Adaptive Non Linear Centroidal MPC ensuring Robust Locomotion for legged robots

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1 Introduction

A simplified model in order to study the Locomotion of a legged robot is the LIP (Linear Inverted Pendulum) in which the dynamics of the robot is described as an inverted pendulum in which the base is given by the the ZMP and the top is given by the CoM of the robot. It turns out that the ZMP pushes the CoM away (i.e. the acceleration of the CoM is directed in the direction of falling for the pendulum).
This model extremely simplifies the dynamics and it will be not usefull for some applications for example to compensate a disturbance given by a payload carried out by the robot.
For this reason we want to investigate another model for the dynamics of the robot.

2 Centroidal Momentum Dynamics

Consider an inertial frame or *world frame* w. We assume that the robot is composed of n+1 rigid bodies(links) connected by **n** joints with one degree of freedom each.

We can characterize the robot configuration space by the *position* and the *orientation* of a frame attached to one robot's link (in generale the torso) that is called **Base frame** $\mathcal{B}$ and the *joint configurations*. So $\mathbb{Q} \in \mathbb{R}^3 \times SO(3) \times \mathbb{R}^n$. An element of $\mathbb{Q}$ is a triplet $q = ({}^w p_{\mathcal{B}}, {}^w R_{\mathcal{B}}, q_j)$ where $({}^w p_{\mathcal{B}}, {}^w R_{\mathcal{B}})$ denotes the origin position and orientation of $\mathcal{B}$ expressed with respect to the world frame w, while q_j denotes the joint angles.

Let's consider the full dynamics of a legged robot, taking into account $\mathbf{n_c}$ environment exchanging distinct wrenches (always present due to the ground reaction force):

$$M(q)\dot{v} + C(q,v)v + G(q) = B\tau + \sum_{k=1}^{n_c} J_k^T f_k \tag{1}$$

where: v= $({}^w \dot{p}_{\mathcal{B}}, {}^w \omega_{\mathcal{B}}, \dot{q}_j) \in \mathbb{V} = \mathbb{R}^3 \times \mathbb{R}^3 \times \mathbb{R}^n$ is the generalized robot's velocity, with ${}^w \dot{p}_{\mathcal{B}}$ linear velocity of the base frame, and ${}^w \omega_{\mathcal{B}} = {}^w \dot{R}_{\mathcal{B}} = S({}^w \omega_{\mathcal{B}})\, {}^w R_{\mathcal{B}}$ angular velocity of the base frame with respect to the world frame w.
$M \in \mathbb{R}^{n+6 \times n+6}$ is the inertia matrix, $C \in \mathbb{R}^{n+6 \times n+6}$ is the Coriolis matrix, $G \in \mathbb{R}^{n+6}$ are the gravity terms and $b_1 = [0_{6\times 6}, 0_{n \times 6}]^T$, $b_2 = [0_{6\times n}, I_{n\times n}]^T$ so $B = [b_1, b_2]$ is the selector matrix (the base frame is not actuated).

Moreover $\tau \in \mathbb{R}^n$ is the torques vector of the actuated joints, and $f_k \in \mathbb{R}^6$ denotes the *k-th* external wrench applied by the environment to the robot in the *k-th* contact point.

These forces are expressed in a frame F_k oriented in the same way as the Base frame but the origin is located at the point of application of the force. So the jacobian $J_k = J_k(q)$ is the map between the robot's velocity v and the linear and angular velocity ${}^w v_{F_k} := ({}^w \dot{p}_{F_k}, {}^w \omega_{F_k})$ of the frame F_k with respect to the base frame i.e. ${}^w v_{F_k} = J_{F_k}(q)v$.
It has the following structure:

$$J_{C_k}(q) = \begin{bmatrix} J^b_{C_k}(q) & J^j_{C_k}(q) \end{bmatrix} \in \mathbb{R}^{6\times(n+6)}, \qquad J^b_{C_k}(q) = \begin{bmatrix} I_{3\times3} & -S(p_{F_k} - p_{CoM}) \\ \mathbf{0}_{3\times3} & I_{3\times3} \end{bmatrix} \in \mathbb{R}^{6\times6}.$$

Remark Note that $S(x) \in \mathbb{R}^{3\times3}$ be the skew-symmetric matrix such that $S(x)y = x \times y$, where $\times$ is the cross product operator in $\mathbb{R}^3$.

Now following [1],[2] we can consider a coordinate trasformation in the state (q,v) that puts the system dynamics (1) into a new form in which the matrix M is block diagonal.

This means that we can decouple the joint and base fras accelerations.
Consider the following partition of the inertia matrix:

$$M = \begin{bmatrix} M_b & M_{bj} \\ M_{jb} & M_j \end{bmatrix} \qquad \text{with } M_b \in \mathbb{R}^{6\times6} \quad , \quad M_{bj} \in \mathbb{R}^{6\times n} \quad , \quad M_j \in \mathbb{R}^{n\times n}$$

and the frame c with origin in the CoM of the robot and with the same orientation of w, we can define this coordinate trasformation:

$$q := q \qquad \bar{v} := T(q)v$$

where

$$T := \begin{bmatrix} {}^c X_{\mathcal{B}} & {}^c X_{\mathcal{B}} M_b^{-1} M_{bj} \\ 0_{n\times6} & I_{n\times n} \end{bmatrix} \qquad {}^c X_{\mathcal{B}} := \begin{bmatrix} I_{3\times3} & -S(p_{F_k} - p_{CoM}) \\ \mathbf{0}_{3\times3} & I_{3\times3} \end{bmatrix}$$

Remark The matrix T cannot be singular, it can always be inverted.

