Homework-III

Chapter 3 Problems

3.3 3.13 3.14 3.18 3.26 3.43

Matlab Simulation

3.46, 3.49, 3.51, 3.53

3.3 Find the Laplace $X(s)$ given that $x(t)$ is

(a) $2tu(t-4)$

(b) $5\cos t\ \delta(t-2)$

(c) $e^{-t}u(t-\tau)$

(d) $\sin 2t\ u(t-\tau)$

(a) $2t\cdot u(t-4)$

$\mathcal{L}\{2t\cdot u(t-4)\}$

$= \int_0^{\infty} 2t\cdot u(t-4)\, e^{-st}\, dt$

$= \int_4^{\infty} 2t e^{-st} dt$

設 $u = 2t$, $du = 2dt$
$dv = e^{-st} dt$, $v = \frac{1}{-s}e^{-st}$

$= uv\Big|_4^{\infty} - \int_4^{\infty} v\, du$

$= 2t\frac{1}{-s}e^{-st}\Big|_4^{\infty} + \frac{2}{s}\int_4^{\infty} e^{-st}\cdot dt$

$= 0 - \frac{8}{-s}e^{-4s} + \frac{2}{s}\left(\frac{1}{-s}e^{-st}\Big|_4^{\infty}\right)$

$= \frac{8}{s}e^{-4s} + \left(\frac{2}{s}\left(0 - \frac{1}{-s}e^{-4s}\right)\right)$

$= \frac{8}{s}e^{-4s} + \frac{2}{s^2}e^{-4s}$

$= e^{-4s}\left(\frac{8}{s} + \frac{2}{s^2}\right)$

(b) $5\cos t\ \delta(t-2)$

$\mathcal{L}\{f(t)\}$

$= 5\int_0^{\infty} \cos t\ \delta(t-2)\, e^{-st} dt$

$= 5\cos 2\, e^{-2s}$

(c) $e^{-t}u(t-\tau)$

$\int_0^{\infty} e^{-t}u(t-\tau)e^{-st}dt$

$= \int_0^{\infty} e^{-(1+s)t} u(t-\tau)\, dt$

$= \frac{1}{-(1+s)} e^{-(1+s)t}\Big|_{\tau}^{\infty}$

$= \frac{1}{s+1} e^{-(1+s)\tau}$

(d) $\sin 2t\cdot u(t-\tau)$

$= \sin(2(t-\tau+\tau))\, u(t-\tau)$

$= \sin(2(t-\tau)+2\tau)\, u(t-\tau)$

$= (\sin 2(t-\tau)\cos 2\tau + \cos 2(t-\tau)\sin 2\tau)\, u(t-\tau)$

$\mathcal{L}\{f(t)\}$

$= \mathcal{L}\{\sin 2(t-\tau)\cos 2\tau\} + \mathcal{L}\{\cos 2(t-\tau)\sin 2\tau\}$

$= e^{-s\tau}\frac{2}{s^2+2^2}\cos 2\tau + e^{-s\tau}\frac{s}{s^2+2^2}\sin 2\tau$

$= \frac{e^{-s\tau}}{s^2+4}(2\cos 2\tau + s\sin 2\tau)$

Find the inverse Laplace transform for

(a) $X(s) = \dfrac{3s+1}{s^2+2s+5}$

(b) $Y(s) = \dfrac{3s+7}{s^2+3s+2}$

(c) $Z(s) = \dfrac{s^2-8}{s^2-4}$

(d) $H(s) = \dfrac{12}{(s+2)^2}$

(a)

$$\mathcal{L}^{-1}\left\{\frac{3s+1}{s^2+2s+5}\right\} = \mathcal{L}^{-1}\left\{\frac{3(s+1)-2}{(s+1)^2+2^2}\right\}$$

$$= \mathcal{L}^{-1}\left\{3\cdot\frac{s+1}{(s+1)^2+2^2} - \frac{2}{(s+1)^2+2^2}\right\}$$

