Instituto Universitario de Microgravedad
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Universidad Politécnica de Madrid

Aerodinámica de Altas Velocidades y Fenómenos de Reentrada

Dinámica de la reentrada

Master Universitario en Sistemas Espaciales
Curso 1º - 2º semestre

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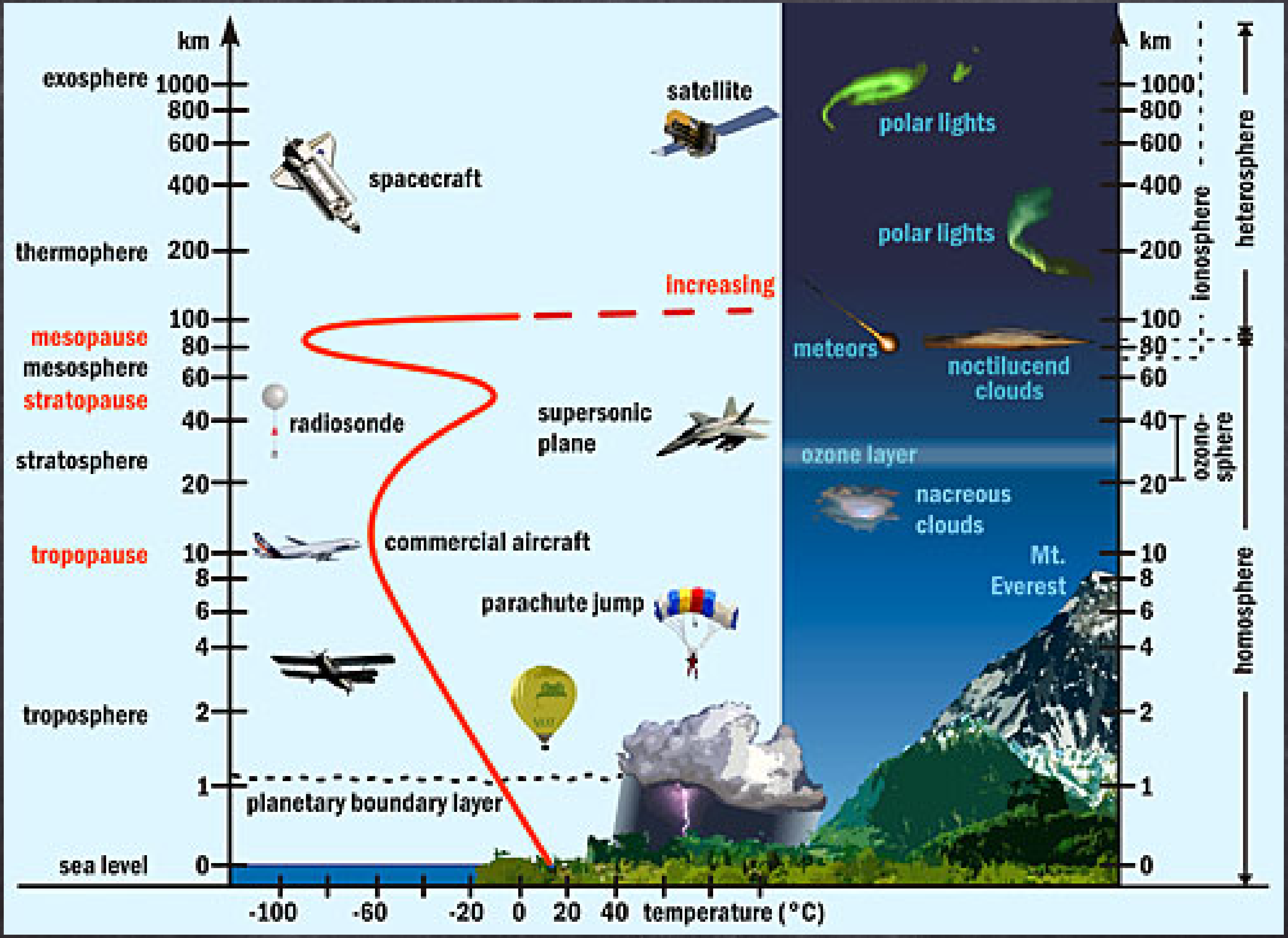
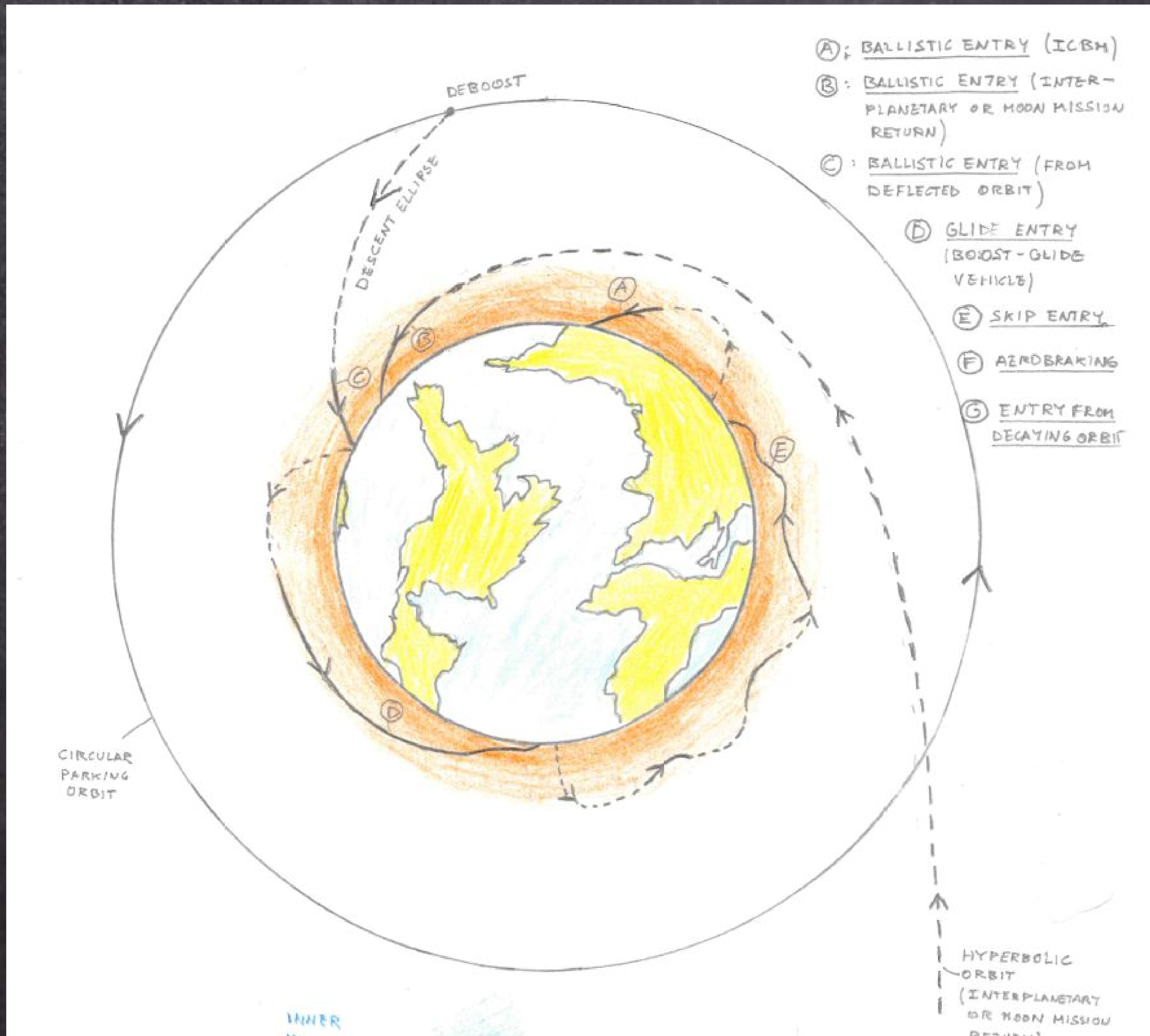
- Introducción
- Decaimiento orbital
- Ecuaciones generales de la reentrada
- Entrada balística
- Entrada en plano

