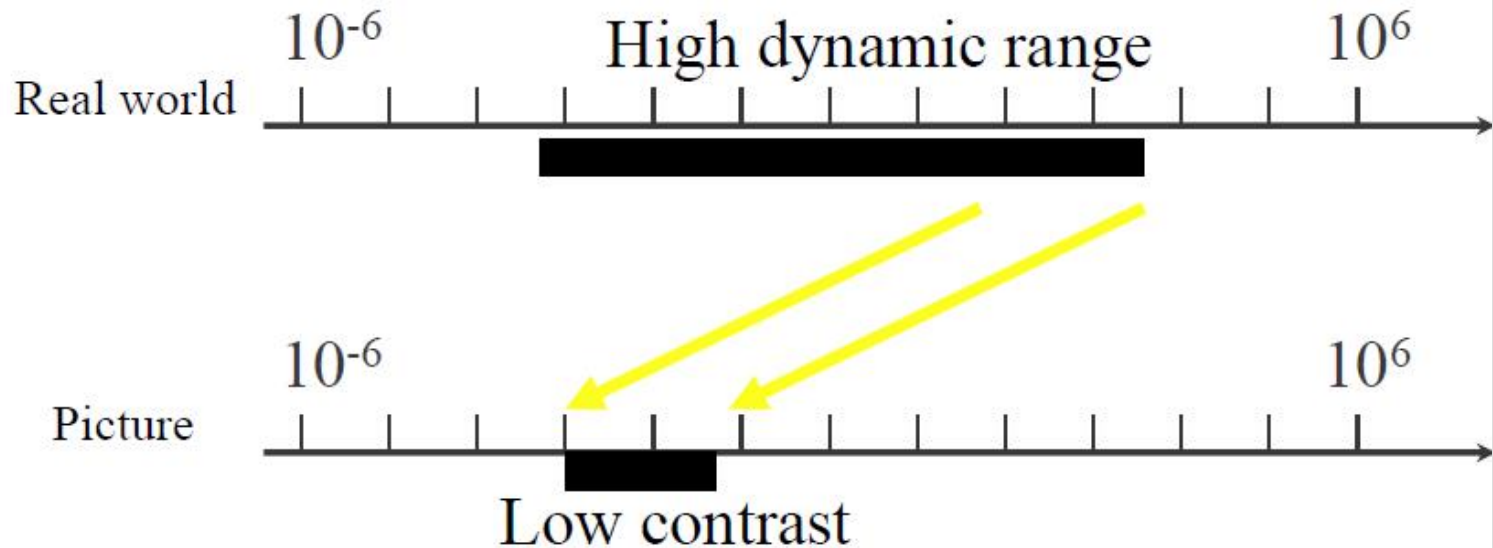


# Sequential Collection of Images

## Multiple exposure photography



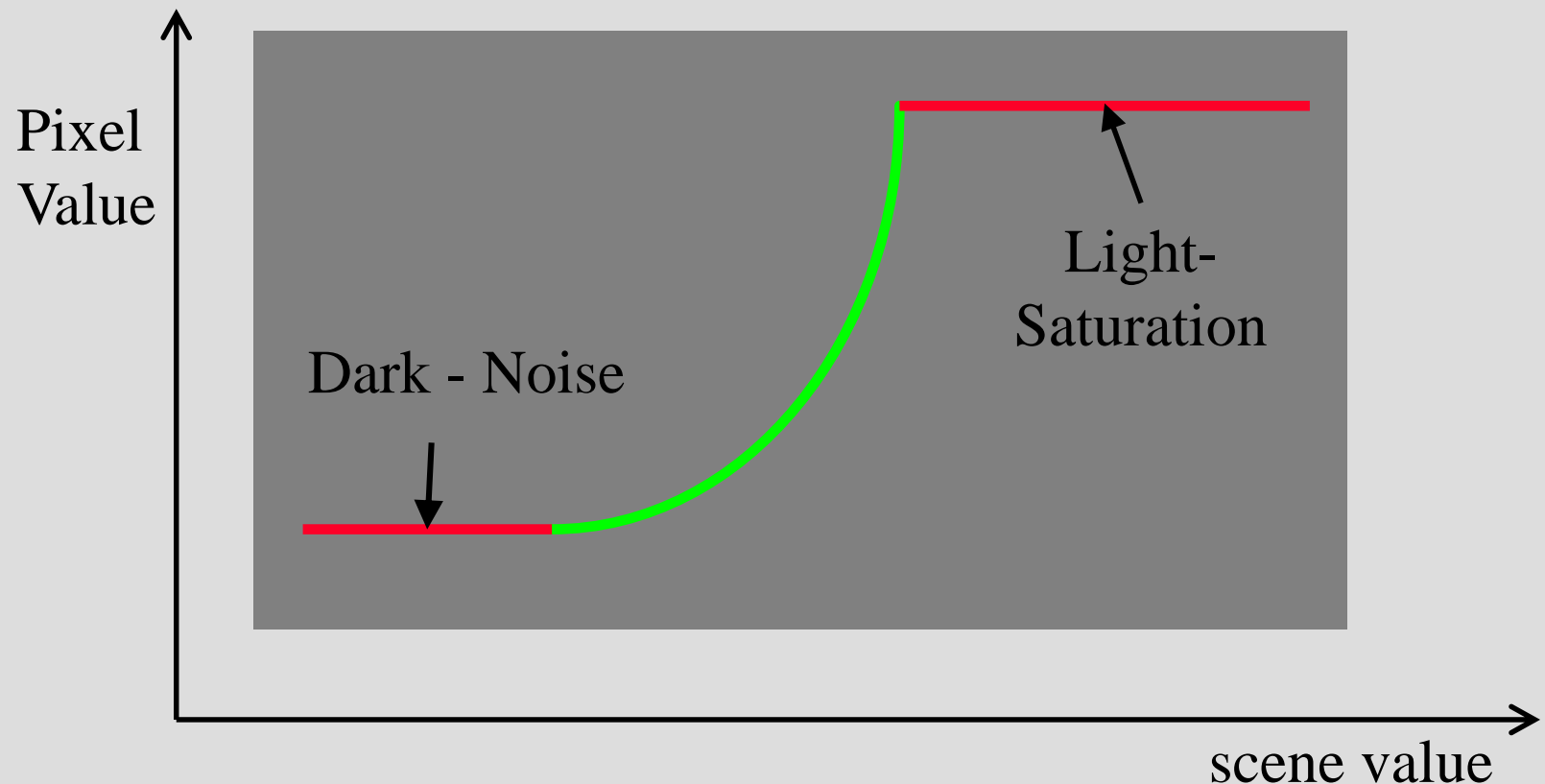
- Sequentially measure all segments of the range



# Response Curve

The relation between the radiance and the pixel values is

- not linear and
- not completely known



# Estimate Camera Response Curve

$$Z_{ij} = f(E_i \Delta t_j) \quad (1)$$

Since we assume  $f$  is monotonic, it is invertible, and we can rewrite (1) as:

$$f^{-1}(Z_{ij}) = E_i \Delta t_j$$

Taking the natural logarithm of both sides, we have:

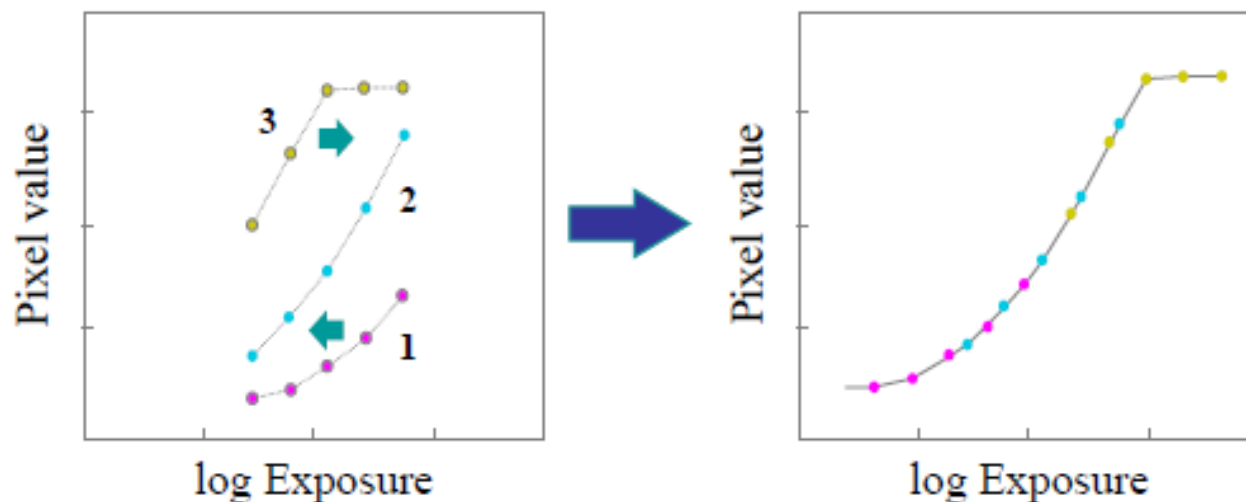
$$\ln f^{-1}(Z_{ij}) = \ln E_i + \ln \Delta t_j$$

