PS3

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0.1 Problem Set 3

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[ ]:
```
# Importing the necessary libraries
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import autograd.numpy as np
from autograd import grad, jacobian
import scipy.optimize as optimize
from scipy.optimize import minimize
```

[ ]:
```
# Import data

df = pd.read_stata("../data/PS2_Data.dta")

# indiv - individual ID
# age
# SchD - schooling decision
# WhCoD - white collar work decision
# BlCoD - blue collar work decision
# SchE - school experience
# WhCoE - white collar work experience
# BlCoE - blue collar work experience
```

0.1.1 Question 1

Replicate Table 1 (choice distribution) and 2 (transition matrix) from Keane & Wolpin (1997). Reproduce a line graph similar to Figure 1, with a line for each career option.

What can you say about the relationship between age and choices? What about persistence and state dependence?

[ ]:
```
# Table 1 simply shows number of observations and percentages of individuals at␣
↪each age that chose school, home, white-collar,  blue-collar, miilitary,␣
↪total

table1 = df.groupby('age') \
```

```
        .agg(School_mean = ('SchD', 'mean'), School_sum = ('SchD', 'sum'),␣
↪White_mean = ('WhCoD', 'mean'), White_sum = ('WhCoD', 'sum'), Blue_mean =␣
↪('BlCoD', 'mean'), Blue_sum = ('BlCoD', 'sum'), Total_mean = ('age',␣
↪'nunique'), Total_sum = ('age', 'count'))

# Wasn't really figuring out how to do the total_mean, but this should just be␣
↪=1, since it's just the sum of all the other %, so that just sums to 1
```

[ ]:
```
table1.columns =['School (Mean)', 'School (Sum)', 'White-Collar (Mean)',␣
↪'White-Collar (Sum)', 'Blue-Collar (Mean)', 'Blue-Collar (Sum)', 'Total␣
↪(Mean)', 'Total(Sum)']

table1
```

[ ]:
```
     School (Mean)  School (Sum)  White-Collar (Mean)  White-Collar (Sum)  \
age
1            0.442         221.0                0.266               133.0
2            0.398         199.0                0.284               142.0
3            0.370         185.0                0.330               165.0
4            0.328         164.0                0.356               178.0
5            0.298         149.0                0.354               177.0
6            0.244         122.0                0.390               195.0
7            0.244         122.0                0.354               177.0
8            0.190          95.0                0.390               195.0
9            0.182          91.0                0.392               196.0
10           0.136          68.0                0.452               226.0

     Blue-Collar (Mean)  Blue-Collar (Sum)  Total (Mean)  Total(Sum)
age
1                 0.292              146.0             1         500
2                 0.318              159.0             1         500
3                 0.300              150.0             1         500
4                 0.316              158.0             1         500
5                 0.348              174.0             1         500
6                 0.366              183.0             1         500
7                 0.402              201.0             1         500
8                 0.420              210.0             1         500
9                 0.426              213.0             1         500
10                0.412              206.0             1         500
```

[ ]:
```
# Table 2 is the transition matrix, so we need lags
df['LagSchD'] = df.groupby('indiv')['SchD'].shift(1)
df['LagWhCoD'] = df.groupby('indiv')['WhCoD'].shift(1)
df['LagBlCoD'] = df.groupby('indiv')['BlCoD'].shift(1)
```

[ ]:
```
# "The first figure in each cell is the percentage
# of transitions from origin to destination (the row percentage) and
```

```
# the second the reverse, that is, the percentage in a particular destination
# who started from each origin (column percentage)."

# Row percentage

def table2_row(t , t_1):
    row = df.loc[(t == 1) & (t_1 == 1),].shape[0] / df.loc[(t_1 == 1) & (df.age␣
 ↪!=1),].shape[0]
    return row

# Column percentage

def table2_col(t , t_1):
    col = df.loc[(t == 1) & (t_1 == 1),].shape[0] / df.loc[(t == 1) & (df.age !
 ↪=1),].shape[0]
    return col
```