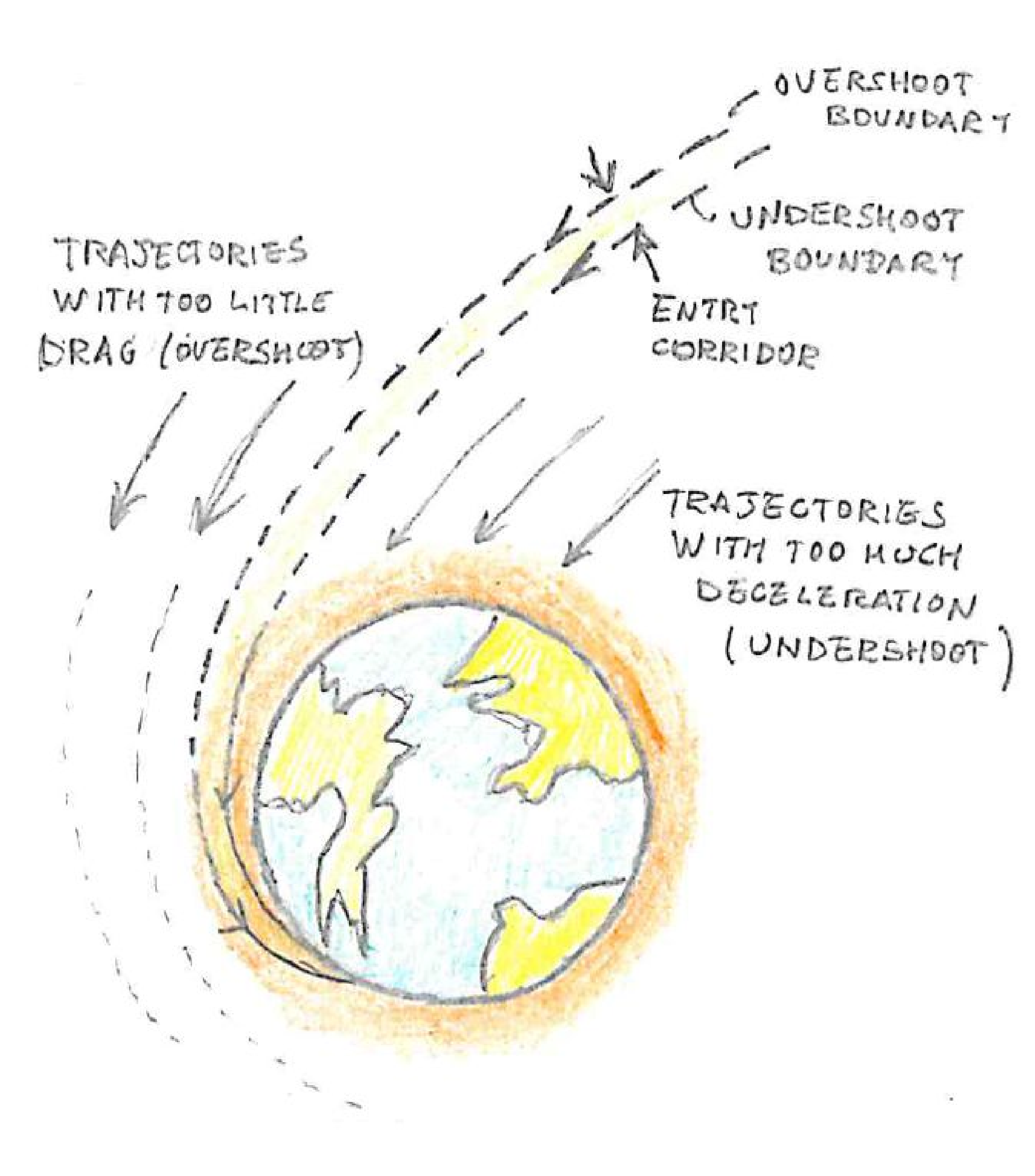
Introducción



Introducción

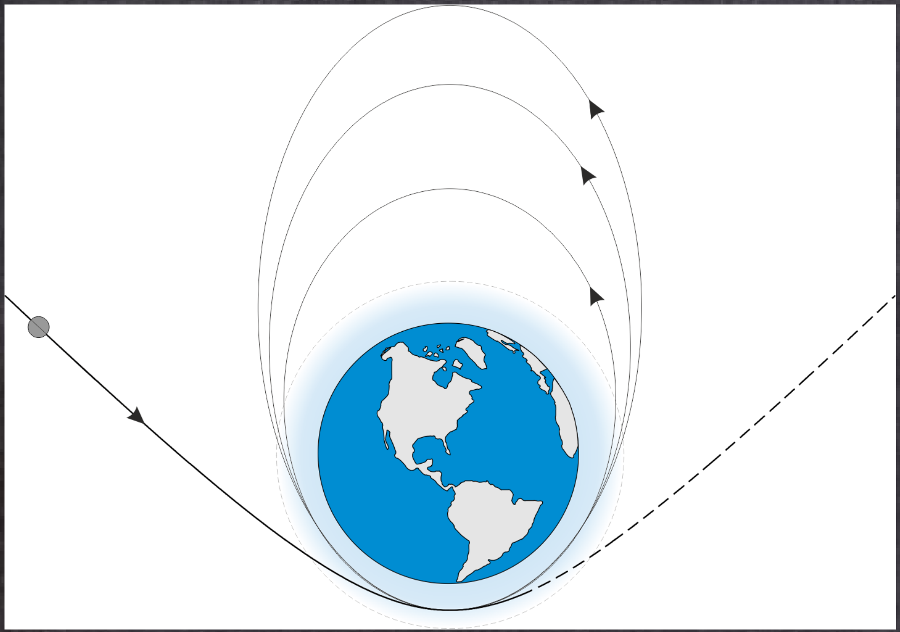


Introducción



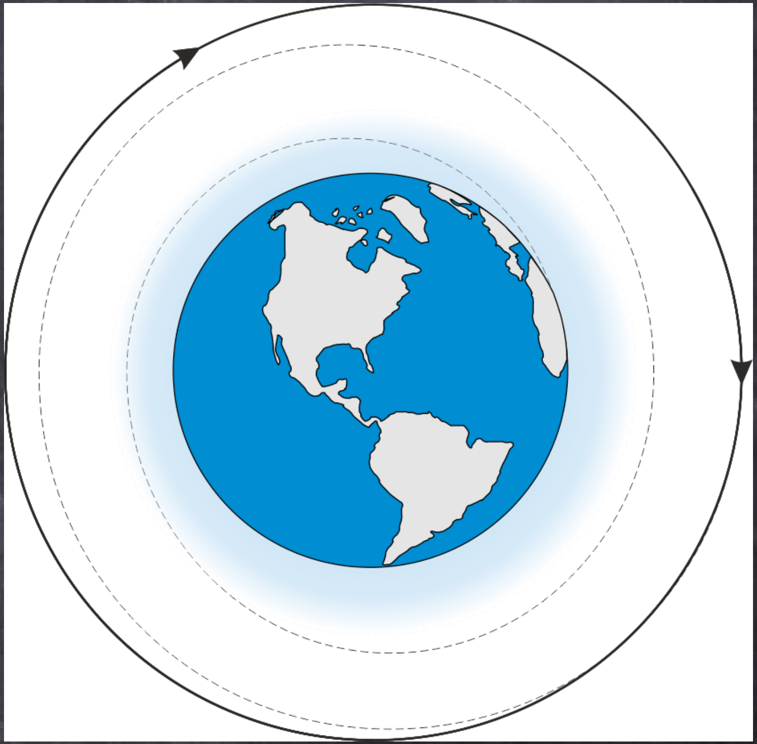
Introducción

Aero-frenado

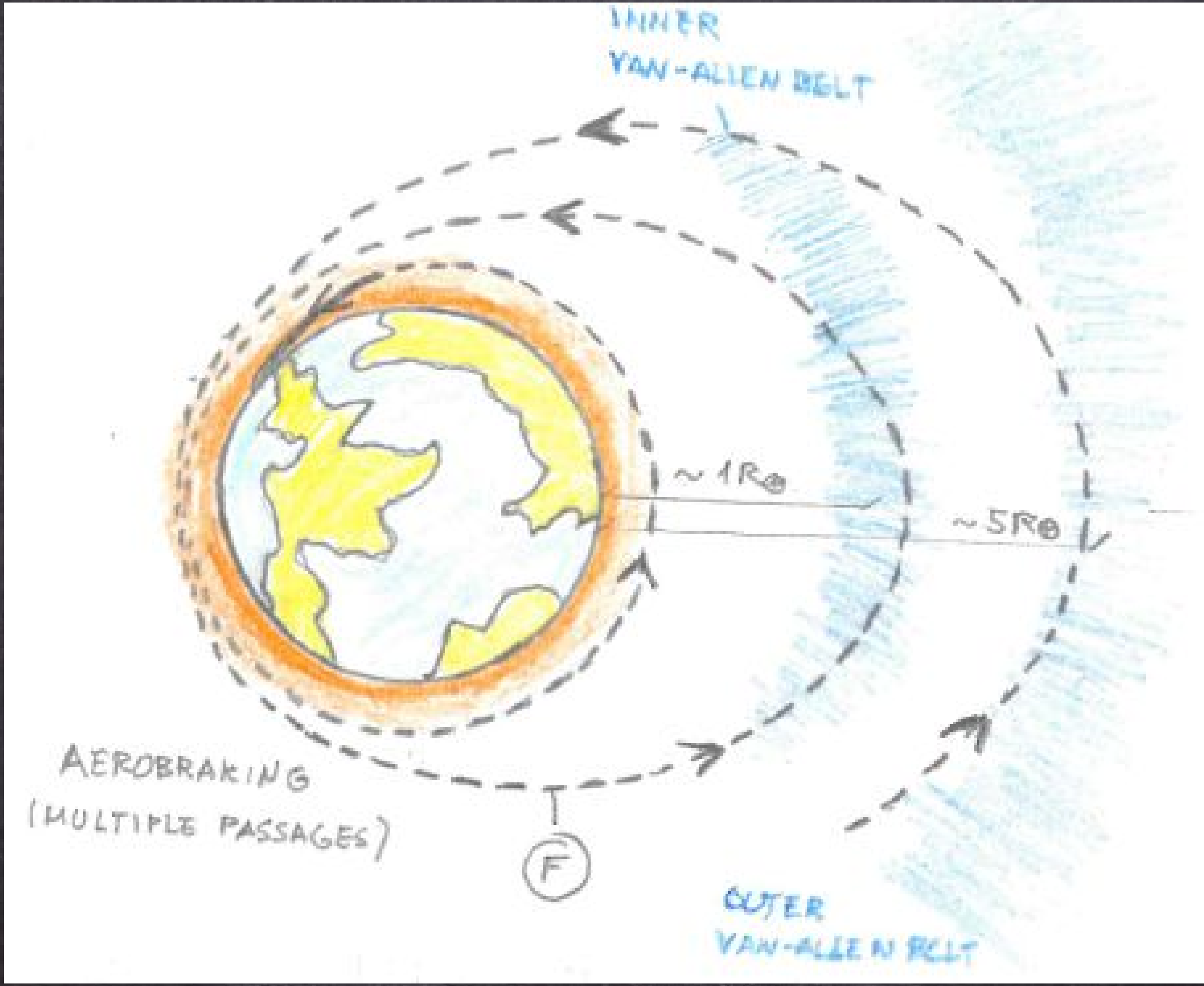


En misiones tripuladas no es factible

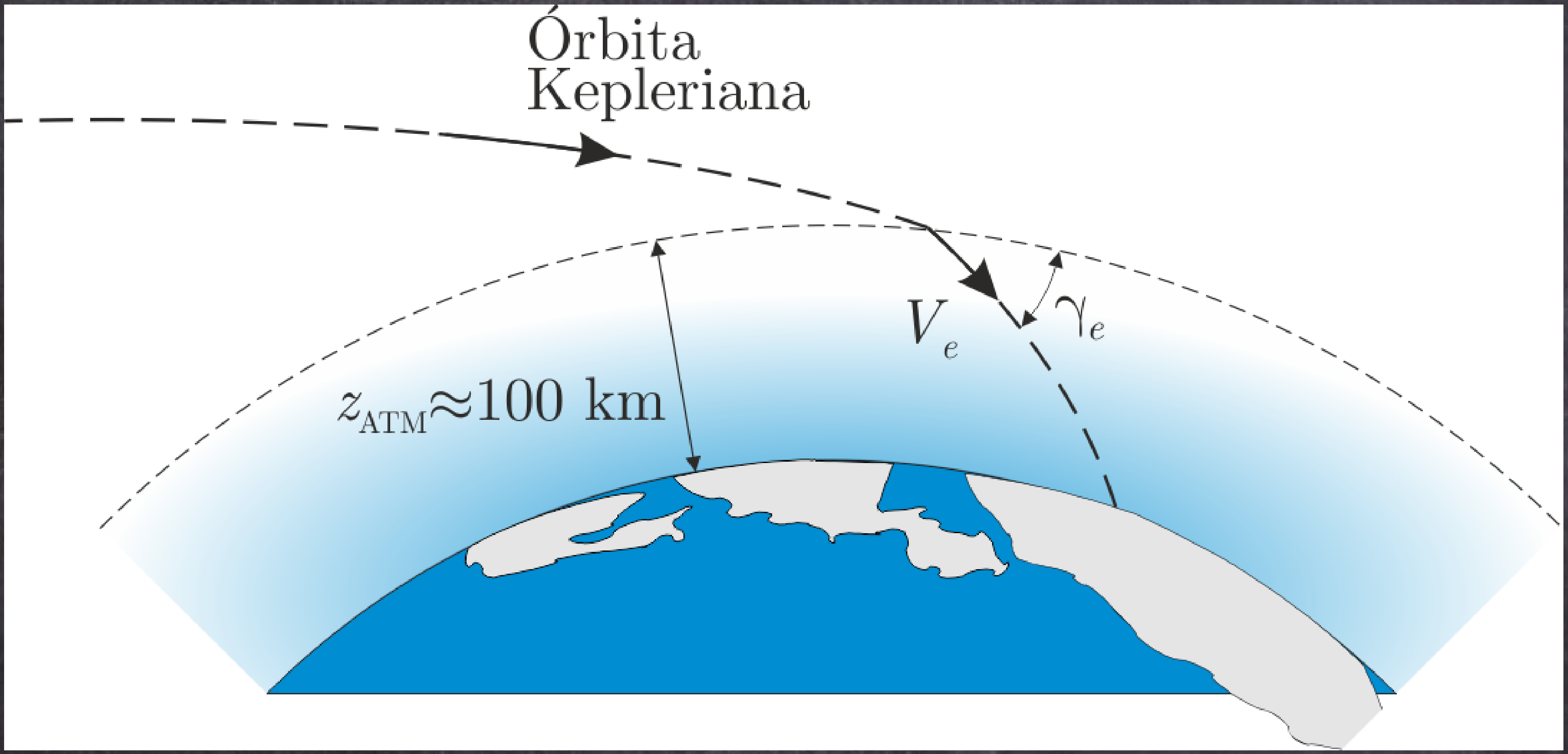
Decaimiento orbital



Introducción

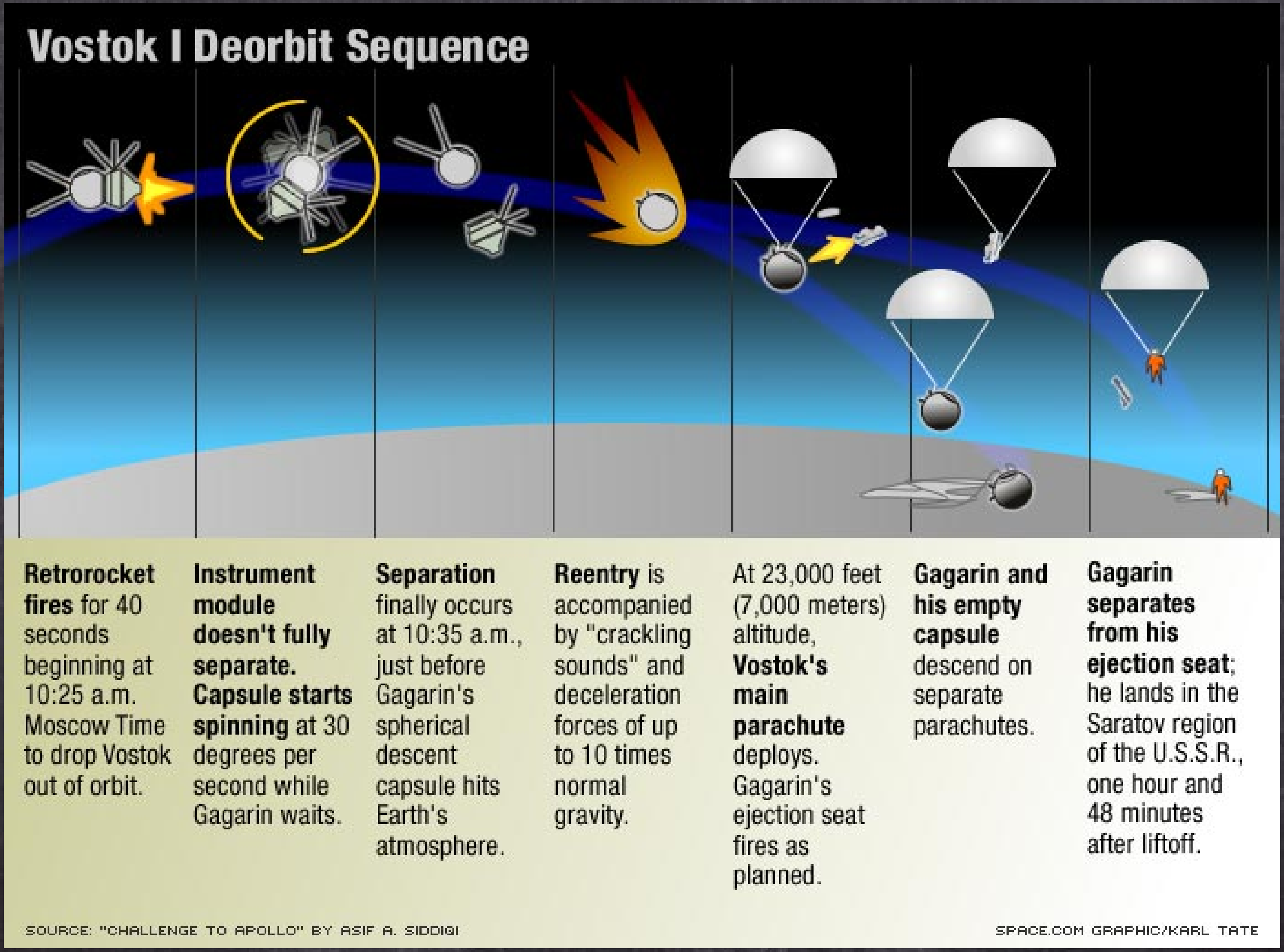


Introducción

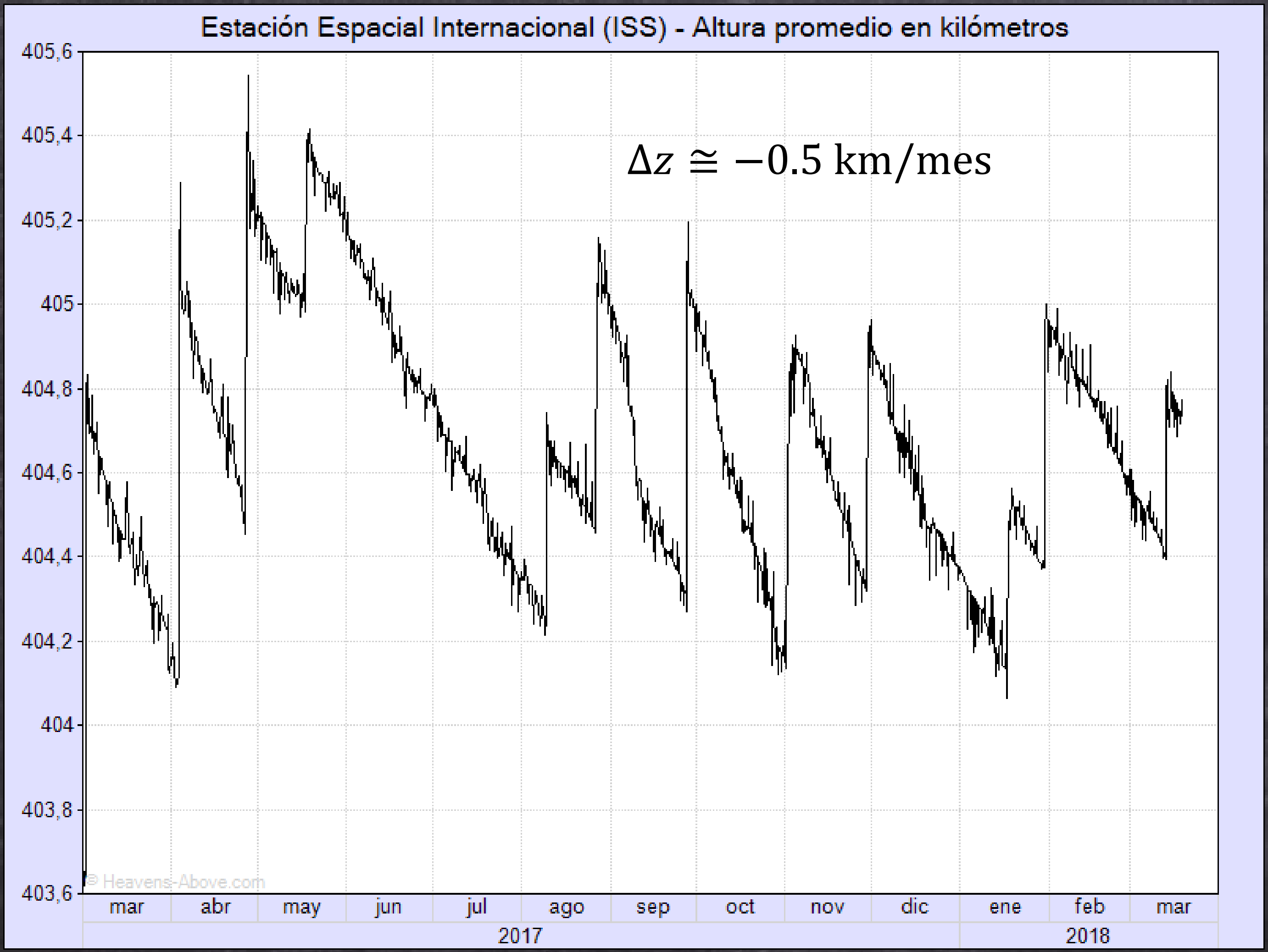


$z_{ATM} \approx 0.01 R_T$

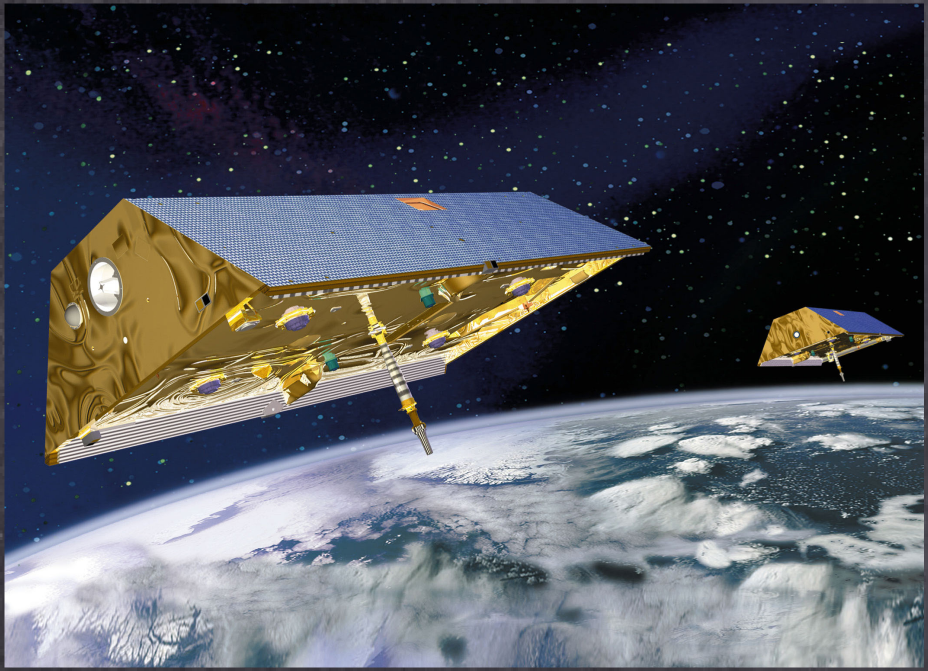
Introducción



Decaimiento orbital



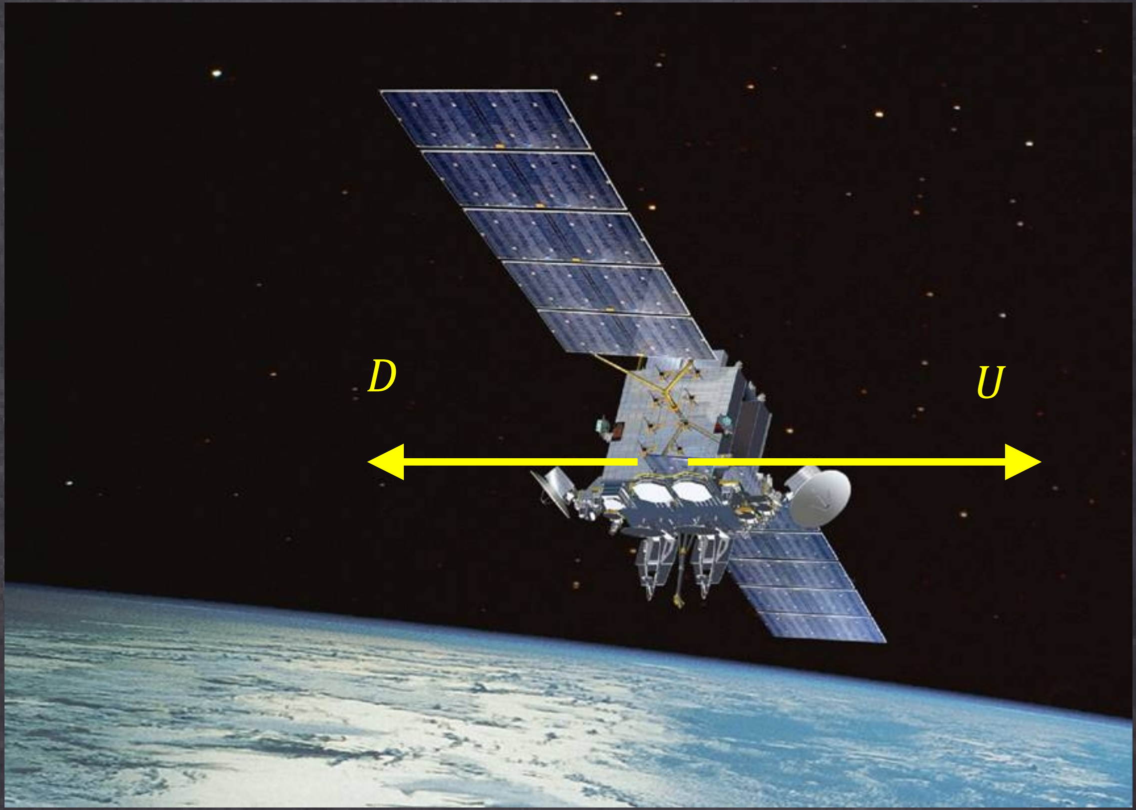
Decaimiento orbital



Gravity Recovery and Climate Experiment (GRACE):

- Misión espacial conjunta entre la NASA y la Agencia Espacial de Alemania
- Objetivo: cartografiar con precisión el campo gravitatorio terrestre.
- Lanzada el 17 de marzo de 2002
- Vida útil: 15 años
- Altitud media 495 km

Decaimiento orbital



$$D = \frac{1}{2}\rho U^2 S c_D$$

Aceleración debida a *D*:

$$a_D = \frac{D}{m} = \frac{1}{2}\rho U^2 \frac{S c_D}{m}$$

$$\beta = \frac{m}{S c_D}$$ Coeficiente balístico

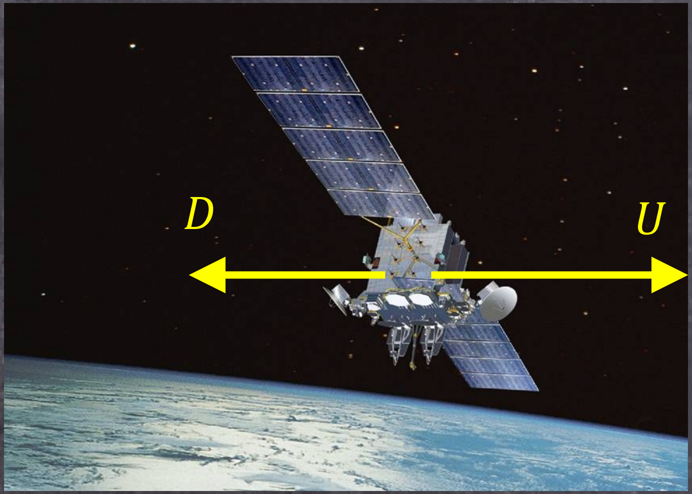
$$\rho = \frac{2 a_D \beta}{U^2}$$

Midiendo la aceleración y la velocidad → densidad

Decaimiento orbital

$$D = \frac{1}{2}\rho U^2 S c_D$$

$$\frac{\mathrm{d}E_D}{\mathrm{d}t} = -\frac{UD}{m} = -\frac{1}{2}\rho U^3 \frac{S c_D}{m}$$



$$E_O = -\frac{\mu}{2a}$$