To simplify notation, let us define function  $g = \ln f^{-1}$ . We then have the set of equations:

$$g(Z_{ij}) = \ln E_i + \ln \Delta t_j \quad (2)$$

where  $i$  ranges over pixels and  $j$  ranges over exposure durations. In this set of equations, the  $Z_{ij}$  are known, as are the  $\Delta t_j$ . The unknowns are the irradiances  $E_i$ , as well as the function  $g$ , although we assume that  $g$  is smooth and monotonic.

# Response Curve



**Figure 10.13** Radiometric calibration using multiple exposures (Debevec and Malik 1997). Corresponding pixel values are plotted as functions of log exposures (irradiance). The curves on the left are shifted to account for each pixel's unknown radiance until they all line up into a single smooth curve.

# Linear equations

$$g(Z_{ij}) = \ln E_i + \ln t_j \quad \text{same as} \quad g(Z_{ij}) - \ln E_i = \ln t_j$$

Fix  $i$  and  $j$  and assume that  $Z_{ij} = m$  with  $0 \leq m < 256$

Define vector  $A_{ij} = (0 \dots 1 \dots 0)$  where the 1 is at position  $m$

Define vector of unknowns:  $x_1 = (g(0) \dots g(255))'$

Then

$$g(Z_{ij}) = A_{ij} x_1$$

Define the vector  $B_i$  as unit vector  $(0 \dots 1 \dots)$  with 1 at position  $i$

Define vector of unknowns:  $x_2 = (\ln E_1, \dots, \ln E_N)'$

Then

$$B_i x_2 = \ln E_i$$

For

$$x' = (x_1 \ x_2), \quad A_j = (A_{ij} \ B_i)$$

$$A_j x = \ln t_j$$

# Linear Algebra

$$\mathcal{O} = \sum_{i=1}^N \sum_{j=1}^P [g(Z_{ij}) - \ln E_i - \ln \Delta t_j]^2 + \lambda \sum_{z=Z_{min}+1}^{Z_{max}-1} g''(z)^2 \quad (3)$$

**Data Term**

**Smoothness Term**

Measurement  $k$

$$A_k x = b_k$$

no weights means elements in  $A_k$  are 0 or 1

“many” measurements give the equation

$$Ax = b$$

with

$$x = A \backslash b$$

as solution

# Weights

Points near the end are not as reliable as those at the center  
Introduce weights  $w(z)$

$$\mathcal{O} = \sum_{i=1}^N \sum_{j=1}^P \{w(Z_{ij}) [g(Z_{ij}) - \ln E_i - \ln \Delta t_j]\}^2 +$$
$$\lambda \sum_{z=Z_{min}+1}^{Z_{max}-1} [w(z)g''(z)]^2$$

# Conclusions/Code

```
%
% Given a set of pixel values observed for several pixels in several
% images with different exposure times, this function returns the
% imaging system's response function g as well as the log film irradiance
% values for the observed pixels.
%
% Assumes:
%
%   Zmin = 0
%   Zmax = 255
%
% Arguments:
%
%   Z(i,j) is the pixel values of pixel location number i in image j
%   B(j)   is the log delta t, or log shutter speed, for image j
%   l      is lambda, the constant that determines the amount of smoothness
%   w(z)   is the weighting function value for pixel value z
%
% Returns:
%
%   g(z)   is the log exposure corresponding to pixel value z
%   lE(i)  is the log film irradiance at pixel location i
%
function [g,lE]=gsolve(Z,B,l,w)

n = 256;

A = zeros(size(Z,1)*size(Z,2)+n+1,n+size(Z,1));
b = zeros(size(A,1),1);

%% Include the data-fitting equations

k = 1;
for i=1:size(Z,1)
    for j=1:size(Z,2)
        wij = w(Z(i,j)+1);
        A(k,Z(i,j)+1) = wij;   A(k,n+1) = -wij;       b(k,1) = wij * B(i,j);
        k=k+1;
    end
end

%% Fix the curve by setting its middle value to 0

A(k,129) = 1;
k=k+1;

%% Include the smoothness equations

for i=1:n-2
    A(k,i)=1*w(i+1);          A(k,i+1)=-2*1*w(i+1);   A(k,i+2)=1*w(i+1);
    k=k+1;
end

%% Solve the system using SVD

x = A\b;

g = x(1:n);
lE = x(n+1:size(x,1));
```



# Problems

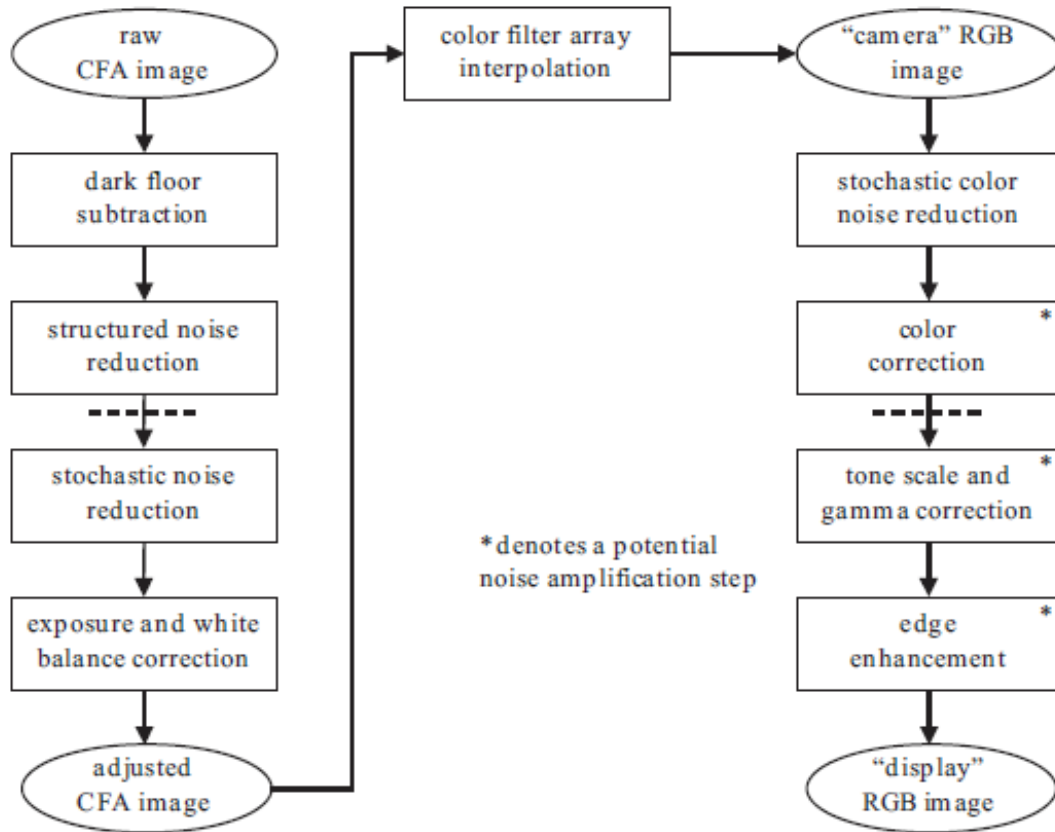
Ghosting:        Scene is changing while taking pictures

Aligning:         SIFT etc.

