Data Mining II / Adv. Topics in Data Science

Association Rules

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Summary

Mining Association Rules

Problem Definition

- Given:
  - data set of transactions D
  - minimal support *minsup*
  - minimal confidence *minconf*
- Obtain:
  - **all** association rules

    $$X \rightarrow Y \quad (s = Sup, c = Conf)$$

    such that

    $$Sup \geq minsup \text{ and } Conf \geq minconf$$

Apriori Algorithm

The **Apriori Algorithm** [Agrawal and Srikant, 1994] works in two steps:

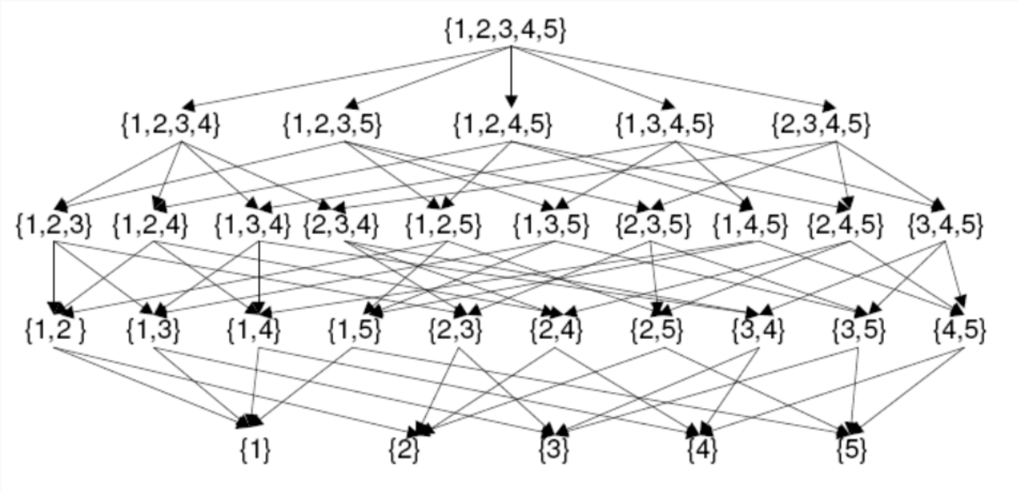
1. **Frequent itemset generation**
   - itemsets with *support* $\geq$ *minsup*
2. **Rule generation**
   - generate all confident association rules from the frequent itemsets, i.e. rules with *confidence* $\geq$ *minconf*

Apriori Algorithm (cont.)

- Problem:
  - there is a very large number of candidate frequent itemsets!
    - for transactions with k items, there are $2^k - 1$ distinct subsets.
- Downward Closure Property
  - every subset of a frequent itemset must also be frequent.
    - ex: if $\{A1, A2, A4\}$ is frequent, so is $\{A1, A2\}$ because every transaction containing $\{A1, A2, A4\}$ also contains $\{A1, A2\}$.
  - thus, every superset of an infrequent itemset is also infrequent.
    - ex: if $\{A1, A2\}$ is infrequent, so is $\{A1, A2, A4\}$.
- Apriori Pruning Principle:
  - if an itemset is below the minimal support, discard all its supersets.

Example - 1

Search Space for 5 items



Example - 1 (cont.)

- Apriori enumerates and counts the support of patterns with increasing length.
- Starts looking for frequent itemsets of size 1 (F_1), assuming *minsup* $= 50\%$ (2 transactions)
- $C_1 = \{\{1\}, \{2\}, \{3\}, \{4\}, \{5\}\}$