$$\frac{\mathrm{d}E_O}{\mathrm{d}t} = \frac{\mu}{2a^2}\frac{\mathrm{d}a}{\mathrm{d}t}$$

$$\frac{\mathrm{d}E_O}{\mathrm{d}t} = \frac{\mathrm{d}E_D}{\mathrm{d}t}$$

$$\frac{\mu}{\cancel{2}a^2}\frac{\mathrm{d}a}{\mathrm{d}t} = -\frac{1}{\cancel{2}}\rho U^3 \frac{1}{\beta}$$

Asumimos:

- Orbita circular $\rightarrow$ $a = r$, $U = \sqrt{\frac{\mu}{r}}$
- Modelo de atm. Exponencial: $\rho = \rho_0 e^{-\frac{z}{z_s}}$

$$\frac{\mu}{r^2}\frac{\mathrm{d}r}{\mathrm{d}t} = -\rho_0 e^{-\frac{z}{z_s}}\left(\frac{\mu}{r}\right)^{3/2}\frac{1}{\beta}$$

$$\frac{e^{\frac{z}{z_s}}}{\sqrt{r}}\mathrm{d}r = -\frac{\rho_0\sqrt{\mu}}{\beta}\mathrm{d}t \quad \leftarrow \quad \left[\begin{array}{l} r = z + R_T \\ \mathrm{d}r = \mathrm{d}z \end{array}\right.$$

$$\frac{e^{\frac{z}{z_s}}}{\sqrt{z + R_T}}\mathrm{d}z = -\frac{\rho_0\sqrt{\mu}}{\beta}\mathrm{d}t$$

$$\sqrt{z + R_T} \rightarrow \sim\sqrt{R_T}$$

$$e^{\frac{z}{z_s}}\mathrm{d}z = -\frac{\rho_0\sqrt{\mu R_T}}{\beta}\mathrm{d}t$$

Decaimiento orbital

$$e^{\frac{z}{z_s}}\mathrm{d}z = -\frac{\rho_0\sqrt{\mu R_T}}{\beta}\mathrm{d}t$$

$$\int_{z_0}^{z(t)} e^{\frac{z}{z_s}}\mathrm{d}z = -\frac{\rho_0\sqrt{\mu R_T}}{\beta}\int_{t_0}^{t}\mathrm{d}t$$

$$e^{\frac{z}{z_s}} z_s\Big]_{z_0}^{z(t)} = -\frac{\rho_0\sqrt{\mu R_T}}{\beta}(t - t_0)$$

$$\boxed{z(t) = z_s \ln\left[e^{\frac{z_0}{z_s}} - \frac{\rho_0\sqrt{\mu R_T}}{\beta z_s}(t - t_0)\right]}$$

El decaimiento orbital depende de:

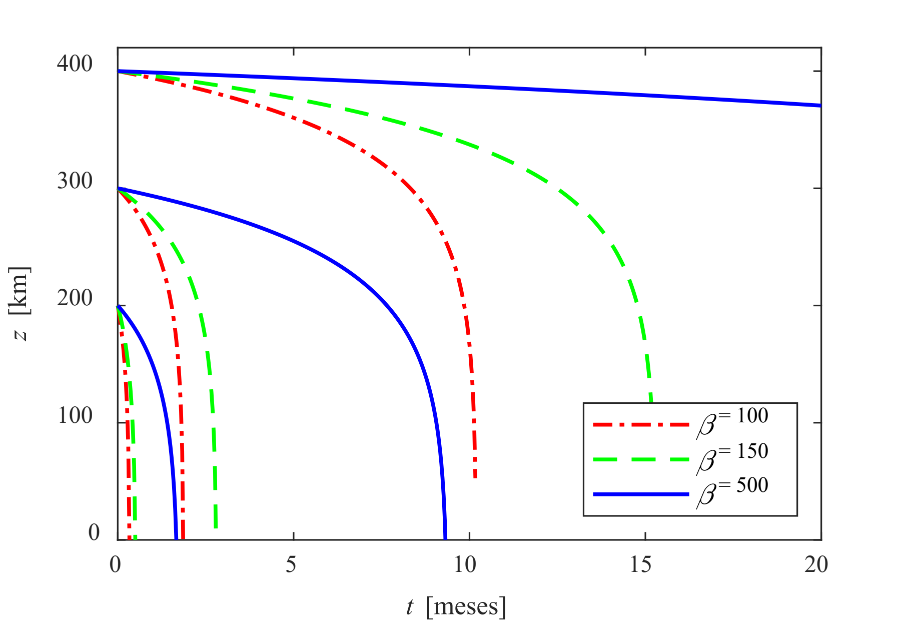
- La densidad a la altitud orbital
- El coeficiente balístico $\beta = \frac{m}{S c_D}$

A veces puede interesar aumentar la masa

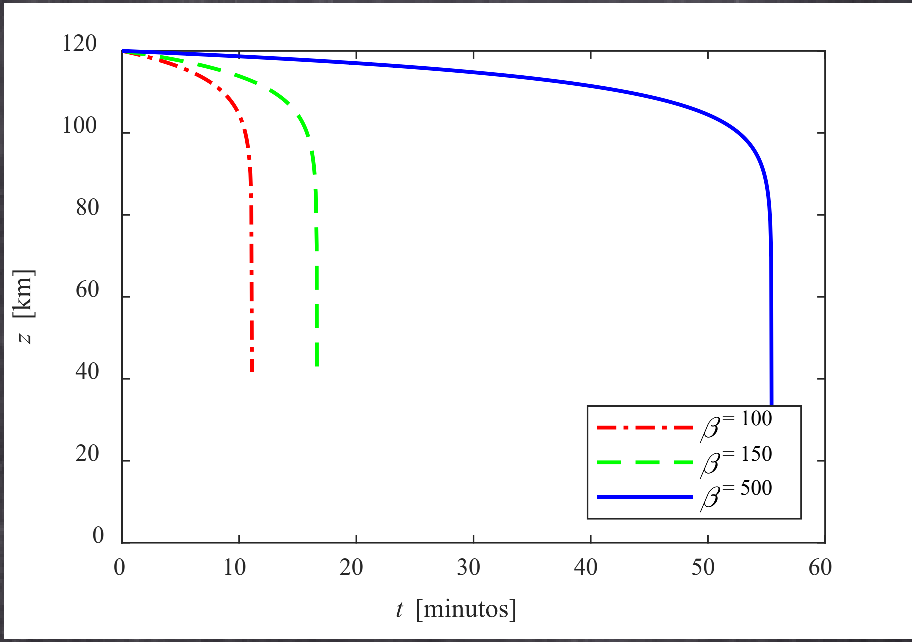
La predicción de este modelo depende del modelo de ATM

Decaimiento orbital

$$z(t) = z_s \ln\left[e^{\frac{z_0}{z_s}} - \frac{\rho_0\sqrt{\mu R_T}}{\beta z_s}(t - t_0)\right]$$



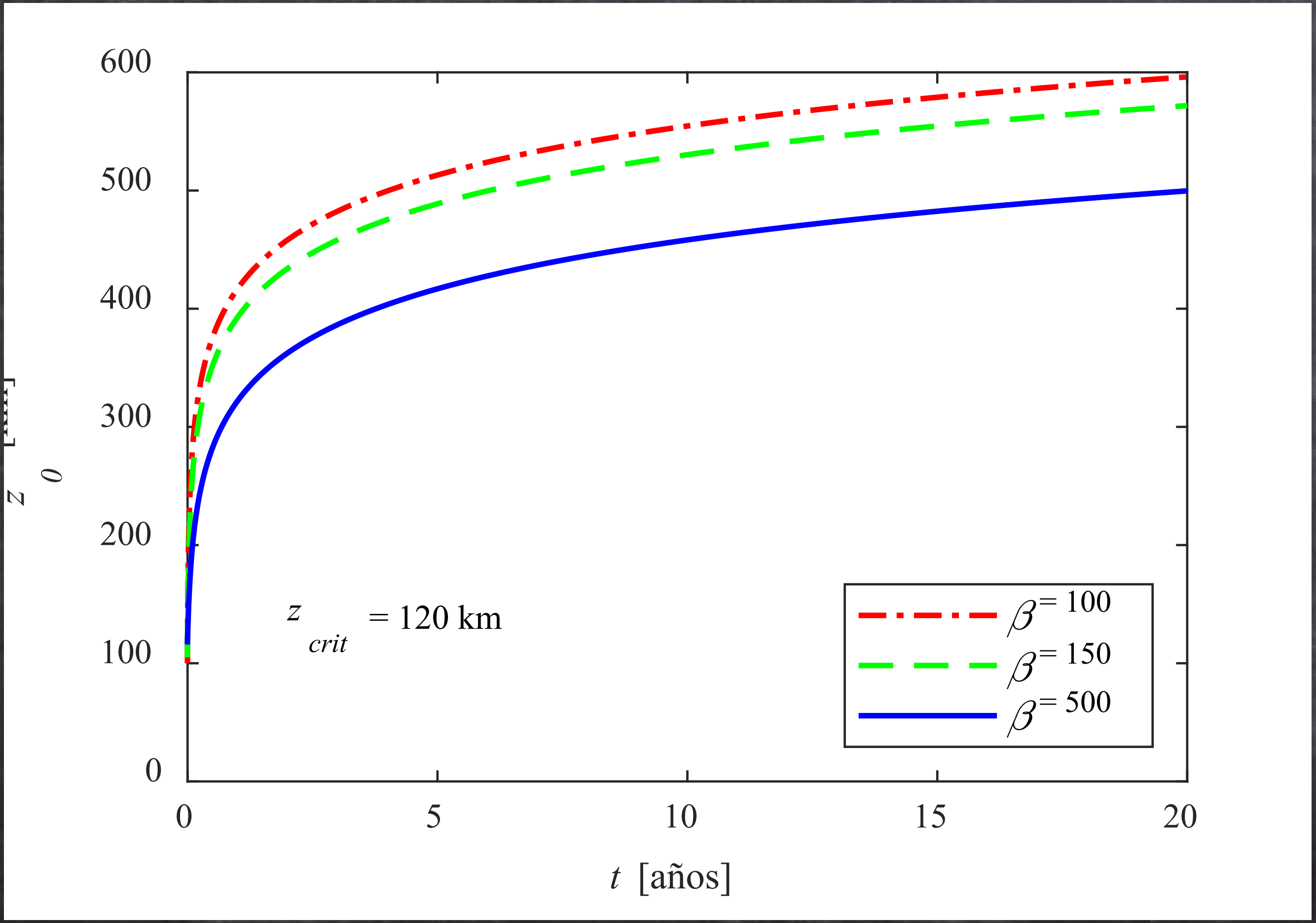
$\rho_0 = 3.9 \times 10^{-9}$ kg/m^3
$z_s = 60$ km



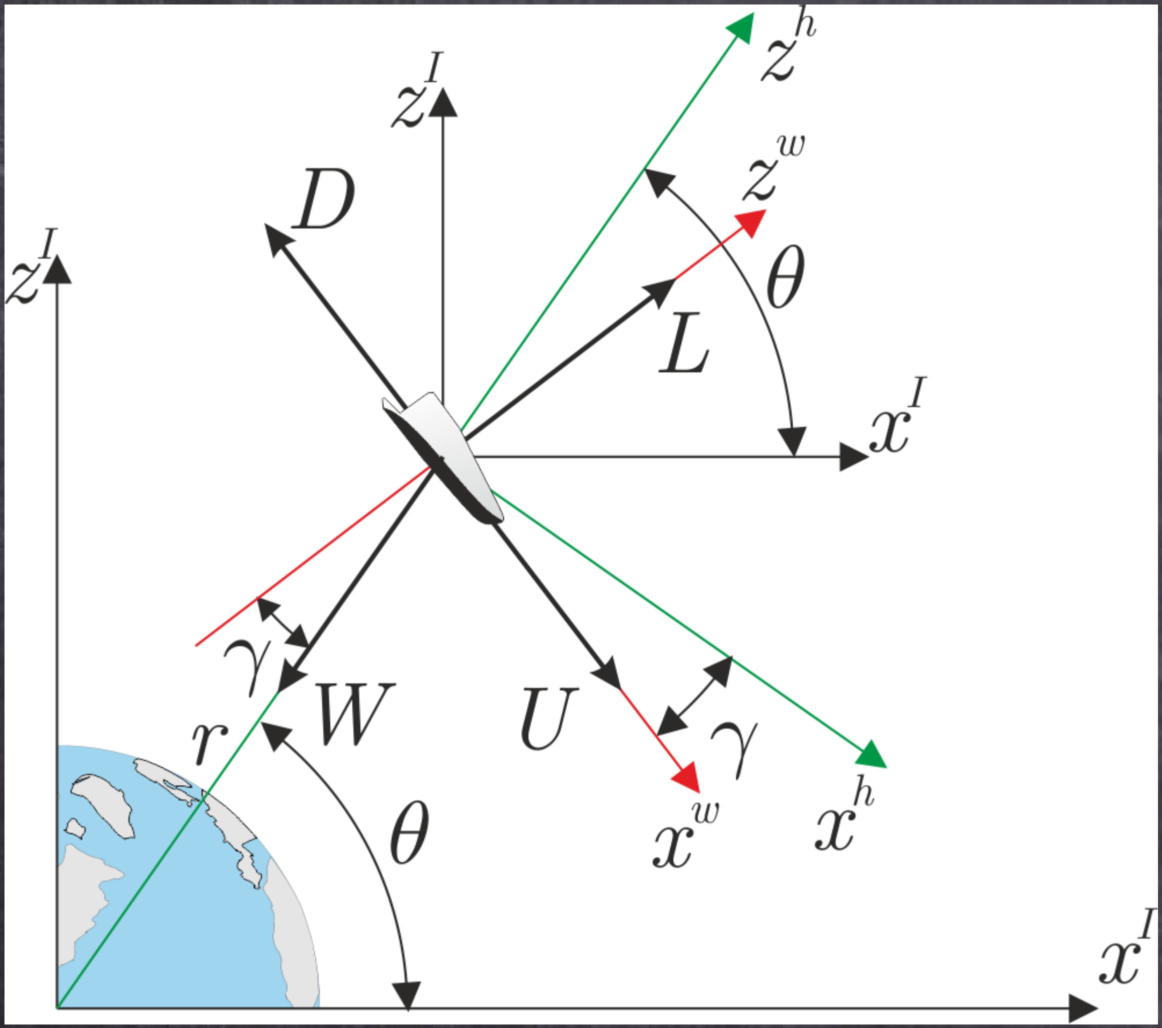
$\rho_0 = 1.2$ kg/m^3
$z_s = 6.7$ km

Decaimiento orbital

$$z(t) = z_s \ln\left[e^{\frac{z_0}{z_s}} - \frac{\rho_0\sqrt{\mu R_T}}{\beta z_s}(t - t_0)\right] \xrightarrow{z_{crit}} t_{crit} = \frac{\beta z_s}{\rho_0\sqrt{\mu R_T}}\left(e^{\frac{z_{crit}}{z_s}} - e^{\frac{z_0}{z_s}}\right)$$



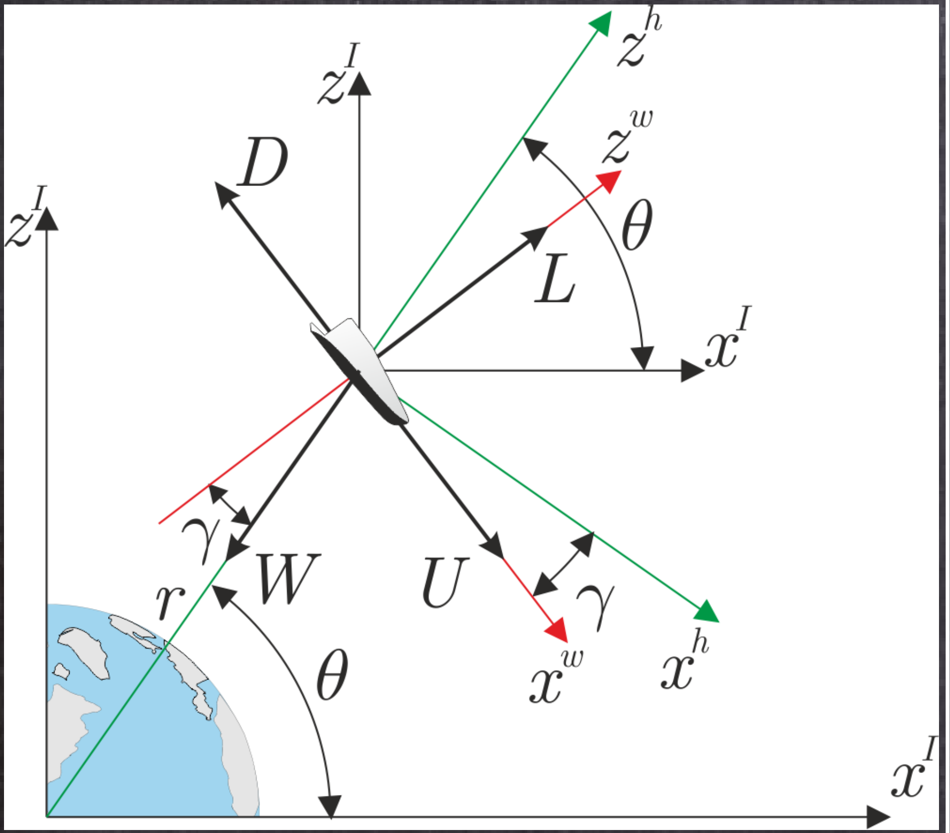
Ecuaciones generales de la dinámica de la re-entrada



