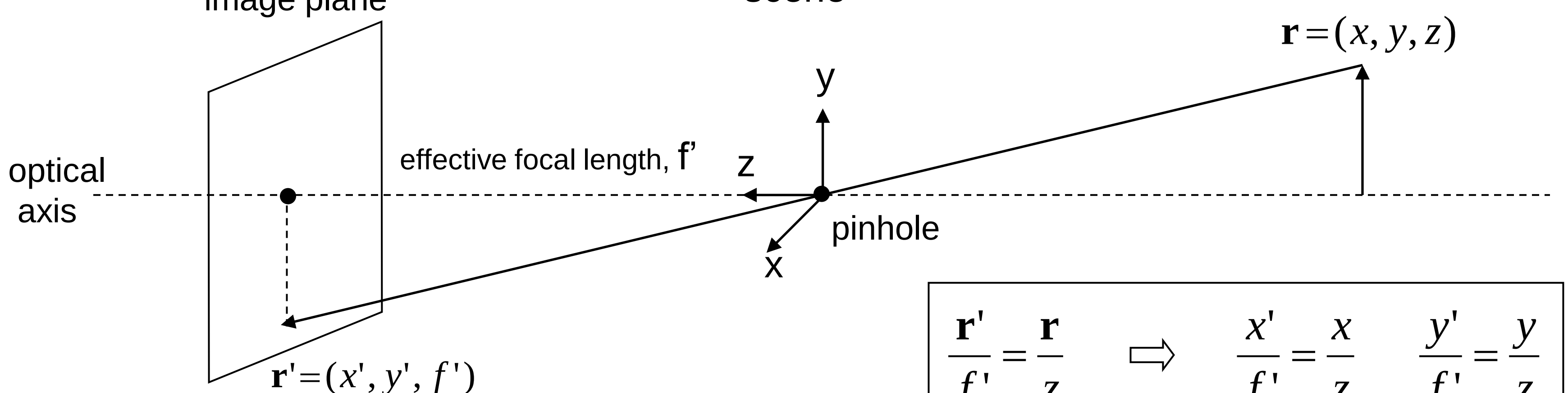
What a computer sees

Pinhole and the Perspective Projection



Is an image being formed on the screen?

YES!   But, not a “clear” one.



$$\frac{\mathbf{r}'}{f'}=\frac{\mathbf{r}}{z} \quad \Rightarrow \quad \frac{x'}{f'}=\frac{x}{z} \qquad \frac{y'}{f'}=\frac{y}{z}$$

