DOC 221 Dinámica orbital y control de actitud

Problems Lecture ADCS - I

Problem 1:

Consider the Earth.

What is the radial acceleration due to gravitational force?

- **(a)** At sea surface.
- **(b)** For an Earth observation satellite at 800 km altitude.
- **(c)** For a GPS satellite at 20200 km altitude.
- **(d)** For a geostationary satellite at 35800 km altitude.

Problem 2:

In lecture I we have seen that the energy of a photon is given by following formula $E = hc/\lambda$.

- **(a)** Please write the energy of the photon as a function of frequency ν.
- **(b)** Calculate the energy in joules for a green (λ = 525 nm) light photon as well as for gamma ray ($\lambda = 10^{-12}$ m) and for radio wave (λ = 1 m).

Problem 3:

In lecture I page 34 we have seen that Planck's distribution describes the variation of the power per unit area (intensity, I) emitted from a thermal source (black body) as a function of the emission wavelength λ and the temperature of the source T.

- **(a)** What is the physical meaning of the integral of the Planck distribution? Please write the formula for the integral.
- **(b)** Wien's law describes the dependence of the wavelength corresponding to maximum emission intensity (λ_{max}) as a function of the source temperature T. Show that Planck´s law

$$I(T,\lambda) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1}$$

Can be used to derive Wien´s law $\lambda_{\max} = \frac{b}{T}$, with $b = 2.898 \times 10^{-3} \mathrm{m\,K}$.

Hint: Use product and chain rule. The calculation gets horrible messy, however you can obtain an expression of the form 5 = (5-*x*)e^x where *x* is a trivial function of *λ* and *T*. The numerical solution of this equation is *x* ≈ 4.96511 (or *x* = 0 not physical).

Note: Planck's law originates from first principles derivation and agrees very well with observations, whereas Wien's law is empirical (means it originates from observations and not from first principles).

Problem 4:

Explain qualitatively the atmospheric loss.

Assume a volume of gas at the top of a planet’s atmosphere. The gas will have a range of velocities described by the Maxwell-Boltzmann distribution. Please write the Maxwell-Boltzmann distribution formula and plot the function for different temperatures. Why do particles/molecules escape particles into space? Which particle/molecules escape first into space? Please compare temperature, escape velocity and atmosphere composition of e.g. Earth, Mars and Jupiter.

Problem 5:

Consider a star with mass *M*, radius *R* and uniform density $\rho_0 = \frac{3}{4\pi} \frac{M}{R^3}$. Use the equation of hydrostatic equilibrium ($\frac{\mathrm{d}p}{\mathrm{d}r} = -g(r)\rho(r)$) and show that the pressure as a function of radius in the star is given by $p(r) = \frac{2\pi}{3} G {\rho_0}^2 (R^2 - r^2)$.