$$= \underline{3e^{-t}\cos 2t - e^{-t}\sin 2t}$$

(b) $$\frac{3s+7}{s^2+3s+2} = \frac{3s+7}{(s+2)(s+1)} = \frac{-1}{s+2} + \frac{4}{s+1}$$

$$\mathcal{L}^{-1}\{Y(s)\} = (-1)\mathcal{L}^{-1}\left\{\frac{1}{s+2}\right\} + 4\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\}$$

$$= \underline{-e^{-2t} + 4e^{-t}}$$

(c) $$\frac{s^2-8}{s^2-4} = \frac{s^2-8}{(s+2)(s-2)} = \frac{\frac{3}{2}s+4}{(s+2)} + \frac{-\frac{1}{2}s}{(s-2)}$$

$$= \frac{3}{2}\cdot\frac{s}{s+2} + 4\frac{1}{s+2} - \frac{1}{2}\frac{s}{s-2}$$

$$\mathcal{L}^{-1}\{Z(s)\} = \frac{3}{2}\left(\delta(t) - 2e^{-2t}\right) + 4e^{-2t} - \frac{1}{2}\left(\delta(t) + 2e^{2t}\right)$$

$$= \underline{\delta(t) + e^{-2t} - e^{2t}}$$

(d)

$$\mathcal{L}^{-1}\left\{\frac{12}{(s+2)^2}\right\}$$

$$= 12\mathcal{L}^{-1}\left\{\frac{1}{(s+2)^2}\right\}$$

$$= 12\cdot e^{-2t}\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\}$$

$$= \underline{12e^{-2t}\cdot t}$$

3.14 Find the inverse Laplace transform of the following functions:

(a) $F(s) = \dfrac{20(s+2)}{s(s^2+6s+25)}$

(b) $P(s) = \dfrac{6s^2+36s+20}{(s+1)(s+2)(s+3)}$

(a)

$$\frac{20s+40}{s(s^2+6s+25)} = \frac{\frac{8}{5}}{s} + \frac{-\frac{8}{5}s+\frac{52}{5}}{s^2+6s+25}$$

$$= \frac{8}{5}\cdot\frac{1}{s} + \frac{-\frac{8}{5}(s+3)+\frac{76}{5}}{(s+3)^2+4^2}$$

$$= \frac{8}{5}\cdot\frac{1}{s} + \frac{-8}{5}\,\frac{s+3}{(s+3)^2+4^2} + \frac{19}{5}\cdot\frac{4}{(s+3)^2+4^2}$$

$$\therefore\ \mathcal{L}^{-1}\{F(s)\} = \underline{\frac{8}{5}u(t) - \frac{8}{5}e^{-3t}\cos 4t + \frac{19}{5}e^{-3t}\sin 4t}$$

(b)

$$\frac{6s^2+36s+20}{(s+1)(s+2)(s+3)} = \frac{a}{s+1} + \frac{b}{s+2} + \frac{c}{s+3}$$

$$= \frac{-5}{s+1} + \frac{28}{s+2} + \frac{-17}{s+3}$$

$$a(s+2)(s+3) + b(s+1)(s+3) + c(s+1)(s+2)$$

$$= (a+b+c)s^2 + (5a+4b+3c)s + (6a+3b+2c)$$

$$\Rightarrow \begin{cases} a+b+c=6 \\ 5a+4b+3c=36 \\ 6a+3b+2c=20 \end{cases} \Rightarrow \begin{cases} a=6-11=-5 \\ b=28 \\ c=11-28=-17 \end{cases}$$

$$\mathcal{L}^{-1}\{P(s)\} = -5\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\} + 28\mathcal{L}^{-1}\left\{\frac{1}{s+2}\right\} - 17\mathcal{L}^{-1}\left\{\frac{1}{s+3}\right\}$$

$$= \underline{-5e^{-t} + 28e^{-2t} - 17e^{-3t}}$$

3.18 Let $F(s) = \dfrac{5(s+1)}{(s+2)(s+3)}$

(a) Use the initial and final value theorems to find $f(0)$ and $f(\infty)$.
(b) Verify your answer in part (a) by finding $f(t)$ using partial fractions.