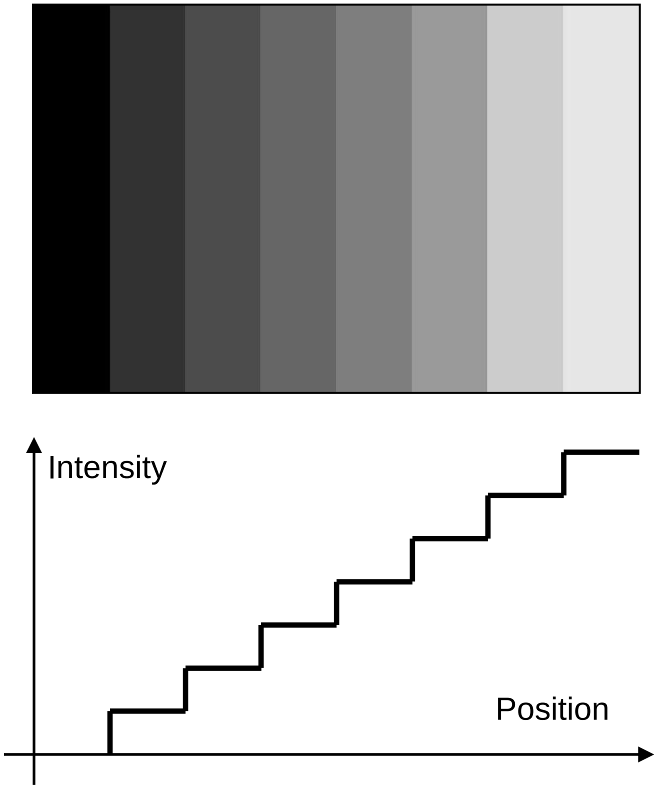
Simple Image Model

- Image as a 2D light-intensity function

$$f(x, y)$$

- Continuous
- Non-zero, finite value

$$0 < f(x, y) < \infty$$

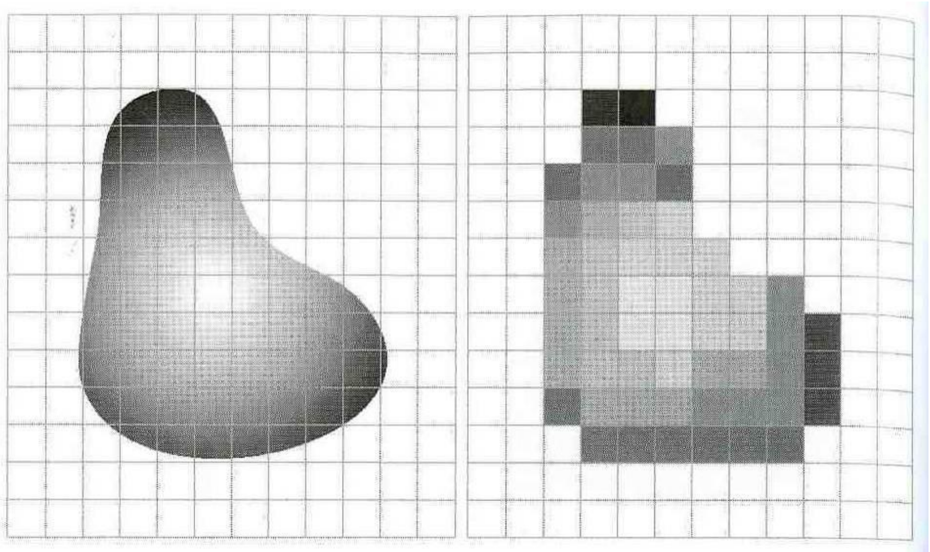


[Gonzalez & Woods]

Analog to Digital

The scene is:

- **projected** on a 2D plane,
- **sampled** on a regular grid, and each sample is
- **quantized** (rounded to the nearest integer)