- Trayectoria contenida en un plano
- Masa constante

- Fuerzas:
  - Gravitatoria
  - Aerodinámicas

Ecuaciones generales de la dinámica de la re-entrada



Cinemática:

$$\dot{r} = U \sin\gamma \qquad \mathbf{V} = U\mathbf{i}$$

$$\dot{\theta} r = U \cos\gamma \qquad \boldsymbol{\omega} = (\dot{\theta} - \dot{\gamma})\mathbf{j}$$

$$\mathbf{F} = m\mathbf{a}$$

Aceleración:

$$\mathbf{a} = \left(\frac{\mathrm{d}\mathbf{V}}{\mathrm{d}t}\right)_I = \left(\frac{\mathrm{d}\mathbf{V}}{\mathrm{d}t} + \boldsymbol{\omega} \times \mathbf{V}\right)_w$$

$$\boldsymbol{\omega} \times \mathbf{V} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & \omega & 0 \\ U & 0 & 0 \end{vmatrix} = -U\omega\mathbf{k}$$

$$\mathbf{a} = \dot{U}\mathbf{i} - U\omega\mathbf{k}$$

Fuerzas:

$$\mathbf{F} = (W \sin\gamma - D)\mathbf{i} + (L - W\cos\gamma)\mathbf{k}$$

En componentes:

$$W \sin\gamma - D = m\dot{U}$$

$$L - W\cos\gamma = -m\omega U$$

Ecuaciones generales de la dinámica de la re-entrada

En componentes:

$$\begin{cases} W\sin\gamma - D = m\dot{U} \\ L - W\cos\gamma = -m\omega U \end{cases}$$

En x:

$$\dot{U} = g\sin\gamma - \frac{D}{m}$$

$$\dot{U} = g\sin\gamma - \frac{\rho}{2\beta}U^2$$

En z:

$$\omega U = g\cos\gamma - \frac{L}{m}$$

$$(\dot{\theta} - \dot{\gamma})U = g\cos\gamma - \frac{\rho E}{2\beta}U^2$$

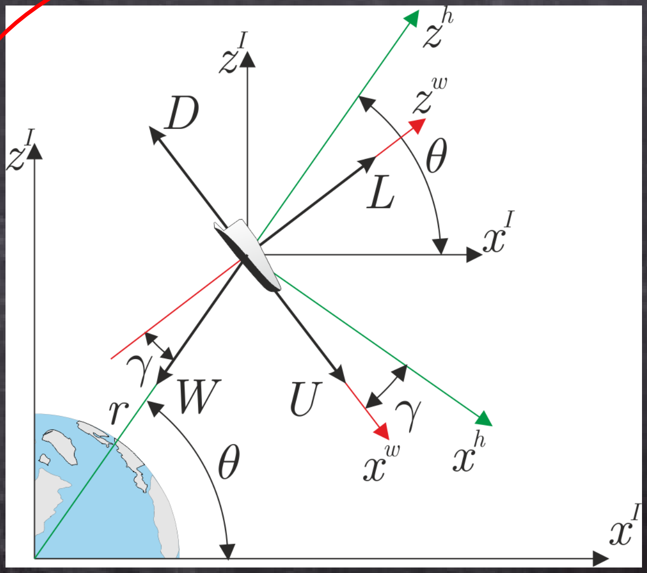
$$\dot{\gamma} = \frac{1}{U}\left[U^2\left(\frac{\rho E}{2\beta} - \frac{\cos\gamma}{r}\right) - g\cos\gamma\right]$$

$$\frac{D}{m} = \frac{1}{2}\rho U^2\frac{SC_D}{m} = \frac{\rho}{2\beta}U^2$$

$$\frac{L}{m} = \frac{1}{2}\rho U^2\frac{SC_L}{m}\frac{C_D}{C_D} = \frac{\rho E}{2\beta}U^2$$

$$\boldsymbol{\omega} = (\dot{\theta} - \dot{\gamma})\mathbf{j}$$

$$\dot{\theta}r = U\cos\gamma \Rightarrow \dot{\theta} = \frac{U}{r}\cos\gamma$$



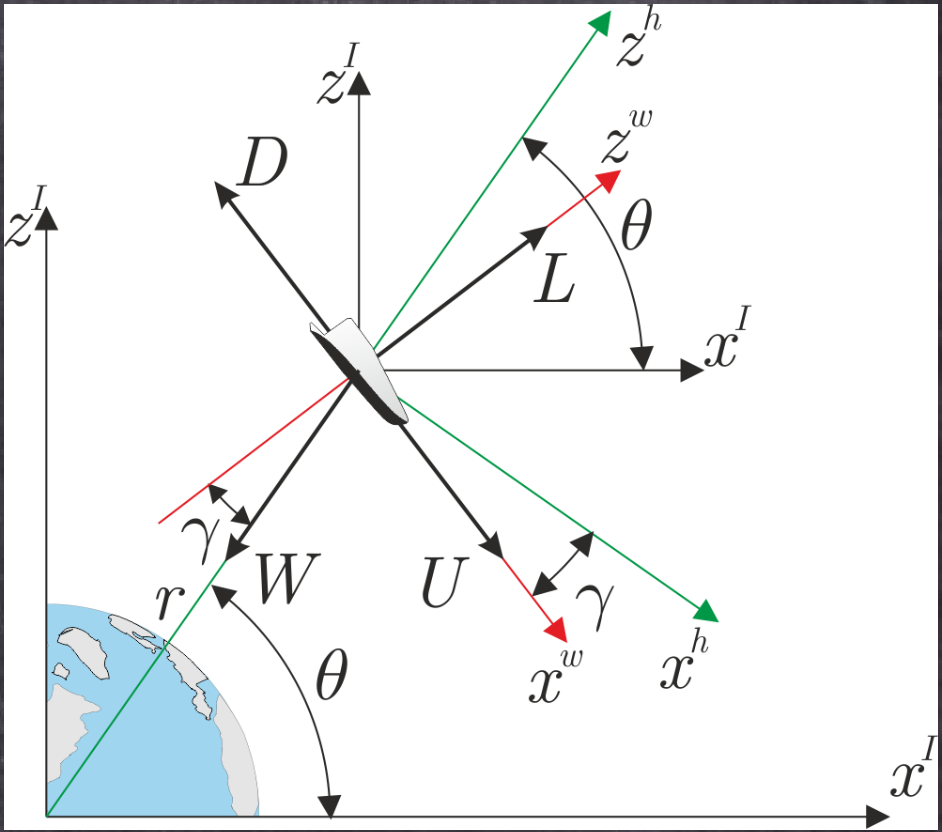
Ecuaciones generales de la dinámica de la re-entrada

Resumen:

$$\begin{cases} \dot{U} = g\sin\gamma - \dfrac{\rho}{2\beta}U^2 \\ \dot{\gamma} = \dfrac{1}{U}\left[U^2\left(\dfrac{\rho E}{2\beta} - \dfrac{\cos\gamma}{r}\right) - g\cos\gamma\right] \\ \dot{r} = U\sin\gamma \end{cases}$$

$$\dot{\theta}r = U\cos\gamma$$

Ecuaciones diferenciales de primer orden acopladas



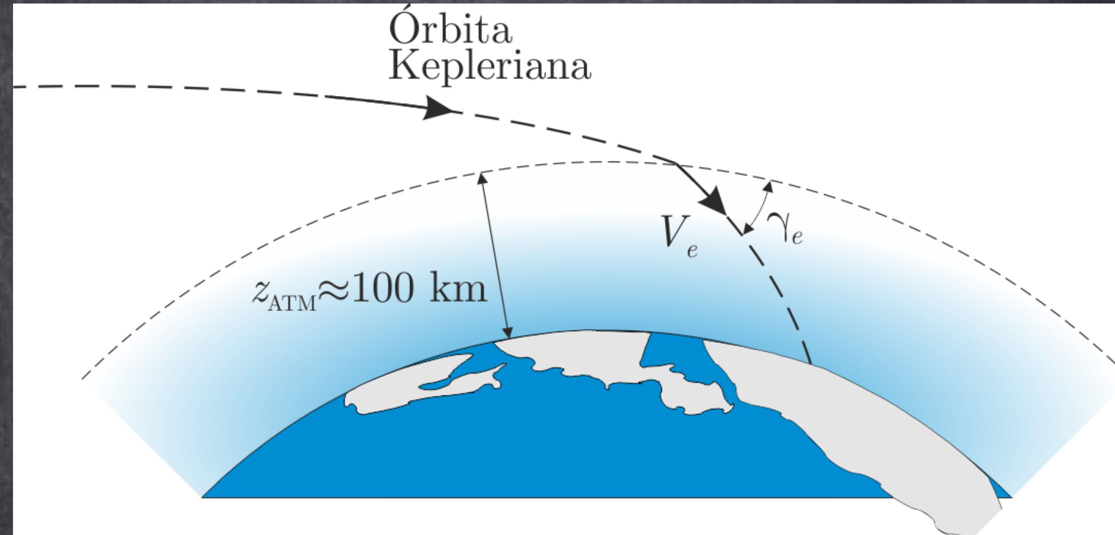
Con:

$$r = z + R_T$$

$$\rho = \rho(z)$$

$$g(z) = g_0\left(\frac{R_T}{z + R_T}\right)^2$$

Re-entrada balística

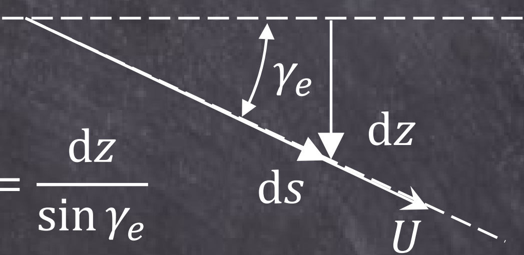


Hipótesis

- La sustentación es $L \approx 0$
- Sólo actúan W y D, siendo $D \gg W$
- $\dot{\gamma} = 0$ ➔ $\gamma = \gamma_e = \text{cte}$

En x:

$$\dot{U} = \frac{\mathrm{d}U}{\mathrm{d}t} = \cancel{g \sin\gamma} - \frac{\rho}{2\beta} U^2$$



$$\mathrm{d}s = \frac{\mathrm{d}z}{\sin\gamma_e}$$

$$\frac{\mathrm{d}U}{\mathrm{d}t} = \frac{\mathrm{d}U}{\mathrm{d}s}\frac{\mathrm{d}s}{\mathrm{d}t} = \frac{\mathrm{d}U}{\mathrm{d}s} U = \underbrace{\frac{1}{2}\frac{\mathrm{d}U^2}{\mathrm{d}s} = -\frac{\rho}{2\beta} U^2}$$

$$\rho = \rho_0 e^{-\frac{z}{z_s}}$$

$$\mathrm{d}\rho = -\frac{\rho_0}{z_s} e^{-\frac{z}{z_s}} \mathrm{d}z = -\frac{\rho}{z_s} \mathrm{d}z$$

$$\frac{\sin\gamma_e}{2}\frac{\mathrm{d}U^2}{\mathrm{d}z} = -\frac{\rho}{2\beta} U^2$$

$$\mathrm{d}z = -\frac{z_s}{\rho} \mathrm{d}\rho$$

Re-entrada balística

$$\frac{\sin\gamma_e}{\cancel{2}}\frac{\mathrm{d}U^2}{\mathrm{d}\rho}\left(-\frac{\cancel{\rho}}{z_s}\right) = -\frac{\cancel{\rho}}{\cancel{2}\beta}U^2$$

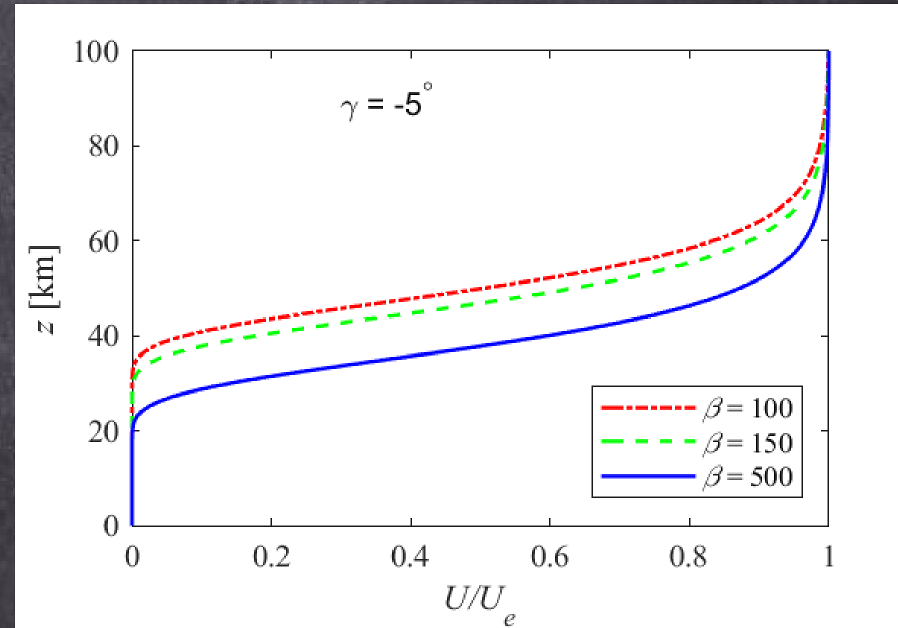
$$\int_{U_e}^{U}\frac{\mathrm{d}U^2}{U^2} = \frac{z_s}{\beta\sin\gamma_e}\int_0^{\rho}\mathrm{d}\rho$$