[ ]:
```
# Let's create our table

table2 = pd.DataFrame()

for t_1 in [df.LagSchD, df.LagWhCoD, df.LagBlCoD]:
    row = []
    col = []

    for t in [df.SchD, df.WhCoD, df.BlCoD]:
        row.append(round(table2_row(t, t_1), 2))
        col.append(round(table2_col(t, t_1), 2))

    table2 = table2.append(pd.DataFrame(row).T)
    table2 = table2.append(pd.DataFrame(col).T)
```

[ ]:
```
# Naming columns and rows
table2.columns = ['School', 'White Collar',
                'Blue Collar']

choices = ['School (Row)', 'School (Col)', 'White Collar (Row)', 'White Collar␣
 ↪(Col)', 'Blue Collar (Row)', 'Blue Collar (Col)']
table2.insert(0, 'Choice (t-1)', choices)
```

[ ]:
```
table2
```

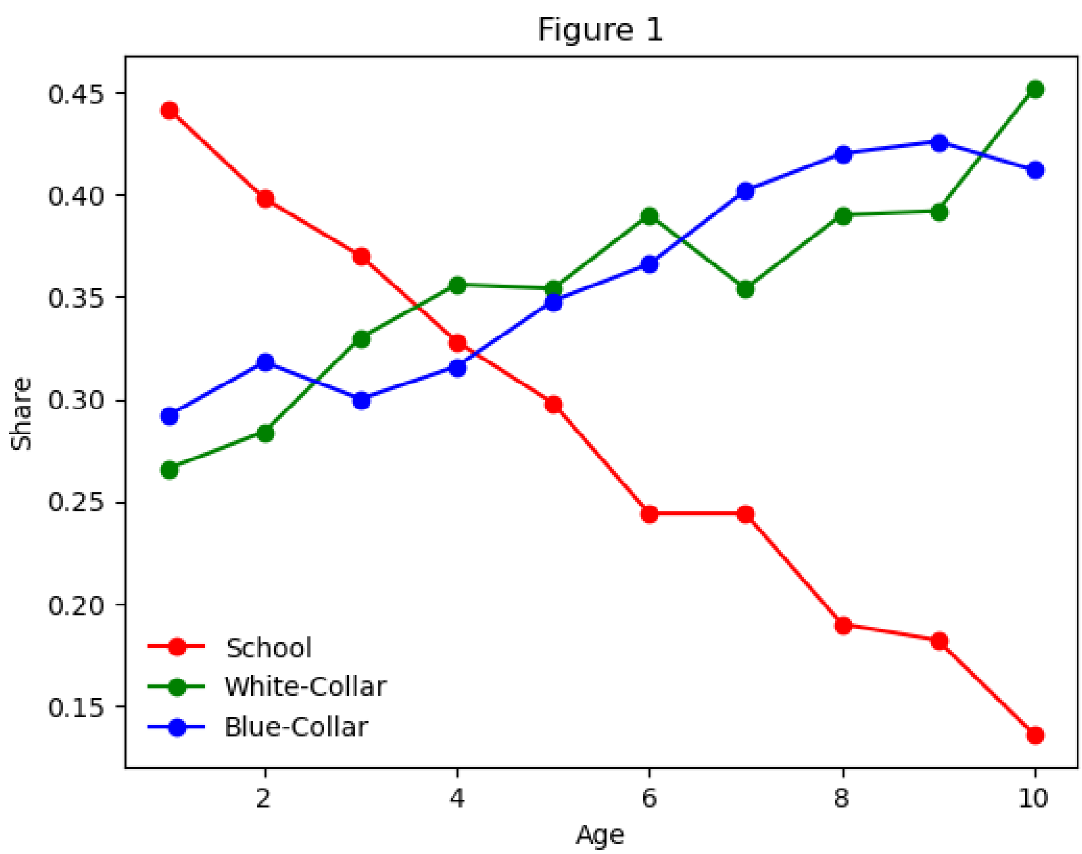
[ ]:

	Choice (t-1)	School	White Collar	Blue Collar
0	School (Row)	0.25	0.37	0.38
0	School (Col)	0.28	0.31	0.31
0	White Collar (Row)	0.27	0.41	0.31
0	White Collar (Col)	0.36	0.39	0.30

```
0   Blue Collar (Row)    0.27          0.32          0.41
0   Blue Collar (Col)    0.36          0.31          0.40
```