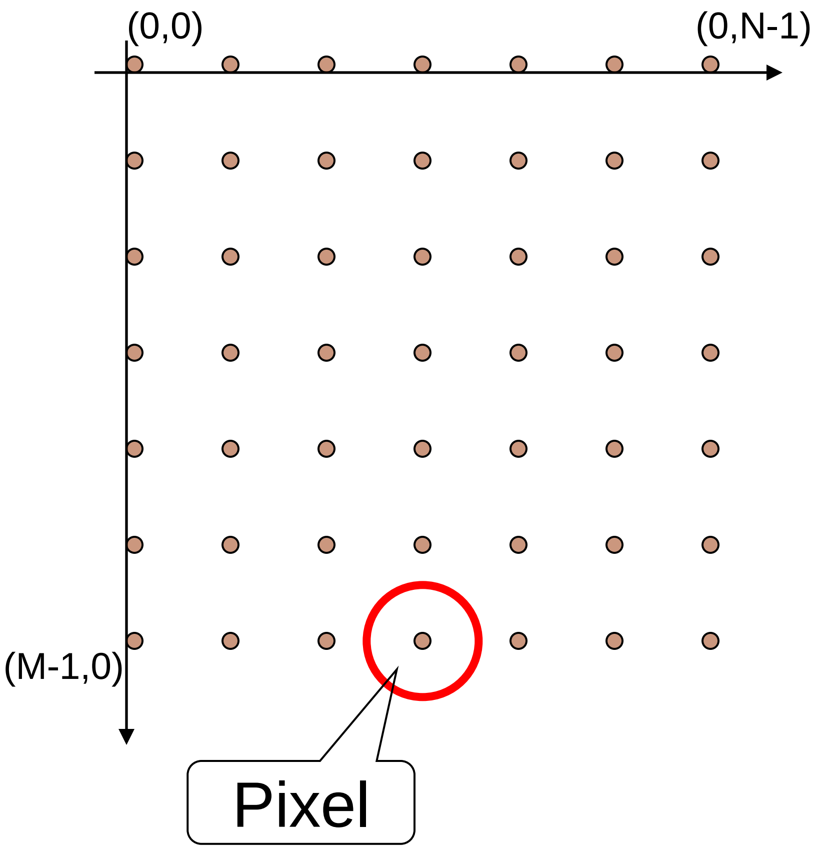


$$f(i, j) = \text{Quantize}\{f(i\Delta, j\Delta)\}$$

Images as Matrices

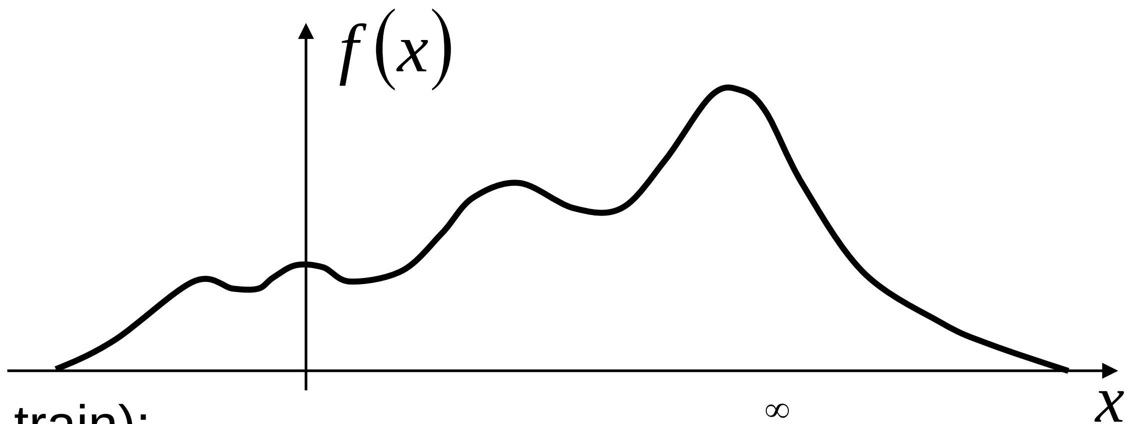
- Each point is a pixel with amplitude:
  - f(x,y)
- An image is a matrix with size N x M

M = [(0,0) (0,1) …
[(1,0) (1,1) …
…



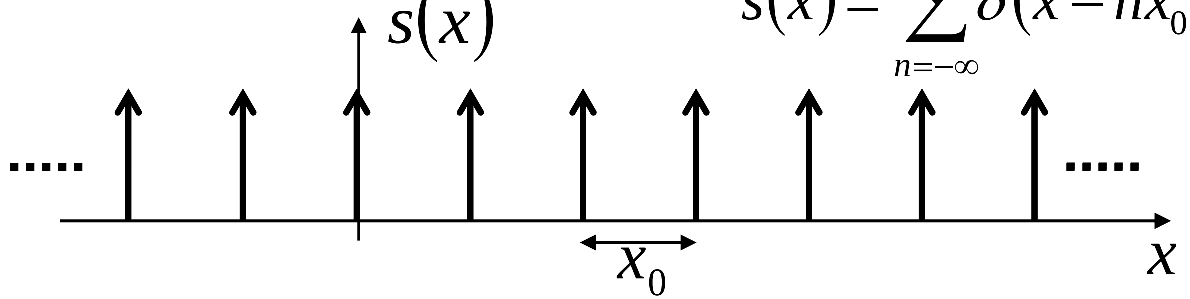
Sampling Theorem

Continuous signal:



Shah function (Impulse train):

$$s(x) = \sum_{n=-\infty}^{\infty} \delta(x - nx_0)$$



Sampled function:

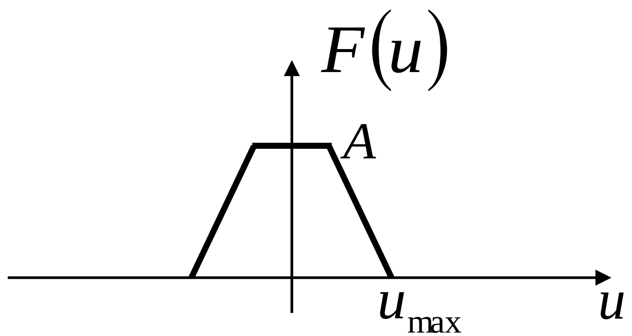
$$f_s(x) = f(x)s(x) = f(x) \sum_{n=-\infty}^{\infty} \delta(x - nx_0)$$