(a)

$$f(0) = \lim_{s\to\infty} sF(s) = \lim_{s\to\infty} \frac{5s^2+5s}{s^2+5s+6} = 5$$

$$f(\infty) = \lim_{s\to 0} sF(s) = 0$$

(b)

$$F(s) = \frac{5(s+1)}{(s+2)(s+3)} = \frac{-5}{s+2} + \frac{10}{s+3}$$

$$\Rightarrow f(t) = -5e^{-2t} + 10e^{-3t}$$

$$\therefore f(0) = -5+10 = 5, \quad f(\infty) = 0+0 = 0$$

3.26 For each of the systems shown in Figure 3.28, use Laplace transform to find $y(t)$.

(a)

$$\mathcal{L}\{e^{-2t}u(t)\} = \frac{1}{s+2}$$

$$\mathcal{L}\{4\} = 4\cdot\frac{1}{s}$$

$$\mathcal{L}\{te^{-t}u(t)\} = \frac{1}{(s+1)^2}$$

$$\therefore H(s) = \frac{1}{s+2}\cdot\frac{4}{s} + \frac{1}{(s+1)^2}$$

$$= \frac{4}{s(s+2)} + \frac{1}{(s+1)^2}$$

$$= \frac{2}{s} + \frac{-2}{s+2} + \frac{1}{(s+1)^2}$$

$$Y(s) = \cancel{X(s)}^{1} H(s) = H(s)$$

$$\Rightarrow y(t) = 2u(t) - 2e^{-2t} + te^{-t}$$

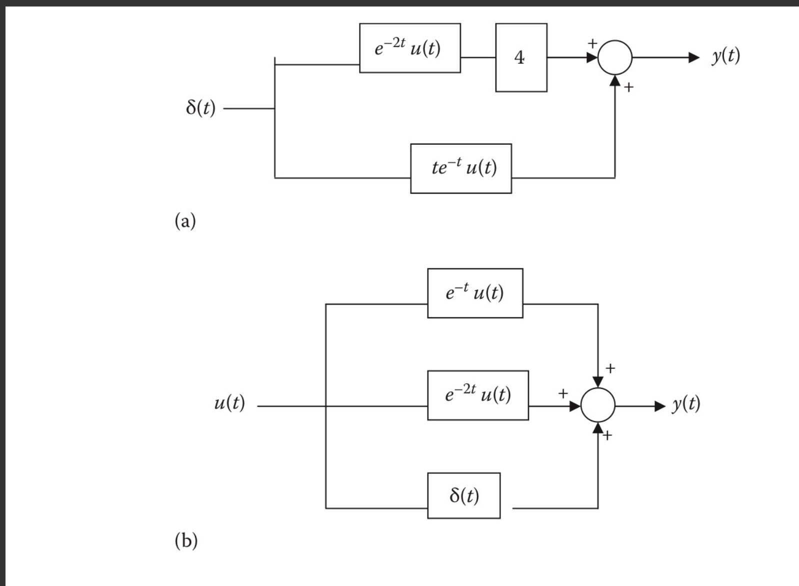


FIGURE 3.28 For Problem 3.26.

(b)

$$Y(s) = X(s)H(s)$$

$$= \frac{1}{s}\frac{1}{s+1} + \frac{1}{s}\frac{1}{s+2} + \frac{1}{s}$$

$$= \frac{1}{s} + \frac{-1}{s+1} + \frac{\frac{1}{2}}{s} + \frac{-\frac{1}{2}}{s+2} + \frac{1}{s}$$

$$\therefore y(t) = \frac{5}{2}u(t) - e^{-t} - \frac{1}{2}e^{-2t}$$

3.43 An LTI system is described by

$$\frac{d^2y(t)}{dt^2}+2\frac{dy(t)}{dt}+2y(t)=\frac{dx(t)}{dt}-3x(t)$$

(a) Determine the transfer function of the system.
(b) Obtain the impulse response of the system.