Lens properties:    Aberations

...

# Internal Processing in a Camera



**FIGURE 3.1**  
Reference image processing chain.

In a real camera a lot of signal processing is done that may lead to problems when the camera response curve is estimated

# Tonemapping

After the HDR reconstruction we have an image with HDR

How can we "see" it?

# The second half: contrast reduction

- **Input: high-dynamic-range image**
  - (floating point per pixel)



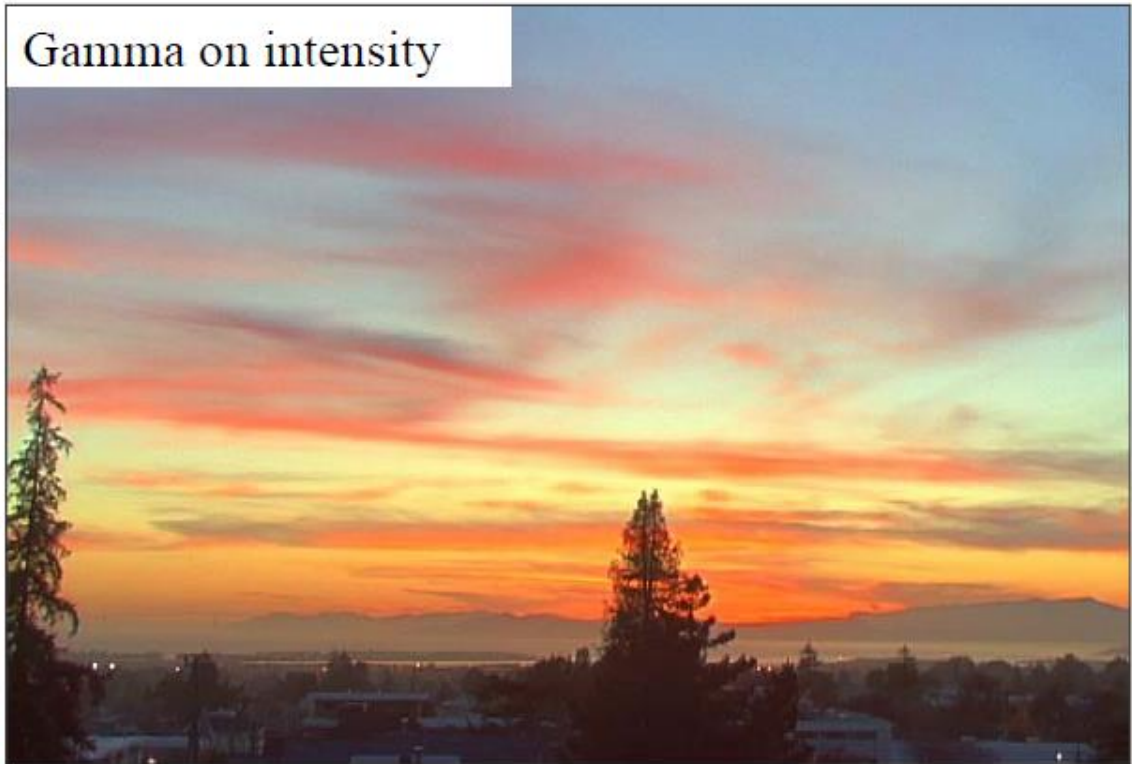
# Gamma compression on intensity

- Colors are OK,  
but details (intensity high-frequency) are blurred

Intensity



Gamma on intensity



Color



# Our approach

- Do not blur across edges
- Non-linear filtering

Large-scale



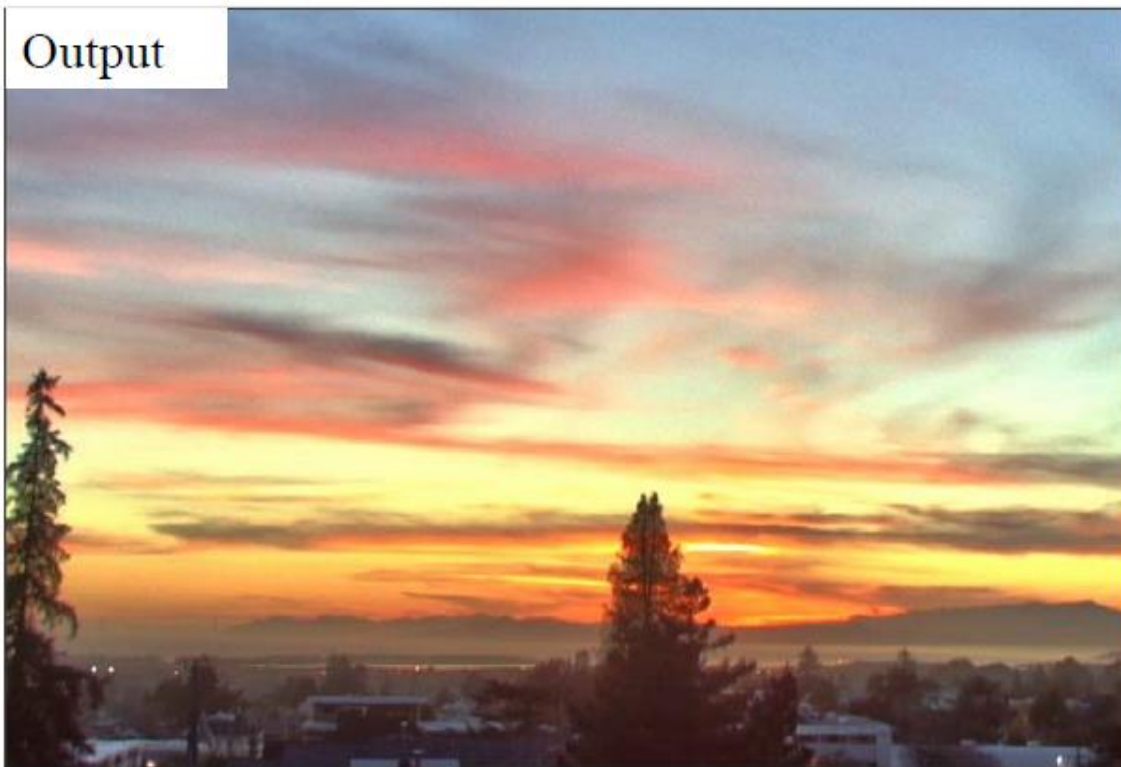
Detail



Color



Output



# Summary

- 1) Relation LDR and HDR and the basic idea behind HDR
- 2) Different ways to vary exposure, major advantages-drawbacks
- 3) Properties of the response curve and its different regions
- 4) Can explain Fig. 10.13 and the relation between different exposures
- 5) Explain the three stages:
  - Estimation of the response curve
  - Estimation of the radiance map
  - Role of tonemapping