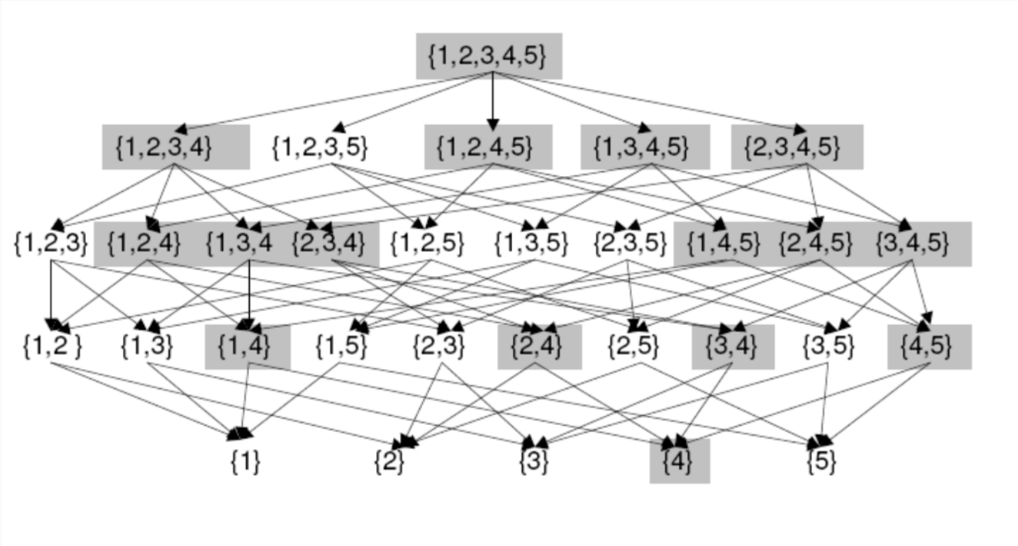
TID	ITEM-SET
100	1 3 4
200	2 3 5
300	1 2 3 5
400	2 5

ITEM-SET	Support
{1}	2
{2}	3
{3}	3
{4}	1
{5}	3

- $F_1 = \{\{1\}, \{2\}, \{3\}, \{5\}\}$

Example - 1 (cont.)

- Filtered Search Space for 5 items (after removing item "4")



Example - 1 (cont.)

- Looks for frequent itemsets of size 2 (F_2) from frequent itemsets of size 1 (F_1)
- Candidates $C_2 = \{\{a, b\} | \{a\} \in F_1 \wedge \{b\} \in F_1\}$
- $C_2 = \{\{1, 2\}, \{1, 3\}, \{1, 5\}, \{2, 3\}, \{2, 5\}, \{3, 5\}\}$

ITEM-SET	Support
{1,2}	1
{1,3}	2
{1,5}	1
{2,3}	2
{2,5}	3
{3,5}	2

- $F_2 = \{\{1, 3\}, \{2, 3\}, \{2, 5\}, \{3, 5\}\}$

Example - 1 (cont.)

- Looks for frequent itemsets of size 3 (F_3) from frequent itemsets of size 2 (F_2)
- Generation:
  $C0_3 = \{\{a, b, c\} | \{a, b\} \in F_2 \wedge \{a, c\} \in F_2\}$
- Filter:
  $C_3 = \{\{a, b, c\} | \{a, b, c\} \in C0_3 \wedge \forall x \in \{a, b, c\}\ S - \{x\} \in F_2\}$
- $C_3 = \{\{2, 3, 5\}\}$

ITEM-SET	Suporte
{2,3,5}	2

- $F_3 = \{\{2, 3, 5\}\}$
- There are no frequent itemsets of size 4

Example - 2

A	B	C	D
1			
1	1	1	
		1	
1	1	1	1
	1		
1			1
1	1	1	
		1	1
1	1	1	

Pass 1

- $minsup = 40\%$
- $C_1 = \{\{A\}, \{B\}, \{C\}, \{D\}\}$
- $F_1 = \{\{A\}, \{B\}, \{C\}\}$

Example - 2 (cont.)

