Design of Unknown Input Observers and Robust Fault Detection Filters

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Abstract

Fault detection filters are a special class of observers which can generate directional residuals for the purpose of fault isolation. This paper proposes a new approach to design robust (in the disturbance de-coupling sense) fault detection filters which ensure that the residual vector, generated by this filter, has both robust and directional properties. This is done by combining the unknown input observer and fault detection filter principles. The paper proposes a new full order unknown input observer, and gives necessary and sufficient conditions for its existence. After the disturbance de-coupling conditions are satisfied, the remaining design freedom can be used to make the residual have the directional property, based on the fault detection filter principle. A nonlinear jet engine system is used to illustrate the robust fault isolation approach presented. It is shown that linearization errors can be treated approximately as unknown disturbances and be de-coupled in the design of a robust fault detection filter. Simulation results show that mis-isolation of faults can be avoided using the robust scheme.

1 Introduction

The process of detecting and isolating system faults has been of considerable interest during the last two decades (Beard, 1971; Jones, 1973; Willsky, 1976; Patton, Frank and Clark, 1989; Frank, 1990; Gertler, 1993). Research is still under way into the development of more effective solutions for fault detection and isolation (FDI) in automatic control systems (Patton and Chen, 1994). A fault is defined as an unexpected change in a system, such as a component malfunction and variations in operating condition, that tend to degrade overall system performance. In the early developmental stages of fault detection techniques, most investigators used the term "failure" (Beard, 1971; Jones, 1973; Willsky, 1976). Recently, most international researchers agree that the term "fault" is more appropriate than the

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term “failure” (Patton et al., 1989; Frank, 1990; Park and Rizzoni, 1993; Patton and Chen, 1994). The term failure suggests complete breakdown, whist a fault may denote something tolerable. We use the term “fault” rather than “failure” to denote a tolerable malfunction rather than a catastrophe.

The purpose of fault detection is to determine that a fault has occurred in the system, whereas fault isolation procedures are used to determine the location of the fault, after detection. The basic principle of model-based FDI is to compare actual and anticipated system responses generated using mathematical models. The most effective model-based FDI approach is the observer-based approach (Beard, 1971; Jones, 1973; Willsky, 1976; Patton et al., 1989; Frank, 1990) in which, the difference between actual and estimated outputs is used a residual vector. This residual is zero when the system is normal and is non-zero when a fault occurs. Hence, the residual can be compared with a threshold for detection purposes. Fault isolation is more complicated than detection and one approach is to design a set of structured residual signals (Patton et al., 1989; Frank, 1990; Gertler, 1993; Patton and Chen, 1994). “Structured” means that each residual is designed to be sensitive to a certain group of faults, whilst insensitive to others. The sensitivity and insensitivity properties make isolation possible. Another approach to isolation is via the design of a directional residual vector, i.e. to make the residual vector lie in a fixed and fault-specific direction in the residual space in response to each fault. With directional residual vectors, the fault isolation can be achieved by determining which of the known fault signature directions the residual vector lies closest to.

The most effective way to generate direction residual vectors is to use the Beard fault detection filter (BFDF) (Beard, 1971; Jones, 1973; Massoumnia, 1986; White and Speyer, 1987; Park and Rizzoni, 1993). It has to be pointed that this class of observers has been referred to in the literature as “failure detection filter” (Beard, 1971; Jones, 1973; White and Speyer, 1987) in the early development of fault (failure) diagnosis. Fault detection filters are a particular class of full-order Luenberger observer with a specially designed feedback gain matrix such that the output estimation error (residual vector) has uni-directional characteristics associated with some known fault directions. To be specific, the residual vector of a fault detection filter is fixed along with a predetermined direction for an actuator fault or lies in a specific plane for a sensor fault. Since the important information require for isolation is contained in the direction of the residual rather than in its time function, the use of a BFDF does not require the knowledge of the fault mode. The fault isolation task can be facilitated by comparing the residual direction with pre-defined fault signature directions (or planes), and only one (or the minimum number of) observer(s) required for fault isolation due to directional characteristics of the residual. This is the main and most appealing feature of fault detection filters.

Note that the robustness issues have not been considered in the context of Beard fault detection filters up to now. Hence, this approach does not account for the effects of disturbances, non-linearities, modelling errors, parameter variations and other uncertain factors in the system. There would be false or missed alarms when this approach is directly applied to industrial systems, in which the uncertain factors are unavoidable in modelling (specially for systems such as mechanical, eletromechanical, thermofluid and aircraft systems). The application of BFDFs has been obstructed by the lack of robustness. Robustness in FDI is defined as the degree to which the FDI performance is unaffected by (or remains insensitive to) uncertainties of the system. An ideal (robust) FDI scheme is such a scheme which is sensitive to faults, whilst insensitive to system uncertainties.

One of the most successful robust fault diagnosis approaches is the use of the disturbance de-coupling principle, in which the residual is designed to be insensitive to unknown disturbances, whilst sensitive to faults. This can be done by using unknown input observers (Watanabe and Himmelblau, 1982; Frank and Wünnenberg, 1989; Chen and Zhang, 1991; Patton and Chen, 1993b), by eigenstructure assignment (Patton, Willcox and Winter, 1987; Patton and Chen, 1992; Patton and Chen, 1993a) or using structured parity equations (Gertler, 1993; Gertler and Kunwer, 1993). Using the disturbance de-coupling approach, the robust FDI with respect to unknown inputs (disturbances) can be solved. However, the robustness problem with respect to modelling errors and other uncertain factors is more difficult. To solve this problem, some investigators have suggested an approach in which modelling errors are represented approximately as unknown disturbances with an estimated distribution matrix (Patton and Chen, 1992; Patton and Chen, 1993a; Patton, Chen and Zhang, 1992; Gertler and Kunwer, 1993; Gertler, 1994). In this way, an optimally robust solution is achievable. This approximate strategy has extended the application domain of disturbance de-coupling-based robust FDI approaches which have been favoured recently in FDI research.

This paper proposes a new strategy to design robust fault detection filters whose residuals have both disturbance de-coupling and uni-directional properties. This filter is designed by combing the unknown input observer (UIO) principle with the Beard fault detection filter principle. Using UIO principles, the residual has been made robust against unknown inputs (disturbances). The uni-directional property is achieved based on BFDF techniques. It should be pointed that there are two approaches to the robust detection filter design. The detection filter can be designed first and then the design freedom can be used to reject disturbances. Alternatively, the UIO can be designed first, the freedom is then used to make the fault signature have uni-directional properties. The choice depends on the design emphasis and the availability of the design procedure. This paper is based upon the second approach.

