Multimedia Coding

Lossy Coding 2

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Vector Quantization

- Vector quantization is an immediate generalization of scalar quantization of a single random variable to quantization of a block or vector of random variables.
- In vector quantization an L-dimensional source vector $\boldsymbol{x} = (x_1, x_2, \ldots, x_L)$ is mapped into a set of reproduction vectors.
- As for the case of entropy coding, where considering blocks of symbols rather than individual symbols increases the efficiency, also in quantization the simultaneous coding of a group of random variables allows a more efficient representation of the source information.

Vector Quantizer

Let $\boldsymbol{x} = (x_1, x_2, \ldots, x_L)$ be an L-dimensional random vector.
A vector quantizer is described by:

- a set of *decision regions* or *cells* $\mathcal{I}_i \in \mathbb{R}^L$, $i = 1, 2, \ldots, K$, such that

$$\mathcal{I}_i \cap \mathcal{I}_j = \emptyset \qquad \text{for} \quad i \neq j \qquad \text{and} \quad \bigcup_{i=1}^{K} \mathcal{I}_i = \mathbb{R}^L$$

  (i.e., the cells are a partition of the L-dimensional real space);

- a set of *reproduction vectors* (or *codevectors*) $\boldsymbol{y}_i \in \mathbb{R}^L$, $i = 1, 2, \ldots, K$ (this set is called *codebook*);

- the quantization rule

$$q(\boldsymbol{x}) = \boldsymbol{y}_i \qquad \text{if} \quad \boldsymbol{x} \in \mathcal{I}_i$$

Multimedia Coding: Lossy Coding

- Also for a vector quantizer, its "quality" is measured by the average distortion between the quantizer input $\boldsymbol{x}$ and the quantizer output $\boldsymbol{y} = q(\boldsymbol{x})$.
- Again, the most common distortion measure is the squared error

$$d(\boldsymbol{x}, \boldsymbol{y}) = \sum_{n=1}^{L} (x_n - y_n)^2$$

and the quality is measured by the *mean squared error* (MSE) per dimension

$$\begin{aligned}
\frac{1}{L} \mathrm{E}\Big[d(\boldsymbol{x}, \boldsymbol{y})\Big] &= \frac{1}{L} \int_{\mathbb{R}^L} \big(\boldsymbol{a} - q(\boldsymbol{a})\big)^2 f_{\boldsymbol{X}}(\boldsymbol{a}) d\boldsymbol{a} \\
&= \frac{1}{L} \sum_{i=1}^{K} \int_{\mathcal{I}_i} \big(\boldsymbol{a} - q(\boldsymbol{a})\big)^2 f_{\boldsymbol{X}}(\boldsymbol{a}) d\boldsymbol{a} \\
&= \frac{1}{L} \sum_{i=1}^{K} \int_{\mathcal{I}_i} \big(\boldsymbol{a} - \boldsymbol{y}_i\big)^2 f_{\boldsymbol{X}}(\boldsymbol{a}) d\boldsymbol{a}
\end{aligned}$$

Optimal vector quantizer

- The design of an optimal vector quantizer for a source with a given statistics, consists in finding the codebook and the partition that minimize the distortion.
- An optimal quantizer must satisfy the following conditions (which are a straightforward extension of those found for the scalar quantizer)
  - *Nearest neighbor condition*
    given the set of codevectors $\boldsymbol{y}_i \in \mathbb{R}^L$, $i = 1, 2, \dots, K$, the optimal partition of $\mathbb{R}^L$ is the one giving the minimum distortion

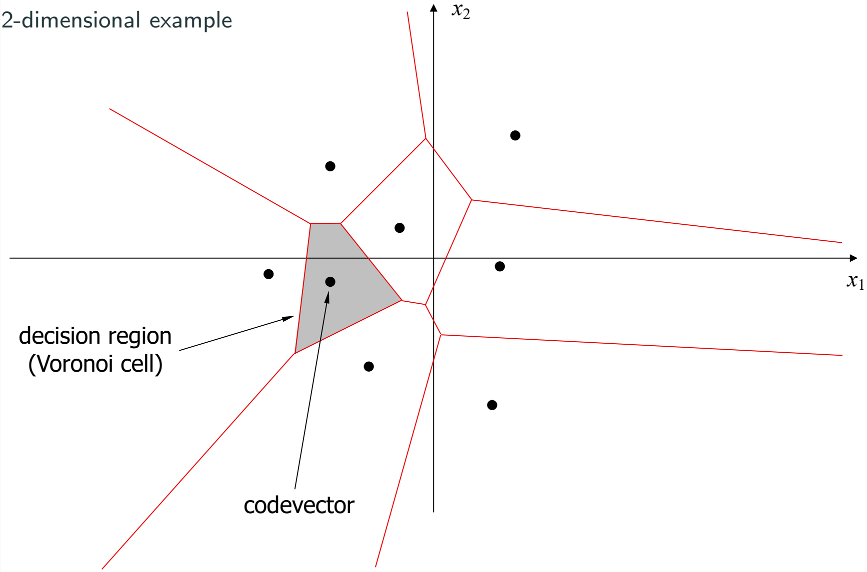
$$\mathcal{I}_i = \left\{ \boldsymbol{x} : \ d(\boldsymbol{x}, \boldsymbol{y}_i) \leq d(\boldsymbol{x}, \boldsymbol{y}_j), \ j \neq i \right\}$$

  - *Centroid condition*
    given the partition $\mathcal{I}_i$, $i = 1, 2, \dots, K$, the codevectors of the optimal codebook are the centroids of the decision regions

$$\boldsymbol{y}_i = \int_{\mathcal{I}_i} \boldsymbol{a} \, f_{\boldsymbol{X}|\boldsymbol{X} \in \mathcal{I}_i}(\boldsymbol{a} | \boldsymbol{x} \in \mathcal{I}_i) d\boldsymbol{a}$$

Vector Quantization

2-dimensional example



- As for scalar quantization, the unbounded cells are called *overload* or *saturation cells*, and their union is called *saturation* or *overload region*.
- The bounded cells are called *granular cells* and their union is called *granular region*.

- Similarly to the Lloyd-Max algorithm in case of scalar quantization, the previous optimality conditions can be used to design a vector quantizer based on the joint pdf of the input vectors.
- Typically, rather than using the joint pdf, the quantizer is designed on the basis of the data to be actually coded (or on a representative *training set*).
- The most popular algorithm for vector quantization is the Linde-Buzo-Gray (LBG) algorithm (which is based on the Lloyd - Max algorithm).

Linde-Buzo-Gray (LBG) algorithm

1. Choose an initial codebook of reconstruction vectors $\boldsymbol{y}_i$;
2. Optimize the partition for the given codebook, which gives the regions $\mathcal{I}_i$;
3. Optimize the codebook for the partition found above;
4. Repeat steps 2. and 3. until convergence is reached.

Properties of the Linde-Buzo-Gray (LBG) algorithm

- The LBG procedure guarantees that at each iteration the distortion does not increase.
- There is no guarantee that the algorithm converges to the *global* minimum of the didtortion, and it can "stuck" on a *local* minimum.
- The final codebook depends on the choice of the initial codebook.

Issues to be addressed using the Linde-Buzo-Gray (LBG) algorithm

- Choice of the initial codebook.
- "Empty cell" problem.