# **2D Filters**

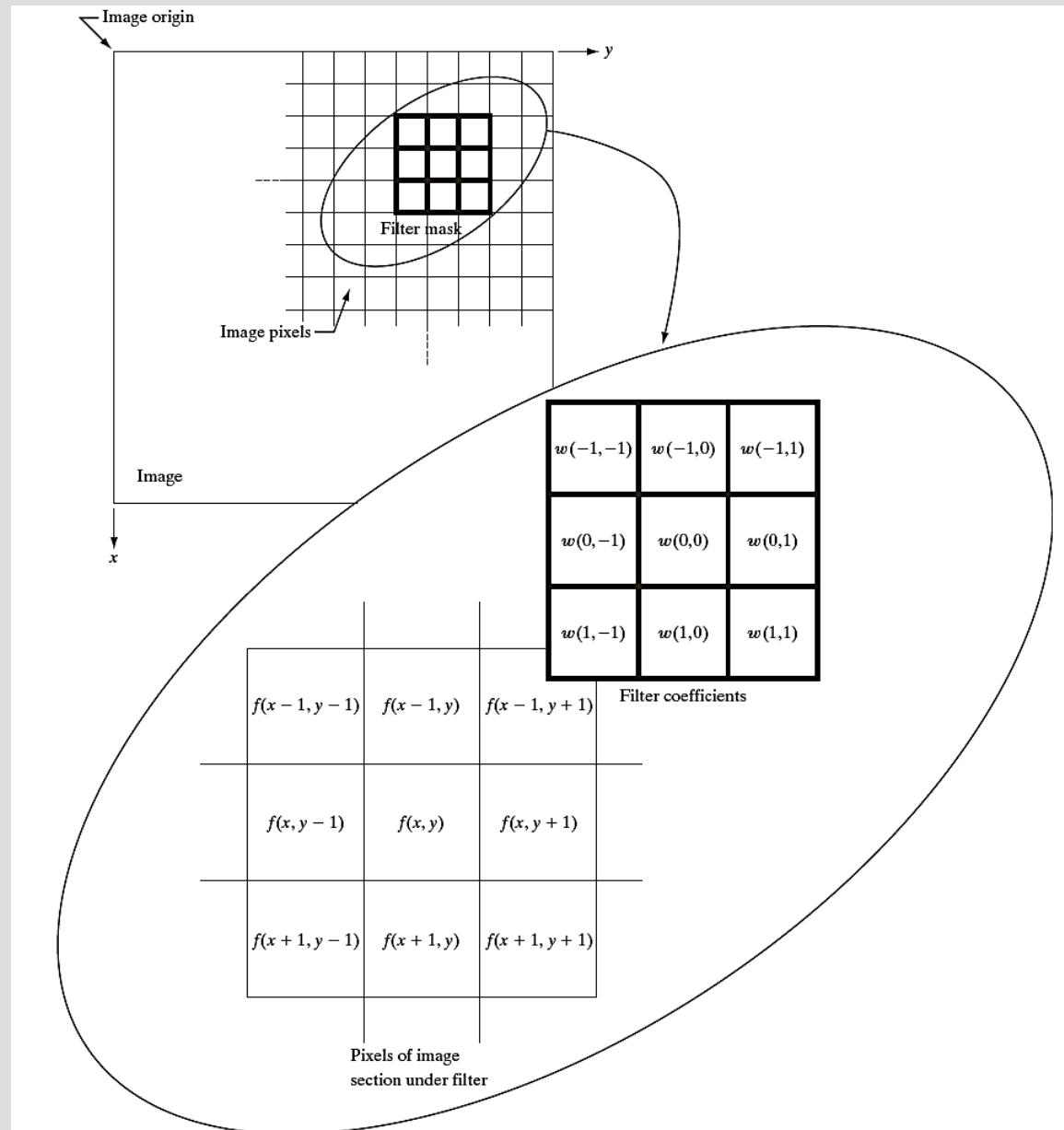
Reiner Lenz

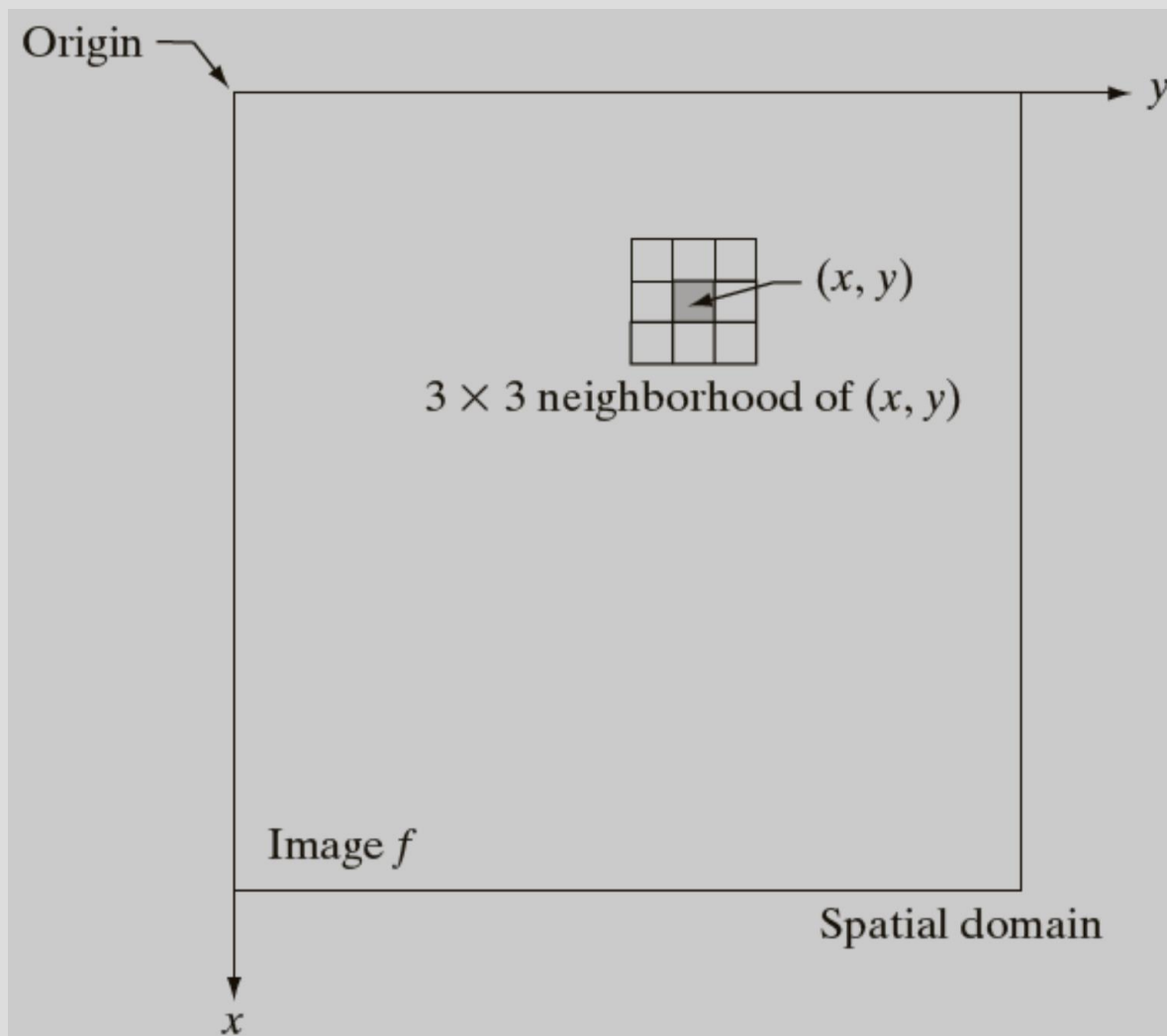
2015



# Filtering in the spatial domain

A mask (filter kernel) slides over the image. The centerpixel's new value (filter response) is a function of the kernel values and the pixels covered by the kernel (the neighbourhood).





**FIGURE 3.1**

A  $3 \times 3$  neighborhood about a point  $(x, y)$  in an image in the spatial domain. The neighborhood is moved from pixel to pixel in the image to generate an output image.

Image origin

y



Mask

Image  $f(x, y)$

x

$w(-1, -1)$	$w(-1, 0)$	$w(-1, 1)$
$w(0, -1)$	$w(0, 0)$	$w(0, 1)$
$w(1, -1)$	$w(1, 0)$	$w(1, 1)$

Mask coefficients, showing  
coordinate arrangement

$f(x-1, y-1)$	$f(x-1, y)$	$f(x-1, y+1)$
$f(x, y-1)$	$f(x, y)$	$f(x, y+1)$
$f(x+1, y-1)$	$f(x+1, y)$	$f(x+1, y+1)$

Pixels of image  
section under mask

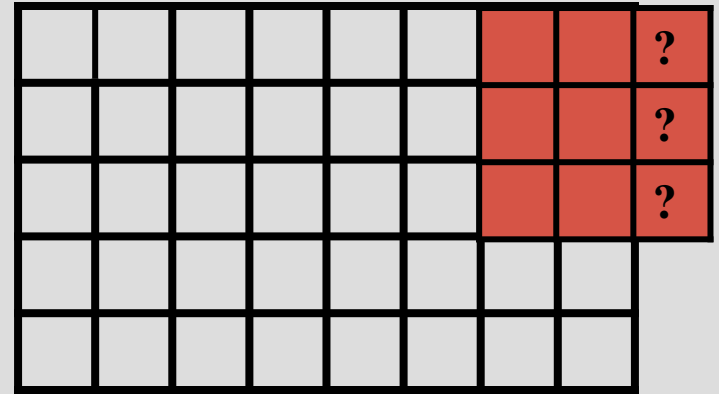
## Filtering in the spatial domain

## What happens when the filter kernel reaches the borders of the image?

- \* Don't let the kernel reach outside  
=> frame of unfiltered pixels in the  
result image

- Give the image a frame of zeroes ("zero- padding") =>

frame of miscalculated pixels in the  
result



# Filtering in the spatial domain

Linear filter:

$w_1$	$w_2$	$w_3$
$w_4$	$w_5$	$w_6$
$w_7$	$w_8$	$w_9$

Kernel

$$g = \sum_{i=1}^9 w_i f_i$$

$g$  : filter respons

$f_i$  : pixels covered by kernel

# Linear filtering - correlation

The *correlation* between two 2D-functions, the image  $f(x,y)$  and the filter kernel  $w(m,n)$ .