(a)

$$\mathcal{L}\{LHS\}=\mathcal{L}\{RHS\}$$

$$\Rightarrow s^2Y(s)+2sY(s)+2Y(s)=sX(s)-3X(s)$$

$$H(s)=\frac{Y(s)}{X(s)}=\frac{s-3}{s^2+2s+2}$$

(b)

$$H(s)=\frac{s-3}{s^2+2s+2}$$

$$=\frac{s+1-4}{(s+1)^2+1^2}$$

$$=\frac{s+1}{(s+1)^2+1^2}-4\frac{1}{(s+1)^2+1^2}$$

$$=e^{-t}\cos t-4e^{-t}\sin t$$

3.46

由附圖程式我們得到

$$H(s) = \frac{0.3125}{s+5} + \frac{0.6875}{s+1} + \frac{1.2500}{(s+1)^2}$$

因此

$$h(t) = 0.3125e^{-5t} + 0.6875e^{-t} + 1.2500te^{-t}$$

Editor - C:\Users\s9745\文件\MATLAB\HW3\hw3_3_46.m

hw3_3_46.m

```
b = [1 6 10];
a = [1 7 11 5];
[r,p,k] = residue(b,a)
```

Command Window

```
>> hw3_3_46

r =

    0.3125
    0.6875
    1.2500


p =

   -5.0000
   -1.0000
   -1.0000


k =

     []
```

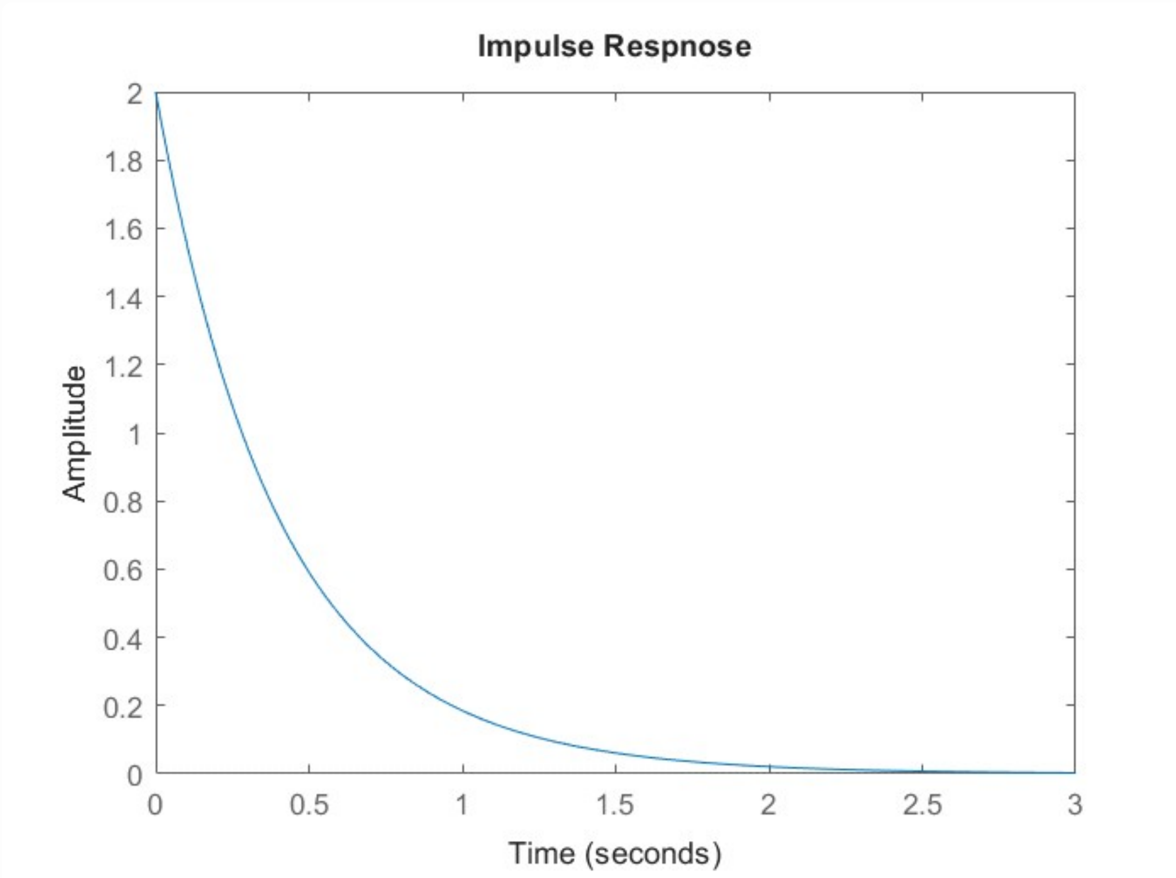
```
b = [1 6 10]; %分子
a = [1 7 11 5]; %分母
[r,p,k] = residue(b,a)
```