A	B	C	D
1			
1	1	1	
		1	
1	1	1	1
	1		
1			1
1	1	1	
		1	1
1	1	1	

Pass 2

- *minsup* $= 40\%$
- $C_2 = \{\{A, B\}, \{A, C\}, \{B, C\}\}$
- $F_2 = \{\{A, B\}, \{A, C\}, \{B, C\}\}$

Example - 2 (cont.)

A	B	C	D
1			
1	1	1	
		1	
1	1	1	1
	1		
1			1
1	1	1	
		1	1
1	1	1	

Pass 3

- *minsup* $= 40\%$
- $C_3 = \{\{A, B, C\}\}$
- $F_3 = \{\{A, B, C\}\}$

Example - 2 (cont.)

Output

- frequent itemsets ($minsup = 40\%$)

$\{A\}$ 66%
$\{A, B\}$ 44%
$\{A, B, C\}$ 44%
$\{B\}$ 55%
$\{A, C\}$ 44%
$\{C\}$ 66%
$\{B, C\}$ 44%

- rules ($minconf = 80\%$)

$\{B\} \rightarrow \{A\}$	(sup = 44%, conf = 80%)
$\{B\} \rightarrow \{C\}$	(sup = 44%, conf = 80%)
$\{B, C\} \rightarrow \{A\}$	(sup = 44%, conf = 100%)
$\{B, A\} \rightarrow \{C\}$	(sup = 44%, conf = 100%)
$\{B\} \rightarrow \{A, C\}$	(sup = 44%, conf = 80%)

Step 1 - identifying frequent itemsets

- It is a **level-wise** algorithm
  - it traverses the itemset lattice one level at a time, from frequent 1-itemsets to the maximum size of frequent itemsets.
- It employs a **generate-and-test** strategy for finding frequent itemsets
  - at each iteration, new candidate itemsets are generated from the frequent itemsets found in the previous iteration; the support for each candidate itemset is then counted and tested against minsup.

Step 1 - identifying frequent itemsets (cont.)

- Candidate generation (Self-Join step)
  - generates new candidate k-itemsets based on the frequent (k-1)-itemsets found in the previous iteration.
- Candidate pruning (Prune step)
  - eliminates some of the candidate k-itemsets using the support-based pruning strategy.

Step 1 - identifying frequent itemsets (cont.)

- Self-Join Example:

  Given the size k candidates
  $\{A, B, C\}$
  $\{A, B, D\}$
  $\{A, C, D\}$
  $\{B, C, D\}$
  $\{A, B, E\}$
  $\{B, C, E\}$
  and assuming that in each itemset the items are lexicographically sorted
- Which are the candidates of size $k + 1$?
- What is the most efficient way of finding them (without repetitions)?

Step 1 - identifying frequent itemsets (cont.)

- Look for pairs of sets with the same prefix of size $k-1$
  $\{A, B, C\}$ and $\{A, B, D\}$
- Combine both, keeping the prefix
  $\{A, B, C, D\}$
- This way
  - No frequent set is unnoticed
  - No candidate is generated more than once

Step 1 - identifying frequent itemsets (cont.)

- Prune Example:

  $F_3 = \{\{A, B, C\}, \{A, B, D\}, \{A, C, D\}, \{A, C, E\}, \{B, C, D\}\}$

  $C_4 = \{\{A, B, C, D\}, \{A, C, D, E\}\}$

  but $\{A, C, D, E\}$ can be pruned away

  because $\{A, D, E\} \notin F_3$

- Note:
  - Prune maintains the completeness of the process

Step 2 - rule generation

- Given a frequent set $\{A, B, C, D\}$
- Which are the possible rules?
  - $\{A, B, C\} \rightarrow \{D\}$
  - $\{A, B, D\} \rightarrow \{C\}$
  - $\{A, B\} \rightarrow \{C, D\}$
- How to generate them systematically?
- How to reduce the search space?

Step 2 - rule generation (cont.)