[ ]:
```
# Figure 1 percentage of people who chose school, wc and bc by age
table1.reset_index(inplace=True)

plt.plot(table1['age'], table1['School (Mean)'], color='red', marker='o')
plt.plot(table1['age'], table1['White-Collar (Mean)'], color='green',␣
  ↪marker='o')
plt.plot(table1['age'], table1['Blue-Collar (Mean)'], color='blue', marker='o')
plt.title('Figure 1')
plt.xlabel('Age')
plt.ylabel('Share')
plt.legend(['School', 'White-Collar', 'Blue-Collar'], frameon = False)
plt.show()
```



The share of people in school decreases with age, naturally. At the same time, the share of people in white-collar and blue-collar jobs increases with age.

0.1.2 Question 2

Assume individuals make decisions based on a standard human capital model, where each career option gives them a period-specific reward. How would you structure these choice-specific reward functions based on the choice distribution and transition matrix you observe in the data?

Not sure what to comment here, but I guess important points should be that rewards to work are increasing with time in the labor force. And probably returns to schooling decreasing.

0.1.3 Question 3

Reward functions:

$$R_s(a) = \beta_0 + \beta_1 I[g(a) \geq 12] + \epsilon_s(a)$$
$$R_w(a) = \alpha_1 g(a) + \alpha_2 x_w(a) + \alpha_3 x_w^2(a) + \epsilon_w(a)$$
$$R_b(a) = \eta_1 g(a) + \eta_2 x_b(a) + \eta_3 x_b^2(a) + \epsilon_b(a)$$

$g(a)$ is the school attainment at age a, $x_w(a)$, $x_b(a)$ are white and blue collar experience, and $\epsilon_k(a)$ are choice specific shocks distributed as type extreme value.

Observed state space: $S(a) = [g(a), x_w(a), x_b(a)]$

Make a random guess of the parameters and find the alternative specific value functions for each possible path of choices and age, defined as:

$$V_k(S(a), a) = R_k(S(a), a) + \delta E[V(S(a+1), a+1)|S(a), d_k(a) = 1], \text{ for } a < A$$
$$V_k(S(A), A) = R_k(S(A), A)$$

A is the last year of age in the sample and d_k is the alternative-specific decision.

[ ]:
```
N = 500

# Schooling years and work experience
df['g_a'] = df.groupby('indiv')['SchD'].transform('sum')
df['x_w'] = df.groupby('indiv')['WhCoD'].transform('sum')
df['x_b'] = df.groupby('indiv')['BlCoD'].transform('sum')
```

[ ]:
```
def Rs(params_s, g_a):
    beta0 , beta1 = params_s
    return beta0 + beta1*(g_a>11)

def Rw(params_w, g_a, x_w):
    alpha1, alpha2, alpha3 = params_w
    return alpha1*g_a + alpha2*x_w + alpha3*x_w**2

def Rb(params_b, g_a, x_b):
    eta1, eta2, eta3 = params_b
    return eta1*g_a + eta2*x_b + eta3*x_b**2
```

```
# Constants
sigma = 0.95
 = 0.57721566490153286061 # mean of extreme value type 1
```

[ ]:
```
value_functions = []

for i in range(10):

    def value_f(g_a, x_b, x_w, params_s, params_w, params_b, i = i):

        # Schooling
        Vs = Rs(params_s, g_a) + sigma*value_functions[i-1](g_a + 1 , x_b, x_w,␣
↪params_s, params_w, params_b)[0] if i > 0 else Rs(params_s, g_a)

        # White collar
        Vw = Rw(params_w, g_a, x_w) + sigma*value_functions[i-1](g_a , x_b, x_w␣
↪+ 1, params_s, params_w, params_b)[0] if i > 0 else Rw(params_w, g_a, x_w)

        # Blue collar
        Vb = Rb(params_b, g_a, x_b ) + sigma*value_functions[i-1](g_a , x_b + 1␣
↪, x_w, params_s, params_w, params_b)[0] if i > 0 else Rb(params_b, g_a, x_b )

        EMax =   + np.log(np.sum(np.exp([Vs, Vw, Vb]), axis = 0)) # slide 73 -␣
↪type 1 extreme value shocks

        return [EMax, Vs, Vw, Vb]

    value_functions.append(value_f)
```