Continuous case :

$$g(x, y) = (f \circ w) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x+a, y+b)w(a,b)dad b$$

Discrete case:

$$g(x, y) = \sum_{(s,t) \in neighbourhood} f(x+s, y+t)w(s,t)$$

# Linear filtering - convolution

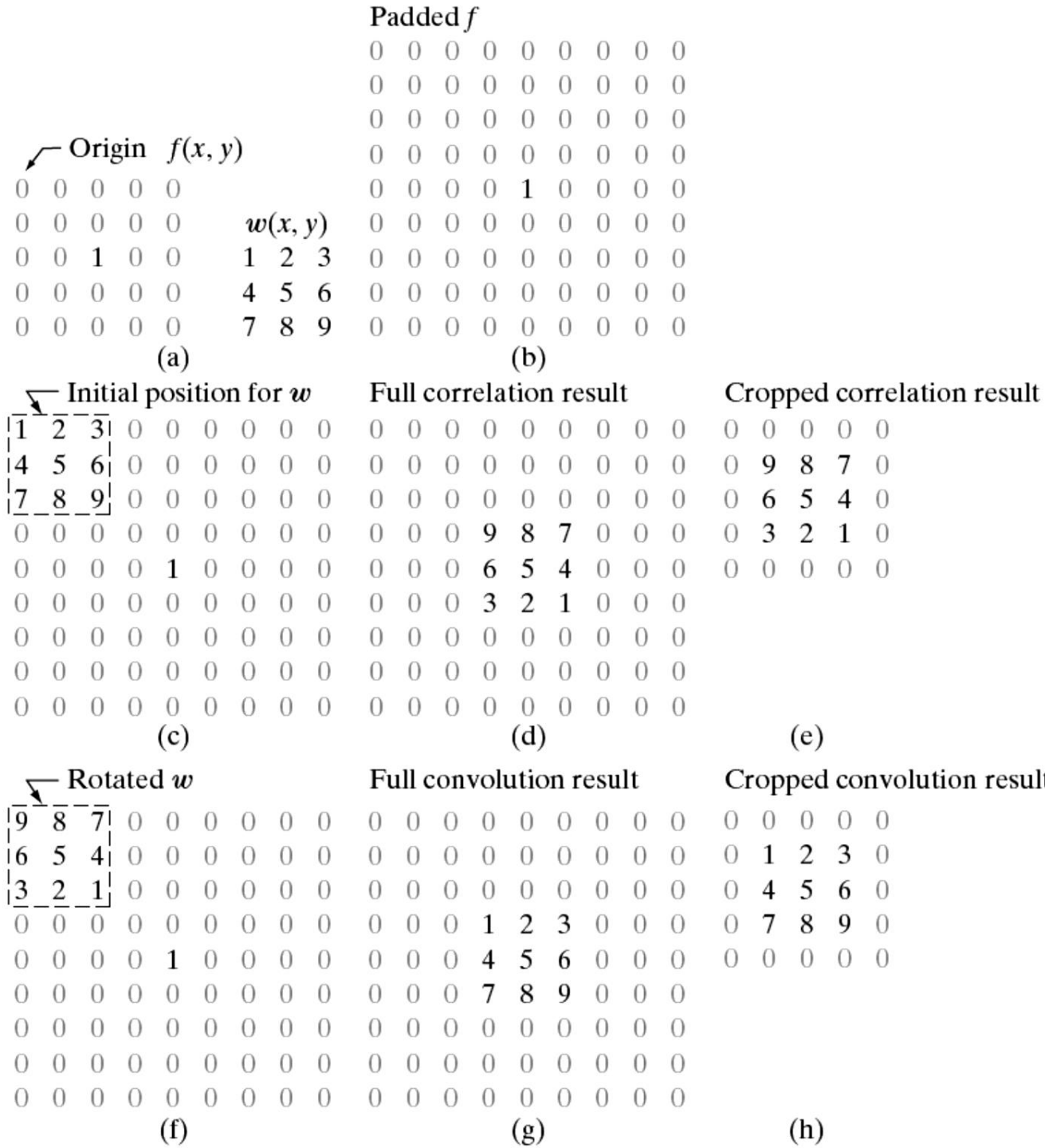
The *convolution* between two 2D-functions: the image  $f(x,y)$  and the filter kernel  $w(m,n)$ .

Continuous case :

$$g(x, y) = (f \bullet w) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x-a, y-b)w(a,b)dad b$$

Discrete case:

$$g(x, y) = \sum_{(s,t) \in neighbourhood} f(x-s, y-t)w(s,t)$$



**FIGURE 3.30**  
Correlation (middle row) and convolution (last row) of a 2-D filter with a 2-D discrete, unit impulse. The 0s are shown in gray to simplify visual analysis.



# Matlab

```

ima = imread(fullfile(impath,'F2_08.TIF'));
subplot(2,1,1),imshow(ima)
subplot(2,1,2),imshow(ima,[])

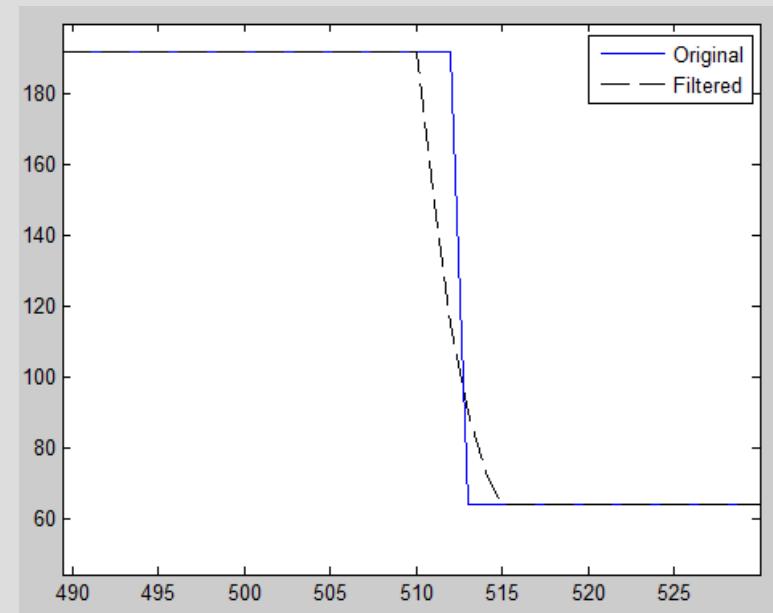
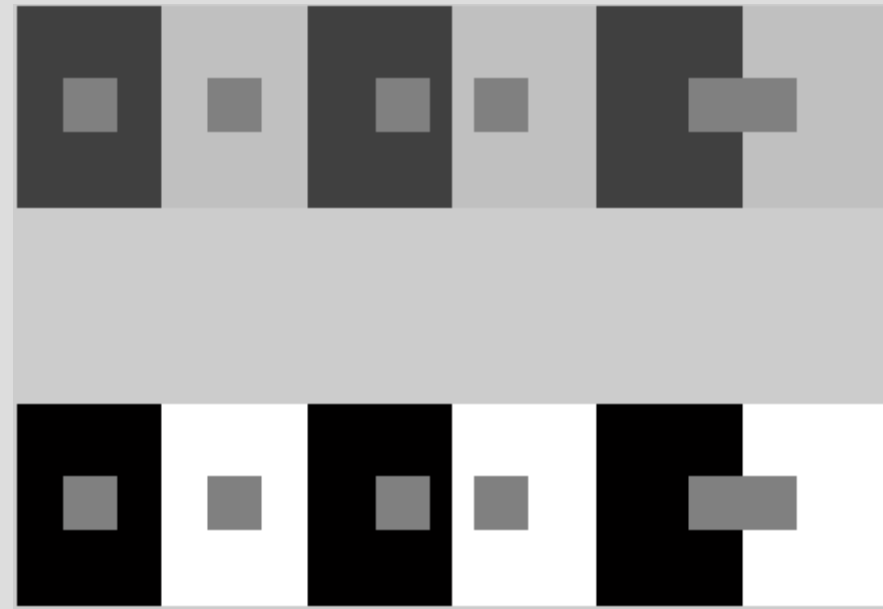
h = (1:5)';
fima= imfilter(ima,h'/sum(h));
figure
plot(ima(90,:),'b')
hold on
plot(fima(90:,:), 'k--')
legend('Original','Filtered')

ima(90,508:517)
192 192 192 192 192 64 64 64 64 64

lima=ones(6,1)*double(ima(90,508:517));
for k = 1:6
    filterresult(k) = lima(k,k:k+4)*h/15;
end

filterresult =
192.0000 149.3333 115.2000 89.6000
72.5333 64.0000
>> fima(90,508:517)
192 192 192 149 115 90 73 64 64 64

