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Disciplina: Cálculo II

LISTA 8

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1- Estude com relação a máximos e mínimos locais a função $f(x, y) =$

$$a) x^2 + 3xy + 4y^2 - 6x + 2y$$

$$\frac{\partial f}{\partial x} = 2x + 3y - 6 = 0$$

$$\frac{\partial f}{\partial y} = 3x + 8y + 2 = 0$$

$$\frac{\partial f}{\partial x} = y = -\frac{2}{3}x + 2$$

$$\frac{\partial f}{\partial y} = y = -\frac{3}{8}x - \frac{1}{4}$$

*Vamos igualar as funções para encontrar o ponto!

$$-\frac{2}{3}x + 2 = -\frac{3}{8}x - \frac{1}{4}$$

$$x = \frac{54}{7}$$

$$-\frac{2}{3}x + \frac{3}{8}x = -2 - \frac{1}{4}$$

$$y = \frac{3}{8}x - \frac{1}{4}$$

$$y = -\frac{3}{8} \cdot \frac{54}{7} - \frac{1}{4}$$

$$-\frac{16}{24}x + \frac{9}{24}x = \frac{8}{4} - \frac{1}{4}$$

$$y = \frac{3}{4} \cdot \frac{27}{7} - \frac{1}{4}$$

$$-\frac{7}{24}x = -\frac{9}{4}$$

$$y = \frac{81}{28} - \frac{7}{28} = \frac{-88}{28} = \frac{44}{14} = \frac{22}{7}$$

$$x = -\frac{9}{4} \cdot \left(-\frac{24}{7}\right)$$

$$\left(\frac{54}{7}, \frac{22}{7}\right)$$

CONTINUAÇÃO

ponto crítico

$$d) -m^2 + y^2 + 2my + 4m - 2y.$$

$$\frac{\partial f}{\partial m} = -2m + 2y + 4 = 0$$

$$\frac{\partial f}{\partial y} = 2y + 2m - 2 = 0$$

$$\frac{\partial f}{\partial m} = y = m - 2$$

$$\frac{\partial f}{\partial y} = y = -m + 1$$

* Igualando a função:

$$m - 2 = -m + 1$$

$$m + m = 2 + 1$$

$$2m = 3$$

$$m = \frac{3}{2}$$

$$y = -\left(\frac{3}{2}\right) + 1$$

$$y = -\frac{1}{2}$$

* ponto critico é $\left(\frac{3}{2}, -\frac{1}{2}\right)$.

$$e) m^3 - 3m^2y + 27y$$

$$\frac{\partial f}{\partial m} = 3m^2 - 6my = 0$$

$$\frac{\partial f}{\partial y} = -3m^2 + 27 = 0$$

$$\frac{\partial f}{\partial m} = 3m(m - 2y) = 0$$

$$\frac{\partial f}{\partial y} = m^2 = 9$$

$$\frac{\partial f}{\partial m} = y = \frac{m}{2}, m \neq 0$$

$$\frac{\partial f}{\partial y} = m = \pm 3$$

* Igualando a função, temos:

$$2y = \pm 3$$

$$y = \pm \frac{3}{2}$$

* ponto critico $\left(3, \frac{3}{2}\right), (-3, \frac{3}{2})$

$$f) m^2 - 4my + 4y^2 - m + 3y + 1$$

$$\frac{\partial f}{\partial m} = 2m - 4y - 1 = 0$$

$$\frac{\partial f}{\partial y} = -4m + 8y + 3 = 0$$

$$\frac{\partial f}{\partial y} = y = \frac{1}{2}m - \frac{1}{4}$$

$$\frac{\partial f}{\partial y} = y = \frac{1}{2}m - \frac{3}{8}$$

* igualando função:

$$\frac{1}{2}x - \frac{1}{4} = \frac{1}{2}x - \frac{3}{8}$$

$$-\frac{1}{4} = -\frac{3}{8} \quad \# \text{ não há ponto crítico} =$$

* Recalculando novamente com f.

$$f(m, y) = m^2 - 4my + 4y^2 - m + 3y + 1$$

$$\frac{\partial f}{\partial m} = 2m - 4y - 1$$

$$\frac{\partial f}{\partial y} = -4m + 8y + 3$$

$$\frac{\partial f}{\partial m \partial y} = \frac{\partial f}{\partial m \partial y} = -4$$

$$\frac{\partial f}{\partial m^2} = 2$$

$$\frac{\partial f}{\partial y^2} = 8$$

$$f(m, y) = \begin{vmatrix} 2 & -4 \\ -4 & 8 \end{vmatrix} = -16 - 16 = 0$$

$$g) \sqrt{m^2 + 2my + 4y^2 - 6m - 12y}$$

$$\frac{\partial f}{\partial m} = (m, y) = 2m + 2y = 6$$

$$\frac{\partial f}{\partial m} = (m, y) = 2m + 8y = 12$$

$$\begin{cases} 2m + 2y = 6 \Rightarrow \\ 2m + 8y = 12 \Rightarrow 0 \end{cases}$$

organizando função

$$\begin{cases} 2m + 2y = -6 \quad \cdot (-1) \\ 2m + 8y = 16 \end{cases}$$

* somando equações

$$6y = 6 \Rightarrow y = 1$$

* substituindo

$$m = 2$$

* pontos críticos
(2, 1)

i) $m^4 + my + y^2 - 6m - 5y$
 $\frac{\partial f}{\partial m} = 4m^3 + y - 6 = 0$

