SIAG/FME Code Quest 2023

DeFi & RoboAdvising Challenge

QuantHub Team

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Summary

1. Introduction

2. Theoretical Background: The SLSQP + KRR Approach

3. Technical Details

4. Conclusion

Introduction

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Notation and Definitions

- We consider $n = 6$ pools and the control variable is the initial wealth distribution: $\boldsymbol{\theta} \in \mathcal{S} = \left\{[0,1]^6 | \sum_{i=1}^{6}(\theta_i) = 1\right\}$. The return distribution at time T can be viewed as $r_T = r_T(\boldsymbol{\theta})$.
- Given a confidence level $\alpha \in [0,1]$, the CVaR is defined as:

$$\text{CVaR}_\alpha(r_T) = \frac{1}{1-\alpha}\int_\alpha^1 \text{VaR}_s(r_T)ds \approx \mathbb{E}[-r_T | -r_T \geq VaR_\alpha]$$

- The target function to minimize is $CVaR_\alpha(\boldsymbol{\theta}) : \boldsymbol{\theta} \rightarrow CVaR_\alpha(r_T(\boldsymbol{\theta}))$. Its evaluation is expensive due to the simulations of 1000 paths for obtaining the empirical $r_T(\boldsymbol{\theta})$ distribution.

Theoretical Background: The SLSQP + KRR Approach

Direct Optimization (only)

First Idea: Direct Optimization

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Sequential Least Squares Programming (SLSQP)

Iterative algorithm for non-linear optimization. At each step, the original problem is approximated by a quadratic programming one. The latter is solved via the Newton method applied to the KKT conditions.

$$\boldsymbol{\theta}^{(k+1)} = \boldsymbol{\theta}^{(k)} + \boldsymbol{d}_k \tag{1}$$

$$\boldsymbol{d}_k = \underset{\boldsymbol{d}}{\operatorname{argmin}}\ CVaR_\alpha(\boldsymbol{\theta}^{(k)}) + \nabla CVaR_\alpha(\boldsymbol{\theta}^{(k)})^T \boldsymbol{d} + \frac{1}{2}\boldsymbol{d}^T \nabla^2_{\theta\theta}\mathcal{L}\left(\boldsymbol{\theta}^{(k)}, \boldsymbol{\lambda}^{(k)}, \sigma^{(k)}\right)\boldsymbol{d} \tag{2}$$

where $\mathcal{L}$ is the Lagrangian function associated to the problem and $\boldsymbol{\lambda}$ and σ are the Lagrange multipliers associated with the inequality and equality constraints.

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Main Pros

- Can handle both linear and non-linear, equality and inequality constraints.
- No assumptions about convexity of the objective function.

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To achieve this goal, we need:

- A dataset of observations $\left\{ \left(\boldsymbol{\theta}^{(i)}, CVaR_\alpha^{(i)} \right) \right\}_{i=1}^{N}$ (generated in parallel).
- A model to learn the target function.

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KRR

It is a Ridge regression (linear regression with $l2$ regularization) where the regressors are obtained by applying a kernel (additive χ^2) transformation, namely:

$$f(\boldsymbol{\theta}) = \sum_{i=1}^{N} \alpha_i K(\boldsymbol{\theta}, \boldsymbol{\theta}^{(i)}) \qquad K(\boldsymbol{\theta}, \boldsymbol{\theta}^{(i)}) = -\sum_{j=1}^{6} \frac{\left(\theta_j - \theta_j^{(i)}\right)^2}{\theta_j + \theta_j^{(i)}} \tag{3}$$

In this setting, with **just N=10 training points**, KRR obtains R^2 score ≈ 0.996.

Two Steps Approach

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Ultimate Idea: Optimization in Two Steps

As standard in the optimization literature, we merge the two approaches previously described by using the faster method to achieve a coarse approximation of the solution. Then, this approximation is used as the starting point for the finer and more expensive method.

