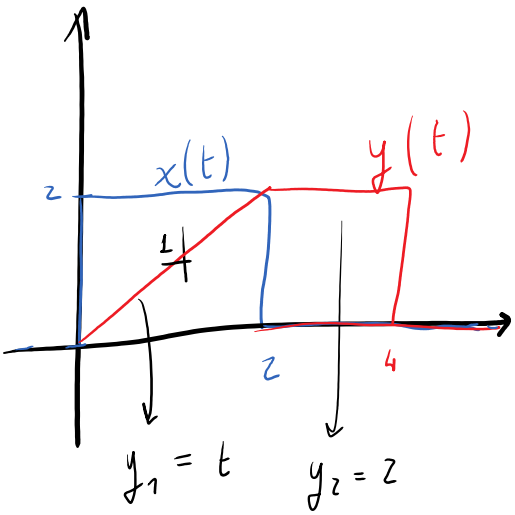
Es. Convoluzione

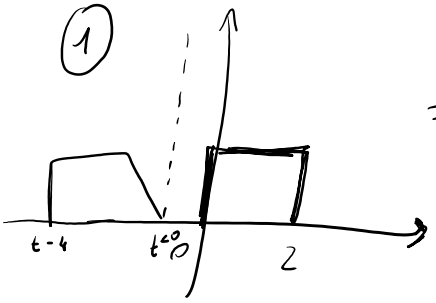
giovedì 2 ottobre 2025 15:49

$$z(t) = x(t) * y(t)$$

$$z(t) = \int_{-\infty}^{+\infty} x(\tau)\, y(t-\tau)\, d\tau$$

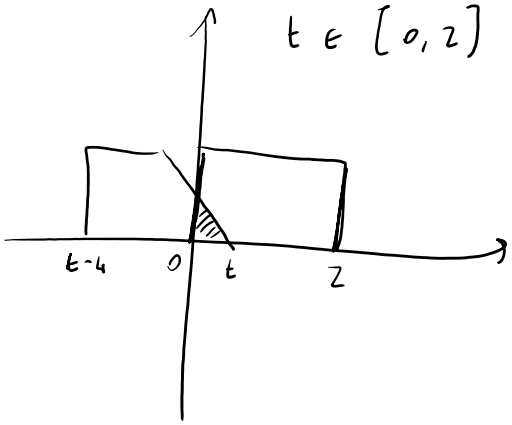


① t



$\Longrightarrow$ $z(t) = 0 \quad \forall\, t < 0$

② $t \in [0,2]$



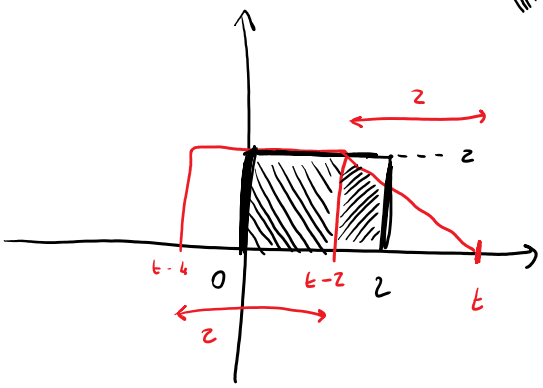
$$\int_0^t x(\tau)\, y(t-\tau)\, d\tau =$$

$$= \int_0^t 2\,(t-\tau)\, d\tau =$$

$$= \int_0^t 2t\, d\tau - 2\int_0^t \tau\, d\tau =$$

$$= 2t^2 - 2\,\frac{\tau^2}{2}\Big|_0^t = \boxed{t^2} \quad \forall\, t \in [0,2]$$

③



$$\int_0^{t-2} x(\tau)\, y_2(t-\tau)\, d\tau +$$

$$+ \int_{t-2}^{2} x(\tau)\, y_1(t-\tau)\, d\tau =$$

$$= \int_0^{t-2} 2 \cdot 2\, d\tau +$$

$$+ \int_{t-2}^{2} 2 \cdot (t-\tau)\, d\tau =$$

$$= 4(t-2) + 2t(2-t+2) - 2\left(\frac{\tau^2}{2}\right)\Big|_{t-2}^{2} =$$

$$= 4t - 8 + 4t - 2t^2 + 4t - 4 + (t-2)^2 =$$

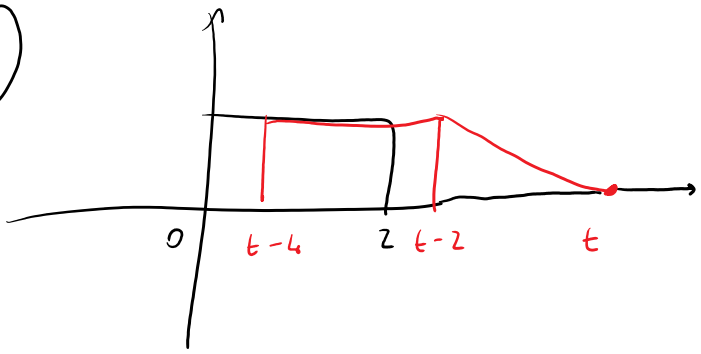
$$= 12t - 2t^2 - 12 + t^2 - 4t + 4 =$$

$$= -t^2 + 8t - 8 \quad \forall\, t \in [2,4]$$

$= 12t - 2t^2 - 12 \cdot t$ [illegible]

$= -t^2 + 8t - 8 \quad \forall\, t \in [2,4]$

(4)



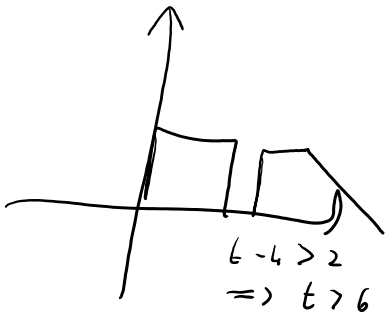
$$\int_{t-4}^{2} x(\tau)\, y(t-\tau)\, d\tau =$$

$$= \int_{t-4}^{2} 2 \cdot 2 \, d\tau =$$

$$= 4\,(2 - t + 4) =$$

$$= 24 - 4t \quad \forall\, t \in [4,6]$$

(5)



$z(t) = 0 \quad \forall\, t > 6$

$$z(t) = \begin{cases} 0 & t < 0 \quad (1) \\ t^2 & t \in [0,2) \quad (2) \\ -t^2 + 8t - 8 & t \in [2,4) \quad (3) \\ 24 - 4t & t \in [4,6) \quad (4) \\ 0 & t \geq 6 \quad (5) \end{cases}$$

$I1 = x^2 \{0 < x < 2\}$

$= \{x^2 (0 < x < 2)\}$

$f = (-x^2 + 8x - 8) \{2 < x < 4\}$

$= \{(-x^2 + 8x - 8) (2 < x < 4)\}$

$I2 = (24 - 4x) \{4 < x < 6\}$

$= \{(24 - 4x) (4 < x < 6)\}$

Inserimento...