0.1.4 Question 4

Use the full solution of the dynamic programming problem you just derived (inner loop) to estimate the parameters of the reward functions by maximum likelihood (outer loop) (note: no need for simulation here due to the distributional assumption of the shocks). Don't estimate the discount factor, just set it to = 0.95. Calculate the standard errors using finite difference approximation of the likelihood function at the estimated parameters (hint: you can use the same code provided for the infinite horizon single agent dynamic problem explained in class). Report your results. How do you interpret these coefficients? How does this compare to your interpretation of the descriptives in questions 1 and 2?

[ ]:
```
# Likelihood Function
def LL(params, df = df):

    beta0, beta1, alpha1, alpha2, alpha3, eta1, eta2, eta3 = params
```

```
    beta= np.array([beta0, beta1])
    alpha = np.array([alpha1, alpha2, alpha3])
    eta = np.array([eta1,eta2,eta3])

    ll = 0

    for i in range(10):

        g_a = np.array(df.loc[df.age == 10 - i,'g_a']).reshape(1,N)
        x_b = np.array(df.loc[df.age == 10 - i,'x_b']).reshape(1,N)
        x_w = np.array(df.loc[df.age == 10 - i,'x_w']).reshape(1,N)

        # numerator
        num = np.array(np.concatenate(value_functions[i](g_a, x_b , x_w , beta,␣
↪alpha, eta )[1:4])).T
        # denominator
        den = np.array(np.concatenate(value_functions[i](g_a, x_b , x_w , beta,␣
↪alpha, eta )[1:4])).T
        # probabiility of decision at age a for each individual
        p_a = np.exp(np.sum(np.multiply(df.loc[df.age == 10 - i, ('SchD',␣
↪'WhCoD', 'BlCoD')], num), axis = 1))  / np.sum(np.exp(den), axis = 1)

        ll += np.sum(np.log(p_a))

    ll = -1*ll

    return ll
```

[ ]:
```
# takes very long to run
guess = [0.0, 0.0, 0.1, 0.1, -0.01, 0.1, 0.1, -0.01]
MLE = minimize(LL, guess, method = 'BFGS', tol = 1e-6)
```

[ ]:
```
# Obtaining standard errors through inverse hessian

# Extract the Hessian matrix
inv_hess = MLE.hess_inv

# Extract the diagonal elements of the covariance matrix
diag_cov = np.diag(inv_hess)

# Compute the standard errors
std_errors = np.sqrt(diag_cov)

# Create a table with the coefficients and standard errors
coefficients = MLE.x
table = np.stack((coefficients, std_errors), axis=-1)
parameters = [" _0", " _1", " _1", " _2", " _3", " _1", " _2", " _3"]
```

```python
results = pd.DataFrame(table, parameters)
results.columns = ['Coefficient', 'SE']

results
```

[ ]:

	Coefficient	SE
_0	0.910235	0.027974
_1	0.000189	0.016914
_1	0.234336	0.016915
_2	0.181244	0.008552
_3	0.004581	0.001315
_1	0.228755	0.018061
_2	0.188102	0.008960
_3	0.004003	0.001298

As expected, both occupations have positive returns to schooling. However, the difference is not a lot between the returns to schooling for white collar vs blue collar workers. The linear component of the returns to experience is also very similiar between the two occupations. The cuadratic component is close to zero, meaning the returns to experience are almost linear in time. The returns to after high school education seems to be close to nothing, so it makes sense that the curve in Figure 1 is downwards sloping.