```



# Averaging filters

Smooths the image. Supression of noise and small objects in the image.

Example:

$\frac{1}{9} \times$ 

1	1	1
1	1	1
1	1	1

$\frac{1}{16} \times$ 

1	2	1
2	4	2
1	2	1

Boxfilter

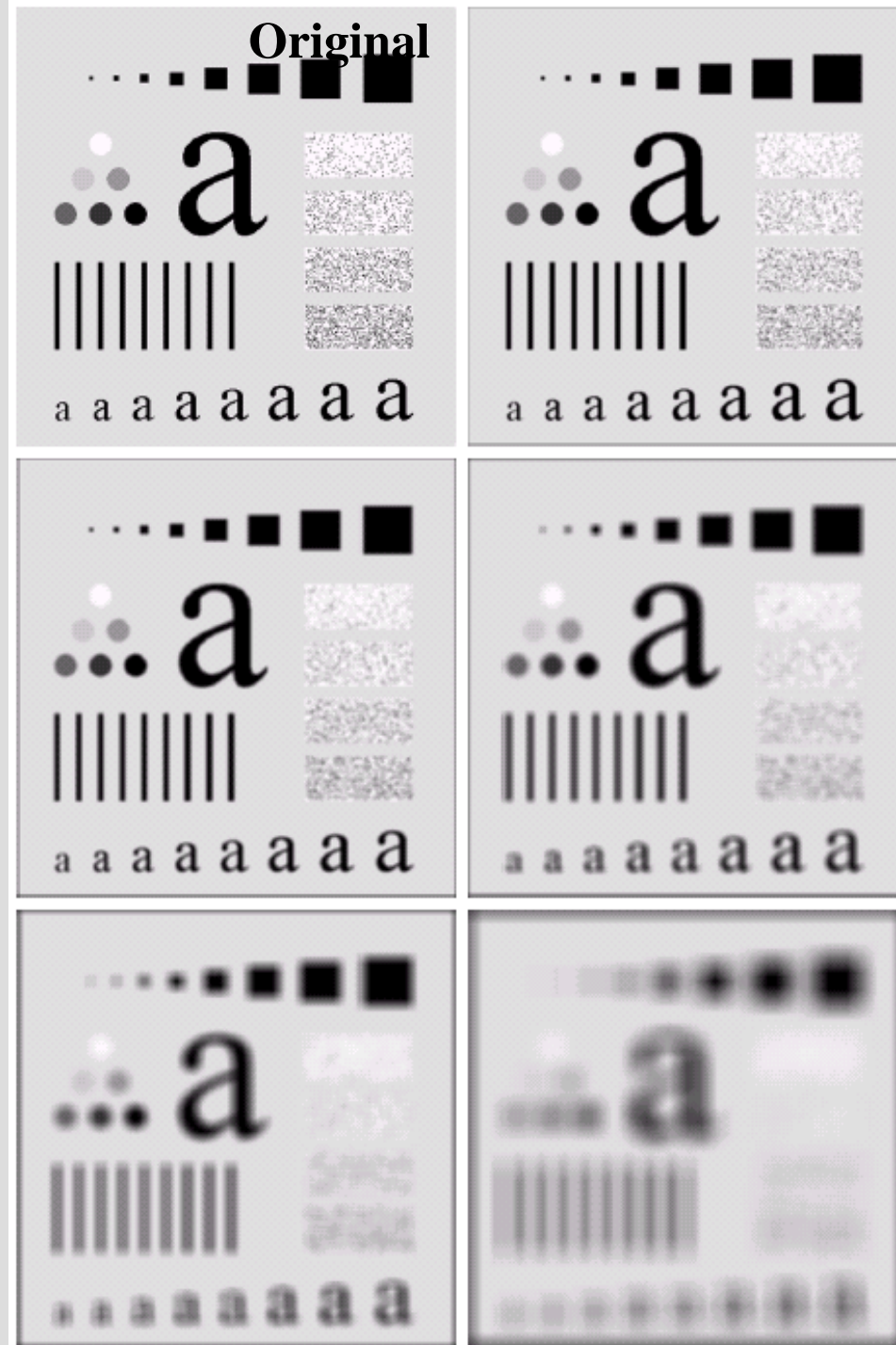
Weighted filter

# Averaging filters

Different kernel sizes:

3x3, 5x5, 9x9, 15x15,

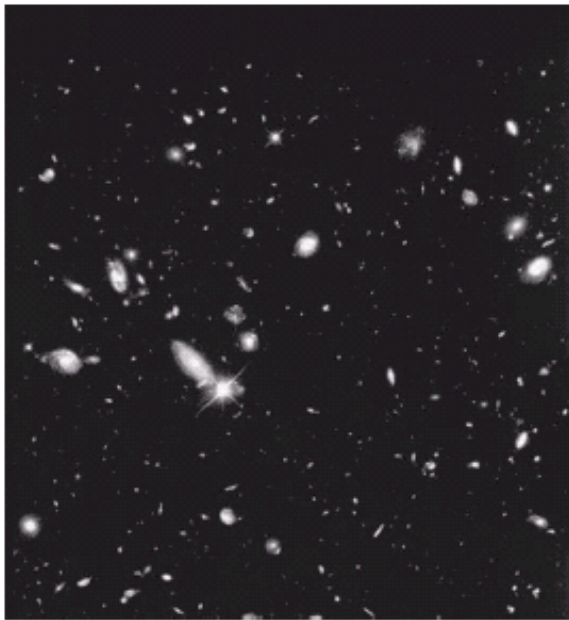
35x35



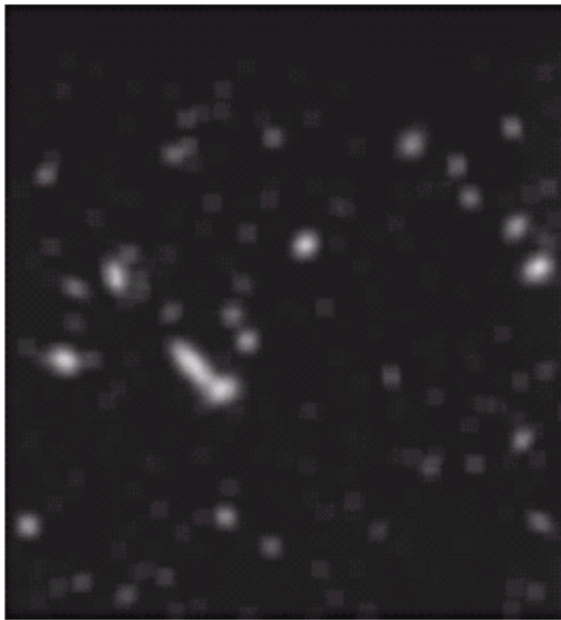
# Averaging filters

Supression of small objects in the image.

