2.4 Given the following signals

$$x(t) = 2\delta(t), \quad y(t) = 4u(t), \quad z(t) = e^{-2t}u(t),$$

Evaluate the following operations.

(a) $x(t)*y(t)$
(b) $x(t)*z(t)$
(c) $y(t)*z(t)$
(d) $y(t)*[y(t) + z(t)]$

(a)

$$\begin{aligned} & x(t) * y(t) \\ &= \int_0^t x(\tau)\, y(t-\tau)\, d\tau \\ &= \int_0^t 2\delta(\tau) \cdot 4u(t-\tau)\, d\tau \\ &= 8 \int_0^t u(\tau)\, \delta(t-\tau)\, d\tau \\ &= 8 \int_0^t \delta(t-\tau)\, d\tau \\ &= \underline{8\, u(t)} \end{aligned}$$

(b)

$$\begin{aligned} & x(t) * z(t) \\ &= \int_0^t x(\tau)\, z(t-\tau)\, d\tau \\ &= \int_0^t x(t-\tau)\, z(\tau)\, d\tau \\ &= \int_0^t 2\delta(t-\tau)\, e^{-2\tau} u(\tau)\, d\tau \\ &= 2\int_0^t e^{-2\tau} \cdot \delta(t-\tau)\, d\tau \\ &= \underline{2e^{-2t} u(t)} \end{aligned}$$

(c)

$$\begin{aligned} & y(t) * z(t) \\ &= \int_0^t z(\tau)\, y(t-\tau)\, d\tau \\ &= \int_0^t e^{-2\tau} u(\tau)\, 4u(t-\tau)\, d\tau \\ &= 4 \int_0^t e^{-2\tau}\, d\tau \\ &= 4 \left(\frac{e^{-2\tau}}{-2} \Big|_0^t \right) u(t) \\ &= \underline{[2 - 2e^{-2t}]\, u(t)} \end{aligned}$$

(d)

$$\begin{aligned} & y(t) * [y(t) + z(t)] \\ &= y(t) * y(t) + y(t) * z(t) \\ &= \int_0^t 4u(\tau) 4u(t-\tau)\, d\tau + [2 - 2e^{-2t}]\, u(t) \\ &= \underline{[16t + 2 - 2e^{-2t}]\, u(t)} \end{aligned}$$

2.14 The impulse response of a low-pass filter is $h(t) = e^{-t}u(t)$. Determine its step response, that is, the output when the input is a unit step.

$$u(t) \rightarrow \boxed{h(t)} \rightarrow y(t)$$

$$\begin{aligned} y(t) &= u(t) * h(t) \\ &= \int_0^t u(\tau)\, h(t-\tau)\, d\tau \\ &= \int_0^t e^{-(t-\tau)}\, u(t-\tau)\, d\tau \\ &= e^{-t} \int_0^t e^{\tau}\, d\tau \\ &= e^{-t} (e^{t} - e^{0})\, u(t) \\ &= \underline{[1 - e^{-t}]\, u(t)} \end{aligned}$$

2.24 Determine the overall impulse response for the system shown in Figure 2.34.

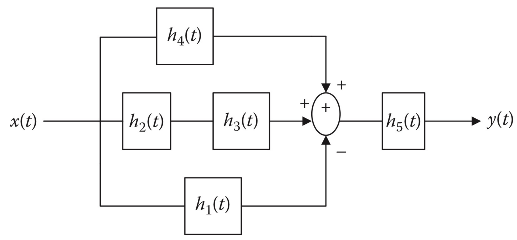


FIGURE 2.34 For Problem 2.24.

$$h'(t) = [h_4(t) + h_2(t) * h_3(t) - h_1(t)] * h_5(t)$$

$$= h_4(t) * h_5(t) + h_2(t) * h_3(t) * h_5(t) - h_1(t) * h_5(t)$$

$$y(t) = x(t) * h'(t)$$

$$= x(t) * h_4(t) * h_5(t) +$$

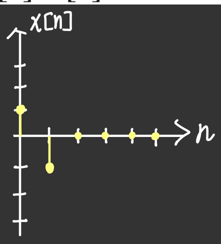
$$x(t) * h_2(t) * h_3(t) * h_5(t) -$$

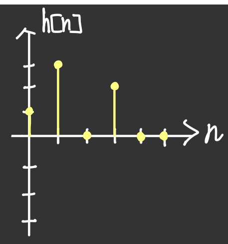
$$x(t) * h_1(t) * h_5(t)$$

2.30 Given that $x[n] = \begin{cases} 1, & n = 0 \\ -1, & n = 1 \\ 0, & \text{otherwise} \end{cases}$, $h[n] = \begin{cases} 1, & n = 0 \\ 3, & n = 1 \\ 2, & n = 3 \\ 0, & \text{otherwise} \end{cases}$