0.1.5 Question 5

Estimate the model now with unobserved heterogeneity. Assume there are 2 types of individuals in the sample, who differ in the effect of schooling on rewards from working. One type has a higher return from schooling in a white collar job and a lower return in a blue collar one, as she specialized in managerial studies, whereas the opposite is true for the other type, who specialized in a more blue collar oriented education. Don't estimate type proportions but just set them equal to 0.5. Report estimated coefficients and standard errors. How do your results compare with the previous question? How much does unobserved heterogeneity matter?

```python
# One type has a higher return from schooling in a white collar job - different␣
↪alpha1
# The other has a higher return from schooling in a blue collar job - different␣
↪eta1
# Other parameters are the same for both types
```

```python
def LL_UH(params, df = df):

    beta0, beta1, alpha1_t1, alpha1_t2, alpha2, alpha3, eta1_t1, eta1_t2, eta2,␣
↪eta3 = params

    beta= np.array([beta0, beta1])
    alpha_v1 = np.array([alpha1_t1, alpha2, alpha3])
    eta_v1 = np.array([eta1_t1,eta2,eta3])
    alpha_v2 = np.array([alpha1_t2, alpha2, alpha3])
```

```python
    eta_v2 = np.array([eta1_t2,eta2,eta3])

    ll_t1 = np.ones(500)
    ll_t2 = np.ones(500)

    for i in range(10):

        ga = np.array(df.loc[df.age == 10 - i,'g_a']).reshape(1,N)
        xb = np.array(df.loc[df.age == 10 - i,'x_b']).reshape(1,N)
        xw = np.array(df.loc[df.age == 10 - i,'x_w']).reshape(1,N)

        # Type 1
        # numerator and denominator
        num_t1 = np.array(np.concatenate(value_functions[i](ga, xb , xw , beta,␣
↪alpha_v1, eta_v1 )[1:4])).T
        den_t1 = np.array(np.concatenate(value_functions[i](ga, xb , xw , beta,␣
↪alpha_v1, eta_v1 )[1:4])).T
        # Probability of decision
        p_a_v1 = np.exp(np.sum(np.multiply(df.loc[df.age == 10 - i, ('SchD',␣
↪'WhCoD', 'BlCoD')], num_t1), axis = 1))  / np.sum(np.exp(den_t1), axis = 1)

        # Type 2
        # numerator and denominator
        num_t2 = np.array(np.concatenate(value_functions[i](ga, xb , xw , beta,␣
↪alpha_v2, eta_v2 )[1:4])).T
        den_t2 = np.array(np.concatenate(value_functions[i](ga, xb , xw , beta,␣
↪alpha_v2, eta_v2 )[1:4])).T
        # Probabiility of decision
        p_a_v2 = np.exp(np.sum(np.multiply(df.loc[df.age == 10 - i, ('SchD',␣
↪'WhCoD', 'BlCoD')], num_t2), axis = 1))  / np.sum(np.exp(den_t2), axis = 1)

        # Log-likelihood
        ll_t1 = np.multiply(ll_t1.reshape(500,1), np.array(p_a_v1).
↪reshape(500,1))
        ll_t2 = np.multiply(ll_t2.reshape(500,1), np.array(p_a_v2).
↪reshape(500,1))

    ll = -1 * np.sum(np.log(0.5 * ll_t1 + 0.5 * ll_t2)) # set type proportions␣
↪equal to 0.5

    return ll
```

[ ]:
```python
guess = [1.0, -1.0, 0.1, 0.1, 0.1, -0.1, 0.1, 0.1, 0.1, -0.1]
MLE_UH = minimize(LL_UH, guess, method = 'BFGS', tol = 1e-3)

# It ran for one hour, no convergence
```

0.1.6 Question 6

Produce 3 graphs similar to the one you did in question 1, one for each career decision. In these graphs plot a line for the actual data, and two the model's prediction both with and without unobserved heterogeneity. Use the predicted probabilities to construct the lines for the two models. Which model seems to fit the data better?

[ ]:
```
def prob(params, df=df):
    beta0, beta1, alpha1, alpha2, alpha3, eta1, eta2, eta3 = params
    Beta = np.array([beta0, beta1])
    alpha = np.array([alpha1, alpha2, alpha3])
    Eta = np.array([eta1, eta2, eta3])

    pred_prob = []
    for i in range(10):
        ga = np.array(df[df['age'] == 10 - i]['g_a'])
        xb = np.array(df[df['age'] == 10 - i]['x_b'])
        xw = np.array(df[df['age'] == 10 - i]['x_w'])

        utility = value_functions[i](ga, xb, xw, Beta, alpha, Eta)[1:]
        prob = np.exp(utility) / np.sum(np.exp(utility), axis=0)
        mean_prob = np.mean(prob, axis=1)
        pred_prob.append(mean_prob)

    return pred_prob
```