Example (15x15-kernel):



Original



Result



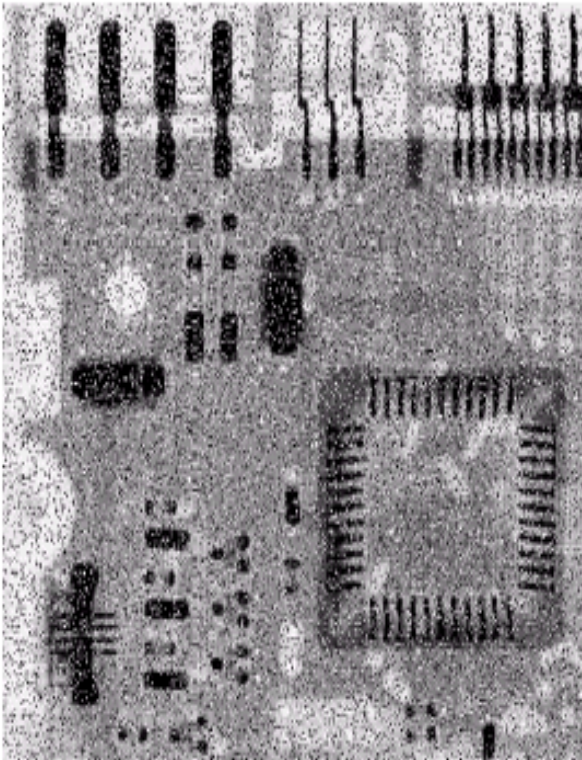
Thresholded result

# Median filter

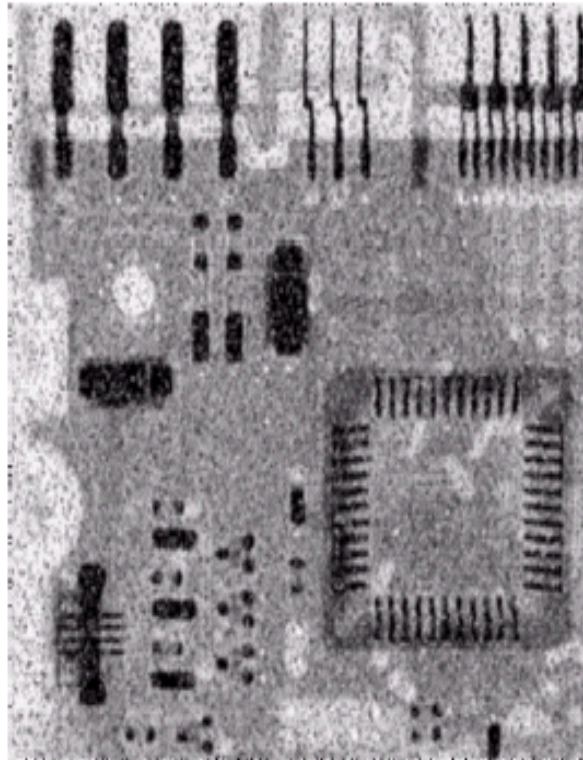
- The center pixel is replaced by the *median* of the pixel values covered by the kernel.
- Non-linear filter.
- Good for removal of ”spiky” noise (salt and pepper noise)
- No smoothing of sharp edges.

# Median filter

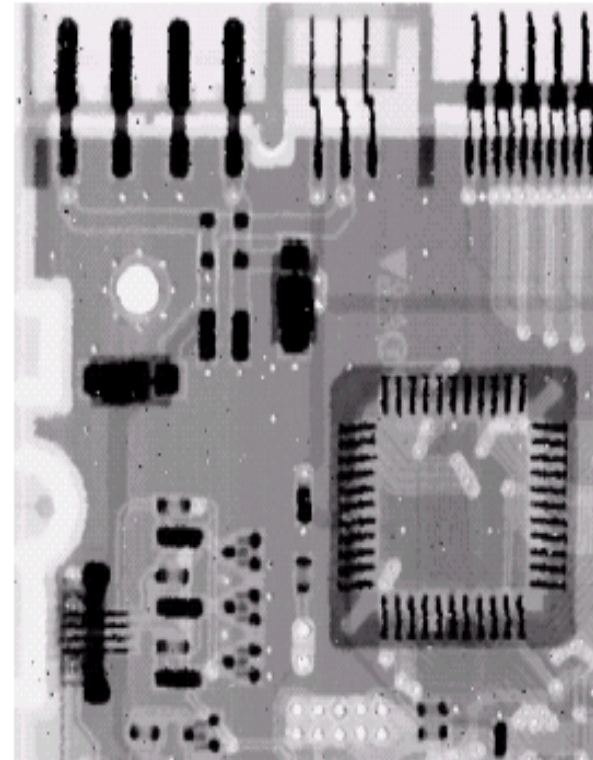
Example:



Original



3x3 average



3x3 median

# Derivating filters

- Averaging (integrating) filters result in smoothing
- Derivating filters result in sharpening.

# 1-D Derivatives

$$\frac{\partial f}{\partial x} \approx f(x+1) - f(x)$$

$$(-1 \ 1)$$

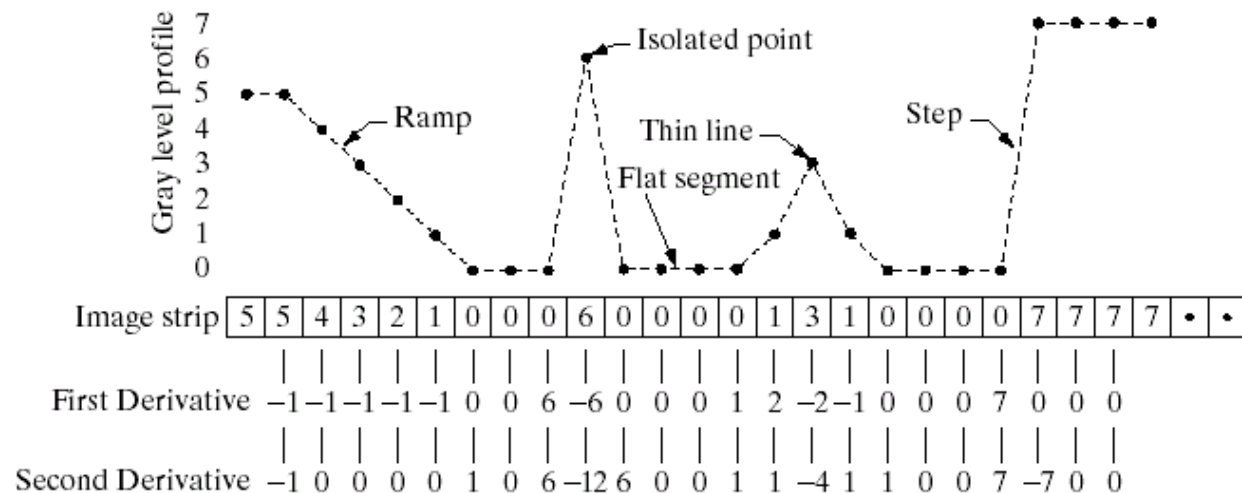
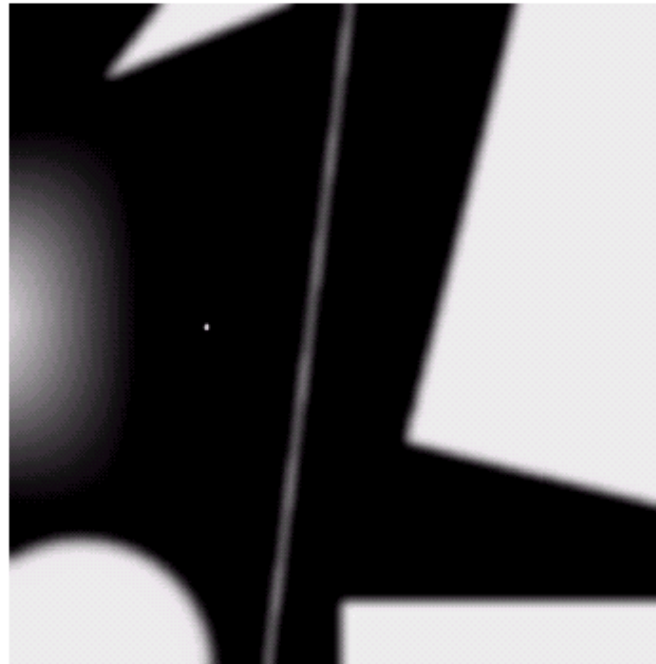
$$\begin{aligned} \frac{\partial}{\partial x^2} &= \frac{\partial}{\partial x} \frac{\partial}{\partial x} \approx g(x+1) - g(x) = \\ & f(x+2) - f(x+1) - (f(x+1) - f(x)) = \\ & f(x+2) - f(x+1) - f(x+1) + f(x) = \\ & f(x+2) - 2f(x+1) + f(x) \end{aligned}$$

$$(1 \ -2 \ 1)$$



# Derivating filters

Effects of 1st and 2nd derivatives on a 1D-signal (intensity profile in image).