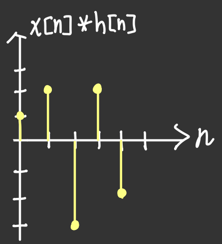
(a) Sketch $x[n]$ and $h[n]$.
(b) Find $x[n]*h[n]$.

(a)





(b)



$$x[n] * h[n] = \begin{cases} 1, & n = 0 \\ 2, & n = 1 \\ -3, & n = 2 \\ 2, & n = 3 \\ -2, & n = 4 \\ 0, & \text{otherwise.} \end{cases}$$

1 -1 0 0
2 0 3 1

1 2 -3 2 -2

2.33 Two systems are described by

$$h_1[n]=(0.4)^n u[n], \qquad h_2[n]=\delta[n]+0.5\delta[n-1]$$

Determine the response to the input $x[n] = (0.4)^n u[n]$ if
(a) The two systems are connected in parallel
(b) The two systems are connected in cascade

(a) $x[n] * (h_1[n] + h_2[n])$

$= x[n] * h_1[n] + x[n] * (\delta[n] + 0.5\delta[n-1])$

$= \sum_{k=0}^{n} 0.4^k u[k]\, 0.4^{n-k} u[n-k] + x[n] * \delta[n] + x[n] * 0.5\delta[n-1]$

$= 0.4^n (n+1) u[n] + 0.4^n u[n] + 0.5 \cdot x[n-1]$

$= 0.4^n (n+1) u[n] + 0.4^n u[n] + 0.5 \cdot 0.4^{n-1} u[n-1]$

$= 0.4^n (n+2) u[n] + 0.5 \cdot 0.4^{(n-1)} u[n-1]$

$= 0.4^n \left[(n+2) u[n] + \frac{5}{4} u[n-1] \right]$

(b) $x[n] * h_1[n] * h_2[n]$

$= 0.4^n (n+1) u[n] * \delta[n] + 0.4^n (n+1) u[n] * 0.5\delta[n-1]$

$= 0.4^n (n+1) u[n] + 0.5 \left(0.4^{n-1} \cdot n \cdot u[n-1] \right)$

$= 0.4^n \left((n+1) u[n] + \frac{5}{4} n \cdot u[n-1] \right)$

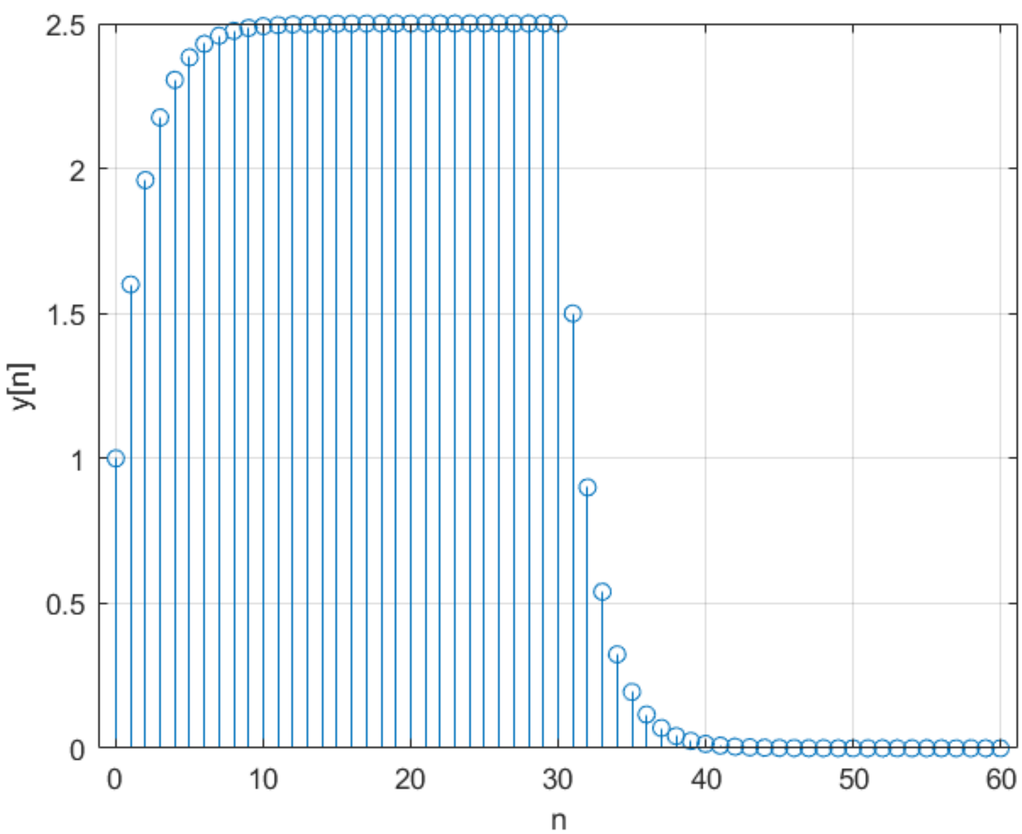
2.36 The input $x[n] = [1\ -1]$ to a system produces the output $y[n] = [4\ 2\ 5\ 1]$. Determine the impulse response.