[ ]:
```
list_pred = list(map(lambda x: [x], prob(MLE.x)))
pred_prob = pd.DataFrame(np.concatenate(list_pred))
pred_prob.columns = ['School', 'White-Collar', 'Blue-Collar']
pred_prob
```

[ ]:

	School	White-Collar	Blue-Collar
0	0.224657	0.386765	0.388578
1	0.237625	0.380145	0.382230
2	0.249497	0.374132	0.376370
3	0.260003	0.368870	0.371127
4	0.269344	0.364324	0.366332
5	0.279313	0.359932	0.360755
6	0.293019	0.353360	0.353621
7	0.311757	0.343430	0.344814
8	0.333075	0.331654	0.335271
9	0.353664	0.320020	0.326316

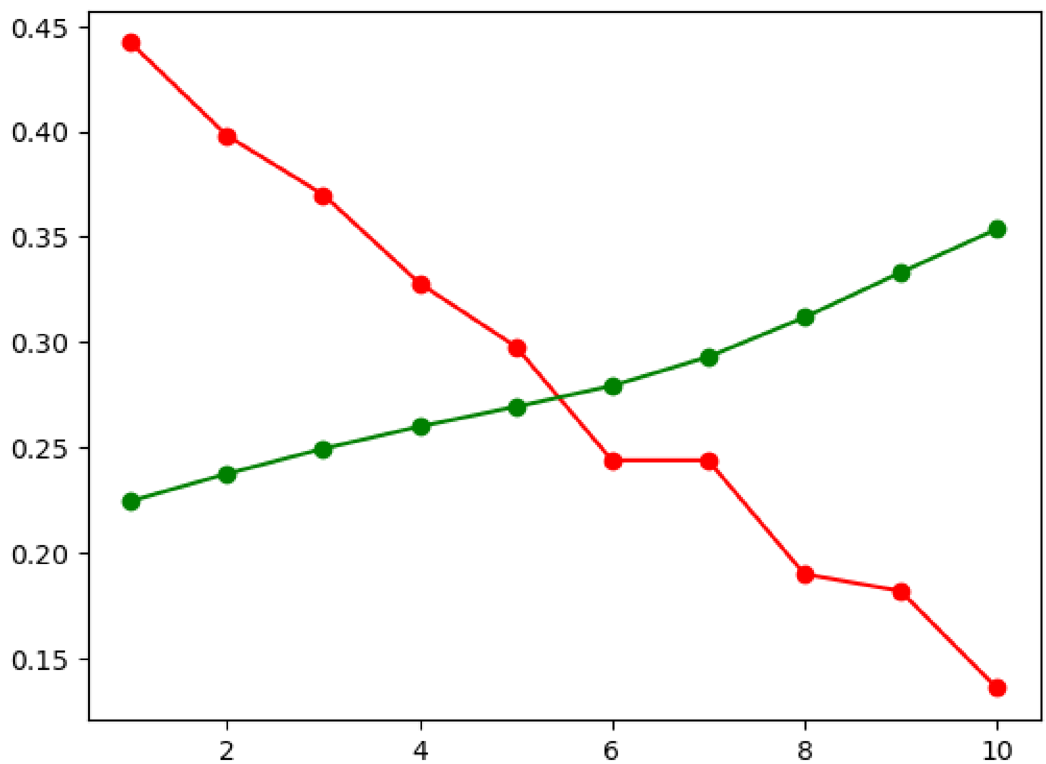
Because the minimization didn't converge for the unobserved heterogeneity case, we're only presenting results for the case without unobserved heterogeneity.