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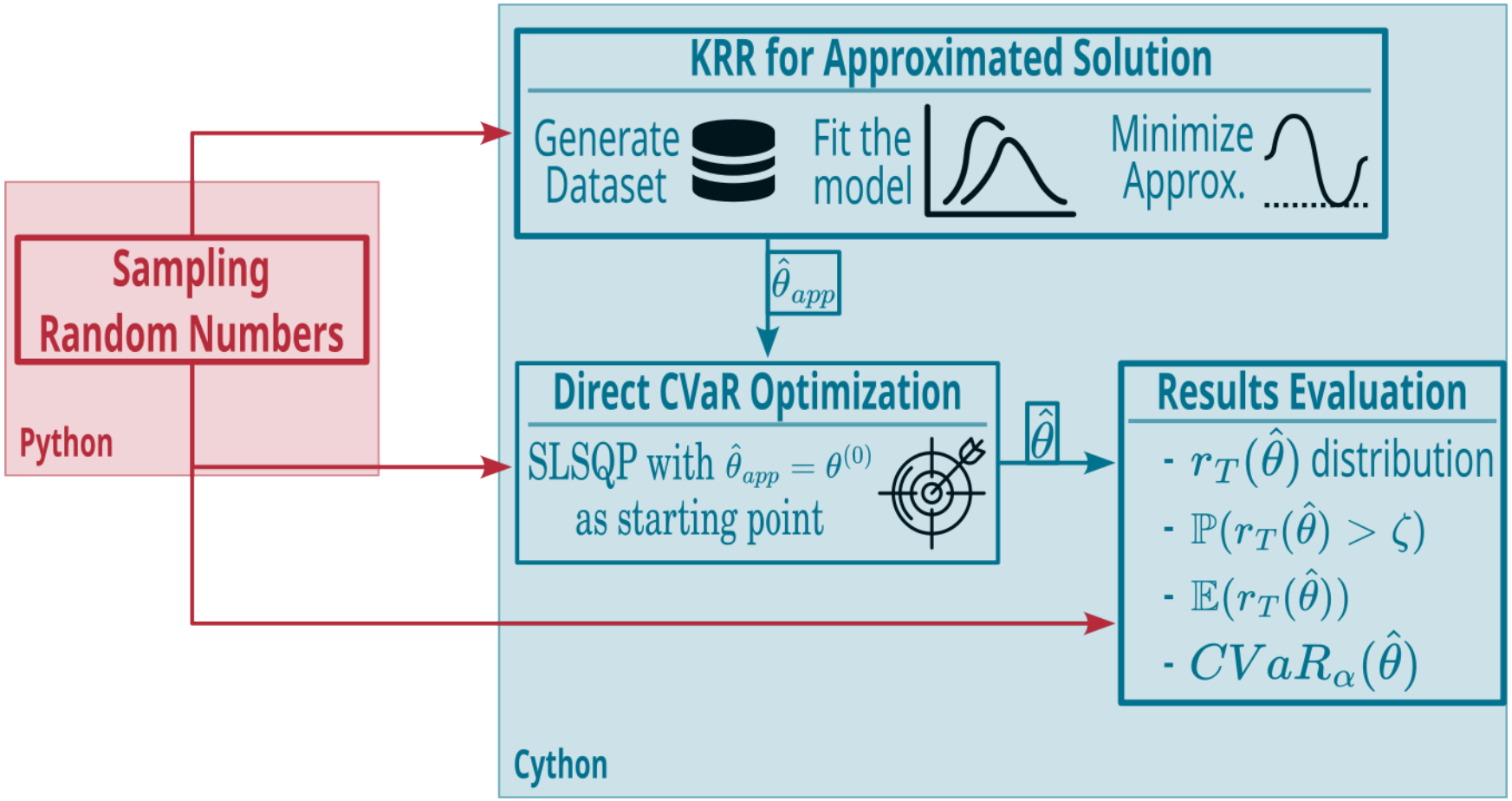
	Grid Search	**KRR**	**SLSQP**	**KRR+SLSQP**
$\mathbb{P}[r_T > \zeta]$	84.50%	84.20%	84.50%	**84.60%**
$\mathbb{E}[r_T]$	16.0985%	**16.1744%**	16.1306%	16.1250%
VaR_α	3.1832%	3.2228%	3.2196%	**3.2237%**
CVaR_α	-0.3693%	-0.3731%	-0.3632%	**-0.3627%**
$\hat{\boldsymbol{\theta}}$	[0.11057, 0.34389, 0.17021, 0.13990, 0.22973, 0.00567]	[0.14408, 0.29763, 0.17649, 0.16160, 0.19700, 0.02320]	[0.12360, 0.29529, 0.19724, 0.15761, 0.2262, 0.00001]	**[0.12848, 0.29556, 0.18971, 0.15680, 0.22943, 0.00001]**