$$\begin{array}{r|rrrr}
 & 4 & 6 & 11 & \\
\hline
1\ -1 & 4 & 2 & 5 & 1 \\
 & 4 & -4 & & \\
\hline
 & & 6 & 5 & \\
 & & 6 & -6 & \\
\hline
 & & & 11 & 1 \\
 & & & 11 & -11 \\
\hline
 & & & & 12
\end{array}$$

System 可能為非LTI系統 或是實際 input 有被干擾

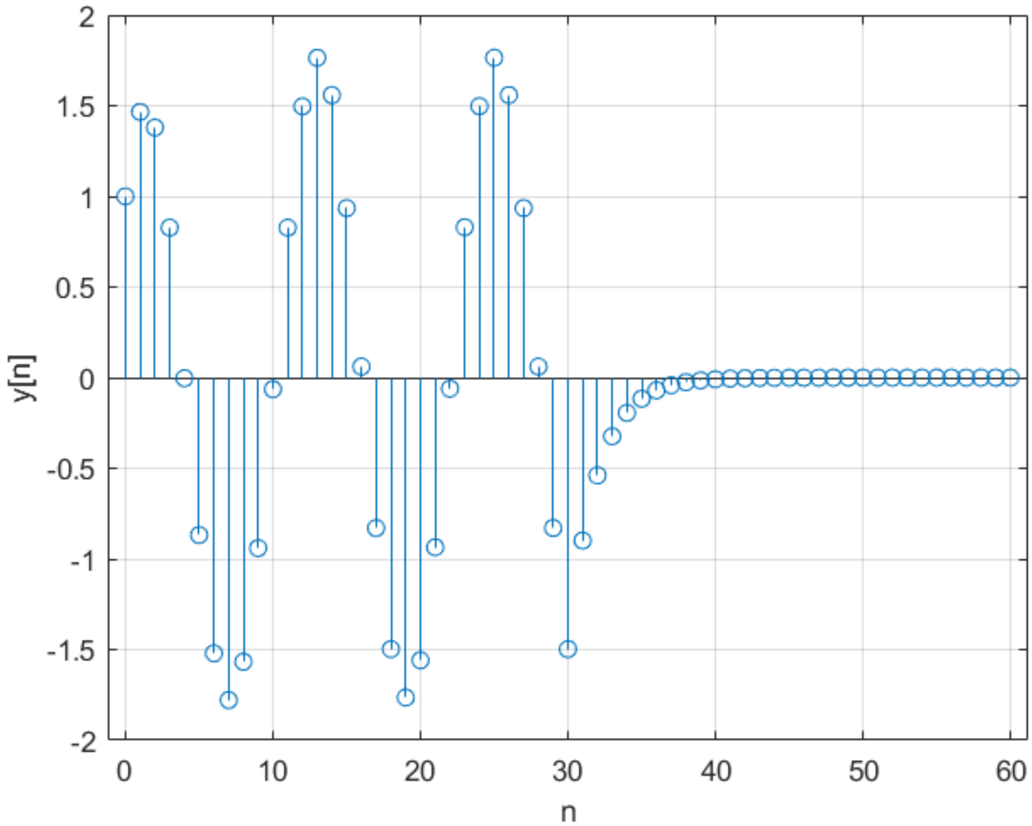
故 題目線索不足以計算出 $h(t)$

2_39



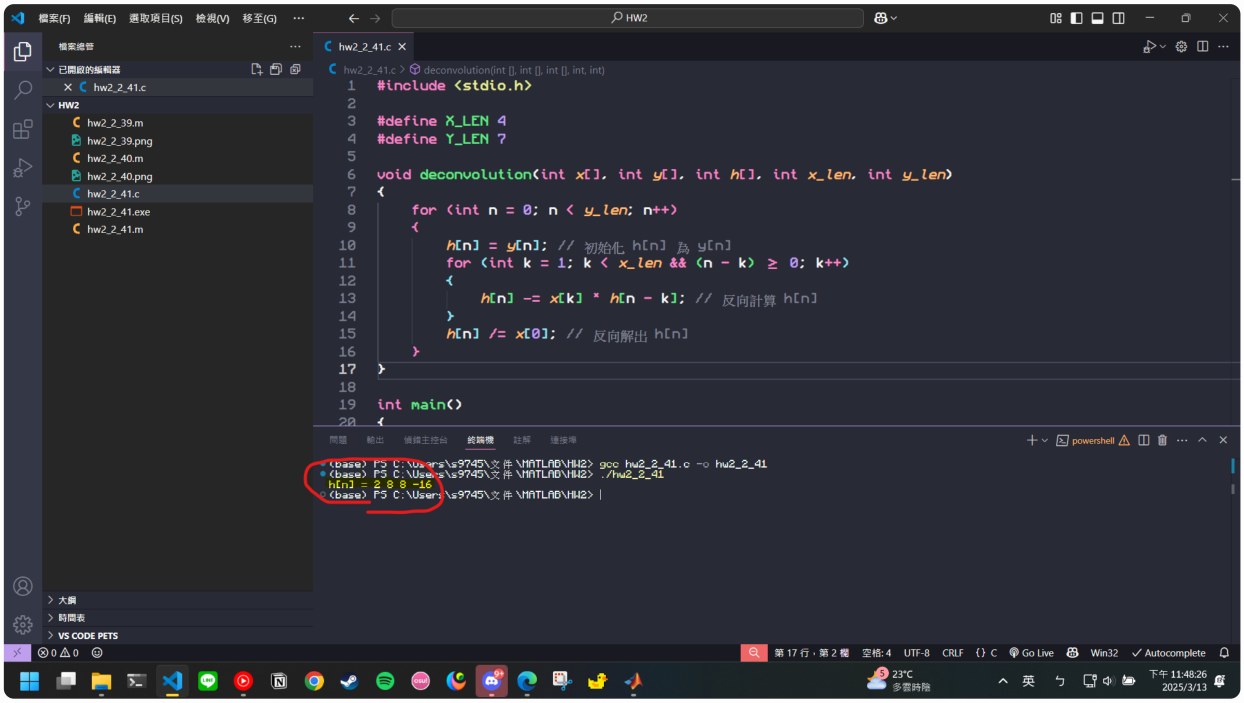

```
n = 0:1:30;
u = ones(size(n));   % u[n]
h = (0.6).^n .* u;   % h[n]
y = conv(u, h);

n_y = 0:length(y)-1;   % y[n]

stem(n_y, y);
xlabel('n');
ylabel('y[n]');
grid;

```

2.40




```
n = 0:1:30;
u = ones(size(n));  % u[n]
x = cos(n.*pi./6).*u; % x[n]
h = (0.6).^n .* u;  % h[n]
y = conv(x, h);

n_y = 0:length(y)-1;  % y[n]

stem(n_y, y);
xlabel('n');
ylabel('y[n]');
grid;