The paper is organized as follows. In Section 2, a new full-order UIO structure is proposed. A rigourous mathematical foundation in designing such an UIO has been laid down and the necessary and sufficient existence conditions for such an UIO are presented and thoroughly proved. The existence conditions are easy to verify and the design procedure is very simple. There is still some design freedom left after the disturbance (unknown input) de-coupling conditions have been satisfied. Section 3 studies how to exploit the remaining design freedom to make the residual have uni-directional characteristics. This section discusses first the basic principles of the Beard fault detection filters. These principles are then used to make the residual, generated by an UIO, take on the required uni-directional properties. A filter which can produce disturbance de-coupled and directional residuals is called a "disturbance de-coupled (robust) fault detection filter". Section 3 also presents a robust fault isolation scheme using robust directional residual vectors. In Section 4, a real example of the isolation of faulty sensors in a jet engine system is presented. This is a nonlinear system and the linearization error can cause mis-isolation problems if the robustness issue is not properly considered. The paper presents a way to represent linearization errors as an unknown input term and its distribution is estimated using a least-squares procedure. This is a new and effective way to tackle the robustness problem. The paper describes the design of a robust fault detection filter for isolating faults in a jet engine system and the simulation results show that faults are correctly isolated.

2 Theory and design of unknown input observers

In this paper, we consider a class of systems, in which the system uncertainty can be summarized as an *additive unknown disturbance* term in the dynamic equation described as follows:

$$\left\{ \begin{array}{lcl} \dot{x}(t) & = & Ax(t) + Bu(t) + Ed(t) \\ y(t) & = & Cx(t) \end{array} \right. \tag{1}$$

where $x(t) \in \mathcal{R}^n$ is the state vector, $y(t) \in \mathcal{R}^m$ is the output vector, $u(t) \in \mathcal{R}^r$ is the known input vector and $d(t) \in \mathcal{R}^q$ is the unknown input (or disturbance) vector. A, B, C and E are known matrices with appropriate dimensions. There is no loss of generality in assuming that the known unknown input distribution matrix E is a full column rank matrix. Otherwise, the following rank decomposition can be applied to the matrix E:

$$Ed(t) = E_1 E_2 d(t)$$

where E_1 is a full column rank matrix and $E_2 d(t)$ can now be considered as a new unknown input. The disturbance term may also appear in the output equation, however this case is not considered here because the disturbance term in the output equation can be nulled by simply using a transformation of the output signal $y(t)$.

The term $Ed(t)$ can be used to describe additive disturbance as well as a number of different kinds of modelling uncertainties. For example, noise, interconnecting terms in large scale systems, nonlinear terms in system dynamics (Watanabe and Himmelblau, 1982; Chen and Zhang, 1991), terms arising from time-varying system dynamics, linearization and model reduction errors, parameter variations can all be represented in this way. A detailed study of this problem can be found in (Patton and Chen, 1993b; Patton and Chen, 1992; Patton and Chen, 1993a; Patton et al., 1992; Gertler and Kunwer, 1993). In the following, two examples in the representation of modelling errors as additive disturbance term $Ed(t)$ are given.

The first example considers that the system dynamics with parameter perturbations represented as:

$$\dot{x}(t) = (A + \Delta A)x(t) + (B + \Delta B)u(t)$$

The parameter perturbations sometimes considered in the robust control literature are approximated as:

$$\Delta A \approx \sum_{i=1}^{N} a_i A_i \qquad \Delta B \approx \sum_{i=1}^{N} b_i B_i$$

where A_i and B_i are known matrices with proper dimensions, a_i and b_i are scalar factors. In this case, the modelling errors can be approximated by the disturbance term as:

$$\begin{aligned} Ed(t) &= \Delta A x(t) + \Delta B u(t) \\ &= [A_1 \; \cdots \; A_N \; B_1 \; \cdots \; B_N] \begin{bmatrix} a_1 x(t) \\ \vdots \\ a_N x(t) \\ b_1 u(t) \\ \vdots \\ b_N u(t) \end{bmatrix} \end{aligned}$$

Now, to consider the second example in which the system matrices are functions of a parameter vector $\alpha \in \mathcal{R}^g$:

$$\dot{x}(t) = A(\alpha)x(t) + B(\alpha)u(t)$$

If the parameter has a perturbation around the nominal condition $\alpha = \alpha_0$, this equation can be expanded as:

$$\begin{aligned} \dot{x}(t) &= A(\alpha_0)x(t) + B(\alpha_0)u(t) \\ &+ \sum_{i=1}^{g}\{\frac{\partial A}{\partial \alpha_i}\delta\alpha_i x(t) + \frac{\partial B}{\partial \alpha_i}\delta\alpha_i u(t)\} \end{aligned}$$

In this case, the distribution matrix and unknown disturbance vector are expressed as:

$$E = [\frac{\partial A}{\partial \alpha_1} \mid \frac{\partial B}{\partial \alpha_1} \mid \cdots \mid \frac{\partial A}{\partial \alpha_g} \mid \frac{\partial B}{\partial \alpha_g}]$$

$$d(t) = [\delta\alpha_1 x(t)^T \mid \delta\alpha_1 u(t)^T \mid \cdots \mid \delta\alpha_g x(t)^T \mid \delta\alpha_g u(t)^T]^T$$

Definition 1: {Unknown Input Observer (UIO)} An observer is defined as an *unknown input observer* for the system (1), if its state estimation error vector $e(t)$ approaches zero asymptotically regardless of the presence of the unknown input (disturbance) in the system.

The problem of designing an observer for a linear system with both known and unknown inputs has been studied for nearly two decades (Wang, Davison and Dorato, 1975). The problem is of considerable importance since in practice there are many situations where there are disturbances present, or when some of the system inputs are inaccessible (or unmeasurable), and therefore a conventional observer which uses all input signals cannot be used. It is more useful to assume no *a priori* knowledge about unknown inputs. Wang et al. (1975) proposed a minimal-order for the system (1). The existence conditions for such an $(n-m)th$-order observer were shown by Kudva, Viswanadham and Ramakrishna (1980). After the work of Wang *et al*, many approaches for designing unknown input observers have been proposed, for example, the geometric method by Bhattacharyya (1978), the inversion algorithm by Kobayashi and Nakamizo (1982), the matrix algebra method by Watanabe and Himmelblau (1982), the generalized matrix inversion approach by Miller and Mukundan (1982), and the singular value decomposition technique by Fairman, Mahil and Luk (1984). Park and Stein (1988) studied the simultaneous estimation problem for both states and unknown input observers. More recently, the problem of designing reduced order unknown input observers has been revisited by Hou and Müller (1992) and Guan and Saif (1991) using algebraic approaches. In their studies, the state vector is divided into two parts, via a linear transformation onto the state equation, a part can be directly obtained from the measurements, and another part has to be estimated using a reduced order disturbance de-coupled observer.

