Problem Set #4 Non-Stationarity and Unit-Root Testing

This problem set deals with unit root non-stationarity and the Dickey Fuller unit root test. Exercise #4.1 asks you to outline the behavior of an AR(1) process in the stationary case and in the case of a unit root. We particularly focus on the interpretation of the constant term. Exercise #4.2 show how to perform unit root tests for some Danish macro time series.

#4.1 Stationary and Unit-Root AR(1) Models

Let U_t denote the unemployment rate in percentage of the labour force and let U_t be given by the following model

$$U_t = \gamma + \theta U_{t-1} + \eta_t, \quad t = 1, 2, ..., T, \tag{4.1}$$

where η_t is a white noise process, $\eta_t | U_{t-1} \overset{d}{=} N(0, \sigma^2_\eta)$, and the initial value, U_0, is given.

(1) Show that the model can be rewritten as

$$\Delta U_t = \delta + \pi U_{t-1} + \epsilon_t, \quad \epsilon_t | U_{t-1} \overset{d}{=} N(0, \sigma^2_\epsilon) \tag{4.2}$$

and state the relationship between the two sets of parameters

$$\{\gamma, \theta, \sigma^2_\eta\} \quad \text{and} \quad \{\delta, \pi, \sigma^2_\epsilon\}.$$

(2) Consider the four cases:

$$\begin{array}{rcc} (i): & -2 < \pi < 0 & \delta \neq 0 \\ (ii): & -2 < \pi < 0 & \delta = 0 \\ (iii): & \pi = 0 & \delta \neq 0 \\ (iv): & \pi = 0 & \delta = 0 \end{array}$$

For each case, $(i) - (iv)$, find an expression for U_t as a function of δ, the innovations, ϵ_i $(i = 1, 2, ..., t)$, and the initial value, U_0. This is the moving average representation for U_t.

(3) For each case, draw a sketch of the appearance of the time series. Which of the models do you think are appropriate representations of the unemployment rate?

(4) Explain how the cases above are related to the Dickey-Fuller $t-$test for the hypothesis $H_0 : \pi = 0$.

(5) Explain how the cases above are related to the LR test for the joint hypothesis $H_0^* : \pi = \delta = 0$.

(6) Which of the tests would you consider to be the most satisfactory for the present case?

(7) Consider the AR(2) case

$$U_t = \gamma + \theta_1 U_{t-1} + \theta_2 U_{t-2} + \eta_t, \quad \eta_t | U_{t-1}, U_{t-2} \overset{d}{=} N(0, \sigma_\eta^2). \tag{4.3}$$

(a) Write the observations you condition on in the likelihood analysis for this case.

(b) Show how the AR(2) model can be rewritten as

$$\Delta U_t = \delta + \pi U_{t-1} + c_1 \Delta U_{t-1} + \epsilon_t, \quad \epsilon_t | U_{t-1}, U_{t-2} \overset{d}{=} N(0, \sigma_\epsilon^2), \tag{4.4}$$

and state the relationship between the two sets of parameters

$$\{\gamma, \theta_1, \theta_2, \sigma_\eta^2\} \quad \text{and} \quad \{\delta, \pi, c_1, \sigma_\epsilon^2\}.$$

#4.2 Unit Roots in Danish Macro-Variables

(1) The file `TimeSeries.in7` also used in Problem Set #2 contains 9 time series for the Danish economy:

EFKRKS	Effective (nominal) exchange rate index.
ENL	Current account (current prices).
FIPMXE	Business machinery investments (constant prices).
FY	Total GDP (constant prices).
IBZ	Bond yield (pro anno yield in ratio).
KP	House price index.
PCP	Consumption deflator.
UNR	Unemployment rate (ratio).
TT	Terms of trade, export price/import price.

Choose one of the time series to analyze.
Take logs if you think it is appropriate and make a time series graph.
Based on the graph and your knowledge of economic theory, what would you suggest as a deterministic specification for the Dickey-Fuller test regression; is the relevant alternative stationarity or trend-stationarity?

(2) Set up an autoregressive model in OxMetrics and find the preferred lag length.
Is the model well specified?

(3) Perform the augmented Dickey-Fuller $t-$test.
What is the conclusion regarding a unit root?
How would you relate the finding to economic theory?

(4) Perform the LR test for the joint hypothesis involving also the relevant deterministic variable. What is the conclusion regarding a unit root?