$\frac{\partial f}{\partial y} = m + 2y - 5 = 0$

$\frac{\partial f}{\partial m} = y = -4m^3 + 6$

$\frac{\partial f}{\partial y} = y = \frac{m}{2} + \frac{5}{2}$

* Igualando a função:

$-4m^3 + 6 = -\frac{m}{2} + \frac{5}{2}$

$-4m^3 + \frac{m}{2} + 6 - \frac{5}{2} = 0$

$8m^3 + m - 7 = 0$

$m \approx 1$

$y = \frac{m}{2} + \frac{5}{2}$

$y = -\frac{1}{2} + \frac{5}{2} \Rightarrow y = 2$

* Pontos críticos é $(1, 2)$

j) $m^4 + y^4 + 4m + 4y$

$\frac{\partial f}{\partial m} = 4m^3 + 4 = 0$

$\frac{\partial f}{\partial y} = 4y^3 + 4 = 0$

$\frac{\partial f}{\partial m} = m^3 = -1$

$\frac{\partial f}{\partial y} = y^3 = -1$

* Ponto crítico é $(-1, -1)$

c) $x^3 + 2xy + y^2 - 5x$

$$\frac{\partial f}{\partial x} \Rightarrow 3x^2 + 2y - 5 = 0$$

$$\frac{\partial f}{\partial y} \Rightarrow 2x + 2y = 0$$

$$\frac{\partial f}{\partial x} \Rightarrow y = -\frac{3}{2}x^2 + \frac{5}{2}$$

$$\frac{\partial f}{\partial y} \Rightarrow y = -x$$

* igualando as funções para encontrar os pontos x e y .

$$-\frac{3}{2}x^2 + \frac{5}{2} - x$$

$$-\frac{3}{2}x^2 + x + \frac{5}{2} = 0$$

$$x \approx -1 \quad x \approx \frac{5}{3}$$

$$\frac{-b \pm \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a}$$

$$\frac{-1 \pm \sqrt{1 + 15}}{3}$$

$$\frac{-1 + 4}{3} = 1$$

$$\frac{-1 - 4}{3} = -\frac{5}{3}$$

$$y = -x$$

$$y = -(-1) = 1$$

$$y = -\frac{5}{3}$$

* Pontos são $(-1, 1)$, $(\frac{5}{3}, \frac{5}{3})$

m) $\frac{1}{x^2} + \frac{1}{y} + xy, x > 0$ e $y > 0$

$$\frac{\partial f}{\partial x} \Rightarrow \frac{2}{x^3} + y = 0$$

$$\frac{\partial f}{\partial y} \Rightarrow -\frac{1}{y^2} + x = 0$$

$$\frac{\partial f}{\partial x} \Rightarrow y = -\frac{2}{x^3}$$

$$\frac{\partial f}{\partial y} \Rightarrow y = \frac{1}{\sqrt{x}}$$

* igualando as funções para encontrar os pontos x e y .

$$\frac{2}{x^3} = \frac{1}{\sqrt{x}}$$

$$x > 0 \quad e \quad y > 0$$

$$\frac{\sqrt{x}}{1} = \frac{x^3}{2}$$

$$0 = x^3 - 2\sqrt{x}$$

$$0 = \sqrt{x}(x^2 - 2)$$

$$x = \sqrt{2^2}$$

$$y = \frac{2}{x^3}$$

$$y = \frac{2}{(\sqrt{2^2})^2}$$

$$y = \frac{1}{\sqrt{2}}$$

Pontos críticos

$$\rightarrow (\sqrt{2}, \frac{1}{\sqrt{2}})$$

3 - Estude com relação a extremantes globais a função (m, y) :

a) $m^2 + 2my + 2y^2 - m + 2y$

$$\frac{\partial f}{\partial m} = 2m + 2y - 1 = 0$$

$$\frac{\partial f}{\partial y} = 2m + 4y + 2 = 0$$

$$\frac{\partial f}{\partial m} = y = -m + \frac{1}{2}$$

$$\frac{\partial f}{\partial y} = y = -\frac{m}{2} - \frac{1}{2}$$

* Igualando a função:

$$-m + \frac{1}{2} = \left(-\frac{m}{2} - \frac{1}{2}\right)$$

$$-m + \frac{1}{2} = -\frac{m}{2} - \frac{1}{2}$$

$$-\frac{m}{2} = -1$$

$$m = 2$$

* Agora, substituindo o m para descobrir o y :

$$y = \left(-\frac{m}{2} - \frac{1}{2}\right)$$

$$y = -\frac{2}{2} - \frac{1}{2}$$

$$y = -\frac{3}{2}$$

* Ponto crítico é $\left(2, -\frac{3}{2}\right)$

b) $m^2 - y^2 - 3my + m + 4y$

$$\frac{\partial f}{\partial m} \Rightarrow 2m - 3y + 1 = 0$$

$$\frac{\partial f}{\partial y} \Rightarrow -2y - 3m + 4 = 0$$

$$\frac{\partial f}{\partial m} \Rightarrow y = \frac{2}{3}m + \frac{1}{3}$$

$$\frac{\partial f}{\partial y} \Rightarrow y = -\frac{3}{2}m + 2$$

* Igualando a função para encontrar o ponto m e y .