Unlike all above mentioned work in which the reduced order observer structure has been used, Kurek (1982) proposed a full order UIO structure. Yang and Richard (1988) gave a direct design procedure of the full order UIO for system (1) and have shown, through an example, that the reduced-order observer may restrict the convergence rate in estimation. However, the design procedure they presented is very complicated and involves some trial-and-error excercises, and the existence conditions are not very easy to verify. It has been shown that the minimal order of an UIO is $(n-m)$, any order between $(n-m)$ to n is possible for an UIO to exist. The disturbance de-coupling conditions for a full-order UIO is not very different from that for a reduced-order UIO. That is to say, there are no significant differences between full-order and reduced-order UIOs, as far as unknown input (disturbance) de-coupling is concerned. However, there is more design freedom available for a full-order UIO to achieve other required performances such as the rate of convergence and minimum variance. This is easy to understand, since the number of free parameters will increase if the observer order is increased.

In this study, a full order UIO structure which is similar to the one in Kurek (1982) is used because extra design freedom is required for generating directional residuals in fault isolation. A rigourous mathematical foundation in designing a full order UIO is presented. The necessary and sufficient conditions for this observer to exist are given and thoroughly proved in this paper. These conditions are easy to verify, the design procedure is systematic and straightforward to implement. Moreover, one of the contributions of this paper is to show that the remaining freedom can be used to make the residual have directional properties, after unknown input (or disturbance) de-coupling has been achieved.

The structure for a full order observer is described as:

$$\left\{ \begin{array}{rcl} \dot{z}(t) & = & Fz(t) + TBu(t) + Ky(t) \\ \hat{x}(t) & = & z(t) + Hy(t) \end{array} \right. \tag{2}$$

where $\hat{x} \in \mathcal{R}^n$ is the estimated state vector and $z \in \mathcal{R}^n$ is the state of this full order observer, and F, T, K, H are matrices to be designed for achieving unknown input de-coupling and other design requirements. The observer described by Eq.(2) is illustrated in Fig.1.

When the observer (2) is applied to the system (1), the estimation error $(e(t) = x(t) - \hat{x}(t))$ is governed by the following equation:

$$\begin{array}{rcl} \dot{e} & = & (A - HCA - K_1C)e + [F - (A - HCA - K_1C)]z \\ & + & [K_2 - (A - HCA - K_1C)H]y \\ & + & [T - (I - HC)]Bu + (HC - I)Ed \end{array} \tag{3}$$

where

$$K = K_1 + K_2 \tag{4}$$

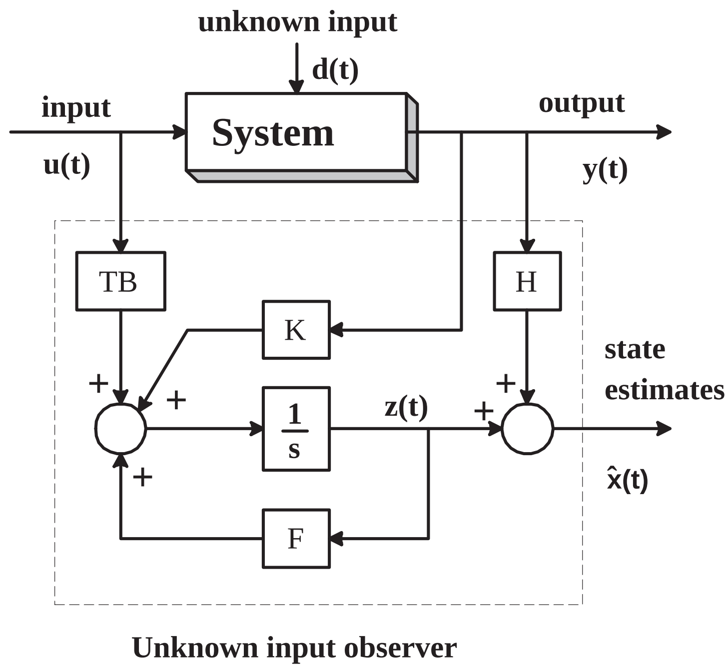

Figure 1: **The structure of a full order unknown input observer**

If one can make the following relations hold true:

$$
\begin{aligned}
(HC - I)E &= 0 && (5)\\
T &= I - HC && (6)\\
F &= A - HCA - K_1C && (7)\\
K_2 &= FH && (8)\\
& && (9)
\end{aligned}
$$

The state estimation error will then be:

$$\dot{e}(t) = Fe(t) \tag{10}$$

If all eigenvalues of F are stable, $e(t)$ will approach zero asymptotically, i.e. $\hat{x} \longrightarrow x$. This means that the observer (2) is an unknown input observer for the system (1) according to the definition 1. The design of this UIO involves solving Eqs. (4) – (8) and ensuring that all eigenvalues of the system matrix F are stable. Before we give the necessary and sufficient conditions for existence of an UIO, two Lemmas are introduced.

Lemma 1: Eq.(5) is solvable *iff*:

$$rank(CE) = rank(E) \tag{11}$$

and a special solution is:

$$H^* = E[(CE)^T CE]^{-1}(CE)^T \tag{12}$$

Proof: See Appendix A.

Lemma 2: Let:

$$C_1 = \begin{bmatrix} C \\ CA \end{bmatrix}$$

then the detectability for the pair (C_1, A) is equivalent to that for the pair (C, A).

Proof: see Appendix B.

Note that the detectability (Chen, 1984) is a weaker condition than observability. A pair (C, A) is detectable when all unobservable modes for this pair are stable.

Theorem 1: Necessary and sufficient conditions for (2) to be an UIO for the system defined by (1) are:

(i) $rank(CE) = rank(E)$

(ii) (C, A_1) is detectable pair, where

$$A_1 = A - E[(CE)^T CE]^{-1}(CE)^T CA \tag{13}$$

Proof: Sufficiency: According to Lemma 1, the Eq. (5) is solvable when condition (i) holds true. A special solution for H is $H^* = E[(CE)^T CE]^{-1}(CE)^T$. In this case, the system dynamics matrix is:

$$F = A - HCA - K_1 C = A_1 - K_1 C$$

which can be stabilized by selecting the gain matrix K_1 due to the condition (ii). Finally, the remaining matrices of observer described in (2) can be calculated using Eqs.(4) – (8). Thus, the observer (2) is an UIO for the system (1).