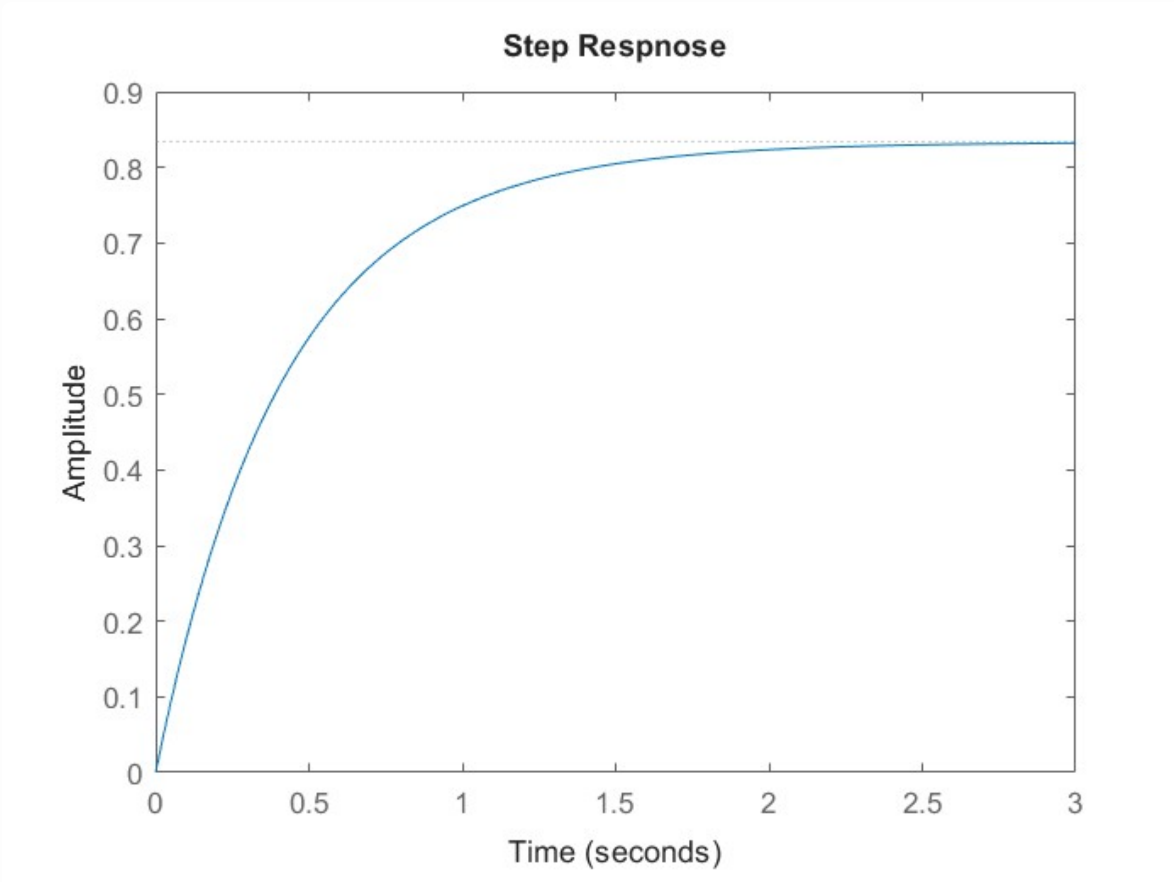
3.49

對於transfer function

$$H(s) = \frac{2s+5}{(s+2)(s+3)} = \frac{2s+5}{s^2+5s+6}$$

利用以下MATLAB程式得到impulse response以及step response：






```
num = [2 5]; %分子係數
den = [1 5 6]; %分母係數

H = tf(num, den); % Transfer function

figure;
impulse(H);
title('Impulse Respnose')

figure;
step(H);
title('Step Respnose')
```