The new dynamics in $(q, \bar{v})$ is given by:

$$\bar{M}(q)\dot{\bar{v}} + \bar{C}(q, \bar{v})\bar{v} + \bar{G}(q) = B\tau + \sum_{k=1}^{n_c} \bar{J}_k^T f_k \tag{2}$$

with:

$$\bar{M}(q) = T^{-\top} M T^{-1} = \begin{bmatrix} \bar{M}_b(q) & 0_{6\times n} \\ 0_{n\times6} & \bar{M}_j(q_j) \end{bmatrix}, \qquad \bar{C}(q, \bar{\nu}) = T^{-\top}\left(M\dot{T}^{-1} + CT^{-1}\right),$$

$$\bar{G} = T^{-\top} G = mge_3, \qquad \bar{J}_{F_i}(q) = J_{F_i}(q)T^{-1} = \begin{bmatrix} \bar{J}^b_{F_i}(q) & \bar{J}^j_{F_i}(q_j) \end{bmatrix},$$

$$\bar{M}_b(q) = \begin{bmatrix} mI_3 & 0_{3\times3} \\ 0_{3\times3} & \mathbb{I}(q) \end{bmatrix}, \quad \bar{J}^b_{F_i}(q) = \begin{bmatrix} I_3 & -S(p_{F_i} - {}^w p_c) \\ 0_{3\times3} & I_3 \end{bmatrix}.$$

Where $\mathbb{I}(q)$ is the inertia matrix computed with respect to the CoM frame c, oriented as w, and $e_3 \in \mathbb{R}^{n+6}$ is the third canonical base vector.

As we can see $\bar{M}$ is block diagonal and so the transformed base acceleration is indipendent from the joint acceleration.

More precisely the transformed robot generalized velocity is given by $\bar{v} = ({}^w\dot{p}_c^T, {}^w\omega_c^T, \dot{q}_j^T)^T$ where ${}^w\dot{p}_c.$ is the velocity of the center-of-mass of the robot, and ${}^w\omega_c$ is the so-called *average angular velocity.*

In particular the equation (2) unifies what in literature is presented with two set of equations: the upper part is the so-called **Centroidal dynamics** when it is expressed in terms of *average angular velocity*, while the bottom part is the *free floating system dynamics.*

3 Control of the Centroidal Momentum Dynamics

The previous section puts in light the **Centroidal dynamics**, i.e. the rate of change of the robot's momentum expressed at the CoM, which then equals the summation of all the external wrenches acting on the multibody system [3].

The *robot's momentum* $h = (h_\ell^T, h_\omega^T)^T \in \mathbb{R}^6$ is given by $h = \bar{M}_b v_{\mathcal{B}} = \bar{M}_b * ({}^w\dot{p}_{\mathcal{B}}^T, {}^w\omega_{\mathcal{B}}^T)^T$. The rate-of-change of the robot momentum equals thenet external wrench acting on the robot,which in the present case reduces to the contact wrench f plus the gravity wrench. This is exactly the first equation of (2) and we can write it as:

$$\dot{h} = \frac{d}{dt}(M_b v_{\mathcal{B}}) = J_b^T f - mge_3 = \sum_{k=1}^{n_c} J_{b_k}^T f_k - mge_3 = \sum_{k=1}^{n_c} \begin{bmatrix} I_3 & 0 \\ -S({}^w p_{F_k} - {}^w p_c)^T & I_3 \end{bmatrix} f_k - mge_3$$

Now we need to consider a main **assumption**:

- We assume that the exchanged forces are only linear forces and so f has the last three component equal to zero, this means that we can image $f \in \mathbb{R}^3$.
- To control the robot momentum, it is assumed that the contact impact forces can be chosen at will, so they can be seen as our **control inputs** $u \in \mathbb{R}^3$

So the dynamics of the robot's momentum can be expres in that way:

$$\dot{h} = J_b^T u - mge_3 = \sum_{k=1}^{n_c} \begin{bmatrix} I_3 \\ S({}^w p_{u_k} - {}^w p_c) \end{bmatrix} u_k - mge_3 \quad = \quad \sum_{k=1}^{n_c} A_k(p_{u_k})\Gamma_k u_k + m\overrightarrow{g}$$

Where $\Gamma_k = 1, 0$ is a variable that captures the status of the contact k. Now in order to have a full rapresentation of the centroidal quantities dynamics we consider also the evolution of the position of the CoM, its dynamics is simply given by $\dot{p}_{CoM} = \frac{1}{m}Bh$ where $B = [I_3 0]^T$ is a selector matrix.

So we will consider this reduced dynamics to study the behaviour of the robot:

$$\begin{aligned} \dot{p}_{CoM} &= \tfrac{1}{m}Bh \\ \dot{h} &= \textstyle\sum_{k=1}^{n_c} A_k(p_{u_k})\Gamma_k u_k + m\overrightarrow{g} \end{aligned} \tag{3}$$

That is the union of the centroidal momentum dynamics and the CoM dynamics.

4 References

[1] Stability Analysis and Design of Momentum-based Controllers for Humanoid Robots (Gabriele Nava, Francesco Romano, Francesco Nori and Daniele Pucci)
[2] S. Traversaro, D. Pucci, and F. Nori, "On the base frame choice in free-floating mechanical systems and its connection to centroidal dynamics," Submitted to Humanoid Robots (Humanoids), 2016, IEEE-RAS International Conference on, 2016. [Online]. Available: https://traversaro.github.io/preprin
[3] D. Orin, A. Goswami, and S.-H. Lee, "Centroidal dynamics of a humanoid robot," Autonomous Robots,2013