- The rules are generated as follows:
  - generates all non-empty subsets s of each frequent itemset l
  - for each subset s computes the confidence of the rule $(l - s) \rightarrow s$
  - selects the rules whose confidence is higher than *minconf*

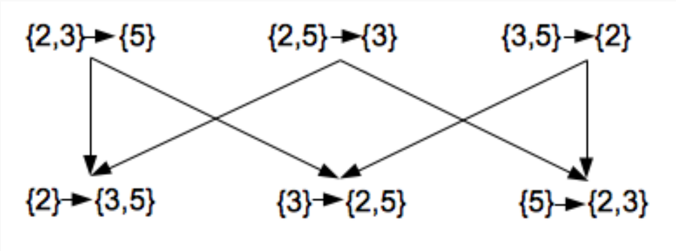
Step 2 - rule generation (cont.)

Consider again

Cliente (TID)	Itens (Item-set)
100	1, 3, 4
200	2, 3, 5,
300	1, 2, 3, 5,
400	2, 5,

and $I = \{2, 3, 5\} (= F_3)$

- Rules generated from the frequent itemset $\{2, 3, 5\}$



- Select rules $(I - a) \rightarrow a$, where $a \subseteq I$, with $minconf = 1$

$$conf((I - a) \rightarrow a) = \frac{sup(I)}{sup(I - a)}$$

Step 2 - rule generation (cont.)

- Rules with 1 consequent

$\{2, 3\} \rightarrow \{5\}$ (conf= 2/2)
$\{2, 5\} \rightarrow \{3\}$ (conf= 2/3) eliminated because $minconf = 1$
$\{3, 5\} \rightarrow \{2\}$ (conf= 2/2)

- Rules with 2 consequents

$\{3\} \rightarrow \{2, 5\}$ (conf= 2/3) eliminated because $minconf = 1$

- we don't need to worry about rules with item 3 in the consequent, because any rule obtained from $\{2, 5\} \rightarrow \{3\}$ will have a $conf < 2/3$

> Moving items from the antecedent to the consequent never changes support and never increases confidence.

Number of DB scans

- 1 to count frequencies of C_1
- C_2 built in memory
- 2 to count frequencies of C_2
- ...
- n to count frequencies of C_n
- Rule generation does not need to scan DB
- Number of scans is n
  - if the size of the largest frequent set is n or $n-1$

Complexity factors

- Number of items
- Number of transactions
- Minimal support
- Average size of transactions
- Number of frequent sets
- Average size of a frequent size
- Number of DB scans
  - k or $k+1$, where k is the size of the largest frequent set

Exercises

1. Consider the following set of transactions:

$$\{\{A, B, C\}, \{A, C\}, \{B, D\}, \{B, C, D\}, \{A\}\}$$

   Using the Apriori algorithm with *minsup* $= 40\%$ and *minconf* $= 70\%$

   - find the frequent itemsets
   - find the set of relevant rules

Exercises (cont.)

2. Consider the following set of transactions:

   Using the Apriori algorithm with *minsup* $= 30\%$ and *minconf* $= 80\%$

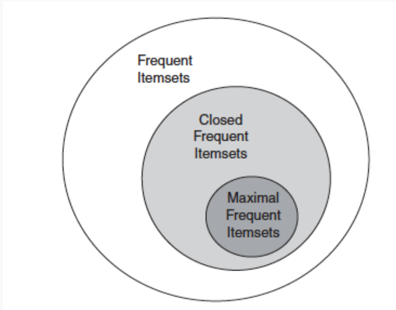
   - find the frequent itemsets
   - find the set of relevant rules

TID	Itemset
1	A D E
2	B C D
3	A C E
4	A C D E
5	A E
6	A C D
7	B C
8	A C D E
9	B C E
10	A D E

Compact Representation of Itemsets

- The number of frequent itemsets produced from a transaction data set can be very large.
- It is useful to identify a small representative set of itemsets from which all other frequent itemsets can be derived.