[ ]:
```
# Graph Schooling - Results look wrong, but we tried :(
```

```python
plt.plot(table1['age'], table1['School (Mean)'], color='red', marker='o')
plt.plot(table1['age'], pred_prob['School'], color='green', marker='o')
plt.plot(table1['age'], pred2['School'], color='blue', marker='o')
plt.title('Schooling Decision')
plt.xlabel('Age')
plt.ylabel('Share')
plt.legend(['Data', 'Without UH', 'With UH'], frameon = False)
plt.show()
```

```
---------------------------------------------------------------------------
NameError                                 Traceback (most recent call last)
Cell In [257], line 5
      3 plt.plot(table1['age'], table1['School (Mean)'], color='red', marker='o')
      4 plt.plot(table1['age'], pred_prob['School'], color='green', marker='o')
----> 5 plt.plot(table1['age'], pred2['School'], color='blue', marker='o')
      6 plt.title('Schooling Decision')
      7 plt.xlabel('Age')

NameError: name 'pred2' is not defined
```

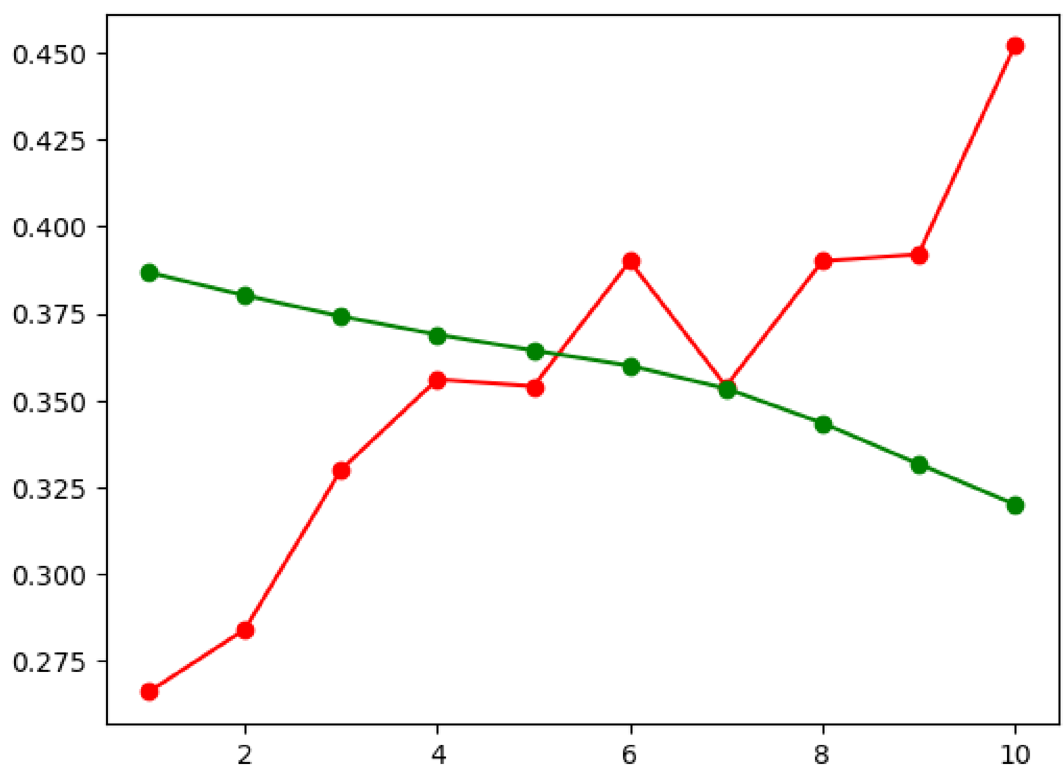


[ ]:
```python
# Graph White-Collar

plt.plot(table1['age'], table1['White-Collar (Mean)'], color='red', marker='o')
plt.plot(table1['age'], pred_prob['White-Collar'], color='green', marker='o')
plt.plot(table1['age'], pred2['White-Collar'], color='blue', marker='o')
plt.title('White-Collar Decision')
plt.xlabel('Age')
plt.ylabel('Share')
plt.legend(['Data', 'Without UH', 'With UH'], frameon = False)
plt.show()
```

```
---------------------------------------------------------------------------
NameError                                 Traceback (most recent call last)
Cell In [258], line 5
      3 plt.plot(table1['age'], table1['White-Collar (Mean)'], color='red',␣
 ↪marker='o')
      4 plt.plot(table1['age'], pred_prob['White-Collar'], color='green',␣
 ↪marker='o')
----> 5 plt.plot(table1['age'], pred2['White-Collar'], color='blue', marker='o')
      6 plt.title('White-Collar Decision')
      7 plt.xlabel('Age')

NameError: name 'pred2' is not defined
```



