

Bing

Here is a summary of the document:

- **Elements of probability:** This section introduces the basic concepts of probability theory, such as sample space, events, and probability measure. It also defines the conditional probability and independence of events. An example of conditional probability is $P(A|B) = P(A \cap B)/P(B)$, which means the probability of event A given that event B has occurred.
- **Random variables:** This section defines random variables as functions that map outcomes to real numbers. It also introduces different types of random variables, such as discrete and continuous, and their corresponding distribution functions, such as PMF and PDF. An example of a discrete random variable is $X \sim \text{Binomial}(n, p)$, which means the number of heads in n independent flips of a coin with heads probability p. An example of a continuous random variable is $X \sim \text{Normal}(\mu, \sigma^2)$, which means a Gaussian distribution with mean μ and variance σ^2 .
- **Expectation and variance:** This section defines the expectation and variance of a random variable as measures of its central tendency and dispersion. It also shows some properties and formulas for calculating them. An example of expectation is $E[X] = \sum_{x \in \text{Val}(X)} x \cdot p_X(x)$ for a discrete random variable, and $E[X] = \int_{-\infty}^{\infty} x \cdot f_X(x) \, dx$ for a continuous random variable. An example of variance is $\text{Var}[X] = E[(X - E[X])^2] = E[X^2] - E[X]^2$.
- **Some common random variables:** This section lists some common random variables and their distribution functions, such as Bernoulli, Binomial, Geometric, Poisson, Uniform, Exponential, and Normal. It also shows their mean and variance values. An example of a common random variable is $X \sim \text{Poisson}(\lambda)$, which means a probability distribution over the nonnegative integers used for modeling the frequency of rare events. Its mean and variance are both λ .
- **Multiple random variables:** This section extends the concepts of probability, distribution, expectation, and variance to the case of multiple random variables. It also introduces the notions of joint, marginal, and conditional distributions, as well as independence and covariance. An example of a joint distribution is $f_{XY}(x, y) = f_X(x) f_Y(y)$ if X and Y are independent. An example of covariance is $\text{Cov}[X, Y] = E[XY] - E[X]E[Y]$.
- **Random vectors:** This section defines random vectors as vectors of random variables. It also introduces the multivariate Gaussian distribution as an important example of a probability distribution over random vectors. An example of a random vector is $X = [X_1 \ X_2 \ \dots \ X_n]^T$, where X_i are random variables. An example of a multivariate Gaussian distribution is $X \sim N(\mu, \Sigma)$, where μ is the mean vector and Σ is the covariance matrix.