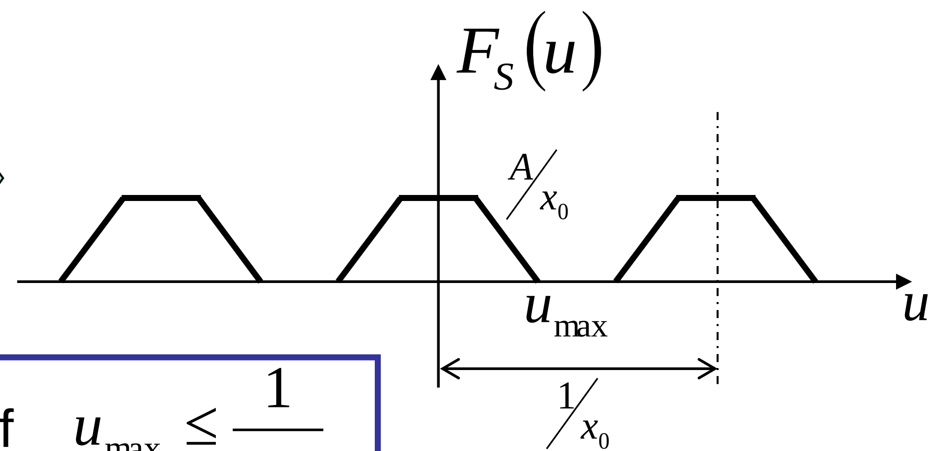
Sampling Theorem

Sampled function: $f_s(x) = f(x)s(x) = f(x)\sum_{n=-\infty}^{\infty}\delta(x - nx_0)$

Sampling frequency $\dfrac{1}{x_0}$

$$F_S(u) = F(u) * S(u) = F(u) * \frac{1}{x_0}\sum_{n=-\infty}^{\infty}\delta\left(u - \frac{n}{x_0}\right)$$