Table 1: Comparison of results obtained with the different approaches.

We have carried out other experiments by changing the random seed in the path simulation process. The results concerning KRR, SLSQP, and KRR+SLSQP are qualitatively the same.

Technical Details

Graphical Abstract



Cython

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- Create a .pyx file
- Write the code as in Python
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```
def simulate(self, cnp.ndarray kappa,
                cnp.ndarray p, cnp.ndarray sigma, double T,
                cnp.ndarray N_list, cnp.ndarray event_type_list,
                cnp.ndarray event_direction_list,
                cnp.ndarray v_list, int batch_size):

        cdef double sum_kappa
        cdef cnp.ndarray pi, mu_vec, event_type, event_direction
        cdef double mu
        cdef int k, j, N, c, random_number
        cdef cnp.ndarray Rx, Ry, v
        cdef list pools, Rx_t, Ry_t, v_t, event_type_t,
                event_direction_t
        ...
```

```
HOW COMPILE THE CYTHON CODE:
python code/setup.py build_ext --inplace
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	Cython	Standard
Time (sec)	207	260

Table 2: Execution time in Google Colab

Sampling procedure

- During the optimization procedure we must simulate price paths multiple times.

```
def simulate(self, kappa, p, sigma, T=1, batch_size=256):
    (...)
    for k in tqdm(range(batch_size)):
      N = np.random.poisson(lam = sum_kappa*T)
      (...)
      for j in range(N):
          event_type[j] = np.random.choice(len(kappa), p=pi)
          event_direction[j] = int(np.random.rand()<p[event_type[j]])
          if event_direction[j] == 0:
              mu = np.zeros(len(pools[k].Rx))
          else:
              mu = np.log(pools[k].Ry/pools[k].Rx)
          if event_type[j] == 0:
              v[j,:] = np.exp((mu-0.5*sigma[0]**2) + sigma[0]*np.random.randn()
                       )
          else:
              v[j,:] = np.zeros(len(pools[k].Rx))
              mu = mu[event_type[j]-1]
              v[j,event_type[j]-1] = np.exp((mu-0.5*sigma[event_type[j]]**2) +
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          if event_direction[j] == 0:
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  return pools, Rx_t, Ry_t, v_t, event_type_t, event_direction_t
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Samplings	One	Each iteration
Time (sec)	260	526

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Conclusion

Summary & Conclusion

We were able to:

- Formulate the minimization problem as a two-step strategy.
- Exploit the efficiency of KRR and the flexibility of SLSQP.
- Reduce the computational load of paths simulation by sampling random numbers just once.
- Leverage the benefits of Cython to reduce the overall computational load.

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Further research

The main idea is to test the proposed approach for the CVaR minimization with other underlying simulation processes. This can be done in two ways:

- **Parametric approach**: Change the Poisson distribution of the events occurrence (e.g. Hawks process)
- **Non parametric approach**: Simulation of future scenarios using a generative neural network (e.g. GAN, Variational Autoencoder ecc.)

In the latter case, as we expect a huge drop in the simulation complexity, the KRR component could be substituted by a neural network to enhance the flexibility of the proposed methodology.

Thanks for your attention!