$$\frac{2}{3}m + \frac{1}{3} = -\frac{3}{2}m + 2$$

$$\frac{2}{3}m + \frac{3}{2}m = -\frac{1}{3} + 2$$

$$\frac{13}{6}m = \frac{5}{3}$$

$$m = \frac{10}{13}$$

$$\begin{aligned} y &= -\frac{3}{2}m + 2 \\ y &= -\frac{3}{2} \cdot \frac{10}{13} + 2 \\ y &= \frac{3}{1} \cdot \frac{5}{13} + 2 \\ y &= \frac{11}{13} \end{aligned}$$

Ponto crítico $\left(\frac{10}{13}, \frac{11}{13}\right)$

c) $m + 2y, -2my - m^2 - 3y^2$

$$\frac{\partial f}{\partial m} = 1 - 2y - 2m = 0$$

$$\frac{\partial f}{\partial y} = 2 - 2m - 6y = 0$$

$$\frac{\partial f}{\partial m} \Rightarrow y = -m + \frac{1}{2}$$

$$\frac{\partial f}{\partial y} \Rightarrow y = -\frac{m}{3} + \frac{1}{3}$$

* Igualando as funções para encontrar o ponto crítico

$$-m + \frac{1}{2} = -\frac{m}{3} + \frac{1}{3}$$

$$-m + \frac{m}{3} = \frac{1}{3} - \frac{1}{2}$$

$$-\frac{2m}{3} = -\frac{1}{6}$$

$$m = \frac{1}{4}$$

* Agora substituindo o m para descobrir o y.

$$y = -m + \frac{1}{2}$$

$$y = -\frac{1}{4} + \frac{1}{2}$$

$$y = \frac{1}{4}$$

* O ponto crítico desta função é $(\frac{1}{4}, \frac{1}{4})$.

d) $3m^2 + y^2 + my - 2m + 2y$

$$\frac{\partial f}{\partial m} \Rightarrow 6m + y - 2 = 0$$

$$\frac{\partial f}{\partial y} \Rightarrow 2y + m - 2 = 0$$

$$\frac{\partial f}{\partial m} \Rightarrow y = -6m + 2$$

$$\frac{\partial f}{\partial y} \Rightarrow y = -\frac{m}{2} + 1$$

* Calculando o ponto de m e y

$$-6m + 2 = -\frac{m}{2} + 1$$

$$-6m + \frac{m}{2} = -2 + 1$$

$$-\frac{11}{2}m = -1$$

$$m = \frac{2}{11}$$

$$y = -6m + 2$$

$$y = -6 \cdot \frac{2}{11} + 2$$

$$y = \frac{10}{11}$$

ponto crítico é $(\frac{2}{11}, \frac{10}{11})$

DOM SEG TER QUA QUI SEX SÁB
DOM LUN MAR MIÉ JUE VIE SÁB

1) $m^2 + 2y^2 + 3my + 2m + 2y$

$$\frac{\partial f}{\partial m} \Rightarrow 2m + 3y + 2 = 0$$

$$\frac{\partial f}{\partial y} \Rightarrow -4y + 3m + 2 = 0$$

$$\frac{\partial f}{\partial m} \Rightarrow y = -\frac{2}{3}m - \frac{2}{3}$$

$$\frac{\partial f}{\partial y} \Rightarrow y = -\frac{3}{4}m - \frac{1}{2}$$

* igualando as funções para encontrar o ponto crítico de m e y .

$$-\frac{2}{3}m \left(-\frac{2}{3}\right) = -\frac{3}{4}m - \frac{1}{2}$$

$$-\frac{2}{3}m + \frac{3}{4}m = \frac{2}{3} - \frac{1}{2}$$

$$\frac{1}{12}m = \frac{1}{6}$$

$$m = 2$$

$$y = -\frac{3}{4}m - \frac{1}{2}$$

$$y = -\frac{3}{4} \cdot 2 - \frac{1}{2}$$

$$y = -2$$

* Pontos críticos $(2, -2)$

1) $m^2 + y^2 - 2m - 4y$

$$\frac{\partial f}{\partial m} \Rightarrow 2m - 2 = 0$$

$$\frac{\partial f}{\partial y} \Rightarrow 2y - 4 = 0$$

$$\frac{\partial f}{\partial m} \Rightarrow m = 1$$

$$\frac{\partial f}{\partial y} \Rightarrow y = 2$$

* Ponto crítico $(1, 2)$

10 - NÃO sei fazer!