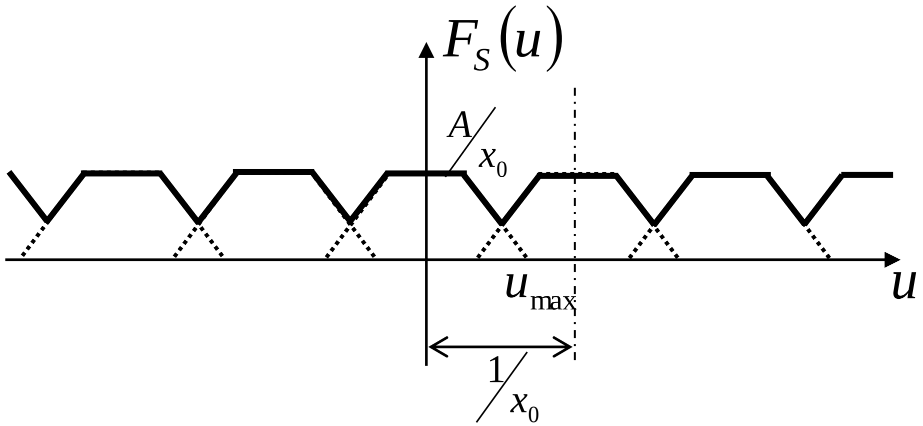




Only if $u_{max} \leq \dfrac{1}{2x_0}$

Nyquist Theorem

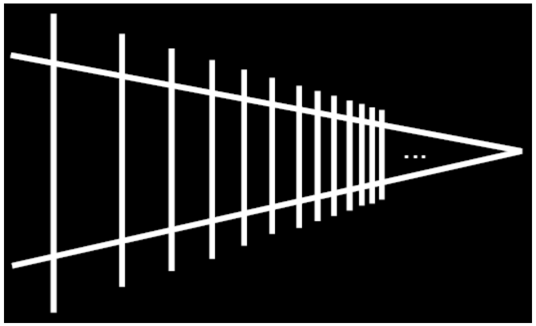
If $u_{\max} > \dfrac{1}{2x_0}$



Aliasing

$\boxed{\text{Sampling frequency must be greater than } 2u_{\max}}$

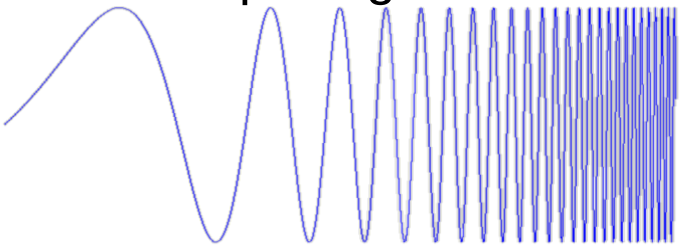
Aliasing



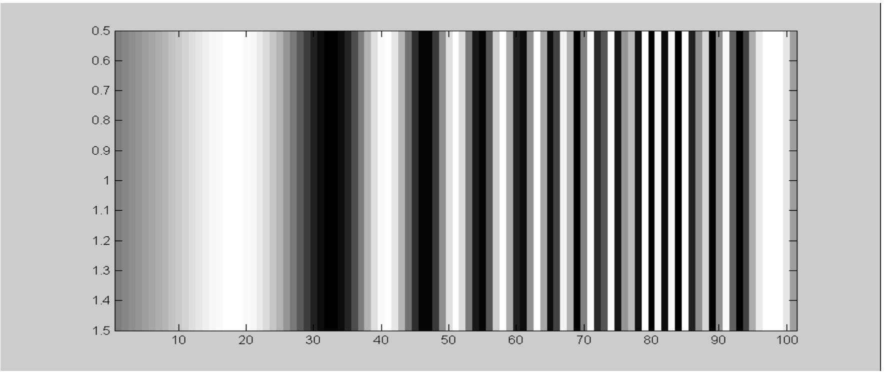
Picket fence receding into the distance will produce aliasing…

WHY?

Input signal:



Matlab output:




```
x = 0:.05:5;  imagesc(sin((2.^x).*x))
```

Sample Question

Consider an image in which the color intensity component of each pixel is represented in the matrix in Figure 1. The image is in analog format, has dimension 3x3 and has values between 0 and 1.

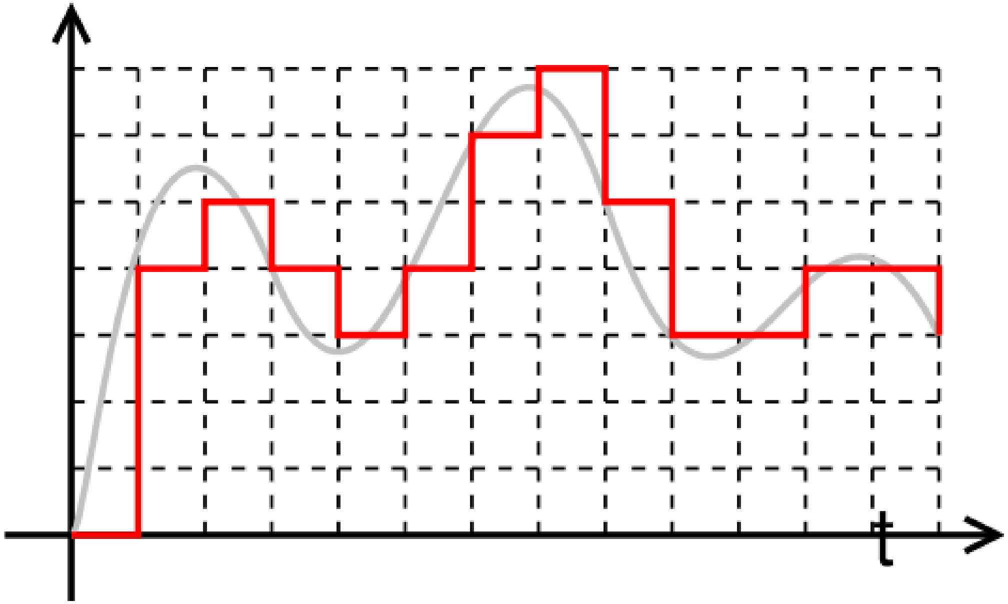
0.1	0.1	0.1
0.5	0.5	0.5
0.9	0.9	0.3

Figure 1

Apply a 2-bit quantization to the matrix depicted in Figure 1. Present the calculations made and the final result in matrix form.