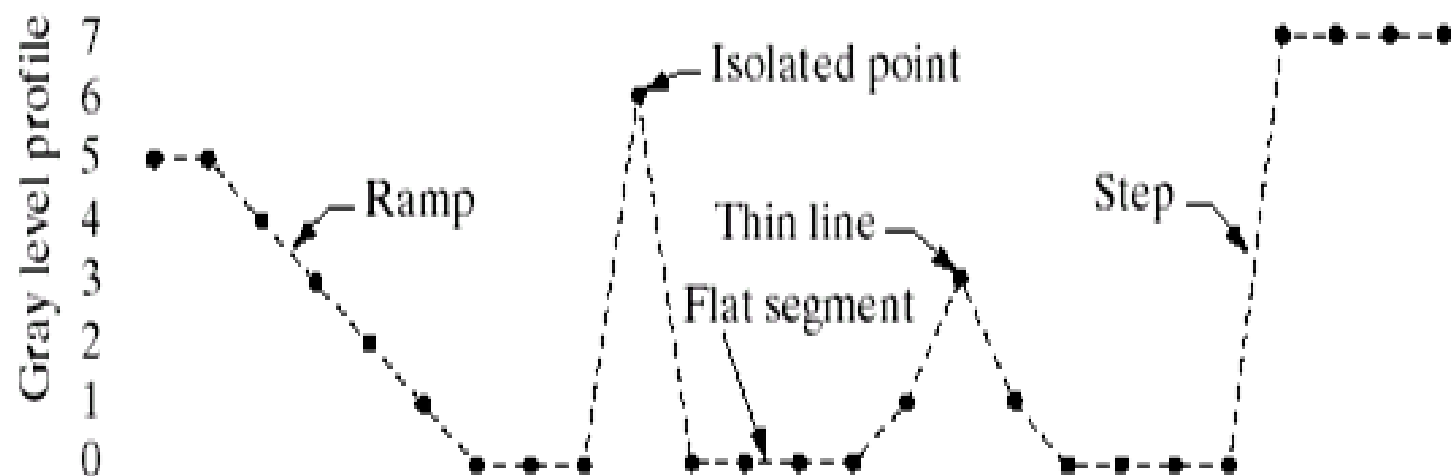


Image strip	5	5	4	3	2	1	0	0	0	6	0	0	0	0	1	3	1	0	0	0	0	7	7	7	7	•	•
-------------	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

First Derivative	-1	-1	-1	-1	-1	0	0	6	-6	0	0	0	0	1	2	-2	-1	0	0	0	7	0	0	0		

Second Derivative	-1	0	0	0	0	1	0	6	-12	6	0	0	0	1	1	-4	1	1	0	0	7	-7	0	0		

# Laplacian filter

- \*Commonly used for sharpening.
- Based on the 2nd derivative
- Isotropic*, i.e. yields the same result independent of the rotation of the image (rotation invariance)
- The Laplace operator:

$$\nabla^2 f(x, y) = \frac{\partial^2}{\partial x^2} f(x, y) + \frac{\partial^2}{\partial y^2} f(x, y)$$

# Laplacian filter

- Discrete Laplace:

$$\nabla^2 f(i, j) = f(i, j-1) + f(i-1, j) + f(i+1, j) + f(i, j+1) - 4f(i, j)$$

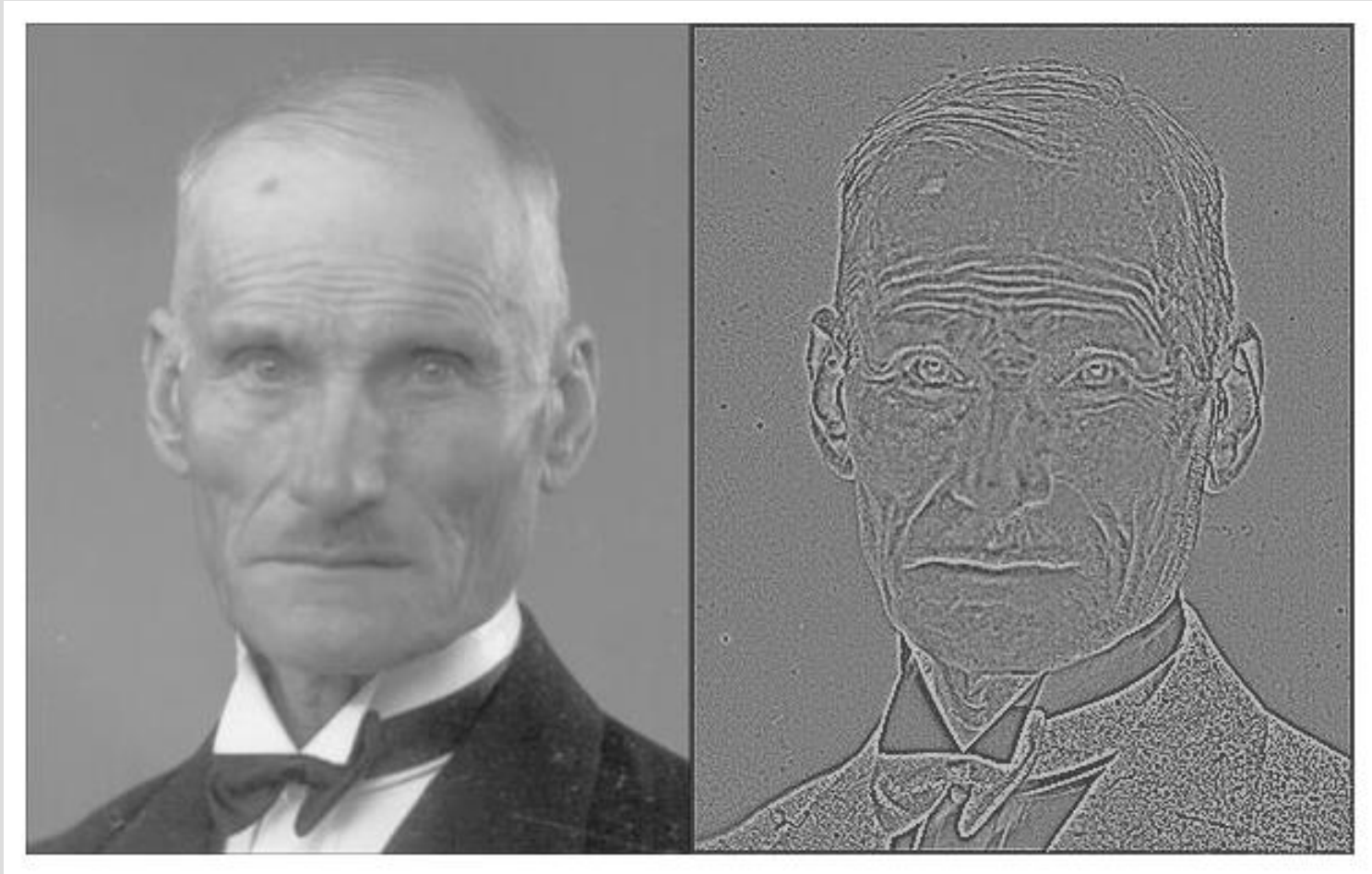
Laplace-kernels

0	1	0	1	1	1
1	-4	1	1	-8	1
0	1	0	1	1	1

0	-1	0	-1	-1	-1
-1	4	-1	-1	8	-1
0	-1	0	-1	-1	-1

# Laplacian filter

Example:



The Laplace image has an offset; 0-level is set to 0.5