Necessity: Since (2) is an UIO for (1), Eq. (5) is solvable. This leads to the fact that condition (i) holds true according to Lemma 1. The general solution for the matrix H for Eq.(5) can be calculated as:

$$H = E(CE)^* + H_0[I_m - CE(CE)^*]$$

where $H_0 \in \mathcal{R}^{n\times m}$ an arbitrary matrix and $(CE)^*$ is the left inverse of CE which is:

$$(CE)^* = [(CE)^T CE]^{-1}(CE)^T$$

Substituting the solution for H into Eq.(7), the system dynamics matrix F is:

$$\begin{aligned} F &= A - HCA - K_1 C \\ &= [I_n - E(CE)^* C]A - [K_1 \quad H_0]\begin{bmatrix} C \\ [I_m - CE(CE)^*]CA \end{bmatrix} \\ &= A_1 - [K_1 \quad H_0]\begin{bmatrix} C \\ CA_1 \end{bmatrix} \\ &= A_1 - \overline{K}_1 \overline{C}_1 \end{aligned}$$

where

$$\overline{K}_1 = [K_1 \quad H_0] \quad \text{and} \quad \overline{C}_1 = \begin{bmatrix} C \\ CA_1 \end{bmatrix}$$

Since the matrix F is stable, the pair $(\overline{C}_1, A_1)$ is detectable, and the pair (C, A_1) also is detectable according to Lemma 2.

$\diamond$ **QED**.

It can be noted that the number of independent row of the matrix C must not be less than the number of the independent columns of the matrix E to satisfy condition (i). That is to say, the maximum number of disturbances that can be de-coupled cannot be larger than the number of the independent measurements. Condition (ii) can be verified in terms of the structural properties of the original system. In fact, this condition is equivalent to the condition that the transmission zeros from the unknown inputs to the measurements must be stable, i.e.

$$\begin{bmatrix} sI_n - A & E \\ C & 0 \end{bmatrix}$$

is of full column rank for all s with $Re(s) \geq 0$. This can be proved as follows:

It can be verified that:

$$\begin{bmatrix} I_n - E(CE)^*C & sE(CE)^* \\ 0 & I_m \\ E(CE)^*C & -sE(CE)^* \end{bmatrix} \begin{bmatrix} sI_n - A & E \\ C & 0 \end{bmatrix} = \begin{bmatrix} sI_n - A_1 & 0 \\ C & 0 \\ -E(CE)^*CA & E \end{bmatrix}$$

As the first matrix in the left side of the above equation is a full column rank matrix, we have:

$$rank \begin{bmatrix} sI_n - A & E \\ C & 0 \end{bmatrix} = rank \begin{bmatrix} sI_n - A_1 & 0 \\ C & 0 \\ -E(CE)^*CA & E \end{bmatrix} = rank \begin{bmatrix} sI_n - A_1 \\ C \\ -E(CE)^*CA \end{bmatrix} + rank(E)$$

We have assumed that E is a full column rank matrix. Hence, condition (ii) is equivalent to the case when the matrix of the left side of the above equation is full column rank for all s with $Re(s) \geq 0$. This is because the condition for pair (C, A_1) to be detectable is equivalent to the following matrix

$$\begin{bmatrix} sI - A_1 \\ C \end{bmatrix}$$

haveing full column rank for all s with $Re(s) \geq 0$.

From the above analysis, it can be seen that K_1 is a free matrix of parameters in the design of an UIO. After K_1 is determined, other parameter matrices in the UIO can be computed by Eqs.(4) – (8). The only restriction on the matrix K_1 is that it must stabilize the system dynamics matrix F. The matrix K_1 which stabilizes the matrix F is not unique due to the multivariable nature of the problem. That is to say that there is still some design freedom left in the choice of K_1, after unknown input disturbance conditions have been satisfied. In Section 3, this freedom is exploited to make the diagnostic residual have directional properties which can be utilized to fulfil the fault isolation task.

3 Disturbance de-coupled fault detection filters and robust fault isolation

3.1 Basic principles of fault detection filters

The Beard fault detection filter was first developed by Beard (1971) using a matrix algebra approach and later reformed by Jones (1973) in a vector space notation. The theory of BFDFs has been extended by many researchers, for example, Massoumnia (1986) used a geometric interpretation, White and Speyer (1987) improved the design procedure using a spectral approach which is suitable for the isolation of multiple faults, and more recently Park and Rizzoni (1993) developed a closed-form expression of BFDFs using eigenstructure assignment.

In order to describe the BFDF theory, let us consider a system without disturbances in the state space format as:

$$\begin{cases} \dot{x}(t) &= Ax(t) + Bu(t) + b_i\xi_i(t) \\ y(t) &= Cx(t) + I_j\eta_j(t) \end{cases} \tag{14}$$

The term $b_i\xi_i(t)$ $(i = 1, 2, \cdots, r)$ denotes that a fault occurs in the ith actuator, $b_i \in \mathcal{R}^n$ is the ith column of the input matrix B and is defined as the fault event vector of the ith actuator fault, and $\xi_i(t)$ is an unknown scalar time-varying function which represents the evolution of the fault. The term $I_j\eta_j(t)$ $(j = 1, 2, \cdots, m)$ denotes that a fault occurs in the jth sensor, $I_j \in \mathcal{R}^m$ is a unit vector corresponding to a fault with the jth sensor. Note that component faults appear in the system equation in the same way as the actuator fault and hence are not discussed further here.

A BFDF is just a full order observer and its structure and the residual can be described as:

$$\begin{cases} \dot{\hat{x}}(t) &= A\hat{x}(t) + Bu(t) + K(y(t) - C\hat{x}(t)) \\ r(t) &= y(t) - C\hat{x}(t) \end{cases} \tag{15}$$

where $r \in \mathcal{R}^m$ is the residual vector, $\hat{x} \in \mathcal{R}^n$ is the state estimation, and $K \in \mathcal{R}^{m\times n}$ is the observer gain matrix which has to be specially designed to make the residual have restricted uni-directional properties in the presence of a particular fault. If the state estimation error is defined as: $e(t) = x(t) - \hat{x}(t)$, the residual and $e(t)$ will be governed by the following error system, when a fault occurs in the ith actuator:

$$\begin{cases} \dot{e}(t) &= (A - KC)e(t) + b_i\xi_i(t) \\ r(t) &= Ce(t) \end{cases} \tag{16}$$

When a fault occurs in the jth sensor, the error system will be:

$$\begin{cases} \dot{e}(t) &= (A - KC)e(t) - k_j\eta_j(t) \\ r(t) &= Ce(t) + I_j\eta_j(t) \end{cases} \tag{17}$$

where k_j is the jth column of the detection filter gain matrix.

The task of BFDF design is to make $Ce(t)$ have a *fixed direction* in the output space responding to either $b_i\xi_i(t)$ or $k_j\eta_j(t)$. Both actuator and sensor fault situations can be considered in the following general error system equation:

$$\begin{cases} \dot{e}(t) & = & (A - KC)e(t) + l_i\eta_i(t) \\ r(t) & = & Ce(t) \end{cases} \tag{18}$$

where $l_i \in \mathcal{R}^n$ is called the *fault event direction*. The definition of the isolability of a fault with known direction l_i is given by Beard (1971) as stated below:

Definition 2: {Isolability of a fault with a given direction}: The fault associated with l_i in the system described by Eq.(18) is *isolable* if there exists a filter gain matrix K such that:

(a) $r(t)$ maintains a fixed direction in the output space, and

(b) $(A - KC)$ can be stabilized.