Quantization

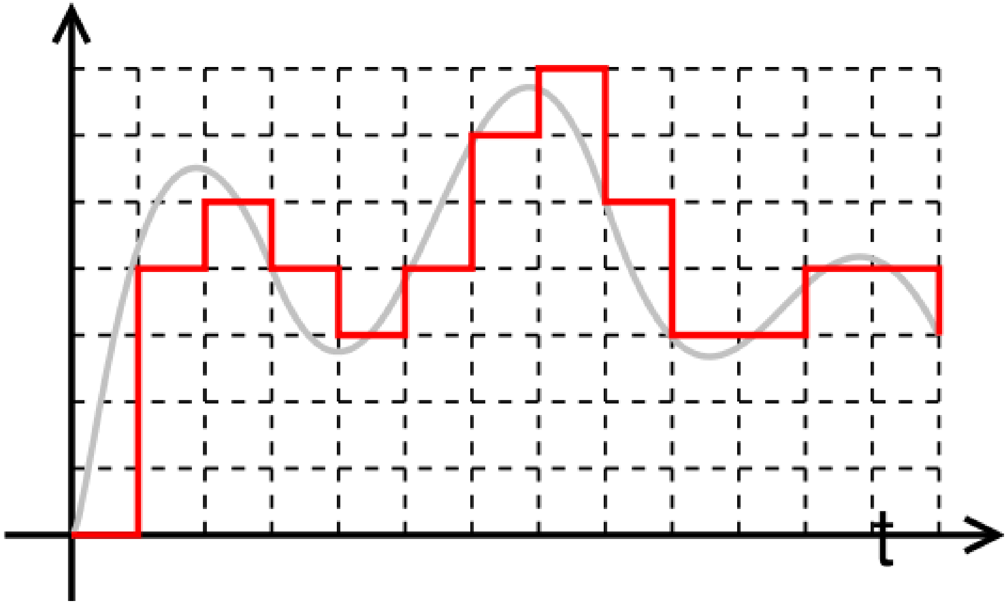
- Analog: $0 < f(x, y) < \infty$
- Digital: Infinite storage space per pixel!
- Quantization



Quantization Levels

- G - number of levels
- m – storage bits
- Round each value to its nearest level

$$G = 2^m$$



Effect of quantization

Effect of quantization

Image Size

- Storage space
  - Spatial resolution: N x M
  - Quantization: m bits per pixel
  - Required bits b:

$$b = N \times M \times m$$

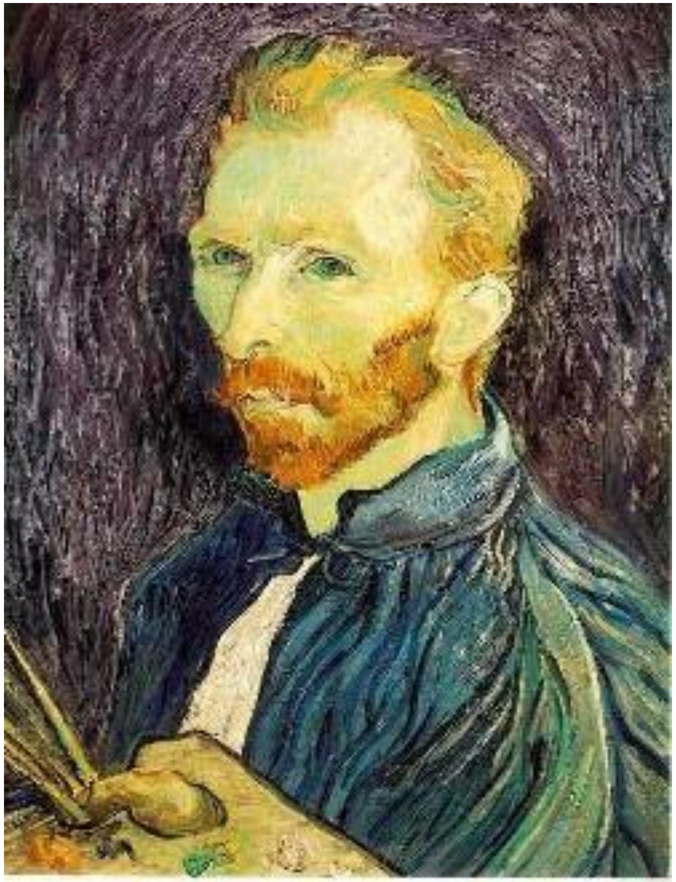
- Rule of thumb:
  - More storage space means more image quality

Image Scaling

This image is too big to fit on the screen. How can we reduce it?

How to generate a half-sized version?