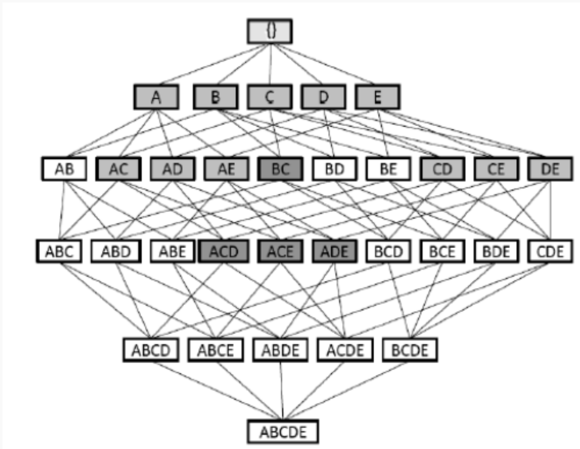
- Two such representations are:
  - closed
  - maximal



Compact Representation of Itemsets (cont.)

- s is a **closed frequent itemset** if it is a frequent itemset that has no frequent supersets with the same support.
- Example: find closed frequent itemsets with $minsup = 30\%$

TID	Itemset
1	A D E
2	B C D
3	A C E
4	A C D E
5	A E
6	A C D
7	B C
8	A C D E
9	B C E
10	A D E



closed frequent itemsets are:
$\{A\}, \{C\}, \{D\}, \{E\}, \{A, C\}, \{A, D\}, \{A, E\}, \{B, C\}, \{C, D\}, \{C, E\}, \{A, C, D\}, \{A, C, E\}, \{A, D, E\}$

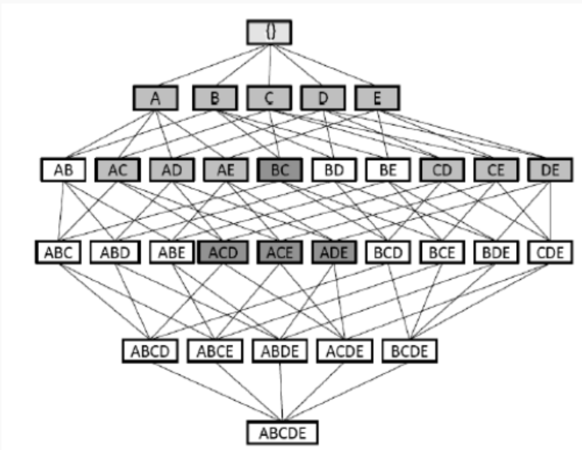
Compact Representation of Itemsets (cont.)

- The set of all **closed frequent itemsets** preserves the knowledge about the support values of all frequent itemsets.
  - $\{D, E\}$ is a non closed frequent itemset. What is its support?
  - As it is not closed, its support must be equal to one of its immediate supersets.
  - look for the most frequent closed itemset that contains $\{D, E\}$: $\{A, D, E\}$
  - $sup(\{D, E\}) = sup(\{A, D, E\})$

Compact Representation of Itemsets (cont.)

- s is a **maximal frequent itemset** if it is a frequent itemset for which none of its supersets is frequent.
- Example: find maximal frequent itemsets with $minsup = 30\%$

TID	Itemset
1	A D E
2	B C D
3	A C E
4	A C D E
5	A E
6	A C D
7	B C
8	A C D E
9	B C E
10	A D E



maximal frequent itemsets are:

$\{B, C\}, \{A, C, D\}, \{A, C, E\}, \{A, D, E\}$

Compact Representation of Itemsets (cont.)

- From the **maximal itemsets** is possible to derive all frequent itemsets (**not their support**) by computing all non-empty intersections.
  - subsets of the maximal frequent itemset $\{A, C, D\}$ are frequent itemsets
  - $\{A\}$, $\{C\}$, $\{D\}$, $\{A, C\}$, $\{A, D\}$, $\{C, D\}$
- There are algorithms that take advantage of this compact representation of frequent itemsets.

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