Condition (a) guarantees that the residual has uni-directional characteristics. This condition is equivalent to the rank of the controllability matrix of (A, l_i) pair is one, i.e:

$$rank[l_i \;\; (A - KC)l_i \;\; \cdots \;\; (A - KC)^{n-1}l_i] = 1$$

Condition (b) ensures the convergency of the filter. In the original definition of Beard (1971), condition (b) requires that the eigenvalues of $(A - KC)$ can be arbitarily assigned. This condition has been modified as the stability requirement is sufficient if the residual response time does not need to specified. This definition was referred to as “fault detectability” by Beard (1971) and others. We feel that the “isolability” is more appropriate, because the directional property of residuals is especially desirable for fault isolation purposes, although it can also be used for fault detection. The BFDF is designed to satisfy fault isolability.

A filter (or an observer) is called as a BFDF if its residual has uni-directional properties. If a fault associated with the direction b_i is isolable, the residual of the BFDF will be fixed in the direction parallel to Cb_i when a fault occurs in the ith actuator. Similarly, the residual will lie somewhere in the plane defined by Ck_j and I_j when a fault occurs in the jth sensor.

In order to isolate faults associated with p isolable fault event directions l_i $(i = 1, 2, \cdots, p)$, the following output separability condition (Beard, 1971) must be satisfied.

Definition 3: {Output Separability of Faults}: The faults associated with p fault event directions l_i $(i = 1, 2, \cdots, p)$ are separable in the residual space if the vectors $Cl_1, Cl_2, \cdots, Cl_p$ are linearly independent.

Output separability is necessary for a group of faults to be isolated in the residual space according to their signature directions. The directions Cl_i $(i = 1, 2, \cdots, p)$ are called the *fault signature directions* in the residual space.

Definition 4: {Mutual Isolability}: The faults associated with the fault event vectors l_i $(i = 1, 2, \cdots, p)$ are *mutually isolable* if there exists a filter gain matrix K which satisfies the isolability conditions of Definition 2 for all l_i $(i = 1, 2, \cdots, p)$, i.e.

$$rank[l_i \;\; (A - KC)l_i \;\; \cdots \;\; (A - KC)^{n-1}l_i] = 1 \qquad \text{for all} \;\; i = 1, 2, \cdots, p$$

A group of mutually isolable faults can be isolated using the residual generated by a single BFDF by comparing the residual direction with fault signature directions, when there are no simultaneous faults. The condition for mutual isolability can be found in references (Beard, 1971; Jones, 1973; White and Speyer, 1987). If a group of faults is not mutually isolable, it can be divided into a number of subgroups and each subgroup is mutually isolable. For such cases, a few fault detection filters are required to fulfil the fault isolation task. In any case, only a minimum number of filters are required for fault isolation. This is the most important and appealing advantage of the fault detection filter approaches.

In conclusion, the task of designing a fault detection filter is to make the residual have an uni-directional property by choosing the gain matrix K. Design techniques for this can be found in the classical literature on fault detection filters (Beard, 1971; Jones, 1973; White and Speyer, 1987).

3.2 Disturbance de-coupled fault detection filters and robust fault isolation

It can be seen that uncertain factors associated with a dynamical system, such as disturbances and modelling errors have not be considered in the design of BFDFs. This is the main disadvantages of BFDFs, because uncertain factors are unavoidable in real systems and any fault detection and isolation scheme has to be made robust against disturbances and modelling errors. Now, consider a system with disturbance term $Ed(t)$ and possible sensor and actuator faults described as:

$$\left\{ \begin{array}{lcl} \dot{x}(t) & = & Ax(t) + Bu(t) + Ed(t) + b_i\xi_i(t) \\ y(t) & = & Cx(t) + I_j\eta_j(t) \end{array} \right. \tag{19}$$

If a standard BFDF described in Eq.(15) is applied to such system, the state estimation error and residual will be:

$$\left\{ \begin{array}{lcl} \dot{e}(t) & = & (A - KC)e(t) + Ed(t) + b_i\xi_i(t) - k_j\eta_j(t) \\ r(t) & = & Ce(t) + I_j\eta_j(t) \end{array} \right. \tag{20}$$

It is clear from Eq.(20) that all faults and disturbances affect on the residual. It is not easy to discriminate between faults and disturbances if this residual is used to detect and isolate faults. Hence, it is necessary to de-couple disturbance effects from the residual for reliable diagnosis.

We have show that the disturbance can be de-coupled from the state estimation error using an unknown input observer. This inspires us to generate the residual using an UIO described in Eq.(2), the residual is thus defined as:

$$r(t) = y(t) - C\hat{x}(t) = (I - CH)y(t) - Cz(t) \tag{21}$$

When this UIO-based residual generator is applied to the system described by the model of Eq.(20), the residual and the state estimation error ($e(t)$) will be:

$$\left\{ \begin{array}{lcl} \dot{e}(t) & = & (A_1 - K_1C)e(t) + Tb_i\xi_i(t) \\ r(t) & = & Ce(t) \end{array} \right. \tag{22}$$

when a fault occurs in the ith actuator.

Similarly,

$$\left\{ \begin{array}{rcl} \dot{e}(t) & = & (A_1 - K_1 C)e(t) - k_{1j}\eta_j(t) - h_j\dot{\eta}_j(t) \\ r(t) & = & Ce(t) + I_j\eta_j(t) \end{array} \right. \tag{23}$$

when a fault occurs in the jth sensor. Where k_{1j} is the jth column of the matrix K_1 and h_j is the jth column of the matrix H. From Eq.(22) & (23), it can be seen that the disturbance effects have been de-coupled from the residual. This robust (in the disturbance de-coupling sense) residual can be used to detect faults according to a simple threshold logic:

$$\left\{ \begin{array}{lllll} \|r(t)\| & < & Threshold & \text{for} & \text{no fault cases} \\ \|r(t)\| & \geq & Threshold & \text{for} & \text{faulty cases} \end{array} \right. \tag{24}$$

As pointed out in the introduction, fault isolation can be facilitated using uni-directional residual vectors. So, we have to make the residual generated by an UIO, have the directional properties in order to achieve robust fault isolation. From the design of UIOs, we know that the matrix K_1 can be designed arbitraily after the robust (in the sense of disturbance de-coupling) conditions have been satisfied. This design freedom can be exploited to make the residual have the uni-directional property.

Comparing the error system Eq. (22) with the Eq. (16), it can be seen that the actuator fault is expressed in the same way for an UIO or a standard BFDF. Hence, the theory for the design a of BFDF (Beard, 1971; Jones, 1973; White and Speyer, 1987) can be directly used to design the matrix K_1, if the vector b_i is replaced by Tb_i and the matrix A is replace by A_1.