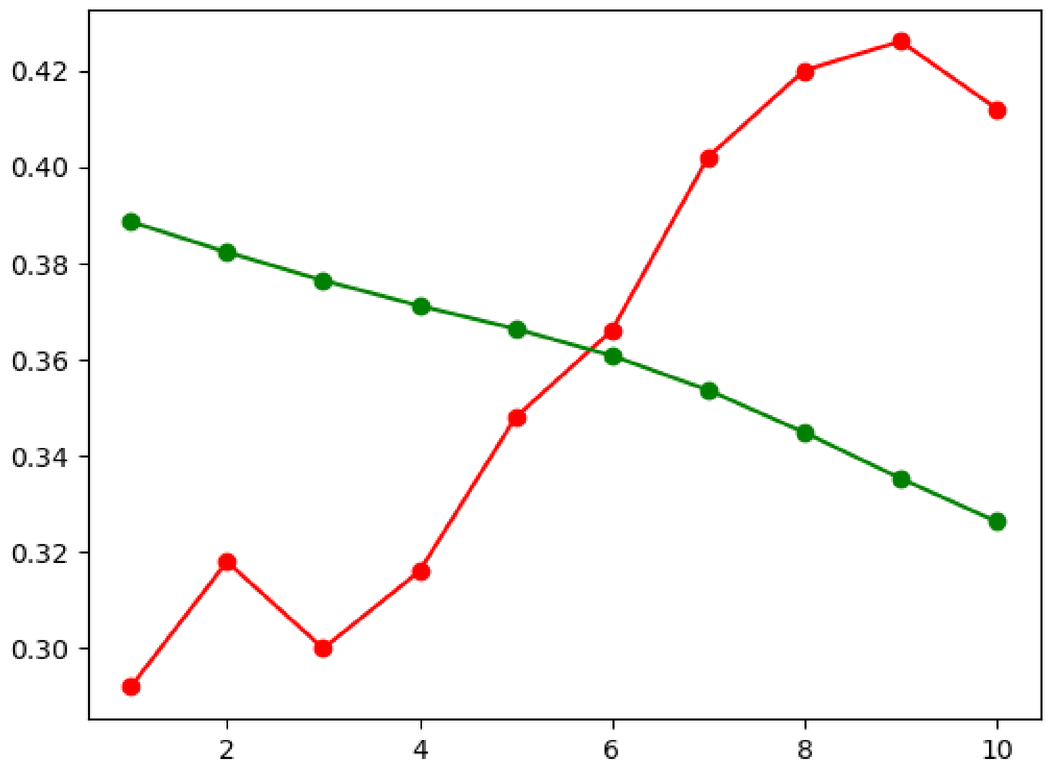
[ ]:

```python
# Graph Blue-Collar

plt.plot(table1['age'], table1['Blue-Collar (Mean)'], color='red', marker='o')
plt.plot(table1['age'], pred_prob['Blue-Collar'], color='green', marker='o')
plt.plot(table1['age'], pred2['Blue-Collar'], color='blue', marker='o')
plt.title('Blue-Collar Decision')
plt.xlabel('Age')
plt.ylabel('Share')
plt.legend(['Data', 'Without UH', 'With UH'], frameon = False)
plt.show()
```

```
---------------------------------------------------------------------------
NameError                                 Traceback (most recent call last)
Cell In [259], line 5
      3 plt.plot(table1['age'], table1['Blue-Collar (Mean)'], color='red',␣
 ↪marker='o')
      4 plt.plot(table1['age'], pred_prob['Blue-Collar'], color='green',␣
 ↪marker='o')
----> 5 plt.plot(table1['age'], pred2['Blue-Collar'], color='blue', marker='o')
      6 plt.title('Blue-Collar Decision')
      7 plt.xlabel('Age')

NameError: name 'pred2' is not defined
```



Clearly, these results don't seem very correct, as the actual data and prediction are almost opposite. We really tried to figure out what was wrong with our code, but we were not able to.