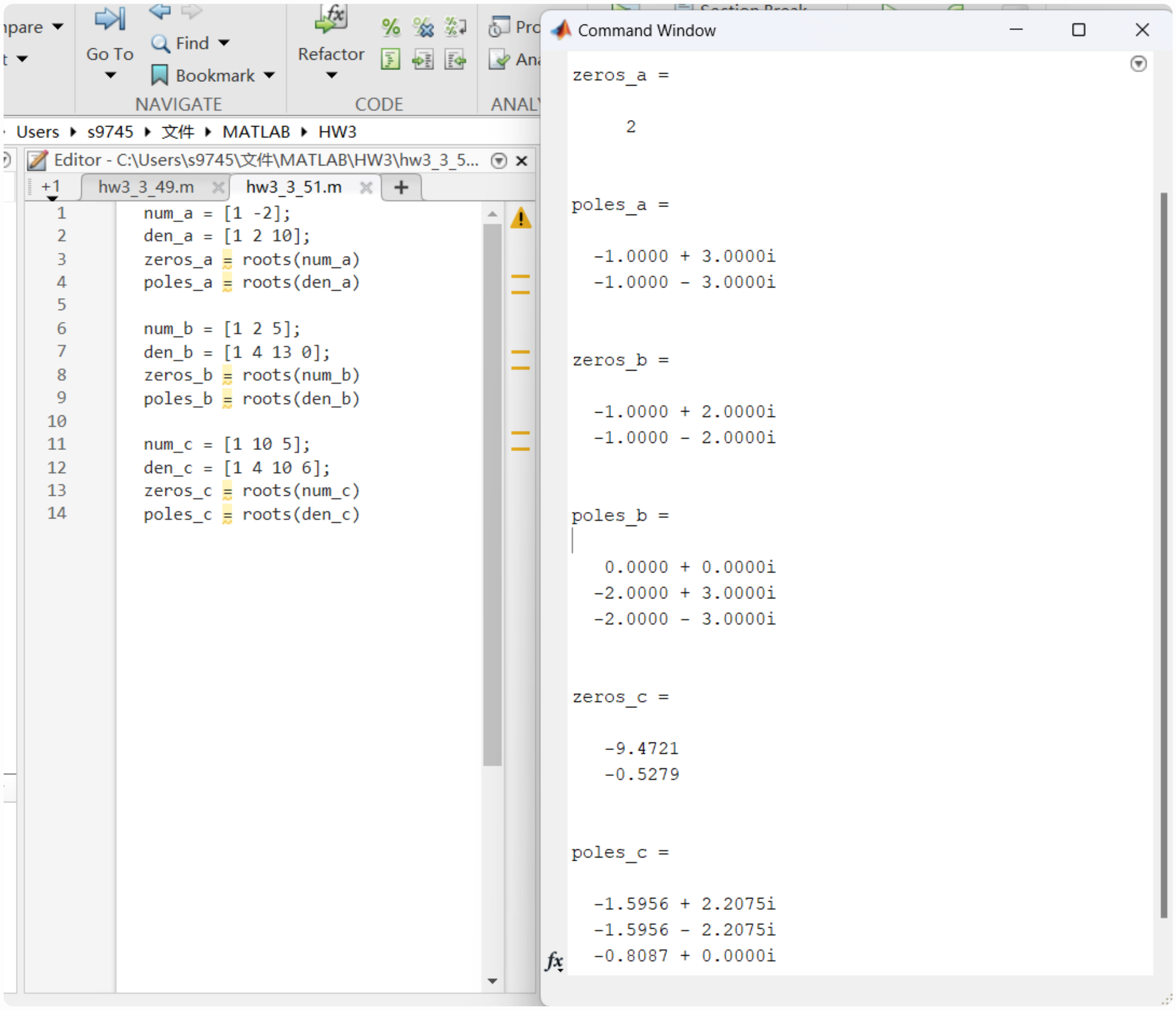
3.51

我們利用roots()來得到各自的zeros和poles，發現：

$$zeros_a = 2, poles_a = -1 \pm 3i$$

$$zeros_b = -1 \pm 2i, poles_b = -2 \pm 3i, 0$$

$$zeros_c = -9.4721, -0.5279, poles_c = -1.5956 \pm 2.2075i, -0.8087$$

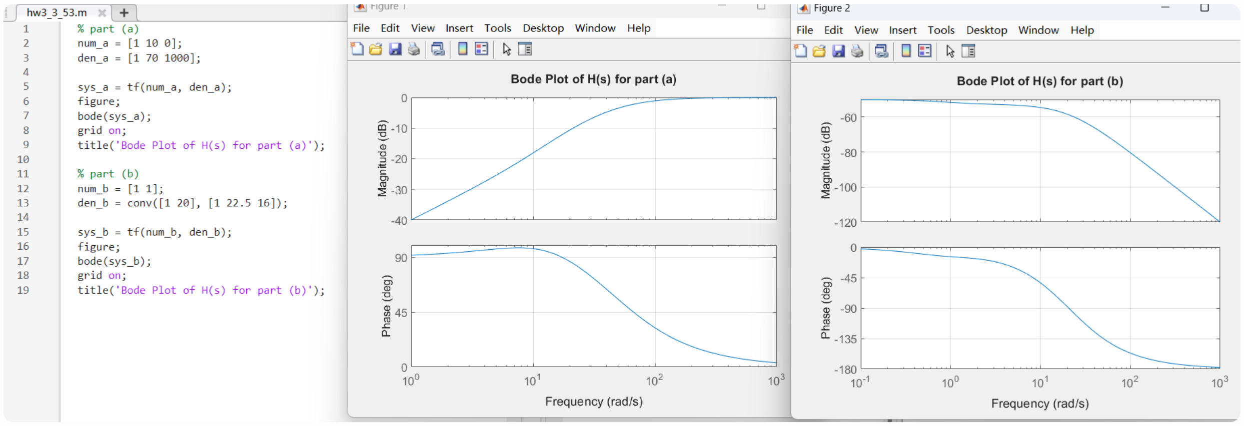



```
num_a = [1 -2];
den_a = [1 2 10];
zeros_a = roots(num_a)
poles_a = roots(den_a)

num_b = [1 2 5];
den_b = [1 4 13 0];
zeros_b = roots(num_b)
poles_b = roots(den_b)

num_c = [1 10 5];
den_c = [1 4 10 6];
zeros_c = roots(num_c)
poles_c = roots(den_c)
```

3.53




```
num_a = [1 10 0];
den_a = [1 70 1000];
sys_a = tf(num_a, den_a);
figure;
bode(sys_a);
grid on;
title('Bode Plot of H(s) for part (a)');


num_b = [1 1];
den_b = conv([1 20], [1 22.5 16]);
sys_b = tf(num_b, den_b);
figure;
bode(sys_b);
grid on;
title('Bode Plot of H(s) for part (b)');
```