Comparing the error system Eq. (23) with the Eq. (17), it can be seen the sensor fault is also expressed in a similar way for *both* BFDF and UIO, except an extra term $h_j\dot{\eta}_j(t)$ occurs in the error equation of the UIO. Fortunately, this term can be treated in the same way as an actuator fault. Hence, the theory of BFDF can be adopted for the design of K_1 in the sensor isolation problem. However, it must be pointed out that the residual will lie in a subspace spanned by vectors I_j, Ck_{1j} and Ch_j when the residual uni-directional property has been satisfied. For constant sensor faults, the term $h_j\dot{\eta}_j(t)$ will disappear from the error system and the residual will lie in the plane spanned by the vectors I_j and Ck_{1j}, this is same as the BFDF.

It should be pointed out that the maximum number of faults which can be mutually isolated using an observer developed in this paper could be less than the numbers which can be isolated by a standard BFDF. This is because some of design freedom has been used to achieve robustness. Hence, the number of detection filters needed to isolate the same group of faults could be increased. However, this price is worth paying if the robustness is an essential property. In fact, this is the normal case for real applications.

To combine the theory of UIOs with the theory of BFDFs, the design procedurefor a robust (disturbance de-coupled) fault detection filter is summarised as follows:

- Compute matrices H and T using Eqs. (12) & (6), to satisfy disturbance de-coupling conditions.

- Compute A_1 using Eq.(13).
- Compute K_1 to satisfy the uni-direction property using the theory of Beard fault detection filters.
- Compute the observer gain matrix K using Eqs.(8) & (4).

The key step is then to design the matrix K_1; once it is available, the computation of other matrices is very straightforward. The design procedure of BFDFs can be found in the literature (Beard, 1971; Jones, 1973; White and Speyer, 1987) and is not presented in this paper. To show the basic idea, we discuss an ideal situation, in which the number of independent measurements is equal to the number of states, i.e. $rank(C) = n$. In this situation, all eigenvalues of the matrix $A_1 - K_1 C$ can be assigned to the same value $\sigma > 0$, i.e.,

$$A_1 - K_1 C = -\sigma I$$

This can be achieved by setting K_1 as:

$$K_1 = (A_1 + \sigma I)C^* \tag{25}$$

where C^* is the pseudo-inverse of C. For this design, the residual will be:

$$\begin{aligned}
r(t) &= Ce(t) + I_j\eta(t) \\
&= I_j\eta(t) + Ce^{-\sigma(t-t_0)}e(t_0) \\
&\quad + C\int_{t_0}^{t} e^{-\sigma I(t-\tau)}[Tb_i\xi(\tau) - k_{1j}\eta(t) - h_j\dot{\eta}(t)]d\tau \\
&= Ce^{-\sigma(t-t_0)}e(t_0) + CTb_i\int_{t_0}^{t} e^{-\sigma(t-\tau)}\xi(\tau)d\tau \\
&\quad + I_j\eta(t) - Ck_{1j}\int_{t_0}^{t} e^{-\sigma(t-\tau)}\eta(\tau)d\tau - Ch_j\int_{t_0}^{t} e^{-\sigma(t-\tau)}\dot{\eta}(\tau)d\tau \\
&= Ce^{-\sigma(t-t_0)}e(t_0) + CTb_i\alpha(t,t_0) \\
&\quad + I_j\eta(t) + Ck_{1j}\beta(t,t_0) + Ch_j\gamma(t,t_0)
\end{aligned}$$

Clearly, the residual is parallel to CTb_i after the transient has settled down, following a fault in the ith actuator. Similarly, the residual will lie in the subspace spanned by vectors I_j, Ck_{1j} and Ch_j, when a fault occurs in the jth sensor.

Due to the residual directional property, the fault can be isolated by comparing the residual direction with the fault signature directions (or subspaces).

Definition 5: The direction of CTb_i is called as the *signature direction* of the ith actuator fault.

The directional relationship between two vectors CTb_i and $r(t)$ can be quantified by the correlation parameter $CORR_i$:

$$CORR_i(t) = \frac{|(CTb_i)^T r(t)|}{\|CTb_i\|_2 \|r(t)\|_2} \tag{26}$$

If $CORR_j > CORR_k$, the fault is more likely in the jth actuator rather than in the kth actuator.

Definition 6: The *signature subspace* of the jth sensor fault is defined as:

$$R_j = Span\{I_j, Ck_{1j}, Ch_j\} \tag{27}$$

The relationship between the vector $r(t)$ with the subspace R_j can be measured by the relationship between the vector $r(t)$ with its projection $r_j^*(t)$ in the subspace R_j. This is quantified by:

$$CORR_j(t) = \frac{|(r_j^*)^T r(t)|}{\|r_j^*\|_2 \|r(t)\|_2} \tag{28}$$

where the projection $r_j^*(t)$ of $r(t)$ in R_j is:

$$r_j^*(t) = \Phi_j(\Phi_j^T \Phi_j)^{-1} \Phi_j^T r(t) \tag{29}$$

where

$$\Phi_j = [I_j \ Ck_{1j} \ Ch_j]$$

If $CORR_j > CORR_k$, the fault is more likely in the jth sensor rather than in the kth sensor. The relationship between a residual vector with the signature subspace can also be judged by the *normalized projection distance* which is defined as:

$$NPD_j = \frac{\|r(t) - r_j^*(t)\|_2}{\|r(t)\|_2} \tag{30}$$

when NPD_{j^*} is the smallest one amongst all NPD_j $(j = 1, 2, \cdots, m)$, we will declare that the j^*th sensor is faulty. The idea of fault isolation by comparing the residual direction with the signature subspace is shown in Fig.2.

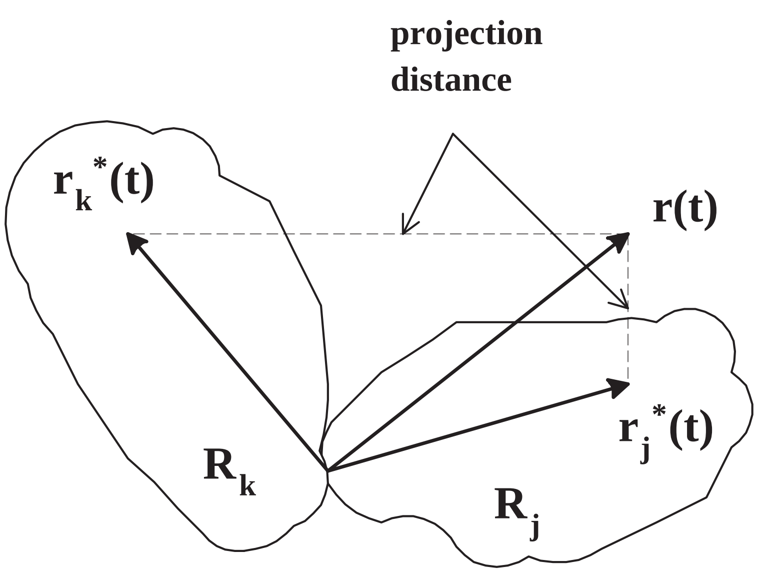

Figure 2: **Fault isolation based on directional residuals**

4 Robust isolation of faulty sensors in a jet engine system

To control a jet engine efficiently and to monitor a jet engine effectively, the sensors have to be perform reliably. However, the sensors in a jet engine work in a very harsh environment and could fail during normal engine operation. Hence, the detection of faulty sensors in jet engine systems is very important and has become an active research field (Merrill, 1989; Merrill, 1990; Meserole, 1981; Patton and Chen, 1992). A simplified nonlinear dynamic model of a jet engine control system can be described as:

$$\begin{cases} \dot{X}_1(t) &= f_1(X_1, X_2, X_3) \\ \dot{X}_2(t) &= f_2(X_1, X_2, X_3) \\ \dot{X}_3(t) &= 10(U - X_3) \end{cases} \tag{31}$$

where:

$$\begin{array}{llll} X_1 &= n_L & \longmapsto & \text{Low pressure rotor speed} \\ X_2 &= n_H & \longmapsto & \text{High pressure rotor speed} \\ X_3 &= W_f & \longmapsto & \text{Main burner fuel flow} \\ U &= W_{fe} & \longmapsto & \text{Fuel flow command} \end{array}$$

The jet engine is a very complicated nonlinear dynamic system. The nonlinear functions such as $f_1(X_1, X_2, X_3)$ and $f_2(X_1, X_2, X_3)$ cannot be written out *analytically*. The system behaviour is normally expressed in a nonlinear dynamic simulation package (Merrill and Leininger, 1981; Merrill, 1990; Meserole, 1981). This package is capable of simulating the entire operating envelope of the engine, and also can generate linearized model for any operating points. Let us define the following non-dimensional variables:

$$x_1 = \frac{X_1 - X_1^0}{X_1^0} \quad ; \quad x_2 = \frac{X_2 - X_2^0}{X_2^0}$$

$$x_3 = \frac{X_3 - X_3^0}{X_3^0} \quad ; \quad u = \frac{U - U^0}{U^0}$$

where superscript "0" denotes the values at equilibrium. The system can be linearized around an operating point. If u is small (e.g. 1%), x_1, x_2 and x_3 will be small, i.e. all variables have a small variation around the equilibrium and the following linear model is derived:

$$\begin{cases} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \end{cases}$$

where the state is $x = [x_1 \ x_2 \ x_3]^T$ and the measurement vector is:

$$y = [x_1 \quad x_2 \quad x_3 \quad p_2 \quad p_4 \quad t_4]^T$$

in which

$$p_2 = \frac{P_2 - P_2^0}{P_2^0} \ ; \ p_4 = \frac{P_4 - P_4^0}{P_4^0} \ ; \ t_4 = \frac{T_4 - T_4^0}{T_4^0}$$

where

$$\begin{array}{lll} P_2 & \longmapsto & \text{High pressure compressor discharge pressure} \\ P_4 & \longmapsto & \text{Turbine discharge pressure} \\ T_4 & \longmapsto & \text{Turbine exit temperature} \end{array}$$

When the equilibrium is set at $n_L = 450(rpm)$, the linear model matrices are:

$$A = \begin{bmatrix} -1.5581 & 0.6925 & 0.3974 \\ 0.2619 & -2.2228 & 0.2238 \\ 0 & 0 & -10 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.55107 & 0.13320 & 0.30603 \\ 0.55217 & 0.13526 & 0.32912 \\ -0.25693 & -0.23625 & 0.61299 \end{bmatrix}$$

A BFDF described by Eq.(15) is designed to isolate sensor faults. If all eigenvalues of the filter are set as -3, the gain matrix can be determined as $K = (3I + A)C^*$ because $rank(C) = 3$. The fault isolation scheme is applied to the nonlinear simulation model. A reliable diagnostic scheme should perform well for a wide range of operating conditions, hence the input is set at $u = 20\%$ in the simulation. The sensor fault is simulated as 2% offset around the normal measurement. In the simulation, we only consider the fault in sensors No.1, No.2 and No.3, i.e. the low pressure rotor speed sensor, the high pressure rotor speed sensor and the main burner fuel flow sensor. After the transient has settled down, the normalized projection distances for different faulty situations are shown in Table 1.

Faulty sensor	No.1	No.2	No.3
NPD_1	**0.37090**	0.77783	0.66389
NPD_2	0.93117	0.95527	0.42455
NPD_3	0.96529	**0.71161**	**0.31559**

Table 1: Fault isolation using Beard fault detection filter

From Table 1, it can be seen that the fault in sensor No.1 (or No.3) can be correctly isolated as the corresponding normalized projection distance NPD_1 (or NPD_3) is the smallest one. However, the fault in sensor No.2 will be mis-reported as a fault in sensor No.3 as NPD_3 is the smallest one amongst all normalized projection distances. Moreover, the smallest NPD is not significantly different from other NPDs, and this could make isolation difficult when there is noise in the system.

The example in Table 1 illustrates the importance of robustness in fault isolation. The mis-isolation problem is possibly caused by the linearized errors, as the fault isolation scheme is based on the linear model and this scheme is applied to the original nonlinear system. In the model linearization, we only considered the first order terms in the Taylor expansion. To model a system more accurately, we can consider the inclusion of second order terms in the system dynamic equation as follows:

$$\dot{x}(t) = Ax(t) + Bu(t) + Ed(x(t)) \tag{32}$$

where the matrices A and B are the same as the linear model. The term $Ed(x)$ represents modelling errors and the vector $d(x)$ consists of the second order terms of $x(t)$ as:

$$d(x) = [x_1^2 \quad x_2^2 \quad x_3^2 \quad x_1x_2 \quad x_1x_3 \quad x_2x_3]^T$$

The distribution E can be obtained using an identification procedure based on the least-squares method. Given a series of values $u^{(1)}$, $u^{(2)}$, $\cdots$, $u^{(N)}$ for input u, we can obtain the corresponding steady reposes $x^{(1)}$, $x^{(2)}$, $\cdots$, $x^{(N)}$ and $d^{(1)}$, $d^{(2)}$, $\cdots$, $d^{(N)}$, which are related by the following steady state equations:

$$\begin{cases} Ax^{(1)} + Bu^{(1)} + Ed^{(1)} & = & 0 \\ Ax^{(2)} + Bu^{(2)} + Ed^{(2)} & = & 0 \\ \cdots & . & . \\ Ax^{(N)} + Bu^{(N)} + Ed^{(N)} & = & 0 \end{cases}$$