Sub-sampling

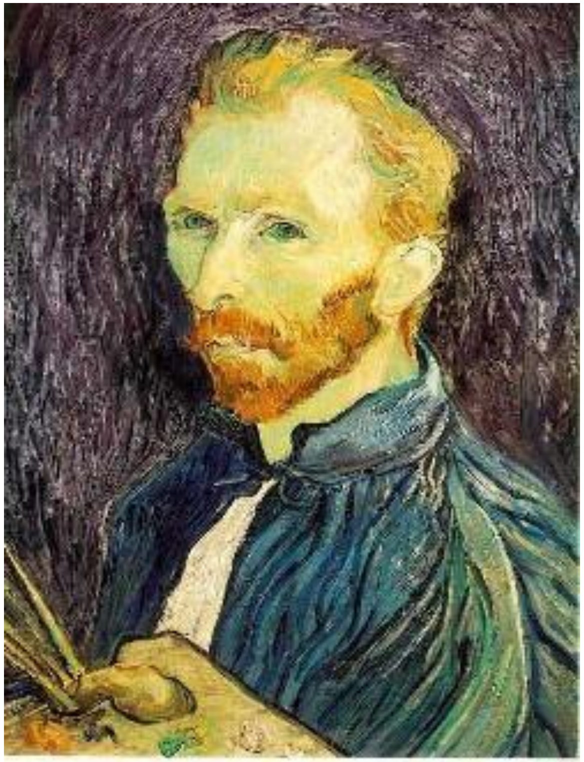


1/4

1/8

Throw away every other row and column to create a 1/2 size image
- called *image sub-sampling*

Sub-sampling



1/2

1/4 (2x zoom)

1/8 (4x zoom)

Sample Question

1. **Digital Image.** Consider the image represented in Figure 1, in analog format, with 3x3 resolution. Each value represents the *intensity* of the colour of that pixel, and varies continuously between 0 and 5.

1.1	1.3	1.1
3.4	4.4	4.9
0.1	4.2	4.8

Figure 1

a) The image in Figure 1 is an example of an image obtained by a computer vision system that you were asked to build, with the objective of measuring finger tremors in patients with Parkinson's disease. The doctors asked that the system could measure tremors with repetition frequencies of up to 10Hz. What is the minimum sampling rate that you would choose for your system? Justify your answer. (2 points)

b) Assuming that your system has an internal memory of only 3MBits, and that you want to record an hour of video, what is the quantization that you would choose for the digitalization of your image? If you did not solve the previous question, assume a sampling rate of 10 Hz. Justify your answer. (2 points)

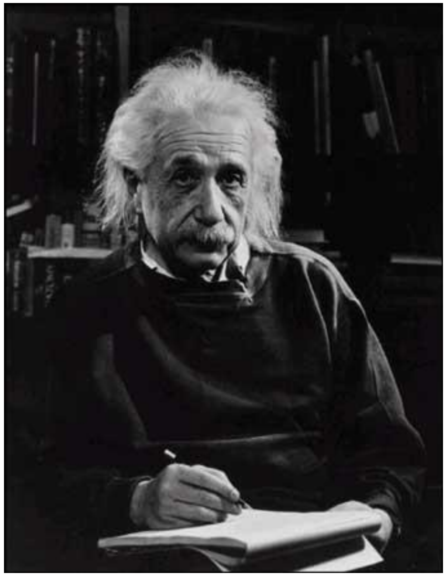
c) What is the result of the application of a 2 bit quantization operation to the image represented in Figure 1? Present your calculations and the final result in the form of a matrix. (2 points)

Topic: Data structures for digital images

- Sampling and quantization
- Data structures for digital images

Data Structures for Digital Images

- Are there other ways to represent digital images?



What we see

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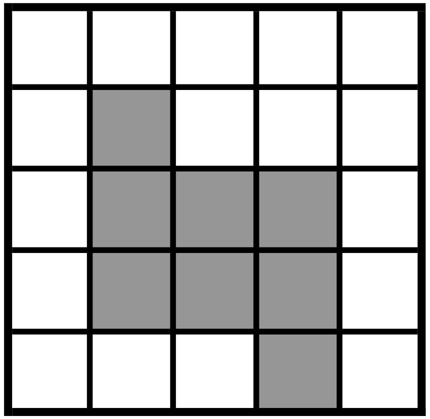
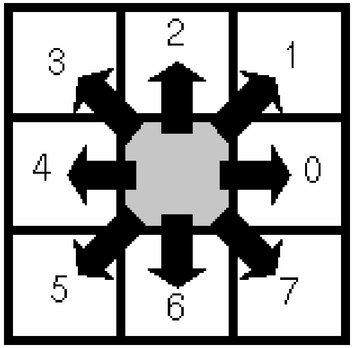
4	3	2	1	0	3	2	5	4
7	4	5	2	3	0	1	2	3
6	5	4	3	2	1	0	3	2
9	6	7	4	5	2	3	0	1
8	7	6	5	4	3	2	1	0

What a computer sees

Chain codes

- Chains represent the borders of objects.
- Coding with *chain codes*.
  - Relative.
  - Assume an initial starting point for each object.
- Needs segmentation!