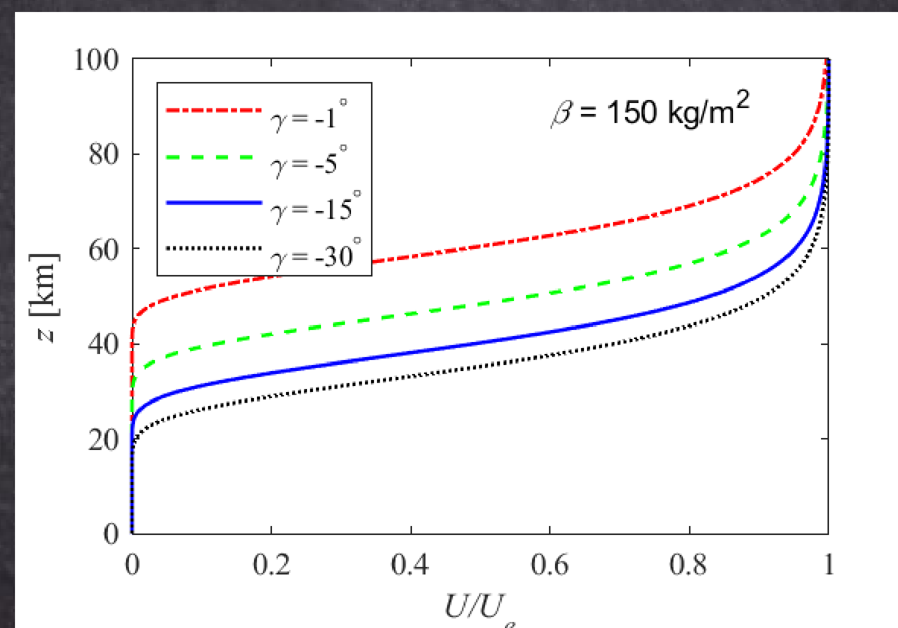
$$\ln\frac{U^2}{U_e^2} = 2\ln\frac{U}{U_e} = \frac{z_s}{\beta\sin\gamma_e}\rho$$

$$\frac{U}{U_e} = e^{\left(\frac{z_s}{2\beta\sin\gamma_e}\rho\right)} \quad \longleftarrow \quad \rho = \rho_0 e^{-\frac{z}{z_s}}$$

$$\boxed{\frac{U}{U_e} = e^{\left(\frac{z_s\rho_0}{2\beta\sin\gamma_e}e^{-\frac{z}{z_s}}\right)}}$$

- Depende de
  - Ángulo de entrada, γ_e
  - Coeficiente balístico, β





Re-entrada balística

Factor de carga

$$\frac{U}{U_e} = e^{Be^{-\frac{z}{z_s}}} \qquad B = \frac{z_s \rho_0}{2\beta \sin\gamma_e}$$

La aceleración es:

$$\frac{dU}{dt} = U_e e^{Be^{-\frac{z}{z_s}}} Be^{-\frac{z}{z_s}}\left(-\frac{1}{z_s}\right)\frac{dz}{dt}$$

$$= -U_e \frac{B}{z_s} e^{Be^{-\frac{z}{z_s}}} e^{-\frac{z}{z_s}} \frac{dz}{dt}$$

$$= -U_e \frac{B}{z_s} e^{Be^{-\frac{z}{z_s}}} e^{-\frac{z}{z_s}} U \sin\gamma$$

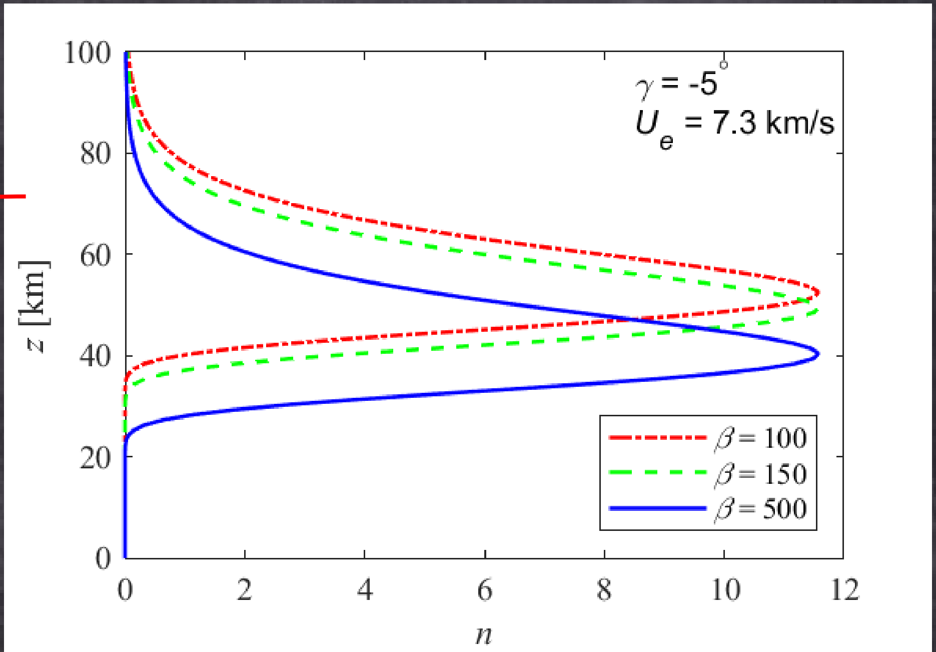
$$= -U_e^2 \frac{B}{z_s} \sin\gamma \, e^{\left(2Be^{-\frac{z}{z_s}} - \frac{z}{z_s}\right)}$$

Recuperamos B: $$\frac{dU}{dt} = -\frac{U_e^2 \rho_0}{2\beta} e^{\left(2Be^{-\frac{z}{z_s}} - \frac{z}{z_s}\right)}$$

Factor de carga

$$\boxed{n = -\frac{dU}{dt}\frac{1}{g_0} = Ce^{\left(2Be^{-\frac{z}{z_s}} - \frac{z}{z_s}\right)}}$$

$$C = \frac{U_e^2 \rho_0}{2\beta g_0}$$



La máxima aceleración no depende del coeficiente balístico, β

Re-entrada balística

Factor de carga máximo

$$B = \frac{z_s \rho_0}{2\beta \sin \gamma_e}$$

$$n = Ce^{\left(2Be^{-\frac{z}{z_s}} - \frac{z}{z_s}\right)}$$

$$C = \frac{U_e^2 \rho_0}{2\beta g_0}$$

Derivando e igualando a cero:

$$\frac{\mathrm{d}n}{\mathrm{d}z} = Ce^{\left(2Be^{-\frac{z}{z_s}} - \frac{z}{z_s}\right)}\left(-\frac{2B}{z_s}e^{-\frac{z}{z_s}} - \frac{1}{z_s}\right) = 0$$

El paréntesis de la derecha debe ser:

$$2Be^{-\frac{z}{z_s}} + 1 = 0 \qquad z_{n,\max} = z_s \ln(-2B)$$

$$z_{n,\max} = z_s \ln\left(-\frac{z_s \rho_0}{\beta \sin \gamma_e}\right)$$

Altitud de factor de carga máximo

Velocidad de Factor de carga máximo

$$\frac{U}{U_e} = e^{Be^{-\frac{z}{z_s}}} \Rightarrow \left(\frac{U}{U_e}\right)_{n,\max} = e^{-\frac{1}{2}}$$

$$(U)_{n,\max} = 0.61U_e$$

Factor de carga máximo

$$n_{\max} = Ce^{\left(2Be^{-\ln(-2B)} - \ln(-2B)\right)}$$

$$= Ce^{2B\left(-\frac{1}{2B}\right)}e^{-\ln(-2B)}$$

$$= -\frac{C}{2Be}$$

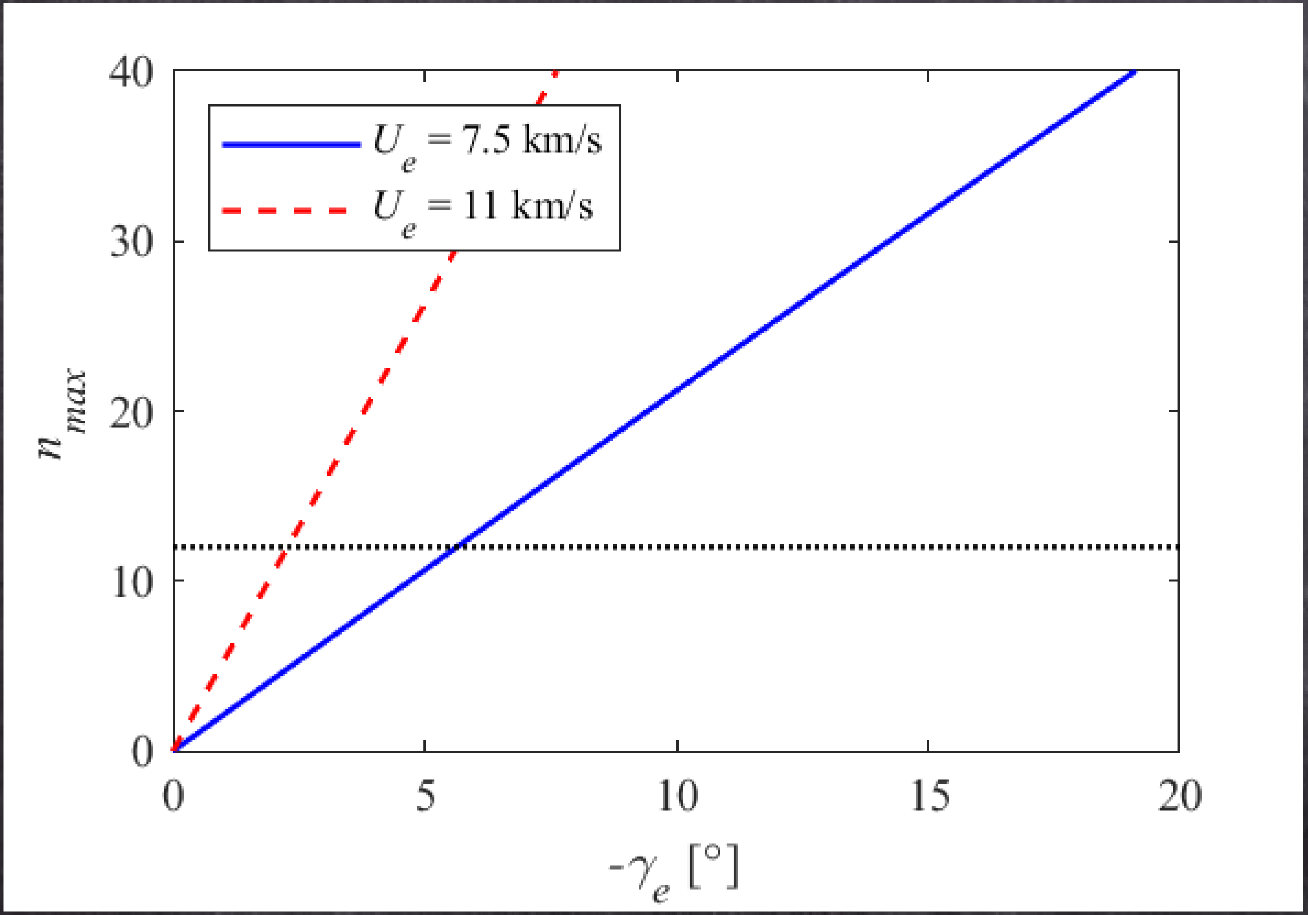
$$\boxed{n_{\max} = \frac{U_e^2 \sin \gamma_e}{2g_0 z_s e}}$$

Analicemos…

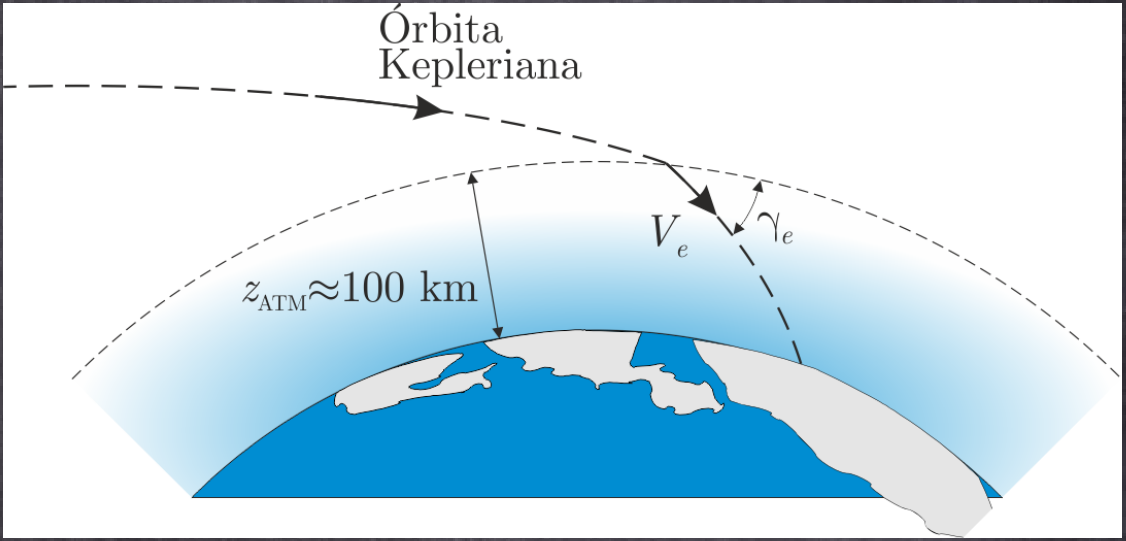
Re-entrada balística

$$n_{\max} = \frac{U_e^2 \sin \gamma_e}{2 g_0 z_s e}$$

- Depende de
  - La velocidad de entrada, U_e
  - Ángulo de entrada, γ_e
  - No depende del coeficiente balístico, β



Re-entrada en planeo



Hipótesis

- Ahora ${}^{L}/_{D} = \text{cte} \neq 0$
- También $D \gg W$
- Fuerzas en z equilibradas ➔ $\dot{\gamma} = 0$
- ➔ $\gamma = \gamma_e = \text{cte}$ y $\gamma_e \ll 1$