If N is greater than the dimension of $d(x)$, the least-squares estimate of the matrix E is given as:

$$E^* = (\Gamma^+\Psi)^T$$

where Γ^+ is the pseudo-inverse of Γ and

$$\Gamma = \begin{bmatrix} (d^{(1)})^T \\ (d^{(2)})^T \\ \vdots \\ (d^{(N)})^T \end{bmatrix} \qquad \Psi = -\begin{bmatrix} (Ax^{(1)} + Bu^{(1)})^T \\ (Ax^{(2)} + Bu^{(2)})^T \\ \cdots \\ (Ax^{(N)} + Bu^{(N)})^T \end{bmatrix}$$

From the simulation,

$$E^* = \begin{bmatrix} 1.3293 & 3.4440 & 0.1375 & -5.1304 & -1.7826 & -1.8719 \\ 5.6812 & -0.5281 & -0.3385 & -1.6193 & 0.5229 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The E^* is *not* a full column matrix ($rank(E^*) = 2$) and should be decomposed as $E^* = E_1E_2$. Here E_1 is a full column matrix and will be used in the robust fault detection filter design.

$$E_1 = \begin{bmatrix} 6.2006 & 2.8639 \\ 4.1048 & -4.3262 \\ 0 & 0 \end{bmatrix}$$

All eigenvalues of the robust fault detection filter are set to -3. Using the design procedure presented in this paper, with E replaced by E_1, the parameter matrices of the robust fault detection filter are obtained as below:

$$H = \begin{bmatrix} 0.6117 & -0.1170 & 0 & 0.3215 & 0.3220 & -0.1295 \\ -0.1170 & 0.9382 & 0 & 0.0605 & 0.0623 & -0.1916 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$T = \begin{bmatrix} 0 & 0 & -0.1251 \\ 0 & 0 & 0.0783 \\ 0 & 0 & 1.0000 \end{bmatrix}$$

$$K = \begin{bmatrix} -0.0708 & 0.0443 & 0.5658 & 0.1400 & 0.1531 & 0.3540 \\ 0.0443 & -0.0277 & -0.3540 & -0.0876 & -0.0958 & -0.2215 \\ 0.5658 & -0.3540 & -4.5229 & -1.1193 & -1.2239 & -2.8297 \end{bmatrix}$$

This robust fault detection filter is also applied to the nonlinear simulation model to isolate faults in sensors 1, 2 and 3. To compare the isolation performance with the standard BFDF, the system and fault simulation have been set as exactly the same. The normalized projection distances for different faulty situations are shown in Table 2.

Faulty sensor	No.1	No.2	No.3
NPD_1	**0.00621**	0.86727	0.90677
NPD_2	0.88625	**0.00213**	0.56602
NPD_3	0.89433	0.02092	**0.00159**

Table 2: Fault isolation using robust fault detection filter

From Table 2, one can seen that NPD_i $(i = 1, 2, 3)$ is the smallest one amongst all normalized projection distances when a fault occurs in the ith sensor. Moreover, the smallest NPD is significantly different from other NPDs. This simulation shows that the fault can be correctly isolated using a robust fault detection filter, even in the presence of modelling errors.

5 Conclusion

This paper has studied the design of robust fault detection filters whose residuals have both disturbance de-coupling and uni-directional properties. This problem is solved using a combination of unknown input observer and Beard fault detection filter principles. To facilitate this combination, a new full-order unknown input observer structure is proposed, and the necessary and sufficient conditions are presented and proved. The design procedure for this observer is very straightforward. After satisfying the disturbance de-coupling conditions, the remaining design freedom can be utilized to make the residual have directional properties, based on Beard fault detection filter techniques. Robust and directional residuals can easily be used in reliable fault isolation. The paper has studied the sensor fault isolation problem for a jet engine control system. This is a highly nonlinear system, and the linearization error causes unreliable isolation if the robustness issues are not considered at the design stage. To cope with this problem, the paper has developed a second model to account for the linearization errors. Based on this model, a robust fault detection filter is designed and applied to the nonlinear jet engine simulation model and the results show the effectiveness of the robust fault isolation strategy developed in the paper. The technique can be applied to robust fault isolation for a wide range of systems with uncertain factors.

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Appendix A: Proof of Lemma 1

Necessity: When Eq.(5) has a solution H, we have $HCE = E$ or

$$(CE)^T H^T = E^T$$

i.e., E^T belongs to the range space of the matrix $(CE)^T$ and this leads to:

$$rank(E^T) \leq rank((CE)^T)$$

i.e.

$$rank(E) \leq rank(CE)$$

However,

$$rank(CE) \leq \min\{rank(C), rank(E)\} \leq rank(E)$$

Hence, $rank(CE) = rank(E)$ and the necessary condition is proved.

Sufficiency: When $rank(CE) = rank(E)$ holds true, CE is a full column rank matrix (as we have assumed that E is a full column matrix), and a left inverse of CE exists:

$$(CE)^+ = [(CE)^T CE]^{-1}(CE)^T$$

Clearly, $H = E(CE)^+$ is a solution to Eq.(5).

$\diamond$ **QED**.

Appendix B: Proof of Lemma 2

If $s_1 \in \mathcal{C}$ is an unobservable mode of the pair (C_1, A), we have:

$$rank\{\begin{bmatrix} s_1 I - A \\ C_1 \end{bmatrix}\} = rank\{\begin{bmatrix} s_1 I - A \\ C \\ CA \end{bmatrix}\} < n$$

This means that a vector $\alpha \in \mathcal{C}^n$ will exist such that:

$$\begin{bmatrix} s_1 I - A \\ C \\ CA \end{bmatrix} \alpha = 0$$

This leads to:

$$\begin{bmatrix} s_1 I - A \\ C \end{bmatrix} \alpha = 0 \qquad \text{or} \qquad rank\{\begin{bmatrix} s_1 I - A \\ C \end{bmatrix}\} < n$$

That is to say that s_1 is also an *unobservable mode* of the pair (C, A).

If $s_2 \in \mathcal{C}$ is an unobservable mode of the pair (C, A), we have:

$$rank\{\begin{bmatrix} s_2 I - A \\ C \end{bmatrix}\} < n$$

This means that a vector $\beta \in \mathcal{C}^n$ can always be found, such that:

$$\begin{bmatrix} s_2 I - A \\ C \end{bmatrix} \beta = 0$$

This leads to:

$$(s_2 I - A)\beta = 0 \qquad C\beta = 0$$

$$CA\beta = Cs_2\beta = s_2 C\beta = 0$$

Hence:

$$\begin{bmatrix} s_2 I - A \\ C \\ CA \end{bmatrix} \beta = \begin{bmatrix} s_2 I - A \\ C_1 \end{bmatrix} \beta = 0$$

i.e., s_2 is also an *unobservable mode* of the pair (C_1, A).

As the pairs (C_1, A) and (C, A) have the same unobservable modes, their detectability is formally equivalent.

$\diamond$ **QED**.

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