```

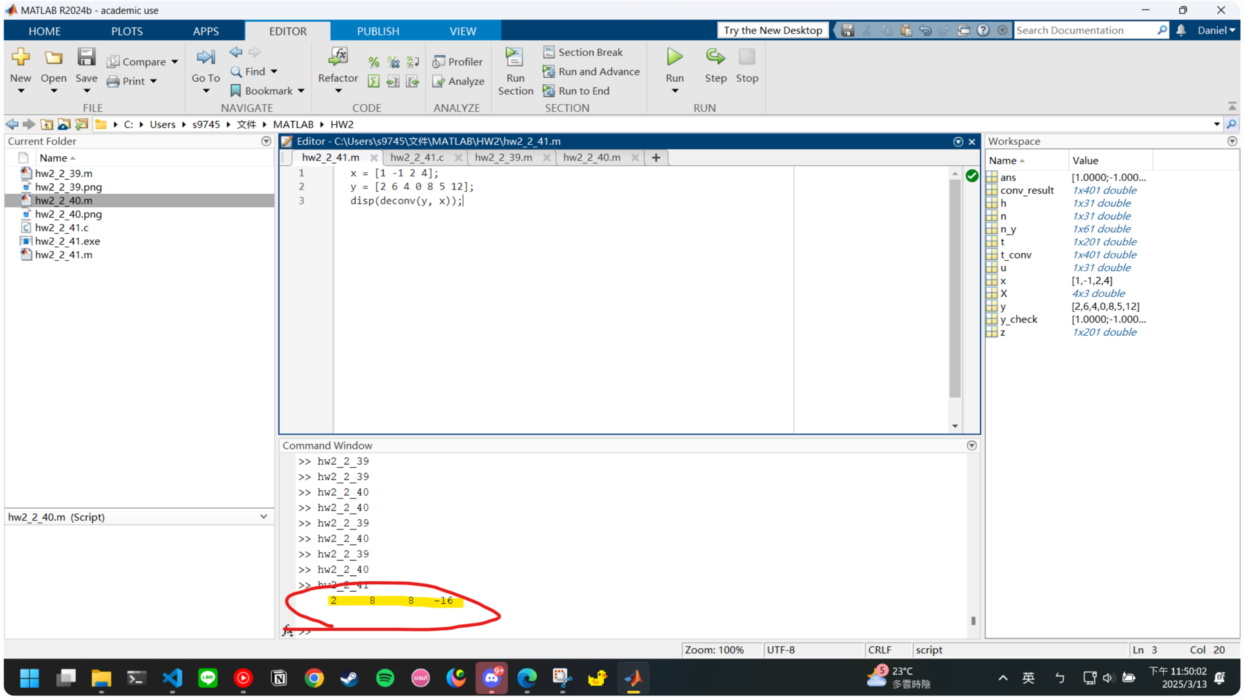
2.41 (a)



hw2_2_41.c
#include <stdio.h>
#define X_LEN 4
#define Y_LEN 7
void deconvolution(int x[], int y[], int h[], int x_len, int y_len)
{
for (int n = 0; n < y_len; n++)
{
h[n] = y[n]; // 初始化 h[n] 為 y[n]
for (int k = 1; k < x_len && (n - k) ≥ 0; k++)
{
h[n] -= x[k] * h[n - k]; // 反向計算 h[n]
}
h[n] /= x[0]; // 反向解出 h[n]
}
}
int main()
{
(base) PS C:\Users\s9745\文件\MATLAB\HW2> gcc hw2_2_41.c -o hw2_2_41
(base) PS C:\Users\s9745\文件\MATLAB\HW2> ./hw2_2_41
h[n] = 2 8 8 -16
(base) PS C:\Users\s9745\文件\MATLAB\HW2>

```c
#include <stdio.h>

#define X_LEN 4
#define Y_LEN 7

void deconvolution(int x[], int y[], int h[], int x_len, int y_len)
{
    for (int n = 0; n < y_len; n++)
    {
        h[n] = y[n]; // 初始化 h[n] 為 y[n]
        for (int k = 1; k < x_len && (n - k) >= 0; k++)
        {
            h[n] -= x[k] * h[n - k]; // 反向計算 h[n]
        }
        h[n] /= x[0]; // 反向解出 h[n]
    }
}

int main()
{
    int x[X_LEN] = {1, -1, 2, 4};           // 輸入信號 x[n]
    int y[Y_LEN] = {2, 6, 4, 0, 8, 5, 12}; // 輸出信號 y[n]
    int h[Y_LEN] = {0};                     // h[n] 初始化為 0

    deconvolution(x, y, h, X_LEN, Y_LEN);

    printf("h[n] = ");
    for (int i = 0; i < 4; i++)
    {
        printf("%d ", h[i]);
    }
    printf("\n");

    return 0;
}


```

2.41 (b)




```matlab
x = [1 -1 2 4];
y = [2 6 4 0 8 5 12];
disp(deconv(y, x));
```