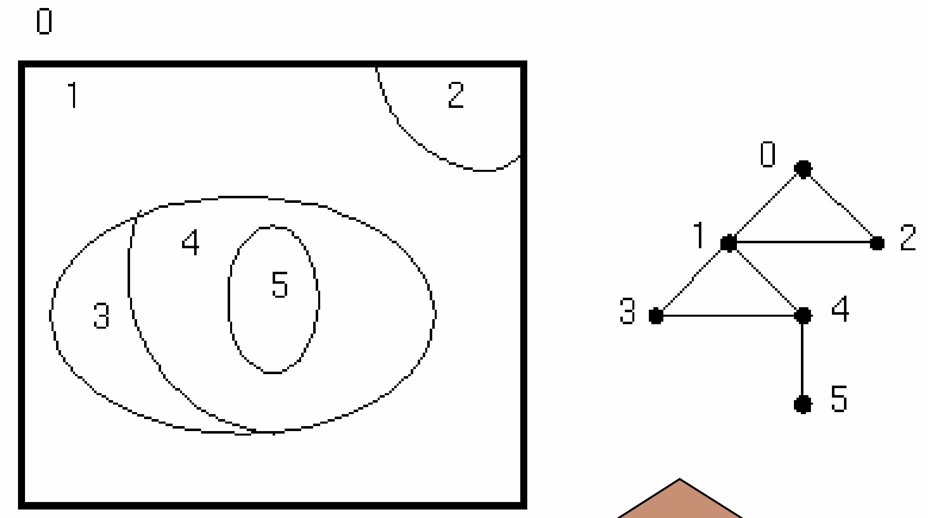


Freeman Chain Code

Using a Freeman Chain Code and considering the top-left pixel as the starting point:
70663422

Topological Data Structures

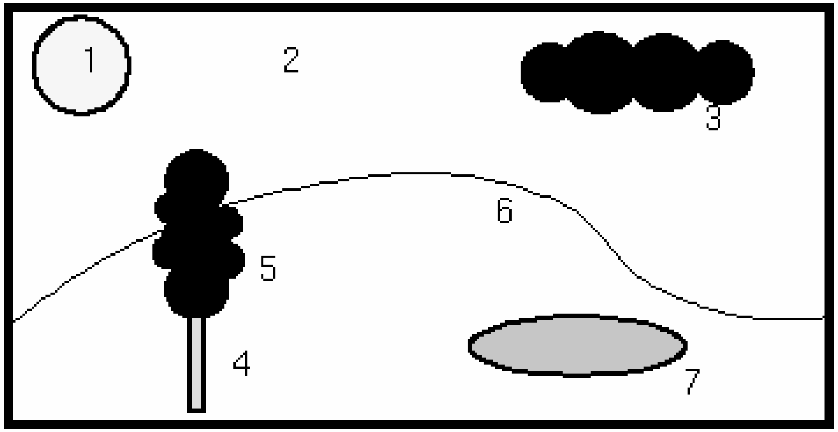
- *Region Adjacency Graph*
  - **Nodes** - Regions
  - **Arcs** – Relationships
- Describes the elements of an image and their spatial relationships.
- Needs segmentation!



Region Adjacency Graph

Relational Structures

- Stores relations between objects.
- Important semantic information of an image.
- Needs segmentation and an image description (features)!



No.	Object name	Colour	Min. row	Min. col.	Inside
1	sun	white	5	40	2
2	sky	blue	0	0	-
3	cloud	grey	20	180	2
4	tree trunk	brown	95	75	6
5	tree crown	green	53	63	-
6	hill	light green	97	0	-
7	pond	blue	100	160	6

Relational Table

Resources

- Szeliski, “Computer Vision: Algorithms and Applications”, Springer, 2022
  – Chapter 2 – “Image Formation”