En z:

$$\frac{U^2}{r}\cancel{\cos\gamma}^{\,1} - \cancel{\dot{\gamma}U} = g\,\cancel{\cos\gamma}^{\,1} - \frac{\rho E}{2\beta}U^2$$

$$U^2\left(\frac{1}{r} - \frac{\rho E}{2\beta}\right) = g_0$$

$$U = \sqrt{\frac{g_0 r}{1 - \frac{\rho E r}{2\beta}}}$$

- Como $r \approx$ cte, El numerador es $\approx$ cte

$$\left.\begin{array}{l} \sqrt{g_0 r} \approx \sqrt{g_0 r_e} \approx U_e \\ \text{O bien} \\ \sqrt{g r} \approx \sqrt{g R_T} \approx U_T \end{array}\right\} \text{1-2\% dif}$$

- En el denominador $r \approx R_T$
- Modelo de atmósfera exponencial:

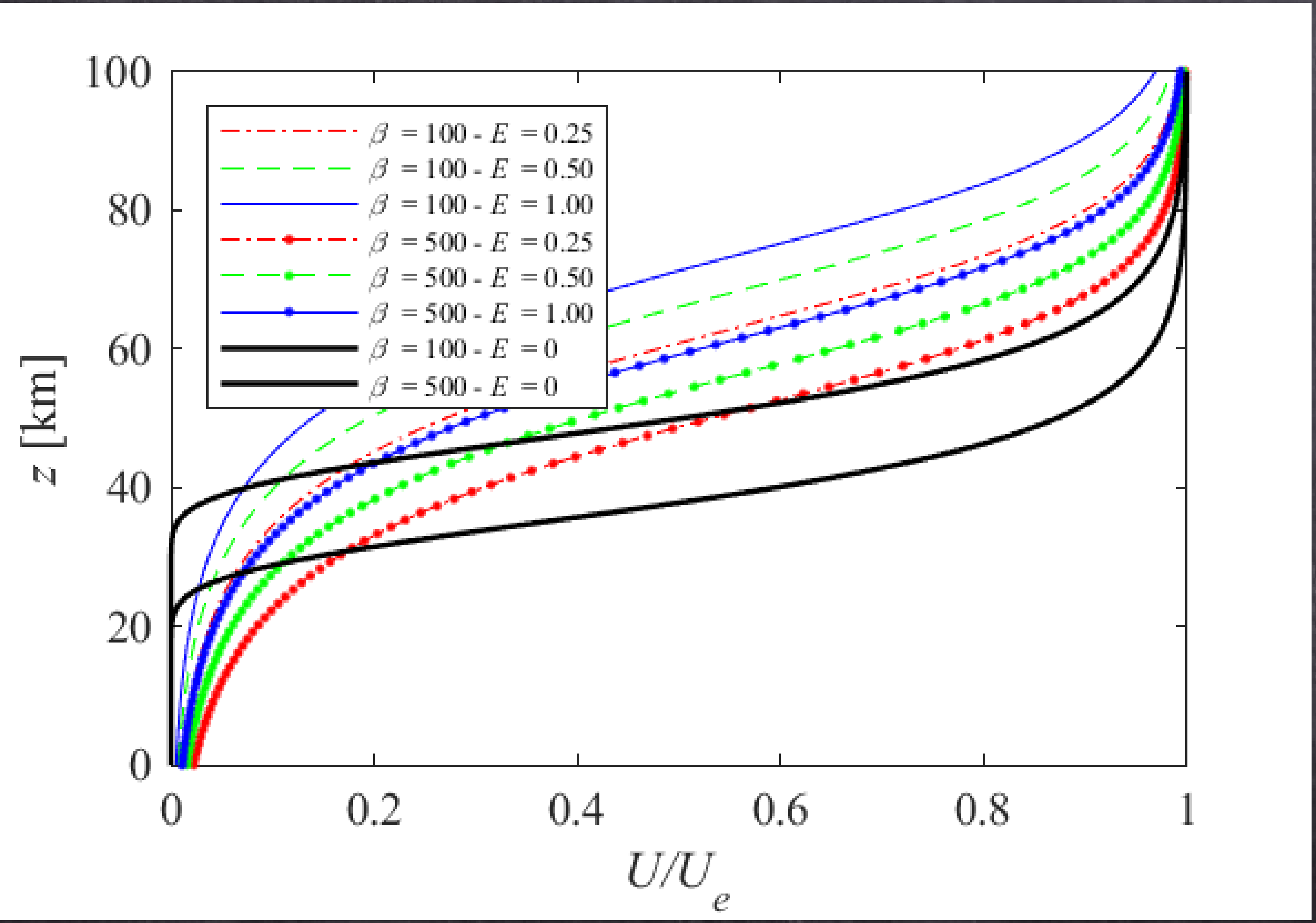
$$\rho = \rho_0 e^{-\frac{z}{z_s}}$$

$$\boxed{\frac{U}{U_e} = \left(1 - \frac{E}{\beta}\frac{\rho_0 R_T}{2}e^{-\frac{z}{z_s}}\right)^{-1/2}}$$

Re-entrada en planeo

- Depende de la relación ${}^{E}/_{\beta}$

$$\boxed{\frac{U}{U_e} = \left(1 - \frac{E}{\beta}\frac{\rho_0 R_T}{2} e^{-\frac{z}{z_s}}\right)^{-1/2}}$$



Re-entrada en planeo

Factor de carga

En z:

$$\frac{U^2}{r}\cancelto{1}{\cos\gamma} - \cancel{\dot{\gamma}}U = g\cancelto{1}{\cos\gamma} - \frac{\rho E}{2\beta}U^2$$

$$\frac{U^2}{r} = g - \frac{L}{m}$$

$$\frac{L}{m} = g_0 - \frac{U^2}{r}\times\frac{g_0}{g_0} = g_0\left[1 - \left(\frac{U}{U_e}\right)^2\right]$$

En x:

$$\dot{U} = \frac{\mathrm{d}U}{\mathrm{d}t} = \cancel{g\sin\gamma} - \frac{D}{m}$$

$$\frac{\mathrm{d}U}{\mathrm{d}t} = -\frac{D}{m}\times\frac{L}{L} = -\frac{1}{E}\frac{L}{m}$$

El factor de carga queda:

$$n = -\frac{\mathrm{d}U}{\mathrm{d}t}\frac{1}{g_0} = \frac{1}{E\,g_0}\frac{L}{m}$$

$$n = \frac{1}{E}\left[1 - \left(\frac{U}{U_e}\right)^2\right]$$

Por otro lado: $$\frac{U}{U_e} = \left(1 - Ef(z)\right)^{-1/2}$$

$$f(z) = \frac{1}{\beta}\frac{\rho_0 R_T}{2}e^{-\frac{z}{z_s}}$$

$$n = \frac{1}{E}\left[1 - \frac{1}{1 - Ef(z)}\right] = \frac{1}{E}\left[\frac{Ef(z)}{1 + Ef(z)}\right]$$

$$= \frac{f(z)}{1 + Ef(z)} = \frac{1}{E + f(z)^{-1}}$$

$$n = \frac{1}{E + \frac{2\beta}{\rho_0 R_T}e^{\frac{z}{z_s}}}$$

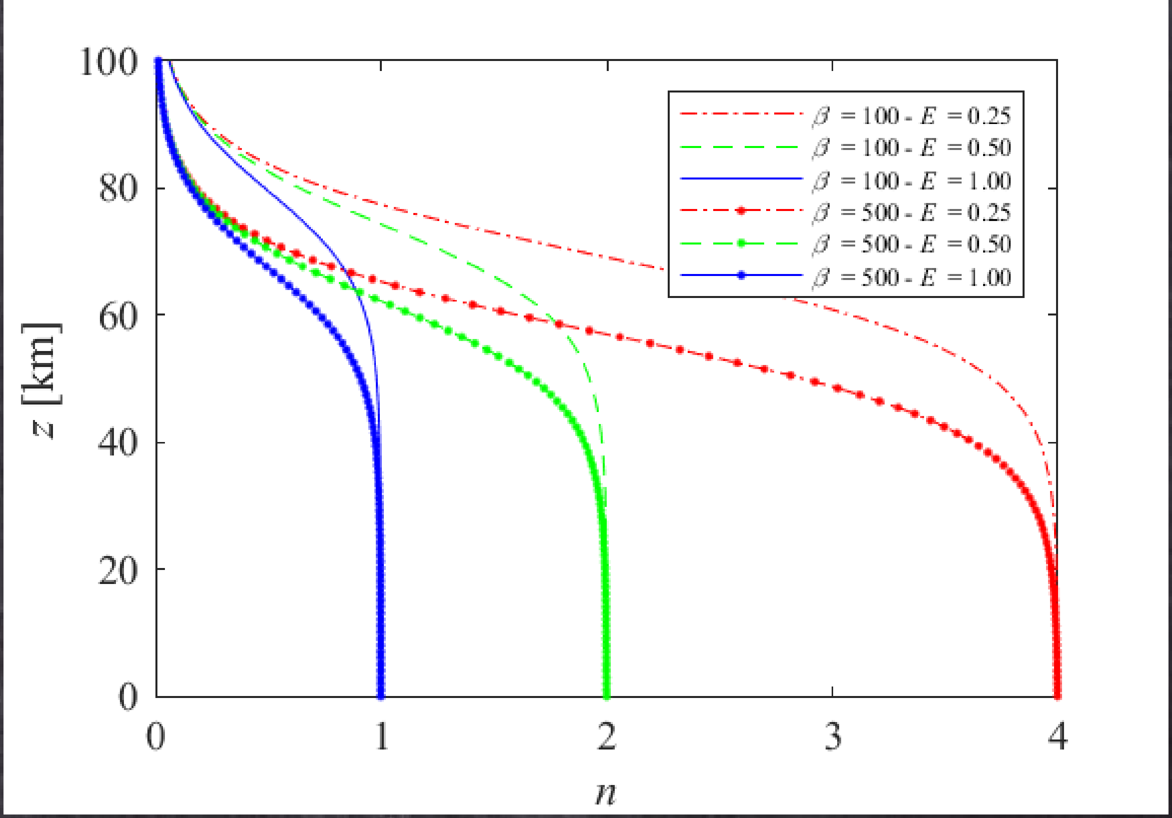
Re-entrada en planeo

Factor de carga

$$n = \frac{1}{E}\left[1 - \left(\frac{U}{U_e}\right)^2\right] = \frac{1}{E + \frac{2\beta}{\rho_0 R_T} e^{\frac{z}{z_s}}}$$

Derivando con respecto a *U*:

$$n_{\max} = \frac{1}{E}$$

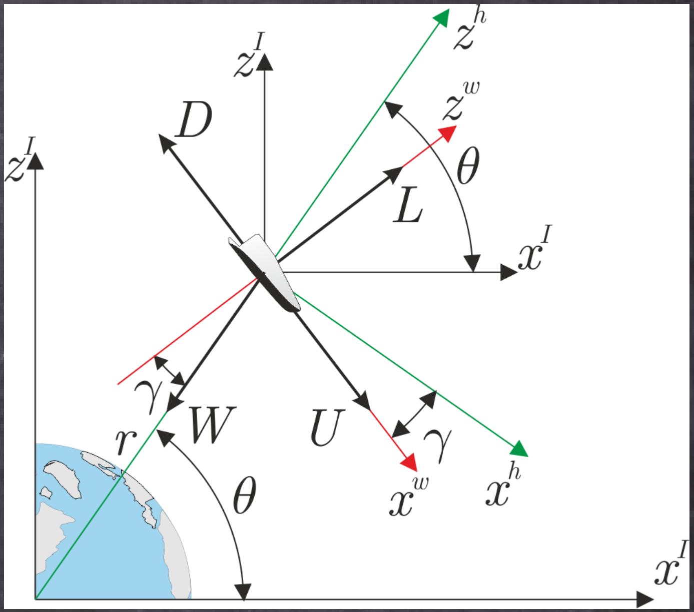


Solución numérica de la dinámica de la reentrada

$$\begin{cases} \dot{U} = g \sin\gamma - \dfrac{\rho}{2\beta} U^2 \\ \dot{\gamma} = \dfrac{1}{U}\left[U^2\left(\dfrac{\rho E}{2\beta} - \dfrac{\cos\gamma}{r}\right) - g\cos\gamma\right] \\ \dot{r} = U \sin\gamma \end{cases}$$

$$\dot{\theta} r = U \cos\gamma$$

Ecuaciones diferenciales de primer orden acopladas



Con:

$$r = z + R_T$$

$$\rho = \rho(z) \qquad g(z) = g_0\left(\frac{R_T}{z + R_T}\right)^2$$

Método de Runge Kutta de 4º orden

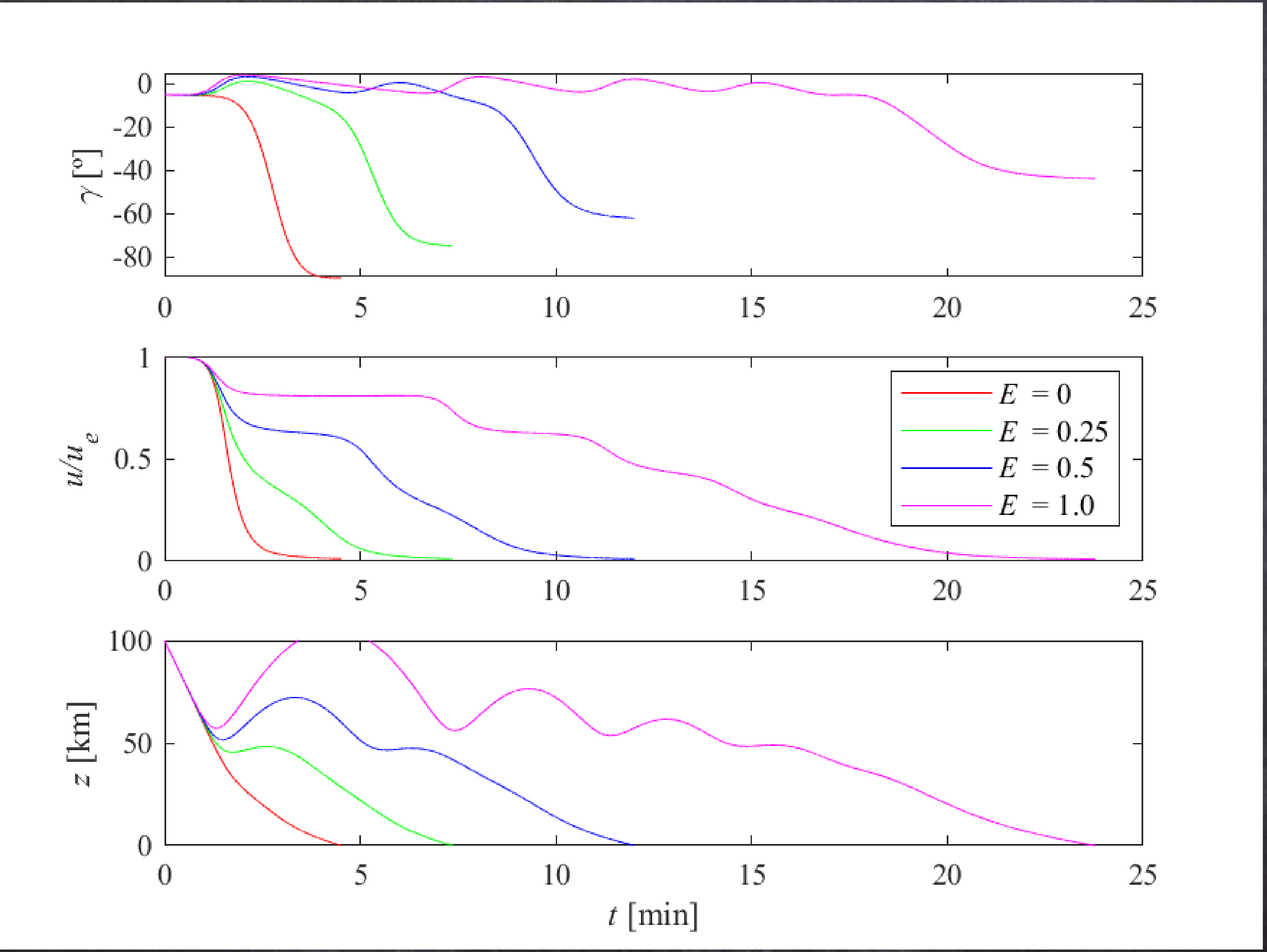
$$\dot{\mathbf{y}} = f(\mathbf{y}, t) \qquad \mathbf{y}(t_0) = \mathbf{y}_0$$

$$\mathbf{y}_{n+1} = \mathbf{y}_n + \frac{\Delta t}{6}(\mathrm{k}_1 + 2\mathrm{k}_2 + 2\mathrm{k}_3 + \mathrm{k}_4)$$

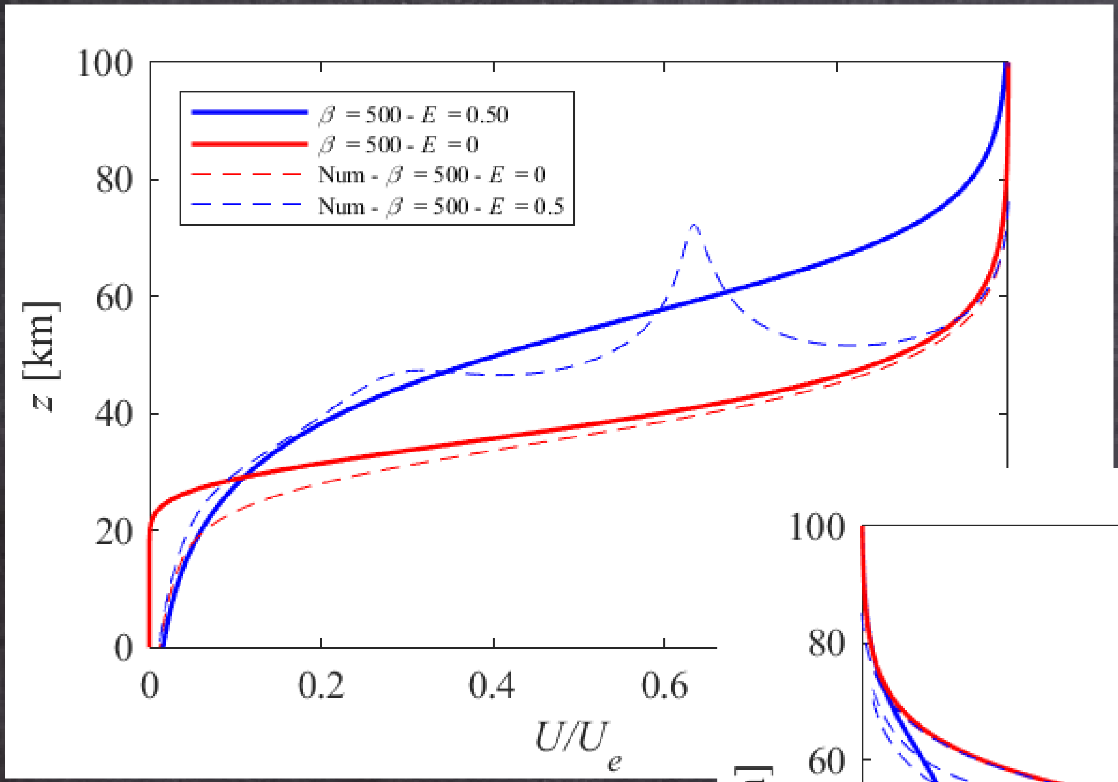
$$t_{n+1} = t_n + \Delta t$$

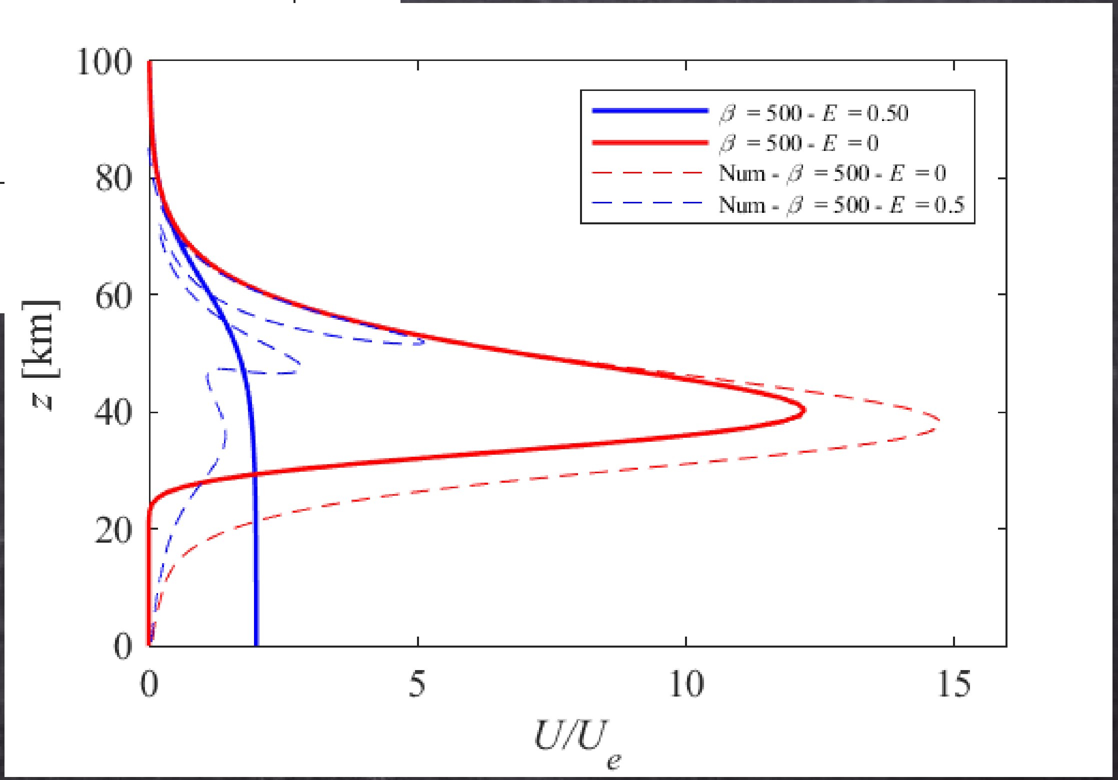
$$\begin{cases} \mathbf{k}_1 = f(\mathbf{y}_n, t_n) \\ \mathbf{k}_2 = f\left(\mathbf{y}_n + \dfrac{\Delta t}{2}\mathbf{k}_1, t_n + \dfrac{\Delta t}{2}\right) \\ \mathbf{k}_3 = f\left(\mathbf{y}_n + \dfrac{\Delta t}{2}\mathbf{k}_2, t_n + \dfrac{\Delta t}{2}\right) \\ \mathbf{k}_4 = f\left(\mathbf{y}_n + \Delta t\mathbf{k}_3, t_n + \dfrac{\Delta t}{2}\right) \end{cases}$$

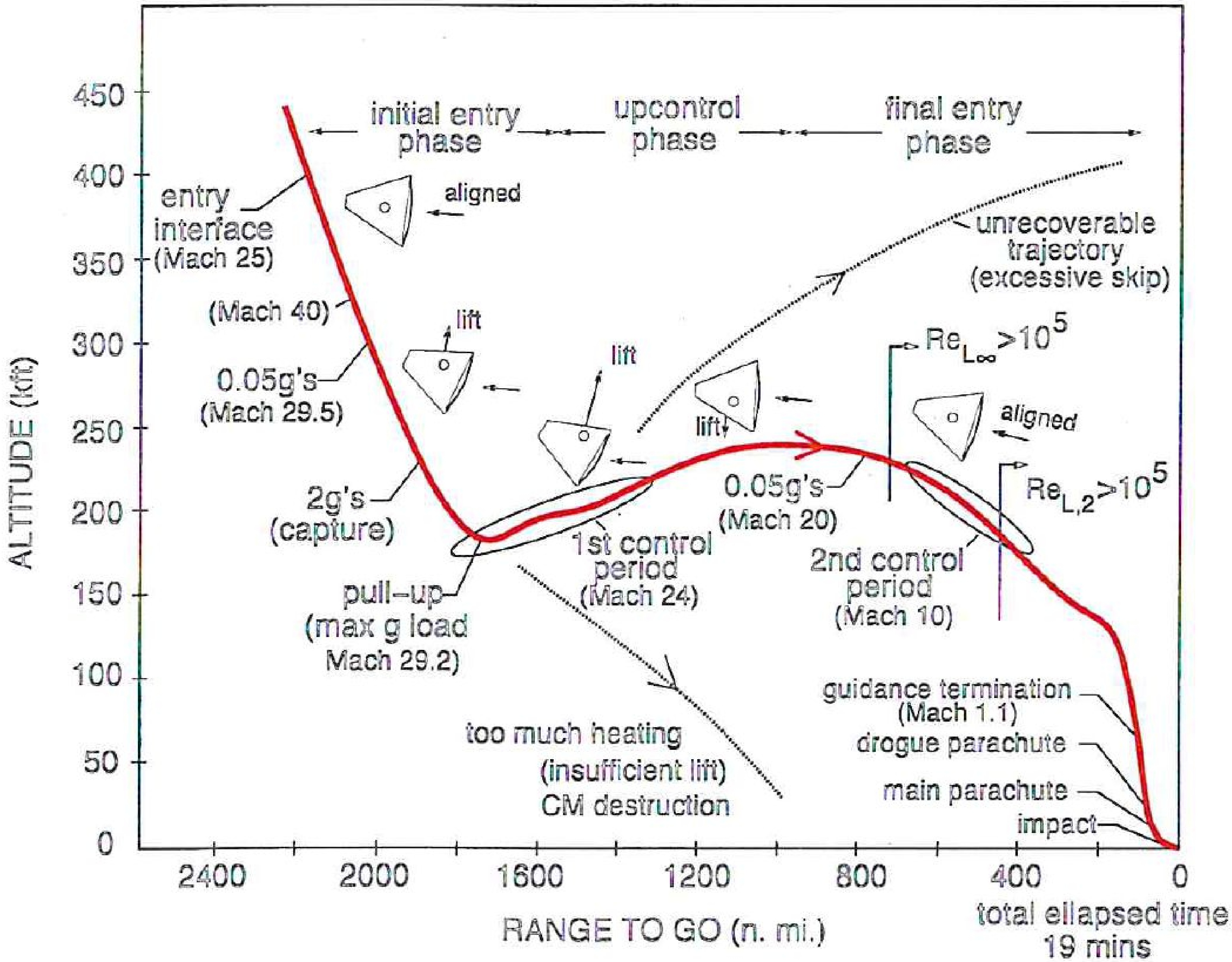
Solución numérica de la dinámica de la reentrada



Solución numérica de la dinámica de la reentrada







initial entry phase
upcontrol phase
final entry phase
entry interface (Mach 25)
aligned
unrecoverable trajectory (excessive skip)
(Mach 40)
lift
0.05g's (Mach 29.5)
lift
lift
$Re_{L\infty} > 10^5$
aligned
2g's (capture)
0.05g's (Mach 20)
$Re_{L,2} > 10^5$
1st control period (Mach 24)
2nd control period (Mach 10)
pull-up (max g load Mach 29.2)
guidance termination (Mach 1.1)
too much heating (insufficient lift) CM destruction
drogue parachute
main parachute
impact
ALTITUDE (kft)
450
400
350
300
250
200
150
100
50
0
2400
2000
1600
1200
800
400
0
RANGE TO GO (n. mi.)
total ellapsed time: 19 mins

¿Preguntas?